A POLICY SENSITIVE MODEL OF FREIGHT DEMAND

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ABSTRACT

This study represents a first attempt to develop a freight demand model which includes the entire set of relevant short-run choices open to a firm in its logistics management process. A disaggregate model of mode, shipment size and origin choice is developed at the level of the individual firm. This approach has allowed explicit consideration of the tradeoffs the firm can make in response to a short-run change in transport level-of-service. The major assumption of this study is that the substitution between transportation and other factors of production such as labor, capital, etc., is relatively inelastic when compared to the substitutions which can take place within the transportation sector itself.

Based on the logistics decision process, a framework was developed and then used to derive a set of disaggregate choice models involving the full set of logistics choices. A general specification of the cost function has been presented. The theoretical portion of the study is then followed by an empirical estimation of the proposed random cost model of freight demand. In order to undertake the empirical estimation, a disaggregate data base has been developed including intercity shipment flows, level-of-service attributes, commodity attributes, receiver attributes, and market attributes.

The resulting models appear to be quite good with coefficients which are, for the most part, logically correct, of the proper sign and statistically significant. The rates of substitution between coefficients and the elasticities which have been developed from the models are intuitively reasonable and quite instructive. There are however innumerable ways in which these estimates can be improved through the use of better data or improved level-of-service models.

Thesis Supervisors: Paul O. Roberts, Professor of Civil Engineering
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Table of Contents

LIST OF FIGURES

(List of Figures, Continued)

CHAPTER 1

INTRODUCTION

1.1. The Need for a Policy Sensitive Model of Freight Demand

The interest in planning and analysis work in freight transportation has increased greatly over the past few years for a variety of reasons. One key factor is the growing complexity of policy issues involving freight. Freight transportation, long a central concern in government highway construction and regulatory policy making, has recently become a significant element in issues involving energy conservation, air pollution, foreign trade, inflation, economic growth and regional development to name only a few.

An important part of any quantitative analysis of freight transport is a capability for forecasting the demand for a certain type of service under a given set of conditions. A policy sensitive model of freight demand to predict the impact on flows resulting from changes in freight transportation services is a critically important element if the full implications of proposed policy issues affecting transportation are to be understood prior to implementation.

To evaluate the consequences of a change in transportation policy quantitatively, a proposed policy action is translated into a set of changes in the level-of-service of the various transport service offerings. The demand model is then used to forecast the resulting changes in demand. Changes in demand will, in turn, result in changes in carrier revenue, energy use or pollution, to

mention just a few impacts. A demand model is policy sensitive if it is capable of satisfactorily forecasting the impact of policies on the demand for freight transport.

It is important to note that a single change in policy can have very different effects on different groups of shippers. For example, an expansion of the unregulated zone for pickup and delivery services of air carriers could greatly affect the demand for air carriage by shippers located just outside current service areas, while the demand generated by more distant shippers would be less affected. A model which predicts the demand for air carriage at the national or even at the regional level would lack the detail required for an analysis of the important differential impacts caused by this policy. Similarly, some policies have impacts which are specific to certain industries. Thus, to function as an effective policy analysis tool, the demand model should be capable of predicting demand at a fairly disaggregate level with respect to both geographic areas and to industries.

The above requirements can be fulfilled by two general approaches: a system of commodity and location specific models or alternatively a single, commodity and location abstract model. The latter is obviously more attractive since it is flexible and involves fewer data requirements. If a demand model is constructed based on the behavior of the individual decision-maker at the disaggregate level (as described by his attributes), it is possible to use it to predict

the impact on demand for any industry or commodity in any region at any time. The transferability over space, time and commodity is thus an important property of a useful freight demand model. From time to time, policy proposals in the freight area involve the introduction of completely new services, or greatly modified versions of existing services. In these cases a model restricted to a certain type of service will not be applicable and since there are no data on which re-estimation of the original model can be performed, the policy cannot be analyzed with existing aggregate approaches. Since the modes are described in terms of their level-of-service attributes, a behaviorally-oriented model has the ability to forecast the demand for a new mode.

1.2. Summary of the State of the Art

The state of the art in freight demand modelling is still rather primitive. It is clear that the firm is the basic decision-making unit in freight transportation. However, the role of the firm in selecting freight transport service has not been explored satisfactorily. No really operational, firm-oriented freight demand model has been developed and implemented for planning in the real world. Most of the existing freight models are correlative rather than explanatory and completely insensitive to changes in transport level-of-service measures. This is due to a number of factors; first, the data limitations. Data which can be used to undertake a careful estimation of a disaggregate behavioral freight demand model are

almost nonexistent. Thus, researchers in the past have been con-
strained to either piecing together useful aggregate data to esti-
mate an aggregate demand model[1] or to using shipper surveys to esti-
mate very limited shipper choice models.[2]

A second limitation comes from the fundamental difficulties
which most researchers have experienced in attempting to apply
economic theories of derived demand to freight demand analysis
without making unattractive simplifying assumptions. One frequently
used assumption is constant transport cost. That is, the freight
rate is assumed not to be influenced by the quantity shipped.
This makes the model policy insensitive to changes in the transpor-
tation level-of-service. In fact, in practice freight rates are
a decidedly decreasing function of shipment size. There are clearly
economies to the shipper to larger shipment sizes.

Finally, the true cost of transport should include inventory
costs as well as tariff charges which results from the logistics
management process and are thus also a function of shipment size.

Existing freight demand studies are also characterized by their
different orientations. Models have been developed by researchers
from many disciplines using many different approaches in an attempt
to solve many different problems. This is just one indication that

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[1] For examples, Morton (1969), Tihansky (1972), Wang and Epstein
(1975) and Sloss (1971).

[2] For examples, Miller (1972), Mathematica (1969), and Watson
et al. (1974).

freight transportation involves a complicated decision-making process. As a matter of fact, the choices that a firm should consider range from the long-run decisions of plant location, technology of production, etc., to the more short-run decisions involving the choice of mode and shipment size for its inputs and outputs. These decisions are all related to the demand for freight transportation. The phenomena involved are too complicated to be represented by a simple model. A simple model can just not cover all the interacting dimensions that are involved. A system of well-structured models is required to be able to describe the transportation-related decision-making process satisfactorily.

1.3. Summary of the Study

The purpose of this study is to develop useful tools for analyzing the impact of alternative policies on the demand for freight transportation. The model developed is disaggregate, explanatory, and considers the firm as the basic decision-making unit. It is disaggregate because its specification reflects the viewpoint of an individual decision-maker. It is explanatory because the variables are designed to represent those values which the firm uses in its decision-making process.

The study begins with a critical review of the economic theories in the literature which have been used to derive factor demand. Transportation is a factor of production for most firms. Consequently, transportation demand should not be analyzed apart

from these theories of factor demand. Fundamental difficulties
in the empirical estimation of demand functions in general and for
freight transportation demand functions in particular are discussed
(Secions 2.4 and 2.5). To resolve the dilemma in modelling freight
demand raised by this examination of the literature, the complica-
ted interrelationships involving freight transport are reformulated
in terms of a hierarchical framework involving a long-run location
choice, an intermediate-run production choice, and a short-run
logistics choice (Section 2.6). Criteria by which one can identify
the level of a given transportation decision in the proposed
hierarchy and guidelines for approaching the modelling of these
choices are discussed. A literature survey is then conducted to review
the previous studies of freight transport demand and to investi-
gate their underlying assumptions in terms of the general frame-
work (Chapter 3).

The short-run logistics choice involves the choice of mode,
shipment size and point of supply given the annual use rate of
inputs. The annual use rate is treated as given. In terms of produc-
tion theory, this is somewhat restrictive, since it allows no factor
substitution between the demand for transportation and other factors,
but it does allow for substitution within the transportation and
logistics cost elements. Therefore, a demand model using the logis-
tics choice process as its basis must be categorized as a short-run
freight demand model.

The study then proceeds to develop the short-run model of freight demand. The theoretical background for a short-run model is presented in Section 4.2. The cost function and its relationship to the tradeoffs in the logistics management process are discussed. Each cost element in the logistics cost function is examined in detail. This examination then serves as the basis for the formulation of the demand model of logistics choice. One can either formulate the model based on a deterministic cost function or as a random cost function (Section 4.3). The limitations of the deterministic formulation are critically examined in Section 4.4. The random cost formulation is more attractive since there is typically a great deal of uncertainty in the information. The random cost model assumes that the logistics cost function contains an observable part and an unobservable random part. Thus, the choice probabilities can only be predicted probabilistically. A general formulation of the random cost model of logistics choice is presented in Section 4.5. Modelling techniques to estimate the random cost function are then investigated and issues of implementation are discussed.

The study is then followed by an empirical estimation of the proposed random cost model. First, a disaggregate data base must be developed. Data to permit the estimation of a disaggregate, commodity and location abstract freight demand model are not available. Chapter 5 describes the procedures and assumptions used to prepare a useful data base. Models formulated in Chapter 4 are then

calibrated against this data set. The models are estimated in the logit model environment involving a joint choice of mode, shipment size, and origin of supply. The estimation results are shown in Chapter 6.

The major findings of the study and the recommendations for further research are summarized in Chapter 7.

CHAPTER 2

FREIGHT DEMAND: A GENERAL FRAMEWORK

2.1 Introduction

This chapter presents a comprehensive framework for describing

the interrelationships among various aspects of a firm's transportation

related decision-making. This framework serves as a basis for

developing a behavioral freight demand model using the firm as the

basic decision-making unit. The choices that a firm should

consider range from the long-run decisions, such as the choice of

plant location, the goods to be produced, the technology of

production and the level of output, to more short-run decisions,

such as the choice of a distributor for its outputs, or supplier

for its inputs, and the logistics strategies for its production

and inventory process. These decisions are all closely related

to the flow of freight transportation in the market. For example,

changes in the output price could influence the manager of the firm

to reconsider the technology to be employed in production and the

level of output to be produced. Factor demand will then be adjusted

accordingly, as different levels of demand for input commodity

affect the inventory control plan. A change in the inventory

strategy might result in a new transportation plan, including a

different transport mode and/or shipment size, or even the selection

of a new origin supply point. To elaborate the point with

another example: suppose that the tariff for common carriers of

general commodities were to be reduced in a particular set of
markets; this will affect the firm's logistics management strategy
not only in terms of a possible shift of mode to truck from other
modes, but also a possible new equilibrium in the tradeoff
between transport cost and storage cost. Therefore, the mode,
the shipment size, the origin of transporting and the input
commodities might all be changed. Moreover, since transportation
is a factor of production, a change in transportation price
might in the long term also affect the technology and the equili-
brium in the market.

Two points can be observed immediately from the complexity
of the choices available to a firm. First, a freight demand model
will not be policy sensitive if the various transportation-related
aspects are not considered. Second, again because of the com-
plexity, it would be impossible to have a simple freight demand
model which included all of the factors simultaneously. It is,
therefore, not surprising to discover that a great many entirely
different approaches have been investigated in the past in attempting
to model the demand for transportation, and yet none of them has
been successful in covering the interacting dimensions satisfactorily.
A systematic approach is indeed required to sort out the complicated
interrelationships involving freight demand into an empirically
tractable framework. This chapter presents such an effort.
The remainder of the chapter will focus on a series of questions
including the following:

(1) Why is the firm considered to hold the central role in the freight demand analysis?

(2) What are the choices available to a firm in its production and logistics process?

(3) How are these choices interrelated with each other?

(4) How can these interrelationships be described in a systematic framework?

(5) What are the modelling implications of the proposed framework?

2.2. The Decision-Making Unit--the Individual Firm

It is self-evident that the firm is the basic decision-making unit in freight transportation. Clearly, it is the manager of a manufacturing plant, a wholesale distributorship, or a retail store who decides upon the transportation plans for the firm's inputs; and sometimes for its outputs too. These decisions are made constantly in response to the changes in the market prices of commodities and transportation services.

The role of the firm in production and inventory control has long been recognized and investigated thoroughly in the neo-classical theory of the firm and as a topic in management science. However, the role of the firm in selecting freight transportation services has only been explored marginally. The limited amount of firm-oriented freight demand studies that have been done have

tended to be very theoretical.[3/] No really satisfactory operational,
firm-oriented freight demand model has even been developed and
implemented for planning in the real world. Many factors account
for this phenomenon. Two main reasons are: 1) a lack of incentive
for developing a freight demand model from the shipper's point of
view; and 2) a lack of suitable modelling techniques by which
the economic theory of the firm is translated into a nice
formulation for empirical estimation. We shall elaborate on the
first point throughout the remainder of this section. The second
point will be discussed in greater length in the following sections
of this chapter.

There are three separate entities who have an interest in
freight demand models--the government, the carrier, and the shipper.
Each of these entities views the problem in a different way,
hence has a different set of requirements for the freight transpor-
tation models. The government is interested in freight demand
models as a planning tool to assist in the decision-making for
facility investment planning or in setting regulatory policy.
The freight models constructed for government facility investment
are typically large-scale and comprehensive, including all transport
modes and involving a range of commodities moving over the entire

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[3/] For example, see Allen (1977). A short review of Allen's model
is given in Section 3.4.

system.[4/] Government-oriented models used to answer policy ques-
tions are typically multi-modal and aggregate over both commodities
and geographical areas.[5/]

A carrier is interested in a freight demand model as a mar-
keting or pricing tool. A typical question to be answered by a
carrier is: What effects will changes in tariff charges, transit
times and/or waiting times have on the shipper's demand, for
transport service? The carrier is basically interested in the
optimal level of service to be offered, and the optimal profits
which can be achieved in the competitive market.

A shipper looks at the problem differently. A shipper is
interested in the selection of points of supply of his input
materials or places to store his finished products, along with
the selection of logistics and transport strategies for these inputs
and outputs.

The government and the carrier are not particularly interested
in the problem from a shipper's point of view. However, it is
clear that it is the shipper who makes the decision on choice of
transportation services. Since a shipper is the ultimate buyer of
transportation services, we should take the shipper's viewpoint

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[4/] For example, see Kresge and Roberts (1971), Transportation
Research Institute, et al. (1968), Chiang (1975).

[5/] For example, see Morton (1969), Perle (1964) and Moore (1972).

regardless of the actor for whom the model is being built (Roberts, 1971). The carrier is concerned with those shippers who are to be his potential customers and the government concern lies with the spectrum of all shippers over all modes. Through **an** appropriate aggregation procedure, a shipper oriented model can be used to answer the specific questions in which the government and the carriers are interested.

2.3. The Location of the Decision-Maker

Having decided that the firm assumes a central behavioral role in the freight demand decision, the next question is how to approach the problem from the viewpoint of a firm. First, we have to address the question of the location of the decision-maker. This is not perceived as a problem in passenger travel demand analysis. The decision unit is the trip-maker who is always located at the origin of a trip or a trip-chain. The location of the decision-maker in the freight demand situation is not quite as clear. In practice, we will find that decision-makers can be located at the origin, at the destination or at both. The location of a decision-maker is influenced by industry structure, ownership, physical distribution channel structure, etc.[6] A variety of different situations exist in the real world and

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[6] See Roberts (1975) for a more complete discussion of this topic.

decision-makers are found at a number of places within the system.

Generally speaking, a firm with monopoly power is likely to exercise control over the distribution of outputs; the decision-maker is therefore located at the origin end. He can decide which markets he is going to participate in and how much of his product to transport to each market and by what mode. In the competitive market, a firm has little power by which it can control the distribution of its product. The firm can, of course, decide where and at what price it will offer to sell the product. By contrast, the firm has the power to control the logistics plan for input commodities: such as where to buy, the amount to buy and by what mode. The location of the decision-maker is thus moved to the destination end. The third case is for large firms with warehouse and/or multiple plants. For these firms, freight can be shipped internally from one place to another; the decision-maker is thus located at both origin and destination.

In practice, goods may be bought either FOB factory or FOB destination. For goods which are purchased FOB destination, the shipper typically selects the mode of transport. Frequently, the FOB destination mode of pricing arises because the shipper ships to his own local warehouse from which he distributes the goods. In this case, both the buyer in the destination and the shipper in the origin play a role in the destination of the flow of goods in the market. In contrast, for goods which are purchased FOB

factory, the decision-maker is clearly located at the destination end.
A freight demand model considering the origin end as the location
of the decision-maker is referred to hereafter as a model with a
downstream orientation. A freight demand model considering
destination end as the location of the decision-maker is referred to
hereafter as an upstream-oriented model.

In modelling freight demand, the upstream orientation has the
advantage over the downstream orientation since the choice set
is relatively small and the choice behavior is relatively easier
to capture in the former approach than in the latter approach.
Many products are consumed as intermediate goods over a large
number of industries, and the possible places of requiring the
product as an input are virtually innumerable. In contrast,
the number of input commodities used by a single firm is typically
much smaller and the points of supply of these inputs are relatively
limited. The attractiveness of the upstream orientation approach
is further strengthened by the fact that FOB factory purchases
appear to be more prevalent in practice. Since the upstream orien-
tation is more attractive, the development of the freight demand
model given in this research is based upon the hypothesis that the
decision-maker is located at the destination and is rational in
deciding the logistics and transportation choices for his input
commodities.
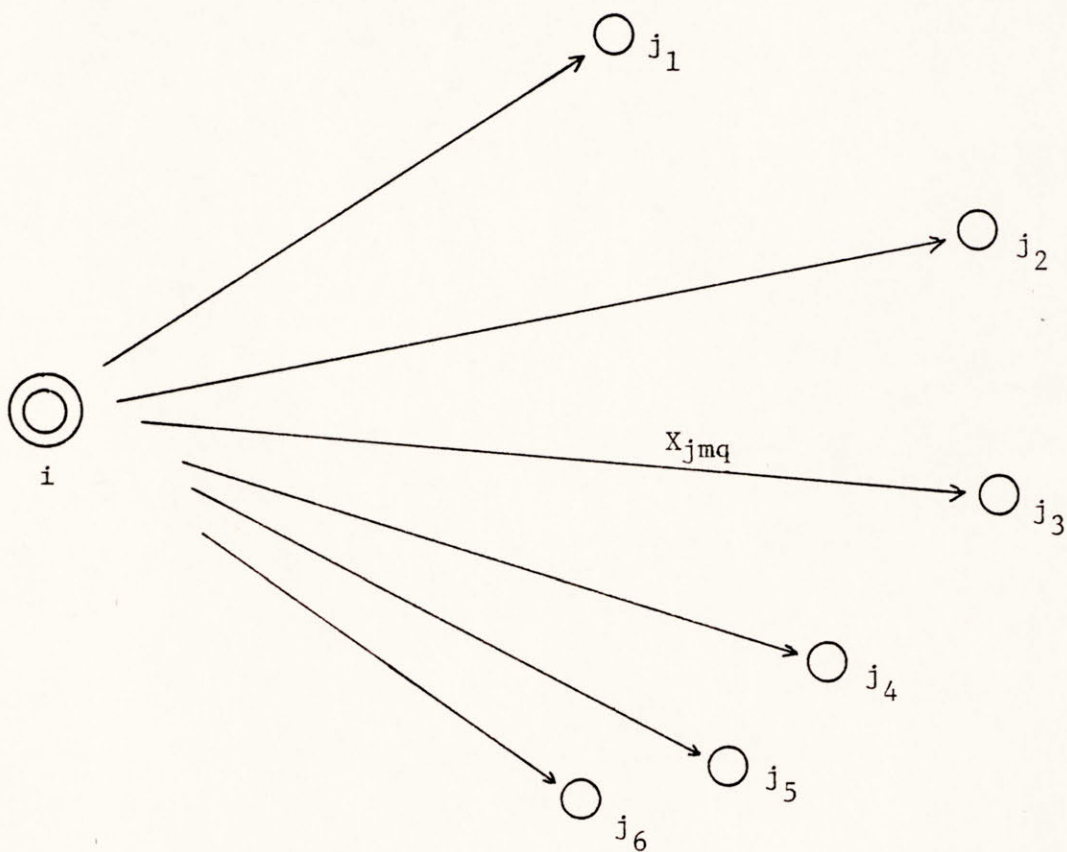
Decision variables: j = destination

m = mode

q = shipment size

Figure 1. A Freight Demand Model with a Downstream Orientation
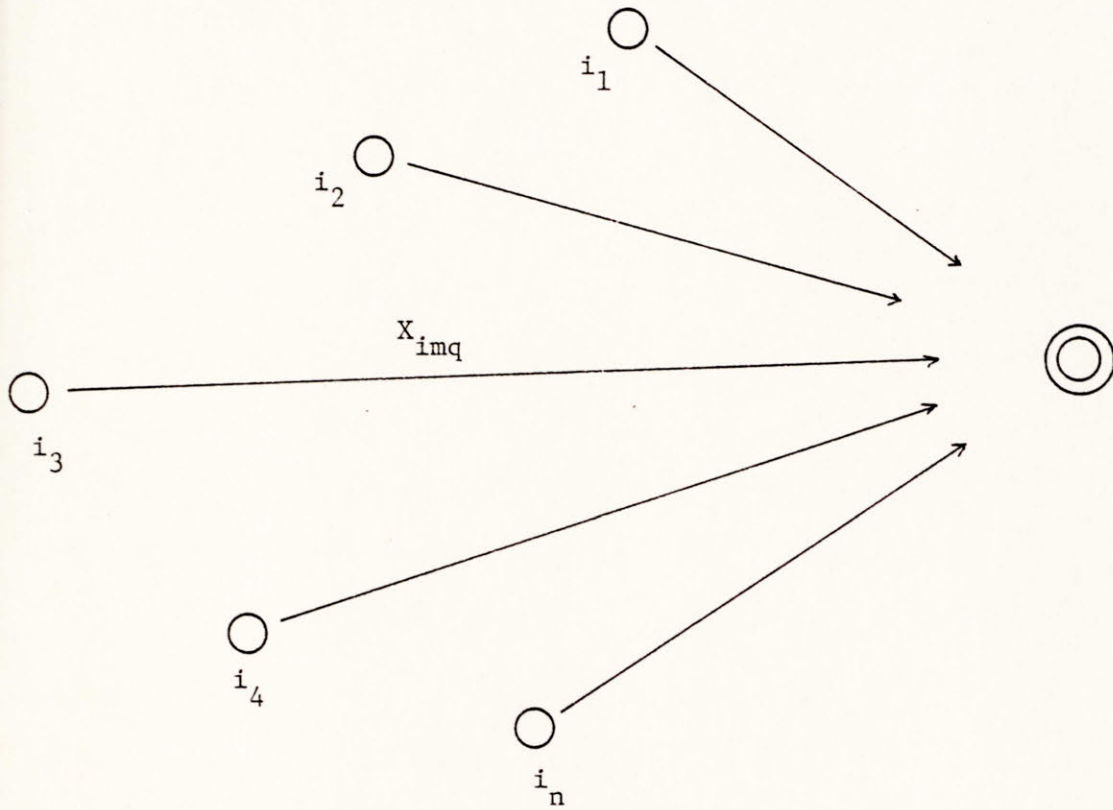
Decision variables: i = origin

m = mode

q = shipment size

Figure 2. A Freight Demand Model with an Upstream Orientation

2.4. Theory of Factor Demand

Transportation is a factor of production. Consequently,
transportation demand should not be analyzed apart from the theory
of production. In this section we review the economic theories
which have been used to derive factor demand. These theories
should serve as the basis for developing the freight demand
function from a shipper's point of view. The review given here
emphasizes the underlying relationships between various factors
that are relevant in the specification of the freight demand function.
This presentation is not formal and many mathematical details are
omitted, but it is suited to the purposes of this exposition.
A much more extensive and rigorous treatment of this and related
subjects can be found in Varian (1978), McFadden (1978), and Hall (1973).

Let us first introduce some notations to be used in subsequent
development.

X : a vector of inputs, $X = (x_1,\ldots,x_M)$

Y : a vector of outputs, $Y = (y_1,\ldots,y_N)$

W : a vector of factor prices, $W = (w_1,\ldots,w_M)$

P : a vector of output prices, $P = (p_1,\ldots,p_N)$

C : the minimum cost of producing Y at factor prices W

π : the firm's profit.

The development here assumes a spaceless environment, i.e., supply
and demand all occur at a single point and the spatial dimension
is not considered. We will relax this assumption in the next section

in order to introduce transportation into the analysis.

A firm produces a vector of outputs, Y, using a vector of inputs X. An input requirement set, V(Y) is defined as the set of X which can be used to produce the vector of outputs of Y:

$$V(Y) = \{X | Y \text{ can be produced with } X\} \qquad (2.1)$$

Since there is free disposal of inputs, we are interested in the set of input bundles that can just produce outputs Y. This can be shown to be the isoquant of input requirement set, Q(Y), given by

$$Q(Y) = \{X | X \in V(Y) \text{ and } X \notin (Y') \quad \text{if } Y' > Y\}$$
$$= \{X | t(X,Y) = 0\} \qquad (2.2)$$

in which $t(X,Y) = 0$ is defined as the maximum output which can be produced from the input bundles. This is called the production function of the technology. Commonly assumed objectives of the firm are either to maximize profits or to minimize the cost of producing a given output. A cost function, C(Y,W), is defined as the minimum cost of producing outputs Y at factor prices W:

$$C(Y,W) = \min_{X} W \cdot X \qquad \text{s.t.} \quad t(X,Y) = 0 \qquad (2.3)$$

The profit maximizing level of outputs, Y^*, which can be sold in a market at fixed prices P and W is given by the solution to the equation:

$$\max_{Y} \pi = P \cdot Y - C(Y,W) \qquad (2.4)$$

The assumption which is required in Eq. (2.4) to solve for
Y^* is that the cost function, $C(Y,W)$, should be convex in Y;
in other words, decreasing returns to scale. The profit function,
$\pi(P \cdot W)$, is then defined as

$$\pi(P;W) = P \cdot Y^* - C(Y^*,W) \tag{2.5}$$

There is a dual relationship that exists between the cost
function and the production function. The theorem of duality
states that a well behaved technology can be described equally well
in terms of the relationships between either the input quantities,
or factor prices, as long as firms are cost-minimizing. Maximizing
either profit or output for a given expenditure level produces
the same factor input ratios as minimizing the cost of producing
a given level of output. Proofs of the theorem are given by
Shephard (1953), Uzawa (1964), and McFadden (1978).

Duality implies that due to a unique set of correspondences
between production function and cost function, the technological
relationships can be inferred from either the cost function or
from the production function. Thus production function and cost
function are equally applicable in the description of the under-
lying technology. However, it is a common practice to use the
cost function in the empirical study of technology, because
1) cost data are more readily available; 2) cost function are more
likely to have independent error terms; and 3) tests of the hypotheses

concerning the underlying technology are more easily performed
on cost functions.

Under the assumption that all markets for factors are competitive, the demand for factor k is equal to the derivative of the
cost function with respect to the price of factor k:

$$x_k = \frac{\partial C(Y,W)}{\partial w_k} = x_k(Y,W) \qquad k = 1,\ldots,M \qquad (2.6)$$

This is known as Shephard's lemma for factor demand. These equations are the conditional demand functions for factors of production,
given the level of outputs Y.

The unconditional demands for factors are simply derived by
substituting the profit-maximizing level of output, $Y^* = Y(P,W)$,
into the conditional demand functions:

$$x_k = x_k[Y(P,W),W] = x_k(P,W) \qquad k = 1,\ldots,M \qquad (2.7)$$

These can also be derived by Hotelling's lemma which states that
the unconditional factor demands are equal to the negative value
of the derivatives of the profit function with respect to factor
prices:

$$x_k = \frac{-\partial \pi(P,W)}{\partial w_k} = x_k(P,W) \qquad k = 1,\ldots,M \qquad (2.8)$$

We can summarize the above equations by a simple example.
Assume a single product firm in a competitive market with a technology characterized by a Cobb-Douglas production function:

$$y = \alpha_0 \, x_1^{\alpha_1} \, x_2^{\alpha_2} \tag{2.9}$$

First, set up the following minimizaton problem:

$$\min_{x_1, x_2} \quad C = w_1 x_1 + w_2 x_2$$

$$\text{s.t.} \quad y = \alpha_0 \, x_1^{\alpha_1} \, x_2^{\alpha_2} \tag{2.10}$$

The first order condition for minimization gives:

$$x_1 = \alpha_0^{-\frac{1}{\alpha_1 + \alpha_2}} \left(\frac{w_1 \alpha_2}{w_2 \alpha_1}\right)^{-\frac{\alpha_2}{\alpha_1 + \alpha_2}} y^{\frac{1}{\alpha_1 + \alpha_2}}$$

$$x_2 = \alpha_0^{-\frac{1}{\alpha_1 + \alpha_2}} \left(\frac{w_2 \alpha_1}{w_1 \alpha_2}\right)^{-\frac{\alpha_1}{\alpha_1 + \alpha_2}} y^{\frac{1}{\alpha_1 + \alpha_2}} \tag{2.11}$$

Substituting into the objective function, we get the following
Cobb-Douglas cost function:

$$C = \alpha_0^{-\frac{1}{\alpha_1 + \alpha_2}} \left[\left(\frac{w_1 \alpha_2}{w_2 \alpha_1}\right)^{-\frac{\alpha_2}{\alpha_1 + \alpha_2}} w_1 + \left(\frac{w_2 \alpha_1}{w_1 \alpha_2}\right)^{-\frac{1}{\alpha_1 + \alpha_2}} w_2\right] y^{\frac{1}{\alpha_1 + \alpha_2}}$$

$$= u \alpha_0^{-\frac{1}{u}} \left(\frac{w_1}{\alpha_1}\right)^{\frac{\alpha_1}{u}} \left(\frac{w_2}{\alpha_2}\right)^{\frac{\alpha_2}{u}} y^{\frac{1}{u}} \qquad u = \alpha_1 + \alpha_2 \tag{2.12}$$

Next, set up the maximization problem to derive the profit
function:

$$\max_{y} \pi = p \cdot y - C$$

s.t. $\quad C = u\alpha_0^{-\frac{1}{u}} \left(\frac{w_1}{\alpha_1}\right)^{\frac{\alpha_1}{u}} \left(\frac{w_2}{\alpha_2}\right)^{\frac{\alpha_2}{u}} y^{\frac{1}{u}}$ \hfill (2.13)

Assuming decreasing returns to scale ($u < 1$), the first order condition for maximization of Eq. (2.13) gives:

$$p = u\alpha_0^{-\frac{1}{u}} \left(\frac{w_1}{\alpha_1}\right)^{\frac{\alpha_1}{u}} \left(\frac{w_2}{\alpha_2}\right)^{\frac{\alpha_2}{u}} \left(\frac{1}{u}\right) y^{\frac{1}{u} - 1} \hfill (2.14)$$

The profit-maximizing level of output, y^*, becomes:

$$y^* = p^{\frac{u}{1-u}} \alpha_0^{\frac{1}{1-u}} \left(\frac{w_1}{\alpha_1}\right)^{-\frac{\alpha_1}{1-u}} \left(\frac{w_2}{\alpha_2}\right)^{-\frac{\alpha_2}{1-u}} \hfill (2.15)$$

The profit function is then derived as follows:

$$\pi = p[p^{\frac{u}{1-u}} \alpha_0^{\frac{1}{1-u}} \left(\frac{w_1}{\alpha_1}\right)^{-\frac{\alpha_1}{1-u}} \left(\frac{w_2}{\alpha_2}\right)^{-\frac{\alpha_2}{1-u}}]$$

$$- u\, \alpha_0^{-\frac{1}{u}} \left(\frac{w_1}{\alpha_1}\right)^{\frac{\alpha_1}{u}} \left(\frac{w_2}{\alpha_2}\right)^{\frac{\alpha_2}{u}} [p^{\frac{1}{1-u}} \alpha_0^{\frac{1}{u(1-u)}} \left(\frac{w_1}{\alpha_1}\right)^{-\frac{\alpha_1}{u(1-u)}} \left(\frac{w_2}{\alpha_2}\right)^{-\frac{\alpha_2}{u(1-u)}}]$$

$$= (1-u)p^{\frac{1}{1-u}} \alpha_0^{\frac{1}{1-u}} \left(\frac{w_1}{\alpha_1}\right)^{-\frac{\alpha_1}{1-u}} \left(\frac{w_2}{\alpha_2}\right)^{-\frac{\alpha_2}{1-u}} \hfill (2.16)$$

We are interested in the demand functions for x_1, x_2.

By Shephard's lemma, the conditional demand functions are derived as

$$x_1 = \alpha_0^{-\frac{1}{u}} \left(\frac{w_2 \alpha_1}{w_1 \alpha_2}\right)^{\frac{\alpha_2}{u}} y^{\frac{1}{u}}$$

$$x_2 = \alpha_0^{-\frac{1}{u}} \left(\frac{w_1 \alpha_2}{w_2 \alpha_1}\right)^{\frac{\alpha_1}{u}} y^{\frac{1}{u}} \tag{2.17}$$

Substituting the profit-maximizing level of output, y^*, into the above conditional demand functions, we have the following unconditional demand functions:

$$x_1 = \alpha_0^{\frac{1}{1-u}} p^{\frac{1}{1-u}} \left(\frac{w_1}{\alpha_1}\right)^{\frac{\alpha_2-1}{1-u}} \left(\frac{w_1}{\alpha_2}\right)^{-\frac{\alpha_2}{1-u}}$$

$$x_2 = \alpha_0^{\frac{1}{1-u}} p^{\frac{1}{1-u}} \left(\frac{w_1}{\alpha_1}\right)^{-\frac{\alpha_1}{1-u}} \left(\frac{w_2}{\alpha_2}\right)^{\frac{\alpha_1-1}{1-u}} \tag{2.18}$$

By Hotelling's lemma, taking the derivatives of the profit function with respect to factor prices, we will get the same result for unconditional demands.

It is perhaps time to point out the critical issue in the empirical derivation of the cost function or demand function. In Eq. (2.2) we define the production function in terms of a transformation function:

$$t(X,Y) = 0 \tag{2.19}$$

A transformation function such as Eq. (2.19) gives the general

interactions between a vector of inputs and a vector of outputs.

For the purposes of econometric estimation, an explicity functional

form for the production function is required. In our above example,

the production function was assumed to be Cobb-Douglas:

$$y = \alpha_0 \, x_1^{\alpha_1} \, x_2^{\alpha_2} \qquad\qquad (2.9) \text{ repeated}$$

In doing so, we have imposed an assumption of separability between

inputs and outputs as expressed by

$$t(X,Y) = g(Y) + f(X) = 0 \qquad\qquad (2.20)$$

Most of the explicit production functions employ the assumption

of separability.[7/] The nice property of separability is to make

the problem mathematically tractable. Unfortunately, it has a

number of strong implications for the underlying technology.

First, for multiple output technology, separability means joint

production. Joint production means that there is no way to

represent any subset of outputs in terms of separate production

functions. Hall (1973) has shown that a separable technology

always implies joint production, but joint production does not

necessarily imply separability.

[7/] For example, in addition to the Cobb-Douglas production function
assumed here, there is also the Constant-Elasticity-of-Substitution
(CES) production function.

An example will illustrate this argument. Keeler (1974) has
developed a methodology to derive long-run cost functions from
short-run cost functions which was a breakthrough in the empirical
costing analysis. Keeler was studying the issue of excess capacity
in U.S. railroads. He considered the railroad as a two-output
technology: one output is passenger service and the other freight
service. He formulated the following Cobb-Douglas production
functions to represent the technology of producing passenger and
freight service.

$$Q_i = A_i T_i^{\alpha_i} R_i^{\beta_i} F_i^{\gamma_i} L_i^{\delta_i} \qquad i = 1,2 \qquad (2.21)$$

where   Q : output (thousands of gross ton-miles)

T : physical plant (track miles)

R : investment in rolling stock

F : fuel consumed

L : labor used per unit of time

A, α, β, γ, δ : parameters

i = 1 : freight service

i = 2 : passenger service

He then used these production functions to derive the short-run
and the long-run cost functions for empirical estimation. The problem
arises that the production functions of Eq. (2.21) utilized
the assumption of separability between input and output. Yet,
passenger service and freight service were specified as separate

production functions. There is therefore a contradiction since
separability implies a joint production; and there do not exist
separate production functions for a technology of joint production.

The second implication of separability is that the marginal
rates of the transformation between outputs are independent of
factor intensities or factor prices. This can be shown as follows.
A necessary and sufficient condition for separability,
$t(X,Y) = -g(X) + f(Y)$, is that the joint cost function be multi-
plicatively separable: $C(Y,W) = h(Y)q(W)$ (Hall, 1973), Denoting
$mc_i = \partial C(Y,W)/\partial y_i$, and $h_i(Y) = \partial h(Y)/\partial y_i$, we have

$$mc_i = h_i(Y)q(W)$$

$$mc_j = h_j(Y)q(W) \tag{2.22}$$

$$\partial(mc_i/mc_j)/\partial w^k = 0 \qquad \forall\, i,j,k$$

Thus, the assumption of separability with multiple outputs implies
that a vector of inputs is used to produce a single homogeneous
output, which can then be transformed into a vector of outputs.
The allocation of the single homogeneous output among final products
depends upon relative market prices of these final products alone,
and is independent of the relative prices of the inputs used to
produce them.[8] This is obviously a strong restriction.

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[8] See Spady and Friedlaender (1976) for a full discussion of this
and related topics.

Another fundamental problem associated with the empirical appli-
cation of the theory of factor demand lies in the assumption of hori-
zontal supply curves for inputs and demand curves for outputs.
Shephard's lemma requires that input prices be competitive, while
Hotelling's lemma requires both the input and output prices be compe-
titive. Thus, the validity of the demand functions derived from
Shephard's lemma or Hotelling's lemma depends upon whether or not
the underlying properties of the theoretical cost function or
profit function are close to reality. In viewing that imperfect
competition does exist and prevail in the real world, it is useful
to examine how closely Shephard's lemma and Hotelling's lemma
hold in the empirical applications.

Unfortunately, in a situation where the quantity of an input
demanded affects its price (i.e., a monopsonist or a firm that
enjoys quantity discounts), the derivative of the cost function
with respect to the factor price is not well defined because the
factor price is endogenous. To examine this formally, Bailey (1978)
developed a simple case of a two-factor production process with
one factor price determined endogenously. The supply functions
were assumed as

$$w_1 = \beta x_1 + \alpha$$
$$w_2 = \text{constant} \tag{2.23}$$

where β is an exogenous parameter. The cost function is derived

by solving

$$C(y,w_2,\beta) = \min w_1 x_1 + w_2 x_2$$

$$\text{s.t. } y = f(x_1,x_2)$$

$$w_1 = \beta x_1 + \alpha \tag{2.24}$$

The derivative of the cost function with respect to the endogenous factor price, w_1, yields

$$\frac{\partial C(y,w_2,\beta)}{\partial w_1} = x_1\left(1 - \beta\frac{dx_1}{dw_1}\right) \tag{2.25}$$

It is clear that $\frac{\partial C}{\partial w_1} \neq x_1$ unless $\beta = 0$ which is the price taker case. The derivative of cost with respect to factor price is thus a function of the underlying supply characteristics, and no generalization can be made.

It is fully possible to develop econometric tests for whether or not factor supply functions are horizontal. For example, Bailey (1978) implemented a test for the hypothesis that electric generating utilities are price takers in input markets. The test involved testing the cross equation coefficient restrictions between the cost function and the associated factor share equations implied by Shephard's lemma. The test was not conclusive, since there are other possible reasons besides sloping supply curves that the restrictions might be rejected. One such reason might be non-cost-minimizing in the firm's resource allocation.

Above we have briefly reviewed the economic theories for deriving

factor demand. By Shephard's lemma, the derivative of the cost

function with respect to the price of a factor gives the conditional

demand function for the factor. By Hotelling's lemma, the negative

of the derivative of the profit function with respect to the price

of a factor gives the unconditional demand function for the factor.

To derive a demand function for econometric estimation, one must

be able to specify an explicit functional form for the underlying

production function or cost function. To make the problem analy-

tically tractable, it is a common practice to assume separability

between input and output. This assumption is rather restrictive

for the technology of multiple outputs, therefore, it is not

generally accepted as valid.

Shephard's lemma requires the supply functions for inputs to

be horizontal, while Hotelling's lemma requires the supply functions

for both inputs and outputs to be horizontal. These requirements

become crucial when a demand function for freight transportation

is being derived. This is shown in the next section.

2.5. Modelling Freight Transportation as Derived Demand

So far we have not focused special attention on the demand

for freight transportation. We have merly undertaken a critical

review of the theories of factor demand in general, and discussed

some fundamental difficulties in the empirical estimation of demand

functions generally. Since freight transportation is only one of the factors of production, the demand function for transportation should be related to the underlying production and cost functions. This and related issues will be discussed in this section.

In Sections 2.2 and 2.3, the decision-maker is said to be the manager of a firm located at the destination or consuming end of a freight movement (denoted as j), and is responsible for making the transportation plans for the firm's input commodities. We noted that transportation is not a physical requirement to be used directly in production. Instead, it is merely a service to move each input commodity from the place where it is available (denoted as i), to the location of the firm (j). Assuming that the firm uses the inputs of capital (K), labor (L) and a vector of intermediate materials (X) to produce a vector of products, and that the firm is in long-run equilibrium, the underlying cost function can be written as:

$$C = C[Y, w_K, w_L, W_X(P_X,T_X)] \tag{2.26}$$

where   C : long-run total costs

        Y : a vector of outputs

        X : a vector of input materials, $X = (x_1,\ldots,x_M)$

        w_K : price for capital K

        w_L : price for labor L

        P_X : a vector of prices for intermediate materials

T_X : a vector of transportation-related costs associated

with the input materials, henceforth referred to

as logistics costs.

The total cost, W_X, of providing physical input includes

both the purchase costs and logistics costs. It is also important

to realize that tariff charges do not reflect the full cost of

transportation. The full cost of transportation includes also

the order and handling cost, capital carrying cost, storage cost

associated with acquiring and storing the inputs, etc. Transport

charges as well as these inventory-associated costs are referred

to as logistics costs.[9/] The simplest assumption for W_X is

thus the sum of purchase costs and total logistics costs:

$W_X = P_X + T_X$. Notice that both purchase costs and logistics costs

are functions of the quantities demanded. For instance, a firm

might well enjoy a lower purchase price per unit as the quantity

of an order increases.

For a given input commodity, the firm considers the tradeoffs

between: purchase price and transport price; large orders with

a low transport rate and high storage costs vs. small orders with

a high transport rate and low storage costs; reliability of delivery

vs. high safety stock costs; transit time vs. perishability; etc.

The decision variables associated with these various logistics

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[9/]The components of total logistics costs will be discussed in
greater length in Chapter 4.

stratigies are basically: choice of a supplier, choice of a ship-
ment size, and choice of a mode for the shipment so that the sum
of purchase costs and logistics costs can be minimized. Mathe-
matically, that is,

$$w_k = \min_{i,m,q} [p_{k,iq} (x_k) + t_{k,imq} (x_k)] \qquad (2.27)$$

$$k = 1,\ldots,M$$

where: w_k : total price for material k

   i : origin of the shipment

   m : mode

   q : shipment size

   $p_{k,iq}$: purchase price for material k at origin i in

        shipment size q

   $t_{k,imq}$: transport and logistics costs for material k

        under a logistics strategy defined by

        choosing i,m,q

   x_k : the use rate of material k.

Notice that logistics costs are expressed as a function of the
use rate of the material. Freight rate is, in general, a decreasing
function of shipment size, while the shipment size is also the key
decision variable in inventory theory. Thus, both the transport
charges and the other logistics costs depend upon the logistics
decisions of where to buy, the amount to buy and by the transport mode.

If we now set up the familiar Shephard's lemma for factor demand and the underlying cost function, we have:

$$C(Y, w_K, w_L, W_X) = \min_{K,L,X} w_k \cdot K + w_L \cdot L + W_X' X$$

$$\text{s.t.} \quad t(K,L,X,Y) = 0 \tag{2.28}$$

$$w_k = \min_{i,m,q} [p_{k,iq}(x_k) + t_{k,imq}(x_k)] \quad k = 1,\ldots,M$$

$$X = (x_1,\ldots,x_k,\ldots,x_M)$$

$$W_X = (w_1,\ldots,w_k,\ldots,w_M)$$

$$\frac{\partial C(Y, w_K, w_L, W_X)}{\partial w_K} = K \tag{2.29}$$

$$\frac{\partial C(Y, w_K, w_L, W_X)}{\partial w_L} = L \tag{2.30}$$

$$\frac{\partial C(Y, w_K, w_L, W_X)}{\partial w_k} = x_k \quad k = 1,\ldots,M \tag{2.31}$$

Several points deserve discussion. First, factor prices for input materials are not exogenously specified. Therefore, the derivative of the cost function with respect to material factor prices as implied by Shephard's lemma given as Eq. (2.31) no longer holds unless the supply functions for these material inputs, given as Eq. (2.27), are horizontal. Second, though prices for materials are endogenous, the demand functions for capital and labor implied

by Shephard's lemma, given as Eqs. (2.29) and (2.30), still hold

as long as the prices for capital and labor are competitive.

However, econometric estimation of the system of equations

(2.28) through (2.31) by full information methods would give

inconsistent results for capital demand and labor demand, since

specification errors in one equation will "spread" to the

rest of the equations in the system. Third, even if the above

problems did not exist, there is still the fundamental difficulty

with the empirical derivation of the demand function or cost func-

tion, i.e., the requirement for an explicit specification of the

underlying production function as discussed in Section 2.4.

The last problem can be solved by a flexible specification of

the cost or production function. In recent years increasing atten-

tion has been paid to a number of general cost and production functions

that are second-order approximations to any given cost or production

function. The best known functional forms of second-order approxi-

mations are the generalized Leontief function proposed by Diewert

(1971), the generalized linear-generalized Leontief joint cost

function proposed by Hall (1973), and the transcendental logarithmic

function proposed by Christensen, Jorgenson and Lau (1973). The

nice property of these approximations is that they impose no

a priori restrictions on the underlying production or cost function.

Instead, they allow econometric tests for the underlying technology

against empirical estimation restults. Among these approximations,

the transcendental logarithmic function (the translog function) seems to be the most flexible and allows for testing virtually all important hypotheses such as homogeneity in factor prices, homogeneity in outputs, separability, and nonjoint production.[10] As commented by Burgess (1975), translog functions are not self-dual, i.e., an estimated translog cost function cannot infer its underlying production function and vice versa, because of the approximation. However, the duality theorem allows one to find the necessary information concerning the technology from the estimated cost or production function.

Although we can solve the problem of specification errors caused by a priori restrictions in the production or cost function, we are not able to solve the problem of endogenous factor prices satisfactorily. Simplifying assumptions are inevitably required. The conventional aggregate demand function assumes transportation separable with the intermediate materials and treats transportation as a factor of production directly.

$$C = C(Y, w_K, w_L, w_M, w_T) \tag{2.32}$$

where: w_M : price for materials

w_T : price for transportation

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[10] See Spady and Friedlaender (1976) for a full discussion of the translog cost function and parameter restrictions to test these hypotheses. An example using translog cost functions can be found in the literature review of this study (Section 3.4).

A demand function for transportation can then be derived from Shephard's lemma by assuming that the supply function for transportation is horizontal:

$$\frac{\partial C(Y, w_K, w_L, w_M, w_T)}{w_T} = T \qquad (2.33)$$

where: T : the demand for transportation.

This model is not attractive since it is not policy sensitive to the changes in the transportation level-of-service. The usefulness of this model is limited to addressing questions such as the elasticity for freight transportation in general, and the cross-elasticities between transportation and other factors.

More elaborate models can be developed following the same rationale. To study the elasticities of demand for different transportation modes, it is possible to specify the cost function as:

$$C = C(Y, w_K, w_L, w_M, w_T, w_R) \qquad (2.34)$$

where: w_T : price for trucking service

w_R : price for rail service.

Moreover, logistics costs can be considered as part of the true cost of transportation.

$$w_T = w_T(p_T, q_T)$$

$$\qquad (2.35)$$

$$w_R = w_R(p_R, q_R)$$

where:

p_T : freight rate for trucking service

q_T : logistics costs associated with trucking service

p_R : freight rate for rail service

q_R : logistics costs associated with rail service.

Specifying functional forms of Eqs. (2.35) in such a manner that the assumption of a horizontal supply curve for transportation is implied, the demand functions for trucking service and rail service are thus developed using Shephard's lemma. As a planning tool, the usefulness of models of this type is still limited. The assumption of a price taker in transportation is a crucial one since freight rates decrease significantly with the size of a shipment. Annual expenditures for freight transportation service assuming a fixed use rate of an input would be different since different shipment sizes are utilized. Moreover, there are additional logistics costs, such as capital carrying cost or storage cost, which are also sensitive to shipment size. To assume the firm acting as a price taker in transportation restricts the capability for exploring interesting transportation questions associated with shipment size and origin choice. In fact, most of the freight transportation problems in the real world are more or less related to the flow of cargoes between city pairs, which apparently involves the choice of mode as well as shipment size and origin. Models that are restricted by the assumption of horizontal

transportation supply curve are clearly paying a price in terms
of less policy sensitivity to transportation options in order to
gain a capability for answering questions involving substitutions
between transportation and other factors.

One natural alternative to simplifying the problem goes,
therefore, in the other direction. Assuming that the factor demand
in a certain long period is fixed, freight demand models can be
developed to address directly the issues of choosing transportation
mode, shipment size, and places to fulfill the factor demand.
This implies that changes in transportation cost have no effect
on factor substitution. Thus, this approach also pays a price
in terms of generality in order to concentrate on the physical
transportation movements.

Generally speaking, there appear to be two basic approaches
by which we can simplify the problem of developing freight demand
relationships as outlined in Eqs. (2.28) through (2.31). Neither
approach considers the complete set of interactions. It is interes-
ting to note that real world data limitations have also restrained
the development of freight demand models in the past to essentially
the same two basic approaches.

2.6. A General Framework for Freight Demand

The dilemma in modelling freight demand can be resolved con-
ceptually by establishing a general framework. The proposed

conceptual framework considers three classes of transportation-related

decisions. These decisions are then represented as a complex

hierarchy of choices. This hierarchy is depicted in Figure 3.

As indicated above, a firm ultimately makes the decisions

on the purchase of transportation services. The longest run

decision that a manager of a firm has to make concerns the location

of the plant. A material-oriented industry is likely to locate

close to the source of raw materials. A consumption-oriented

industry would tend to locate close to popoulation or other

industries requiring his output as intermediate inputs. The firm

attempts to select a location for the plant such that an expected

profit-maximizing or a cost-minimizing objective can be achieved.

Transportation cost is merely a component of the total cost of

operation. The location decision is made with a general knowledge

of both the suppliers of inputs and the markets for output,

along with the quality of transport services available. However,

the location decision is usually not predicated on the choice

of a single individual supplier, transport mode, or shipment size,

though these factors are important overall. The choice of supplier,

mode, and shipment size are relatively short-run decisions concerned

with a single input. They may be altered from time to time, but

the plant location decision will be changed only if there are

major changes in regional markets for inputs, outputs, or transpor-

tation services. Meanwhile, the plant location may give a
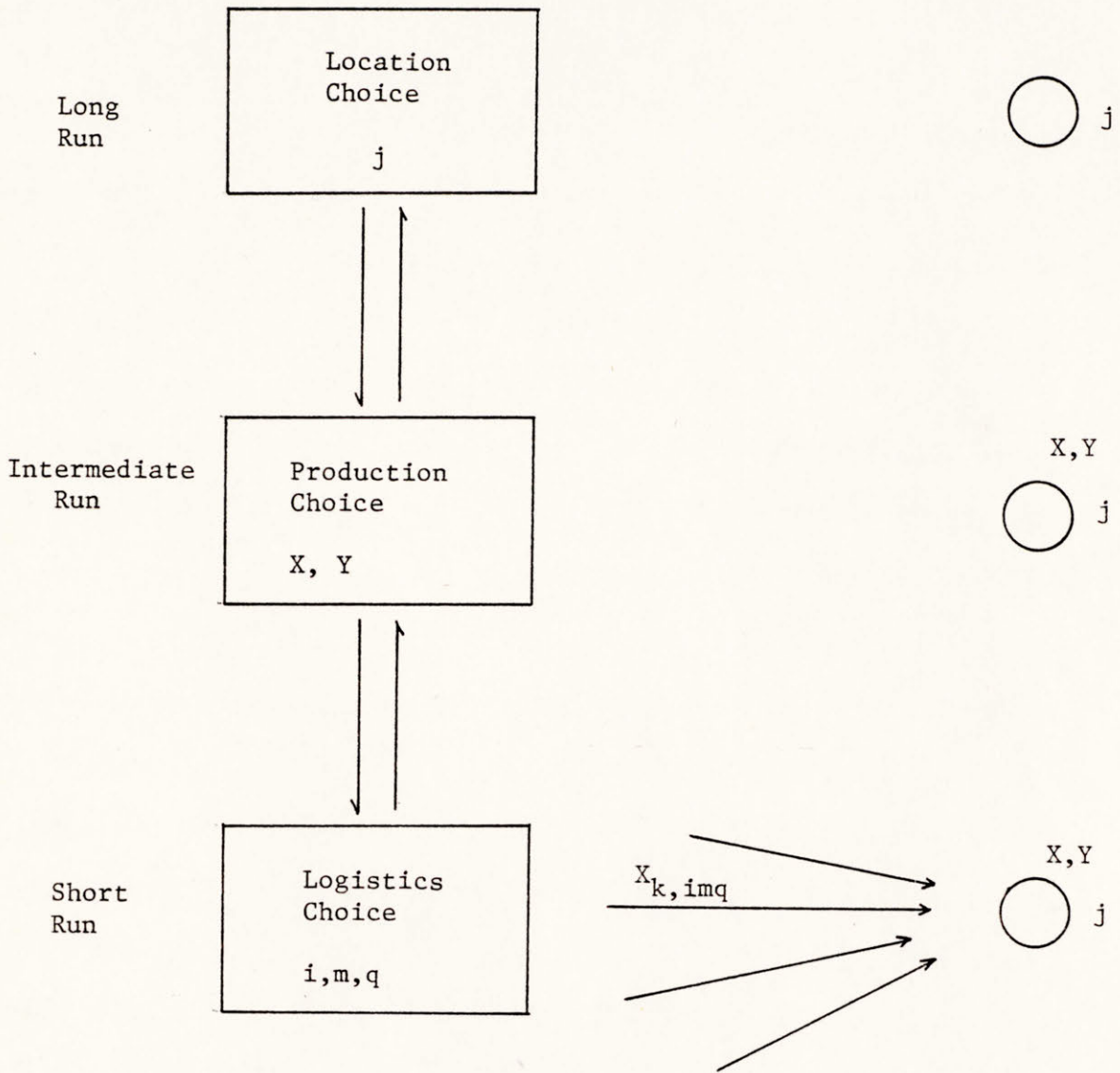
Figure 3. Hierarchy of Choices

commanding advantage to some subset of suppliers and carriers.[11]

Conditional on the long-run location, there are intermediate
decisions concerning production involving the choices of technology
of production and level of output. The firm must consider factor
prices for inputs: capital, labor, raw materials and transportation,
as well as the market prices for outputs in order to determine the
optimal plant size and input/output combinations. If the firm
is able to adjust factor utilization easily, it can attain a long-
run equilibrium; otherwise, it can only attain a short-run equilib-
rium. Since firms typically cannot adjust factor usage in an optimal
fashion, it is likely that they are operating along a short-run
cost function instead of a long-run cost function. In practice,
the production decisions for most products may be less flexible
than those involving the short-run transportation choices. The
choice of factor usage may put an upper and lower bound on the
quantity of inputs required. This will, in turn, place some
broad limits on the set of feasible shipment sizes. This may even
preclude the use of certain modes which specialize in very large or
very small shipments. Furthermore, the level of output decision
may emanate from a consideration that some suppliers will not be
able to fill orders at a rate compatible with the volume of production.

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[11] Some comments on these choices here are taken from Terziev (1976).

Conditional on the choices of plant location, production techno-
logy and output level, there are short-run logistics and transporta-
tion choices. In this choice bundle, the manager of the firm has
to consider a supplier or suppliers and a reorder point for each of
his input materials along with the associated transportation
arrangements. In many cases, the purchaser will have to enter into
a multi-order contract with a supplier. Thus, the choice of a
supplier is in some respects more of an intermediate-run decision
than are either the mode or shipment size choices. Similarly, the
choice of a shipment size (for example, the choice between 50 pounds
and 100,000 pounds) is a more intermediate-run decision than the
choice of a mode between shipments of the same size (i.e., full
truckload and TOFC), since handling equipment and storage facilities
are likely to be different for different shipment sizes.
Nevertheless, the choices of supplier, shipment size and mode of
transport are very closely related to each other in both the short
run and long run.

Choices of supplier, reorder point, shipment size and mode
could be viewed as the outcomes of a product inventory control
strategy. This strategy will be designed to give the required level
of protection against stockouts deemed desirable by the manager.
The choice of a risk of stockout will roughly define the range of
feasible shipment sizes and the minimum required storage facilities
used for stockpiling input materials. It will also give a guideline

for the minimum acceptable reliability of the transport mode.

It is important to note that this hierarchy does not imply one-way causality. There is feedback from short-run decisions to long-run decisions. Changes in freight rates will affect the inventory control strategy and the choice of both shipment size and suppliers. Changes in purchase and logistics costs will then influence the factor utilization. This feedback will eventually affect the long-run decision of plant location. Thus, the causality runs in both directions in the long run. Although the choice hierarchy reflects a time-staging of decisions, the time scale associated with the hierarchy is not rigidly defined. In some cases the short-run choices of mode and shipment size may be flexible on a daily basis, while in other cases several years might be required.

Reformulation of the transportation-related decisions open to a firm in terms of a hierarchical framework prompts the question of whether this line of reasoning leads to the development of satisfactory methodologies by which these choices can be modelled. First, let us define the problem formally in terms of this choice hierarchy. We suggest the following criteria by which we mean to determine the level of a given transportation problem:

(1) If the changes in transportation level-of-service have no significant effects on factor substitution and plant location, the problem is said to be a short-run one.

(2) If the changes in transportation level-of-service do have

significant effects on factor substitution but no effect on plant location, the problem is said to be an intermediate one.

(3) If the changes in transportation level-of-service have significant effects on both factor substitution and location the problem is said to be a long-run one.

It is important to notice that the level of a problem in the choice hierarchy is not determined by the time scales involved, but rather by the consequences that would arise.

There are well developed economic theories by which we can derive the factor demand. However, as shown in the previous section, these theories are difficult to apply to demand analysis in the real world directly without making unattractive simplifying assumptions. This is espeically true when they are applied to the analysis of problems involving freight flows. As previously indicated in Section 2.5, there are methodological difficulties as well as data limitations in applying the theory. The fundamental difficulties and assumptions usually made are summarized in Figure 4. First, modelling freight transportation as a derived demand, the underlying technology has to be either explicitly specified or implied. Problems with specification error could occur when the technologies involve multiple outputs. Thus, the flexible, second-order approximations have been commonly used to avoid a priori restrictions imposed on the technology being modelled. Because of approximation, there are approximation errors that could also occur.

Figure 4. Fundamental Difficulties of Modelling Transportation as Derived Demand

Difficulties	Assumptions Usually Made	Possible Problems
1. Underlying technology has to be specified or implied	Separability	Specification error
	Flexible functional form	Approximation error
2. Supply function is not horizontal	Horizontal supply function for transportation	Specification error Loss of policy sensitivity
	No substitution between transportation and other factors	Limitations in use
3. Disaggregate, time series data are not available	Use of aggregate, cross-sectional data	Aggregation bias Specification error

Second, according to inventory theory, firms exercise alterna-
tive strategies to minimize the sum of purchase costs and logistics
costs. Therefore, the factor price for transportation is not fixed,
implying that Shephard's lemma no longer holds. The practical
assumptions usually made are to simplify the relationship either
by ignoring the substitution between transportation and other
factors or by assuming that the supply function for transportation
is horizontal. The assumption of a horizontal supply curve
restrains the model in such a fashion that it is no longer sensitive
to micro-transportation policies and likely to invite specification
errors. On the other hand, the assumption that there can be no
factor substitution also limits the use of the model to short-run
transportation problems where transportation changes have no impact
on factor substitution or this impact can be ignored.

The third difficulty, as shown in Figure 4, is the data limi-
tation. To estimate the demand function for freight transportation
from a firm's point of view requires time-series data at a dis-
aggregate level for the firm. Unfortunately, these data are not
typically available. In practice, data to be used are either
disaggregate/cross-sectional, aggregate/cross-sectional or
aggregate/time-series. Use of cross sectional data to estimate
demand functions or the underlying cost function implies the
assumption that all firms have the same production technology.
This assumption becomes crucial if the demand functions is estimated

at a more aggregate level. Firms producing different outputs are
unlikely to have the same technology of production. Moreover, using
aggregate data to estimate a demand model is likely to cause
aggregation bias when the model is nonlinear in parameters.[12]

In viewing these fundamental difficulties and the capabilities
that are required to solve transportation demand problems at each
level in our choice hierarchy, it is suggested that a freight
demand model based on inventory theory which treats the factor
demand as fixed is the best candidate for the short-run choices.
By our criteria, a short-run transportation problem is defined as
one in which there is no significant substitution between transpor-
tation and other factors, therefore, the problem of specification
error is bypassed. Models of mode choice, shipment size choice and
origin choice, conditional on a fixed amount of commodity demanded
in a certain period of time, are conceptually the solution to our
short-run problem.

As to the intermediate-run and long-run problems in our choice
hierarchy, there appear to be no methodological solutions which are
operational at this moment. For intermediate range problems,
production choice and logistics choice should be considered jointly,

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[12]
For a detailed discussion of aggregation bias, see, for example,
Theil (1971).

while for long-range problems, locational choice, production choice
and logistics choice should be considered jointly. Future research
is required to develop suitable techniques to translate theory into
feasible, empirical modelling methodologies. However, a preliminary
guideline is proposed as follows.

As mentioned before, the causality of the choice hierarchy
runs in both directions: short-run decisions are conditional on
long-run decisions while the long-run decisions are also affected
by the feedback from the short-run decisions. In practice, longer
range decisions are always less flexible than short-range decicions.
Production decision will only be adjusted where there are significant
changes in the lower level logistics decisions. Similarly, plant
location will only be reconsidered where there are major changes in
the lower level production and logistics decisions. In terms of the
conceptual framework, it is expected that utilities from lower
level choices enter into the higher level decisions in aggregated form.
This implies that models in the three levels are linked sequentially.

In the logistics choices, a firm is basically trying to minimize
its total purchase and logistics costs over all of the input
materials by choosing an optimal combination of mode, shipment size,
and origin for each. That is,

$$w_k = \min_{i,m,q} [p_{k,iq}(x_k) + t_{k,imq}(x_k)]$$

$$k = 1,\ldots,M \qquad\qquad (2.27) \text{ repeated}$$

For intermediate-run decisions, the expected minimum factor price for transportation, denoted as $\overline{w}_k$, is considered in the production process.

$$C = C(Y, w_K, w_L, \overline{W}_X)$$

$$\overline{W}_X = (\overline{w}_1, \ldots, \overline{w}_k, \ldots, \overline{w}_M) \tag{2.36}$$

$$\overline{w}_k = E_{x_k} \{ \min_{i,m,q} [p_{k,iq}(x_k) + t_{k,imq}(x_k)] \} \quad k = 1, \ldots, M$$

Similarly, the expected operating costs and profit are considered in the model of locational choice.

A sequential structure among choices suggests that a set of sequential models is a candidate approach for modelling the freight transportation-related decisions at different levels. Considerable research has been done recently to address the choice with multiple dimensions in a disaggregate modelling framework.[13] The approach that has been developed allows us not only to model the choice dimensions in a sequential manner, but also to test the degree to which the multi-dimensional choices can be modelled sequentially instead of jointly. Following this approach, models of production choice and locational choice could hopefully be developed from the theories of production and location, once a successful model

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[13] For example, see Ben-Akiva (1973), Ben-Akiva and Lerman (1977).

for logistics choice becomes available.[14/] To use a disaggregate

modelling approach has the additional advantage that it will minimize

the possible aggregation bias and specification error likely to be

encountered in using aggregate/cross-sectional data in the estima-

tion of a freight demand model.

[14/] For example, if the logistics choices are modelled as a logit
model, the production and logistics choices become a set of
sequential logit models.

CHAPTER 3

LITERATURE REVIEW

3.1. Introduction

This chapter reviews the previous studies on freight trans-
portation demand and investigates their underlying assumptions in
terms of the general framework described in the previous chapter.

It is not surprising to discover that very little has been
done in the past on the area of freight demand in view of the fact
that there are methodological difficulties as well as data limita-
tions that prevented researchers from empirically developing the
demand function. Besides, there has been little incentive in the past
for the development of a freight model from the viewpoint of the
individual firm.

The literature review presented in this chapter emphasizes
the theories that have been used to derive the model. Special
attention has been placed on their underlying assumptions. In some
cases, they are the assumptions originally made in the derivation
of the model. In other cases, they are assumptions implied by
their formulations. We will review these assumptions in terms of
the theories of factor demand. This is not intended to be an
exhaustive review. In fact, a great portion of the previous work
in freight modelling consists of correlative statistical models which
are more or less imported from the conventional urban passenger
transportation models. Since there is no economic rationale

behind these models, they will not be reviewed here.[14/] Also

excluded are those purely theoretical models.

The approaches reviewed in this chapter include input-output

theory, spatial equilibrium theory, the theory of the firm,

consumer theory and inventory theory.

3.2. Input-Output Theory

The economic theory underlying input-output analysis has a

long history, dating from the publication of Francois Quesnay's

Tableau Economique in 1758, through Leon Walras' multisector, general

equilibrium model of production in 1877, to Wassily Leontief's

simple theoretical structure in 1936 (Polenske, 1978). The funda-

mental assumptions of a static, open input-output model are:

(1) Homogeneous product (i.e., each industry produces only

one product; each product is only produced by one industry;

no joint production exists)

(2) Constant returns to scale

(3) Total effects are the sum of individual effects

(4) Firms within an industry have identical technologies

(5) Technical coefficients remain constant over time

The amount of inputs required from each producing industry to produce

one dollar's worth of the output for sale to a given purchasing

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14/ A good survey of this type freight models can be found in
P.T.R.C. Symposium (1971).

industry is given by the technical coefficients denoted as a_{ij},
with i representing the producing industries in the economy and
j representing the purchasing industries in the economy. Technical
coefficients are usually expressed in terms of monetary value,
although they may also be expressed in terms of physical quantities.
Let X be the vector of gross output, Y be the vector of final
demand, A be the matrix of technical coefficients and I be the
identity matrix, we have:

$$X - AX = Y$$

$$\text{or} \quad X = (I-A)^{-1} Y \tag{3.1}$$

Thus, given a vector of final demand and holding the technical
coefficients unchanged in the future, future production can be
determined by Eq. (3.1).

The restrictions involved in the input-output model can be
shown using the generalized Leontief function proposed by Diewert
(1971). First we define the cost function as Diewert's second-order
approximation function:

$$C(y,W) = \sum_{h} \sum_{\ell} a_{k\ell} \sqrt{w_k w_\ell} \; y$$

$$a_{k\ell} = a_{\ell k} \qquad \neq \ell, k \tag{3.2}$$

Thus, the first assumption of the technology is a single output.
Using Shephard's lemma and taking the derivative of the cost function
with respect to factor price w_k gives

$$\frac{\partial C(y,W)}{\partial w_k} = \sum_{\ell} a_{k\ell} \sqrt{w_\ell / w_k} \ y \qquad (3.3)$$

Finally, by imposing the restrictions of nonjoint production:
$a_{k\ell} = 0 \ \forall \ k \neq \ell$, we have the Leontief production function of an
input-output model.

The input-output model was extended to include space first
by Isard in 1951. Since that effort, many theoretical and empirical
regional and multiregional input-output studies have been published.
Most of the multiregional studies were based upon the theoretical
framework developed by Moses (1952) and Chenery (1953). The
Chenery-Moses model, referred to as the column trade coefficient
input-output model, was used to formulate the multiregional input-
output (MRIO) model by Polenske (1972). This model has 51 regions
and 79 industries for the United States. The column trade coeffi-
cient input-output model uses the regional column trade coefficients
as well as regional technical coefficients. The regional column
trade coefficients are defined as

$$t_i^{gh} = r_i^{gh} / R_i^{oh} \qquad (3.4)$$

where: t_i^{gh} : regional column trade coefficient of industry i

between regions g and h

r_i^{gh} : regional trade of industry i from region g to

region h

R_i^{oh} : region h's total purchase of industry i.

Thus, given the regional technical coefficients, the regional column trade coefficients and the future regional final demand, the future regional production as well as the resultant interregional transportation flows can be determined as follows:

$$X - TAX = TY$$

or $X(I-TA) = TY$ (3.5)

or $X = (I-TA)^{-1}TY$

where   X : the vector of outputs by industry by region

Y : the vector of final demands by industry by region

A : the regional technical coefficient matrix

T : the regional trade coefficient matrix

I : an identity matrix.

Besides the basic assumptions of a national input-output model, the Chenery-Moses model requires additional assumptions to account for the trading relationships between regions. The current version of the MRIO model employs the following additional assumptions (Polenske, 1978):

(1) Constant trade relationships among regions over the given time period;

(2) Uniform import (trade) percentages for all industries in a region for a given good or service in a given year and, because of (1), over a given time period as well.

Thus, production technologies and interregional trading patterns are assumed to be unresponsive to shifts in final demands. For constant trading patterns to occur, the following conditions must exist:

(1) Regional costs of production must be constant and there must be excess production capacity in each industry in each region.

(2) The average cost of transport must be fixed for each commodity and region, and there must be excess capacity in the transport network.

(3) The supply curve of labor must be infinitely elastic at given regional factor prices.

The MRIO model is a comprehensive tool that can be used for systematic studies of a variety of regional economic policies. However, the assumptions of constant regional trade coefficients and technical coefficients make it unable to predict a realistic transportation impact. Thus, to be a transportation planning tool, the usefulness of this form of input-output model is limited.

Recently, Hudson and Jorgenson (1974) have developed a model that attempts to yield interindustrial coefficients from the profit-

maximizing behavior of the firm rather than from pure technological requirements. They treat input-output coefficients as endogenous variables which are determined by the relative prices of factor inputs. The impacts of a change in transportation policies can then be evaluated through changes in the price of transportation relative to other inputs. Unfortunately, the attempt has not been fully successful.

3.3. Spatial Equilibrium Theory

A number of optimization approaches have been used to study the spatial equilibrium of commodity demands and supplies.[15] The spatial equilibrium model developed by Samuelson is basically a linear programming transportation problem with a commodity supply function and demand function at each node. Consider a system of N regions for a given commodity and assume also that there exists a commodity excess supply curve for each region as

$$ES_i = f_i(p_i) \qquad i = 1,\ldots,N \qquad (3.6)$$

where: ES_i : excess supply curve for region i

   p_i : price of the commodity in region i.

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[15] For example, Samuelson (1952), Stevens (1961), Silberman (1969), and Harris (1970).

The system is said to be in equilibrium when the following conditions hold:

$$\sum_{j=1}^{N} (x_{ij} - x_{ji}) \leq ES_i \qquad i = 1,\ldots,N$$

$$p_j - p_i = t_{ij} \qquad \text{if } x_{ij} > 0 \qquad i,j = 1,\ldots,N \qquad (3.7)$$

$$p_j - p_i < t_{ij} \qquad \text{if } x_{ij} = 0$$

where: t_{ij} = unit transportation cost from region i to region j. The demand for transportation, d_{ij}, which is defined as the amount of the commodity that producers in region i want to ship to consumers in region j given the whole matrix of transportation costs, depends upon the shapes of the excess supply curves in each region and is thus claimed to be a "derived" demand.

In a two-region case, let ES_1 be Region 1's excess supply curve and ED_2 by Region 2's excess demand curve. Each curve is expressed in terms of the local price in each region. With no trade allowed between the two regions, the equilibrium prices in Regions 1 and 2 would be p_1^0 and p_2^0 respectively (see Figure 5). However, with trade permitted at transport cost t_{12}, the producers in Region 1 are willing to ship an amount of $x_{12} = ES_1$ from Region 2. Plotting these quantities for shipment for each value of t_{12} gives the transportation demand curve d_{12} (Silberman, 1969).
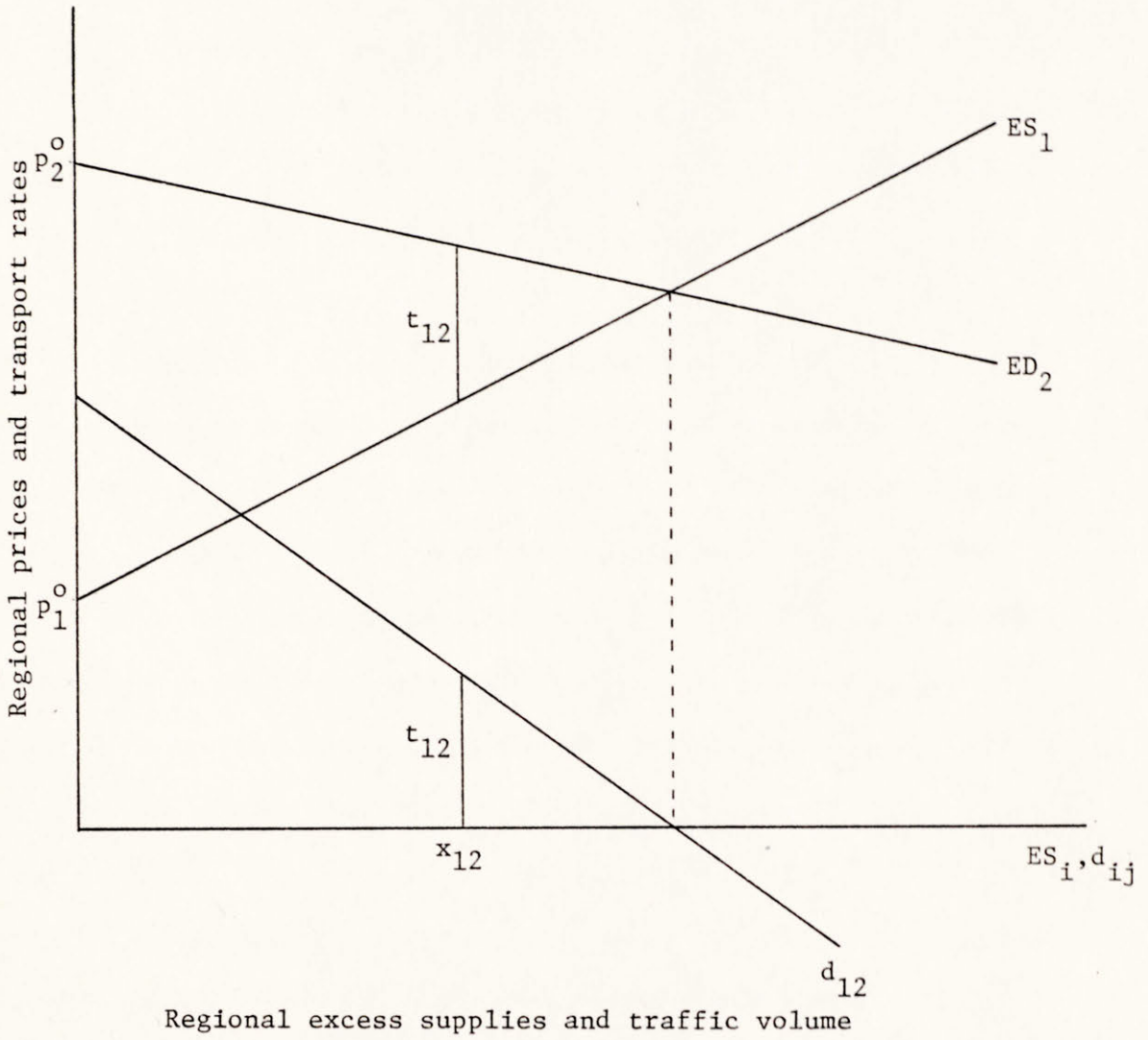
Figure 5.   The Demand for Transportation in a Two-Region Economy[*]

[*]This figure is taken from Silberman (1969), page 111.

The objective function for the system in the maximization of
net social payoff, NSP, which is defined by Samuelson as the sum
of social payoff in all regions minus total transport cost.
NSP can be shown to be the area under the transport demand function
less transportation costs. This area is maximized when the marginal
willingness to pay for transportation is equated with the unit trans-
port cost t_{ij}. Maximizing NSP thus produces the equilibrium
conditions shown as Eqs. (3.7).

Samuelson has proven that this system reduces to a linear
programming transportation problem when total net exports of each
region are given. Thus, if one is willing to make the assumption
of fixed net exports, the linear programming algorithms can be
used to solve the problem analytically. Silberman (1969) used this
basic approach to implement a two-step model to study the demand for
inland waterway transportation. First, he econometrically estimated
the regional imports and exports by barge using barge and rail freight
rates as well as other economic activity variables as explanatory
variables in regression models. Then, these regional imports and
exports were treated as fixed in the second-stage linear programming
transportation problem. Thus, the effect of changes in barge rates
and rail rates on interregional flows of barge traffic can be pre-
dicted through changes in net regional exports.

Because of the way the problem is formulated, linear programming
models are useful to address problems involving equilibrium in a

network. However, the normative nature of the model along with
its simplifying assumptions limits its usefulness in real world
transportation applications. System optimization is just not rele-
vant in many practical situations. The assumption of constant
transportation cost is also a crucial one. As indicated above, the
true transportation cost function is concave with respect to the
amount transported. Also, there are still methodological diffi-
culties in solving a cost-minimizing transportation programming
problem where the unit transport cost decreases with increasing flows.

3.4. Theory of the Firm

Very little work has been done on modelling freight demand using
the theory of the firm. Recently, Allen (1977) developed a model for
freight transportation demand using the classical theory of the
firm as a base. In this model the production and transportation
processes have been assumed to be interdependent for the first time.

In terms of our terminology, Allen's model is a downstream
oriented freight demand model; the decision-maker is located
at the origin of a shipment. Assume that the firm is a price taker
who is at place A and there is only one market. It is located at
place B. The objective of the firm is to maximize profit given as

$$\pi = [p - T(Q,\alpha)]Qe^{-i\alpha} - f(Q) \qquad (3.8)$$

where: π = producing firm's profit

$\quad$ p = market price of the firm's product at B

$\quad$ Q = quantity the firm produces and ships

$\quad\quad$ α = time required to ship the goods from A to B in days

$T(Q,\alpha)$ = transportation change per unit of product from A to B

$\quad\quad$ i = interest rate/day (opportunity cost of funds)

$\quad$ f(Q) = producing firm's cost function.

The transportation rate is assumed to be a function of the quantity shipped (reflecting a quantity discount) and the time in transit. This latter variable acts as a proxy for the service level provided by the carrier. Goods delivered are paid for on a C.O.D. basis and inventory holding is not considered. The first-order condition for maximization gives:

$$[p - T(Q,\alpha) - QT_Q(Q,\alpha)]e^{-i\alpha} - f'(Q) = 0 \qquad (3.9)$$

where: $\quad T_Q(Q,\alpha) = \dfrac{\partial T(Q,\alpha)}{\partial Q}$

$\quad\quad\quad\quad f'(Q) = \dfrac{\partial f(Q)}{\partial Q}$

The term $[p - T(Q,\alpha) - QT_Q(Q,\alpha)]e^{-i\alpha}$ is the marginal revenue and is referred to as $\hat{p}$ for the sake of simplicity.

$\quad$ Assuming a U-shaped average variable cost curve, the optimal output of the shipping firm under different transportation rates is shown as Figure 6(a). When there is no tariff charge, marginal revenue equals $pe^{-i\alpha}(\hat{p}_0)$; thus, the corresponding Q_0 is the maximum

output as marginal revenue equals the minimum point on the average variable cost curve. Q_1, Q_2 are the optimal outputs corresponding to marginal revenues $\hat{p}_1$ and $\hat{p}_2$ respectively. Thus, the demand for transportation with respect to transportation rate can be derived as shown in Figure 6(b), where T_0, T_1, T_2, T_M are transportation rates corresponding to $\hat{p}_0$, $\hat{p}_1$, $\hat{p}_2$ and $\hat{p}_M$ respectively.

Other level-of-service variables can also be considered. For example, assuming that there are loss and damages in transportation, and that all loss and damage costs accrue to the shipper, the shipping firm's profit function becomes:

$$\pi = (1 - \beta)[p - T(Q,\alpha,\beta)]Qe^{-i\alpha} - f(Q) \qquad (3.10)$$

where β = the expected percentage of loss and damage.

The above analysis is based upon a one-mode, one-market situation. It is possible to consider this model in a multimarket-multimodal context. The mode and market choice are determined by maximizing $\hat{p}_{jm}$, where

$$\hat{p}_{jm} = [p_j - T^{jm}(Q,\alpha^{jm}) - QT_Q^{jm}(Q,\alpha^{jm})]e^{-i\alpha^{jm}} \qquad (3.11)$$

j = market

m = mode.

To implement Allen's model involves the explicit specification of the cost function $f(Q)$ and the transportation rate function $T(Q,\alpha)$. Allen formulated the cost function with output as the
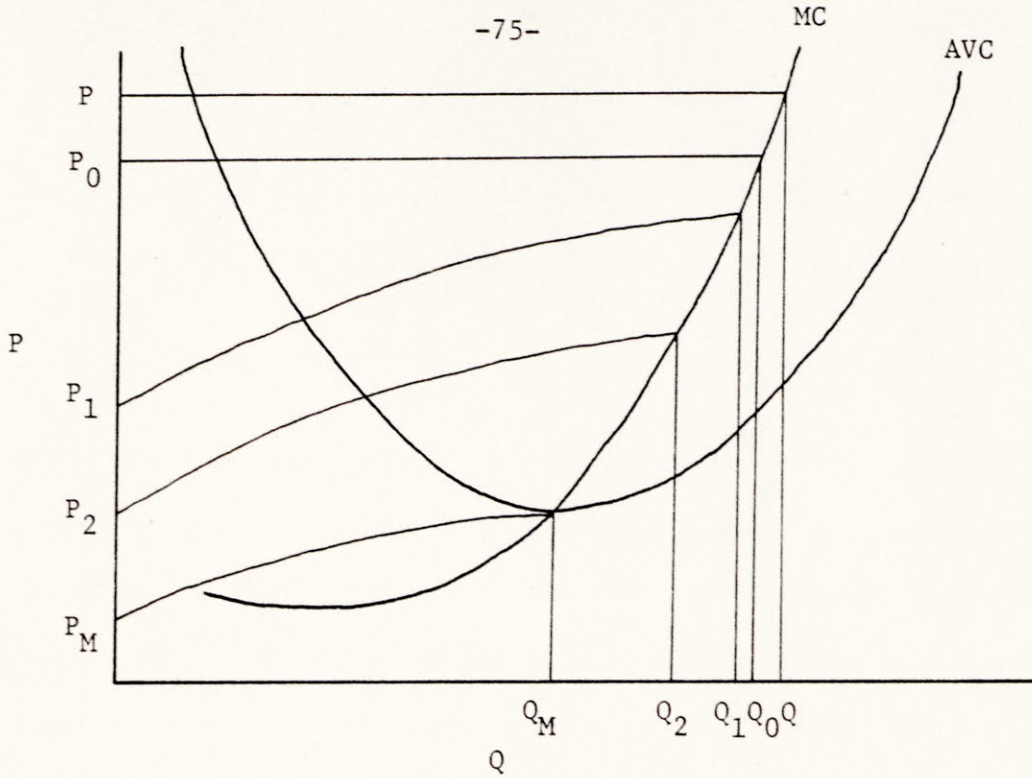
Figure 6(a). Output of the Shipping Firm under Different
              Transport Rates*
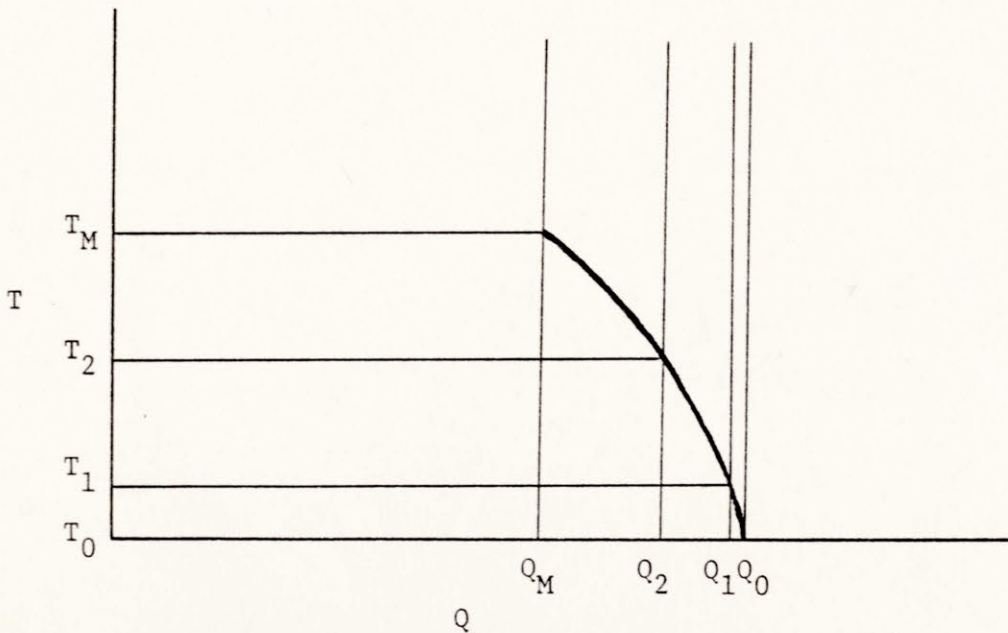
Figure 6(b). The Demand Function for Transportation*

*These figures are taken from Allen (1977), pp. 11-12.

only argument. There is the problem of missing variables since
cost is a function of output as well as factor prices. Allen
suggested a functional form for transportation rate as

$$T(Q,\alpha) = kQ^{b-1}\alpha^{a-1} \qquad 0 < a,b < 1 \qquad (3.12)$$

The condition $0 < a,b < 1$ is required to ensure that the derivatives
of the transportation rate, $T(Q,\alpha)$ and transportation charges
$T(Q,\alpha)Q$ will behave correctly. Eq. (3.12) is clearly restrictive
and unrealistic.[16/] To assume no inventory holding also over-
simplifies the problem since freight rates typically do not reflect
the true cost of transportation.

Allen's model is based upon the assumptions that the producing
firms are located at the raw material site and that goods delivered
are paid for on a C.O.D. basis. Thus, the model is quite restric-
tive in its application. Although the model is still in the
theoretical stage, it could be a candidate for addressing our
intermediate transportation problems for those cases which satisfy
the above two conditions.

Friedlaender and Spady (1977) developed an aggregate demand
function for freight transportaiton. The unique features of their

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[16/] Recently Samuelson (1977) has developed a set of freight rate
equations for various modes in the United States. He found
that freight rate can be modelled as a function of shipment
size and distance as well as commodity attributes. Samuelson's
study concluded that freight rate is indeed a high variance process.

model are that the model is derived from an underlying cost function and inventory costs are considered together with freight rate as the price for transportation. The cost function and its associated demand function for rail and truck were of the following form:

$$C_s = C_s(Y, K, M, p_L, p_T, p_R)$$

$$X_R = \frac{\partial C_s(Y, K, M, p_L, p_T, p_R)}{\partial p_T} \qquad (3.13)$$

$$X_T = \frac{\partial C_s(Y, K, M, p_L, p_T, p_R)}{\partial p_R}$$

where C_s = short-run variable costs

   Y = output

   K = capital

   M = materials

   p = factor price

   X = demand for transportation

   L = labor

   T = truck transportation

   R = rail transportation.

The price of transportation service was hypothesized as a hedonic function of freight rate and shipment characteristics. Shipment characteristics were used to reflect the inventory costs that are associated with a shipment.

$$P_i = r_i \cdot \phi^i(q_1^i, q_2^i, q_3^i) \qquad i = T,R \qquad (3.14)$$

where: r = freight rate

q_1 = the value or density of the commodity

q_2 = the average length of haul

q_3 = the average shipment size.

To avoid imposing a priori restrictions on the underlying technology, the translog function was used to approximate the cost function in Eq. (3.13).   The translog cost function is a second-order approximation to $\ln C_s$, thus the associated factor share equations for rail and truck implied by Shephard's lemma become:

$$\frac{P_T X_T}{C_s} = \frac{\partial \ln C_s}{\partial \ln p_T} = \alpha_T + A_{yT} \ln y + A_{kT} \ln K + A_{MT} \ln M$$

$$+ A_{LT} \ln p_L + A_{TT} \ln p_T + A_{RT} \ln p_R$$

$$(3.15)$$

$$\frac{P_R X_R}{C_s} = \frac{\partial \ln C_s}{\partial \ln p_R} = \alpha_R + A_{yR} \ln y + A_{kR} \ln K + A_{MR} \ln M$$

$$+ A_{LR} \ln p_L + A_{TR} \ln p_T + A_{RR} \ln p_R$$

where α's and A's are parameters.

The hedonic price functions were also approximated as translog functions:

$$\ell np_i = \ell nr_i + b_0^i + \sum_{h=1}^{3} b_h^i \ell n \, q_h^i + 1/2 \sum_{h=1}^{3} \sum_{\ell=1}^{3} \beta_\ell^i \ell n \, q_h^i \ell n \, q_\ell^i$$

$$i = T,R \qquad (3.16)$$

where b's and β's are parameters.

Substituting Eqs. (3.16) into Eqs. (3.15) and adding multivari-
ate normal disturbances, the factor share equations were estimated
econometrically.[17] The model was applied to manufacturing industries
in the United States, 1972. To capture technological differences
among industries and regions, commodity and regional dummies were
added to the hedonic price function. The own price elasticities
and cross price elasticities of demand as well as the elasticities
of demand with respect to shipment characteristics for rail and truck
were thus calculated from the estimated coefficients.

Friedlaender et al. attempt to model transportation as derived
demand explicitly and incorporate shipment characteristics to
reflect inventory costs. In these respects, it has merit over the
conventional aggregate demand functions. However, the usefulness
of the model is still limited. It is not sensitive to micro-
transportation policies where a network is involved. Besides, the
hedonic price function given in Eqs. (3.14) did not utilize the
a priori knowledge that the relationship between freight rate and

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[17] Since factor share equations sum up to one, the covariance
matrix is singular. One share equation has to be dropped.
See Friedlaender et al. (1977) for details.

inventory costs is additive.[18] However, another argument arises
from the use of Shephard's lemma in the derivation of the demand
function, as a consequence of the assumption that the supply
functions for transportation are horizontal.[19]

3.5. Consumer Theory

Consumer theory has been used to model freight demand of mode
choice in conventional abstract-mode models as well as the proba-
bilistic shipper choice models developed more recently.

Quandt and Baumol (1966) define a transport mode in terms of
the type of service it provides to the traveller rather than in
terms of the physical equipment it employs. This abstract-mode
concept is parallel to consumer theory as developed by Lancaster
(1966). Baumol and Vinod (1967) extended the abstract-mode model
to the analysis of freight transportation demand. The theory of
abstract mode is typically embodied in the formulation of a conven-
tional, aggregate, direct demand model.

The general form of the direct demand model can be written as:

$$T_{ijm} = \alpha_0 \; \prod_{m,k} (LOS_{ijmk})^{a_{mk}} \; \prod_p (E_{ip})^{b_p} \; \prod_q (E_{jq})^{c_q} \tag{3.17}$$

---

[18] Of course, if quantity discounts were considered, Eqs. (3.14)
would become a relevant general specification. Also, since the
translog approximation was utilized later on, the matter of speci-
fication error was, in fact, bypassed.

[19] See discussions of this in Section 2.5.

where: T_{ijm} = shipments from location i to location j

by mode m

LOS_{ijmk} = k^{th} level of service attribute for mode m,

from location i to location j

E_{ip} = p^{th} economic activity measurement for location i

E_{jq} = q^{th} economic activity measurement for location j

α, a, b, c are parameters.

Direct demand models attempt to treat transportation as an economic good; however, they fail to model freight transportation as derived demand. As indicated by Friedlaender et al. (1977), the formulation of an abstract-mode model is quite restrictive since it implies that the underlying technology can be characterized by a Cobb-Douglas production function and thus assumes an elasticity of substitution equal to one between any two factors. To understand this, suppose an industry is characterized by a Cobb-Douglas pro-duction function and transportation is separable. The associated cost function is given by:

$$C(y,W) = u a_0^{-\frac{1}{u}} \left(\frac{w_T}{a_T}\right)^{\frac{a_T}{u}} \left(\frac{w_R}{a_R}\right)^{\frac{a_R}{u}} \left(\frac{w_K}{a_K}\right)^{\frac{a_K}{u}} \left(\frac{L}{a_L}\right)^{\frac{a_L}{u}} y^{\frac{1}{u}} \qquad (3.18)$$

where: T = truck transportation

R = rail transportation

K = capital

L = labor

$$u = a_T + a_R + a_K + a_L.$$

The demand function for rail implied by Shephard's lemma is thus:

$$x_T = \frac{\partial C(y,W)}{\partial w_T} = a_0 \; (\frac{w_T}{a_T})^{-\frac{1}{u} \frac{a_T}{u} - 1} (\frac{w_R}{a_R})^{\frac{a_R}{u}} (\frac{w_K}{a_K})^{\frac{a_K}{u}} (\frac{w_L}{a_L})^{\frac{a_L}{u}} y^{\frac{1}{u}} \qquad (3.19)$$

Assuming that 1) rail and trucking price are hyberbolic functions of level-of-service attributes; 2) firms have the same technology and factor prices; 3) the aggregate output of a zone is a function of economic activity measurements, Eq. (3.19) yields an expression similar to the direct demand model given as Eq. (3.17).

Recently, a number of researchers have modelled mode choice using consumer theory in a probabilistic modelling framework.[20] This approach usually defines a set of mode alternatives and characterizes each mode by a vector of level-of-service attributes. Shippers are postulated to select among these alternatives such that their utilities can be maximized. Models have been estimated at either disaggregate or aggregate levels using different modelling techniques. A summary of these models is given in Figure 7.

The utility function of a shipper was in every case assumed as a linear additive function of mode attributes and shipment characteristics. Level-of-service attributes that have been considered include freight rate, mean transit time and reliability.

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[20] For example, Kullman (1973), Watson et al. (1974), Boyer (1977), Levin (1978) and Winston (1978).

Figure 7. Summaries of Shipper's Choice Models

Developed by	Mode	Variable	Data	Modelling Technique
Kullman (1973)	1. rail 2. truck	1. freight rate 2. mean transit time 3. standard deviation in transit time 4. value of commodity 5. mileage 6. annual tonnage	aggregate market share from 1967 Census of Transportation	logit model, estimated by ordinary least squares
Watson et al. (1974)	1. rail 2. truck	1. freight rate 2. mean transit time 3. standard deviation in transit time 4. value of commodity	waybills of household appliances	logit model, estimated by maximum likelihood
Boyer (1977)	1. rail 2. truck	1. freight rate 2. mileage 3. tons per vehicle 4. value of commodity 5. commodity dummies	aggregate market share from 1967 Census of Transportation	logit model, estimated by ordinary least squares and weighted least squares
Levin (1978)	1. truck 2. rail boxcar 3. piggyback	1. freight rate 2. mean transit time 3. standard deviation in transit time 4. value of commodity	aggregate market share from 1972 Census of Transportation and 1972 Carload Waybill Sample	logit model, estimated by ordinary least squares

Figure 7, Continued

Developed by	Mode	Variable	Data	Modelling Technique
Winston (1978)	1. rail 2. truck	1. freight rate 2. mean transit time 3. standard deviation in transit time 4. reliability (3/2) 5. value of commodity 6. shipment size	perishable agricultural	probit model, estimated by Weighted Exogenous Sampling Maximum Likelihood

Reliability was typically measures as the standard deviation of mean transit time. Shipment characteristics were entered into the utility function to reflect commodity attributes and market attributes. These attributes that have been used include value of commodity, shipment distance and quantities. Most of these models were binary mode choice models, except Levin's model where three modes were considered.

Let us use Levin's model as an example for illustration. Levin (1978) tried to model the allocation in surface freight transportation in order to investigate rate distortions that have occurred as the consequence of ICC rate regulation. He postulated a multinomial logit model involving the choice among common carrier truck, rail boxcar and piggyback. He then transformed the logit model into the following regression equations and estimated them by ordinary least squares.

$$\log \frac{P_2}{P_1} = a_{21} + b_1(R_1 - R_2) + b_2 V(T_1 - T_2) + b_3[2V(\sigma_1 - \sigma_2)] + u_1$$

$$\log \frac{P_3}{P_1} = a_{31} + b_1(R_1 - R_3) + b_2 V(T_1 - T_3) + b_3[2V(\sigma_1 - \sigma_3)] + u_2$$

$$(3.20)$$

where: P_i = market share by mode i

        R_i = freight rate by mode i

        V = value of commodity

        T_i = mean transit time for mode i

σ_i = standard deviation in transit time for mode i

u = disturbance term.

Thus, Levin's model is typical of those based on consumer theory.

Consumer theory maximizes the utility to a consumer of a given choice.. In the freight case, firms purchase transportation for moving their inputs and outputs. There is an explicit objective function, that is, either to maximize profit or to minimize costs, etc. Thus, consumer theory is less relevant than production theory as a basis for the derivation of a freight demand model. As a matter of fact, models of shipper choice have been limited only to the choice of mode. To develop a shipper choice model including shipment size by maximizing shippers' utility would be irrelevant, since shipment size is at the core of the logistics process and as such is part of the firm's production costs.

3.6. Inventory Theory

Stenason (1960) applied conventional inventory theory to the choice of mode . He considered the total variable costs of inventory holdings as the sum of order costs, transport costs, capital carrying costs and storage costs. Thus, the "service indifference functions" between two modes can be developed under given shipment sizes.

Baumol and Vinod (1970) reformulated this model by adding safety stock costs to reflect uncertainty in demand and transportation

service. Baumol considered order costs, transport costs, capital carrying costs and safety stock costs as the components of the total inventory costs. Under some assumptions for the distributions of demand and transit time, Baumol formulated the safety stock costs as $k \ (s + \bar{t} + k \ \bar{t})T$, where T is the annual demand of the commodity, s is the time between two orders, $\bar{t}$ is the mean transit time and k is a parameter associated with the risk level and other factors.

Elaborate models have been developed by Roberts (1971) and Roberts et al. (1976) using the inventory theoretic approach. Roberts' model was formulated in a probabilistic environment. Purchase costs, order costs, transport costs, capital carrying costs, storage costs and stockout costs were considered in the model. Both the demand and transit time were assigned by empirical distributions. Another unique feature of his model is the explicit treatment of stockout costs. The calculation of expected stockout costs requires that a matrix of probabilities p(m,n) be calculated for each possible condition of stockout of m items for n days, and that a unique cost be associated with each stockout condition. A set of probabilistic birth and death equations has been developed to calculate the matrix p(m,n) given the demand and transit time distributions and the reorder point. This model has been implemented on a computer to study the choice among a set of modes and shipment sizes for various types of commodities in given intercity markets.[21]

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[21] Roberts et al. (1976), referred to as FEA model.

Terziev (1976) attempted to develop a freight demand model from Roberts' inventory theoretic framework using disaggregate modelling technique. He developed a simple mathematical form to approximate the safety stock carrying costs and stockout costs. He then specified a linear additive logistics cost function which includes terms for purchase cost, order and handling cost, capital carrying cost in transit, loss of value during transit, packaging cost and loss and damage, transportation charges, capital carrying cost in storage, safety stock carrying cost and stockout cost. He assumed a logit form of the model and tried to estimate the model against the observations found in the Commodity Transportation Survey, Census of Transportation, 1967. The empirical results from Terziev's study were disappointing. He estimated a conditional mode choice model and a mode-shipment size joint choice model. A large portion of the coefficients were estimated with either counterintuitive signs or irrelevant magnitudes. However, this was mainly due to the poor quality of the data set used.

To conclude, the basic assumption of the inventory theoretic approach to freight demand analysis is that factor substitution will not occur. Changes in transportation costs, therefore, are assumed to have no effect on factor substitution. This restricts a model developed from inventory theory to only the short-run problems which are defined in Section 2.6. Models that have been developed from inventory theoretic approach are currently limited to the choice

of mode or shipment size. Origin choice models have not yet been
undertaken.

CHAPTER 4

DEVELOPMENT OF A SHORT-RUN FREIGHT DEMAND MODEL

4.1. Introduction

As indicated in the general framework presented earlier,
a demand model using logistics choice as its basis must be cate-
gorized as a short-run freight demand model. In a short-run freight
demand model, the annual demand for an input material is treated
as fixed and only transportation-related choices, such as the origin
of the supplier, the shipment size and/or the mode of transport,
are considered as open to choice. In terms of production theory,
this is moderately restrictive, since it allows no factor substi-
tution between the demand for transportation and the demand for other
factors, but it does allow the substitution within the transportation
and logistics cost elements.

This chapter presents an outline for modelling short-run freight
demand. Section 4.2 describes the logistics process that is behind
the formulation of the short-run model. The inventory control
systems are briefly examined and the objective function for logistics
strategy is discussed in terms of the complete range of logistics
costs. Section 4.3 gives two general approaches to modelling these
logistics processes--a deterministic cost model and a random cost model.
The formulation and the limitations underlying a deterministic
cost model are discussed in Section 4.4. Section 4.5 presents a
framework for a random cost model of short-run freight demand.

The assumptions behind a random cost formulation are then discussed and a general specification for the choice of mode, shipment size and origin of supplier is proposed.

4.2. The Logistics Process

To examine the process involved in selecting an appropriate logistics strategy, one must first know what the choice variables are. The apparent objective of logistics decisions in most cases is to minimize the purchase and associated logistics costs for input materials. This can be shown as the following mathematical expression:

$$w_k = \min_{i,m,q} [p_{k,iq}(x_k) + t_{k,imq}(x_k)] \qquad k = 1,\ldots,M \qquad (2.27) \text{ repeated}$$

In practice, logistics strategy may be approached as either a single-item inventory control process or a multi-item inventory control process. In a single-item inventory process, the inventory control plan is designed item by item; while in a multi-item inventory process, items are ordered, shipped and possibly stored together. A multi-item inventory process is somewhat more complicated than a single-item process, yet the tradeoffs and objective function in both processes are virtually identical. Thus, we proceed to outline our subsequent developments in terms of a single-item inventory process. We can start by rewriting Eq. (2.27) as:

$$w = \min_{i,m,q} [p_{iq}(x) + t_{imq}(x)] \qquad\qquad (4.1)$$

in which x is the annual use rate of an input material.

Defining a logistics strategy involves: 1) where to buy;
2) how often to buy; 3) how much to buy; and 4) by what mode to
transport; given the annual use rate of commodity x. Notice that
how often and how much are interdependent and only one of them can be
selected as a decision variable. Let us denote f as the frequency
of purchase and q as the amount of each purchase. We then have the
identity: $x = f \cdot q$ in the decision-making process. To use q as
the decision variable in inventory control is usually referred to as
a "trigger system" since reorder is initiated at that point in time
when the number in stock approaches a predetermined or "trigger"
number. To use f as the decision variable instead is said to use
a periodic inventory strategy. From the view point of stockouts,
the periodic system has the disadvantage over the trigger system
because more safety stocks would be required if an order is not
decided according to the actual numbers on hand through time.
Note also that the periodic system fits nicely with the assumptions
required to operate a multi-item inventory control system while the
trigger system lends itself more easily to the single-item approach.

For the purpose of transportation demand analysis, it is useful
to represent the decision variables defining alternative logistics
strategies as the choice of origin i, shipment size q, and mode of
transport m. By using the identity, $x = f \cdot q$, the decision process of
a periodic system also involves the choice of shipment size.

The single-item inventory process of the trigger system can therefore
serve as the underlying framework for a short-run freight demand
model in this work.[22/]

To begin, the following definitions are made:

I = the number of items in stock

u = daily use rate of the commodity

r = reorder point

q = size of an order

ℓ = lead time

s = safety stock.

A single-item, trigger actuated inventory system is illustrated in
Figure 8. The inventory level of a commodity, I, changes through time
due to the demand for the commodity and the resupply of the commo-
dity. The level of inventory peaks as a new shipment arrives and
declines steadily through time until the arrival of another shipment.
The daily use rate of the commodity, u, fluctuates through time.
For wholesale/retail industries, the use rate varies with the random
arrival of customers. For manufacturing industries, use rate varies
with the fluctuation in the demand for materials, machine breakdowns,
absenteeism, etc. Notice that the sum of daily use rate over the
whole year becomes the annual use rate which is denoted as x in Eq. (4.1).

[22/] For firms using a periodic system or multi-item system, we can show
that respecification will involve only part of the variables in the
logistics cost function. Most remain identical.
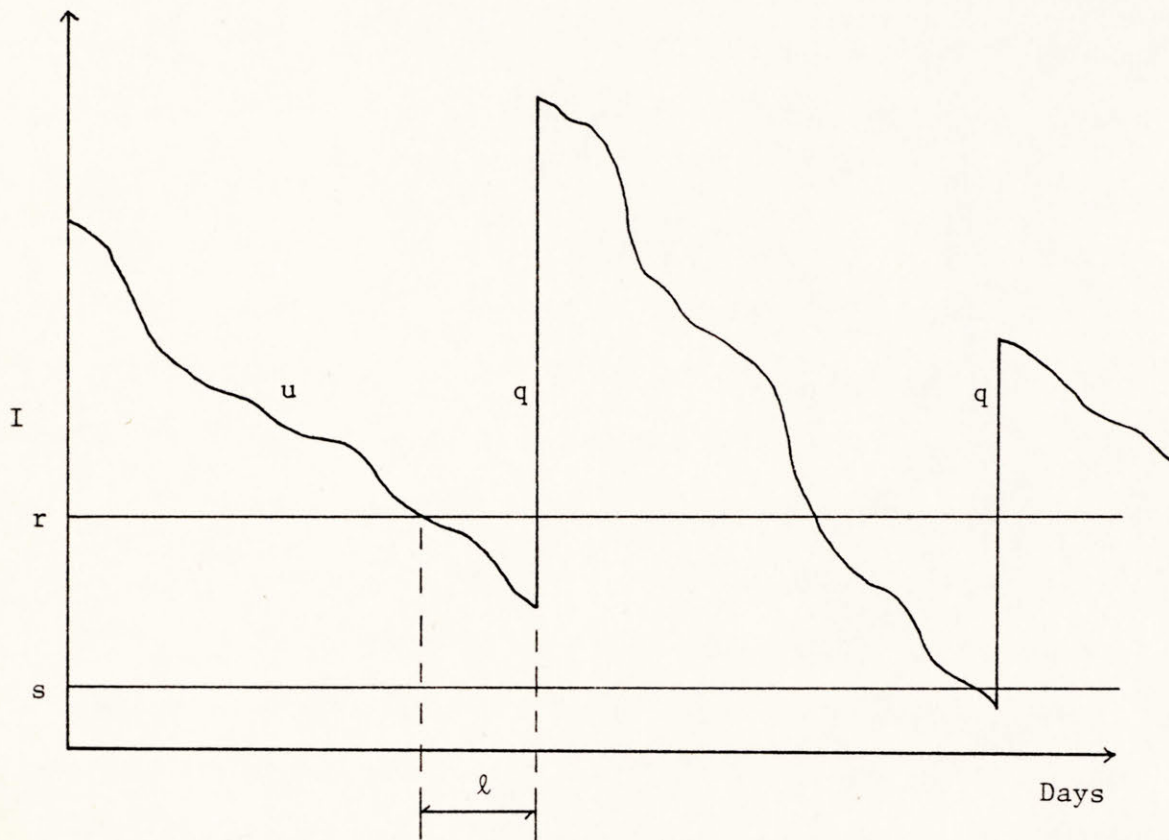
u = daily use rate

r = reorder point

s = safety stock

q = size of an order

ℓ = lead time

Figure 8. A Trigger Actuated Inventory System

The time that elapses from the placement of an order until the
arrival of the shipment is referred to as the lead time (ℓ). The
lead time is usually uncertain due in part to unreliability in trans-
portation services. In viewing the uncertainty in both use rate and
lead time, a reorder point (r) is set at a sufficiently high inven-
tory level so that the stock will not fall to zero before a new
shipment arises. When the inventory level reaches the reorder point,
an order is thus placed. A reorder point is normally set equal to
at least the expected use of the commodity during the expected lead
time. In practice, a buffer is designed to protect against stockout
in considering the fluctuation in demand and transportation services.
This buffer is sometimes referred to as safety stock. We will
denote the safety stock variable as s. Thus, safety stock can be
expressed as the actual reorder point minus the expected use rate
multiplied by the expected lead time: $s = r - E(u) \cdot E(\ell)$. A given
level of safety stock will then provide an associated level of pro-
tection against stockout to the firm. Obviously, whether this level
of protection is adequate is closely related to the consequences of
a possible stockout.

A reasonable objective function for the firm's logistics strategy
is to minimize the total costs of purchasing and inventory holding
of input commodities; or equivalently, to minimize the costs of
purchasing and inventory holding per unit of input. The cost components
of the objective function are: purchase cost, order cost, transport

cost, storage cost, capital carrying cost, and stockout cost.

The cost components of Eq. (4.1) can be detailed as:

$$w = \min_{i,m,q} (P_{iq} + ODC_q + TC_{imq} + STC_{mq} + CCC_{iq} + STOUT_q) \quad (4.2)$$

where: W = true factor price for an input material

   P = purchase price per unit

   ODC = order cost per unit

   TC = transport cost per unit

   STC = storage cost per unit

   CCC = capital carrying cost per unit

   $STOUT$ = stockout cost per unit.

Note that Eq. (4.2) gives the unit cost for an input commodity, k, under a given annual use rate x. Both k and x are omitted from the equation for the sake of simplicity. The subscripts for these cost components are the associated decision variables. A short discussion of these costs is now presented.[23]

Purchase Cost per Unit (P)

   This is the cost to acquire the product. It is a function of the choice of origin since different markets may involve differential prices. Sometimes firms may enjoy quantity discounts with larger purchases; therefore, purchase cost also becomes a function of

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[23] Based largely upon the logistics cost framework developed in Roberts (1971), Roberts et al. (1976) and Terziev (1975).

shipment size.

Order Cost (ODC)

Order cost is the cost to place, process and receive the order.
Basically, it represents all those administrative and handling costs
associated with an order. Order cost is usually related to the choice
of shipment size. The larger an order, the more a firm is willing
to spend to locate an appropriate supplier and place and handle the
order.

Transport Cost (TC)

Transport cost is defined as the cost incurred during and as
the result of transport. It includes packaging cost, freight charges,
including any special charges, handling costs, loss and damage
during transport, capital carrying cost on the money tied up during
transit, and loss of value due to reduction of shelf life during
transit.

Freight tariff charges and special charges are the direct trans-
port cost paid to the carrier. Freight rates are shown to be a
function of mode, shipment size,and length of haul as well as
commodity attributes. Thus, the freight rate is interrelated
to the choice of mode, shipment size and origin.

Handling cost is also a function of shipment size. Different
shipment sizes may reflect different handling equipment required.
For example, manual handling is sufficient for a shipment of 50 pounds,

while mechanical handling equipment could become necessary for a shipment of 100,000 pounds or more. Handling cost is also affected by the choice of mode. For instance, the use of rail multiple carload shipments is very likely to involve a private railroad siding. Thus, there are both fixed and variable handling costs.

The cost of packaging is highly correlated with the loss and damage that sometimes occurs during shipment.[24] The better the packaging, the smaller the costs of loss and damage. There appear to be four cost components that arise as the result of loss and damage. First, there is the direct cost of the loss product. Second, there is the cost of filing a claim. Third, there are capital carrying costs which are tied up while the claim is being investigated. Evidence shows that a considerable amount of time is frequently required to complete such an investigation and reach a settlement. Finally, the receipt of a damaged shipment may cause costly disruptions in the firm's inventory control system.

In most cases, the cost of direct loss and damage to the shipment is paid by the carrier. In this situation, only the second through the fourth cost components are to be considered.

Capital carrying cost in transit is the interest on capital tied up from the time an order is shipped until the shipment arrives. This cost depends on the dollar amount of a purchase and the total time

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[24] See Colquitt (1965).

it will take to receive the order. With FOB factory purchases,
the legal ownership of the product passes at the time of loading on
the transport vehicle and the payment is due accordingly. Thus,
the time element involved is the actual transit time of the carrier.
On the average, this is the mean transit time plus the reliability.
In most cases, the buyer does not bear the capital carrying cost during
the time spent waiting at the origin.

Loss of value during transit is important for time sensitive
goods such as newspapers and perishable goods such as fruits and
vegetables. Thus, the loss of value is a function of the shelf
life of the commodity. This cost is expected to be the dominant
factor for those commodities with short shelf lives.

Storage Cost (STC)

Storage cost is the cost associated with warehousing, stocktaking,
shelving and obsolescence during storage. It includes the cost
associated with non-safety stock as well as safety stock. Storage
cost per unit is very commodity dependent involving the level of
protection, height of stacking, etc. Storage cost also depends on
the level of inventory; thus it is directly related to the risk
of stockout and is also a function of shipment size. In some cases,
mode choice could also affect the storage cost. For instance,
shipment by rail carload may be chosen over truck in some cases
because it allows the receiver to use the rail cars for short-term

storage. The use of rail also allows the handling and storage work
to be performed at the convenience of the receiver, thus increasing
labor efficiency.

Capital Carrying Cost (CCC)

Capital carrying cost is the interest on capital tied up in
inventory. It depends on the average level of inventory and its
value. Thus capital carrying cost is a function of both shipment size
choice and origin choice.

Stockout Cost (STOUT)

This is the cost associated with being out of stock. Cost of
a stockout is, of course, very difficult to measure directly.
The problem of the retailer who faces the possible loss of a sale
if he stocks out is not the same as that of the manufacturer who
must shut down a production line if a critical component or material
is not available. Most approaches to inventory control attempt to
rationalize the problem by adopting a "service level" approach to
stockouts. That is, instead of facing up squarely to the question
of how much it costs to incur a stockout, a policy is adopted that
will prevent stockouts from happening with a certain probability.[25]
A model to compute stockout costs has also been developed.[26]

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[25] See Brown (1967).

[26] See Section 3.6.

The annual cost of stockouts depends on the cost of a stockout
and the risk of stocking out during a reorder period. The manager
can select a stockout risk by adjusting the size of the safety stock.
The higher the level of safety stock, the lower the possibility of
stocking out. Stockout occurs because of the uncertainty in daily
use rate and transportation service. It is thus affected by the
choice of shipment size, mode, and origin.

The main tradeoffs in logistics strategy involve the economies
of scale involved in making larger shipments vs. storage and capital
carrying costs. From a strictly transport viewpoint, the larger the
shipment, the lower the transport cost. The cost of ordering is
also minimized by this strategy. However, the cost of storing and
carrying this inventory could in some cases be prohibitive. There
would be costs due to pilferage, spoilage, and obsolescence as
well as the opportunity cost associated with the capital tied up in
the inventory. Another important tradeoff involves the cost of storing
and carrying safety stock vs. the cost of stocking out. Tradeoffs
also exist between the loss of value during transit versus a higher
transport rate, packaging cost versus loss and damage, and
purchase cost versus transport cost, etc.

The relative importance of each logistics cost component is
highly dependent on the type of commodity, the annual use rate, the
manager's risk of stocking out, the market of the input material,
and the transport level-of-service.

To review these costs, some hypothetical cases are developed
and shown in Figure 9. Cost figures are calculated using the Logistics
Strategy Analyzer developed at MIT. One can see that the percentage
distribution of logistics costs over the components is highly variable
in each case. Nevertheless, freight rate and capital carrying costs
in inventory are typically the two most important logistics costs.

4.3. Modelling Logistics Choice

Having outlined the logistics process and the costs involved
in the selection of logistics strategies, we will now investigate
the techniques that can be used to model logistics choice. For the
purposes of this analysis, a logistics strategy is designated by the
choice of a supplier at origin i, a shipment size q and a mode m.
Thus, a possible combination of i, m, q is one of the possible
alternative logistics strategies open to a firm. An optimal strategy
is said to be one with the total lowest purchase and logistics costs.
The criteria for an alternative i,m,q to be the optimal logistics
strategy is satisfaction of the following condition:

$$w_{imq} \leq w_{i'm'q'} \qquad \forall \, i'm'q' \in A \qquad (4.3)$$

where A is the set of possible origin, shipment size and mode
combinations.

There appear to be two general approaches one can take in
modelling the logistics decision of a firm. The difference between

Figure 9. Hypothetical Logistics Costs[*]

	Case 1	Case 2	Case 3
Commodity	Drugs	Iron Bars	Tabacco Leaf
Density (lbs/ft^3)	20	200	20
Value ($/lb)	11	0.1	2
Annual Use Rate (tons)	50	500	5
Shipment Size (tons)	2.5	6	2.5
Mode	TOFC	CL	LTL
Transport Distance (miles)	200	400	800
Purchase Cost	1100	10	200
Order Cost	0.04	0.017	0.4
Freight Rate	1.204	1.388	5.594
Capital Carrying Cost in Inventory	45.632	0.167	6.186
Storage Cost	0.733	0.008	0.309
Stockout Cost	0.272	0.000	0.034
Total Logistics Cost	49.183	1.628	12.874
Total Cost	1149.183	11.628	212.874

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[*] Costs are calculated by Logistics Analyzer developed in FEA model
(Roberts et al., 1976). All costs are represented as dollars
per 100 pounds unless otherwise indicated.

the two approaches lies in the assumptions involved in the level of
certainty concerning the information in the logistics cost function.

In the first approach, the logistics cost function given in
Eq. (4.2) is assumed to be fully observable. Therefore, the
alternative defined by the choice of i,m,q is selected with certainty
if $w_{imq} \leq w_{i'm'q'}$, $\forall i'm'q' \in A$. A model developed upon this assump-
tion will be referred to as a deterministic cost model of short-run
freight demand. The model states:

$$P_r(imq|A) = 1, \text{ if } w_{imq} \leq w_{i'm'q'} \quad \forall i'm'q' \in A$$

$$= 0, \text{ otherwise} \tag{4.4}$$

where w_{imq}, $w_{i'm'q'}$ are all fully observed.

The second approach assumes that the logistics cost function
is not fully observable. Only part of the cost function is observed.
Denoting w^o_{imq} as the observable part of the logistics cost function
and w^u_{imq} as the unobservable part of the logistics cost function,
we have

$$w_{imq} = w^o_{imq} + w^u_{imq} \quad \forall imq \in A \tag{4.5}$$

The optimal strategy has been selected if $w_{imq} \leq w_{i'm'q'}$, $\forall i'm'q' \in A$.
However, since w^u_{imq}, $\forall imq \in A$ are not observable, we are unable to
predict the actual choice with certainty. Only the probability of
choosing each alterantive can be predicted. This probability is

related to the unobservable part of the logistics cost function.
That is,

$$P_r(imq|A) = P_r[w^o_{imq} + w^u_{imq} \leq w^o_{i'm'q'} + w^u_{i'm'q'}, \; \forall \, i'm'q' \in A]$$

$$= P_r[w^u_{imq} - w^u_{i'm'q'} \leq w^o_{i'm'q'} - w^o_{imq}, \; \forall \, i'm'q' \in A]$$

$$(4.6)$$

In Eq. (4.6), the unobservable logistics cost can be treated as random
variable. Thus, probabilistic models can be derived by assuming
appropriate distributions for w^u_{imq} and integrating Eq. (4.6) mathe-
matically. A model of this type we will refer to as a <u>random cost</u>
<u>model</u> of short-run freight demand.

Models of each of these approaches are detailed in turn in the
subsequent sections.

4.4. Deterministic Cost Model

The deterministic cost model assumes that the logistics cost
function of Eq. (4.2) is fully observable. Thus, the best candidate
for a deterministic cost model is the analytical solution to Eq. (4.2)
for the optimal choice of i, m, and q. Since modes and origins are
discrete alternatives, this involves solving for the optimal q
given i and m. Unfortunately, the logistics cost function in the
real world is by its very nature a complicated function of shipment
size. No closed form solution can be found without simplifying
the function with respect to shipment size.

This can be demonstrated by the following example. Consider the
case where: 1) market price of the input material is fixed, i.e.,
there is no quantity discount; 2) cost per order is the same for all
sizes of orders; 3) both packaging cost and handling cost are
proportional to shipment size and may be different by mode;
4) freight rate is not a function of shipment size; 5) the input
is not a seasonal or perishable good; 6) loss and damage are
negligible; 7) daily use rate of the input is quite stable;
and 8) fixed percentages of annual use rate are used as safety
stock to protect against stockout; these percentages are different
by mode to reflect the variation of reliability. Next, we define
some notations to be used:

OC = order cost per order

PKC_m = packaging cost per pound of shipment by mode m

$RATE_m$ = freight rate of mode m

TT_m = mean transit time in days of mode m

i = interest rate per year

HC_m = handling cost per pound of shipment on mode m

SC = storage cost per ft^3

a_m = percentage of annual use rate for safety stock as
mode m is used

DEN = density of the commodity.

Thus we have a simplified logistics cost function defined as the
sum of the following elements.

purchase cost per unit $= P_i$

order cost per unit $= [(x/q) \cdot OC]/q = OC/q$

packaging cost per unit $= [(x/q)(PKC_m \cdot q)]/x = PKC_m$

tariff charge per unit $= [(x/q)(RATE_m \cdot q)]/x = RATE_m$

capital carrying cost in transit per unit $=$

$$\{[q \cdot (TT_m/365)](x/q) \cdot P_i \cdot i\}/x = (TT_m/365) \cdot P_i \cdot i$$

handling cost per unit $= [(x/q)(HC_m \cdot q)]/x = HC_m$

non-safety stock storage cost per unit $=$

$$\{[(q/2)/DEN] \cdot SC\}/x = (q \cdot SC)/(2 \cdot DEN \cdot x) \quad \underline{27/}$$

non-safety stock carrying cost per unit $=$

$$[(q/2) \cdot P_i \cdot i]/x = (q \cdot P_i \cdot i)/(2 \cdot x)$$

safety stock storage cost per unit $=$

$$\{[(a_m \cdot x/2)/DEN] \cdot SC\}/x = (a_m \cdot SC)/(2 \cdot DEN) \quad \underline{27/}$$

safety stock carrying cost per unit $=$

$$[(a_m \cdot x/2) \cdot P_i \cdot i]/x = (a_m \cdot P_i \cdot i)/2$$

Assuming that the objective function of the firm is to minimize total costs, this can be written as:

$$w_{imq} = P_i + \frac{OC}{q} + PKC_m + RATE_m + \frac{TT_m \cdot P_i \cdot i}{365} + HC_m$$

$$+ \frac{q \cdot SC}{2 \cdot DEN \cdot x} + \frac{q \cdot P_i \cdot i}{2 \cdot x} + \frac{a_m \cdot SC}{2 \cdot DEN} + \frac{a_m \cdot P_i \cdot i}{2} \qquad (4.7)$$

$\underline{27/}$ Under assumption 7), the average level of non-safety stock and safety stock through time are $q/2$ and $a_m \cdot x/2$ respectively.

Taking the derivative of the costs with respect to q produces

$$\frac{\partial w_{imq}}{\partial q} = -\frac{OC}{q^2} + \frac{SC}{2 \cdot DEN \cdot x} + \frac{P_i \cdot i}{2 \cdot x} = 0 \qquad (4.8)$$

Thus, the optimal shipment size for a given combination of i and m, denoted as $q^*(i,m)$, is

$$q^*(i,m) = \sqrt{\frac{2 \cdot x \cdot DEN \cdot OC}{SC + P_i \cdot i \cdot DEN}} \qquad (4.9)$$

Notice that $q^*(i,m)$ is not a function of m in this case. Examining the derivatives of $q^*(i,m)$ with respect to each argument reveals that the optimal shipment size is increasing but it does so at a decreasing rate with respect to annual use rate x, density DEN, and order cost OC. In contrast, there is a decreasing and convex function of storage cost SC, purchase P_i and interest rate i. Substituting Eq. (4.9) into Eq. (4.7) yields the optimal cost given i and m, $w^*(i,m)$, as

$$w^*(i,m) = P_i + OC \sqrt{\frac{SC + P_i \cdot i \cdot DEN}{2 \cdot x \cdot DEN \cdot OC}} + PKC_m + RATE_m + \frac{TT_m \cdot P_i \cdot i}{365} +$$

$$+ HC_m + \frac{SC}{2 \cdot DEN \cdot x} \sqrt{\frac{2 \cdot x \cdot DEN \cdot OC}{SC + P_i \cdot i \cdot DEN}} + \frac{P_i \cdot i}{2 \cdot x} \sqrt{\frac{2 \cdot x \cdot DEN \cdot OC}{SC + P_i \cdot i \cdot DEN}} +$$

$$+ \frac{a_m \cdot SC}{2 \cdot DEN} + \frac{a_m \cdot P_i \cdot i}{2} =$$

$$= P_i + PKC_m + RATE_m + \frac{TT_m \cdot P_i \cdot i}{365} + HC_m$$

$$+ \frac{a_m \cdot SC}{2 \cdot DEN} + \frac{a_m \cdot P_i \cdot i}{2} + \sqrt{\frac{2(SC + P_i \cdot i \cdot DEN) \cdot OC}{DEN \cdot x}} \qquad (4.10)$$

Therefore, the optimal strategy can be determined by comparing $w^*(i,m)$ for all i and m and selecting that i and m for which the minimum w^* is observed.

The tradeoffs between transport cost and transit time implied by the above model are linear. For each mode, the indifference curves of rate and transit time are given by

$$RATE_m + \frac{TT_m \cdot P_i \cdot i}{365} = C \qquad (4.11)$$

where C is a constant. Varying interest rate i, a set of linear relationships can be plotted as Figure 10. Denoting the subscript R as rail and T as truck, the indifference curves between rail and truck become

$$PKC_R - PKC_T + RATE_R - RATE_T + (TT_R - TT_T) \frac{P_i \cdot i}{365} + HC_R - HC_T$$

$$+ (a_R - a_T)(\frac{SC}{2 \cdot DEN} + \frac{P_i \cdot i}{2}) = 0 \qquad (4.12)$$

The assumptions involved in the above development are fairly realistic with one exception. That is, the assumption of constant
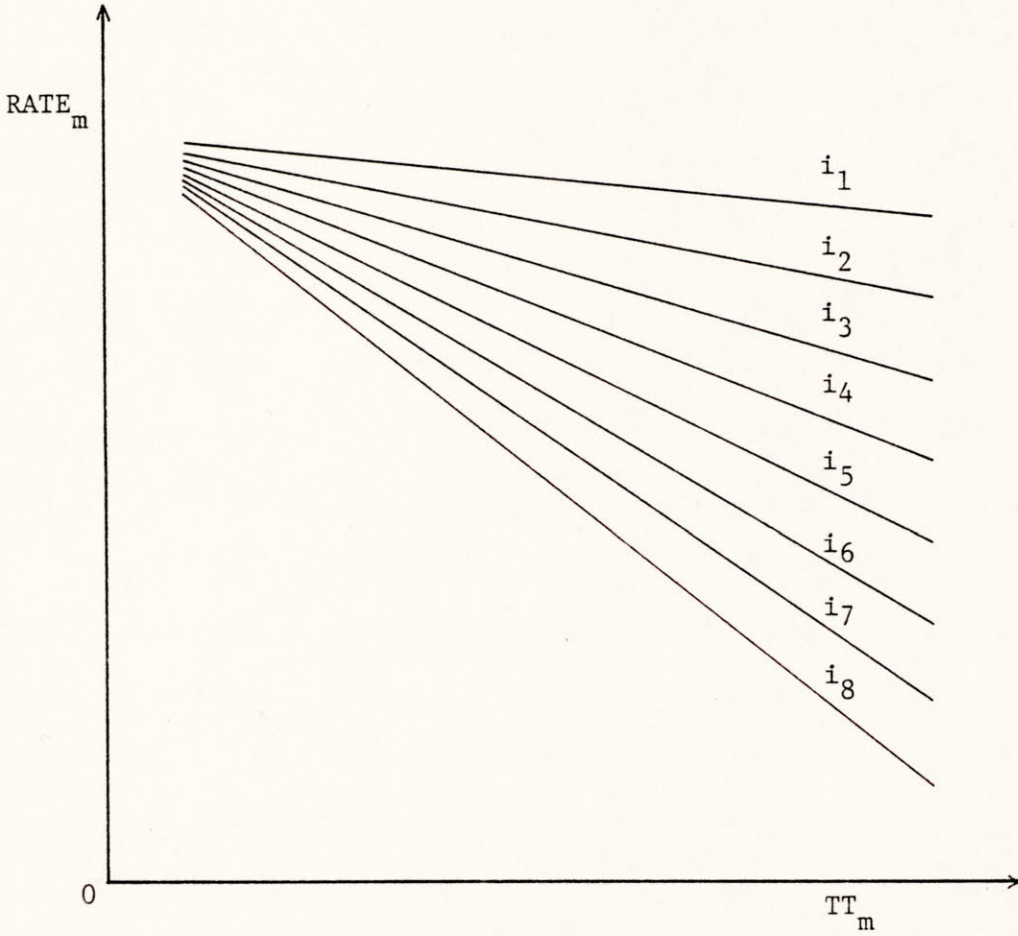
Figure 10. Indifference Curves between $RATE_m$ and TT_m at Different Interest Rate (i), $i_n > i_{n-1}$

freight rates. As a matter of fact, in practice freight rates
are a decidedly decreasing function of shipment size. There are
clearly economies of larger shipment sizes. One of the central
roles played by the logistics management process has been making
the tradeoffs between the several factors involved. Thus, to
assume constant freight rates is totally unrealistic. Unfortunately,
constant freight rates are the crucial assumption required to be
able to solve for q^* analytically. Without this assumption, the
first order conditions of Eq. (4.8) would involve a polynomial
of q and no general solution can be found. Conventional determinis-
tic inventory theoretic models usually employ this assumption and
thus are unrealistic when compared to real world results.[28/]

To bypass these difficulties, a simulation method can be
used to construct a deterministic cost model. The Logistics
Strategy Analyzer model is of this type. It explicitly considers
each cost component that would occur for each logistics strategy
by a set of models which measure the consequences of the strategy
and its associated costs.

In summary then, logistics decisions can be predicted at a
very disaggregate level. As the example in Figure 11 shows for
a firm using wood posts at an annual use of 49 tons per year,
shipping from a supplier 1300 miles away, the best solution would
be to order rail carloads with a shipment size of 60,000 pounds.

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[28/] For example, Stenason (1960), Baumol (1967).

Figure 11.   Total Logistics Costs as a Function of Mode and
             Shiment Size*

Commodity : Wood Posts
Value   : 3¢/lb
Density   : 33 lbs/ft^3
Distance : 1300 miles
Annual Use: 49 tons

Shipment Size lbs × 10^3
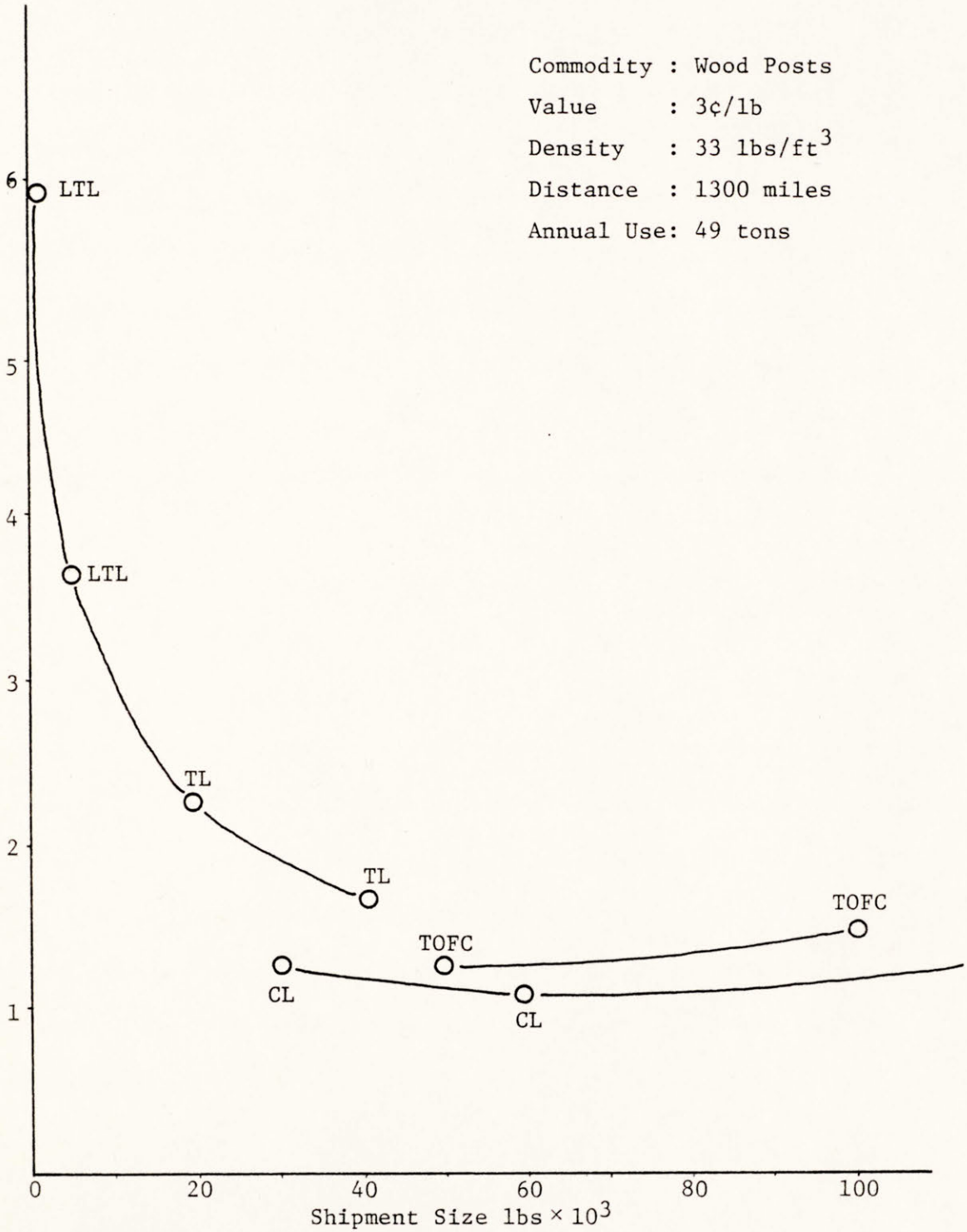
*Taken from Roberts and Lang (1978), p. 27.

There are two major limitations of this approach. First, it has
to enumerate all possible alternatives in order to search for the
optimal one; thus it is expensive in use. More importantly,
a deterministic cost model, by definition, assumes that all
information is observable. In fact, there is a great deal of the
basic information that is usually not available. Thus, it is
not possible to predict the logistics choice with certainty.
To explicitly assume unobservable information in a deterministic
cost model invites the possibility of biased results. Conse-
quently, a random cost formulation becomes more relevant in this
respect.

4.5. Random Cost Model

The random cost model assumes that the purchase and logistics
costs, w_{imq}, contain an observable part w^o_{imq} and an unobservable
part, w^u_{imq}. The unobservable part is assumed to be a random variable
denoted as ε_{imq}. The randomness in the specification of the cost
function is due to omission of the unobservable cost components or
to measurement error, as well as to other possible sources of errors.
Since ε_{imq}'s are not observed, only choice probabilities can be
predicted as follows:

$$p_r(imq|A) = p_r[w^o_{imq} + \varepsilon_{imq} \leq w^o_{i'm'q'} + \varepsilon_{i'm'q'} \, , \, \forall i'm'q' \in A]$$

$$= p_r[\varepsilon_{imq} - \varepsilon_{i'm'q'} \leq w^o_{i'm'q'} - w^o_{imq} \, , \, \forall i'm'q' \in A] \quad (4.13)$$

Therefore, one can assume a probability density function for ε_{imq}'s to derive the choice probabilities as the cumulative distribution of ε_{imq}'s. Making different assumptions for ε, random cost models of different functional forms will result. For example, assuming that ε_{imq}, $\forall imq$ A are independently and identically Weibull distributed, the following multinomial logit model results from the integration of Eq. (4.13):

$$p_r(imq|A) = \frac{e^{-w^o_{imq}}}{\sum\limits_{i'm'q' \in A} e^{-w^o_{i'm'q'}}} \quad (4.14)$$

Alternatively, assuming that ε_{imq}'s are distributed as a multivariate normal distribution, a probit model results.[29] Note that the derived choice models are functions of the observed cost components only.

We will start to formulate the logistics choice process using the random cost model in detail. First, let us introduce some new notation. Variables defined in the previous section are still valid.

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[29] For details of the derivation of logit and probit models, see McFadden (1974), Ben-Akiva (1973), Daganzo et al. (1976), Lerman and Manski (1977).

SPC = special tariff charges per unit

WT = wait time in days

LDP = percentage lost and damaged

IT = time required to finish the investigation of a claim and
pay the claim

CLP = probability of a loss/damage

PGC = packaging cost per shipment

SHELF = shelf life in days

HT = height of stacking in storage

R = reliability of transit time in days.

A general specification of the logistics cost function for the choice of origin, mode and shipment size for a random cost model is proposed as follows.[30]

Purchase Cost per Unit

For practical reasons the purchase cost is defined as the FOB factory price. For the present time there will be no quantity discount. Thus purchase cost per unit can be specified as $\beta_1 \cdot P_i$. The coefficient β_1 serves as a scaling factor. It can be used to normalize all of the other coefficients. If this is done, then the resulting coefficients will be expressed in units which are compatible with costs measured in dollars. If quantity discount exists and is known

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[30] Reformulated from the work done in Terziev (1975), and Roberts (1976).

to be $p_i(q)$, the term can be respecified as $\beta_1 \cdot P_i(q)$.

Order Cost per Unit

Order costs are extremely difficult to specify precisely. However, it can be developed in such a manner that the unknown unit cost can be estimated as part of the coefficient. Assuming that the order cost per order is constant, order cost per unit can be formulated as $\beta_2 \cdot \frac{x}{q} \cdot \frac{1}{x} = \beta_2 \cdot \frac{1}{q}$. Thus the estimated β_2 divided by β_1 becomes the average cost per order for all commodities by all firms in all shipment sizes. This specification is simple but not very attractive, since order cost is likely to be a function of shipment size. A better assumption would be to assume order cost per order is $\beta_2 (P_i \cdot q)^n$, where n is a parameter of the choice model to be determined before estimation. We know n is constrained to values between 0 and 1. Thus order cost per unit can be specified as $\beta_2 \cdot (P_i \cdot q)^n \cdot \frac{x}{q} \cdot \frac{1}{x} = \beta_2 \cdot P_i^n \cdot q^{n-1}$. This specification allows order cost to vary with the amount of a purchase considering the fact that one is usually willing to spend more to process an order for a large purchase than for a small one.

Handling Cost per Unit

Handling cost data are usually not available. If handling cost per shipment can be assumed as fixed, one can specify handling cost per unit as $\beta_3 \cdot \frac{1}{q}$. In this case, handling and order costs will become one variable and only one coefficient will be estimated. However, this assumption is not realistic since handling equipment require-

ments are closely related to shipment size. It is possible to specify

handling cost per unit as several shipment size specific variables:

$\beta_{3n} \cdot \frac{1}{q}$, where n denotes the shipment size block. Thus, handling

cost per shipment could be estimated differently with different

shipment size blocks. At least three shipment size blocks can be

considered: the shipment sizes where no mechanized handling equip-

ment is required, those for which semi-automatic equipment such as

forklifts or cargo cranes are required, and fully automatic equipment

such as vacuum unloading, air slide or conveyor equipment for bulk

unloading are required.

Transport Charges per Unit

This can be specified simply as $\beta_4 \cdot (\text{RATE}_{imq} + \text{SPC}_{imq})$.

If there are no special charges (or data on special charges are not

available), the specification is simplified to $\beta_4 \cdot \text{RATE}_{imq}$. Notice

that β_4 serves only as a scaling factor.

Capital Carrying Cost in Transit per Unit

Capital carrying cost in transit is measured as the interest rate

times the value of the capital tied up in a shipment during its transit

time. It is extremely difficult to obtain the stated interest rates

used by firms in their planning; therefore, it is useful to estimate

the implied interest rate as a coefficient. This can be done by

specifying capital carrying cost per unit in transit as $\beta_5 \cdot \frac{\text{TT}_{imq}}{365} \cdot P_i$.

Dividing β_5 by β_1 will produce an estimate of the interest rate in

dollars per dollar per year. In those cases where purchase costs are paid during the time spent waiting for the vehicle, capital carrying cost per unit in transit becomes $\beta_5 \cdot (\frac{TT_{imq}}{365} + \frac{WT_{imq}}{365}) \cdot P_i$.

Capital Carrying Cost per Unit Tied Up with a Claim

This is the capital cost tied up with a loss or damage claim from the time of arrival until the claim is settled. This can be specified as $\beta_6 \cdot LDP_{imq} \cdot \frac{IT_m}{365} \cdot P_i$. The time required to complete an investigation, IT, varies with the carrier and is thus a function of the mode. This information is not currently available. One can assume either that this time is constant for all carriers and for all modes, or that this time is constant for all carriers of the same mode. The specification for the first assumption becomes $\beta_6 \cdot LDP_{imq} \cdot P_i$. Dividing β_5 by β_1 gives the product of the interest rate and the average time of investigation. For the second assumption, one needs a set of mode-specific variables expressed as $\beta_{6n} \cdot LDP_{imq} \cdot P_i$, where n is the modal subscript.

Cost of Filing a Claim per Unit

The cost to file a claim can reasonably be assumed to be a constant. Thus, this cost item can be specified as $\beta_7 \cdot CLP_{imq} \cdot \frac{x}{q} \cdot \frac{1}{x} = \beta_7 \cdot CLP_{imq} \cdot \frac{1}{q}$. Dividing β_7 by β_1 will produce an estimate of the average cost of filing a claim.

Packaging Cost per Unit

If packaging cost data were available, this cost term could be specified as $\beta_8 \cdot PGC \cdot \frac{x}{q} \cdot \frac{1}{x} = \beta_8 \cdot PGC \cdot \frac{1}{q}$. Unfortunately, information on packaging cost is frequently not available. Since it is not reasonable to assume identical packaging cost for all shipments, one should in this case ignore this cost item.

Loss of Value per Unit during Transit or Storage

The loss of shelf life during transit or storage is only significant for time-sensitive or perishable goods. Therefore, this cost item should be set equal to zero if the commodity being shipped has an indefinitely long shelf life. For time-sensitive or perishable goods, loss of value in transit per unit can be formulated as $\beta_9 \cdot (WT_{imq} + TT_{imq}) \cdot \frac{1}{SHELF} \cdot P_i$. This specification assumes that the loss of value is proportional to the percentage of the total life of the good which has expired by the time the order arrives at the destination. A more complicated formulation which accounts for storage as well is $\beta_9 \cdot [q - u(SHELF - TT_{imq} - WT_{imq})] \cdot P_i \cdot \frac{1}{q}$, in which the term $SHELF - TT_{imq} - WT_{imq}$ is the time available to use the time-sensitive or perishable goods. Thus, $u(SHELF - TT_{imq} - WT_{imq})$ is the maximum shipment size for these goods if there is to be no loss due to spoilage or time loss of utility. Therefore, $\beta_9 \cdot [q - u(SHELF - TT_{imq} - WT_{imq})] \cdot P_i \cdot \frac{1}{q}$ reports the loss of value per unit associated with the shipment size q.

Note that $q - u(SHELF - TT_{imq} - WT_{imq})$ is restrained to be non-
negative. For shipment sizes smaller than $u(SHELF - TT_{imq} - WT_{imq})$,
there is no loss of value. β_9 serves merely as a scaling factor.

Capital Carrying Cost per Unit in Inventory

It is almost impossible to obtain data on daily use rate fluc-
tuations for the various inputs used by industry generally. Although
it is possible to assume a probability distribution for use rate,
one can for practical purposes simply assume that use rates per day
are constant. Thus, the average inventory level for non-safety
stock becomes one-half of shipment size. On the average, one-half
of the stock is held in storage for the length of time between
orders. Therefore, we specify capital carrying cost per unit in
inventory to be $\beta_{10} \cdot \frac{q}{2} \cdot P_i \cdot \frac{1}{x}$. Dividing β_{10} by β_1 will produce
an estimate of the interest rate which can be used to express
the carrying cost in terms of dollars.

Non-Safety Stock Storage Cost per Unit

If storage cost data were available , this cost term could be
specified as $\beta_{11} \cdot \frac{q}{2} \cdot STC \cdot \frac{1}{x}$. Unfortunately, information on storage
cost is usually unobserved. There are, however, several possible
ways to specify storage cost per unit without observing it. If we
assume constant storage cost per pound for all commodities we will
have $\beta_{11} \cdot \frac{q}{2} \cdot \frac{1}{x}$. Dividing β_{11} by β_1 gives an estimate of storage
cost per pound in terms of dollars. Alternatively assuming a constant

storage cost per cubic foot for all commodities gives

$\beta_{11} \cdot \frac{q}{2} \cdot \frac{1}{x} \cdot \frac{1}{DEN}$. In this case, the coefficient represents

the storage cost per cubic foot of storage space. More elaborate

specification might assume a constant storage cost per square foot,

in which case this cost term can be specified as $\beta_{11} \cdot \frac{q}{2} \cdot \frac{1}{x} \cdot \frac{1}{DEN} \cdot \frac{1}{HT}$

with the coefficients representing the average storage cost per square

foot of storage space. This specification is more reasonable since

the height of stacking in storage varies by commodity.

Safety Stock Carrying Cost, Storage Cost and Stockout Cost per Unit

This term reflects the capital cost of goods kept on hand to

protect against stockouts, and the cost of stockouts when they occur.

The exact specification of this term is rather complicated because

of the probabilistic relationships which must be taken into account.

The situation is further complicated by the constraints imposed by

data availability.

The safety stock carrying cost/storage cost and the cost of

stockouts are counterbalancing factors. A compromise between the two

is established by the stockout risk chosen by the plant manager.

The probabilistic equations developed by Roberts (1971) are too

copmlicated to be used directly in our random cost model.

Terziev (1975) developed a simple equation to approximate safety

stock, carrying and stockout cost from Roberts' probabilistic system

as follows:

$$C(\gamma) = \beta_{12} \cdot (a_0 \cdot \gamma^{-a_1}) \cdot \frac{PRICE}{x} + \beta_{13} \cdot (a_2 \cdot \gamma) \frac{P_i}{q} \tag{4.15}$$

where: $C(\gamma)$ = safety stock carrying and stockout cost

γ = stockout risk

PRICE = price of the commodity being produced from the input

a_0, a_1, a_2 = parameters derived from Roberts' system.

He then solved for the optimal cost $C(\gamma^*)$. This resulted in an

expression which can be used as the specification of safety stock

carrying and stockout cost per unit.[31]

The above specification requires a knowledge of the commodities

to be produced from the input material and their market prices;

information which is not usually available. An alternative specifica-

tion is proposed as follows. We know that the firm tries to provide

safety stock to protect against stockout. To provide more safety

stock will increase capital carrying and storage cost. At the optimal

amount of safety stock, the marginal safety stock carrying and storage

costs would be equal to the negative of the marginal stockout cost.

The safety stock carrying and storage costs can be modelled as

a function of reliability. Assuming that the daily use rate

is constant and the firm's stockout risk is set at a con-

fidence level of n percent, the safety stock can be expressed

as $u \cdot (R_{imq}^n - TT_{imq})$, where R^n is the reliability (expressed in days)

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[31] See Terziev (1975) for details.

at a confidence level of n percent. Thus, if we assume that all firms

have the same stockout risk, the total safety stock carrying cost

becomes $i \cdot u \cdot (R_{imq}^{n} - TT_{imq}) \cdot P_i$. In viewing that firms may not have

the same risk and their risks are not observed, we thus try to specify

the safety stock carrying cost per unit as a commodity-specific

variable. This becomes $\beta_{12k} \cdot u \cdot (R_{imq}^{99} - TT_{imq}) \cdot P_i \cdot \frac{1}{x} =$

$\beta_{12k} \cdot (R_{imq}^{99} - TT_{imq}) \cdot \frac{P_i}{365}$, where k denotes commodities and R^{99}

denotes the reliability at a level of confidence of 99 percent.

Dividing β_{12k} by $2 \cdot \beta_1$ gives an estimate of the term

$i \cdot \dfrac{(R_{imq}^{n} - TT_{imq})}{(R_{imq}^{99} - TT_{imq})}$, which is assumed to be constant for each

commodity group. In general, final consumption goods, capital goods

and raw materials are likely to have different stockout consequences,

implying that their associated sotckout risk will be correspondingly

different. Notice that this form of specification is close to the

form used for capital carrying cost in transit, except that here

we have a set of commodity-specific variables as opposed to one generic

variable defined as capital carrying cost in transit. Similarly,

the safety stock storage cost can be specified as

$\beta_{13k} \cdot u \cdot (R_{imq}^{99} - TT_{imq}) \cdot \frac{1}{DEN} \cdot \frac{1}{x} = \beta_{13k} \cdot (R_{imq}^{99} - TT_{imq}) \cdot \frac{1}{DEN} \cdot \frac{1}{365}$.

Having introduced the general specification of the cost function

for a random cost model of freight demand, we begin now to examine

carefully the issues involved in the implementation of the random

cost model. First, we will investigate the choice variables more closely. So far, we simply denote an (i,m,q) combination as an alternative to be considered for logistics choice. Although the choice of i, m, and q are made interdependently, basically there are three separate choice dimensions involved, namely origin, mode and shipment size. These three choices are, in the long run, made jointly. However, in many cases these decisions might be made sequentially. For example, if the purchaser enters into a multi-order contract with a supplier, the choice of a supplier becomes a longer term decision than the choice of mode and shipment size. Similarly, the choice of shipment size might, in some circumstances, be more of an immediate choice than the choice of mode. Thus our problem is a multi-dimensional choice problem. We need a model which is capable of addressing the three dimensions in a joint-choice manner as well as in a sequential manner.

This requirement is further complicated as the nature of the choice set is considered. It is important to note that there is a discrete set of modal alternatives available but there is a continuous set of supplier and shipment size possibilities open to the firm. For intercity freight demand analysis, it is practical to assume the choice set of suppliers is also discrete. However, there are an infinite number of shipment sizes that are actually possible to use.

All of the commonly used joint-choice models require that the set of alternatives be completely discretized or completely continuous.

They cannot be mixed. Recently, attention has been paid to addressing
the choice problems involving qualitative alternatives and continuous
alternatives.[32/] Unfortunately, these models usually place restrictions
on the functional form and type of variables which may be used.
These restrictions make them difficult to apply in our case. We will
examine this in terms of Westin's model.

Westin's model involves a five-step procedure. In terms of the
joint choice of mode and shipment size, we can rewrite his steps
as follows:

(1) Define a multivariate probabilistic distribution of the desired
shipment sizes associated with all of the available modes.

(2) Define a model of the probability of choosing each mode given
the desire of shipment on all modes.

(3) Combine the results of steps (1) and (2) above to obtain the
joint probability distribution for all modes and shipment sizes.

(4) For each decision-maker, integrate the joint probability
distribution over all unchosen shipment sizes to obtain the marginal
probability distribution of the chosen mode and shipment size.

(5) Combine the results of step (4) for all observations in the
sample to obtain the likelihood function from which the model parameters
can be estimated.

In a two-mode case, Westin's model can be stated as:

[32/] For example, Heckman (1975), Westin (1975), Hauseman and Wise (1978).

$$q_1 = \beta_1' X_1 + \varepsilon_1 \tag{4.16}$$

$$q_2 = \beta_2' X_2 + \varepsilon_2 \tag{4.17}$$

$$I = \alpha_1 q_1 + \alpha_2 q_2 + \gamma'W + \eta \tag{4.18}$$

$$P_r(m_1|q_1,q_2) = P_r(I \geq 0) \tag{4.19}$$

$$P_r(m_1,q_1) = \int_{q_2} P_r(m_1|q_1,q_2) P_r(q_1,q_2) dq_2 \tag{4.20}$$

where: q_i = the continuous dependent variable q associated with the choice of mode i

I = an indicator function for the discrete choice of mode

X_1, X_2, W = vectors of explanatory variables

$\alpha_1, \alpha_2, \gamma, \beta_1, \beta_2$ = parameters

$\varepsilon_1, \varepsilon_2, \eta$ = disturbances.

Assuming that $(\varepsilon_1, \varepsilon_2)$ and u are normally distributed as

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \sim N \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{4.21}$$

$$\eta \sim N(0,1)$$

and η is independent of $\varepsilon_1, \varepsilon_2$, we have:

$$P_r(m_1|q_1,q_2) = \Phi(\alpha_1 q_1 + \alpha_2 q_2 + \gamma'W)$$

and

$$P_r(q_1 q_2) = N\left[\begin{pmatrix} \beta_1' X \\ \\ \beta_2' X \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \\ \sigma_{12} & \sigma_{22} \end{pmatrix}\right] \qquad (4.22)$$

Since in each observation, only the chosen mode and shipment size are observed, the marginal distribution of m_1 and q_1 is arrived at by the integration shown in Eq. (4.20). Note that the original statement of Westin's model specified Eqs. (4.16) and (4.17) as linear. However, the use of nonlinear expressions is also possible. Westin has also shown that a logit model may be used in place of the probit model, although this complicates the mathematics considerably. The important points are that Westin's model requires the derivation of equations for q_1 and q_2 as functions of exogenous variables, and the derivation of a cost function that is linear in shipment size. Unfortunately, these derivations cannot be assumed from our logistics cost function because the application of the first-order minimization condition yields a very nonlinear functional form for q that in general cannot be solved for the optimal shipment size, unless unrealistic assumptions are made.[33/]

Still another possilbity for modelling shipment size as a continuous variable has also failed due to essentially the same difficulties. This method can be stated as follows.[34/] Assume that q_1, q_2

[33/] The use of Westin's functional specification has been investigated and documented in Roberts et al. (1977), Appendix C.

[34/] The author is indebted to Prof. Daniel McFadden for this formulation.

and C are specified as:

$$q_1 = q_1(X) + \varepsilon_1 \tag{4.23}$$

$$q_1 = q_2(X) + \varepsilon_2 \tag{4.24}$$

$$C = C(X, q_1, q_2) + \varepsilon_3 \tag{4.25}$$

where: q_i = the continuous dependent variable q associated with the choice of mode i

   C = the difference between the logistics cost of using mode 1 (C_1) and mode 2 (C_2), $C = C_1 - C_2$

   X = the explanatory variables

   ε_i = disturbances.

Next, assuming that the disturbances ε_1, ε_2 and ε_3 are distributed as the general extreme value distribution $F(\varepsilon_1, \varepsilon_2, \varepsilon_3)$,[35] we can derive the marginal distribution of mode 1 and shipment size q_1 by

$$P_r(m_1, q_1) = \int_{q_2} F_2[q_1 - q_1(X), q_2 - q_2(X), -C(X, q_1, q_2)] dq_2 \tag{4.26}$$

where $F_2(\cdot)$ is the derivative of the general extreme value distribution with respect to ε_2. One can see that this formulation also requires the derivation of equations for q_1 and q_2 as functions of exogenous variables. Thus, to carry out the integration in Eq. (4.26), crucial assumptions are also needed.

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[35] See McFadden (1977) for the definition of a general extreme value distribution.

Generally speaking, to model logistics choice involving dis-
crete choice of mode and origin as well as continuous choice of ship-
ment size requires the solution of our logistics cost function for
optimal shipment size. The assumptions which will enable us to
develop this derivation would include constant freight rates, etc.,
which have been reviewed previously and found too critical to be
accepted. A formulation which is free from this constraint has
not yet been successfully found.

A solution which allows us to bypass this difficulty is the
division of the ranges of shipment sizes into a set of discrete
segments and the modelling of shipment size choice as discrete choices.
The major disadvantages of this approach are that 1) if the segments
are made too small, then the resulting model is likely to violate
the assumption of independence between the random elements in the
logistics cost function; 2) on the other hand, if the segments are
made too large, then it becomes difficult to describe the alternatives
adequately, especially with respect to variables like transport
rate (Terziev, 1975).

As a matter of fact, the disadvantages of modelling shipment size
as discrete choices are not as numerous as they appear to be at
first glance. Shipment data very often are available only as discrete
descriptions of shipment size.[36] Thus, it is not possible to model
choice of shipment size as continuous in the very beginning using

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[36] For example, the Commodity Transportation Survey, Census of Transpor-
tation.

these shipment data. Also, the choice of shipment size in the real
world does not exhibit a continuous distribution. The tariff rates
are quoted for a given modal service offering along with a minimum
shipment size. Thus, there are certain natural break points that
the shipper must normally choose between if he is to enjoy lower rates.

As a consequence, it appears that both logit and probit models
can be used to model discrete choice of mode, shipment size and origin.
Logit models require the assumption of independence in the error terms.
The logit model is also characterized by the independence from irrele-
vant alternatives property (IIA). The IIA property states that the
relative probabilities of any two alternatives are independent of
the presence of all other alternatives. Thus there is a tendency
of the multinomial logit model to over-predict the choice proba-
bilities for alternatives which are perceived by decision-makers to
be similar. In this respect, a probit model seems more attractive
than a logit model. However, the computational difficulties of the
multinomial probit model clearly limit its usefulness in modelling
the logistics choice. The computational difficulties arise from the
fact that the maximum of the bivariate normal variables is not a normal
variable. Thus either a Monte Carlo or an approximation procedure
is required to derive the probit model for more than two alterna-
tives.[37]

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[37] See Daganzo et al. (1976) and Lerman and Manski (1977) for the
details of these computational procedures.

Consequently, the computations are extremely inefficient and approximation errors would be significant if there were many alternatives involved. Since the logistics choice process is characterized by a choice set involving a large number of possible modes, shipment sizes and supplier origins, it is almost impossible for practical purposes to use a probit formulation. By contrast, the IIA property of a logit model enables us to calibrate the model based on the conditional choice in a small subset of the full set of alternatives. As indicated by McFadden (1976), segmentation of heterogeneous populations, more complete specifications of explanatory variables to reduce the unexplained effects caused by unobserved variables, and the general robustness properties of the multinomial logit form permit reduction of bias due to IIA failure to tolerable limits. We thus conclude that the multinomial logit model is the most attractive formulation for the random cost model of freight demand found so far. Therefore, we will proceed to the estimation using the random cost model in the logit framework.

CHAPTER 5

DEVELOPMENT OF A DISAGGREGATE DATA BASE

5.1. Introduction

This chapter describes the procedures used for developing a
data base to estimate the random cost model of freight demand outlined
in Section 4.5. The data required to implement the random cost
model involve intercity freight shipment data at a completely
disaggregate level along with the associated information concerning
the commodity being transported, the characteristics of the decision-
maker, the transport level-of-service available, etc. As suggested
by Chung (1975) and Terziev (1975), this information can be classi-
fied into four broad categories: level-of-service attributes, commo-
dity attributes, receiver attributes and market attributes. The
first category of variables describes the transportation level-of-
service offered by each mode for various commodities, shipment
sizes, and origin-destination pairs. These level-of-service
variables include waiting time, transit time and reliability, tariff
charges, loss and damage, etc. The second category of variables
describes the characteristics of the commodity being transported.
This includes such variables as value, density, shelf life, perisha-
bility, etc. The third category of variables describes the character-
istics of the receiving firm. Important receiver attributes include
the annual use rate of the commodity being ordered, the variability in
the use rate, risk of stockout, the type of inventory system used, and

the storage cost, capital carrying cost, etc. The fourth category

of variables describes the market for the commodity being modelled.

This has special relevance to the chocie of supplier. Variables include

price, supply availability, etc. Unfortuantely, good data for many of

these variables are not available. As a consequence, models and/or

assumptions are inevitably required to estimate useful data. This

chapter gives the procedures and assumptions that were used to prepare

the data base. Section 5.2 describes the sources and methods used to

develop the intercity freight flow data. These data are used as the

sample to estimate the models. Section 5.3 discusses the model to

derive annual use rates for each observation in the sample. Investi-

gation by Roberts and Lang (1978) using the deterministic models seems

to indicate that use rate is the key receiver attribute required.

Section 5.4 describes the details involved in estimating transport

level-of-service variables. Data on commodity attributes and market

attributes are described in Section 5.5.

5.2. The Intercity Shipment Data

To study urban passenger travel choice at a fully disaggregate

level, the conventional home interview surveys provide almost all of

the information that would be needed. Unfortunately, there is no data

source analogous to a home interview survey which exists in the freight

area. Although disaggregate shipment size data are available from

several sources, almost all of the existing data sets have one or more

of the following problems which diminish their usefulness in the estima-

tion of a freight demand model at the disaggregate level.[38]

1) Failure to include the name of the reciever, thus making it difficult to accurately determine receiver attributes.

2) Lack of geographic detail in the description of shipment origin and destination. This makes it difficult to accurately determine level-of-service and market attributes.

3) Failure to cover a wide range of commodities, modes, and origin-destination markets. This tends to limit the usefulness of the data set in the estimation of a general model.

Most of the available shipment data are from existing shipper surveys. The most comprehensive shipper survey is the Commodity Transportation Survey of the Census of Transportation. The Census of Transportation is one of the Economic Censuses conducted by the U.S. Bureau of the Census once every five years. There are three independent but related surveys in the Census of Transportation: the National Travel Survey, the Truck Inventory and Use Survey, and the Commodity Transportation Survey. The largest part of the Commodity Transportation Survey is the Shipper Survey, which collects intercity freight flows for all manufactured commodities. The data base of commodity flows used in this study is based upon the Shipper Survey of the 1972 Census of Transportation.

The data sources collected in the survey were bills of lading or other shipping documents pertaining to individual shipments.

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[38] Surveys of available data sources can be found in Terziev (1975) and Chung (1975).

These data were collected in a stratified sample from the universe of
manufacturing establishments with 20 employees or more excluding SIC 19
(ordnance and accessories, excluded for security reasons), SIC 27
(printing and publishing, excluded because they are covered by the
Mail Survey), and some local industries. The Public Use Tape
reports the information collected in the Shipper Survey as individual
records. A record is a summary of the shipping documents for each
commodity by mode, shipment size and origin-destination pair. There
are two files available to the public; one shows the origin-destination
flow from 27 production areas to 50 market areas, while the other shows
the origin-destination data for state-to-state commodity flows.
Each record gives the following information: 1) Origin Production
Area or State; 2) Destination Market Area or State; 3) STCC commodity
code at the 2-, 3-, 4- and 5-digit level; 4) mode, coded as rail,
common carrier truck, private truck, air, barge or other; 5) shipment
size, coded for 20 weight blocks; 6) estimated total tonnage in the
population represented by this record; 7) estimated total ton-miles
represented by this record; and 8) actual number of shipments with
the same shipment characteristics in the total sample represented by
this record. Details of these data are given in Appendix A.

A data set can be synthesized for use in the disaggregate estimation
by using the number (n) of shipments represented by each record to
expand the record into n identical shipments; each shipment is
assumed to carry the identical characteristics described for the record.

For example, a record representing 3 waybills of medicine by air from
Chicago to Boston in shipment sizes ranging from 50 to 99 pounds can
be developed into three individual 75-pound shipments with the same
origin, destination and mode.[39]/

Each commodity is reported at the 2-, 3-, 4- and 5-digit STCC
level. However, in order to protect the identity of individual
shippers, the Census Bureau has disclosed origin-destination flows at
the 5-digit STCC level only where there are five or more firms manu-
facturing the commodity in the origin area in the Census Sample.
As a consequence, most of the flow data are only available at the
4-digit, 3-digit, or in many cases even the 2-digit STCC level.
Only about 12 percent of the records on the 1972 tape are available
at the 5-digit level. Thus, one way to develop the data base is to use
only those records at the 5-digit level and to expand them into a
quasi-disaggregate data set. The major problem with this approach occurs
because of the disclosure rule. The problem is the possibility for
bias toward commodities which are manufactured by a large number of
firms.

An alternative way to build the data base is to use only the records
at the most disaggregate level of commodity classification. That is,
for commodities not reported at a 5-digit level, the lowest digit

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[39]/ An equivalent, but more computationally efficient, way to do this
would be to weight each record by the number of shipments in the estima-
tion. This involves no physical expansion of the record. One would need
to assume the use rate of these shipments are identical, which is not
attractive.

level reported would be used. This leads to a larger sample size at the expense of a much higher level of aggregation of decision-makers. The record of a 4-digit commodity movements very often represents as many as thirty or even fifty individual shipments. This proved to be so difficult that the approach became unattractive and therefore the first method was employed. The most critical problem, however, is the lack of receiver attributes. Census data show that for shipments of the same commodity, from the same Production Area to the same Market Area, two more modes have typically been used, and for each mode, five or more shipment sizes are usually chosen. There are, on the average, more than 10 diffierent mode-shipment size combinations which have been selected for a single commodity flowing in a given intercity market. By inventory theory, this suggests that there are a variety of firms with various use rates ordering the commodity between the same origin-destination pair. Thus, to explain the decision-making of a firm satisfactorily, use rates must be taken into account.

5.3. Developing Information Concerning Annual Use Rate

As indicated earlier, a short-run freight demand model attempts to model a firm's logistics choice given the annual use rate of the input material. Since data on annual use rate are not generally available, a systematic way of developing relevant use rate information would be required. Thus a short-run model involves not only the model to predict choice of shipment size, mode and origin given use rate,

but also a method by which we can estimate use rate itself.

Procedures to estimate use rate for each shipment datum have been developed which involve basically two steps. The first step is to estimate the use rate distribution of each 5-digit STCC commodity in each Market Area. The second step is then to estimate a probable use rate from these use rate distributions for each observation obtained from the Census of Transportation.

The approach used to develop the use rate distributions for each product in each city involves simple the basic input-output relationships. It starts by estimating the output level of a firm. Firm output can be developed from the summary of firms by type presented in the County Business Patterns. Then, by using the input-output technical coefficients from a large input/output table for the country the inputs required to produce a given output can be derived. This is done by 1) developing the make-up of industry in each market area 2) estimating outputs for each producing industry; 3) translating outputs into the required inputs using the I/O technical coefficients; 4) estimating final demand for commodities by the population in each market area; and 5) calculating the use rate distribution by commodity and by market area. Each step is described in some detail as follows:[40/]

[40/] This part of the work is heavily based upon the original research done in Chiang and Roberts (1976).

Presenting Industry Structure

A simple way to represent the population and industry structure
of a given geographical unit, a region, or Market Area, an SMSA,
a county or a city, is to use an industry/firm-size distributions matrix.
Data to develop this matrix for the United States are found in the
County Business Patterns prepared by the U.S. Bureau of the Census.
See Figure 12. This matrix shows the number of firms in each industry
by each size category. Firms are classified into eight size categories
based on employment. Industries have been aggregated from a 4-digit
SIC code into 62 industry codes, denoted as IND codes.

Estimating Outputs for the Industry Structure

With the industry/firm-size matrix in hand, we can estimate
the output distributions by industry. This is done by multiplying
information on the productivity per employee from the Census of
Manufactures, Census of Retail Trade, Census of Wholsale Trade, etc.,
by the average number of employees in the firm. The number of firms
in an industry of a given size times the average number of employees
in that size cateogry times the average productivity per employee
in that industry, becomes the estimated total output of all the firms
in the given industry/size category. This can be stated precisely as:

$$PVS(IND,SIZE,MA) = NFS(IND,SIZE,MA) * AVEMPL(SIZE,MA)$$

$$* PVEM(IND,MA) \qquad (5.1)$$

Figure 12.   Urban Industry/Firm-Size Distribution Matrix

FIRM SIZE (No.Employees)

	1	2	3	4	5	6	7	8
1								
2								
3								
.								
.								
.			number of firms					
.								
.								
62								

INDUSTRY (IND Code)

1-3   4-7   8-19   20-49   50-99   100-249   250-499   500+

where:　　IND = industry

　　　　SIZE = firm size

　　　　　MA = Market Area

　　　　　PVS = total output value for all firms in an industry/size

　　　　　　　　category

　　　　　NFS = number of firms

　　　AVEMPL = number of employees per firm

　　　　PVEM = productivity per employee.

In estimating the average employment for each size firm using the
County Business Patterns, special attention must be paid to the last
cell of the industry firm/size matrix. This cell contains the number
of firms with 500 employees or more. Firms in this category are big
firms for which the number of employees could go from 500 to several
thousand. To use a simple average number of employees is somewhat
dangerous. A better way to determine the average number of employees
in this cell would be to estimate the employment size of this
category indirectly. That is, the employees of the first seven size
categories can be estimated and subtracted from the total number of
employees in the industry.

Translating Outputs into Inputs

　　　The input-output technical coefficients are developed from a
6-digit 1967 input-output table prepared by the U.S. Bureau of Economic
Analysis (BEA). It has 484 rows and 494 columns. For our purposes,

the technical coefficients have been aggregated into a 355×62 matrix
with the 62 columns representing the buying industries, IND. The first
351 rows represent the selling industries. These are directly match-
able to STCC 4-digit commodities. The final 4 rows represent the
service and value added sectors. With these technical coefficients,
inputs to the firms in each industry/firm-size category can be derived
as follows:

$$DFIRMS(IND,SIZE,K4,MA), = PVS(IND,SIZE,MA) * TC(K4,IND) \qquad (5.2)$$

where: K4 = 4-digit STCC commodities

     DFIRMS = annual use rate of all firms in an industry/firm-size

           category

     TC = technical coefficients.

The use rate of an individual firm is developed by simply
dividing the estimated total use rate in each industry/firm-size
category by the number of firms. The 4-digit STCC demands can then
be further disaggregated to the 5-digit STCC level by applying the
proportion of 5-digit flows found in each 4-digit flow in the 1972
Census of Transportation at the national level.

Estimating Final Demands

The goods consumed by processing industries usually represent
only a portion of the total consumption of a city. The balance is
consumed as final demand by the hinterland population and the government.

In estimating the consumption by final demand, expenditures for personal

consumption and government are considered. These two sectors represent

about 85 percent of total final demand consumption. Exports are

excluded here since we are only interested in the logistics decision

for input materials. Investment and inventory are also not included

since they are aggregated over all firms and all industries and there

is no way to classify them by firm size or industry.

The personal and government consumption of the individual market

areas are estimated by applying a regional share ratio to national final

demands, or by computing a per capita figure. This per capita figure

for personal and government consumption is then applied to the popu-

lation of the area on the assumption that consumption per capita is

the same for all urban areas. However, the per capita consumption

could be different from urban to rural areas, of from high income

areas to low income areas. If desired, the computations could be

elaborated further to take these factors into consideration, provided

that the data are available. In addition, a ratio based on such

regional factors as government employment, etc., could also be derived

to estimate the regional govenrment consumption if this were felt

to be necessary.

Deriving the Use Rate Distribution

The demand for intermediate goods and final goods should normally

be combined to obtain the total flow of commodities. However, there

are some situations that are a bit more complex. For example, the

intermediate demands for retailers or wholsalers calculated from input-
output technical coefficients are only the products used in the

production process. They do not include the products carried for the

purpose of selling to customers. A scheme for treating these inter-

sectoral flows has been developed, but it will not be elaborated

upon here.[41]

The commodity by commodity demands for input materials for each

region derived from the above procedures are stored in a 4-dimensional

matrix. Each element contains the demand for a 5-digit STCC commodity,

by industry, by firm size, and by market area. Using simple mathe-

matical manipulations, the use rate distributions of each 5-digit

STCC commodity can be developed for each market area. Cumulative

distributions, showing the probabilities of using each input material

in various amounts per year by all firms in each market area have

been developed using the procedure. See Figure 13.

After developing the use rate distributions for each market area,

the next major step is to estimate a likely use rate for each shipment

in the original disaggregate sample from the use rate distributions.

As mentioned earlier, there are no receiver attributes associated

with an observation in the Census public use tapes that can be used to

infer the characteristics of the receiving firm. The simplest way to

associate a use rate to each record would be to sample from the esti-

mated use rate distribution. This is not particularly attractive

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[41] For details, see Chiang and Roberts (1976).
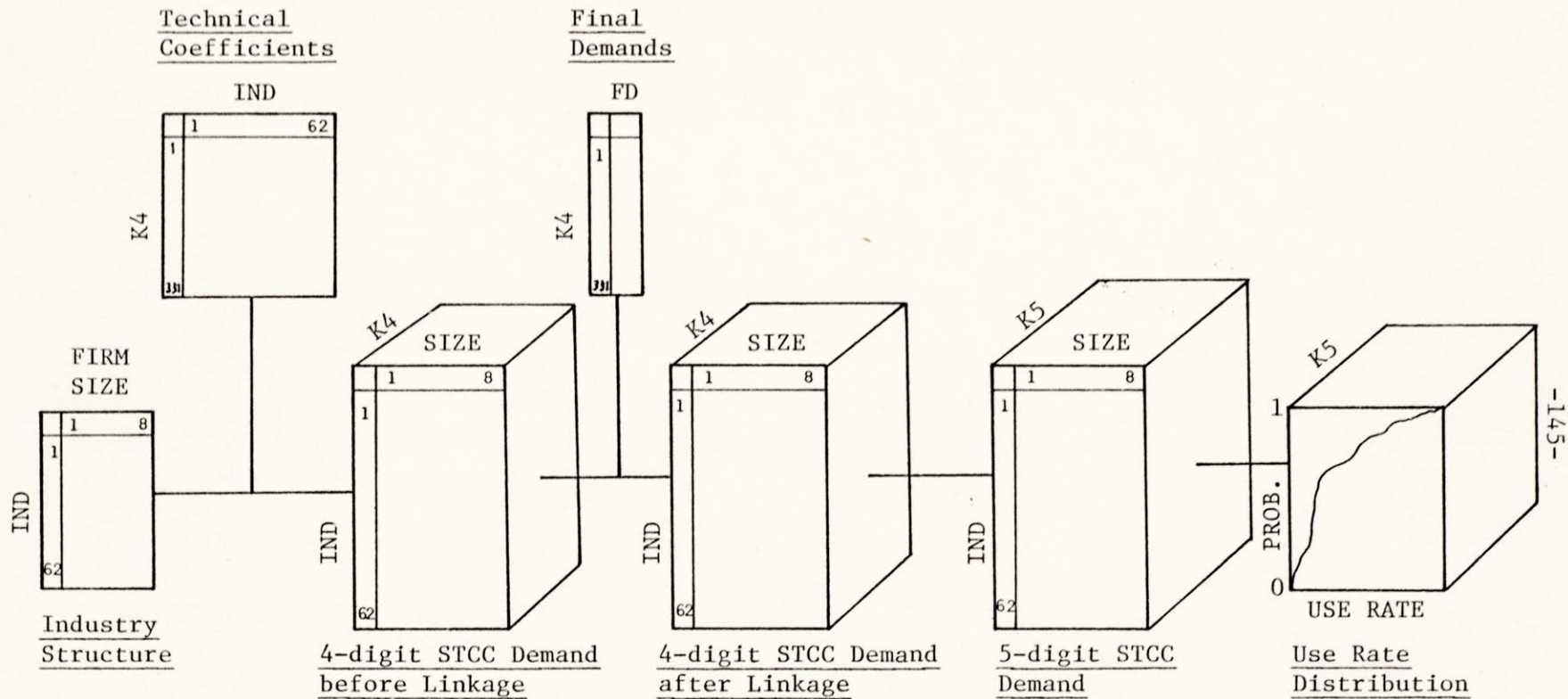
Figure 13. Procedures Used to Develop Commodity Use Rate Distributions for Each Market Area

since there will be no information gained from this process statistically.

Two strategies have been proposed in this study to estimate representative use rates for each record from the use rate distribution. Each strategy involves certain assumptions concerning the underlying relationship between annual use rate and shipment size.

The first method is based upon the assumption that a firm with a higher use rate will <u>always</u> select a larger shipment size. Under this assumption, there is a one-to-one relationship between the use rate distribution and the shipment size distribution for a commodity in a particular market area as shown in Figure 14. The shipment size distribution can be derived from the Census data separately for each commodity in every market area. Thus, using the observed shipment size in each record (q^*), a use rate (x^*) can be derived from these two distributions. This approach is not very attractive, since shipment size is one of the choice variables to be modelled. The proposed process would therefore involve generating an explanatory variable from a dependent variable. As a consequence, if this approach were followed, the model estimation would be likely to end up with inconsistent results.

The second approach solves this problem by sampling a use rate from a conditional distribution of use rate given shipment size. This requires the development of conditional distributions for each commodity in each market area. In order to do this the aggregate joint
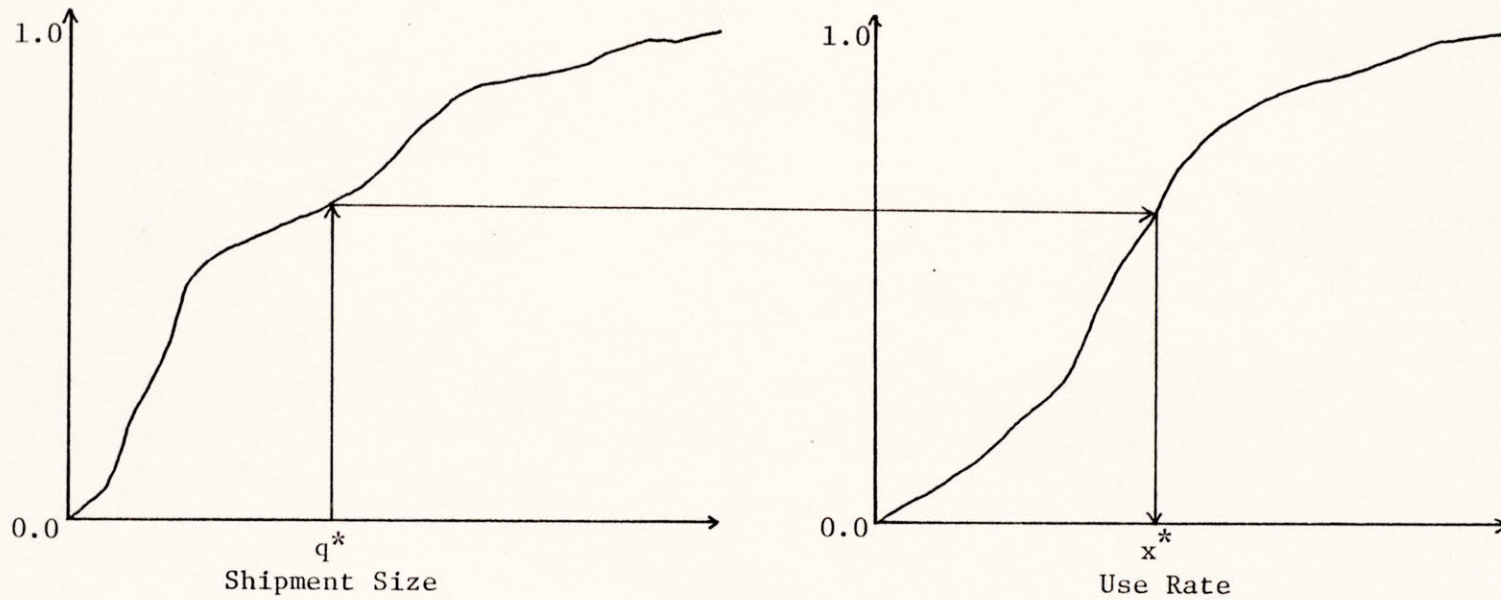
Figure 14. Procedures to Estimate Annual Use Rate (Method 1)

distribution of use rate ahd shipment size was first derived from
waybills collected by the Freight Study Group at MIT. This joint
distribution is shown in Figure 15. With this aggregate joint
distribution of use rate and shipment size, the disaggregate joint
distribution of use rate and shipment size for each commodity by
market area can be produced by an iterative scaling procedure
using the individual use rates and shipment size distributions
(by commodity by market area) as marginal distributions. Thus, the
conditional distributions of use rate given shipment size are developed
for each commodity in each market area. The above proceudres are
summarized in Figure 16.

Note that the final figure used for the use rate is then sampled
from the final conditional distribution using Monte Carlo procedures.

5.4. Level-of-Service Attributes

The key transport level-of-service variables include freight
rate, mean transit time, transit time reliability, and loss and damage.
Each will be discussed.

The setting of freight rates in the transport industry is a
process which produces highly variable results. Actual rates are not
only specific to origin, destination, mode and commodity but also to
shipment size and method of packaging. The level of specificity at
which actual freight rates are quoted precludes the possibility of
obtaining and storing individual tariffs for use with the Census of

Figure 15. Joint Probability Distribution of Annual Use Rate and Shipment Size[*]

Annual Use Rate (Tons/Year)

Shipment Size (Tons)

Shipment Size	0.5	1.5	2.5	12.5	30	50	100	250	500	1,000	2,500	5,000
	65											
0.1	65	65	65	131								
0.5		65	65	65								
1.5						131						
2.5				65				131				
5				131		196	196	196				
10							196	326	65			
15				131	196	392	654	984	326			
20				65				523	65			
25						326	392	458	131			
30							131	131				
45							131	131		65		
50							131	131	262	131		
60						65	131	131	392	131		
75							65	131	196	131	131	

[*]Probabilities are shown in 10^4.

Use Rate Distribution (J,K)

Shipment Size Distribution(J,K)

Aggregate Joint
Distribution of Use
Rate and Shipment
Size

Iterative
Scaling →

Use Rate (J,K)

Sampling Use Rate

Conditional Distribution

Shipment Size (J,K)

Joint Distribution of
Use Rate and Shipment
Size by Commodity (K)
and by Market Area (J)
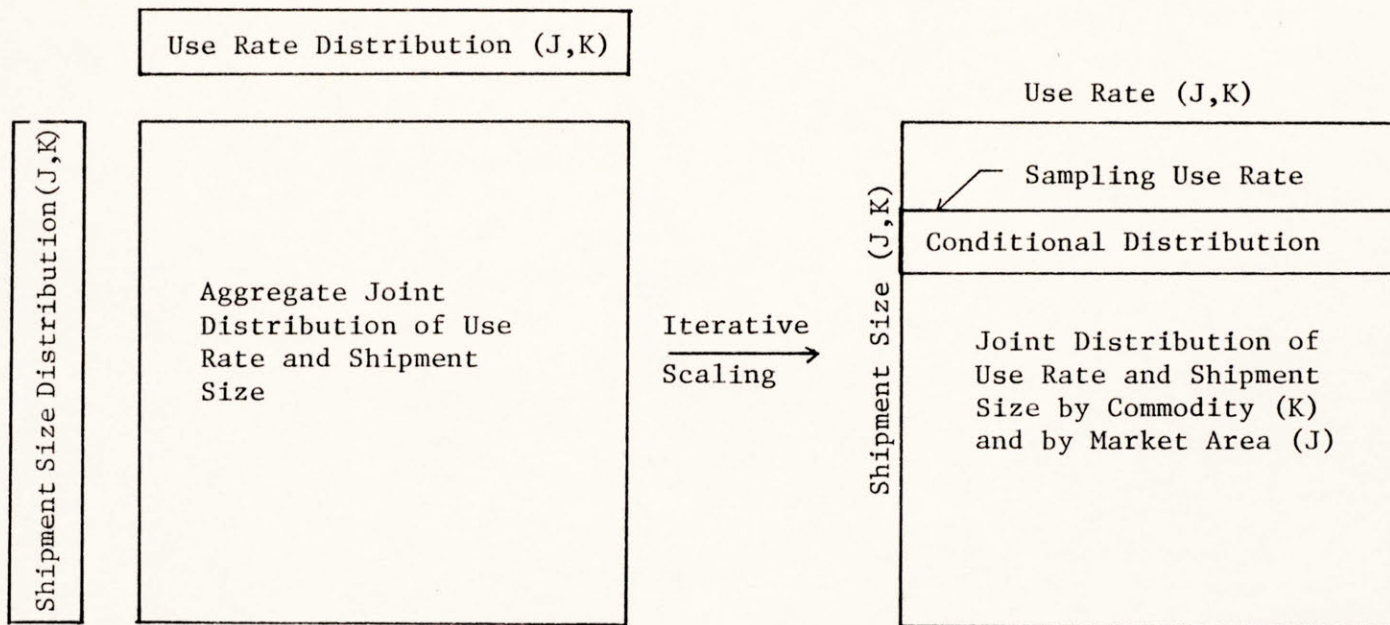
Figure 16.   Procedures to Estimate Annual Use Rate (Method 2)

Transportation data in the estimation process described earlier.

It is essential therefore to have models to predict the rate structure

as a function of the observable characteristics of a shipment.

A set of rate models has been developed at MIT as a result of previous

research. These models predict rates for less-than-truckload,

full truckload, truck minimum charge, trailer-on-flatcar, rail carload,

multiple carload and air transport service offerings, using commodity

as well as shipment characteristics as explanatory variables.[42]

The accuracy of these models has been verified in this study using

actual point to point rates for specific commodities obtained from

Transportation Distribution Systems, Inc., a traffic auditing and

consulting firm located in Lexington, Massachusetts. Most of

the critical weaknesses found in this comparison have been strengthened

by slight changes to the original rate models (particularly boundary

conditions). Considerable effort was involved in developing

and verifying rate models for rail freight forwarders and air minimum

charges which were not available in the previous studies and in

modifying a cost model developed by Roberts (1977) for private truck

for use in lieu of a rate model. Sample outputs from the freight rate

models used in this study are shown in Figure 17.

Transit time, waiting time, and reliability are another set of

important variables defining the level-of-service measures of a

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[42] See Samuelson (1977), Wilson et al. (1976) for details of these
models.
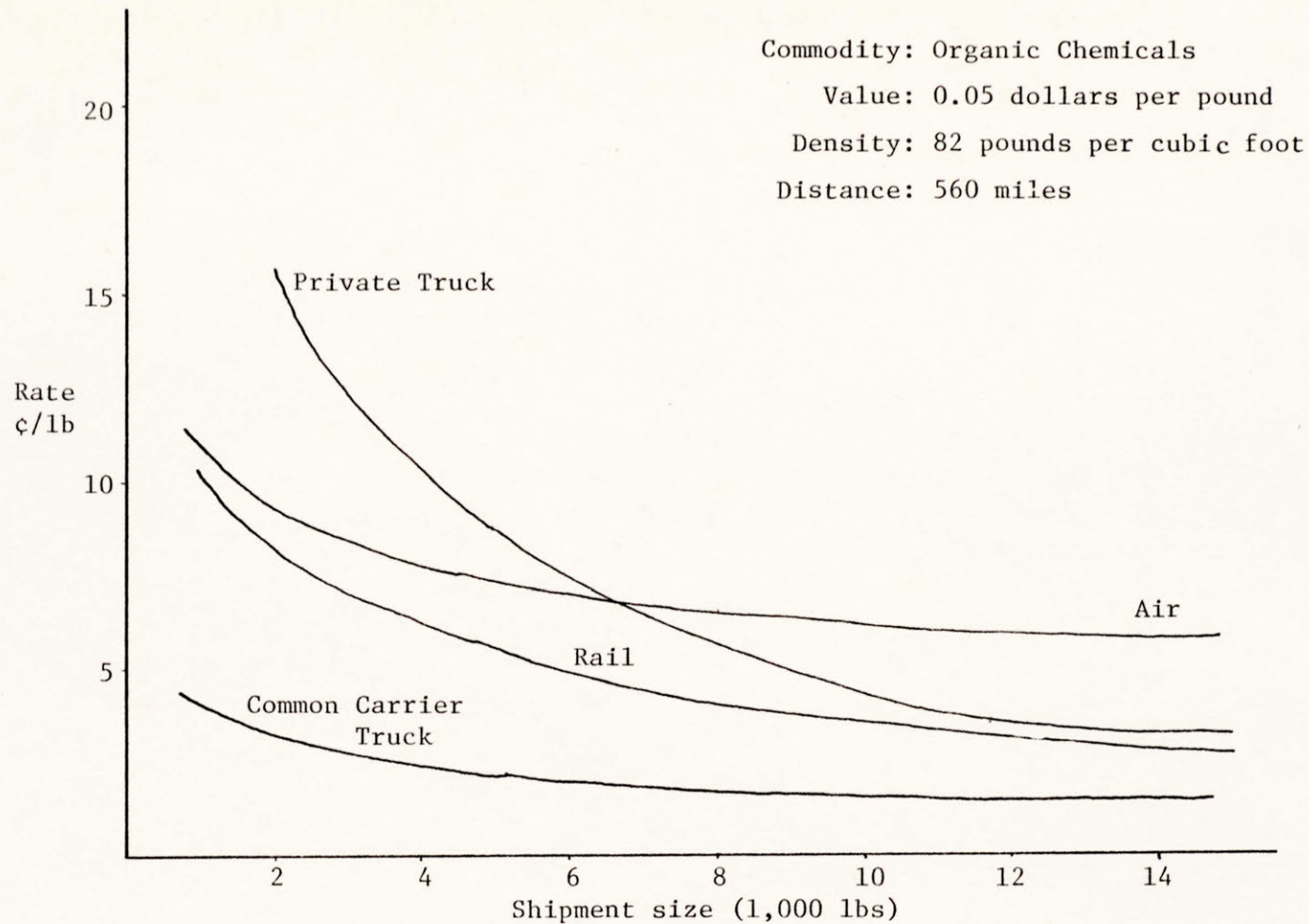
Figure 17. Rate Structure as a Function of Shipment Size

transport mode.

There are obviously a number of ways to measure travel time and reliability. Average travel time in number of hours or number of days is frequently used as a measure of travel time. Reliability is somewhat more complex. The railroad reliability project at MIT has explored several measures including the standard deviation of transit time, the N-day percent, and the left and right tails of the transit time distribution (Lang and Martland, 1972).

The most general way to describe transit time and reliability is to use the full transit time distribution. An empirical model for predicting the full transit time distribution has been developed for regular-route (LTL) trucking as a part of this study. The same approach is currently being applied to rail. Basically, the model involves a family of shifted Gamma distributions. The Gamma function is postulated for the distribution of transit time based upon observations made in the real world. A data set of 8,170 shipments observed on 36 city-pair markets, ranging in distance from 106 to almost 3,000 miles, shows that the distribution tends to be skewed toward the longer transit time in typical Gamma shape. We thus propose the following Gamma functions to replicate these observations:

$$f(\overline{DAY}_\ell) = \frac{1}{GAMMA(b_\ell)} (\overline{DAY}_\ell)^{b_\ell-1} \cdot e^{-(\overline{DAY}_\ell)}, \ \overline{DAY}_\ell \geq 0 \qquad (5.3)$$

$$= 0, \text{ otherwise}$$

DAY

$\overline{DAY}_\ell$

$\overline{DAY}_{\ell}'$

mean transit time

$D \left\{ f_\ell (\overline{DAY}_{\ell}') \right.$

$D \left\{ f_\ell (\overline{DAY}_\ell) \right.$

$DIST_{\ell}' \cdot TAN(\theta)$

$DIST_\ell \cdot TAN(\theta)$

θ
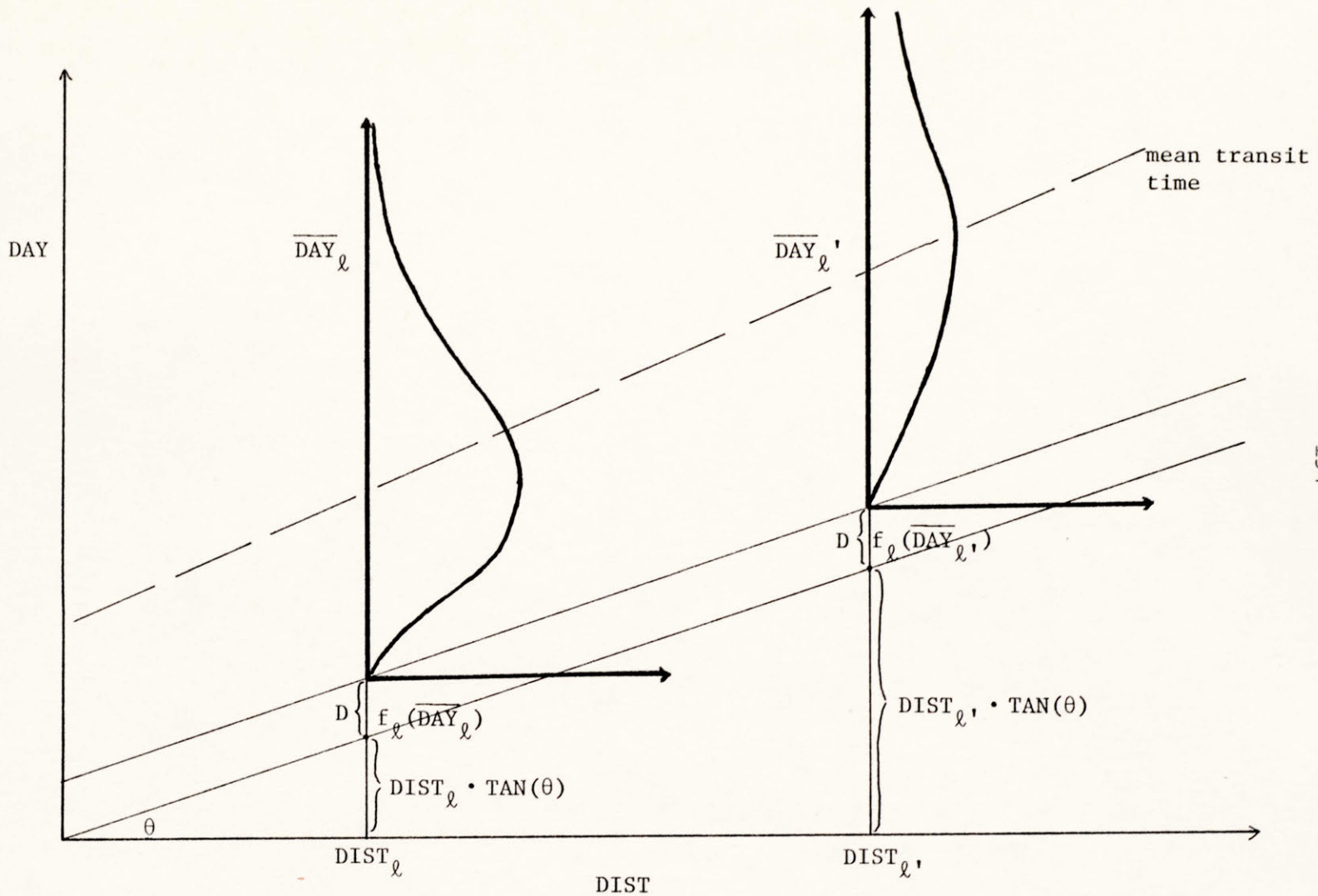
$DIST_\ell$

$DIST_{\ell}'$

DIST

Figure 18.   The Total Transit Time Distribution

where: $\text{GAMMA}(b_\ell) = \int_0^\infty x^{b-1} e^{-x} dx$

$\overline{\text{DAY}}_\ell$ = transit time

b = parameter of the Gamma function

ℓ = subscript to denote a specific length of haul.

The parameter of the Gamma function, b, is assumed to be length-of-haul-dependent. As we mentioned earlier, the transit time distribution is skewed with long haul. Generally, b can be expressed as a hyperbolic function of length of haul:

$$b_\ell = B_0 \cdot \text{DIST}_\ell^{B_1} \tag{5.4}$$

The shift in transit time is hypothesized to be a function of distance in the following manner:

$$\overline{\text{DAY}}_\ell = \text{DAY}_\ell - D - \text{DIST}_\ell \, \text{TAN}(\theta) \tag{5.5}$$

where: DAY_ℓ = shifted transit time

D, θ = parameters of the shift of the transit time.

The reuslting total transit time distribution is shown in Figure 18. Combining Eqs. (5.3) through (5.5), we have the following empirical model for transit time and reliability for regular-route, less-than-truckload trucking:

$$f(\overline{\text{DAY}}) = \frac{1}{\text{GAMMA}(B_0 \cdot \text{DIST}^{B_1})} \overline{\text{DAY}}^{(B_0 \cdot \text{DIST}^{B_1} - 1)} \cdot e^{-\overline{\text{DAY}}} \tag{5.6}$$

where: $\overline{DAY} = DAY - D - DIST \cdot TAN(\theta)$

$$GAMMA(B_0 \cdot DIST^{B_1}) = \int_0^\infty x^{(B_0 \cdot DIST^{B_1} - 1)} \cdot e^{-x} dx$$

The mean and variance of this distribution are then given by:

$$E(DAY) = B_0 \cdot DIST^{B_1} + D + DIST \cdot TAN(\theta) \qquad (5.7)$$

$$Var(DAY) = B_0 \cdot DIST^{B_1} \qquad (5.8)$$

B_0, B_1, D and θ are parameters to be estimated. We require B_0 and
$(DAY - D - DIST \cdot TAN(\theta))$ to be positive.

The model was estimated using the method of minimum χ^2. The
complex procedure of Box[43] was used as the algorithm to search for
the minimum. The initial value for the parameters can be derived
easily by looking at the properties of the Gamma function. In the
Gamma distribution function, the shape of the distribution is deter-
mined by the parameters "b". With a larger b, the distribution will
be skewed to the right. Parameters D and θ are merely used to shift
the axes of the Gamma functions. From the observed distribution of
transit time we can immediately assume initial D to be zero. Then, a
simple linear regression was estimated using distance as the exogenous
variable and mean transit time as the endogenous variable:

---

[43] See Box (1965), Kuester and Mize (1973).

$$\text{DAY} = \quad 1.7279 + \quad 0.00236 \text{ DIST} \qquad\qquad (5.9)$$
$$(t = 10.61) \qquad (18.38)$$

From the slope of the regression model, we can infer the value of θ

to be no less than $\tan^{-1}(0.00236)$, since the Gamma distribution is

skewed with long haul. Having initial values for D and θ, reasonable

values for B_0 and B_1 can be derived from the mean and variance of the

Gamma functions as given in Eqs. (5.7) and (5.8). The

minimum χ^2 estimates are consistent and asymptotically efficient.

Figure 19 shows the final estimates of the estimated parameters.

Three sets of estimation results are shown in the table. Model I

is the unrestricted model, using a full set of parameters as defined

in Eq. (5.6). Model II restricts parameter D to be zero, i.e., the

Gamma functions are shifted only by a rotation of angle θ. Model III

restricts parameter B_1 to be zero, i.e., the Gamma functions are with

identical shape in distribution and without skew. The χ^2 statistic

of Model I is 85.01 which is significantly below the critical region

at the 90 percent level of confidence. The χ^2 statistic of Model II

is marginally below the critical region. Both Models I and II are

not rejected in the hypothesis testing. The transit time distributions

predicted by the model as compared to observed values are shown in

Figure 20.

Transit times for the other modes are estimated using a set

of somewhat more simple-minded simulation models which consider the

distance between the origin and destination, the geographical region,

Figure 19. Coefficient Estimates and Statistics for LTL Transit Time Distributional Model

Model	B_0	B_1	D	θ	χ^2	χ^2_{n-k-1}*	Remarks
I	0.211	0.000295	0.128	0.00272	85.01	107.6	Unrestricted
II	0.203	0.0105	--	0.00284	107.29	107.6	Restricts $D = 0$
III	0.5	--	0.00001	0.00271	151.69	107.6	Restricts $B_1 = 0$

*Critical region at 90 percent level of confidence, n is the number of categories, k is the number of parameters to be estimated.

Figure 20. Comparisons between Observed and Predicted Transit Time Distribution
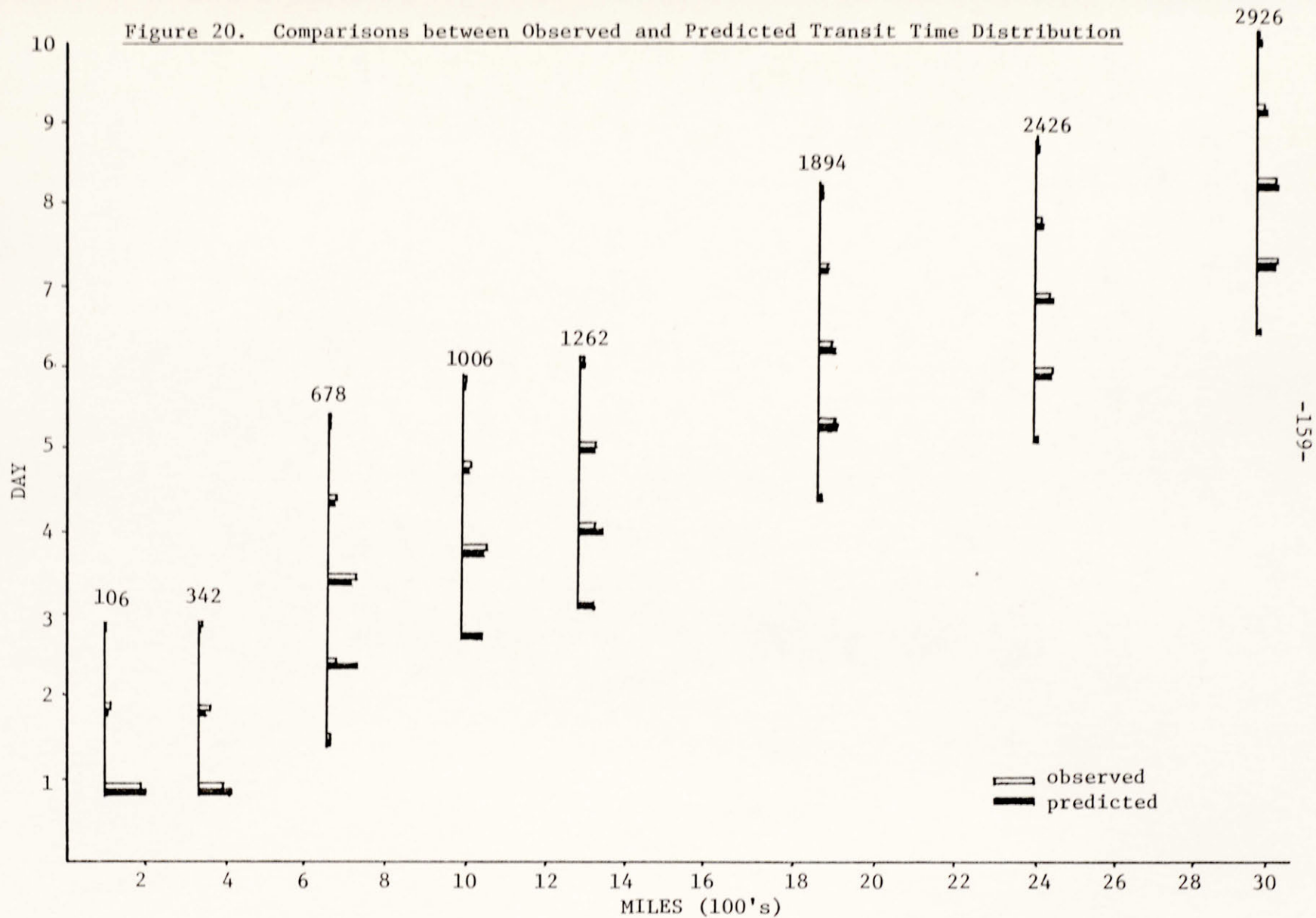
and the general type of transport service being offered. A model of predict highway and rail distances between any two points measured by their longitudes and latitudes has also been developed and is available for use in forecasting.

Wait time has been shown to be less important in its influence of shippers' mode choice. A survey was done by Robicheaux (1976) to study freight service expectations and the tradeoffs made by shippers of various commodities. Questionnaires were mailed to traffic managers to identy five criteria from a list of ten which are most important to them in their selection of a carrier. The rank order of service criteria is shown as Figure 21. Out of 402 usable returns, frequency of mention for pickup service speed (wait time) is comparatively lower than that for total service time, rate or loss and damage. Note that this rating was developed based on the shipper's choice among motor carriers. Between-mode variations in wait time are probably more significant than within-mode variations. Since data to allow careful estimation of a wait time model are not available, a simple model based on assumptions is used in this study.

Models to estimate freight loss and damage are regression equations with percent loss and damage as dependent variables, and value, density, temperature control requirements as explanatory variables (Wilson et al., 1976). The model covers only common carrier truck and rail. It has been extended to private truck and air. This model will predict the percentage of a shipment which will be lost or damaged. This informa-

Figure 21. Importance Ratings of Service Criteria for Motor Carriers[*]

Rank Order of Service Criteria	Frequency of Mention
1. Total Service Time	277
2. Rate per Hundred Weight	230
3. Expediting[**]	184
4. Percent of Successful Shipment Traking	159
5. Percent of Loss and Damage	147
6. Pickup Frequency	138
7. Damage Claims Settlement Time	123
8. Pickup Consistency	115
9. Pickup Service Speed[***]	97
10. Special Equipment	75

[*] Taken from Robicheaux (1976), p.8.

[**] The percent of shipments for which the carrier is willing to expedite their shipment requests.

[***] Time between a request for pickup is placed until the carrier actually makes the pickup.

tion is useful in calculating the capital cost tied up by a claim.
As indicated earlier, another cost associated with loss and damage
is the cost to file a claim. To estimate this cost requires infor-
mation on the probability that a loss/damage would occur to a shipment.
A study by Shepard (1977) gives useful information on this for
common carrier truck with shipment sizes of less than 1,000 pounds.
Unfortunately, this covers only a small subset of our mode/shipment-
size alternatives.

5.5. Commodity and Market Attributes

The commodity attributes for each shipment are taken from a
file assembled at MIT from data provided by the Transportation Systems
Center of the U.S. Department of Transportation and a number of other
sources.[44] This file includes the value, density, shelf time, state
and special handling requirements for each of the 1200 5-digit STCC
commodities. The file also includes an estimated national average
FOB price. Additional attributes available in the file include
minimum shipment size and maximum height of stacking in storage.
Both have been added to the file since its original compilation.

Regional price differentials are a key piece of information to
explain the choice of a supplier. Useful regional price information
by commodity at disaggregate level has not yet been located.
Therefore, the aggregate regional price developed in an input-output

---

[44] Samuelson and Roberts (1975).

study by Rodgers (1973) is used. It provides average prices by industry by Census regions. The industry classification is approximately matched to a 2-digit STCC. From this information, we derive a matrix of relative price index by commodity and by market area. Applying these price indices to our average regional prices given in the commodity attributes file produces the price information needed.

The market availability of each commodity can be measured by the total amount of a commodity sold in the market area. This has been developed from the Census of Manufactures. A matrix of market availability by commodity by market area is derived in the study using the 1972 Census of Manufactures.

CHAPTER 6

ESTIMATION OF A RANDOM COST MODEL

6.1. Introduction

This chapter describes an empirical estimation of the random

cost model of short-run freight demand formulated in Chapter 4.

The model treats shipment size and point of supply as discrete choices

in the same discrete fashion as mode choice in a logit model environ-

ment. The main purpose of the empirical study is to investigate the

feasibility of estimating the coefficients for a model in which the

choice of mode, shipment size and origin make use of logistics manage-

ment theories as the basis. Section 6.2 discusses the preparation of

the working sample and the definitions used to define each alterna-

tive--including both the alternative which is chosen as well as

the alternatives which are not chosen. Section 6.3 describes the choice

dimensions available and the structure of the models to be estimated.

Section 6.4 gives the estimated conditional mode choice model given

shipment size and origin. Section 6.5 presents the estimated condi-

tional choice of shipment size given chosen origin. The marginal

origin choice model is shown in Section 6.6. The elasticities and

marginal rate of substitutions implied by these models are developed

in Section 6.7.

6.2. Preparation of the Working Sample

We define mode as the type of vehicle used to transport the
shipment under consideration. Transport services are in fact dif-
ferentiated in terms of both type of vehicle and shipment size.
For example, "LTL" means to transport by truck in a shipment size
of less than a truckload; "rail carload" means to use rail vehicles
with a shipment size between roughly 25 and 50 tons. We thus define
the alternatives for transport service in terms of a mode-shipment
size matrix with one side representing the type of vehicle and the
other side representing the size of the shipment. This matrix is
shown in Figure 22.

The Commodity Transportation Survey of the Census of Transpor-
tation classifies mode of transport into rail, common carrier, truck,
private truck, air, water and "other." The "other" category cannot
be considered in our model since it covers many entirely different
transport services (i.e., railway express, United Parcel Service,
etc). We also exclude water shipments since level-of-service models
for water transportation are not currently available. Therefore, we
have four modes of transport: rail (RAIL), common carrier truck (CT),
private truck (PT) and air (AIR). Shipment size is reported in the
Census by means of twenty weight blocks. We cannot use the entire
set of Census classifications directly as the discrete choice set of
mode and shipment size. As mentioned in Section 4.5, the logit model
requires the assumption that the random cost component of each alter-

Shipment Size

Mode	1	2	3	4	5	6	7	8
RAIL 1	Rail Freight Forwarder			Trailer on Flatcar		Carload		Multiple Carload
CT 2	Less than Truckload (Common Carrier Truck)					Full Truckload (Common Carrier Truck)		
PT 3	Less than Truckload (Private Truck)					Full Truckload (Private Truck)		
AIR 4	Air Individual Shipment	Air Container				Air Charter		

100 lbs   3000 lbs   20,000 lbs   40,000 lbs   80,000 lbs   120,000 lbs   160,000 lbs

Figure 22. Definition of Alternatives

native be independent of the random cost component of all other
alternatives. To use the original weight blocks directly is likely
to violate this requirement since there are too many weight blocks
and some weight blocks are too close to others. We therefore group
the twenty weight blocks into eight shipment size categories.
Each shipment size category with its mode of transport is defined as
transportation service alternative to be chosen by the shipper.
This leads to a choice set with thirty-two alternatives of mode and
shipment size, namely:

    (1) Rail freight forwarder, minimum charge

    (2) Rail freight forwarder, small shipment

    (3) Rail freight forwarder, large shipment

    (4) Trailer-on-flatcar, one trailer

    (5) Trailer-on-flatcar, two trailers

    (6) Rail carload, small shipment

    (7) Rail carload, large shipment

    (8) Rail multiple carload

    (9) Less-than-truckload, minimum charge

  (10) Less-than-truckload, small shipment

  (11) Less-than-truckload, large shipment

  (12) Full truckload, one truck

  (13) Full truckload, two trucks

  (14) Full truckload, three trucks

  (15) Full truckload, four trucks

  (16) Full truckload, more than four trucks

(17) Private truck, small shipment

(18) Private truck, medium shipment

(19) Private truck, large shipment

(20 Private truck, one truckload

(21) Private truck, two truckloads

(22) Private truck, three truckloads

(23) Private truck, four truckloads

(24) Private truck, more than four truckloads

(25) Air individual shipment, minimum charge

(26) Air individual shipment

(27) Air container, small shipment

(28) Air container, medium shipment

(29) Air container, large shipment

(30) Air charter, small shipment

(31) Air charter, medium shipment

(32) Air charter, large shipment

The shipment size for each alternative can be found in Figure 22.

The working sample is prepared from the Census of Transportation Public Use Tape using the following procedures: 1) Skim the records in the Census tape looking for records which are complete at the 5-digit STCC level; 2) extract these records and expand them into a quasi-disaggregate data set as described in Section 5.2; 3) sample a representative annual use rate for each shipment from the use rate distribution as described in Section 5.3; 4) determine the non-chosen

alternatives which are to be considered for each shipment; and

5) develop transport level-of-service attributes for both chosen and non-chosen alternatives.

We have used a one-percent sample of the expanded disaggregate data set for the calibration of the model. In order to study the stability of the estimated coefficients, three working samples have been prepared. These samples will be referred to as SAMPLE1, SAMPLE2 and SAMPLE3, each with 1078 quasi-disaggregate data points. Figures 23 through 25 give a statistical picture of the characteristics of the distribution of chosen mode and chosen shipment size for each sample. The distributions of chosen mode by distance are given in Figures 26 through 28.

Within each mode of transport, a shipper can typically purchase different types of service. For example, a shipper using rail can choose rail freight forwarder, TOFC or carload. These different service offerings are characterized by differences in their pick-up and delivery systems, loading and unloading practices, consolidation systems, etc; all of which impact the service quality and price to the shipper. A differentiated service offering is referred to as a sub-mode. Each sub-mode requires its own level-of-service models to describe service characteristics such as tariff rate, transit time, or loss and damage. The rate information for each chosen and non-chosen alternative has been prepared based on separate rate models for each of the following sub-modes:

Figure 23. Distribution of Shipments by Chosen Mode and Shipment

Size (SAMPLE1)

	Number of Shipments (percentage)			
	Mode			
Shipment Size	RAIL	CT	PT	AIR
1	2 (0.19%)	49 (9.1%)	10 (0.92%)	37 (3.44%)
2	18 (1.67%)	494 (45.87%)	58 (5.39%)	39 (3.62%)
3	17 (1.58%)	90 (8.36%)	23 (2.14%)	1 (0.09%)
4	20 (1.86%)	104 (9.66%)	30 (2.79%)	--
5	26 (2.41%)	16 (1.49%)	1 (0.09%)	--
6	23 (2.14%)	1 (0.09%)	--	--
7	13 (1.21%)	2 (0.19%)	--	--
8	3 (0.28%)	1 (0.09%)	--	--

Figure 24. Distribution of Shipments by Chosen Mode and Shipment

Size (SAMPLE2)

| | Number of Shipments (percentage) | | | |
| | Mode | | | |
Shipment Size	RAIL	CT	PT	AIR
1	5 (0.46%)	62 (5.75%)	15 (1.39%)	45 (4.17%)
2	15 (1.39%)	463 (42.95%)	49 (4.55%)	36 (3.34%)
3	20 (1.86%)	102 (9.46%)	31 (2.88%)	4 (0.37%)
4	16 (1.48%)	79 (7.33%)	34 (3.15%)	--
5	25 (2.32%)	21 (1.95%)	6 (0.56%)	--
6	28 (2.60%)	4 (0.37%)	1 (0.09%)	--
7	9 (0.83%)	1 (0.09%)	1 (0.09%)	--
8	5 (0.46%)	1 (0.09%)	--	--

Figure 25. Distribution of Shipments by Chosen Mode and Shipment

Size (SAMPLE3)

| | Number of Shipments (percentage) | | | |
| | Mode | | | |
Shipment Size	RAIL	CT	PT	AIR
1	1 (0.09%)	37 (3.43%)	8 (0.74%)	32 (2.97%)
2	26 (2.41%)	473 (43.88%)	32 (2.97%)	54 (5.01%)
3	15 (1.39%)	114 (10.58%)	26 (2.41%)	11 (1.02%)
4	10 (0.93%)	93 (8.63%)	19 (1.76%)	--
5	18 (1.67%)	28 (2.60%)	11 (1.02%)	--
6	35 (3.25%)	7 (0.65%)	4 (0.37%)	--
7	10 (0.93%)	4 (0.37%)	--	--
8	6 (0.56%)	3 (0.28%)	1 (0.09%)	--

Figure 26. Distribution of Shipments by Chosen Mode and Distance

(SAMPLE1)

MILE (200's)	Mode			
	RAIL	CT	PT	AIR
1	14	202	78	5
2	12	172	21	10
3	17	85	5	16
4	13	84	5	10
5	13	53	4	6
6	7	36	2	2
7	9	30	--	3
8	2	18	2	4
9	5	12	--	1
10	1	3	--	3
11	1	22	3	3
12	5	10	--	2
13	7	6	--	2
14	2	21	--	8
15	3	4	2	2
16	8	1	--	--
17	3	--	--	--
18	--	--	--	--

Figure 27. Distribution of Shipments by Chosen Mode and Distance

(SAMPLE2)

MILE (200's)	Mode			
	RAIL	CT	PT	AIR
1	11	217	84	3
2	16	164	18	14
3	18	72	7	14
4	16	88	11	8
5	10	42	8	9
6	9	24	1	6
7	12	38	2	4
8	6	15	--	7
9	5	10	1	--
10	--	9	--	4
11	3	13	2	2
12	1	14	1	4
13	4	8	--	6
14	1	15	1	3
15	1	1	--	1
16	4	2	1	--
17	5	1	--	--
18	1	--	--	--

Figure 28. Distribution of Shipments by Chosen Mode and Distance
(SAMPLE3)

| MILE | Mode | | | |
(200's)	RAIL	CT	PT	AIR
1	16	187	54	2
2	10	156	15	17
3	17	93	4	11
4	16	101	11	9
5	8	38	7	12
6	12	41	1	4
7	8	29	1	5
8	3	24	1	2
9	6	18	--	2
10	1	11	1	6
11	--	8	1	5
12	2	21	2	5
13	6	9	1	5
14	4	18	2	4
15	3	4	--	3
16	4	--	--	--
17	2	1	--	--
18	3	--	--	--

(1) Rail minimum charge

(2) Rail freight forwarder

(3) TOFC

(4) Rail carload

(5) Multiple carload

(6) Truck minimum charge

(7) LTL

(8) Full truckload

(9) Private truck

(10) Air minimum charge

(11) Air general shipment

Transit time data are prepared using separate transit time models

for each of the following sub-modes:

(1) Rail freight forwarder

(2) TOFC

(3) Rail carload

(4) Rail multiple carload

(5) LTL

(6) Full truckload

(7) Private truck

(8) Air

Due to data limitations, loss and damage information is developed using

loss and damage models at the mode level:

(1) Rail

(2) Common carrier truck

(3) Private truck

(4) Air

For practical purposes, it is not relevant to consider the full
set of alternatives for each shipment observed in the sample.
We have reduced the number of alternatives in two basic ways.
First, only alternatives which are feasible are treated as unchosen
alternatives. There is a minimum shipment size for each commodity.
Therefore, an unchosen alternative cannot involve a shipment size
smaller than this minimum. It seems highly improbable that the shipper
will choose a shipment size so small that it involves excessive ship-
ments. For example, a 50-pound shipment size for a shipper with a
use rate of 50 tons a year implies 40 orders per week. However,
we cannot conclude that this is physically impossible. Rather, it
is treated as an emergency shipment. A shipment size greater than
annual use rate is also treated as an emergency shipment. We will
return to the emergency shipment problem in the subsequent discussion
of model specification.

The second way to reduce the size of the choice set is to use the
IIA property of the logit model. Using the IAA property, it is
possible to sample randomly a subset of alternatives from the full
set of alternatives. This is especially useful for origin choice.
The Census reported the origin of shipment over twenty-seven separate
production areas. This is obviously too large a set to work with
in estimation. For practical purposes, a smaller number of produc-
tion areas can be sampled as the alternative points of supply. The

sampling procedure can be designed either to give each production area
the same weight or each production area can be weighted by the
size of its market. It is important to keep the sampling procedure
consistent with the model specification. For instance, if production
areas are sampled using market size as weight, it would be incon-
sistent to have market size as an explanatory variable in the model.
Instead, we randomly sample nine production areas, giving each pro-
duction area the same weight. Thus, the maximum number of alterna-
tives in the choice set becomes the chosen alternative plus 287
unchosen alternatives (4 modes × 8 shipment sizes × 9 origins).

6.3. The Structure of the Models

As described earlier, the choices of mode, shipment size and
origin supplier are the outcomes of the logistics management process.
The choices of mode, shipment size and supplier are assumed to be
made interdependently. The choice of origin, for example, is affected
by the choice of mode and shipment size not only because a higher
purchase price in a given production area will influence the firm
to consider other points of supply, but it will also cause the firm
to reevaluate its choices of mode and shipment size. There are trade-
offs which must be considered among purchase cost, transport cost and
inventory cost. The dependency is in both directions, e.g., the
choice of mode depends on the choice of shipment size and the choice
of shipment size depends on the choice of mode, etc. The two-directional

dependency exists between any two of the choice dimensions. This dependency can be modelled using a joint choice logit model:

$$p(imq) = \frac{e^{-w^o_{imq}}}{\sum\limits_{i'm'q'} e^{-w^o_{i'm'q'}}} \qquad \begin{array}{c}(4.14)\\ \text{repeated}\end{array}$$

Model (4.14) requires that the ε_{imq}'s be independently and identically distributed as Weibull distributions with scale parameter $\mu = 1$:

$$p_r[\varepsilon \le \varepsilon^*] = e^{-e^{-\mu(\varepsilon^* + \alpha)}} \qquad (6.1)$$

In some cases, one would suspect that there are time lags between these three logistics choices. For industries using automobile parts for assembly, the choice of supplier is very often made initially with only a vague composite picture of the transport and inventory costs. Thus, the choices of mode and shipment size are in some sense conditional on the outcomes of origin choice. It is also possible for a firm to choose the transport mode conditional on both shipment size and point of supply. This suggests that the three logistics choices might be modelled as sequential logit models.

The sequential formulation is more complicated than the joint one. It involves a conditional choice model as well as a marginal choice model. For example, to model the joint choice of mode and shipment size conditional on origin choice we will need a model to describe the conditional choice of mode/shipment size given origin and a model

to describe the marginal choice of origin. To specify a sequential
model system consistently, an expected minimum cost must be calcu-
lated from the lower level choice for use in the higher level choice
model. Let us define:

w_{imq} = the observed purchase and logistics costs. Note that we are
dropping the superscript "o" for simplicity.

w_i = the variables in the cost function related to the choice
of origin which are independent of mode and shipment size.

$w_{q|i}$ = the variables in the cost function related to the choice
of shipment size given that a supplier is chosen which
are independent of mode.

$w_{m|iq}$ = the variables in the cost function related to the choice
of mode given that a supplier and a shipment size
are chosen.

I, M, Q = the choice set for origin, mode and shipment size res-
pectively.

By definition, we have $w_{imq} = w_i + w_{q|i} + w_{m|qi}$.

An example for the variable w_i is the purchase cost. If there is
no quality discount in purchase, then purchase cost per unit is a
function only of origin choice and is independent of mode and
shipment size. Ordinary capital carrying cost is an example
of the variables given as $w_{q|i}$. Conditional on purchase cost,
the capital-carrying cost for goods in storage (not including
safety stock) is a function of the shipment size only and is

independent of mode choice. Transport related costs are variables belonging to the variable type $w_{m|qi}$. They are a function of mode choice given that a shipment size and origin have been previously chosen.

The conditional and marginal probabilities in the sequential formulation can be derived as follows.[45/]

$$p(m|iq) = \frac{e^{-w_{m|iq}}}{\sum\limits_{m' \in M_{iq}} e^{-w_{m'|iq}}} \tag{6.2}$$

$$p(q|i) = \frac{e^{-\mu_1 w_{q|i} - \mu_1 \ln \sum\limits_{m \in M_{iq}} e^{-w_{m|iq}}}}{\sum\limits_{q' \in Q_i} e^{-\mu_1 w_{q'|i} - \mu_1 \ln \sum\limits_{m \in M_{iq'}} e^{-w_{m|iq'}}}} \tag{6.3}$$

$$p(i) = \frac{e^{-\mu_2 w_i - (\mu_2/\mu_1) \ln \sum\limits_{q \in Q_i} e^{-\mu_1 w_{q|i} - \mu_1 \ln \sum\limits_{m \in M_{iq}} e^{-w_{m|iq}}}}}{\sum\limits_{i' \in I} e^{-\mu_2 w_{i'} - (\mu_2/\mu_1) \ln \sum\limits_{q \in Q_{i'}} e^{-\mu_1 w_{q|i'} - \mu_1 \ln \sum\limits_{m \in M_{i'q}} e^{-w_{m|i'q}}}}} \tag{6.4}$$

[45/] For the mathematical details involved in deriving sequential choice models see Ben-Akiva (1973) or Ben-Akiva and Lerman (1976).

We require $0 < \mu_1 \leq 1$ and $0 < \mu_2 \leq \mu_1$ to ensure that the cross elasticities will behave correctly.

It has been shown that Model (4.14) can be decomposed mathematically into Models (6.2), (6.3) and (6.4) with $\mu_1 = \mu_2 = 1$. This property provides an alternative way to estimate the joint logit model given in (4.14) by estimating Models (6.2) through (6.4) with the coefficient restricted such that $\mu_1 = \mu_2 = 1$. Amemiya (1976) proved that direct estimation of the joint choice model gives more efficient results than an indirect sequential estimation. However, the indirect, sequential estimation is sometimes very useful for empirical reasons. First, logistics choices are typically characterized by a large number of alternatives. It becomes infeasible to consider all alternatives and to estimate the joint logit model directly. A sequential estimation will reduce the size of the choice set significantly. However, the tradeoff between the loss of efficiency of a sequential estimation and a direct estimation based on a small choice set using the IIA property is not yet clear. Secondly, it is quite possible that there is multicollinearity between variables in the logistics cost function. Thus, it appears that a sequential estimation will give a more correct estimate of the coefficients when multicollinearity is severe.

A sequential estimation in which the scale parameter μ is estimated to be significantly equal to unity indicates that the ε_{imq}'s are independent. By contrast, if μ is estimated to be

significantly different from unity, this indicates that the ε_{imq}'s

are not independent. For purposes of illustration, a two-mode, two-

shipment size case is considered. We write the cost function as

$$w_{mq} = v_q + v_{mq} + \eta_q + \eta_{mq} \tag{6.5}$$

where $\quad w_{mq}$ = the cost function associated with the choice of

mode and shipment size

$\quad\quad v_q$ = the variable in the cost function related to shipment

size only

$\quad\quad v_{mq}$ = the variable in the cost function related to both

mode and shipment size

$\quad\quad \eta_q, \eta_{mq}$ = the random cost associated with v_q and v_{mq} respec-

tively.

Note that there is no variable v_m which is related only to mode in

the logistics cost function. Now, if we estimate (6.5) sequentially

in the order of mode and shipment size, the cost function in the

conditional mode choice model is

$$w_{m|q} = v_{mq} + \eta_{mq} \tag{6.6}$$

The marginal cost function over shipment size choice becomes

$$w_q = v_q + \eta_q + \underset{m}{\text{Max}}[w_{m|q}] \tag{6.7}$$

Thus, the scale parameter in the marginal shipment size choice model

will not be estimated to be unity if $\eta_q \neq 0$. In terms of the corre-

lation matrix between cost functions, we have

$$
\begin{array}{c}
\begin{array}{cccc}
q_1 m_1 & q_1 m_2 & q_2 m_1 & q_2 m_2
\end{array} \\
\begin{array}{c}
q_1 m_1 \\
q_1 m_2 \\
q_2 m_1 \\
q_2 m_2
\end{array}
\left[
\begin{array}{cc|cc}
1 & 1-\mu & 0 & 0 \\
1-\mu & 1 & 0 & 0 \\
\hline
0 & 0 & 1 & 1-\mu \\
0 & 0 & 1-\mu & 1
\end{array}
\right]
\end{array}
$$

When $\eta_q = 0$, the random error in the joint cost function reduces to η_{mq} and a joint choice model results.

It is important to notice that one cannot conclude the underlying decision-making process to be either joint or sequential from the estimated scale parameter μ. A sequential decision-making process is defined as one in which causality is only one-way. If the choice of shipment size is independent of the mode that is actually chosen, and the choice of mode is dependent on the chosen shipment size, the decision-making process can be considered to be sequential. The mode choice is conditional on the shipment size choice; the reverse is not true. By inventory theory, there is a two-way causality which exists among the three logistics choices. Thus, the logistics decision-making process is not a sequential process.

The question left is to what degree one choice is more dominant than the other choices. This is related to the question of how impor-tant is the η_q in comparison with η_{mq}. Thus, the estimated μ should

serve as an indicator of the existence of η_q. A small μ will indicate
that η_q is significantly different from zero. Unfortunately, a formal
test cannot be developed to infer the degree to which the underlying
decision-making process is a joint one or a sequential one. The
magnitude of the estimated μ is also affected by various data problems
such as sampling error, measurement error and multicollinearity
between variables.

We will however proceed in our estimation of the random cost
model using the indirect, sequential estimation procedure since it
is significantly easier to apply. First, the conditional choice of
mode, given shipment size and origin will be estimated. Next, the
conditional choice of shipment size given origin will be estimated.
Finally, the marginal choice of origin of supplier will be developed.

6.4. The Conditional Mode Choice Model

The conditional mode choice model predicts the probabilities
of selecting alternative modes of transport given the chosen ship-
ment size and origin. The model is expressed as follows:

$$p(m|iq) = \frac{e^{-w_{m|iq}}}{\sum_{m' \in M_{iq}} e^{-w'_{m|iq}}} \qquad (6.2)\ \text{repeated}$$

$w_{m|iq}$ contains the variables in the logistics cost function which
are related to mode given that a supplier and a shipment size are

chosen. Using the general specification of the cost function

presented in Section 4.5., the following cost items are included

in the cost function $w_{m|iq}$:

transport charges per unit

capital carrying cost in transit per unit

capital carrying cost per unit tied up with a claim

cost of filing a claim per unit

loss of value per unit during transit

safety stock carrying cost, storage cost and stockout cost

per unit.

Transport charges include tariff rates and special charges

such as pickup and delivery charges for TOFC or terminal charges

for AIR. Data on the time required to undertake the investigation

of a claim and make a payment (IT) are not available; however,

an average of 60 days is assumed. This assumption enables the cost

function to include loss and damage which has been shown with a

moderate impact on choice of carrier. Cost of filing a claim

is excluded since information on the probability of incurring a

loss/damage claim (CLP) are not available. Also excluded is the loss

of value during transit because there are no time-sensitive or

perishable goods in our sample.

The specifications of the safety stock carrying and storage costs

require data on reliability measured in days by mode for any given

city pair. The empirical transit time distribution model described

in Section 5.4 is capable of giving this required information.
Unfortunately, this model has so far only been applied to regular-
route (LTL) trucking. Simplifying assumptions have been made to
derive usable modally differentiated reliability data as an average
factor multiplied times mean transit time for the sub-modes other than
LTL truck. Trial estimation of the conditional mode choice model based
upon these assumptions for reliability were not successful. The
safety stock carrying and storage costs were therefore not included
in the final estimation.

The definitions of variables used in the estimated conditional
mode choice model are given in Figure 29. The first variable is
transport charges measured as dollars per pound. The second and third
variables are the transit and waiting time for emergency shipments.
An emergency shipment is defined by either of the following two
criteria:

(1) The annual use rate divided by shipment size is greater than
52, i.e., more than one order per week would be required if this
shipment size were chosen regularly.

(2) The annual use rate is less than the shipment size, i.e.,
the entire annual use is being shipped at once which suggests that
the shipment is irregular and may be a spare part, etc.
Note that these criteria can also be implemented easily in the pre-
diction process. The coefficient estimated for transit time for
emergency shipments indicates the time value per day per pound of

Figure 29. Definitions of Variables for the Conditional Mode Choice

Model Given Shipment Size and Origin

Variable	Definition

RATE — Transport rate and special charges

EMRG1 — Transit time for emergency shipment of raw materials

$(TT + WT) \cdot E \cdot COMM$

EMRG2 — Transit time for emergency shipment of other goods

$(TT + WT) \cdot E \cdot (1 - COMM)$

CCCIT — Capital Carrying Cost in transit or tied up with a loss/damage claim

$(TT + WT + LDP \cdot IT) \cdot \dfrac{1}{365} \cdot p \cdot (1 - E)$

DIST — $\dfrac{1}{distance}$ for private truck, 0 otherwise

RAIL — 1 for rail, 0 otherwise

CT — 1 for common carrier, 0 otherwise

AIR — 1 for air, 0 otherwise

Note: 1. $E = 1$, if $\dfrac{x}{q} > 52$ or $\dfrac{x}{q} < 1$; $= 0$ otherwise

2. COMM = 1, if the commodity is raw materials; = 0 otherwise

3. transport charges are in dollars per pound.

4. TT, WT, IT are in days

5. distance is in miles

shipment. In view of the fact that different types of commodities are likely to have different stockout consequences, we specify this variable as two commodity-specific variables; one for raw materials and another for final or capital goods.

The fourth variable in the cost function is capital carrying cost for goods in transit or tied up in a loss/damage claim. These two cost items are combined together to estimate the interest rate perceived by the shipper. Note that this variable is specified only when it is not an emergency shipment.

A prerequisite for using private truck is its ownership by either the shipper or receiver. Thus, the choice of private truck may involve the decision of truck ownership. Unfortunately, there is no information that can be used to take this factor into consideration. A dummy variable measured as the reciprocal of distance is used for private truck to adjust this effect on the assumption that private truck is usually justified only for short-haul transport.

Finally, alternative-specific constants are specified to measure "pure alternative" effects, i.e., the net effect of all attributes of an alternative which are not measured by the other variables. A model which does not include a constant term will give inconsistent estimates if the constant term exists in the true model. In contrast, a constant term will be estimated asymptotically zero when it is not part of the true model. Thus, there are advantages to using constant terms in the statistics sense. For the logit model, it can be shown

that alternative-specific constants will be estimated with the magnitude of the overall market share of the alternatives.

The specification of the cost functions for each alternative are shown in Figure 30. All logistics cost items have been specified generically. This allows the estimated model to be applied to policy changes involving an entirely new alternative. Figure 31 gives the estimation results of the conditional model choice model. Coefficients of the variables and their t-statistics are shown in the figure for each sample. $L^*(0)$ is the value of the log of the likelihood function when all parameters are zero, i.e., when every alternative has the same probability. $L^*(\hat{\theta})$ is the value of the log of the likelihood function at the maximum likelihood coefficients value. Thus $\chi^2 = -2(L^*(0) - L^*(\hat{\theta}))$ is a statistic asymptotically distributed as chi square with the number of degrees of freedom equal to the number of parameters estimated. This statistic provides a test against the null hypothesis that all parameters are zero. ρ^2 is a measure analogous to R^2 in a linear regression model which is calculated as $\rho^2 = 1 - L^*(\hat{\theta})/L^*(0)$. Thus ρ^2 is equal to the ratio of the explained log of likelihood to the total log likelihood, and it lies between 0 and 1.

The coefficients of all cost items: RATE, EMRG1, EMRG2, CCCIT are estimated with expected negative signs, indicating that there are disutilities associated with increasing each cost element in the logistics decision process. The coefficients of RATE are -1.658,

Figure 30. Specifications of the Cost Functions for the Conditional

Mode Choice Model Given Shipment Size and Origin

| Coefficients | Specification | | | |
	RAIL	CT	PT	AIR
β_1	RATE	RATE	RATE	RATE
β_2	EMRG1	EMRG1	EMRG1	EMRG1
β_3	EMRG2	EMRG2	EMRG2	EMRG2
β_4	CCCIT	CCCIT	CCCIT	CCCIT
β_5	0	0	DIST	0
β_6	RAIL	0	0	0
β_7	0	CT	0	0
β_8	0	0	0	AIR

Figure 31. Maximum Likelihood Estimates for the Conditional

Mode Choice Model Given Shipment Size and Origin

| Variable | Coefficient and t-statistics[*] | | |
	SAMPLE1	SAMPLE2	SAMPLE3
RATE	-1.658	-1.497	-1.644
	(-3.32)	(-3.57)	(-3.06)
EMRG1	-0.465	-0.369	-0.505
	(-1.14)	(-1.52)	(-1.46)
EMRG2	-0.104	-0.126	-0.048
	(-0.65)	(-0.94)	(-0.88)
CCCIT	-2.746	-2.485	-2.974
	(-2.35)	(-2.16)	(-3.48)
DIST	0.662	0.554	0.216
	(7.82)	(6.44)	(6.85)
RAIL	0.215	0.185	0.234
	(1.92)	(1.74)	(1.45)
CT	2.231	2.221	2.419
	(13.81)	(15.24)	(10.97)
AIR	0.0327	0.0165	0.0653
	(0.69)	(0.24)	(0.45)
$L^*(0)$	-1474.25	-1459.83	-1470.66
$L^*(\hat{\theta})$	-961.47	-993.16	-1023.81
χ^2	1025.56	1005.34	893.7
ρ^2	0.35	0.32	0.30
No. of Observations	1078	1078	1078

[*] t-statistics are given in parentheses.

-1.497, and -1.644 respectively for SAMPLE1, SAMPLE2 and SAMPLE3.
All are significantly greater than one. This implies that the shippers
are concerned with transport charges in the choice of mode for their
shipments. Since the coefficients of the logit model are estimated
as a multiplier of the scale parameter μ, the magnitudes of the
coefficients are better interpreted relatively. We thus normalize
all coefficients by the coefficients of RATE; the results are shown
in Figure 32. The normalized coefficients of EMRG1 give the value of
time per day per pound for an emergency shipment of raw materials.
The normalized coefficients of EMRG2 give the same value of time for
other goods. The value of time for raw materials is estimated to be
much higher than that for final goods or capital goods as one would
expect, since the late arrival of a raw material in an emergency
implies that possibility of an interruption of the production process.

The coefficients of CCCIT give the normalized interest rate
perceived by the shipper. Note that the results are estimated to
be significantly higher than the normal market cost of capital.
This indicates that shippers in the real world have over-emphasized
the importance of transit time. This might also be explained by the
omission of safety stock carrying and stockout costs in the model
specification.

The common carrier truck constant is statistically significant
and with a large value. The rail constant is relatively small and
moderately significant. The air constant is not significantly

Figure 32. The Normalized Conditional Model of Mode Choice[*]

Variable	Coefficient		
	SAMPLE1	SAMPLE2	SAMPLE3
RATE	1.0	1.0	1.0
EMRG1	0.281	0.246	0.307
EMRG2	0.063	0.084	0.029
CCCIT	1.656	1.66	1.809
DIST	0.399	0.363	0.131
RAIL	−0.13	−0.124	−0.142
CT	−1.346	−1.484	−1.471
AIR	−0.0197	−0.011	−0.0398

[*]Normalized by the coefficient of RATE.

different from zero. These imply that there are pure alternative

effects to the revealed preference for rail and common carrier truck,

and especially the latter. One reasonable explanation would be that

there are additional supply effects involved. In choosing the mode

of transport, the ready availability (or perhaps the available

capacity) of some transport service offerings should play an influencing

role in the choice which has not been considered in the model spe-

cification. This effect can be incorporated into the model by a

variable to describe the modal availability analogous to the size

of the market in the origin choice model.[46/] Unfortunately, no suit-

able data for this purpose have been found.

Comparing the model coefficients for different data sets, we

found that most of the coefficients have very stable estimates over

the three samples. The coefficient for RATE is close to identical

in SAMPLE1 and SAMPLE3 and is only slightly smaller in absolute

magnitude in SAMPLE2. Once all coefficients have been normalized

by the coefficients of RATE, most of the variations in coefficients

across samples are within a ± 10 percent level excpet EMRG2 and AIR.

EMRG2 is estimated with a lower coefficient in SAMPLE3 than in SAMPLE1

and SAMPLE2. The air constant is estimated with a lower coefficient

in SAMPLE2 but a much higher coefficient in SAMPLE3. However,

these two coefficients are only barely significantly different

from zero.

[46/] The origin choice model is shown in Section 6.6.

6.5. The Conditional Shipment Size Choice Model

The conditional shipment size choice model predicts the probabilities of choosing a particular shipment size, q, given that an origin of supply has already been selected. The model is written as:

$$p(q|i) = \frac{e^{-\mu_1 w_{q|i} - \mu_1 \ln \sum_{m \in M_{iq}} e^{-w_{m|iq}}}}{\sum_{q' \in Q_i} e^{-\mu_1 w_{q'|i} - \mu_1 \ln \sum_{m \in M_{iq}} e^{-w_{m|iq'}}}} \qquad \begin{matrix}(6.3)\\ \text{repeated}\end{matrix}$$

The term $\ln \sum_{m \in M_{iq}} e^{w_{m|iq}}$ is called the <u>log sum of the denominator</u> of the conditional mode choice model. It is treated as a single variable in the cost function for the conditional choice of shipment size. By our general specification, variables belonging to the type of variable $w_{q|i}$ include order cost, handling cost, packaging cost, capital carrying cost for regular stock, and storage cost for regular stock. Both handling and packaging costs are omitted due to unavailability of data. Attempts to estimate handling cost as a coefficient using a set of shipment size specific variables as proposed in Section 4.5 have not been successful. Thus, the conditional shipment size model has been estimated with the following variables:

LOGM = log sum of the denominator of the conditional mode
        choice model

 STC = storage cost

CCC = capital carrying cost on goods in storage (except
        safety stock or goods being investigated for
        damage claims)

  OC = order cost.

The definitions for these variables are shown in Figure 33. All variables are specified generically for the eight shipment size categories.

The maximum likelihood estimates of Model (6.3) without the parameter restriction $\mu_1 = 1$ are shown in Figure 34. The coefficient of LOGM is estimated as 0.681, 0.816 and 0.554 in SAMPLE1, SAMPLE2 and SAMPLE3 respectively. CCC has the expected negative sign in each sample and is statistically significant. The coefficients for STC in SAMPLE2 and OC in SAMPLE3 are positive which is counterintuitive. However, these two variables are not significant in any of the samples. This suggests that we have not used empirical information concerning storage cost and order cost in the model specification. The simple formulations used assume an average storage cost per ft^3 and a constant order cost proportional to the value of the shipment. Collection of data concerning storage and order costs in the real world is deemed desirable for the development of a better disaggregate freight demand model.

Figure 33. Definitions of Variables for the Conditional Shipment

Size Choice Model Given Origin

Variable	Definition
LOGM	Log of the denominator of the mode choice model
STC	Storage Cost $$\frac{q}{2} \cdot \frac{1}{DEN} \cdot \frac{1}{x}$$
CCC	Nonsafety stock carrying cost $$\frac{q}{2} \cdot p_i \cdot \frac{1}{x}$$
OC	Order cost $$(q \cdot p)^{0.4} \cdot \frac{1}{q}$$

Note: 1. x, q are in pounds

2. DEN is in pounds per cubic foot

3. p is in dollars per pound

Figure 34. Maximum Likelihood Estimates for the Conditional Shipment

Size Choice Model Given Origin (Unrestricted)

| Variable | Coefficient and t-Statistics[*] | | |
	SAMPLE1	SAMPLE2	SAMPLE3
LOGM	0.681 (4.74)	0.816 (3.65)	0.554 (3.89)
STC	-0.317 (-0.54)	0.141 (0.28)	-0.462 (-0.39)
CCC	-0.525 (-9.41)	-0.491 (-8.85)	-0.605 (-7.99)
OC	-0.864 (-0.72)	-0.218 (-0.43)	0.582 (0.15)
$L^*(0)$	-2241.63	-2241.86	-2240.74
$L^*(\hat{\theta})$	-1836.49	-1872.05	-1817.25
χ^2	810.28	739.62	846.98
ρ^2	0.37	0.33	0.38
No. of Observations	1078	1078	1078

[*] t-statistics are given in parentheses.

Restricting μ_1 to unity and reestimating the conditional

shipment size choice model gives the results shown in Figure 35.

Both coefficients and t-statistics are changed as expected but

their values are still quite close to the unrestricted estimates.

Capital carrying costs are very significant and with the coefficients

of reasonable magnitude. Storage and order costs are still not statis-

tically significant. However, unlike the unrestricted model, the

coefficient of OC is estimated with the expected signs in all samples.

To test the relevance of modelling shipment size choice as a

discrete choice rather than as a continuous variable, a simulation is

carried out to investigate whether the estimated coefficients were

affected by an arbitrary change in shipment size classes. We found

in general that the coefficients are stable and do not change signi-

ficantly. The coefficients which are estimated for each case are

almost identical if the changes in shipment size classes occur only

in the larger shipment sizes. The coefficients seem to be less stable

as the changes in shipment size involve smaller shipments. Two

extreme cases are shown in Figure 36.

This phenomenon is probably due to the "edge" effect of the

rate models used. Rate models give a nonlinear but monotonically

decreasing prediction of rates for each transport service offering.

It has been found that the difference in predicted and observed

Figure 35. Maximum Likelihood Estimates for the Conditional Shipment
Size Choice Model Given Origin (Restricted)

| Variable | Coefficient and t-statistics[*] | | |
	SAMPLE1	SAMPLE2	SAMPLE3
LOGM	1.0	1.0	1.0
STC	-0.479	0.094	-0.778
	(-0.15)	(0.41)	(-0.27)
CCC	-0.574	-0.516	-0.841
	(-7.63)	(-8.93)	(-7.23)
OC	-1.135	-0.231	-0.148
	(-0.67)	(-0.58)	(-0.21)
$L^*(0)$	-2241.63	-2241.86	-2240.74
$L^*(\hat{\theta})$	-1936.33	-1902.58	-1994.27
χ^2	610.6	678.56	492.94
ρ^2	0.27	0.30	0.22
No. of Observations	1078	1078	1078

[*] t-statistics are given in parentheses.

Figure 36. Test of the Stability of Coefficients for Different Ship-
ment Size Categories (SAMPLE1)

| Variable | Coefficient and t-statistics[*] | | |
	Original	Test-A	Test-B
LOGM	0.681	0.516	0.723
	(4.74)	(3.44)	(5.38)
STC	-0.317	-0.148	-0.326
	(-0.54)	(-0.42)	(-0.23)
CCC	-0.525	-0.405	-0.551
	(-9.41)	(-8.86)	(-9.77)
OC	-0.864	-0.332	-0.819
	(-0.72)	(-0.15)	(-0.56)

[*]1. The following shipment size classes are used in Test-A:
less than 49 lbs; 50 ∿ 499 lbs; 500 ∿ 4,999 lbs;
5,000 ∿ 19,999 lbs; 20,000 ∿ 39,999 lbs; 40,000 ∿ 79,999 lbs;
80,000 ∿ 119,999 lbs; 120,000 ∿ 15,999 lbs; 160,000 or more lbs.

2. The following shipment size classes are used in Test-B:
less than 99 lbs; 100 ∿ 2,999 lbs; 3,000 ∿ 9,999 lbs;
10,000 ∿ 29,999 lbs; 30,000 ∿ 59,999 lbs;
60,000 ∿ 99,999 lbs; 100,000 ∿ 149,999 lbs; 150,000 or more lbs.

3. t-statistics are given in parentheses.

rates sometimes are large for some commodities, especially when
shipment sizes are small. Comparing predicted and observed rates
indicates that the commodity attributes developed by the Freight
Study Group at the Massachusetts Institute of Technology are too
aggregate when they are used to predict tariff charges.[47/] Rates
in practice are far more commodity-specific.

6.6. The Marginal Origin Choice Model

This model predicts the marginal choice of point of supply over
all modes and shipment sizes. The model is expressed as follows:

$$p(i) = \frac{e^{-\mu_2 w_i - (\mu_2/\mu_1)\ln \sum\limits_{q \in Q_i} e^{-\mu_1 w_{q|i} - \mu_1 \ln \sum\limits_{m \in Q_{iq}} e^{-w_{m|iq}}}}}{\sum\limits_{i' \, I} e^{-\mu_2 w_{i'} - (\mu_2/\mu_1)\ln \sum\limits_{q \in Q_{i'}} e^{-\mu_1 w_{q|i} - \mu_1 \ln \sum\limits_{m \in Q_{iq}} e^{-w_{m|iq}}}}}$$

<div align="right">

(6.4)
repeated

</div>

The variables and their definitions are as shown in Figure 37.
LOGQ denotes the log sum of the denominator of the conditional
shipment size choice model. p is the purchase price per unit in the
Production Area. This piece of information is developed as the product

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[47/] See Wang (1978).

Figure 37. Definitions of Variables for the Marginal Origin
          Choice Model

Variable	Definition
LOGQ	Log sum of the denominator of the shipment size choice model
P	Purchase price
MKT	Market availability, total value of shipments of the commodity produced in the production area
RC	= 1 if the production area is the regional center for the receiving firm; = 0 otherwise

---

of the average wholesale FOB price and the relative regional price
index as described in Section 5.5. Market availability (MKT) is
measured as the total value of shipments of the 5-digit STCC commodity
produced in a Production Area. RC is a dummy variable used to
characterize the production area in central place terms. If the
production area is the regional center for the receiving firm it is
set to one, zero otherwise. The assumption is that firms are more
likely to purchase their requirements in their regional centers.

The estimation results of the marginal origin choice model
are shown in Figure 38. The variable LOGQ is statistically signi-
ficant in all samples. It has a value close to unity in SAMPLE1
and SAMPLE2, but significantly different from one in SAMPLE3.
Purchase price estimates with a small coefficient which is not
significant in all cases. This indicates that the regional price
information is poor and too aggregate. Both MKT and RC are estima-
ted to be highly significant and stable across different samples.
This appears to provide evidence that a central place orientation
does exist in the regional economy of the United States.

The maximum likelihood estimates of the restricted origin
choice model are given in Figure 39 (restricting $\mu_1 = \mu_2 = 1$).
Comparing restricted and unrestricted models, the differences are
mainly in SAMPLE3. The restricted model estimates a wrong positive
sign for purchase price in SAMPLE3. Note that it was negative in
the unrestricted model. However, in both cases the t-statistic

Figure 38. Maximum Likelihood Estimates for the Marginal Origin

   Choice Model (Unrestricted)

Variable	Coefficient and t-statistics[*]		
	SAMPLE1	SAMPLE2	SAMPLE3
LOGQ	0.872 (5.46)	1.131 (3.48)	0.427 (3.15)
P	-0.247 (-0.15)	0.143 (0.11)	-0.184 (-0.03)
MKT	1.112 (28.73)	1.263 (25.53)	0.944 (23.92)
RC	0.446 (6.14)	0.392 (6.75)	0.448 (7.97)
$L^*(0)$	-3552.91	-3552.91	-3552.91
$L^*(\hat{\theta})$	-3021.52	-3074.17	-3008.67
χ^2	1062.78	957.48	1088.48
ρ^2	0.30	0.27	0.31
No. of Observations	1078	1078	1078

[*] t-statistics are given in parentheses.

Figure 39. Maximum Likelihood Estimates for the Marginal Origin

Choice Model (Restricted)

| Variable | Coefficient and t-statistics* | | |
	SAMPLE1	SAMPLE2	SAMPLE3
LOGQ	1.0	1.0	1.0
P	-0.352 (-0.34)	0.152 (0.13)	0.235 (0.19)
MKT	1.149 (25.28)	1.084 (27.32)	1.476 (20.41)
RC	0.523 (6.77)	0.458 (7.42)	0.467 (5.38)
$L^*(0)$	-3552.91	-3552.91	-3552.91
$L^*(\hat{\theta})$	-3148.57	-3189.64	-3276.01
χ^2	808.68	726.54	553.8
ρ^2	0.23	0.20	0.16
No. ob Observations	1078	1078	1078

*t-statistics are in parentheses.

is very low.

6.7. Elasticities and Marginal Rates of Substitution

Our empirical estimations seem to support our a priori expecta-
tions of a joint decision-making process in the firm's logistics
management strategy. The models have estimated more satisfactorily
for SAMPLE1 than for other samples. Thus, we will use the restricted
model of SAMPLE1 as the example to discuss the implied elasticities
and marginal rates of substitution.

The joint choice model of mode, shipment size and origin estimated
for SAMPLE1 is shown in Figure 40. The marginal rates of substitu-
tion between logistics cost components can be calculated. Figure 41
shows the substitution rates for the cost items which have been
significantly estimated in the model. The marginal rates of sub-
stitution between RATE and EMRG1 and EMRG2 are 0.299 and 0.063 respec-
tively, implying that for a hundred pounds of emergency shipment,
the time value will be 29.9 and 6.3 dollars per day respectively for
raw materials and final or capital goods. The interest rate associa-
ted with goods in transit is 165.6% which is about five times higher
than the interest rate for goods in storage: 34.6%. The rate of
substitution between capital carrying cost (CCC) and the value of
emergency shipments are estimated to be 1.16 for raw materials and
5.519 for final or capital goods.

Figure 40.   The Joint Choice Model of Mode, Shipment Size and Origin
        (SAMPLE1)

Variable	Coefficient
P	-0.352
OC	-1.135
RATE	-1.658
EMRG1	-0.495
EMRG2	-0.104
CCCIT	-2.746
CCC	-0.574
STC	-0.479
DIST	0.662
RAIL	0.215
CT	2.231
AIR	0.037
MKT	1.149
RC	0.523

	RATE	EMRG1	EMRG2	CCCIT	CCC
RATE	1	0.299	0.063	1.656	.346
EMRG1		1	--	--	1.16
EMRG2			1	--	5.519
CCCIT				1	.209
CCC					1

Figure 41. Marginal Rate of Substitution

The sensitivity of the shipper to changes in the choice of mode, shipment size or origin resulting from the changes in transport level-of-service can be expressed in terms of elasticities. The direct elasticities of demand with respect to a change in a level-of-service variable can be derived as:

$$E^{p(imq)}_{LOS_{imq,\ell}} = [1 - p(imq)] \frac{\partial w_{imq}}{\partial LOS_{imq,\ell}} \cdot LOS_{imq,\ell} \qquad (6.8)$$

$$E^{p(mq)}_{LOS_{mq,\ell}} = \sum_i E^{p(imq)}_{LOS_{imq,\ell}} \cdot \frac{p(imq)}{p(mq)} \qquad (6.9)$$

$$E^{p(m)}_{LOS_{m,\ell}} = \sum_i \sum_q E^{p(imq)}_{LOS_{imq,\ell}} \cdot \frac{p(imq)}{p(m)} \qquad (6.10)$$

where   LOS = level-of-service variable

$E^{p(imq)}_{LOS_{imq,\ell}}$ = the direct elasticity of demand for mode m, shipment size q and origin i with respect to a change in the ℓth level-of-service variable

$E^{p(mq)}_{LOS_{mq,\ell}}$ = the marginal direct elasticity of demand for mode m and shipment size q with respect to a change in the ℓth level-of-service variable

$E^{p(m)}_{LOS_{m,\ell}}$ = the marginal direct elasticity of demand for mode m with respect to a change in the ℓth level-of-service variable.

Similarly, the cross elasticities are expressed as:

$$E^{p(imq)}_{LOS_{i'm'q'},\ell} = -p(i'm'q') \frac{\partial w_{i'm'q'}}{\partial LOS_{i'm'q',\ell}} \cdot LOS_{i'm'q',\ell} \quad (6.11)$$

$$E^{p(mq)}_{LOS_{m'q'},\ell} = \sum_i E^{p(imq)}_{LOS_{i'm'q'},\ell} \cdot \frac{p(imq)}{p(mq)} \quad (6.12)$$

$$E^{p(m)}_{LOS_{m'},\ell} = \sum_i \sum_q E^{p(imq)}_{LOS_{i'm'q'},\ell} \cdot \frac{p(imq)}{p(m)} \quad (6.13)$$

There are four level-of-service variables in the final estimation of the model, namely tariff charges, transit time, wait time, and percentage lost and damaged (LDP). To calculate elasticities of demand with respect to each of these variables, the following proto-typical firms are considered:

FIRM1 - using canned fruits at an annual use of 5,000 pounds;

FIRM2 - using data processing equipment at an annual use of 800 pounds;

FIRM3 - using fabricated metals at an annual use of 1,000,000 pounds.

The marginal direct and cross elasticities of demand for each mode with respect to each level-of-service variable are calculated and given in Figures 42 and 43. It can be shown that the elasticities are highly related to the value of the commodity and annual use rate of the receiving firm.

Figure 42. Direct Elasticities of Demand for Mode of Transport

Level-of-Service Variable	Mode	Elasticities		
		FIRM1	FIRM2	FIRM3
RATE	RAIL	-0.22	-0.29	-0.10
	CT	-0.12	-0.08	-0.10
	PT	-1.42	-4.06	-0.19
	AIR	-0.48	-0.72	-0.30
TT	RAIL	-0.04	-1.66	-0.19
	CT	-0.02	-0.27	-0.10
	PT	-0.02	-0.71	-0.07
	AIR	--	-0.08	-0.01
WT	RAIL	--	-0.08	-0.01
	CT	--	-0.01	--
	PT	--	-0.02	--
	AIR	--	--	--
LDP	RAIL	--	-0.10	--
	CT	--	-0.05	--
	PT	--	-0.16	--
	AIR	--	-0.25	--

*
 -- denotes insignificance.

Figure 43. Cross Elasticities of Demand for Mode of Transport

Level-of-Service Variable	Mode	Elasticities		
		FIRM1	FIRM2	FIRM3
RATE	RAIL	0.01	0.02	--
	CT	0.07	0.26	0.01
	PT	0.01	0.01	--
	AIR	0.01	0.12	--
TT	RAIL	--	0.11	--
	CT	0.01	0.91	0.01
	PT	--	--	--
	AIR	--	0.01	--
WT	RAIL	--	0.01	--
	CT	--	0.05	--
	PT	--	--	--
	AIR	--	--	--
LDP	RAIL	--	0.01	--
	CT	--	0.15	--
	PT	--	--	--
	AIR	--	0.04	--

*
 -- denotes insignificance.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

7.1 Summary of Fingings

This study represents a first attempt to develop a freight
demand model which includes the entire set of relevant short-run
decisions open to a firm in its logistics management process.
The disaggregate model of mode, shipment size and origin choices
is developed at the level of the individual firm. This approach
has allowed an explicit consideration of the tradeoffs the firm can
make in response to a short-run change in transport level-of-service.
The major assumption of this study is that the substitution between
transportation and other factors of production such as labor, capital,
etc. is relatively inelastic when compared to the substitutions
which can take place within the transportation sector itself.
This allows us to develop an hierarchical choice structure for viewing
the transportation-related decision-making open to a firm. The
criterion, "no substitution between transportation and other produc-
tion factors," can be used to categorize a transportation problem
to be either a long-run or short-run one. The short-run problem
is assumed to conform to the assumption that transportation level-
of-service has no significant effect on factor substitution. In the
short run, the demand for the various input materials is fixed.
The firm thus exercises logistics strategies to minimize its pur-
chase and logistics costs for these inputs. A logistics strategy

is characterized by its associated choice of mode, shipment size
and point of supply. These serve as the theoretical basis for the
development of the disaggregate model described in the previous
chapter.

In terms of the proposed choice hierarchy, most previous freight
demand models can also be categorized as short-run demand models.
However, none of them has considered the full set of decisions
available to a firm over the short run, namely the mode, the shipment
size and the origin. Inventory theory, however, suggests that these
three choices are highly interdependent. The causality runs in both
directions between any two of these logistics choices. As a con-
sequence there is a joint decision-making process in the logistics
strategies. This implies that conventional mode choice models give
only the conditional probabilities of choosing a mode under the already
chosen shipment size and origin. In response to an increase in
freight rates, the shipper will consider not only shifting to other
modes with a lower rate but also adjusting his inventory system to
a higher shipment size or even a chosen point of supply. Thus, a
conditional mode choice model will over-predict the change in mode
share if these effects are not considered.

Based on the logistics decision process hypothesized here,
a framework to derive a set of disaggregate choice models invol-
ving the full set of logistics choices has been developed by this
study. A general specification of the cost function has been pre-
sented.

A common limitation in the modelling of freight demand is that proper data do not exist. The available data also suffer from poor quality. An empirical question is thus the extent to which current data sources can be used to implement any proposed freight demand model. A data set has been assembled for this purpose from existing sources. This has required a careful synthesis of existing data from several possible sources and the squeezing of all useful information from them. A disaggregate data base is thus established which allows us to test our random cost model of freight demand. The results from the empirical estimations are very encouraging. The information in the public use tape of the Census of Transportation is the primary source of the data used to calibrate the disaggregate freight demand model described here. In preparing the data base, assumptions involving the appropriate values for commodity attributes, receiver attributes, and market attributes as well as level-of-service attributes have been made. This suggests that the model would have performed even better if more accurate data had been available at the disaggregate level.

The estimated models indicate that transport changes and capital carrying costs are the two key factors in determining the choice of mode and shipment size. Emergency shipments also play an important role in explaining mode choice. This is especially true for raw materials used as input to production. The willingness on the part of the firm to pay for emergency shipments is estimated to be

much lower for final and capital goods than for raw materials.
The interest rate associated with capital carrying cost in transit is
estimated to be higher than the market interest rate that one would
normally expect. This indicates that the shipper in the real world
seems to over-emphasize the importance of transit time. This could
also be due to the omission of reliability in the model specification
used in the final estimations. Storage cost and order cost are both
estimated to be "not significant" in the choice of mode and shipment
size, mainly due to the poor quality of the data used. However,
they were expected a priori to have less influence in the determination
of logistics choices.

The regional price data used in the study are too aggregate
over both commodity and location; therefore, their estimated coef-
ficients have a low significance. Our estimations however do seem
to provide evidence that a central place orientation does exist in
the spatial economy of the United States.

The inventory theoretic approach to freight demand models
logistics choices gives a certain assumed period of demand of the
input to a more or less continuous production process. The annual
use rate is one of the key variables required. This piece of infor-
mation is not available in the shipment data of the Census of
Transportation. We have established some rather sophisticated
procedures for developing a firm's annual use rate of a given input.
These procedures have proven to be worthwhile. These procedures also
have the potential for generating data which would be useful in

a variety of urban and regional, transportation and economic activity studies. Our empirical estimations also indicate that better freight models will result as true use-rate information becomes available.

7.2. Recommendations for Further Research

At the present time, the data problem is the most imposing constraint on the further development of the short-run freight demand model described in this study. The most promising existing data source for the estimation of a general, flexible demand model at the disaggregate level is the Census of Transportation. However, this data set does not contain the name of the receiver. This makes it very difficult to derive the receiver attributes. Origin, destination and commodity coding are also too aggregate in their reporting. Even more important, mode of transport is reported by type of vehicle instead of type of service offering; also, the survey covers only manufactured goods. Clearly, a new set of Census procedures is desirable if these deficiencies are to be overcome.

Our empirical estimation as performed here can be extended to include barge. Barge is of course more of a competing mode to rail than to air and private truck. Research should also be performed which investigates the role of truck ownership in the transportation-related logistics management process. It is also useful to segment the market by commodity and to study the impact on the model's coefficients of different types of segmentation.

Use of disaggregate demand models to evaluate transport policies requires that the disaggregate predictions be aggregated to the appropriate level for policy making. This requires development of formal aggregation procedures. Aggregation prediction using the model developed in this study involves the employment of a method by which the entire use rate distribution for all commodities in all market areas can be developed as well as a method for aggregating the predicted individual choices. The procedures for deriving a firm's annual use is of course the first requirement. Research should be undertaken to develop an efficient aggregation algorithm for carrying out the second requirement.

Methodologically, there is still a need for the development of a modelling technique which is capable of treating shipment size as a continuous rather than a discrete choice. This model should be able to handle more than two alternatives efficiently. Also, it should use the full a priori knowledge of the logistics cost function as developed in Chapter 4.

Finally, since the freight demand model using logistics choice theory as its basis must be categorized as a short-run freight demand model, research should be initiated to address the transportation-related long-run choices. As outlined in Chapter 2, there appear to be possibilities to model the firm's production and location choices as the higher level decisions in the full set of transport-related decision-making processes available to a firm.

APPENDIX A

CENSUS OF TRANSPORTATION

A.1 Mode Definitions

1. Rail - including combinations such as piggyback in which the
    major distance was by rail (railway express is included
    under other).

2. Motor Carrier - including combinations in which the major dis-
    tance was by motor carrier. (This includes all highway
    transport, except by private truck.)

3. Private Truck - trucks operated by the shipper or the customer.

4. Air - including air freight and air express and combinations
    in which the major distance was by air.

5. Water - including combinations in which the major distance
    was by water.

6. Other - including Railway Express, United Parcel Service, bus,
    freight when major means of transport (such as rail and
    air are not known, messenger service, etc.

A.2. Weight Block   (1bs)

1.   under 50

2.   50-99

3.   100-199

4.   200-499

5.   500-999

6.   1,000-2,999

7.   3,000-4,999

8.   5,000-9,000

9.   10,000-19,999

10.   20,000-29,999

11.   30,000-39,999

12.   40,000-49,999

13.   50,000-59,999

14.   60,000-79,999

15.   80,000-89,999

16.   90,000-99,999

17.   100,000-119,999

18.   120,000-149,999

19.   150,000-199,999

20.   over 200,000

A.3. Production Areas and Market Areas

Code*	SMSA Included
1	Boston, Worcester, Providence, Warwick-Pawtucket, Brockton, Larence-Haverhill, Lowell
2	Hartford, New Britain, Meriden, Waterbury, New Haven, Bridgeport, Norwalk, Stamford, Springfield-Chicopee-Holyoke
3	New York
4	Newark, Jersey City, Paterson-Clifton-Passaic, Middlesex County, Somerset County
5	Philadelphia, Wilmington, Trenton
6	Baltimore
7	Allentown-Bethlehem-Easton, Reading
8	Harrisburg, Lancaster, York
9	Syracuse, Utica-Rome, Albany-Schnectady-Troy
10	Buffalo, Rochester
11	Cleveland, Akron, Canton, Loraine-Elyris, Youngstown-Warren, Erie
12	Pittsburgh, Steubenville-Weirton, Wheeling
13	Detroit, Flint, Toledo, Ann Arbor
14	Cincinnati, Dayton, Hamilton-Middletown, Springfield
15	Chicago, Gary-Hammond-East Chicago
16	Milwaukee, Kenosha, Racine
17	Minneapolis-St. Paul
18	St. Louis
19	Atlanta
20	Dallas, Fort Worth

Code[*]	SMSA Included
21	Houston, Beaumont-Port Arthur-Orange, Galveston-Texas City
22	Denver
23	Seattle-Everett, Tacoma
24	SanFrancisco-Oakland, Vallego-Napa, San Jose
25	Los Angeles-Long Beach, Anaheim-Santa Ana-Garden Grove, Riverside-San Bernardino-Ontario
26	Indianapolis, Muncie, Anderson
27	Kansas City, St. Joseph, Topeka
31	Scranton, Wilkes-Barre-Hazleton, Binghamton
32	Washington
33	Newport News-Hampton, Norfolk-Portsmouth
34	Columbus (Ohio)
35	Grand Rapids, Muskegon-Muskegon Heights
37	Louisville
38	Nashville
39	Memphis
40	Augusta, Columbia
41	Ft. Lauderdale-Hollywood, Miami, West Palm Beach
42	Birmingham, Tuscaloosa
43	Tampa-St. Petersburg
44	Mobile, Pensacola
45	New Orleans
46	Omaha, Lincoln
48	Oklahoma City, Tulsa

Code[*]	SMSA Included
49	San Antonio, Austin
50	Salt Lake, Provo-Orem, Ogden
51	Phoenix, Tucson
52	Portland, (Oregon)
53	Sacramento, Stockton
54	Fresno, Bakersfield
55	San Diego

[*]Regions 1 through 27 are Production Areas, Regions 1 through 55 are Market Areas.

APPENDIX B

IND CODE FOR USE RATE DISTRIBUTION

1   Agricultural Services, Forestry & Fisheries

2   Construction

3   Ordnance & Accessories

4   Food & Kindred Products

5   Tobacco Manufactures

6   Textile Mills

7   Textile Products

8   Fabric Mills

9   Misc. Fabricated Textile Products

10 Veneer & Plywood

11 Wood Products

12 Household Furniture

13 Other Furniture

14 Pulp & Paper Products

15 Paperboard Containers & Boxes

16 Printing & Publishing

17 Agricultural Chemicals

18 Plastic Materials & Synthetics

19 Soap, Cleaners & Toilet Goods

20 Paints & Allied Products

21 Petroleum & Coal Products

22   Rubber Products

23   Leather & Leather Products

24   Stone & Stone Products

25   Glass and Glass Products

26   Cement

27   Basic Steel Products

28   Primary Nonferrous Metals

29   Metal Cans

30   Plumbing & Heating Equipment

31   Screen Machine Products

32   Fabricated Wire Products

33   Steam Engines & Turbines

34   Farm Machinery

35   Mining Machinery

36   Machine Tools

37   Food Products Machinery

38   Pumps & Compressors

39   Other Industrial Machinery

40   Electronic Components & Equipment

41   Refrigerators & Freezers

42   Electric Measuring Instruments

43   Household Laundry Equipment

44   Electric Lamps

45   Telephone & Telegraph Apparatus

46   Semiconductors

47   Other Electrical Equipment & Supplies

48   Motor Vehicles & Equipment

49   Aircraft & Parts

50   Ship & Boat Building & Repairing

51   Instruments & Supplies

52   Photographic Equipment & Supplies

53   Musical Instruments & Parts

54   Agriculture

55   Mining

56   Retail

57   Wholesale

58   Transportation Services

59   Public Utilities

60   Services Except Auto Services

61   Auto Services

62   Governmental Enterprises

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