THE EFFECT OF COUPLING IN AUTOMOTIVE ACTIVE SUSPENSIONS

by

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In this study, a 7 degree-of-freedom vehicle model is used to analyze how the coupling, which is present between the various vehicle modes, affects the ride quality and roadholding performance when sprung mass absolute body velocity feedback is used in the control strategy. This full-vehicle model makes it possible to study the response in one mode when the vehicle is subjected to an input in another mode. It also allows the study of the tire normal load distribution between the four corners.

Beside providing absolute body velocity feedback, the active system is also used to make the suspension fully interconnected. This allows the independent specification of all the suspension rates (stiffnesses, absolute and relative damping rates), as well as the coupling rates.

Frequency response results obtained using existing vehicle data show that when absolute damping is used, decoupling the heave and pitch sprung mass modes both improves body isolation, and reduces fore-aft tire load transfer as the vehicle traverses an uneven road. This indicates that "flat-ride" turning may not be desirable when absolute body damping is used. Absolute body roll damping, while improving body isolation, increases the effect of road disturbances on lateral load transfer distribution. It is also found that for a road warp input, increased relative damping is superior to absolute body damping.

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Finally, this page would not be complete without a posthumous mention of the late "lemon" (yellow) 914 which got me interested in vehicle dynamics in the first place. It lived by Trailing-Throttle Oversteer, and died by it.
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CHAPTER 1
INTRODUCTION

1.1 BACKGROUND

Of all the numerous inventions since the industrial revolution, the automobile or motor vehicle has undoubtedly had one of the greater impacts on society in general. Recent studies have indicated that despite the energy shortages of the last decade and the accelerating reduction in the world oil reserves, and despite the ever-tightening emission regulations against both noise and atmospheric pollution, the automobile is not likely to be replaced soon by alternate modes of transportation. In fact, it is expected that the automobile, in one form or another, will remain the main mode of personal transportation well into the next century.

Over the one hundred years of its existence, the motor vehicle, as an engineering system, has been gaining in sophistication and complexity at an exponential pace. This has been due to various reasons, including advances in technical knowledge and analytical capability on the part of the designer, an increased demand from the consumer for a better product, and fierce competition among the manufacturers to satisfy (or create) that demand. Needless to say, the microprocessor has been playing a major role in making the car a more sophisticated system. This trend is expected
to continue, especially in view of the fierce competition seen today in the automobile market, and the resulting need to provide a constantly improved product. Because the suspension, as a major subsystem of the automobile, directly influences many of its qualities and performance aspects, both subjective and objective, (ride quality, noise and vibration, handling, cornering, active safety, etc...), it has received increasing attention from the automotive engineer, and will continue to do so.

1.2 THE AUTOMOBILE SUSPENSION

The suspension is "the system of devices (such as springs) supporting the upper part of a vehicle on the axles" (Webster's Seventh New Collegiate Dictionary). From that definition it is clear that the term "suspension" is a misnomer, and that "support" would be a better one. The term has been carried over from the days of the horse-drawn carriage, when the body was actually suspended from flexible leaf springs which rose over their attachment points to be body.

The "upper part of a vehicle" in the above definition refers to the body or sprung mass; this consists of the structure housing the passenger and cargo compartments, as well as their contents, the engine and gearbox, and other mechanical parts. The "axles" refer to the unsprung masses of the vehicle; these consist of the tires and wheels, the
wheel hubs, the brake discs and calipers in the case of outboard brakes, and part of the driveshafts and of the suspension and steering linkages. Depending on the particular suspension design, there may be a solid axle, or there may only be an imaginary line as in the case of an independent suspension system. In the case of a live rear axle, the rear unsprung mass includes the differential unit and its housing. Strictly speaking, the unsprung masses are not unsprung since the tires have finite stiffnesses. Figure 1.1 schematically shows a vehicle and its suspension system.

The suspension system has to perform two distinct functions. The first is to support and isolate the sprung mass (mainly the occupants and the cargo) from any external disturbances, such as road irregularities and wind gusts. The second is to control the motion of the unsprung masses so as to keep the tires in firm contact with the road despite any disturbing inputs.

1.2.1 Ride Quality

The effectiveness of the suspension system in performing the first function, namely isolating the body, determines the ride quality and comfort of the vehicle. Although vibrations in the vehicle can be caused by the various sub-systems of the vehicle, for instance the drivetrain, or by aerodynamic disturbances, those induced by road surface
irregularities account for most of the discomfort perceived by the vehicle occupants. As with anything that depends on human perception, it is impossible to objectively determine precise measures for ride quality and comfort limits. This is caused by the variations in sensitivity from one individual to another, and by the different methods used by the various investigators who have undertaken such studies. Nevertheless, there exists a large body of knowledge dealing with the response of humans to vibration.

Of the many ride comfort criteria which have been proposed, the best known are the Janeway comfort criteria published by the Society of Automotive Engineers [1], and the International Standard ISO 2631, 1974 [2], which deals with whole body vibration. Janeway divided the sinusoidal frequency range from 1 to 60 Hz into three ranges, and suggested acceptable vertical vibration amplitude limits for each range. From 1 to 6 Hz, maximum values for the jerk (or rate of change of acceleration) amplitude are given; from 6 to 20 Hz, peak values of vertical velocity amplitude are given; and from 20 to 60 Hz, limits are given for the peak vertical velocity amplitude. These limits are shown in Figure 1.2.

The ISO standards deal only with RMS acceleration resulting from sinusoidal vibration over the 1 to 80 Hz frequency range; however, they not only consider vertical vibration, but also longitudinal and lateral ones. They provide guidelines for the exposure times which would result
in various levels of discomfort as a function of frequency and acceleration amplitude. These guidelines are the "exposure limits" beyond which safety would be impaired, the "fatigue or decreased proficiency boundaries", and the "reduced comfort" boundaries. Figure 1.3 shows the limits for whole-body vibration for fatigue or decreased proficiency in the vertical direction.

Although the ISO standards are stricter for lateral and longitudinal accelerations, experiments conducted by the Motor Industry Research Association (MIRA) of England in 1960 and 1964 found very good correlation between ride comfort as perceived by the occupants, and vertical acceleration. More specifically, in the 7-50 Hz range, the vertical acceleration of the floor correlated with the comfort ratings given by the test subject, whereas in the 0.75 - 6 Hz the vertical acceleration of the seat showed better correlation. Pitch and roll were not found to play any significant role in the ride quality. For these reasons, vertical acceleration of the passenger is generally considered to be a good measure of ride comfort.

It should be noted that although the Janeway criteria and the ISO standards only consider vibrations above 1 Hz, the range from 0.5 to 1 Hz is also important, as it covers the frequencies at which motion sickness is felt when large amplitudes are present.

Finally, it should be remembered that ride quality is
very subjective for a reason not mentioned so far, which is the fact that even if two persons are equally sensitive to vibrations, the two might have different notions of what constitutes good ride. Where one might prefer a softly sprung well-isolated luxury car, the other might find a harder ride, which allows more road inputs to filter through, more desirable, as it provides a greater feeling of control.

1.2.2 Handling

Vehicle handling is an all-encompassing term which reflects objective measures of the vehicle performance, as well as the subjective "feel" it conveys to the driver during maneuvers. The maneuvers in question may involve acceleration or braking, turning, or any combination thereof. The cornering ability of the vehicle is one aspect of handling which can be measured objectively, using a circular skidpad. There are other tests used to measure the road-holding capacity and steering response of the vehicle, for instance the slalom course, the J-turn, and others, but these rely more on the skill and sensitivity of the driver. Other more subjective aspects of handling include the feedback received by the driver through the steering wheel (on-center feel, cornering force feedback), the brake pedal feel, and more. Because of all these parameters, it is not possible to trace the handling qualities of a vehicle to one
single subsystem. Moreover, it is not possible to quantify handling.

For the purpose of this study, the aspect of handling that is of interest is roadholding, which can be considered the ability of the suspension system to successfully maintain the tires in firm contact with the road.

The need to maintain the contact force at the tire/road interface stems from the fact that the forces needed to guide and control the vehicle are generated at that interface. These forces include the tractive effort needed for braking and accelerating, and the lateral tire forces needed for maintaining a given course despite external disturbances, or for changing direction. The capacity of a pneumatic tire to generate a longitudinal, lateral, or combined force is a nonlinear function of the normal load acting on it. Figure 1.4 presents data corresponding to a typical pneumatic tire. It shows families of curves, each family corresponding to a different tire normal load $F_z$. In each family, the x-axis corresponds to the longitudinal force or tractive effort $F_x$, while the y-axis corresponds to the lateral or cornering force $F_y$. Each curve, which is a quarter of what is known as the tire ellipse, corresponds to a constant angle of sideslip. The angle of sideslip is defined as that between the wheel plane and the actual direction of travel of the wheel. For a given slip angle, the points on the curve represent the maximum combination of
tractive effort and lateral force that the tire is capable of generating. It should be noted that as the normal load increases, both forces increase for a given slip angle. This trend levels off beyond a certain point. All the above applies when there is no gross relative motion between the tire and the road. At higher levels of slip, either longitudinal or lateral (sideslip), for instance when the tire is locked, its force-generating potential is considerably reduced.

From the above, it is clear that a good suspension system should attempt to reduce as much as possible any variation in the tire contact-force that results from road irregularities. That would make the vehicle handle better over rough roads, other things (such as the amount of toe and camber variation with suspension compression) being invariant.

However, it is not enough for the suspension to maintain the total tire contact-force at the four corners of the vehicle constant. It is more important to maintain the desired distribution of the contact-forces between the 4 tires, despite any disturbing input. It is this distribution that will determine the basic handling characteristics of a vehicle, other parameters being equal (weight distribution, relative tire sizes, inflation pressures, suspension geometry, wheel alignment, and many others) and that will
also affect the stability and directional control of the vehicle.

Before further explanation, a few terms will be defined. The terms understeer and oversteer have been the subject of several definitions but a simple one will be used here. Understeer refers to the condition during steady-state cornering where the front tires are experiencing larger sideslip angles than the rear ones. Oversteer refers to the opposite situation [2]. In general, oversteer is considered undesirable, as it makes the vehicle unstable above a certain critical speed, when subjected to lateral disturbances [2,4].

Whether a vehicle will understeer or oversteer during steady-state cornering is mainly determined by the relationship between the lateral transfer of tire normal load at the front axle and that at the rear axle. This is due to the fact that for a given angle of sideslip, the lateral force generated by the tire is a highly nonlinear function of the tire normal load, as shown in Figure 1.5. The total lateral force generated by the axle is the sum of the two forces generated by each tire, but because of the decreasing slope of the curves in Figure 1.5, two equally loaded tires will generate more lateral force than two tires with the same total load divided unequally between them. Therefore, if the tire load transfer is greater at the front than at the rear in an otherwise symmetric (front to rear)
vehicle, a larger angle of sideslip will be needed at the front to generate the same lateral force as the rear, and the vehicle will understeer. The vehicle will oversteer if the opposite is true. This is explained in more details in reference [5], from which Figure 1.5 is taken.

During a combined turning and braking, or turning and accelerating maneuver, the understeer or oversteer characteristics of the vehicle are affected by the relative distribution of braking forces between the front and rear axles, as well as by the front-to-rear tire load transfer.

Generally, the suspension, steering, and braking systems are designed to provide specific handling characteristics when traversing smooth roads. Ideally, these characteristics should be maintained over rough roads. The level to which the handling characteristics are unaffected by road roughness is a measure of the success of a given suspension design. For instance a very stiffly sprung sports car that handles well over smooth roads, but is so unsettled by bumps as to become difficult to manage cannot be considered successful.

1.3 THE PASSIVE SUSPENSION

One of the simpler models used to study automotive suspensions, known as the De Carbon model, is shown in Figure 1.6. It is a linear, two degree-of-freedom (d.o.f.), lumped element model, with motion in the vertical or heave direc-
tion only. It consists of two lumped masses \( M \) and \( m \), the first representing the sprung mass or body, and the second representing the unsprung or axle masses (the nomenclature used follows standard SAE nomenclature [6] whenever possible). The tire is modeled as a simple spring with constant \( K_T \) equal to the tire rate. Tire damping, due to hysteresis effects in the tire itself, is neglected. The supporting media are the spring \( K \) and the damper \( D \). The road disturbances are modeled as a displacement input to the tire.

This simple ride model can be used to gain some qualitative feel as to how a suspension works, and what the compromises involved are.

The data given in Figure 1.6 are typical of a high-performance luxury GT car. It can be seen that the sprung mass is an order of magnitude larger than the unsprung one, whereas the suspension spring rate is smaller than the tire rate by more than an order of magnitude. Because of that, the frequency of the mode of vibration associated with motion of the sprung mass is separated by more than a decade from the frequency of the mode consisting mainly of unsprung mass vibration. The first is about 1 Hz whereas the second is above 20 Hz. The suspension system thus acts as a low-pass filter, as far as the sprung mass is concerned. Figure 1.7 is a frequency response plot of the sprung mass motion amplitude per unit input amplitude, over the frequency range from 0.1 Hz to 100 Hz, for the system shown in Figure 1.6.
Two curves are shown, corresponding to sprung mass damping ratios of 0.25 and 0.5 respectively. It can be seen that for input frequencies below that of the body resonance, the body follows the input. The response peaks at the body resonant frequency, after which it decreases steadily, except for another local peak at the frequency corresponding to the unsprung mass resonance. The body is effectively isolated from high frequency inputs.

One of the compromises facing the suspension designer is selecting the optimum damping. From Figure 1.7 it can be seen that using the higher damping ratio reduces the motion transmissibility at the low frequency unsprung mass resonance, and would thus provide better passenger isolation around that frequency. The trade-off however occurs at higher frequencies, where the increased damping results in worsening isolation up to the high frequency resonant peak which is not affected much. Beyond that peak, the roll-off rate is decreased also. This is due to the fact that the higher frequencies result in an ever-increasing relative velocity across the damper which consequently "hardens". As mentioned above, the second peak is not reduced by the increased damping, some tire damping would be needed to achieve such an improvement.

The increased damping is also beneficial as far as tire normal load is concerned. Figure 1.8 shows the variation in that load from the static equilibrium value, as a function
of input frequency, for a unit input. The increased damping results in smaller variation at the two resonant peaks, as well as more uniform variation between these two frequencies.

In order to circumvent the low-frequency versus high-frequency isolation, as well as other compromises of the passive suspension, the active suspension system was proposed.

1.4 THE ACTIVE SUSPENSION

In the previous section, we saw one aspect of the limitations of passive suspensions in isolating the body from road disturbances. That limitation was due to the reliance on a simple damper in parallel with the spring as supporting media. Since the 1930's, suspension systems which would eliminate the shortcomings of that arrangement have been considered. Significant work on these active suspension systems has been carried out since the 1950's, following advances in the development of high-performance servo-mechanisms using feedback control.

In their 1975 review of the state-of-the-art of active suspensions for ground vehicles [7], Hedrick and Wormley define an active suspension as one that may rely on externally-supplied power for its operation. In a typical active suspension (Fig. 1.9), the spring and damper are
replaced by an active force actuator, which could be either hydraulic, pneumatic, electric, or any combination.

Unlike the passive suspension which, by virtue of its spring and dampers can only temporarily store, return, or dissipate the energy input due to road disturbances, the actuator force \( F_A \) in the active suspension does not depend upon the input time-history. The energy input to the active suspension can be modulated according to any arbitrary control law. This frees the suspension from the limitations imposed by the constitutive relations of the spring and the damper. The suspension force is no longer function of the relative displacement and the relative velocity only, but it can be function of any variable, either local or remotely measured. Also, these measured variables can be arbitrarily processed before being used to generate the suspension force. The active suspension can thus be made to adapt to varying inputs, road conditions, or just driver taste.

Because of its need for an external power source, the active suspension suffers from disadvantages relating to cost, complexity, and safety in the case of a power failure. Moreover, as with any complex system, maintenance becomes an important requirement, especially in view of the rough operating environment involved.

The heart of an active suspension system is the control strategy used to generate the suspension force. Many control system design and optimization techniques have been
applied to that problem. The performance parameters used reflect factors such as body isolation, suspension stroke requirement (rattlespace), actuator force or power requirements. Again, the state-of-the-art review by Hedrick and Wormley [7], or a later one by Goodall and Kortüm [8], cover the various developments of active suspension systems.

One of the first applications of active suspension in the automotive field is the system designed and built by the Westinghouse Research Laboratories in the late 1950's [9]. The control strategy used is based on the sprung mass acceleration, mostly in the vertical direction, but also in the horizontal plane. The system relied on acceleration feedback to increase the effective sprung-mass inertia several-fold, thus reducing the sprung-mass resonance frequency and improving ride quality. Additional passive damping is used to control the increased displacement at the reduced frequency. The controllers, installed at each corner of the sprung-mass, consist of a spring-damper mounted masses with feedback compensation, and rather complicated fluidic circuits (Fig. 1.10). These controllers can be tilted in the longitudinal or frontal plane to provide inertial stabilization against pitch and roll, or they can be turned around to cause banking of the sprung mass during cornering.
Thompson [10] uses a pole-zero synthesis technique to design an active suspension system, and considers two different arrangements, both based on a 2 d.o.f., heave-only model. One arrangement consists of the actuator mounted parallel to the spring, whereas in the other both are mounted in series. A sensor arrangement which allows banking control of the body is also presented.

A system which may appear similar to the above-mentioned Westinghouse design, but which is actually quite different, is that developed by Automotive Products of England, and tested in a Rover automobile [11]. This system is a high-gain self-levelling system which counteracts body motions due to dynamic body forces generated during cornering, and acceleration or braking. It does not respond to road disturbance inputs, which are left for the passive component of the suspension to deal with.

Another control strategy often used is full state variable feedback. It has been used both in the context of pole placement techniques [12,13,14], as well as optimal control [15,16]. Hullender et al. [15] show that for a one d.o.f. suspension system subjected to a random input, the optimal suspension consists of a spring supporting a mass, and of an absolute damper, i.e. an active force actuator responding to absolute body velocity feedback (Fig. 1.11). That suspension is optimal with respect to a quadratic performance index penalizing suspension stroke as well as
body acceleration. Hedrick [16] uses such an arrangement, combined with body acceleration feedback, to improve the vertical and lateral ride quality, as well as the lateral stability of railway cars.

Absolute body velocity feedback, or absolute damping has received increased attention lately [14,17,18], and is the control strategy of interest in the present work. One of the attractions of absolute damping is its relative ease of implementation, as only one sensor is needed to measure body acceleration, and a single integration of that signal yields the absolute body velocity. Another advantage lies in the possibility of extending the results obtained with an active absolute damper to a semi-active one, with little loss of system performance, using a modulable passive damper [14]. A semi-active system is one which does not require external power input, but in which the passive forces are modulated according to some control strategy.

In Ref. [17], the concept of absolute velocity feedback is extended to the unsprung mass in a 2 d.o.f. heave-only model in an attempt to control wheel hop. It is shown that even an active suspension system still presents a trade-off as far as providing body isolation while at the same time controlling the motion of the unsprung masses. This conclusion, similar to the one Thompson reaches in [10], is caused by a limitation due to the structural layout of the
system, which results in the active element applying equal and opposite forces to the sprung and unsprung masses. Karnopp [18] reaches similar conclusions, but also develops a simple criterion to help determine whether active damping will bring any significant performance improvements in a given application.

1.5 OBJECTIVES OF THE STUDY

Most studies reviewed in the previous section utilize Heave-only, 1 or 2 d.o.f. models to design or to analyze the performance of active suspensions. Because of their relative simplicity, these models are very useful for understanding active suspensions and for designing control strategies, as demonstrated by the vast body of knowledge that has been built around them. In designing a full suspension system, one could use a one-dimensional two-mass model for each of the vehicle modes, namely heave, pitch, roll and warp. Such an approach however would not reflect the fact that there is coupling between those modes. Static coupling between heave and pitch is usually unavoidable because it may be necessary to fit springs of unequal stiffness at the front and at the rear, even in a vehicle with equal front-to-rear weight distribution. This may be done for various reasons:

- vehicles without load-levelling devices may require stiffer springs under the cargo compartment,
order to maintain a reasonable static deflection under full load;

- the designer may choose specific locations for the pitch and bounce centers of the sprung mass, in order to obtain a given ride characteristic, a "flat ride" at a given speed for instance [2,19];

- the choice may be made in order to obtain the desired handling characteristics. Stiffer springs at the rear could be used to alter the front-to-rear suspension roll couple distribution and cause more tire load transfer at the rear, resulting in less understeer (or more oversteer) [5,20].

The suspension roll and warp modes are usually coupled because of the desirability of having unequal roll couples at the front and at the rear.

As the above indicates, coupling between the various modes is a very important tool used by the designer to confer on the vehicle specific handling and ride characteristics, and therefore cannot be neglected.

Although both Thompson [10], and Hanna and Osbon [9] consider a full vehicle with banking control, they only provide an analysis of the steady-state banking moment needed. The effect that active suspension control has on the handling characteristics as the vehicle traverses an uneven road is not discussed.
Margolis [14] does consider a vehicle with coupled heave and pitch motions, but the model he uses does not include unsprung masses, and therefore cannot predict the effect of the control strategy on tire normal loads, and thus on handling.

From the point of view of ride comfort, Margolis found that active damping results in decreased body isolation in certain cases. Specifically, the body pitch response to a heave input, and vice versa, increased compared to the passive system. This effect was more marked in the case of full state variable feedback.

The objective of the present study is to analyze active suspension systems using a model which reflects the coupling present between the various degrees of freedom in a real vehicle. Such a model should also allow the effect of active control on vehicle handling to be assessed.

In a first step, a model including 7 d.o.f. is developed. In that model the remote sensing capability of the active suspension is used to provide full interconnection between the four corners of the vehicle, and thus to develop the most general suspension system, while maintaining the symmetry requirement. An example of suspension interconnection are the anti-roll bars, or stabilizers which are common in today's cars. Another is the Moulton front-to-rear interconnected suspension [21].

The model is then used in a parametric study, using
data from a current production vehicle, to understand the effect of coupling on vehicle ride quality and handling, when body absolute velocity feedback is used. That control strategy was chosen for the advantages which have been described previously.

A partially decoupled suspension configuration is used to eliminate the disadvantages found by Margolis [14], while at the same time leaving the handling characteristics unaffected.

1.6 SCOPE OF THE STUDY

This first chapter has presented some necessary background material concerning suspension systems in general, as well as some ride confort and handling considerations.

Active suspensions were presented, along with a review of previous work of particular interest to the present study.

The vehicle model used, along with the corresponding equations of motion, are presented in Chapter 2.

Chapter 3 discusses the various suspension configurations, including coupling and absolute damping.

Results corresponding to the various configurations, and based on parameters from a current production automobile, are presented in Chapter 4.

Finally, the conclusions of the study and recommendations for future work are summarized in Chapter 5.
Fig. 1.1  A Typical Vehicle Suspension Model
(From Ref. [2])
Fig. 1.2 Limits of Vertical Vibration, as Recommended by Janeway (From Ref. [1])
Fig. 1.3  Fatigue or Decreased Proficiency Limits Due To Whole-Body Vibration in the Vertical Direction, as Recommended by the ISO (From Ref. [2])

Fig. 1.4  The Effect of Normal Tire Load (Fz) on the Available Tractive Effort (Fx) and Lateral Force at Various Slip Angles (From Ref. [3])
Fig. 1.5  Lateral Cornering Force as a Function of Normal Tire Load, for Various Sideslip Angles  (From Ref. [5])
Fig. 1.6 Simple Suspension Model (De Carbon), With Typical Values Shown on the Side

\[ M = 1400 \text{ Kg} \]
\[ k = 80 \text{ KN/m} \]
\[ m = 140 \text{ Kg} \]
\[ k_T = 3,200 \text{ KN/m} \]
\[ \zeta_{\text{body}} = 0.25 - 0.40 \]
Fig. 1.7  Transmissibility of Road Input to the Sprung Mass, for the System Shown in Fig. 1.6
Fig. 1.8  Variation in Tire Normal Load, Due to Road Input, for the System Shown in Fig. 1.6
Fig. 1.9  Simple Active Suspension Model
Fig. 1.10  Schematic Diagram of One of Four Controllers Used in the Westinghouse Inertially Stabilized Automobile
(From Ref. [9])
Fig. 1.11 Simple Active Suspension Model, With Absolute Body Velocity Feedback

$F_A$ : active control force
2.1 INTRODUCTION

The automobile is a rather complex engineering system. From a system dynamics point of view, an accurate model representing all the subsystems that can vibrate would involve multiple degrees of freedom. To start with, such a model would include the dynamic modes of the vehicle body. Besides the 6 d.o.f.'s needed to describe the gross motions of the vehicle, an infinite number would be required to model the vibratory modes of the body structure itself, such as torsion and bending. The sub-assemblies such as the engine and transmission, the propeller shaft (if present) and the driveshafts, the various components of the suspension, the steering system (including the steering column and steering wheel), as well as others, should be included. These various assemblies are not rigidly mounted to the vehicle structure, and can thus vibrate relative to it. In fact, such vibrations are very often the cause of occupant discomfort in actual automobiles, and are the subject of extensive analysis by the manufacturers. Unfortunately, each of these assemblies would require several d.o.f.'s for a complete description. A complete dynamic model of the vehicle should include the rotating and reciprocating inertias in the engine and transmission, instead of treating
these assemblies as masses only, because the gyroscopic torques that result from these rotating inertias can affect handling, not to mention the vibrations they generate. Moreover, the occupants are supported by the seats which are relatively compliant, and they should be treated as sub-assemblies of the whole system, even if the human body is assumed a rigid mass instead of the mass-spring-damper system that it is. Clearly, such an accurate model would be difficult to generate and even more difficult to analyze. Besides requiring sizeable computational power, its complexity will result in the loss of any physical insight into the system being analyzed.

As a general rule, the best model is the simplest one which can represent the phenomena being analyzed, and provide the answers sought. For instance, a more complex model would be needed if accurate numerical results, as opposed to general trends, are needed.

2.2 OVERVIEW OF VEHICLE MODELS

In general, vehicle models tend to fall into one of two categories: ride models, and handling models. In the higher order models, this distinction is no longer valid however.

The simplest ride model is the one-mass, heave-only model used, for example, by Hullender et. al. [15]. Because of the large difference in bandwidth between the sprung and the unsprung masses, this model can be used to gain some
qualitative understanding of the sprung mass isolating properties of the suspension. One step above in complexity is the two-mass, heave-only model, which includes both sprung and unsprung masses. This model, which was described in Chapter 1, has been widely used \([2,10,13,17,18]\). Another two d.o.f. model is the heave-pitch model described by Wong [2], and used by Margolis [14]. This model is closer to the one-mass heave model, since it does not include the unsprung masses. However, by treating the body as a non-symmetric beam supported by two unequal springs, it accounts for the coupling present between the heave and the pitch degrees of freedom of the vehicle. A model combining the characteristics of the last two models mentioned can be used to analyze the unsprung mass vibration at the same time as body heave and pitch. A seven d.o.f. model, involving three for the body, and one for each of the four unsprung masses is described by Wong [2]. It allows the study of vehicle response to inputs with different profiles under the right and the left tracks. Such a model is used by Mercier [22], but only in the case of a vehicle which is symmetric with respect to a longitudinal and a lateral vertical planes. The next step of refinement, as far as ride models are concerned, would be to include the seat suspension and the mass of the occupant.

Figure 2.1 shows the various models mentioned above. It is worth noting that all these models treat the vehicle as a linear lumped-parameter system, and they all assume the
vehicle travelling at a constant forward velocity, over a
given road profile.

The main difference between ride models and handling
models, is that the formers do not account for the lateral
displacement of the vehicle, or for its yaw or rotation
around the heave axis, they thus assume that the road
traversed is straight. The simplest handling model, known
as the bicycle model \([2,3]\), consists of two degrees of
freedom, one for vehicle sideslip, and one for vehicle yaw.
The unsprung masses are not modeled separately, and the
vehicle is not allowed to roll; the tires are modeled as
lateral stiffnesses. The model permits investigating the
effect of the location of the center of gravity, and the
front and rear tire stiffnesses, on the understeer or over-
steer characteristics of the vehicle during constant speed
maneuvers. A more refined handling model is the one pro-
posed by Segel \([23]\), and used widely. The model considers
the vehicle as a three-dimensional system, consisting of an
unsprung mass representing the whole vehicle, with freedom
to yaw and sideslip, as well as a sprung mass constrained to
roll about an axis fixed to the unsprung mass. Another
degree of freedom could be added for the steering system, to
account for the inertia and compliance there. In the latter
case, it becomes possible to study the "free control" of the
vehicle, where a torque input is applied to the steering
wheel, as opposed to "fixed control" where the steering
angle at the front wheels is specified.

A handling model using a totally different approach is the one developed by Ellis [24]. In this model, no roll axis is assumed, and the unsprung masses are not given separate degrees of freedom, but they are assumed "geared" to the sprung mass. Therefore, they cannot vibrate independently, and the model is only valid over smooth roads with no disturbances, although degrees of freedom for body heave and body pitch are provided. The six degrees of freedom are those of the whole vehicle. Variable forward speed, as well as coupling effects due to the particular suspension geometry are accounted for.

Higher order models, which can be used to analyze ride quality and handling at the same time, have been developed, and are commonly used. The simplest has ten degrees of freedom, six for the body and one for each of the four unsprung masses. Eleven degrees of freedom can be used, if the steering system dynamics are of interest. Some models include a rotational degree of freedom for each of the unsprung masses, normal to the wheel plane. An example of such a model is that used at the University of Michigan [25].

2.3 MODEL DEVELOPMENT

The model used in this study is a seven d.o.f. ride model. The model is first described below, and its advant-
ages and limitations are analyzed. This is followed by the
description of a coordinate transformation which is of
importance in deriving the equations of motion and in
analyzing the system. Finally, in a third section, the
equation derivation is outlined.

2.3.1 Model Description

The model used is shown schematically in Figure 2.2.

It has seven degrees of freedom:

- Three degrees of freedom for the sprung mass which
  is assumed rigid. Relative to a reference origina-
ting from the center of gravity of the sprung mass,
these are the heave $H$, or vertical translation, the
roll $R$, or rotation about the longitudinal axis, and
pitch $P$, or rotation about the lateral axis, as
shown in Fig. 2.2;

- One degree of freedom for each unsprung mass. The
  four unsprung masses are lumped independently. Each
  is assumed to move in a direction normal to the
  plane of the body. Furthermore, the unsprung masses
  can only transmit forces through the suspension
  springs, dampers, and active elements. These forces
  are assumed normal to the plane of the sprung mass.

It should be noted that the coordinates chosen are only
valid for small displacements and rotations, since the
geometric nonlinearities that result from large rotations
are not accounted for, as would be the case if Euler angles were used to locate the body in space. The rotations have to be small enough for the small angle approximation to be valid.

The whole system is assumed linear, and the stiffnesses, both of the suspension and of the tires, as well as the damping rates, whether relative or absolute, are constant. Although a real vehicle exhibits many non-linearities, both geometric and constitutive, the linear model is justifiable since the object of the study is not to obtain accurate numerical results, but rather to understand the general behavior of the system.

The small amount of damping present in the tires is neglected, and the tires are thus modeled as linear springs. In terms of energy dissipation, tire damping has a secondary effect compared to the damping present in the suspension [2]. In fact, if the body is assumed inertially fixed, and the unsprung masses are allowed to vibrate, tire hysteresis would provide only about 10% of critical damping for the unsprung masses, according to results obtained by Overton, Mills, and Ashley [26].

Another assumption concerning the tires is that they remain in contact with the road at all time. The road itself is assumed to provide the only inputs to the system, as the vehicle travels at a constant speed in a straight line.
Finally, the vehicle is assumed to be symmetric with respect to a longitudinal vertical plane through its middle.

Although the model described above is a ride model, it is adequate for understanding the effect of road disturbances on the handling characteristics of the vehicle, because it is a full three-dimensional model. It is possible to keep track of all four tire normal loads individually, and thus to analyze how their distribution varies as the vehicle traverses a bumpy road. The extent to which the normal load distribution is affected by the disturbances, can be used to qualitatively compare various suspension designs, or control strategies. This approach presents certain limitations however, when quantitative estimates are desired: a given change in normal load distribution could have no appreciable effect at low vehicle acceleration, yet result in a dramatic change in handling at higher accelerations, because the effect of normal load on the available tire forces becomes more pronounced closer to the tire limit of adhesion (ref. Fig. 1.5). Because of this and other non-linear behavior, simple superposition does not apply.

2.3.2 Coordinate Transformation

It is desirable to obtain the equations of motion in a form that lends itself to simple physical interpretation. A crucial step in this direction involves the transformation of all the variables and parameters at the four corners into
new ones which reflect the modes of the sprung mass (heave, pitch, and roll). This can be accomplished via linear combinations of these "local" variables, as described in this sub-section. It should be noted that this transformation, as well as the equation derivation which follows in the next sub-section, are loosely based on those of Mercier [22], although definitions may differ, partly because the vehicle is not assumed symmetric, and for other reasons as well.

The transformation will be first derived for the road inputs.

The four corners of the vehicle are numbered from 1 to 4, starting at the left front corner and moving clockwise. Referring to Figure 2.3, the problem could be stated as follows:

Given the road vertical displacements \( Z_{R_i} \), \( i = 1 \) to 4, under each wheel, taken positive downward, a transformation matrix \([TF]\) is needed, such that:

\[
\{QR\} = [TF] \{ZR\};
\]

where \( \{QR\} \) is the new road input vector consisting of road heave \( HR \), road pitch \( PR \), road roll \( RR \), and road warp \( WR \);

\[
\{QR\} = \begin{bmatrix} HR \\ PR \\ RR \\ WR \end{bmatrix}.
\]
The transformation matrix is obtained easily once the new variables are defined. This is facilitated by the introduction of the mid-point displacements $ZR_{12}$, $ZR_{34}$, $ZR_{14}$, and $ZR_{23}$ shown in Figure 2.3, where $ZR_{ij}$ is the weighted average vertical displacement of the road plane at the points corresponding to the intersections of the x- and y-axes with the vehicle perimeter:

$$ZR_{12} = \frac{ZR_1 + ZR_2}{2}; \quad (1)$$

$$ZR_{34} = \frac{ZR_3 + ZR_4}{2}; \quad (2)$$

$$ZR_{14} = \frac{b \cdot ZR_1 + a \cdot ZR_4}{(a + b)}; \quad (3)$$

and

$$ZR_{23} = \frac{b \cdot ZR_2 + a \cdot ZR_3}{(a + b)}; \quad (4)$$

where $a$ is the distance from the center of gravity of the sprung mass, or from its projection onto the road plane, to the front axle, and $b$ is the distance from the same point to the rear axle.

The heave of the road plane is now defined as the vertical distance between the projection of the sprung mass center-of-gravity onto the static or zero-input road plane, and its projection onto the actual displaced road plane.
\[ HR = \frac{b \cdot ZR_{12} + a \cdot ZR_{34}}{(a + b)} \]  

or upon substituting for \( ZR_{12} \) and \( ZR_{34} \) from Eqns. (1) and (2):

\[ HR = \frac{b \cdot ZR_1 + b \cdot ZR_2 + a \cdot ZR_3 + a \cdot ZR_4}{2(a + b)} \]  

The pitch of the road plane is defined as the angle formed by the projections of the x-axis on the static, and on the displaced road planes:

\[ PR = \frac{-ZR_{12} + ZR_{34}}{(a + b)} \]  

or

\[ PR = \frac{-ZR_1 - ZR_2 + ZR_3 + ZR_4}{2(a + b)} \]  

Similarly, road roll is defined as the angle between projections of the y-axis onto a static and a displaced road planes:

\[ RR = \frac{ZR_{23} - ZR_{14}}{2t_m} \]  

where \( t_m \) is the half track of the vehicle, measured along the y-axis. From Fig. 2.3, it can be seen that:

\[ t_m = \frac{b \cdot t_f + a \cdot t_r}{a + b} \]
where $t_f$ is the front half track (measured at the front axle), and $t_r$ is the rear half track. Thus road roll becomes:

$$RR = \frac{-b \cdot ZR_1 + b \cdot ZR_2 + a \cdot ZR_3 - a \cdot ZR_4}{2(b \cdot t_f + a \cdot t_r)}$$  (11)

after substituting for $t_m$, and for $ZR_{14}$ and $ZR_{23}$.

Finally, road plane warp is defined as the conjugate to road roll. It gives an indication of the torsional deflection of the road plane:

$$WR = \frac{-b \cdot ZR_1 + b \cdot ZR_2 - a \cdot ZR_3 + a \cdot ZR_4}{2(b \cdot t_f + a \cdot t_r)}$$  (12)

Based on the above definitions, $[TF]$ can be obtained:

$$[TF] = \begin{bmatrix} \frac{b}{2w} & \frac{b}{2w} & \frac{a}{2w} & \frac{a}{2w} \\ \frac{-b}{2q} & \frac{b}{2q} & \frac{a}{2q} & \frac{-a}{2q} \\ \frac{-1}{2w} & \frac{-1}{2w} & \frac{1}{2w} & \frac{1}{2w} \\ \frac{-b}{2q} & \frac{b}{2q} & \frac{-a}{2q} & \frac{a}{2q} \end{bmatrix} ;$$  (13)

where $w$ is the wheelbase:

$$w = a + b$$  (14)

and

$$q = b \cdot t_f + a \cdot t_r.$$  (15)
The same coordinate transformation can be used to define tire and suspension compressions and unsprung mass displacements.

The tire compression terms $ZT_i$, $i = 1$ to $4$, corresponding to the four corners, can be combined into tire heave $HT$, tire pitch $PT$, tire roll $RT$, and tire warp $WT$:

$$
\begin{bmatrix}
HT \\
PT \\
RT \\
WT
\end{bmatrix} = [TF] \cdot \begin{bmatrix}
ZT_1 \\
ZT_2 \\
ZT_3 \\
ZT_4
\end{bmatrix}
$$

Similarly for suspension compression terms $ZS_i$:

$$
\begin{bmatrix}
HS \\
PS \\
RS \\
WS
\end{bmatrix} = [TF] \cdot \begin{bmatrix}
ZS_1 \\
ZS_2 \\
ZS_3 \\
ZS_4
\end{bmatrix}
$$

and for the unsprung mass displacements:

$$
\begin{bmatrix}
HU \\
PU \\
RU \\
WU
\end{bmatrix} = [TF] \cdot \begin{bmatrix}
ZU_1 \\
ZU_2 \\
ZU_3 \\
ZU_4
\end{bmatrix}
$$

The advantage of using the transformed coordinates and variables will become apparent later on in the study. The transformations make the system more transparent as far as analyzing and understanding the coupling between the various modes, and its effect on the response of the system.
2.3.3 Equation Derivation

The equations of motion are derived by applying Newton's second law to the various masses and inertias. The derivation is outlined in this section, with some typical equations; the full derivation is shown in Appendix A.

In a first step, the equations of motion for the unsprung masses are derived. First, the net force acting on each of the four unsprung masses is evaluated in terms of the transformed system variables. Then the motion of each unsprung mass is expressed in terms of the new variables. Next, the tire compression terms are eliminated, using the constraint that the tires are always in contact with the road. Newton's law is then applied to each unsprung mass. Finally, the four equations thus obtained are combined linearly, using the transformation matrix \([TF]\), to obtain four new equations which are dynamically uncoupled.

The second step is to derive the sprung mass equations of motion. The individual suspension forces at each corner are combined into a suspension heave force, a pitching moment, a rolling moment, and a warping moment. Then Newton's second law is used to obtain the equations.

In a last step, the equations of motion of the sprung mass are used to eliminate the sprung mass acceleration terms from the equations of the unsprung masses.
First Step:

The net downward force acting on the unsprung mass at each corner is given by:

$$F_i = F_{Si} - F_{Ti} \; ; \; \; i = 1 \text{ to } 4 \; ; \; (19)$$

where \(F_S\) is the local suspension force (positive when the suspension is in compression), and \(F_T\) is the local tire force (positive when the tire is in compression). In deriving the equations, the suspension force is going to be assumed as general as possible, within the constitutive limitations previously imposed, namely that only relative stiffnesses, relative damping, and absolute damping will be considered.

The resulting expression for the local suspension forces is:

$$F_{Si} = AD_{Hi} \cdot \dot{H} + AD_{Pi} \cdot \dot{P} + AD_{Ri} \cdot \dot{R}$$

$$+ DH_i \cdot \ddot{H} + DP_i \cdot \ddot{P} + DR_i \cdot \ddot{R} + DW_i \cdot \ddot{W}$$

$$+ KH_i \cdot HS + KP_i \cdot PS + KR_i \cdot RS + KW_i \cdot WS \; ;$$

for \(i = 1 \text{ to } 4 \; ; \; (20)$$

where (Q referring to the transformed d.o.f.'s H, P, R, and W):

- \(AD_{Qi}\) is the absolute damping coefficient in the "Q" mode, at the \(i\)th corner; i.e. \((AD_{Hi} \cdot H)\) is the suspension force component at corner one, resulting
from the absolute body heave velocity. It should be noted that no warp term is included for the absolute damping component of the suspension force. Since the body is assumed rigid, we identically have:

\[ W = \ddot{W} = \dddot{W} = 0 \quad ; \quad (21) \]

- \( DQ_i \) is the relative damping coefficient in the "Q" suspension mode, for the \( i \)th corner;
- \( KQ_i \) is the relative stiffness coefficient in the "Q" suspension mode, for the \( i \)th corner again.

It should be noted that symmetry requirements place restrictions on these coefficients. For instance:

\[ HK_1 = HK_2 \quad ; \]
\[ RK_1 = -RK_2 \quad ; \quad (22) \]

etc....

This fact will be used in Chapter 3.

The tire force at each corner is given by:

\[ FT_i = KT_i \cdot ZT_i \quad ; \quad i = 1 \text{ to } 4 \quad . \quad (23) \]

Using the inverse of the transformation matrix \([TF]\), it is possible to express the tire forces in terms of the whole system coordinates. Since:
\[ \{ZT_i\} = [TF]^{-1} \cdot \{QT\} \]

\[ = [ITF] \cdot \{QT\} ; \]  
then:

\[ FT_i = KT_i \cdot \{ITFi,1 \cdot HT + ITFi,2 \cdot PT \]
\[ + ITFi,3 \cdot RT + ITFi,4 \cdot WT\}; \]

\[ \text{for } i = 1 \text{ to } 4 \ . \]  

(25)

The net vertical downward displacement of each unsprung mass is equal to the road displacement under that mass, added to the tire compression at that corner. Again, using the inverse transformation matrix \([ITF]\), it is possible to express that displacement in terms of the transformed system variables:

\[ (ZR_i + ZT_i) = ITFi,1 \cdot (HR + HT) \]
\[ + ITFi,2 \cdot (PR + PT) \]
\[ + ITFi,3 \cdot (RR + RT) \]
\[ + ITFi,4 \cdot (WR + WT) ; \]

\[ i = 1 \text{ to } 4 \ . \]  

(26)

It is possible to eliminate the tire compression terms \((QT)\) from the net force expression (Eqn. 19) and from the displacement expression above, because the tire is not allowed to lose contact with the ground. This restriction
can be expressed as:

\[ Z_i = ZR_i + ZT_i + ZS_i \quad ; \quad \text{for } i = 1 \text{ to } 4. \quad (27) \]

Because matrix multiplication is distributive, it is possible to go from the above equation to the following one:

\[ Q = QR + QT + QS \quad ; \quad Q = H, P, R, W \quad ; \quad (28) \]

or:

\[ QT = Q - QS - QR \quad ; \quad Q = H, P, R, W \quad . \quad (29) \]

Eqn. 29 can be used to substitute for the tire deflection terms.

Now, Newton's Second Law is used:

\[ \frac{d^2}{dt^2} (ZR_i + ZT_i) = \frac{1}{m_i} \cdot F_i \quad ; \quad \text{for } i = 1 \text{ to } 4; \quad (30) \]

where \( m_i \) is the unsprung mass of the \( i \)th corner.

Because the four equations of motion contain only linear terms, it is possible to combine the equations linearly. This is shown schematically below:

\[
\begin{bmatrix}
\text{eqn. H} \\
\text{eqn. P} \\
\text{eqn. R} \\
\text{eqn. W}
\end{bmatrix}
= [TF] \cdot 
\begin{bmatrix}
\text{eqn. 1} \\
\text{eqn. 2} \\
\text{eqn. 3} \\
\text{eqn. 4}
\end{bmatrix}
\]

Distributivity is again used to make this possible.

The unsprung mass heave equation only involves the body heave and the suspension heave in the acceleration terms.
(H and HS). Similarly, the unsprung mass pitch equation has only body pitch and suspension pitch acceleration terms (P and PS), and so on for the roll and the warp equations.

**Second Step:**

In this step, the equations of motion of the sprung mass are derived.

The suspension forces $F_{S1}$ are combined to obtain the total vertical force $F_S$, the total pitch moment $M_{PS}$, the total roll moment $M_{RS}$, and the total warp moment $M_{WS}$, which act on the sprung mass. These force and moments are given by:

$$F_S = F_{S1} + F_{S2} + F_{S3} + F_{S4};$$  \(32\)

$$M_{PS} = -a \cdot (F_{S1} + F_{S2}) + b \cdot (F_{S3} + F_{S4});$$  \(33\)

$$M_{RS} = t_f \cdot (-F_{S1} + F_{S2}) + t_r \cdot (F_{S3} + F_{S4});$$  \(34\)

$$M_{WS} = t_f \cdot (-F_{S1} + F_{S2}) - t_r \cdot (F_{S3} - F_{S4}).$$  \(35\)

After substituting for the suspension forces, using Eqn. (20), and combining terms, the suspension forces and moments can be written as:

$$F_S = ADH_H \cdot \ddot{H} + ADP_H \cdot \ddot{P} + ADR_H \cdot \ddot{R}$$  \(36\)

$$+ DH_H \cdot \dddot{H} + DP_H \cdot \dddot{P} + DR_H \cdot \dddot{R} + DW_H \cdot \dddot{R} + K_H \cdot HS + KP_H \cdot \dddot{P} + K_H \cdot \dddot{R} + KW_H \cdot \dddot{R} + KW_H \cdot \dddot{W};$$
\[ MPS = ADH_p \cdot H + ADP_p \cdot P + ADR_p \cdot R \]
\[ + DH_p \cdot H + D_P_p \cdot P + D_R_p \cdot R + D_W_p \cdot W \]
\[ + KH_p \cdot H + KP_p \cdot P + KR_p \cdot R + KW_p \cdot W \; ; \]

and so on for MRS and MWS. The various coefficients are the suspension absolute damping rates \((ADQQ_v)\), the suspension relative damping rates \((DQQI)\), and the suspension stiffnesses \((KQQ_s)\). Because of symmetry, they are not all independent, and several are equal to zero.

Using the suspension force and moments, the equations of motion of the sprung mass are derived by applying Newton's Second Law as follows:

\[ \ddot{H} = - \frac{1}{M} \cdot FS \; ; \]  
\[ \ddot{P} = - \frac{1}{I_y} \cdot MPS \; ; \]  
\[ \ddot{R} = - \frac{1}{I_x} \cdot MRS \; ; \]

and

\[ \ddot{W} = 0 \; . \]  

The last equation, as previously explained, is due to the assumed infinite torsional rigidity of the sprung mass.
Third Step:

Using Eqns. (38) - (41), it is possible to substitute for $\ddot{H}, \ddot{P}, \ddot{R}$ and $\ddot{W}$ in Eqn. (31).

The equations obtained can be expressed, in matrix form, as follows:

$$[M]\{\ddot{X}\} + [D]\{\dot{X}\} + [K]\{X\} = [BR]\{XR\} \quad ; \quad (42)$$

where the system is described by the variables in vector $\{X\}$:

$$\{X\}^T = \{H, P, R, HS, PS, RS, WS\} \quad . \quad (43)$$

The road disturbance input vector $\{XR\}$ is:

$$\{XR\}^T = \{HR, PR, RR, WR\} \quad . \quad (44)$$

The inertia matrix $[M]$ is diagonal:

$$[M] = \begin{bmatrix}
M & I_y & I_x & 0 \\
I_y & I_x & 0 & 0 \\
0 & 0 & IU_H & IU_P \\
0 & 0 & IU_R & IU_W
\end{bmatrix} \quad ; \quad (45)$$

where $IU_Q$ is the combined inertia of the unsprung masses in the $Q$ mode, ($Q$ being either heave $H$, pitch $P$, roll $R$, or warp $W$). These inertias are linear combinations of the front and rear unsprung masses $m_f$ and $m_r$:

$$IU_H = \frac{2(a+b)^2 \cdot m_f \cdot m_r}{a^2 \cdot m_f + b^2 \cdot m_r} \quad ; \quad (46)$$
Finally, the matrices \([D]\) and \([K]\) are the damping matrix and the stiffness matrix, respectively.

This chapter reviewed the various vehicle models used to study ride quality and handling. A seven degree-of-freedom model, which is used in this study was presented, and an outline of the derivation of the equations of motion was given. The full equations are presented in Appendix A.

The next chapter looks at the coupling present between the various modes, and at how active control can be used to affect it.
Fig. 2.1 Various Vehicle Models:
(a) One-Mass Heave Model
(b) Two-Mass Heave Model
Fig. 2.1 (cont.) Various Vehicle Models:
(c) Two-Mass Heave-Pitch Model
(d) Four-Mass Heave-Pitch Model
Fig. 2.1 (cont.) Various Vehicle Models:
(e) Seven-Mass Model
(f) Heave-Pitch Model with Seat-Driver Dynamics
Fig. 2.2 Seven d.o.f. Vehicle Model Used in Present Study
Fig. 2.3 Coordinate Transformation
CHAPTER 3

VEHICLE CONFIGURATIONS

The previous chapter provided a description of the vehicle model used for the present study, as well as an outline of the derivation of the equations of motion. This chapter takes a closer look at that system. In a first section, the effect of symmetry and the coupling present between the different modes are considered. In a second section, the control strategy used for the active suspension is presented. The third section is a discussion of some implementation considerations of the system. In the last section, the various vehicle configurations used to obtain the results of the next chapter are described.

3.1 SYMMETRY AND COUPLING

3.1.1 Symmetry

Beside the linearity assumption, the side-to-side symmetry inherent to the automobile results in the most important set of constraints affecting the system and its equations of motion. The effect of symmetry is best understood by following the outline of the equation derivation given in the previous chapter, starting with the suspension forces at each corner as given by Eqn. (20). To simplify the discussion, only the relative stiffness components of these forces are considered, although similar
statements could be made concerning relative damping, as well as absolute damping. The relative stiffness components of the suspension force at each corner is given by:

\[ F_{KS_i} = K_{Hi} \cdot HS + K_{Pi} \cdot PS + K_{Ri} \cdot RS + K_{Wi} \cdot WS; \]

for \( i = 1 \) to \( 4 \). \hfill (49)

Lateral symmetry requires that a given heave displacement results in equal forces at both front corners, as well as equal forces at both rear ones, giving the following constraints:

\[ K_{H1} = K_{H2} \hfill (50) \]
\[ K_{H3} = K_{H4} \hfill (51) \]

The same requirement applies for a given pitch angle resulting in the following constraints:

\[ K_{P1} = K_{P2} \hfill (52) \]
\[ K_{P3} = K_{P4} \hfill (53) \]

For roll and warp displacements, the linear constitutive relations of the springs combined with lateral symmetry require that forces at the front corners be equal in magnitude but opposite in sign, with the same requirement for the rear corners. The resulting constraints are:

\[ K_{R1} = -K_{R2} \hfill (54) \]
\[ K_{R3} = -K_{R4} \hfill (55) \]
and

\[ \text{KW}_1 = -\text{KW}_2 \quad ; \]  \hspace{1cm} (56) \\
\[ \text{KW}_3 = -\text{KW}_4 \quad . \quad \]  \hspace{1cm} (57)

When the suspension forces are combined using the transformation matrix \([TF]\), as done in Eqn. (31) for the derivation of the equations of motion of the unsprung masses, the roll and warp terms cancel each other in the heave and pitch equations, and vice versa. This stems directly from the above constraints (Eqns. (50-57)) and from \([TF]\) (Eqn. (13)).

Furthermore, the roll and warp terms present in the tire force expressions (Eqn. (25)) also cancel each other when the unsprung mass heave and pitch equations are assembled using Eqn. (31), and vice versa. To show that, the inverse transformation matrix \([ITF]\) is needed. It is given here, but further proof is left for the reader.

\[
[\text{ITF}] = \begin{bmatrix}
1 & -a & -\frac{q}{2b} & -\frac{q}{2b} \\
1 & -a & \frac{q}{2b} & \frac{q}{2b} \\
1 & b & \frac{q}{2a} & -\frac{q}{2a} \\
1 & b & -\frac{q}{2a} & \frac{q}{2a}
\end{bmatrix}.
\]  \hspace{1cm} (58)

The elimination of the roll and warp terms in the heave and pitch equations, and vice versa, also takes place in the derivation of the equations for the sprung mass. This is clearly seen from the expressions for the total suspension
force and moments given by Eqns. (32-35). In the expressions for the heave force and the pitch moment, the forces at laterally opposite corners are added, whereas they are subtracted in the expressions for the roll and the warp moments. This, combined with the constraints (Eqns. (50-57)) results in the elimination of roll and warp terms from the expressions for the suspension heave force and pitch moment (Eqns. (36-37)):

\[
\begin{align*}
ADR_H &= ADR_p = 0 \\
DR_H &= D\dot{W}_H = DR_p = DW_p = 0 \\
KR_H &= K\dot{W}_H = KR_p = KW_p = 0
\end{align*}
\]

(59) \hspace{1cm} (60) \hspace{1cm} (61)

Similarly, the roll and warp moments are not function of heave or pitch.

The symmetry considerations presented above result in a system of equations that is composed of two entirely independent subsystems, one for heave and pitch, the other for roll and warp, as is apparent from the equations of motion in Appendix A. The first subsystem is for heave and pitch and consists of four equations of motion. The second is for roll and warp and consists of three. The coupling within these two independent subsystems is discussed below.

3.1.2 Heave-Pitch Coupling

Because of the lack of fore-aft symmetry, the equations of motion in heave and pitch are coupled. As mentioned before, this coupling is sometimes used to fine-tune the
ride characteristics of the vehicle [19]. The flat-ride characteristic is based on the premise that humans are more negatively affected by the fore-aft acceleration due to pitch rotation, than by the vertical acceleration due to heave. However, not all cars are tuned to achieve a flat-ride, because of conflicting requirements (static deflection under load, etc...). In fact, the designer disposes of only two parameters, namely the front and rear spring stiffnesses, to affect three variables: the heave stiffness, the pitch stiffness, and the coupling between the two modes. Similar limitations apply to the damping. To clarify the previous points the following notation is introduced for the local spring rate at a corner: $k_{ji}$ is the relative stiffness at the $i$th corner in response to a relative displacement at the $j$th corner. The same convention applies to the local relative damping rates $d_{ji}$ and local absolute damping rates $a_{ji}$. In terms of the local stiffnesses or spring rates, the designer can only specify the front and rear stiffnesses $k_{11} = k_{22}$ and $k_{33} = k_{44}$. However, it can be easily shown that these two local stiffnesses determine all three of the distinct non-zero heave and pitch stiffness terms of the whole suspension, as they appear in Eqns. (36-37):

$$K_{HH} = 2 \cdot k_{11} + 2 \cdot k_{33} ;$$  \hspace{1cm} (62)

$$K_{PP} = 2 \cdot a^2 \cdot k_{11} + 2 \cdot b^2 \cdot k_{33} ;$$  \hspace{1cm} (63)
\[ K_{PH} = K_{HP} = -2 \cdot a \cdot k_{11} + 2 \cdot b \cdot k_{33} \]  

It should be noted that the coupling stiffnesses \( K_{PH} \) and \( K_{HP} \) are equal because of reciprocity.

From the point of view of handling and roadholding, the coupling between the heave and pitch modes is expected to indirectly affect the way the vehicle responds when performing a maneuver on irregular roads.

### 3.1.3 Roll-Warp Coupling

Unlike heave-pitch coupling which affects ride quality primarily, with vehicle roadholding only to a secondary degree, the coupling between the roll and warp modes plays a primary role in determining the handling characteristics of the vehicle.

During steady-state cornering maneuvers, the behavior of the vehicle (understeer or oversteer) is significantly affected by the difference between the lateral transfer of tire load at the front axle, and that at the rear axle. Other parameters that determine vehicle understeer or oversteer are its weight distribution (fore-aft), the relative tire sizes and inflation pressures between the front and rear axles, and the roll-steer effects built into the suspension geometry.

The fore-aft distribution of lateral load transfer is directly related to the roll-couple distribution in the suspension [5]. During steady-state, the roll-couple is
only function of the suspension stiffnesses, and not the damping rates. In the particular case corresponding to the simplified geometry of the model used in this study, the tire load difference and the suspension roll couple at one end of the vehicle are only related by the track width at that end. In terms of the total suspension stiffness terms, the front and rear roll couples are respectively given by:

\[ t_f \cdot (-FS_1 + FS_2) = t_f \cdot (-FT_1 + FT_2) = \]
\[ = \frac{1}{2} \cdot (KR_R \cdot RS + KW_R \cdot WS + KR_W \cdot RS + KW_W \cdot WS); \]
\[ (65) \]

and

\[ t_r \cdot (FS_3 - FS_4) = t_r \cdot (FT_3 - FT_4) = \]
\[ = \frac{1}{2} \cdot (KR_R \cdot RS + KW_R \cdot WS - KR_W \cdot RS - KW_W \cdot WS); \]
\[ (66) \]

which are only valid in steady-state.

For a vehicle with a conventional passive suspension system using springs and anti-roll bars at each end, reciprocity results in further simplification of the above equations:

\[ t_f \cdot (-FS_1 + FS_2) = \frac{1}{2} \cdot (KR_R + KR_W) \cdot (RS + WS); \]
\[ (67) \]

\[ t_r \cdot (FS_3 - FS_4) = \frac{1}{2} \cdot (KR_R - KR_W) \cdot (RS - WS). \]
\[ (68) \]

In the simple vehicle model chosen for this study, the roll centers of both front and rear suspensions are at the
same height (in this case ground level). Therefore, during steady-state cornering on a smooth flat road, suspension warp (WS) will be negligible, or zero if the vehicle has equal front-rear weight distribution \((a=b)\). In such a case, Eqns. (67-68) clearly show the important role played by the roll-warp coupling stiffness \((K_{RW} = K_{WR})\) in determining the roll-couple distribution. Although the front and rear tracks can be altered to change that distribution, it is only possible to a limited extent. The fact that \((K_{RW})\) is preceded by different signs in Eqns. (67) and (68) allows greater latitude in choosing the roll-couple distribution.

From the above, it is seen that any changes to the roll-warp coupling terms would affect the basic handling characteristics of the vehicle. Since this is beyond the scope of this study, these coupling terms are left as on the original baseline vehicle for all the different vehicle configurations considered.

3.2 CONTROL STRATEGY

Before presenting the various vehicle configurations analyzed, the control strategy that makes them achievable is presented here.

In order to derive the general form of the control law, the generic form of the equations of motion, represented by Eqn. (42), is rewritten to show more detail. The vector of variables used to describe the system is extended to include
body warp (w), although this is assumed to be identically zero, as seen earlier. This step makes for a more concise derivation. Furthermore, the suspension force at each corner is broken into a "passive" and an "active" component. The former corresponds to that provided by the suspension of the base vehicle, which consists of a spring and damper at each corner, and an anti-roll bar at each end of the vehicle. The latter is that force provided by the active actuator. It should be noted that the terms "passive" and "active" are used loosely here, and thus appear within quotation marks. As it will be seen, the active element could be used to generate those passive forces, and some of the forces generated by the active element could be realized by passive means. The equations of motion are rewritten so that the damping and stiffness matrices include only the "passive" terms of the baseline vehicle, with the "active" forces added as a vector \{U\} of control forces. Furthermore, all matrices and vectors are written in an expanded form:

\[
\begin{bmatrix}
MB & 0 \\
0 & MS
\end{bmatrix} \cdot \ddot{X} + \begin{bmatrix}
AD1 & D1 \\
AD2 & D2
\end{bmatrix} \cdot \frac{\ddot{X}}{XS} + \begin{bmatrix}
AK1 & K1 \\
AK2 & K2
\end{bmatrix} \cdot \frac{XB}{XS} = \begin{bmatrix}
B1 \\
B2
\end{bmatrix} \cdot \{U\} + \begin{bmatrix}
BR1 \\
BR2
\end{bmatrix} \cdot \{XR\};
\]

where all submatrices are square.

It should be noted that in the passive baseline system, [AK1] and [AD1] are zero except for the terms corresponding
to the body torsional stiffness and internal damping. \([AD2]\)
is zero because the tire radial damping is assumed negligi-
gible. Also, \([BR1]\) is zero. It is also worth mentioning
that absolute body velocity feedback (absolute damping)
affects both \([AD1]\) and \([AD2]\) because the forces used to
provide absolute damping act as action-reaction pairs on
both sprung and unsprung masses.

Now, it is possible to isolate the equations of motion
of the sprung mass, as follows:

\[
\ddot{MB} \cdot \{XB\} + [AD1] \cdot \{XB\} + [D1] \cdot \{XS\} + \\
+ [AK1] \cdot \{XB\} + [K1] \cdot \{XS\} = [B1] \cdot \{U\}; \quad (70)
\]

Since \([AK1]\) is null except for the body warp terms, it
can be dropped once body warp is identically set to zero.
If \([AD1^*]\), \([D1^*]\), and \([K1^*]\) are the desired absolute
damping, relative damping, and relative stiffness matrices,
then by choosing the control forces to be equal to:

\[
\{U\} = [B1]^{-1} \cdot \left\{ ([AD1]-[AD1^*]) \cdot \{XB\} + ([D1]-[D1^*]) \cdot \{XS\} + \\
+ ([K1]-[K1^*]) \cdot \{XS\} \right\}, \quad (71)
\]

the equations of motion of the body become:

\[
\ddot{MB} \cdot \{XB\} + [AD1^*] \cdot \{XB\} + [D1^*] \cdot \{XS\} + [AK1] \cdot \{XB\} + \\
+ [K1^*] \cdot \{XS\} = \{0\} \quad . \quad (72)
\]

The possibility of arbitrarily specifying the \([AD1^*]\),
\([D1^*]\), and \([K1^*]\) matrices allows the natural frequencies and
damping ratios of the various body modes to be altered at will. Beside altering the eigenvalues, it is also possible to modify the eigenvectors; in particular the coupling present between the various modes can be eliminated by setting the appropriate off-diagonal terms in the above matrices to zero. However, this is only possible if the matrix \([B_l]\) is invertible. Fortunately, this is the case as demonstrated below:

\[
[B_l] = \begin{bmatrix}
-1 & -1 & -1 & -1 \\
a & a & -b & -b \\
t_f & -t_f & -t_r & t_r \\
t_f & -t_f & t_r & -t_r
\end{bmatrix},
\]

therefore the determinant of \([B_l]\) is:

\[
|B_l| = -8 \cdot t_f \cdot t_r \cdot (a+b)
\]  

(74)

This determinant is never zero for a physically realizable car. This is a controllability requirement: if the determinant is equal to zero, this implies that two or more of the actuators are located at the same point, with the resulting loss of the ability to apply the required forces and moments on the body.

Using the active force actuators to alter the dynamics of the sprung mass necessarily entails changes in the dynamics of the unsprung masses, as mentioned earlier. The new equations of motion of the unsprung masses are given by
the following equation:

\[
\begin{align*}
\ddot{\mathbf{MS}} \cdot \dot{\mathbf{XS}} + \mathbf{AD2} \cdot \dot{\mathbf{XB}} + \mathbf{D2} \cdot \dot{\mathbf{XS}} + \mathbf{AK2} \cdot \mathbf{XB} + \\
+ \mathbf{K2} \cdot \dot{\mathbf{XS}} - \mathbf{B2} \cdot \mathbf{U} = \mathbf{BR} \cdot \dot{\mathbf{XR}},
\end{align*}
\] (72)

where \( \mathbf{U} \) is given by Eqn. (71).

3.3 IMPLEMENTATION

From Eqn. (71), it can be seen that implementing the general control law requires full-state feedback, with the exception of absolute body position. This is also clear from Eqn. (20) which gives the expression for the suspension force at each corner of the vehicle. The two would be equal if the active actuators are relied upon to provide all suspension forces.

Using the inverse coordinate transformation \([\text{ITF}]\), the suspension forces can be easily expressed as:

\[
\mathbf{FS}_i = \mathbf{f}_i(Z_j, ZS_j, ZS_j; j = 1, 4);
\]

for \( i = 1 \) to 4 .

(76)

Eqn. (76) shows that the suspension force at each corner is a function of the inertial or absolute body velocity at each of the four corners, as well as the suspension compression and compression rate at each corner. In other words, this requires a fully interconnected suspension, not only in terms of relative damping and stiffness, but also extending to absolute damping.
Theoretically, the active force actuators are only required for providing the absolute damping component of the suspension force, since it should be possible to provide the relative damping and relative stiffness components by purely passive means. Current production cars already have suspensions providing a force which is a function of the local relative displacement and of its rate at a given corner, via the spring and damper at that corner, as well as a force which is a function of the relative displacement at the laterally opposite corner, via anti-roll bars or stabilizers. Some older cars even had suspensions providing front-to-rear interconnection, either via "anti-pitch" bars or via an arrangement such as that of the Renault 2 C.V. It should be noted however that those interconnections provide stiffness forces only, without relative damping. One exception is the Moulton Hydragas [21] arrangement, where the front-to-rear interconnection was done via hydraulic connections providing combined damping and stiffness forces.

The reader will appreciate the difficulties involved in providing both relative damping and relative stiffness interconnection in a vehicle, not only side-to-side and fore-aft, but also diagonally. The complexity of the linkages and/or hydraulic circuits needed would make such an arrangement prohibitive for most applications, from the point of view of bulk, weight, and possibly cost. Since the active force actuators are needed to provide the absolute
damping forces, they could also be relied upon to provide the various interconnections in the suspension. It would be desirable however to use regular springs to support the static weight of the vehicle, and thus reduce the size and power requirement of the actuators. This would also make the vehicle serviceable in case of failure of the active system. To make the vehicle fail-safe, the actuators should be designed to act like dampers (albeit less than optimal) in case of failure, instead of locking up. Furthermore, it would also be desirable to use anti-roll bars to further reduce the load on the active system, and also to fine-tune the handling balance of the vehicle to make it safe in case of failure.

Another advantage of using the active system to provide the various force components is the ability to modify the suspension parameters instantaneously, to conform to the driving conditions or to the driver's requirements.

3.4 VEHICLE CONFIGURATIONS

As was seen in Section 3.2, the control strategy used allows independent and arbitrary specification of all damping (absolute and relative) and stiffness terms in the equations of motion of the sprung mass. This capability is used in the present investigation to compare several vehicle configurations, which are presented in this section. When referring to Eqn. (69), the following assumption is made:
since body warp and its time rate of change are assumed identically zero, the damping and stiffness terms corresponding to that degree-of-freedom, namely \((ADl)_{3,4}\), \((ADl)_{4,3}\), \((ADl)_{4,4}\), \((AKl)_{3,4}\), \((AKl)_{4,3}\), and \((AKl)_{4,4}\), which are normally finite, are hereby assumed to be zero.

The first vehicle configuration considered is one with a passive suspension system, used as a baseline for comparison. The parameters used in this case correspond roughly to a current production GT automobile. These parameters are given in the next chapter, along with the results.

The second case considered is again a passive suspension, differing from the baseline case only by having twice the relative damping rates. This results in the damping matrix (Eqn. 69) equaling twice that of the baseline case, all other system parameters being the same.

The third configuration considered is where the active suspension is used to provide absolute damping. This configuration corresponds to the simplest absolute damping implementation: a fictitious "skyhook" damper at each corner of the vehicle provides the absolute damping force. The rates for the absolute dampers, i.e. the feedback gains corresponding to the absolute body velocity at each corner, are chosen numerically equal to the relative damping rate at that corner. Again referring to Eqn. (69), we have:

\[
[ADl] \text{ABSOLUTE DAMPING} = [Dl] \text{BASELINE} \quad ;
\] (77)
which results in:

\[
[AD2]_{\text{ABSOLUTE DAMPING}} = [D2]_{\text{BASELINE}} ; 
\]  

(78)

Clearly, such a choice for the absolute damping rates is less than optimal, and is only intended to provide qualitative results. Ref. [18] contains guidelines as to how the absolute and relative damping rates should be chosen relative to the vehicle parameters, in order to provide optimum performance.

The other configurations considered are those where the equations of motion of the sprung mass are decoupled. More specifically, the heave and pitch equations are decoupled. For the reasons presented in the first section of this chapter, the roll-warp coupling is not altered. The first of these configurations is the simple decoupled system where the submatrices \([D1]\) and \([K1]\) are made diagonal, with the terms on the diagonal remaining unchanged from the baseline case. Since no absolute damping is used, \([AD1]\) is zero (as is \([AK1]\)). It should be noted that the active actuators are not required in order to implement this configuration. In practice however, they would be needed to achieve the decoupling under varying load conditions. The second configuration is the decoupled (heave-pitch) suspension, with absolute damping. In this case, \([AD1]\) duplicates the \([D1]\) matrix of the simple decoupled case, similarly for \([AD2]\) and \([D2]\).
Finally, a configuration where the equations of the sprung mass are left coupled, but where decoupled absolute damping is used, is also investigated. In this case, the submatrix \([D_1]\) of the baseline case is used along with the submatrix \([AD_1]\) of the decoupled case with absolute damping.

Results corresponding to these various configurations are presented in the next chapter.
CHAPTER 4
RESULTS

In this chapter, some results which correspond to the different vehicle configurations presented in the previous chapter are given. These results aim at giving a qualitative understanding of the effect that using absolute body velocity feedback has on vehicle performance, in the presence of coupling between the different degrees-of-freedom. Vehicle performance is considered both from the point-of-view of body isolation, and that of roadholding.

In order to generate the results that follow in this chapter, some vehicle parameters were taken from a current production GT automobile. In some cases, data that were not available were estimated. These data are presented in Table 4.1.

Two types of results are presented: first the eigenvalues and eigenvectors of certain configurations are compared; secondly, frequency response plots for different input-output combinations are given.

4.1 EIGENVALUES, EIGENVECTORS

The eigenvalues and eigenvectors are presented in order to show the effect of the suspension configuration on the basic dynamics of the vehicle.
In Table 4.2, the damped natural frequencies and the damping ratios of the various natural modes of the vehicle are presented. Two cases are considered: first the baseline vehicle, and secondly the vehicle with absolute body velocity feedback. However, unlike the configuration for absolute body velocity feedback which was presented in Section 3.4, the feedback gains used for this particular case are equal to only one-half the numerical value of the passive damper rate at the corresponding corner.

The results in Table 4.2 illustrate the effect of absolute damping: the damping ratios of the modes consisting of motion of the sprung mass increase, whereas those of the modes of the unsprung masses are not affected. The damped natural frequencies of the body modes decrease despite the fact that the stiffnesses of the system are not altered, because of the increase in the corresponding damping ratios.

Table 4.3 shows some results corresponding to the heave-pitch subsystem (4 d.o.f.). The damped natural frequencies and damping ratios of the various natural modes are given, along with the amplitudes of the corresponding eigenvectors. The cases considered are the baseline vehicle, and the vehicle with the heave-pitch decoupled suspension.

Referring to the baseline vehicle, it can be seen that the suspension modes consist mainly of unsprung mass motion,
with a negligible amount of body displacement (ref. the two high frequency modes: at 26.1 Hz and 23.5 Hz). These two modes consist of combined heave and pitch (100% and 80% respectively), out-of-phase in one case, and in-phase in the other. The other two modes, with the low frequencies (1.16 Hz, and 1.20 Hz), consist mostly of sprung mass motion: roughly equal amounts of downward body heave and of suspension heave compression result in the unsprung masses remaining practically stationary. These two modes also consist of heave and pitch, (100% and 70-80% respectively), out-of-phase, and in-phase.

In the second configuration, where the equations of motion of the sprung mass have been decoupled, it can be seen that the body modes consist almost exclusively of heave or pitch. In the former, only 1% of pitch motion remains, whereas in the latter the remaining heave motion is only 4%. It should be noted that the natural frequencies and damping ratios are slightly changed, compared to the baseline case. This is due to the simple approach used, which is to eliminate the coupling stiffnesses and damping rates, without correcting the remaining terms to obtain the same natural frequencies and damping rates as in the coupled (baseline) case. This is not necessary for the purpose of this study however. As far as the eigenvector amplitudes of the modes of the unsprung masses are concerned, they are practically unaffected by the decoupling. Although the
damping ratios are slightly different, the natural frequencies are almost unchanged. The reason is that most of the stiffness in these modes comes from the tires, and not from the suspension springs, whereas the damping comes only from the suspension dampers. When the body equations of motion are decoupled, the tire stiffnesses are not altered.

With this basic understanding of the dynamics of the system, we now turn to the frequency response analysis.

4.2 FREQUENCY RESPONSE

Frequency response analysis is used to evaluate the performance of the different suspension configurations. The measures of performance used fall into two categories, those relating to ride quality and body isolation, and those relating to roadholding, or more exactly the effect that traversing a rough road would have on the roadholding ability of the vehicle.

To obtain the frequency response of the vehicle, it is alternately subjected to simple road heave, road pitch, road roll, and road warp displacement inputs. The output-to-input amplitude ratio is computed for each input-output pair, for an assumed unit input amplitude: 1 m. for a heave input (HR), and 1 rad. for a pitch, roll, or warp input (PR, RR, or WR). It should be noted that specifying a unit pitch input, for instance, does not specify the inputs under the four tires, but only a relationship between them, given by
Eqn. (6) (similiarly for the other inputs).

The frequency range selected for calculating the frequency response results spans three decades, from 0.1 Hz to 100 Hz. This latter frequency is higher than is strictly needed; the range more than generously covers the majority of road input frequencies, even at high vehicle forward velocity.

The results that follow below fall into two categories: ride quality, and roadholding.

4.2.1 Ride Quality Performance

To study ride quality, the vertical acceleration of a point coinciding with the location of the driver's head is used. This acceleration is computed by linearly combining the acceleration of the body in the three degrees-of-freedom that it is allowed, with the weighing factors being: unity for heave acceleration; the longitudinal distance from the center-of-gravity to the driver's location for pitch acceleration; and the lateral distance between those two points for roll acceleration. Pitch and roll, therefore, are considered in terms of their contribution to vertical acceleration, which is used as a measure of discomfort, and not in terms of the sensitivity of the driver to accelerations in these d.o.f.

The reader should also note that, although the acceleration of the driver's head is considered, the dynamics of the seat and of the human body itself are not taken into
account in the model used here.

With the above in mind, the amplitude ratio of input displacement to the vertical acceleration of the driver point, when the vehicle is subjected to a pure heave sinusoidal input, is shown in Figure 4.1. Three cases are considered as shown: the baseline passive case, the case with increased relative damping, and the case with simple absolute damping. The results underline the tradeoff inherent in choosing the suspension damping rate, and show how absolute damping eliminates the trade off. The first peak at around 1.2 Hz corresponds to the sprung mass resonance in the coupled heave and pitch modes (roll not being excited because of lateral symmetry). The second peak at around 25 Hz corresponds to the unsprung mass resonance. When relative damping is increased to better control the sprung mass motion at its resonant frequency, the isolation at higher frequencies deteriorates, and the roll-off rate beyond the second resonance is reduced. Both effects result in a deteriorated ride at high frequency, and increased harshness. The added damping has only a minimal effect at the high frequency resonance; tire damping would be needed to control that peak.

By using absolute body velocity feedback instead of additional relative damping, that situation is eliminated: the body resonance is better controlled, without compromising high frequency isolation.
It should be mentioned here that the high frequency resonance occurs at a frequency which is higher than that normally found in real vehicles (25 Hz instead of 15 Hz approximately). This is due to the use of a higher than usual tire radial stiffness.

The next two figures serve to illustrate the effect of heave-pitch coupling. These show the amplitude ratios of body heave displacement and pitch displacement to a sinusoidal heave displacement input. Four cases are shown: the baseline case, the simple absolute damping case, the decoupled case, and the decoupled case with absolute damping. In order to obtain comparable displacement ratios, the pitch angle is multiplied by the distance from the C.G. to the front axle, thus yielding the approximate corresponding vertical displacement at the front axle line.

The body heave response to a heave input (Fig. 4.2) exhibits the typical behavior of a two-mass, one-dimensional suspension system, namely good roadway following properties at very low frequencies, up to the low frequency body resonance (around 1.2 Hz), followed by body isolation above that frequency, and by a sharper roll-off rate at higher frequencies, beyond the second resonant peak (around 25 Hz). Again, the use of absolute damping provides better resonance control around 1 Hz, without negatively affecting the desirable high frequency isolation. Decoupling the system has practically no effect on the heave response,
because both input and output are of the same type, heave in this case.

The unsprung mass pitch response to heave input (Fig. 4.3) exhibits different behavior. Although the addition of absolute damping controls the first resonant peak without affecting the response at higher frequencies, the response at frequencies lower than the first peak shows a marked deterioration. This agrees with the results reported by Margolis [14]. However, by decoupling the heave and pitch sprung mass equations, the pitch response to a heave input is reduced dramatically, up to the unsprung mass resonant frequency. In this particular case, a transfer function zero is introduced around 0.2 Hz, and its effect is to bring down the frequency response curve. When absolute damping is added to the decoupled system, the low resonance is reduced further. However this is of academic value since the amplitude ratio at the resonant peak is already very small (3x10E-3). One negative aspect of the decoupled system pertains to the loss of the transfer function zero around 30 Hz.

Next, the suspension compression response, in heave (Fig. 4.4) and in pitch (Fig. 4.5), is considered, again to heave displacement input. It is important to note that in none of the active systems does the suspension deflection (heave or pitch) exceed the maximum values reached by the passive (baseline) system. Therefore, none of the configur-
ations would require more rattlespace than the passive one. The suspension heave response exhibits more activity at frequencies below the first resonant peak, when absolute damping is used; the resonant peak itself is better controlled. Decoupling does not affect the heave response, again because both input and output are of the same type. The suspension pitch response (Fig. 4.5) exhibits some similarities to the body pitch response: absolute damping results in more suspension travel at low frequencies, while better controlling the first resonance. However, by decoupling the system, suspension travel is dramatically reduced at low frequencies. Above approximately 8 Hz, neither absolute damping nor decoupling shows any significant effect, as expected. It is worth noting that even in the decoupled system, the suspension pitch response deteriorates at frequencies below the first peak when absolute damping is added. Again, since the amplitudes involved are very small, this effect is unimportant.

Having considered the various vehicle responses when subjected to a sinusoidal heave input, it is noted that the responses to a pitch input are similar when the corresponding input-output pairs are considered. The responses of the sprung mass and suspension in pitch to a pitch input are similar to their heave responses to a heave input. Also, the heave responses to a pitch input exhibit the same behavior as the pitch responses to a heave input presented
above. For that reason, and because of the redundancy involved, the pitch input results are not shown here.

Having analyzed the heave-pitch subsystem, the roll-warp subsystem is considered next. In this case however, the roll and warp equations are not to be decoupled, as was explained in the previous chapter. This, and the fact that body warp is identically zero, make the roll-warp subsystem simpler to analyze.

Figures 4.6, 4.7, and 4.8 show the responses to roll displacement input. The cases considered here are passive baseline case, the coupled case with coupled absolute damping, and the coupled case with uncoupled absolute damping. The first two figures exhibit the expected behavior of a simple suspension with absolute body velocity feedback: better resonance control at the low resonant frequency for both the body and the suspension displacement, no detrimental effects at high frequency. The trade-off is increased suspension activity below the first resonant peak. Whether absolute damping is coupled or not has no effect on the response, because both the input and the outputs correspond to the same degree-of-freedom. It is worth noting that the body resonance in roll occurs around 3 Hz, due to the high stiffness of the anti-roll bar used.

The suspension warp compression to roll input (Fig. 4.8) shows that if decoupled absolute damping is used,
instead of simple (coupled) absolute damping, the increase in suspension activity at low frequency is reduced by a fair amount (keeping in mind that the scale is logarithmic), but not eliminated by any means. This indicates that this behavior is mostly due to the use of absolute body velocity feedback.

The last set of results concerning ride quality are the responses to warp displacement input, in Figures 4.9 to 4.11. These show that in the case of warp input, added relative damping actually presents advantages over absolute damping at high frequencies. Specifically, whereas absolute damping does not change the response of the sprung mass, compared to the baseline case, beyond the low frequency resonance, added relative damping actually reduces the amplitude at the high frequency resonance (Fig. 4.9). Furthermore, the detrimental effect of added relative damping between the resonances, and at higher frequencies is rather small.

The suspension warp response to warp input (Fig. 4.10) shows that the low frequency resonance is almost non-existent in this case, due to the fact that the body is assumed torsionally infinitely stiff. However, the high frequency resonance is present, and it is seen that added relative damping reduces the amplitude at that resonance, whereas absolute damping does not affect it. Similar behavior at the high frequency resonance is exhibited by the suspension roll response to a warp input (Fig. 4.11).
This behavior could be explained by the fact that the rigid sprung mass presents a fixed support for the dampers to act against in the warp mode, thereby increasing their effectiveness. Since that support is fixed, more of the input motion goes into relative motion across the dampers, instead of rigid body translation of those, thus increasing the effective damping ratio. In contrast, absolute damping can only respond to body roll velocity, since body warp is zero.

4.2.2 Roadholding Performance

Two performance measures are used to analyze the effect of the different vehicle configurations on roadholding. These are the fore-aft tire load transfer of FALT, and the lateral tire load transfer difference between the front and rear axles, or LLTD. Both measures concern the effect that road disturbance input has on the distribution of tire-to-road contact forces. It is assumed that the suspension system in the baseline vehicle has been optimized to take full advantage of the tires used, and that an increase in the tire contact force differences would result in decreased roadholding ability.

Fore-aft tire load transfer is defined as:

\[
FALT = \frac{FT_1 + FT_2 - FT_3 - FT_4}{FALT_{\text{max}}} ;
\]  

(79)
where $FT_i$ is the tire contact force at the $i$th corner, and $FALT_{max}$ is the fore-aft load transfer experienced by the vehicle when braking (or accelerating) at 0.85g. This is done in order to make FALT a dimensionless measure. It should be noted that although most production cars can achieve deceleration levels of 0.85g, only high-powered race cars can accelerate at that level.

A good suspension system should minimize FALT in response to road disturbances, in order for the vehicle to maintain its built-in smooth road handling and roadholding characteristics, even when maneuvering over rough roads. A sudden decrease in the normal load on one axle is accompanied by a similar decrease in the overall cornering or braking stiffness of that axle (force generated versus slip). If this were to happen during a hard cornering maneuver, larger slip angles would suddenly be required by the tires at the axle affected, in order to generate the needed lateral forces. This would result in an increase in the amount of understeer or oversteer at which the vehicle is operating, depending on whether the front or rear end is unloaded. In the extreme case where the tires have reached their cornering limit, the vehicle would either plow or spin out. During hard braking, with the tires near or at the impending lockup point, a sudden reduction of the tire normal load at the front or at the rear end could result in the front wheels locking, with the accompanying loss of
directional control, or the rear wheels locking, resulting in the loss of directional stability. In a combined braking and cornering maneuver, the above-mentioned problems would be compounded.

The FALT response to a pure heave input is shown in Figure 4.12, for the four vehicle configurations considered previously. The passive baseline vehicle shows a general increase in amplitude with frequency, with the expected low and high frequency peaks (around 1.2 Hz and 25 Hz respectively). The first peak is preceded, and the last one followed by sharp drops corresponding to zeroes in the transfer function.

Compared to the baseline case, absolute damping does not cause any deterioration in FALT except between 5 Hz and 8 Hz due to the loss of the sharp dip there. However, the loss of that transfer function zero, combined with the marked reduction of FALT at the low frequency resonance, has a smoothing effect on the FALT response, which should at least make the vehicle more predictable. Beyond the first peak, absolute damping has no effect on FALT.

When the vehicle heave and pitch modes are decoupled, the result is a higher FALT amplitude up to about 0.7 Hz, beyond which there is a marked reduction in FALT, up to the high frequency peak. The response of the decoupled system does not exhibit the sudden drop beyond that peak. The effect of adding absolute damping to the decoupled case is
to further reduce the amplitude of the low-frequency peak, making it almost critically damped. Because of the relatively low amplitudes of FALT at low frequencies (below 0.7 Hz), and because of the sizeable improvement in the important 1 Hz to 10 Hz range, as well as the overall smoothing effect throughout the frequency range, the decoupled suspension with absolute damping should be the best choice from a roadholding point of view.

That decoupling heave and pitch reduces FALT at the sprung mass resonant frequency, when the vehicle is subjected to a heave input is expected: decoupling reduces body pitch under heave input, and consequently reduces the resulting out-of-phase loading and unloading of the axles. Similar reasoning can explain why decoupling body heave and pitch has no effect on FALT (Fig. 4.13): decoupling reduces the body heave response to pitch input; however, heave of the sprung mass causes only a negligible amount of fore-aft load transfer, compared to pitch of the sprung mass. Again, absolute damping shows a beneficial effect on FALT.

The second measure for the effect of suspension configuration on roadholding, lateral load transfer difference, is given by:

$$\text{LLTD} = \frac{(-FT_1 + FT_2) - (FT_3 - FT_4)}{LLTD_{\max}}$$

\hspace{1cm} (80)
where \( LLTD_{\text{max}} \) is the total lateral load transfer experienced by the vehicle when cornering in steady state at 0.85g of lateral acceleration. Since LLTD is directly related to the roll couple distribution, it is clear why a good suspension system should minimize LLTD in response to road disturbance inputs.

Figure 4.14 shows the LLTD response to a pure road roll displacement input. The cases considered here, beside the baseline passive one, are the simple absolute body velocity feedback case, and the case where decoupled absolute body velocity feedback is used. For the reasons previously explained, the roll-warp stiffness and relative damping coupling terms are not eliminated. The response shows that absolute damping has the usual beneficial effect of better controlling the low-frequency resonance, which falls around 3 Hz in this case, without affecting the response at higher frequencies. However, it causes a sizeable increase in the LLTD amplitude below the 3 Hz resonance, compared to the baseline case. The best that can be done to reduce this effect is to decouple the roll-warp absolute damping, which brings a slight improvement. Another approach would be to use a high-pass filter to attenuate the body roll feedback signal at frequencies below about 2 Hz.

The LLTD response to road pitch input is shown in Figure 4.15 for the same three cases as above. In the baseline case, the response is almost flat up to low-frequency
resonance (around 3 Hz); this reflects the fact that for a pitch input, LLTD is mostly due to the static warp stiffness of the suspension, and not to any dynamic effect, up to a frequency of about 1 Hz. As the input frequency gets closer to the body roll resonance, the coupling between roll and warp starts playing a role in the LLTD response. Only for large body roll displacement does the LLTD caused by the coupling approach the magnitude of the LLTD caused by suspension warp. Absolute damping reduces the amplitude of body roll at resonance, and thus results in a reduction in LLTD amplitude at that peak. Finally, the effect of removing the absolute damping coupling term is minimal, since that coupling only plays a secondary role, for the reasons mentioned above.
Table 4.1 Vehicle Parameters

Inertias:

- **Sprung Mass:**
  - Mass \( (M) \): 1,385 Kg
  - Roll Moment of Inertia \( (I_x) \): 400 Kg.m\(^2\)
  - Pitch Moment of Inertia \( (I_y) \): 2,120 Kg.m\(^2\)

- **Unsprung Mass:**
  - Front \( (m_f) \): 30 Kg
  - Rear \( (m_r) \): 40 Kg

Dampers:

- **Damper Rate (at wheel):**
  - Front \( (b_f) \): 922 N.S/m
  - Rear \( (b_r) \): 1,747 N.S/m

Stiffnesses:

- **Spring Rate (at wheel):**
  - Front \( (k_f) \): 18.63 KN/m
  - Rear \( (k_r) \): 22.55 KN/m

- **Anti-Roll Bar Rate (at wheel):**
  - Front \( (k_{sf}) \): 83.4 KN/m
  - Rear \( (k_{sr}) \): 10.2 KN/m

- **Tire Radial Stiffness:**
  - Front \( (k_{tf}) \): 800 KN/m
  - Rear \( (k_{tr}) \): 880 KN/m

Dimensions:

- **Distance from C.G. to Axle:**
  - Front \( (a) \): 1.275 m
  - Rear \( (b) \): 1.225 m

- **Half-Track:**
  - Front \( (t_f) \): 0.7725 m
  - Rear \( (t_r) \): 0.7570 m

- **Distance from C.G. to Driver:**
  (Coordinates of driver's location):
  \( x_d \): -0.450 m
  \( y_d \): -0.335 m
  \( z_d \): -0.630 m
Table 4.2 Damped Natural Frequencies (Hz) and Damping Ratios of the Various Vehicle Modes.

<table>
<thead>
<tr>
<th>Vehicle Configuration</th>
<th>( \omega_d ) (Hz)</th>
<th>( \xi )</th>
<th>Natural Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Vehicle</td>
<td>1.20 0.30</td>
<td>Body Heave/Pitch (in-phase)</td>
<td></td>
</tr>
<tr>
<td>(Passive)</td>
<td>1.16 0.18</td>
<td>Body Heave/Pitch (out-of-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.78 0.13</td>
<td>Body Heave/Roll</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.5 0.15</td>
<td>Suspension Heave/Pitch (in-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.1 0.09</td>
<td>Suspension Heave/Pitch (out-of-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28.8 0.09</td>
<td>Suspension Roll/Warp (in-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.8 0.15</td>
<td>Suspension Roll/Warp (out-of-phase)</td>
<td></td>
</tr>
<tr>
<td>Simple Absolute Damping (Coupled)</td>
<td>1.15 0.42</td>
<td>Body Heave/Pitch (in-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.12 0.31</td>
<td>Body Heave/Pitch (out-of-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.74 0.21</td>
<td>Body Heave/Roll</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.5 0.15</td>
<td>Suspension Heave/Pitch (in-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>26.1 0.09</td>
<td>Suspension Heave/Pitch (out-of-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>28.7 0.09</td>
<td>Suspension Roll/Warp (in-phase)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23.8 0.15</td>
<td>Suspension Roll/Warp (out-of-phase)</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.3 Damped Natural Frequencies, Damping Ratios, and Eigenvector Amplitudes for the Heave-Pitch Subsystem.

<table>
<thead>
<tr>
<th></th>
<th>$\omega_d$ (Hz)</th>
<th>$\zeta$</th>
<th>$\nu$</th>
<th>$\zeta$</th>
<th>$\omega_d$ (Hz)</th>
<th>$\zeta$</th>
<th>$\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Vehicle</td>
<td>26.1</td>
<td>.09</td>
<td>&lt;.01</td>
<td>.01</td>
<td>26.0</td>
<td>.13</td>
<td>&lt;.01</td>
</tr>
<tr>
<td>Vehicle (Passive)</td>
<td>23.5</td>
<td>.15</td>
<td>&lt;.01</td>
<td>.01</td>
<td>23.7</td>
<td>.11</td>
<td>&lt;.01</td>
</tr>
<tr>
<td></td>
<td>1.16</td>
<td>.18</td>
<td>.13</td>
<td>.13</td>
<td>1.18</td>
<td>.24</td>
<td>.13</td>
</tr>
<tr>
<td></td>
<td>1.20</td>
<td>.30</td>
<td>.13</td>
<td>.12</td>
<td>1.19</td>
<td>.24</td>
<td>.01</td>
</tr>
<tr>
<td>Uncoupled</td>
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<td>.13</td>
<td>&lt;.01</td>
<td>.01</td>
<td>26.0</td>
<td>.13</td>
<td>&lt;.01</td>
</tr>
<tr>
<td>Suspension</td>
<td>23.7</td>
<td>.11</td>
<td>&lt;.01</td>
<td>.01</td>
<td>23.7</td>
<td>.11</td>
<td>&lt;.01</td>
</tr>
<tr>
<td></td>
<td>1.18</td>
<td>.24</td>
<td>.13</td>
<td>&lt;.01</td>
<td>1.18</td>
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<td>1.19</td>
<td>.24</td>
<td>.01</td>
<td>.13</td>
<td>1.19</td>
<td>.24</td>
<td>.01</td>
</tr>
</tbody>
</table>
Fig. 4.1 Vertical Acceleration at Driver's Location, Under Heave Input
Fig. 4.2 Sprung Mass Heave Response to Heave Input
Fig. 4.3 Sprung Mass Pitch Response to Heave Input
Fig. 4.4 Suspension Compression in Heave Due to Heave Input
Fig. 4.5 Suspension Compression in Pitch Due to Pitch Input
Fig. 4.6 Sprung Mass Roll Response to Roll Input
Fig. 4.7 Suspension Compression in Roll Due to Roll Input
Fig. 4.8 Suspension Compression in Warp Due to Warp Input
Fig. 4.9 Sprung Mass Roll Response to Warp Input
Fig. 4.10 Suspension Compression in Warp Due to Warp Input
Fig. 4.11 Suspension Compression in Roll Due to Warp Input
Fig. 4.12 Fore/Aft Tire Load Transfer Due to Heave Input
Fig. 4.13 Fore/Aft Tire Load Transfer Due to Pitch Input
Fig. 4.14 Tire Lateral Load Transfer Difference (LLTD) Between Front and Rear Axles, Due to Roll Input
Fig. 4.15 Tire Lateral Load Transfer Difference (LLTD) Between Front and Rear Axles, Due to Warp Input
5.1 CONCLUSIONS

The purpose of this study was to consider active suspensions in the context of a complete vehicle, instead of relying on a simple quarter-vehicle model. The reason being that such a simplified model does not reflect the coupling present between the various vehicle modes, and thus cannot predict the effect of the active suspension control strategy on the heave and pitch interaction from the ride comfort point of view, or on the roll and warp interaction from the roadholding point of view. Therefore, a three-dimensional, seven d.o.f. vehicle model was used to analyze an active suspension system. The control strategy used provides for full suspension interconnection between the four corners, as well as absolute sprung mass velocity feedback (absolute damping).

The fully interconnected suspension permits the arbitrary and independent specification of all the suspension parameters, namely stiffnesses, relative damping rates, and absolute damping rates, for each mode. It also allows the arbitrary specification of the coupling parameters between the different modes.

This capability was used to compare the frequency response of different outputs of interest, both for ride
quality and for roadholding, for different suspension configurations. The configurations considered were: the baseline passive case, the sprung mass heave-pitch decoupled case, the case where simple absolute damping was added, the heave-pitch decoupled case with absolute damping, and the case with decoupled roll-warp absolute damping. The sprung mass roll-warp coupling stiffness and relative damping were not altered, in order to maintain the handling balance of the vehicle.

Decoupling the sprung mass heave and pitch equations of motion was found to eliminate a shortcoming of absolute damping, namely increased body pitch displacement to a heave input, and vice versa, at frequencies below the sprung mass resonance. The above-mentioned shortcoming is one of the effects of active suspensions that cannot be predicted by a quarter-vehicle model. Furthermore, heave-pitch decoupling, combined with absolute damping, was also found to reduce fore-aft tire load transfer, which should have a beneficial effect on roadholding while the vehicle is traversing bumpy roads. However, these improvements were at the expense of increased suspension activity at low frequencies. This might require the suspension designer to pay more attention to the deflection steer effect, as well as the camber changes with suspension deflection, in order to realize the improvements in roadholding mentioned above. This is especially critical at low lateral accelerations, where the tires have not
reached their non-linear cornering stiffness versus normal load range yet.

Absolute body roll damping was found to offer the expected improvements from the body isolation point of view. However, it was found to increase the effect of road roll disturbance inputs on the lateral load transfer difference between the front and rear axles, at frequencies below the body roll resonance, with the expected detrimental effect on rough-road handling and roadholding. It was proposed to high-pass filter the body roll rate measurement before using it to provide absolute damping, in order to eliminate that low-frequency problem.

In the case of a warp input, increased relative damping was found to have advantages over absolute damping for the following reason: in warp, the body can be thought of as having infinite inertia, therefore countering the high frequency "locking effect" of the dampers.

5.2 RECOMMENDATIONS FOR FURTHER INVESTIGATION

Future research should aim at ascertaining whether the performance improvements found in this study, for certain suspension configurations, would still be significant when the vehicle is subjected to combined inputs, in particular a random road profile. The effect of the vehicle parameters should also be analyzed.

Furthermore, a handling model including a yaw and
sideslip degrees-of-freedom should be used to analyze the effect of absolute body roll damping on the vehicle response to a steering input. Absolute body roll damping may cause some sluggishness in the steering response. If this were to prove true, and in view of the limitations that were found to result from relying on body roll to generate a LLTD, then it may be advantageous to synthetically cause a LLTD in the suspension in response to variables other than the state or its derivative.
REFERENCES


20. Ibid., Ch. 4.


APPENDIX A
The equations of motion of the system are expressed in matrix form as follows:

\[ [M] \{X\} + [D]\dot{\{X\}} + [K]\{X\} = [BR]\{XR\} \quad ; \quad (A.1) \]

where the vector \{X\} of variables describing the system is:

\[ \{X\}^T = \{H, P, R, HS, PS, RS, WS\} \quad ; \quad (A.2) \]

and the road disturbance input vector \{XR\} is:

\[ \{XR\} = \{HR, PR, RR, WR\} \quad . \quad (A.3) \]

The inertia matrix \([M]\) is diagonal:

\[ [M] = \begin{bmatrix}
M & I_y & 0 \\
I_y & I_x & I_{UH} \\
0 & I_{UP} & I_{UR} \\
0 & 0 & I_{UR} \\
\end{bmatrix} \quad ; \quad (A.4) \]

where:

\[ I_{UH} = \frac{2(a+b)^2 \cdot m_f \cdot m_r}{a^2 \cdot m_f + b^2 \cdot m_r} \quad ; \quad (A.5) \]

\[ I_{UP} = \frac{2(a+b)^2 \cdot m_f \cdot m_r}{m_f + m_r} \quad ; \quad (A.6) \]
and
\[ IU_R = IU_W = \frac{4 t_f \cdot t_r (b \cdot t_f + a \cdot t_r) \cdot m_f \cdot m_r}{a \cdot t_f \cdot m_f + b \cdot t_r \cdot m_r}. \quad (A.7) \]

The elements of the damping matrix \([D]\) can be easily expressed if the following vehicle parameters are first defined:

\[ a_H = \frac{IU_H}{M} + 1 \quad ; \quad (A.8) \]
\[ a_P = \frac{IU_P}{I_y} + 1 \quad ; \quad (A.9) \]
\[ a_R = \frac{IU_R}{I_x} + 1 \quad ; \quad (A.10) \]
\[ a_W = 1 \quad ; \quad (A.11) \]

and also:
\[ \beta_H = \frac{a \cdot m_f - b \cdot m_r}{a^2 \cdot m_f + b^2 \cdot m_r} \quad ; \quad (A.12) \]
\[ \beta_P = \frac{a \cdot m_f - b \cdot m_r}{m_f + m_r} \quad ; \quad (A.13) \]
\[ \beta_R = \beta_W = \frac{-a \cdot t_f \cdot m_f + b \cdot t_r \cdot m_r}{a \cdot t_f \cdot m_f + b \cdot t_r \cdot m_r}. \quad (A.14) \]

Using these parameters, the non-zero damping terms are:
D_{11} = \text{ADHH} \quad ; \\
D_{12} = \text{ADPH} \quad ; \\
D_{14} = \text{DHH} \quad ; \\
D_{15} = \text{DPH} \quad ; \\
D_{21} = \text{ADHp} \quad ; \\
D_{22} = \text{ADPp} \quad ; \\
D_{24} = \text{DHp} \quad ; \\
D_{25} = \text{DPp} \quad ; \\
D_{33} = \text{ADRR} \quad ; \\
D_{36} = \text{DRR} \quad ; \\
D_{37} = \text{DWR} \quad ; \\
D_{41} = \alpha_H \cdot \text{ADHH} + \beta_H \cdot \text{ADHp} \quad ; \\
D_{42} = \alpha_H \cdot \text{ADPH} + \beta_H \cdot \text{ADPp} \quad ; \\
D_{44} = \alpha_H \cdot \text{DHH} + \beta_H \cdot \text{DHp} \quad ; \\
D_{45} = \alpha_H \cdot \text{DPH} + \beta_H \cdot \text{DPp} \quad ; \\
D_{51} = \alpha_P \cdot \text{ADHP} + \beta_P \cdot \text{ADHH} \quad ; \\
D_{52} = \alpha_P \cdot \text{ADPP} + \beta_P \cdot \text{ADPH} \quad ; \\
D_{54} = \alpha_P \cdot \text{DHp} + \beta_P \cdot \text{DHH} \quad ; \\
D_{55} = \alpha_P \cdot \text{DPp} + \beta_P \cdot \text{DPH} \quad ; \\
D_{63} = \alpha_R \cdot \text{ADRR} + \beta_R \cdot \text{ADRw} \quad ; \\
D_{66} = \alpha_R \cdot \text{DRR} + \beta_R \cdot \text{DRW} \quad ; \\
D_{67} = \alpha_R \cdot \text{DWR} + \beta_R \cdot \text{DWR} \quad ;
\[ D_{73} = \alpha_W \cdot ADR_W + \beta_W \cdot ADR_R \quad ; \quad (A.37) \]
\[ D_{76} = \alpha_W \cdot DR_W + \beta_W \cdot DR_R \quad ; \quad (A.38) \]

and
\[ D_{77} = \alpha_W \cdot DW_W + \beta_W \cdot DW_R \quad . \quad (A.39) \]

The following additional parameters are needed to express the elements of the stiffness matrix \([K]\) concisely:

\[ \gamma_H = 2(a+b) - \gamma = 2(a+b) - (A.37) \]
\[ \gamma_P = 2(a+b) - \gamma = 2(a+b) - (A.38) \]
\[ \gamma_R = \gamma_W = IU_R \cdot \left( \frac{KT_f}{2 \cdot m_f} + \frac{KT_r}{2 \cdot m_r} \right) \quad ; \quad (A.42) \]

and

\[ \delta_H = 2(a+b) - \gamma = 2(a+b) - (A.39) \]
\[ \delta_P = 2(a+b) - \gamma = 2(a+b) - (A.40) \]
\[ \delta_R = \delta_W = IU_R \cdot \left( \frac{KT_f}{2 \cdot m_f} - \frac{KT_r}{2 \cdot m_r} \right) \quad ; \quad (A.45) \]

The non-zero elements of \([K]\) are:
\[
\begin{align*}
K_{14} &= K_{HH} \quad ; \\
K_{15} &= K_{PH} \quad ; \\
K_{24} &= K_{HP} \quad ; \\
K_{25} &= K_{PP} \quad ; \\
K_{36} &= K_{RR} \quad ; \\
K_{37} &= K_{WR} \quad ; \\
K_{41} &= -\gamma_H \quad ; \\
K_{42} &= -\delta_H \quad ; \\
K_{44} &= \alpha_H \cdot K_{HH} + \beta_H \cdot K_{HP} + \gamma_H \quad ; \\
K_{45} &= \alpha_H \cdot K_{PH} + \beta_H \cdot K_{PP} + \delta_H \quad ; \\
K_{51} &= -\delta_P \quad ; \\
K_{52} &= -\gamma_P \quad ; \\
K_{54} &= \alpha_P \cdot K_{RP} + \beta_P \cdot K_{HH} + \delta_P \quad ; \\
K_{55} &= \alpha_P \cdot K_{PP} + \beta_P \cdot K_{PH} + \gamma_P \quad ; \\
K_{63} &= -\gamma_R \quad ; \\
K_{66} &= \alpha_R \cdot K_{RR} + \beta_R \cdot K_{RW} + \gamma_R \quad ; \\
K_{67} &= \alpha_R \cdot K_{WR} + \beta_R \cdot K_{WW} + \delta_R \quad ; \\
K_{73} &= -\delta_W \quad ; \\
K_{76} &= \alpha_W \cdot K_{RW} + \beta_W \cdot K_{RR} + \delta_W \quad ; \\
\text{and} \\
K_{77} &= \alpha_W \cdot K_{WW} + \beta_W \cdot K_{WR} + \gamma_W \quad .
\end{align*}
\]
Finally, the non-zero elements of the input distribution matrix \([BR]\) are:

\[
\begin{align*}
BR_{41} &= -\gamma_H \\
BR_{42} &= -\delta_H \\
BR_{51} &= -\delta_P \\
BR_{52} &= -\gamma_P \\
BR_{63} &= -\gamma_R \\
BR_{64} &= -\delta_R \\
BR_{73} &= -\delta_W \\
\text{and} \\
BR_{74} &= -\gamma_W
\end{align*}
\]