THE DYNAMIC BEHAVIOR OF BUSINESS SYSTEMS

BY THE SIMULATION TECHNIQUE

HENRY WEBER

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Certified by ____________________
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ABSTRACT

Title: THE DYNAMIC BEHAVIOR OF BUSINESS SYSTEMS BY THE SIMULATION TECHNIQUE

Author: Henry Weber

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This paper is an attempt to demonstrate the simulation of the production and management control in a manufacturing operation. Part I of Chapter I is a discussion of the simulation method. The basic approach is outlined along with some of the future benefits available to management through simulation. Several problems unique to this technique are treated in an effort to show processes which can now be simulated and those which are still beyond the scope of the technique thus far. Part II deals with concepts of Operations Resdarch and some of the original applications of the simulation technique. Part III concerns itself with problems of interest in the development of this particular paper. Parameter effect on transient response and analogies between physical and business systems are also discussed.

Chapter II is the development of the model for a manufacturing operation. To facilitate as rapid as possible an understanding of the model two run-throughs or explanations are given. From pages 16 to 22 each section of the system is treated with regard to its effect on the overall system. Between pages 22 and 45 a detailed analysis of each subsystem with block diagrams and mathematical formulation in difference equation form is presented.

Chapter III mentions some future projects that would be interesting extensions of what has already been completed.

Thesis Supervisor: Jay W. Forrester

Title: Professor of Industrial Management

Date: May 20, 1957.

The author is indebted to many for their invaluable aid during the development of this thesis. However, special thanks are due Professor J. W. Forrester for his guidance and constructive criticism throughout each stage of the model. Also Prof. Joe Yance was unusually generous with time and effort in giving a penetrating appraisal of the organization and factual accuracy of this paper.

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CHAPTER I

THE SIMULATION TECHNIQUE

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PART I

This paper is an attempt to demonstrate the simulation of the production and management control in a manufacturing operation. A selection of the most salient variables characterizing the system of the firm is made; then, beginning with a set of initial conditions and functional relationships for the group of variables selected, the value of each variable at progressive instants in time can be determined. This process if known as simulation.

If a business simulator with results approximating those of the actual system can be devised, it would provide an expedient tool to management. The simulation process necessarily starts in the past, so that simulated data can be compared to historic results as a validity check; however, there is no reason why the time variable cannot be projected beyond the present into the future. The simulation process then becomes one of prediction. The results of alternative management policies can be calculated in advance, after which management could pick the optimum policy to follow.

The long run purpose of this study is to provide the field of management science with more accurate criteria in decision-making. For its initial objective, the disposition of this paper is to shed more light on existing business structures and methods by which decisions are currently formulated.

Let us analyze the problem by direct approach. First, the functional structure of a typical firm is developed to determine the major operational sections, such as the production process, the inventory process, labor flow, etc. These blocks are closely related to the conventional departmental classifications found in most organizations.

The second step undertakes to investigate the inter-relationships existing between sections of the firm. The set of variables selected is important from several standpoints. Not only must the operational system be completely specified, but these variables must have functional dependencies upon one another capable of being defined mathematically.

To illustrate: assume that product sales increase by 10% over last month's rate. The change in this one variable (sales sector) will induce fluctuations in practically all other variables within the system. As sales increase, stock of finished-goods inventories must necessarily fall. Similarly, the production group will receive demands for higher production which in turn causes increases to occur in the purchasing block and in the labor hiring rate.

The important concept to grasp is that the effects of even a small change in one variable are likely to be felt throughout the firm. The problem can become serious, as will be shown later; small perturbations of one variable can cause rather large oscillations of another variable when the system tends to be unstable. It is this kind of relationship which demands careful consideration.

There are two immediate objectives to be realized from this type of analysis leading to simulation models. Close scrutiny of an existing system brings forth inconsistencies, defects, and illogical management methods which bear improving to produce a more effective system. Secondly, one is made aware of repetitive decisions effected by management personnel under the present system, which, for maximum efficiency, should be processed by machine.

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To synthesize a device which can simulate the activities of the firm will require a duplication of decisions made by the managerial staff. A thorough investigation into the mental processes that take place prior to these decisions establishes the operating relationships between the different sections of the business.

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For a starting point, consider a few of the decision-making qualifications prerequisite for a human being to participate in the coordination of a company's activities. General fundamental knowledge of the business field is basal; familiarity with company policy and experience with former precedents set by others are indispensable; sensitivity and "feel" for the company's current situation in respect to the market and competition are paramount. These qualifications represent information stored in the human memory. This process of absorbing pertinent information - this training period for an executive - is extensive, costing both time and money.[1.]

Individual acquisition of the required data takes a long time, although processing the "stored" information to arrive at a solution or decision seems to occur automatically. Logically, there must be some

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1. H. A. Simon, Administrative Behavior; A Study of Decision-Making Processes in Administrative Organizations.

program of operation by which the original data is manipulated to produce the answer, regardless of the illusory, seemingly intuitive process. Most people, after making a decision, are unable to identify the steps through which they obtain their answer. The sequence of mental steps we use in everyday thinking handles large amounts of input information and somehow correlates it to produce a reasonable answer; but we really don't know how we get this answer. This built-in programming ability possessed by fortunate people is what is termed judgment.

Before it is feasible to construct a machine which will simulate business operations, there must be a scheme to include within the machine a sense of business judgment. Is it actually such a hopeless task? The problem is not so baffling as it would seem if we clarify the area to which the proposed simulator will apply. The category of unprogrammed decision processes which are not routine and cannot yet be built into a model of the firm will not be discussed. The manufacturing system presented in this paper is constructed around repetitive, routine decisions. A sequence of models must be built---each model characterizing that portion of the real business system equivalent to a specific process from which man-made decisions have evolved. With the transfer function for each subsystem known and the correct relationship between subsystems known, the dynamic behavior of the firm can be computed; this is the technique of simulation.

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The human brain stores prodigious amounts of information, produces it in fast, random fashion; it generally cannot reproduce its own program by which we treat data. Individuals often arrive at conflicting solutions. Whenever dealing with fallible mental processes, one is bound to experience inconsistency and irrationality.[2]

The electronic computer has many similarities to the human thinking process. It holds information available; it can process quantitative information into a unified result. The computer memory is much smaller in capacity, but the important difference is in programming. Each step or operation in its program must be precisely determined. Once the program has been formulated for the machine, a given set of input information uniquely specifies the output solution. The machine is guiltless of the ambiguity found in humans.

Most progressive businessmen will agree that for data processing the speed and versatility of a computer are highly desirable to increase the efficiency of the operation. However, it is always necessary to justify the use of a computer from a cost argument. The more diverse applications to which a computer can be applied, the more feasible the installation becomes financially. The primary application is straight forward data handling (viz. payroll, inventory accounting, etc.). The last two or three years have seen digital machines gradually expand into the realm of simple decision-making, such as inventory re-ordering and market analysis problems. But the ultimate use of a digital computer as a business simulator would consummate its varied applications as an invaluable aid to management and thereby completely justify its expense.[3]

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2. Woolridge, Dean E., "Trends in Electronic Business Data Systems Development," Trends in Computers: Automatic Control and Data Processing, Proceedings of Western Computer Conf. Los Angeles, Cal. 1954, New York: A.I.E.E.

3. Hurd, Cuthbert C., Computing in Management Science, Management Science, Vol. 1 No. 2, January 1955.

   Berkeley, Edmund C., "Avenues for Future Development in Computing Machinery," Computers and Automation April, 1953.

The most workable approach to the simulation problem is through models of the firm. The fundamental programming information is the formulation of the model leading to the mathematical expressions for the structure of the firm. The next section of this chapter will review previous ideas for models which have dealt primarily with the general economy.

PART II

Brief mention of the type work that has been done leading towards a model of the firm should be useful as an aid in understanding the model to be described in Chapter II. Literature on a realistic, working model of the firm in its entirety as a system is virtually non-existent, but similar problems have been discussed especially in economics. Preliminary attempts in economics were made around the turn of the century. Typically, most formulations were linear and highly simplified. The desire to achieve something simple was so great that the resulting models became too general. Although they were unrealistic and of little immediate practical value, it was this beginning research which stimulated interest in the use of mathematics to describe economic processes. In 1937 Marshall actually proposed a non-linear model; Marshall's work was subsequently analyzed and extended in a paper by Frisch;[5] in 1950 W. W. Cooper made his contribution to the expanding collection of literature on this subject.[6]

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5. Frisch, Ragnar, Notes on Time Series. Q.J.E. 53, August 1939.

6. Cooper, W. W., Extending the Theory of the Firm, Q.J.E. 55, February 1951

By 1952 Arnold Tustin, whose main line of endeavor had been electrical engineering, demonstrated striking similarities between proposed economic models and electrical systems. Tustin, using models that had been developed by others, indicated the parallel between the behavior of closed-loop servo-systems and economic behavior in the modern society.[7] He applied techniques devised for automatic control systems to the economic model. Tustin's significant contribution to the field is this recognition of the possibility to merge techniques between electrical engineering and theory of the firm.

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In the last ten years largely as an offshoot of work done in military logistics during World War II, a new science known as Operations Research has evolved. Workers in the operations research field are generally concerned with specific and localized problems within the firm in contrast to the emphasis in this paper on an entire system. Like simulation, mathematical models describing the functional relationship of several variables of interest are constructed; the sub-system in question is then optimized in terms of highest efficiency and profit for the over-all operation.[8]

To illustrate the operations research method there is an important problem that recurs in most firms carrying inventory. What quantity of each item should be ordered from the supplier to minimize expenses?

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7. Tustin, Arnold, Mechanism of Economic Systems, 1953, London, Heinemann

8. Vazsonyi, Andrew, Industrial Operations Research and Mathematical Models, Ramo-Woolridge, Los Angeles, 1954.

Churchman, Ackoff and Arnoff, Introduction to Operations Research, Wiley, New York, 1956.

Cost factors to be considered in this decision are divided into two groups: one favoring small purchase lots and the second suggesting large orders. Discounts given on quantity purchases, advantageous timing with the market, and fixed clerical costs associated with each order suggest large orders and high inventories. Conversely, large inventories effect an increase in the required working capital, warehouse charges, spoilage, pillferage, and in clerical control costs----all of which boost overhead. There is some optimum compromise between these two possible extremes. By a summation of each cost component a function for total yearly cost can be derived. If the derivative of total cost in respect to order size is set equal to zero, the minimum value of the cost versus order size function is specified.[10]

This example was mentioned not for the sake of its result, but because it illustrates one of the simplest operations research techniques. A model is formulated, optimized, and solved. Operations research has been described as the use of mathematics to aid the manager in making wise decisions.

For the same purpose of background for the reader another tool of O.R. is cited. "Linear programming", proved useful in the study of industrial operations, employs a large number of linear equations to describe a process. The number of variables exceeds the number of equations; hence, a unique solution is impossible. By maximizing or minimizing some function of the variables one obtains a "best" solution. These sets of equations derived from linear programming typically involve a large number of unknowns.

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10. Canning, Richard G., Electronic Data Processing for Business and Industry, New York, John Wiley, 1956

Solution on a computer is the only practical method of handling the large number of operations required.[9]

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Simulation is still another technique to study industrial operations, but there is a fundamental difference between the operations research methods mentioned and simulation. The latter recreates mathematically the behavior of a system over a period of time. It is this time dependence that is particularly characteristic of the simulation process.

Here are some typical simulations. As the field of chemistry has become better understood in recent years, it has been possible to describe the chemical processes occurring within a plant mathematically. It is possible to carry out a series of trial experiments by simulation. One writes a set of equations describing the plant and then solves the system repeatedly for a number of varying conditions. Although not as accurate as operating the plant itself, it is a good deal cheaper to experiment on paper than by building plants and later discarding them.

Simulation has also been used extensively in aeronautical engineering and missile research. In designing specifications for a new aircraft, a selection of optimized parameters is extremely complex because of the necessary best compromise between a number of critical factors; viz. top speed must be balanced against other conflicting desirables like armor, electronic gear, range, etc.[3] The record of the Japanese Zeros is a good example of the balance between safety and performance in aircraft design.

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9. Churchman, Ackoff and Arnoff, Introduction to Operations Research, Wiley, New York, 1956.

3. Hurd, Cuthbert C., Computing in Management Science, Management Science, Vol. 1 No. 2, January 1955.

The flight characteristics of an airplane can be described in terms of differential equations into which enter such parameters as weight, amount of fuel, size of power plant, etc. Such equation sets for flight characteristics have no unique solution; neither is it known how to supply an additional condition which leads one directly to an optimum solution.[11] It has been found both economical and practical to calculate the solution to these differential equations for a large number of choices of parameters, to supply the resulting solutions to the design engineers in order that they may choose the appropriate combination of parameters for best design. Effectively, one simulates the flight characteristics of many different airplanes through the use of a computer. This method is used because no unique or optimizing direct solution exists.

Similarly, in the development of radar defense systems, the simulated flight paths of approaching aircraft have been used as the input information for evaluation of the defense system under study.

Simulation has been used extensively in the scientific and engineering fields, but only recently has there been much thought on applying simulation to the study of business operations. William Fair of Stanford Research Institute in 1951 was one of the first to attempt a simulation of the business enterprise.[12] He envisioned an analogue machine set up in terms of servo theory concepts much as Tustin had done with economics. Fair believes that the important aspect of the closed-loop system synthesis approach is that actual business decisions are normally reached by a process of synthesis rather than analysis. The synthesis approach

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11. Proceedings of the National Simulation Conference, Dallas, Southern Methodist University, Dallas, 1956

12. Fair, William, Analogue Computation of Business Decisions, Journal of Operations Research Society of America, Vol. I, No. 4, Aug. 1953.

is to be given input and output functions appropriate to the system. From these one must synthesize the network structure or transfer function. In the typical analysis approach one calculates output response given input and transfer functions.

Fair carries his analogy even further by writing, "The problem bears a close resemblance to that facing a process-control engineer. Part of his system is fixed and therefore measurable. The other part is to be designed at his option, but he does not know the values of the variables his apparatus must control. He must design equipment that takes action when it receives input information in the form of a changed value of a variable. Similarly the decision-making unit of a business firm must synthesize rules for taking action when it receives information, i.e., it must lay down policy."

His proposed computer is an analogue simulator which must be set up by the entrepreneur about to make a policy decision. The machine is set up for each alternative policy under consideration. Every alternative can be evaluated in terms of a "dollar profit rating". The business manager continues to try different policies on his machine. The number of tests evaluated is limited only by the amount of time available before a decision must be made.

In theory the foregoing is fine, but there is one practical problem. In synthesizing one of many possible structures of a firm, a great deal of time is consumed. The writer can speak from intimate experience; building even one model is a lengthy, tedious process. It seems questionable that the manager "consulting" this machine will have time to test out many policies.

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It is hoped that the preceding discussion has stimulated the reader's interest along two lines: the potential usefulness of simulation as a management tool, and the background thinking that has been done in the past on related problems.

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PART III

The most important single consideration throughout the development of the model in Chapter II will be the dynamic behavior of the firm. It is this dynamic aspect which has so frequently been neglected in past models; and yet it is obviously of crucial importance to know not only what the effect of a present policy will be upon the operation, but when this effect will manifest itself. Indeed, it seems impossible to have a realistic idea of policy effect without an understanding of its time dependence. This model will be concerned with a sequence of events in time rather than a static set of relationships. The variable time is always significant and considered to be the independent variable of the system.

With a study of the dynamic behavior of any system, one becomes especially interested in the transient response to different types of disturbances.[13] What are the significant factors which cause instability? On what parameters depend the natural frequencies of oscillation? For most closed loop systems one finds time delays closely associated with stability. What will take place when a business converts its information handling procedures from manual to fast electronic methods? Some time

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13. Campbell, D. P., Dynamic Behavior of Linear Production Systems, A.S.M.E. Mech. Eng. April 1953.

delays will be important; others unimportant. Only the significant delays will be worth modification. Such questions can be answered by analyzing the equations derived from the model. Like electrical networks this system must be characterized by a set of natural frequencies which can be derived from the system functions.[14] Techniques such as the s-plane representation and the Nyquist diagram are useful in electrical network theory to show the effects of the natural frequencies on system response. After a set of equations has been formulated for the business firm, one of the methods used in servo or network theory will undoubtedly be invaluable to portray parameter effect on dynamic behavior.

Past models were frequently based on a set of linear relationships. A small amount of reflection on almost any portion of a business is enough to realize that this restriction of linearity is highly unrealistic. It seems too bad to compromise the validity of the model from its inception. When studying the dynamics of physical systems, one talks of quantities like mass, acceleration, and force for mechanical systems, or voltage, derivatives of current and energy storage elements in electrical engineering. Similarly, for the dynamics of a manufacturing process there are comparable variables. Instead of physical inertia there is a "resistance to change", that human inertia which opposes new ideas, clings to old traditions, and hampers the learning process. Instead of a storage element in the form of a condenser, we deal with inventory, a different type of storage reservoir to be sure, but still the concept of accumulation is similar. Most important, the mathematical formulation is the same. The electrical delay line finds its counterpart in accounting lags, transportation time, and delays in information flow channels. The only difference is one of scale factor. Whereas delays in the electrical network

14. Guillemin, E. A., Network Analysis and Synthesis, M.I.T.E.E. notes unpublished 1956.

might be in the order of microseconds, the businessman may well find that his time lags run into years.

Since the basic functions of these two fields have many similarities, transfer and response characteristics undoubtedly show related behavior patterns. Differential equations are particularly well suited to describe the dynamic relations in the conventional physical systems. Their use would be equally desirable for an analogue formulation such as Fair had in mind. The model discussed in this paper will be based on variables changing sequentially as in a time series. This type of formulation will be more convenient to deal with from the numerical analysis standpoint. The differential equations become difference equations; this technique lends itself to a step-by-step solution implemented on a digital computer.

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The manufacturing process is a closed-loop system. The main feed-forward path is production, converting raw material and labor into a useful product, while proceeds from sales are converted into labor and raw materials, hence, the feedback in the system. A closer examination will uncover the existence of many smaller feedback paths and will disclose the fact that the business entity we are concerned with is really a small subsystem within the complex national economy.

As suggested earlier, not only do many of the variables of the business world have their counterpart in the more formalized sciences, but behavior characteristics are related to those of servomechanisms. While constructing this model some of the concepts developed by the electrical engineers in the servo field have been borrowed. Most notable of these is the convenient tool for portraying the interrelationship of components within the system known as signal flow graphs.[13] The structure of the system is mapped out in terms of flow graphs which lead to the mathematical equations formalizing the system.

To analyze the operating structure of the business firm there are two steps. First, one must select the operating sections of the business. These will later be isolated and broken down into subsystems. The second step requires that the input and output quantities of each section be specified; i.e., sales, optimum production level, labor hiring rate, etc. In picking these variables several factors should be considered. It must be possible to relate the variables mathematically, but this is not the only criterion of selection. To have a usable model these variables should be measurable quantities already familiar to management. If some unusual quantity is selected, the ultimate usefulness of the simulation to management will be greatly reduced, and comparison of the simulated results with known conditions will be difficult.

The same process can be carried on at the subsystem level. For example, starting with the inputs and outputs of the labor block, the internal flow of labor is determined.

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Before proceeding into the development of the model, the reader's attention is directed to the appendix where conventions used in constructing signal flow graphs and the notation used for the various parameters and variables are explained.

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13. Campbell, D. P., Dynamic Behavior of Linear Production Systems, A.S.M.E. Mech. Eng. April 1953.

CHAPTER TWO

FORMULATION OF THE MODEL

The model in this chapter has been developed in terms of six component sections:

a) Finished goods inventory

b) Production Control

c) Labor

d) Raw materials

e) Production

f) Flow of Funds

This is a hypothetical model in that it is built around a common firm. It consolidates into one system relations that seem common to many firms. This model is actually an exploration into making a model with specific functions.

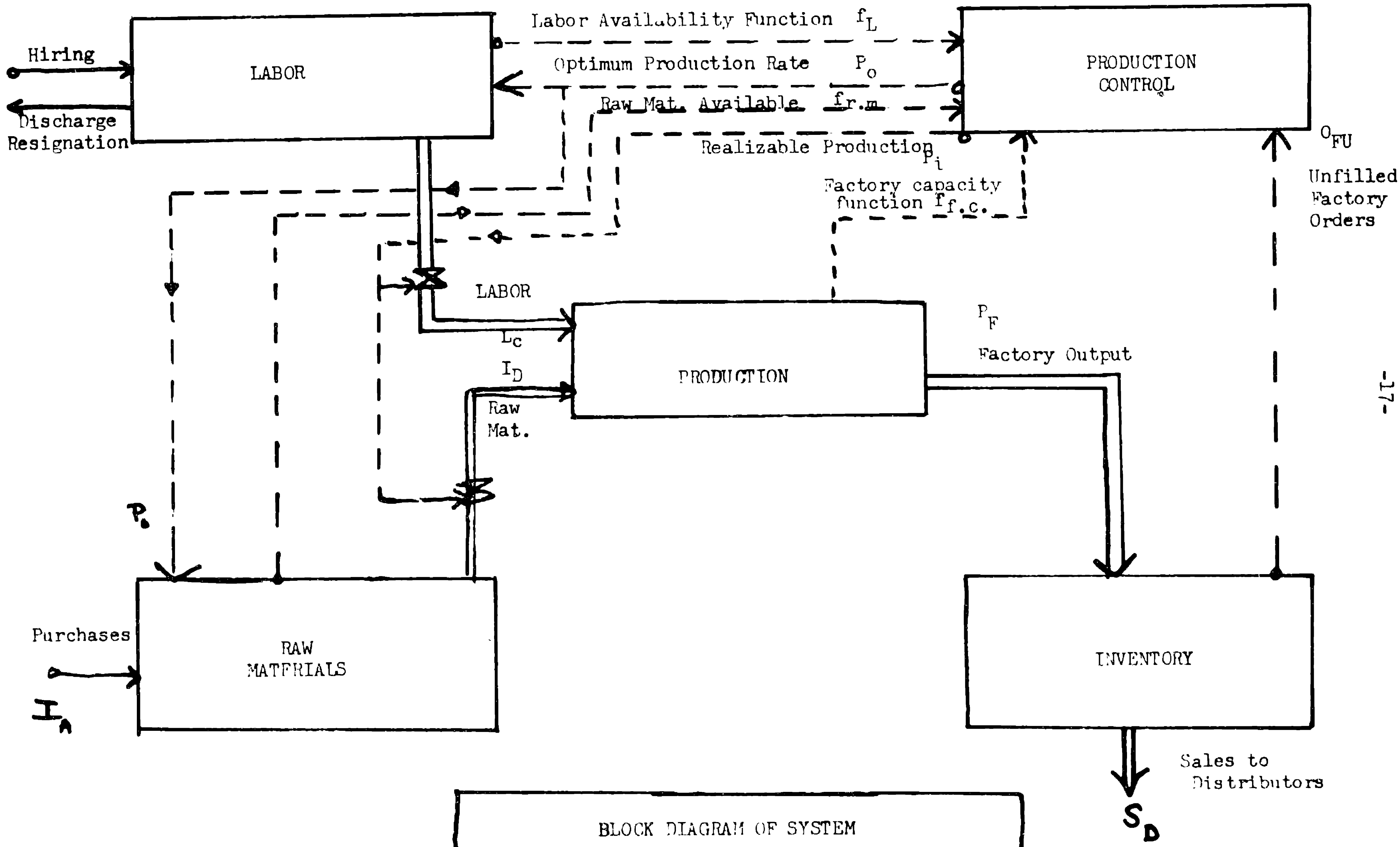
The exposition of the model is presented in two parts. Part I explains the purpose and external relationships linking each of the six component blocks. Part II is a detailed analysis of each block with the functions in difference equation form. Figure 1 shows the complete system in block diagram form.

PART I

Inventory[15]

The finished goods inventory block stores all goods received from factory production. Realistically, there are usually several stages of

15. Dynamic Model of Consumer Goods Distribution, S.I.M., M.I.T., Jan. 8, 1957, by J.W. Forrester

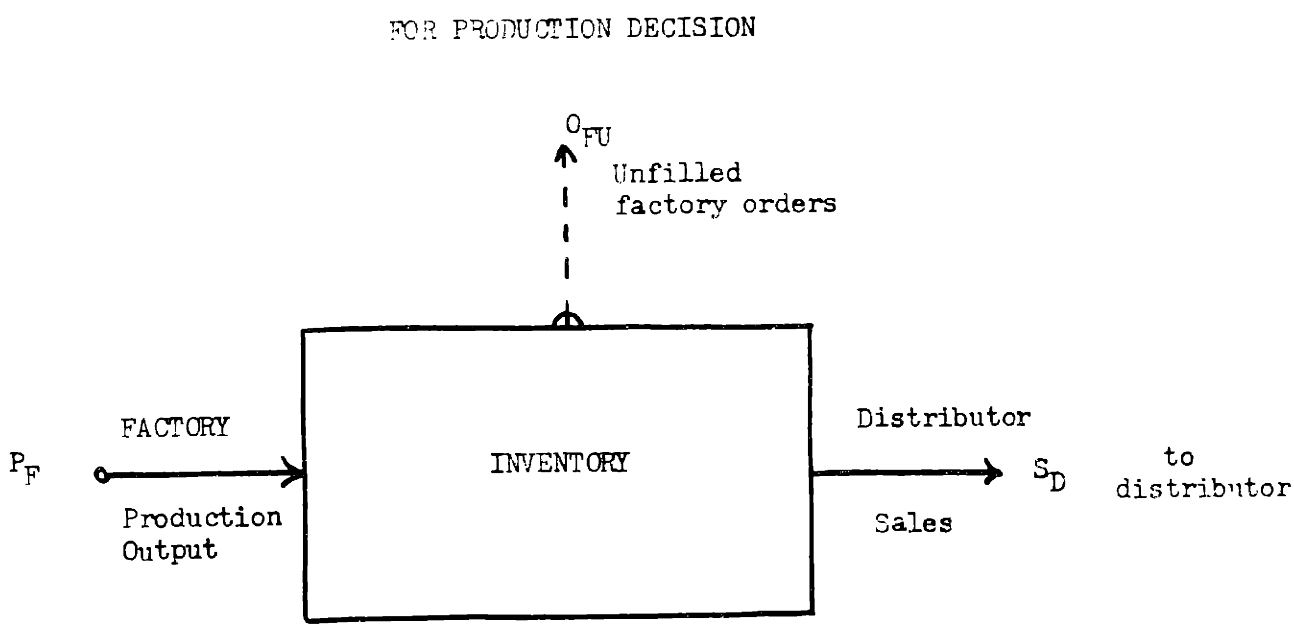


BLOCK DIAGRAM OF SYSTEM

Figure 1

inventory. Goods from the factory are initially stored in a factory warehouse which supplies regional or distributor warehouses. The distributor, in turn, serves a number of retail outlets. The output of this multi-stage inventory system consists of retail sales to customers throughout the country.

In conjunction with the system described in this paper only the factory warehouse stage of inventory need be considered. The flow of goods into the system is factory output. ——————————————— ———(Units are number of product units off assembly line.) The discharge flow is a consolidation of all distributor sales. There is an additional information output line carrying production control data. Within the production control block, the level of unfilled factory orders is a portion of the data used to establish optimum production level.

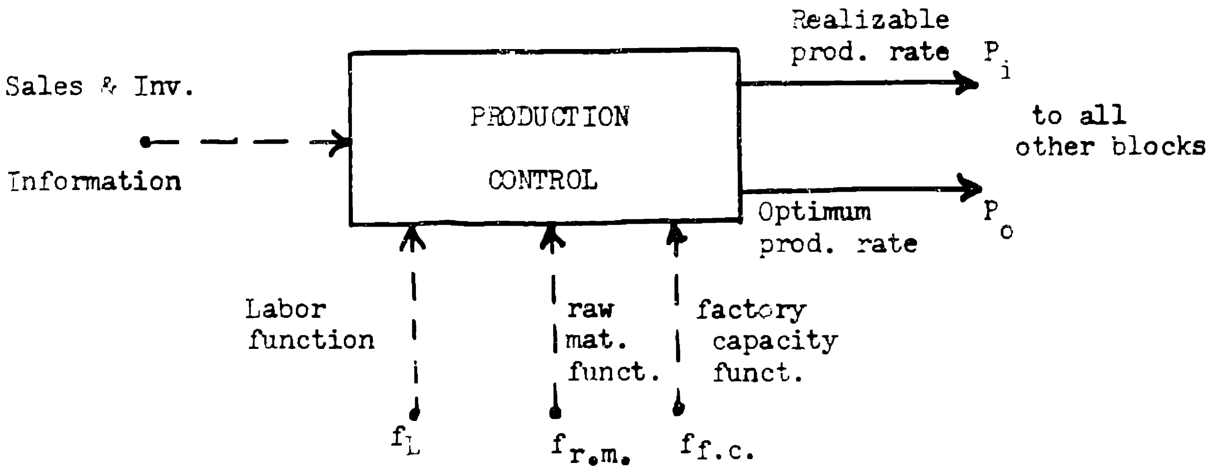


BLOCK DIAGRAM OF INVENTORY SECTION

Figure 2

Production Control

The production control block handles information exclusively; there is no flow of goods through this block. Input information is analyzed and processed to determine two decisions: the optimum rate of production and the rate of production which can be realized with all constraints acting. There is an important distinction in meaning between these two variables. The optimum production rate is related to sales, market acceptance, promotion and similar quantities outside the factory. It is the proper rate to provide the sales force with as many product units as can be sold. In the formulation P_0 is a function of inventory level and unfilled orders at the factory. Realizable production is based upon internal factory conditions which impose constraints on the operating capacity of the production facility. Typical quantities influencing the realizable production rate are size of labor force, ratio of new employees to trained employees, size of raw material stocks, machines and space available in factory. The input information to the production control block comes from three other sectors. From labor comes an information function expressing the fraction of optimum production possible based on the existing labor force. Similarly, a function stating the ratio of raw materials to the optimum level arrives from the raw materials block. Finally, the ratio of rated factory capacity to optimum production completes the necessary information to make the realizable production rate decision. It is the production control section which regulates all other processes within the factory.



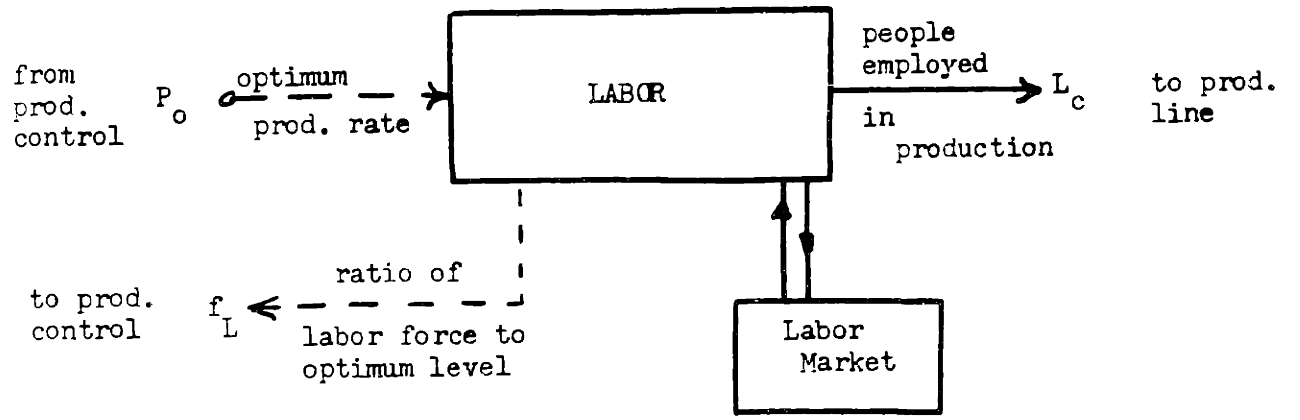
INTERNAL INFORMATION

BLOCK DIAGRAM OF PRODUCTION CONTROL

Figure 3

Labor

The labor block handles all personnel functions -- hiring, discharge, and turnover. In addition, records are maintained of the size of the labor force and number of employees in training. There is a pair of inputs and outputs for both information and actual labor flow. Information input is the optimum production rate. By comparing this ideal level with the existing labor reservoir, hiring decisions can be initiated. A similar comparison forms the information output sent to production control to determine the realizable production rate. (P_i) There are two flows of labor. The output from the system is labor employed by production. An interchange of people also exists between the factory labor force and the outside labor market to cover hiring and discharge.



BLOCK DIAGRAM OF LABOR

Figure 4

Raw Materials

The function of the raw material inventory block is to supply the factory with materials. Decisions when to buy and how much to buy are based upon the rate of optimum production. Like the labor block there are two inputs and two output lines, a pair for flow quantities and a pair for information. Purchase decisions are based on P_o, an input line. Data relating actual raw material level to optimum level is sent to production control. Input flow is, of course, the purchasing rate; while discharge consists of materials requisitioned by the production operation.

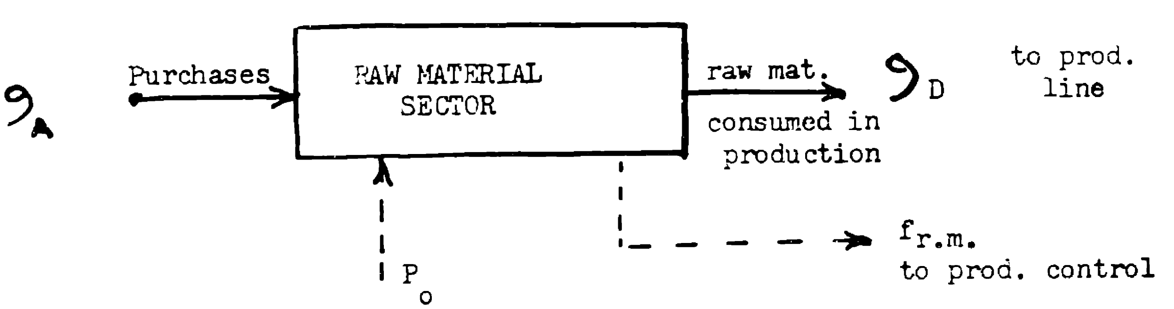


Figure 5

Production Process

Here is the heart of the entire operation; all other sectors can be considered as supporting operations for production. Externally the process is very simple. There are two input flows and one output. Raw materials and labor are combined by the production process into the product.

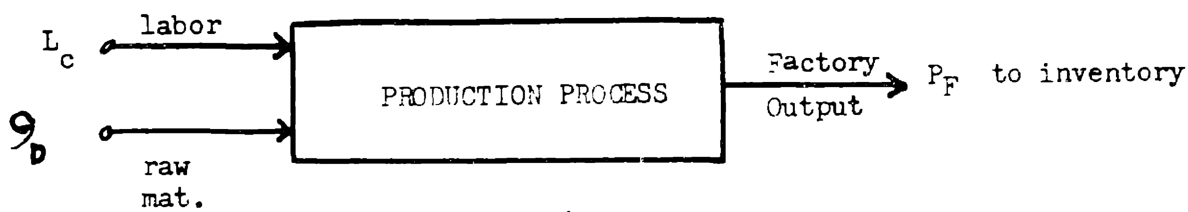


Figure 6

Flow of Funds

The flow of funds is somewhat divorced from other blocks. Although the information controlling the funds flow is generated within other sections, there is no actual flow of a physical quantity between this section and any of the others. One might consider the flow of funds statement to be a link between conventional accounting methods and the presentation of the business system in the model form. Some of the ordinary accounting variables are extracted from the operating model and fitted together in such a way that current assets balance current liabilities. A model for funds flow with accompanying equations is shown later in the chapter.

PART II

Labor

The labor sector will now be treated in detail. The primary function of this portion of the system is to maintain a properly regulated reservoir of labor. An overly large labor pool for a given rate of production is inefficient; but insufficient manpower throttling an otherwise higher production can result in just as many unnecessary expenses through overhead, unused factory capacity, and lost sales. The labor block is supplied with the optimum production rate; with this information the ideal size of the labor force is determined. A comparison between the ideal and the actual numbers produces an error function (positive or negative) representing the number of men to be added or subtracted from the labor force. Hiring is, of course, the method by which additions are realized; while discharge and turnover result in a reduction of manpower.

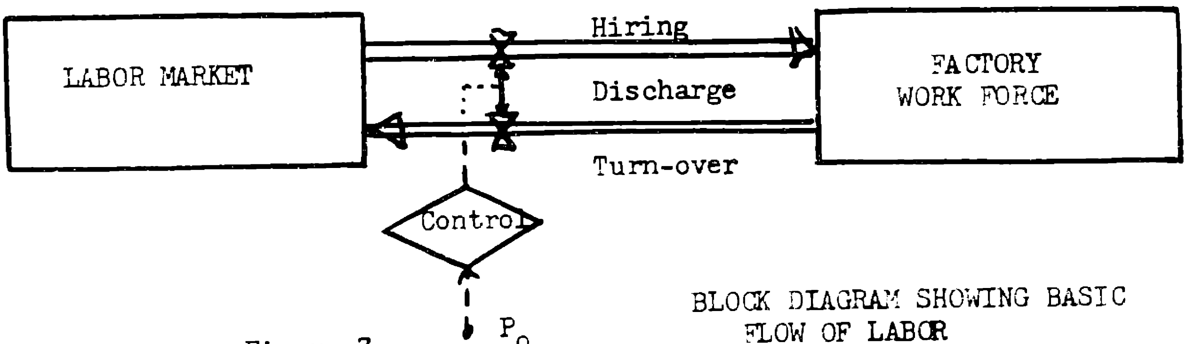
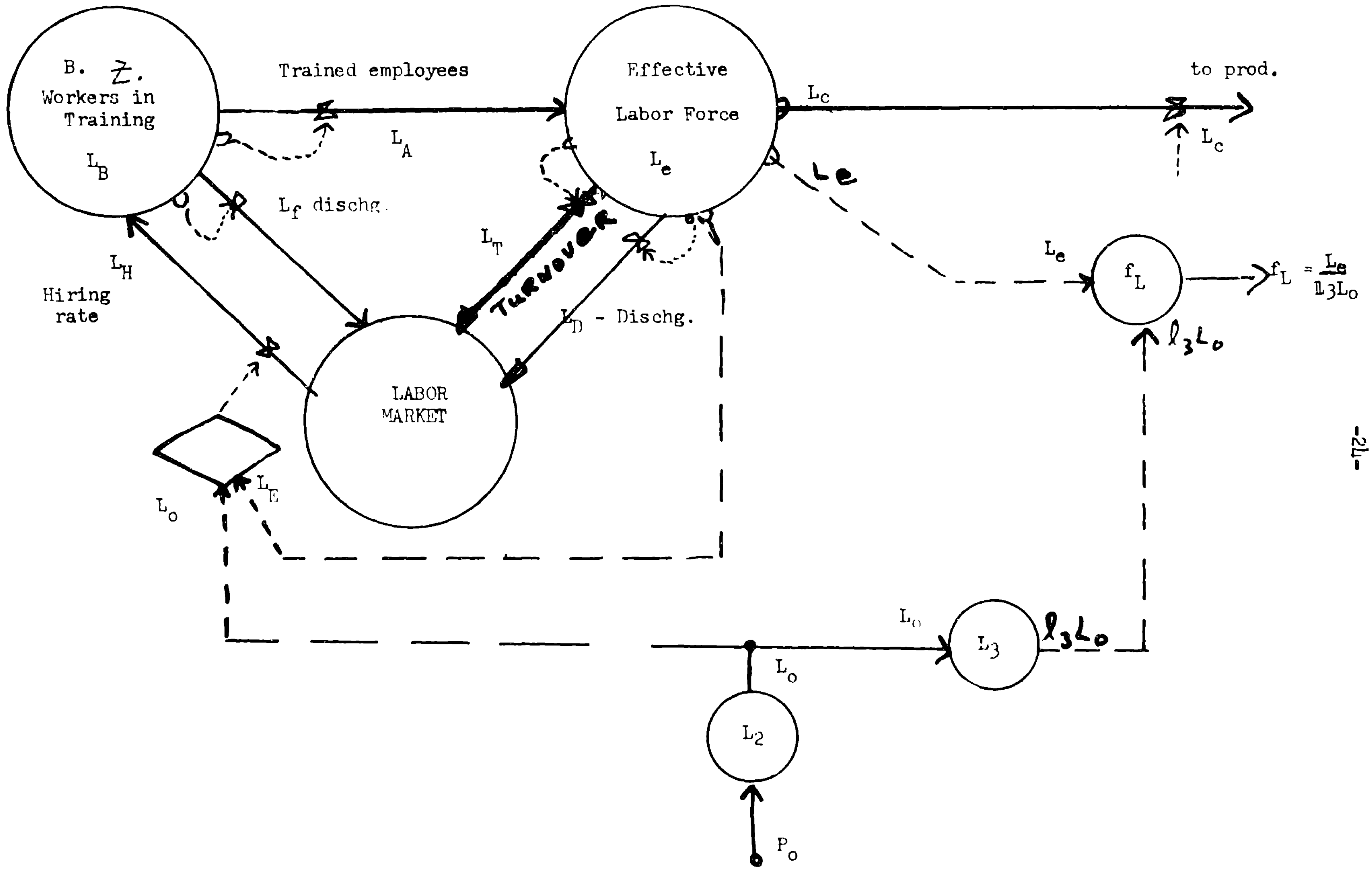


Figure 7 BLOCK DIAGRAM SHOWING BASIC FLOW OF LABOR

There are several other factors which cause the labor control problem to become more complex. For many operations a training period is involved before a new employee is able to perform his job efficiently. This period produces a time lag between hiring and the time that the new employee can be considered a useful member of the labor force. Frequently unionized companies find it difficult to discharge employees; consequently, a probationary period is used before a new worker is accepted as permanent to the group. Discharge is much higher during the probationary period than later. Some companies in an attempt to provide labor stability refrain from layoffs except in emergency. In such cases normal turnover and discharge



LABOR SECTION FLOW DIAGRAM

provide the only means to shrink an oversized labor force.

Figure 8 shows the flow diagram of the labor block. Broken lines are information flows; solid lines represent the transfer of people; and the three large circles are reservoirs. Note that there are three reservoirs connected by flow lines. Within the factory system the labor force has been split into two pools. The effective labor force of the factory (L_e) is made up of people on the permanent payroll who have completed the training necessary for their job. The buffer zone of employees in training (L_b) consists of new employees on probation -- still learning their jobs. The output from a worker in the buffer zone is less than his potential capacity which should be realized when he is transferred into L_e. All additions to the effective factory force must pass through the buffer zone. Flow L_a is the rate of additions to the effective labor pool linking L_b to L_e. Reductions in the entire labor force ($L_b + L_e$) can be effected through several paths. L_f, the discharge rate of employees on probation, reduces the size of the buffer zone pool. Shrinkage in L_e results from the normal turnover rate, L_t, and the permanent employee discharge rate L_d.

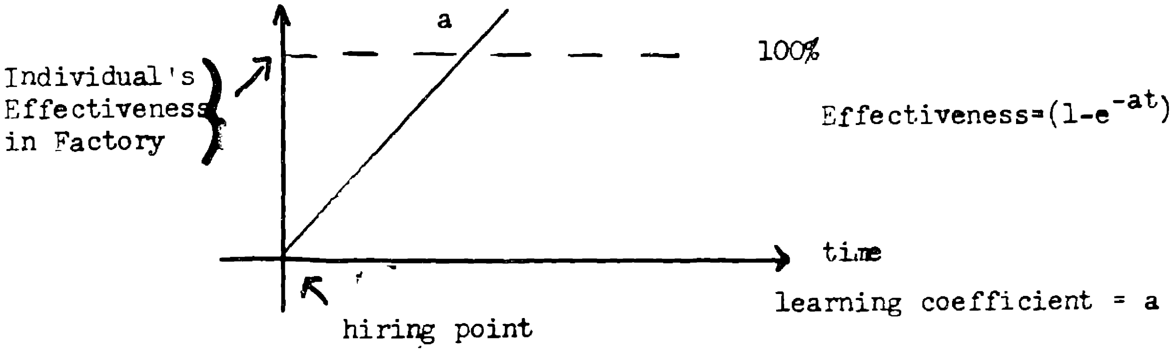
Equations expressing the size of the labor pools are:

$$L_{b(t)} = L_{b(t-1)} + L_{h(t-1)}\Delta t - L_{f(t-1)}\Delta t - L_{a(t-1)}\Delta t$$

$$L_{e(t)} = L_{e(t-1)} + L_{a(t-1)}\Delta t - L_{t(t-1)}\Delta t - L_{d(t-1)}\Delta t$$

Total factory force on payroll = $(L_b + L_e)$

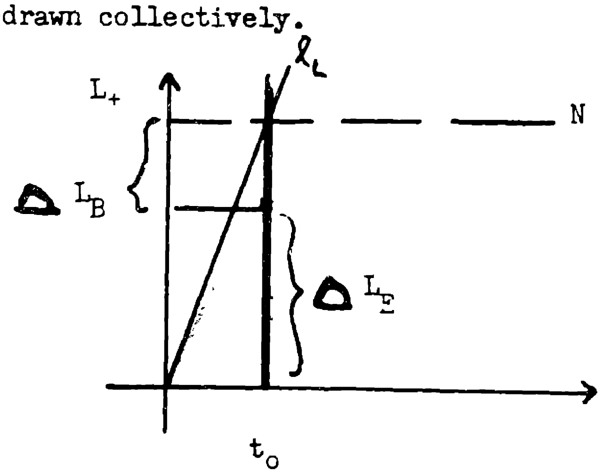
The mathematical connotations of L_b, L_a and L_e in the model are slightly different than what happens in reality. An employee in training is represented as going through a learning phase similar to that shown below.



LEARNING PROCESS OF A WORKER

Figure 9

This learning process is gradual and continuous unlike the discontinuous method depicted in the model. Nevertheless, instead of one large group of partially trained people, one can visualize two component subgroups -- one hypothetical group consisting of an equivalent number of people with no training, the other group containing the equivalent number of completely trained people. The composite efficiency of the two hypothetical groups equals that of the real group. This artifice provides a technique for separating non-productive manhours from the productive, yet still retaining labor flows in terms of people. The model is incorrect when considering an individual event, but true from a statistical outlook. This same concept can be expressed in slightly different form. Visualize N new employees hired simultaneously. The learning process figure is redrawn collectively.



$L_{+\,max}$ = N men

L_+ = additions to effective labor pool L_E

$L_+ = L_{+max}\,(1-e^{-\ell t})$

$\dot{L}_{A(t)} = \left.\frac{dL_+}{dt}\right|_t = \ell_L L_B$

Figure 10

Figure 10

At any instant in time t_o, the distance ΔL_E shows the equivalent number of additions to the effective labor pool; while ΔL_B represents the equivalent number of workers still untrained.

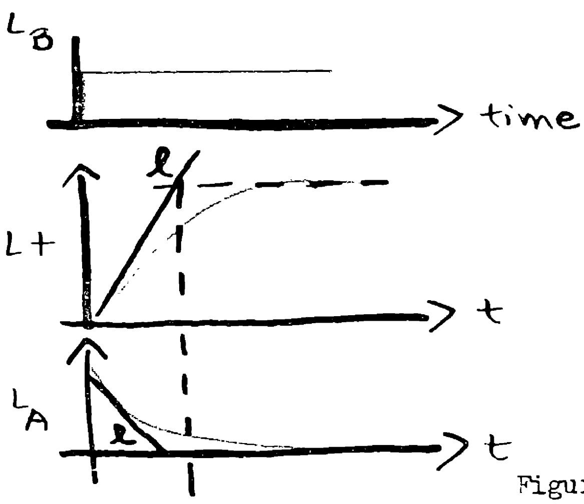
The rate of transfers into L_e is considered proportional to the number of people in the buffer reservoir.

$$L_{A}(t) = l_e \; L_{B}(t-1)$$

Units are people/wk

l_e is group learning coefficient.

For a step function of people added to the buffer zone the following shaped curves result.



N = no. of people added

L_+ = no. of additions to L_E

$L_A = \frac{dL_t}{dt}$ = rate of additions to L_E

Figure 11

The normal turn-over rate L_t is conceptually easier to grasp than L_a, but mathematically it is also a proportional rate function

$$L_T(t) = l_T \; L_E(t-1)$$

Units are people/wk

l_T = turnover proportionality constant.

Both discharge functions are similar to the others:

$L_{D(t)} = l_D \; L_{E(t-1)}$ $\qquad$ l_D = discharge prop. const.

$L_{f\,(t)} = l_f \; L_{B\,(t-1)}$ $\qquad$ l_f = discharge prop. const. untrained people.

Now the effective work force L_e can be expressed in terms of $(L_E + L_B)$ in the preceding period.

$$L_{E\,(t)} = L_{E\,(t-1)} + l_e L_{B\,(t-1)} \Delta t - l_T L_{E\,(t-1)} \Delta t - l_D L_{E(t-1)} \Delta t$$

residual $\qquad$ additions $\qquad$ turnover $\qquad$ discharge

All flow channels have been discussed except L_h, the hiring rate of input to the buffer zone. As mentioned before, an error function is generated expressing any discrepancy between $L_o + L_E$. The effort applied to recruiting new employees is proportional to the manpower deficiency with an appropriate time lag included. Ideal labor level L_o is directly proportional to P_o. The proportionality constant l_2 relates the number of workers needed to product units per week.

$$L_o = \ell_2 \; P_o$$

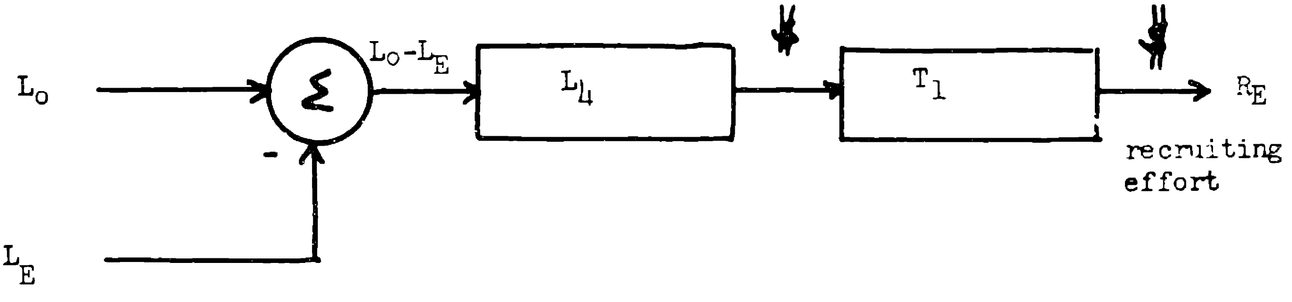


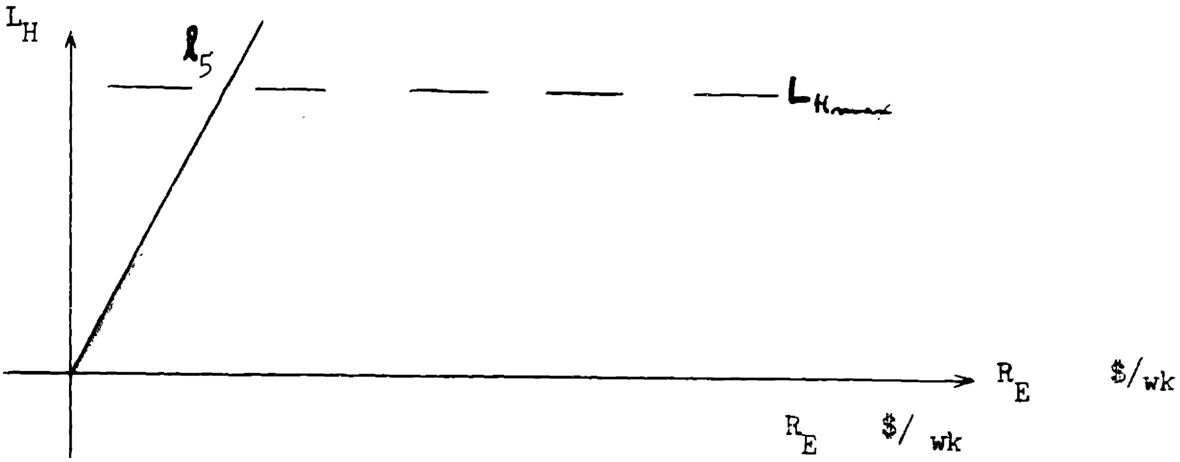
Figure 12

l_4 = constant relating need for people to recruiting cost.

T_1 = appropriate time lag for personnel department to act.

$$R_{E(t)} = \left[L_o - L_E \right] l_4 \left(t - 1 - \frac{T_1}{\Delta t}\right)$$

To reflect the finite availability of workers in the local labor market, an exponential function has been proposed relating the hiring rate secured for a given level of recruiting effort. Recruiting effort is measured in dollars; the hiring rate is in number of men per week.



$$L_{H(t)} = L_{H\,max} \left(1 - e^{-l_5 R_{E(t-1)}}\right)$$

Figure 13

SUMMARY OF LABOR DEFINITIONS

L_E effective labor level in factory

L_B employees in training (buffer zone)

L_A rate of additions to effective labor force (men/week)

L_T normal turn-over rate

L_D discharge rate (trained employees)

L_f discharge rate (probationary employees)

L_H hiring rate

L_o optimum level of effective labor

L_c labor used in factory (a rate: manhours/week)

R_E recruiting effort (in dollars)

l_T normal turn-over constant

l_D discharge constant (trained)

l_f discharge constant (untrained)

P_o optimum production rate

l_1 group learning rate

l_2 ideal no. of employees per weekly product unit

l_3 overtime correction factor

l_4 recruiting cost per person

l_5 effectiveness coefficient of recruiting effort

T_1 time lag for personnel dept. to hire people

Raw Materials

The purpose of the raw materials section, like labor, is to supply the production operation. To provide an adequate flow of materials there must be an inventory. Too high a level of inventory is wasteful and, of course, with too low a level one risks the danger of running out. The optimum inventory level decision is based on ideal production P_o and the previous inventory depletion rate. The ideal inventory level is compared to actual stores; any discrepancy results in a change of the purchase order rate.

Additions are made to the inventory reservoir I_m through flow channel I_a, the purchasing rate. Depletions are drawn off by the factory through I_d. The level of inventory at any moment is given by:

$$q_{m(t)} = q_{m(t-1)} + q_{A(t-1)} \Delta t - q_{D(t-1)} \Delta t .$$

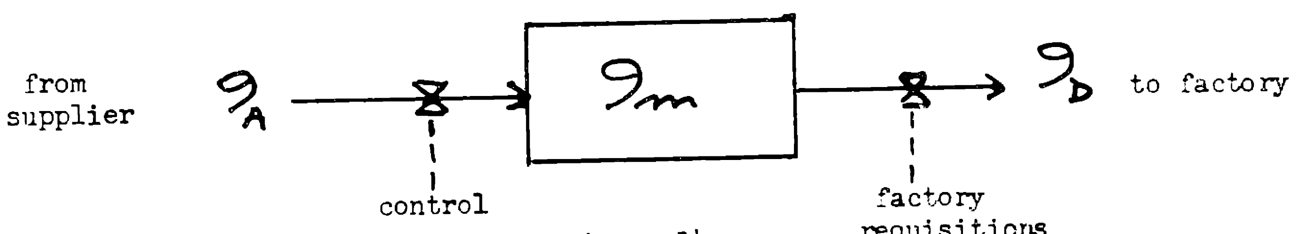


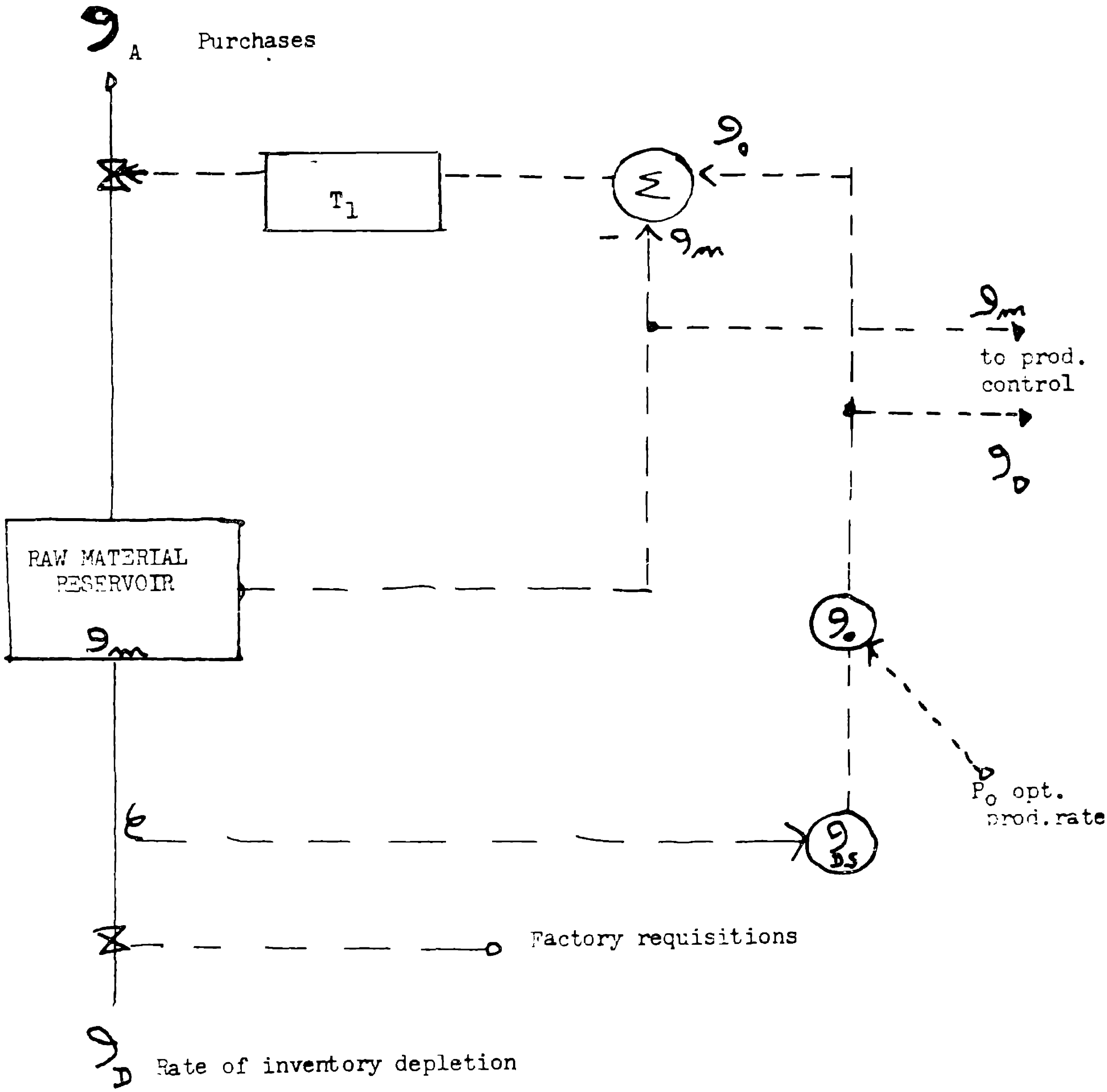
Figure 14

Criteria for ordering decisions are slightly more complicated. Three quantities are sampled for decision input data:

a) rate of raw material depletion I_d

b) level of raw material inventory I_m

c) optimum factory production rate P_o



FLOW CHART OF RAW MATERIAL INVENTORY

Figure 15

Weighted values of the equivalent P_o and the smoothed inventory depletion rate are used to formulate I_o , the ideal inventory level.

$$I_{DS(t)} = I_{DS(t-1)}^{*}\, a + I_{D(t-1)}(1-a)$$

Smoothed rate

$I_{DS(t-1)}$	smoothed inventory depletion for last time interval
a	smoothing constant (less than 1).
$\mu_1 P_o$	necessary level of raw material inventory to maintain a production rate of P_o.
μ_2	weighting constant -- relative decision-making importance between P_o and I_{ds}.

$$I_{o(t)} = \mu_2 I_{DS(t-1)} \Delta t + \mu_1 P_{o(t-1)} (1-\mu_2)$$

Purchases are based on the difference of I_o and I_m. This difference represents a number by which present stocks should be increased or decreased. The number of weeks over which this correction should be spread is constant μ_3.

The change in purchase rate to make up the deficit is:

$$\left[\frac{I_o - I_m}{\mu_3} \right]$$

Thus the new purchase rate is:

$$q_{A(t)} = \frac{\left[q_{O(t+\frac{T_m}{\Delta t})} - q_{m(t+\frac{T_m}{\Delta t})}\right]}{w_3} + q_{A(t-1)}$$

SUMMARY OF RAW MATERIALS DEFINITIONS.

q_m	actual level of inventory
q_o	optimum level of inventory
q_A	rate of additions and purchases
q_D	rate of depletion of stock from factory requisitions
q_{DS}	smoothed rate of inventory depletion
a	smoothing constant
T_m	time lag between purchase orders and additions to inventory.
w_1	constant relating production rate to comparable raw materials level.
w_2	decision criteria weighting constant
w_3	number of weeks to spread purchasing deficit

Production Control

The optimum rate of factory production, P_o, is derived from two other quantities; the number of unfilled factory orders originating from the factory warehouse, and the amount of inventory in process in the factory. Throughout the construction of the model an effort has been made to make the formulation coincide with existing business practices. It is recognized that P_o represents the present optimum production rate and that because of numerous cascaded time lags and factory lead time, P_f lags P_o by as much as several months. Undoubtedly, some sort of predictor for establishing an advance value of P_o would tighten the system considerably. The fact is that many businesses operate by just the method described in the model. Therefore, for this initial model no attempt at prediction has been made.

Unfilled factory orders are divided by a filling rate per week constant to give the value of P_o with one slight modification. It is recognized that factory work in process will soon be transferred into a finished product. Therefore, goods in process are deducted from unfilled factory orders.

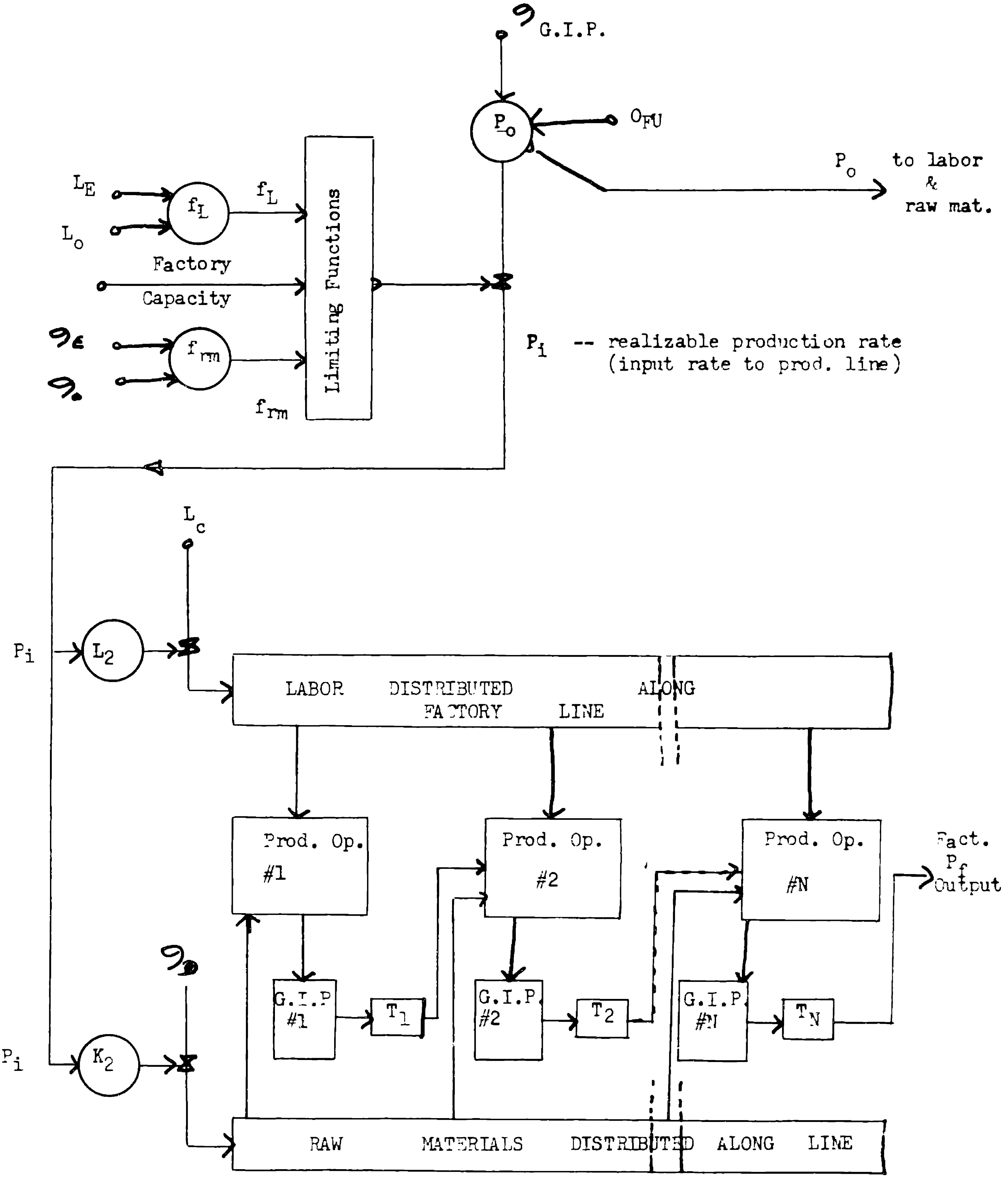


Figure 16

FLOW CHART FOR PRODUCTION CONTROL AND PRODUCTION SYSTEMS

FORMULATIONS FOR PRODUCTION CONTROL

$$P_{o(t)} = \frac{[O_{Fu} - g_{G.I.P.}]_{(t-1)}}{K_3}$$

P_o = ideal production rate (units/wk)

O_{Fu} = unfilled factory orders in units

K_3 = no. of weeks of product orders normally held between O_{fu} and I_{gip}.

$g_{G.I.P.}$ = consolidation of goods in process in factory (derived from labor and raw material in process)

$$P_{i(t)} = P_{o(t-1)} \times [f_L ; f_{r.m.}]$$

$P_i \leq$ factory capacity.

P_i production rate which is realizable when one considers labor force, raw materials and factory capacity

$$g_{D(t)} = P_{i(t-1)} \times K_2$$

$$L_{c(t)} = l_2 \times P_{i(t-1)}$$

$$h_{(t)} = \frac{l_2 \times P_{i(t-1)}}{L_{E(t-1)}}$$

g_D = raw materials consumer per week (a rate)

L_c = labor consumed in manhours/week

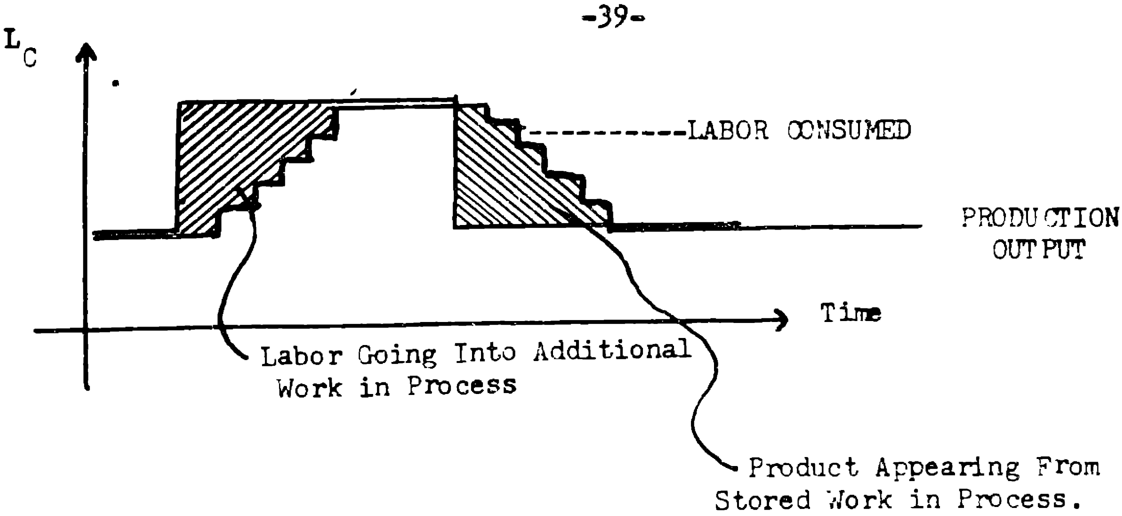
K_2 = consumption of raw materials per product unit.

l_2 = labor consumption coefficient (manhours/wk.)

h = length of work week for labor force.

The production process has been treated in regard to its external effect on the rest of the system. The input and output functions have been formulated quantitatively. However, the internal structure of production has only been suggested by a feasible structure in the diagram. A process is assumed which requires a number of individual operations to complete a finished product. Both labor and raw materials can be added at each operation. Between successive stages there will be a buildup of goods partially completed. Also there is a time lag between each operation. For most situations the waiting time exceeds actual fabrication time; therefore, only waiting time lags have been shown explicitly.

To obtain a transfer function for the production network only the static behavior is considered. This assumption is an approximation, but barring events like major machine failure or labor walk-outs, it should prove valid for this model. For a steady state goods in process inventory level, product output can be considered proportional to a moving average of the labor input. The labor consumption is averaged over the length of the production process. Thus, increases or decreases in output lag the input. This model causes goods in process inventories to follow the production rate. Additional input labor must go either into output or into goods in process.



RELATIONSHIP BETWEEN LABOR USED AND FACTORY OUTPUT

Figure 17

$$P_{F(t)} = \frac{\Delta t}{b} \times \frac{L_{c(t-1)} + L_{c(t-2)} + \cdots L_{c(t-\frac{b}{\Delta t})}}{\ell_2}$$

$$g_{L.I.P.(t)} = g_{L.I.P.(t-1)} + L_{c(t-1)} \Delta t - \ell_2 P_{F(t-1)} \Delta t$$

$$g_{m.i.p.(t)} = g_{m.i.p.(t-1)} + g_{D(t-1)} \Delta t - \ell_2 P_{F(t-1)} \Delta t$$

P_F	Factory output rate (units per week)
b	No. of weeks of production cycle
$g_{l.i.p.}$	Labor in process (units in manhours)
$g_{m.i.p.}$	Raw materials in process.

Inventory

This segment of the model, the finished goods inventory subsystem, is only treated superficially, since an extensive model of the dynamic behavior of inventory systems has already been formulated by J. W. Forrester.[15] The important variable from the inventory model affecting our system is the value of unfilled factory orders due to inadequate inventory levels. O_{fu} is one of the determining factors for the optimum production rate decision. The factory stage of Forrester's inventory model is shown with accompanying equations.

FORRESTER'S DYNAMIC INVENTORY MODEL--FACTORY STAGE

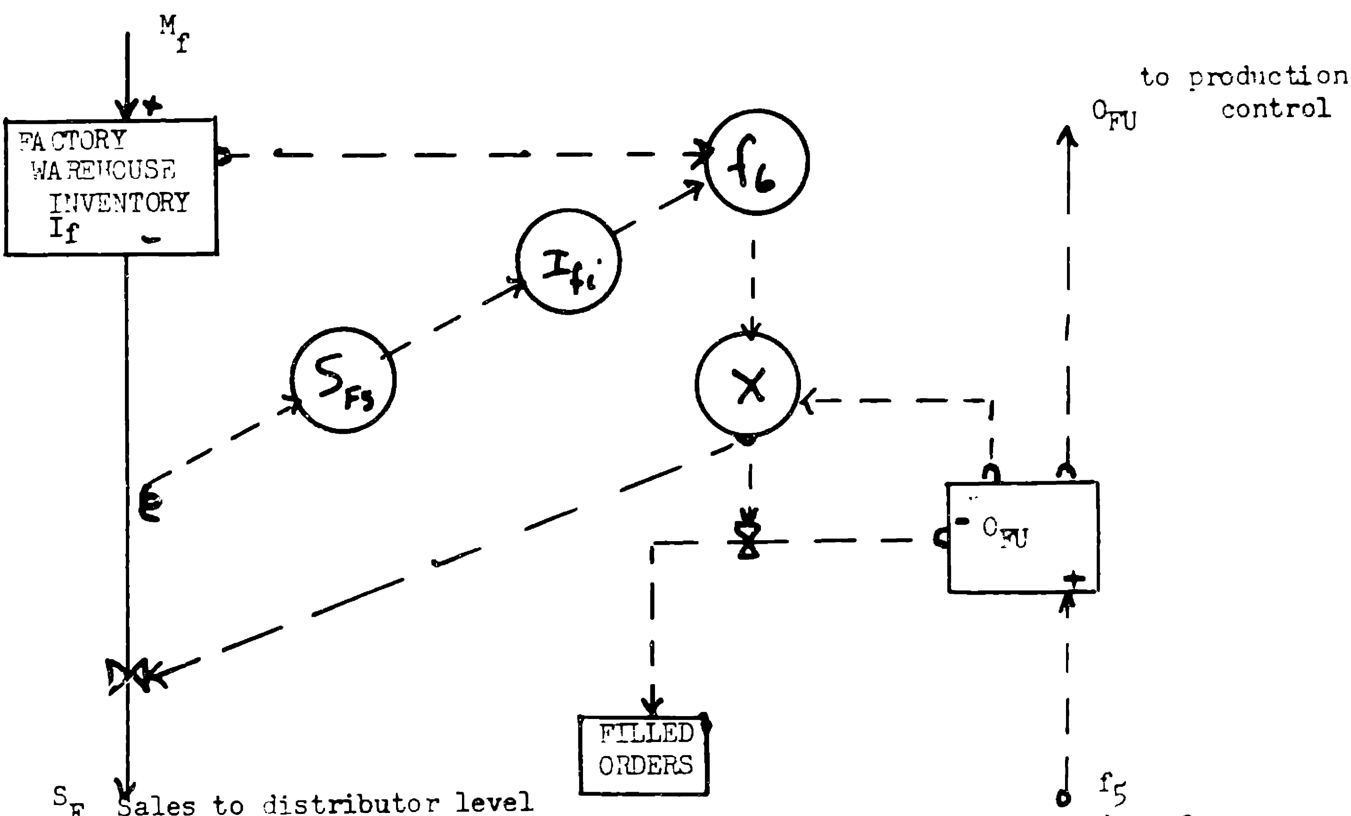


Figure 18

15. Dynamic Model of Consumer Goods Distribution, S.I.M., M.I.T., Jan. 8, 1957

Equations for Forrester's Model:

Smoothed factory sales:

$$S_{FS(t)} = (1-a_5)\, S_{F(t-1)} + a_5\, S_{FS(t-1)}$$

$\frac{1}{1-a_5}$ is the smoothing time constant.

Ideal factory inventory:

$$I_{fi(t)} = a_6\, S_{FS(t-1)}$$

$$f_{6\,(t)} = \left[\frac{9_{F(t-1)}}{9_{Fi(t-1)}}\right]^2 \geq 0$$

$$S_{F(t)} = \left[f_6 \times O_{FU} \right]_{(t-1)} \qquad \text{Units/Week}$$

$$O_{FU(t)} = O_{FU(t-1)} + f_5\, \Delta t - S_{F(t-1)}\, \Delta t$$

$$9_{F(t)} = 9_{F(t-1)} + M_{F(t-1)}\, \Delta t - S_{F(t-1)}\, \Delta t$$

Flow of Funds

In determining functional relationships for the flow of funds diagram, several of the variables are derived from the preceding sections, while others must be somewhat arbitrary assumptions.

Wages paid to factory workers F_L (units in dollars/wk) must be based on the total factory force $(L_b + L_e)$ and the length of the work week h.

$$F_{L_{(t)}} = \left[\left(L_{E_{(t-1)}} + L_{E_{(t-2)}} + \cdots - L_{E_{(t-\frac{1}{\Delta t})}}\right)\Delta t + \left(L_{B_{(t-1)}} + \cdots L_{B_{(t-\frac{1}{\Delta t})}}\right)\Delta t\right] h$$

Delay of 1 week

Funds paid to suppliers are proportional to raw material purchases:

$$F_{r.m._{(t)}} = k\left[I_{A_{(t+\frac{T_A}{\Delta t})}} + I_{A_{(t+\frac{T_A}{\Delta t}+1)}} + \cdots - I_{A_{(t+\frac{1}{\Delta t}+\frac{T_A}{\Delta t})}}\right]\Delta t$$

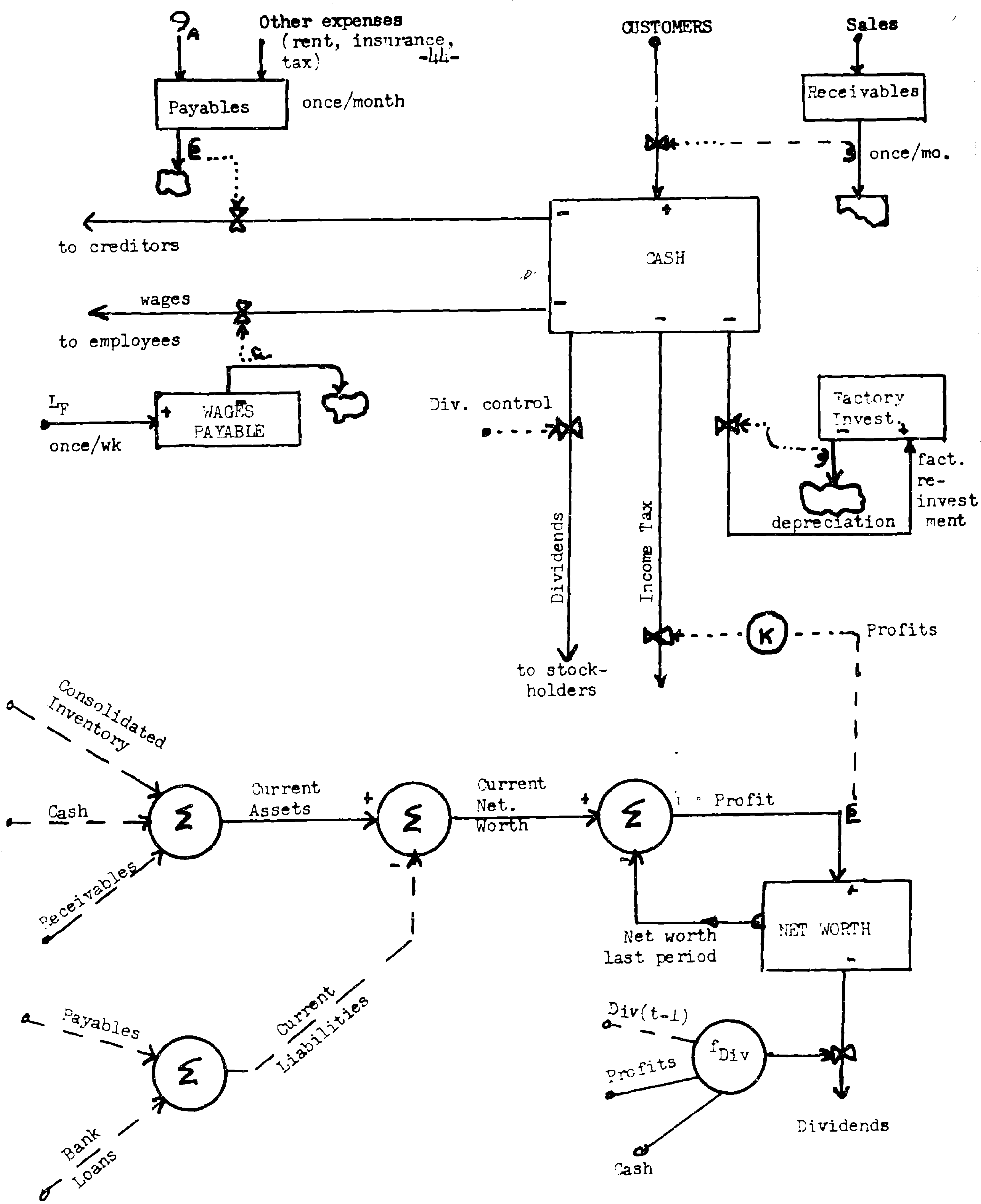
where k is the proportionality constant

I_a is the ordering rate in units per week

T_a is the accounting delay in weeks.

Income tax is assumed proportional to earnings for the period. Dividends paid out are based on three factors: the previous dividend rate, the profit for the period, and the current cash level. As each sale is made, an appropriate amount is added to the receivables storage reservoir. A deduction from cash is made equal to the amount of factory depreciation. These funds are then reinvested into the plant and equipment; thus, factory investment remains constant.

Profits for the period are normally derived from the difference between revenue from sales and cost of goods sold. An elaborate costing system is required if correct valuation of inventory is to be made. For this exploratory model of funds flow, a simplified method is used. The difference of current assets and liabilities provides a current net worth value. (Since the entire model is based on short-term factors, long-term assets and liabilities are considered constant.) Comparison of current net worth for the present period with that of the preceding period establishes a change in value of the firm or the profit value. This value is apt to fluctuate more than a profit figure computed from a true inventory costing system because short-term investments in inventory are subtracted from the profit variable.



FUNDS FLOW

Figure 19

CHAPTER THREE

CONCLUSION

This concluding section of my paper discusses several problems associated with the model worthy of investigation. In fact, it is this unfinished work that will determine the value of what has been done so far. The construction of the model forms the platform from which a number of interesting studies can commence.

In this field, as in the aircraft industry, one of the reasons for the use of the simulation technique is to facilitate testing of systems with different parameters or even altered structure "on paper". Rarely is it possible to try several alternatives on an actual system; these experiments are only feasible on simulated systems. To make use of the model envolves writing a program for a digital computer from the set of difference equations already formulated. It is very important that the programmer be conscious of the purpose of his program. Rather than strive for a "clever", efficient program, it will be of far greater concern to remember that the reason for simulating is to enable us to test the effectiveness of different system parameters. The program should be made as flexible as possible. Parameter values should be kept separate from the program itself. In this way the proper data can be read into the machine before each run, while still using the same general-purpose program. It might even be possible to build the program in blocks or sub-routines (like the subsystems of the model); then, when the structure of the model is changed only a small portion of the over-all program need be altered.

Concurrently with the programming, it will be necessary to establish numerical values for the various coefficients and time lags. No doubt the problem will have to be narrowed to a particular company or at least to a particular type of manufacturing concern.

For the first run on the computer the precise value of these constants is not critical, but even rough approximations may be difficult to determine, since so little previous work has been done in the field. This combination problem of programming and selecting parameter values might be a worth while thesis for someone interested in continuing this new study.

Throughout the model formulation the timing interval Δt appears. For the first runs an interval of about a week would seem reasonable, but it will be important to optimize the interval length. There is a correlation between the size of the time constants in the network and the best value for Δt. To find out how large Δt can grow before significant accuracy is lost can best be determined experimentally.

Two broad categories remain to be explored. First, an evaluation of the present model formulation should be made by comparing simulated data with actual statistics collected from a real industry. The second area is possibly a bit more abstract but of even greater importance. It is here that big rewards for our labors can be found. A critical appraisal of the system should be made to uncover the origins of instability leading to fluctuations in production or inventory. From such a study one might expect to learn which information channels must be speeded up by modern data handling methods to stabilize the system or which decision-making functions are inadequate. In this area the simulator can spotlight poor management practices causing a sub-optimum system.

An evaluation of the model should be made by comparison of the simulated behavior with corresponding data from an actual industry. This analysis can be conducted at two levels. First, the validity of model structure should be established, and next, a comprehensive check of coefficients and final results is in order. These two categories are analogous to the qualitative and quantitative types of analysis. To clarify what is meant by model structure, each decision made by the model is based on some function of one or more variables. The optimum raw material inventory level is dependent upon two other variables---the optimum production rate and the smoothed factory consumption of materials. The fact that I_o depends on P_o and I_{Dsm} rather than on some other set of variables is an expression of system structure; the weighting factors μ_1 and μ_2 are merely coefficients pertinent to a particular system. The structure of the model is more basic than coefficient values. Therefore, before much effort is directed towards checking final results, a rather critical examination should be made to assure compatibility of present model structure with the industry being studied.

Assuming that the model conforms in structure with its industrial counterpart, the next step is to set coefficients and time lags realistically. Establishing the exact values for these parameters is apt to be a lengthy job. One must find a company willing to supply the pertinent set of statistics and then this data will have to be analyzed to extract the proper parameter values.

With a realistic working model of the firm developed, the system can be reappraised to find inefficiencies. Delay lengths, information handling rates, and decision criteria should all be challenged to ensure compatibility with a given level of system performance.

....

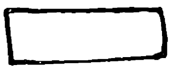
The model proposed in this thesis has been an exploration into the realm of management decision criteria. As more interest on the part of management centers around this field of simulation, many tangible benefits will begin to appear. A better understanding of the firm's dynamic behavior will result in efficiency and reduced costs. Routine decisions implemented automatically will save valuable executive time. Machine evaluation of policy alternatives by simulation will improve the judgment by which policy decisions are made. The simulation technique may well develop into one of the important tools of management science.

APPENDIX

FLOW GRAPH CONVENTIONS

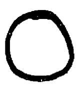
Solid lines indicate the flow of some quantity such as goods, money, units of labor, etc.

Dotted lines indicate the flow of information.

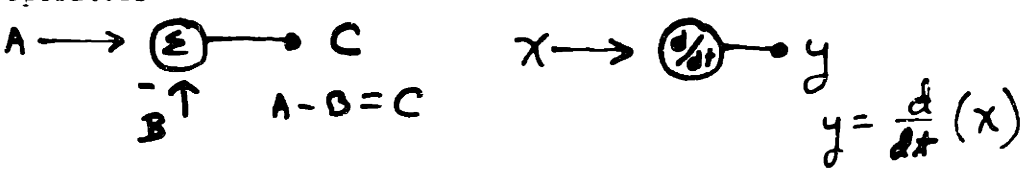


A rectangular box designates:
   a) a separate sub-system; i.e. production process.

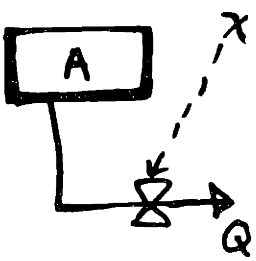
   b) a reservoir; i.e. --- Labor pool.



Circles show functional relationships or mathematical operators



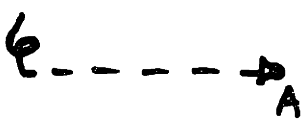
$$A - B = C \qquad y = \frac{d}{dt}(x)$$



X is a controlling or modulating function on the flow of A. Thus the discharge flow Q from the reservoir containing A is

$$A \cdot X = Q$$

This symbol denotes a sampling operation of the flow A.



NOTATION FOR VARIABLES

Each variable is designated by a principal letter with several subscripts:

$$L_{H(t-1)}$$

The L denotes a variable from the labor sector. Subscript H determines which variable from the labor group: while the subscript $(t-1)$ indicates the appropriate time interval. The above symbol represents the number of men hired to the labor force during time interval $(t-1)$.

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