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THE SOLUTION OF COMPLEX STEADY-STATE HEAT-CONDUCTION PROBLEMS BY THE USE OF AN ELECTRIC ANALOGUE

By

Samuel W. Ing, Jr.

Submitted in Partial Fulfillment

of the Requirements for the Degree of

Master of Science in Chemical Engineering Practice

from

Massachusetts Institute of Technology

May 24, 1954

Signature of Author — Signature redacted

Samuel W. Ing, Jr.

Signature of Supervisor — Signature redacted

Professor E. W. Merrill

Signature of Head of Department

Professor W. G. Whitman

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ABSTRACT

Title: "The Solution of Complex Steady-state Heat-conduction Problems by the use of an Electric Analogue"

Author: Samuel W. Ing, Jr.

Submitted to the Department of Chemical Engineering on May 24, 1954 in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering.

In many two-dimensional steady-state heat conduction problems, where the geometrical shapes are more complicated, the analytical method often fails to solve these problems.

Other methods have been developed. The most important three are, the relaxation method, the curvilinear square graphical method and the electrical analogue method. The relaxation and the graphical method involves tedious trial-and-error work, therefore becomes impractical as the geometrical shape of the system gets more complicated. The electrical analogue method was claimed to be the best available method by the previous investigators. The purpose of this thesis is to investigate the electrical method more thoroughly and intensively and to develope the technique.

The fundamental principle of the electrical analogue method is based on the similarity between the heat flow by conduction and the flow of electric current by conduction. Two methods were used in this investigation. The first method was the electric mapping method. This was done by obtaining a map of equipotential lines on a conductivity paper corresponding to the isothermal and adiabatic lines obtained from curvilinear square graphical method. The second method was the electric

shape-factor method. The shape-factor, A/X, was calculated by substituting the measured current and the total potential drop across the conductivity paper, which was cut into a shape representing the heat conduction system, into the equation:

$$A/X = (I/\Delta E)(f)$$

where f is the resistivity of the paper.

Six systems were investigated and a technique of solving the heat flow rate through two mediums with different conductivity was developed. It was concluded that the electric analogue method is no doubt the simplest and the best method. For simpler systems the electric shape-factor method is recommended and for complicated systems the electric mapping method is recommended.

Thesis Supervisor: Professor E. W. Merrill

Title: Assistant Professor of Chemical Engineering

Department of Chemical Engineering
Massachusetts Institute of Technology
Cambridge 39, Massachusetts
May 24, 1954

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

The thesis entitled "The Solution of Complex Steady-state Heat-conduction Problems by the Use of An Electric Analogue" is hereby submitted in partial fulfillment of the requirements for the degree of Master of Science in Chemical Engineering Practice.

Respectfully,

Signature redacted

Samuel W. Ing, Jr.

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SUMMARY

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where f is the resistivity of the paper.

Six systems were investigated and a technique of solving the heat flow rate through two mediums with different conductivity was developed. It was concluded that the electrical analogue method is no doubt the simplest and the best method. For simpler systems the electric shape-factor method is recommended and for complicated systems the electric mapping method is recommended.

INTRODUCTION

Heat transfer by conduction is the transfer of heat from one part of a body to another part of the same body, or from one body to another in physical contact with it, without appreciable displacement of the particles of the body. In the great majority of cases arising in engineering practice, heat flows from some medium into and through a solid retaining wall and out into some other medium. The resistance to the conduction of heat through the retaining wall is only one of a series of resistances, but a solution of this resistance is often-times essential to the designers in designing a heat-transfer process. In many mechanical and chemical engineering proceses, the heat flow through a retaining wall involves only a two dimensional steady-state type of heat conduction, that is the heat flows only in the directions of the cross-sectional plane and the temperature at every point inside the solid wall does not vary with time. This type of heat conduction is what this thesis is concerned with.

The basic equation for thermal conduction in the steady state is:

$$q = -KA \frac{dt}{dx}$$

wherein A/dx may be interpreted as a shape factor, entirely governed by the geometrical shape of the system or the retaining wall. In few cases this shape factor can be can be evaluated rather simply by some analytical methods, such as heat-flow through the walls of a cylindrical pipe. But in many other cases the geometry of the systems are too complicated for analytical solutions.

Various other methods of solving these problems have been developed. The most important three are: (1) the numerical method or the relaxation method, (2) the mapping method, (3) the electrical analogue method. It must be mentioned that all these methods were developed in assuming the thermal conductivity, K, is constant through out the solid wall.

The relaxation method involves a great deal of numerical trial and error and is extremely tedious as the geometry of the system becomes more complex. Therefore it has limited application. Emmons (4) used this method to calculate the heat loss from a furnace wall. For 5 percent deviation from the experimental result, 0.75-hr. was spent in calculating, while for a deviation of 2.8 percent 1.75-hr. was required.

The mapping method, which can be carried out by trial and error graphical solutions, using the method of curvilinear squares (7), is also very tedious and is relatively inaccurate. A detailed explanation of the method is shown in the Appendix.

The electrical analogy method seems to be the most promising one. People have long realized the similarity between the heat flow and the electric current flow (6). In order to solve a complicate two dimensional steady-state heat conduction problem by actually set up the heat transfer system and apply the temperature differences, a most complicated and elaborate set up will be realized. But if an electrical analogy method is used, the problem can be solved in a much simipler manner with good accuracy, provided, that one can obtain a thin sheet of material with relatively high resistance to electric current and uniformly conductive to electricity. Not until recently has such a sheet been developed, which explains why this method has not been widely used. Investigators (1)(2) had used the electrical analogue to predict the rate of heat flow in the two dimensional systems. The results obtained checked closely with the results obtained from the analytical, the numerical and the graphical methods. The procedures used by the previous investigators is as follows:

The conduction paper was cut into the shape determined by the cross-sectional geometry of the system under investigation, divided by a line of symmetry, which was also an adiabatic line. The dimensions of the paper-shape were proportional to the exact dimensions of the system.

An electric potential difference was set up across the paper in the same pattern in which a temperature gradient would be applied. Using a potentiometer, the line of symmetry was divided into a number of equipotential points. Equipotential lines were then plotted. These lines are analogous to isotherms in heat flow. This can be shown by comparing the Ohm's equation with Fourier's equation:

$$\Delta E = IR = I\left(\frac{1}{r}\right)\left(\frac{X}{A}\right)$$

$$\Delta t = q\left(\frac{1}{K}\right)\left(\frac{X}{A}\right)$$

These two equations are analogous if the current flow by conduction and the heat flow by conduction are assumed identical. By dividing the first equation by the second equation, it is possible to see that the ratio of the potential difference to the temperature difference is equal to a constant times the ratio of the current flow to the heat flow.

Using a second piece of paper, with the electrodes located on the lines of symmetry, equipotential lines representing adiabatics were presented.

The lines representing adiabatics and isotherms were superimposed by tracing from the two experimental plots, to obtain a map. This map was equivalent to the map obtained by the graphical method.

The purpose of this thesis is to continue the work of the previous investigators, and to apply the electrical analogue method more intensively in solving the complex two dimensional steady-state heat conduction problems.

Six different heat conduction proglems have been investigated. They are:

Problem 1: heat flow through a rectangular flue duct.

Problem 2: heat flow through a flat plate with longitudinal bar fins on the top surface.

Problem 3: and 4 heat flow from longitudinally finned condensors.

Problem 5: heat flow in a cored, steam heated platen of a vulcanizing press.

Problem 6: heat flow thru two mediums with different conductivety.

PROCEDURE

Two electrical analogue methods were used in this work. identical apparatus was used for the two methods, as shown in figure 1.

The first method, that is the graphical method, which is essentially the same as used by the previous investigators (2).

Two pieces of conductivity paper were used to solve each problem. Both were cut into the proper shape to represent the heat flow problem. Metal bus bars were used as the electrodes. Aquadag, a colloidal suspension of graphite, was used as a glue to attach the metal bars on to the paper. Across the first piece of the paper an electric potential difference was applied, in the same pattern in which a temperature gradient would be applied. Equipotential lines were then plotted. These lines are analogous to isotherms in heat flow.

On the second piece of the paper, with the electrodes located on the lines of symmetry, equipotential lines representing adiabatics were presented.

The lines representing adiabatics and isotherms were superimposed by tracing from the two experimental plots, to obtain a map.

In problem five, only the lines representing the isotherms were plotted. It was not possible to plot the adiabatic lines by the electrical method because the

APPARATUS

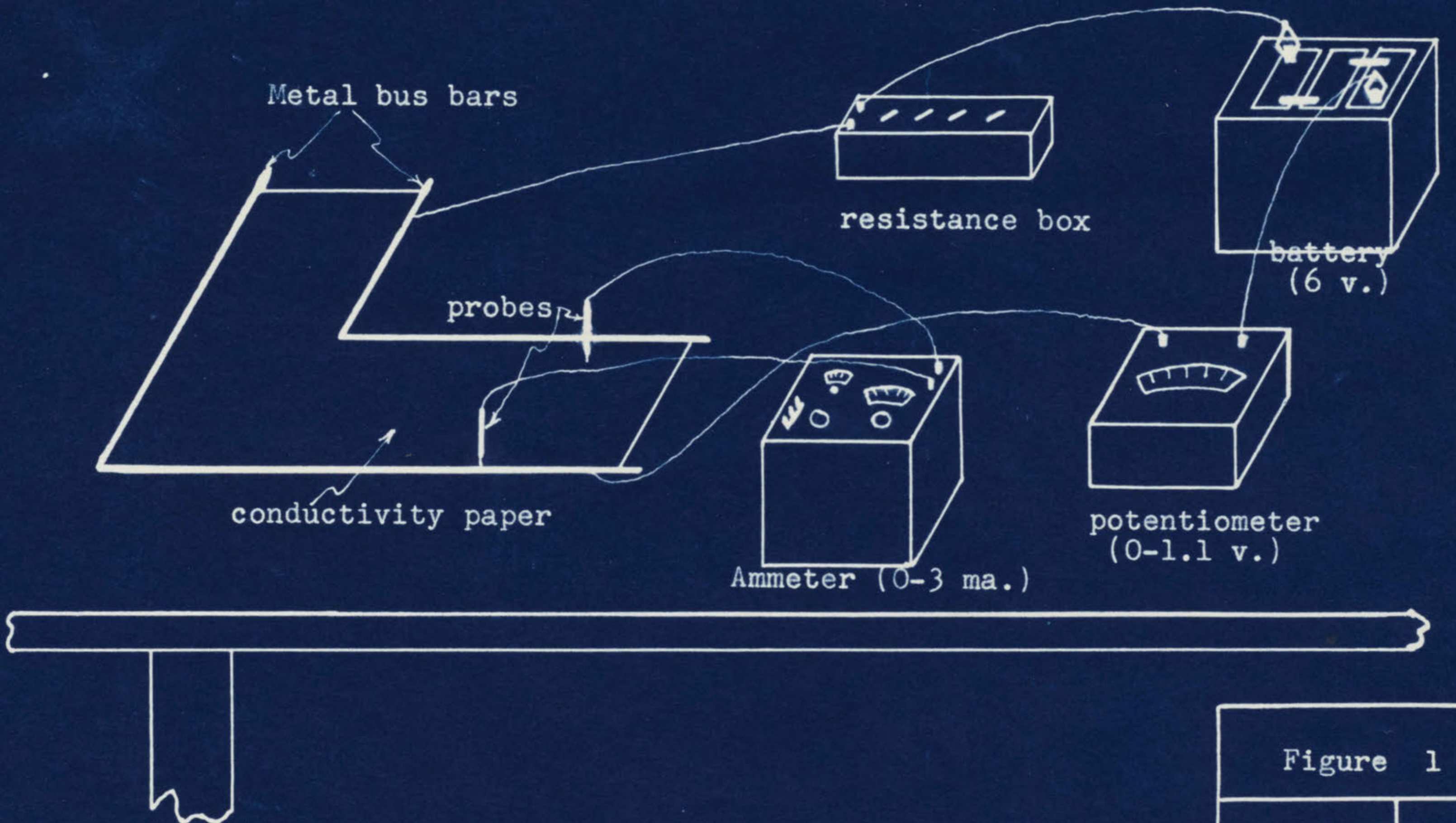


Figure 1

5-14-1954 S. I.

geometrical shape were not symmetrical. Therefore the adiabatic lines were drawn in by forming curvilinear squares with the experimentally determined isotherms.

In problem six, since a portion of the conduction material has a different conductivity constant than the rest of the material, therefore a special set up was necessary. The minor portion, which has lower conductivity was cut out from the conductivity paper , of which the entire cross-section of the system was represented. Another piece of the conductivity paper was cut out into the same shape as the portion with lower conductivity. It had dimensions equal to the ratio of the conductivity of the major portion (portion with the higher conductivity) to the conductivity of the minor portion multiplied by the original dimensions of the minor portion. This piece of the paper was laid within easy reach from the paper representing the major portion. Small metal electrodes of the same sizes were attached along the sides of the major portion where originally these were the sites of the minor portion. A small distance was neccessary to saparate one electrode from the other. Along the sides of the enlarged minor portion, equal numbered small electrodes were attached. On each edge, the total effective length of these electrodes was equalled to the length of the respective edge of the original minor portion. The small electrodes of the major portion were connected with copper wires to the respective electrodes of the enlarged minor portion. The isothermal edge of the enlarged minor portion was attached to a metal bus bar which has the effective length again equal to the length of the respective edge of the original minor portion. This isothermal edge had the same temperature as the isothermal edges of the major portion adjacent to the minor portion. Hence, these edges must have the same potential. Subsequently, they were connected to the same potential source. Figure 11 can further clarify this set up. The isothermal lines were plotted in the same way as in the previous systems. The adiabatic lines were drawn in due to the fact that the second and the third isothermal lines were parallel to the isothermal edge. (See Fig. 10) The positions of the adiabetic lines can be easily located to form the curvilinear squares with the isothermal lines.

The second method, the shape-factor method, is simpler than the first method. For each problem only one piece of the conductivity paper was required. The electric conductivity constant of the paper, which equals the reciprocal of resistivity, was first obtained by applying a potential difference across a piece of the paper with known shape-factor. Then another piece of the same paper was cut into a shape representing the heat flow problem under investigation. The metal bus bars were arranged in the same manner as in the case of plotting the isothermal lines. An electric potential difference was applied in the same pattern as a temperature gradient would be applied. The potential difference across the paper and the current flow through the paper were accurately measured. By substituting these quantities into the equation:

$$I = \frac{F}{f}(\Delta E)$$

Where I = current in m.a.
F = shape-factor
f = resistivity
ΔE = potential difference in volts

The shape-factor was obtained for that particular shape. This method was not used in determing the shape -factor for problem No. 6 becaue the ammeter was damaged during the test.

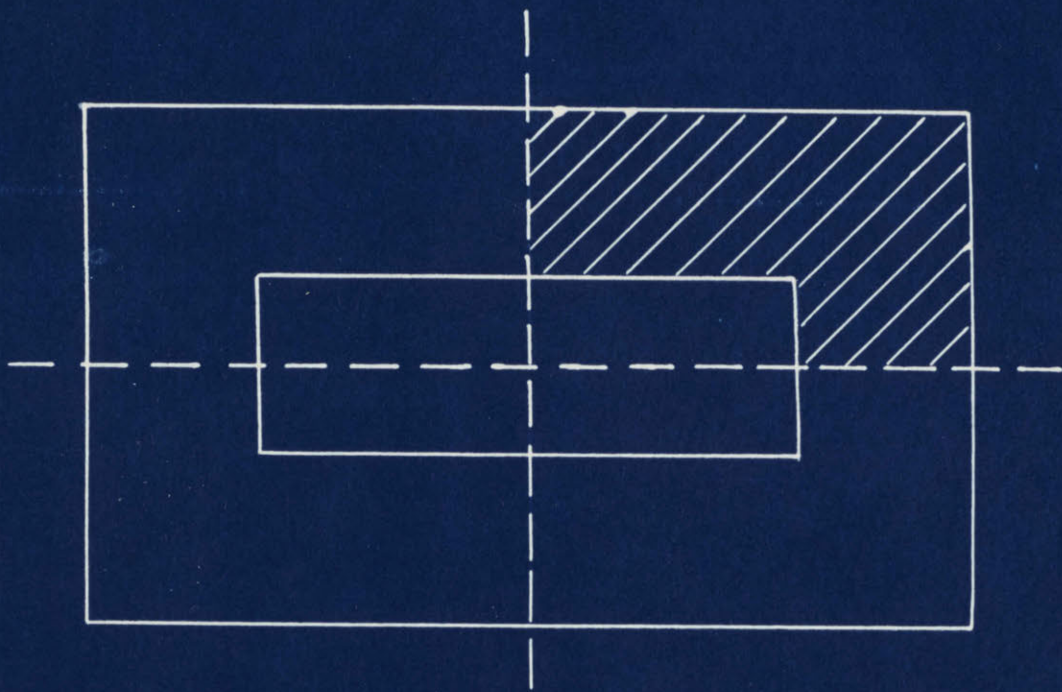
RESULTS

Altogether six systems were investigated. The first system was a rectangular flue duct with hot flue gas flowing inside the duct and the outer surface of the duct exposed to the air. The temperature of the hot flue gas would change as it flows along the duct. But this change in temperature was neglected in this investigation. The second system was a flat piece of metal with bar fins on its top. The third and the fourth system were longitudinally finned condenser tubes of two different types. The temperature along the surfaces of the fins is assumed uniform, although in practice a slight temperature difference would occur along these surfaces. The fifth system was a cored, steamheated platen of a vulcanizing press. Three sides of the metal block were insulated with asbestos and were assumed adiabatic. Hot steam is presumed to condense inside the circular bores of the metal block to heat up the top surface of the metal block where unvulcanized rubber would be placed. The metal surface in contact with rubber was assumed isothermal.

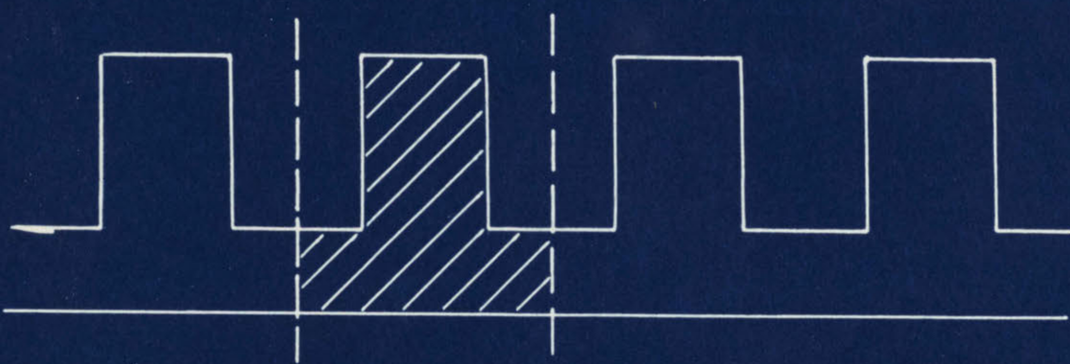
The sixth system consisted of a long square block made of two different kinds of metals. Two sides of this block were insulated and other two sides were exposed to two different temperature levels. It has no practical use, but was investigated just to develope the method of evaluating heat flow through two material mediums of different conductivity.

The cross-sectional shapes of these systems were sketched in Figures 2, 3 and 4.

CROSS-SECTIONAL AREAS
(The shaded areas are the areas investigated)



SYSTEM 1



SYSTEM 2

Figure 2

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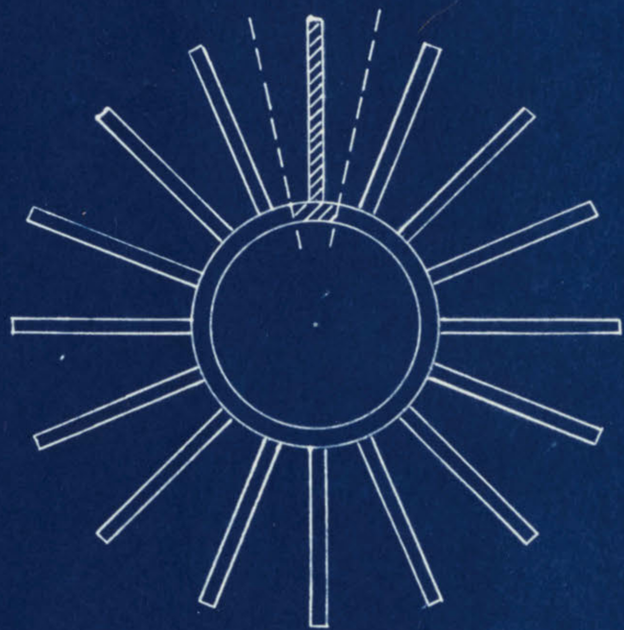
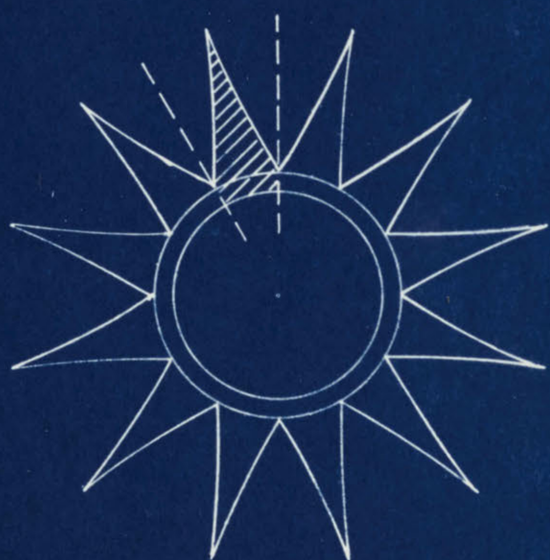
CROSS-SECTIONAL AREAS
(The shaded areas are the areas investigated)



SYSTEM 3



SYSTEM 4

Figure 3	
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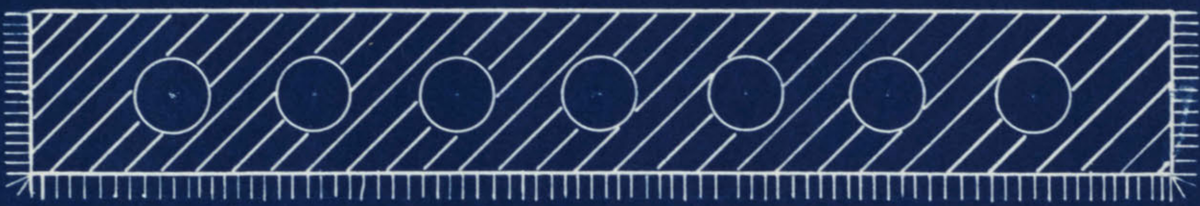
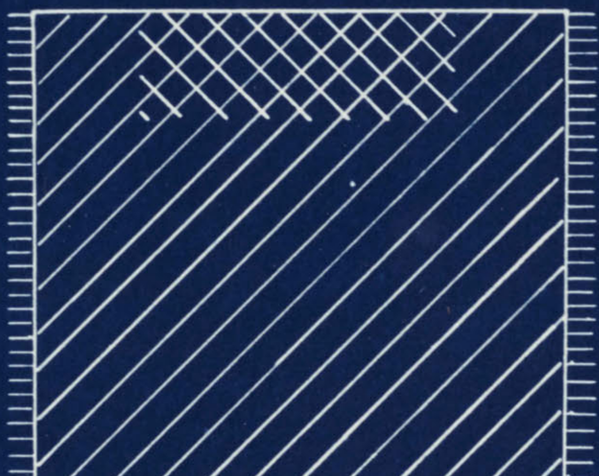
CROSS-SECTIONAL AREAS
(The shaded areas are the areas investigated)



SYSTEM 5



SYSTEM 6

Figure 4

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The results of the investigation were given in terms of shape factor in table I. For practical use K, L, and Δt must be specified in order to evaluate the rate of heat flow.

The actual maps were also presented in Figures 5, 6, 7, 8, 9 and 10. In Figure 9 only half of the complete map was shown. This was because the other half would just be the mirror image of the part presented in the figure.

TABLE I

System	Shape Factor - F	
	Electric Mapping Method	Electric Shape Factor Method
1	10.25 ± 0.55	10.8 ± 0.57
2	2.33 ± 0.09 (Per fin)	2.68 ± 0.14 (Per Fin)
3	3.0 ± 0.15 (" ")	3.2 ± 0.17 (" ")
4	3.0 ± 0.15 (" ")	3.12 ± 0.17 (" ")
5	23.3 ± 2.2	16.90 ± 2.0
6	0.89 ± 0.067	

$$q = \frac{A}{X} K(\Delta t) = FLK \Delta t$$

DISTRIBUTION OF ISOTHERMAL AND ADIABATIC LINES IN A RECTANGULAR FLUE DUCT

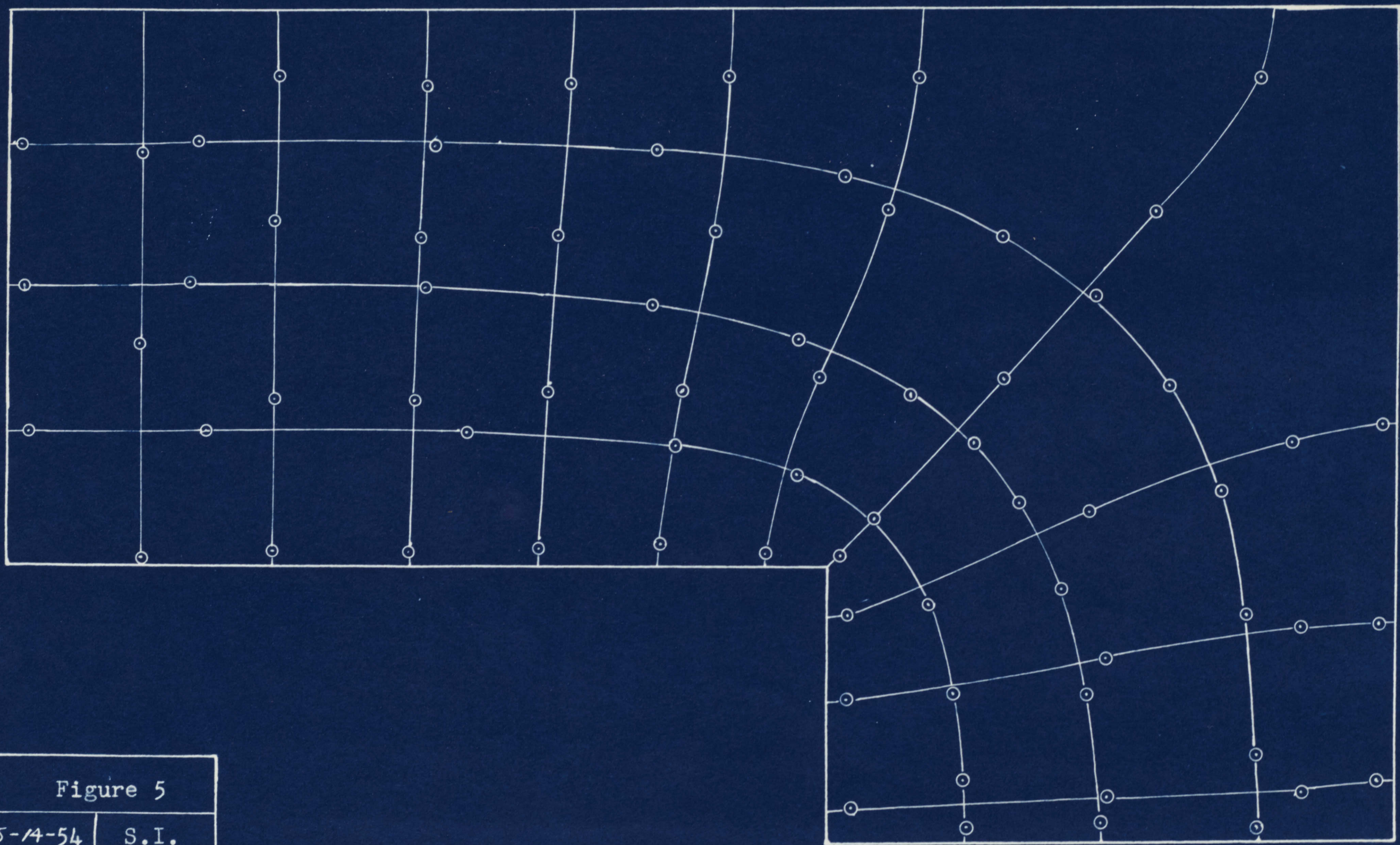

Figure 5

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DISTRIBUTION OF ISOTHERMAL AND ADIABATIC LINES IN A FLAT LONGITUDINAL BAR FIN SYSTEM

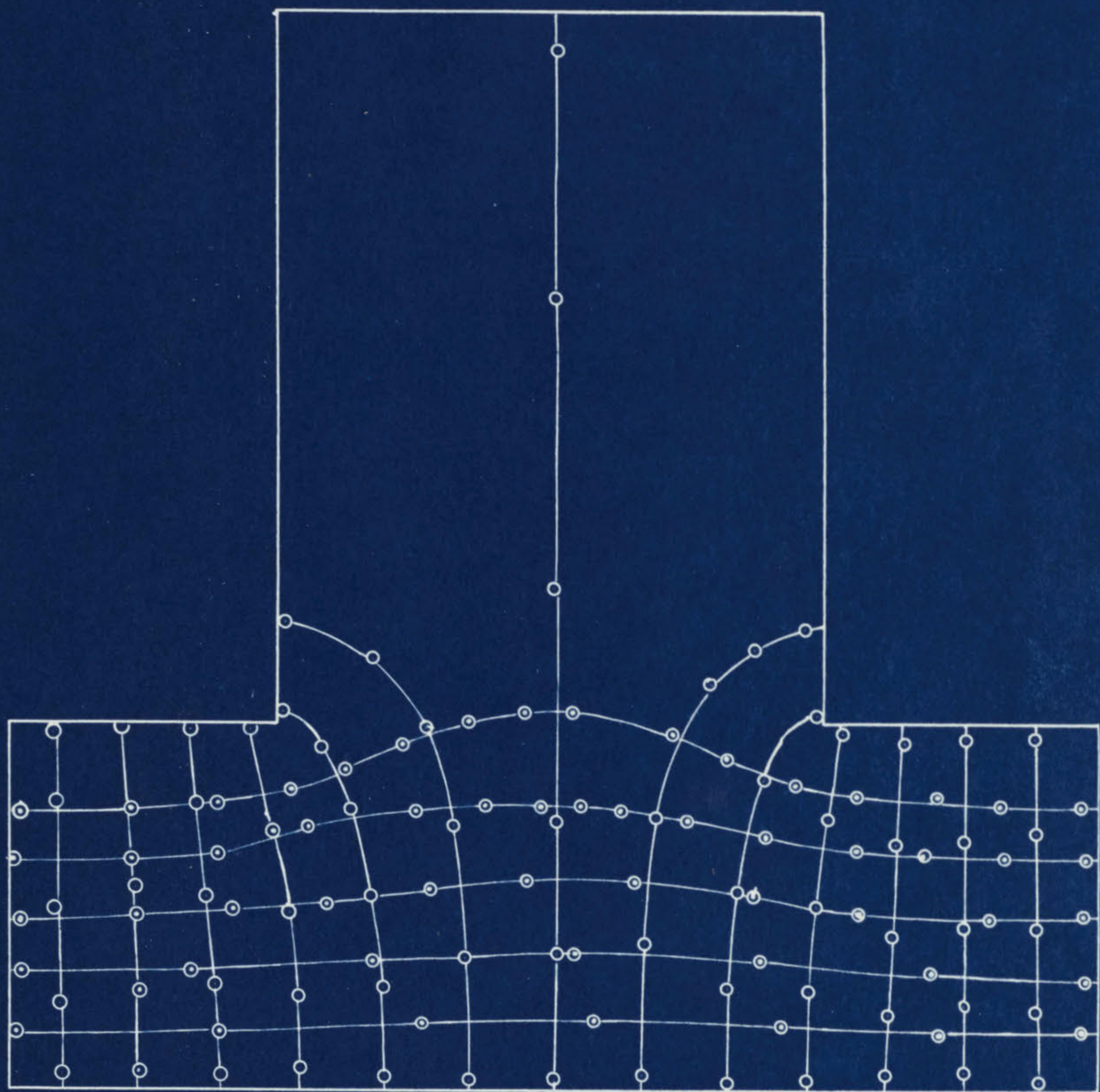

Figure. 6

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DISTRIBUTION OF ISOTHERMAL AND ADIABATIC LINES IN A LONGITUDINALLY FINNED CONDENSER

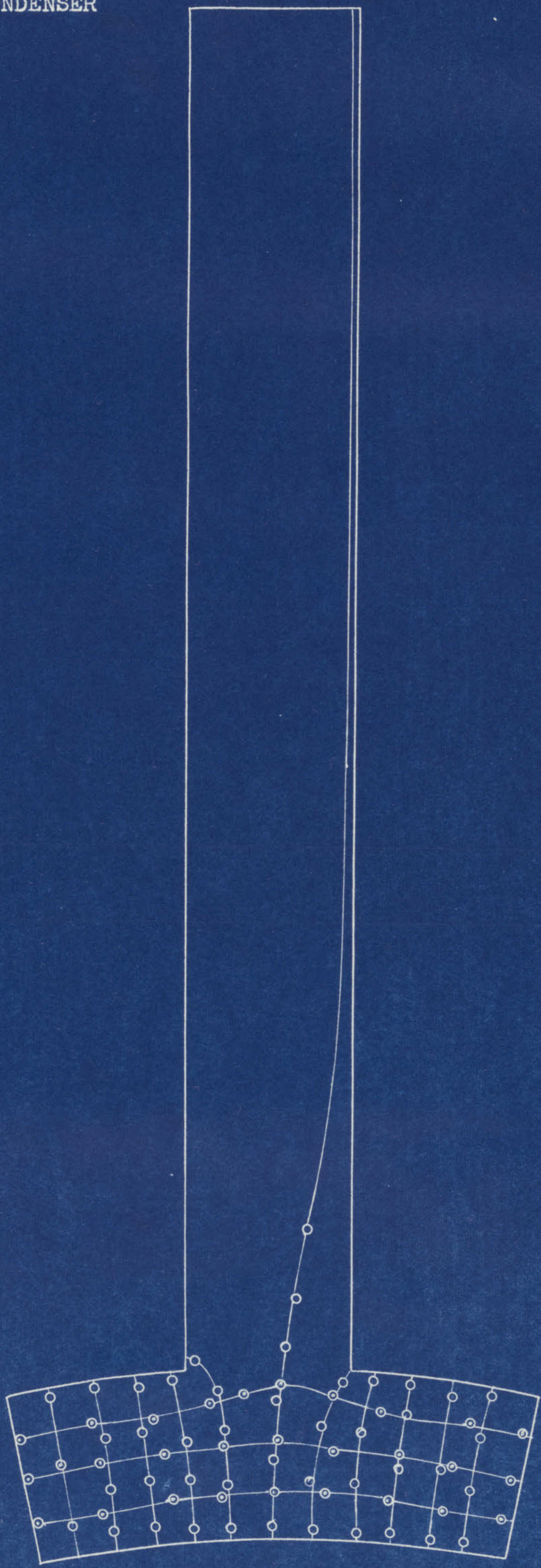

Figure 7

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DISTRIBUTION OF ISOTHERMAL AND ADIABATIC LINES
IN A LONGITUDINALLY FINNED CONDENSER

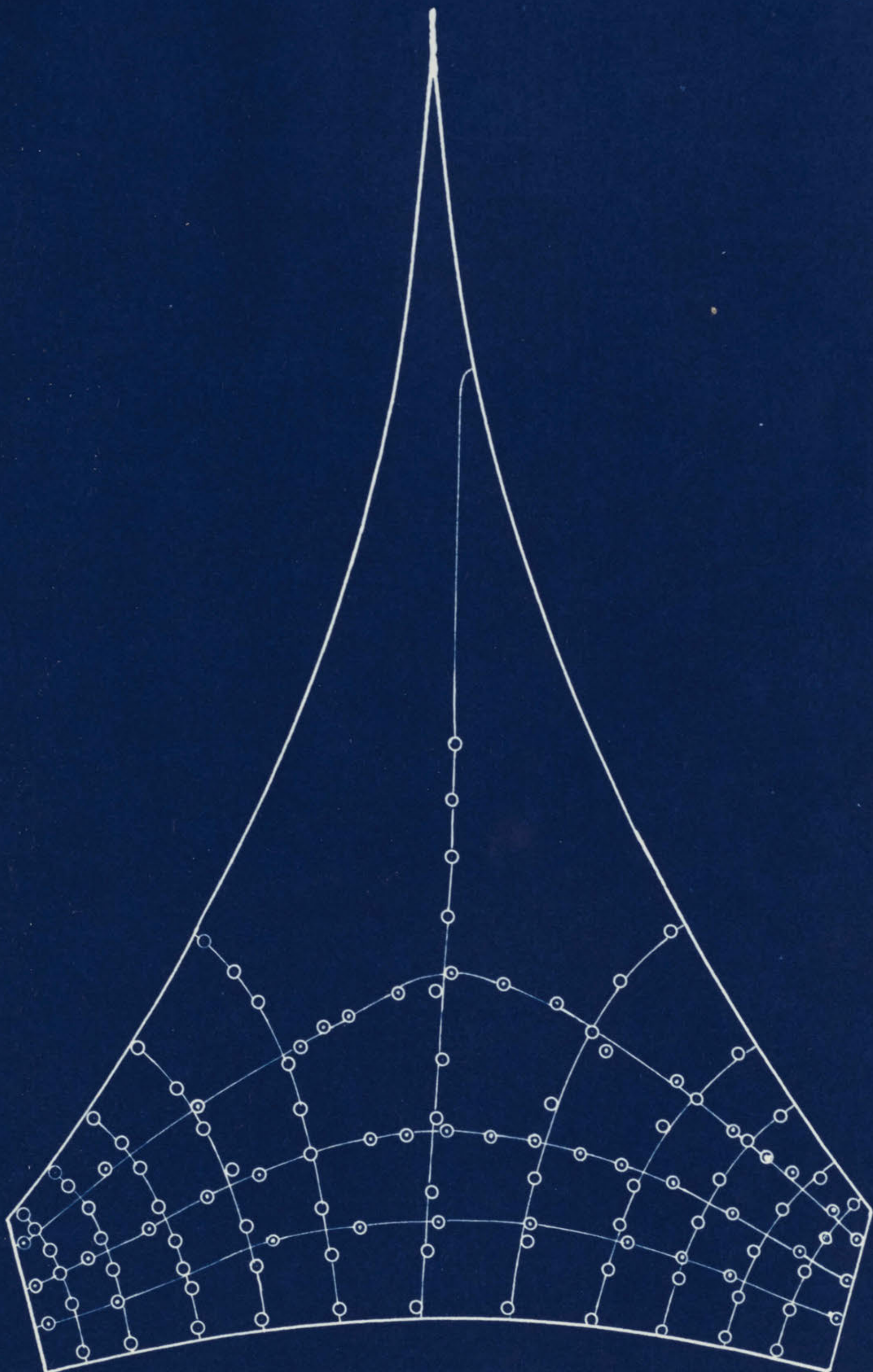

Figure 8	
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DISTRIBUTION OF ISOTHERMAL AND ADIABATIC LINES IN STEAM
PLATEN OF A VULCANIZING PRESS

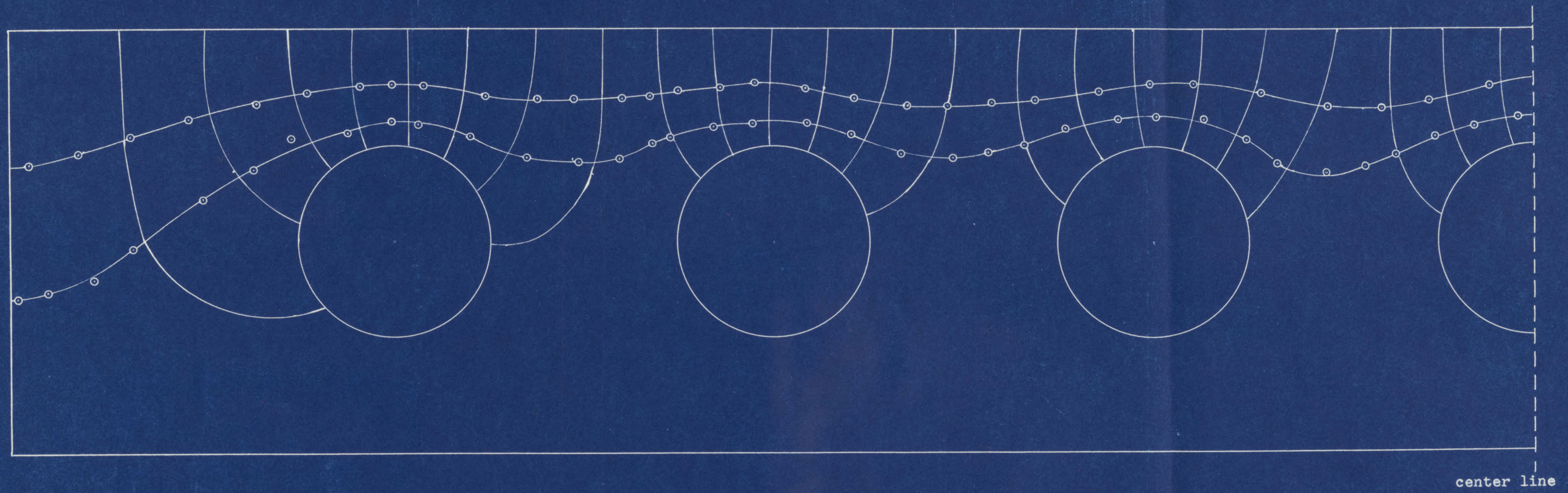

Figure 9

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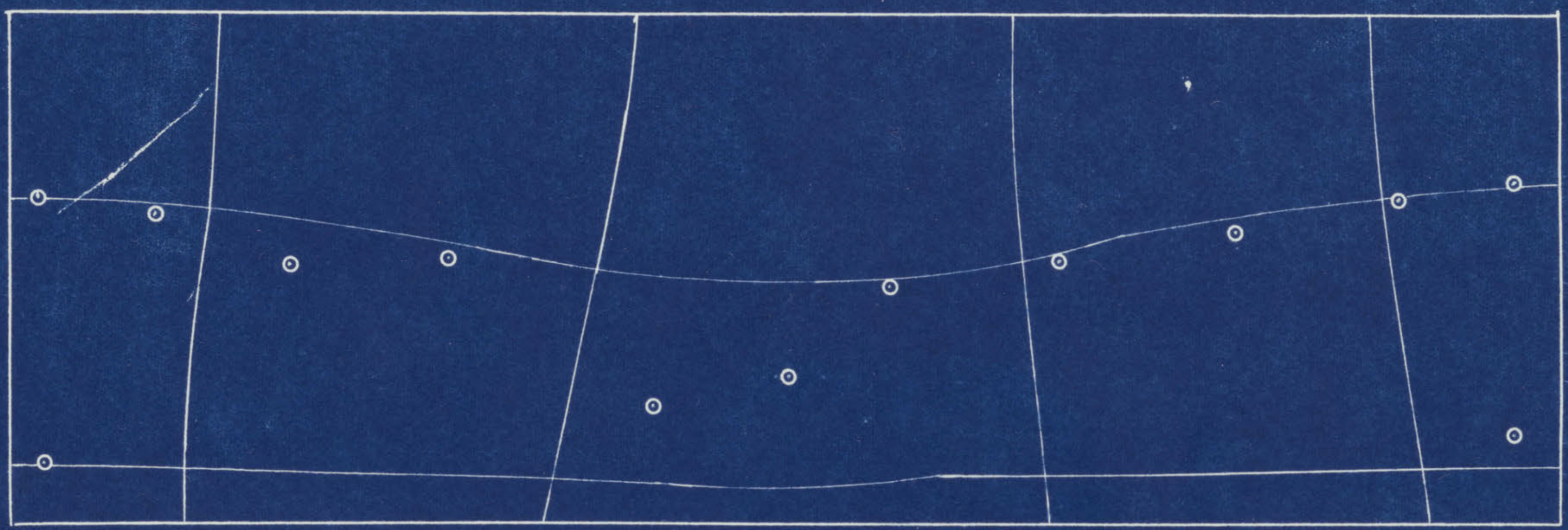
DISTRIBUTION OF ISOTHERMAL AND ADIABATIC LINES IN A SYSTEM CONSISTS OF TWO MEDIUMS WITH DIFFERENT CONDUCTIVITY



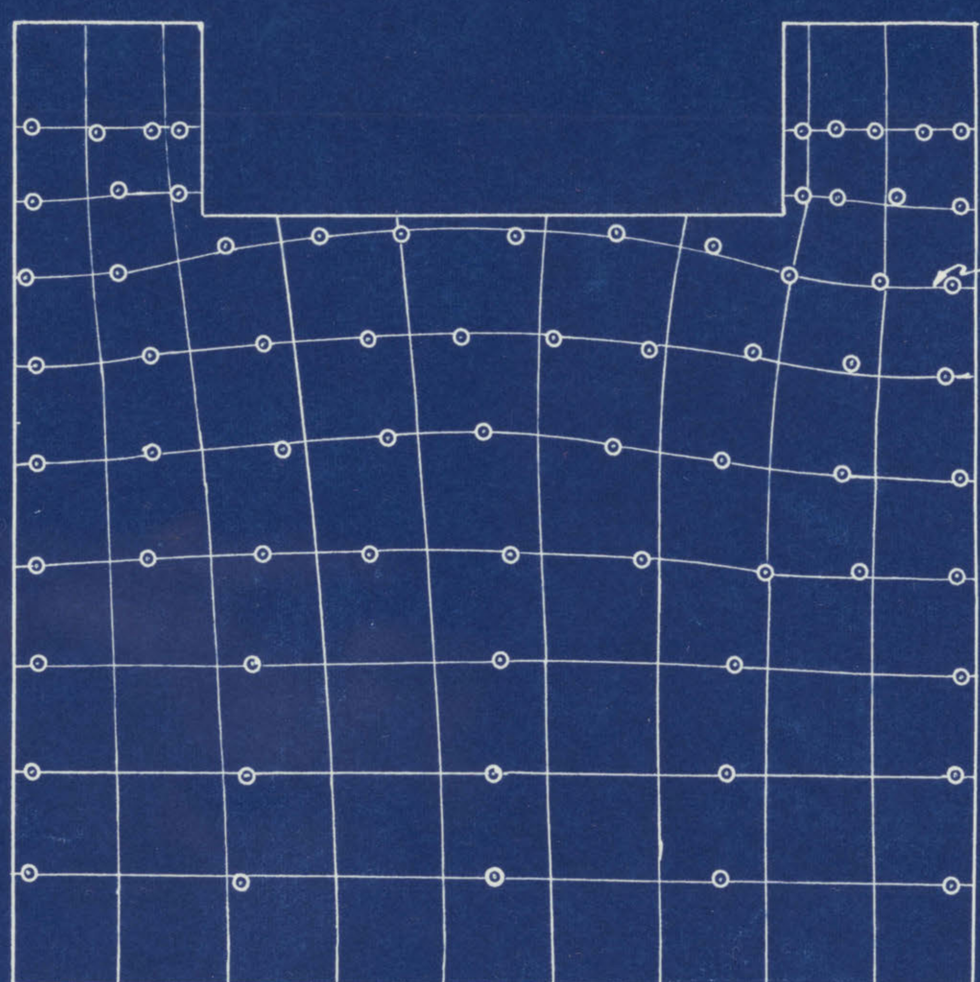


Figure 10

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DISCUSSION

The results of the two methods for the first system checked closely to within 5%. The error estimated for each of the method was about 5%. By using the analytical approximation of Awbery and Schofield (see sample calculations in the Appendix), F of 10.23 was obtained. This checked to within 0.2% deviation from the result obtained by the mapping method.

For systems 2, 3 and 4, the error estimated for both methods was 4 - 5%. The deviation of the results between the two methods for system 2 was about 15%, which exceeded the estimated error. No definite explanation can be given to this large deviation. As for systems 3 and 4 the results checked closely between the two methods and are within the estimated error.

The estimated error for system 5, was about 9% for the mapping method and 12% for the shape factor method. The deviation between the results of these two methods was about 30%. This large deviation was due to the following reasons: (1) The adiabatic lines were drawn in to form curvilinear squares instead of by electrical plotting. (2) The long contact length between the electrodes and the paper, might cause the potential drop across the "aquadag" paint to be non-uniform along the entire length of the electrodes.

From Table I, it is possible to see that the estimated errors were slightly lower for the mapping method than the shape-factor method. This was because the shape-factor method required very accurate measurements of both the current and the potential and also the conductive paint used to attach the electrodes on to the paper should be spreaded extremely uniformly along the length of contact in order to insure a uniform potential drop across this layer of paint. The mapping method is less sensitive to the potential deviations along the isothermal edges, because each isothermal line was drawn though a great number of points which automatically averaged out the small potential differences. Therefore the shape-factor method is proposed to use only in investigating a relatively simple system; even so, a number of measurements of the total potential drop along the isothermal edges is necessary in order to obtain a more representative average value. Another advantage to the mapping method is that it shows the pattern of heat flow through the rigid wall. Constant checks can be made during the mapping process. If the isotherms or the adiabatics do not go in the directions expected, or the isotherms and the adiabatics do not form curvilinear squares, then the circuit and the connections will be inspected in order to locate the flaws and to make necessary adjustments.

The new development of evaluating the heat flow through two different mediums in a two dimensional system presented clearly in system 6. (See Appendix in regards to the exact set up and procedure.) There are several important facts in regard to this development is worth mentioning here.

(1) This kind of set up and procedure allows the two different solid mediums to have a wide range of difference in the resistance to heat flow. This can be done by just varying the dimensions of the enlarged portion. But as the difference in conductivity between the two mediums becomes too large this method may fail. It is because of the fact that the dimensions of the enlarged portion will increase proportionally to the ratio of the conductivities. As the lengths of the sides of the enlarged portion become too great as compared to their original lengths, the small electrodes along the sides of the enlarged portion will be so scattered and wide apart from each other that this set up will cause the system to deviate tremendously from its original conditions. In the original system, the current (which is analogue to heat), by conduction, flows across the entire area of this section of low conductivity. With the small electrodes saparated apart along the sides of the enlarged portion, the current tends to by-pass certain areas. This undesirable effect becomes more significant as the distance between the elctrodes increases and the set-up becomes less and less representative to the original system.

(2) This technique can also be applied to the relatively complicated systems. For instance, in the case of system 5, if the shape-factor for both the vulcanizing rubber and the steamheated platen is required, with known relative conductivity of the rubber and the platen, a set-up similar to the set-up employed in the system No. 6 can be used. Small electrodes can be attached along the top edge, representing the surface of the steamheated platen, and are connected to the respective electrodes along the bottom side of the enlarged rubber portion, representing the rubber surface adjacent to the platen. The long metal electrode should now be attached to the top edge of the enlarged portion, representing the top surface of the vulcanizing rubber. The long metal electrode is connected directly to a potential source. Some difficulty may be encountered in the above described set-up. Since the surface of the platen is so long a great number of small electrodes will be used. Imperfect contact between the electrodes and the conduction paper can hardly be avoided. This difficulty has even been experienced in preparing the set-up for problem 6. The way to minimize this difficulty is to decrease the number of electrodes by increasing the size of the electrodes. But this improvement is made on the expense of the accuracy which will be mentioned in the next paragraph. Therefore a balance should be made in selecting the size of the electrodes.

(3) The number of small electrodes used is directly proportional to the accuracy because in the actual case the potentials (same as the temperature) along the interface of the two mediums vary continuously. This is true until a critical number is reached where the accuracy will drop off as the number of electrodes increases. This is because more space will be left uncovered by the electrodes along the edge of the small rectangle (see figure 11), since the total space necessary to saparate one electrode from the other will be increased as the number of electrodes increases.

(4) In calculating the rate of heat flow, if the shape-factor F is evaluated by dividing N from total N_L, the conductivity constant K of the major portion will be used. From Figure 10 it is apparent that in the large rectangle the areas included by the isothermal and the adiabatic lines are not curvilinear squares. In this system the Y/X is equal to 3, since the material of the major portion has the conductivity constant 1/3 that of the major portion. This can be shown further by just looking at the major portion below the isothermal line No. 8 as shown in Figure 10. Here N/N_L is equal to the total N/N_L divide by 0.7, and the Δt equals to the total Δt multiplied by 0.7. The two effects cancel each other.

In this work an important modification in plotting the isotherms and adiabatics was made. The previous workers (2) assumed the potential drop across the aquadag paint was negligible, so only one probe was used. The other lead from the potentio-meter was attached directly to the metal bus bar. Whereas in this investigation it was found that there was a significant amount of potential drop across the aquadag paint layer. Therefore two probes were used. The potential drops measured were the actual drops across the paper between the two probes. By this modification the paint with relatively high resistance can also be used provided it must be spread very uniformly along the edge. This is because that by using two probes the potential drop across the paint layer is not included in the potential measurements.

Previous investigators (2) had recommended a short-cut way to arrive at a solution by the electric mapping method. The recommendation is as follows:

Number of isothermal lanes, N, is set arbitrarily, and short sections of two isotherms are plotted in a region where nearly perfect squares will be formed. The distance between these isotherms is measured. At the same position on the adiabatic plot, the potential necessary to make Y/X = 1 is measured. The total voltage across the paper is then divided by the Y potential. This gives the number of lanes, and the necessary data for the calculations have been obtained.

This short-cut method certainly simplified the procedure. But there are certain disadvantages. Firstly, after going through setting up the electrodes and the wire attachments for both the isothermal and the adiabatic plots, it will certainly be a great loss not to obtain the entire plot of the heat flow pattern. Secondly, in laying out the short sections of two isotherms only very few points can be plotted. In some complicated systems there are no good ways of checking these points. Thirdly, this short-cut method cannot be applied to some systems where no lines of symmetry can be selected, such as in the case of problem 5 where the adiabatic lines have to be drawn in. In view of the above disadvantages this short-cut way is not recommended. If a fast solution is required in a relatively simple system, the shape-factor method is more advisable to use.

CONCLUSION AND RECOMMENDATION

In conclusion the electrical analogy method is the simplest method in solving the two dimentional steady-state heat transfer problems and still gives good accuracy.

The elctrical shape-factor method is recommended for simpler systems and for the more complicated systems the electrical mapping method is recommended.

The method for evaluating rate of heat flow through different solid mediums as developed in this investigation is satisfactory. Although this method can be applied to systems with relatively wide range of difference in the conductivity of the two mediums, it is still limited. Some more work should be done in further testing this method and to simplify the present tedious set-up.

APPENDIX

A. Detailed Procedure

1. The first method or the mapping method:

In the first problem the conduction paper was cut into a shape represent 1/4 of the cross-section of the duct. The four sides adjacent to the corner represent the inner and the outer surfaces of the duct. The other two sides were the lines of symmetry or the adiabatic lines. As for the fin problems, the papers were cut into shapes represent only one of the multiple fins of the systems. The lines to seperate one fin from its adjacent fins were the lines of symmetry. As to the rubber vulcanizer, no lines of symmetry were selected. Therefore the paper was cut into the same shape as the entire cross-section of the system. (The dimensions of the sides do not affect the shape-factor in anyway as long as all the sides are increased or decreased proportionaly from the original systems.)

The isothermal lines were first plotted by applying electric potential on the sides which represent the inner and the outer surfaces of the systems and were assumed to be isothermal. Flat and polished metal bus bars, were carved so as to fit the edges of the isothermal sides of the paper. These bars were glued on to edges with the "Aquadag" paint. The electric wire with either a positive or a negative charge was attached to the metal bars glued to one of the isothermal edges and the electric wire with an oposite charge was attached to the other isothermal edge. The total voltage drop across the paper was measured by two probes (one on each isothermal edge) which were connected to the two terminals of a potentialmeter. In measuring this total voltage drop, the needle of the probes were held as close to the metal bars as possible without touching them. In order to test whether the bus bars had good contact with the conduction paper, the probes were moved along the isothermal edges. The total voltage drop should be the same througout the whole lengths of the isothermal edges. A resistance box was connected in series with the conduction paper. It was used to regulate the voltage drop across the paper to a desired range. After the total potential drop was obtained, equipotential lines or the isothermal lines were than plotted. The procedure can be illustrated by a simple example. If a total voltage drop of 4/10 volts was obtained (this voltage drop can be regulated to any desired value by adjusting the resistance) and four isothermal lanes seemed to be adequate for the map, the 4/10-volt was divided by four which gave 1/10-volt for each lane. One probe was always held in the original position, that is on the isothermal edge and the other probe was moved over the paper to locate the points where the voltage drops were 3/10, 2/10 and 1/10 volts.

Smooth-lines were drawn through the points with the same potential and this gave a plot of isotherms.

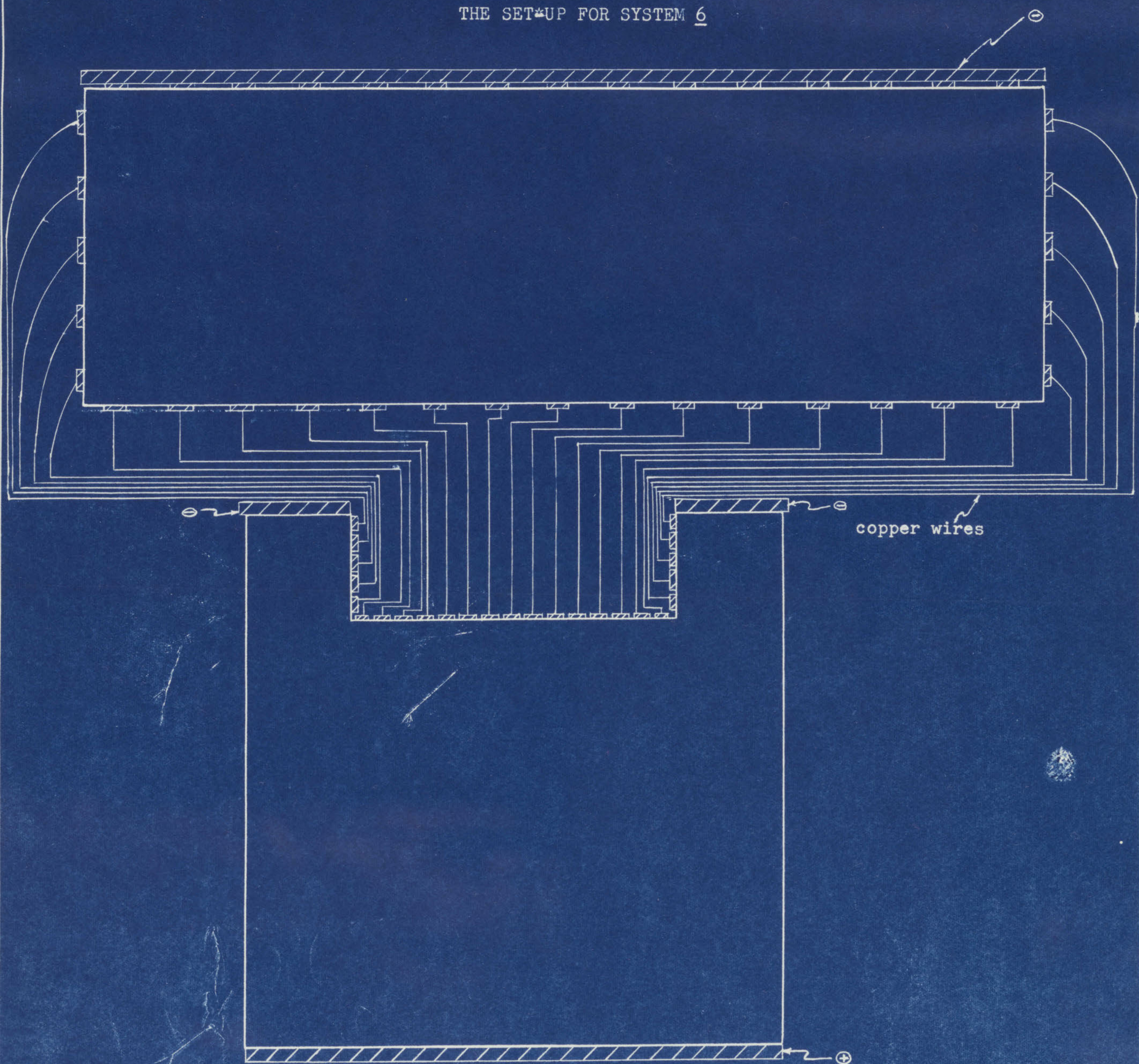
As to the adiabatic lines, a second piece of the paper with the same shape and dimension was used. The bus bars were now glued to the adiabatic edges. Again the contact between the bars and the paper was tested. On the first piece of the paper, a curvalinear square was carefully constructed between two isothermal lines and an adiabatic edge. The distance between the adiabatic edge and the newly constructed line was measured. Then this distance was laid out on the second piece of the paper in the same manner as on the first plot. The potential drop for this distance was measured. Then the adiabatic lines were plotted with each adiabatic lane of the above measured potential drop. In plotting the adiabatic lines, the plot always started from one end of the paper to the other. For most cases the last adiabatic lane had potential drop less than for the other lanes. The potential was measured and an average value was recorded. In counting the number of adiabatic lanes for the calculation, the last lane was counted as a fraction of one lane. This fraction factor was equal to the ratio of the average potential drop of the last lane and the potential drop for the other single lane.

In plotting the isothermal lines for the rubber volcanizer, mercury was used as the electrode on the long continuous isothermal edge. A dam was built along the edge to confine the mercury into a long and narrow line touched the paper uniformly along the edge. This was done because of the fact that it was impossible to have smooth and uniform contact between the metal bar and the paper for such a long edge. Also in this system, due to the lack of lines of symmetry the adiabatic lines could only be obtained by drafting these lines on to the plot.

As for system No. 6, where a portion along the surface of the system has different resistances to heat flow from the main body. It has a cross-section as shown in Figure 4. It was arbitrarily chosen that the resistance to heat flow for this portion was three times greater than the main body or in another words in this portion heat has to flow through a path three times longer than the apparant path, if this portion was made out of the same material as the main body. Therefore by keeping this in mind, a method was developed.

Two pieces of conduction paper were used in making the isothermal plot. One piece of the paper was cut into the shape of the main portion which has the higher conductivity. Along the three sides where the interfaces of the two mediums were located, a number of small electrodes were attached. These electrodes were of the same size and were saparated from each other by a very small distance. They were set as close to each otcer as possible but without touching. Then another piece of the conduction paper was cut into a shapeof the minor portion (the portion with lower conductivity). It had dimensions equal to the ratio of the conductivity oftne minor portion to the conductivity of the main portion multiplied by the original dimensions of the minor portion. In this case all the dimensions of this enlarged minor portion were three times the original dimensions. (see Figure 11) Small electrodes were attached to the threesides of this big rectangle where originally these sides were in contact with the main body. The number of electrodes on each side corresponded to the number of electrodes attached to the respective side of the major portion. The total effective length of these electrodes on each side was equal to the length of the same side with original dimension. The small electrodes along the sides of the large rectangle were connected with copper wires to the respective small electrodes along the sides of the major portion. As should be noticed the electrodes attached to the enlarged minor portion were distributed evenly along the sides. Silver paint (an excellent conductor when dried) was used to glue the electrodes on to the paper because any potential drop between the electrodes and the paper should be avoided. The fourth side of the large rectangle was part of the isothermal edge of the system. Metal bus bar was attached to this edge and again the effective length of this bus bar must equal to the original length of the same side. This

THE SET-UP FOR SYSTEM 6



The shaded areas are the locations of metal bus bars.

Fig. 11

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This isothermal edge must have the same temperature as the isothermal edges of the major portion adjacent to the minor portion. Therefore the bus bars along these isothermal edges were connected to the same potential source. Another bus bar,which was at a different potential level, was attached to the base of the major portion. The isothermal lines were plotted in the same way as in the previous system. The adiabatic lines were drawn in due to the fact that the second and the third isothermal lines were parallel to the isothermal edge. (see Figure 10) The positions of the adiabatic lines can be easily located to form the curvilinear squares with the isothermal lines.

2. The second method or the shape-factor method:

The set-up for this method was essentially the same as the set-up used in the first method when plotting the isothermal lines. The only difference was that a very accurate milliammeter was connected in series with the conduction paper.

The conductivity constant, or the reciprocal of the resistivity, of the conduction paper was first obtained. This was done by using a piece of the conduction paper which bore a square shape. Bus bars were attached to the two opposite edges of the square. The total potential drop across the paper and the current passed thru the circuit were accurately measured. By substituting these values into the following equations

$$I = \frac{A}{X}\left(\frac{1}{f}\right)(\Delta E)$$

In this case $I = (L)\left(\frac{1}{f}\right)(\Delta E)$

or $f = \Delta E(L)/I$

f was obtained. The above equation is analogues to

$$q = (A/X)\ (K)\ (\Delta t)$$

Then another piece of the conduction paper was cut out into the shape of the system under investigation. Metal bus bars were attached to the isothermal edges and potential difference was applied as in the case of plotting the isothermal lines. The total potential drop and the current were again accurately measured. By substituting these values into the above equation with f already known, the shape-factor A/X or F(L) could be calculated.

B. SUMMARY OF DATA

TABLE II

	Electric Mapping Method			Electric Shape-Factor Method ($\frac{f}{L}$ = 1.9196 ± 1%)		
System	N	N_L	$(F = \frac{N}{N_L})$	I in ma.	ΔE in v.	$F = \frac{f}{L}(\frac{I}{E})$
1 (¼ of Total Area)	10.25 ± 2.9%	4.0 ± 2.5%	2.57 ± 5.4%	1.52 ± 1.3%	1.08 ± 3%	2.7 ± 5.3%
2 (One fin)	14.0 ± 2.1%	6.0 ± 1.7%	2.33 ± 3.8%	1.52 ± 1.3%	1.09 ± 3%	2.68 ± 5.3%
3 (One fin)	12.0 ± 2.5%	4.0 ± 2.5%	3.0 ± 5%	1.22 ± 1.2%	0.732 ± 3%	3.2 ± 5.2%
4 (One fin)	12.0 ± 2.5%	4.0 ± 2.5%	3.0 ± 5%	0.65 ± 3.1%	0.4 ± 1.3%	3.12 ± 5.4%
5 (Entire Area)	46.6 ± 4.3%	2.0 ± 5%	23.3 ± 9.3%	2.64 ± 0.76%	0.30 ± 10%	16.9 ± 11.8%
6 (Entire Area)	8.9 ± 5.6%	10.0 ± 2%	0.89 ± 7.6%	------	-----	------

C. Derivation of the equation for the graphical method.

In deriving the equation a simple system is used for illustration. The system consists of a square block with two opposite sides insulated and the other two sides expose to two different temperatures. Cross-section of the system is shown in Fig. 12. There is no heat flow in the longitudinal direction L, measured at right angles to the drawing of Fig. 12.

In applying the method, one may arbitrarily subdivide the integral $\int_{t_1}^{t_2} K\, dt$ into any convenient number N of equal parts, $K\, t_x$. The heat may be visualized as flowing in series through a narrow lane starting at the isotherm AB, representing t_1, and ending somewhere along the isotherm CD, representing t_2 (see Fig. 12). Heat flows at the steady rate q_L through each such lane. For any small part, such as OPQR, of any lane, the heat is flowing at right angles to the area Ly, where L is the length of the body, and y is the mean width of the quadrilateral OPQR, i.e., y equals (PQ+OR)/2. The conduction equation is

$q_L = KA_m \Delta t_x/x = K(Ly)(\Delta t_x)/x$, and since $K \Delta t_x$ from one isotherm to the next is the same, and q_L and L are also constant throughout the lane, the equation shows that the ration y/x must be constant throughout the lane although both y and x may vary. If the construction used in drawing each lane is such that y equals x, as is the case in Fig. 12, the heat flow per lane is $KL(t_1-t_2)/N$, and the total flow carried by the total number of lanes, N_L, is

AN ILLUSTRATION FOR DERIVING THE EQUATION FOR THE GRAPHICAL METHOD

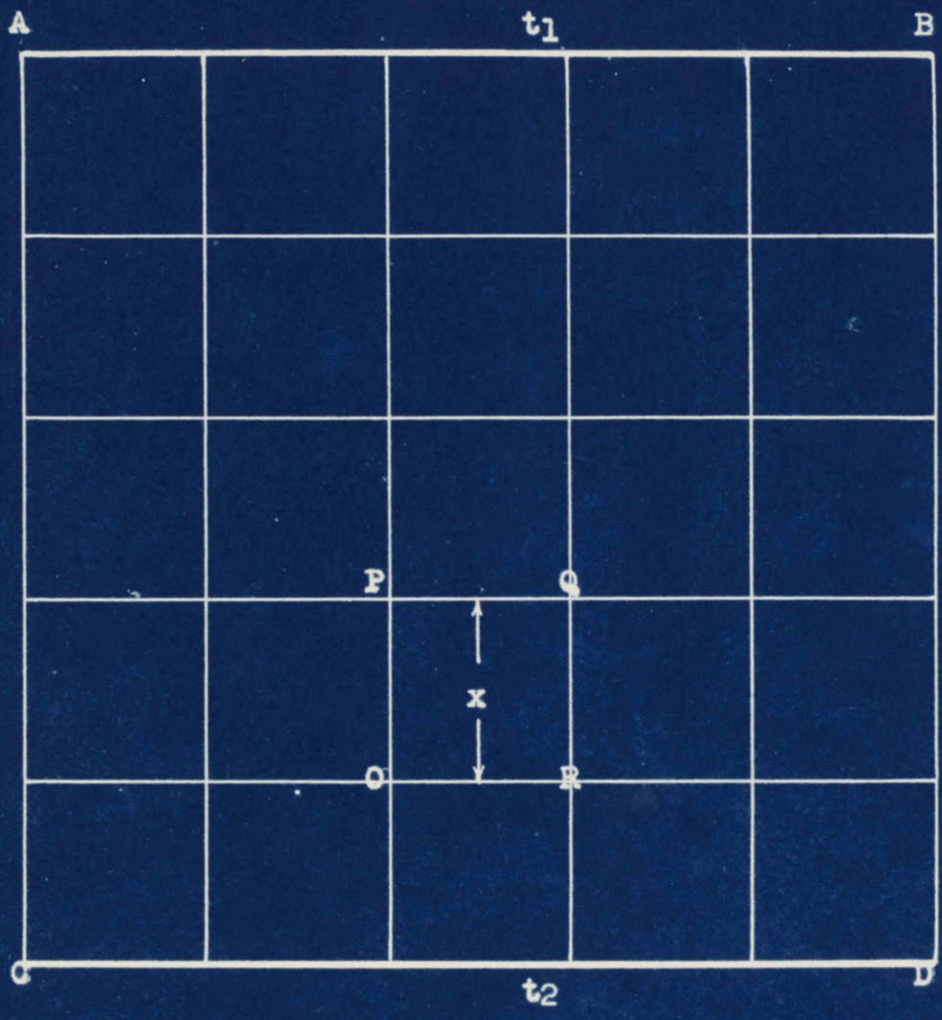

Figure 12	
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$NLKL(t_1-t_2)/N$. A constant K is assumed in deriving this equation.

One may fix N and by trial, so locate the isotherms and the flow lines they intersect at right angles to form quadrilaterals such that Y is substantially equal to X, that is, the ratio of the sums of opposite sides closely approaches unity.

D. Sample Calculations

The first system is used for illustration.

1. The calculation for the electric mapping method.

$$q = \frac{N}{N_L} (L)\left(\frac{Y}{X}\right)(K)(\Delta t) = \frac{N}{N_L} (L)(K)(\Delta t) \quad \text{since } \frac{Y}{X} = 1$$

$N = 10.25 \pm 0.3$

(For 1/4 of the total area)

$N_L = 4 \pm 0.1$

$$F_{(total)} = \frac{4xN}{N_L} = \frac{4x(10.25 \pm 2.9\%)}{4 \pm 2.5\%} = 10.25 \pm 5.4\%$$

$$= 10.25 \pm 0.55$$

$$q = (10.25 \pm 0.55)(L)(K)(\Delta t)$$

2. The calculations for the electrical shape factor method.

A square shape conduction paper was used to evaluate f.

$I = 0.5$ m.a. $\pm$ .005

$E = 0.9598$ v. $\pm$ .0002

$$f = \frac{0.9598 \pm .02\%}{0.5 \pm 1.0\%} (L) = (1.9196 \pm 1\%)(L)$$

Now for the rectangular duct.

$I = 1.52 \pm 0.02$ ma.

$E = 1.08 \pm 0.03$ v.

$$\left(\frac{A}{X}\right) = F(L) = \frac{f(I)}{\Delta E} = (L)\frac{(1.9196 \pm 1\%)(1.52 \pm 1.3\%)}{1.08 \pm 3\%}$$

or $F = 2.7 \pm 5.3\%$ (For 1/4 of the duct)

$$q = (4)(F)(K)(L)(\Delta t) = (10.8 \pm 0.57)(K)(L)(\Delta t)$$

3. Calculations based on the approximation by Awbery & Schofield. (1)

$$q = \frac{KL}{B} (p + CB)(\Delta t)$$ (For one corner)

$$q \text{ total} = \frac{4\,kL}{B}(p + C\,B)(\Delta t) = \frac{4(K)(L)}{4}\left[(6+2)+0.558(4)\right](\Delta t)$$

$$= 10.232\,KL\Delta t.$$

E. TABLE OF NOMENCLATURE

q = Quantity of heat transferred Btu/(hr)(ft.)of the body.

K = Thermal conductivity Btu/(hr)(ft)(°F).

L = Length of the system. (ft.)

N_L = The number of adiabatic lanes carrying heat from source to sink.

N = The number of zones into which the lanes were divided by isotherms to form curvilinear rectangles.

I = current in milli-amperes.

E = Potential difference in volts.

f = Resistivity in (volts)(ft) / (milli-amperes).

R = Resistance in volts / milli-amperes.

A = Area through which heat flows at right angles, ft^2.

X = Length of conduction path, ft.

F = Shape factor A/X, in ft.

Y = Length of any inside edge of a parallelepiped, ft.

P = Length of the internal boundary of the corner of an square edge, ft.

B = Thickness of the rectangular flue duct, ft.

C = A dimensionless constant of the order of, but necessarily greater than, 0.477.

F. Literature Citations

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