

Essays in Financial Economics

by

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Abstract

This dissertation is comprised of three essays. The first analyzes Treasury auctions when modeled as common-value share auctions. The strategic interaction between the auction and the surrounding trading in forward contracts is studied. Ample evidence is found that trading before the auction mitigates problems of information asymmetry and increases competitiveness in the auction. The effects of “quantity uncertainty” caused by non-competitive bidding are also analyzed; these bids are assured fulfillment, leaving an unknown quantity to the competitive bidders. Surprisingly, the expected auction price increases as the quantity uncertainty rises. This result is in contrast to the reduction in expected price experienced when uncertainty about underlying value increases — a well-known result from the auction literature. Policy implications conclude the essay.

The second essay investigates empirically the link between Treasury-bill auctions and the degree to which a specific bill is traded “special” in the repurchase and reverse market. For auctions with relatively aggressive bidding, a strong positive link is documented between “specialness” and unexpectedly high auction prices. In less aggressive auctions, the opposite result is found. Some additional findings relating overall uncertainty about a particular issue to “specialness” are also presented.

In the third essay, the impact of real options in a firm’s asset mix on its stock-return characteristics is investigated. The volatility elasticity with respect to price, γ , for assets with option features is studied analytically. Simulations are used to assess the small-sample properties of γ . The well-known empirical fact that stock returns display negative γ ’s, and this finding’s link to leverage, are documented in the sample. In addition, there is strong empirical evidence that proxies for options, especially research and development expenditures, help to explain this effect for profitable companies.

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Et veniam pro laude peto: laudatus abunde,
non fastiditus si tibi, lector, ero.

Ovid; *Tristia, I, vii, 31*

Contents

1	Price Formation in the Treasury Market — Quantity Uncertainty and Interaction across Markets	13
1.1	Introduction	13
1.2	Institutional Features	15
1.3	Interaction across Markets: The Model	18
1.4	Price Formation: Analysis	25
1.4.1	Price Impact	26
1.4.2	“Quantity” and “Value” Uncertainty	29
1.4.3	Interaction across Markets	32
1.5	Policy Implications	35
1.6	Concluding Remarks	36
1.7	References	37
1.8	Figures	40
2	Treasury-Bill Auctions and the Repurchase and Reverse Market	55
2.1	Introduction	55
2.2	The Market for Treasury Bills	57
2.2.1	The Primary Market	57
2.2.2	The Repurchase and Reverse Market	61
2.3	The Data	63
2.4	Auction Bidding and Issues Traded Special	66
2.5	Uncertainty and the Repo Market	71
2.6	Concluding Remarks	72
2.7	References	73
2.8	Tables	75

3	Stock Returns and Real Options: An Exploratory Study	85
3.1	Introduction	85
3.2	Options and Conditional Volatilities	88
3.3	The Data	93
3.4	Cross-Sectional Analysis of the Elasticity γ	96
	3.4.1 Equity Returns for Individual Firms	97
	3.4.2 Equity Returns for Groups of Firms	99
	3.4.3 Returns on Total Assets for Individual Firms	100
3.5	Concluding Remarks	103
3.6	References	103
3.7	Figures and Tables	106

Chapter 1

Price Formation in the Treasury Market — Quantity Uncertainty and Interaction across Markets

1.1 Introduction

Probably no other concept is more at the heart of modern finance theory than “price formation;” we try to price assets and to establish what the determinants of equilibrium prices truly are. Despite its fundamental nature — or just because of it — our understanding of these issues is still far from complete. The market structure does have an impact on how rational economic agents behave and on the ways information is compounded into prices. We think important insights can be made by studying specific markets, and by understanding the strategic and informational linkage between them. In this respect the Treasury auctions and the surrounding trading in the forward market in securities to be delivered when issued (the so-called when-issued market) offer a challenging example; exactly the same commodity is traded in two very different markets, between which an interesting strategic interaction safely can be assumed to take place.¹ In this paper we try to analyze the impact of the surrounding trading opportunities on the price formation in the Treasury auction. This being one of the largest and most liquid financial markets in the world, we think it is of particular importance to understand how

¹Empirical evidence of such strategic interaction can be found in Bikhchandani-Edsparr-Huang (1994) and Simon (1993). See also Chapter 2.

this market structure affects the auction.

The theoretical auction literature is one of the most successful results of the game-theoretic approach to economic problems.² Unfortunately, there are several characteristics of Treasury auctions that do not fit the assumptions of most of the traditional auction literature. First, most auction models are based on the single-unit assumption, i.e., there is only one unit for sale (say, a painting or a bottle of wine), which cannot be divided. Even if multiple units are allowed, each bidder can, usually, acquire at most one of these units.³ As bidders in Treasury auctions are allowed to submit any number of bids and the over-all quantity restrictions rarely are binding, the standard assumptions seem inadequate to capture the essence of Treasury-auction bidding: the trade-off between price and quantity. Second, some fraction of the securities in Treasury auctions is sold to smaller investors, the Fed, and foreign central banks through a non-competitive bidding procedure, effectively reducing the quantity available to the participants in the regular (competitive) bidding. As the quantity being set aside for non-competitive bidders is not known in advance, the regular bidders are in fact faced with a random quantity rather than the fixed number that is the customary assumption throughout the auction literature. Third and maybe most important, a Treasury auction is not an isolated event. Trading in surrounding markets may convey information relevant to the auction and change the strategic behavior of the bidders;⁴ the most clear-cut example is the when-issued market, in which an identical good is traded. We also think that the trading in Treasury securities before the secondary market opens is best captured in a finite-horizon model. Having an infinite horizon is tantamount to saying that each period of trading is strategically the same.⁵ We think this is a good description of the stock market, but Treasury auctions and the when-issued market are different. The auction is a singular event, and once the securities are delivered, the when-issued market ceases to exist. It thus seems more appropriate to use a finite-horizon approach with its explicit time dependency. Once the sec-

²For a detailed account of auction theory and its implications for Treasury auctions, see Bikhchandani-Huang (1993).

³One exception to this one-unit-per-bidder assumption is the setup in Tenorio (1993). The only strategies he allows, however, force the bid price per unit to be the same, even if a bidder tries to acquire many units; in Treasury auctions, a bidder can effectively submit an entire demand function. He uses a private-values model.

⁴This is what Bikhchandani-Huang (1989) find when they allow for a resale market after the auction.

⁵One such model can be found in Wang (1994), who analyzes trading volume and stock returns under asymmetric information and with competitive agents.

ondary market opens, the market dynamics change radically, and we think that it makes a reasonable end point of the strategic interaction displayed by the primary dealers.

We analyze the primary Treasury-securities market within a framework that maintains the above characteristics. Our approach is based on the concept of share auctions, as analyzed in the seminal paper Wilson (1979), i.e., auctions of a perfectly divisible good in which the bidders submit demand schedules rather than discrete bids. To capture the effect of non-competitive bidding, we also allow for quantity uncertainty in the auction, and trading can take place before as well as after the auction.

In Section 1.2 we outline the institutional features pertinent to our setup, and Section 1.3 presents the model and characterizes the equilibrium strategies. In Section 1.4, we first analyze the impact of introducing additional trading opportunities on the bidding (demand) strategies; the effects of quantity uncertainty are also studied and compared to those of value uncertainty. We then carry this analysis one step further and determine the impact on price formation. First, we focus on the quantity uncertainty generated by the non-competitive bidding in Treasury auctions. Quantity uncertainty is radically different from the far better understood value uncertainty, and its impact in a strategic setting with asymmetric information is to force the bidders to behave more competitively. In this respect we can think of non-competitive bidding as “noise trading,” and when it becomes less predictable, the market “liquidity” improves. Its impact is similar to that of having less risk-averse bidders or having a larger number of bidders, i.e., it tends to raise the expected auction prices. In a competitive setting, increased quantity uncertainty has the opposite effect of *lowering* the expected auction prices. Second, we find that pre-auction when-issued trading fosters competitiveness in the auction and reduces information asymmetries. The former of these effects is accentuated when the bidders enter the auction with, on average, net-short positions. Furthermore, In Section 1.5 we analyze the policy implications of our results, and Section 1.6 concludes.

1.2 Institutional Features

In this paper we try to model the pre-delivery trading of Treasury securities, i.e., we have one real and very specific market structure in mind. It is not intended to be as general as possible, but rather to be as close a description of the real

Treasury-securities markets as feasible while maintaining tractability. To provide the necessary background information, we will use this section to outline the structure of the institutional features. Our model is a uniform-auction model, i.e., all winning bidders pay the highest losing bid, regardless of their own bids. Currently, this is the auction format used for 2- and 5-year Treasury notes, whereas the other maturities are sold using a discriminatory auction. There are also plans to experiment with ascending-price auctions.⁶ Both uniform and ascending-price auctions are commonly believed to raise higher revenues for the seller than discriminatory auctions.⁷ We believe that our assumption of a uniform-auction format also serves as an approximation of an ascending-price auction. As our model applies to all Treasury securities, we try to describe each of the maturities currently being auctioned by the Treasury. Further details of the market structure can be found in Section 2.2.1 of Chapter 2.

Treasury bills, 3-, 7- and 10-year Treasury notes, and 30-year Treasury bonds are sold by the Department of Treasury in a discriminatory auction. The 13- and 26-week bills are auctioned every Monday, or, when it is a holiday, the first trading day thereafter. The 52-week bills are sold every fourth Tuesday, and the 3-, 7-, and 10-year notes and 30-year bonds are auctioned quarterly. In a discriminatory auction, the bidders submit sealed bids, and the auctioneer allocates the goods according to the demand curve generated by the bids. (The highest bidder gets whatever amount he⁸ requested at that price, the second-highest bidder the amount he asked for, and so on until the entire quantity has been distributed.) Each bidder pays the price he submitted. If there is excess demand at the lowest winning rate, the securities are pro-rated among the bidders at that price.

Once a month the Treasury sells 2- and 5-year notes in a uniform auction. In a uniform auction, the bidders also submit sealed bids that are being met from the highest bidder down, but the successful bidders pay only the highest losing bid. The pro-rationing at the lowest winning rate is done as in a discriminatory auction.

In the Treasury auctions, there is one additional feature: *non-competitive* bids.

⁶See the Joint Report on the Government Securities Market (1992).

⁷This issue is thoroughly discussed in Bikhchandani-Huang (1993). A classic reference in the single-unit case is Milgrom-Weber (1982). Back-Zender (1993) and Reinhart (1992) address the related issues of collusive bidding strategies and susceptibility to manipulations.

⁸The use of the masculine as the default third-person pronoun is merely a choice of convenience, not a description of factual or implied gender; this remark applies to the remainder of the dissertation as well.

The auction procedures described above apply only to those submitting *competitive* bids, mainly the approved *primary dealers* and their institutional clients. Bidders unwilling to specify a price can submit bids that will be fulfilled at the quantity-weighted average winning price of the competitive bids. In other words, these bidders cannot condition their allocation on the price, but they are assured fulfillment. In a typical T-bill auction, the amount of non-competitive bids accounts for 15–20% of the total; for notes and bonds, the amount of non-competitive bids tends to be somewhat less. No one is allowed to submit a non-competitive bid for a quantity larger than five million dollars, except the Fed, which submits non-competitive bids for its own account as well as for other central banks. The competitive bidders submit price-quantity⁹ pairs. An over-all limit of 35% of the issue size — auction bids and the position in the when-issued market before the auction combined — applies to all competitive bidders. All competitive bids must be submitted by 1:00 p.m., and the outcome of the auction is announced around 2:00 p.m.¹⁰ The securities are usually delivered the following Thursday morning, the issue date, at which time the secondary market opens.

An integral part of the trading in Treasury securities is the forward market, in which securities to be delivered when issued are traded. This market is usually referred to as the “when-issued” market.¹¹ Trading in the when-issued market begins when the Treasury has announced the quantity of securities to be auctioned off in the following week, and it continues until the secondary market opens. The when-issued market is an over-the-counter market, in which principally the primary dealers and large institutional investors participate. Like all forward markets, the when-issued market is a zero-net supply market.

Institutional investors with demand larger than the five-million limit for non-competitive bids, use the when-issued market to ascertain that their demands are filled. In this market the primary dealers play the role of market makers. Since it is a zero-net supply market, the primary dealers usually have a net-short position when they submit their auction bids. If a primary dealer does not succeed in acquiring enough securities in the auction to cover his short position, he is exposed

⁹When bidding for bills, the primary dealers actually submit discount rates that are whole basis points (fractions thereof are not allowed), rather than prices. The auctions for coupon securities are based on yield.

¹⁰Until 1991 the outcome was announced after 3:30 p.m.

¹¹For an empirical investigation of when-issued rates relative to auction rates, see Bikhchandani-Edsparr-Huang (1994) for 13- and 26-week bills, and Simon (1993) for coupon securities.

to being “short squeezed,” i.e., forced, irrespective of the cost, to cover the position, either through purchases in the when-issued market or through transactions (loans) in the repurchase (“repo”) and reverse market. (The latter market, however, is *not* part of our model. A detailed description of the repo and reverse market is found in Section 2.2.2 of Chapter 2.) In this market a holder of a security can borrow short-term with the security as collateral (“obtain a repo”). The opposite side of such a transaction (a “reverse repo”) can be taken by someone who needs to cover a short position in that particular security. Securities acquired in the repo market can be used as collateral in further transactions or sold in the secondary market; to sell a security obtained as collateral in a reverse repo is in fact the standard way of short selling Treasury securities.

1.3 Interaction across Markets: The Model

Our model consists of N primary dealers, or *dealers* for short, who can trade on three different occasions before the public trading in the secondary market begins and all uncertainty is resolved. These three trading dates try to model the trading in the when-issued market immediately preceding the weekly auction, the auction itself, and the when-issued trading after the auction but before delivery. All three trading dates are modeled as Walrasian auctions, out of which we interpret the middle one as a share auction, as in Wilson (1979). We are interested in the interaction between these three markets as the prices are formed. Despite their sequential nature, we are not trying to model a gradual release of information *per se*, but rather how the existing private information is compounded into the price and becomes public. Especially the first two trading dates are thought to be virtually simultaneous. The last trading date tries to capture the when-issued trading between the auction and the opening of the secondary market. This setup also endogenizes the costs of being short squeezed. If a dealer cannot cover his position in the auction, he has to buy either in the when-issued market or in the secondary market; these costs will obviously affect the equilibrium auction price.

We let \tilde{v} denote the true value of the security, i.e., the price it will fetch in the secondary market, and assume that this value is not influenced by the preceding trading. This assumption corresponds to our belief that the secondary market is more competitive and less affected by information asymmetries than the when-issued market and the auction. The larger number of participants and the less important role played by the primary dealers in the secondary market are factors

that point in this direction. Clearly, these features are more a question of degree than of different qualitative natures. We believe, however, that they are strong enough to justify our approach of studying the preceding trading without explicitly modeling the secondary market as well. For simplicity, we normalize $\mathbf{E}[\tilde{v}]$ to be zero, and let σ_v^2 denote $\mathbf{V}[\tilde{v}]$, where $\mathbf{E}[\cdot]$ and $\mathbf{V}[\cdot]$ are the unconditional expectation and variance operators. Dealer i receives a signal $\tilde{s}_i = \tilde{v} + \tilde{\epsilon}_i$, where we assume that \tilde{v} and the $\tilde{\epsilon}_i$'s are all jointly normal and independent of each other. We let $\sigma_{\epsilon_i}^2$ denote $\mathbf{V}[\tilde{\epsilon}_i]$ for $i = 1, 2, \dots, N$. This information structure makes the private signals positively correlated. Most of the private information is likely to come from the order flow that a primary dealer observes in the course of his usual activities. We think it is reasonable to assume that this information is positively correlated across dealers; if one dealer experiences an extraordinarily high demand from his clients, the other dealers probably do too. Our model is designed to capture that feature. The trading in period one is modeled as a share auction, in which the dealers submit linear demand functions:

$$x_i^1(p) = \kappa_1 + \zeta_1 \tilde{s}_i - \pi_1 p. \quad (1.1)$$

In addition to the N primary dealers, we also have regular investors who want to ensure a certain quantity of the securities soon to be auctioned off; buying in the auction is always associated with uncertainty, either about the price (as a non-competitive bidder) or about the quantity (as a competitive bidder). In this model we treat their demand, \tilde{z} , as exogenous, distributed normally with variance σ_z^2 and mean \bar{z} . All winning bidders pay the price that equates supply and aggregate demand, \tilde{p}^1 , which is the price that solves:

$$\sum_{i=1}^N x_i^1(p) + \tilde{z} = 0, \quad (1.2)$$

where we note that this is a market with zero-net supply.

The trading in period two, the ‘‘auction,’’ is modeled in a similar way, but since the dealers now have additional information from the when-issued market, the demand schedules take the form:

$$x_i^2(p) = \kappa_2 + \zeta_2 \tilde{s}_i + \pi_2^1 \tilde{p}^1 + \eta_2 x_i^1 - \pi_2^2 p. \quad (1.3)$$

As the bidders have observed the price in period one, they will use this information to update their expectation of \tilde{v} . They have also established a position in the when-issued market, x_i^1 . (Usually this position is negative, which is to say that on

average the dealers sell short in the when-issued market as part of their market making.) Once a dealer has established a position, it will affect his attitude toward additional trades; for example, if he has a short position when submitting bids, a purchase in the auction will actually *reduce* his over-all risk (variance). Thus, x_i^1 and \tilde{p}^1 enter the demand schedule (1.3).

In real life, the quantity to be auctioned off is announced publicly. The amount of non-competitive bids, however, is stochastic, so the amount available to the dealers is *de facto* stochastic. We try to capture this feature by letting the quantity to be auctioned off, \tilde{q} , be stochastic. We let \tilde{q} be normally distributed with variance σ_q^2 and mean \bar{q} . Everyone pays the price that clears the market, \tilde{p}^2 , i.e., the price that solves:

$$\sum_{i=1}^N x_i^2(p) = \tilde{q}. \quad (1.4)$$

The trading in the when-issued market after the auction is modeled in the same way as before the auction. All dealers submit demand schedules of the form:

$$x_i^3(p) = \kappa_3 + \zeta_3 \tilde{s}_i + \pi_3^1 \tilde{p}^1 + \pi_3^2 \tilde{p}^2 + \eta_3^1 x_i^1 + \eta_3^2 x_i^2 - \pi_3^3 p. \quad (1.5)$$

For the same reasons as before, \tilde{p}^2 and x_i^2 will now also enter the demand schedule. We allow for an exogenous demand \tilde{w} , which is normally distributed with variance σ_w^2 and mean \bar{w} . The trading in the third period allows us to study the impact of a potential squeeze on the price formation; a dealer who has sold a larger quantity short in the pre-auction when-issued trading than he has been able to acquire in the auction will be hard pressed to buy sufficient quantities in the when-issued trading before delivery has to take place, usually on the following Thursday. As before, everyone pays the the price, \tilde{p}^3 , that clears the market:

$$\sum_{i=1}^N x_i^3(p) + \tilde{w} = 0. \quad (1.6)$$

As in period one, this round of trading has a zero-net supply.

Since x_i^1 and x_i^2 are functions of \tilde{s}_i , \tilde{p}^1 , and \tilde{p}^2 , it may seem possible to eliminate the dependence on x_i^1 and x_i^2 in the demand schedules. To derive an equilibrium we need, however, for the equilibrium strategy to be optimal in future periods, even if the bidder happened to deviate in previous periods. (This requirement can be thought of as a condition necessary to ensure that a bidder is not fooled by his own price impact when he deviates.) So *in equilibrium* it is perfectly true that we

could eliminate the explicit dependence on x_i^1 and x_i^2 , but when solving for the equilibrium parameter values, we need the specified form.

All the dealers have constant absolute risk aversion, and they maximize:

$$\mathbf{E} \left[-e^{-\rho((\tilde{v}-\tilde{p}^1)x_i^1+(\tilde{v}-\tilde{p}^2)x_i^2+(\tilde{v}-\tilde{p}^3)x_i^3)} \middle| \mathcal{I}_{i,t=1} \right], \quad (1.7)$$

where the coefficient of absolute risk aversion, ρ , is equal across dealers, and $\mathcal{I}_{i,t=1}$ is all information available to investor i in period one. The three terms in the exponent are simply the profits originating from positions taken in periods one, two and three, respectively.

In the maximization problem each dealer takes into account his own price impact, or, in other words, he behaves strategically. To capture this, we generalize the residual-supply-curve technique used by Kyle (1989) to a multi-period setting:

$$\begin{aligned} p^1 &= \tilde{p}_{-i}^1 + \gamma_1 x_i^1 \\ p^2 &= \tilde{p}_{-i}^2 + \gamma_2^1 x_i^1 + \gamma_2^2 x_i^2 \\ p^3 &= \tilde{p}_{-i}^3 + \gamma_3^1 x_i^1 + \gamma_3^2 x_i^2 + \gamma_3^3 x_i^3, \end{aligned} \quad (1.8)$$

where \tilde{p}_{-i}^1 , \tilde{p}_{-i}^2 , and \tilde{p}_{-i}^3 are the (random) components of the price in the three periods that dealer i has no control over. Expressed in a different way, those are the prices that would prevail if dealer i chose not to make any trades. It is also worth pointing out that the reason x_i^1 enters into the period-two and period-three prices is the informational content of a trade. A large trade by dealer i , in period one pushes up the price not only in that period, but also in the next one, as the dealers use the market clearing prices to infer what \tilde{v} is, which in turn will affect the price in periods two and three.

We solve the model by deriving a set of fifteen characterizing equations, one for each coefficient in the three symmetric demand schedules. But first we need to express \tilde{p}_{-i}^1 , \tilde{p}_{-i}^2 , and \tilde{p}_{-i}^3 , as well as the six γ 's, in terms of the other parameters. By substituting the conjectured demand schedules (1.1), (1.3), and (1.5) into the market-clearing conditions (1.2), (1.4), and (1.6), and identifying coefficients, we find that:

$$\begin{aligned}
\tilde{p}_{-i}^1 &= \frac{\kappa_1}{\pi_1} + \frac{\zeta_1}{\pi_1} \tilde{v} + \frac{\zeta_1}{(N-1)\pi_1} \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\epsilon}_j + \frac{1}{(N-1)\pi_1} \tilde{z} \\
\tilde{p}_{-i}^2 &= \frac{\kappa_2}{\pi_2} + \frac{\pi_1^1}{\pi_2} \tilde{p}_{-i}^1 + \frac{\zeta_2}{\pi_2} \tilde{v} + \frac{\zeta_2}{(N-1)\pi_2} \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\epsilon}_j - \frac{\eta_2}{(N-1)\pi_2} \tilde{z} - \frac{1}{(N-1)\pi_2} \tilde{q} \\
\tilde{p}_{-i}^3 &= \frac{\kappa_3}{\pi_3} + \frac{\pi_1^1}{\pi_3} \tilde{p}_{-i}^1 + \frac{\pi_2^2}{\pi_3} \tilde{p}_{-i}^2 + \frac{\zeta_3}{\pi_3} \tilde{v} + \frac{\zeta_3}{(N-1)\pi_3} \sum_{\substack{j=1 \\ j \neq i}}^N \tilde{\epsilon}_j \\
&\quad - \frac{\eta_3^1}{(N-1)\pi_3} \tilde{z} + \frac{\eta_3^2}{(N-1)\pi_3} \tilde{q} + \frac{1}{(N-1)\pi_3} \tilde{w},
\end{aligned} \tag{1.9}$$

and:

$$\begin{aligned}
\gamma_1 &= \frac{1}{(N-1)\pi_1}; \quad \gamma_2^1 = \frac{1}{(N-1)\pi_2} \left(\frac{\pi_1^1}{\pi_1} - \eta_2 \right); \quad \gamma_2^2 = \frac{1}{(N-1)\pi_2}; \\
\gamma_3^1 &= \frac{1}{(N-1)\pi_3} \left(\frac{\pi_1^1}{\pi_1} + \frac{\pi_2^2}{\pi_2} \left(\frac{\pi_1^1}{\pi_1} - \eta_2 \right) - \eta_3^1 \right); \\
\gamma_3^2 &= \frac{1}{(N-1)\pi_3} \left(\frac{\pi_2^2}{\pi_2} - \eta_3^2 \right); \quad \gamma_3^3 = \frac{1}{(N-1)\pi_3}
\end{aligned} \tag{1.10}$$

Our next step is to solve for a symmetric Nash equilibrium through backward induction. Let us first introduce some notation regarding the information that the dealers can condition their demand schedules on. We have: $\mathcal{I}_i^1 = \{\tilde{s}_i, \tilde{p}^1\}$, $\mathcal{I}_i^2 = \{\tilde{s}_i, \tilde{p}^1, \tilde{p}^2; x_i^1\}$, and $\mathcal{I}_i^3 = \{\tilde{s}_i, \tilde{p}^1, \tilde{p}^2, \tilde{p}^3; x_i^1, x_i^2\}$. Let $U(x_i^1, x_i^2, x_i^3)$ denote the utility function from equation (1.7). In period three, we are solving:

$$\max_{x_i^3} \mathbf{E}[U(x_i^1, x_i^2, x_i^3) | \mathcal{I}_i^3]. \tag{1.11}$$

Let $\hat{x}_i^3(x_i^1, x_i^2)$ denote the solution to this optimization problem. To keep the notation simple, we have suppressed the other arguments — the private signal and previous and present prices — but keep in mind that before period three \hat{x}_i^3 is a stochastic variable. The optimal demand schedule in period two can now be stated as the solution to:

$$\max_{x_i^2} \mathbf{E}[U(x_i^1, x_i^2, \hat{x}_i^3(x_i^1, x_i^2)) | \mathcal{I}_i^2], \tag{1.12}$$

and we call it $\hat{x}_i^2(x_i^1)$, which also will be a stochastic variable in period one. Finally, in period one the optimal demand schedule will be the solution, \hat{x}_i^1 , to the following program:

$$\max_{x_i^1} \mathbf{E}[U(x_i^1, \hat{x}_i^2(x_i^1), \hat{x}_i^3(x_i^1, \hat{x}_i^2(x_i^1))) | \mathcal{I}_i^1]. \tag{1.13}$$

Although these conditional expectations may seem simple in principle, the explicit calculations can easily become intractable. To keep the calculations simple, we introduce some additional notation. In what follows, we let boldface minuscules denote vectors, boldface majuscules denote matrices, and “ τ ” indicates the

transpose of any given matrix. The computationally less interested readers may want to skip directly to the concluding proposition.

Let us first define:

$$\begin{aligned}
\boldsymbol{\psi}^{1\top} &= \{1, \tilde{s}_i, \tilde{p}_{-i}^1, \tilde{p}_{-i}^2, \tilde{p}_{-i}^3, \tilde{v}\} \\
\boldsymbol{\psi}^{2\top} &= \{1, x_i^1, \tilde{s}_i, \tilde{p}_{-i}^1, \tilde{p}_{-i}^2, \tilde{p}_{-i}^3, \tilde{v}\} \\
\boldsymbol{\psi}^{3\top} &= \{1, x_i^1, x_i^2, \tilde{s}_i, \tilde{p}_{-i}^1, \tilde{p}_{-i}^2, \tilde{p}_{-i}^3, \tilde{v}\} \\
\boldsymbol{\phi}^{1\top} &= \{1, \tilde{s}_i, \tilde{p}_{-i}^1\} \\
\boldsymbol{\phi}^{2\top} &= \{1, x_i^1, \tilde{s}_i, \tilde{p}_{-i}^1, \tilde{p}_{-i}^2\} \\
\boldsymbol{\phi}^{3\top} &= \{1, x_i^1, x_i^2, \tilde{s}_i, \tilde{p}_{-i}^1, \tilde{p}_{-i}^2, \tilde{p}_{-i}^3\},
\end{aligned}$$

and

$$\mathbf{E}^1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{E}^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{E}^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

We can now define $\mathbf{F}^{t\top}$, ($t = 1, 2, 3$), through the equation $\mathbf{F}^{t\top} \boldsymbol{\phi}^t = \mathbf{E}[\boldsymbol{\psi}^t | \mathcal{I}_i^t]$, and let $\boldsymbol{\Sigma}^t = \mathbf{V}[\mathbf{E}^{t\top} \boldsymbol{\psi}^t | \mathcal{I}_i^t]$. Thanks to our assumption of joint normality, $\boldsymbol{\Sigma}^t$ is independent of the realization of \mathcal{I}_i^t . We can finally define $\boldsymbol{\epsilon}^t$ so that $\boldsymbol{\psi}^t = \mathbf{F}^{t\top} \boldsymbol{\phi}^t + \mathbf{E}^{t\top} \boldsymbol{\epsilon}^t$. The last equation is simply a decomposition of $\boldsymbol{\psi}^t$ into its conditional expectation $\mathbf{F}^{t\top} \boldsymbol{\phi}^t$, given \mathcal{I}_i^t , and the remaining uncertainty, $\mathbf{E}^{t\top} \boldsymbol{\epsilon}^t$, where $\boldsymbol{\epsilon}^t$ is distributed normally with zero mean and variance-covariance matrix $\boldsymbol{\Sigma}^t$. It is now easy to verify that we can write:

$$\begin{aligned}
&\mathbf{E}[U(x_i^1, \hat{x}_i^2(x_i^1), \hat{x}_i^3(x_i^1, \hat{x}_i^2(x_i^1))) | \mathcal{I}_i^1] = \\
&\int_{\epsilon_1^1} \int_{\epsilon_2^1} \int_{\epsilon_3^1} -e^{-c^1(x_i^1)^2 - x_i^1 \mathbf{k}^{1\top} \boldsymbol{\psi}^1 - \frac{1}{2} \boldsymbol{\psi}^{1\top} \mathbf{V}^1 \boldsymbol{\psi}^1 - \frac{1}{2} \boldsymbol{\epsilon}^{1\top} (\boldsymbol{\Sigma}^1)^{-1} \boldsymbol{\epsilon}^1} d\epsilon_1^1 d\epsilon_2^1 d\epsilon_3^1, \quad (1.14)
\end{aligned}$$

$$\begin{aligned}
&\mathbf{E}[U(x_i^1, x_i^2, \hat{x}_i^3(x_i^1, x_i^2)) | \mathcal{I}_i^2] = \\
&\int_{\epsilon_1^2} \int_{\epsilon_2^2} -e^{-c^2(x_i^2)^2 - x_i^2 \mathbf{k}^{2\top} \boldsymbol{\psi}^2 - \frac{1}{2} \boldsymbol{\psi}^{2\top} \mathbf{V}^2 \boldsymbol{\psi}^2 - \frac{1}{2} \boldsymbol{\epsilon}^{2\top} (\boldsymbol{\Sigma}^2)^{-1} \boldsymbol{\epsilon}^2} d\epsilon_1^2 d\epsilon_2^2, \quad (1.15)
\end{aligned}$$

$$\mathbf{E}[U(x_i^1, x_i^2, x_i^3)|\mathcal{I}_i^3] = \int_{\epsilon_1^3} -e^{-c^3(x_i^3)^2 - x_i^3 \mathbf{k}^3 \top \boldsymbol{\psi}^3 - \frac{1}{2} \boldsymbol{\psi}^3 \top \mathbf{V}^3 \boldsymbol{\psi}^3 - \frac{1}{2} \boldsymbol{\epsilon}^3 \top (\boldsymbol{\Sigma}^3)^{-1} \boldsymbol{\epsilon}^3} d\epsilon_1^3, \quad (1.16)$$

where c^1, c^2, c^3 are scalars, $\mathbf{k}^1, \mathbf{k}^2, \mathbf{k}^3$ are vectors of constants, and $\mathbf{V}^1, \mathbf{V}^2, \mathbf{V}^3$ are symmetric matrices of constants.

By evaluating the integrals in equations (1.14), (1.15), and (1.16), we can differentiate and form a set of first-order conditions for the maximization problems (1.11), (1.12), and (1.13). Since these equations were derived under the assumption of parameter values according to equations (1.1), (1.3), and (1.5), the derived optimal values must be equal to the ones conjectured. Therefore, we can summarize our results as:

Proposition 1 *A linear symmetric Nash equilibrium in our model is characterized by the following set of fifteen algebraic equations:*

$$\mathbf{b}^t = (\mathbf{k}^{t\top} \mathbf{E}^t \boldsymbol{\Omega}^{-1} \mathbf{E}^{t\top} \mathbf{k}^t - 2c^t)^{-1} (\mathbf{k}^{t\top} \mathbf{F}^t - \mathbf{k}^{t\top} \mathbf{E}^t \boldsymbol{\Omega}^{-1} \mathbf{E}^{t\top} \mathbf{V}^t \mathbf{F}^t), \quad t = 1, 2, 3,$$

where

$$\begin{aligned} \boldsymbol{\Omega} &= \mathbf{E}^{t\top} \mathbf{V}^t \mathbf{E}^t + (\boldsymbol{\Sigma}^t)^{-1}, \\ \mathbf{b}^1 &= \frac{1}{1 + \gamma_1 \pi_1} \{\kappa_1, \zeta_1, -\pi_1\}, \\ \mathbf{b}^2 &= \frac{1}{1 + \gamma_2^2 \pi_2^2} \{\kappa_2, \eta_2 + \pi_2^1 \gamma_1 - \pi_2^2 \gamma_2^1, \zeta_2, \pi_2^1, -\pi_2^2\}, \\ \mathbf{b}^3 &= \frac{1}{1 + \gamma_3^3 \pi_3^3} \{\kappa_3, \eta_3^1 + \pi_3^1 \gamma_1 + \pi_3^2 \gamma_2^1 - \pi_3^3 \gamma_3^1, \eta_3^2 + \pi_3^2 \gamma_2^2 - \pi_3^3 \gamma_3^2, \zeta_3, \pi_3^1, \pi_3^2, -\pi_3^3\}, \end{aligned}$$

and when there exists a solution to these equations, the strategies used by the dealers are given by equations (1.1), (1.3), and (1.5).

It is also easy to see how the above technique can be used to solve one- and two-period versions of the full-blown three-period model — something we will do to assess the impact of opening new markets. The single period in the one-period model will then correspond to period two of the three-period model. Similarly, the two-period model refers to the above setup without period three. The demand schedules, residual supply curves, etc. are adjusted accordingly.

Unfortunately, these algebraic equations cannot be solved analytically, so most of the following sections will deal with the properties of this equilibrium through use of numerical techniques.¹²

¹²We use a discretized Newton-Raphson method; see, e.g., Dahlquist-Björk (1974).

1.4 Price Formation: Analysis

Before presenting the simulation results, it is worth spending some time on the main driving forces in the model. By carefully studying the dealers' utility function in equation (1.7), we can note that when $x_i^1 + x_i^2 + x_i^3 = 0$, \tilde{v} drops out of the argument. This is what we refer to as the "market-making" aspect of a dealer's business; he wants to buy cheap and sell dear, which in this case typically can be rephrased as selling dear, in order to buy cheap in the auction, and finally sell some more after the auction. If market making were the sole source of profits and the dealer could ascertain that the above equality always held, he would not care about the absolute level of prices, only the spread between the prices at which he buys and those at which he sells. The dealer's expectation of \tilde{v} would in this case be irrelevant. The spread is in this case also a direct measure of profits per unit. It is *not* enough that the equality holds in expectation. Even if $\mathbf{E}[x_i^1 + x_i^2 + x_i^3] = 0$, there will be with probability one¹³ a non-zero amount that has to be sold or bought in the secondary market. It is then clear that \tilde{v} will play a role in determining the equilibrium prices. (Usually, we consider cases in which this equality does not hold at all, as dealers on average carry positive inventories for resale in the secondary market. In terms of the model, this corresponds to $\bar{q} > \bar{z} + \bar{w}$, i.e., the average quantity sold in the auction exceeds the average net exogenous demand in the when-issued trading.)

If there were no private information, all dealers would have identical demand schedules and they would all end up with an N 'th of the aggregate supply (positive or negative) in each period. Having some private information, however, the dealers in our model have different conditional expectations of \tilde{v} ; this is the driving force behind all strategic interaction in the model. A dealer whose signal is very strong (positive) will be willing to pay more than one whose signal is weak (negative). But as he submits a more aggressive demand schedule, he will move prices and reveal at least some of this information to the other dealers. Being savvy traders, the dealers take that effect into account. The very essence of strategic interaction between dealers and across markets is to be found in deliberations like these. A thorough understanding of what determines a trade's impact on price, is therefore a first step toward understanding the price-formation process, and that is the topic to which we now turn.

¹³We do not consider degenerate probability distributions.

1.4.1 Price Impact

The scenario we have in mind in this section includes pre- and post-auction trading in the when-issued market, as well as the auction itself. For now we need not be concerned with the levels (averages) of the exogenous demand and the quantity for sale in the auction, because all coefficients in the demand schedules (see equations (1.1), (1.3), and (1.5)), except the constants (κ_1, κ_2 , and κ_3), are independent of the levels (\bar{q} , \bar{z} , and \bar{w}), and those constants do not affect the price impact, as defined below.¹⁴ We will also look at one- and two-period versions of the model. In the one-period case, the single trading round will be interpreted as a share auction without any surrounding trading; when there are two periods, we think of the first period as the pre-auction when-issued market (zero-net supply), and the second one as the auction itself. In all cases \tilde{q} is the quantity for sale in the auction, and \tilde{z} the exogenous demand in the pre-auction when-issued trading.

Let us also define *price impact* as the γ 's in equations (1.8). For instance, the price impact of a trade in period two on the period-two price is γ_2^2 , and on the period-three price it is γ_3^2 .

In Figure 1.1 (all figures are found in Section 1.8), we see how the γ 's change in response to a change in σ_v . First, we notice that the price impact is always greater in the period when the trade occurs than in subsequent trading rounds ($\gamma_1^1 > \gamma_2^1$; $\gamma_1^1 > \gamma_3^1$; $\gamma_2^2 > \gamma_3^2$). This is what we expect, as a trade affects the coincident price both because of a pure demand effect and because of the signal the trade sends to the market, whereas only the latter effect is present in its price impact on subsequent trading. The price impact is also gradually diminished in later trading rounds, as more information (previous prices) is available and the signal therefore is relatively less powerful ($\gamma_2^1 > \gamma_3^1$, and $\gamma_1^1 > \gamma_2^2 > \gamma_3^2$).

We now focus on the effect of changes in σ_v , the variance of the true value. This has the over-all effect of making the dealers less sure of the true value of \tilde{v} , i.e., the conditional variance of \tilde{v} is increased. It also means that the variance of \tilde{v} “dwarfs” the variance of the noise (the ϵ_i 's and the exogenous demand/supply, \tilde{z} , \tilde{q} , and \tilde{w}), which will increase the weights the dealers put on the signals, private as well as public.¹⁵ This result is quite intuitive; a larger variance of \tilde{v} means that extreme observations of the signal are more likely to stem from an extreme

¹⁴See equations (1.10).

¹⁵In a one-dimensional problem, $\mathbf{E}[\tilde{v}|\tilde{s}] = (\text{Cov}[\tilde{v}, \tilde{s}]/\mathbf{V}[\tilde{s}]) \cdot \tilde{s}$, which means that when the covariance between the signal and the true value increases faster than the variance of the signal, one will attribute greater weight to the signal. So even if the conditional variance of the true value goes up, one will “pay more attention” to the signal.

realization of \tilde{v} than from the noise. In particular, the dealers will rely more on the market price as a signal than before. When the dealer observes a high price, he will then infer that it probably also means a high value. The dealer will reduce his demand less than if he thought that the high price were not associated with a high value. Thus, anyone wishing to acquire a certain quantity in the market is forced to be even more aggressive when the price serves as a good signal — the price impact is larger of a given trade! Figure 1.1 shows that this effect is pervasive in all periods. The reason that γ_1 grows faster than the rest is that in the first period, the market price is the main source of information about \tilde{v} and thus most exposed to the argument above; in later periods there is more information available, which reduces the impact of the effect.

We do not show the analogous graph for changes in σ_ϵ , the variance of the noise in the private signal, because it looks very much the same as Figure 1.1. From the above line of reasoning, however, it is tempting to predict that the effect of changes in σ_ϵ would be the opposite of changes in σ_v , as both the private (the ϵ_i 's) and the public signals (the prices) become noisier. This is true for the private signal — the weights are reduced — but *not* for the prices. It is true that the price becomes noisier, but much less so than the private signal. The reason can be seen in equations (1.9); only the “average” noise matters in the price ($(\sum_{j=1, j \neq i}^N \tilde{\epsilon}_j)/(N - 1)$, from dealer i 's vantage point), and since the noise is independent across agents, the variance of the “average” noise is only a fraction of the individual signals' noise ($1/(N - 1)$ in this case). Although the price also becomes noisier, it looks increasingly attractive relative to the dealer's private information. In fact, this shift in emphasis from the private signal toward the market prices is much stronger than the direct impact of the noise itself, and the net effect on the price impact is positive.

As we have seen, the dynamics differ in the above two cases, but the net effect on price impact is the same. For this reason, we will sometimes refer to these cases jointly as “value uncertainty,” since they both are directly related to the conditional variance of \tilde{v} .

We finally look at “quantity uncertainty,” which here is defined as the variance of the quantity for sale to competitive bidders in the auction, σ_q . Remember that the non-competitive bids have precedence, making the quantity left for the competitive bidders stochastic. From our previous discussion, it is easy to see what an effect a change in σ_q will have on price impact, as both of the driving forces work in the same direction. First, the price becomes a noisier signal about true value. Second, this additional noise does *not* affect the private signal. Thus, the price is

a less attractive signal in both an absolute and a relative sense. As the dealers will infer less about value, they will reduce their demands more when they observe a high price and the quantity uncertainty is large. The price impact is therefore smaller when quantity uncertainty is large. We illustrate this phenomenon in Figure 1.2, where we only show γ_2^2 and γ_3^2 , the price impact in the auction and in the post-auction when-issued trading (all the other γ 's are virtually unaffected by changes in σ_q). It is worth pointing out that this reduction in price impact is similar to what happens when the number of bidders increases. We can summarize our findings as:

Observation 1 *When value uncertainty, σ_v^2 or σ_c^2 , increases, the price impact of a trade goes up. When quantity uncertainty, σ_q^2 , increases, however, the price impact is reduced.*

Let us now turn to the issue of surrounding trading, and see how that affects the price impact in the auction. To do this we rely on our one- and two-period versions of the model. In Figure 1.3, we show for our three models the price impact of demanding one more unit in the auction, as σ_v changes. In this and the following two figures, "1" refers to a share auction without any surrounding trading, "2" to an auction with pre- but not post-auction when-issued trading, and "3" to our usual model. We are now concerned with their *relative* positions, rather than the absolute levels, which we already have discussed. The price impact is largest when there is no surrounding trading at all; this reflects the relative importance of the concurrent price as a signal. When the concurrent price is the only available information besides the private signal, it automatically becomes the *best* signal and therefore carries a lot of weight. If there already has been a round of trading, another market price has already been established, and the concurrent price carries less weight. The price impact is therefore the largest when there are no surrounding trading opportunities.

If there are trading opportunities in the future, however, one does not want to trade too aggressively on one's private information right away, as that would move prices more than necessary; a better strategy is to hold back and spread out the trading over all available periods. In terms of the demand schedules, this can be interpreted as putting less weight on the private signal (ζ_2) and more on the public information (the π 's). If there are no more trading opportunities, there are no benefits to be made by holding back, and the market price will carry less weight. For this reason, the price impact is smaller when there are no more trading opportunities. We can also see in Figure 1.3 that the differences between

the market structures grow as σ_v increases and these informational motives become more important. In Figure 1.4, we show what happens when σ_c changes. As an illustration of the above argument, we see that the introduction of post-auction when-issued trading has less of an impact when σ_c is large. As σ_c increases, the private information becomes gradually less valuable relative to the public signals, and the strategic impact of an additional trading opportunity is correspondingly less important. The same argument applies when the auction price becomes very noisy and virtually eliminates the concerns about price impact in future trading periods. This is illustrated in Figure 1.5, where we can see “2” and “3” almost converge as σ_q increases.

Having established what determines the price impact, we now focus on the price formation.

1.4.2 “Quantity” and “Value” Uncertainty

The “benchmark” three-period case we have in mind for the following analysis is a scenario in which the dealers, on average, are short after the pre-auction when-issued trading ($\bar{z} > 0$). The dealers then cover their positions in the auction and continue selling in the post-auction when-issued market. We usually also assume that they, on average, hold a net-long position when delivery occurs and the secondary market opens ($\bar{q} > \bar{z} + \bar{w}$).

In our setup the expected auction prices are proportional to the seller’s expected revenues. This is so because \bar{q} is not the total amount for sale, but only what remains for the competitive bidders. Usually, the quantity is negatively correlated with price, i.e., a high quantity tends to go with a low price. When that is the case, the expected revenues are less than the expected quantity times the expected price. As the price in the competitive bidding also applies to the non-competitive bids, price is in this model correlated only with the fraction remaining for the competitive bidders. The *total* quantity and price are independent, as the total quantity is known before the auction. The expected revenues are therefore equal to the expected price *in the competitive bidding* times the total quantity. All of the ensuing analysis is done directly in terms of expected prices.

We now turn to the link between prices and the price impact of a trade, the key feature of the model’s strategic nature. The dealers participate in a game in which they maximize their utility, assuming that all other participants adhere to the equilibrium strategies; this is the very definition of a Nash equilibrium. As part of their maximization they take their own price impact into account, the direct

impact in the trading round at hand as well as the indirect impact in subsequent rounds. In equilibrium, the potential gain in volume by deviating and bidding more aggressively is exactly offset by the reduced expected profit per unit caused by the price impact. The greater the price impact of a trade, the easier it is to maintain a profitable equilibrium, as the cost of deviating goes up. *Ceteris paribus* a larger price impact is translated into higher equilibrium profits. This is of course the reason why the equilibrium profits in an oligopoly tend to decrease as the number of competitors goes up. We simply want to emphasize the similarity between more competitors and a (relatively) less informative price in their respective impact on competition in strategic models with asymmetric information. Although these ideas are well established and explored in the game-theoretic literature, we think they are worth repeating with respect to this particular model, as a lot of “gut feeling” intuition about capital markets is based upon competitive models. For instance, in a competitive model with risk-averse participants, increased uncertainty (defined, e.g., as the conditional variance of the item for sale) translates into a higher risk premium, which in turn lowers the price. This need not be the case in a strategic model.

In Figure 1.6, we show the impact of increased uncertainty about the underlying value, σ_v , on expected prices.

Observation 2 *When the variance of \tilde{v} , σ_v^2 , increases, the expected price goes up in periods when the dealers on average act as net sellers, and down when they on average are net buyers.*

Remember that each dealer receives a signal $\tilde{s}_i = \tilde{v} + \tilde{\epsilon}_i$, where \tilde{v} is the “true” underlying value that will be revealed once the secondary market opens and $\tilde{\epsilon}_i$ is the “noise” in the signal, which is assumed to be independent of \tilde{v} and across dealers. In periods with positive exogenous demand, the expected prices will be positive as the dealers act as net sellers. In the auction, where they act as buyers, the expected price is negative. In Figure 1.1, we saw what happened to the price impact in the different periods when σ_v rises: the price impact gets larger. From the above discussion, we also saw that a higher price impact facilitates a more profitable equilibrium; in periods where the dealers act as net buyers (sellers), this means lower (higher) prices. In Figure 1.6, we see this illustrated. The main reason the change in the period-three price is so much smaller than in the other prices, is that the average quantity traded (\bar{w}) is smaller than in the other periods. When the average traded quantity increases, so does the average quantity each dealer will sell or buy. But that means that the price hit from his price impact if he

deviates becomes increasingly costly, as he must take the hit on *all* units he sells in equilibrium. The price impact and average quantity traded in a period therefore affect the expected price in a multiplicative fashion (see equations (1.9)). When the quantity is lower, it also causes changes in price impact (due to, say, changes in σ_v) to have a smaller effect on the expected price. Figure 1.7 shows the effects of changes in σ_ϵ on expected prices.

Observation 3 *When the variance of the $\tilde{\epsilon}_i$'s, σ_ϵ^2 , increases, the expected price goes up in periods when the dealers on average act as net sellers, and down when they on average are net buyers. In short, the effects of changes in σ_ϵ^2 are very similar to those of changes in σ_v^2 .*

Given our discussion about the price impact when the noise in the private signals changes, the net effects on expected prices from changes in σ_ϵ are very similar to those caused by changes in σ_v . The results in the one- and two-period models are more or less identical and therefore not included.

We will now focus our attention on the effects of changing variance in \tilde{q} , the stochastic quantity offered for sale to the competitive bidders in the auction. The quantity uncertainty stems from the presence of non-competitive bids, which are guaranteed fulfillment at the quantity-weighted price of the competitive bids. A more volatile flow of non-competitive bids from small investors (or the Treasury itself), would increase σ_q . Figure 1.8 shows the impact of increasing quantity uncertainty on expected prices.

Observation 4 *When quantity uncertainty, σ_q , increases, so does the expected auction price (and the Treasury's revenues).*

The auction price rises in line with the decreasing price impact (see Figure 1.2). We can also note that market-making spreads, $p^1 - p^2$ and $p^3 - p^2$, go down, i.e., the per-unit profit from market making is also reduced by the increased quantity uncertainty. In short, the market becomes more competitive when quantity uncertainty rises.

The above result may seem counterintuitive, as increased uncertainty usually *reduces* the seller's expected revenues ("the winner's curse"). To more fully understand the driving forces behind this effect, we will spend some time in a one-period setting. In Figure 1.9, the same result is shown for the one-period version of our general model. Let us now consider what happens in a competitive setting, i.e., when the dealers do not take their own price impact into account. All other assumptions remain the same. When this is the case, a noisier price simply means

that the uncertainty about \tilde{v} increases. As all dealers behave competitively, the increased uncertainty is translated into a higher risk premium, i.e., a lower price. Figure 1.10 shows that the expected auction price indeed is reduced when σ_q increases in this competitive setting. We could also have shown what happens if the agents act strategically and the information structure is symmetric (say, all dealers receive the same signal, $\tilde{s}_i = \tilde{v} + \tilde{\epsilon}$), but in this case, the variance of \tilde{q} does not even enter the problem. It is thus clear that the quantity-uncertainty results hinge upon the dual assumption of asymmetric information and strategic behavior. These results are also supported by the theoretical findings in Edsparr (1993), who studies quantity uncertainty in one-period share auctions with different information structures.¹⁶ One particular finding is that with perfect information, the equilibrium in Wilson (1979) where the sale price is one half of the true value breaks down, if quantity uncertainty is introduced. In this sense, quantity uncertainty restores the results from the single-unit case, for which the equilibrium sale price must equal the true value; this is an example of how quantity uncertainty forces the bidders to act perfectly competitively.

Thus, we want to stress the qualitative difference between value and quantity uncertainty. We have provided ample evidence for the increase in competitiveness in a share auction when the uncertainty about the quantity rises. We next look at the importance of trading in surrounding markets.

1.4.3 Interaction across Markets

Let us first think about the effects of trading *before* the auction, the pre-auction when-issued market. The first effect is the very existence of a market and an accompanying market-clearing price, which can serve as a (noisy) signal about \tilde{v} in future trading and thereby reduces the uncertainty dealers face. The second effect arises when the exogenous demand is non-zero ($\bar{z} \neq 0$). If this is the case, the dealers, on average, enter the subsequent trading with a long or short position. As we think of noise traders primarily as investors who want to ascertain a quantity above the limit for non-competitive bids, we focus on the case when the average dealer position is negative ($\bar{z} > 0$). The dealers have then already committed to deliver on the issue date, and if they cannot cover their positions, they will have to buy the missing securities in the secondary market regardless of price. In the auction, there is therefore an additional hedging motive, as the dealers already

¹⁶See also Müller (1990), who has related results for a model of a discriminatory private-values auction.

have established a position and a purchase in the auction will reduce that exposure. In Figure 1.11, the impact of this hedging motive on the expected prices in later periods, p^2 and p^3 , is shown.

Observation 5 *The greater the average short position a dealer holds when the bidding in the auction starts, the higher the expected auction price.*

The fact that p^1 rises, too, is exactly what we would expect, as the dealers are risk averse and the expected profit has to rise to make them take on additional positions; in addition, they exploit the noise traders, whose demand is not sensitive to price.

The assumption of price insensitivity is clearly one to ensure tractability rather than realism. Note that the price paid in period one has no impact *per se* on future prices. Only the net position enters into the maximization problem in future periods (see the utility function (1.7)), not the cost of establishing the position. A graph showing the price impact of increased noise-trader demand in the post-auction when-issued trading (\bar{w}) would look very similar, but the driving economic forces are quite different. The increased (price-insensitive) demand in the last period drives up the expected price in that period for the same reasons as before, but now it is that very price that drives up the the expected prices in preceding periods, as it is mainly the expected “spreads” that determine the market-making profits, not the price levels themselves.¹⁷ Since we are somewhat sceptical of this extreme form of “noise trading,” we believe this effect would be much smaller if we had modeled the exogenous demand in a more realistic way. To support the claim of a significant hedging motive as a driving force in the price formation, we show in Figure 1.12 how the price “spreads,” $p^2 - p^1$ and $p^3 - p^2$, change when the dealers’ risk aversion, ρ , goes up. (The levels offer no surprise: p^1 and p^3 go up, and p^2 goes down; the results in the one and two-period versions of the model are the same.) We can see here how $p^2 - p^1$ decreases much faster than $p^3 - p^2$, as the auction is much more exposed to the hedging motive than the post-auction when-issued trading, in which most of the previous trades have canceled each other out. Since we are looking at differences, most of the adjustment in price levels for higher risk aversion has been eliminated, and in addition to the hedging

¹⁷As pointed out in the section on price impact, no coefficients in the demand schedules, except the constants (κ_1 , κ_2 , and κ_3), change when any of \bar{z} , \bar{q} , and \bar{w} changes. The means of the exogenous supply/demand simply determine the *level* of prices (and profits), but the strategic interaction between private and public information is left unaffected.

motive only the changes due to the successive improvement of the information available are picked up.

In what follows, we will analyze the introduction of trading opportunities, before as well as after the auction. To eliminate the impact on expected auction prices from non-dealer demand, we let \bar{z} and \bar{w} be zero, i.e., the noise traders are as likely to be net buyers as net sellers and the market making is on average zero. This allows us to focus on the information-disseminating aspect of trading, and the question we have in mind is: do increased trading opportunities help reduce the dispersion of the bidders' valuations and thereby raise the expected auction price?

The base case is now the one-period model (denoted "1" in the figures). The two-period model with $\bar{z} = 0$ in period one tries to capture the effect of pre-auction trading, but no trading between the auction and the secondary market (denoted "2"). Finally, the full three-period model allows pre- as well as post-auction trading (denoted "3"). In Figures 1.13 and 1.14, the expected auction price is shown for the three models as we change σ_v and σ_q , respectively.¹⁸

Observation 6 *The expected price is always lowest in the isolated auction and highest when there is pre-auction trading but no trading after the auction.*

This result corresponds to the amount of information that has been revealed through the trades preceding the auction. When there is an additional trading opportunity after the auction, the dealers bid less aggressively, as the private information still has some value; if there is no such opportunity, there is no reason to hold back. All these results carry over from our previous analysis of the price impact.

In Figure 1.14 we see that the difference in expected price between the different market structures is reduced as σ_q increases, whereas the opposite happens in Figure 1.13 when σ_v grows. Once again we see the difference between quantity and value uncertainty. The former fosters competition, whereas the latter not only reduces the expected price so as to compensate the bidders for the well-known winner's curse, but amplifies the value of pre-auction trading. If we vary σ_e , instead of σ_v , the corresponding results are similar.

¹⁸To make the models comparable, we use exactly the same parameter values in corresponding periods, i.e., the one-period model corresponds to period two in the other two models. Given that and our assumption of zero average exogenous demand, the only remaining parameters are σ_z and σ_w , but, as noted before, quantity uncertainty has only a very limited impact on the price formation in other periods. In all our simulations, the absolute levels may change slightly, but all the qualitative results seem robust.

In an attempt to summarize our findings, we emphasize the benefits of pre-auction trading. When the average exogenous demand is zero, the very presence of this market increases the expected auction price, thanks to better dissemination of private information to the market as a whole. If this non-dealer demand is positive, the increase in the expected auction price is even higher. Post-auction trading, on the other hand, reduces the expected auction price, as it introduces an additional trading opportunity in which the private information has some value; the dealers simply bid less aggressively on their information when it still can be used.

1.5 Policy Implications

In the previous section, we have dealt with a variety of comparative statics, many of which are beyond the control of the Department of Treasury, and therefore without direct policy implications. Quantity uncertainty, however, is different. Since the Treasury submits non-competitive bids on behalf of itself as well as foreign central banks, it is at the Treasury's discretion whether to convey that information before the auction. (So is, of course, the exact issue size, which could be given as a range rather than an exact number or not at all.) Throughout this paper we have found a dramatic difference between quantity and value uncertainty. A difference that has largely been ignored or simply not studied in the traditional auction literature. What also shows that "intuition" based on results for value uncertainty would be very misleading when applied to uncertainty about quantity. Our finding of a reduction in the expected price in the auction (p^2) when the quantity uncertainty is reduced (σ_q) therefore leads to :

Claim 1 *The Treasury may be able to increase the expected revenues from its securities auctions by reducing the information about the amount of non-competitive bids it reveals before the auction.*

We have also found that pre-auction trading reduces value uncertainty, which is translated into higher expected auction prices. This is the case when the average noise-trader demand is zero, but if it is positive, the increase is amplified. The reason for this additional price increase in the auction is *not* the unsophisticated behavior displayed by the noise traders in this model and the elevated price level thereby created in the pre-auction when-issued trading. The increase is rather due to the increased competitiveness in the auction, which is caused by the, on average,

short positions established by the dealers in the pre-auction trading. Entering the auction with such a short position increases the cost of an unexpectedly small allocation in the auction, as the dealer then has to cover his position in the secondary market, where the prices are higher. The dealers, therefore, bid more aggressively. A higher level of non-dealer demand in the post-auction when-issued trading also raises expected auction prices, but we think that effect is largely driven by the aforementioned price insensitivity of the noise traders. The purely informational impact of post-auction when-issued trading lowers the expected auction price, as it creates an additional trading opportunity that makes the bidding in the auction less aggressive. To summarize these results, we state:

Claim 2 The Treasury has good reason to encourage a liquid pre-auction when-issued market that attracts buyers of Treasury securities that otherwise would buy in the secondary market, as it fosters competitiveness in the auction as well as reduces information asymmetries.

Given that the informational and the demand-driven effects of post-auction trading work in opposite directions, we think its net impact (whether positive or negative) on auction prices is very small. There is no way to assess the value of liquidity within this framework, but it is another factor that would increase the value of post-auction when-issued trading.

We finally think that our results show the need for Treasury-securities research that is based on the entire market structure (as opposed to just the auction in splendid isolation), and that policy advice based solely on results from the pure auction literature is inadequate. We do not claim to have “the” answer to these questions, but we do hope that we have highlighted some possible fallacies and added to our understanding of these important markets.

1.6 Concluding Remarks

This paper has tried to address the issue of price formation in the Treasury market, and especially the interaction across markets. The impact of additional trading opportunities on the price formation in the auction has been our main focus. We think that our results have helped shed some light on these complex but fascinating issues. We have also shown the pervasive effects of the quantity uncertainty generated by non-competitive bidding; its effects have been contrasted to those of the far better understood value uncertainty. The present work has not addressed

the classic question of choosing an optimal auction format: uniform or discriminatory (first- or second-price)? Analyzing the issues in this paper for discriminatory share auctions is an interesting issue for future research.

In Bikhchandani-Edsparr-Huang (1994), we have studied some related empirical issues regarding the strategic interaction between Treasury-bill auctions and the when-issued market. The link between the primary market and the repurchase and reverse market is investigated empirically in Chapter 2. We also hope that some of the implications of the results in this paper can be tested empirically. For instance, the average open interest in the when-issued market before the auction should have a positive effect on expected auction prices, and auctions with less variable non-competitive bidding should experience lower prices than those with more.¹⁹ As always for over-the-counter trading, data are hard to come by, but if the practical problems can be overcome, we hope to pursue at least some of these issues in future research.

1.7 References

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¹⁹If we believe that the variability of non-competitive bidding increases with the average quantity, this claim may seem supported by the fact that "underpricing" in the auction (see Cammack (1991), Jegadeesh (1993), and Spindt-Stolz (1992)) is more of an issue for notes and bonds than for bills, as non-competitive bids make up a much smaller fraction of the total issue size in the longer maturities. To make any such formal test, however, there is a host of differences between different auctions that have to be taken into account.

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1.8 Figures

Figure 1.1: Price Impact Versus Variance of \tilde{v}
 $\sigma_q^2 = 2.5; \sigma_z^2 = 3; \sigma_w^2 = 1.5; \sigma_\epsilon^2 = 0.4; \rho = 1; N = 4$

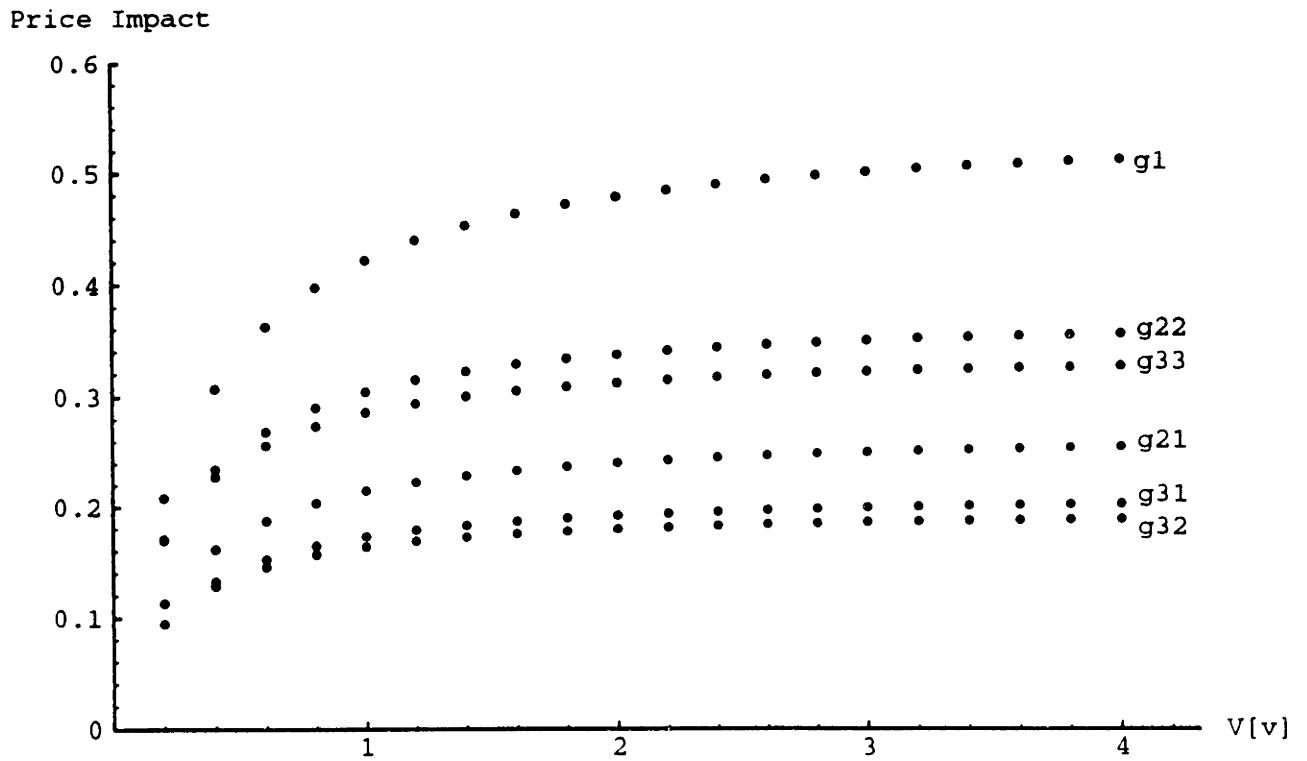


Figure 1.2: Price Impact Versus Variance of \tilde{q}

$$\sigma_z^2 = 3; \sigma_w^2 = 2; \sigma_v^2 = 1; \sigma_\epsilon^2 = 0.5; \rho = 1; N = 4$$

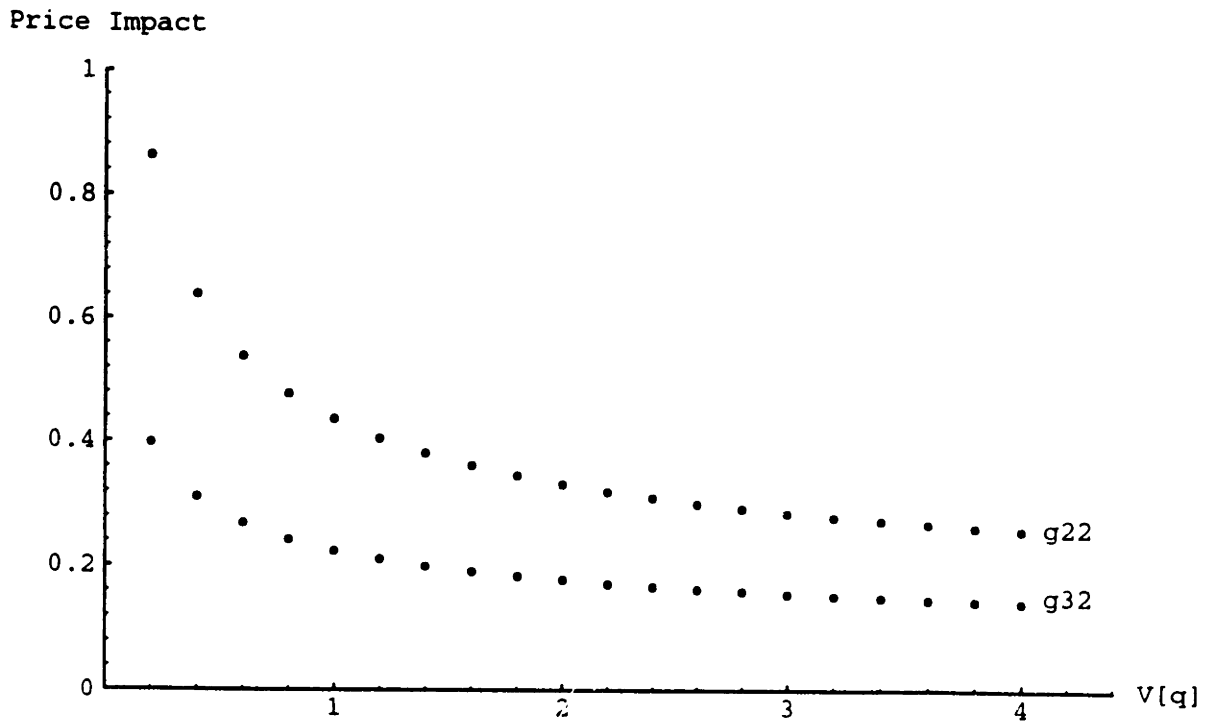


Figure 1.3: Price Impact for Different Market Structures Versus Variance of \tilde{v}

(“1” — Isolated Auction; “2” — Auction with When-Issued Trading Before;
 “3” — Auction with Surrounding When-Issued Trading)
 $\sigma_q^2 = 3; \sigma_z^2 = 3; \sigma_w^2 = 2; \sigma_c^2 = 0.5; \rho = 1; N = 4$

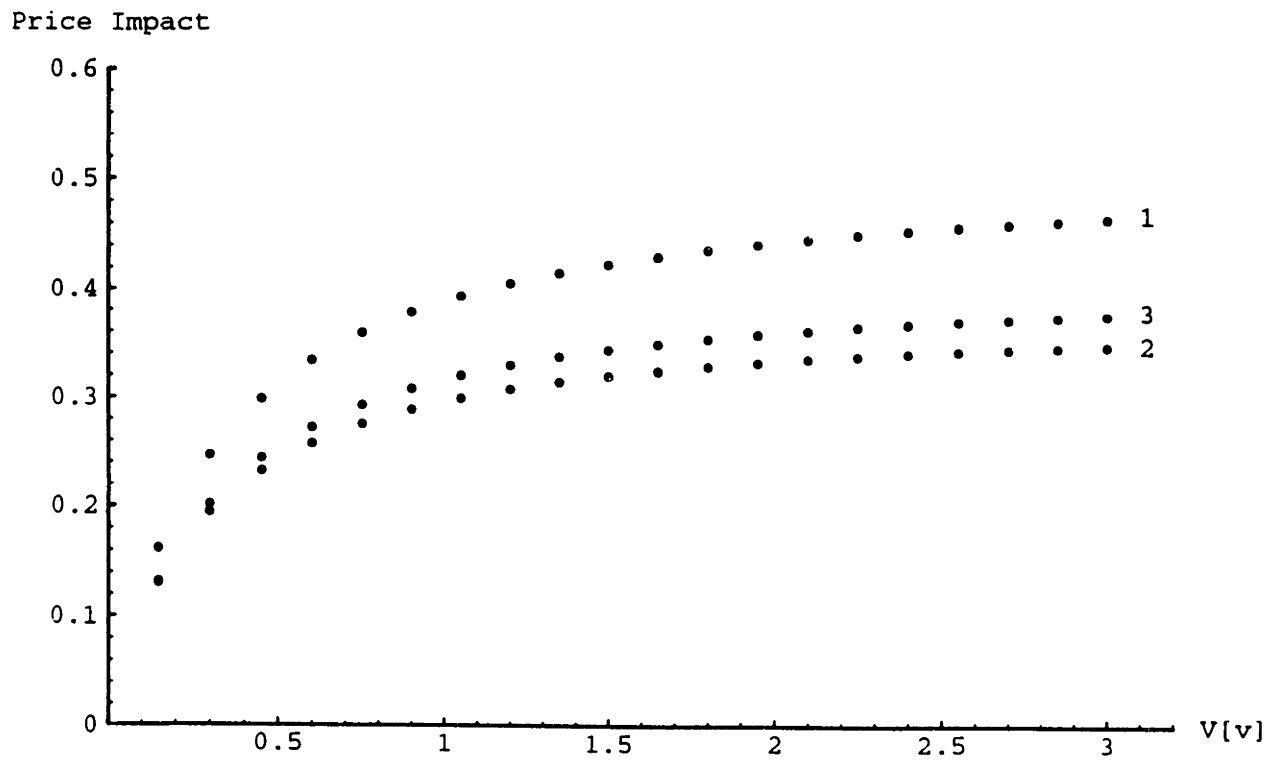


Figure 1.4: Price Impact for Different Market Structures Versus Variance of the $\tilde{\epsilon}_i$'s

(“1” — Isolated Auction; “2” — Auction with When-Issued Trading Before;
 “3” — Auction with Surrounding When-Issued Trading)
 $\sigma_q^2 = 3; \sigma_z^2 = 3; \sigma_w^2 = 2; \sigma_v^2 = 1; \rho = 1; N = 4$

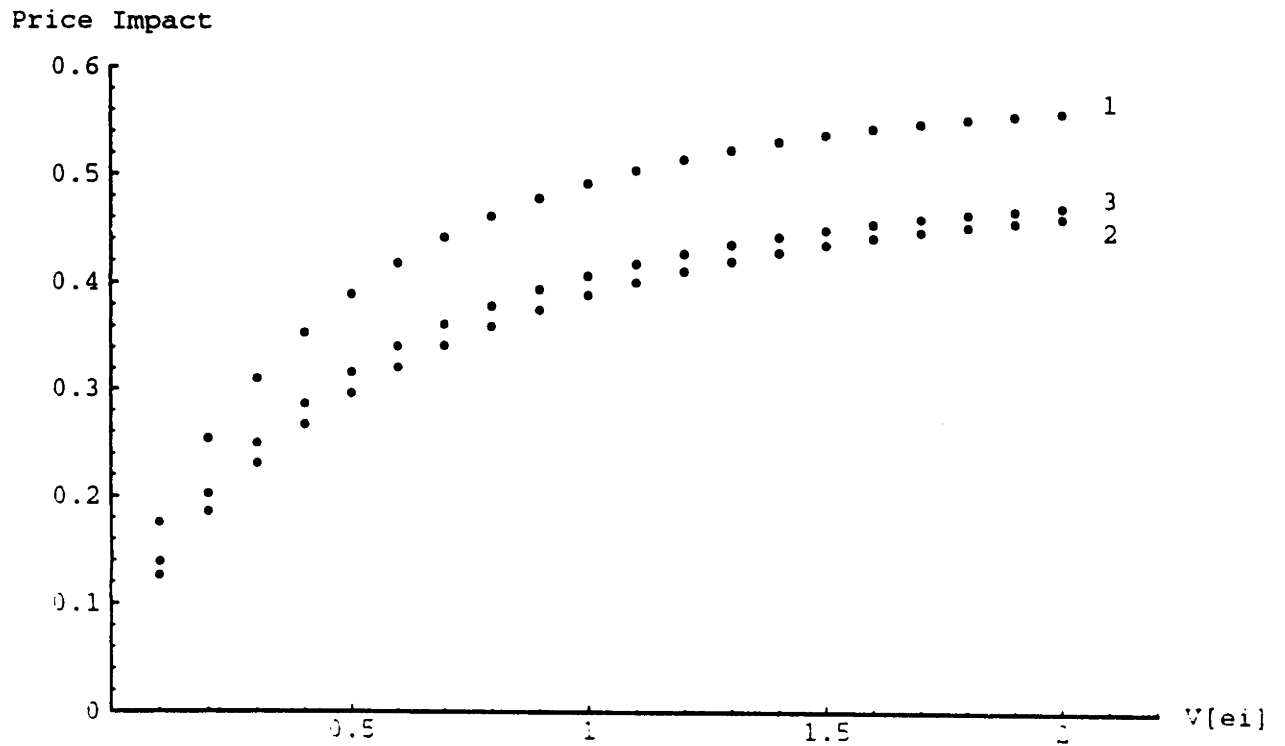


Figure 1.5: Price Impact for Different Market Structures Versus Variance of \tilde{q}

(“1” — Isolated Auction; “2” — Auction with When-Issued Trading Before;
 “3” — Auction with Surrounding When-Issued Trading)
 $\sigma_z^2 = 3; \sigma_w^2 = 2; \sigma_v^2 = 1; \sigma_\epsilon^2 = 0.5; \rho = 1; N = 4$

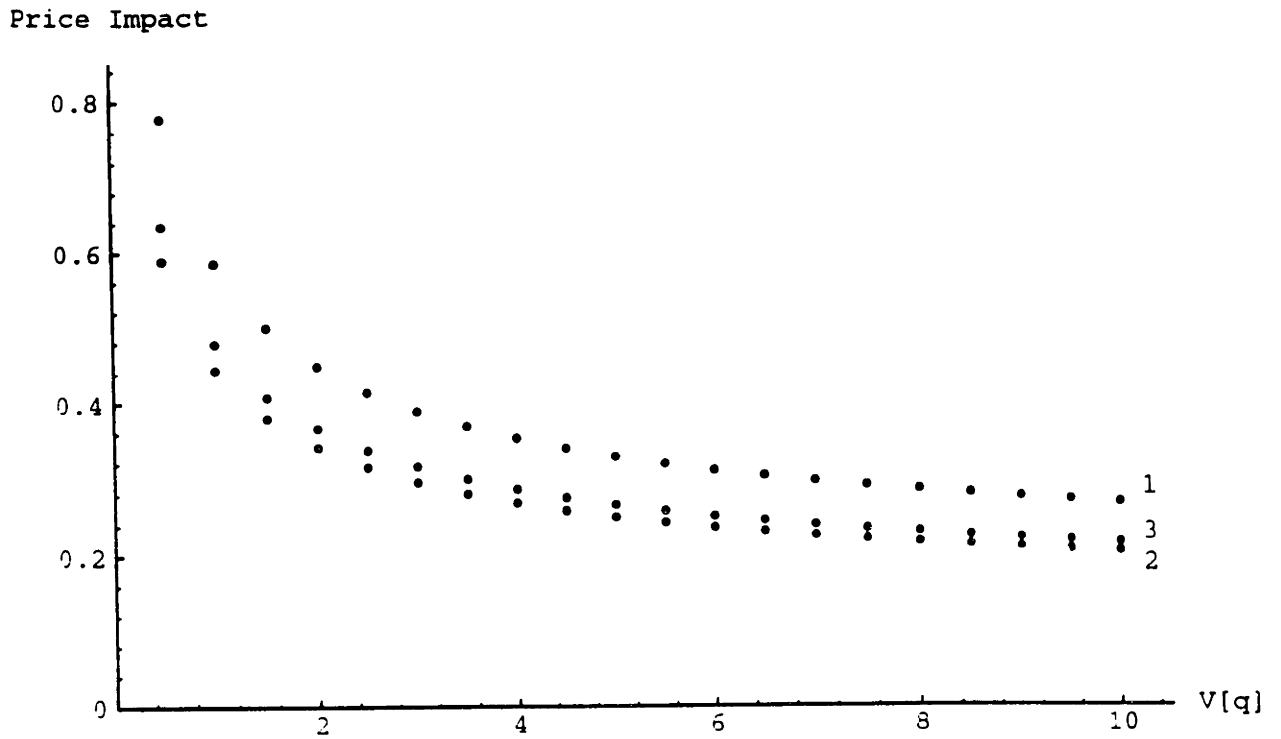


Figure 1.6: Expected Prices Versus Variance of \tilde{v}

$$\begin{aligned} \sigma_q^2 &= 2.5; \sigma_z^2 = 3; \sigma_w^2 = 1.5; \sigma_\epsilon^2 = 0.4; \\ \bar{z} &= 7; \bar{q} = 10; \bar{w} = 2; \rho = 1; N = 4 \end{aligned}$$

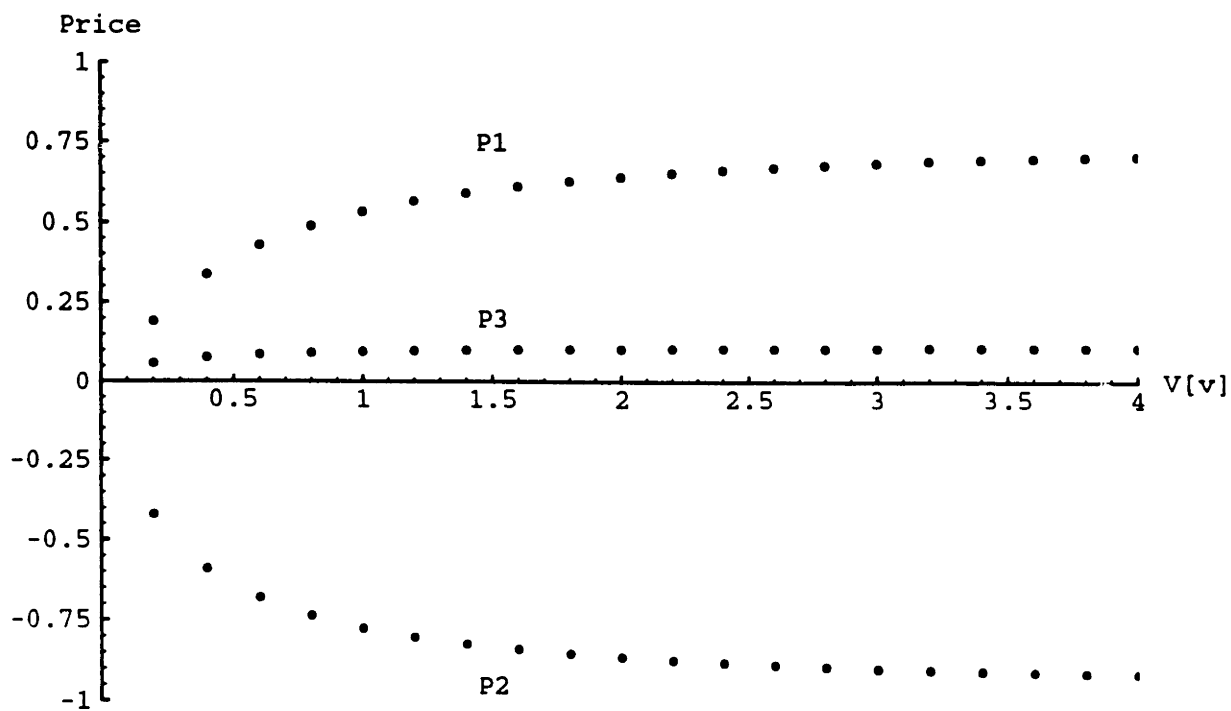


Figure 1.7: Expected Prices Versus Variance of the $\tilde{\epsilon}_i$'s

$$\begin{aligned} \sigma_q^2 &= 2.5; \sigma_z^2 = 3; \sigma_w^2 = 1.5; \sigma_v^2 = 1; \\ \bar{z} &= 7; \bar{q} = 10; \bar{w} = 2; \rho = 1; N = 4 \end{aligned}$$

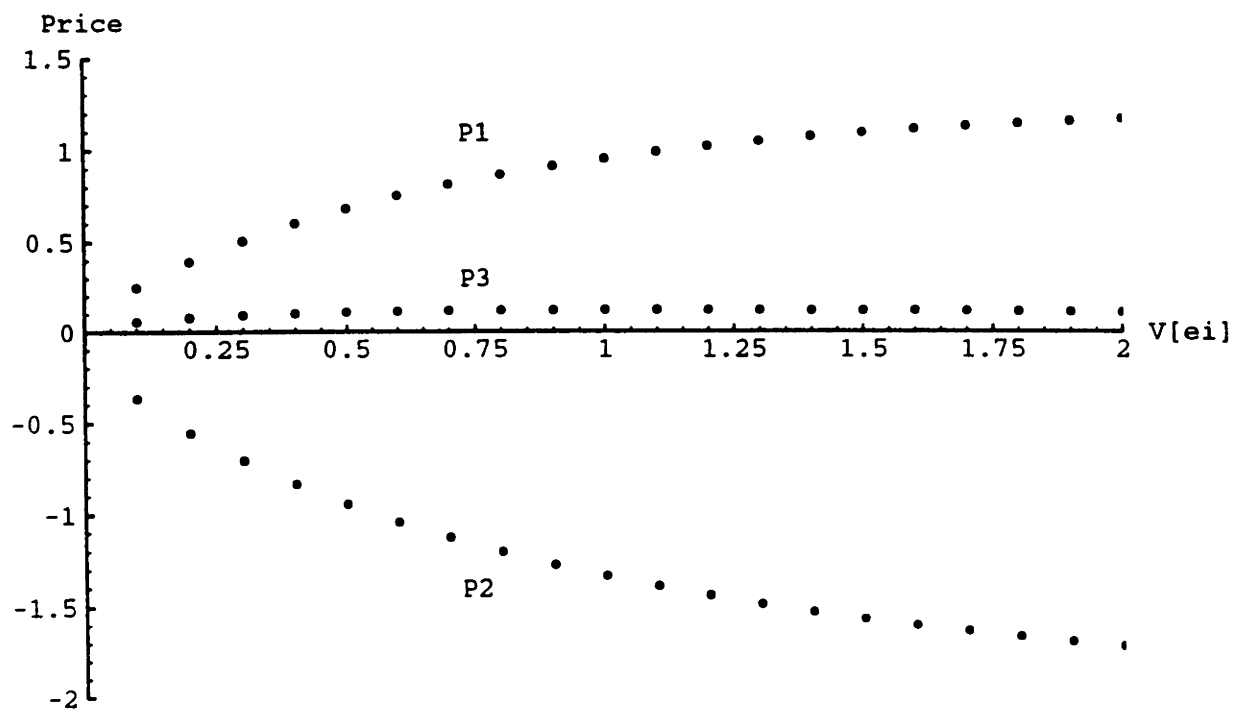


Figure 1.8: Expected Prices Versus Variance of \tilde{q}

$$\begin{aligned} \sigma_z^2 &= 3; \sigma_w^2 = 2; \sigma_v^2 = 1; \sigma_\epsilon^2 = 0.5; \\ \bar{z} &= 7; \bar{q} = 10; \bar{w} = 3; \rho = 1; N = 4 \end{aligned}$$

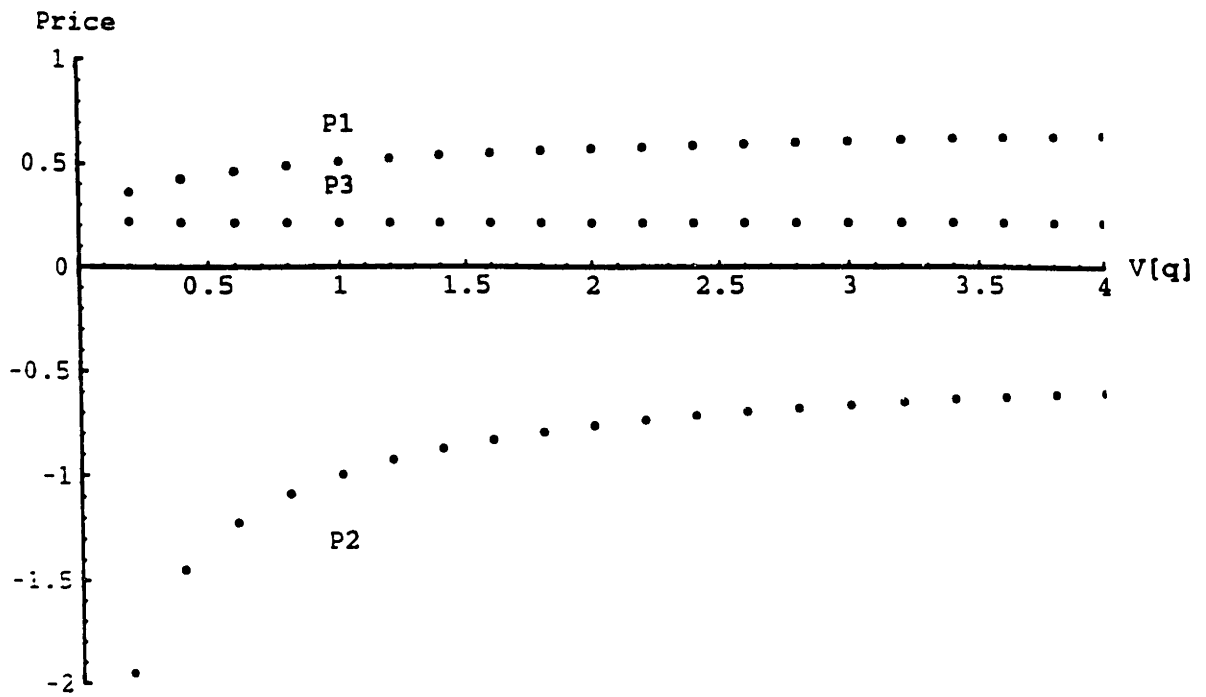


Figure 1.9: Expected Price Versus Variance of \tilde{q} in Isolated Auction

$$\sigma_v^2 = 0.8; \sigma_\epsilon^2 = 0.4; \bar{q} = 10; \rho = 1; N = 4$$

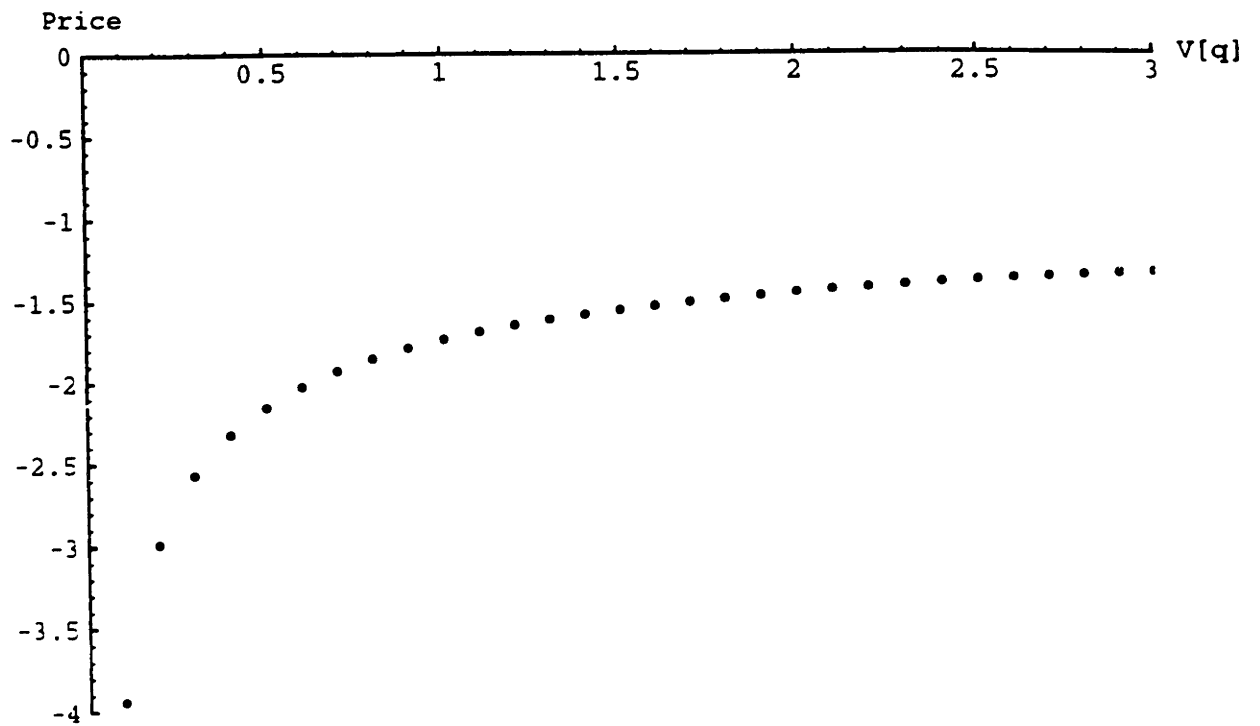


Figure 1.10: Expected Price Versus Variance of \tilde{q} with Competitive Agents

$$\sigma_v^2 = 0.8; \sigma_\epsilon^2 = 0.4; \bar{q} = 10; \rho = 1; N = 4$$

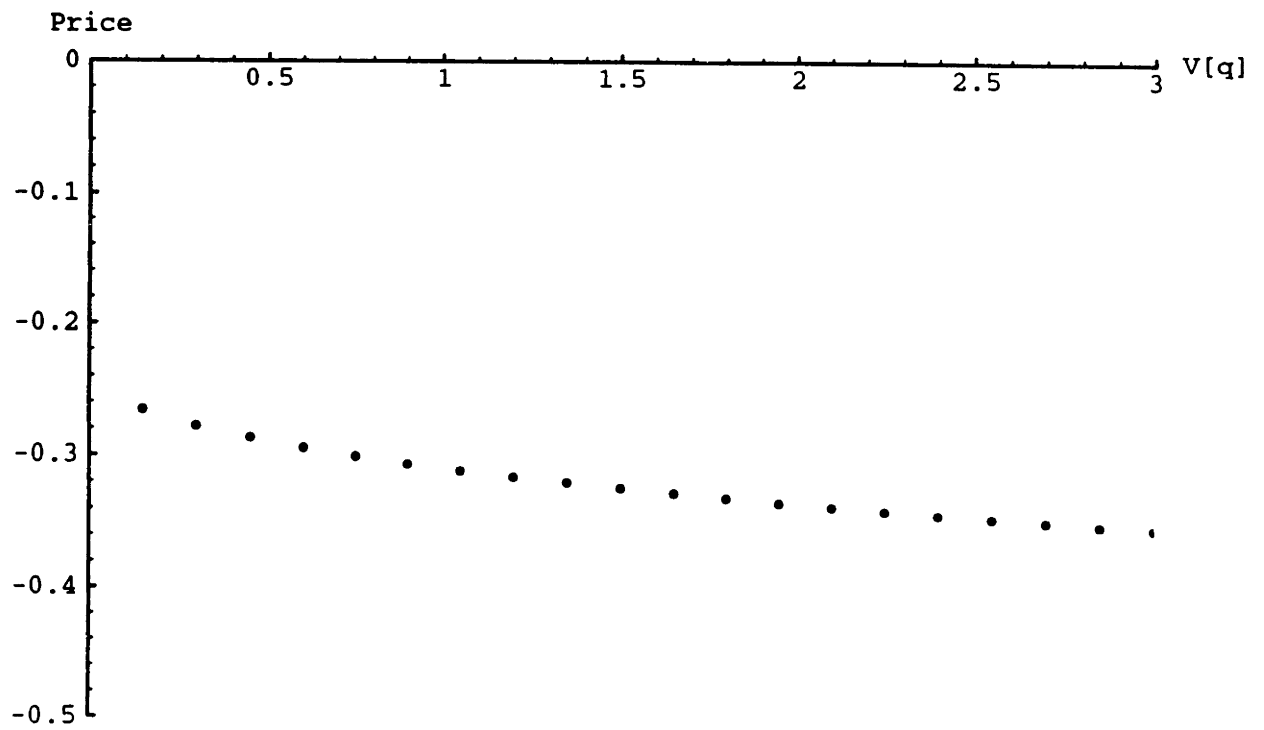
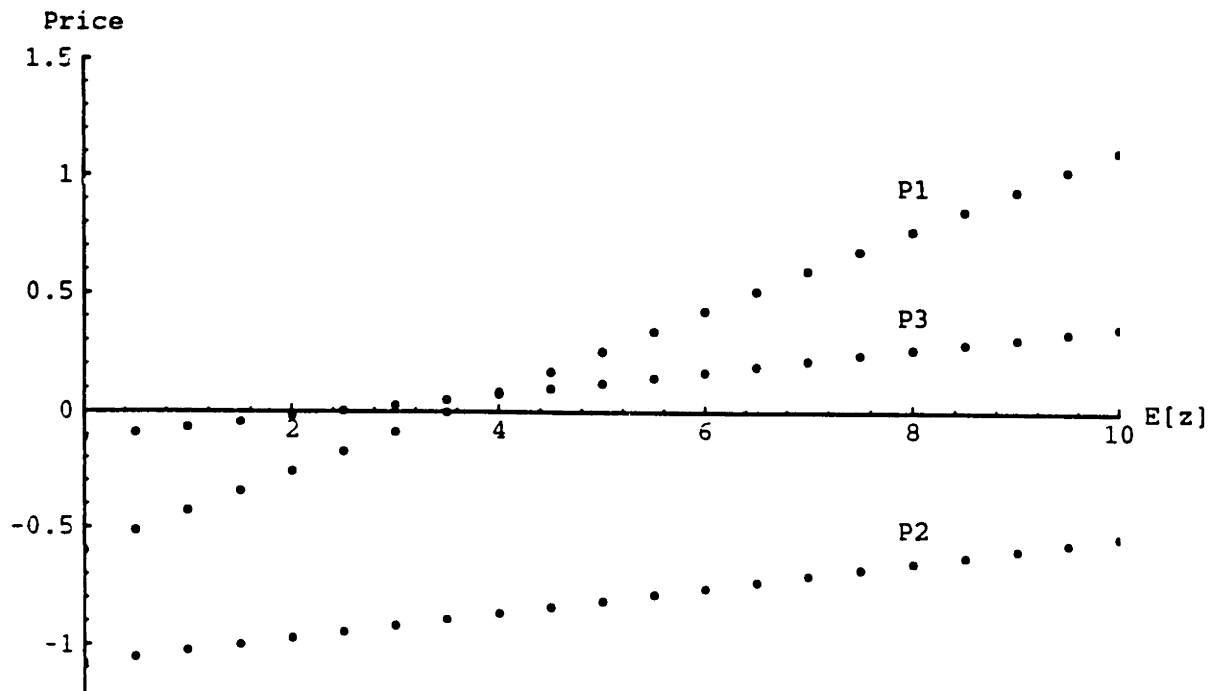


Figure 1.11: Expected Prices Versus Expectation of \tilde{z}

$$\begin{aligned} \sigma_q^2 &= 2.5; \sigma_z^2 = 3; \sigma_w^2 = 1.5; \sigma_v^2 = 1; \sigma_c^2 = 0.4; \\ \bar{q} &= 10; \bar{w} = 3; \rho = 1; N = 4 \end{aligned}$$



:

Figure 1.12: Expected "Spreads" Versus ρ

$$\sigma_q^2 = 2.5; \sigma_z^2 = 3; \sigma_w^2 = 1.5; \sigma_v^2 = 1; \sigma_\epsilon^2 = 0.4;$$
$$\bar{z} = 7; \bar{q} = 10; \bar{w} = 3; N = 4$$

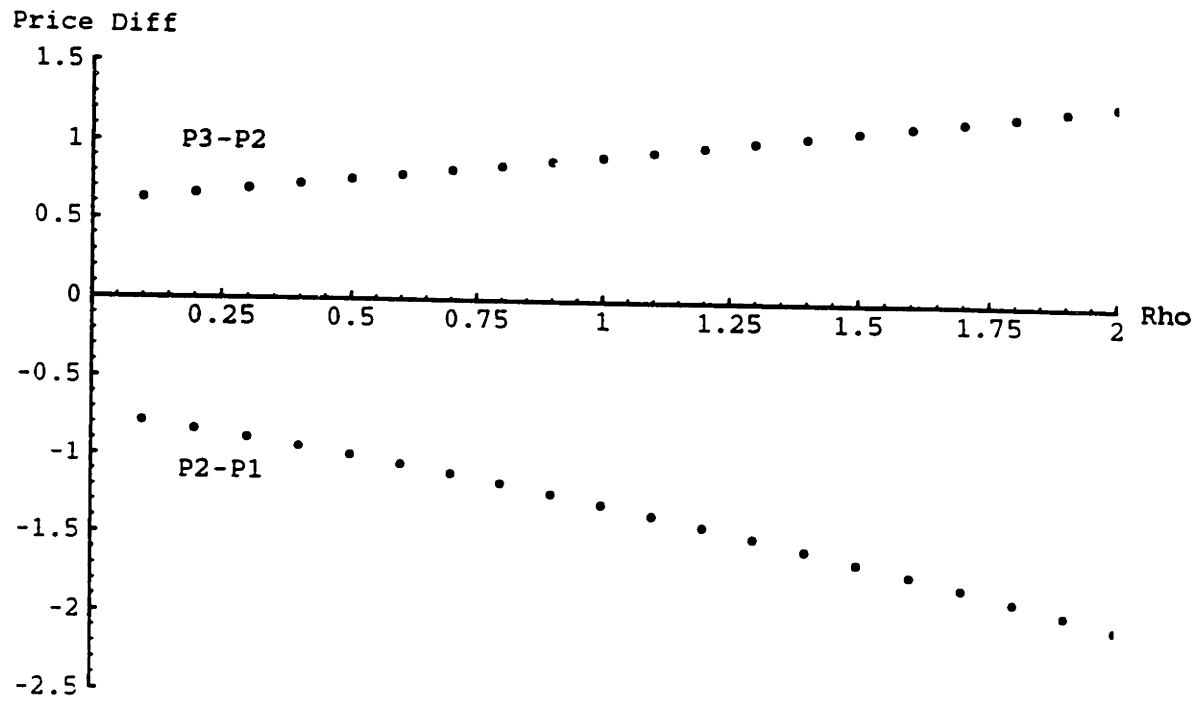


Figure 1.13: Expected Prices for Different Market Structures Versus Variance of \tilde{v}

(“1” — Isolated Auction; “2” — Auction with When-Issued Trading Before;
 “3” — Auction with Surrounding When-Issued Trading)

$$\sigma_q^2 = 3; \sigma_z^2 = 3; \sigma_w^2 = 2; \sigma_\epsilon^2 = 0.5;$$

$$\bar{z} = 0; \bar{q} = 10; \bar{w} = 0; \rho = 1; N = 4$$

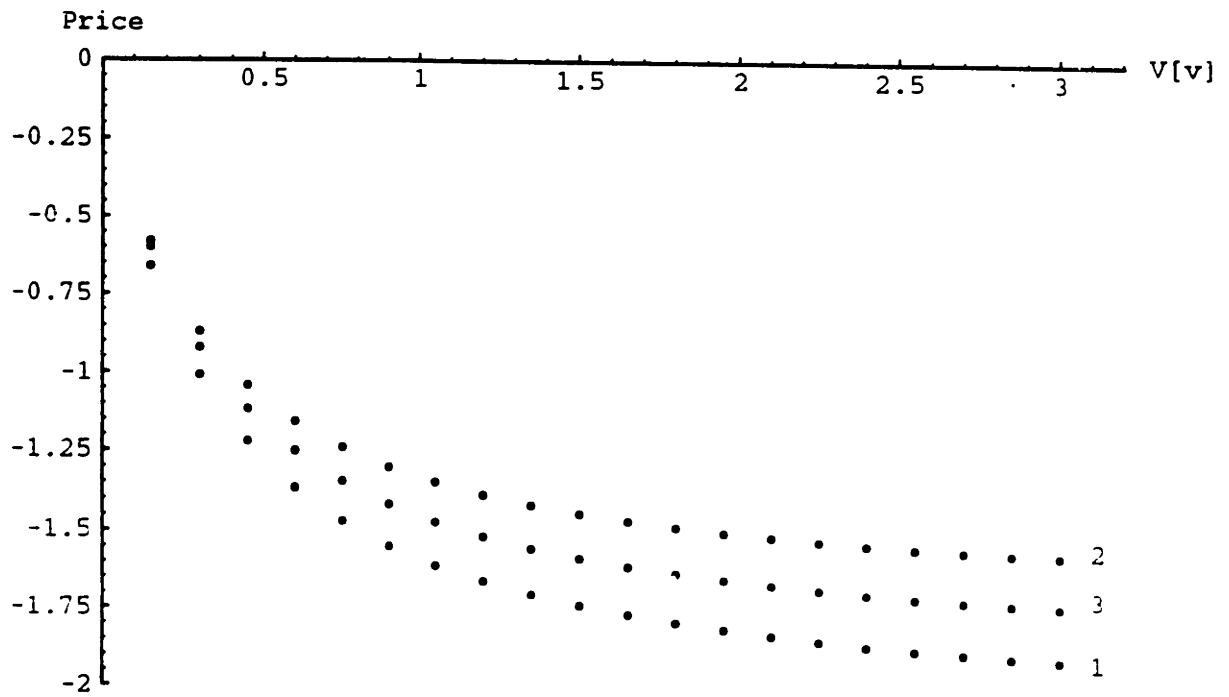
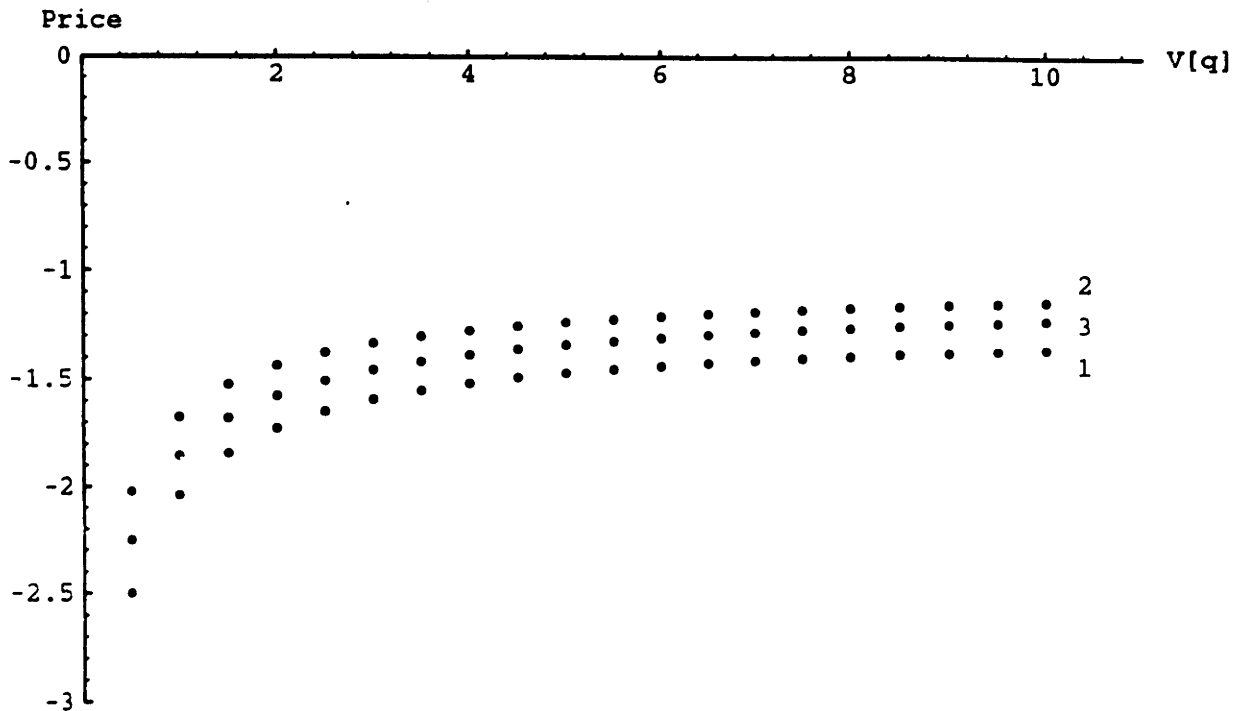


Figure 1.14: Expected Prices for Different Market Structures Versus Variance of \tilde{q}

(“1” — Isolated Auction; “2” — Auction with When-Issued Trading Before;
 “3” — Auction with Surrounding When-Issued Trading)

$$\sigma_z^2 = 3; \sigma_w^2 = 2; \sigma_v^2 = 1; \sigma_\epsilon^2 = 0.5;$$

$$\bar{z} = 0; \bar{q} = 10; \bar{w} = 0; \rho = 1; N = 4$$



Chapter 2

Treasury-Bill Auctions and the Repurchase and Reverse Market

2.1 Introduction

What is usually referred to as *the* market for U.S. Treasury securities is in reality a plethora of interconnected markets, in which you can buy, sell, lend, and borrow individual issues. These markets are normally very liquid and often serve as benchmarks for other risky securities. Despite the markets' size, they are susceptible to manipulation attempts, as the revelation of infractions by Salomon Brothers¹ illustrates. A few key players dominate these markets, and there is evidence that they use the market structure to play strategically. Given their importance, we believe these markets provide a unique opportunity to study market interaction. So far, the research has been limited in this area.

The relationship between the primary and secondary markets is the focus in empirical work by Cammack (1991) and Spindt-Stolz (1991), who study the prices attained in 13-week T-bill auctions and their closest substitutes in the secondary market. They both find systematic underpricing in the auction relative to the seasoned securities. Bikhchandani-Edsparr-Huang (1994) confirm these findings for both 13- and 26-week bills. In addition, they explore the informational link between T-bill auctions and the forward market in issues not yet delivered, the so-called when-issued market. The *innovation* in the auction is defined as the

¹This event is explicitly studied in Jegadeesh (1992). It also prompted a regulatory review; see the Joint Report on the Government Securities Market (1992) and Mullins (1993).

residual from a linear forecasting rule, given the information available before the auction in the when-issued and secondary markets. Their most striking finding is that a large portion of the innovation in the auction is reflected in the when-issued market before the auction results are announced. Simon (1993) reports somewhat similar results for Treasury coupon auctions. In Chapter 1 we have analyzed the impact of surrounding trading opportunities on price formation in the auction. The practical case in point for the model is the primary market for U.S. Treasury securities.

In this paper, we attempt to extend the analysis of Bikhchandani et al. to the repurchase and reverse market, usually referred to as the "repo market." In this market specific issues can be borrowed short term. Conversely, holders of a security can borrow money using it as collateral. In most transactions, it does not matter exactly which Treasury security is used as collateral, as they all are safe and have approximately the same liquidity. Sometimes, however, there is an excess demand for a certain issue. This excess demand is mainly driven by short-sellers in the when-issued market who did not acquire a sufficient amount of it in the auction, and who need it to make delivery. (This situation is called a *short squeeze*.) Holders of such an issue can then borrow money much more inexpensively using the sought-after security as collateral; i.e., the interest rate charged is lower than that for "general" collateral. In such a case, that issue is said to be traded "special." The corresponding interest rates are generally referred to as general and special repo rates. Most on-the-run issues tend to be traded special.

One of the reasons for the sparse academic interest most of these markets have received is the lack of publicly available data. We have been able to obtain proprietary repo-market data for on-the-run 13- and 26-week Treasury bills during the period 1986-88. In fact, we have two data sets: one which quotes the rates transaction by transaction, the other recording a single quote. The two data sets originate from two different firms.

We say that the bidding in an auction is *aggressive* when the ask rate in the when-issued market is higher than the lowest winning rate. We present evidence that the difference between the special and general repo rates, the *specialness*, can be linked to the innovation in the auction. For auctions with aggressive bidding, the specialness tends to increase when the auction prices are unexpectedly high. Our results are robust across both data sets and maturities. These findings support the idea that short squeezes tend to originate when the rates are unexpectedly low in the auction and persist after the delivery date when the repo market opens. We cannot, however, distinguish between random occurrences and collusive attempts

to artificially create a shortage of a particular issue. The impact on specialness of uncertainty in general, as measured by the dispersion of winning bids in the auction and the bid-ask spread in the when-issued market, is also studied. We find evidence of a positive link between the auction dispersion and specialness for the first set of repo data, but not for the second.

In Section 2.2, we present an overview of the T-bill auctions, the when-issued market, and the ensuing trade in the repo market. We also discuss previous empirical results. Section 2.3 discusses the data and provides some descriptive statistics. In Section 2.4, we investigate empirically the link between aggressive auction bidding and issues traded special in the repo market. In particular, we find that for aggressively bid auctions, specialness in the repo market tends to increase when the rates in the preceding auction are unexpectedly low. The opposite result holds for auctions not so aggressively bid. Section 2.5 analyzes the relationship between specialness and overall uncertainty, as measured by the dispersion of accepted bids in the auction and the bid-ask spread in the when-issued market. We find some evidence that the dispersion of winning bids in the auction is positively related to specialness in the repo market. Some concluding remarks are found in Section 2.6.

2.2 The Market for Treasury Bills

In this section we will outline the institutional structure of the primary market for Treasury bills and the repurchase and reverse market, a market for collateralized short-term lending and borrowing. A fairly detailed understanding of this structure is needed for the empirical investigation in subsequent sections. Here, we are only dealing with 13- and 26-week bills. In Section 1.2 of Chapter 2, a description of the primary market for other maturities is found.

2.2.1 The Primary Market

The Department of Treasury holds weekly auctions in which 13- and 26-week bills are sold. These auctions are held every Monday, unless Monday is a holiday, in which case they take place on the first trading day thereafter. The deadline for submitting bids is 1:00 pm. There are two types of bids that can be submitted in the auction: *competitive* and *non-competitive*. All bids are sealed. A non-competitive bid only specifies a quantity, and the price is set as the quantity-weighted average of the winning competitive bids. There is a cap of \$5 million

per bidder through the non-competitive procedure, but any quantity below that limit is assured fulfillment. Usually, the non-competitive bids account for 15-20% of the total amount sold in a given T-bill auction. The non-competitive bidders are primarily individual investors and the Federal Reserve, which submits non-competitive bids for its own account and for foreign central banks. The Federal Reserve is not bound by the \$5 million limit.

The competitive bidding is dominated by the *primary dealers*, currently 38 in number. These are designated by the Federal Reserve. In implementing its open market operations, the Federal Reserve is only trading with dealers having this status. The key criteria to be designated primary dealer are a substantial volume of trading in Treasury securities (at least one percent of the total activity), and sufficient capital reserves. Primary dealers are expected to bid in every Treasury auction for at least three percent of the issue and to be active participants in the secondary market. A firm that wishes to become a primary dealer must provide information about its trading volume and capital reserves to the Federal Reserve. If this information is satisfactory, the firm will be granted *reporting-dealer* status. When the Federal Reserve is convinced that the firm will maintain the required level of activity, it can be designated a primary dealer.

A competitive bid specifies one or more discount rate-quantity pairs, indicating what quantity the bidder is willing to buy at a given price.² The bidders can only submit discount rates which are whole basis points; a basis point is one hundredth of one percent. Although primary dealers may submit as many discount rate-quantity bids as they wish, they usually submit only one or two. The primary dealers buy Treasury bills for their institutional clients and for resale on the secondary market. Although anybody may bid competitively, the competitive bidding is dominated by the primary dealers, who have easy access to the information necessary for effective bidding. There is also a financial motive for placing bids through a primary dealer, as dealers without primary-dealer status have to pay up front to guarantee their bids; a requirement that does not apply to primary dealers. The Treasury stipulates that the sum of the amount of the bills won in the auction and the long position in the when-issued market cannot exceed 35% of the total amount auctioned. In addition, nobody can submit bids in an auction larger in quantity than the total amount auctioned. When the non-competitive bids have been filled, competitive bidders compete for the remainder in a discrimi-

²For a 13-week bill with a face value of \$1,000, a discount rate of, say, 5.98% translates into a price of $\$1000 / (1 + 5.98\%(91/360)) = \984.88 .

natory auction.³ That is, the demands of the bidders, starting with the lowest-rate bidder, are met until all the bills are allocated. The winning competitive bidders pay the unit prices implied by the discount rates they submitted.

After the auction, the Department of Treasury announces summary statistics about the bids submitted. These include total tender amount received, total tender amount accepted, lowest winning rate (highest winning price), highest winning rate (lowest winning price), proportion of bids accepted at the highest winning rate (lowest winning price),⁴ quantity-weighted average of winning rates, and the split between competitive and non-competitive bids. Treasury bills are delivered to the winning bidders on the following Thursday.⁵ Once delivery has taken place, the trading in the secondary market begins. The most recently issued bills of each maturity are said to be *on the run*. On-the-run bills are usually the most actively traded. They also tend to be traded at prices slightly higher than other comparable issues, which usually is ascribed to this superior liquidity.⁶ Another explanation may be that on-the-run issues tend to be traded “special” in the repurchase and reverse market, as described below, providing cheap financing; see Duffie (1993).

There is also a forward market for Treasury bills. Every Tuesday, the Treasury announces the amount of bills to be auctioned the following Monday. The primary dealers begin trading, for themselves and for their institutional clients, forward contracts on the bills to be auctioned.⁷ The maturity date of these contracts is the Thursday of the auction week, the same day the newly auctioned Treasury bills are delivered. As the bills are to be delivered “when issued,” this forward market is usually referred to as the “when-issued market.” After the auction, trading continues in the when-issued market until the when-issued contracts mature, after which the bills are traded in the secondary market. Like all forward markets, the when-issued market has a zero net supply. The absolute value of the total short (or long) positions is called the *open interest*. In the when-issued market, the open interest varies from a small amount to several times the amount auctioned.

Thus there are two different markets for acquiring Treasury bills around auction

³For a review of the implications of auction theory for the Treasury securities markets, see Bikhchandani-Huang (1993).

⁴That is, amount of bids accepted at the highest winning rate divided by the amount of bids received at this rate.

⁵The Treasury bills are not physically delivered to the winning bidders. Instead, they are registered to the winning bidders in the Federal Reserve’s computerized records, the so-called book-entry form; see Fabozzi (1991) and Stigum (1989).

⁶See Amihud-Mendelson (1991).

⁷A standard forward contract is for a principal amount of \$ 5 million.

time — the Treasury auction and the when-issued market. Accordingly, a primary dealer can acquire Treasury bills in three different ways: buy in the when-issued market before the auction, submit bids in the auction, or buy in the when-issued market or in the secondary market after the auction. If a primary dealer buys in the when-issued market or the secondary market, he is sure to acquire the bills, whereas he faces the uncertainty of losing in the auction.

However, there is an advantage to buying in the auction. A primary dealer who has a large demand for the bills to be auctioned risks revealing this information if he buys before the auction in the when-issued market. This will increase the when-issued ask price and encourage aggressive bidding in the auction. If, on the other hand, he bids in the auction, this private information may be revealed only after the auction. Bikhchandani et al. find empirical support for this kind of game playing between the auction and the when-issued market. In about 80% of the auctions there are bids submitted at rates lower than the coincident ask rate in the when-issued market — i.e., a bidder is willing to pass up a trade in the hope of winning in the auction at a *higher* price. This behavior is explained by the informational costs of trading in the when-issued market. It is also shown that this phenomenon is economically significant and not just due to occasional errors by the bidders.

Bikhchandani et al. define the *innovation* in the auction results as the residual from a linear prediction of the auction rate, given the information in the when-issued and secondary markets at the deadline for submitting auction bids. They find that a large fraction of the innovation in the auction results is reflected in the when-issued market before these results are announced. This is also consistent with the hypothesis that informed market participants hold back in the when-issued market until the deadline for submitting bids has passed, and then start trading on their information in the when-issued market. As a bidder cannot go short in the auction, this also explains why the link is stronger when rates go down than when they increase. Having established these patterns consistent with strategic behavior, Bikhchandani et al. also perform some indirect tests for collusion but find no evidence of it. In contrast, Simon (1993) argues that a similar stronger link in aggressively bid coupon auctions is indirect evidence of collusion.

Besides partially aggregating the participants' information, the when-issued market serves as a forward market. Many primary dealers are short in the when-issued market before the auction, as they will have sold these contracts to those institutional clients who want to be certain of obtaining the bills to be auctioned. Of course, some institutional clients may also be short in the when-issued market.

A *short squeeze* occurs when many of the short-sellers fail to acquire bills in the auction. In this event, they have two alternatives. They can buy back in the when-issued market after the auction or they can “borrow” the newly auctioned bills. This borrowing can take place in two different ways: either against some other collateral plus a fee in the securities lending market, or as a *reverse* in the *repurchase and reverse market*, also known as repo and reverse market.

2.2.2 The Repurchase and Reverse Market

The repo and reverse market is a large over-the-counter market for short-term borrowing and lending that is collateralized by securities.⁸ These transactions, however, are structured as sales and subsequent repurchases of securities, rather than as loans in the traditional sense. If, for instance, an individual who possesses securities needs to borrow funds overnight, he can “sell” the securities to a counter party and at the same time sign an agreement to repurchase them the next day at a predetermined price. This price may be equal to the selling price paid on the previous day by the counterparty. In this case, the counter party is also paid an explicit repo rate as interest on the money invested. Alternatively, the purchase price is set to be different from the selling price so as to incorporate the interest. In either case, the return earned by the counter party is called the “repo rate” for the securities used as collateral. The counterparty in a repo agreement is said to be engaged in a “reverse” — borrowing securities while lending funds.

Most of the transactions in the repo market are overnight, but 1-, 3-, and 6-month transactions also take place on a regular basis. All repo agreements that are longer than overnight are called “term repos.” In addition, there are *open* repos without a definite term. In an open repo, the agreement is renewed on a daily basis, but it can be terminated at any time by either party.

Securities dealers and money-market banks tend to be the largest net borrowers in the repo market. Corporations, smaller banks with excess deposits, and municipal bodies are the major suppliers of funds. The repurchase and reverse market also serves as a way of conducting open-market operations by the Federal Reserve.⁹ In fact, it is the primary vehicle for short-term adjustments of bank reserves.

⁸For a detailed account of current market practices, see Stigum (1989) and also Sundaresan (1992), who provides a wide range of descriptive statistics for Treasury auctions and repo and reverse markets.

⁹Because of regulatory restrictions, the Federal Reserve cannot do regular reverses. Instead, the it uses “matched/sale purchases” (MSP’s), which in essence correspond to reverses.

In the spirit of a collateralized loan, there is usually a margin requirement, or *haircut*, to protect the lender from changes in the market value of the collateral. That is, the borrower is not allowed to borrow 100% of the market value of the collateral. Today the margin requirement for Treasury bills is usually 1/4 of a basis point per month of the time to maturity. The Federal Reserve takes a larger margin. In term repos, the lender can call for additional margin if the market value changes.

When there is a short squeeze, say in the when-issued market for 26-week Treasury bills, the repo rate using the newly auctioned 26-week Treasury bills as collateral might decrease dramatically and could even become negative. This is because these bills become scarce commodities and one can borrow money cheaply using them as collateral. The bills are said to be traded “special,”¹⁰ and the rate on loans with that particular security as collateral is called the *special* or *specific* repo rate. The repo rate charged for loans collateralized by issues not traded special is called the *general* repo rate, or equivalently the repo rate for *stock collateral*. In addition to banks, savings and loan associations, some municipalities, and a few corporations are large suppliers of collateral in the market for special issues.

The main use of the repo and reverse market in specific collateral is to cover short positions in the cash market. The overnight general repo rate is normally somewhat below the Federal-funds rate, the rate charged for loans of excess funds in the reserve account at the Federal Reserve. This is what one would expect, as the overnight repo market and the Federal funds market are very close substitutes. The main difference is that the borrowing of excess Federal funds is an unsecured loan, whereas a repo is effectively a collateralized loan. The special repo rate, however, is driven by the scarcity of a certain security, and is not directly related to any other money-market rates. Shortages or short squeezes in a particular on-the-run security stem primarily from uncovered short positions in the when-issued market. Most of this paper is focused on the interaction between the auction and the special repo and reverse market. We refer to the difference between the general and special repo rates as the *specialness* of a particular issue.

¹⁰For an example of an apparent anomaly in the pricing of a 30-year Treasury bond in the secondary market as well as in the repo market, see Cornell and Shapiro (1989).

2.3 The Data

The data used are for the period from October 6, 1986 to November 28, 1988. We have for this period:

1. 99 observations of the bid and ask (discount) rates for 13- and 26-week when-issued contracts at 1:00 pm on auction days.¹¹
2. Summary statistics of the 99 auctions that correspond to the when-issued rates at 1:00 pm on auction days in our data set. These statistics include quantity-weighted average winning rate, lowest and highest winning rates, and the total amount tendered and accepted.
3. The overnight repurchase rate for general collateral for transactions on the first trading day the secondary market is open after the auction, normally Thursday.
4. The special overnight repo rates, transaction by transaction, for on-the-run 13- and 26-week bills on the first trading day the secondary market is open after the auction, taken from one dealer in the repo market. These data also include the number of contracts traded at each rate, but not the time for the transaction.
5. The overnight repo rate for the on-the-run 13- and 26-week bills on the first trading day the secondary market is open after the auction taken from a *second* dealer. These quotes represent a somewhat subjective daily average as recorded by the traders at the end of the day.

The when-issued rates at 1:00 pm on auction days are not publicly available. These data were collected by calling a dealer at Shearson Lehman at 1:00 pm on auction days, who in turn got them from Fundamental Brokers Inc. (FBI).¹² The summary statistics of the weekly auction are announced by the Department of Treasury and published in *The Wall Street Journal*. The 1:00 pm when-issued data will

¹¹The when-issued contract is quoted by its discount rate. The bid rate is the discount rate at which one sells and the ask rate is the discount rate at which one buys.

¹²This market is extremely efficient, and the quotes hardly ever vary across the different brokers. These data are a subset of the sample used in Bikhchandani et al.; approximately thirty observations in the beginning of 1986, for which we lack observations from the repo market, are excluded in this paper.

primarily be used in this paper to calculate the average of the bid and the ask rate, which we denote B_{Ann} , $nn=13$ and 26 , and the spread between the bid and ask rates, $WISPRD_{nn}$. The summary auction statistics are used to form the difference between highest and lowest winning rates, $DISP_{nn}$, and the ratio of the total amount tendered to the amount accepted, $COVER_{nn}$, $nn=13$ and 26 . We also use the quantity-weighted average winning rate, AR_{nn} , $nn=13$ and 26 .

From the two repo samples, we construct three different variables to measure the degree of specialness in a particular issue. The first is a quantity-weighted average difference between the special and general repo rates. This variable is denoted $1SPAV_{nn}$, $nn=13$ and 26 , for the 13- and 26-week bills, respectively. Similarly, we use the maximum difference between the special and general repo rates among all transactions in a given day, and denote it $1SPM_{nn}$. Finally, a simple difference is calculated between the special and general repo rates from the second firm. This variable is called $2SP_{nn}$, $nn=13$ and 26 . In addition, we use a measure of dispersion in the special repo rates used in a given day. It is calculated as the maximum difference between all special repo rates at which transactions have taken place through the first dealer. We denote this variable $SPSPRD_{nn}$.

There are about 15 missing observations in the first repo sample and 29 in the second, but only rarely is an observation missing in both, so no bias should be introduced. Only dates for which we have when-issued data are included. During our sample period, approximately ten auction days are missing from the sample. This is because the person providing the data was out of the office on the days in question. As he was not a trader in this market, we do not believe any selection bias resulted.

Table 2.1 lists definitions of the variables used. To facilitate comparison between 13- and 26-week bills, we represent the data in (discount) rates rather than prices. Sample statistics are presented in Table 2.2 (this and the following tables are found in Section 2.8), and the correlation coefficients for the different measures of specialness are found in Table 2.3.

First we look at the repo data from the first dealer, the transactions data. We can see that, on average, the on-the-run 13-week bill is traded in the repo market at a rate of 70 basis points below that for general collateral, and the variability from one week to the other is very high, a standard deviation of about 74 basis points. Similarly, the on-the-run 26-week bill has an average repo rate of 76 basis points below the general rate, and a standard deviation of 84 basis points. The differences between special and general repo rates for the second sample are somewhat higher, 109 and 105 basis points, respectively. We can also see that

Table 2.1: List of Variables

ARnn:	The quantity-weighted average winning rate in the auction for nn week bills, nn=13 or 26.
BAnn:	The average of the ask and bid rates in the when-issued market at 1:00 pm on auction days for nn week bills, nn=13 or 26.
WISPRDnn:	The difference between the bid and ask rates for nn week bills, nn=13 or 26, quoted on the when-issued market at 1:00 pm on auction days.
DISPnn:	The difference between the highest and lowest winning rate in the auction for nn week bills, nn=13 or 26.
COVERnn	The ratio of the total amount tendered and the amount accepted in the auction for nn week bills, nn=13 or 26.
1SPAVnn:	The quantity-weighted average difference between the special repo rates and the rate for general collateral for all transactions in the first repo sample for the on-the-run nn week bill, nn=13 or 26.
1SPMnn:	The maximum difference between the special repo rates and the rate for general collateral among all transactions in the first repo sample for the on-the-run nn week bill, nn=13 or 26.
2SPnn:	The difference between the special repo rate from the second dealer and the rate for general collateral for the on-the-run nn week bill, nn=13 or 26.
SPSPRDnn:	The maximum difference among the special repo rates for all transactions in the first repo sample for the on-the-run nn week bill, nn=13 or 26.
Dnn:	A dummy variable whose value is one if the when-issued ask rate for the nn week bill, nn=13 or 26, is greater than the lowest winning rate in the concurrent auction, and zero otherwise.

the rates vary considerably from one transaction to the next within the same day: SPSPRD13 has a mean of 38 basis points and SPSPRD26 of about 58 basis points.

By looking carefully at Table 2.3, we also see that the special repo rates in the two samples have correlation coefficients of less than 0.6. As the special repo rates can fluctuate considerably within a single day, we think the low correlation coefficient reflects that the dealers' transactions take place at different times.

2.4 Auction Bidding and Issues Traded Special

In this section, we investigate the link between bidding in the auction and the subsequent repo market in specific issues. It seems plausible that the allocation generated by the auction is the driving force behind specialness in the repo market. If a large fraction of the short positions in the when-issued market remain uncovered after the auction, the dealers are forced to either buy the missing securities or borrow them. If, in addition, the issue is perceived to be overpriced in the post-auction when-issued market, the solution is to engage in a reverse transaction until the price in the cash market goes down. Even if the special repo rate is substantially below that for stock collateral, it may be a small price to pay compared to buying the sought-after security at a temporarily inflated price. Naturally, those borrowing the missing securities hope that prices in the secondary market will come down eventually. The empirical results in Simon (1993) and Bikhchandani et al. indicate that auctions in which the bidding is aggressive are qualitatively different from other auctions. This might be because of collusive attempts to squeeze the market, or simply because there is no way of short selling in the auction. In either case, they require special attention.

Our first step toward finding such a connection is to assess the surprise in the auction results. We use a linear specification to forecast the average rate of the winning bids in the auction:

$$ARnn^i = \theta^{CONST} + \theta^{13} \cdot BA13^i + \theta^{26} \cdot BA26^i + \tilde{\epsilon}^i, \quad (2.1)$$

where $\tilde{\epsilon}^i$ is an error term, and BA13 and BA26 are the when-issued rates at 1:00 pm for 13- and 26-week bills, respectively. The θ 's are constants. The results presented in Bikhchandani et al. show that there is no additional information contained in the secondary-market prices beyond what is already incorporated in the when-issued market. Therefore, only the when-issued rates are used to forecast the average winning auction rate. We define the *innovation*, or surprise, in the auction

results as the residuals from this regression. These residuals are denoted RES13 and RES26, respectively. As all the variables are expressed in discount rates rather than prices, the larger the residuals, the lower the prices in the auction relative to the forecast.

The forecasting regressions are found in Table 2.4. The adjusted R^2 's in these regressions show that there are rarely any big surprises in the auction results, given the information in the when-issued market. All coefficients in this paper are estimated using the ordinary-least-squares procedure, and standard errors are calculated allowing for heteroskedasticity by using White's (1980) procedure.

We now ask:

1. Is specialness in the repo market related to the innovation in auction results?

To answer this question, we regress the residuals, RES13 and RES26, on the different variables for specialness. As we believe that auctions with aggressive bidding are qualitatively different, we also use a specification with dummies for aggressive bidding ($nn=13$ and 26):

$$RES_{nn}^i = \kappa + \zeta^{SP} \cdot 1SPAV_{nn}^i + \zeta^{AGG} \cdot D_{nn} \cdot 1SPAV_{nn}^i + \tilde{\epsilon}^i. \quad (2.2)$$

Analogous specifications are used for $1SPM_{nn}$ and $2SP_{nn}$. The ζ 's and κ are constants. These dummies, $D13$ and $D26$, take the value one if the lowest rate in the auction is lower than the ask rate in the when-issued market at 1:00 pm, and zero otherwise. This dummy is intended to control for at least one bidder being willing to pass up an opportunity to buy in the when-issued market at a lower price than he is prepared to pay in the auction. It may seem possible that this is just a mistake by some bidders. Bikhchandani et al., however, show that this variable is economically significant. We use it as an analytically appealing way of defining aggressive bidding relative to the when-issued market.

The regressions for 13-week bills are presented in Table 2.5, and those for 26-week bills in Table 2.6. Without the dummy for aggressiveness in the auction, the adjusted R^2 's are roughly zero. The coefficients are -0.004 , -0.004 , and 0.001 for the different specialness variables in the case of 13-week bills, and 0.003 , 0.003 , and -0.002 in the case of 26-week bills. None of the coefficients is significantly different from zero, indicating that there is no significant link for the sample as a whole. When the dummy is introduced, a very clear pattern emerges in all the regressions

regardless of maturity and variable used to measure specialness: the coefficient on the dummy variable is negative, the one on the specialness variable without the dummy is positive, and the sum of the two is negative. The adjusted R^2 's are around 0.25 for the 13-week bill, and slightly less than 0.10 for the 26-week bill. For the 13-week bill, the coefficients on the variables for specialness without the dummy are around 0.015 with t -statistics of roughly 2.0, and the dummy variables have coefficients between -0.018 and -0.029 with t -statistics around -4.0 . The results for the 26-week bill are very similar: the coefficients on the specialness variables are all in the range 0.009 – 0.025 with t -statistics between 1.7 and 2.4; the dummy variables have coefficients between -0.014 and -0.025 with t -statistics around 2.5.

The negative sum of the two coefficients indicates that a surprisingly low rate in the auction (i.e., high price) goes with a high degree of specialness when the bidding is aggressive relative to the pre-auction when-issued market. The intuition is straightforward. A surprisingly low rate in the auction is likely to leave many of the short-sellers in the when-issued market with a less than sufficient allocation to cover their short positions. This short squeeze is then translated into a larger than usual demand for that issue in the repo market, which in turn pushes down the repo rate. It is interesting to see that this line of reasoning applies *only* when someone is willing to submit bids at prices higher than those at which one could have bought in the when-issued market at the time of the deadline for submitting bids in the auction.

We believe that the results can be explained by two different effects working in opposite directions. The first effect, outlined above, is significant only when there is a true surprise in the auction, i.e., most of the bidders do not anticipate the low rate, which leaves them vulnerable to one or a few bidders who deliberately conceal their intentions in the when-issued market. (This reading does not, however, differentiate between attempts to short squeeze the market and strategic trading on private information.)

The second effect stems from the perceived price level in the when-issued market. We suggest that when no one is bidding at prices above the coincident level in the when-issued market, the residuals from the auction prediction primarily measure the perceived under- or overpricing in the when-issued market at the time of the auction. It is also reasonable to believe that the dealers' short positions grow with the degree of perceived overpricing in the when-issued market, as that would justify a greater risk exposure in their short positions. If this is the case, the positive coefficients when the dummy is zero, would correspond to the increased

risk of being short squeezed as the open interest rises. In other words, a particular T-bill's specialness in the repo market depends on the surprises in the auction as well as the overall level of short positions in the when-issued market.

The dummy variable is intended to dichotomize the sample according to the level of surprise in the auction. When there is little surprise in the auction itself, the latter effect dominates. If there is a surprise, as here indicated by someone's willingness to abstain from trading in the when-issued market in order to conceal the information until the auction, the former dominates. These results seem robust, as they hold for all the different measures of specialness. We have also verified that the results are not driven by outliers.

Next, we want to know whether overall demand in the auction is related to specialness in the repo market. Attempts to surprise or deliberately squeeze the market may, for example, be more successful if the total amount of bidding is relatively small. We therefore ask:

2. Is specialness in the repo market related to the overall demand in the auction?

To test this, we use $COVER_{nn}$, $nn=13$ or 26 , to measure the aggregate demand in the auction. Remember that $COVER_{nn}$ is the ratio of total amount tendered to the amount accepted in an auction. We regress the different variables for specialness on $COVER_{nn}$:

$$1SPAV_{nn}^i = \kappa + \zeta^{COVER} \cdot COVER_{nn}^i + \tilde{\epsilon}^i, \quad (2.3)$$

and similarly for $1SPM_{nn}$ and $2SP_{nn}$. The results in Table 2.7 show that the null hypothesis that no such link exists cannot be rejected in four of the six cases. In the two regressions with significant coefficients at the 10% level, the coefficient on $COVER_{13}$ is approximately -0.25 , indicating that a lot of bids in the auction is negatively related to specialness. All the other coefficients on $COVER_{nn}$ are positive, making it impossible to discern any consistent pattern. We have also used dummies to see whether there is an effect in the aggressively bid auctions but not in the rest, or *vice versa*. Since these additional regressions show no such link, they are not included.

The first set of repo data provides an opportunity to see what determines the intra-day variability in repo rates, transaction by transaction. The variable $SPSPRD_{nn}$, $nn=13$ or 26 , measures the maximum difference between the special

repo rates at which transactions took place through the dealer. We think this is a better measure than some kind of variance estimator, as the number of transactions is relatively small and no time is recorded. If there were a time recorded for each transaction, we could trace the rate changes through the day. Given the limitations of the available data, we find it more useful to see within what bounds the transactions take place.

As we found above, the level of special repo rates is related to the surprise in the auction results. Thus, it seems logical to ask:

3. Is the maximum difference between the special repo rates related to the surprise in the auction?

To test this, we mimic the procedure above, regressing the residuals from Table 2.4 on $SPSPRD_{nn}$, with and without the dummy variable D_{nn} ; i.e., with the dummy:

$$RES_{nn}^i = \kappa + \zeta^{SPR} \cdot SPSPRD_{nn}^i + \zeta^{AGG} \cdot D_{nn} \cdot SPSPRD_{nn}^i + \tilde{\epsilon}^i. \quad (2.4)$$

Table 2.8 presents the results. Without the dummies, we can reject the null hypothesis of no significant relationship at the 10%-level for the 13-week bills, but not for the 26-week bills. This tells us that for the 13-week bills, it is more likely to observe a big difference from one special repo transaction to the next when the rates are unexpectedly low in the auction. When we introduce the dummies, D_{13} and D_{26} , we see that a similar pattern is present for both maturities for the aggressively bid auctions. The coefficient on the dummy variable is -0.029 for the 13-week bills and -0.023 for the 26-week bills, and both are significantly different from zero at the 1% level. For the-not-so aggressively bid auctions, the coefficient is positive for both maturities (0.014 and 0.019 , respectively), but significant (at the 10% level) only for the 26-week bills.

We think this fits the pattern established above, in which two different effects are at work simultaneously. A low rate in the when-issued market leads to larger short positions, which leads to a positive relationship between the innovation in the auction and both the degree and variability of specialness in the ensuing repo market. When there is no surprise in the auction, this effect dominates. When there is a surprise, as measured by D_{nn} , the surprise itself creates a short squeeze, increasing both the specialness and its variability.

Having established a close link between the auction and both the degree and

variability of specialness in the repo market, we now focus on the role played by uncertainty in general in explaining issues traded special.

2.5 Uncertainty and the Repo Market

In this section, we will investigate whether specialness can be related to overall uncertainty in the market. Volatile market conditions are usually conducive to speculation by the best informed participants. It is not a question of arbitrage in the academic sense of the word, but of risky positions that earn superior returns. If we assume that a particular T-bill issue is traded special primarily because short-sellers in the when-issued market have failed to cover their positions in the auction, it is plausible to think that specialness in the repo market is linked to the amount of speculative activity. From the theory of market microstructure, one measure of divergence of beliefs is the bid-ask spread.¹³ In the auction context, the difference between highest and lowest winning bids is a measure similar in spirit. This is what is used by Bikhchandani et al., and Cammack (1991) uses the closely related difference between the lowest and average winning rate.

We then ask:

4. Is specialness in the repo market related to the overall uncertainty in the market?

To test this hypothesis, we regress the variables for specialness on the bid-ask spread in the when-issued market immediately preceding the auction, $WISPRD_{nn}$, and the difference between highest and lowest winning rate in the auction, $DISP_{nn}$, where $nn=13$ and 26 :

$$1SPAV_{nn}^i = \kappa + \zeta^{DISP} \cdot DISP_{nn}^i + \zeta^{WISP} \cdot WISPRD_{nn}^i + \epsilon^i, \quad (2.5)$$

and analogously for $1SPM_{nn}$ and $2SP_{nn}$. The results are found in Table 2.9. The coefficient for $DISP_{nn}$ is significant in half of the regressions, for two of three variables for specialness in the case of the 13-week bill, and for one of the three for the 26-week bill. All the specialness variables formed by the data from the first firm are positive and rather large. Those from the second firm are roughly zero. For the 13-week bill, the significant coefficients on $DISP_{13}$ are around 8.0, telling

¹³See, e.g., Copeland-Galai (1983) and Glosten-Milgrom (1985).

us that one basis point in the dispersion of winning auction bids on average causes the specialness to increase by eight basis points. In the regressions of 1SPAV26 and 1SPM26 on DISP26, the coefficients are somewhat smaller: 5.79 and 4.54, respectively. Only the former is significant (at the 10% level). We regard this as weak evidence in support of overall uncertainty being related to specialness. It is clear, however, that the spread in the when-issued market is not related to specialness, regardless of the variable used.

Finally, we want to see if the fluctuations in special repo rates from one transaction to the other are related to market uncertainty, so we ask:

5. Is the maximum difference in special repo rates related to the overall uncertainty in the market?

To answer this question, we regress $SPSPRD_{nn}$, on $DISP_{nn}$ and $WISPRD_{nn}$, $nn=13$ and 26 :

$$SPSPRD_{nn}^i = \kappa + \zeta^{DISP} \cdot DISP_{nn}^i + \zeta^{WISP} \cdot WISPRD_{nn}^i + \tilde{\epsilon}^i. \quad (2.6)$$

As we can see from the results in Table 2.10, the coefficients on $DISP_{nn}$ and $WISPRD_{nn}$ are 0.68 and -0.26 for the 13-week bill, and -4.74 and -2.15 for the 26-week bill. None of the coefficients is significant, and thus we cannot reject the null hypothesis that no such relationship exists for both maturities.

In short, we have found some evidence in favor of overall uncertainty at the time of the auction being related to the degree of specialness in the repo market. No link between $SPSPRD_{nn}$ and market uncertainty has been found.

2.6 Concluding Remarks

We have tried in this paper to link the repo market to the preceding Treasury-bill auctions. The evidence seems very strong that such a link exists, and that there are two distinct effects working in opposite directions. In aggressively bid auctions, unexpectedly low rates in the auction tend to go with a high degree of specialness. For auctions in which the bidding is less aggressive, we also find a strong link between the innovation in the auction and the special repo rates, but in the opposite direction. We have suggested a possible explanation for this phenomenon.

The results presented in this paper as well as those in Bihchandani et al. seem to contradict the widespread notion that the Treasury market is perfectly competitive, and suggest that there is a close link between the institutional structure of the market and the price formation. At the very least, the results highlight the need for further research that explicitly takes market structure into account. An obvious next step would be to enlarge the sample, and preferably include transactions from several dealers. We also think that time-stamped data could provide valuable insights into the intra-day dynamics of the repo market.

2.7 References

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2.8 Tables

Table 2.2: Sample Statistics[†]

	AR13	BA13	WISPRD13	COVER13	DISP13
Sample Size	99	99	99	99	99
Max	8.05	8.035	0.04	635	0.15
Min	5.08	5.065	0.00	298	0.00
Mean	5.99	5.98	0.0101	398	0.045
Std. Dev.	0.63	0.63	0.006	62	0.029
	1SPAV13	1SPM13	2SP13	SPSPRD13	
Sample Size	82	82	72	82	
Max	3.57	3.90	5.00	2.94	
Min	-0.25	-0.10	0.00	0.00	
Mean	0.70	0.84	1.09	0.38	
Std. Dev.	0.74	0.78	1.26	0.49	
	AR26	BA26	WISPRD26	COVER26	DISP26
Sample Size	99	99	99	99	99
Max	8.13	8.13	0.08	532	0.15
Min	5.13	5.14	0.00	279	0.00
Mean	6.20	6.20	0.012	372	0.037
Std. Dev.	0.67	0.67	0.010	44	0.024
	1SPAV26	1SPM26	2SP26	SPSPRD26	
Sample Size	86	86	72	86	
Max	5.26	5.65	4.75	5.12	
Min	-0.28	-0.20	-0.08	2.98	
Mean	0.78	1.07	1.05	0.58	
Std. Dev.	0.84	1.14	1.30	0.90	

[†] The rates are in percent. For example, 2.50 means 2.5%.

Table 2.3: Sample Correlation Coefficients for Measures of “Specialness”

	1SPAV13	1SPM13	2SP13	1SPAV26	1SPM26	2SP26
1SPAV13	1.00	0.95	0.48	0.28	0.18	0.27
1SPM13		1.00	0.40	0.20	0.08	0.22
2SP13			1.00	0.37	0.33	0.27
1SPAV26				1.00	0.91	0.57
1SPM26					1.00	0.55
2SP26						1.00

Table 2.4: Regression of Auction Rates on When-Issued Rates

Independent Variables	Dependent Variables	
	AR13	AR26
Constant	-0.0394* (-1.98)	0.0293 (1.17)
BA13	0.9096*** (47.65)	-0.0018 (-0.04)
BA26	0.0960*** (5.10)	0.9976*** (23.76)
Sample Size	99	99
Adjusted R^2	0.9978	0.9965

† The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 2.5: Regression of Residuals from the Regressions of Table 2.4 on Measures of “Specialness”†

Independent Variables	Dependent Variable					
	RES13					
Constant	0.004 (1.14)	0.006* (1.75)	0.005 (1.26)	0.007* (1.88)	-0.003 (-0.69)	-0.003 (-0.61)
1SPAV13	-0.004 (-1.03)	0.015** (2.01)	—	—	—	—
D13·1SPAV13	—	-0.029*** (-3.97)	—	—	—	—
1SPM13	—	—	-0.004 (-1.18)	0.014* (1.98)	—	—
D13·1SPM13	—	—	—	-0.028*** (-3.94)	—	—
2SP13	—	—	—	—	0.001 (0.46)	0.012** (2.59)
D13·2SP13	—	—	—	—	—	-0.018*** (-3.84)
Sample Size	82	82	82	82	72	72
Adjusted R^2	0.00	0.23	0.00	0.26	-0.01	0.17

† RES13 is the residual from the regression of Table 2.4. It represents the innovation from the prediction of AR13 given the 1:00 pm quotes in the when-issued market. The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 2.6: Regression of Residuals from the Regressions of Table 2.4 on Measures of “Specialness”[†]

Independent Variables	Dependent Variable					
	RES26					
Constant	-0.002 (-0.43)	-0.004 (-0.62)	0.003 (1.33)	-0.003 (-0.56)	-0.002 (-0.68)	0.003 (0.45)
1SPAV26	0.003 (1.15)	0.025** (2.40)	—	—	—	—
D26·1SPAV26	—	-0.025*** (-2.69)	—	—	—	—
1SPM26	—	—	0.003 (1.33)	0.018* (1.97)	—	—
D26·1SPM26	—	—	—	-0.019** (-2.28)	—	—
2SP26	—	—	—	—	-0.002 (-0.68)	0.009 (1.66)
D26·2SP26	—	—	—	—	—	-0.014*** (-2.73)
Sample Size	86	86	86	86	72	72
Adjusted R^2	-0.01	0.07	0.00	0.09	-0.01	0.02

[†] RES26 is the residual from the regression of Table 2.4. It represents the innovation from the prediction of AR26 given the 1:00 pm quotes in the when-issued market. The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 2.7: Regression of Measures of “Specialness” on the Ratio of Tendered to Accepted Bids in the Auction[†]

Independent Variables	Dependent Variables					
	1SPAV13	1SPM13	2SP13	1SPAV26	1SPM26	2SP26
Constant	1.66*** (2.73)	1.92*** (3.14)	0.10 (0.09)	0.06 (0.09)	-0.30 (-0.30)	-0.93 (-0.49)
COVER13	-0.24* (-1.69)	-0.27* (-1.91)	0.25 (0.93)	—	—	—
COVER26	—	—	—	0.19 (1.01)	0.37 (1.36)	0.54 (1.04)
Sample Size	82	82	72	86	86	72
Adjusted R^2	0.03	0.04	0.00	0.00	0.01	0.01

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 2.8: Regression of Residuals from the Regressions of Table 2.4 on Maximum Difference of in Special Repo Rates[†]

Independent Variables	Dependent Variables			
	RES13		RES26	
Constant	0.005 (1.21)	0.005 (1.20)	-0.001 (-0.16)	-0.001 (-0.12)
SPSPRD13	-0.009* (-1.78)	0.014 (1.32)	—	—
D13·SPSPRD13	—	-0.029*** (-2.73)	—	—
SPSPRD26	—	—	0.002 (0.64)	0.019* (1.71)
D26·SPSPRD26	—	—	—	-0.023*** (-2.10)
Sample Size	82	82	86	86
Adjusted R^2	0.01	0.07	-0.01	0.05

[†] RES13 and RES26 are the residuals from the regression of Table 2.4. They represent the innovation from the prediction of AR13 or AR26 given the 1:00 pm quotes in the when-issued market. The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 2.9: Regression of Measures of “Specialness” on Proxies for Uncertainty in the Market[†]

Independent Variables	Dependent Variable					
	1SPAV13	1SPM13	2SP13	1SPAV26	1SPM26	2SP26
Constant	0.36** (2.42)	0.47*** (2.92)	0.83*** (2.77)	0.60*** (4.40)	0.98*** (4.47)	1.04*** (3.70)
DISP13	7.95* (2.46)	8.38** (2.64)	-1.41 (-0.29)	—	—	—
DISP26	—	—	—	5.79* (1.80)	4.54 (1.18)	-0.06 (-0.01)
WISPRD13	-1.47 (-0.16)	-0.06 (-0.01)	28.89 (1.40)	—	—	—
WISPRD26	—	—	—	-3.61 (-0.62)	-6.83 (-0.96)	1.18 (0.08)
Sample Size	82	82	72	86	86	72
Adjusted R^2	0.08	0.08	-0.01	0.00	-0.01	-0.03

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 2.10: Regression of Maximum Difference in Special Repo Rates on Proxies for Uncertainty in the Market[†]

Independent Variables	Dependent Variable	
	SPSPRD13	SPSPRD26
Constant	0.35*** (4.29)	0.78*** (3.66)
DISP13	0.68 (0.46)	—
DISP26	—	-4.74 (-1.27)
WISPRD13	-0.26 (-0.41)	—
WISPRD26	—	-2.15 (-0.45)
Sample Size	82	86
Adjusted R^2	-0.02	0.00

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Chapter 3

Stock Returns and Real Options: An Exploratory Study

3.1 Introduction

The statistical properties of stock returns constitute one of the classic research areas in capital markets theory. Since Bachelier's work at the turn of the century,¹ the random-walk hypothesis and its continuous-time counterpart, Brownian motion, have been the theoretical benchmarks against which empirical findings are measured and analyzed. The failures of the analytically appealing Brownian-motion model is crucial to many areas in financial economics, e.g., optimal portfolio selection, hedging strategies, and asset pricing. For example, it has been well established that individual stock returns display "fat tails" (a leptokurtic distribution) and that the returns for stock indices are serially correlated. Most of this literature is descriptive, and relatively few attempts have been made to explain the statistical properties from the underlying economics.

In this paper, we will focus on the driving forces behind another such finding, first documented by Black (1976): The volatility of stock returns tends to decline when the price goes up. This effect has been studied in numerous papers, e.g., Beckers (1980), French-Schwert-Stambaugh (1987), and MacBeth-Merville (1980), and it seems to persist throughout a wide variety of samples and statistical specifications. Most of the work, however, has been done on stock indices rather than at the level of individual stocks.

¹See Bachelier (1900).

Two possible explanations have been advanced in the literature: leverage and “volatility feedback,” both of which were originally suggested by Black (1976). The first explanation relies on the assumption that the volatility of the returns on the firm’s total assets is constant (or changes only slowly), so that, in the presence of debt, positive earnings make the equity less volatile as the debt-equity ratio falls (approaching the asset volatility as the debt-equity ratio goes to zero). Although leverage explains some of the effect, empirical evidence suggests that it is not large enough to be its sole origin; see Christie (1982) and Schwert (1989).

The second explanation links expected returns to expected volatility. When the market anticipates higher volatility, the stock price falls immediately to provide expected future returns commensurate with the new level of risk. Because the change in expected volatility sometime in the future causes the stock prices to fall now, and not *vice versa*, this effect is usually called volatility feedback.² Pindyck (1984) has stressed this explanation. Volatility feedback, however, seems important only during periods of extreme volatility, such as October 1987; see Campbell-Hentschel (1992). Poterba-Summers (1986) also argue that volatility feedback could not explain this effect, because changes in volatility do not last long enough to have any substantial impact on stock prices. Furthermore, if we believe that only non-diversifiable risk is prized in market, this explanation makes more sense for the market as a whole, than for individual stocks. Changes in the idiosyncratic component of a stock’s volatility should have no impact on its price. Thus volatility feedback is a statement about future changes in systematic risk, not about volatility *per se*.

In this paper, we suggest a third explanation that focuses directly on the asset mix of the company: growth opportunities and real options, especially those related to research and development. We know that returns on options have different statistical properties than returns of the underlying asset. In many cases, options can be used to achieve the same objectives as a levered position in the underlying assets. It thus seems plausible that the stocks of companies whose main assets are options on future growth opportunities will have statistical characteristics different from those of a manufacturing firm in a mature industry.

An extreme example would be an upstart biotech company whose only asset was an idea for a new drug. Under the best possible scenario, this drug could be on the market in five years. The company has the option to proceed with each new phase of development, testing, production and marketing of the new product.

²Black (1976) used the term “reverse causation effect.”

In the early stages of this venture the ultimate outcome is likely to be completely unpredictable. The assets in such a company can be likened to a portfolio of options.

Myers-Shyam-Sunder (1991) and Shyam-Sunder (1991) find supporting evidence in the U.S. pharmaceutical industry, where research and development expenditures and CAPM β 's are positively correlated. This is what we would expect if the market treats research and development projects as call options, because β is larger for an option than for the asset on which it is written. This effect is driven by the option feature of future investments, and not by the high variance of the research project itself as that is almost exclusively idiosyncratic. Myers and Shyam-Sunder use the analogy of a lottery ticket on a call option, where the lottery corresponds to success in the research project and the call option to the ensuing decision on commercial introduction.

We use a sample of 600–700 firms per year for the period 1988–92 — i.e., all companies in COMPUSTAT and CRSP for which both the necessary accounting data and the daily stock returns are available. For firms that are profitable, we find a significant relationship between the magnitude of volatility decline in response to positive stock returns and some proxies for real options and growth opportunities; research and development expenditures, in particular, prove to be a significant explanatory variable. To control for leverage, a return series is constructed to approximate the returns on a firm's total assets. The "option" effect is even more pervasive for these leverage-adjusted returns. Our results show that real options in a firm's asset mix are reflected in the behavior of its stock price. They also illustrate that stock returns are not merely statistical objects but closely tied to the underlying economic structure.

The remainder of the paper is organized as follows. In Section 3.2, we analyze the statistical properties of option returns, to acquire a better understanding of what to expect empirically. A continuous-time formulation is used to derive a closed-form expression for volatility conditional on past returns. We denote this volatility elasticity with respect to price γ . Simulations for a discrete "binomial-tree" model are performed to assess the small-sample properties of the γ estimates. Section 3.3 discusses the data and presents some descriptive statistics. Our empirical results are presented in Section 3.4, where we report a significant relationship between γ and proxies for real options, as well as leverage. The sample is also divided into groups according to their level of research and development expenditures. A statistical analysis performed group by group yields similar results. Finally, we adjust the returns for leverage and perform the equivalent analysis for

the new return series. The proxies for real options remain statistically significant. Section 3.5 concludes.

3.2 Options and Conditional Volatilities

In this section we explore the statistical properties of option returns, to better assess how assets with option features affect stock returns. When dealing with real options, it can be difficult to specify the underlying asset, since it is usually unobservable; this is in sharp contrast to options on assets traded in liquid markets, where we can observe the prices directly. Here, we simply seek to capture qualitatively what can be expected when a firm's assets are of an option nature. Our ultimate objective is to investigate empirically whether these statistical patterns can be linked to real options in a firm's asset mix.

We will work with the volatility elasticity of stock returns with respect to price, γ . Our empirical specification is:

$$\ln \sigma_{t+1}^i - \ln \sigma_{t-1}^i = \kappa^i + \gamma^i \cdot r_t^i + \tilde{\epsilon}^i, \quad (3.1)$$

where σ_t^i and r_t^i are the volatility and return, respectively, of firm i in period t , and κ^i is a constant. The γ^i captures how sensitive the company's stock volatility is to changes in the price of its stock. Everything else that might affect the stock volatility is modeled as a random variable, $\tilde{\epsilon}^i$.

Changes in macroeconomic conditions such as interest rates, consumer expectations, and political factors, as well as turmoil in currency and raw-materials markets, might all give rise to changes in volatility that are unrelated to the past performance of the company. These are all captured by $\tilde{\epsilon}^i$. Firm-specific uncertainty that is unrelated to past performance *per se*, such as labor and environmental problems, is also captured by $\tilde{\epsilon}^i$. Market inefficiencies could also show up in the error term. It is possible that some of these other factors might be correlated across time or across firms. As we are not concerned with the significance level of an individual firm's estimated γ , the exact covariance matrix does not matter. If we wanted to calculate standard errors of the γ estimates, we would be concerned with the covariance matrix of the error term as it could affect the efficiency of these standard errors. For our purpose, it suffices to know that the ordinary-least-squares procedure yields unbiased estimates regardless of the true covariance structure. Measurement errors in the returns, however, would bias the absolute value of the estimates downwards. Particular types of market inefficiencies could also bias the

estimates. For instance, if volatility reflected new information instantaneously but returns displayed negative serial correlation, we would underestimate the true γ 's.

We use the difference of logged volatility, as it approximates the fractional change in the variable. This can be illustrated by a first-order Taylor expansion:

$$\ln \sigma_{t+1}^i - \ln \sigma_{t-1}^i \approx \frac{\sigma_{t+1}^i - \sigma_{t-1}^i}{\left(\frac{\sigma_{t+1}^i + \sigma_{t-1}^i}{2}\right)}. \quad (3.2)$$

Equation (3.1) (or some slight variation of it) is the one most commonly used in earlier literature, e.g., Black (1976). In our empirical work, and in the simulations, we have to estimate the volatility, since it cannot be observed directly. The measurement errors introduced thereby do not bias the estimates, as they will also be part of the error term $\tilde{\epsilon}^i$. They do, however, reduce the explanatory power.

To see how γ behaves when the firm's assets are options, we now turn to the standard option-pricing model. We assume that the underlying asset's price (or, more correctly in this setting, value) on the time interval $[0, T]$, $S(t)$, follows a geometric Brownian motion:

$$dS(t) = \mu S(t)dt + \sigma S(t)dw(t), \quad (3.3)$$

where $w(t)$ is a standard Brownian motion, and μ and σ are the instantaneous expected rate of return and standard deviation, respectively. It is worth stressing that this price process has a constant volatility of the returns, i.e., the γ of equation (3.1) is zero. The asset does not pay any dividends, and there is a risk-free security that yields an instantaneous rate of return of r . Using Itô's lemma, we can calculate the standard deviation of the rate of return of a call option, with an exercise price of X , written on the asset:

$$\sigma_C(S(t), t) = \frac{\sigma C'_S(S(t), t)S(t)}{C(S(t), t)}, \quad (3.4)$$

where $C(S(t), t)$ is the price of the call option at time t with an underlying asset price of $S(t)$. Straightforward calculations show that:

$$\frac{d\sigma_C(S(t), t)}{\sigma_C(S(t), t)} = \frac{\partial}{\partial S} \left(\frac{C'_S(S(t), t)S(t)}{C(S(t), t)} \right) \frac{C(S(t), t)}{C'_S(S(t), t)} \frac{dS(t)}{S(t)}, \quad (3.5)$$

from which we can identify the parameter γ from equation (3.1). As $C(S(t), t)$ can be expressed in closed form (the Black-Scholes formula; see Black-Scholes (1973)),

we can explicitly calculate γ as a function of $S(t)$, t , and the parameters σ , r , and X :

$$\gamma(S(t), t) = \frac{\partial}{\partial S} \left(\frac{C'_S(S(t), t)S(t)}{C(S(t), t)} \right) \frac{C(S(t), t)}{C'_S(S(t), t)} \quad (3.6)$$

$$= 1 + \frac{N'(d_1)}{N(d_1)\sigma\sqrt{T-t}} - \frac{S(t)N(d_1)}{C(S(t), t)}, \quad (3.7)$$

where $N(\cdot)$ is the cumulative distribution function (CDF) of a standard Gaussian distribution, and:

$$C(S(t), t) = S(t)N(d_1) - Xe^{-r(T-t)}N(d_2), \quad (3.8)$$

$$d_1 = \frac{\ln(S(t)/X) + r(T-t) + \frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}, \quad (3.9)$$

$$d_2 = d_1 - \sigma\sqrt{T-t}. \quad (3.10)$$

Finally, we define the option's time to expiration as $\tau = T - t$.

The above result provides a convenient way of exploring the characteristics of γ as a function of the asset price as well as the other underlying parameters. By using L'Hospital's rule, we find that:

$$\lim_{S \rightarrow \infty} \gamma(S) = \lim_{S \rightarrow 0} \gamma(S) = 0. \quad (3.11)$$

The first part is intuitive. As a call option gets deeper in the money, it behaves increasingly like the underlying asset, and γ is zero for this asset by construction. To show what happens in between these extremes, we have plotted $\gamma(S)$ for some different values of σ , the volatility, in Figure 3.1. (All figures and tables are found in Section 3.7.) In the part of the graph with a positive slope, which corresponds to the option's being *at* or *in* the money, the option returns behave like a levered position in the underlying asset—something we will return to below. As we will estimate the magnitude of γ rather than its slope, it does not matter for our purpose in which part of the graph we really are. It is interesting to see that the slope around the exercise price gets steeper as the volatility goes down, and that the curves never cross. We also note that γ can take on values less than minus one. In other words, the fractional change in volatility is larger when the overall volatility is lower.

In Figure 3.2, we show the analogous graph for different expiration dates. The effect becomes more pronounced the closer to the expiration date we are. The

overall shape in both these graphs can be understood intuitively by looking at the usual pay-off function of a call option (as a function of the asset price). In Figure 3.3, the pay-off on the maturity date is shown along with option values for different volatility levels. As the volatility goes down, the option price displays more and more of the typical kink in the pay-off function at maturity. The same effect could be illustrated by a graph showing how the option-price function changes as the time to maturity approaches zero. We see that the trough in Figures 3.1 and 3.2 becomes increasingly deep as the option-price function approaches this kinked shape. This is intuitive, since an option differs most radically from the underlying asset when it is soon to expire and it is at the money: at that point a small change, up or down, has a relatively small effect on the value of the underlying asset, but a large one on the option. When the expiration date lies farther in the future, the difference is much less striking. Similarly, a change in the underlying asset's price has a larger effect on the option price when the volatility is low and the change can be expected to be more persistent. Changes in the risk-free rate, r , have virtually no effect on γ .

One previously proposed explanation for negative γ 's is leverage (see, e.g., Black (1976) and Christie (1982)), although it does not seem sufficiently large to be the sole reason. To show how leverage could generate negative γ 's, we assume that the rate of return on the firm's total assets has a constant standard deviation (volatility), σ_A , and D is the fixed amount of debt the firm has outstanding. For simplicity, we treat D as if it were risk free. Let E denote the value of the firm's equity and $\sigma_E(E)$ the standard deviation of the equity returns. We then have:

$$\sigma_E(E) = \sigma_A \frac{E + D}{E}, \quad (3.12)$$

and:

$$\frac{d\sigma_E(E)}{\sigma_E(E)} = -\frac{D}{E + D} \frac{dE}{E}, \quad (3.13)$$

which shows that for constant asset volatility and a fixed amount of debt, D :

$$\gamma(E) = -\frac{D}{E + D}. \quad (3.14)$$

As a function of leverage ($D/(E + D)$), γ is obviously just a straight line, and in Figure 3.4 we show $\gamma(E)$ for a few different values of D . It is also immediately apparent that:

$$\lim_{E \rightarrow \infty} \gamma(E) = 0; \quad \lim_{E \rightarrow 0} \gamma(E) = -1. \quad (3.15)$$

Our main conclusion so far is that the elasticity of the volatility with respect to returns, γ , behaves very similarly for an at-the-money call option and a levered position of an asset with constant volatility. In addition, option returns do not display any lower bound of minus one for γ , as a pure leverage effect would indicate.

Before turning to our empirical analysis, we want to establish a sense of the statistical problems involved in estimating γ for option returns with the ordinary-least-squares procedure. The specification is the one in equation (3.1), where we make the additional assumption that γ is a constant. As we know from the analysis above, this is not strictly true. However, as long as the sampling period is short relative to the expiration date and the price movements are fairly small, it should be a reasonable approximation. We model the underlying asset's price movements with a binomial tree and price the options using the implied risk-neutral probabilities.³

We believe that most of a firm's growth opportunities lie at least a couple of years in the future, whereas we are forced to work with daily data to get a sufficient number of observations. In our simulations, the daily returns are sampled over a two-month period, and the volatility estimate and summed returns, $\hat{\sigma}_t$ and r_t , are subsequently calculated for each ten-day period. Every two-month period gives us four observations of $\hat{\sigma}_t$ and r_t , from which we can construct two observations consisting of $(\ln \hat{\sigma}_{t+1} - \ln \hat{\sigma}_{t-1})$ and r_t . This procedure is then repeated until the desired sample size is achieved. The $\hat{\gamma}$ is calculated by regressing $(\ln \hat{\sigma}_{t+1} - \ln \hat{\sigma}_{t-1})$ on r_t . Because the sample period is kept so short, we believe that the assumption of a constant γ is appropriate. To estimate the standard deviation of our estimate $\hat{\gamma}$, the above procedure is repeated many times (usually 1,000). The standard deviation of the estimate is then calculated as the sample standard deviation of $\hat{\gamma}$.

Tables 3.1 and 3.2 present these results for different sample sizes as well as different levels of the underlying asset price, S , i.e., different parts of the $\gamma(S)$ function as shown in Figures 3.1 and 3.2. In the first set of simulations the step sizes (u and d) are chosen to illustrate clearly the characteristic shape. The second set of simulations is performed with parameter values as realistic as possible, i.e., standard deviation $\approx 22\%$, risk-free rate $\approx 5\%$, and risk premium $\approx 8.5\%$ in annual terms. In this case, the trough is somewhat less pronounced but still quite distinct.

These simulations illustrate a few important points. First, we can see that the overall shape of γ , as a function of S , corresponds to the one established in

³For an extensive treatment of this technique, see Cox-Rubinstein (1985), and Cox-Ross-Rubinstein (1979).

the continuous-time framework above.⁴ Second, the estimation errors tend to be worse for higher values of S . Most importantly, all estimation errors are large in small samples. With a sample size of 25, the standard deviation of the estimates is roughly the same size as the coefficient, γ , itself. This means that the estimated γ for an individual company would almost never be significantly different from zero in a statistical test. Not even with a sample size of 1,000 are all γ 's more than two standard deviations from zero. These simulations illustrate the properties of a single call option. This would correspond to the extreme case of a firm's assets being a single option. For the vast majority of companies, the assets consist of both options and "regular" assets without option features. The firm's γ is then a function of the γ 's of the different pieces. Obviously, the size of γ will depend on both the properties of its real options and the amount of other assets. Thus the simulations underestimate the problems in estimating γ 's for real firms.

From a practical point of view, there is not very much that can be done to correct this. We are dealing with a limited number of observations (roughly 250 per year), and the sample period cannot be too long if γ is assumed to be constant. (If we calculate the $\hat{\sigma}_t$'s over ten-day periods and use a one-year sample period, we have sample sizes of approximately 25.) It does mean, however, that we have to work with as large a cross-section of firms as possible, since an individual firm's estimated γ , $\hat{\gamma}$, will contain large measurement errors. Alternatively, the γ 's can be estimated for groups of firms that are formed according to *a priori* presumed size of γ . Under the null hypothesis that all firms in a group have roughly the same γ , the estimation errors in $\hat{\gamma}$'s will be much smaller thanks to the larger sample size.

Our objective is to empirically link the observed γ 's to the presence of assets of option type, in particular research and development activities, and this is the topic to which we now turn.

3.3 The Data

To estimate equation (3.1), we use the daily return data from CRSP. We try to capture the presence of "options" in a firm's assets by using a number of prox-

⁴We have in fact tried this simulation procedure for a wide variety of parameter values, all generating results analogous to the ones presented in Figures 3.1 and 3.2; as the continuous-time results are untainted by estimation errors and contain all the theoretical insights, these additional simulation results are not included in this paper.

ies which are constructed with data from COMPUSTAT, including all firms in a given year for which both the CRSP and COMPUSTAT data are available. The study is based on the five-year period 1988–1992. This procedure leaves us with approximately 600–700 firms per year. The annual Treasury-bill and long-term corporate-bond rates are from the 1993 Ibbotson yearbook.

As we outlined above, options can arise in many different ways. A company may simply be in an industry that is rapidly expanding, providing immense opportunities if the right product or service is found and the market develops in a favorable way. In this case, the option stems from possible future growth and not necessarily from specific projects initiated by the firm. Examples might be cellular phones and portable computers today, or video rentals ten years ago. Another example is the option intrinsic to any major research and development (R&D) project. When the R&D project is launched, its ultimate outcome and economic significance are rarely known. If it is successful, it can be the first step in a long chain of investments and follow-on projects. Few observers could have anticipated the avalanche of products and services made possible by the discovery of plastic or the semiconductor. Even when the success is less conspicuous, there will be important sequential decisions regarding production, marketing, and refinement of the product or service. On the other hand, the project may lead nowhere and be discontinued shortly after its inception. A project with these characteristics is most appropriately modeled as an option.

We hope to capture the pure R&D options by using a firm's R&D expenses, normalized by its sales. This proxy is denoted RD. The distinction between R&D and other investments may be somewhat ambiguous. To allow for this possibility, capital expenditures (normalized by sales), CAPX, is also used as a proxy for real options. The objective is to find a proxy for the present value of the options created by a firm's R&D activities, both now and in the past. As long as R&D expenditures are at steady-state level, this proxy works fairly well. If the R&D expenditures are very high relative to past levels, the proxy would tend to overestimate the value of all options created by R&D. Similarly, RD will underestimate the total option value if R&D expenditures has been recently cut. To reduce this problem, we use an average of the last two years' R&D expenditures. We have found that proxies of R&D and of capital expenditures normalized by total assets, rather than sales, yield virtually identical results; thus these results are not included in the paper.

Options that are not created by R&D are much harder to link to specific accounting numbers. Instead, we use the firm's price-earnings and market-to-book ratios as they measure the difference between the earning power and asset value

today and the market's assessment for the future. Both these proxies are widely used in the academic literature as well as by practitioners; see, e.g., Fama-French (1992) and Standard & Poor's Stock Guide (1993). These are denoted PE and MB, respectively. In the same spirit, we use the text-book valuation of the present value of future growth opportunities (PVGO):

$$PVGO = P - \frac{EPS}{r}, \quad (3.16)$$

where P is the stock price, EPS is the present earnings per share, and r is the appropriate expected rate of return for the firm's risk characteristics. In other words, this measures the difference between the present value of the present level of earnings in perpetuity and the stock price. If this difference is large, the market expects the firm's earnings to grow substantially in the future. The discount rate, r , is calculated from the CAPM, where we use the β 's from COMPUSTAT,⁵ the risk-free rate from the Ibbotson yearbook (the T-bill rate), and an expected risk premium of 8.5% (a long-term historical average). Finally, we normalize it by dividing by the stock price and subtracting one. This proxy is denoted PVGO.

All these proxies have severe flaws. If present earnings are temporarily inflated, PE and PVGO will tend to underestimate the growth options. The opposite is true if a firm has a particularly bad year. As PE and PVGO are clearly inappropriate when a firm is losing money, we will use these proxies only for profitable companies. This subsample is simply referred to as "profitable companies." MB is directly tied to accounting issues. (What can be treated as an expense, and what is considered investments that have to be depreciated over many years? Is the difference between historical cost and replacement value small or large?) As accounting rules affect industries and individual companies differently, MB can at best serve as a rough approximation of future growth opportunities. To mitigate some of these problems, we use the average of the values at the beginning and the end of the year.

We will also use leverage as an independent variable, so we introduce LEV to denote the ratio of debt to debt-plus-equity. Equity is valued at the market price, whereas the book values are used for debt. Because we are interested in the average leverage during a given year, LEV is also calculated as the average of the value at the beginning and the end of the year.

The γ 's are estimated using the specification of equation (3.1) and the ordinary-least-squares procedure, with estimation periods of ten and twenty days. These

⁵These β 's are calculated for monthly data over the most recent five-year period. The S&P 500 stock index is used as proxy for the market.

estimates are denoted $\hat{\gamma}^{10}$ and $\hat{\gamma}^{20}$, respectively. All variables are listed and defined in Table 3.3. Tables 3.4 and 3.5 present some sample statistics for the full sample, as well as for profitable companies. Worth noting is the large variability in the estimates of γ , and a few extreme price-earnings and R&D-to-sales ratios. Table 3.6 gives a year-by-year listing of $\hat{\gamma}$'s for the full sample. Most of the average $\hat{\gamma}$'s are negative, but the variation from year to year is considerable. This is partly driven by a few extreme outliers. We also believe that it illustrates the measurement errors analyzed in Section 3.2.

It may seem plausible that some of these option proxies are highly correlated. The sample correlation coefficients are presented in Tables 3.7 and 3.8. It is noteworthy that the pairwise correlation coefficients between PE, MB, and PVGO, respectively, are only in the 0.12–0.21 range, despite the fact that they all try to capture a similar concept. We think this illustrates the measurement problems associated with the proxies discussed above. The signs of the coefficients, however, are all what one might expect. That less profitable (or unprofitable) firms tend to have higher leverage is a well-established empirical fact, which would explain the negative correlation between LEV on one hand, and MB, PE, and PVGO on the other. It is also a plausible rationale for the negative correlation between LEV and RD, since less profitable companies are likely to reduce their R&D expenditures. Finally, it is possible that companies with high R&D expenditures choose to operate with less debt because of higher costs of financial distress. These costs are usually thought to be more of a problem for firms with largely intangible assets. Similar arguments apply to the correlation between LEV and CAPX.

Let us now proceed to a cross-sectional analysis of the parameter γ , the elasticity of stock return volatility with respect to the stock price.

3.4 Cross-Sectional Analysis of the Elasticity γ

Our objective is to explain a firm's γ in terms of its underlying assets. As shown in Section 3.2, call options have negative γ 's even if the underlying asset does not. The presence of debt also causes stock returns to display negative γ 's. If the option proxies described above really capture the amount of real options in a company's assets, we would expect a negative relation between the $\hat{\gamma}$'s and these proxies. Similarly, leverage ought to be negatively related to $\hat{\gamma}$.

We saw in Section 3.2 that a firm's γ will not be constant over time. As the stock price changes, so will the γ . The γ is also affected whenever the value of

the options changes or new debt is raised. We are therefore faced with a trade-off between sample size and the assumption that γ is constant. We believe that one year is a reasonable compromise.⁶ A plausible explanation is that the true γ 's move around enough over five years to make the $\hat{\gamma}$'s meaningless.

To ensure robustness, we approach this problem in several ways. First, we regress the $\hat{\gamma}$'s estimated from stock returns on leverage and the option proxies. This is done both for the full sample and for the profitable firms alone. Second, we divide the firms into groups based on their R&D expenditures. The γ 's are then estimated group by group. We expect these groupwise $\hat{\gamma}$'s to decrease with growing R&D. Third, a leverage-adjusted return series is constructed for each company. These returns correspond to the returns on the company's total assets. If leverage were the sole reason for the observed negative $\hat{\gamma}$'s in stock returns, this procedure would generate a return series with a zero γ .

3.4.1 Equity Returns for Individual Firms

The first step is to estimate the γ 's for each firm year by year. In doing this we use estimation periods for the volatility of ten and twenty days. These $\hat{\gamma}$'s are then regressed on leverage, market-to-book value, R&D, and capital expenditures; i.e., for firm i , $i = 1, \dots, N$, in a given year, we assume that:

$$\hat{\gamma}^i = \theta^{CONST} + \theta^{LEV} \cdot LEV^i + \theta^{RD} \cdot RD^i + \theta^{CAPX} \cdot CAPX^i + \theta^{MB} \cdot MB^i + \tilde{\epsilon}^i, \quad (3.17)$$

where the θ 's are constants and $\tilde{\epsilon}^i$ is a stochastic variable uncorrelated across firms and different years. It has zero mean. We treat each $\hat{\gamma}$ as a separate observation, i.e., most firms will appear five times in the sample, once for each year. It seems most likely that a firm's γ will be correlated from one year to the other. This does not pose any problem, however, as our econometric specification states only that the measurement errors are not correlated across time, an assumption we find quite plausible. The coefficients are estimated using the ordinary-least-squares (OLS) procedure, and the standard errors of the estimated coefficients are calculated allowing for heteroscedasticity by using White's procedure.⁷ Regardless of the true covariance structure of $\tilde{\epsilon}^i$, the estimates will be unbiased. The standard errors, however, could be misleading. Thus we will pay attention to the absolute

⁶In fact, we also used five-year periods but without much success. These results are not included in the paper.

⁷See, e.g., White (1980).

size of the estimates and not just to the t -statistics, although we do not see any reason for the covariance matrix not to be diagonal.

For each of the independent variables there are some outliers in the upper end of the distribution. To reduce the risk of having the results driven by a few extreme observations, quite possibly tainted by human errors in the reporting process, an observation is eliminated from the sample whenever any of the right-hand side variables belong to the top 1% of the sample. (No observation is deleted because of the left-hand side variable, since this could cause biased estimates and inflated t -statistics.) All regressions are run for both $\hat{\gamma}^{10}$ and $\hat{\gamma}^{20}$.

Table 3.9 shows the estimates and the corresponding t -statistics. A negative coefficient means that an increase in that variable tends to make $\hat{\gamma}$ smaller, i.e., on average more negative. All the option proxies are defined so that a larger value is assumed to correspond to a larger fraction of options among the company's assets. The coefficients on LEV for the two different estimation periods are -0.535 and -0.320 with corresponding t -statistics of -2.67 and -1.72 . The coefficients are thus significantly different from zero at the 1% and 10% levels, respectively. As expected, this shows that more highly levered firms tend to have more negative γ 's. The RD coefficients are -0.711 and -0.848 with t -statistics of -1.38 and -1.74 , respectively. They have the expected sign, but only the coefficient for the 20-day estimation period is significant at the 10% level. The absolute magnitudes of the RD coefficients are larger than the ones on LEV, suggesting that a one-percent change in R&D expenditures has a larger impact on γ than a change of equal size in leverage. The coefficients for the remaining option proxies, CAPX and MB, are both insignificant.

In using the earnings-based option proxies, only the subsample of profitable firms is considered. The above specification is augmented in the obvious way and estimated:

$$\hat{\gamma}^i = \theta^{CONST} + \theta^{LEV} \cdot LEV^i + \theta^{RD} \cdot RD^i + \theta^{CAPX} \cdot CAPX^i + \theta^{MB} \cdot MB^i + \theta^{PE} \cdot PE^i + \theta^{PVGO} \cdot PVGO^i + \tilde{\epsilon}^i. \quad (3.18)$$

In Table 3.10, the regression results show that the coefficients on LEV are -0.793 and -0.455 with t -statistics of -2.58 and -1.63 . Thus only the coefficient for the 10-day sampling period is significant (at the 1% level). The coefficients are of roughly the same size as in the regressions for the full sample. From the theoretical treatment in Section 3.2, we expected a coefficient around minus one. Given that leverage is known to be measured with some error, a problem which tends to bias

the absolute size of the coefficients downwards, the coefficients seem very close to values suggested by the theory.

The RD coefficients are now significant at the 5% level in both regressions. Their absolute sizes, -2.495 and -2.482 , are also distinctly larger than those for the full sample. PVGO enters with the expected sign, -5.078 and -1.066 , respectively. It is only significant for the 10-day estimation period. The other proxies are all insignificant, except CAPX which is significant at the 10% level for the 20-day estimation period. It has a positive sign, and the most plausible explanation would seem to be that it captures expenditures with no option features, thus tending to increase γ . As profitable firms generally have lower leverage, it is not surprising that leverage might play somewhat less of a role in this subsample. We also believe that RD might be a better proxy for the option content in the asset mix when a company is profitable, as companies making losses tend to cut back on (at the very least) new R&D projects. Such behavior would make the present R&D expenditures less revealing about the total value of options embedded in present and past R&D projects. Because of this, we focus on profitable firms for the next test.

3.4.2 Equity Returns for Groups of Firms

To increase the sample size in our γ estimates, the firms are separated into five groups according to their R&D expenditures. One group consists of firms with no R&D, and the rest are divided into four groups of equal size, out of which the first quartile represents the group with the least (non-zero) R&D. Even though all four non-zero quartile groups have the same number of firms, the number of missing stock returns is not identical. Therefore, the sample size varies somewhat across subsamples. For the 10-day estimation period, the sample size is approximately 10,000 per group, and for the 20-day estimates, around 5,000.

The results in Table 3.11 display a clear pattern: $|\hat{\gamma}|$ tends to increase with R&D except, notably, in the zero-R&D group. For the 10-day estimation period, the coefficients are -0.385 , -0.202 , -0.282 , -0.308 , and -0.386 in order of growing R&D expenditures. The corresponding numbers for the 20-day estimation period are -0.580 , -0.306 , -0.450 , -0.378 , and -0.456 . All the $\hat{\gamma}$'s are significantly different from zero; all but one are at the 1% level. The $\hat{\gamma}$'s for the zero-R&D firms are of a size roughly equivalent that of the 4th quartile, i.e., the group with the highest R&D expenditures. Except for the zero-R&D firms, these results confirm the findings in our first test, i.e., that R&D and γ are negatively related. As the

zero-R&D firms seem different from other companies, we redid the tests in the previous section without them. The regression coefficients and t -statistics were virtually identical, so they are not included in the paper.

But why do the zero-R&D firms have such negative γ 's? We have carefully examined the zero-R&D group and found only one truly distinguishing characteristic: industry. By looking at the SIC codes,⁸ we found that most of these companies were in retail trade or in finance, insurance, and real estate. (In fact, the vast majority of all the firms in these categories reported no R&D expenditures.) It is hardly surprising that most restaurant chains and insurance companies have no R&D expenses, and by definition R&D does not contain any information pertaining to options among the assets for such firms. It is striking, however, that these industries have a stock-price behavior similar to that of the firms with the highest R&D levels.

We can think of two possible explanations. First, these industries may simply happen to have plenty of other assets with option features. Attempts to explain the $\hat{\gamma}$'s within this group using the other proxies do not provide answers, largely because of the limited sample size. It also seems likely that some of the proxies may be especially poor for these industries. Second, companies in these industries may be more highly levered. This is true (average leverage of the zero-R&D firms is 0.28 compared to 0.23 for the full sample of profitable firms), although the estimated impact of leverage, as shown in Table 3.10, seems to indicate that differences in leverage can explain only part of the difference, not its full magnitude. In short, we believe that these industries both have a lot of assets with option features and are somewhat more levered, and that these two reasons combined explain the full effect. Unfortunately, the limited sample size does not allow any conclusive answers.

3.4.3 Returns on Total Assets for Individual Firms

Although our main interest in this paper is R&D, the above results show the need to more carefully disentangle leverage from the option effect. To adjust for leverage, we construct "asset returns." The returns on a firm's assets, r_A , can be expressed as:

⁸Standard Industrial Classification codes; see the Standard Industrial Classification Manual (1987).

$$r_A = \frac{D}{D+E}r_D + \frac{E}{E+D}r_E, \quad (3.19)$$

where r_D and r_E are returns on debt and equity, respectively. Every day, the leverage is recalculated based on the previous day's stock returns. If the γ 's were driven solely by leverage, the resulting asset returns would have constant volatility. Ideally, we would like to obtain the daily returns on the firm's outstanding debt. Since this is impossible, we treat r_D as deterministic, and use the long-term corporate-bond rates from the Ibbotson series. As long as the firm's leverage is not too high, this approximation seems reasonable, as most of the volatility in that case stems from the stock returns and changes in leverage.

When the leverage is high, however, the argument is no longer valid. First, since most of the weight is on the debt, its volatility will have a large impact on asset returns, even if it is much smaller than the stock's volatility. Second, when leverage goes up, so does the riskiness of the debt. This approximation will therefore overcompensate for highly levered firms. With this in mind, we can turn to the main question: Will the γ 's for these leverage-corrected asset returns be explained by the option proxies?

First, we re-estimate the γ 's in equation (3.1) using the new series of asset returns. These estimates are denoted $\hat{\gamma}_A^{10}$ and $\hat{\gamma}_A^{20}$ for 10- and 20-day estimation periods, respectively. Having eliminated the observations with right-hand side variables in the top 1%, we run the regressions analogous to equation (3.17) for the full sample of firms:

$$\hat{\gamma}_A^i = \theta^{CONST} + \theta^{LEV} \cdot LEV^i + \theta^{RD} \cdot RD^i + \theta^{CAPX} \cdot CAPX^i + \theta^{MB} \cdot MB^i + \epsilon^i. \quad (3.20)$$

The results from these regressions are reported in Table 3.12. As expected, the θ^{LEV} 's are now significantly positive: 1.143 and 0.925 with corresponding t -statistics of 2.05 and 1.92. The overcompensation in the construction of asset returns is thus significant. According to the argument above, this is of major importance only to companies with high leverage. To verify that, we re-estimate the coefficients after eliminating the top 5%, rather than the usual 1%. These regressions are also found in Table 3.12. We do find that the LEV coefficient is no longer significantly different from zero. As for equity returns in the full sample, the coefficients on RD are not significant (though they do have the right sign). They are also of approximately the same magnitude as those for equity returns, i.e., about -0.8 . None of the other variables is significant.

Next, we focus on the profitable firms and run the regressions analogous to equation (3.18), but this time for asset returns:

$$\hat{\gamma}_A^i = \theta^{CONST} + \theta^{LEV} \cdot LEV^i + \theta^{RD} \cdot RD^i + \theta^{CAPX} \cdot CAPX^i + \theta^{MB} \cdot MB^i + \theta^{PE} \cdot PE^i + \theta^{PVGO} \cdot PVGO^i + \tilde{\epsilon}^i. \quad (3.21)$$

Table 3.13 presents these regressions. As we can see, there is no need in this case to eliminate more than 1% of highly levered firms; this is not surprising, since some of the very highly levered companies are taken out of the sample anyway because they are unprofitable. Having performed this specification test, we re-estimate the coefficients excluding LEV, as leverage is already adjusted for in the estimation of γ_A . These regressions are also found in Table 3.13. Below, we focus on the regressions that do not include LEV.

The RD coefficients are -3.165 and -3.789 with t -statistics of -2.14 and -2.66 corresponding to a significance level of 5% and 1%, respectively. Clearly, the option effect of R&D persists after the leverage correction. In fact, the absolute magnitude of these coefficients is larger than that for the raw equity returns. For both equity and asset returns, the RD coefficients are in the range of (-2.4) to (-3.8) . We remember the theoretical lower bound of minus one for γ 's exclusively driven by leverage. From our empirical results, we can say that the option effect from R&D is at least twice as large, in the sense that a one-percent change in R&D expenditures has twice the effect on γ as a similar change in leverage.

In addition to θ^{RD} , θ^{PVGO} is significant at the 10% level for the 10-day estimation period. The magnitude of the coefficient is large, -7.624 , but so is the standard error. The other variables all have insignificant regression coefficients. These results indicate that option-like assets do have a pervasive impact on stock returns that is quite distinct from that of leverage. R&D seems to be the best proxy for these options, and PVGO also seems to have some explanatory power.

It is noteworthy that PE and MB do *not* seem to capture options embedded among the assets. CAPX was included, as we thought the boundary between R&D and capital expenditures was somewhat arbitrary. The empirical results, however, do not seem to link capital expenditures to the magnitude of γ in any robust way. In short, R&D seems to be the main source of real options for a company. Naturally, there may be other proxies that capture non-R&D related options much better than the ones investigated here. That is an issue remaining to be explored.

3.5 Concluding Remarks

We have made an attempt to explore the link between stock-return characteristics and firm-specific factors such as leverage and asset base, in order to better explain stock returns cross-sectionally. Our main focus has been the presence of real options in a firm's asset mix. The effect of these real options on the volatility elasticity with respect to stock price, γ , has been explored. A closed-form expression has been derived for γ of a call option in a continuous-time setting. We have used simulations to assess the estimation properties of γ in small samples. Our empirical results provide evidence that options *do* matter when explaining observed γ 's. The results seem robust across several specifications and they are not driven by leverage alone. In fact, the results seem even stronger when we first adjust for leverage.

Obviously, there may be other properties of returns that could also be better explained by looking at real options, and we hope that this paper may serve as motivation for further work in this area. One example is the skewness of the return distribution. Quite possibly skewness could be linked to the amount of assets with option characteristics. It would also be interesting to see whether an individual firm's returns change as the firm evolves, e.g., as an upstart biotech firm matures into a full-scale pharmaceutical company.

We believe that our results highlight the need to integrate corporate-finance issues with traditional capital markets theory, in order to gain a deeper understanding of the underlying forces that govern capital markets and the securities traded there.

3.6 References

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3.7 Figures and Tables

Figure 3.1: The Elasticity of Volatility with Respect to Returns, γ , as a Function of the Underlying Asset Price for Some Selected Volatilities

"1" — $\sigma=0.5$; "2" — $\sigma=0.4$; "3" — $\sigma=0.3$; "4" — $\sigma=0.2$; "5" — $\sigma=0.1$;
 $X=40$; $r=0.01$; $\tau=1.0$

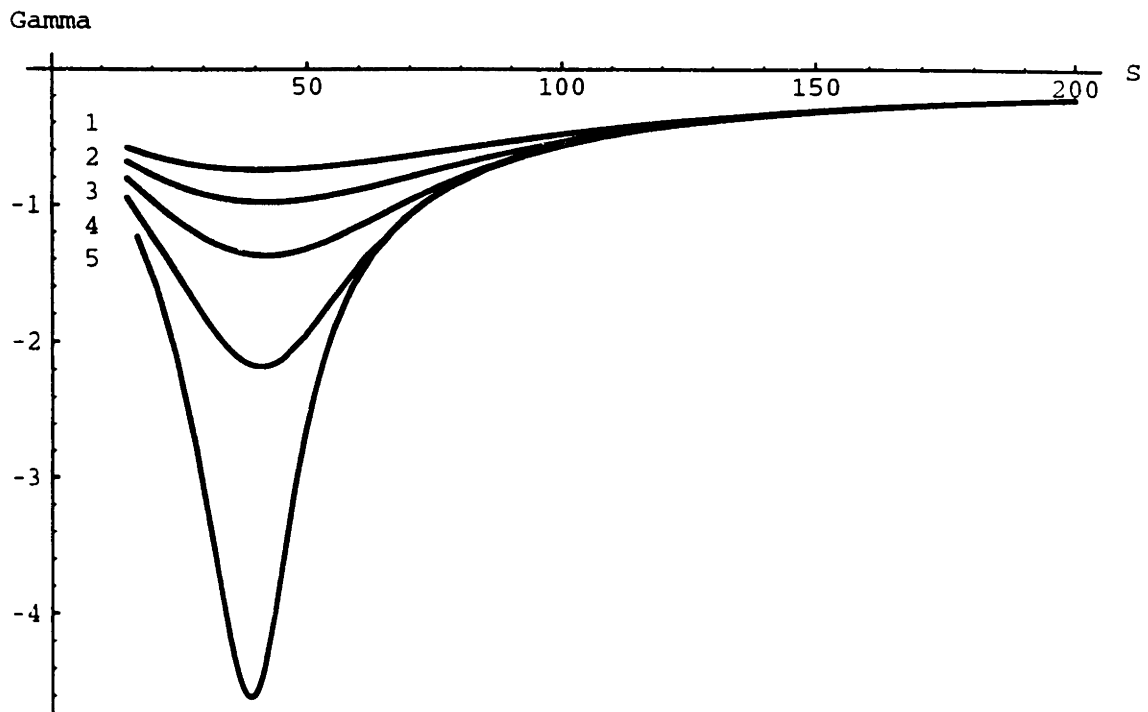


Figure 3.2: The Elasticity of Volatility with Respect to Returns, γ , as a Function of the Underlying Asset Price for Some Selected Expiration Dates

"1" — $\tau=10$; "2" — $\tau=7$; "3" — $\tau=5$; "4" — $\tau=4$; "5" — $\tau=3$; "6" — $\tau=2$; "7" — $\tau=1$;
 $X=40$; $r=0.01$; $\sigma=0.2$

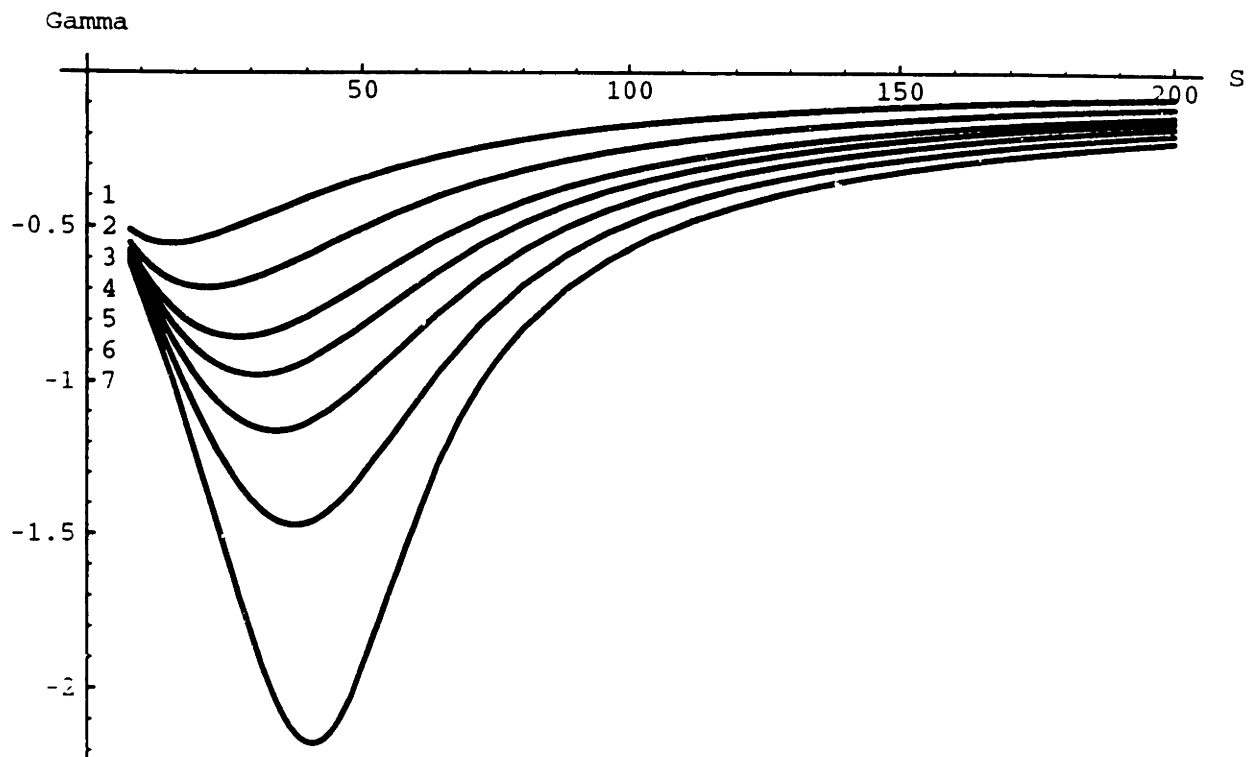


Figure 3.3: Option Price as a Function of Underlying Asset Price for Different Volatility Levels and at Maturity

$$X=40; r=0.01; \tau=1; \sigma=0.1 - 0.7$$

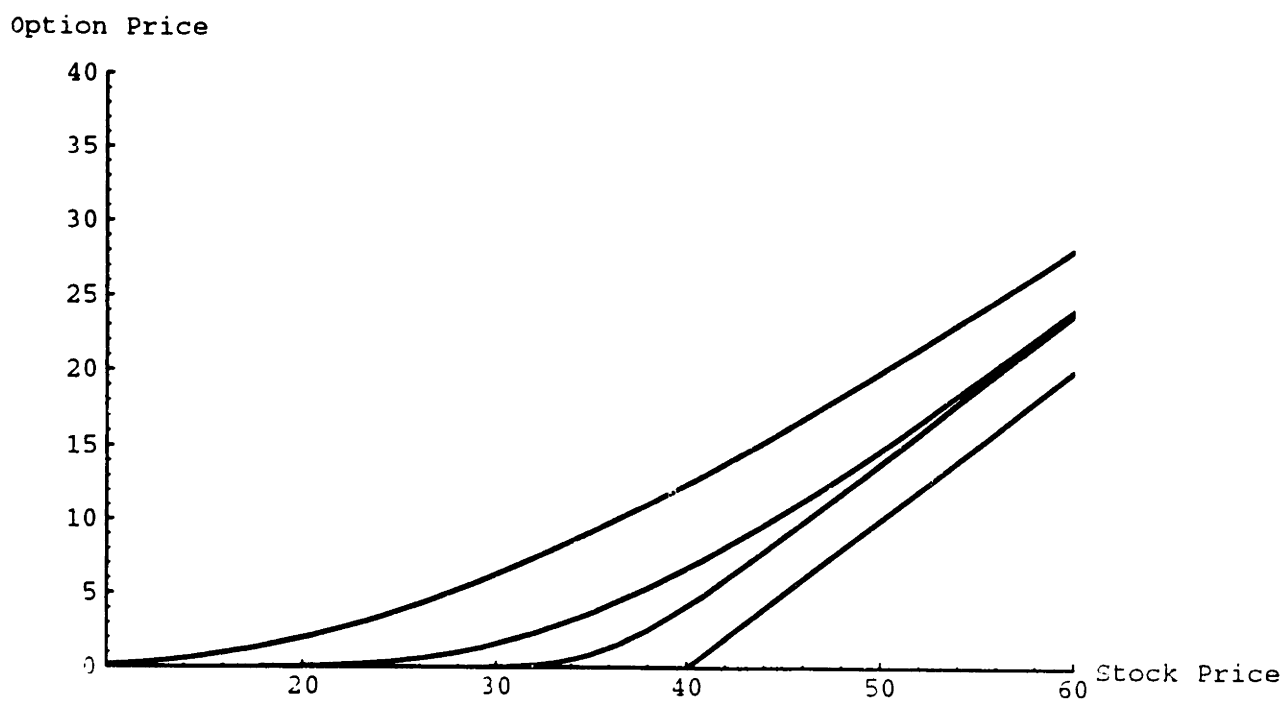


Figure 3.4: The Elasticity of Volatility with Respect to Returns, γ , as a Function of the Stock Price for Some Selected Debt Levels

"1" — $D=10$; "2" — $D=40$; "3" — $D=70$

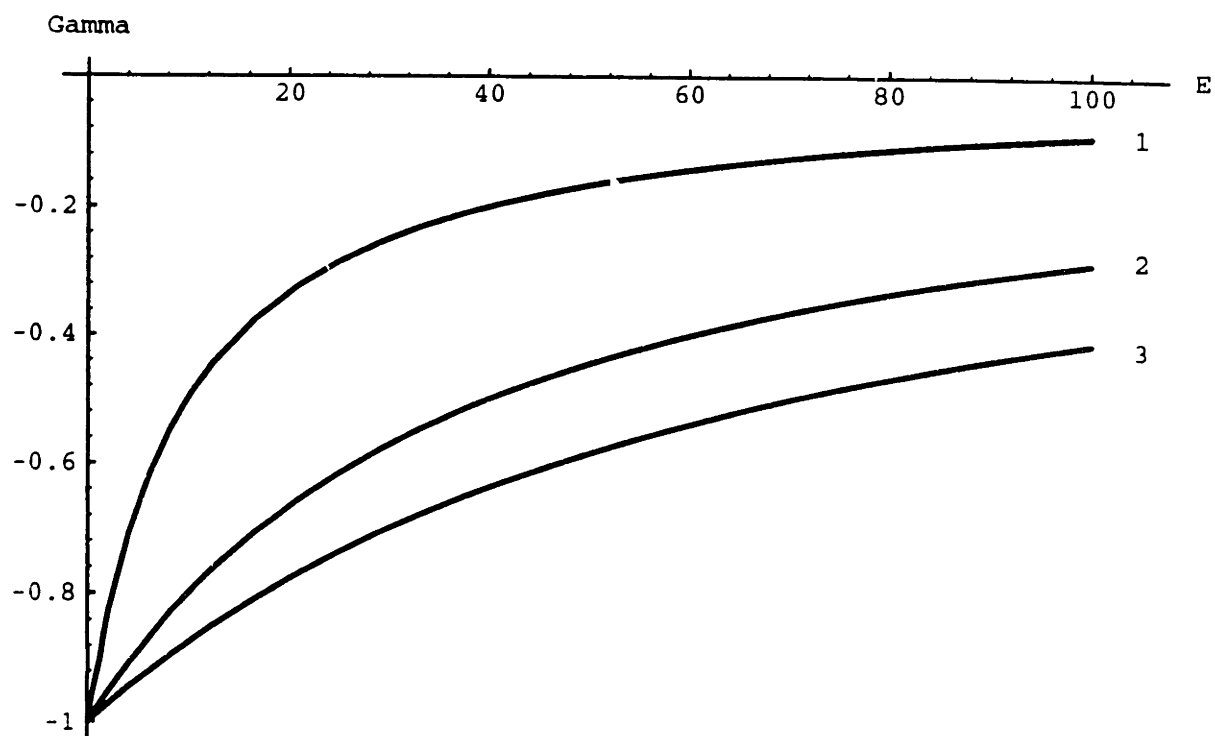


Table 3.1: Simulation 1; Standard Deviation of the γ Estimates, $\hat{\gamma}$, for Different Sample Sizes[†]

S	γ	Sample Size			
		25	100	1,000	10,000
20	-0.016	0.046	0.025	0.0081	0.0027
40	-0.045	0.073	0.040	0.013	0.0041
60	-0.11	0.11	0.060	0.020	0.0059
80	-0.23	0.18	0.087	0.030	0.0097
100	-0.40	0.26	0.15	0.046	0.014
120	-0.53	0.40	0.21	0.070	0.020
140	-0.57	0.56	0.27	0.091	0.027
200	-0.45	0.86	0.48	0.16	0.047
300	-0.31	1.24	0.64	0.23	0.068

[†] These simulations are based on options priced using a binomial tree with 500 periods, approximately equivalent to two years, each period being one trading day. The parameters used are: $u = 1.01$, $d = 0.9912$, $r = 0.0002$, and $X = 100$. In annual terms, they correspond to a risk premium of 8.5%, a standard deviation of 2.5%, and a risk-free rate of 5%.

Table 3.2: Simulation 2; Standard Deviation of the γ Estimates, $\hat{\gamma}$, for Different Sample Sizes[†]

S	γ	Sample Size			
		25	100	1,000	10,000
10	-0.051	0.076	0.040	0.013	0.0045
40	-0.13	0.13	0.069	0.022	0.0072
70	-0.19	0.16	0.088	0.030	0.0087
100	-0.23	0.20	0.098	0.034	0.011
140	-0.26	0.23	0.13	0.041	0.013
200	-0.27	0.29	0.15	0.051	0.015
300	-0.24	0.37	0.18	0.060	0.018
400	-0.20	0.38	0.21	0.071	0.021

[†] These simulations are based on options priced using a binomial tree with 500 periods, approximately equivalent to two years, each period being one trading day. The parameters used are: $u = 1.031$, $d = 0.971$, $r = 0.0002$, and $X = 100$. In annual terms, they correspond to a risk premium of 8.5%, a standard deviation of 22%, and a risk-free rate of 5%.

Table 3.3: List of Variables

RD:	Research and development expenditures over sales.
CAPX:	Capital expenditures over sales.
PVGO:	Present value of growth opportunities, defined as stock price minus a perpetual stream of earnings at the present level, all normalized by dividing with the current stock price and then subtracting one. The discount rate is calculated from the CAPM.
MB:	The ratio of the firm's market value of equity to the book value.
PE:	The price-earnings ratio.
LEV:	The ratio of debt to debt plus equity. The equity is valued at the market price, and the debt at the book value.

All the above variables are calculated as the average of their values at the beginning and the end of the year.

Table 3.4: Sample Statistics

	$\hat{\gamma}^{10}$	$\hat{\gamma}^{20}$	RD
Sample Size	3,287	3,287	3,514
Max	18.75	16.10	276.98
Min	-20.37	-11.97	0.00
Mean	-0.294	-0.418	0.144
Std. Dev.	2.30	2.17	4.69
	CAPX	MB	LEV
Sample Size	3,514	3,526	3,526
Max	60.43	126.92	0.98
Min	0.00	0.034	0.00
Mean	0.080	2.43	0.273
Std. Dev.	1.02	4.16	0.22

Table 3.5: Sample Statistics for Profitable Firms

	$\hat{\gamma}^{10}$	$\hat{\gamma}^{20}$	RD	CAPX
Sample Size	2,367	2,334	2,528	2,528
Max	12.29	16.10	0.66	0.73
Min	-20.37	-11.97	0.00	0.00
Mean	-0.241	-0.375	0.028	0.057
Std. Dev.	2.45	2.31	0.045	0.057
	MB	LEV	PVGO	PE
Sample Size	2,528	2,528	2,528	2,518
Max	45.82	0.98	-0.0008	2487.5
Min	0.04	0.00	-1.56	0.58
Mean	2.305	0.234	-0.067	33.08
Std. Dev.	2.14	0.197	0.056	93.57

Table 3.6: Estimates of γ Year by Year

$\hat{\gamma}^{10}$:	1988	1989	1990	1991	1992
Sample Size	578	606	667	686	750
Max	12.29	18.75	9.35	9.83	8.54
Min	-20.37	-11.19	-10.76	-8.85	-10.37
Mean	0.05	0.30	-1.01	-0.19	-0.49
Std. Dev.	2.55	2.69	1.93	1.92	2.19
$\hat{\gamma}^{20}$:	1988	1989	1990	1991	1992
Sample Size	564	601	658	682	740
Max	9.70	11.43	13.09	16.10	8.45
Min	-11.97	-9.88	-10.28	-7.95	-9.49
Mean	-0.33	0.01	-0.88	-0.33	-0.50
Std. Dev.	2.19	2.71	1.95	1.90	2.00

Table 3.7: Sample Correlation Coefficients[†]

	RD	CAPX	MB	LEV
RD	1.00	0.19	0.30	-0.24
CAPX		1.00	0.11	-0.09
MB			1.00	-0.27
LEV				1.00

Table 3.8: Sample Correlation Coefficients for Profitable Firms^{††}

	RD	CAPX	MB	LEV	PVGO	PE
RD	1.00	0.25	0.23	-0.24	0.14	0.07
CAPX		1.00	0.14	-0.10	0.13	0.01
MB			1.00	-0.29	0.21	0.12
LEV				1.00	-0.23	-0.01
PVGO					1.00	0.18
PE						1.00

[†] These calculations were based on 3,459 observations. One extreme outlier, MB>100, was eliminated.

^{††} These calculations were based on 2,514 observations. One extreme outlier, PE>2000, was eliminated.

Table 3.9: Regression of Estimated γ 's on Leverage and Proxies for Asset Type[†]

Independent Variables	Dependent Variables	
	$\hat{\gamma}^{10}$	$\hat{\gamma}^{20}$
Constant	-0.164 (-1.49)	-0.379*** (-3.78)
LEV	-0.535*** (-2.67)	-0.320* (-1.72)
RD	-0.711 (-1.38)	-0.848* (-1.74)
CAPX	-0.344 (-0.42)	0.915 (1.18)
MB	0.03 (0.98)	0.02 (0.66)
Sample Size	3,188	3,150

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 3.10: Regression of Estimated γ 's on Leverage and Proxies for Asset Type for Profitable Firms[†]

Independent Variables	Dependent Variables	
	$\hat{\gamma}^{10}$	$\hat{\gamma}^{20}$
Constant	-0.396 (-1.49)	-0.364 (-1.55)
LEV	-0.793*** (-2.58)	-0.455 (-1.63)
RD	-2.495** (-2.11)	-2.482** (-2.18)
CAPX	0.573 (0.54)	1.742* (1.68)
MB	0.022 (0.56)	-0.008 (-0.24)
PE	0.000 (0.00)	0.001 (-0.41)
PVGO	-5.078* (-1.77)	-1.066 (-0.51)
Sample Size	2,275	2,242

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 3.11: Estimated γ 's for Different Subsamples Ranked According to R&D Expenditures[†]

R&D Expenditures	$\hat{\gamma}^{10}$	Sample Size	$\hat{\gamma}^{20}$	Sample Size
None	-0.385*** (-5.54)	13,432	-0.580*** (-9.76)	5,840
1st quartile	-0.202** (-2.16)	10,226	-0.306*** (-3.86)	4,444
2nd quartile	-0.282*** (-3.12)	10,159	-0.450*** (-5.39)	4,408
3rd quartile	-0.308*** (-2.93)	10,105	-0.378*** (-5.35)	4,388
4th quartile	-0.386*** (-5.27)	9,927	-0.456*** (-7.28)	4,300

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 3.12: Regression of Estimated Asset Return γ 's on Leverage and Proxies for Asset Type[†]

Independent Variables	Dependent Variables			
	$\hat{\gamma}_A^{10}$	$\hat{\gamma}_A^{20}$	$\hat{\gamma}_A^{10}, -5\%$	$\hat{\gamma}_A^{20}, -5\%$
Constant	-0.349* (-1.78)	-0.468*** (-2.93)	-0.083 (-0.44)	-0.409*** (-2.91)
LEV	1.143** (2.05)	0.925** (1.92)	0.043 (0.09)	0.590 (1.41)
RD	-0.536 (-0.80)	-0.789 (-1.19)	-0.862 (-1.32)	-0.836 (-1.39)
CAPX	-0.753 (-0.55)	-0.522 (-0.34)	-0.415 (-0.32)	-0.298 (-0.22)
MB	0.051 (1.33)	0.043 (1.38)	0.024 (0.65)	0.041 (1.36)
Sample Size	3,188	3,150	3,057	3,019

[†] The “-5%” remark indicates that the top 5% of firms with the highest leverage have been eliminated from the sample.

The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

Table 3.13: Regression of Estimated Asset Return γ 's on Leverage and Proxies for Asset Type for Profitable Firms[†]

Independent Variables	Dependent Variables			
	$\hat{\gamma}_A^{10}$	$\hat{\gamma}_A^{10}$	$\hat{\gamma}_A^{20}$	$\hat{\gamma}_A^{20}$
Constant	-0.603 (-1.40)	-0.489 (-1.31)	-0.433 (-1.24)	-0.301 (-0.94)
LEV	0.513 (0.64)	— —	0.589 (0.86)	— —
RD	-2.770* (-1.75)	-3.165** (-2.14)	-3.336** (-2.32)	-3.789*** (-2.66)
CAPX	0.776 (0.50)	0.902 (0.59)	1.030 (0.62)	1.176 (0.71)
MB	0.031 (0.59)	0.016 (0.32)	0.018 (0.42)	0.000 (0.00)
PE	0.0002 (0.10)	0.0004 (0.18)	-0.0009 (-0.45)	-0.0007 (-0.34)
PVGO	-7.069* (-1.69)	-7.624* (-1.73)	-1.996 (-0.57)	-2.616 (-0.73)
Sample Size	2,275	2,275	2,242	2,242

[†] The numbers in the parentheses are the test statistics under the null hypothesis that the coefficient is zero. These statistics follow a standard normal distribution asymptotically (see, for example, Fuller (1976)), and *, **, and *** denote statistical significance at 10%, 5%, and 1% levels according to the asymptotic distribution.

