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Citation: Woerner, Kyle et al. "Quantifying Protocol Evaluation for Autonomous Collision Avoidance." *Autonomous Robots* (May 2018): 1-25 © 2018 Springer Science+Business Media, LLC, part of Springer Nature

As Published: <http://dx.doi.org/10.1007/s10514-018-9765-y>

Publisher: Springer-Verlag

Persistent URL: <http://hdl.handle.net/1721.1/116295>

Version: Author's final manuscript: final author's manuscript post peer review, without publisher's formatting or copy editing

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Quantifying Protocol Evaluation for Autonomous Collision Avoidance

Toward Establishing COLREGS Compliance Metrics

Kyle Woerner · Michael R. Benjamin · Michael Novitzky · John J. Leonard

Received: 12 November 2016 / Accepted: date

Abstract Collision avoidance protocols such as COLREGS are written primarily for human operators resulting in a rule set that is open to some interpretation, difficult to quantify, and challenging to evaluate. Increasing use of autonomous control of vehicles emphasizes the need to more uniformly establish entry and exit criteria for collision avoidance rules, adopt a means to quantitatively evaluate performance, and establish a “road test” for autonomous marine vehicle collision avoidance. This paper presents a means to quantify and subsequently evaluate the otherwise subjective nature of COLREGS thus providing a path toward standardized evaluation and certification of protocol-constrained collision avoidance systems based on admiralty case law and on-water experience. Notional algorithms are presented for evaluation of COLREGS collision avoidance rules to include overtaking, head-on, crossing, give-way, and stand-on rules as well as applicable entry criteria. These rules complement and enable an autonomous collision avoidance road test as a first iteration of algorithm certification prior to vessels operating in human-present environments. Additional COLREGS rules are discussed for future development. Both real-time and post-mission protocol evaluation tools are introduced. While the motivation of these techniques applies to improvement of autonomous marine collision avoidance, the concepts for

protocol evaluation and certification extend naturally to human-operated vessels. Evaluation of protocols governing other physical domains may also benefit from adapting these techniques to their cases.

Keywords COLREGS · autonomous collision avoidance · human-robot collaboration · marine navigation

1 Introduction

The ability to quantify and subsequently evaluate collision avoidance performance allows society to more uniformly assess capability and risk of the driver. In the case of autonomous collision avoidance, a means to validate the underlying algorithms to standards consistent with human expectations necessitates a first step toward quantification of performance. Improving algorithms and their evaluation techniques incrementally in real-world environments may then contribute to successively increasing collision avoidance performance standards throughout the world.

The methods discussed in this paper are intended to be a first step toward a more robust and standardized autonomous collision avoidance evaluation process. These methods can further serve to standardize literature regarding collision avoidance compliance, especially under protocol constraints such as COLREGS¹ [26, 31].

K. Woerner, M. R. Benjamin, M. Novitzky, & J. J. Leonard
Massachusetts Institute of Technology
Department of Mechanical Engineering
& Computer Science and Artificial Intelligence Laboratory
77 Massachusetts Avenue 32-220
Cambridge, Massachusetts, 02139 USA
E-mail: k.w@mit.edu, mikerb@mit.edu, novitzky@mit.edu, jleonard@mit.edu *This work was supported by the U.S. Office of Naval Research (Code 33: Robert Brizzolara; Code 311: Behzad Kamgar-Parsi and Don Wagner) and Battelle (Mike Mellott). Portions of this paper first appeared in [31].

¹ COLREGS refers to the international rules as formalized at the Convention on the International Rules for Preventing Collisions at Sea, developed by the International Maritime Organization, and ratified as an international treaty by Congress. These rules were further formalized by the U.S. International Navigational Rules Act of 1977 [26], and are sometimes referred to as the Collision Regulations outside the United States.

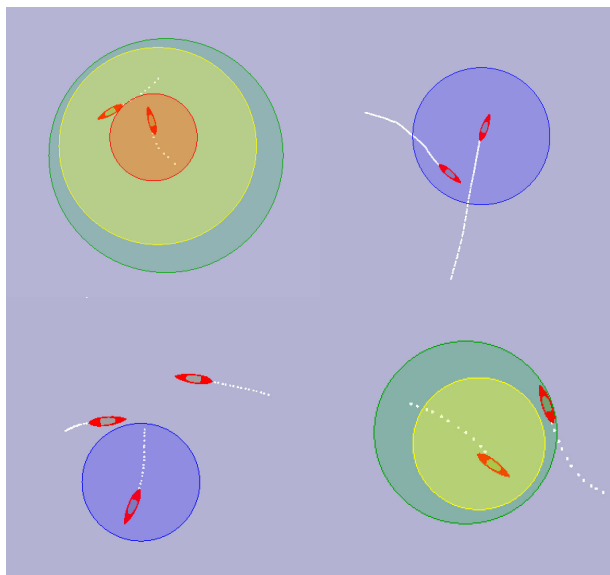


Fig. 1. Algorithms of this paper use autonomous surface vessel collision avoidance track data to evaluate each vehicle’s safety (top left and bottom right) and protocol compliance (top right and bottom left) based on the COLREGS rules, at-sea experience, maritime case law, and case studies of past collisions. Evaluation algorithms may be tuned to localized requirements using a library of functions and configuration parameters. Both real-time and post-mission analysis tools are presented giving evaluators and drivers a means to objectively quantify the safety and protocol compliance using established metrics and standards. This paper provides the metrics necessary to complete a recently introduced “road test” for protocol-constrained collision avoidance for certifying autonomous marine vehicles to operate at sea in the vicinity of human-operated vessels. Here, blue marker rings indicate an encounter whose COLREGS score fell below a threshold level. Other markers are shown when violations of configurable safety parameters occur including minimum acceptable range at CPA (green), range at which a near-miss occurs (yellow), and range at which a collision is assumed (red).

Figure 1 demonstrates example safety and protocol compliance evaluation during on-water experimentation.

Government certifying agencies such as the International Maritime Organization and its nation-state agents (e.g., U.S. Coast Guard, etc.) may rely on real-world performance and track data before issuing licenses to operate in open ocean or other environments where non-test vessels operate. Past performance and accident reconstruction may be objectively evaluated using real-world track data and a standardized set of expectations quantified by the algorithms of this paper.

Insurance companies can use these performance scores when issuing policies for autonomous vessels or determining fault in an accident. Regulatory bodies can use these metrics for an autonomous or remotely operated vehicle’s “road test” before certification as presented in [33]. Humans and machines alike may train with

real-time feedback using measured performance of track data. Autonomous collision avoidance systems can use machine learning to improve behavior using aggregated data from both human-operated and autonomous vessels around the world. This may lead to specific tuning of collision avoidance systems or collision avoidance evaluation standards in different areas of the world depending on local customs or standards.

With the methods of this paper, conversations in future literature can be more exact in their meaning of compliance in protocol-constrained collision avoidance research. This paper is organized as follows:

- review of collision avoidance in the literature
- introduction of collision avoidance evaluation algorithms and function libraries addressing overtaking, head-on, give-way, and stand-on rules
- discussion of other collision avoidance rules as they relate to future development of evaluation algorithms
- introduction of real-time and post-mission analysis tools for overtaking, head-on, give-way, and stand-on rules
- discussion of recommendations to alter the Rules to be more inclusive of autonomous vessels
- conclusion with remarks to support other collision avoidance domains such as ground and air vehicles

A means to quantify the power driven rules is presented to include a numeric scale of compliance (0-100%) for each applicable rule and its subsequent contribution to the applicable categories of rules. Detailed numeric evaluation of scenarios may accompany an overall score to provide additional feedback and amplification of penalties assessed. Tuning of each algorithm’s parameters allows evaluators to control the scores corresponding to specific performance. The scores and thus mapping of numeric values to actionable results (e.g., pass, fail, etc.) are largely dependent on the evaluator’s tuning decisions. The assignment of scoring thresholds such as pass, fail, etc. are therefore reserved for the evaluation algorithm tuner or the appropriate certifying agency. The tuner in a developmental stage is likely the algorithm designer with input from standardized, published requirements as set by an evaluation authority (e.g., U.S. Coast Guard). The algorithm tuner in certification phases is likely the certification organization or its designated agent.

COLREGS collision avoidance evaluation within this paper consists of two primary metrics: safety and protocol compliance. Safety is based on a combination of range and pose of the vessels at closest point of approach (CPA). CPA is defined as the point on ownship’s track where the distance to the contact reaches its minimum value within the context of an encounter. Protocol compliance is based on collision avoidance rule-specific

requirements. Pose in this context refers to the relative angles of two vessels with respect to the other and is introduced in Section 3. Pose at CPA, specific values of range at CPA, complexity of simultaneous contact geometries, and total contact picture are complicating factors that are not directly quantified in the written COLREGS except in limited circumstances or as a consideration without specific definition. Each of these factors are, however, important to collision avoidance decision making.

Examination of past encounters and collisions can be used to help train algorithms to understand safe and unsafe characteristics of interactions. Protocol compliance and safety are related yet can provide additional value when observed in context of the other. For example, a sufficiently compliant maneuver with respect to the written rules might have varying degrees of safety depending on how a designer configures certain collision avoidance configuration parameters such as acceptable range at CPA. By examining the components of safety and compliance to include range, speed, pose, and similar quantities, much insight into true performance can be inferred. Section 2 introduces COLREGS compliance as found in the literature. Section 3 discusses the safety metrics of evaluation. Section 4 discusses protocol compliance metrics and considerations for power-driven vessels under Rules 13-17. Section 5 provides narrative for other COLREGS rules that would benefit from development of similar evaluation techniques. Section 6 discusses specific COLREGS testing and evaluation techniques including a function library and both pre- and post-mission analysis tools. Conclusions are presented in Section 7.

2 COLREGS Compliance in the Literature

Collision avoidance protocols are prevalent in many physical domains where explicit negotiation or communication is either impractical or infeasible. In common practice, these protocols are often communicated simply as having “right of way.” In ground transit, drivers are taught to yield to the driver on the right when arriving simultaneously at an intersection with stop signs [5]. Airplanes use the Rules of the Air to determine right of way and appropriate maneuvers—including altitude deviation—when not under active control of an air traffic controller [10]. Surface vessels similarly abide by the COLREGS to determine right of way and appropriate maneuvers without explicit communication [26]. Special rules within each protocol have evolved from real-world feedback; one such example is the traffic separation schemes of COLREGS when entering or exiting a harbor [4, 25]. While the Rules of the Air and COLREGS are largely

similar, differences in the physical domains manifest as differences between the collision avoidance protocol requirements, such as maintaining altitude separation for aerial vehicles.

Collision avoidance using COLREGS has been incorporated on autonomous vessels using various approaches since first demonstrated with on-water experimentation in [2]. Throughout maritime literature discussing COLREGS, the term “compliance” arises with varying context and meaning. Power-driven collision avoidance implementations of COLREGS (Rules 13-18) dominate the COLREGS-related collision avoidance literature. Other non-collision avoidance rules of COLREGS arise as being compliant within the literature when discussing light configurations [6].

Testing in the literature predominately fails to define the term compliance in any quantifiable fashion with respect to COLREGS. Several authors claim compliance with these protocols without specifying the degree or scope of compliance [3, 11, 17, 18, 23, 24, 32]. In [11], the head-on rule was shown to appropriately eliminate all turns to port. It did not, however, appear to prefer courses that were “readily apparent” (COLREGS Rule 8) when finding a turn to starboard. Case law defines apparent course maneuvers to consist of a minimum turn of 35° while common practice often requires no less than 30° of heading change [1, 4, 27, 28]. Courts have found that head-on maneuvers with insufficient turns (i.e., not readily apparent) are in fact non-compliant and, when a collision occurs, partly to blame [1]. With velocity vector cost functions that favor maintaining course and speed such as [11], improper selection of costing weights may easily result in less than apparent course changes. Other authors consider breaches of COLREGS that “may be in the USV’s best interest” such as turning to port to avoid a collision when explicitly prohibited by the COLREGS [17]. Many authors such as [32] simply claimed COLREGS compliance without any quantification or definition of scope.

This trend in claiming unquantified compliance likely stems from a combination of three factors including:

- the vagueness of the rules as written for human usage
- the unspecified scope of each author’s work with respect to the numbered COLREGS rules
- the tacit assumption that the COLREGS rules as written fully encompass all collision avoidance requirements

2.1 Intentional Vagueness of COLREGS

This intentional vagueness might allow the autonomous collision avoidance designer to assume some liberty to

interpret the vast array of complex collision avoidance scenarios without being overly restricted from a common sense yet safe approach. However, case law and common practice greatly influence the requirements of COLREGS despite not being found anywhere within the written rules. Examples of on-water collisions and case law provide relevant insight into nuances of the COLREGS and their evolution over the years. Areas for increased scrutiny in autonomous collision avoidance solutions can be derived from problematic past encounters of human ship drivers. The intentional vagueness of the COLREGS including their underlying meaning as derived from the evolution of protocol-constrained collision avoidance in maritime environments, analysis of real-world examples, critiques of experienced mariners, and relevant rulings from Courts of Admiralty are presented in detail in [1, 4, 9, 25, 35] with examples presented as appropriate in this paper.

2.2 Categories for COLREGS Scope

A further complicating factor results from the disconnect between experienced mariners and autonomous designers: few designers of marine autonomous collision avoidance algorithms have demonstrated significant experience using COLREGS in open ocean navigation for non-academic purposes. The varying scope of what authors claim as compliant largely depends on the scope of interest of a particular researcher. For example, a perception and sensing author might claim COLREGS compliance if day shapes or vessel types are correctly identified using vision sensing algorithms. An acoustician might claim compliance for properly identified sound signals. The notion of compliance, however, should be amplified with the applicable scope of the COLREGS within each author's work.

Collision avoidance compliance in the most general sense involves maneuvering one's vessel to properly interact with a contact for a given initial geometry with appropriate caveats for vessel type, maneuvering restrictions, etc. To counter the disparity between claims and actual performance, the scope and requirements of COLREGS compliance were quantified in [31] and [33] as part of an autonomous collision avoidance approach. The COLREGS rules were separated into categories to allow a vehicle to demonstrate compliance of appropriate COLREGS subsets. International Maritime Organization guidance, US Coast Guard's local issuance of inland-specific requirements, and other local guidelines can be adopted as appropriate.

A few notable exceptions to this trend exist including work with a restricted visibility (Rule 19) compliance factor in [19] to give a preliminary estimate of fitness.

A traffic separation scheme compliance factor in [21] measures fitness of a reference track with respect to the ship's track. A fitness scheme was used in [20] to penalize maneuvers which grossly violated COLREGS such as maneuvering to port when turns to starboard were required.

Several categories of scope were presented in [31] and [33] as a first pass means of grouping similar research and subsequent evaluation. Table 1 reproduces this grouping for reference and discussion throughout this paper. With the development of metrics and evaluation techniques within each category, performance can be reliably demonstrated to a certifying body to a required degree of satisfaction. A means would then exist to properly combine work of differing categories to produce more fully compliant solutions prior to achieving the next level of operational or testing certification.

Table 1. **Categories of Scope for COLREGS Compliance Evaluation from [33]**

I	General Rules (Rules 1-3)
II	General Conduct of Vessels (Rules 4-8)
III	Special Traffic Schemes (Rules 9-10)
IV	Sailing in Sight of Another Sailing Vessel (Rule 12)
V	Vessel Encounters in Sight of One Another (Rules 13-17)
VI	Responsibilities in Sight of One Another (Rules 11, 18)
VII	Restricted Visibility (Rule 19)
VIII	Lights and Shapes (Rules 20-31)
IX	Sound and Light Signals (Rules 32-37)
X	Inter-vehicle Communications
XI	Cumulative Performance Including Local Customs

Rule categorization allows one designer to claim compliance within one or more categories (for example, maneuvering requirements of power driven vessels) while deferring evaluation of rules related to other areas (for example, sound identification and response) to other authors. By defining the scope of applicable rules and demonstrating quantifiable levels of compliance within each category, autonomous collision avoidance algorithm designers can more sufficiently articulate their contributions to the literature. It should be noted that evaluation within the scope of one category may rely on compliance of another category to some degree. For example, because Category II includes maintaining a lookout, determining safe speed, determining risk of collision, and taking action to avoid a collision, it heavily influences evaluation of Categories III-VII. This paper primarily focuses on Category V scenarios while discussing necessary aspects of future development of the other categories.

2.3 Collision Avoidance and Risk

Several approaches have been taken with respect to collision avoidance and risk in the literature. Each physical domain offers its unique challenges with modeling and assessing risk. In the maritime domain, collision risk assessments have often studied single vehicle pairs such as vessels in a traffic lane [13] or a vessel in a similar open ocean scenario [14]. A study of the reliability of quantitative risk analysis through a case study of ship-ship collision risk is presented in [7], which showed that probability and indicator based risk perspectives do not necessarily provide the same risk picture. A framework for measuring ship collision risk was presented in [8], though no evaluation of protocol compliance was formally considered. A methodology for assessing the collision risk without consideration of protocol requirements in an electronic navigation environment with vessel state uncertainties was presented in [16]. A multi-ship anti-collision decision support formulation is presented using simulations in [34]. A domain violation problem was presented in [22] that considered both the degree of domain violation and time of domain violation aspects. A non-linear model for risk estimation which attempts to capture mariners judgment was presented in [15], largely based on data analysis from a questionnaire to experienced mariners.

3 Evaluating Safety Using CPA Range and Pose

Safety has traditionally been viewed in the collision avoidance literature as a measure of the number of collisions relative to the number of encounters. This, however, is inconsistent with how human-operated ships make decisions, especially in scenarios of complex multi-contact collision avoidance. By considering several quantities important to making collision avoidance decisions, safety performance can be quantified in a way that is more meaningful than simply declaring a collision or collision-free encounter. By approaching the evaluation problem in the same way that ships make maneuvering decisions [27, 28], closer ranges are considered to be higher risk even if not resulting in a collision.

Within the scope of COLREGS, a collision avoidance encounter is defined to be from first detection of a contact that is assessed to have a risk of collision until the contact is past CPA, opening range, and no longer considered a risk of collision. Within the scope of this evaluation, vessels are assumed to commence a collision avoidance encounter once the other vessel is detected. Vessels detect each other at a nominal threshold range of R_{detect} .

While many factors are important to the CPA vernacular, the two most important values are the range between vessels at CPA (“CPA range”, r_{cpa}) and the time until or at which CPA occurs (“time of CPA”, t_{cpa})². A third value important to CPA calculations is the pose (“pose at CPA”, Θ_{cpa}). Pose in this context refers to a vector $\Theta_{\text{cpa}} = \langle \alpha_{\text{cpa}}, \beta_{\text{cpa}} \rangle$ denoting values of contact angle (α) and relative bearing (β). Relative bearing denotes the bearing of a contact relative to ownship’s bow as 0° . Contact angle³ refers to the relative bearing of ownship as seen from the perspective of the contact in question and serves as a clear means to distinguish between the two relative bearings: one from the perspective of ownship, the other from the perspective of the contact. Consistent with on-ship conventions, relative bearing is considered on the domain $[0,360)$ and contact angle on the domain $[-180,180)$. A graphical representation of relative bearing and contact angle is presented in Figure 2.

3.1 Range at CPA

Maneuvers that are otherwise compliant with required turn direction and speed but maneuver in a way that results in unnecessarily close range at CPA are penalized in the safety score. The resulting range and pose at closest point of approach are considered when penalizing unsafe maneuvers. A safety score for evaluating autonomous collision avoidance considers four primary configurable range thresholds as shown in Table 2 and Figure 3. Range notation uses upper case R to denote a threshold or nominal value and a lower case r to denote a measured or actual value. Using these range thresholds, a collision avoidance decision is considered to desire ranges at CPA greater than some value R_{pref} while accepting a known value of risk for ranges as close as R_{min} . Ranges closer than R_{min} are considered unsafe and abnormal operating conditions. An encounter is considered to have a high risk of imminent collision if its range violates a “near miss” threshold R_{nm} . A physical collision is considered to occur for any violation of the R_{col} threshold.

3.2 Pose at CPA

Using contact angle, relative bearing, range, and speed, a complete contact geometry can be realized. Contact

² Range at CPA sometimes appears in the literature as distance at CPA (DCPA). Both may be used interchangeably. Similarly, t_{cpa} sometimes appears as TCPA.

³ The term originated in World War II submarine operations under the names of “angle on the bow” and “target angle.”

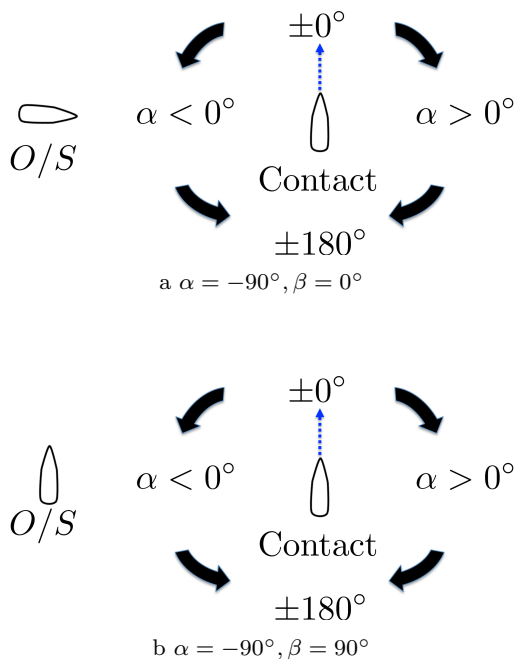


Fig. 2. Contact angle α represents how ownship’s (labelled “O/S”) relative bearing is seen from the perspective of the contact. Contact angle is 0° if the contact is pointing its bow at ownship. Similarly $\alpha = 180^\circ$ if the contact’s stern is facing ownship. Starboard-facing aspects assume positive values while port-facing aspects assume negative values. In both of these examples, ownship is far to the left and the contact is on course North. In (a) ownship is pointing the contact while in (b) the ships are on parallel tracks, both at the same location in 2-D space (x,y). Contact angle remains unchanged while relative bearing changes for the two cases.

Table 2. Primary Threshold Ranges for Evaluation

Range	Description
R_{pref}	preferred range at CPA
R_{min}	minimum acceptable range at CPA
R_{nm}	range considered as a “near miss” encounter
R_{col}	range considered as a physical collision

angle assumes positive increasing values clockwise from the starboard bow and negative values counterclockwise from the port bow, such that $\alpha = 0^\circ$ represents the contact’s bow and $\alpha = \pm 180^\circ$ represents the contact’s stern as shown in Figure 2. Pose at CPA is therefore not a single quantity, but rather a vector of two angles that give great insight into the collision avoidance problem. The pose at time of sighting or detection (θ_0) often defines which rule(s) of a protocol applies. The pose at CPA when combined with r_{cpa} and relative speed gives considerable insight into the degree of risk at t_{cpa} . Figure 4 shows relative bearing and contact angle for an arbitrary initial geometry and geometry at CPA. Figure 5 shows the importance of considering pose for two different encounters of the same range at CPA.

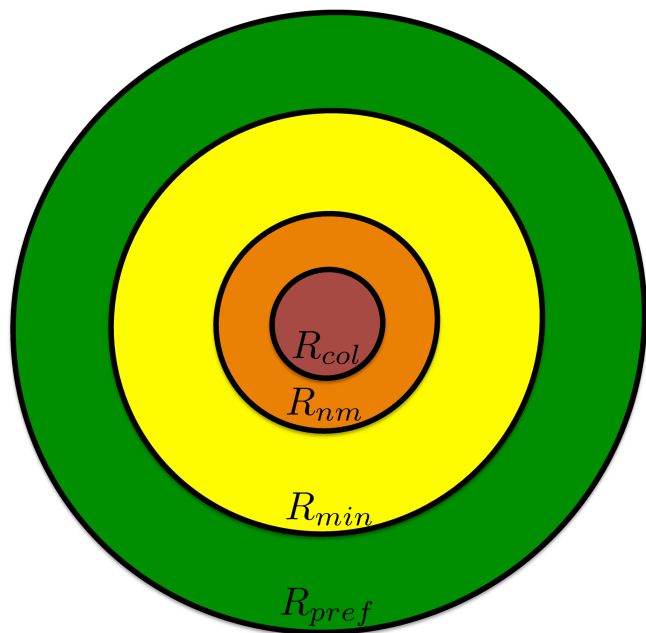


Fig. 3. Concentric range rings represent configurable threshold range values that may be used in evaluation of safety of a collision avoidance encounter. While vessels nominally prefer CPA ranges greater than R_{pref} , maneuvers with $R_{pref} > r_{cpa} > R_{min}$ (inside the green area) are considered allowable if certain precautions are taken [27, 28]. Maneuvers with $R_{min} > r_{cpa} > R_{nm}$ (inside the yellow area) should be considered unsafe encounters and examined more closely to improve future performance. Any encounter violating a closer R_{nm} (inside the orange area) threshold is considered to be a near miss. R_{col} denotes the range at which an actual collision is assumed to occur (inside the red area). Safety scores may be quantified using evaluator-defined functions that map specific values of r_{cpa} using these threshold values. While the range rings of this figure only consider r_{cpa} , pose may be factored into the safety score to create shapes other than concentric circles. Evaluation ranges shown here are not necessarily to scale.

3.3 Safety Functions

A tiered range approach allows for maneuverability considerations and quantification of a safety score between the minimum acceptable CPA range and preferred CPA range. This technique also produces a safety score for r_{cpa} values closer than the minimum acceptable CPA range in a format more meaningful than standard binary evaluation (collision or non-collision). While any r_{cpa} closer than R_{min} is undesirable, quantifying each encounter allows more thorough insight into the overall effectiveness of a collision avoidance algorithm, collision avoidance configuration parameters, performance under certain rule constraints, and similar considerations. User-defined safety functions map threshold ranges to safety scores. In a basic example, a linear function maps values between each of the configuration ranges with a

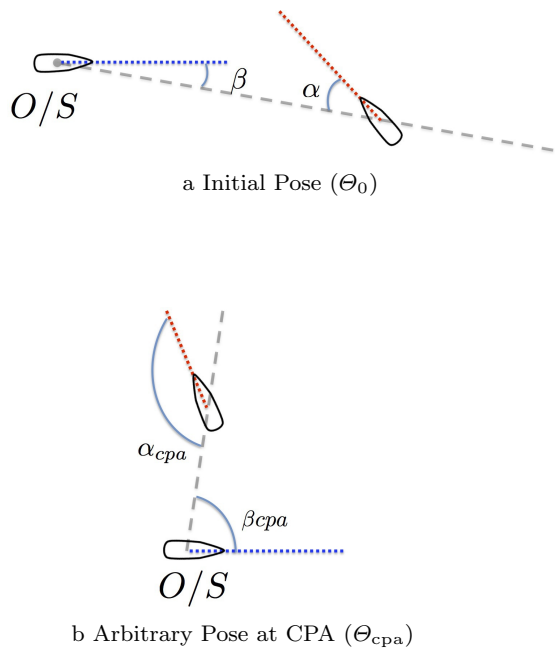


Fig. 4. Ownship (labeled “O/S”) is traveling east and first sights a contact at relative bearing β with contact angle α in (a). Speed is represented by the length of the colored lines from each vessel (red for contact, blue for ownship). From the perspective of the contact looking at ownship, α and β are simply interchanged. These two angles give great insight into the collision avoidance picture and quickly aid in determining the applicable protocol constraints. Combined with CPA range and time (r_{cpa} , t_{cpa}), pose at CPA ($\theta_{cpa} = \langle \alpha_{cpa}, \beta_{cpa} \rangle$) gives important information as to risk of collision, collision avoidance protocol compliance, and overall safety of a maneuver. Relative bearing and contact angle at CPA are shown in (b). CPA (b) occurs when the range between contacts reaches its minimum value.

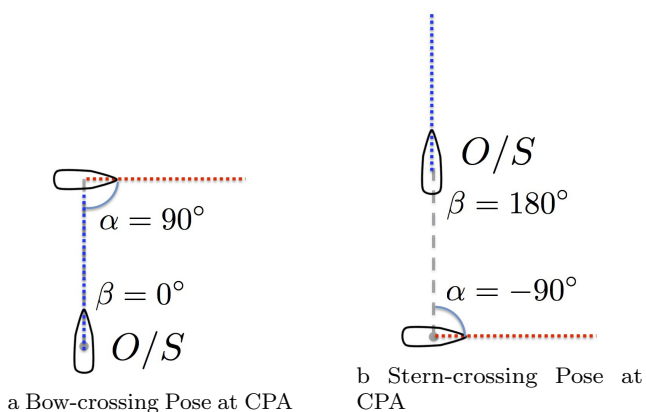


Fig. 5. Ownship (labeled “O/S” and traveling north) encounters two scenarios of a canonical track crossing with identical ranges at CPA. The bow crossing scenario of (a) demonstrates a much more dangerous encounter than the stern crossing of (b). While many techniques treat all ranges equally, this canonical example of equivalent ranges demonstrates the necessity of incorporating pose into calculations of risk and performance.

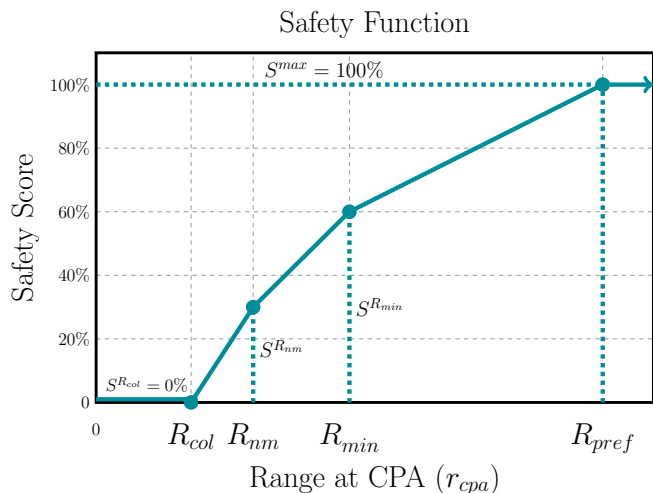


Fig. 6. Safety scores map range and pose at CPA to a numeric value. This figure demonstrates a piecewise linear mapping of range at CPA (r_{cpa}) to a range-based safety score S_r using the primary threshold ranges of Table 2. Additional penalty or reward may be assigned for pose considerations according to the evaluator’s preference of a safety function. Threshold evaluation ranges give safety scores at preferred range at CPA ($S_r(R_{pref}) = S^{R_{pref}}$), minimum acceptable range at CPA ($S_r(R_{min}) = S^{R_{min}}$), range at which a near-miss occurs ($S_r(R_{nm}) = S^{R_{nm}}$), and the nominal range at which a physical collision is assumed ($S_r(R_{col}) = S^{R_{col}}$).

collision having a safety score of $S = 0$ and any range greater than preferred CPA range having a safety score of $S = S^{max} = 100\%$. Safety functions can be tailored by the evaluator to create specific results based on regulations or experience. A piecewise linear safety function (Figure 6) demonstrates the adaptability of evaluating performance using multiple range thresholds.

Figure 5 demonstrated that two identical ranges are not necessarily equally dangerous. For example, a ship crossing in front of another (ownship’s bow pointing a contact’s beam at CPA) may be considerably more dangerous than two vessels passing at the same range in a port-to-port or stern-to-beam arrangement. For this reason, incorporation of both pose and range at CPA may prove valuable to an evaluator of collision avoidance algorithms.

A safety function (S) may include CPA values of both pose ($\theta_{cpa} = \langle \alpha_{cpa}, \beta_{cpa} \rangle$) and range (r_{cpa}) as shown in Equation (1). While most often evaluated at CPA, the safety function concept may be applied at any point in a collision avoidance encounter to achieve a real-time measurement of safety as shown in Equation (2). The safety function may take both range and pose components directly or as a combination of a range-based safety function (S_r) and a pose-based safety function (S_θ). If desired, an evaluator might consider only the

range-based safety function or only the pose-based safety function.

$$S = S(r_{\text{cpa}}, \Theta_{\text{cpa}}) = S(S_r, S_\Theta) \quad (1)$$

$$S = S(r, \Theta) \quad (2)$$

Equation (3) defines an example range-based safety function S_r . This may take the form of a piecewise-defined function corresponding to each range threshold (Figure 6) or a single function defined across the entire domain. Safety functions might take other arbitrary shapes such as quadratic, logarithmic, or step-wise designs depending on the needs of the evaluator.

$$S_r = S_r(S^{R_{\min}}, S^{R_{\text{nm}}}, R_{\text{col}}, R_{\text{nm}}, R_{\min}, R_{\text{pref}}) \quad (3)$$

Pose-based safety functions (Equation (4)) are used to account for the risk associated with the degree which contacts are pointing each other. By combining a pose factor for contact angle using Equation (5) and relative bearing using Equation (6), an assessment may be made to reward beam or stern aspects at CPA up to a maximum value of S_Θ^{\max} . Contact angles at CPA aft of a configurable cutoff value α_c are given a uniform reward value. Similarly, relative bearings at CPA aft of a configurable cutoff value β_c are given a uniform reward value. Figure 7 demonstrates the pose-based safety scoring scheme of Equation (5) that favors non-bow pointing contact angles at CPA using $\alpha_c = 80^\circ$.

$$S_\Theta = S_\Theta^{\max} \cdot S_\Theta^\alpha \cdot S_\Theta^\beta \quad (4)$$

$$S_\Theta^\alpha = \begin{cases} \frac{1 - \cos(\alpha_{\text{cpa}})}{1 - \cos(\alpha_c)}, & \text{if } |\alpha_{\text{cpa}}| < \alpha_c \\ 1, & \text{if } |\alpha_{\text{cpa}}| \geq \alpha_c \end{cases} \quad (5)$$

$$S_\Theta^\beta = \begin{cases} \frac{1 - \cos(\beta_{\text{cpa}})}{1 - \cos(\beta_c)}, & \text{if } |\beta_{\text{cpa}}| < \beta_c \\ 1, & \text{if } |\beta_{\text{cpa}}| \geq \beta_c \end{cases} \quad (6)$$

An evaluator may find particular value in one form of a safety function over others for a given use case. These safety functions include range-only, pose-only, weighted summation, multiplicative, reward-only multiplicative, and effective range.

The range-only method of Equation (3) improves the precision of standard approaches in collision avoidance: most literature currently measures safety performance as either a collision or a success. By approaching the evaluation problem in the same way that ships make maneuvering decisions [27, 28], additional risk may be

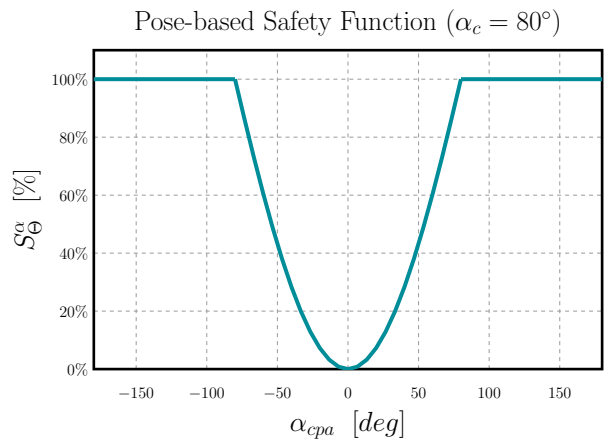


Fig. 7. COLREGS pose-based safety functions can be used to give preference to passing contacts with relative bearing and contact angle that are least likely to increase risk of collision. This safety function shows S_Θ^α that favors beam and stern contact angles at CPA (α_{cpa}) using Equation (5). Here, a cutoff angle $\alpha_c = 80^\circ$ gives equal preference to beam and stern aspects. Pointing the bow (0°) results in a pose-based safety score of $S_\Theta = 0$.

assumed to be present at closer ranges even if they do not result in a collision. Even without incorporating pose, this technique provides evaluators with a means to examine encounters with higher precision than simply determining whether or not a collision occurred.

The pose-only method of Equation (4) allows for testing safe poses at the time of encounter. On its own, this method might prove valuable to an evaluator interested only in determining how consistently a vessel passes in non-bow pointing aspects. Typically, the pose-based method appears with the range-based method in the form of a combined safety function.

The weighted summation method of Equation (7) provides a means for an evaluator to consider both range and pose. This method limits the influence of the range and pose components using the weights s_r and s_Θ , respectively. Evaluators may choose weights to emphasize the appropriate balance of range and pose as necessary.

$$S = s_r \cdot S_r + s_\Theta \cdot S_\Theta \quad (7)$$

$$s_r + s_\Theta = 1$$

The multiplicative safety function of Equation (8) requires high scores for both range and pose components to have an overall high safety score. Poor performance in either range or pose immediately results in a low safety score. An evaluator's choice of S_r and S_Θ is therefore particularly important. When using a safety function such as Equation (8) that requires both satisfactory performance of both range and pose, $S_\Theta^{\max} \equiv 1$.

$$S = S_r \cdot S_\Theta \quad (8)$$

As an alternative to the penalty nature of pose in Equation (8), an evaluator may choose to use pose to reward vessels for passing astern, for example, up to some reasonable percentage defined by S_Θ^{\max} using Equation (9). Reward-only multiplicative safety functions (Equation (9)) allow for the pose component to add a reward value to the original range-based safety score. In this reward case, an evaluator may choose to limit the pose reward to $S_\Theta^{\max} < 1$ (e.g., 20% maximum reward).

$$S = \max\left(S_r \cdot (1 + S_\Theta), 100\%\right) \quad (9)$$

The effective range method of Equation (10) directly adds some range value to the actual r_{cpa} based on a pose-based safety score S_Θ . This effective range r_{eff} (Equation (11)) is then used as the input of a range-based safety function to compute the overall safety score at CPA for $r = r_{\text{eff}}$ rather than $r = r_{\text{cpa}}$. The true r_{cpa} value may not be compensated more than some maximum possible pose-reward r_Θ . This technique allows for pose to effectively improve the r_{cpa} which is seen by the safety function as though range were actually at r_{eff} . In contrast, a technique such as the reward-only multiplicative of Equation (9) uses a proportional reward rather than the absolute maximum reward of the effective range method.

$$S = S(r_{\text{eff}}) \quad (10)$$

$$r_{\text{eff}} = r + S_\Theta \cdot r_\Theta \quad (11)$$

Algorithm 1 demonstrates assessment of safety as a function of both range and pose. Table 3 summarizes the different means of computing safety scores.

Algorithm 1 General Approach of Safety Evaluation

```

1: procedure PSEUDOCODE FOR ANALYZESAFETY()
2:   Input: range thresholds and associated penalty values
3:   Input: safety functions and shapes
4:   for each encounter do
5:      $\Theta_{\text{cpa}} \leftarrow$  pose at CPA
6:      $r_{\text{cpa}} \leftarrow$  range at CPA
7:      $S \leftarrow S(r_{\text{cpa}}, \Theta_{\text{cpa}})$ 
        $\triangleright$  combine range and pose components using Table 3
8:   end for
9: end procedure

```

Table 3. Summary of Collision Avoidance Safety Functions

Eqn.	Likely Use Case
(3)	Range-only solution desired.
(4)	Pose-only solution desired.
(7)	Preserves strong performance in either range or pose.
(8)	Requires high performance in both range and pose.
(9)	Rewards favorable pose without penalizing poor pose.
(10)	Rewards advantageous pose by increasing the “effective” range; limits reward to an absolute quantity rather than relative increase of (9). Easy to visualize as graph of $S_r(r_{\text{eff}})$.

4 Evaluating Protocol Requirements Using Rule-specific Algorithms and Considerations

Standardized measures of collision avoidance algorithm performance and effectiveness enable consistent evaluation of COLREGS for both human-operated and autonomous vessels. Creating rule categories, assigning each rule to a category, and defining metrics for each applicable rule enables consistent evaluation and allows for more clear scientific conversation regarding the advancement of autonomous collision avoidance. Incorporation of the appropriate case law, localized nuance, and knowledge of the evolution of the COLREGS are vital to ensuring appropriate behavior in nuanced situations. While the Rules give general guidance, actions generally consistent with human behavior and expectations must be the objective when integrating autonomous systems into human-present environments. Appropriately modeling and accounting for human intuition, common practice, and human expectations are among the many factors not found on any written page of [26]. For example, the colloquial Law of Gross Tonnage states that a small craft generally stays out of the way of a large vessel such as an intercontinental merchant even when, strictly by the protocol requirements, the small vessel might have right of way. Adherence to custom and human expectation must be considered and honored in order to integrate autonomous platforms into a human-dominated environment.

Algorithm 2 demonstrates the general approach of evaluating COLREGS compliance including calculation of the safety score of Algorithm 1. Collision avoidance encounters commence at a threshold detection range $r = R_{\text{detect}}$. Protocol compliance functions take the symbol \mathfrak{R} with a superscript denoting the applicable COLREGS rule(s) (e.g., \mathfrak{R}^{14} denotes evaluation of head-on compliance). Each subsection below presents a protocol requirement and brief discussion of evaluation technique and nuance as appropriate.

Algorithm 2 General Approach of Evaluation Technique

```

1: procedure PSEUDOCODE FOR EVALUATEENCOUNTER()
2:   Input: positions (x,y), courses ( $\theta$ ), and speeds ( $v$ ) from track data
3:   Input: configurable threshold ranges and angles
4:   Input: configurable penalty values and functions
5:   for each encounter commencing at  $r = R_{\text{detect}}$  do
6:     Calculate: initial pose ( $\Theta_0$ )
7:     Calculate: pose at CPA ( $\Theta_{\text{cpa}}$ )
8:     Calculate: CPA range ( $r_{\text{cpa}}$ )
9:     Calculate: changes in speed ( $\Delta v, v_{\text{min}}, v_{\text{max}}$ )
10:    Calculate: changes in course ( $\Delta\theta$ )
11:    Determine  $\mathfrak{R}$  using Algorithm 3  $\triangleright$  entry criteria
12:     $\mathfrak{R} \leftarrow \mathfrak{R}(\Theta_0, \Theta_{\text{cpa}}, r_{\text{cpa}}, \Delta v, \Delta\theta)$ 
13:     $\triangleright$  evaluate for each contact with respect to its rule set  $\mathfrak{R}$ 
14:     $S \leftarrow S(r_{\text{cpa}}, \Theta_{\text{cpa}})$ 
15:     $\triangleright$  evaluate safety for each contact using AnalyzeSafety()
16:  end for
17: end procedure

```

This section presents methods corresponding to Category V of Table 1 (vessel encounters in sight of one another (Rules 13-17)) with the understanding that other categories (e.g., I, II, VI) may additionally apply. Each collision avoidance rule for vessels in sight of one another except Rule 14 (COLREGS Rules 11-13, 15-18) allows for entry criteria that assign one vessel to be stand-on (maintain course and speed⁴) and the other give-way (keep out of the way of the other) based on geometry, ship type or maneuverability restrictions, and environmental (wind) conditions for the specific case of two sailing vessels. Rules 13-17 are presented with considerations below. Rule 18 is discussed in Section 5.1. Open ocean vessel behavior assumes that maneuvers are most likely due to collision avoidance requirements unless otherwise specified for navigational or operational necessity.

4.1 Entry Criteria

Entry criteria for Rules 13-17 largely depend on a combination of relative geometry, relative speeds, and an assessed risk of collision for two power driven vessels assuming no special precedence of Rule 18. While relative bearing is specified explicitly in the COLREGS for Rule 13, ambiguity exists for Rule 14. Contact angle offers significant insight into the appropriate rule and helps discriminate risk of collision before making more computationally costly calculations. Algorithm designers and evaluators evaluating entry criteria must show due regard for the written rules of [26], appropriate case law, and local custom [1, 4].

⁴ Maintaining course and speed gives appropriate latitude to normal actions required per case law [1, 4, 35].

Algorithm 3 COLREGS Entry Criteria: Determining the Appropriate Rule Set

```

1: procedure PSEUDOCODE FOR COLREGS ENTRY CRITERIA
2:   if  $r_{\text{cpa}} > R_{\text{detect}}$  then
3:      $\mathfrak{R} \leftarrow \mathfrak{R}^{\text{cpa}}$   $\triangleright$  no risk of collision
4:      $\triangleright R_{\text{detect}}$  is nominal range of detection
5:      $\triangleright$  continue tracking  $r_{\text{cpa}}$ , range-rate, and bearing-rate and evaluate subsequent rule entry
6:   return
7:   end if
8:   risk of collision assumed
9:    $\alpha_{\text{crit}}^{13} \leftarrow$  overtaking tolerance
10:   $\triangleright$  default  $45^\circ$ ; tolerance for “coming up with” pose
11:   $\triangleright$  also require closing range and risk of collision
12:   $\alpha_{\text{crit}}^{14} \leftarrow$  head-on tolerance  $\triangleright$  default  $13^\circ$ 
13:   $\triangleright$  tolerance for “reciprocal or nearly reciprocal courses”
14:   $\triangleright$  “When...in any doubt...assume...[head-on].”
15:   $\alpha_{\text{crit}}^{15} \leftarrow$  crossing aspect limit  $\triangleright$  default  $10^\circ$ 
16:   $\triangleright$  all  $\alpha_{\text{crit}}$  values are configurable by evaluator
17:   $\alpha_0 \leftarrow$  initial contact angle ( $\alpha \in [-180^\circ, 180^\circ]$ )
18:   $\triangleright \alpha_0^{360^\circ}$  maps  $\alpha_0$  from  $[-180^\circ, 180^\circ] \rightarrow [0^\circ, 360^\circ]$ 
19:   $\beta_0 \leftarrow$  initial relative bearing ( $\beta \in [0^\circ, 360^\circ]$ )
20:   $\triangleright \beta_0^{180^\circ}$  maps  $\beta_0$  from  $[0^\circ, 360^\circ] \rightarrow [-180^\circ, 180^\circ]$ 
21:  if ( $\beta_0 > 112.5^\circ$ ) && ( $\beta_0 < 247.5^\circ$ ) && ( $|\alpha_0| < \alpha_{\text{crit}}^{13}$ ) then
22:     $\mathfrak{R} \leftarrow \mathfrak{R}^{13/17}$   $\triangleright$  vessel is overtaken (stand-on)
23:  else if ( $\alpha_0^{360^\circ} > 112.5^\circ$ ) && ( $\alpha_0^{360^\circ} < 247.5^\circ$ ) && ( $|\beta_0^{180^\circ}| < \alpha_{\text{crit}}^{13}$ ) then
24:     $\mathfrak{R} \leftarrow \mathfrak{R}^{13/16}$   $\triangleright$  vessel is overtaking (give-way)
25:  else if ( $|\beta_0^{180^\circ}| < \alpha_{\text{crit}}^{14}$ ) && ( $|\alpha_0| < \alpha_{\text{crit}}^{14}$ ) then
26:     $\mathfrak{R} \leftarrow \mathfrak{R}^{14}$ 
27:     $\triangleright$  vessel is head-on; tolerance is configurable
28:  else if ( $\beta_0 > 0$ ) && ( $\beta_0 < 112.5^\circ$ ) && ( $\alpha > -112.5^\circ$ ) && ( $\alpha < \alpha_{\text{crit}}^{15}$ ) then
29:     $\mathfrak{R} \leftarrow \mathfrak{R}^{15/16}$   $\triangleright$  vessel is crossing give-way
30:     $\triangleright$  crossing aspect limit is configurable
31:  else if ( $\alpha_0^{360^\circ} > 0^\circ$ ) && ( $\alpha_0^{360^\circ} < 112.5^\circ$ ) && ( $\beta_0^{180^\circ} - 112.5^\circ && \beta_0^{180^\circ} < \alpha_{\text{crit}}^{15}$ ) then
32:     $\mathfrak{R} \leftarrow \mathfrak{R}^{15/17}$   $\triangleright$  vessel is crossing stand-on
33:     $\triangleright$  crossing aspect limit is configurable
34:  else
35:     $\mathfrak{R} \leftarrow \mathfrak{R}^{\text{cpa}}$ 
36:     $\triangleright$  detectable but likely no risk of collision (not yet in a dedicated rule)
37:     $\triangleright$  continue tracking  $r_{\text{cpa}}$ , range-rate, and bearing-rate and evaluate subsequent rule entry
38:  end if
39: end procedure

```

A configurable critical contact angle (α_{crit}) shown in Figure 8 for Rules 13-15 helps to specify whether a vessel should take action per the COLREGS. The ability to configure α_{crit} gives flexibility to the evaluator. When self-evaluating for the purposes of autonomous collision avoidance algorithm development and improvement, a designer might tune α_{crit} to best mimic human ship driving practice. Algorithm 3 demonstrates entry criteria for Rules 13-17 assuming no overriding Rule 18 precedence. Specific considerations of entry criteria for each rule are presented in the following subsections.

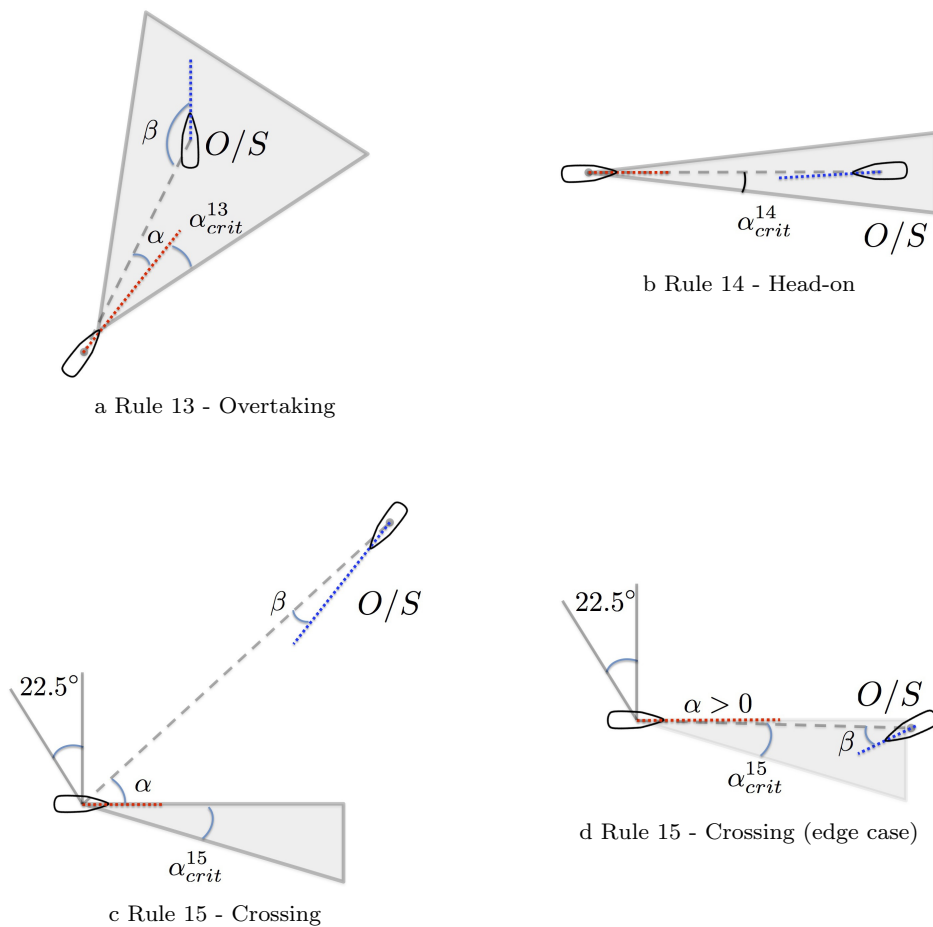


Fig. 8. Entry criteria for Rules 13-15. All critical angles are relative to ownship's heading. Critical angles of Rules 13 and 14 represent half-angles of the shaded region. All critical contact angles are configurable to the evaluator as they have no prescribed value in the COLREGS.

The introduction and configuration of α_{crit} allows more context for discussion and evaluation of entry criteria compared to the basic consideration of range and relative bearing alone. For example, algorithms might struggle to determine the appropriate context to assign a contact as “coming up on” within the context of Rule 13 [1, 4]. Using α relative to α_{crit} as a first pass filter to determine potential applicability of Rule 13 may give insight to appropriate values of α_{crit} in different regions of the world where local case law differs on “coming up on” with respect to Rule 13 entry criteria.

Entry criteria for Rule 14 must be carefully considered within the context of the local environmental conditions. Context of a ship's course over ground as observed by radar and its heading as observed visibly must be reconciled within collision avoidance algorithms to avoid inappropriate rule entry. That is, a contact's course as observed visually may be quite different than

that observed by relative motion on radar. Selection of entry criteria using a contact's heading for both collision avoidance and its subsequent evaluation must account for this environmental influence.

Further research is required to determine the appropriate use of α_{crit} for edge case scenarios such as Figure 8d. Rule 15 specifies a contact off the starboard side as being a crossing give-way. This does not relieve the vessel of her duties to give-way if a risk of collision exists in an edge case geometry that might not be within the canonical examples of $\beta \in [0^\circ, 112.5^\circ]$. This is especially true if strong environmental conditions might dominate a slow contact speed causing a risk of collision if no action were taken.

Entry into many of the numbered collision avoidance rules requires adherence to give-way (Rule 16) or stand-on (Rule 17) requirements. Evaluation algorithms for give-way requirements are presented in Section 4.2

with amplification in later sections as necessary for rule-specific requirements (e.g., power-driven crossing vessels in Rule 15). Similarly, evaluation algorithms for stand-on vessel requirements are presented in Section 4.3 with appropriate amplification in later sections.

4.2 Rule 16 – Give-way

Give-way vessels are to take early action, to take substantial action, and to keep well clear as shown in Algorithm 4. This yields three measurable criteria for all give-way vessels:

- range at time of maneuver relative to the ranges at times of detection, determination of collision risk, and CPA (Algorithm 5)
- determination of substantial action as measured by the size and direction of the maneuver (turn or speed change consistent with Rule 8) (Algorithms 6, 7, and 8)
- range and pose at CPA

Algorithm 4 Rule 16: Give-way Vessels

```

1: procedure PSEUDOCODE FOR GIVE-WAY VESSELS
2:    $\mathfrak{R}^{16} \leftarrow \mathfrak{R}^{\max}$ 
3:    $\mathfrak{R}^{16} \leftarrow \text{AnalyzeSafety}()$  (Rules 8,16,18)
                                      $\triangleright$  “keep well clear”
4:    $\mathfrak{R}^{16} \leftarrow$  penalize for delayed action
                                      $\triangleright$  (Algorithm 5)
5:    $\mathfrak{R}^{16} \leftarrow$  penalize for non-apparent maneuvers
                                      $\triangleright$  (Algorithms 6, 7, and 8)
                                      $\triangleright$  “take early and substantial action”
6:    $\mathfrak{R}^{16} \leftarrow$  penalize for hindrance of stand-on vessel
                                      $\triangleright$  “keep well clear”
7: end procedure

```

It should be noted that Rule 16 does not apply exclusively to power-driven vessels nor does it apply exclusively to crossing situations [1, 4]. Rather, Rule 16 may be invoked as a result of Rules 12, 13, 15, or 18. Claims of “compliance” with Rule 16 have been implicitly made in autonomous collision avoidance literature with a scope limited to power-driven crossing give-way situations (Rule 15) without discussion of its wider implications. Full Rule 16 compliance claims must, however, specify that they include the scope of Rule 12 (sailing vessels), Rule 13 (overtaking), Rule 15 (power-driven crossing), and Rule 18 (precedence) to be complete and truly compliant. Detailed discussion of the applicability of Rule 16 to each of these give-way situations is presented in the appropriate subsections.

Algorithm 5 Penalize for Delayed Action

```

1: procedure PSEUDOCODE FOR PENALIZEDelayedAction()
2:    $r_{\text{detect}} \leftarrow$  range to contact at time of detection
    $\triangleright$  if  $r_{\text{detect}}$  not explicitly known, set  $r_{\text{detect}} = R_{\text{detect}}$ 
    $\triangleright$  assumes collision risk assessed immediately
3:    $r_{\text{maneuver}} \leftarrow$  range at time of ownship’s maneuver
4:    $r_{\text{maneuver}} \leftarrow \min(r_{\text{maneuver}}, r_{\text{detect}})$ 
5:    $\mathfrak{R}^{\text{delay}} \leftarrow$  maximum score deduction (percent)
6:    $\mathfrak{R}^{\text{delay}} \leftarrow \mathfrak{R}^{\text{delay}} \cdot \left( \frac{r_{\text{detect}} - r_{\text{maneuver}}}{r_{\text{detect}}} \right)$ 
7:    $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} \cdot (1 - \mathfrak{R}^{\text{delay}})$ 
8: end procedure

```

Algorithm 6 Penalize for Non-Readily Apparent Maneuver

```

1: procedure PSEUDOCODE FOR PENALIZENonReadilyApparentManeuver()
2:    $\mathfrak{R}^{\Delta\theta_{\text{app}}} \leftarrow \text{Non-ApparentCourseChange}()$ 
    $\triangleright$  (Algorithm 7)
3:    $\mathfrak{R}^{\Delta v_{\text{app}}} \leftarrow \text{Non-ApparentSpeedChange}()$ 
    $\triangleright$  (Algorithm 8)
4:    $\text{thresh} \leftarrow$  threshold penalty before non-apparent maneuver deducts from score
    $\triangleright$  default 30%
5:   if ( $\mathfrak{R}^{\Delta\theta_{\text{app}}} < \text{thresh}$ ) || ( $\mathfrak{R}^{\Delta v_{\text{app}}} < \text{thresh}$ ) then
6:     return;
7:   else if ( $\mathfrak{R}^{\Delta v_{\text{app}}} < \text{thresh}$ ) then
8:      $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} \cdot (1 - \mathfrak{R}^{\Delta\theta_{\text{app}}})$ 
9:   else
10:     $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} \cdot (1 - \mathfrak{R}^{\Delta v_{\text{app}}})$ 
11:     $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} \cdot (1 - \mathfrak{R}^{\Delta\theta_{\text{app}}})$ 
12:   end if
13:    $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} \cdot (1 - \mathfrak{R}^{\Delta\theta_{\text{app}}})$ 
14: end procedure

```

Algorithm 7 Check for Non-Readily Apparent Course Change

```

1: procedure PSEUDOCODE FOR NON-ReadilyApparentCourseChange()
2:    $\mathfrak{R}^{\Delta\theta_{\text{app}}} \leftarrow$  max penalty for non-apparent course maneuver
    $\triangleright \mathfrak{R}^{\Delta\theta_{\text{app}}} \in [0, 1]$ , default 50%
3:    $\triangleright \mathfrak{R}^{\Delta\theta_{\text{app}}} \in [0, 1]$ , default 50%
4:    $|\Delta\theta| \leftarrow$  absolute course deviation
5:    $\Delta\theta_{\text{app}} \leftarrow$  apparent course deviation threshold
    $\triangleright$  default 30°
6:    $\Delta\theta_{\text{md}} \leftarrow$  minimum detectable course deviation
    $\triangleright$  default 0°
7:   if  $|\Delta\theta| > \Delta\theta_{\text{app}}$  then
8:     return ( $\mathfrak{R}^{\Delta\theta_{\text{app}}} \leftarrow 0$ )
9:   end if
10:   $\mathfrak{R}^{\Delta\theta_{\text{app}}} \leftarrow \mathfrak{R}^{\Delta\theta_{\text{app}}} \cdot \left( \frac{\Delta\theta_{\text{app}} - |\Delta\theta|}{\Delta\theta_{\text{app}} - \Delta\theta_{\text{md}}} \right)$ 
11: end procedure

```

4.3 Rule 17 – Stand-on

Stand-on vessels are by definition the vessel not assigned give-way responsibilities for an encounter requiring one vessel to keep clear (i.e., Rule 16 give-way). Stand-on vessels are not necessarily limited to situations of a crossing encounter with two power-driven vessels.

Algorithm 8 Check for Non-Readily Apparent Speed Change

```

1: procedure PSEUDOCODE FOR NON-APPARENTSPEEDCHANGE()
2:    $\mathfrak{R}^{\Delta v_{\text{app}}} \leftarrow$  max penalty for non-apparent speed maneuver
3:    $\triangleright \mathfrak{R}^{\Delta v_{\text{app}}} \in [0, 1]$ , default 50%
4:    $\Delta v_{\text{app}} \leftarrow$  apparent speed reduction threshold
    $\triangleright \Delta v_{\text{app}} \in [0, 1]$ , default 50%
5:    $v_0 \leftarrow$  initial ownship speed at time of detection
6:    $v_{\text{min}} \leftarrow$  speed after slowing
7:    $\Delta v \leftarrow \left( \frac{v_0 - v_{\text{min}}}{v_0} \right)$ 
8:   if ( $\Delta v \geq \Delta v_{\text{app}}$ ) then
9:     return ( $\mathfrak{R}^{\Delta v_{\text{app}}} \leftarrow 0$ )
    $\triangleright$  sufficiently apparent speed change
10:  end if
11:   $\mathfrak{R}^{\Delta v_{\text{app}}} \leftarrow \mathfrak{R}^{\Delta v_{\text{app}}} \cdot \left( \frac{\Delta v_{\text{app}} - \Delta v}{\Delta v_{\text{app}}} \right)$ 
12: end procedure

```

The stand-on vessel is required by Rule 17 to maintain course and speed as demonstrated in Algorithm 9. A penalty should thus be assessed for changing course (Algorithm 10) and another penalty assessed for changing speed (Algorithm 11) with some reasonable tolerance for environmental conditions and noise. Consideration must be given, however, to stand-on vessels maneuvering when invoking their obligation to avoid collision when *in extremis* under Rule 17.a.ii. Stand-on vessels failing to maneuver prior to a collision have repeatedly been found partially (usually 25%) at fault by admiralty courts when not invoking this clause [1]. Environmental and contact picture-specific variables heavily influence the determination of when to maneuver as a stand-on vessel.

Further, obligations of a stand-on vessel simultaneously assigned responsibilities as a head-on or give-way vessel with another contact must take care to understand the obligations of case law as it applies to maintaining course and speed. Courts have repeatedly ruled that maintaining course and speed implies those navigational maneuvers consistent with a “steady, predictable maneuver.” This includes maneuvers for avoidance of danger or other navigational requirements that the stand-on vessel would otherwise perform (e.g., slowing to take on a pilot, maneuvering for another COLREGS obligation, etc.) [1, 35].

Stand-on vessels that determine themselves to be *in extremis* are allowed by the COLREGS to take action subject to certain restrictions. Reasonable and consistent criteria are required for determination of when to take action under Rule 17.a.ii. Once entry criteria are established, evaluation of stand-on vessels deemed to be *in extremis* should focus on safely avoiding a collision subject to the power-driven restriction of Rule 17.c.

Algorithm 9 Rule 17: Stand-on Vessels

```

1: procedure PSEUDOCODE FOR STAND-ON VESSELS
2:    $\mathfrak{R}^{17} \leftarrow \mathfrak{R}^{\text{max}}$ 
3:    $\mathfrak{R}^{17} \leftarrow$  AnalyzeSafety()  $\triangleright$  Rules 8, 17, 18
    $\triangleright$  “she shall take such action as will best aid to avoid collision”
4:    $\mathfrak{R}^{17} \leftarrow$  penalizeCourseChange()  $\triangleright$  (Algorithm 10)
5:    $\mathfrak{R}^{17} \leftarrow$  penalizeSpeedChange()  $\triangleright$  (Algorithm 11)
    $\triangleright$  “shall keep her course and speed”
6:    $\mathfrak{R}^{17} \leftarrow$  compensate for maneuvers required of normal navigation (do not penalize a stand-on vessel for maneuvering for reasons other than this contact)
    $\triangleright$  case law [1, 4, 25]
7:   if in extremis then
8:      $\mathfrak{R}^{17} \leftarrow$  compensate for maneuvers required in extremis
    $\triangleright$  do not penalize stand-on if  $r_{\text{cpa}} <$  configurable value
    $\triangleright$  “take action to avoid collision by her maneuver alone”
9:     if power-driven crossing then
10:       $\mathfrak{R}^{17} \leftarrow$  penalize port maneuvers for port contacts
    $\triangleright$  “...not alter course to port for a vessel on her own port side”
11:     end if
12:   end if
13: end procedure

```

With the exception of Rule 17.c, evaluation of evasive action should use the safety score as a primary metric for rule compliance of the stand-on vessel.

Algorithm 10 Penalize Course Change

```

1: procedure PSEUDOCODE FOR PENALIZECOURSECHANGE()
2:   if  $t_{\text{maneuver}} > t_{\text{cpa}}$  then
3:     return;
4:   end if
5:    $\mathfrak{R}^{\text{max}} \leftarrow$  maximum penalty for changing course
    $\triangleright$  default 50%
6:    $|\Delta\theta| \leftarrow$  maximum heading deviation
7:    $\Delta\theta_{\text{app}} \leftarrow$  apparent turn threshold  $\triangleright$  default 30°
8:    $\Delta\theta_{\text{md}} \leftarrow$  minimum detectable heading deviation
    $\triangleright$  default 2°
9:   if ( $|\Delta\theta| < \Delta\theta_{\text{md}}$ ) then
10:    return;
11:   else if ( $|\Delta\theta| > \Delta\theta_{\text{app}}$ ) then
12:    return ( $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} - \mathfrak{R}^{\text{max}}$ )
13:   end if
14:    $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} - \mathfrak{R}^{\text{max}} \cdot \left( \frac{|\Delta\theta| - \Delta\theta_{\text{md}}}{\Delta\theta_{\text{app}} - \Delta\theta_{\text{md}}} \right)$ 
15: end procedure

```

For stand-on vessels, a change in speed is a violation of Rule 17 within the aforementioned caveats. To quantify speed change, the speed at the declaration of entry into the stand-on obligation must be identified. A penalty can then be assigned for any subsequent speed up or slow down relative to this initial speed value. A speed change that is likely undetectable by the contact or insignificant to the collision avoidance scenario

Algorithm 11 Penalize Speed Change

```

1: procedure PSEUDOCODE FOR PENALIZESPEEDCHANGE()
2:   if  $t_{\text{maneuver}} > t_{\text{cpa}}$  then
3:     return;
4:   end if
5:    $\mathfrak{R}^{\text{max}} \leftarrow \text{max penalty for slowing}$   $\triangleright$  default 50%
6:    $\Delta v_{\text{fast}} \leftarrow v_{\text{max}} - v_0$ 
7:    $\Delta v_{\text{slow}} \leftarrow v_0 - v_{\text{min}}$ 
8:    $\Delta v_{\text{max}} \leftarrow \text{max}(\Delta v_{\text{fast}}, \Delta v_{\text{slow}})$ 
9:    $\Delta v_{\text{md}} \leftarrow \text{min detectable speed change}$   $\triangleright$  default 0.2m/s
10:  if  $\Delta v_{\text{max}} < \Delta v_{\text{md}}$  then
11:    return;
12:  end if
13:   $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} \cdot \left(\frac{v_0}{v_{\text{max}}}\right)^2$   $\triangleright$  penalize speeding up
14:   $\mathfrak{R}^{\text{rule}} \leftarrow \mathfrak{R}^{\text{rule}} - \mathfrak{R}^{\text{max}} \cdot \left(\frac{\Delta v_{\text{slow}}}{v_0}\right)$ 
    $\triangleright$  penalize slowing down (not mutually exclusive)
15: end procedure

```

should be disregarded. Speeding up or slowing down by appreciable amounts without navigational necessity, however, violates Rule 17 and can result in unnecessary complication of the collision avoidance scenario.

Similarly, course changes greater than some threshold noise level (say, 2°) must be penalized for stand-on vessels not otherwise invoking Rule 17.a.ii. Some small heading change up to the generally accepted substantial value of 30° must be increasingly penalized. An example metric uses a linear or quadratic mapping between minimum detectable and substantial course changes ($2^\circ - 30^\circ$) with a plateau of penalty outside the linear region. Several small turns resulting in a larger effective turn should also be penalized accordingly. Figure 9 demonstrates a stand-on vessel course change penalty.

4.4 Rule 13 – Overtaking

Collision avoidance routines for overtaking vessels (Figure 8a) may rely on explicit entry criteria specified in the COLREGS with respect to initial pose: a contact must be more than 22.5° abaft the other vessel’s beam. Different countries have interpreted the “coming up with” phrase to take different meanings including a notable admiralty case in England involving *Nowy Sacz* and the *Olympian* [1]. Most courts contend, however, that the overtaking rule applies when the appropriate encounter geometry exists, the astern vessel has a higher speed than the overtaken vessel, the vessels are closing range, and an expected range at CPA would reasonably require prudence.

The overtaking (higher speed) vessel is defined as a give-way vessel by Rule 16 [1, 4, 26]. Pose becomes an important aspect of measuring performance for the

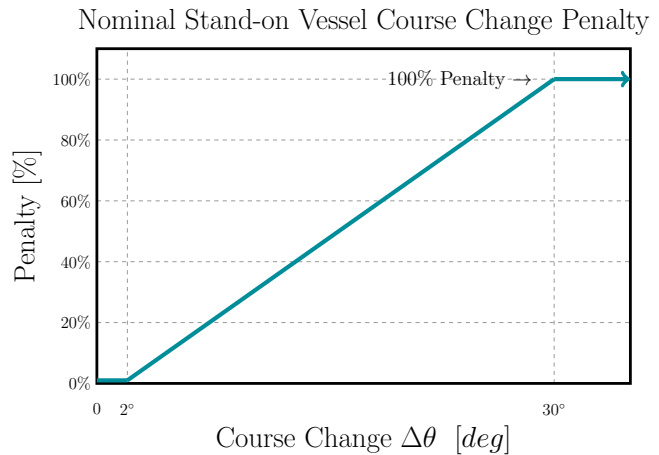


Fig. 9. The stand-on vessel is required by Rule 17 to maintain course and speed. Without a justifiable reason to alter course, a penalty may be assessed. This figure demonstrates a linear penalty for unwarranted course changes by a stand-on vessel. No penalty is assessed until a minimum detectable heading deviation is exceeded, here $\Delta\theta_{\text{md}} = 2^\circ$. In this example function, a linearly increasing penalty is invoked until a maximum penalty is reached at the apparent turn threshold of $\Delta\theta_{\text{app}} = 30^\circ$.

overtaking vessel due to both common practice and specific requirements in the Rules including her “duty of keeping clear ... until past and clear.” Overtaking on near-parallel tracks (such as in a merchant transit lane) allows for safe pose at CPA and accounts for a significant and mostly trivial case in the absence of other collision avoidance, environmental, or navigational constraints. A reasonable set of entry criteria for Rule 13 generally include a contact angle (α) within the exclusive sternlight region, a sufficient speed and relative bearing (β) for closing range, and a CPA range and CPA pose consistent with a risk of collision.

When the contact situation or initial geometry requires overtaking on non-parallel tracks such as in Figure 10, preference should be given to overtaking astern of the overtaken vessel when possible. Passing track in front of the overtaken vessel creates an encounter with higher risk and less evasive maneuverability for the overtaken vessel. Passing in front of the overtaken vessel within a range considered a risk of collision further degrades the overtaken vessel’s ability to maintain its course and speed. Therefore, a penalty is assessed for overtaking vessels who cross ahead of track of an overtaken vessel within a certain range.

The vessel being overtaken is, by the definition of Rule 17, a stand-on vessel and must keep her course and speed [1, 4, 26]. This nuance is often unknowingly neglected by autonomous collision avoidance authors and emphasizes the need for incorporation of practical at-sea experience of those involved in designing and evaluating

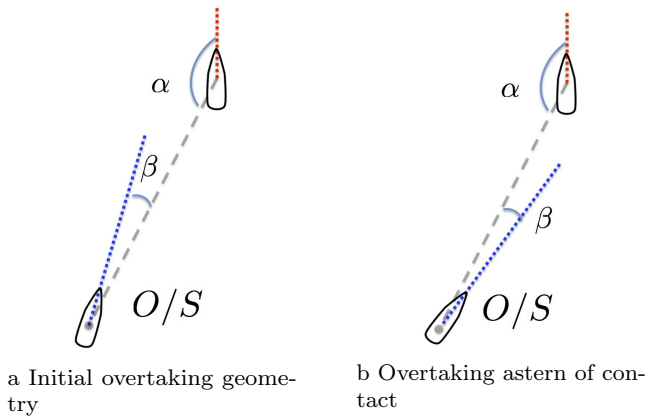


Fig. 10. Ownship's (O/S) initial encounter geometry, closing range, and proximity at CPA require action under an overtaking scenario of Rule 13. Dotted lines indicate the blue (O/S) and red (contact) speeds and demonstrate a closing range given the initial contact pose $\langle \alpha, \beta \rangle$ in (a). By appropriately altering course to starboard early in the collision avoidance encounter (b), O/S will pass to the contact's stern without causing the stand-on vessel to maneuver for a risk of collision.

collision avoidance algorithms for autonomous vessels [11, 12]. Vessels deemed to be overtaken must therefore demonstrate their obligation to maintain course and speed within the context of their contact-free intentions [1, 35].

Overtaking algorithms must be validated for correct contact angle, relative bearing, and speed considerations to verify mode entry criteria and algorithm robustness. A final necessary check in evaluation of overtaking collision avoidance algorithms is to ensure that modes do not shift from overtaking to crossing. Any mode changes from overtaking to crossing should be deemed a failure of the overtaking collision avoidance algorithm, as it violates an explicit clause of Rule 13.

A general approach for evaluating overtaking vessels under Rules 13 and 16 is shown in Algorithm 12 as amplified by Algorithm 4 and includes the following attributes:

- penalize for unnecessary crossing of contact's bow at close ranges
- penalize for unnecessary hindrance of overtaken vessel's desired maneuvers
- penalize for delayed action (range of maneuver relative to detection range and CPA range if a maneuver is required) (Algorithm 5)
- penalize for safety violations including sufficient range and early action (Rules 7-8)

A general approach for evaluating overtaken vessels under Rules 13 and 17 is shown in Algorithm 13 as amplified by Algorithm 9 and includes the following attributes:

Algorithm 12 Rule 13/16: Overtaking Vessels

```

1: procedure PSEUDOCODE FOR OVERTAKING VESSELS
2:    $\mathfrak{R}^{13/16} \leftarrow \mathfrak{R}^{16}$  ▷ (Algorithm 4)
   ▷ overtaking vessels are give-way vessels (Rules 13 & 16)
   ▷ See [1, 4, 26]
3: end procedure

```

Algorithm 13 Rule 13/17: Overtaken Vessels

```

1: procedure PSEUDOCODE FOR OVERTAKEN VESSELS
2:    $\mathfrak{R}^{13/17} \leftarrow \mathfrak{R}^{17}$  ▷ (Algorithm 9)
   ▷ overtaken vessels are stand-on vessels (Rules 13 & 17)
   ▷ See [1, 4, 26]
3: end procedure

```

- penalize the overtaken vessel in accordance with requirements of a stand-on vessel (Rule 17)
- penalize for safety violations resulting from neglecting to invoke Rule 17.a.ii
- compensate for changes in course or speed required as a result of being *in extremis*

4.5 Rule 14 – Head-on

Head-on situations (Figure 8b) provide arguably the most ambiguous entry criteria of the rules for power-driven vessels. The definition of “reciprocal or nearly reciprocal courses” is vague and left to interpretation. The compass course is required to be used when assessing course difference due to the ship-fixed masthead light and sidelight definition of ship's course in Rule 14. Confusion arises when environmental parameters greatly affect the course-over-ground; non-visual means (e.g., radar, lidar, etc.) measure course-over-ground, so care must be taken in evaluating contact geometry for proper entry criteria and resolution of ambiguity. Similarly, a consistent entry criterion for “nearly reciprocal course” should be configurable and set in accordance with local customs, case law, or certifying agency requirements. Environmental conditions such as sea-state, current, or fluctuating wind might also warrant a change to the entry criteria angle ($\alpha_{\text{crit}}^{14}$) tolerance or use of a filter.

Evaluation scenarios should incorporate sufficient set and drift to realize an appreciable distance between course-over-ground and compass heading before certification as compliant with Rule 14. Small sequential maneuvers should also be penalized, as a single, readily apparent maneuver is required (Rule 8). The size of a readily apparent maneuver is not explicitly defined in the COLREGS, though turns of 30° have been determined by custom to be sufficient [27]. Some texts suggest a minimum of 35° for a sufficient turn [1]. The intention of the rule is to ensure that turns are apparent by both radar and visual observation; the single large

turn clearly communicates to the other vessel that a risk of collision has been assumed and the vessel is taking appropriate early action in accordance with the COLREGS. This is especially important for the majority of collision avoidance encounters that do not involve explicit communication but rather rely on observation of the contact's maneuver alone.

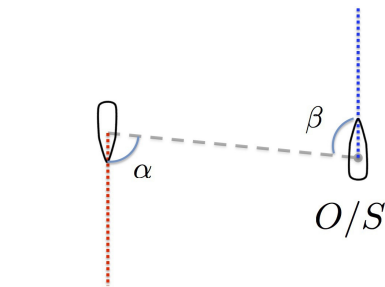
When evaluating maneuvers for a head-on scenario, both vessels must maneuver to starboard in an appreciable and timely way. Maintaining course or turning to port should be viewed as a failure to maneuver in accordance with Rule 14. Rule 14 further specifies that passing pose must be port-to-port. Pose should therefore enter into the protocol compliance metric for head-on encounters. Equation (12) demonstrates an arbitrary Rule 14 pose function ($\mathfrak{R}_{\Theta_{cpa}}^{14}$) accounting for both relative bearing and contact angle at CPA.

$$\mathfrak{R}_{\Theta_{cpa}}^{14} = \mathfrak{R}_{\alpha_{cpa}}^{14} \cdot \mathfrak{R}_{\beta_{cpa}}^{14} \cdot \mathfrak{R}^{\max} \quad (12)$$

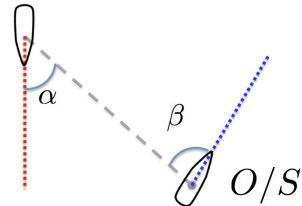
Equation (13) uses specific pose functions for both contact angle and relative bearing to give large preference to near-canonical port aspects. This example pose function uses combinations of sinusoidal functions of relative bearing (β) and contact angle (α) at CPA. A true port-to-port passage will be a relative bearing of $\beta = 270^\circ$ and a contact angle of $\alpha = -90^\circ$ as seen in Figure 11. Within an allowable tolerance, large deviations from port-to-port passage in open-ocean scenarios likely indicate insufficient or delayed maneuvers by one or both vessels. In Figure 11a, a nearly canonical head-on CPA geometry gives a high pose score. In Figure 11b, a likely late maneuver by ownship and a subsequent narrow contact angle at CPA results in a smaller pose score.

$$\mathfrak{R}_{\Theta_{cpa}}^{14} = \left(\frac{\sin(\alpha_{cpa}) - 1}{2} \right)^2 \left(\frac{\sin(\beta_{cpa}) - 1}{2} \right)^2 \mathfrak{R}^{\max} \quad (13)$$

Algorithm 14 demonstrates an approach to evaluate head-on encounters including appropriate penalties for delayed action (Algorithm 5) and non-apparent turns (Algorithm 7). Alternative functions to the specific $\mathfrak{R}_{\alpha_{cpa}}^{14}$ and $\mathfrak{R}_{\beta_{cpa}}^{14}$ of Equation (13) are available in the evaluation library discussed in Section 6. Figure 12 shows an example port-to-port pose function while Figure 13 shows a more severe preference to port angles. Possible scores range from 0 to the maximum possible protocol compliance score ($\mathfrak{R}^{\max} = 100\%$).



a Near-canonical head-on geometry at CPA.



b Head-on geometry at CPA resulting from delayed action of ownship (O/S).

Fig. 11. Rule 14 requires head-on contacts to maneuver to starboard and pass port-to-port. The geometry of a nearly canonical case (a) shows the preferred CPA geometry including relative bearing β and contact angle α . Using Equation (13), a nearly maximum pose score would result. In (b), a delayed maneuver from ownship (“O/S”) results in a port relative bearing at CPA; contact angle α_{cpa} , however, accounts for the less than ideal CPA geometry. Equation (12) would reduce the overall performance score for the situation in (b).

Algorithm 14 Rule 14: Head-on Vessels

- 1: **procedure** PSEUDOCODE FOR HEAD-ON VESSELS
 - 2: Input: α_{cpa}
 - 3: Input: β_{cpa}
 - 4: $\mathfrak{R}^{14} \leftarrow \mathfrak{R}^{\max}$
 - 5: $\mathfrak{R}^{14} \leftarrow$ assess non-starboard turn penalty
 ▷ “each shall alter her course to starboard”
 - 6: $\mathfrak{R}^{14} \leftarrow$ assess delayed action penalty
 ▷ “made in ample time” (Algorithm 5)
 - 7: $\mathfrak{R}^{14} \leftarrow$ assess non-apparent turn penalty
 ▷ “be large enough to be readily apparent” (Algorithm 7)
 ▷ 30° to 35° (configurable) minimum per case law
 ▷ (default linear penalty; configurable)
 - 8: $\mathfrak{R}^{14} \leftarrow$ assess $\mathfrak{R}_{\Theta_{cpa}}^{14}$ penalty if not port-to-port
 ▷ “each shall pass on the port side of the other”
 ▷ Equation (12); configurable per library of Section 6.1
 - 9: **end procedure**
-

4.6 Rule 15 – Power-driven Crossing

Rule 15 assigns give-way and stand-on responsibilities to each of two crossing power-driven vessels with a risk of collision (Figure 8c). The geometric entry criteria are derived from eliminating head-on and overtaking geometries while retaining a risk of collision. Relative bearing therefore spans $\{\beta : (\beta < 112.5^\circ) \text{ or } (\beta > 247.5^\circ)\}$ with an appropriate contact angle (α) such that a risk of

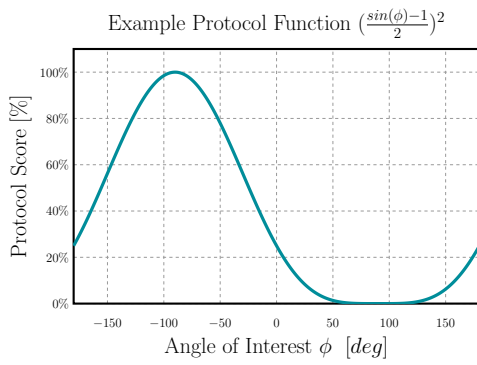
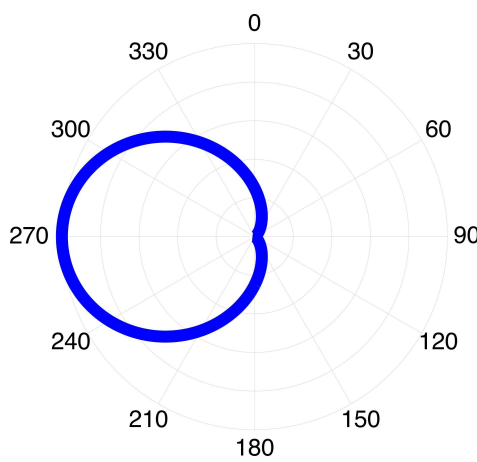
a Example protocol function $\left(\frac{\sin(\phi)-1}{2}\right)^2$ b Polar plot of $\left(\frac{\sin(\phi)-1}{2}\right)^2$

Fig. 12. The example protocol evaluation function $\left(\frac{\sin(\phi)-1}{2}\right)^2$ allows strong reinforcement of port-to-port passage when substituting both relative bearing and contact angle for ϕ as shown in Equation (13). The plot of the Rule 14 cost function (a) demonstrates high reward for near-port angles. The polar plot representation (b) demonstrates the same reward function in a top-down view more natural to a collision avoidance encounter. The radius of the polar plot indicates the percentage of \mathfrak{R}^{\max} ; the origin corresponds to zero while the outer ring corresponds to $\mathfrak{R}^{\max} = 100\%$.

collision exists without inducing head-on or overtaking obligations.

Crossing give-way vessels are specifically required to not cross ahead of the stand-on vessel; this notion has been reinforced in admiralty courts [1]. Note that a risk of collision must exist for Rule 15 to apply. Therefore a risk of collision invoking crossing give-way actions requires a stern crossing. Verification that a vessel crossed astern of the stand-on vessel is possible using Θ_{cpa} . For example, a stern crossing will result in a large negative contact angle at CPA (typically $\alpha_{\text{cpa}} < -90^\circ$) if the stand-on contact does not maneuver as shown in Figure 14. If aggressively regaining course after a stern

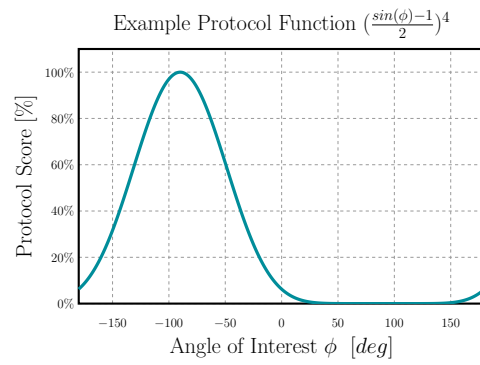
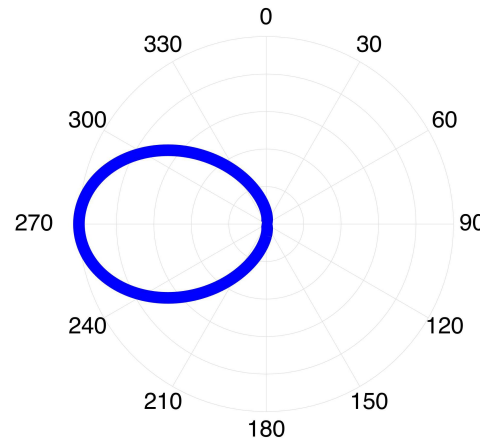
a Example protocol function $\left(\frac{\sin(\phi)-1}{2}\right)^4$ b Polar plot of $\left(\frac{\sin(\phi)-1}{2}\right)^4$

Fig. 13. A function such as this fourth-order sinusoidal function allows for more severe penalties than those of Figure 12.

crossing, the give-way vessel may reach CPA at a large positive value of contact angle (e.g., $\alpha > 165^\circ$) though this would be an exception to the norm. If the stand-on vessel determines that an *in extremis* situation exists and maneuvers to starboard, the give-way vessel should similarly be penalized for failure to act in accordance with the COLREGS.

A general approach to evaluating a crossing give-way power-driven vessel under Rule 15 can be seen in Algorithms 4 and 15 and includes the following attributes:

- penalize crossing ahead (e.g., $-80^\circ < \alpha_{\text{cpa}} < 165^\circ$ (configurable) where α_{cpa} is the stand-on vessel's contact angle if no action is taken under Rule 17.a.ii)
- penalize forcing an *in extremis* maneuver by the stand-on vessel in accordance with Rule 17.a.ii
- penalize give-way requirements of Rule 16 (Section 4.2)
- include safety penalty for early and substantial action clause of Rule 16

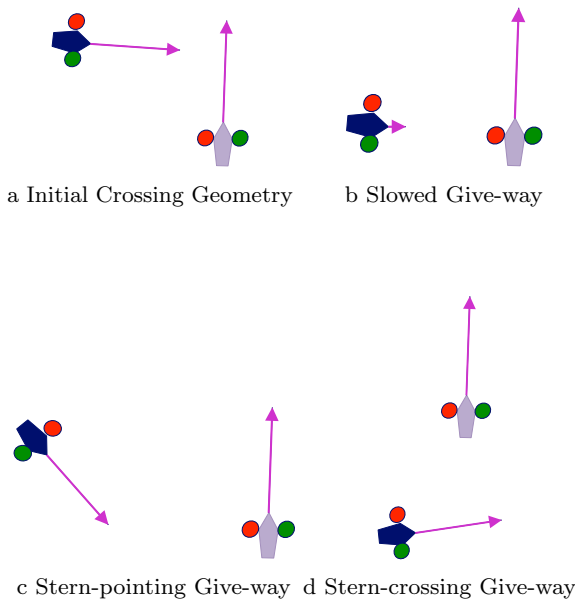


Fig. 14. Crossing astern is required of a crossing give-way vessel with a risk of collision [1]. For an initial Rule 15 crossing scenario (a), a give-way contact might slow considerably (b) resulting in a contact angle at CPA near $\alpha_{cpa} = -90^\circ$. In (c), the give-way contact has maneuvered to starboard and maintained speed likely resulting in a contact angle at CPA near $\alpha_{cpa} = -135^\circ$. In (d), the give-way contact has turned and is about to cross the stern of the stand-on vessel. Because of the forward velocity of the stand-on vessel, CPA has likely already occurred near a contact angle $\alpha_{cpa} = -165^\circ$.

Requirements of the stand-on vessel in a power-driven crossing situation are discussed in Section 4.3 and Algorithm 9.

Algorithm 15 Rule 15: Power-driven Crossing

```

1: procedure PSEUDOCODE FOR POWER-DRIVEN CROSSING
   VESSELS
2:   if crossing give-way vessel then
3:      $\mathfrak{R}^{15/16} \leftarrow \mathfrak{R}^{16}$  ▷ (Algorithm 4)
4:      $\mathfrak{R}^{15/16} \leftarrow$  assess pass-ahead penalty
5:     ▷ penalize for crossing bow of contact
6:     ▷ “avoid crossing ahead of the other vessel”
7:     ▷ see case law [1, 4, 25]
8:   else if crossing stand-on vessel then
9:      $\mathfrak{R}^{15/17} \leftarrow \mathfrak{R}^{17}$  ▷ (Algorithm 9)
10:    ▷ “crossing stand-on vessel must obey Rule 17”
11:  end if
12: end procedure

```

5 Discussion of COLREGS Rules Requiring Further Development of Evaluation Algorithms

While Section 4 presented algorithms for initial evaluation of Rules 13-17, this section presents narratives

of other rules within the protocol that require future development. Section 5.1 presents amplification of methods for Category VI of Table 1. Section 5.2 presents Category I considerations, Section 5.3 presents Category II, Section 5.4 presents Category IV, Section 5.5 presents Category VI, Section 5.6 presents Category X, and Section 5.7 presents Category IX all of Table 1.

5.1 Responsibilities of Vessels within Sight: Rules 11, 18

Identification of the contact’s type (e.g., power-driven, sailing, etc.) gives necessary knowledge for determining precedence under Rule 18. Certain vessels yield right-of-way to others by the nature of their vessel type; similarly, other vessels expect and are afforded right-of-way. To be compliant with Rule 18, autonomous vessels must be able to correctly classify vessel types and properly assign give-way hierarchy.

Detection of another vessel being under sail is insufficient for some scenarios involving multiple sailing craft in the vicinity of a power-driven autonomous vessel. In order to anticipate the likely movements of a sailing give-way to avoid a sailing stand-on, each autonomous vessel should be able to identify which sailing vessel is stand-on and which is give-way to the other. By determining environmental conditions such as wind, a power-driven autonomous vessel can anticipate a likely maneuver of a sailing give-way vessel that might interfere with ownship’s intentions to give-way to both sailing vessels.

5.2 General Rules (Rules 1-3)

Much debate exists as to whether an autonomous vessel without a human physically present constitutes a “vessel” under international law. This paper assumes that the definition accorded in Rules 1-3 apply equally to any floating structure (or “watercraft”) as if a human were physically present and operating it. Rule 3 defines the scope of COLREGS to include any “vessel” without specification of control, be it human, machine, or some combination thereof. The only distinctions drawn by the COLREGS are related to propulsion (e.g., sail, power-driven, etc.) and maneuverability (e.g., fishing, not under command, etc.) constraints. This is consistent with case law dating back to the 19th century [9, 29, 32]. As recently as 2013, the U.S. Supreme Court rejected a permanently moored house boat meeting the definition of a vessel. In doing so, the court affirmed that the definition of a vessel is met if a reasonable observer would consider it designed to a practical degree for carrying people or things over water citing the house

boat’s absence of a rudder or steering mechanism as well as a lack of capacity to generate or store electricity [30].

Accordingly, COLREGS must apply to autonomous vessels as though they were human controlled and performing the same tasks. Rule 3 further stipulates that “Vessels shall be deemed to be in sight of one another only when one can be observed visually from the other.” Various work to emulate a human lookout by use of on-board sensors and sensor processing enables “sight,” including cameras, infra-red sensors, and other similar technologies. The COLREGS deliberately address a visual requirement when two vessels are in “sight” of each other, though this does not exclude non-sight sensors (e.g., radar, lidar, sonar) from assisting with initial detection, classification, or queuing of sight sensors. Similarly, the “restricted visibility” definition of Rule 3 must consider the limitations of human-operated vessels especially as it relates to the human-visible spectrum of light; the inherent safety implications of entering a restricted visibility constraint even if an autonomous vessel’s sensors allow greater detection range than that of a human operator must be considered.

The intention of the restricted visibility sections of COLREGS are two-fold:

1. increase detectability to other contacts to maximize detection range
2. reduce allowable speed while further limiting maneuver directions to account for and partially mitigate limited detection distances

Autonomous designers require clarification from the international governing bodies as to what officially constitutes “sight” of a non-human controlled vessel.

5.3 General Conduct of Vessels and Special Traffic Schemes (Rules 4-10)

Rules 4-10 address the requirements of all vessels including the stationing of a look-out, use of safe speed, determining risk of collision, the action required to avoid collision, behavior in narrow channels, and behavior in traffic separation schemes [26]. One point of contention is the requirements of Rule 5 to maintain a look-out. Several boards have been formed in the international community to address the perceived discrepancy in what, if any, non-human means may constitute a look-out in accordance with the COLREGS. This paper assumes that any means of “sight and hearing” whether human or machine may constitute a look-out so long as it sufficiently functions within the spirit of the COLREGS and to the standards of a qualified human lookout.

5.3.1 Rule 5 – Lookout

Rule 5 requires a look-out to be stationed “by sight and hearing as well as by all available means appropriate in the prevailing circumstances.” Evaluation should prefer coordination between sight and hearing algorithms consistent with a reasonably trained human look-out. Metrics for Rule 5 under the assumption of machine-based lookout include:

- listening with on-board auditory sensors at all times while underway. Above-waterline sensors must always be functional. Sonar may supplement if installed but must never replace a surface vessel’s above-waterline auditory sensor requirement
- observing with a sufficient combination of on-board non-auditory (visual, radar, lidar, infrared, etc.) sensors at all times when underway
- conditionally supplementing with additional on-board sensors (e.g., radar), off-board sensors (e.g., accompanying aerial vehicle), and externally provided data (e.g., AIS) as necessary

5.3.2 Rule 6 – Safe Speed

Environmental factors and ship dynamics predominantly enter with Rule 6. Rule 6 specifically identifies 12 areas – assuming that the autonomous vessel has radar – requiring evaluation when determining a safe speed. The state of visibility, contact density, stopping distance, turning ability, sea state, and draft are just some of the parameters identified when determining a safe speed. Autonomous vessels must be able to independently determine their effective time-distance capabilities, turning kinematics and dynamics, and effects of contact density when selecting a maximum allowable speed to be fully compliant with Rule 6.

5.3.3 Rule 7 – Risk of Collision

As in human-operated ship driving, autonomous marine vehicles enjoy widely varying interpretation of what a “risk of collision” means based on operating style and design. Several factors allow mariners to make assumptions about the other vessel’s level of tolerance when assessing a risk of collision including vessel type, cargo, primary mission, maneuverability, and pose. For example, merchant vessels often have similar desired ranges at CPA based on common training, similar ship maneuverability characteristics, and maritime customs. A liquid Nitrogen gas tanker might have a tendency for larger, more conservative ranges at CPA than say a transiting fishing trawler who is more accustomed to

high contact density environments with greater maneuverability. Pose becomes a highly relevant consideration for determination of collision risk. Both pose at CPA and initial pose must be considered in conjunction with speed and range when assessing risk of collision.

Another consideration is the underlying flexibility of the collision avoidance algorithms. Human operators often use multiple CPA range thresholds to determine risk of collision and necessary actions. To determine risk of collision, one must know the conditions present in the decision space of the vessel as well as the vessel's current capabilities. For example, certain crew members offer greater levels of experience, while certain machinery conditions or watch-stander configurations allow for greater maneuverability or performance. Requiring the vessel's Captain or additional watch officers on the bridge for certain encounter scenarios is one example of a modified watch-stander configuration [28]. These factors directly contribute to the level of conservativeness of the subsequent maneuver.

A vessel master's policy often dictates that certain precautionary measures must be in place before taking contacts closer than certain ranges [27, 28]. This might include certain qualified watch-standers present on the bridge, certain machinery configurations, or certain environmental conditions. Similarly, restricted visibility and other detectability considerations must be considered in the determination of risk of collision.

Two considerations are specifically required as components of determination of collision risk, namely, 1) contacts with constant compass bearing with corresponding decreasing range, and 2) approaching large vessels, tows, or close range contacts. If either of these two areas are not explicitly considered in risk determination, an immediate failure score is warranted for Rule 7. Additional metrics should include the appropriate configuration of range thresholds, the tolerance for determining a constant bearing/decreasing range scenario, early warning capabilities, and analysis of "scanty information". Such scanty information [26] might be considered with appropriate weight based on radar return strength, fusion of other sensor data, and sensor filter settings.

5.3.4 Traffic Separation Schemes (Rule 10)

Traffic separation schemes comprise a special subset of vessel interactions similar to those of driving on a highway with a car. The most prominent solution to driving in this scheme is [21]. Development of an evaluation algorithm for protocol compliance using only track data in future work would be well served to consider the approaches of algorithms such as this.

5.4 Sailing in Sight of Another Sailing Vessel (Rule 12)

Sailing vessels must be properly identified in order to discriminate precedence per Rule 18 and, in the case of ownship also being a sailing vessel, determine stand-on and give-way status per Rule 12. In the case of both vessels being under sail, proper identification of the windward side of both vessels is required (both wind direction as well as the location of the mainsail or the largest fore-and-aft sail). Failure to properly identify other sailing vessels must result in a failure of Rule 12. Further evaluation using the requirements of Rules 16 and 17 applies as appropriate.

5.5 Restricted Visibility (Rule 19)

In addition to the discussion of Section 5.2, Rule 19 addresses situations of reduced visibility, i.e., when vessels cannot see the other due to the environmental reasons prescribed in Rule 3. Specific checks should be made during algorithm testing to ensure restrictions are in place to limit speed consistent with Rules 6 and 19. The two specific cases addressed in Rule 19.d should be explicitly tested in conditions emulating restricted visibility, including:

- ensuring a vessel does not alter course to port for a vessel forward of the beam, except in cases of overtaking
- ensuring a vessel does not alter course toward a vessel abeam or abaft the beam

Testing should also consider the cases of auditory detection of fog signals ahead of the beam to ensure invocation of the bare-steerage clause of Rule 19.e.

5.6 Lights and Shapes (Rules 20-31)

There are two main areas of scope in the lights and shapes section of the COLREGS. First, designers must properly display the required lights and shapes on ownship according to certain ship characteristics. This requires self-awareness of whether a particular section of the COLREGS which requires special lights and shapes applies. In addition, the ability to actually transmit the appropriate signal for the correct duration of time is required. Second, a vehicle must be able to properly identify lights and shapes of other vessels including assignment of proper meaning. This recognition and application to the contact directly complements Rule 18 requirements of precedence with respect to ownship's collision avoidance role *viz* stand-on or give-way. Special lights and shapes may be necessary for autonomously

operated vessels to display. The community would be well served by international governing bodies issuing guidance stating whether a special day shape or light signal is required to identify autonomously operated vehicles, and if so, making such display or signal standard across the world⁵.

5.7 Sound and Light Signals (Rules 32-37)

Quite similarly to the lights and shapes requirements of Section 5.6, vessels must be able to properly communicate using sound and light signals in accordance with the COLREGS. Autonomous vessels are in need of clarification of any special sound or light signals required for autonomous vessels. To avoid *ad hoc* signals intended to indicate an autonomous vessel encountering a human vessel, international governing bodies should provide articulated guidance. Advances in the field within the scope of this section would focus on multiple areas including:

- receiving a contact’s light and sound signals
- interpreting these light and sound signals then influencing ownship’s autonomous collision avoidance behaviors appropriately
- transmitting light and sound signals to a contact when ownship autonomously determines necessity in accordance with the COLREGS

6 COLREGS Testing and Evaluation

A COLREGS testing and evaluation software program was designed to be used from a third party neutral “shoreside” observer with assumed perfect sensing data of the vessels under observation in [31]. The purpose of the testing and evaluation program is to act as a neutral grader of a ship’s performance in complying with the COLREGS, especially in the absence of human intervention. Third party perfect sensing represents a reasonable assumption for a road test or other evaluator entity, as the vessel autonomy could be evaluated in a well-sensored testing range with verified GPS-based location data recorded for all vessels. A position reporting protocol such as AIS may prove satisfactory if reports can be deemed trustworthy.

Future work could incorporate sensor fusion and imperfect sensing scenarios that would enable this concept to be used outside the realm of certification-focused testing and evaluation. Scope of the testing and evaluation

⁵ Submarines operating on the surface are currently the only special signal not contained as a requirement within the numbered international rules.

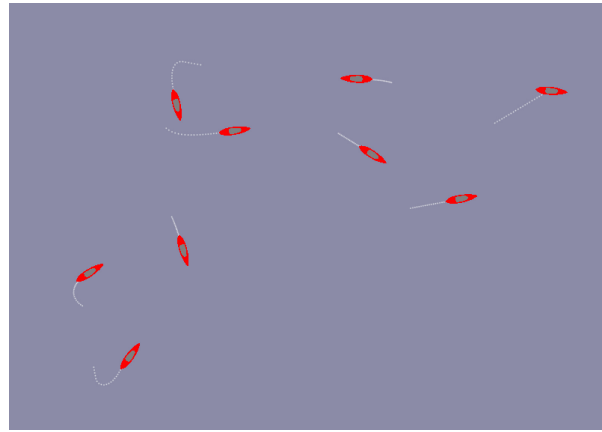


Fig. 15. COLREGS evaluation will be capable of evaluating complex scenarios such as this non-canonical geometry, multi-rule encounter. Note that multiple collision avoidance rules exist simultaneously between these power-driven vessels.

program was limited to power-driven vessel rules, specifically Rules 13-18. A library was developed to allow for both real-time (Section 6.3) and post-mission analysis (Section 6.4). Complex multi-vehicle encounters such as the one shown in Figure 15 are capable of real-time or post-mission analysis.

The testing and evaluation program for a multi-contact power-driven scenario includes the ability to:

- identify that the geometry of two vehicles requires action per the COLREGS
- identify the specific rules assigned to each vessel
- quantify the actions of each vessel with respect to the identified rules
- generate a report of each vessel’s actions at the conclusion of the encounter
- populate a scoring system for each vehicle and a cumulative performance assessment based on various scenarios and interactions over a specified duration
- provide quantified data to support determination of a vessel’s scope of COLREGS compliance after performing specified encounters. Sufficient interactions in various multi-vessel, multi-rule scenarios are necessary as part of the “road test” described in Section 6.5 and presented in [33].

Multi-contact scenarios often involve sets of rules that require differing action. Priority must be assigned as to what action should be taken given the larger scope of the navigation and collision avoidance pictures. Evaluation algorithms must account for the requirements of individual rules while also considering the larger contact picture and more overarching rules of the protocol.

6.1 Library

The protocol-constrained collision avoidance evaluation library allows a common repository for evaluation algorithms. The library enables expansion of functionality to multiple programs using a common set of algorithms while maintaining standardized configuration of collision avoidance parameters and adaptability to other protocol rule sets. This allows real-time and post-mission analysis programs to use equivalent means of evaluation; however, it also allows post-mission evaluation using different penalty functions or configuration settings according to the evaluator's preference. The library of algorithms allows configuration parameters to properly tune weights and metrics to local customs or requirements of certification authorities.

Users may use Equations (14)-(17) as an initial library to construct relevant evaluation functions based on pose angles. The input angle ϕ may be configured to use the contact angle α or relative bearing β . A steering angle ϕ_0 allows tailorable directionality for alternative use of the same functions. An alternative use case might represent a passing arrangement agreed via bridge-to-bridge radio, such as a rare and non-conforming starboard-to-starboard passage. Linear and quadratic functions of range and speed are also available within the initial evaluation library release. Incorporation of other functions and input parameters is reserved for future work. These functions are introduced to promote continued dialogue of alternative evaluation functions and approaches, including those rules whose evaluation algorithms have yet to be fully developed.

$$\sin^2(\phi + \phi_0) \quad (14)$$

$$\text{step}(\phi) - \text{step}(\phi_0) \quad (15)$$

$$\left(\frac{\sin(\phi + \phi_0) - 1}{2} \right)^2 \quad (16)$$

$$\left(\frac{\sin(\phi + \phi_0) - 1}{2} \right)^4 \quad (17)$$

6.2 Configurability of Programs

The evaluation programs are configurable for several parameters of interest to a designer or evaluator including:

- preferred range at CPA
- minimum acceptable range at CPA
- range at which a near-miss occurs
- range at which a collision is assumed

- threshold COLREGS rule compliance score below which instantaneous reports should be made
- threshold safety score below which instantaneous reports should be made
- vessel types to consider (allows knowledge of aerial, ground, and undersea vehicles without interference of collision avoidance evaluation)
- range at which contact detection likely occurs
- maximum time threshold allowed for comparison of a contact's position report and ownship's position report
- display of visual indicators when configuration ranges or minimum rule compliance scores are violated
- sounding of audible alerts when configuration ranges or minimum rule compliance scores are violated

6.3 Real-time Analysis

Using the protocol library for COLREGS presented in this paper, a real-time collision avoidance evaluation program gives instantaneous feedback to vessel designers and a means of real-time evaluation to any certification entity. This can be used to assign penalties or warnings to vessels violating the COLREGS, especially but not limited to training and design verification scenarios. Notifications can be sent to vessels in the vicinity of non-compliant actors to allow increased caution while operating. Reports of egregious actions can be passed to designers, insurance agencies, or enforcement entities as appropriate or required by statute.

Within the scope of the current work, the real-time protocol evaluation tool was used to display important information at the shoreside observation center including:

- COLREGS compliance scores for power-driven rules
- safety scores after an encounter
- rules required as determined by the observer
- range at CPA
- time of CPA
- vessel names and types

A real-time text report is posted to the mission console including summaries of overall performance (e.g., safety, protocol compliance, type of interaction) as shown in Figure 16. To assist a shoreside observer with several vehicles underway, a series of visual and audible indicators were incorporated to provide real-time warning of dangerous or inappropriate action. Colored range rings (Figure 17) appeared whenever violations occurred including:

- green - r_{cpa} less than minimum acceptable CPA range

```

=====
uFldProtocolEval shoreside                                0/0(554)
=====
Delay to Clear Messages [sec]: 500
Checking for Compliance: ON
Vehicle Types Checked: kayak,sailboat
Pulse on Violations: true

Collision Range: 3
Near-Miss Range: 5
CPA Violation Range: 15
Preferred CPA Range: 25
Nominal Detection Range: 30

COLREGS Violations: 2 (Threshold = 80)
Safety Violations: 2 (Threshold = 80)
Collisions: 0
Near-Misses: 0
CPA Violations: 2
Tolerable CPAs: 0
Greater than Preferred CPAs: 0

UID  Ownship  Contact  Rule#  Safety  COLREGS  CPA  Type  Tcpa
-----
0    abe      ben     15/17  60.70  0.00    13.2 min_cpa_violation  7.8
0    ben      abe     15/16  60.70  76.30   13.2 min_cpa_violation  7.8

```

Fig. 16. Scoring of COLREGS collision avoidance rules allow for real-time evaluation of vehicle performance at the shoreside observation center. Configurable range parameters include nominal detection range, preferred range at CPA, minimum acceptable range at CPA, threshold range at which a near-miss occurs, and the range at which a collision is assumed. An aggregate tally of COLREGS violations (scores below a configurable threshold value) and of each configuration range are displayed. Vehicle types as specified by each vessel are used as a filter to allow consideration of only certain entities within the “visibility” of the shoreside observation center. This allows underwater and aerial vehicles to share the shoreside observation display without unnecessarily being considered as COLREGS compliance candidates.

- yellow - r_{cpa} less than near-miss range
- red - r_{cpa} less than collision range
- blue - COLREGS score less than the threshold value

6.4 Post-mission Analysis

A post-mission analysis tool was constructed to provide detailed insight into collision avoidance performance of vessels. The post-mission analysis requires only vehicle position logs; the real-time assessment program was not required to be running to conduct post-mission analysis.

A report is generated for each run of the post-mission analysis tool with a configurable scale of verbosity. In more verbose modes, detailed explanations of cause for score deduction allows designers and operators to understand the rationale for evaluation scores. This can be used to provide feedback and tune future actions. In addition to the verbosity option, all configuration parameters of Section 6.1 are available in the post-mission analysis tool. Evaluation data are exported to a comma-separated value report for ease of meta analysis in a user’s favorite data analysis program. This data can then be used for performance analysis by vehicle, by rule combination, or by other parameters of interest to the evaluator.

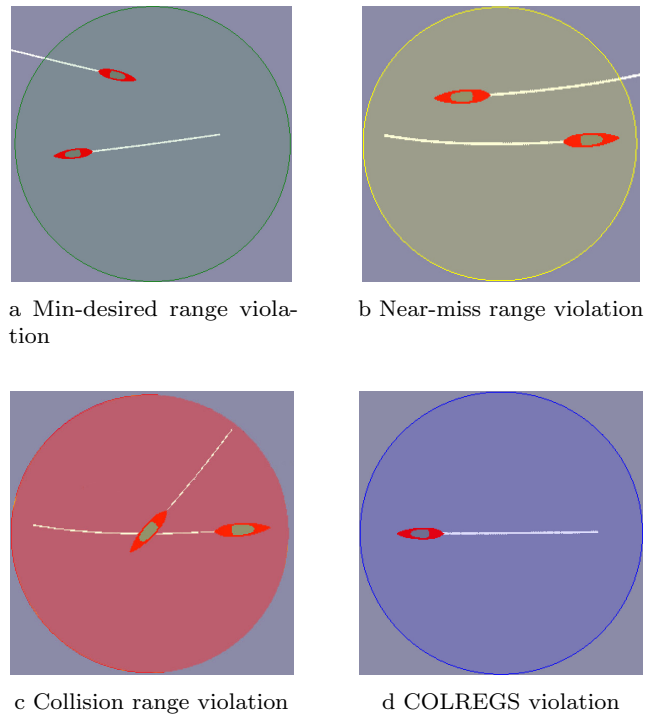


Fig. 17. Violations of range below configurable threshold values resulted in displaying range rings including: (a) green for violating minimum desired range, (b) yellow for violating near-miss range, and (c) red for violating the collision range. Violations of COLREGS collision avoidance rules below a configurable threshold value result in display of a blue ring (d) around the vehicle. Sounding of an optional audible indicator was also possible for each violation. This COLREGS evaluation system allows real-time warning to an evaluator that a dangerous collision situation or egregious violation of the rules is occurring.

6.5 Informing the COLREGS Certification Road Test

In order to certify autonomous collision avoidance algorithms for on-water use outside of a testing environment, a road test comprised of a comprehensive scope of examination and quantifiable metrics of performance was described in [33]. To be compliant with the appropriate protocol rule set of COLREGS, a satisfactory level of performance must be met across each category of evaluation. The categories of Section 2 comprise the evaluation areas for this test and are the principal means for the road test of [33] to be performed. Differing degrees of road tests may be possible for various levels of certification for operation.

7 Conclusion

This paper defined metrics and algorithms to quantify protocol compliance and safety for autonomous collision avoidance. A library of functions was proposed

to allow configuration of the protocol evaluation tools in both real-time and post-mission analysis. Specific instantiation of protocol quantification and evaluation was demonstrated for the rules of the road for sea-going vessels, i.e., COLREGS.

Real-time on-vehicle instantiations of the COLREGS evaluation program in this paper are possible to detect non-compliance of other vessels and adjust collision avoidance parameters accordingly. Future work will allow a vessel to detect vessels with compliance scores less than a threshold value and choose to maneuver sooner than normal or seek a more conservative range or pose at CPA.

Future work will enable third-party evaluation of the full rule sets rather than limitation to power-driven vessels. Further work is required to more fully model contact-free intentions when maintaining course and speed of a stand-on vessel in complex scenarios such as slowing to pick up a pilot. Further discussion and research is needed to fully incorporate local customs and laws within COLREGS (including U.S. Inland Rules), alternative protocols such as Rules of the Air, and special arrangements such as those made by bridge-to-bridge radio. Alternative approaches to evaluating pose may show cases where safety functions are more limited at times other than the closest point of approach.

Before integrating human controlled and autonomous systems outside of laboratory environments, the common practices, customs, and interpretations of the COLREGS by manned operators must be fully understood. Autonomous designs that incorporate expectations and norms of human operators will achieve solutions that more naturally integrate autonomous and human-operated vessels. The categorization of scope and the incorporation of the nuance, applicable case law, and customs related to COLREGS allows appropriate and quantifiable assessment of autonomous collision avoidance performance. The International Maritime Organization, and other governing bodies, may choose to include these metrics as a means to inform both regulation and policy in maritime collision avoidance protocols.

7.1 Abbreviations and Acronyms

α	- contact angle
α_0	- contact angle at detection
α_{cpa}	- contact angle at CPA
α_c	- cutoff contact angle to define reward functions
α_{crit}	- critical cutoff angle for entry criteria
β	- relative bearing
β_0	- relative bearing at detection
β_{cpa}	- relative bearing at CPA

β_c	- cutoff rel. bearing to define reward functions
ϕ	- arbitrary angle for generic functions
ϕ_0	- arbitrary steering angle for generic functions
CPA	- closest point of approach; point of min. range
r	- current range to contact
r_{cpa}	- predicted or actual CPA range (distance)
r_{detect}	- range at which a contact is detected
r_{eff}	- effective range after adding pose component
$r_{maneuver}$	- range at which vessel maneuvers
R_{col}	- range that assumes physical collision
R_{detect}	- assumed range of contact detection
R_{min}	- minimum acceptable CPA range
R_{nm}	- range considered a near-miss encounter
R_{pref}	- preferred CPA range
\mathfrak{R}	- COLREGS rule compliance function
\mathfrak{R}^{max}	- max possible COLREGS compliance score
\mathfrak{R}_Θ	- pose component of COLREGS compliance
S	- safety function for an encounter
S_r	- range component of safety function
S_Θ	- pose component of safety function
S_Θ^{max}	- maximum possible pose score
t_{cpa}	- time of CPA
$t_{maneuver}$	- time of maneuver
θ	- candidate or actual course ($\theta \in [0, 360^\circ)$)
$ \Delta\theta $	- absolute change in course
$\Delta\theta_{app}$	- apparent course deviation threshold
$\Delta\theta_{md}$	- minimum detectable course deviation
Θ	- pose consisting of $\langle\alpha, \beta\rangle$
Θ_o	- initial relative pose (determines rule set)
Θ_{cpa}	- relative pose at CPA (measure of safety)
v	- speed
v_0	- initial speed at time of detection
Δv	- change in speed during maneuver
Δv_{app}	- apparent speed reduction threshold
Δv_{slow}	- slow down of vessel during maneuver
Δv_{fast}	- speed up of vessel during maneuver
Δv_{max}	- greater of absolute speed up or slow down
Δv_{md}	- minimum detectable speed change
$_{-180^\circ}$	- angle converted to $[-180^\circ, 180^\circ)$
$_{-360^\circ}$	- angle converted to $[0, 360^\circ)$

References

1. Allen, C.: Farwell's Rules of the Nautical Road. Blue and gold professional library series. Naval Institute Press (2005)
2. Benjamin, M.R., Leonard, J.J., Curcio, J.A., Newman, P.M.: A Method for Protocol-based Collision Avoidance Between Autonomous Marine Surface Craft. Journal of Field Robotics **23**(5), 333 – 346 (2006)
3. Campbell, S., Abu-Tair, M., Naeem, W.: An automatic colregs-compliant obstacle avoidance system for an unmanned surface vehicle. Proceedings of the Institution of Mechanical Engineers, Part M: Journal of Engineering for the Maritime Environment p. 1475090213498229 (2013)

4. Cockcroft, A.N., Lameijer, J.N.F.: Guide to the Collision Avoidance Rules. Butterworth-Heinemann (2012)
5. Commonwealth of Massachusetts: Right-of-way at Intersecting Ways; Turning on Red Signals, Part 1 Title XIV Chapter 89 Section 8 edn.
6. Fisher, M.: Evaluation of The Vertical Sector Light Requirements for Unmanned Barges; Final Rept. Tech. rep., US Coast Guard (1991)
7. Goerlandt, F., Kujala, P., 2014. On the reliability and validity of ship-ship collision risk analysis in light of different perspectives on risk Safety Science 62, 348-365.
8. Goerlandt, F., Montewka, J., Kuzmin, V., Kujala, P., 2015. A risk-informed ship collision alert system: framework and application Safety Science. 77, 182-204.
9. Henderson, A.: Murky Waters: The Legal Status of Unmanned Undersea Vehicles. Naval Law Review pp. 55-72 (2006)
10. International Civil Aviation Organization: Rules of the Air. Convention on International Civil Aviation (2005)
11. Kuwata, Y., Wolf, M., Zarzhitsky, D., Huntsberger, T.: Safe Maritime Autonomous Navigation With COLREGS, Using Velocity Obstacles. Oceanic Engineering, IEEE Journal of **39**(1), 110-119 (2014). DOI 10.1109/JOE.2013.2254214
12. Kuwata, Y., Wolf, M.T., Zarzhitsky, D., Huntsberger, T.L.: Safe Maritime Navigation with COLREGS Using Velocity Obstacles. IEEE Conference on Intelligent Robots and Systems pp. 4728-4734 (2011)
13. H.-J. Lee and K. P. Rhee. Development of Collision Avoidance System by Using Expert System and Search Algorithm. International Shipbuilding Progress 48.3 (2001), pp. 197212.
14. Y.-i. Lee and Y.-G. Kim. A Collision Avoidance System for Autonomous Ship Using Fuzzy Relational Products and COLREGs. Intelligent Data Engineering and Automated Learning. Springer, 2004, pp. 247252.
15. Lopez-Santander, A., Lawry, J. 2017. An Ordinal Model of Risk Based on Mariner's Judgement. Journal of Navigation. 70(2), 309-324.
16. Perera, L.P., Guedes Soares, C., 2015. Collision risk detection and quantification in ship navigation with integrated bridge systems. Ocean Engineering. 109, 344-354.
17. Shah, B.C.: Trajectory Planning with Adaptive Control Primitives for Autonomous Surface Vehicles Operating in Congested Civilian Traffic. IEEE International Conference on Intelligent Robots and Systems (2014)
18. Švec, P., Shah, B., Bertaska, I., Alvarez, J., Sinisterra, A., von Ellenrieder, K., Dhanak, M., Gupta, S.: Dynamics-Aware Target Following for An Autonomous Surface Vehicle Operating Under COLREGs In Civilian Traffic. In: International Conference on Intelligent Robots and Systems, pp. 3871-3878 (2013). DOI 10.1109/IROS.2013.6696910
19. Szlapczynski, R.: Evolutionary Planning of Safe Ship Tracks in Restricted Visibility. Journal of Navigation **68**, 39-51 (2015). DOI 10.1017/S0373463314000587
20. Szlapczynski, R., Szlapczynska, J.: Colregs compliance in evolutionary sets of cooperating ship trajectories. Electronic journal of international group on reliability, Reliability: Theory and Applications **2**(1), 127-137 (2011)
21. Szlapczynski Rafalapczyński, R.: Evolutionary Sets of Safe Ship Trajectories Within Traffic Separation Schemes Journal of Navigation (2013)
22. Szlapczynski, R., Szlapczynska, J., 2016. An analysis of domain-based ship collision risk parameters. Ocean Eng. 126, 47-56.
23. Tam, C., Bucknall, R.: Collision Risk Assessment for Ships. Journal of Marine Science and Technology **15**(3), 257-270 (2010)
24. Tam, C., Bucknall, R.: Path-planning algorithm for ships in close-range encounters. Journal of marine science and technology **15**(4), 395-407 (2010)
25. Thomas, D.: The Fatal Flaw: Collision at Sea and the Failure of the Rules. Phaiacia, Carmarthenshire, United Kingdom (2001)
26. United States Coast Guard: Navigation Rules, International-Inland. 1977-19. United States Department of Transportation (1999)
27. United States Coast Guard: Standing Orders for The Officers of The Deck (Freedom of Information Act). USCGC HEALY Instruction M1603.1C (2006)
28. United States Navy: Standing Orders for USS Greeneville (Freedom of Information Act). USS Greeneville Instruction C3120.25 (1999)
29. U.S. Supreme Court: Stewart v. Dutra Construction Co. 543 U.S. 481 (2005)
30. U.S. Supreme Court: Lozman v. City of Riviera Beach, Fla. (2013)
31. Woerner, K.: Multi-Contact Protocol-Constrained Collision Avoidance for Autonomous Marine Vehicles. Ph.D. thesis, Massachusetts Institute of Technology (2016)
32. Woerner, K.L.: COLREGS-Compliant Autonomous Collision Avoidance Using Multi-Objective Optimization with Interval Programming. Master's thesis, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139 (2014)
33. Woerner, K.L., Benjamin, M.R., Novitzky, M., Leonard, J.J.: Collision Avoidance Road Test for COLREGS-Constrained Autonomous Vehicles. In: OCEANS 2016 MTS/IEEE Monterey (2016)
34. Zhang, J., Zhang, D., Yan, X., Haugen, S., Guedes Soares, C., 2015. A distributed anti-collision decision support formulation in multi-ship encounter situations under COLREGs. Ocean Engineering. 105, 336-348.
35. Zhao, J.: The Legal Interpretation of Keeping Course and Speed. Journal of Maritime Law & Commerce **41**, 85 (2010)