DISCHARGED FLOW OSCILLATION IN A LONG HEATED TUBE

by

HIROSHI ANDOH

Submitted in partial fulfillment of the
requirements for the Degree of
Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

August, 1964

Signature of Author . . . .
Department of Mechanical Engineering

Certified by . .
Thesis Supervisor

Accepted by. . . . .
Chairman, Department Committee
on Graduate Students

ABSTRACT

DISCHARGED FLOW OSCILLATION

IN A LONG HEATED TUBE

by

HIROSHI ANDOH

Submitted to the Department of Mechanical Engineering on August, 1964 in partial fulfillment of the requirements for the degree of Master of Science.

This thesis is undertaken as an experimental and analytical investigation of two phase flow oscillation.

Experiments have been performed with distilled water at atmospheric pressure to examine the fluid conditions such as inlet temperature, flow rate, inlet subcooling and oscillating frequency at the inception of flow oscillations in a long uniformly heated horizontal pipe. Experimental results reveal:

a) Compressibility of the steam generated in the test section acts as the spring force of oscillating system.

b) Tendency for oscillation increases with subcooling.

c) There is some relation between pressure resistance of two phase flow and flow instability.

d) Frequency of the oscillation varied from 1 to 7 seconds.

The analysis is performed from the three conservation equations for two phase flow in the heated test section. Some model and the perturbation method are applied to solve these equations. A vibration equation of the second order relating to mass velocity may be determined.

A comparison of the experimental data with the analytical results shows that the analysis can predict hydraulic stability boundary and agree with frequency of oscillation in the regime of low steam quality. But the prediction of conditions at high steam quality is not adequate for understanding of the experimental data. It will be considered that the inception of fluid oscillation is defined by some correlation between the spring force due to compressibility of steam void and negative resistance of pressure drop.

Thesis Supervisor: Peter Griffith

Title: Associate Professor of Mechanical Engineering

ACKNOWLEDGMENTS

The author wishes to thank Professor Peter Griffith and Mr. John Maulbetsch for their encouraging and helpful assistance and Mr. E. Hastel for this help in the construction of various parts of the apparatus.

TABLE OF CONTENTS

1. Introduction

The principal purpose of this study is to investigate experimentally and analytically two-phase flow instabilities occurring in a long heated pipe. In recent times the instability problems have been studied intensively in connection with the performance of conventional powered boilers and nuclear power stations, especially since the operation of Boiling Water Reacters is restricted by hydraulic instability coupled with nuclear kinetic characteristics. Several analyses and observations have been made to date of oscillating phenomena in natural, and forced convection systems at conditions of high and low system pressures, and varying subcooling and heat flux.

Lottes(1) reported, the readings taken on natural convection system indicated that instability occurred at exit void fractions in excess of 74 percent and were stopped by closing the down-comer value. Levy(2) and Beckjord(2) reported readings taken on similar natural convection apparatus. These measurements were made at a variety of conditions. The instabilities were observed even at 60 percent exit void fraction; oscillations were damped by restriction of the down-comer value; the flow oscillations increased rapidly with inlet subcooling; most of the observed periods of oscillation were of the order of 2 to 4 seconds. Comparison of several analytical models to the experimental results were made by a number of investigators(3)(4)(5).

Fleck(5) assumed a linear void distribution in the channel to analyze a set of conservation equations and presented the correlation of exit void fraction versus inlet velocity in order to investigate the non-linear equation by method of phase analysis.

The model which was developed by Chilton was negative resistance model. Reference 3 reported that decreasing pressure drop with increasing mass velocity produced instabilities in two-phase flow. Quandt[4] reported instabilities were generated even in positive resistance regimes and Chilton's model could not explain all the two-phase flow instability phenomena. Quandt then concluded an analytical study using a linearized perturbation treatment of conservation equations to get the transfer-function between perturbed flow rate and perturbed heat flux. With these experimental observations and analyses in mind, these investigations were performed. In particular, observations were made to detect oscillations at various heat fluxes.flow rates and subcooling, using a long heated test section consisting of a horizontally installed Nickel-pipe.

As the experiment was undertaken to map out a stability regime, not to verify a particular transfer function in two-phase flow, the apparatus was operated with a constant heat input at atomosphere pressure. Experimental results have shown that with constant heat input the flow rate of liquid varied periodically with time and a small change in one of variable parameters could lead to increasing or decreasing the amplitude of flow oscillation. Sometimes it was observed that there was no liquid in pipe, only vapor, at conditions at which burn-out might have occurred.

A number of possible mechanisms have been considered as the cases of observed flow oscillations but were too complicated to analyze.

In order to explain the physical phenomena analytically, one simplified model is assumed in this study. Comparison of observed data with the analysis indicated that the analysis will be adequate to describe the inception of flow oscillation under some conditions.

2. Test apparatus.

The test apparatus was designed, erected and instrumented to observe and measure the phenomena of two-phase flow oscillations. The apparatus as depicted in Figure 1 is primarily comprised of two loops, one of which is 1 inch diameter pipe closed-loop to maintain the pressure of the water supplied from the storage tank.

The other loop is a single-pass open loop to supply distilled water to the test section after passing the preheater.

Subcooling is achieved by regulating the amount of cold water through the preheater-bypass line passing from the pressurized loop to the exit of test loop. A suitable transformer for voltage control provides electrical power to the test section. The necessary indicating and recording instruments are mounted on the apparatus.

(a) Loops.

The test apparatus consists primarily of the pressurized loop and the test loop. The pressurized loop is 1 inch diameter carbon steel pipe which includes the storage tank, pump and pressure control valve. Distilled water is circulated in the closed system and pressurized by the pump at flow rate 1 GPM at 100 psi and back to the storage tank. Pressure control in the system can be made by throttling a needle type pressure control valve at the exit of pump. The water supplied from the pressurized loop to 1/4 inch copper pipe the test loop is pre-heated in the steam heated type preheater. Therefore, it is impossible to increase the inlet temperature of test section saturation temperature. A bypass line on the preheater is used to regulate the subcooling manually. This water mixing method is sufficient to set subcooling temperature. The amount of flow to the test section is set by needle type control valve. The hot water and steam discharged from the test section is condensed, and then dumped into a discharge tank.

(b) Test section.

The single-pass test section as shown in Figure 2 is horizontally installed. The test section directly connected to power supply system is heated electrically.

The heated tubes are made of nickel-steel of 0.1805 inch. I.D. 0.21 in. O.D. and 10 feet long, and held by five support-plates to prevent vibration. Thermocouples to measure wall temperature are installed on the heated tube. Three copper-constantan thermocouples are fixed by asbestos strings at locations of 1 foot from the exit, the center, and 1 foot from the inlet.

(c) Electric power supply.

Current resistance heating is used to heat up the test section. Direct current generator of M.I.T. Heat Transfer Laboratory is utilized as power supply system. The regulation of power input to test section is made by manual control panel.

(d) Measurement and control.

The apparatus is instrumented to indicate and record power supplied to the test section, test section wall temperature and fluid temperature, exit pressure of pump, and inlet pressure of the test section.

Electric power measurement

The current in the test section was measured by reading the voltage drop across a calibrated shunt in series with the test section. This voltage drop was displayed on the Honeywell recorder.

Temperature measurement

Four thermo-couples, made of copper-constantan, were used to measure temperature, one was for fluid temperature, the others were to measure wall temperatures. Thermo-couple readings were indicated on potentiometer. However, when the heat input was increased readings of wall temperature were difficult to obtain because of the temperature oscillations.

Pressure measurement

A pressure transducer was used to measure the pressure drop across the test section. The transducer was of the resistance strain gage type. Measuring range is 0 to 30 psi. G. Transducer readings were recorded on the Sanborn recorder or the Honeywell recorder.

However, as the transducer was insensitive to low pressure drop, a marcury manometer was connected to the entrance of test section.

Flow rate measurement

Flow rate in this study was too small to be measured by instruments. Therefore, water flow leaving the cooler was measured by collecting the flow in a measuring cylinder for one minute. This method was not suitable for measurement of high steam quality water, because condensation of exit steam is not sufficient.

3. Experimental procedure.

The pressurized loop was first brought close to operating conditions with the pump. A water pressure was set to 100 psi and manually regulated by means of throttling the pressure control valve. Water supply to the test section was made after passing the preheater. The inlet-water temperature of the test section was then adjusted to the desired value. The adjustment was made by means of regulating the subcooling control valve which was mounted on a bypass line, and this method led to satisfactory results. After an adjustment of pressure, subcooling temperature and the desired flow rate were satisfactorily obtained, the electric generator was started. The desired flow rate was set by means of the flow control valve. Before these operations it is necessary to supply cooling water to the cooler. After steady-state conditions were established, heat input was increased until oscillation began, and then all required readings, such as electric power, pressure, wall temperature and inlet water temperature, flow rate and frequency of exit flow, were taken. For flow rate measurement when the steady flow has been achieved after valve-setting (took about 2 minutes), the water flow rate was measured by timing the filling in the measuring cylinder. The test section power was next increased at constant subcooling and constant pressure of pressurized loop. Readings and recordings were taken at different higher steps in power. For measurement of flow rate at conditions of high heat flux and very small flow rate (very high void fraction) it is difficult to measure the exact flow rate, because condensation of steam in the cooler was not sufficient. Therefore, the flow rate was assumed to be the same as that which occurred at a slightly lower heat flux than the actual values. This is sufficiently accurate so long as the pressure drop across the upstream restriction is large compared to that across the test section. Frequency of exit flow oscillation was counted for a minute using a stopwatch. When heat flux and flow rate were highly increased, it was difficult to measure the frequencies. Therefore, some corrections of frequency were made by comparison with pressure oscillation recorded on recorders as described in the previous section. Similar runs were next repeated at different subcoolings. In these runs it is important to note that the inlet temperature varied as the flow rate was changed. It was then necessary to readjust the inlet temperature. An additional problem was the prevention of random fluctuations of the pressure in the system. This fluctuation was 5 psi. It was found that the fluctuations were completely eliminated by filling the storage tank more than half-full.

4. Experimental results and discussion.

Experimental results

Forced-convection runs were performed with pressure 100 psi at the exit of the pump. Measurements were taken at various inlet subcoolings, heat inputs and flow rates.

Representative data of test results are shown in Table 1. The flow rate was determined from measurements of the filling time of water in the measuring cylinder as described in previous section. The inlet temperature was measured by the thermocouple for water temperature (TI-1), and heat fluxes were corrected by heat loss from the test section wall to atomosphere. Heat losses were obtained by heat transfer of natural convection using difference of test section wall temperatures (average values of 3 points TI-2, TI-3, TI-4) and room temperature. The frequencies noted in the Table are exit flow oscillations of the test section. (periods in bracket). For calculations of exit steam qualities it is assumed that the slip ratio is unity. Run numbers 40 through 45 are readings taken for various inlet temperatures, at the fixed flow rate and the fixed heat flux. In order to investigate the effect of subcooling this special test was performed. Patterns of pressure oscillations at the different inlet temperatures traced on the Sanborn recorder are given in Figure 8. Stability maps of the test results are plotted in Figures 3, 4, 5, and 6. The stability maps show stable and unstable regions at a fixed inlet temperature for various flow rates and heat fluxes. Solid lines in figures represent the threshold of instability.

For the low flow rate and high heat flux region it is impossible to make a clear boundary-line because of the limitations of the apparatus. However, it is noted that the flow oscillations were observed even at very small flow rates below 10 lb/hr and heat fluxes 1000 BTU/hr.ft^2. Figure 9 is the pressure drop curve through the test section.at an inlet temperature of 170^{o}F for several heat fluxes. The readings were made with a marcuey manometer installed at the inlets of the test section.

Data in the regime of pressure oscillations were representatives of average pressure drops during the oscillation.

Discussion of experimental results

The basic purpose in this study is to observe the region of instability and investigate the causes of oscillation. Detailed reports of flow patterns during oscillation was presented by Gouse and Andrisiak[6]. Recently, Weatherhead[7] reported the detailed flow patterns for various heat flux and flow rate. They concluded that since flow oscillations occur for various flow regimes when there is boiling, but do not occur for various flow regimes of air-water mixtures with no boiling. Flow patterns alone are not the cause of the flow oscillations and boiling or heat transfer is an essential part of the mechanism that causes the flow oscillations. The works of Reference88 also concluded that boiling is the cause of oscillation.

An observation of flow patterns in this study is impossible. However, satisfactory hydraulic oscillations were obtained at several different conditions. For different values of the inlet subcooling, it is found that the instability regime is varied over a large range of exit steams qualities as reported in reference (2) and (5).

The tests, therefore, were performed over a large range of steam qualities for each of the different fixed inlet subcoolings. Experimental results indicated that the threshold of instability could not be expressed by a simple correlation such as exit steam quality. Figure 3 through 6 showed the complicated correlation including many parameters. The effects of each of parameters to hydraulic instability will be discussed with reference to the experimental results.

Heat input and flow rate

In order to discuss the effects of these variables it must be noted that flow in this test is forced-convection and the inlet flow remains constant even though oscillations of exit flow rate occurs.

As shown in Figure 3 through 6, a similar tendency was found for each of the inlet subcoolings. For example, using Figure 5 the exit flow oscillation started at the void fraction in excess of 60 percent. However, the void fraction at which oscillations occur, is slowly increased as the inlet flow rate is increased. If the flow rate exceeds about 50 lb/hr, no heat input will produce flow oscillations. At high flow rate the upper boundary of the unstable region on a heat flux versus flow rate plot has negative slope.

The most interesting phenomena are in the vicinity of a flow rate of 20 lb/hr. Amplitudes of exit flow rate and pressure drop fluctuation are increased and the heat input of the threshold line is raised to higher values. Figure 7 shows the pressure drop amplitude recordings on the Honeywell recorder at different heat inputs. Maximum pressure drop amplitude is about 1 psi. Such a hydraulic oscillation is noticeable throughout the different subcooling tests. It is also clear that the burn-out would eventually occur due to these physical phenomena, because complete absence of water in the test section is observed as the heat input is raised. At higher heat inputs, the test section would become quite hot as a result of the flow oscillations and periodic buckling would occur because of the thermal expansion.

<u>Subcooling</u>

As shown in Figure 3 through 6, the subcooling is the important variable in determining the hydraulic instability, independent of other parameters. The effects of inlet subcooling was extensively studied by Gouse and Andrysiak. They reported that subcooling has important effects on thr boundaries of the unstable region, amplitude and period of oscillation. In particular, for the analysis of natural convection systems, the driving force due to gravity which is effected by the subcooling, should be considered. Such a driving force is not considered for horizontal tube forced-convection, but a variation of the total void fraction has a large effect on the conditions of hydraulic instability. The oscillation area is increased with the inlet subcoolings.

In a special test, runs at fixed flow rate and heat input for different subcooling temperatures were performed. The method utilized was to set the flow rate at about 10 lb/hr by the flow control valve and regulate the inlet subcooling temperatures by the subcooling control valve with heat flux of about 5640 BYU/hr. ft^2. Runs 40 through 45 show the results of this special test. The effect of subcooling on amplitude and frequency of pressures drop fluctuation are also presented in Figure 8. A comparison of traces recorded on the Sanborn recorder shows that the amplitude of the pressure fluctuations is maximum at inlet temperature 185°F and decreases with increasing and decreasing subcooling. These results are different from the experimental results in Levy and Beckjord's report[2] in which the amplitude of fluctuations increases rapidly with inlet subcooling. However, these differences can be explained by the fact that the driving head in natural-convection is largely dependent on the inlet subcooling. It is important to note that throughout the subcooling test an increased subcooling reduces stability of ststem, but the effect is small compared to the case of natural-convection.

Frequency of Oscillation

As already described in a previous section, the frequencies of oscillations can be obtained from counting the exit flow oscillations and inlet pressure traces. Exit flow frequency data are presented in Table 1. with heat inputs, flow rates and subcoolings and traces shown in Figure 7 illustrate the measured frequencies and amplitude with above parameters. Oscillating periods were found to be in the range of about 1 second to 7 seconds. However, a definite trend of the periods with power input, flow rate and subcooling could not be found. The general tendency is that the increased flow rate reduces the oscillating period. Effects of heat input to period can not be detected from data taken in this test. In addition, transit times, which are the time required for the fluid to pass through the boiling section, are tabulated in Table 1. It is found that the period of the oscillation will be approximately equal to the transit time. This result means that the compressibility of the steam voids have significant effects on the oscillations.

Test section pressure drops

Pressure drop curves through the test section are shown in Figure 9. These pressure drops were calculated from the data measured at a fixed inlet temperature of 170°F for several heat fluxes. The curves are slightly different, at flow rates in excess of 40 lb/hr, from the calculated results using Martinelli-Nelson's[10] data.

A comparison of the pressure drop curve to the hydraulic instability map presents a very interesting correlation. As shown in Figure 5 the boundary line of instability map at low steam quality agrees with the dotted line which is plotted from minimum points (CD line) of the pressure drop curves in Figure 9. On the other hand, maximum amplitude line in Figure 5 is approximately equal to values computed from maximum points (line AB) in Figure 9. From this it will be concluded that the shape of two-phase pressure drop curve is an important factor in hydraulic instability. Therefore, it is felt that problems of hydraulic instability and the determination of the natural frequency can be explained by correlation between driving head due to the compressibility of the steam void fraction and the pressure drop through the test section. An analysis of these phenomena will be performed in the next section.

5. Theoretical analysis.

Theoretical analysis of the two phase flow instability has been performed by previous investigators. However, definite criteria for hydraulic instability have not been proposed. In order to analyze such a complicated problem a simple physical model is essential. The physical model used here is shown in Figure 10. It can be understood from the test results that there is a complicated correlation between the instability and other variables. Discussions of experimental data showed that the pressure drops through the test sections and the compressibility of steam void could define the behavior of oscillation. From their results the following assumptions are made:

(1) Driving head is due to compressibility of steam void fraction in the test section; agreement of oscillating period and transit time was found experimentally. The compressibility of steam void can be considered to be a kind of spring force.

(2) Constant inlet mass flow rate: the pressure drop through the flow control valve is very high compared to the test section pressure drop. Therefore, mass velocity through the flow control valve is constant, and independent of the fluctuation of the inlet

pressure of the test section.

(3) The exit pressure of the test section is constant atmospheric pressure.

The fluid in the test section can be considered as a thermodynamic system which depend on the conservation equations. The partial differential equations of two-phase flow are,

Continuity:

$$\frac{\partial}{\partial t}\left[\rho_f(1-R) + \rho_g R\right] + \frac{\partial}{\partial x}\left[\rho_f(1-R)V_f + \rho_g R V_g\right] = 0 \qquad (1)$$

Energy:

$$\frac{\partial}{\partial t}\left[\rho_f(1-R)h_f + \rho_g R h_g\right] + \frac{\partial}{\partial x}\left[\rho_f(1-R)V_f h_f + \rho_g R V_g h_g\right] = \frac{\partial}{\partial x}\left(\frac{Q}{A}\right) \qquad (2)$$

or

$$\frac{\partial}{\partial t}\left(\rho_g R h_{fg}\right) + \frac{\partial}{\partial x}\left[\rho_g R h_{fg} V_g\right] = \frac{\partial}{\partial x}\left(\frac{Q}{A}\right)$$

Momentum:

$$\frac{1}{g}\frac{\partial}{\partial t}\left[\rho_g R V_g + \rho_f(1-R)V_f\right] + \frac{\partial}{\partial x}\left[\rho_g R V_g^2 + \rho_f(1-R)V_f^2\right]$$

In order to solve directly these non-linear equations a digital computer would be required.

As this analysis is made to examine the overall characteristics of hydraulic instability, the integral method is applied throughout the test section. Integrating Equations (1), (2), and (3) along the test section, the continuity equation is,

$$-\rho_f \frac{\partial}{\partial t} \int_{\ell} R dx = (\rho_f V_f)_i - \left[\rho_g R V_g + \rho_f (1-R) V_f\right]_e \qquad (4)$$

where, it is assumed that $\rho_g R$ is negligibly small compared to $\rho_f(1-R)$.

The energy equation is:

$$A \rho_g \frac{d}{dt} \int_{\ell} R dx = \frac{Q}{h_{fg}}\left(1 - \frac{\ell_1}{\ell}\right) - A(\rho_g R V_g)_e \qquad (5)$$

where, Q is the total input to the fluid and ℓ_1 is the subcooled length.

Using the slip ratio $S = \frac{V_g}{V_f}$, total volume in the test section

$$V = A \int_{\ell} R, \text{ and } \frac{\ell_1}{\ell} = \frac{\Delta h W_i}{Q},$$

the following equation can be obtained from Equation (4) and (5).

$$\frac{dV}{dt} = \frac{Q}{\rho_g h_{fg}} (1-\bar{R}) - \left[\frac{\bar{R}}{\rho_f} + (1-\bar{R}) \frac{\Delta h}{\rho_g h_{fg}}\right] W_i \qquad (6)$$

where, $\bar{R} = \dfrac{R_e}{R_e + \dfrac{\rho_g}{\rho_f} R_e + \dfrac{1-R_e}{s}} \simeq \dfrac{R_e}{R_e + \dfrac{1-R_e}{s}}$

If it is assumed that the slip ratio is unity, $\bar{R}$ is equal to the exit void fraction R_e.

Equation (6) shows the correlation of heat input, inlet subcooling and inlet flow rate. For a fixed subcooling and a heat input, the increased flow rate decreases steam void fraction in the test section. This equation is physically reasonable.

Integration of momentum equations is

$$\frac{1}{g} \frac{d}{dt} \int_{\ell} (\rho_g R V_g + \rho_f (1-R) V_f) = P - \phi \tag{7}$$

where, P is the inlet pressure and ϕ is total resistance due to momentum change and friction loss throughout the test section.

If the heat input and the subcooling are constant, P and ϕ can be expressed as a function of mass velocity and time. The average mass velocity in the test section may be defined as

$$\hat{G} = \frac{1}{\ell} \int_{\ell} (\rho_g R V_g + \rho_f (1-R) V_f) dx \tag{8}$$

where, an assumption $G_i = \hat{G} = G_e$ would be established for simplification of the calculation.

In this analysis the time delay of the fluid throughout the test section is neglected.

From this relation, Equation (7) is,

$$\frac{l}{g}\frac{dG}{dt} = P\,(G) - \phi(G) \tag{8}$$

In the steady-state condition, the momentum equation states that $dG/dt = 0$; therefore, $P = \phi$.

As discussed in the previous section, there is a correlation between the inlet pressure and the total void fraction in the test section.

If the correlation can be expressed by $P\,V^n = C$, the following relation is obtained.

$$\frac{dV}{dt} = -\frac{V}{nP}\frac{dP}{dt} \tag{9}$$

where, n is unity for isothermal change and 1.3 for adiabatic change.

It is seen from the given model that the decreased steam volume (Eq. 6 and 9) is equal to the increased mass velocity. Then, the inlet pressure fluctuation is expressed as,

$$\frac{dP}{dt} = -\,cG - d \tag{10}$$

where,

$$C = A\left[\frac{(1+\bar{R})}{\rho_f} + (1-\bar{R})\,\frac{\Delta h}{\rho_g h_{fg}}\right]\frac{nP}{V}$$

$$d = -\left[\frac{Q}{h_{fg}}\,(1-\bar{R}) + AG_o\right]\frac{nP}{V}$$

Equation (10) shows that the input pressure fluctuation is decreased with the increased mass velocity.

From Eqation (8),

$$\frac{\ell}{g}\,\frac{d^2G}{dt^2} + \frac{d\phi}{dG}\,\frac{dG}{dt} - \frac{dP}{dt} = 0 \tag{11}$$

Defining $b = \frac{d\phi}{dG}$, $a = \frac{\ell}{g}$ and substituting Equation (10) into Equation (11),

$$a\,\frac{d^2G}{dt^2} + b\,\frac{dG}{dt} + CG + d = 0 \tag{12}$$

Equation (12) is a typical non-linear equation with variable coefficients. A direct analytical solution is impossible with present mathematical knowledge. In order to solve this equation,

(1) reduction of the non-linear system to a linear one by making simplifying assumption.

(2) use of perturbations about initial steady state.

(3) finite-difference methods with a high-speed electronic computer or electric analog computer.

are considered,

Using a perturbation technique, the variables may be written as follows:

$$G = G_o + \Delta G \qquad \rho = \rho_o + \Delta \rho \tag{13}$$

Perturbations of void fraction may be considered to be negligible small.
Equation (12) is

$$a \frac{d^2 \Delta G}{dt^2} + b \frac{d \Delta G}{dt} + C \Delta G = 0 \tag{14}$$

wher.,

$$a = \ell$$

$$b = \left.\frac{d\phi}{dG}\right)_{G=G_o}$$

$$c = A \left[\frac{(1+\bar{R})}{\rho_f} + (1-\bar{R}) \frac{\Delta h}{\rho_g h_{fg}} \right] \frac{n\rho_o}{V}$$

Equation (14) is vibration equation, with a damping term and third term CG expressing a kind of the spring force. From such a vibration equation, the stability criteria is obtained as follows.

$b > 0$ stable

$b < 0$ unstable

This criteria represents fairly that if the intersection of the pressure drop and the driving force (or spring force) is in a regime of negative slope of the pressure drop, i.e. ("B" point in Figure 11) the system is stable and if the intersection ("A") is in a regime of positive slope, the system is stable. In this case a natural frequency is expressed as,

$$\omega_n = \sqrt{\frac{f}{g}\left[\frac{(1+\bar{R})}{\rho_f} + (1-\bar{R})\frac{\Delta h}{\rho_g h_{fg}}\right]\frac{nAP_o}{V}} \qquad (15)$$

It will be concluded that the negative resistance model explained by Chilton is reasonable. However, it is noted that this analysis assumes the small perturbation of the variables. The actual relation between the damping coefficient and the amplitude varies from case to case, but it is difficult to give an analytical solution of such a problem as described before. The case of simple damping coefficient which is available for this study is discussed in Reference 11. Such a damping force is,

$$\text{Second term} = (C_2G^2 - C_1)\frac{dG}{dt} \qquad (16)$$

Namely, Equation (14) can be expressed as

$$\frac{d^2G}{dt^2} - \frac{C_1}{a\omega_n}\left(1 - \frac{C_2}{C_1}x^2\right)\frac{dG}{dt} + G = 0 \tag{17}$$

The amplitude of vibration is given,

$$\text{amplitude} = 2\sqrt{\frac{C_1}{C_2}} \tag{18}$$

and periods approaches to the natural frequency of system as a single parameter $\epsilon = C_1/a\omega_n$ is close to zero, and for large ϵ period is expressed as

$$\text{period} = 2\sqrt{\frac{C_1}{a\omega_n}} \text{ seconds} \tag{19}$$

6. Comparison of experimental results and analysis.

The analysis performed in the previous section is based on a linearized treatment of differential equations for small perturbations about some steady state value.

As already illustrated in Figure 5, the lower boundary of the stability map is equal to the minimum pressure drop line. These pressure drops are the values corresponding to b = 0 in the analysis.

The experimental results have indicated that the maximum pressure drop line corresponding to b = 0 does not agree with the upper boundary of instability regime and in fact is equal to a curve on which the most violent oscillation occur. Chilton's model clearly cannot predict such a hydraulic instability. Stability criteria performed in this study also do not explain it. It is considered that the cause of the disagreement is the use of the integral method throughout the test section. This method is not capable of incorporating the damping effects and the compressibility of the steam void fraction at each location of the test section. If the exit steam condition of the test section is at the positive resistance, the steam condition at the other location may be at the negative resistance region. The agreement of the instability boundary line of experimental results and analysis at the low steam quality indicates that the effects of the up-stream is very small.

From these discussion, in order to get the exact boundary of the instability map it is important to examine the damping effects each other location. This calculation takes many time and the digital computer should be used. The integral method given in the analysis have been successful for oscillation analysis provided the damping effects each other locations can be incorporated. The application of the perturbation technique to the integral method is not sufficient at the other conditions except the low steam quality region. The curves of Chilton's model as shown in Figure 12 represents a pressure-drop curve "A" which is monotonically increasing with mass flow is stable. Curve "B" possesses the negative resistance region and allegedly "unstable." This criteria would be satisfied, if the flow condition is near the intersection of curve "A" and no steam generation curve in the subcooling region. A mathematical prediction might be possible by applying the analytical results of the special case.in the previous section. The damping coefficient fitted on the pressure drop curve near the maximum points in Figure 11 is a function of very large C_1, and C_1 fitted near the minimum point is small. Therefore, the amplitude near the maximum points is very very large, and the amplitude near the minimum point is small. Since this behavior is noted experimentally, the perturbation method is satisfied in the regime of small change of pressure drop, but is not sufficient in regime with large pressure drop change. This qualitative analysis proves the lower instability line is sufficiently close to the analytical results (see Fig. 5), and the frequencies approach the natural frequencies (see the calculated results for inlet temperature 170°F) as the intersection of the driving head and the pressure drop approaches a point on the pressure drop curve of small curvatures. Runs no. 40 through 45 show that the frequency increases with subcooling. This experimental tendency can be explained by Equation (15). Because the steam volume in the test section decreases with increasing subcooling, but the exit steam void fraction is not sensitive to variations of the subcooling at the fixed heat input and flow rate.

Reference 2 reported that periods increase with the subcooling which is a reverse tendency to the experimental results in this study. This difference means that the effect of the driving head due to the gravity is large compared to the compressibility of steam void.

It is impossible to define oscillating frequency by a simple expression from Equation (15). There is a complicated correlation of parameters. The amplitude of flow rate calculated by Equation (18) is about 15 lb/hr at 7200 BTU/hr ft^2 near the maximum pressure drop. This amplitude causes the absence of water in the test section. It is noted that this behavior was observed experimentally by reducing the flow rate. Hydraulic instability in this regime can be qualitatively understood by considering the total energy stored in system.

Energy stored in the system is an integration of a damping term in Equation (14).

$$E = -\int b \frac{dG}{dt}\, dG = -\int b \left(\frac{dG}{dt}\right)^2 dt \qquad (20)$$

Negative sloping pressure drop ($b<0$) supplies the energy to the system and positive sloping pressure drop ($b>0$) discharges energy from the system. Oscillation should be sustained when the stored energy approaches zero ($E=0$). Oscillations are increased in the negative resistance and reduced in the positive resistance. Since oscillations are determined by the total energy balance stored in the system, it is considered that oscillations occur at steady state flow rate and heat flux with positive resistance. The experimental results indicated that the amplitude of oscillation near the rate of 20 lb/hr was very large, and the oscillation was sustained for the conditions corresponding to the large positive resistance. The hydraulic instability with large oscillation can be explained by the energy stored in the system. In addition to the calculated periods by Equation (19) are close to experimental periods at conditions corresponding to maximum pressure drops. If an exact solution of non-linear differential equation (Eq. 12) is obtained, the instability criteria would be established.

From this discussion, it is concluded that the inception of oscillation is determined by a correlation between the spring force or the driving force and the negative resistance. For the low steam qualities Chilton's model is capable of predicting the hydraulic instability. The criteria of Chilton's model are in satisfactory agreement with the test results in the low steam quality region. If the mechanics of flow are assumed to conform to the Martinelli-Nelson pressure drop correlation, test data of low steam quality correspond to the value near the minimum pressure drop.

7. Conclusions.

1) Driving head (spring force) is due to the compressibility of steam void generated in the test section.
2) The period of the oscillation is approximately equal to the transit time in the test section.
3) Tendency for flow oscillations increases with subcooling. However, the effect is small compared to the case of natural convection.
4) The criteria for the existence of flow oscillations cannot be simply stated. But, causes of oscillation are the negative sloping of pressure drop and the driving forces.

5) The period of the flow oscillations varies over a wide range. The period is close to the natural frequency as the system condition approaches to minimum pressure drop line.

The amplitude of oscillation becomes large as the point of maximum pressure drop is approached to the minimum pressure drop line.

6) In stability criteria obtained by perturbation method is sufficient to describe the inception of flow oscillation at minimum pressure drop. However, this criteria has not been established near maximum pressure drop.

Nomenclature

A	-	Cross section area of the test section
E	-	Total energy stored in the system
G	-	Mass velocity
G_o	-	Mass velocity through the flow control valve
g	-	Gravitation constant
h_{fg}	-	Latent heat of liquid
h	-	Enthalpy of liquid
ℓ	-	Length of the test section
ℓ_1	-	Subcooled length
P_o	-	Inlet pressure
Q	-	Total heat input
R	-	Steam void fraction
$\bar{R}$	-	Effective steam void fraction
V	-	Total steam volume in the test section
Δh	-	Subcooling
ρ	-	Density
s	-	Slip ratio
x	-	Length coordinate in direction of flow
t	-	Time
ϕ	-	Total resistance through the test section

subscripts -

f	-	Water
g	-	Steam
e	-	Exit of the test section
i	-	Inlet of the test section

BIBLIOGRAPHY

1. P.A. Lottes et al, <u>2nd U. N. International Conference on Peaceful Uses of Atomic Energy, (1958). p/1983</u>

2. S. Levy and E. S. Beckjord, <u>Hydraulic Instability in a Natural Circulation Loop with Net Steam Generation at 1000 psi.</u> Trans. ASME 60-HT-27(1960)

3. H. Chilton, <u>A Theoretical Study of Stability in Water Flow through Heated Passages</u>, J. Nuclear Energy, 1957, Vol. 5. p273

4. E. R. Quandt, <u>Analysis and Measurement of Flow Oscillations. (1960)</u>

5. J. A. Fleck, <u>The Dynamic Behavior of Boiling Water Reactors</u>, J. Nuclear Energy, Part A, 11, 114-30 (1960)

6. S. W. Gouse, Jr. and C. D. Andrysiak, <u>Flow Oscillations in a Closed Loop with Transparent Parallel, Vertical, Heated Channels.</u> (1963) M.I.T.

7. R. J. Weatherhead, <u>Heat Transfer, Flow Instability, and Critical Heat Flux for Water in a Small Tube at 200 psia</u>. (1963) ANL-6715

8. Research Laboratories, Ramo-Woolridge Division, Thompson Ramo-Wooldridge, <u>Kinetic Studies of Heterogeneous Water Reactors</u>, Progress Report 24, RWD-RLM-236, (1961)

9. R. C. Martinelli and D. B. Nelson, <u>Prediction of Water</u>, Trans. ASME Vol. 70 (1948)

10. H. S. Isbin, R. H. Moen and D. R. Mosher, <u>Two-Phase Pressure Drops</u>, AECU-2994 (1954)

11. Den Hartog, <u>Mechanical Vibration</u>,

12. S. Hayama, <u>A study on the Hydrodynamic Instability in Boiling Channels</u>, Trans. JSME (1962)

13. J. E. Meyer and R. P. Rose, Application of Momentum Integral Model to the Study of Parallel Channel Boiling Flow Oscillation, Trans. ASME. 62-HT-41 (1962)

14. S. W. Gouse, Jr. Two Phase Gas-Liquid Flow Oscillations, Note for special summer program, M.I.T. (1964)

15. Peter Griffith, Two Phase Flow in Pipes, Memorandum, M.I.T.

Table 1. Experimental Data

Run No.	Flow Rate (lb/hr)	Inlet Mass Vel. (lb/hr ft²) × 10^5	Inlet Temp (°F)	Heat Flux (BTU/hr ft²) × 10^3	Exit Qual. (% w)	Frequency (cps)	Transit Time (sec)	Cal. Period (sec)
1	19.72	1.108	169.0	3.060	4.45	0.6 (1.7)	2.16	1.06
2	19.72	1.107	169.5	1.220	-	*	-	-
3	28.91	1.624	172.0	3.318	2.16	0.9 (1.1)	1.37	0.628
4	26.95	1.514	171.5	5.867	8.00	0.4 (2.5)	1.90	1.31
5	24.97	1.403	171.5	9.158	16.44	0.35 (2.9)	2.02	1.65
6	23.66	1.329	171.0	13.423	27.79	0.35 (2.9)	1.44	1.55
7	31.28	1.757	173.0	17.761	27.82	*	1.10	-
8	31.28	1.757	172.0	5.490	5.61	0.85 (1.2)	1.40	0.930
9	45.74	2.570	171.0	4.071	1.23	*	0.43	-
10	19.06	1.071	190	1.560	2.16	0.23 (4.35)	3.5	
11	16.82	0.945	190	3.228	8.39	0.33 (3.00)	3.73	
12	15.77	0.886	190	5.835	18.47	0.27 (3.71)	2.30	
13	13.14	0.738	190	9.034	36.46	0.39 (2.56)	1.93	
14	13.14	0.738	190	13.041	53.72	0.43 (2.33)	1.74	
15	27.42	1.540	190	2.980	3.699	0.40 (2.50)	2.46	
16	26.29	1.477	190	5.792	16.0	0.42 (2.30)	2.37	
17	22.34	1.255	190	9.203	20.85	0.50 (2.00)		
18	9.86	0.554	190	5.716	0.609	0.13 (7.5)	5.00	
19	9.2	0.517	190	2.343	11.95	0.19 (5.46)	6.19	
20	8.15	0.457	190	1.537	10.27	0.17 (6.00)	7.60	
21	8.14	0.457	190	[illegible]	91.63	0.20 (5.00)	1.11	
22	11.8	0.662	197.5	1.261	4.48	0.20 (5.00)	6.22	
23	10.3	0.578	197.4	1.998	9.38	0.20 (5.00)	6.25	

24	10.3	0.578	197.1	4.215	21.5	0.17 (5.88)	4.51
25	9.2	0.517	197.0	5.579	32.5	0.2 (5.00)	4.62
26	9.2	0.517	196.0	7.107	42.2	0.233 (4.3)	3.62
27	9.2	0.517	195.0	8.887	53.1	0.33 (3.0)	2.68
28	9.2	0.517	195.6	12.773	77.2	0.4 (2.50)	2.47
29	23.6	1.326	197.00	2.098	3.47	0.22 (4.55)	3.16
30	23.6	1.326	197.0	4.208	8.50	0.18 (5.55)	3.00
31	23.0	1.292	197.0	5.541	12.20	0.20 (5.00)	2.76
32	20.35	1.143	197.0	7.101	18.11	0.23 (4.34)	2.16
33	21.00	1.180	197.8	10.801	27.68	0.25 (4.00)	2.15
34	19.70	1.107	197.8	14.792	41.07	0.25 (4.00)	1.74
35	19.70	1.107	197.6	27.555	54.74	0.25 (4.00)	1.57
36	31.30	1.769	198.2	3.035	4.01	0.266 (3.76)	1.79
37	30.20	1.697	198.2	7.067	14.93	0.266 (3.76)	1.79
38	25.00	1.405	198.5	12.617	27.13	0.300 (3.33)	1.81
39	21.03	1.181	205	5.641	14.4	0.2 (5.0)	2.53
40	21.03	1.181	195	5.641	13.4	0.2 (5.0)	
41	22.08	1.240	185	5.641	11.58	0.2 (5.0)	
42	21.03	1.181	171.0	5.641	10.96	0.33 (3.0)	
43	21.29	1.196	165	5.641	10.35	0.5 (2.0)	
44	21.03	1.181	153	5.641	9.11	0.714 (1.4)	
45	21.69	1.219	141	5.641	7.83	0.83 (1,2)	

Note:

1) The table do not include data in the stable region.
2) * indicates the measurements of the oscillating frequency is difficult.
3) Run No. 40 through 45 are data of the subcooling test.
4) Exit pressure of pump is 100 psi.

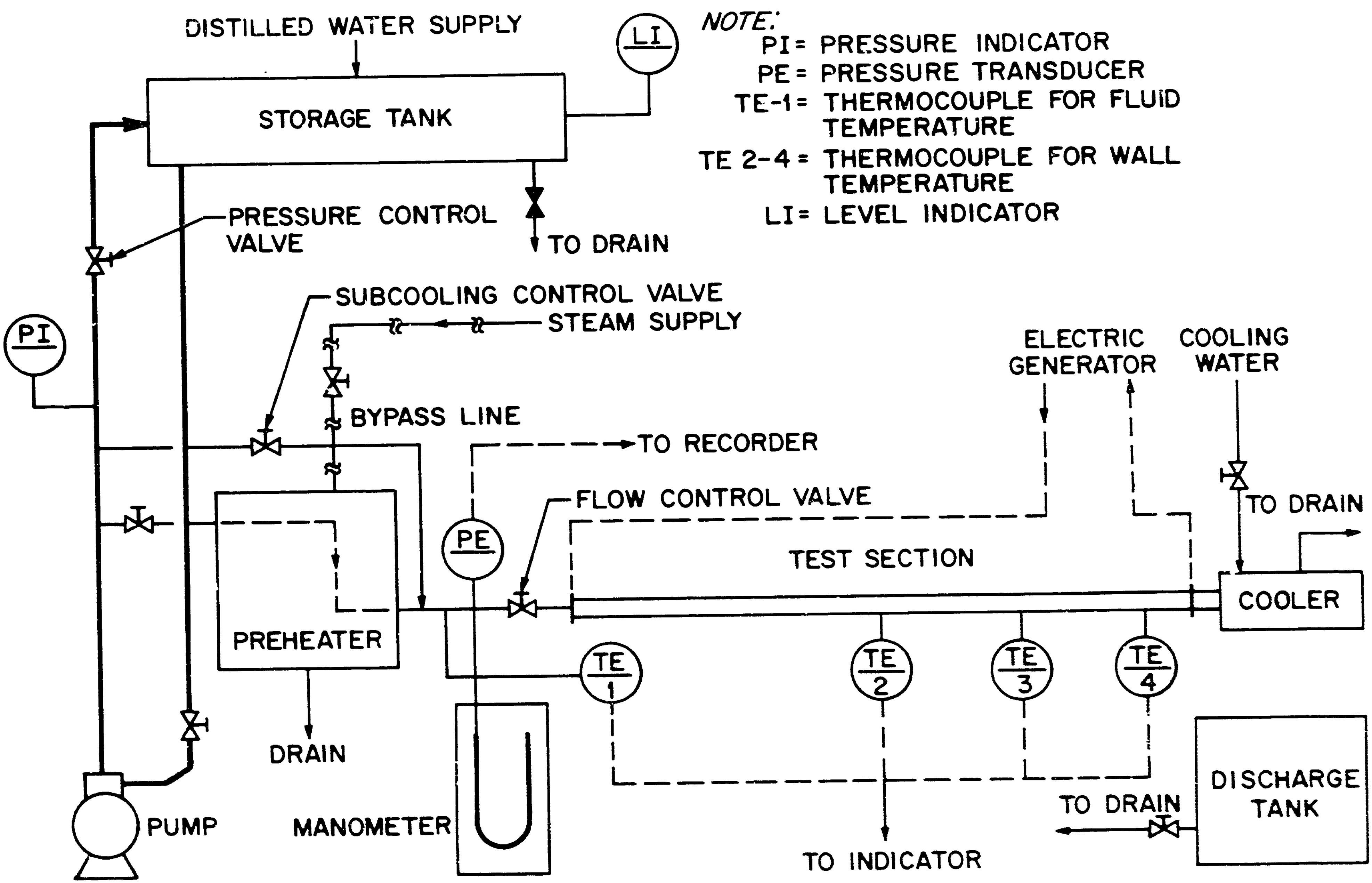


FIG. 1 SCHEMATIC DIAGRAM OF THE APPARATUS

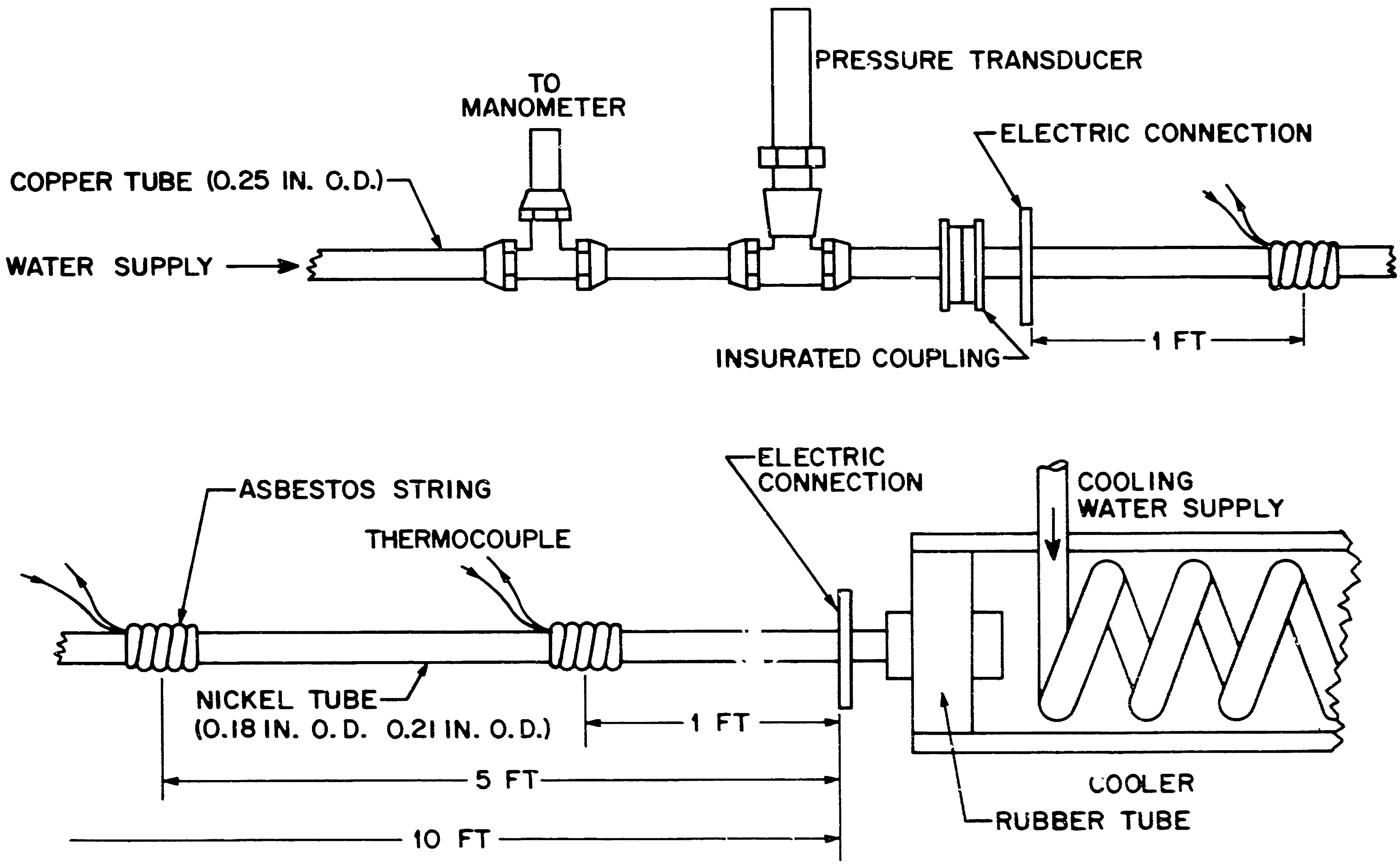


FIG. 2 DETAIL OF TEST SECTION

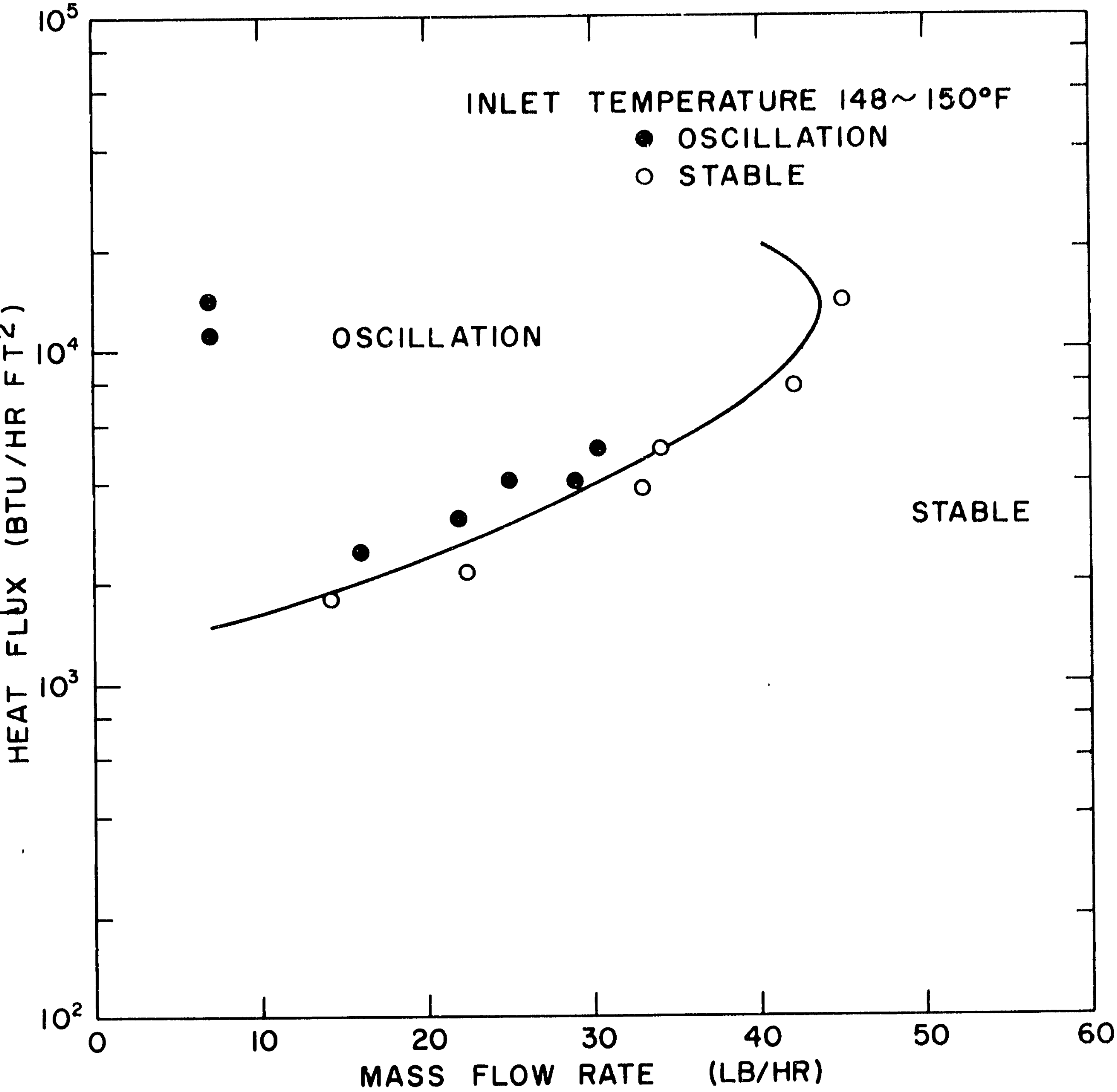


FIG. 3 STABILITY MAP

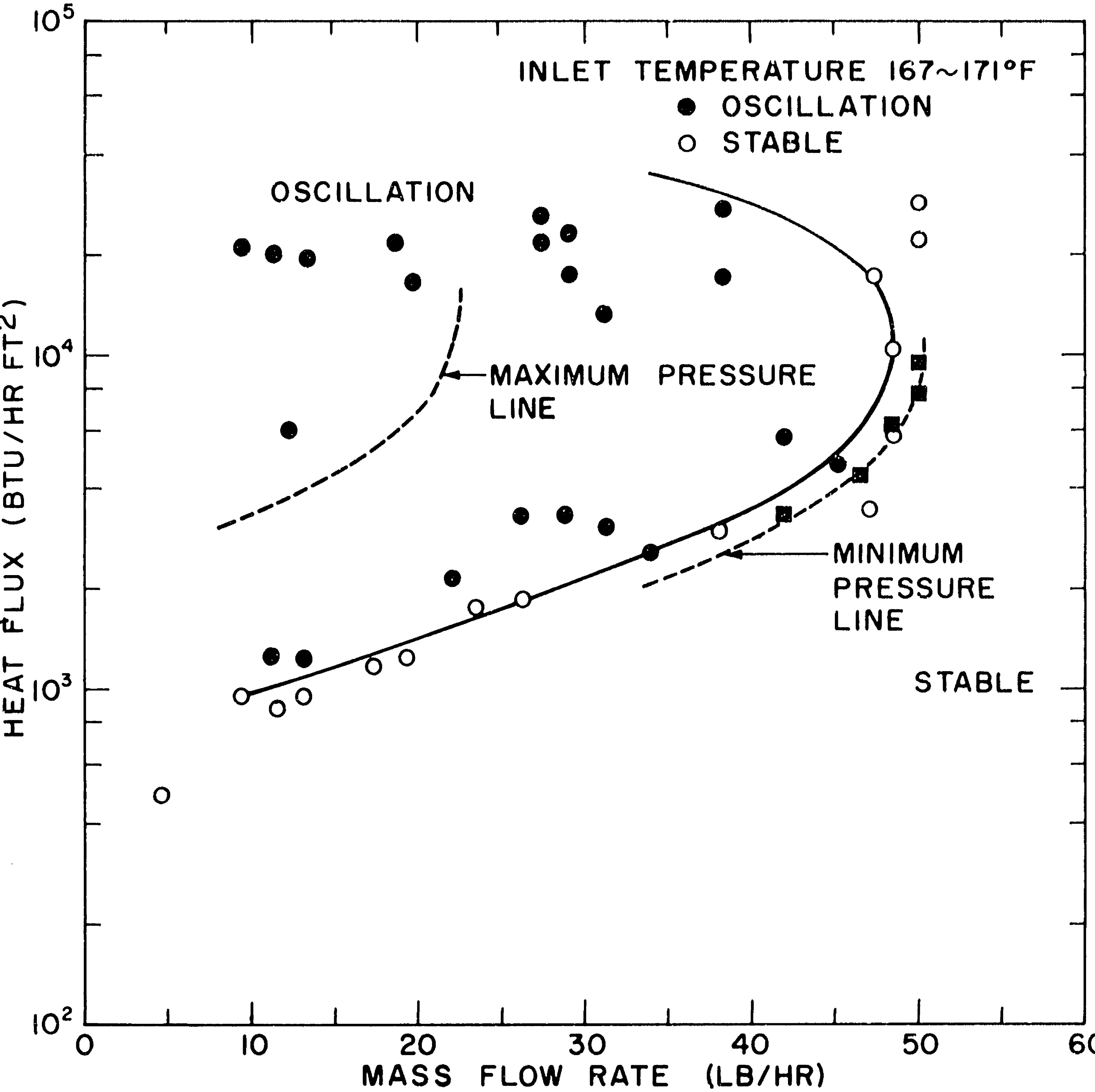


FIG. 4 STABILITY MAP

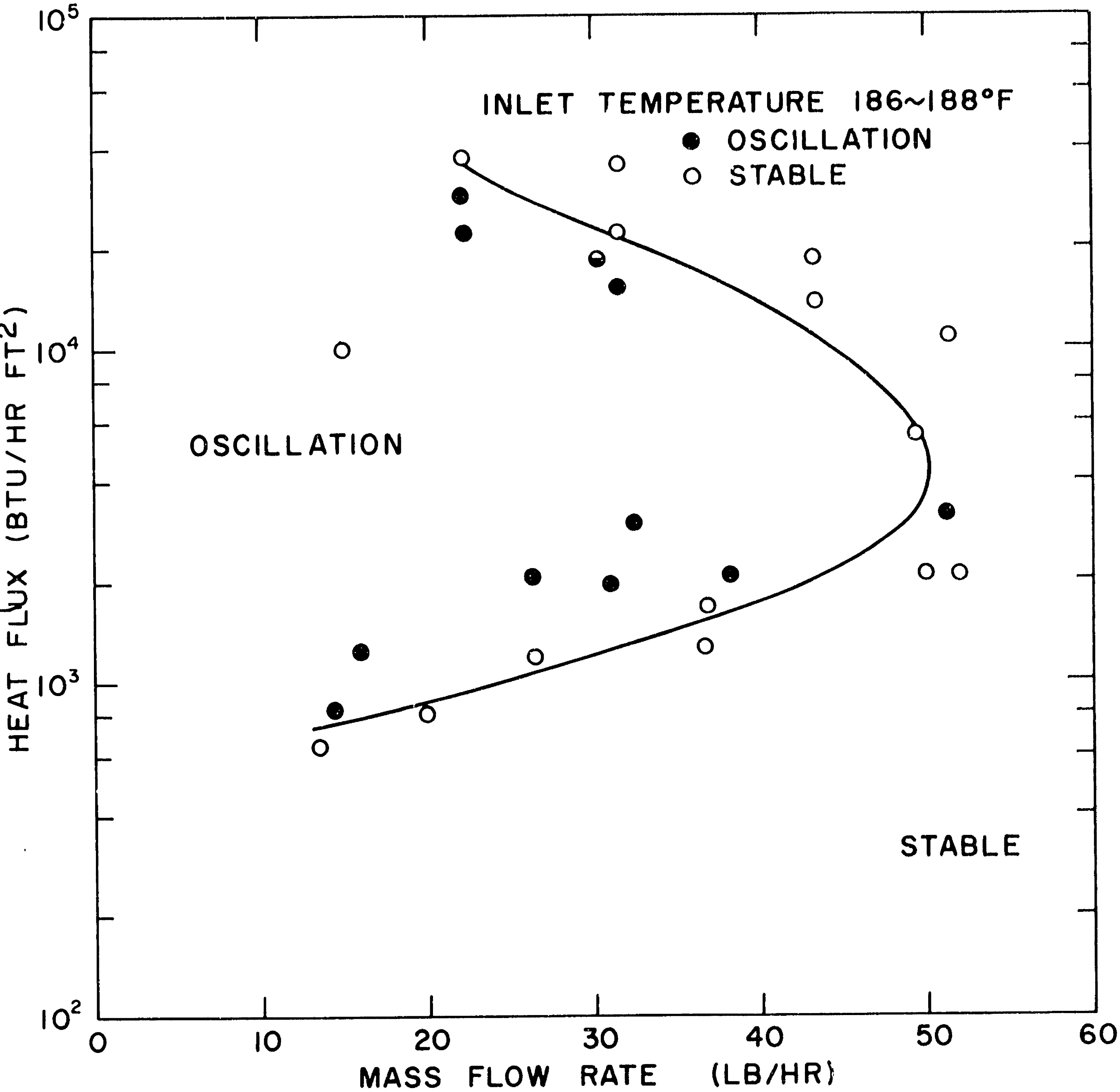


FIG.5 STABILITY MAP

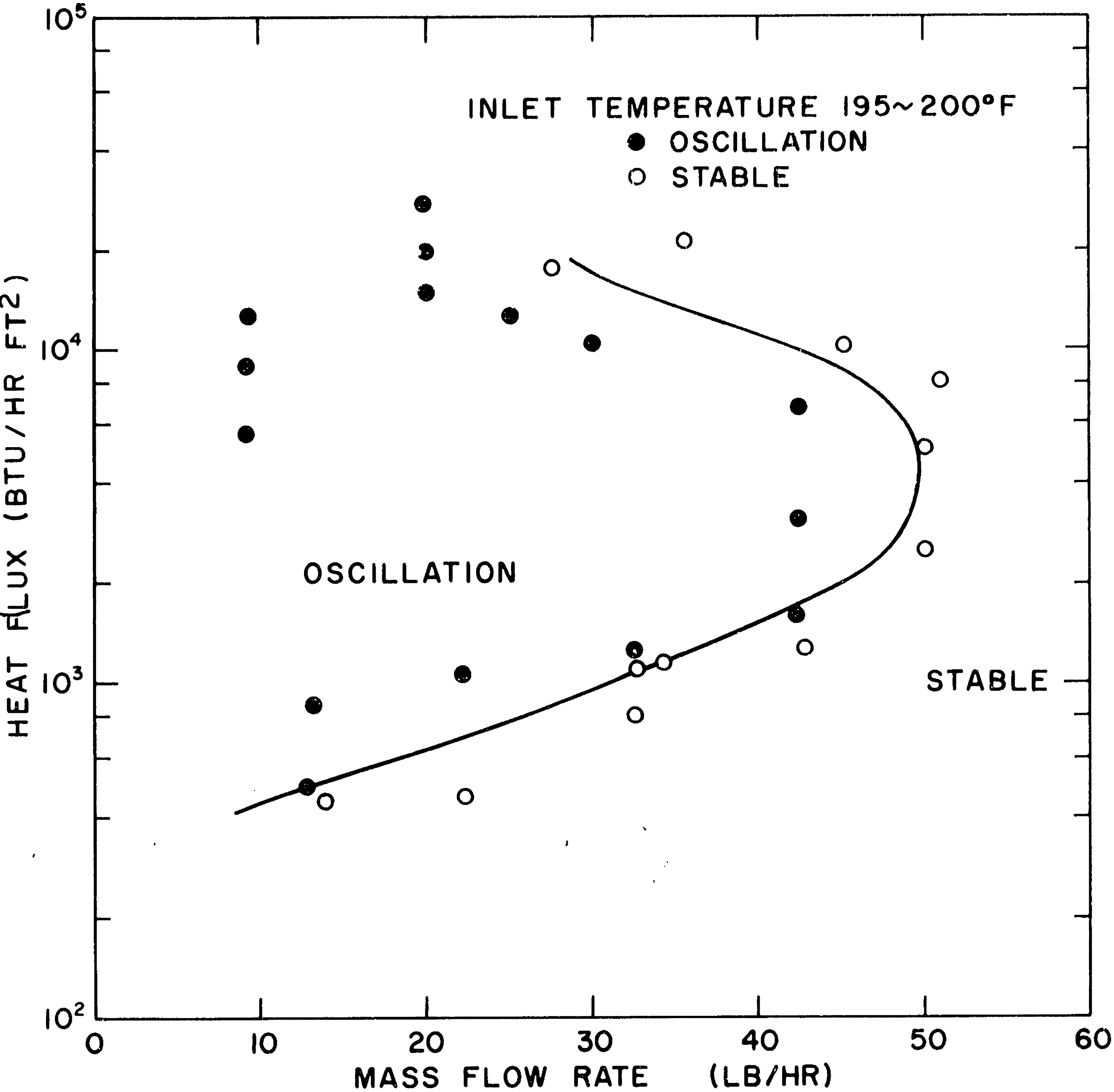
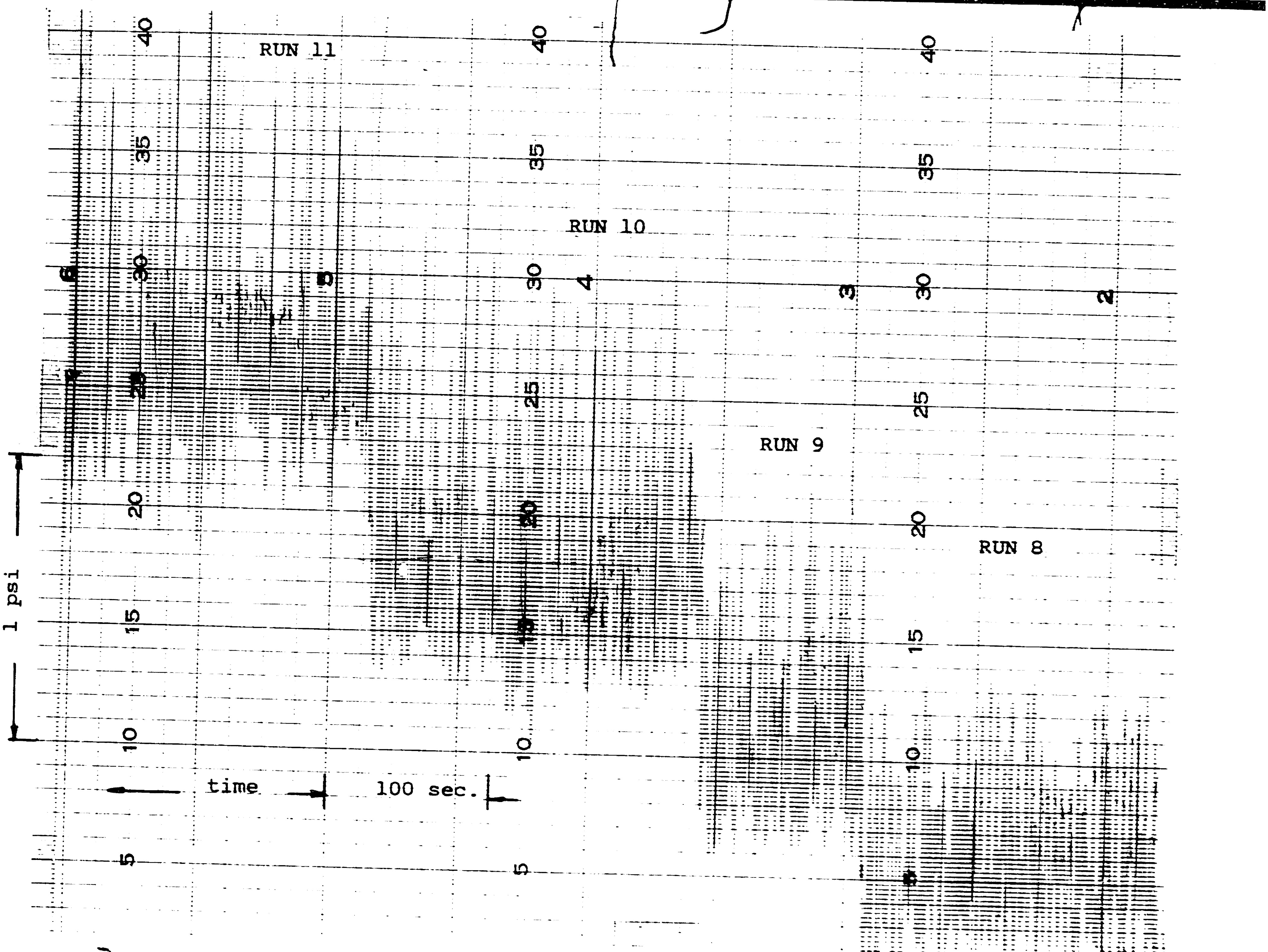
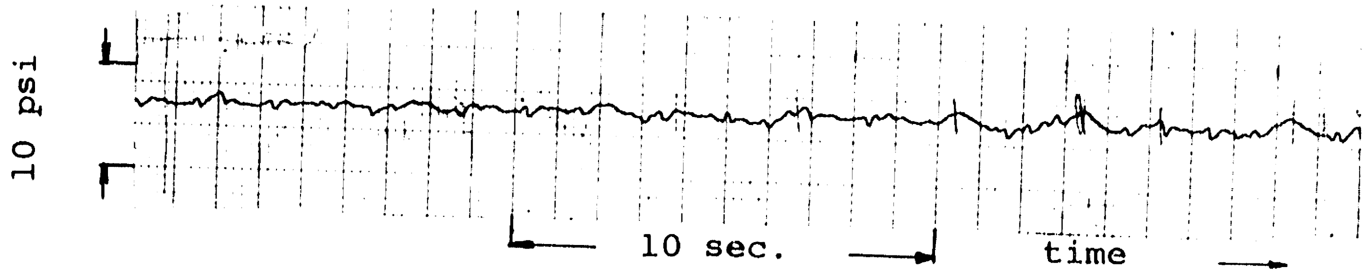
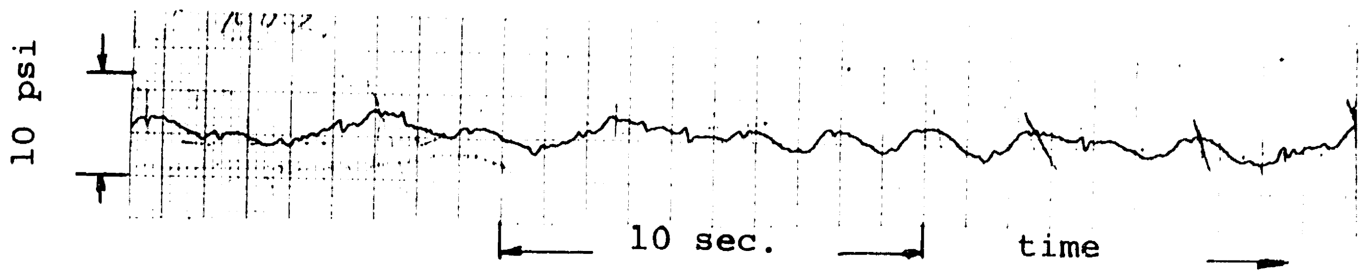


FIG. 6 STABILITY MAP

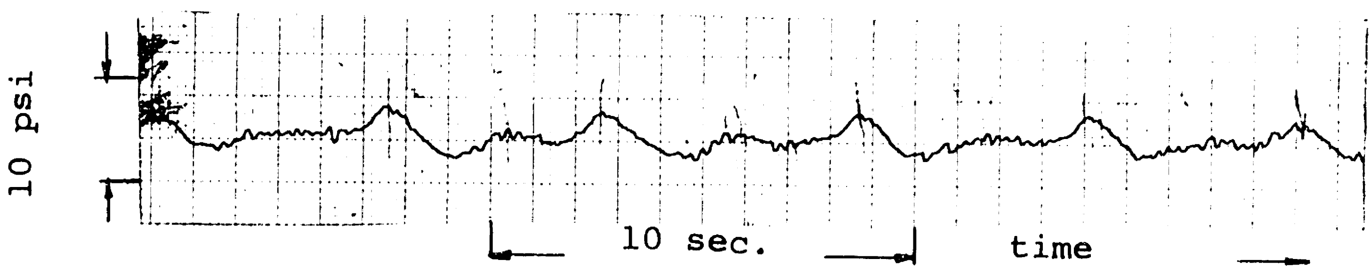

RUN 11
RUN 10
RUN 9
RUN 8
40
35
30
25
20
15
10
5
1 psi
time
100 sec.



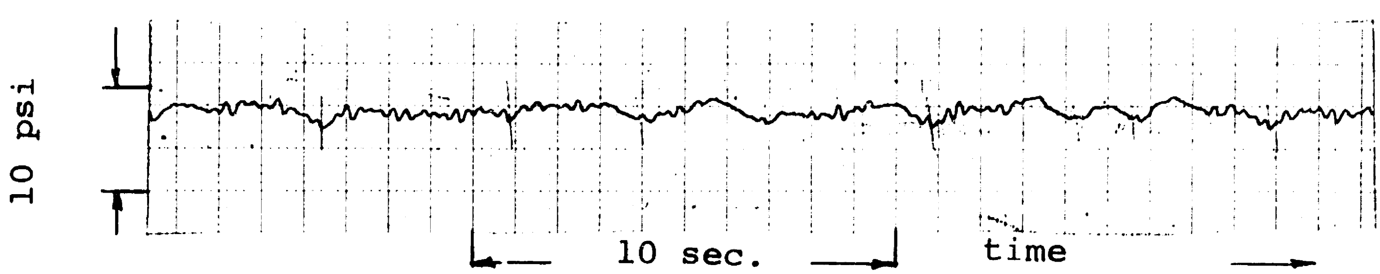
RUN 39



RUN 40



RUN 41



RUN 42

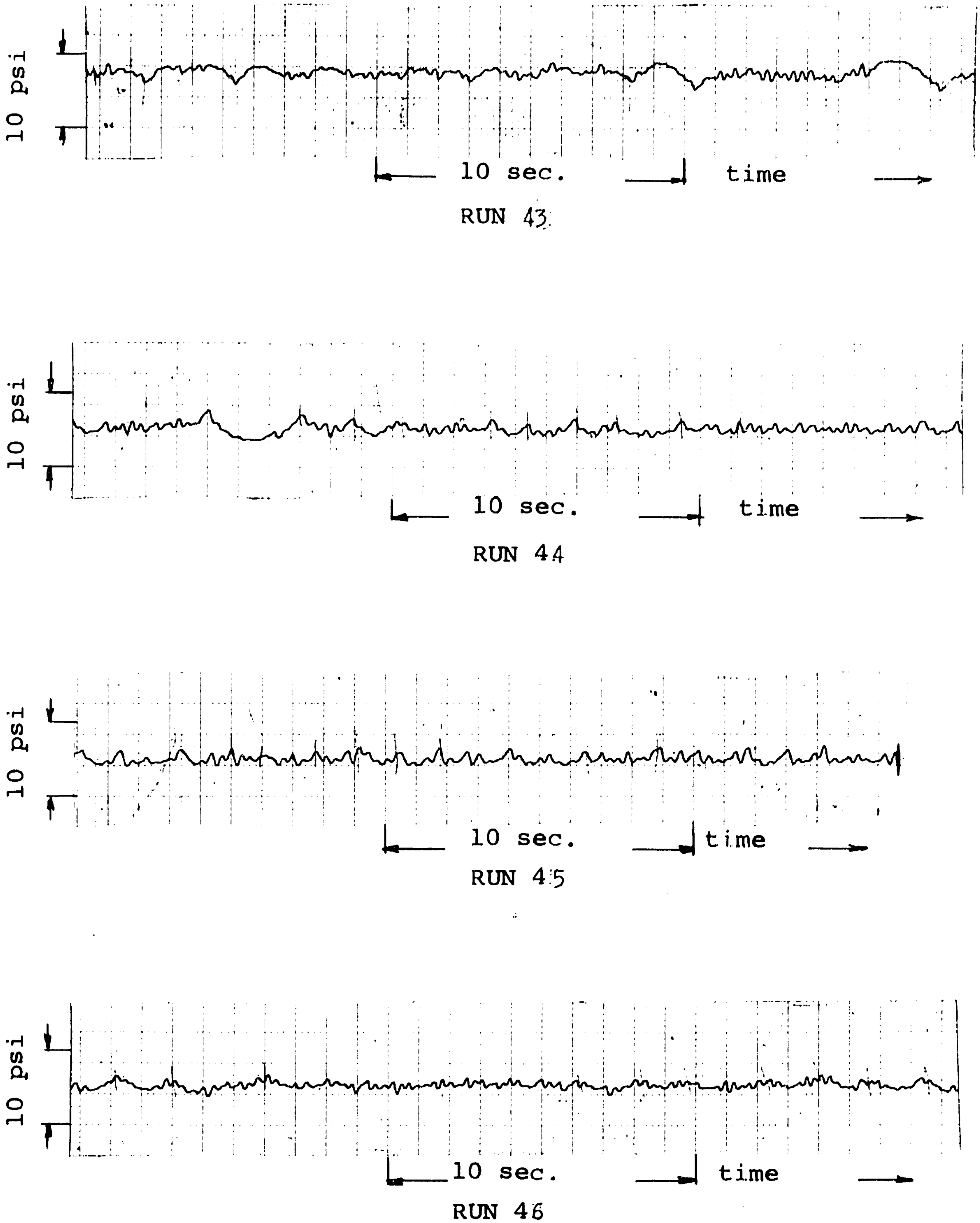


FIG. 8 PRESSURE OSCILLATION

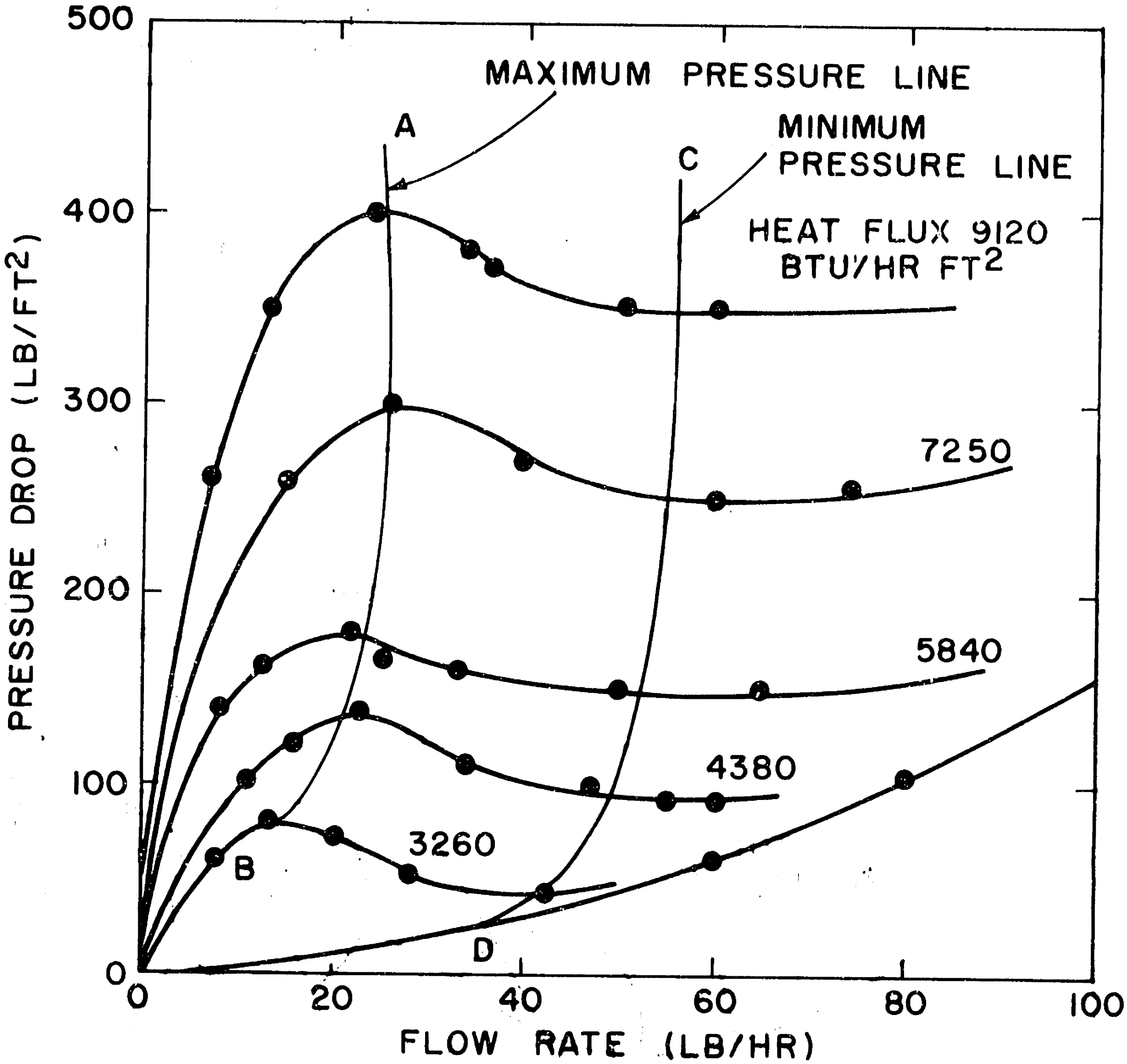


FIG. 9 PRESSURE DROP FOR SUBCOOLING OF 170°F

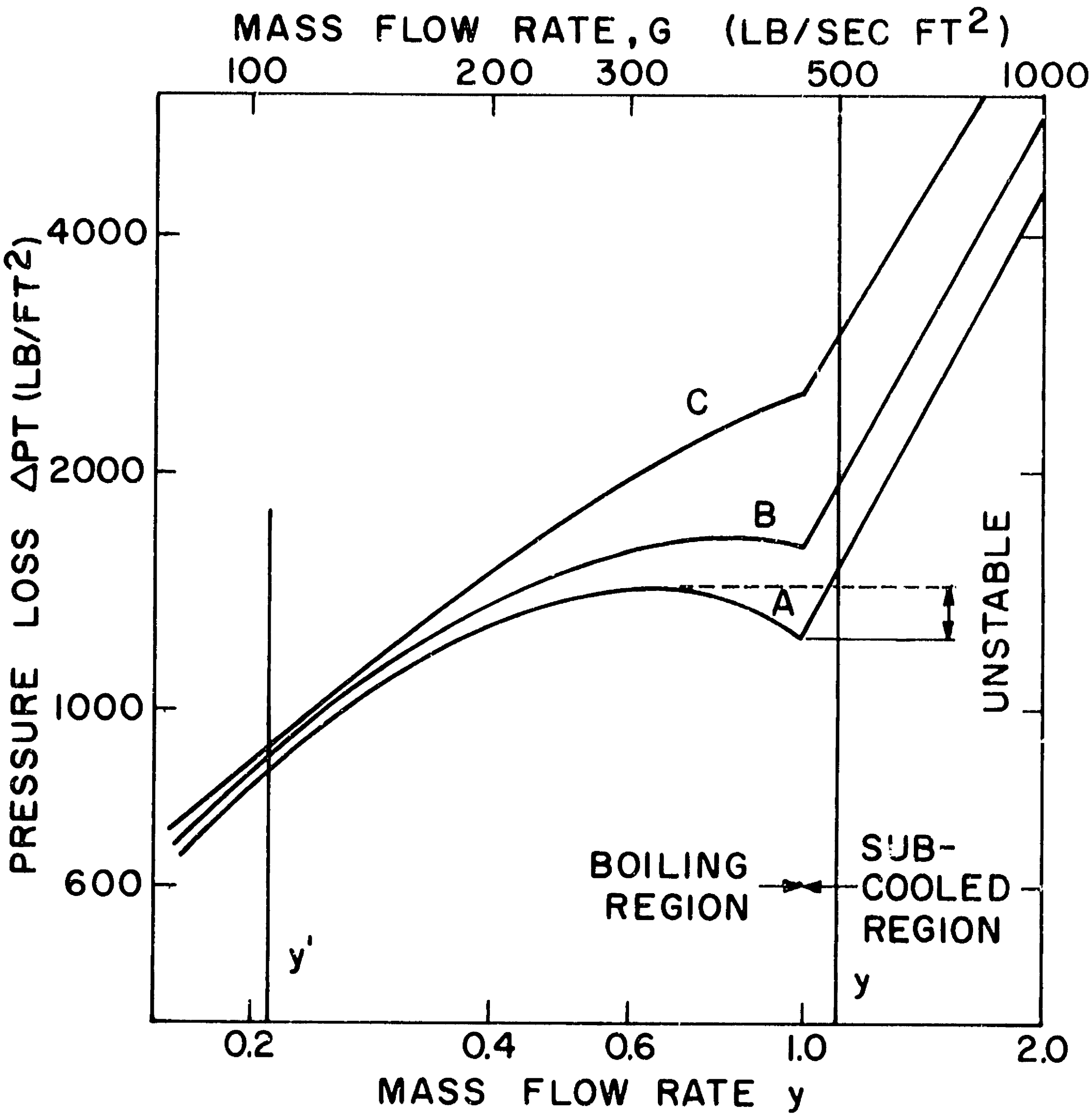


FIG. 12 CHILTON'S MODEL