A MAGNETIC ACCELERATOR

by

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ABSTRACT

The thesis discusses the use of magnetic fields to accelerate an iron slug. The purpose of the investigation was to design and construct an accelerator which is efficient and capable of producing high velocities. The accelerator makes use of the high permeability of iron to obtain the electro-mechanical energy conversion. The accelerators were designed to be used in tandem to lower the energy requirements of one unit. This system also allows for a better velocity control.

The accelerators operate by passing current through the coils to establish magnetic fields. After the acceleration has taken place, the current is rapidly switched off. The magnetic field places stringent requirements on the switching device. The thesis discusses a method of switching which is designed to withstand the high voltage developed and fulfill the time requirements.

To accelerate an object of any appreciable size to a high velocity requires a large amount of energy. Particular emphasis is therefore given to the amount of energy obtainable from each accelerator. The design of the accelerator which was constructed for the thesis is discussed in detail. Then it is shown that with suitable tools and careful construction the energy conversion efficiency may be greatly increased. The requirements of the power supply to be used are also discussed.

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CHAPTER I

INTRODUCTION

1.1 Purpose of the Investigation

The use of magnetic fields to accelerate a small object was studied with particular interest in the maximum velocity obtainable by the method. In recent years efforts have been directed toward the atomic aspects of matter with little attention given to particles of visible size travelling at high velocities. The system described here was investigated in the hope of facilitating such a study.

Such a system would enable the study of collisions with high energy . It would also be useful in the study of air flow and other problems related to objects moving at high speed in air. The particular advantage of the method investigated is that it affords controlled velocities.

1.2 Description of the System

A sketch of the accelerating system is shown in Fig. 1 on the following page. A drawing of a disassembled accelerating unit is shown in Fig. 2. Four accelerating units were constructed for the study. If the system was put to actual use, more units would probably be necessary. To operate the system, 115 VDC is applied to all coils and relays. The coils are connected in series with the relay contacts which are placed between the rails. Current begins to flow through the relay contacts before they separate, establishing a copper arc between them. The iron slug is drawn into the coil, preceded by a fiber slug. When the slugs have gained maximum energy, the iron slug being aligned in the coil, the fiber passes between the contacts and disrupts the arc.

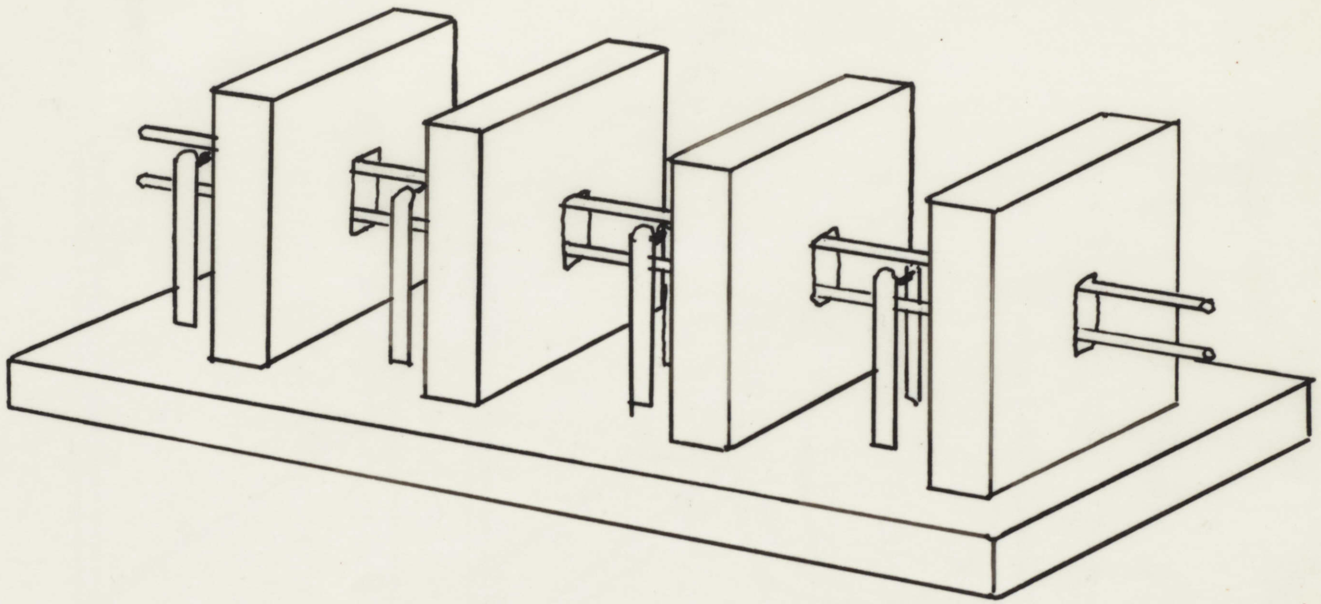


Fig. 1-1 Accelerating System

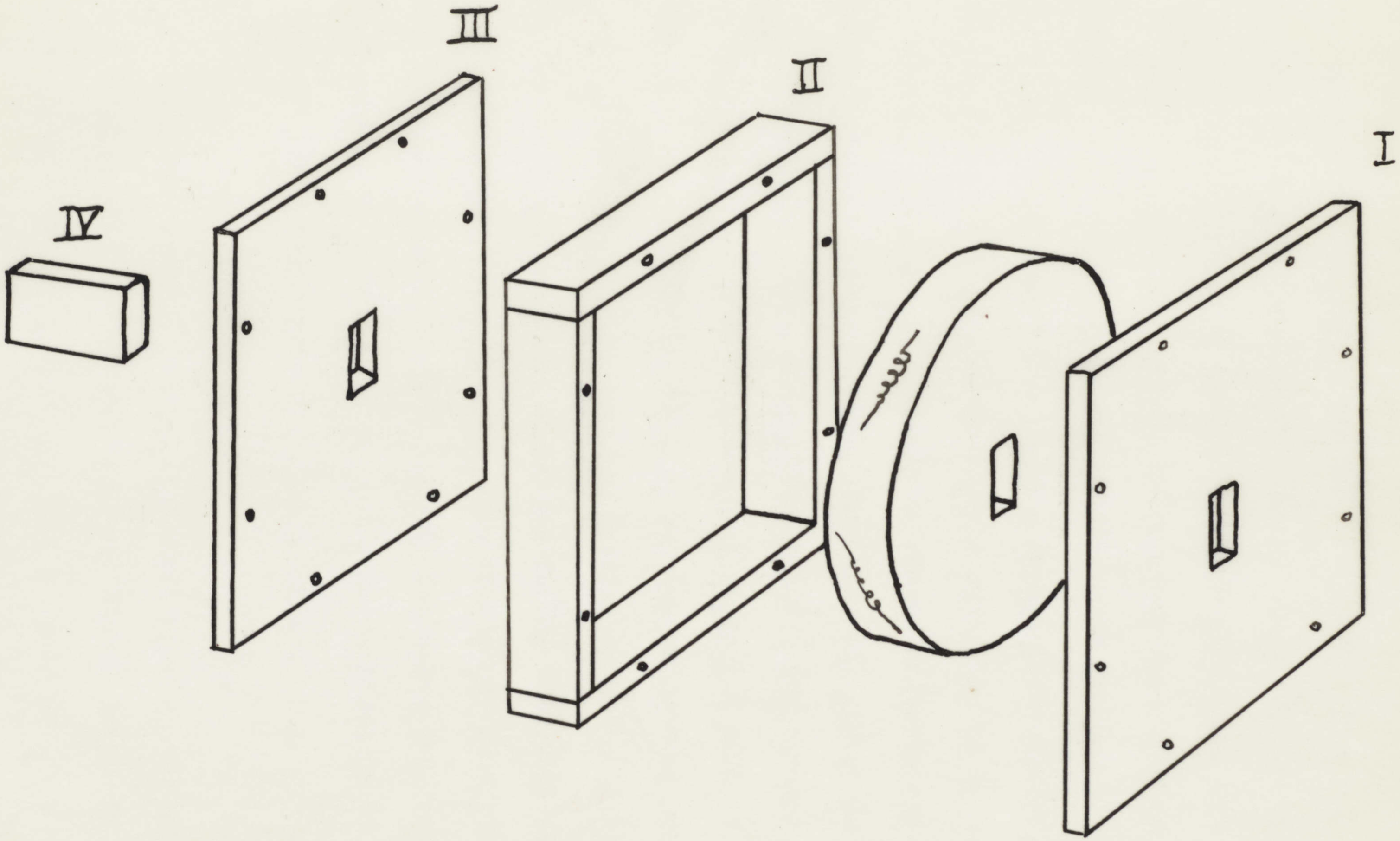


Fig. 1-2 Accelerating Unit

1.3 Limitations of the System

In order that such a system as this will be efficient, it is important that the current through the coil be disrupted as quickly as possible after the slug has been drawn into the coil. Otherwise, the slug will be retarded upon departure, giving energy back to the system. Since $E = L\, di/dt$, an abrupt change in the current is accompanied by a voltage impulse. This places stringent requirements on the switching. The copper arc and fiber slug switching was designed to withstand this impulse. As the slug moves faster, more rapid switching is automatically provided by the slug itself. Of course, the stored magnetic energy must be dissipated somewhere. It is expected that this will take place in an arc over. As long as the energy dissipation takes place through a large resistance, the switch-off time will be small since L/R will be small.

To accelerate a visible particle to high velocities requires large amounts of energy. To make the accelerating system practical, the efficiency should be made as large as possible. The following two chapters discuss the amount of energy conversion obtainable from the system for reasonable input power. The final chapter discusses the requirements of the power supply. It is shown there that a current source is the most suitable type of power supply.

CHAPTER II

DESIGN OF THE ACCELERATING UNIT

2.1 Design Considerations

The greatest efficiency will be obtained from the coil if the magnetic energy storage is confined mainly to the air gap which will be occupied by the iron slug. The strength of the field should be no greater than that required to saturate the slug. By using the high mu region of the iron's B-H curve, greater energy conversion takes place for a given amount of I^2R loss in the coils. Therefore, a series of smaller units is more preferable than one large unit.

The magnetic energy storage in a coil is always accompanied by the I^2R loss which limits the obtainable current through the coil. In fact, for a given conductivity of the wire and dimensions for the coil, the two are directly proportional. The size of the wire then determines the amount of input power and energy storage. To obtain a certain amount of energy storage, the independent variables may be considered to be the coil dimensions, terminal voltage and terminal current. It has been assumed, of course, that the permeance of the circuit has remained constant. The permeance controls the proportionality between the I^2R loss and the magnetic energy storage. This should be made as large as possible, being limited by the permissible bulk of the iron core.

In these accelerating units the dimensions were chosen arbitrarily, being determined by the available materials. For power 115 VDC at 40 amperes was available. It was decided that each coil should draw about 8 amperes, requiring 14.5 ohms DC resistance for each coil.

2.2 Wire Size for the Coil

The diameter of the coil wire may be closely determined by considering a coil of inner radius a, outer radius b, and length ℓ. The diameter of the wire will be denoted by d and the effects of insulation will be neglected. R is the required DC resistance of the coil. ρ is the resistivity of copper.

a = 8 mm. b = 40 mm. ℓ = 22 mm. R = 14.5 ohms $\rho = 1.77 \times 10^{-5}$ ohm-mm.

The number of turns is given by:

$$N = \frac{b - a}{d} \times \frac{\ell}{d} = \frac{705}{d^2} \qquad (2\text{-}1)$$

The mean circumference for a winding is:

$$c = \pi(a + b) = 150 \text{ mm.} \qquad (2\text{-}2)$$

This gives a total length for the wire of:

$$L = Nc = \frac{1.06 \times 10^5}{d^2} \qquad (2\text{-}3)$$

The resistance of the coil as a function of d is:

$$R = \frac{4 \rho L}{\pi d^2} \qquad (2\text{-}4)$$

Substituting R, L, and ρ in the above equation, d is found as follows:

$$d = \left[\frac{4 \times 1.77 \times 10^{-5} \times 1.06 \times 10^5}{3.14 \times 14.5}\right]^{\frac{1}{4}} \qquad (2\text{-}5)$$

$$d = .637 \text{ mm.}$$

The nearest AWG size tb this diameter is #22. The number of turns is found by substituting into Eq. 2-1.

$$N = 1740 \text{ turns} \qquad (2\text{-}6)$$

When the coils were wound, the required dimensions were obtained with 1600 turns. The DC resistance was measured to be 13.6 ohms. There are two reasons for the smaller values. First, insulation was neglected. Secondly, the turns were more loosely packed than what was allowed for in the calculation. This was the case because the available coil winder was not provided with a feeding arm for the wire.

2.3 Energy Conversion

The amount of energy transferred to the slug by each accelerating unit will be the difference in magnetic energy storage with and without the slug inserted in the coil, subject to certain restrictions which will be discussed later. To calculate this value, it is necessary to know the field configuration within the coil. An exact analysis is practically impossible because of non-linearities and the complex shape of the field. However, by considering ideal cases and making a few approximations, the order of magnitude for the energy may be determined.

2.31 Field Pattern and Inductance Without the Slug

For a coil of the dimensions in the unit, the center hole may be neglected without introducing much error because the energy is stored mostly in the leakage flux through the windings. Consider for the present that the coil is imbedded in a medium of infinite permeability. A cross section is shown in Fig. 2-1 a. Using Ampere's Law as indicated in the figure, the magnitude of H will vary in the manner shown by Fig. 2-1 b, with a value expressed in polar coordinates as:

$$H_r = \frac{Ni}{\ell}\left(1 - \frac{r}{b}\right) \qquad (2\text{-}7)$$

Even the above expression is an approximation because the lines of H will

(a)

(b)

(c)

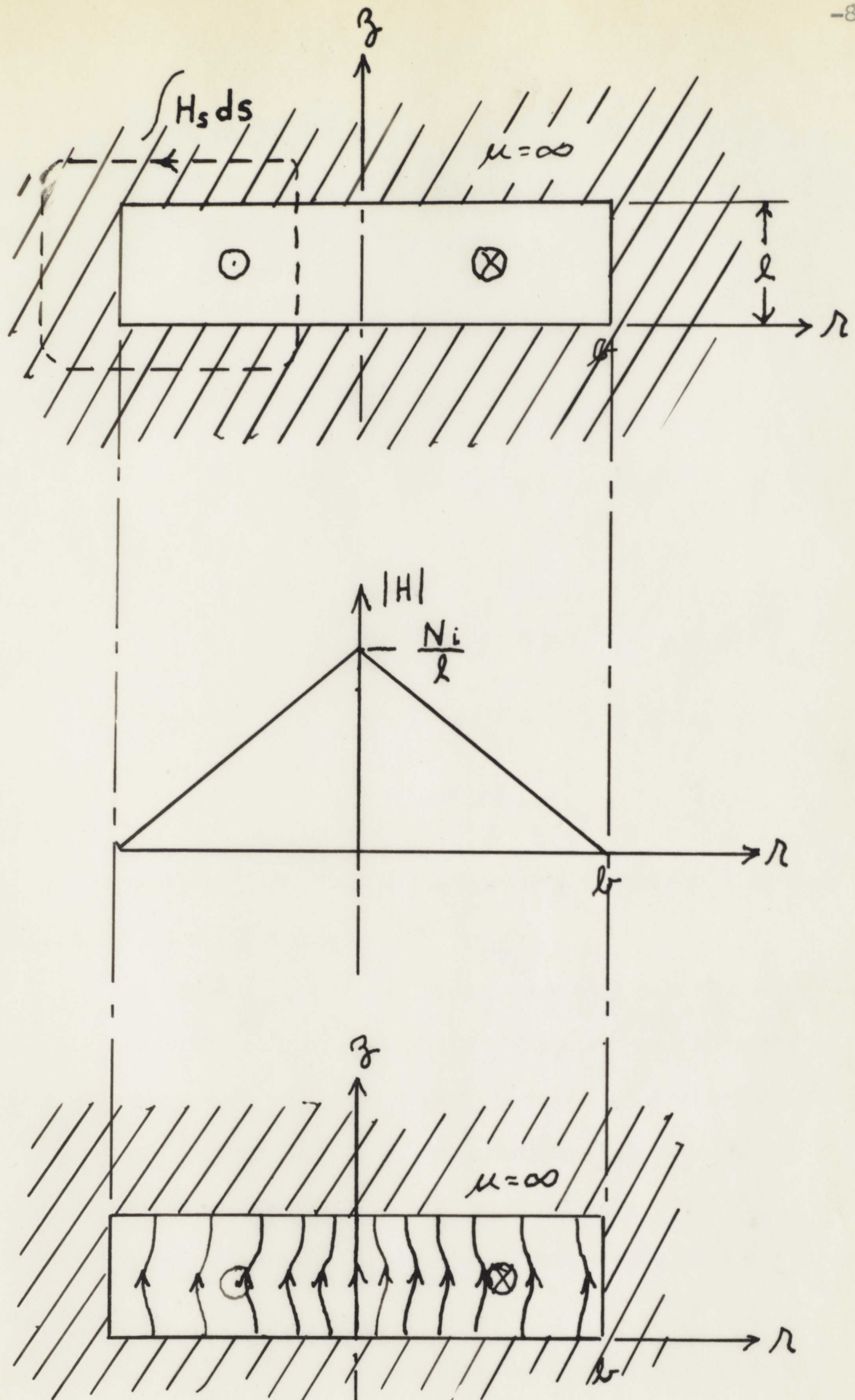


Fig. 2-1 H Field in the Coil

bulge as shown in Fig. 2-1 c. Assuming the H field of Eq. 2-7, the inductance may be found easily by performing the integration shown below and equating to $\frac{1}{2}Li^2$.

$$\frac{1}{2}u_o \int_0^V H^2 \, dV = \frac{1}{2}Li^2 \tag{2-8}$$

Substituting Eq. 2-7 for H, and $2\pi r \ell \, dr$ for dV,

$$\frac{1}{2} \frac{2\pi u_o N^2 i^2}{\ell} \int_0^b (1 - \frac{r}{b})^2 \, r \, dr = \frac{1}{2}Li^2 \tag{2-9}$$

$$\frac{1}{2} \frac{2\pi u_o N^2 i^2}{\ell} \frac{b^2}{12} = \frac{1}{2}Li^2$$

N = 1600 b = 40 mm. ℓ = 22 mm.

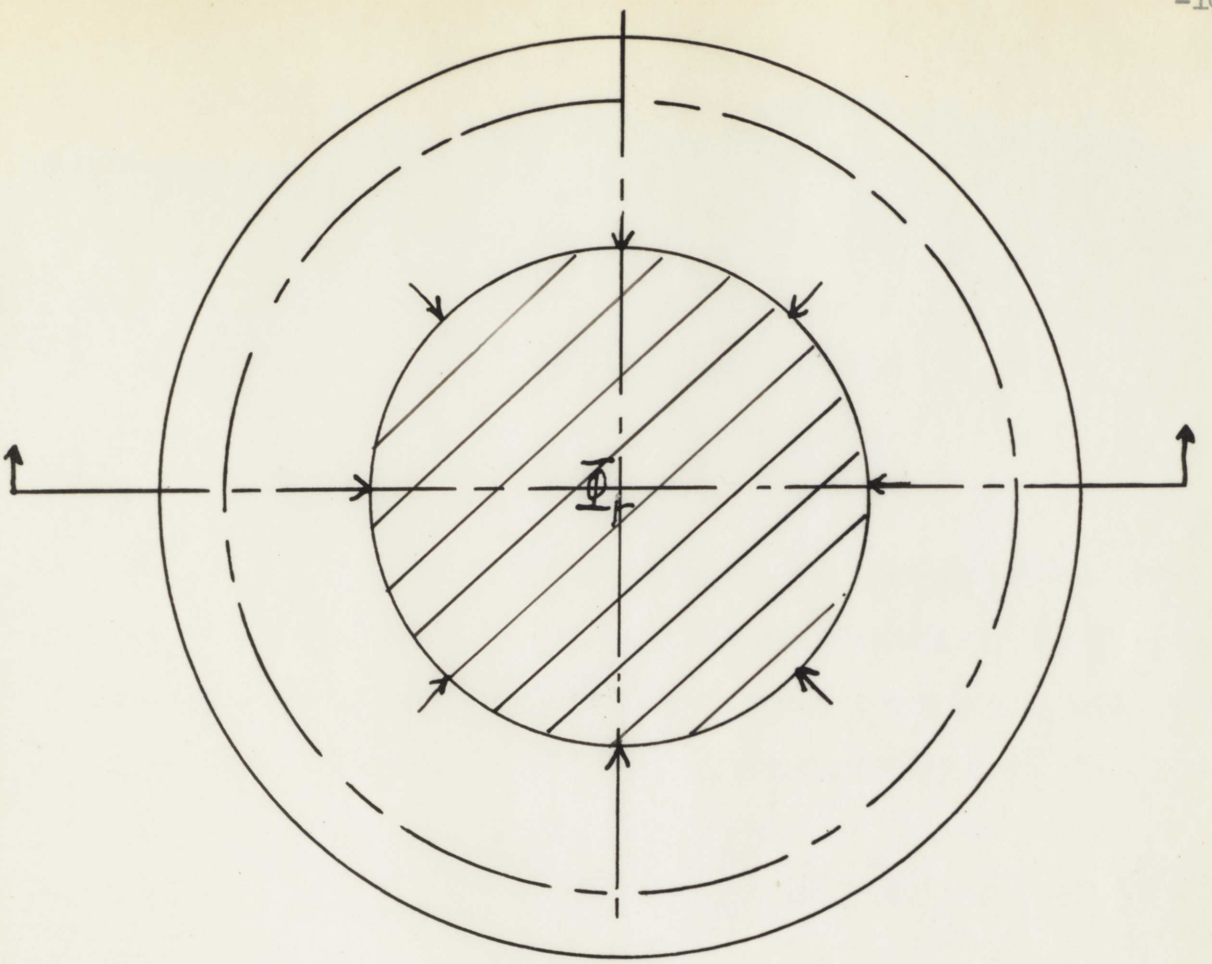
L = .125 henries

In view of the approximations, this value compares favorably with the measured value of L = .10 henries.

It is possible to justify the assumption that the region around the coil may be considered to be infinitely permeable. The magnetic structure was shown in Chapter I. For simplicity, assume that the structure is cylindrical as shown in Fig. 2-2. The flux distribution required to give the assumed H field is known. It will now be shown that the H field required in the magnetic structure is negligible compared with the H field in the winding space.

Using Eq. 2-7, B as a function of r in the winding space is given by:

$$B_r = \frac{u_o Ni}{\ell} (1 - \frac{r}{b}) \tag{2-10}$$



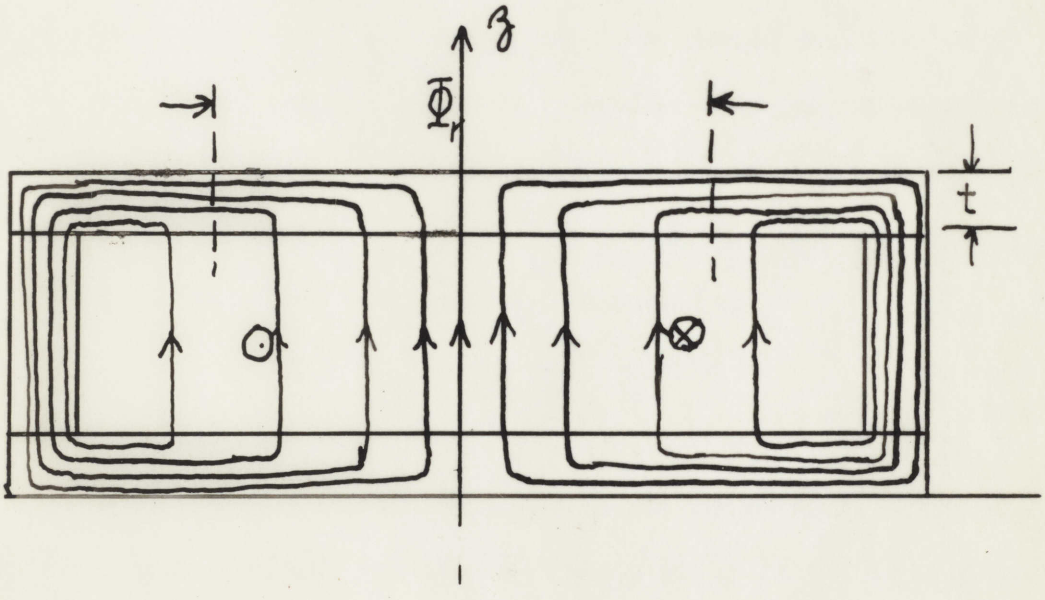


Fig. 2-2 Flux Distribution in Magnetic Core

The total flux passing through any circle of radius r, as in Fig. 2-2, will be:

$$\Phi_r = \int_0^A B_r \, dA = \int_0^r B_r \, 2\pi r \, dr \qquad (2\text{-}11)$$

$$\Phi_r = \frac{u_o Ni}{\ell} \int_0^r \left(1 - \frac{r}{b}\right) 2\pi r \, dr$$

$$\Phi_r = \frac{2\pi u_o Ni}{\ell} \left(\frac{r^2}{2} - \frac{r^3}{3b} \right)$$

This flux, which is in the z-direction, must pass through ribbon shaped surfaces in the circular side pieces in the r-direction. If Φ_r is divided by the area of these surfaces, B and H are given. Let t be the thickness of the sections. The area is then given by $2\pi rt$. H will be:

$$H_r = \frac{B_r}{u} = \frac{\Phi_r}{uA_r} = \frac{u_o}{u} \frac{Ni}{\ell t} \left(\frac{r}{2} - \frac{r^2}{3b} \right) \qquad (2\text{-}12)$$

The original H distribution had been found by making the Ampere path around the outside winding. In the path one should include the H field in the outer ring and the H in the circular side pieces whose value is given above. The maximum value that the above H_r can have is that when $r = b$. If b is substituted for r in the above equation and b/6 is substituted for t, which is nearly the case, the result is:

$$H_r = \frac{u_o}{u} \frac{Ni}{\ell} \qquad (2\text{-}13)$$

This will also be the value for H in the outer ring. Since u/u_o is much greater than 1 except at complete saturation, it is obvious that the H field in the magnetic structure can be completely ignored in this case. If one were interested in knowing the minimum thickness for the magnetic structure which was permissible, the above analysis is useful.

2.32 Calculation of Converted Energy

It has been stated that the converted energy will be equal to the increase in stored magnetic energy. When the slug is inserted, this increase will take place near the center of the coil. The energy stored in the winding space will be nearly unaffected because of the high permeance of the magnetic structure. The field pattern with the slug inserted is shown in Fig. 2-3. The important change is the flux passing through the clearance gaps. It will now be shown that the magnetic energy stored in these gaps is large and is the region of primary interest.

The symbols used in the following analysis are shown in Fig.2-4. For the present the mmf drop in the core will be neglected, although it can become significant for sufficiently small gaps. Taking an Ampere path around the coil and through the center, the H field will be given by:

$$H_g = \frac{Ni}{2g} \qquad (2\text{-}14)$$

The volume of each gap is given closely by:

$$V_g = 2(w + h)\ gt \qquad (2\text{-}15)$$

Since there are two gaps, the total energy storage in the gaps is given by:

$$E = \tfrac{1}{2}u_o H^2 V = \frac{u_o N^2 i^2 (w + h)\ t}{2g} \qquad (2\text{-}16)$$

Substituting the following values into Eq. 2-16:

w = 6 mm. h = 19 mm. g = 1 mm. t = 6 mm. N = 1600

$$E = .241\ i^2$$

Since it was assumed that the energy in the winding space remained

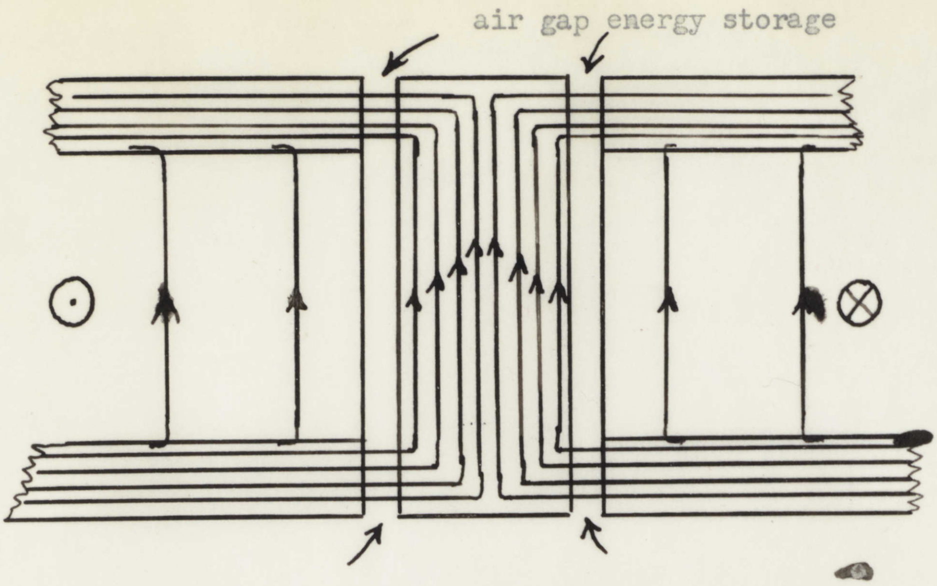


Fig. 2-3 Flux distribution with Slug

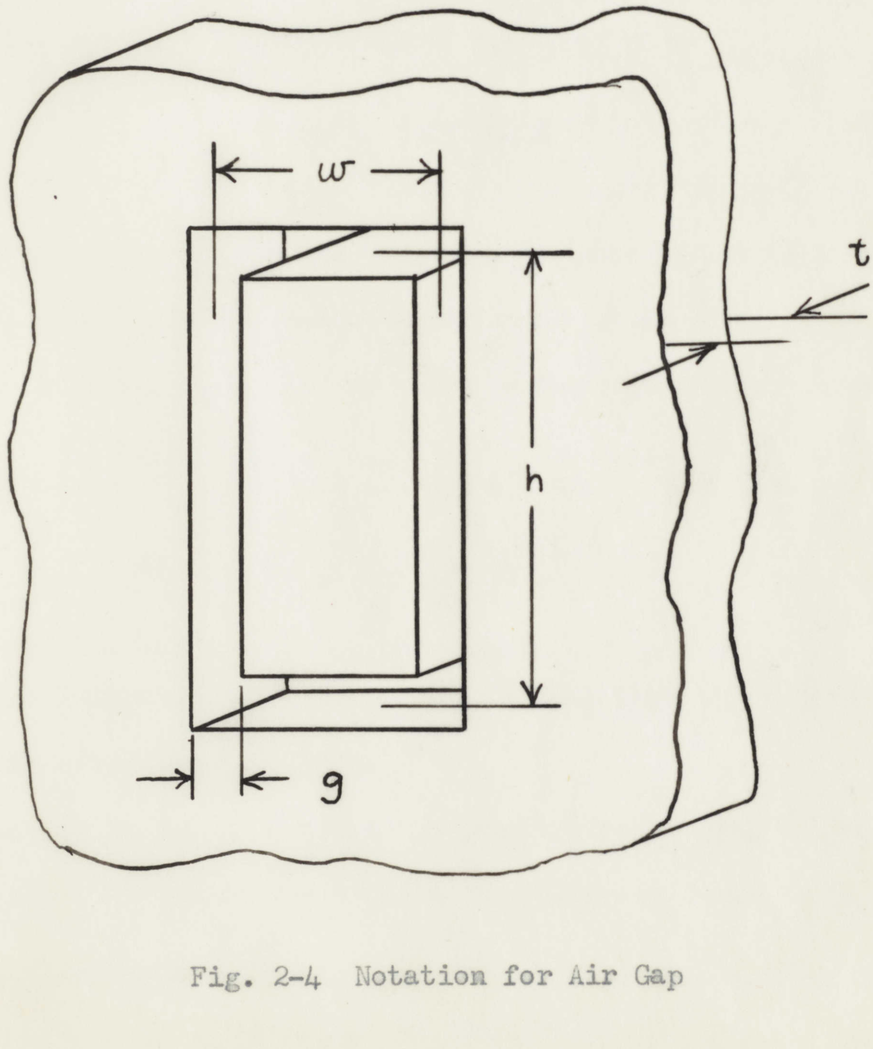


Fig. 2-4 Notation for Air Gap

unchanged, Eq. 2-17 represents the converted energy.

Again it is necessary to show that the H field in the magnetic materials is negligible. Starting with the H field of Eq. 2-14, B_g is given by:

$$B_g = \frac{u_0 Ni}{2g} \qquad (2\text{-}18)$$

The total flux is obtained through multiplying by the area of the gap which is normal to the field.

$$\Phi = B_g\, 2(w+h)\, t \qquad (2\text{-}19)$$

To obtain the B field in the slug, Eq. 2-19 is divided by the area of the slug. H_s in the slug is given through dividing by u.

$$H_s = \frac{u_0 Ni}{u\, 2g} \; \frac{2(w+h)\, t}{wh} \qquad (2\text{-}20)$$

The H field in the slug will be the largest anywhere in the iron because the field is the most highly concentrated there. If Eq. 2-20 is evaluated it will give an estimation of the mmf drop in the magnetic structure.

w = 6 mm. h = 19 mm. g = 1 mm. t = 6 mm.

$$H_s = \frac{Ni}{2g} \; \frac{150\, u_0}{114\, u} \qquad (2\text{-}21)$$

By comparing Eq. 2-21 with Eq. 2-14 it may be seen that the H field in the magnetic structure is negligible.

The energy in Eq. 2-17 should be compared with that which was stored previously. The inductance without the slug was found to be .125 h. This gives a stored energy of:

$$E = \tfrac{1}{2}Li^2 = .0625\, i^2 \qquad (2\text{-}22)$$

The fact that the gap energy is almost four times this value further justified the neglect of small changes in winding space energy while finding the converted energy.

2.33 Measurement of Converted Energy

The amount of energy converted may be easily measured by finding the force as a function of slug position, plotting the result, and integrating graphically to find the total energy. This was done using a spring scale to measure the force on the slug. A low value of current was used because of the range of the scale and also to avoid coil heating during the measurements.

The results in Fig. 2-5 were obtained with a current of .4 amps. The maximum force was also measured using currents of .2 amps. and .8 amps. The force was found to vary as the square of the current, which is what should be expected. When the area under the curve in Fig. 2-5 is measured, it is found that the converted energy is:

$$E = .0309 \text{ joules} \quad \text{at} \quad i = .4 \text{ amps.} \qquad (2\text{-}23)$$

Since $E = ki^2$, k may be determined as:

$$k = .193 \text{ joules/amp}^2 \qquad (2\text{-}24)$$

Thus the converted energy may be expressed as:

$$E = .193\ i^2 \qquad (2\text{-}25)$$

This may be compared with the calculated value in Eq. 2-17 of $.241\ i^2$.

2.34 Velocity of the Slug

The velocity obtainable from the four accelerating units may be

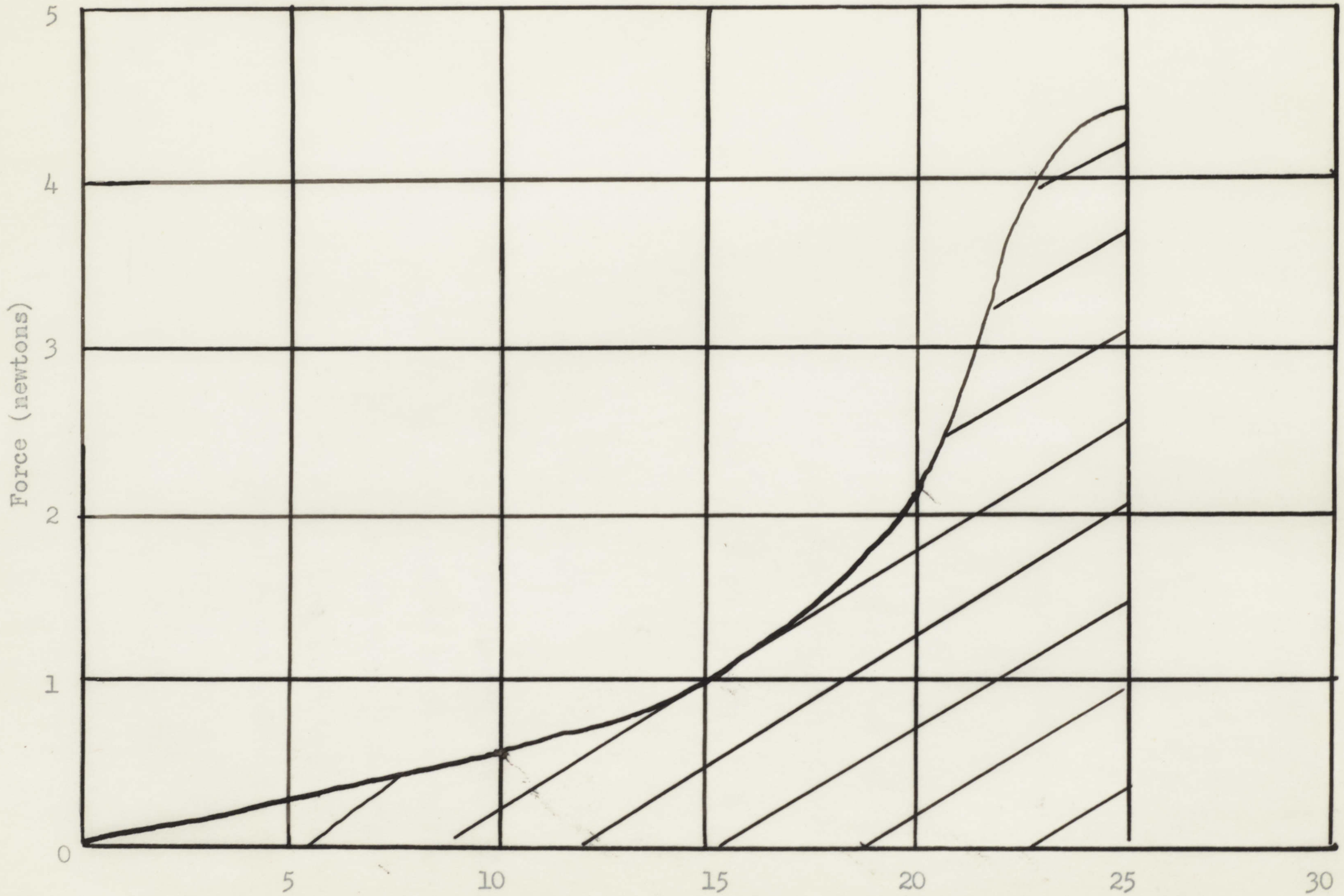


Fig. 2-5 Slug Force vs. position for i = .4 amps.

found by a simple application of energy conservation. Neglecting friction, the converted energy will all go into kinetic energy of motion which is given by $\frac{1}{2}mv^2$. Since each accelerating unit gives equal increments of energy, the velocity will increase only as the square root of the number of accelerating units.

The mass of the slug may be calculated easily as ρV, where ρ is the density of iron and V = whl.

$$\rho = 7.8 \text{ gm/cc.} \quad w = 7 \text{ mm.} \quad h = 20 \text{ mm.} \quad l = 30 \text{ mm.}$$

$$m = \rho whl = 32.7 \times 10^{-3} \text{ Kg.} \tag{2-26}$$

Using the measured value of converted energy, the energy given to the slug by all four coils with 8 amperes through each coil is:

$$E = 4 \times .193 \times 8^2 = 49.5 \text{ joules} \tag{2-27}$$

The velocity is now given by:

$$v = \left[\frac{2E}{m}\right]^{\frac{1}{2}} = 55.2 \text{ m/sec.} \tag{2-28}$$

The velocity given above is not very impressive in view of the fact that the primary purpose of the investigation is to obtain velocities at least above that of sound. It is not worth while to pursue these values any further. It is shown in the next chapter that about twenty times as much energy storage and conversion may be had by decreasing the clearance gap size. In addition, the device first constructed did not perform satisfactorily because of excessive side thrust on the slug. This will now be discussed.

2.4 Side Thrust

The side thrust on the slug as it is drawn into the coil is important in the design of the runners for the slug. The following discussion will be based on the method of analysis in paragraph 2.32 where it is assumed that all the energy is stored in the gaps. Referring to Fig. 2-4, assume that the slug is displaced a distance x horizontally. The stored energy as a function of x will now be found and then differentiated with respect to x to find the force.

The energy stored in the top and bottom gaps will be independent of x and drop out in the differentiation. Therefore, only the energy in the side gaps need be considered. The two gaps which increase in size will have a total volume of:

$$V_a = 2\ ht\ (g+x) \qquad (2\text{-}29)$$

The H field in these gaps will be:

$$H_a = \frac{Ni}{2(g+x)} \qquad (2\text{-}30)$$

Similarly, the two gaps which decrease in size have a total volume of:

$$V_b = 2\ ht\ (g-x) \qquad (2\text{-}31)$$

The H field here is:

$$H_b = \frac{Ni}{2(g-x)} \qquad (2\text{-}32)$$

The total energy storage will be $\frac{1}{2}u_o H_a^2 V_a$ $\frac{1}{2}u_o H_b^2 V_b$ which is:

$$E = \frac{u_o N^2 i^2 htg}{2} \left(\frac{1}{g^2 - x^2}\right) \qquad (2\text{-}33)$$

Differentiating with respect to x to find the force:

$$F = \frac{u_o N^2 i^2 ghtx}{(g^2-x^2)^2} \tag{2-34}$$

If x is zero, the coil being centered, there is no force. If x is equal to g, the force becomes infinite according to the equation. Of course, this is no more true than the energy of Eq. 2-16 being infinite when g equals zero. This condition violates the assumption that the H field in the magnetic structure is negligible. Under such a condition the B field becomes limited by the finite permeance of the magnetic structure.

To appreciate the magnitude of the side thrust, the values given in Eq. 2-17 will be used and a value of x equal to .5 mm. For the current 8 amperes will be substituted. The result is:

$$F = 2.08 \times 10^4 \text{ newtons} \tag{2-35}$$

$$= 4{,}670 \text{ lbs.}$$

At first thought, a side thrust in excess of two tons seems unreasonable. Further discussion is postponed to Chapter III where other calculations make the value seem more reasonable.

CHAPTER III

IMPROVEMENT OF THE ACCELERATING UNIT

3.1 Size of the Clearance Gap

The analysis of Chapter. II showed that the clearance gaps are extremely important in determining the amount of converted energy. It appears from Eq. 2-16 that if the gap width were made smaller, the converted energy would increase for the same input current. Of course, if the gaps were made to be zero, the converted energy would not be infinite. For sufficiently small gaps, Eq. 2-16 is no longer true because the mmf in the magnetic structure becomes too large to be ignored. For an efficient system the clearance gaps should be made as small as possible, being limited only by the tolerances which can be maintained in the construction.

The calculation of converted energy for extremely small gaps is necessarily complicated because most of the energy storage will take place in the magnetic materials which have non-linear B-H characteristics. Also, the B field varies continually throughout the magnetic structure. However, it is possible in this case also to approximate the converted energy by considering some ideal cases.

3.2 Calculation of Converted Energy with No Gap

Consider the case in which the clearance gap is zero and the magnetic materials are operated in a region of low field strength where the permeability is a constant. Refer to Fig. 1-2 in this discussion. The stored energy with the slug inserted will now be found by calculating the reluctance of the four sections, finding the inductance, and $\frac{1}{2}Li^2$.

The reluctance of sections I and III in Fig. 1-2 may be found by pieces as shown in Fig. 3-1. Referring to Fig. 3-2, the equation of the line ab is:

$$y = kx \qquad (3\text{-}1)$$

If the section has a thickness t, the reluctance from the face at x_1 to the face at x_2 is:

$$R = \frac{\ell}{uA} = \int_{x_1}^{x_2} \frac{dx}{u_I\,(2yt)} = \int_{x_1}^{x_2} \frac{dx}{u_I\,(2kxt)} \qquad (3\text{-}2)$$

$$R = \frac{\ln x_2/x_1}{2ktu_I}$$

Pieces 2 and 3 in Fig. 3-1 may be fitted to Fig. 3-2 if:

$x_1 = 10.9$ mm. $x_2 = 62.4$ mm. $k = .92$ $t = 6$ mm.

Substituting these values into Eq. 3-2 gives:

$$R_2 = R_3 = \frac{158}{u_I} \qquad (3\text{-}3)$$

Pieces 1 and 4 in Fig. 3-1 may be fitted to Fig. 3-2 if:

$x_1 = 3.2$ mm. $x_2 = 50.7$ mm. $k = 1.095$ $t = 6$ mm.

Substituting these values into Eq. 3-2 gives:

$$R_1 = R_4 = \frac{212}{u_I} \qquad (3\text{-}4)$$

Since sections I and III are identical, their relucyances may be calculated as follows:

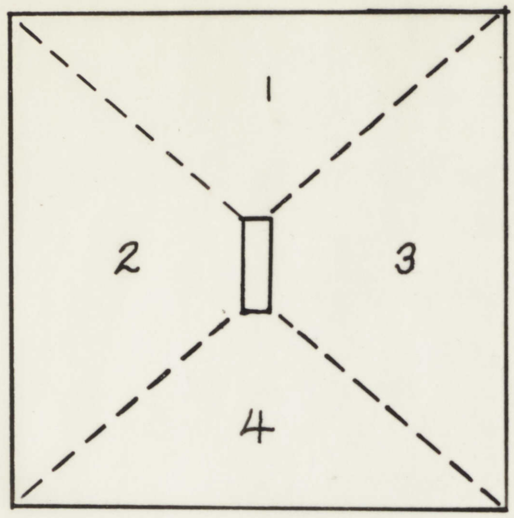


Fig. 3-1 For Reluctance Calculation

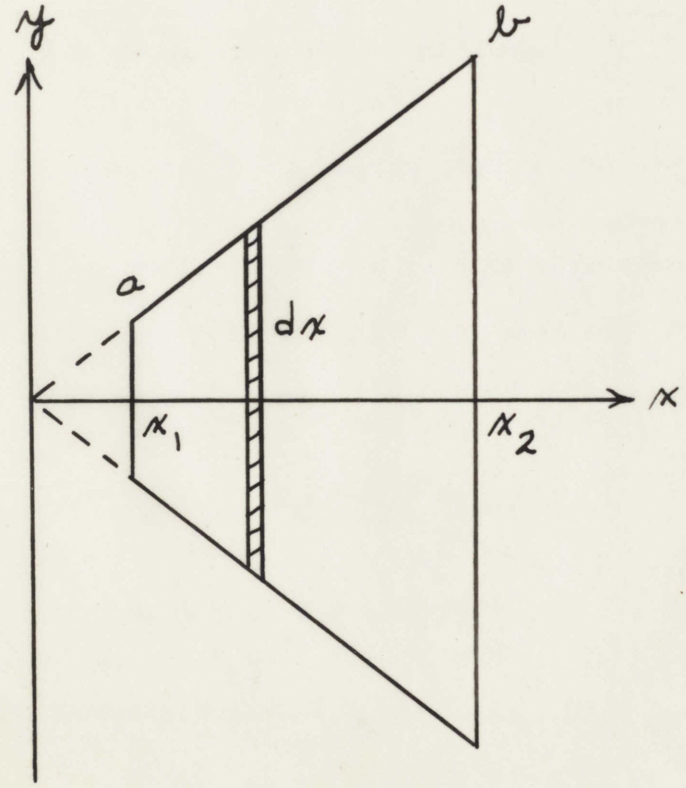


Fig. 3-2 For Reluctance Calculation

$$\frac{1}{R_I} = \frac{1}{R_{III}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \qquad (3\text{-}5)$$

$$R_I = R_{III} = \frac{45.3}{u_I}$$

The reluctance of section II in Fig. 1-2 is simply:

$$R_{II} = \frac{\ell}{u_{II}A} \qquad (3\text{-}6)$$

where A is the total area of the edge faces.

$$\ell = 25 \text{ mm.} \qquad A = 9\ (110 + 110 + 97 + 97)\ \text{mm.}^2$$

$$R_{II} = \frac{6.7}{u_{II}} \qquad (3\text{-}7)$$

The reluctance of section IV, the slug, is given by Eq. 3-6 where:

$$\ell = 30 \text{ mm.} \qquad A = 7 \times 20 \text{ mm.}^2$$

$$R_{IV} = \frac{214}{u_{IV}} \qquad (3\text{-}8)$$

To estimate the energy which can be converted with the gap equal to zero, the above values of reluctance will be used with a tentative value for $u = 1000\ u_o$ to find the inductance. The total reluctance is:

$$R_T = R_I + R_{II} + R_{III} + R_{IV} \qquad (3\text{-}9)$$

$$R_T = 2.48 \times 10^5 \text{ henry}^{-1}$$

L may now be found remembering that N = 1600.

$$L = \frac{N^2}{R_T} = 10.4 \text{ henries} \qquad (3\text{-}10)$$

The stored magnetic energy for this case is:

$$E = \tfrac{1}{2}Li^2 = 5.2\ i^2 \qquad (3\text{-}11)$$

To find the converted energy, the energy stored in the coil without the slug should be subtracted from the above value. This was given in Eq. 2-22 as .065 i^2 and is seen to be negligible. The above energy should be compared with Eq. 2-17 and Eq. 2-25, the calculated and measured values of converted energy with a clearance gap of 1 mm. It will be seen that the converted energy has increased by a factor of about 20.

For small values of current, the above methods which are based on linearity and $u = 1000\ u_o$ are justified. For large currents, however, the upper regions of the iron's B-H curve are used. In this case the stored energy is more accurately given by:

$$E = \int_0^V \int_0^B (H \cdot dB)\ dV \qquad (3\text{-}12)$$

A graphical solution of the energy density, given by the inner integral above may be had by finding the area on the B-H characteristics which is bounded by the B-H curve, the B-axis, and the line $B = B_1$ where B_1 is the field strength.

If a value of 8 amperes is used in Eq. 3-11, the converted energy becomes:

$$E = 332 \text{ joules} \qquad (3\text{-}13)$$

Eq. 3-12 was also used for the same value of current.and for a typical B-H curve. The method was only approximate and the details are not given here. The result is:

$$E = 200 \text{ joules} \qquad (3\text{-}14)$$

3.3 Velocity of the Slug

The velocity of the slug will now be determined using the value of converted energy given by Eq. 3-14. The total energy from the four accelerating units will be about 800 joules. The mass of the slug has already been found to be 32.7×10^{-3} Kg. The velocity will be:

$$v = \left[\frac{2E}{m}\right]^{\frac{1}{2}} = 222 \text{ m/sec.} \qquad (3\text{-}15)$$

$$= 730 \text{ ft/sec.}$$

This velocity compares more favorably with the desired results.

3.4 Side Thrust

The value of side thrust calculated in Chapter II may now seem more reasonable when it is realized how much more energy is stored as the gap, or only part of it, is made to be zero. If the slug with a 1 mm. clearance gap is brought into the coil and kept centered in the hole, and then is allowed to move to the side, all of this additional energy conversion takes place while the slug is moving 1 mm. When this energy is divided by only 1 mm. to estimate the force, it is understandable that the force can be so large.

Large side thrusts need not be expected with gaps of a value near .05 mm. because it has been shown that the energy storage is much less a function of changes in the gap size. In this case most of the mmf is consumed in the iron.

CHAPTER IV

DYNAMIC BEHAVIOUR OF THE ACCELERATING UNIT

4.1 Introduction

In Chapters II and III effort had been given to determining the initial and final stored magnetic energy in the coil. It had been stated that, subject to certain restrictions, the difference between the above energies was also equal to the converted energy, twice the value being supplied by the power source. It should be realized that there must be a restriction on the time in which the source can supply this energy, unless it is capable of infinite power. If these units are used with a voltage source, the only way that infinite power could be drawn from the source is with nearly a short circuit, or zero impedance at the coil terminals. But this is impossible because of the 14.5 ohms DC resistance.

The theorem that the converted energy is equal to the increase in stored magnetic energy is based on virtual work where it is assumed that the changes take place very slowly. Consideration of what is meant by slowly reveals that the transition time from one state to the other is long compared with the time constant of the coil. Therefore, it is necessary to investigate the coil behaviour at it's electrical terminals to find velocity limitations on the accelerating units when connected to a voltage source.

4.2 The Time Function of Current

The complete solution of the dynamic operation of the accelerating unit is unnecessary since the velocity limitation is all that is required. To simplify the calculations, it will be assumed that the slug is passed through the coil at constant velocity. When this is the case, the converted energy will be given to the restraining agent of the slug. Nevertheless, This condition is sufficient to give the required information. The assumption is not so unrealistic because this will nearly be the case in the last accelerating unit where the initial velocity is important. The two assumptions used in the following analysis are stated below.

a. The slug is passed through the coil with constant velocity.

b. The change in the coil inductance is a linear function of the slug's penetration into the coil.

Summing the voltages at the coil terminals gives the following equation:

$$L\frac{di}{dt} + i\frac{dL}{dt} + Ri = E_s \qquad (4\text{-}1)$$

Since dL/dx = const. and dx/dt = const., dL/dt must also be constant. Setting:

$$\frac{dL}{dt} = K \qquad (4\text{-}2)$$

$$L = Kt + L_o$$

Substituting:

$$(Kt + L_o)\frac{di}{dt} + (R + K)i = E_s \qquad (4\text{-}3)$$

The above equation may be rearranged and solved by direct integration. The general solution is:

$$i = \frac{C\,(Kt + L_o)^{-\frac{R+K}{K}} + E_s}{R+K} \qquad (4\text{-}4)$$

Substituting the initial conditions that $i = E_s/R$ at $t = 0$, Eq. 4-4 becomes:

$$i = \frac{E_s}{R+K}\left[1+\frac{K}{R}\left(\frac{I_o}{Kt+I_o}\right)^{\frac{R+K}{K}}\right] \qquad (4\text{-}5)$$

It is seen from this equation that as t increases, i approaches $E_s/R+K$, with a transient determined by the bracketed term. If K is large, the transient takes place more rapidly and i approaches a lower value. Physically, the following takes place: A certain mmf is associated with the current. The incoming slug increases the circuit permeance. The flux begins to increase, causing a positive $d\lambda/dt$. This generates a back emf which decreases i and λ, the flux linkages. The current in Eq. 4-5 is the result.

It may be seen that little energy conversion can take place with the accelerating units connected to a voltage source, if the slug velocity is very large. To show this, let v = 10 m/sec. The slug will change the circuit permeance from its initial to final value in 3 cm. This gives a time interval of 3×10^{-3} sec. The maximum energy that the power source could supply in this interval is:

$$E = 115 \times 8 \times 3 \times 10^{-3} = 2.76 \text{ joules} \qquad (4\text{-}6)$$

It is known that not even this much is supplied because of Eq. 4-5. Furthermore, some of the energy is consumed in the DC resistance. Thus, even for 10 m/sec., the converted energy can only be a small fraction of that calculated in the preceding chapters.

4.3 Conclusion

Returning to the discussion in Paragraph 4.1, it is stated that the transition from one state to the other must be long compared with the time constant of the coil. This condition must be fulfilled so that the current may remain nearly constant. Therefore, the accelerating units may still be used to satisfaction if they are provided with a current source. The thought now arises as to whether or not the required energy can be obtained from a current source in a short time interval.

To obtain infinite power from a current source, it must be open circuited. When this is the case, the current source develops infinite voltage. This condition can be fulfilled with the accelerating units which develop a large back emf at high slug velocities. Thus, the necessary impulse of energy may be supplied.

A suitable current source may be had by placing a large inductance in series with a voltage source. Therefore, the concept may be thought of as having the accelerating coils connected to a reserve of magnetic energy storage. Such an arrangement would be plausible except for the I^2R loss which would be encountered in such a system. It is possible that a more suitable current source may be developed in the future. Useful application of the accelerating system must await the development of such a device.