Behavioral Models of Trade in Financial Markets.

by

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ABSTRACT

This thesis consists of three essays on the nature of trade in financial markets. The common theme throughout each paper is the idea that individuals are heterogeneous. Investor heterogeneity is a necessary condition for trade in financial markets.

In the first study, we develop a theory of trade in financial markets based on assumptions that market participants frequently revise their demand prices and randomly encounter potential trading partners. The model describes two distinct ways informational events affect trading volume. One method is that investor disagreement leads to increased trading. But, the observation of abnormal trading volume does not necessarily imply disagreement, and volume can increase even if investors interpret information identically, if they also have divergent prior expectations.

The theoretical framework developed here is mapped into an empirical test of the conjectures we make concerning the effects of information flows, return volatility, and market liquidity on trading intensity. We compare the implications of our model with the findings of various other theories of intraday trading patterns, order placement, and block trading. We confirm our hypothesis that information flows strongly affect intraday trading intensity, however, security-specific information is dominated by market-wide information flows. This finding is consistent with our theory yet inconsistent with several leading theories of intraday trading patterns. These finding are most acute for the type of trader. Both buyer-initiated and seller-initiated traders use market-wide signals rather than security-specific signals to decide to trade. Furthermore, we find that while market-market trades are influenced by security-specific information, market-limit and block trades are not. In addition, we control both for changes in the distribution of beliefs with return volatility and changes in market liquidity and we find that these variables are significant determinants of aggregate trading intensity and the sub-sample trading intensity.

In a second paper, we study the impact of market centralization on its performance, examining four alternative models of exchange: a consolidated clearing house, fragmented clearing houses, a monopoly dealer market, and an interdealer market. The effects of the market mechanism on the expected quantity traded, the price variance faced by individual traders, the quality of market signals, the expected gains from trade, and the exchange implementation costs are studied.

The third essay presents a new theory of bubbles, or discrepancies between the market clearing price and the fundamental value of an asset. In our setting, Bayesian traders, oriented towards long-term gains, receive private information ('news') and make inferences from noisy price signals. Price exhibits higher variance than the *fundamental value* (the latter defined as the fully-aggregated expected value) especially when news is informative but infrequent. The corresponding bubbles are self-limiting, but exhibit momentum and overshooting.

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ESSAY 1:

INTRADAY BELIEFS, INFORMATION, AND TRADING INTENSITY.

0. Introduction.

Economic theory posits that exchange occurs when market participants assign different values to an asset. Theory can quickly become complicated when we attempt to explain the number of asset exchanges in a market with diverse traders. Many financial market models implicitly assume away trading volume by assuming away heterogeneity. For example, Grossman (1976) proves that in a speculative market populated with rational traders with homogeneous priors who communicate their personal knowledge through their willingness to trade, exchanges will not take place in equilibrium.[1] Yet there is active trading in most financial markets, indicating that individuals are heterogeneous and operate in a changing environment.

We develop a theory of trade based on heterogeneous investors who periodically and idiosyncratically revise their demand prices. A better understanding of trade is important for at least three reasons. First, there is an inconsistency between the widespread use of the homogeneous investor assumption and the observation of positive trading volume. Our model presumes that investors have a demand to trade even in the absence of new information because of unique speculative or liquidity desires. Second, we explicitly model the effects of information flows and changes in traders' beliefs on intraday volume. We establish two distinct channels by

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[1] This famous result is known as the *No Trade Theorem*. Numerous authors have applied this idea in a variety of contexts; see Tirole (1982) and Milgrom and Stokey for a discussion of the foundations of this result.

which information affects trading volume: an increase in volume can indicate that investors interpret the information differently or that they interpret the information identically but begin with diverse prior expectations. Furthermore, we empirically measure the effects of information flows, return volatility, and market liquidity on trading intensity.

We examine the relationship between volume and information by treating volume as the number of transactions, or trading intensity, between buyers and sellers who are randomly paired in the trading period. This measure best characterizes participants' willingness to trade in a given period. Trading opportunities arise because both potential buyers and potential sellers revise their demand prior to the market period according to idiosyncratic speculative or liquidity desires, which appear random to the outside observer who does not have specific data on each trader. We utilize our formal framework to describe the intraday relationship between trading intensity and information flows, the dispersion of beliefs among traders, and market liquidity. Finally, we derive an empirically tractable test of our model and compare and contrast our results with the predictions made by other models of intraday trade.

This approach, employing a random pairing assumption, clearly has some drawbacks. First, we do not examine the market's role in aggregating private information and, therefore, the model is less general then the approach of Pfleiderer (1984). In addition, the random pairing assumption is a crude representation or mechanism of market exchange, as evidenced by the existence of markets that economize on the search and transaction costs of buyers and sellers who purposely seek each other out. Another drawback of the random pairing assumption is that the model does not yield a unique market price or a one period equilibrium. We expect to address some of these issues in later research.

However, the random pairing assumption has several compensating advantages. First, the model yields a simple closed form and empirically tractable solution for expected volume. Second, the model's results concerning trading volume are not drawn by the random pairing assumption, and we are able to apply the model's predications about trading volume to multiple periods with costly market clearing. Third, the model avoids the more restrictive assumptions of constant absolute risk aversion or behavioral distinctions between groups of investors that researchers commonly apply in models of trading volume. Fourth, the model is simple and yields straightforward predications about positive volume in non-event periods and about the bid-ask spread, as well as predictions about the effects of informational events. Lastly, the model's predictions are largely consistent with our empirical evidence and provide a theoretical framework for evaluating transactions volume data. Information increases trading volume if it causes investors to revise their demand prices heterogeneously or if investors partially, but not homogeneously, anticipate the information.

We structure this study as follows. Section 1 presents the model of the intraday trading process. We introduce a basic framework for understanding trading intensity and explicitly model the effects of information flows, return volatility, and market liquidity. The section concludes with a comparison of our model and several other competing theories of intraday trading patterns. The data and empirical specification are outlined in Section 2. In Section 3, we describe the appropriate statistical assumptions and methodology for this problem. We present our empirical results in Section 4. We report findings from the full sample and sample splits by type of trader and time-of-day. The results from a broad range of specifications, samples, and securities are largely consistent with the predictions of our theoretical model. The conclusion (Section 5) discusses the implications of these findings.

1. A Model of Intraday Trading Intensity.

1.1. Description of the Market Process.

Consider a two-period exchange market for some asset with fixed supply I. Given an initial stock equilibrium at the close of the previous trading period t_0, we examine volume at t_1. Assume that the costs of transacting in the market are zero[2]; however, market participants can hold, at most, one unit of the asset and cannot take short positions.[3] Therefore, only current asset owners can be sellers and current nonowners are the only potential buyers. While not descriptively accurate of financial markets, this assumption isolates the population of buyers and sellers and avoids the problem of grouping investors according to *ad hoc* behavioral characteristics.

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[2] Trading may not be costless in other markets.

[3] This is not a restrictive assumption. Obviously, one could create a short position with a nonowner who borrows the asset from an owner and sells it at market.

Market participants are heterogeneous in their personal valuation of the asset. Demand price differentials indicate different expectations or beliefs across investors. Thus, we assume independent, non-random individual behavior; traders' willingness to hold positions in the asset are a function of their expectations. Individuals revise their demand prices between market periods with the revision following what appears to the econometrician to be a stochastic process with mean μ and variance σ^2. The econometrician can observe security-specific and market-wide information as the price process aggregates the expected revision μ of all traders in the market. However, an individual's revision contains a random element (the variance of which we measure as σ^2) simply because the econometrician lacks trader-specific information.

During the market period t_1 each current asset owner randomly encounters a single, unique nonowner; an exchange occurs if the nonowner's revised demand price exceeds that of the current owner. We blindly pair each asset owner with a prospective buyer.[4,5] The random pairing model

[4] This trading process is similar to that of Diamond (1982) and Akerlof (1985). It implies that each paring of a prospective buyer and sellers creates a temporary bilateral monopoly situation. This market structure does not determine a single market clearing price, but rather, a set of pairwise transaction prices. While the random pairing assumption implies that the number of potential buyers is greater than the number of potential sellers, the demand price of many nonowners can be zero, so this is not a restrictive assumption.

[5] Alternatively, we could have assumed that traders arrive sequentially and declare their type, buyer or seller, and their reservation price. A specialist could maintain a market quote and queue mechanism. If the buyer's reservation price exceeded that of the current owner with the lowest reservation price, a transaction would occur. Otherwise, the individual would join a queue of other buyers waiting for sellers. This framework would achieve the same outcome as random pairing but needlessly complicates the problem of solving for trading volume.

approximates real-world continuous markets except that these markets provide information on current quotes and transaction prices, which makes the pairing non-random. With random pairing, participants may not make some mutually preferred exchanges because the appropriate trading partners do not "find" each other. The market generally does not clear at t_1, and the analysis would change if extended to the next period.[6] These assumptions are useful in deriving a tractable solution that yields predictions about trading intensity.

The market consists of I potential sellers (one for each unit of the asset, and J potential buyers. The (finite) set of market participants is $S = \{I, J\}$, and the random pairing assumption implies that $I \leq J$. Since traders have heterogeneous beliefs, in general, $p_k \neq p_h$ for any $k \neq h$ pair such that $k \in S$ and $h \in S$. We characterize stock equilibrium at time t_0 by $p_i \geq p_j$, for all $i \in I$ and $j \in J$. The isolation of buyers and sellers implies that $I \cap J = \{\}$.

The revision of individual demand prices between periods indicates that an individual's demand price p_{k0} will change between t_0 and t_1 by some amount δ_{k1}. For notational ease, p_{k0} will refer to an individual's demand price after trading in period t_0; δ_{k1} is the change between periods; p_{k1} is the corresponding demand price before trade in period 1. Hence, p_{i1} can be less that p_{j1}, and exchanges in period 1 are possible.

Let T_{ij} be a binary variable that indicates an exchange between an

[6] Akerlof (1985) defends the random pairing assumption as a reasonable approximation, given the prevalence of non-contractual relations between buyers and sellers, which indicates the absence of a Walrasian auctioneer market. However, his argument is better suited to the labor and real asset markets than to financial markets.

(i, j) pair:

$$T_{ij} = \begin{cases} 1 & \text{iff } i \in I \text{ and } j \in J \text{ and } p_{i1} > p_{j1} \\ 0 & \text{otherwise} \end{cases} \qquad (1.1)$$

As each market participant can conduct, at most, one trade per period. The total volume of trades is $T = \Sigma_i T_{ij} = \Sigma_j T_{ij}$.

Each (i, j) pairing represents an independent opportunity for an exchange. For a given (i, j) pair, the probability of an exchange $\pi_{ij} = \Pr\{T_{ij} = 1\}$, is the probability that the demand price revisions of individuals i and j are sufficient to overcome the original demand price differential $(p_{i0} - p_{j0})$. In general, π_{ij} will differ for different (i, j) pairs. Before the pairings of each asset holder with a potential buyer at the beginning of the market period, it is a random variable with mean $\pi = E(\pi_{ij})$, the "average" probability at t_0.

Therefore, T is a binomially distributed random variable with parameters I, the number of independent "trials" in the market exchange process, and π, the average *ex ante* probability of an exchange prior to the (i, j) pairings. The binomial density function is

$$b(T) = T!/((I - T)!I!)\pi^T(1 - \pi)^{(1-T)} \qquad (1.2)$$

and

$$\mu_T = E(T) = \pi I. \qquad (1.3)$$

μ_T is a strictly increasing function of π, so, for a given I, the size of π determines the level of trading intensity.

1.2. Expected Trading Intensity.

In this section, we derive expressions for π and μ_T. As indicated above, we define the demand price revisions δ_{i1} and δ_{j1} by $p_{i1} = p_{j0} + \delta_{j1}$. From equation (1.1), $T_{ij} = 1$ if and only if $(\delta_{j1} - \delta_{i1}) \geq (p_{i0} - p_{j0})$. The individual's demand revision, δ_{k1} $(k \in S)$, can be written as

$$\delta_{k1} = \mu_k + \sigma\varepsilon_k, \tag{1.4}$$

where ε_k is a mean-zero random variable.[7] To derive closed-form solutions, ε_k is assumed to be unit normal with independence across investors:

$$E(\varepsilon_k\varepsilon_h) = 0 \text{ for all } k \neq h. \tag{1.5}$$

In equation (1.4), μ_k is the expected demand-price revision from t_0 to t_1, while σ is the standard deviation of the revision process. We can think of the $\sigma\varepsilon_k$ term as resulting from the modeler's ignorance about the changing constraints facing an individual k; σ is a measure of the degree to which individuals in this market idiosyncratically revise their demand prices in response to changed expectations. By construction, these factors are unknown for any person k. On the other hand, the μ_k term represents expectations, given whatever is known about k. In the absence of known influences (e.g., new public information), μ_k is the long-term expected return from the asset.

Apply equation (1.4) to a pair of investors $i \in I$ (owner) and $j \in J$ (nonowner), and define θ as

$$\theta = \delta_{j1} - \delta_{i1} = (\mu_j - \mu_i) + \sigma(\varepsilon_j - \varepsilon_i), \tag{1.6}$$

$$\mu_\theta = E(\theta) = \mu_j - \mu_i, \tag{1.7}$$

$$\sigma_\theta^2 = E(\theta - \mu_\theta)^2 = 2\sigma^2. \tag{1.8}$$

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[7] We also could have derived Equation (1.4) from a discrete process with $\Delta t = t_1 - t_0 = 1$. In a model of "representative" participants with appropriate definitions of μ_k and σ, this would imply a log-normal return distribution.

The probability of an exchange between any given (i, j) pair is

$$\pi_{ij} = \int_{p_{i0}-p_{j0}}^{\infty} f_\theta(x)dx = 1 - F_\theta(p_{i0} - p_{j0}), \quad (1.9)$$

where $f_\theta(x)$ is the normal density function with mean μ_θ and variance σ_θ^2 and $F_\theta(p_{i0} - p_{j0})$ is the associated distribution function evaluated at $(p_{i0} - p_{j0})$.

The probability π_{ij} is specific to the difference $(p_{i0} - p_{j0})$, and so it will generally vary across (i, j) pairs once the pairings are made. The random pairing assumption indicated that the probabilities that a given element of the set J will be paired with any single i are equal, and vice versa. So the average *ex ante* probability π is

$$\pi = (1/IJ)\Sigma_i\Sigma_j\pi_{ij}. \quad (1.10)$$

Substituting (1.9) and (1.10) into (1.3),

$$\mu_T = (1/J)\Sigma_i\Sigma_j[1 - F_\theta(p_{i0} - p_{j0})]. \quad (1.11)$$

Since we assume that the ε_k are normal, this is

$$\mu_T = (1/J)\cdot$$

$$\Sigma_{i=1}^{I}\Sigma_{j=1}^{J} \int_{p_{i0}-p_{j0}}^{\infty} [1/((2\pi)^{1/2}\sigma_\theta)]\exp\text{-}\{1/2[(x - \mu_\theta)/\sigma_\theta]^2\}dx. \quad (1.12)$$

Equation (1.2) describes the market process as a binomial experiment that generates a level of trading volume, given a number of "trials" in the experiment, I, and the *ex ante* probability of "success" in each trial, π. Equation (1.12), in turn, determines the expected number of "successes" (trades).

This model provides an explanation of positive exchange volume in a pure exchange market without exogenous shocks. Define "normal" trading intensity as the state in which no unanticipated information enters the market. μ_k is the asset's expected return for all k. An outside observer, while recognizing that many demand prices will change, has no prior beliefs regarding the direction of change for any single one. Yet, even when $\mu_j = \mu_i$, a sufficient condition for $\pi_{ij} > 0$ is $\sigma > 0$. In turn, $\pi_{ij} > 0$ for any (i, j) pair insures $\pi > 0$. As long as at least one individual has idiosyncratic demand price adjustments, the expected number of exchanges is positive.

1.3. Information Flow and Trading Intensity.

Suppose that an information arrival has a mean effect μ on all traders' demand prices. If each investor receives slightly different information or interprets identical information differently, then expected volume increases.

<u>Proposition 1</u>: *If market participants revise their demand prices in unpredictable ways that do not correlate with the subsets* I *and* J *or with investors' idiosyncratic demand price revisions, expected trading volume increases.*

Proposition 1 is proven in the Appendix. The intuition behind the proposition is that the new information, interpreted differently by market traders, adds to the normal "jumbling" of demand prices that comes from investors' liquidity and speculative trading. This increases the variance of the demand-price revision process σ^2, and therefore increases σ^2_θ. The resulting reallocation of assets to higher valued owners increases the expected intensity of trade.[8]

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[8] Several researchers have examined trading volume around an event to determine whether the event has "information content." See Beaver (1968), Morse (1980), or Bamber (1986). The presumption, usually stated informally, is that an informational event causes more trades as investors disagree about the meaning of the information and revise their portfolios

However, this is not the only mechanism by which trading intensity increases. This model has used the artifice of the assumptions distinguishing buyers and sellers and the random pairing to distinguish conceptually buyers from sellers, but such a distinction can be descriptively accurate. Events that have systematically different effects on buyers' and sellers' demand prices have volume implications since $\mu_i \neq \mu_j$ implies $\mu_\theta \neq 0$.

Proposition 2: *If potential buyers and sellers revise their demand prices in systematically different ways, expected trading volume increases if* $\mu_j > \mu_i$, *and decreases if* $\mu_i > \mu_j$.

The proof of Proposition 2 is in the Appendix. Cases in which $\mu_\theta \neq 0$ are very likely. Consider a practically anticipated event about which investors have different prior expectations. After t_0 but before trading at t_1, information is publicly revealed about the asset's value ζ, where ζ is a random variable with mean μ_ζ. All investors obtain the information and interpret it identically. Before t_1, investors' demand prices at t_0 reflect private information at t_0, described by $z_k = \mu_\zeta + \gamma_k$, where γ_k is a zero-mean random variable unique to investor k and z_k describes investor k's anticipation of the "true" value ζ. So $p_{k0} = g(E(\zeta | z_k))$, where $g' > 0$, and investors with relatively high z_k become owners of the asset at t_0. When the information is fully revealed at t_1, each investor's demand price adjusts to reflect the information:

$$\delta_{ik} = \mu_k + \sigma_{\varepsilon_k} = g(\zeta) - g(E(\zeta | z_k)) + \sigma_{\varepsilon_k}. \qquad (1.13)$$

The effect of the public release is to pull all investors' demand prices

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accordingly. We formalize this intuition with Proposition 1.

toward $g(\zeta)$, which implies $\mu_\theta > 0$. To illustrate, suppose the information is "good," e.g., someone makes a tender offer that investors anticipated with probability less than one. We can describe current owners as investors with relatively high z_h at t_0. They will revise their demand prices by a small amount compared with current nonowners, who were relatively "pessimistic," that is, who had demand prices at t_0 characterized by low z_k. As a result, $\mu_i < \mu_j$ and $\mu_\theta > 0$. Similarly, if the news is "bad," all expected demand price revisions are negative. Current owners' demand price revisions are, as a group, greater in absolute value, and the reversion of investor's demand prices toward the mean implies $\mu_i < \mu_j$.[9] Thus, we expect to observe that buyers and sellers have an asymmetric reaction to new information.

Note that $\mu_i > \mu_j$ implies a decrease in expected trading intensity. This case can arise if the informational event causes further divergence in the demand prices of owners and nonowners. This is a characteristic of models based on behavioral differences between buyers and sellers. A more plausible story is that the information confirms traders' prior beliefs. We cannot speculate on whether this is a common occurrence, but it is sufficient for us to note that information can decrease volume.

So far we have developed two distinct ways in which trading intensity can increase in response to an informational event; however, both situations can occur simultaneously. Information revealing events that

[9] There are other reasons why μ_θ may be different from zero. For example, information asymmetries can arise, say, if current asset owners have access to different information from potential buyers. However, this is likely only in closely held corporations where current owners also are corporate insiders.

induce asymmetries between group means affect μ_θ, and heterogeneity among members of the same group—either buyers or sellers—affects σ_θ. Propositions 1 and 2 indicate that simultaneous positive changes in both μ_θ and σ_θ complement each other; the effect is to increase expected volume. However, an event that simultaneously decreases μ_θ and increases σ_θ has an ambiguous effect on volume. The total differential of π_{ij} is

$$d\pi_{ij} = (\partial\pi_{ij}/\partial\mu_\theta)d\mu_\theta + (\partial\pi_{ij}/\partial\sigma_\theta)d\sigma_\theta. \tag{1.14}$$

Setting this equal to zero and substituting from equations (A.4) and (A.5) in the Appendix yields

$$d\mu_\theta/d\sigma_\theta|_{d\pi_{ij}=0} = \left(-\frac{p_{i0} - p_{j0} - \mu_\theta}{\sigma_\theta}\right)\left(\frac{f_\theta(p_{i0} - p_{j0} - \mu_\theta)}{f_\theta(p_{i0} - p_{j0})}\right) \tag{1.15}$$

For $(p_{i0} - p_{j0}) > \mu_\theta$, positive changes in μ_θ and σ_θ are substitute methods to increase π_{ij}.[10] Event studies that record positive volume reactions to new information cannot distinguish between the causal effects $\mu_\theta > 0$ and $\sigma_\theta > 0$. In contrast, observations of decreases in volume would not only imply that $\mu_i > \mu_j$, but also that the difference outweighs any effect of increases in σ_θ.

1.4. Market Liquidity and Trading Intensity.

Market liquidity depends both on the market's depth (the number of traders waiting to transact) and on the bid-ask spread (the cost of transacting). We examine both elements of market liquidity.

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[10] For $\mu_\theta = 0$, equation (1.15) indicates that an isoquant mapping of $(\mu_\theta, \sigma_\theta)$ combinations that yield a given π_{ij} is characterized by nonparallel straight lines, each with slope equal to the negative of the "z-score" (standardized) value of $(p_{i0} - p_{j0})$ and converging to the point $(\mu_\theta, 0)$ on the μ_θ axis (at which $\pi_{ij} = 0.5$).

Trivially, expected trading intensity increases proportionally with the number of outstanding units of the asset (and therefore, with the number of asset holders),

$$\partial\mu_T/\partial I = \pi > 0. \tag{1.16}$$

While the proportionality is an artifact of this model, we expect to observe a positive relationship between volume and the market size.

Momentarily relax the no transaction cost assumption to introduce a positive transaction cost c. Equation (1.9) becomes

$$\pi_{ij}^c = \int_{p_{i0}-p_{j0}+c}^{\infty} f_\theta(x)dx = 1 - F_\theta(p_{i0} - p_{j0} + c), \tag{1.17}$$

where π_{ij}^c is the probability of an exchange, given the transaction cost. Differentiating,

$$\partial\mu_T/\partial c = -(1/J)\Sigma_i\Sigma_j\, f_\theta(p_{i0} - p_{j0} + c) < 0. \tag{1.18}$$

We predict that expected trading intensity is a decreasing function of the transaction costs c since $(\delta_{j1} - \delta_{i1})$ must now overcome both the original difference $(p_{i0} - p_{j0})$ and the transaction cost.

1.5. Trading Intensity in a Multiperiod Market.

We can extend this model to examine the intraday dynamics of trading intensity. We must fix the length of each period, and assume that only one trade can occur at a given instant. Individuals who are unable to find a match in a given period will revise their beliefs and attempt to trade in subsequent periods. Moreover, we approximate this behavior by an independent stochastic process because the population of owners and nonowners is continually changing. The following result is based on the well-known Poisson approximation to the Bernoulli process:

Proposition 3: *Trade counts from independent draws of the market process over fixed intervals will have a Poisson distribution with intensity parameter λ.*

The Appendix contains the proof of Proposition 3. This result motivates our empirical modeling strategy that we discuss in Section 3.

1.6. Relation to the Literature on Intraday Trade.

Theoretical treatment of trading volume has focused on the relationships among information flows, price changes, and trading. One important contribution of this study is that our model synthesizes some of these results and empirically explores several interesting hypotheses that are relevant to this literature.

Initially, trading volume was important in the mixture of distributions models of Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983), Harris (1986), and, most recently, Gallent, Rossi, and Tauchen (1992), that provide statistical explanations of the leptokurtosis in the empirical distributions of daily stock prices. These models predict a positive relationship between volume and the magnitude of the corresponding price change over daily intervals. Yet, these models are often informally motivated, statistically based, or lack an economic specification that is readily implementable.

The model by Pfleiderer (1984) considers price and volume in a noisy rational expectations equilibrium. This model implies that no correlation exists between the magnitude of price changes and trading by speculators with private information but there is a positive relationship between price changes and trading by liquidity-motivated investors. Thus there exists a negative association between the strength of the correlation between absolute price changes and volume and the existence of private information.[11] We can directly test this proposition with our empirical exercise by controlling for both security-specific and market-wide information in our specification. Private information should be most valuable to those with security-specific information. Hence, a testable implication of Pfleiderer's model is that security-specific information should have a greater influence on trading intensity then does market-wide information.

Pfleiderer also considers the informativeness of prices. The market price does not fully reveal aggregate information in this model. Each investor receives information about the value of a risky asset that includes both a common and a unique component. When there is no common information error (i.e., random deviations in the information from the true price are unique to each investor), the model yields the surprising result that expected volume is a decreasing function of the variance of the idiosyncratic error. This implies that volume is a decreasing function of disagreement between investors (i.e. the variance of the return process), a result that we can explicitly test by including return volatility in the

[11] Campbell, Grossman, and Wang (1992) find an empirical relationship between stock market volume and the *autocorrelation* of daily stock index returns. With our intraday model of trading intensity, we are able to examine whether this pattern is due to transitory shifts in liquidity trading as they hypothesize.

specification.[12,13]

Similar models of intraday trading are advanced in Admati and Pfleiderer (1988, 1989). These papers develop theories in which liquidity traders and informed traders interact strategically and, hence, we observe endogenously concentrated trading patterns. Admati and Pfleiderer boldly make several specific empirical predictions that our model nests and tests. First, they predict an inverse relationship between market depth and trading intensity. If we interpret market depth more generally, then their proposition also implies a positive association between the cost of trade (i.e., the bid-ask spread) and volume.[14] Since our model suggests that the opposite relation should exist, we include several measures of market liquidity in our empirical specification to test these conflicting hypotheses. Second, Admati and Pfleiderer posit a positive relationship between trading volume and the informativeness of prices. We examine this proposition with various time-of-day sub-samples. Given that the market open and close are the most active trading periods, we should observe the strongest relationship between trading intensity and security-specific information during these periods according to Admati and Pfleiderer's

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[12] This occurs because, as private information becomes more certain, investors take larger speculative positions based on their private information. When the common information error is positive, expected volume is at first an increasing, then a decreasing function of the precision of private information. Kazemi (1991) extends Pfleiderer's model to study how the distribution of beliefs affects equilibrium asset prices.

[13] Also note that our model predicts the opposite relationship: greater investor disagreement raises trading intensity in our model.

[14] Easley and O'Hara (1992a) explicitly make the this prediction. They use a sequential arrival asymmetric information rational expectations intraday trading model to predict theoretically a positive relationship between bid-ask spreads and trading volume.

theory.

Several researchers have attempted to model heterogeneous traders with different opinions. Varian (1985, 1989), Morris (1990), and Harris and Raviv (1991) all examine the effects of private information and the market's aggregation of information on volume. Varian uses a Bayesian framework to distinguish between opinions (priors) and information (likelihoods). He argues that trading volume depends only on differences of opinion, even when investors receive different information, because the market price adjusts to reveal all information in the economy and thus negates the values of unique information to any single investor.[15] Harris and Raviv (1991) assume that traders share common prior beliefs and receive common information but differ in the way in which they interpret this information, and obtain the now familiar theoretical result on the relationship between price changes and information. However, this research direction yields relatively few insights or testable implications concerning intraday trading intensity.

Little is known about the effect of order type or placement strategy on trading intensity. Cohen et. al. (1978) assert that limit orders do not reflect changes in the aggregate information set pertaining to a particular security because informed traders are unlikely to use this order type since it reveals their information to the market. However, using a simple asymmetric information trading model, Easley and O'Hara (1991) theorize that informed traders will use market orders rather then limit or stop

[15] One drawback of this model is that it implies that prices decrease while trading volume increases, a prediction inconsistent with most empirical evidence. See Karpoff (1987) for a survey.

orders to maximize the return on their information. Although our model does not specifically address the type of trade, we can draw inferences about what we should observe in our model by reinterpreting the trader type to be those that wish to transact immediately against limit traders, and those that shop their transaction among various floor brokers to trade within the bid-ask spread. Investors with security-specific information could often receive better execution if they shop their transaction among floor brokers. Thus, based on our model, we expect to observe greater trading intensity within the bid-ask spread when traders have more security-specific information. Trades at the bid-ask spread should be less sensitive to changes in security-specific information.

Many researches have observed that trade size has a significant impact on security prices.[16] Two competing theories explain this observation. One explanation, first posited by Stoll (1979) is that block transactions have a significant *liquidity effect* on the market by causing the specialist's inventory to deviate from his preferred position. Easley and O'Hara (1987, 1992b) develop an alternative explanation: that there exists a positive correlation between trade size and private information about the security's true value and therefore, an adverse selection problem arises when an investor wishes to make a block transaction. Thus, they argue that large trades have an important *informational effect* on market prices. While our model limits an individual to trade a single share of the security, we can suspend that assumption and posit that a block trader must find enough

[16] See Kraus and Stoll (1972) and Dann, Mayers, and Raab (1977) for empirical verification of this assertion.

people willing to transact as many times as he wishes in a given period. Obviously, it follows from our model that block trades require sufficient liquidity to carry out the transaction. The only informational impact of a large trade would be negligible because those people who where unable to transact in the current period would not be certain whether their beliefs had deviated from the population or the aggregate information set had changed. Within our empirical framework, we can identify the intensity of block transactions and test these alternative explanations.

2. Data and Specification.

2.1. The Sample.

This study uses stock transactions data from the New York Stock Exchange (NYSE) during the year of 1988 collected by the Institute for the Study of Securities Markets (ISSM) on eighteen randomly selected stocks.[17] The only conditions on stock selection that we imposed were that all stocks in our sample must be members of the S&P 500 Index on the first day of trading in 1988 and their primary market must be the NYSE.

Table 1 offers a numerical description of the eighteen securities with sample statistics on market value, the maximum and minimum transaction price, the average NYSE and non-NYSE volume, the average NYSE and non-NYSE number of transactions, and the average NYSE and non-NYSE number of quotes. 88.5 percent of total transactions volume of the stocks we study occurs on the NYSE. The firms in our sample exhibit substantial heterogeneity with

[17] We excluded the opening and closing trades from our analysis.

respect to trade within the year and across stocks. For example, the Upjohn Company (UPJ) averages 5246.6 lots per day with some 348.2 trades while Fleetwood Enterprises Inc. (FLE) averages 496.9 lots per day in 35.2 trades.

We divided the transactions sample into equally spaced fifteen minute intervals beginning with the daily market open. Theoretically, the length of the interval is irrelevant since the compositions of event count processes are additive. Implicitly, we assume that the trading intensity process adjusts to changes in information and traders' beliefs within a fifteen minute period.[18]

2.2. Empirical Representation.

Trading Intensity. Our model of intraday trading intensity described the aggregate movements in the net demand for a particular security. We measure trading intensity empirically as the number of transactions (NTRADE) between buyers and sellers within a given fifteen minute period.

An important implication of our theory is that different traders react to new information differently. We study three separate transaction classifications to test this proposition. First, we classified transactions by type of trader. For a given fifteen minute period, we counted the number of buyer-initiated trades (NBUY) and the number of seller-initiated trades (NSELL) using the classification rules developed in

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[18] Empirical evidence on the speed of adjustment of stock prices to economic news is sparse. Available evidence from Jain (1988) using hourly stock data indicates that stock prices rapidly adjust to reflect new information over the course of several hours.

Lee and Ready (1991) that compare the trade price with the prevailing quote. Second, we divided transactions by type of order. A *limit-market trade* occurs when an individual desires immediate execution and hits an existing limit order (on the bid or ask side) at the prevailing quote. A *market-market trade* occurs within the bid-ask spread. Frequently, this type of transaction occurs when a customer wants his broker to "work" or advertise his trade on the exchange floor. We accumulated the number of market-limit trades (NMLT) and the number of market-market trades (NMMT) within each fifteen minute period. A final transaction type we studied are block trades. Following Madhaven and Smidt (1991), we adopted a stock-specific definition for a block trade. We define a block trade as a transaction volume that exceeded the 95th percentile of the order size distribution.[19] This definition has the advantage of varying by market liquidity; so, what may be a "large" trade in one market may not be a significant event in another market. We counted the number of block trades (NBLOCK) within each fifteen minute interval.

We provide the sample statistics for our measures of trading intensity in Table 2. For example, the stock Great Northern Nekoosa (GNN) averaged 3.09 trades per fifteen minute interval, 1.24 buyer-initiated trades, 1.11 seller-initiated trades, and 0.16 block trades. Furthermore, limit-market trades account for 1.82 trades per interval, while market-market trades account for 1.27 trades per interval. As an illustrative case, we present a frequency count distribution of NTRADE for GNN in Figure 1. The median

[19] We obtained similar results by analyzing transactions in the 99th percentile of the order size distribution.

of the distribution of NTRADE for GNN is two trades per period and this frequency count declines monotonically for values greater than one.

Figures 2 and 3 illustrate the intraday pattern of trading intensity for the stock Great Northern Nekoosa. We plot the time between transactions and the cumulative number of trades for a single trading day (1/3/88) for GNN. There is a pronounced effect that the rate of trade is above average at the opening and closing of the trading day. For GNN, Figure 3 presents each of our measures of trading intensity averaged into half hour periods: the mean and standard deviation of NTRADE are plotted against time in Panel A; NBUY and NSELL are graphed in Panel B; NMLT and NMMT are illustrated in Panel C; and in Panel D we present the block trade count. Notice that each of our trading intensity measures exhibits the U-shape first noted by Wood, McInish, and Ord (1985). Trading activity declines after the market open until roughly 1 PM, where it levels off and begins to rise again until the market close.

The theoretical model suggests that three classes of variables should be important in determining trading intensity: information flows, return volatility, and liquidity. We lag by one fifteen minute interval each of these variables in our specification to minimize possible simultaneity bias.[20]

Information Flows. Information flows affect an individual's estimate of the value of the security. We focus on two sources of information: security-specific returns and market-wide returns. As developed in Section

[20] In preliminary tests, we were unable to identify separate effects from longer lags or moving averages.

1, positive and negative information can affect different classes of traders in different ways. According to our model, we expect to observe a positive relationship between trading intensity and our measures of information flows.

We define transaction security-specific returns within each interval as the log ratio of the last trade price to the first trade price. We further divided this series into positive returns (PTRET) and the absolute value of negative returns (NTRET).[21] We calculate market-wide returns using intraday transaction prices of the nearest-term S&P 500 futures contract.[22] This series was also used to create positive and negative return series, (PMRET) and (NMRET), respectively.

Return Volatility. Return volatility proxies for the extent of disagreement among traders within each period. Again, we estimate both security-specific and market-wide volatility to examine their differential effects on trading intensity. Transaction security-specific return volatility (TVOL) is the time-weighted standard deviation of returns within a given period. We made a similar computation to determine the market-wide return volatility (MVOL).[23]

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[21] We also computed security-specific return series using prices derived from the midpoint of market quotes. This variable made little difference to the results we report.

[22] Tick Data Inc. collect and disseminate these data from the Chicago Mercantile Exchange.

[23] Since we estimated this variable, it is subject to errors in variables bias. To assure the robustness of our analysis, we also estimated the model below with several different measures of volatility. First, we estimated the standard deviation of returns from the last twenty trades (in transaction time). Second, we estimated an intraday GARCH (1,1) model of fifteen minute returns and used the estimated standard deviation from this procedure. Third, we used the high-low range within each period as a measure of volatility. Our qualitative conclusions remain unchanged using

Liquidity. Liquidity is measured with three security-specific variables: the bid-ask spread, the depth, and the total volume transacted. The bid-ask spread (SPREAD) is a cost of transacting immediately. We measure the time-weighted mean bid-ask spread in percentage terms within a period.[24] High trading costs should discourage trading activity. We define the depth (DEPTH) as the time-weighted mean of the number of lots on the best quote. The more lots available within the market, the higher the probability that a match will occur. Finally, we also measure liquidity as the number of shares (VOLUME) that transacted within an interval. A liquid market is characterized by high transactions volume. We divided both DEPTH and VOLUME by 1000 for computational convenience.

We include intraday half-hour time dummies (where T1 = 9:30 AM to 9:59 AM, T12 = 3:00 PM to 3:29 PM, and T13 is excluded) to capture any remaining idiosyncratic time-specific variation in trading intensity.

In summary, our basic trading intensity specification is:

$$\begin{aligned} INTENSITY_t = \beta_0 &+ \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} \\ &+ \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} \\ &+ \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} \\ &+ \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t \\ &+ \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \varepsilon_t, \qquad (2.1) \end{aligned}$$

which we estimate for six measures of INTENSITY: $NTRADE_t$, $NBUY_t$, $NSELL_t$, $NMLT_t$, $NMMT_t$, and $NBLOCK_t$. We provide summary statistics for all security-

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these measures.

[24] For a given quote the percentage bid-ask spread is:

$$BA_t = (ASK_t - BID_t)/((ASK_t + BID_t)/2).$$

SPREAD is time-weighted average of the spread within an interval.

specific variables in Table 3.[25]

3. Statistical Assumptions and Methodology.

3.1. Modeling Event Counts.

The Poisson probability distribution provides a natural stochastic specification for trading intensity. This distribution captures the discrete nature of trading and researchers have applied it extensively as a model of event count processes.[26] Let Y_t $(t = 1, \ldots, T)$ be the random dependent event count so the values 0, 1, 2, ..., n occur with positive probability. We observe the realization of y only at the end of each observation period t. To derive a specific probability distribution, one nevertheless needs to make assumptions about the *unobserved process* within each observation period generating the *observed count* at the end of the period. Suppose we make the following assumptions about the process during observation period t:

Assumption 1: *More than one event cannot occur at the same instant.*

Assumption 2: *The probability of an event occurring at any instant is* constant *within period t and* independent *of all previous events during that observation period.*

Assumption 3: *Zero events have occurred at the start of the period.*

Assumption 4: *The length of each observation period t is identical.*

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[25] The mean market-wide positive return (PMRET) is 0.062, standard deviation of 0.103; the mean of NMRET is 0.062, standard deviation equal to 0.116; and the mean of market-wide return volatility is 0.198, standard deviation of 0.038.

[26] An early analysis of Poisson regression is Jorgenson (1961). More recently, Hausman, Hall, and Griliches (1984) have analyzed models of event count data with application to the patents-R&D relationship.

From these first principles, one can derive a form of the Poisson probability distribution for the random variable Y_t:[27]

$$f(y|\lambda,t) = \begin{cases} \dfrac{e^{-\lambda_t}(\lambda_t)^{y_t}}{y_t!} & \text{for } \lambda_t > 0 \text{ and } y = 0, 1, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

We specify an event count regression model by letting the expected count, $E(Y_t) \equiv \lambda_t$ vary over observations according to a specific function of a vector of explanatory variables. A general form of this equation is $\lambda_t = \lambda_t(X_t,\beta)$, where X_t is a vector of k exogenous variables and β is a $k \times 1$ parameter vector. We can easily specialize this functional form to $E(Y_t) \equiv \lambda_t = \exp(X_t\beta)$.

To estimate β, the effect of the explanatory variables on the dependent variable, we use the method of maximum likelihood. By assuming the absence of autocorrelation, we may write the likelihood function as

$$\mathcal{L}(\beta|y) = \prod_{t=1}^{T} f_p(y_t|\lambda_t) = \prod_{t=1}^{T} \frac{e^{-\lambda_t}(\lambda_t)^{y_t}}{y_t!} \tag{3.2}$$

with $\lambda_t = \exp(X_t\beta)$. The log-likelihood, reduced to sufficient statistics, is then

$$\ln\mathcal{L}(\beta|y) = \sum_{t=1}^{T}\left\{ -\exp(X_t\beta) + y_t(X_t\beta) \right\}. \tag{3.3}$$

The applicability of the four assumptions concerning stochastic process generating our trade counts is an important issue. Assumption 1 is a technical requirement without many real consequences. Assumption 3 is more of a notational convenience, and Assumption 4 enables one to

[27] Feller (1968, Chapter 17) derives this result.

parameterize the distribution in the form of equation (3.1). However, for modeling a market's dynamics and trading intensity, Assumption 2 is very important since it has two important implications that might not be consistent with our application. First, we must assume that the probability of a trade is constant for every period. This homogeneity assumption appears implausible since either changes in the distribution of market beliefs or variations in the flow of information are likely to vary over time, resulting in a heterogeneous probability of event occurrence. Second, we assume that the occurrence of a trade at one point in time is independent of all previous trades.[28] This part of the assumption may fail in this application because each trade presumably furthers the price discovery process towards equilibrium and may thus draw additional traders into the market.

These assumptions about the unobserved process generating the observed counts have consequences for the variance in the event count regression. The variance of Y_t under the Poisson distribution in equation (3.1) is equal to its expected value:

$$V(Y_t) = E(Y_t) \equiv \lambda_t. \tag{3.4}$$

However, this result relies on micro level independence and homogeneity assumptions about the process generating the counts. If these assumptions do not apply, then the Poisson distribution does not result, and the variance is not equal to the mean. In this case, the log likelihood in equation (3.3) will yield consistent estimates, but they will be

[28] This assumption must hold both within a given interval and across periods.

inefficient, and the standard errors inconsistent.

More generally, let

$$V(Y_t) = \lambda_t \sigma^2 \tag{3.5}$$

for $\lambda_t > 0$ and $\sigma_t > 0$; σ_t is called the *dispersion parameter*. When individual events are independent with homogeneous rates of occurrence, the case of Poisson dispersion results with $\sigma^2 = 1$ and $V(Y_t) = \lambda_t$. Alternative assumptions lead to other values for σ^2. For example, $\sigma^2 > 1$, we term the data *overdispersed* and, if $0 < \sigma^2 < 1$, we define the data as being *underdispersed*. In general, $\sigma^2 = \sigma^2(X_t,\beta,\gamma)$, where γ is now the ancillary parameter. We estimated the variance of the arrival process using the following functional form:

$$\sigma^2 = 1 + \exp(\gamma). \tag{3.6}$$

We use a specialized specification in which σ^2 from equation (3.5) is a scalar parameter; however, numerous other functional forms of $V(Y_t)$ are possible.[29] Furthermore, "even relatively substantial errors in the assumed functional form of [the variance] generally have only a small effect on the conclusions." (McCullagh and Nelder (1983, p. 132)).

Panel A of Figure 3 presents both the mean and standard deviation of the number of trades per fifteen minute period (NTRADE) across the trading day by half-hour for the stock Great Northern Nekoosa (GNN). Note that the standard deviation of NTRADE is always strictly less (and the variance is always greater) than the mean. While this is not statistical evidence of dispersion, we are lead to use additional tests. Collings and Margolin

[29] See Cameron and Trivedi (1986). We may easily alter this form by substituting another form in for σ^2 before differentiation.

(1985) note that one can easily test the Poisson model against the model with extra-Poisson variation by testing H_0: $\gamma = 0$ versus H_1: $\gamma > 0$ from equation (3.6).[30] According to tests reported in Section 4, most of the stocks in our sample exhibit overdispersion based on this hypothesis test. We examine why trading intensity exhibits overdispersion in the following section.

3.2. Models with Overdispersion.

Heterogeneity and *contagion*, two very different and substantively interesting unobserved processes, can produce identical models for overdispersion. We consider these explanations in turn.

If individual trades within observation t are heterogeneous (i.e., traders have diverse opinions), λ_t will vary across individual trades (within observation t), and overdispersion will result.[31] Thus, since we are modeling a fundamentally heterogeneous process, different rates of trading intensity for individual market participants will induce overdispersion.

To model heterogeneous processes, those resulting in overdispersed event counts, we drop the assumption that λ_t is constant within observation t. Instead, we assume that λ_t is a random variable. In order

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[30] The literature on testing for overdispersion is extensive. We also tried various tests contained in Cameron and Trivedi (1990) and Smith and Heitjan (1993). Another form of these tests developed by Mullahy (1986) involves the application of specification tests to this problem. None of these methods altered our qualitative conclusion that the transaction counts in our sample are overdispersed.

[31] Measurement error in the explanatory variables or the omission of relevant explanatory variables (uncorrelated with the ones included) also can cause apparent overdispersion (Prentice (1986)).

to build a stochastic model for heterogeneous processes, we must make some assumption about the distribution of λ_t (e.g., the rate of trading) across traders within each observation period. The usual assumption is that λ_t follows a gamma distribution.[32] Under the gamma distribution $[f_\gamma(\lambda_t|\phi_t,\sigma^2)]$, the random variable λ_t takes on only nonnegative real numbers and is assumed to have mean $E(\lambda_t) \equiv \phi_t$ and variance $V(\lambda_t) \equiv \sigma^2$. The form of this distribution is quite flexible and not overly restrictive, but one must recognize that this is a particular assumption about the nature of unobserved heterogeneity.[33]

Greenwood and Yule (1920) first derived the new distribution, called the *negative binomial*, by adding this additional first principle (λ_t following a gamma distribution) to the initial four assumptions. The procedure is as follows: First, derive the joint distribution (f_j) of Y_j and λ_t (both now random variables) using the basic rule for conditional probability $[\Pr(AB) = \Pr(A|B)\cdot\Pr(B)]$:

$$f_j(y_t,\lambda_t|\phi_t,\sigma^2) = f_p(y_t|\lambda_t)\cdot f_\gamma(\lambda_t|\phi_t,\sigma^2). \tag{3.7}$$

Then, derive the negative binomial distribution (f_{nb}) by collapsing this joint distribution over λ_t:

$$f_{nb}(y_t|\phi_t,\sigma^2) = \int_{-\infty}^{\infty} f_j(y_t,\lambda|\phi_t,\sigma^2)d\lambda. \tag{3.8}$$

In the negative binomial distribution, the left-hand-side of equation (3.8), the parameter ϕ_t represents the mean rate of event count occurrence, as λ_t does in the Poisson distribution. Thus, to maintain comparability,

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[32] See Johnson and Kotz (1970, Chapter 17).

[33] Other assumptions are possible, but do not reduce to closed form, and thus require more complicated estimation procedures.

we reparameterize by substituting λ_t for each occurrence of ϕ_t and write out the entire distribution:

$$f_{nb}(y_t|\lambda_t,\sigma^2) = \frac{\Gamma(-\lambda_t/(\sigma^2 - 1) + y_t)}{y_t!\Gamma(-\lambda_t/(\sigma^2 - 1))} \left\{\frac{\sigma^2 - 1}{\sigma^2}\right\}^{y_t} (\sigma^2)^{-\lambda_t/(\sigma^2 - 1)} \tag{3.9}$$

where $\lambda_t > 0$, $\sigma^2 > 1$, and $\Gamma(\cdot)$ is the gamma function.

The result in equation (3.9) is a probability distribution with an additional parameter. We are able to model the expected number of events as before, $E(Y_t) \equiv \lambda_t = \exp(X_t\beta)$. However, the variance is now greater than the mean since

$$V(Y_t) = \lambda_t\sigma^2 = \exp(X_t\beta)\sigma^2 \tag{3.10}$$

where $\sigma^2 > 1$ and equation (3.6) parameterizes σ^2. As σ^2 approaches one, this distribution approximates the Poisson distribution. Larger values of σ^2 produce a distribution with larger and larger amounts of overdispersion in counts, resulting from more heterogeneity within each observation.

Contagion is a second process that generates overdispersion. Contagion occurs when the expected number of events at one time is dependent on the realized number of events at some previous time. For example, we might hypothesize that large block trades are likely to stimulate a series of future trades as the block trade is broken down into smaller pieces or traders return the price process to the prevailing equilibrium. Since with event count data we only observe the number of events at the end of the period, contagion, like heterogeneity, is unobserved, within the observation period.

Two distributions to model this sort of contagion are the continuous Polya-Eggenberger distribution and Neyman's contagious distributions. A result due to Thompson (1954) is that a limiting form of both distributions is the same negative binomial that we derived above for a heterogeneous event count process. For research problems where both heterogeneity and contagion are plausible, the different underlying processes are not distinguishable with aggregate event count data because they lead to an equivalent probability distribution for the counts.[34] One can still use this distribution to derive fully efficient and consistent estimates, but this analysis is only suggestive of the underlying process.

A negative binomial maximum likelihood solution yields consistent and fully efficient parameter estimates in the case of overdispersion due to contagion or heterogeneity. The log-likelihood is as follows:

$$\ln \mathcal{L}(\beta,\sigma^2|y) = \sum_{t=1}^{T}\left\{ \ln\Gamma\left(\frac{-\lambda_t}{\sigma^2 - 1} + y_t\right) - \ln\Gamma\left(\frac{-\lambda_t}{\sigma^2 - 1}\right) + y_t\ln(\sigma^2 - 1) + \ln(\sigma^2)\left(\frac{-\lambda_t}{\sigma^2 - 1} + y_t\right)\right\} \tag{3.11}$$

where $\lambda_t = \exp(X_t\beta)$, for $y_t = 0, 1, 2, \ldots$ and $\sigma^2 > 1$.

Whereas in the Poisson regression model one maximizes the log-likelihood with respect to β, we maximize this log-likelihood with respect to both β and σ^2. We interpret the information from the maximum likelihood estimate, $\hat{\beta}$, as in the Poisson model and the estimate $\hat{\sigma}^2$ provides information about the overdispersion of the data.

[34] See Neyman (1965), p. 5.

4. Empirical Results.

4.1. Methodological and Specification Robustness.

This section reports estimates of the model of the number of trades per fifteen minute interval (NTRADE) presented in equation (2.1) for the stock Great Northern Nekoosa (GNN). We choose to present results on only one security in this section to illustrate that the estimation methodology and model specification we selected dominate other possible choices.[35] In succeeding sections, we report results for all eighteen securities. We expect to observe that the number of trades is positively associated with information flows (PTRET, NTRET, PMRET, and NMRET) and return volatility (TVOL and MVOL), negatively related to bid-ask spreads (SPREAD), and positively related to market depth (DEPTH) and share volume (VOLUME).

In Table 4, we document the robustness of the estimation methodology. Using the stock GNN, we present results using the following four estimation methods: ordinary least squares (OLS), non-linear least squares (NLLS)[36], Poisson maximum likelihood (Poisson), and negative binomial maximum likelihood (NegBin). Except for the security-specific information variables (PTRET and NTRET), the coefficients are robust across the various estimation methodologies. Coefficients β_3 thru β_9 have the correct sign, appropriate magnitudes, and are statistically significant for each of the methodologies. We observe that OLS achieves a higher likelihood than NLLS,

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[35] A full set of comparable results for all eighteen of the securities we studied are available from the author.

[36] In particular, we estimated the following model: $Y_t = \exp(X_t\beta) + \varepsilon_t$.

while on a log-likelihood basis, NegBin dominates Poisson.[37] Also note that the scalar dispersion parameter is statistically significant, indicating the presence of overdispersion.[38,39]

We examine the robustness of the model specification in Table 5 by varying the set of independent variables using negative binomial maximum likelihood estimation. The estimates in Column 5 report results for the complete model and all variations are compared to this specification with a likelihood ratio test reported on the lines marked χ^2 and p-value.[40] Only the coefficients on market-wide return volatility are unstable across specifications.

In column 0 of Table 5, we estimate the model with a constant and twelve time dummies. The time-of-day pattern observed in Panel A of Figure 3 is evident in these results.

We exclude both security-specific and market-wide information in Column 1. Note that the value of market-wide return volatility (1.393) is 2.4 times greater than the values in specifications 2 through 5. We

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[37] We used White's (1982) information matrix test of the null hypothesis that the Poisson specification is a correct characterization of the data generating process, relative to the negative binomial specification. We failed to reject the null hypothesis of no misspecification for most stocks.

[38] As discussed in Section 3, in the presence of overdispersion, the point estimates are consistent but the standard errors are inefficient and inconsistent. Furthermore, we attempted to model the overdispersion with factors we hypothesized would affect the variance rather than the mean. These factors included the lagged number of trades, the lagged number of quotes, the lagged market-wide return volatility, and the lagged number of trades that resulted in a price change in the S&P 500 futures market. None of these factors were consistently significant in our sample stocks.

[39] We also tested for the presence of first-order serial correlation and could reject this hypothesis at conventional significance levels.

[40] See Amemiya (1985, Chapter 4) for a discussion of the appropriate likelihood ratio test statistic constructed for this application.

interpret this finding to mean that market-wide information flows and return volatility both act to increase the dispersion of beliefs among traders in this market.

In Column 2 we only exclude security-specific information from the specification. We performed a likelihood ratio test of the hypothesis that both coefficients are equal to zero (H_0: $\beta_1 = \beta_2 = 0$). We fail to reject this hypothesis when tested against the two-sided alterative that both coefficients, jointly, are not equal to zero.[41] Compared to the full specification, we may reject this hypothesis because positive security-specific returns may induce trade whereas negative security-specific returns may not. Our model predicts that positive and negative information flows may affect different classes of traders differently.

We exclude security-specific and market-wide return volatility in Column 3 and we exclude the three liquidity variables in Column 4. We can comfortably reject the hypothesis that these sets of coefficients are jointly equal to zero.

While we estimate a statistically significant scalar dispersion parameter across all six specifications, notice that the degree of dispersion declines from -0.150 (standard error, 0.045, $\hat{\sigma}^2 = 1.861$) in Column 0 to -0.422 (standard error, 0.051, $\hat{\sigma}^2 = 1.656$) in Column 5, a 24 percent reduction in estimated overdispersion. As noted in Section (3.2), the additional independent variables may proxy for either heterogeneity or contagion in the arrival process.

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[41] This statement is true for significance levels greater than 0.28.

4.2. Full Model and Sample Results.

We report full model estimates and hypothesis tests for all of the eighteen stocks we analyze in Tables 6A and 6B. The coefficients in Table 6A are estimated using negative binomial maximum likelihood. The elements of the coefficient vector have the interpretation that a one-unit change in variable x_j will lead to a $\beta_j \times 100$ percent change in the trading intensity probability. In Table 6B, we evaluate alternative model specifications using likelihood ratio statistics that compare the full model with partial specifications excluding various sets of coefficients.

Our model of the determinants of the number of trades is strikingly robust across the eighteen securities that we study. While we cannot reject the hypothesis (H_2) that both security-specific and market-wide information flows are statistically distinguishable from zero, security-specific information flows are only significant in nine stocks (this is hypothesis test H_1). We attribute most of the effect of security-specific information to positive information flows (PTRET) stimulating trading intensity. Furthermore, note the relative magnitude of the security-specific and market-wide information flow coefficients. In all eighteen cases, market-wide information has a much stronger effect on trading intensity than security-specific information. For example, a one standard deviation increase of both PMRET and NMRET will raise the expected probability of trading intensity in GNN by 20.0 percent. A comparable one standard deviation increase in both positive and negative security-specific return raises the expected probability of trading intensity by 3.0 percent. This finding is inconsistent with the prediction made by Pfleiderer (1984) that the correlation between absolute price changes and volume is negatively related to the existence of private information. We find that changes in market-wide information flows strongly dominate variations in security-specific information flows in determining trading intensity.[42]

The opposite story is evident when we analyze the effects of security-specific and market-wide return volatility on trading intensity. We can reject the hypothesis (H_3) that β_5 and β_6 (the coefficients on TVOL and MVOL) are equal to zero in fifteen stocks. However, we find that β_5 is positive and significant in fifteen cases while β_6 is positive and significant in only eight cases. Since return volatility proxies for the dispersion of beliefs among traders in our model, we can interpret these results to mean that disparate beliefs about the particular security are more likely to generate trade then are disparate beliefs about the entire market for stocks. For example, a one standard deviation increase in security-specific return volatility in GNN will increase the expected probability of trading intensity by 5.3 percent, while a similar increase in market-wide volatility will increase expected trading intensity by only 2.1 percent. These findings are further evidence against the Pfleiderer (1984) model. His model predicts that volume should be a decreasing function of disagreement among investors. We find only one case (stock KMB) where either β_5 or β_6 is negative and statistically significant.

The three variables that measure market liquidity (SPREAD, DEPTH, and VOLUME) are jointly statistically significant in all eighteen stocks and

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[42] Proposition 1 predicts that information flows will be important in determining trading intensity but does not distinguish among different potential sources of information.

usually have the theoretically expected effect on trading intensity. For example, a 0.254 percent increase in SPREAD (one standard deviation) for GNN decreases the expected probability of NTRADE by 11.4 percent. This result confirms the theory advanced in Section (1.4). Furthermore, this evidence is counter to the propositions of Admati and Pfleiderer (1988) and Easley and O'Hara (1992) that there should be a positive relationship between volume and bid-ask spreads. While SPREAD is statistically significant in all eighteen stocks, DEPTH is significant in ten and VOLUME is significant in fourteen.

Finally, note that we find statistically significant evidence of overdispersion in fifteen stocks. Based on the estimated γ coefficient, the magnitude of overdispersion ranges from 3.102 (for MHP) to 1.442 (for SBC).

4.3. Trading Intensity by Trader Type.

When someone places an order, they commonly tell their broker how much they would like to transact at what price and how they would like the order to be executed. Proposition 2 posits that information flows should affect different classes of traders in an asymmetric fashion. To empirically evaluate this statement, we separate orders into several types: buyer-initiated and seller-initiated trades, market-limit or market-market trades, and block trades. We will attempt to use these types of trades to quantify the differential impact of information flows on trading intensity.

Tables 7 and 8 present model estimates and hypothesis tests for buyer-initiated (NBUY) and seller-initiated (NSELL) trades. This sample split highlights how information flows have differential effects on individuals' motives to transact.[43] We find that market-wide information is an important determinant of traders' decisions either to buy or sell.

In Table 7 we present the results for buyer-initiated trades. A one standard deviation increase in positive market-wide returns raises the expected probability of buyer-initiated trading by 18.5 percent, while a comparable increase in positive security-specific returns raises the expected trading intensity by only 2.2 percent. Negative market-wide returns work in the opposite direction. In all eighteen stocks, an increase in negative market-wide returns either decreases or has no effect (statistically) on the number of buyer-initiated trades.

The results in Table 8 for seller-initiated trades tell the opposite story. Negative market-wide information strongly affects the intensity of these trades. β_4 is positive and statistically significant in seventeen cases (the exception is NCB) and β_3 is either negative or zero in all eighteen stocks. Again, we observe that market-wide information has a much stronger effect on seller-initiated trading than does security-specific information.

We find that both security-specific return volatility (TVOL) and the market liquidity variables are significant determinants of NBUY and NSELL, though no particularly striking pattern emerges relative to the full sample results.

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[43] Interestingly, we find little correlation between NBUY and NSELL for most stocks. A seemingly unrelated negative binomial maximum likelihood estimation did not yield either a significant cross-correlation between these variables or qualitatively different coefficient estimates.

In Tables 9 and 10, we present model estimates and hypothesis tests for market-limit and market-market trades. Comparing the tests in Panel B of Tables 9 and 10, we find that, for market-limit trading intensity, security-specific information is jointly significant in only seven stocks. For market-market trading intensity, security-specific information is significant in sixteen of the eighteen stocks we analyze. For example, a one standard deviation increase in both positive and negative security-specific returns in GNN raises expected market-market trades by roughly 6.0 percent, but, a comparable increase in security-specific returns raises the expected probability market-limit trading intensity by only 0.5 percent. For both types of trades, market-wide information is statistically important in determining the trading intensity. However, the magnitude of the effect differs across order types: for GNN, a given negative change in market-wide returns raises the probability of market-limit trading intensity by 1.008 (standard error 0.107) versus only 0.661 (standard error 0.136) for market-market trades. These findings are largely consistent with our model and the theories developed in Cohen et. al. (1981) and Easley and O'Hara (1991).

We report model estimates and hypothesis tests for the number of block trades in Table 11. Our estimates demonstrate a strong link between the number of block trades and the liquidity conditions prevailing in the market before the trade. As observed in Panel B, we can reject hypothesis H_4, that the liquidity variables are jointly equal to zero, for all eighteen stocks. We contrast this finding with hypotheses H_1 (security-specific information is zero), which we fail to reject in eleven stocks, and H_2 (all return variables are equal zero), which we fail to reject in four stocks. Furthermore, we decompose the effects of security-specific information and market liquidity on the probability of observing the mean number of block trades. A one standard deviation increase in the security-specific return variables for GNN has a negligible effect on the expected probability of observing a block trade. A comparable one standard deviation increase in the liquidity variables raises the expected probability of block trading by over 53.0 percent. Our evidence strongly supports the liquidity hypothesis: block traders react to the liquidity conditions of the market rather than security-specific information flows.

4.4. Trading Intensity by Time-of-Day.

A cursory examination of intraday trading patterns reveals that trading intensity varies by time-of-day. The results from our basic specification in Table 6A reveal that our time-varying independent variables were unable to explain the intraday pattern of the half-hour dummies, coefficients δ_1 to δ_{12}. Information events may arrive discretely and cause people to revise their beliefs more radically during certain times of the day, such as after the market open or shortly before the market closes. Admati and Pfleiderer (1988) theorize that there should be a positive relationship between intraday trading volume and the informativeness of prices. We test this proposition within the context of our model by dividing the trading day into three periods: the opening (9:30 to 10:29 AM), the midday (10:30 AM to 2:59 PM), and the close (3:00

to 4:00 PM).[44] This methodology will enable us to evaluate what factors may be important in determining trading intensity by time-of-day.

In Table 12, we present model estimates and hypothesis tests for the first hour of trading for all eighteen stocks. Panel B reveals the striking result that trading intensity during the opening hour is driven neither by information flows nor return volatility. Security-specific information is jointly significant in only three cases. Both security-specific and market-wide information are jointly significant in only seven stocks. Finally, security-specific and market-wide return volatility are jointly significant in only five markets. We do find strong evidence that the opening is primarily determined by market liquidity. The three liquidity variables are jointly significant in sixteen cases; however, market depth plays an inconsequential role (β_8 is only significant in one stock, UPJ). While asymmetric information models of the bid-ask spread contend that a positive relationship should exist between trading intensity and the bid-ask spread,[45] we find that this relationship is negative and statistically significant in sixteen stocks. Our evidence suggests that investors generally are not using signals present in the securities prices during the opening period. In addition, note that γ is greater than the full sample estimate in fourteen stocks meaning that heterogeneity or contagion among investors is higher during this period.

The midday pattern of trade that we present in Table 13 is largely consistent with the full sample estimates reported in Tables 6A and 6B. We

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[44] We continue to analyze the number of trades in intervals of fifteen minutes during these intraday periods.
[45] For example, see Easley and O'Hara (1992a) and related references.

find that market-wide information flows, security-specific return volatility, and market liquidity (including depth) are all important determinants of trading intensity during the midday period.

Table 14 provides model estimates and hypothesis tests from the final hour of the trading day. During this period we note that security-specific information flows are jointly significant in only four stocks and both security-specific and market-wide return volatility are jointly significant in only three cases. Trading intensity during the closing hour is determined by market-wide information in all eighteen stocks and market liquidity in fifteen stocks.

Thus, we observe that our model performs best during the midday period when information flows, return volatility, and market liquidity are all important. Trading intensity on the market open does not appear to be a resolution of the uncertainty that developed overnight, and trade during this early period is dependent upon market liquidity. Trading intensity just before the market closes depends both on market-wide information and liquidity.

5. Interpretation and Conclusions.

We have developed a theory of trading volume based on assumptions that market participants frequently revise their demand prices and randomly encounter potential trading partners. The model describes two distinct ways by which informational events affect trading volume: first, investor disagreement leads to increased trading; second, trading volume can increase even if investors interpret information identically, as long as they also have divergent prior expectations.

We map our theoretical framework into an empirical test of our conjectures concerning the effects of information flows, return volatility, and market liquidity on trading intensity. In addition, we compare the implications of our model with the findings of various other theories of intraday trading patterns, order placement, and block trading. We confirm our hypothesis that information flows strongly affect intraday trading intensity. However, the effect of security-specific information is dominated by the effect of market-wide information flows. This finding is consistent with our theory and inconsistent with several leading theories of intraday trading patterns, most notably those based on asymmetric information. Furthermore, we control for changes in the distribution of beliefs (using return volatility) as well as changes in market liquidity, and we find that these variables are significant determinants of trading intensity.

Our findings are especially interesting when we separate the sample by trader type and time-of-day. As predicted by our model, both buyer-initiated and seller-initiated traders respond to market-wide signals rather than security-specific signals to decide when to trade and this decision occurs in an asymmetric manner. In addition, we find that while market-market trades are influenced by security-specific information, market-limit and block trades are not. Finally, when we divide the sample by time-of-day, we observe that trading intensity during the opening hour is largely liquidity driven, while trade during the closing hour is determined by market-wide information and market liquidity.

Our analysis raises several interesting questions. Perhaps we could better explain intraday trading patterns if we modeled the interaction of information revelation and price discovery. How do traders react when they observe the actions of others? Does trade arise from the heterogeneity of beliefs among trades or does trade generate trade through a feedback process of contagion? We endeavor to explore these questions in future research.

APPENDIX

Proof of Proposition 1: Individual k's ($k \in S$) demand-price revision due to the information is $\mu_k = \mu + \phi_k$, where ϕ_k is the information effect idiosyncratic to trader k (and is over and above the idiosyncratic demand-price revision due to liquidity and speculative desires). To the outside observer, ϕ_k is a random variable with zero mean and variance σ^2_ϕ. Independence from the idiosyncratic liquidity or speculative demand-price revisions implies $E(\epsilon_k \phi_h) = 0$ for all k and h.

Each individual k, whether a buyer or seller, has an expected demand price revision $E(\delta_{k1}) = \mu$. But the variance of this process is now

$$E(\delta_{k1} - E(\delta_{k1}))^2 = \sigma^2_\phi + \sigma^2. \tag{A.1}$$

For the parameter $\theta = \delta_{j1} - \delta_{i1}$,

$$E(\theta) = 0, \tag{A.2}$$

$$E(\theta - E(\theta))^2 = 2(\sigma^2_\phi + \sigma^2) > 2\sigma^2. \tag{A.3}$$

So the process that describes the probability of an exchange (equation 1.9) is characterized by an increased variance. Differentiating equation (1.12) with respect to σ_θ,

$$\partial\mu_T/\partial\sigma_\theta = (1/J)\Sigma_i\Sigma_j(1/\sigma_\theta)(p_{i0} - p_{j0} - \mu_\theta)f_\theta(p_{i0} - p_{j0} - \mu_\theta). \tag{A.4}$$

Since $(p_{i0} - p_{j0}) > 0$ and $(\mu_i - \mu_j) = 0$, $\partial\mu_T/\partial\sigma_\theta > 0$, and the effect of heterogeneous reactions to new information is to increase expected volume. ■

Proof of Proposition 2: Differentiating (1.12) with respect to μ_θ,

$$\partial\mu_T/\partial\mu_\theta = 1/J\Sigma_i\Sigma_j f_\theta(p_{i0} - p_{j0}). \quad \blacksquare \tag{A.5}$$

Proof of Proposition 3: This result follows directly from the following Proposition derived by Katz (1963):

Proposition A1 (Poisson Approximation to the Bernoulli Process): *For a Bernoulli process* $X = \{X_n: n = 1, 2, ...\}$ *with success probability* p, *suppose the* n-th *trial occurs at time* nh. *For fixed* $h > 0$, *let* N(t; h) *be the number of successes in the interval* [0, t]. *Let* N(t) *be the limit of* N(t; h) *as both* $h \to 0$ *and* $p \to 0$ *in such a way that* $p/h = \lambda$. *Then for every* $\lambda > 0$,

$$P\{N(t) = k\} = e^{-\lambda t}(\lambda \tau)^k/k!, \quad k = 0, 1, ...;$$

that is, N(t) *has the Poisson distribution with parameter* λt.

Proof of Proposition A1: For fixed h, the number of trials in [0, t] is [t/h] where [x] is the integer part of x. Since the number of successes in n trials is N_n and the number of success in [0, t] is N(t; h), then for each realization ω,

$$N(t, h)(\omega) = N_{[t/h]}(\omega). \tag{A.6}$$

Thus N_n is binomial with parameters n and p and, so,

$$P\{N(t; h) = k\} = \binom{[t/h]}{k} p^k(1 - p)^{[t/h]-k}, \quad k = 0, ..., [t/h],$$

$$= \binom{[rt]}{k} (\lambda/r)^k(1 - \lambda/r)^{[rt]-k}, \quad k = 0, ..., [rt], \tag{A.7}$$

where $r = 1/h$ and thereby, $p = \lambda/r$. Since $r \to \infty$ as $h \to 0$, then

$$P\{N(t) = k\} = \lim_{r\to\infty} \binom{[rt]}{k} (\lambda/r)^k(1 - \lambda/r)^{[rt]-k}, \quad k = 0, ..., [rt], \tag{A.8}$$

For r large, [rt] is approximately rt and

$$\binom{[rt]}{k}(\lambda/r)^k \cong \frac{(rt)!}{k!(rt-k)!}(\lambda/r)^k = \frac{(rt)(rt-1)\ldots(rt-k+1)}{k!}(\lambda/r)^k$$

$$= \frac{(\lambda t)^k}{k!}\, 1\cdot(1 - 1/r)\ldots(1 - (k-1)/r). \tag{A.9}$$

So,

$$\lim_{r\to\infty}\binom{[rt]}{k}(\lambda/r)^k = \frac{(\lambda t)^k}{k!}\,. \tag{A.10}$$

Also,

$$\lim_{r\to\infty}(1 - \lambda/r)^{[rt]-k} = \lim_{r\to\infty}(1 - \lambda/r)^{[rt]} = \lim_{r\to\infty}\sum_{j=0}^{[rt]}\binom{[rt]}{j}(-\lambda/r)^j$$

$$= \sum_{j=0}^{\infty}\frac{(-\lambda t)^j}{j!} = e^{-\lambda t}. \tag{A.11}$$

Applying equations (A.10) and (A.11) to equation (A.8) gives that N(t) is Poisson with parameter λt. Furthermore, the number of successes in any interval depends only on the length of the interval, independent of the beginning time of the interval. For any s, t > 0,

$$P\{N(t+s) - N(t) = k\} = e^{-\lambda t}\frac{(\lambda t)^k}{k!}\,. \quad \blacksquare \tag{A.12}$$

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FIGURE 1: FREQUENCY COUNT OF THE NUMBER OF TRADES, GNN.

This figure presents the frequency count of the number of trades per 15 minute period for the stock Great Northern Nekoosa (GNN). All NYSE transactions between 3 January 1988 and 31 December 1988 were included. An observation is the number of trades in a given 15 minute period. The frequency count is computed over 6312 observations.

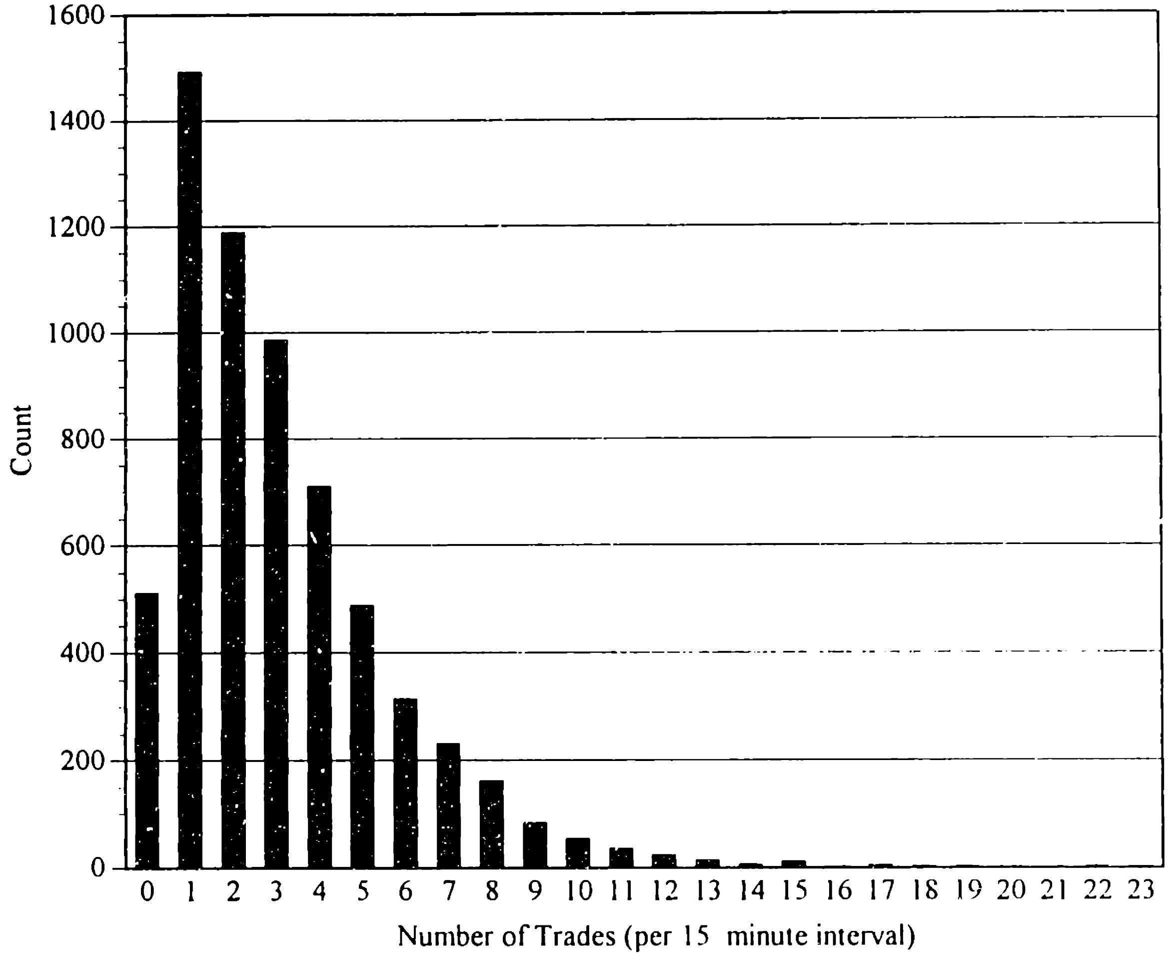

FIGURE 2: TRADING ARRIVALS, GNN (01/03/88).

The solid line on this figure presents the cumulative NYSE trading arrivals for the stock Great Northern Nekoosa (GNN) on 3 January 1988. Time is measured in seconds from 9:30 AM. The dashed line represents the average time between trading arrivals on this date.

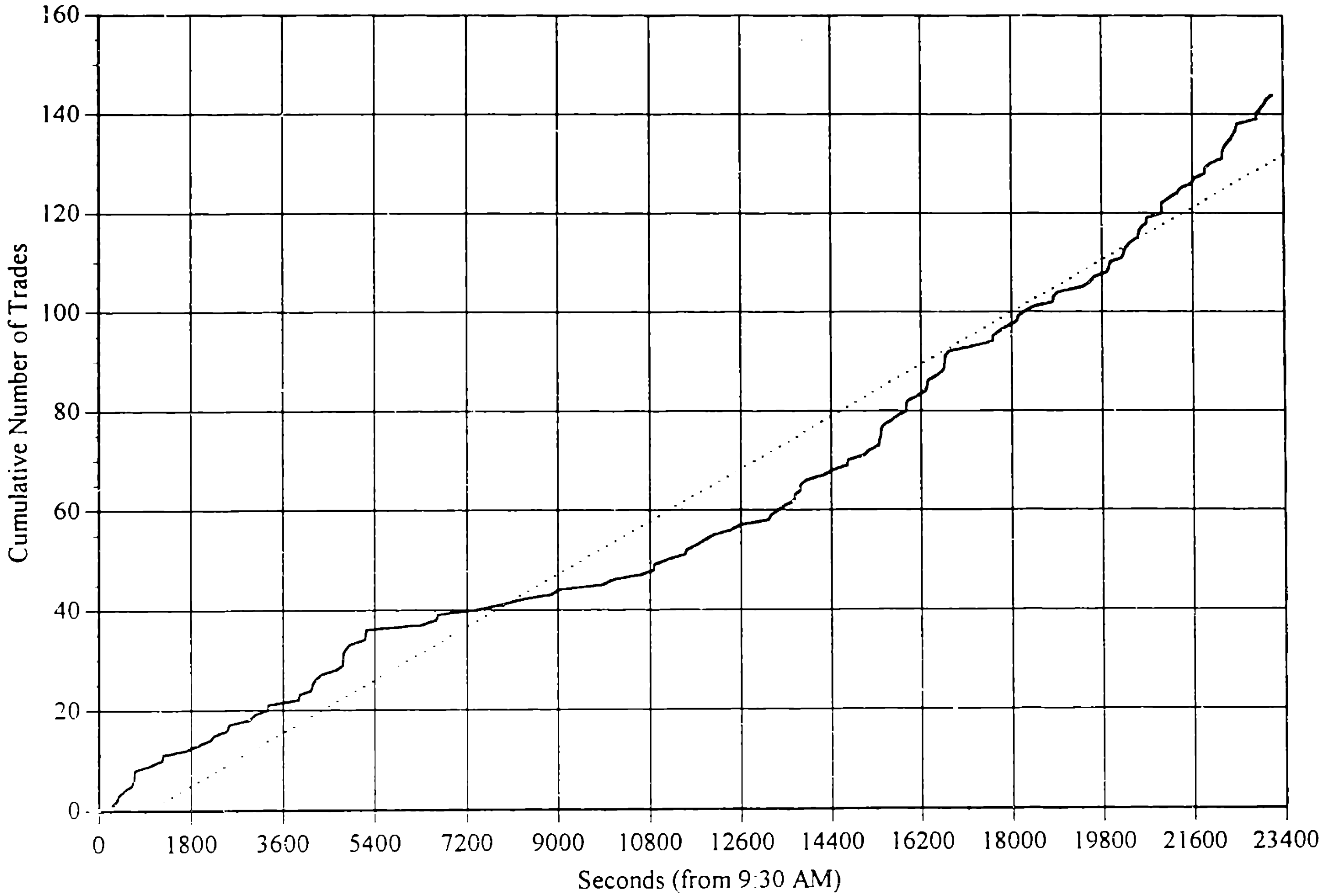

FIGURE 3: INTRADAY TRANSACTIONS COUNT, GNN.

Using intraday half hour intervals, this figure illustrates the average fifteen minute number of trades (NTRADE), number of buyer-initiated trades (NBUY), seller-initiated trades (NSELL), and trades at the bid-ask spread midpoint (NMID), the number of market-limit trades (NMLT) and market-market trades (NMMT), and the number of block trades (NBLOCK) on the NYSE for the stock Great Northern Nekoosa (GNN) in 1988.

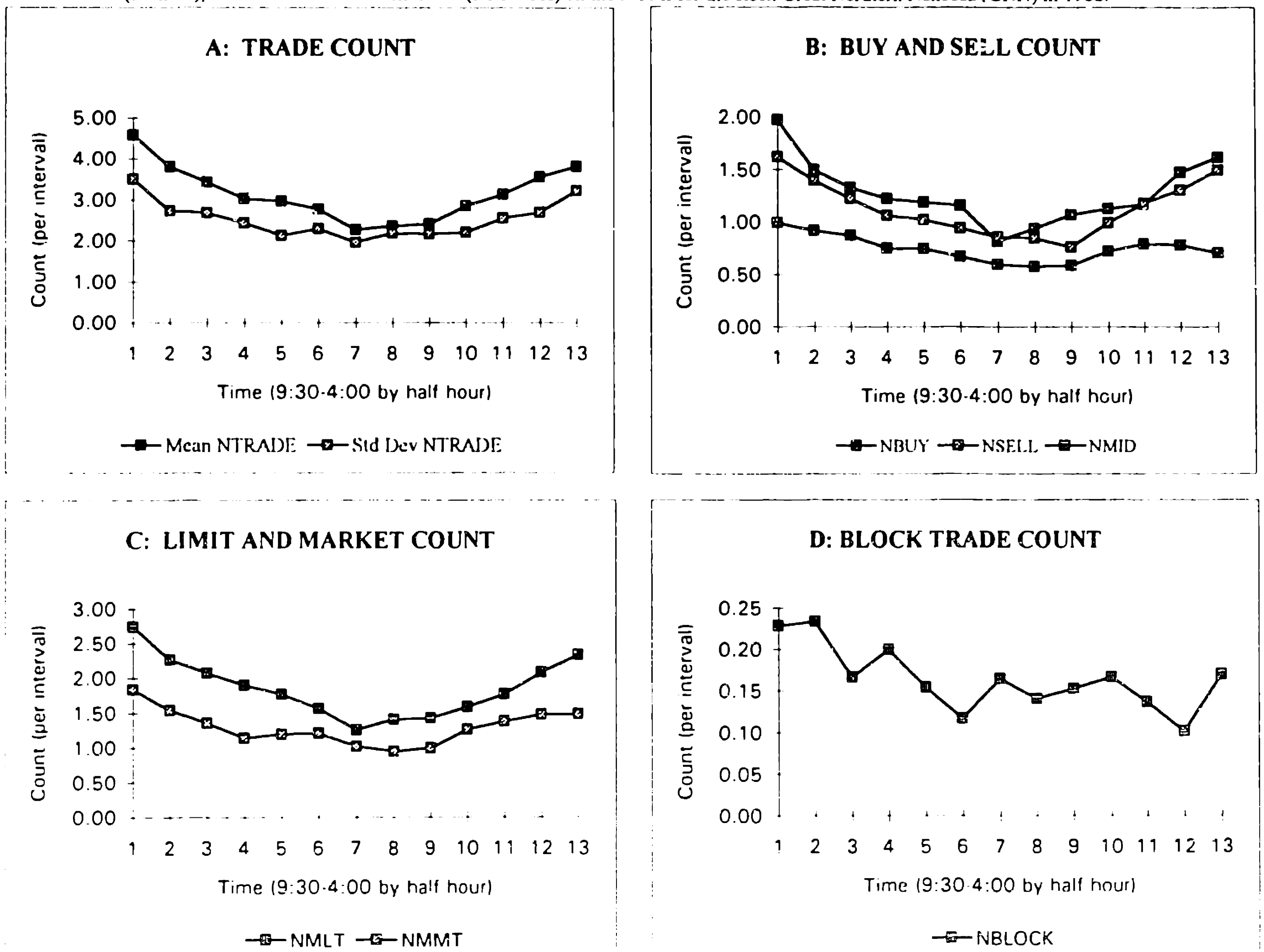

TABLE 1: DESCRIPTION OF SAMPLE STOCKS.

This table reports the ticker symbol, the company name, the 1988 market value, the minimum and maximum price, the average daily NYSE and non-NYSE volume in round lots, the average daily NYSE and non-NYSE number of trades, and the average daily NYSE and non-NYSE number of quotes for the 18 stocks in our sample based on quotes and transactions form the 1988 ISSM database.

Ticker Symbol	Company	Market Value (Millions $)	Minimum Price	Maximum Price	Daily NYSE Volume	Daily non-NYSE Volume	Daily NYSE # Trades	Daily non-NYSE # Trades	Daily NYSE # Quotes	Daily non-NYSE # Quotes
BA	Boeing Co.	5801.27	37.375	67.625	4755.2	554.2	193.4	75.1	112.1	55.4
BC	Brunswick Corp.	1305.85	14.500	24.125	2713.4	379.9	160.0	42.4	180.3	50.9
BCC	Boise Cascade Corp.	1868.44	36.000	71.375	1023.5	109.3	56.3	11.9	60.4	27.7
BMY	Bristol Myers Co.	11955.20	38.125	46.500	4499.4	553.2	273.6	98.7	102.9	83.1
FLE	Fleetwood Enterprises Inc.	406.12	17.000	26.625	496.9	59.9	35.2	6.8	23.6	15.7
GNN	Great Northern Nekoosa Corp.	2544.93	35.000	53.000	2015.2	287.6	83.9	23.6	74.8	36.8
JCP	J.C. Penney Inc.	6571.33	38.000	55.750	3218.6	396.9	111.0	38.1	111.0	38.7
JPM	J.P. Morgan Inc.	6522.50	30.750	40.250	3826.0	512.6	126.0	32.4	75.2	28.8
KMB	Kimberly Clark Corp.	4306.05	46.125	65.750	2050.8	291.8	122.9	31.0	122.9	36.5
MA	May Department Stores Co.	4443.72	28.750	40.000	3356.3	446.3	117.9	50.5	69.5	69.5
MHP	McGraw-Hill Inc.	2438.17	46.875	76.000	2309.9	238.5	133.5	28.7	153.9	34.6
NCB	NCNB Corp.	1474.85	17.500	29.000	1726.9	147.0	60.6	17.0	37.7	38.0
RAD	Rite Aid Corp.	1486.40	29.125	40.875	817.6	91.4	50.6	11.3	55.6	34.8
SBC	Southwestern Bell Corp.	10322.09	33.000	42.625	3246.2	589.3	133.6	71.9	69.5	46.1
UNP	Union Pacific Corp.	5683.39	51.000	70.125	2045.7	202.2	127.7	29.8	96.6	55.5
UPJ	Upjohn Co.	5676.72	26.875	35.350	5246.6	817.9	348.2	113.9	167.9	110.4
USW	U.S. West Inc.	9712.57	48.750	59.625	2574.3	305.9	114.5	44.2	54.4	40.7
VO	Seagram Ltd.	5168.67	50.000	61.875	1071.6	111.5	69.4	13.1	108.5	23.6

TABLE 2: TRADING INTENSITY SUMMARY STATISTICS.

This table illustrates the means and standard deviations (in parenthesis) of the trading intensity measures we estimate with our model for each stock per fifteen minute period. NTRADE is the number of trades, NBUY is the number of buyer-initiated trades, NSELL is the number of seller-initiated trades, NMMT is the number of market-market trades, NMLT is the number of market-limit trades, and NBLOCK is the number of block trades. NOBS is the number of fifteen minute periods used for each security.

STOCK	NTRADE	NBUY	NSELL	NMMT	NMLT	NBLOCK	NOBS
BA	7.216 (4.514)	2.430 (2.687)	2.931 (2.862)	2.107 (2.336)	5.110 (3.903)	0.416 (0.809)	6324
BC	5.975 (4.824)	2.479 (2.882)	2.332 (2.616)	1.450 (2.117)	4.524 (4.041)	0.300 (0.690)	6323
BCC	2.054 (2.032)	0.826 (1.353)	0.699 (1.193)	0.977 (1.357)	1.077 (1.520)	0.105 (0.359)	6293
BMY	10.411 (5.295)	4.154 (3.279)	3.229 (2.758)	3.188 (3.051)	7.223 (4.296)	0.531 (0.879)	6323
FLE	1.309 (1.468)	0.551 (1.097)	0.476 (0.936)	0.513 (0.878)	0.797 (1.228)	0.068 (0.290)	5972
GNN	3.087 (2.569)	1.244 (1.683)	1.108 (1.604)	1.268 (1.531)	1.819 (2.087)	0.161 (0.450)	6312
JCP	4.087 (3.427)	1.314 (1.924)	1.495 (1.947)	1.838 (2.134)	2.250 (2.538)	0.244 (0.585)	6319
JPM	4.699 (3.109)	1.397 (1.806)	1.563 (1.863)	2.178 (2.076)	2.521 (2.416)	0.286 (0.651)	6324
KMB	4.559 (4.571)	1.820 (2.475)	1.228 (1.715)	1.993 (2.673)	2.566 (2.850)	0.350 (0.795)	6316
MA	4.423 (3.288)	1.331 (1.930)	1.216 (1.619)	2.077 (2.229)	2.346 (2.533)	0.301 (0.664)	6309
MHP	4.878 (5.387)	1.947 (2.848)	1.718 (2.374)	2.343 (2.948)	2.535 (3.266)	0.420 (0.943)	6314
NCB	2.217 (2.415)	0.863 (1.456)	0.915 (1.391)	0.662 (1.126)	1.555 (2.041)	0.116 (0.387)	6201
RAD	1.828 (1.951)	0.788 (1.292)	0.683 (1.196)	0.591 (1.084)	1.237 (1.569)	0.092 (0.341)	6279
SBC	4.983 (2.936)	1.597 (1.901)	1.970 (2.033)	1.629 (1.917)	3.354 (2.682)	0.245 (0.543)	6323
UNP	4.825 (3.474)	1.758 (2.144)	2.154 (2.187)	2.094 (2.066)	2.731 (2.543)	0.243 (0.583)	6314
UPJ	13.203 (8.288)	6.252 (5.608)	4.176 (3.735)	3.299 (3.807)	9.904 (7.256)	0.662 (1.102)	6318
USW	4.271 (2.851)	1.245 (1.796)	1.985 (2.031)	1.483 (1.817)	2.788 (2.426)	0.215 (0.525)	6318
VO	2.529 (2.619)	1.064 (1.624)	0.931 (1.572)	0.743 (1.144)	1.787 (2.147)	0.161 (0.464)	6303

TABLE 3: SECURITY-SPECIFIC SUMMARY STATISTICS.

This table illustrates the means and standard deviations (in parenthesis) of the security-specific variables in our model for each stock per fifteen minute period. PTRET is the positive security-specific percentage return, NTRET is the negative security-specific percentage return, TVOL is the return volatility, SPREAD is the time-weighted mean percentage bid-ask spread, DEPTH is the time-weighted mean market depth in lots divided by 1000.0, and VOLUME is the total number of lots transacted divided by 1000.0.

STOCK	PTRET	NTRET	TVOL	SPREAD	DEPTH	VOLUME
BA	0.093 (0.174)	0.087 (0.164)	0.120 (0.075)	0.168 (0.155)	0.079 (0.099)	0.163 (0.239)
BC	0.188 (0.360)	0.190 (0.365)	0.303 (0.211)	0.268 (0.293)	0.059 (0.099)	0.095 (0.200)
BCC	0.085 (0.201)	0.086 (0.208)	0.075 (0.116)	0.396 (0.270)	0.057 (0.064)	0.036 (0.089)
BMY	0.116 (0.203)	0.115 (0.206)	0.189 (0.076)	0.195 (0.156)	0.074 (0.095)	0.166 (0.211)
FLE	0.123 (0.303)	0.118 (0.307)	0.065 (0.160)	1.050 (0.447)	0.058 (0.053)	0.018 (0.057)
GNN	0.109 (0.217)	0.117 (0.234)	0.127 (0.131)	0.345 (0.254)	0.051 (0.052)	0.072 (0.152)
JCP	0.091 (0.201)	0.091 (0.206)	0.099 (0.099)	0.215 (0.191)	0.061 (0.074)	0.113 (0.242)
JPM	0.121 (0.226)	0.124 (0.227)	0.172 (0.125)	0.346 (0.239)	0.083 (0.095)	0.135 (0.776)
KMB	0.110 (0.224)	0.110 (0.209)	0.105 (0.092)	0.221 (0.196)	0.027 (0.040)	0.072 (0.120)
MA	0.108 (0.239)	0.110 (0.224)	0.135 (0.116)	0.348 (0.240)	0.093 (0.134)	0.119 (0.197)
MHP	0.122 (0.262)	0.122 (0.249)	0.107 (0.100)	0.200 (0.204)	0.025 (0.035)	0.081 (0.131)
NCB	0.146 (0.318)	0.140 (0.304)	0.139 (0.206)	0.759 (0.472)	0.080 (0.092)	0.062 (0.363)
RAD	0.103 (0.227)	0.107 (0.245)	0.083 (0.138)	0.500 (0.332)	0.044 (0.049)	0.029 (0.093)
SBC	0.093 (0.182)	0.094 (0.183)	0.148 (0.109)	0.310 (0.213)	0.151 (0.169)	0.110 (0.245)
UNP	0.111 (0.201)	0.109 (0.191)	0.134 (0.096)	0.198 (0.155)	0.032 (0.039)	0.076 (0.126)
UPJ	0.155 (0.280)	0.153 (0.283)	0.251 (0.101)	0.184 (0.192)	0.060 (0.101)	0.192 (0.265)
USW	0.077 (0.154)	0.074 (0.149)	0.107 (0.095)	0.272 (0.165)	0.098 (0.104)	0.091 (0.752)
VO	0.073 (0.152)	0.073 (0.158)	0.070 (0.087)	0.167 (0.146)	0.034 (0.039)	0.037 (0.067)

TABLE 4: ROBUSTNESS OF THE ESTIMATION METHOD, GNN.

This table presents results of various estimation methods of our model for the stock Great Northern Nekoosa (GNN). OLS is ordinary least squares; NLLS is non-linear least squares; Poisson is Poisson maximum likelihood estimation; and Neg Bin is negative binomial maximum likelihood estimation. The estimating equation is:

$$NTRADE_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \varepsilon_t$$

See Tables 2 and 3 for variable descriptions. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. $\mathcal{L}$ is the log-likelihood. $\bar{R}^2$ is the adjusted coefficient of variation for each specification. * indicates that the coefficient is significant at the 5% level, ** at the 10% level.

Parameter	OLS	NLLS	Poisson	Neg Bin
β_0	1.395*	1.097*	1.030*	1.002*
	(0.287)	(0.056)	(0.058)	(0.058)
β_1	0.506*	0.078	0.091**	0.102*
	(0.179)	(0.050)	(0.047)	(0.044)
β_2	0.439*	-0.003	0.042	0.036
	(0.177)	(0.066)	(0.052)	(0.051)
β_3	4.276*	0.942*	1.030*	0.983*
	(0.413)	(0.123)	(0.093)	(0.091)
β_4	4.396*	0.786*	0.899*	0.887*
	(0.429)	(0.101)	(0.089)	(0.089)
β_5	1.010*	0.441*	0.434*	0.402*
	(0.278)	(0.098)	(0.085)	(0.081)
β_6	7.716*	0.352*	0.505*	0.543*
	(1.215)	(0.188)	(0.183)	(0.182)
β_7	-1.262*	-0.494*	-0.463*	-0.450*
	(0.130)	(0.056)	(0.050)	(0.046)
β_8	2.848*	0.851*	0.903*	0.889*
	(0.645)	(0.223)	(0.221)	(0.196)
β_9	1.183*	0.293*	0.280*	0.259*
	(0.292)	(0.054)	(0.053)	(0.054)
δ_1	0.712*	0.195*	0.177*	0.190*
	(0.238)	(0.063)	(0.057)	(0.056)
δ_2	-0.026	-0.058	-0.025	0.005
	(0.175)	(0.055)	(0.048)	(0.047)
δ_3	-0.135	-0.041	-0.041	-0.020
	(0.176)	(0.053)	(0.049)	(0.049)
δ_4	-0.347*	-0.126*	-0.119*	-0.090**
	(0.171)	(0.055)	(0.051)	(0.049)
δ_5	-0.432*	-0.154*	-0.146*	-0.102*
	(0.163)	(0.052)	(0.048)	(0.047)
δ_6	-0.568*	-0.191*	-0.189*	-0.163*
	(0.167)	(0.056)	(0.051)	(0.050)
δ_7	-1.022*	-0.377*	-0.374*	-0.332*
	(0.159)	(0.057)	(0.052)	(0.050)
δ_8	-0.933*	-0.300*	-0.336*	-0.305*
	(0.161)	(0.063)	(0.053)	(0.051)
δ_9	-0.906*	-0.320*	-0.323*	-0.291*
	(0.163)	(0.058)	(0.053)	(0.051)
δ_{10}	-0.538*	-0.166*	-0.168*	-0.136*
	(0.164)	(0.052)	(0.053)	(0.048)
δ_{11}	-0.361*	-0.096**	-0.103*	-0.081
	(0.172)	(0.053)	(0.049)	(0.049)
δ_{12}	-0.125	-0.011	-0.025	0.008
	(0.172)	(0.049)	(0.050)	(0.046)
γ	-	-	-	-0.422*
	-	-	-	(0.051)
$\mathcal{L}$	-14311.2	-14358.0	0.543	0.621
$\bar{R}^2$	0.170	0.161	-	-

TABLE 5: ROBUSTNESS OF THE MODEL SPECIFICATION, GNN.

This table presents results of various specifications of our model using negative binomial maximum likelihood estimation for the stock Great Northern Nekoosa (GNN). The estimating equation is:

$$NTRADE_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \varepsilon_t$$

See Tables 2 and 3 for variable descriptions. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. ℓ is the log-likelihood. Each specification is compared to the full model, denoted column 5, with a likelihood ratio test reported on the lines marked χ^2 and p-value. * indicates that the coefficient is significant at the 5% level; ** at the 10% level.

Parameter	0	1	2	3	4	5
β_0	1.232*	1.017*	1.004*	1.137*	0.881*	1.002*
	(0.040)	(0.048)	(0.060)	(0.045)	(0.056)	(0.058)
β_1	-	-	-	0.147*	0.126*	0.102*
	-	-	-	(0.044)	(0.044)	(0.044)
β_2	-	-	-	0.109*	0.063	0.036
	-	-	-	(0.049)	(0.052)	(0.051)
β_3	-	-	1.013*	1.117*	1.008*	0.983*
	-	-	(0.090)	(0.087)	(0.091)	(0.091)
β_4	-	-	0.908*	0.935*	0.921*	0.887*
	-	-	(0.094)	(0.090)	(0.091)	(0.089)
β_5	-	0.514*	0.441*	-	0.590*	0.402*
	-	(0.078)	(0.077)	-	(0.077)	(0.081)
β_6	-	1.393*	0.585*	-	0.544*	0.543*
	-	(0.120)	(0.191)	-	(0.190)	(0.182)
β_7	-	-0.497*	-0.454*	-0.488*	-	-0.450*
	-	(0.047)	(0.046)	(0.046)	-	(0.046)
β_8	-	0.771*	0.856*	0.787*	-	0.889*
	-	(0.198)	(0.194)	(0.195)	-	(0.196)
β_9	-	0.311*	0.279*	0.311*	-	0.259*
	-	(0.046)	(0.050)	(0.049)	-	(0.054)
δ_1	0.250*	0.152*	0.190*	0.208*	0.226*	0.190*
	(0.061)	(0.058)	(0.057)	(0.058)	(0.056)	(0.056)
δ_2	0.099*	-0.018	0.003	0.036	0.028	0.005
	(0.050)	(0.048)	(0.048)	(0.048)	(0.048)	(0.047)
δ_3	-0.010	-0.078	-0.022	0.000	-0.009	-0.020
	(0.052)	(0.050)	(0.049)	(0.049)	(0.049)	(0.049)
δ_4	-0.109*	-0.151*	-0.091**	-0.074	-0.080**	-0.090**
	(0.051)	(0.049)	(0.049)	(0.049)	(0.049)	(0.049)
δ_5	-0.112*	-0.168*	-0.105*	-0.079**	-0.090**	-0.102*
	(0.049)	(0.047)	(0.047)	(0.048)	(0.048)	(0.047)
δ_6	-0.197*	-0.224*	-0.163*	-0.146*	-0.160*	-0.163*
	(0.052)	(0.050)	(0.050)	(0.050)	(0.050)	(0.050)
δ_7	-0.367*	-0.398*	-0.335*	-0.319*	-0.335*	-0.332*
	(0.052)	(0.051)	(0.051)	(0.051)	(0.051)	(0.050)
δ_8	-0.356*	-0.353*	-0.308*	-0.290*	-0.313*	-0.305*
	(0.054)	(0.052)	(0.051)	(0.052)	(0.052)	(0.051)
δ_9	-0.328*	-0.331*	-0.293*	-0.280*	-0.291*	-0.291*
	(0.053)	(0.051)	(0.051)	(0.051)	(0.051)	(0.051)
δ_{10}	-0.163*	-0.161*	-0.135*	-0.124*	-0.141*	-0.136*
	(0.051)	(0.048)	(0.048)	(0.049)	(0.048)	(0.048)
δ_{11}	-0.092**	-0.097*	-0.082	-0.071	-0.086**	-0.081**
	(0.052)	(0.049)	(0.049)	(0.050)	(0.049)	(0.049)
δ_{12}	0.032	0.001	0.007	0.018	0.002	0.008
	(0.051)	(0.048)	(0.046)	(0.047)	(0.046)	(0.046)
γ	-0.150*	-0.311*	-0.418*	-0.405*	-0.365*	-0.422*
	(0.045)	(0.048)	(0.051)	(0.050)	(0.050)	(0.051)
ℓ	0.570	0.601	0.620	0.618	0.610	0.621
χ^2	316.862	120.559	2.525	17.674	68.801	-
p-value	(0.000)	(0.000)	(0.283)	(0.000)	(0.000)	-

TABLE 6A: MODEL ESTIMATES--NUMBER OF TRADES, ALL STOCKS.

This table presents the coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NTRADE_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \epsilon_t$$

Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. $\mathcal{L}$ is the log-likelihood. * indicates that the coefficient is significant at the 5% level; ** at the 10% level.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	1.981*	1.595*	0.686*	2.325*	0.421*	1.002*	1.287*	1.368*	1.924*
	(0.043)	(0.067)	(0.086)	(0.039)	(0.086)	(0.058)	(0.061)	(0.039)	(0.122)
β_1	0.086*	0.110*	0.281*	0.068*	0.055	0.102*	0.309*	0.011	0.370*
	(0.042)	(0.030)	(0.060)	(0.031)	(0.048)	(0.044)	(0.047)	(0.037)	(0.048)
β_2	0.066	0.093*	0.113*	0.096*	-0.056	0.036	0.271*	0.030	0.276*
	(0.045)	(0.025)	(0.057)	(0.032)	(0.058)	(0.051)	(0.060)	(0.038)	(0.057)
β_3	0.622*	0.659*	1.256*	0.662*	1.029*	0.983*	0.632*	0.769*	0.530*
	(0.066)	(0.084)	(0.109)	(0.055)	(0.133)	(0.091)	(0.091)	(0.073)	(0.091)
β_4	0.614*	0.576*	1.009*	0.469*	0.830*	0.887*	0.525*	0.726*	0.478*
	(0.057)	(0.099)	(0.115)	(0.056)	(0.104)	(0.089)	(0.092)	(0.066)	(0.099)
β_5	0.039	0.266*	0.400*	0.244*	0.632*	0.402*	0.541*	0.274*	-0.286*
	(0.095)	(0.038)	(0.111)	(0.074)	(0.084)	(0.081)	(0.103)	(0.063)	(0.113)
β_6	0.267	1.047*	0.003	0.274**	0.096	0.543*	0.643*	0.868*	-1.060**
	(0.180)	(0.265)	(0.338)	(0.155)	(0.296)	(0.181)	(0.239)	(0.114)	(0.623)
β_7	-0.816*	-0.527*	-0.273*	-0.234*	-0.280*	-0.450*	-0.571*	-0.277*	-1.609*
	(0.064)	(0.047)	(0.046)	(0.045)	(0.030)	(0.046)	(0.064)	(0.040)	(0.067)
β_8	0.554*	0.300*	-0.264	0.074	0.316	0.889*	0.080	0.582*	0.037
	(0.089)	(0.127)	(0.213)	(0.072)	(0.258)	(0.196)	(0.154)	(0.092)	(0.299)
β_9	0.384*	0.302*	0.075	0.225*	0.123	0.259*	0.156*	0.013**	1.059*
	(0.027)	(0.042)	(0.112)	(0.030)	(0.245)	(0.054)	(0.057)	(0.007)	(0.099)
δ_1	0.126*	0.087**	0.146*	0.057**	-0.042	0.190*	0.208*	0.124*	0.137*
	(0.038)	(0.051)	(0.067)	(0.031)	(0.079)	(0.056)	(0.052)	(0.044)	(0.054)
δ_2	-0.012	-0.085**	0.037	-0.077*	-0.084	0.005	-0.015	0.044	-0.046
	(0.032)	(0.044)	(0.057)	(0.027)	(0.073)	(0.047)	(0.043)	(0.036)	(0.046)
δ_3	-0.088*	-0.089*	-0.035	-0.076*	-0.015	-0.020	-0.020	0.006	-0.097*
	(0.031)	(0.041)	(0.058)	(0.027)	(0.074)	(0.049)	(0.043)	(0.036)	(0.047)
δ_4	-0.090*	-0.096*	-0.073	-0.091*	-0.129**	-0.089**	-0.083**	0.009	-0.100*
	(0.032)	(0.042)	(0.057)	(0.027)	(0.075)	(0.049)	(0.045)	(0.036)	(0.046)
δ_5	-0.146*	-0.088*	-0.087	-0.170*	-0.123**	-0.102*	-0.117*	-0.060	-0.089**
	(0.033)	(0.041)	(0.057)	(0.026)	(0.075)	(0.047)	(0.046)	(0.037)	(0.047)
δ_6	-0.206*	-0.209*	-0.119*	-0.237*	-0.112	-0.163*	-0.175*	-0.189*	-0.209*
	(0.034)	(0.041)	(0.057)	(0.027)	(0.073)	(0.050)	(0.045)	(0.037)	(0.047)
δ_7	-0.330*	-0.361*	-0.222*	-0.384*	-0.198*	-0.332*	-0.226*	-0.338*	-0.295*
	(0.035)	(0.043)	(0.059)	(0.028)	(0.076)	(0.050)	(0.045)	(0.039)	(0.051)
δ_8	-0.344*	-0.387*	-0.219*	-0.446*	-0.135**	-0.305*	-0.303*	-0.397*	-0.303*
	(0.035)	(0.043)	(0.059)	(0.030)	(0.075)	(0.051)	(0.045)	(0.042)	(0.051)
δ_9	-0.345*	-0.321*	-0.244*	-0.389*	-0.208*	-0.291*	-0.331*	-0.323*	-0.261*
	(0.035)	(0.041)	(0.060)	(0.028)	(0.077)	(0.051)	(0.048)	(0.039)	(0.050)
δ_{10}	-0.210*	-0.196*	-0.160*	-0.252*	-0.062	-0.136*	-0.233*	-0.151*	-0.120*
	(0.034)	(0.042)	(0.060)	(0.027)	(0.076)	(0.048)	(0.046)	(0.038)	(0.051)
δ_{11}	-0.171*	-0.149*	-0.113**	-0.177*	-0.004	-0.081**	-0.204*	-0.110*	-0.138*
	(0.033)	(0.041)	(0.059)	(0.026)	(0.080)	(0.049)	(0.047)	(0.037)	(0.049)
δ_{12}	-0.077*	-0.048	0.058	-0.080*	0.026	0.008	0.013	-0.038	0.015
	(0.033)	(0.041)	(0.058)	(0.027)	(0.079)	(0.046)	(0.044)	(0.037)	(0.045)
γ	0.051	0.512*	-0.562*	0.068**	-1.371*	-0.422*	0.100*	-0.433*	0.339*
	(0.039)	(0.037)	(0.058)	(0.038)	(0.121)	(0.051)	(0.041)	(0.049)	(0.040)
$\mathcal{L}$	7.496	5.375	-0.403	14.424	-0.877	0.621	2.070	2.795	3.233

TABLE 6A (continued).

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	1.435• (0.047)	1.725• (0.053)	1.011• (0.076)	0.509• (0.088)	1.621• (0.040)	1.431• (0.048)	2.438• (0.036)	1.513• (0.052)	0.785• (0.065)
β_1	0.242• (0.035)	0.257• (0.051)	-0.024 (0.048)	-0.071 (0.060)	0.089• (0.042)	0.312• (0.051)	0.072• (0.036)	0.083• (0.057)	0.082 (0.091)
β_2	0.003 (0.041)	0.213• (0.043)	-0.101• (0.044)	0.058 (0.072)	-0.008 (0.042)	0.229• (0.048)	0.031 (0.026)	0.015 (0.066)	0.237• (0.081)
β_3	0.722• (0.077)	0.631• (0.106)	0.849• (0.121)	1.031• (0.121)	0.723• (0.075)	0.719• (0.083)	0.512• (0.066)	0.715• (0.078)	1.008• (0.121)
β_4	0.695• (0.073)	0.631• (0.079)	0.916• (0.083)	0.921• (0.116)	0.607• (0.062)	0.590• (0.073)	0.414• (0.052)	0.725• (0.077)	0.925• (0.116)
β_5	0.442• (0.079)	-0.197 (0.139)	0.366• (0.057)	0.526• (0.093)	0.346• (0.065)	0.152• (0.089)	0.264• (0.074)	0.238• (0.091)	0.490• (0.146)
β_6	-0.119 (0.180)	-0.134 (0.190)	0.038 (0.273)	0.298 (0.361)	0.373• (0.141)	0.455• (0.163)	0.390• (0.122)	0.113 (0.192)	0.236 (0.228)
β_7	-0.306• (0.040)	-1.438• (0.072)	-0.506• (0.027)	-0.409• (0.041)	-0.234• (0.039)	-0.626• (0.072)	-0.542• (0.046)	-0.392• (0.054)	-0.758• (0.117)
β_8	0.001 (0.067)	0.461 (0.348)	0.387• (0.101)	1.068• (0.236)	-0.017 (0.047)	0.947• (0.245)	0.904• (0.068)	0.173• (0.083)	0.791• (0.432)
β_9	0.308• (0.041)	1.302• (0.089)	0.093• (0.014)	0.671• (0.115)	-0.005 (0.021)	0.424• (0.097)	0.440• (0.040)	-0.002 (0.004)	1.196• (0.182)
δ_1	0.132• (0.054)	0.162• (0.055)	0.032 (0.075)	0.234• (0.071)	-0.049 (0.041)	0.155• (0.049)	0.088• (0.039)	0.065 (0.045)	0.277• (0.068)
δ_2	0.093• (0.040)	-0.064 (0.048)	0.061 (0.060)	0.114• (0.061)	-0.075• (0.033)	0.098• (0.038)	-0.008 (0.032)	-0.031 (0.038)	0.125• (0.054)
δ_3	-0.013 (0.043)	-0.050 (0.046)	0.032 (0.060)	0.080•• (0.062)	-0.131• (0.034)	0.008 (0.040)	-0.065• (0.033)	-0.050 (0.037)	-0.020 (0.057)
δ_4	0.001 (0.042)	-0.185• (0.049)	0.013 (0.060)	-0.004 (0.065)	-0.122• (0.033)	0.002 (0.040)	-0.088• (0.033)	-0.098• (0.038)	-0.084 (0.059)
δ_5	-0.052 (0.042)	-0.140• (0.050)	0.012 (0.061)	-0.046 (0.062)	-0.106• (0.033)	-0.033 (0.041)	-0.089• (0.032)	-0.109• (0.038)	-0.089 (0.056)
δ_6	-0.070•• (0.041)	-0.245• (0.053)	-0.103•• (0.060)	-0.066 (0.064)	-0.164• (0.034)	-0.117• (0.042)	-0.219• (0.033)	-0.194• (0.039)	-0.155• (0.058)
δ_7	-0.270• (0.043)	-0.363• (0.053)	-0.244• (0.060)	-0.186• (0.064)	-0.289• (0.035)	-0.218• (0.042)	-0.382• (0.033)	-0.282• (0.041)	-0.252• (0.059)
δ_8	-0.240• (0.043)	-0.365• (0.051)	-0.314• (0.059)	-0.257• (0.064)	-0.415• (0.037)	-0.307• (0.043)	-0.343• (0.034)	-0.340• (0.042)	-0.239• (0.060)
δ_9	-0.187• (0.043)	-0.347• (0.051)	-0.220• (0.060)	-0.186• (0.062)	-0.404• (0.036)	-0.327• (0.043)	-0.322• (0.034)	-0.344• (0.041)	-0.205• (0.060)
δ_{10}	-0.079•• (0.042)	-0.280• (0.052)	-0.149• (0.061)	-0.042 (0.063)	-0.244• (0.035)	-0.161• (0.042)	-0.215• (0.034)	-0.185• (0.039)	-0.132• (0.059)
δ_{11}	-0.078•• (0.042)	-0.164• (0.048)	-0.096 (0.061)	-0.019 (0.063)	-0.173• (0.035)	-0.059 (0.041)	-0.159• (0.034)	-0.110• (0.039)	-0.084 (0.057)
δ_{12}	0.078• (0.039)	-0.068 (0.049)	0.058 (0.059)	0.082 (0.064)	-0.030 (0.034)	0.034 (0.040)	-0.089• (0.032)	0.013 (0.038)	0.044 (0.058)
γ	-0.102• (0.044)	0.743• (0.037)	-0.295• (0.057)	-0.553• (0.060)	-0.816• (0.063)	-0.047 (0.042)	0.639• (0.040)	-0.479• (0.053)	0.041 (0.045)
l	2.460	4.086	-0.177	-0.558	3.173	3.095	22.006	2.097	0.150

TABLE 6B: MODEL HYPOTHESIS TESTS-- NUMBER OF TRADES, ALL STOCKS.

Based on the negative binomial regressions reported in Table 6A, this table presents likelihood ratio test statistics and p-values on the robustness of the model over different specifications. Our first test examines the hypothesis that H_0: $\beta_1 = \ldots = \beta_9 = 0$. The second test statistic studies the hypothesis that security-specific returns (both $PTRET_{t-1}$ and $NTRET_{t-1}$) have no effect on trading intensity ($NTRADE_t$), i.e., H_1: $\beta_1 = \beta_2 = 0$. The proposition that neither security-specific nor market-wide returns ($PMRET_{t-1}$ and $NMRET_{t-1}$) affect trading intensity is explored in H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Next, we tested whether security-specific volatility ($TVOL_{t-1}$) and market-wide volatility ($MVOL_{t-1}$) affect trading intensity, H_3: $\beta_5 = \beta_6 = 0$. Finally, we examine the hypothesis of whether the market liquidity variables ($SPREAD_{t-1}$, $DEPTH_{t-1}$, and $VOLUME_{t-1}$) affect trading intensity, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis. See Table 1 for a description of the stock ticker symbols.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	396.515	385.071	236.617	329.428	189.312	316.862	331.748	212.486	888.661
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	4.427	18.969	14.474	11.381	2.986	2.525	36.018	1.897	58.107
p-value	(0.109)	(0.000)	(0.001)	(0.003)	(0.225)	(0.283)	(0.000)	(0.387)	(0.000)
H_2	82.212	75.876	137.817	92.316	54.942	120.559	107.423	99.919	97.266
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	0.632	32.247	8.181	12.646	31.652	17.674	16.429	27.826	23.369
p-value	(0.729)	(0.000)	(0.017)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_4	266.873	154.281	23.913	110.653	48.970	68.801	73.300	36.679	564.019
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0	323.652	843.550	335.474	240.486	190.955	280.342	539.557	138.364	285.526
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	34.069	34.727	3.101	3.140	3.161	37.253	10.741	1.264	4.412
p-value	(0.000)	(0.000)	(0.212)	(0.208)	(0.206)	(0.000)	(0.005)	(0.532)	(0.110)
H_2	94.004	81.451	62.010	78.487	94.213	112.389	27.799	78.343	109.672
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	13.880	4.420	18.603	16.325	10.749	0.631	5.686	6.318	5.673
p-value	(0.001)	(0.110)	(0.000)	(0.000)	(0.005)	(0.729)	(0.058)	(0.042)	(0.059)
H_4	97.790	573.943	204.633	94.185	25.924	90.290	418.252	30.958	66.812
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 7: MODEL ESTIMATES AND HYPOTHESIS TESTS– NUMBER OF BUYER-INITIATED TRADES, ALL STOCKS.

This table presents in Panel A abbreviated coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NBUY_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \epsilon_t$$

Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. Estimates of the coefficients δ_1 to δ_{12} are not presented. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. l is the log-likelihood. * indicates that the coefficient is significant at the 5% level; ** at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = ... = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	1.228*	0.784*	-0.315*	1.386*	-0.344*	0.069	-0.025	0.168**	1.192*
	(0.102)	(0.091)	(0.102)	(0.080)	(0.142)	(0.093)	(0.108)	(0.093)	(0.204)
β_1	0.205*	0.147*	0.222*	-0.024	0.267*	0.103	0.318*	-0.054	0.333*
	(0.072)	(0.039)	(0.092)	(0.049)	(0.066)	(0.078)	(0.080)	(0.072)	(0.065)
β_2	0.066	0.103*	0.312*	0.100**	-0.372*	0.141**	0.342*	-0.117	0.217*
	(0.081)	(0.041)	(0.073)	(0.051)	(0.123)	(0.074)	(0.072)	(0.078)	(0.079)
β_3	1.197*	0.985*	2.103*	0.985*	1.555*	1.798*	1.225*	1.443*	1.104*
	(0.118)	(0.132)	(0.179)	(0.102)	(0.210)	(0.143)	(0.150)	(0.139)	(0.139)
β_4	-0.267*	-0.134	-0.083	0.110	-0.593*	-0.140	0.002	-0.053	-0.128
	(0.123)	(0.116)	(0.182)	(0.080)	(0.245)	(0.143)	(0.137)	(0.172)	(0.158)
β_5	0.282	0.434*	0.610*	0.694*	0.788*	0.444*	1.280*	0.824*	-0.399*
	(0.176)	(0.057)	(0.170)	(0.130)	(0.124)	(0.129)	(0.174)	(0.139)	(0.174)
β_6	-0.627	0.750**	0.615**	0.301	-0.445	0.570	0.936*	1.077*	-1.478
	(0.478)	(0.391)	(0.349)	(0.367)	(0.510)	(0.353)	(0.454)	(0.340)	(1.025)
β_7	-1.502*	-0.760*	-0.321*	-0.691*	-0.213*	-0.357*	-0.651*	-0.475*	-1.312*
	(0.134)	(0.076)	(0.082)	(0.082)	(0.053)	(0.078)	(0.125)	(0.084)	(0.108)
β_8	0.562*	0.452*	-0.440	0.852*	0.166	0.108	-0.430	0.611*	-0.665
	(0.188)	(0.178)	(0.384)	(0.117)	(0.427)	(0.357)	(0.343)	(0.197)	(0.480)
β_9	0.296*	0.279*	-0.001	0.190*	0.377	0.101	0.167*	0.015**	0.917*
	(0.047)	(0.050)	(0.150)	(0.055)	(0.439)	(0.089)	(0.057)	(0.009)	(0.099)
γ	0.386*	0.555*	-0.088	0.238*	-0.347*	-0.047	0.418*	0.135	0.378*
	(0.039)	(0.037)	(0.055)	(0.038)	(0.081)	(0.050)	(0.041)	(0.043)	(0.042)
l	0.146	0.264	-0.810	2.093	-0.750	-0.762	-0.612	-0.700	-0.231

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	270.667	279.477	164.247	148.590	148.106	189.360	226.220	165.689	478.121
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	6.956	10.117	7.552	2.529	16.124	4.418	8.847	0.632	20.211
p-value	(0.031)	(0.006)	(0.023)	(0.282)	(0.000)	(0.110)	(0.012)	(0.729)	(0.000)
H_2	101.184	68.288	100.688	58.172	96.149	124.346	60.031	78.418	76.424
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	3.794	28.454	3.147	8.220	16.722	3.787	31.595	29.723	9.474
p-value	(0.150)	(0.000)	(0.207)	(0.016)	(0.000)	(0.151)	(0.000)	(0.000)	(0.009)
H_4	144.187	106.859	29.577	61.333	11.347	13.886	34.755	17.707	283.588
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.010)	(0.003)	(0.000)	(0.001)	(0.000)

TABLE 7 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	0.058	0.649*	0.067	-0.279*	0.401*	0.343*	1.682*	0.141	-0.198**
	(0.094)	(0.085)	(0.102)	(0.130)	(0.101)	(0.088)	(0.059)	(0.098)	(0.109)
β_1	0.264*	0.259*	0.090	0.091	0.272*	0.457*	0.148*	0.276*	0.178**
	(0.060)	(0.067)	(0.062)	(0.087)	(0.083)	(0.069)	(0.043)	(0.106)	(0.109)
β_2	0.008	0.172*	-0.102	0.205*	0.257*	0.478*	0.003	0.122	0.319*
	(0.075)	(0.061)	(0.068)	(0.096)	(0.080)	(0.078)	(0.036)	(0.112)	(0.109)
β_3	1.256*	1.211*	1.549*	1.780*	1.296*	1.436*	0.677*	1.647*	1.711*
	(0.144)	(0.143)	(0.186)	(0.173)	(0.131)	(0.134)	(0.095)	(0.146)	(0.172)
β_4	-0.090	-0.003	-0.433*	-0.344	-0.262**	-0.256**	0.025	-0.198	-0.122
	(0.155)	(0.132)	(0.170)	(0.219)	(0.145)	(0.139)	(0.083)	(0.149)	(0.152)
β_5	1.205*	0.038	0.656*	0.585*	1.134*	0.700*	0.299*	0.884*	0.791*
	(0.151)	(0.172)	(0.089)	(0.148)	(0.134)	(0.154)	(0.102)	(0.165)	(0.211)
β_6	0.383	0.114	-0.061	0.311	0.851	0.252	0.457*	0.182	0.600
	(0.310)	(0.342)	(0.355)	(0.526)	(0.397)	(0.346)	(0.219)	(0.377)	(0.459)
β_7	-0.484*	-1.388*	-0.499*	-0.471*	-0.824*	-0.392*	-1.130*	-0.499*	-0.597*
	(0.083)	(0.109)	(0.048)	(0.064)	(0.099)	(0.125)	(0.082)	(0.115)	(0.178)
β_8	-0.054	0.496	0.602*	0.660**	0.284*	-0.567	1.360*	-0.008	0.499
	(0.140)	(0.539)	(0.164)	(0.388)	(0.091)	(0.522)	(0.106)	(0.169)	(0.678)
β_9	0.257*	1.406*	0.080*	0.256	-0.132	0.426*	0.464*	-0.027	1.209*
	(0.061)	(0.116)	(0.016)	(0.160)	(0.214)	(0.115)	(0.045)	(0.022)	(0.228)
γ	0.471*	0.594*	0.068	-0.288*	0.095*	0.263*	1.021*	0.268*	0.036
	(0.041)	(0.042)	(0.054)	(0.063)	(0.042)	(0.040)	(0.031)	(0.045)	(0.053)
l	-0.615	0.014	-0.770	-0.825	-0.610	-0.452	6.231	-0.710	-0.767

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0	206.304	521.536	215.795	170.161	242.803	244.983	376.314	162.373	192.872
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	8.833	17.048	2.480	2.512	7.588	33.464	24.625	2.527	2.521
p-value	(0.012)	(0.000)	(0.289)	(0.285)	(0.023)	(0.000)	(0.000)	(0.283)	(0.283)
H_2	66.875	78.925	79.993	90.418	93.580	138.277	70.717	100.456	94.545
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	38.485	0.631	22.944	9.419	38.570	10.102	15.154	15.163	7.564
p-value	(0.000)	(0.729)	(0.000)	(0.009)	(0.000)	(0.006)	(0.001)	(0.001)	(0.023)
H_4	27.129	299.915	78.133	40.186	48.687	29.676	281.604	14.531	27.733
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)

TABLE 8: MODEL ESTIMATES AND HYPOTHESIS TESTS-- NUMBER OF SELLER-INITIATED TRADES, ALL STOCKS.

This table presents in Panel A abbreviated coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NSELL_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \varepsilon_t$$

Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. Estimates of the coefficients δ_1 to δ_{12} are not presented. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. ℓ is the log-likelihood. • indicates that the coefficient is significant at the 5% level; •• at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = ... = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	0.985•	0.729•	-0.173	1.231•	-0.607•	0.013	0.521•	0.315•	0.305•
	(0.076)	(0.086)	(0.153)	(0.092)	(0.213)	(0.107)	(0.140)	(0.094)	(0.143)
β_1	-0.310•	0.058	0.379•	0.144•	-0.340•	-0.080	0.142•	-0.040	0.243•
	(0.075)	(0.039)	(0.097)	(0.050)	(0.096)	(0.087)	(0.073)	(0.065)	(0.068)
β_2	0.117••	0.074•	0.161	0.066	0.177•	-0.129	0.205•	0.096	0.090
	(0.071)	(0.032)	(0.109)	(0.056)	(0.066)	(0.082)	(0.091)	(0.068)	(0.095)
β_3	-0.200••	-0.115	-0.158	-0.154•	-1.088•	-0.269	0.018	-0.076	-0.286••
	(0.115)	(0.118)	(0.192)	(0.106)	(0.276)	(0.171)	(0.149)	(0.145)	(0.157)
β_4	0.940•	1.011•	1.509•	0.898•	1.216•	1.435•	0.941•	1.130•	1.110•
	(0.108)	(0.102)	(0.164)	(0.106)	(0.218)	(0.129)	(0.144)	(0.118)	(0.141)
β_5	0.617•	0.205•	0.361••	0.461•	0.849•	0.771•	0.667•	0.674•	0.485•
	(0.167)	(0.061)	(0.187)	(0.144)	(0.134)	(0.131)	(0.162)	(0.106)	(0.180)
β_6	0.650•	0.720•	-0.048	0.231	0.151	1.036•	0.121	0.935•	0.033
	(0.289)	(0.334)	(0.652)	(0.389)	(0.936)	(0.357)	(0.629)	(0.353)	(0.661)
β_7	-1.033•	-0.753•	-0.560•	-0.822•	-0.170•	-0.549•	-1.016•	-0.541•	-1.127•
	(0.113)	(0.070)	(0.085)	(0.094)	(0.055)	(0.084)	(0.116)	(0.077)	(0.114)
β_8	0.576•	1.148•	-0.592••	0.615•	-0.162	0.486	0.458••	1.001•	0.044
	(0.171)	(0.139)	(0.324)	(0.147)	(0.445)	(0.376)	(0.253)	(0.165)	(0.500)
β_9	0.238•	0.280•	-0.132	0.088••	-0.626	-0.025	0.040	-0.030•	0.889•
	(0.045)	(0.050)	(0.163)	(0.046)	(0.427)	(0.145)	(0.041)	(0.015)	(0.135)
γ	0.423•	0.264•	-0.299•	0.021	-0.653•	-0.077	0.199•	-0.010	-0.038
	(0.036)	(0.044)	(0.061)	(0.043)	(0.092)	(0.051)	(0.043)	(0.044)	(0.051)
ℓ	0.613	0.005	-0.810	0.831	-0.739	-0.782	-0.622	-0.647	-0.724

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	239.680	259.243	164.247	196.645	150.494	215.239	189.570	195.412	284.852
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	8.221	1.897	7.552	3.162	17.916	0.631	6.951	1.265	10.106
p-value	(0.016)	(0.387)	(0.023)	(0.206)	(0.000)	(0.729)	(0.031)	(0.531)	(0.006)
H_2	129.642	96.110	100.688	99.271	103.316	117.403	74.564	102.449	89.687
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	7.589	8.852	3.147	5.691	17.916	16.411	10.110	22.766	3.158
p-value	(0.022)	(0.012)	(0.207)	(0.058)	(0.000)	(0.000)	(0.006)	(0.000)	(0.206)
H_4	83.477	101.168	29.577	52.481	10.152	23.986	53.712	32.252	106.740
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.017)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 8 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	0.234*	0.785*	0.185	-0.355*	0.860*	0.753*	1.286*	1.098*	-0.072
	(0.094)	(0.111)	(0.141)	(0.163)	(0.002)	(0.084)	(0.064)	(0.144)	(0.126)
β_1	0.284*	0.264*	0.435*	-0.286*	-0.013	0.271*	0.035	-0.063	0.101
	(0.077)	(0.050)	(0.151)	(0.095)	(0.077)	(0.070)	(0.040)	(0.087)	(0.159)
β_2	-0.055	0.207*	-0.058	0.117	-0.115	0.193*	0.018	0.028	0.317*
	(0.076)	(0.056)	(0.075)	(0.097)	(0.077)	(0.064)	(0.040)	(0.101)	(0.129)
β_3	-0.170	-0.278**	-0.095	-0.383**	-0.328*	0.017	-0.271*	-0.377*	-0.311
	(0.165)	(0.149)	(0.059)	(0.202)	(0.142)	(0.127)	(0.114)	(0.131)	(0.207)
β_4	1.124*	0.985*	-0.417*	1.240*	1.216*	1.025*	0.777*	0.990*	1.443*
	(0.144)	(0.130)	(0.190)	(0.167)	(0.103)	(0.104)	(0.085)	(0.118)	(0.163)
β_5	0.709*	-0.196	1.500*	0.434*	0.871*	0.095	0.339*	0.101	0.481*
	(0.150)	(0.177)	(0.124)	(0.156)	(0.125)	(0.127)	(0.101)	(0.138)	(0.232)
β_6	0.384	0.028	0.219*	0.552	-0.129	0.250	0.769*	-0.299	0.218
	(0.372)	(0.489)	(0.083)	(0.665)	(0.272)	(0.327)	(0.268)	(0.684)	(0.518)
β_7	-0.998*	-1.357*	-0.651	-0.500*	-0.748*	-0.471*	-0.926*	-0.731*	-1.196*
	(0.086)	(0.121)	(0.693)	(0.067)	(0.082)	(0.098)	(0.079)	(0.087)	(0.196)
β_8	1.064*	-0.581	-0.588*	0.545	0.148**	-0.310	1.212*	-0.014	0.540
	(0.097)	(0.682)	(0.043)	(0.421)	(0.082)	(0.372)	(0.105)	(0.144)	(0.692)
β_9	-0.145**	1.149*	0.178	0.331	-0.436*	0.314*	0.381*	0.001	0.749*
	(0.084)	(0.095)	(0.178)	(0.220)	(0.122)	(0.101)	(0.044)	(0.007)	(0.268)
γ	0.010	0.360*	0.029	-0.334*	0.096*	-0.108*	0.498*	-0.097*	0.046
	(0.045)	(0.042)	(0.076)	(0.068)	(0.039)	(0.046)	(0.035)	(0.045)	(0.056)
l	-0.778	-0.303	-0.837	-0.810	-0.422	-0.286	2.324	-0.432	-0.745

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0	213.244	429.352	248.040	155.091	202.336	175.529	298.841	151.000	240.775
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	11.987	18.942	7.441	2.512	3.162	11.997	1.895	0.632	9.454
p-value	(0.002)	(0.000)	(0.024)	(0.285)	(0.206)	(0.002)	(0.388)	(0.729)	(0.009)
H_2	65.614	92.816	106.037	91.673	96.110	109.864	90.347	97.297	148.751
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	14.511	1.894	6.821	5.023	26.557	1.263	8.213	2.527	2.521
p-value	(0.001)	(0.388)	(0.033)	(0.081)	(0.000)	(0.532)	(0.016)	(0.283)	(0.283)
H_4	99.051	236.775	120.920	35.162	65.127	25.887	168.691	48.649	39.079
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 9: MODEL ESTIMATES AND HYPOTHESIS TESTS– NUMBER OF MARKET-LIMIT TRADES, ALL STOCKS.

This table presents in Panel A abbreviated coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NMLT_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \epsilon_t$$

Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. Estimates of the coefficients δ_1 to δ_{12} are not presented. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. ℓ is the log-likelihood. * indicates that the coefficient is significant at the 5% level; ** at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = ... = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	1.766*	1.341*	0.348*	2.003*	0.437*	0.618*	0.862*	0.805*	1.406*
	(0.051)	(0.062)	(0.083)	(0.046)	(0.112)	(0.041)	(0.087)	(0.053)	(0.111)
β_1	-0.013	0.081*	0.124	0.035	0.035	0.018	0.132*	-0.144*	0.171*
	(0.052)	(0.034)	(0.086)	(0.035)	(0.059)	(0.060)	(0.063)	(0.055)	(0.054)
β_2	0.049	1.414*	-0.006	0.027	-0.037	0.005	0.185*	-0.128*	0.066
	(0.054)	(0.061)	(0.078)	(0.038)	(0.059)	(0.066)	(0.074)	(0.055)	(0.063)
β_3	0.624*	0.079*	1.223*	0.572*	0.775*	1.056*	0.518*	0.735*	0.596*
	(0.084)	(0.033)	(0.156)	(0.073)	(0.159)	(0.128)	(0.124)	(0.112)	(0.110)
β_4	0.659*	0.055**	0.864*	0.503*	0.656*	1.008*	0.544*	0.703*	0.505*
	(0.068)	(0.029)	(0.181)	(0.066)	(0.147)	(0.107)	(0.118)	(0.087)	(0.102)
β_5	0.433*	0.587*	0.101	0.458*	0.490*	0.228*	0.470*	0.445*	-0.264**
	(0.124)	(0.095)	(0.160)	(0.096)	(0.099)	(0.111)	(0.137)	(0.102)	(0.139)
β_6	-0.114	0.627*	1.000*	0.188	0.201	0.790*	0.710**	1.125*	-0.517
	(0.206)	(0.083)	(0.246)	(0.177)	(0.399)	(0.214)	(0.374)	(0.147)	(0.543)
β_7	-1.573*	0.282*	-1.302*	-0.730*	-0.762*	-1.218*	-1.764*	-0.742*	-2.257*
	(0.088)	(0.045)	(0.077)	(0.060)	(0.043)	(0.072)	(0.103)	(0.063)	(0.092)
β_8	1.089*	0.728*	0.567*	0.782*	0.383	2.712*	1.579*	1.458*	0.709**
	(0.113)	(0.240)	(0.251)	(0.088)	(0.326)	(0.329)	(0.192)	(0.135)	(0.361)
β_9	0.282*	-0.865*	-0.591*	0.148*	-0.705*	0.057	0.105*	-0.015	0.862*
	(0.032)	(0.058)	(0.229)	(0.040)	(0.351)	(0.104)	(0.038)	(0.010)	(0.095)
γ	0.394*	-0.119*	-0.161*	0.152*	-0.690*	-0.013	0.344*	0.103*	0.197*
	(0.035)	(0.048)	(0.052)	(0.036)	(0.085)	(0.039)	(0.038)	(0.039)	(0.040)
ℓ	3.729	2.913	-0.806	7.430	-0.858	-0.451	-0.036	0.073	0.379

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	356.674	344.604	253.608	215.614	252.018	338.323	307.735	193.514	675.812
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	0.632	8.852	3.146	0.632	0.597	0.631	5.055	2.530	8.842
p-value	(0.729)	(0.012)	(0.207)	(0.729)	(0.742)	(0.729)	(0.080)	(0.282)	(0.012)
H_2	57.548	49.952	50.973	48.055	23.888	77.006	32.859	47.430	34.106
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	1.265	28.453	3.776	6.323	10.750	9.468	10.110	24.664	6.948
p-value	(0.531)	(0.000)	(0.151)	(0.042)	(0.005)	(0.009)	(0.006)	(0.000)	(0.031)
H_4	278.256	185.264	172.428	115.079	175.577	186.835	201.576	93.595	497.701
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 9 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	0.956• (0.084)	1.086• (0.057)	-1.454• (0.171)	0.398• (0.095)	1.337• (0.062)	0.982• (0.056)	2.120• (0.044)	1.474• (0.080)	0.592• (0.071)
β_1	0.166• (0.058)	0.206• (0.059)	0.867• (0.085)	-0.116•• (0.068)	-0.004 (0.059)	0.207• (0.066)	0.074• (0.036)	-0.099 (0.079)	0.032 (0.109)
β_2	-0.124• (0.061)	0.077 (0.066)	-0.081 (0.059)	0.051 (0.078)	-0.054 (0.055)	0.185• (0.061)	-0.040 (0.029)	-0.024 (0.070)	0.137 (0.089)
β_3	0.692• (0.127)	0.747• (0.128)	-0.152• (0.053)	0.853• (0.161)	0.502• (0.103)	0.699• (0.110)	0.437• (0.084)	0.505• (0.107)	1.131• (0.138)
β_4	0.599• (0.120)	0.618• (0.100)	0.820• (0.138)	0.773• (0.132)	0.663• (0.085)	0.677• (0.093)	0.403• (0.061)	0.590• (0.088)	1.121• (0.125)
β_5	0.994• (0.119)	-0.473• (0.146)	0.901• (0.111)	0.569• (0.113)	0.921• (0.094)	0.025 (0.114)	0.472• (0.086)	0.317• (0.122)	0.344• (0.171)
β_6	-0.354 (0.326)	0.455• (0.178)	0.202• (0.070)	-0.054 (0.386)	0.286 (0.230)	0.697• (0.169)	0.461• (0.137)	-0.621•• (0.362)	0.191 (0.249)
β_7	-0.902• (0.066)	-1.981• (0.103)	0.346 (0.287)	-0.897• (0.054)	-0.972• (0.063)	-1.288• (0.101)	-1.062• (0.062)	-1.280• (0.077)	-1.409• (0.144)
β_8	0.825• (0.082)	1.900• (0.430)	-0.994• (0.038)	2.448• (0.268)	0.403• (0.060)	2.363• (0.315)	1.605• (0.091)	0.611• (0.108)	2.029• (0.497)
β_9	0.082 (0.059)	1.324• (0.098)	0.924• (0.110)	0.190 (0.135)	-0.378• (0.112)	0.275• (0.105)	0.383• (0.042)	-0.053 (0.053)	1.036• (0.204)
γ	0.451• (0.036)	0.541• (0.040)	0.106 (0.070)	-0.497• (0.061)	0.089• (0.038)	0.055 (0.040)	1.011• (0.031)	-0.053 (0.042)	0.046 (0.047)
l	0.024	0.551	-0.547	-0.817	0.953	0.286	13.982	0.285	-0.457

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0 p-value	254.253 (0.000)	616.246 (0.000)	547.548 (0.000)	270.625 (0.000)	271.257 (0.000)	232.355 (0.000)	440.365 (0.000)	200.912 (0.000)	279.223 (0.000)
H_1 p-value	11.356 (0.003)	11.997 (0.002)	4.341 (0.114)	3.140 (0.208)	0.632 (0.729)	9.471 (0.009)	6.318 (0.042)	1.264 (0.532)	1.891 (0.389)
H_2 p-value	33.438 (0.000)	46.724 (0.000)	50.228 (0.000)	35.162 (0.000)	35.409 (0.000)	69.454 (0.000)	25.272 (0.000)	25.272 (0.000)	98.957 (0.000)
H_3 p-value	32.176 (0.000)	6.945 (0.031)	3.101 (0.212)	13.814 (0.001)	41.100 (0.000)	3.788 (0.150)	8.213 (0.016)	4.423 (0.110)	3.782 (0.151)
H_4 p-value	128.973 (0.000)	470.393 (0.000)	450.813 (0.000)	180.835 (0.000)	146.061 (0.000)	110.495 (0.000)	374.657 (0.000)	163.004 (0.000)	90.763 (0.000)

TABLE 10: MODEL ESTIMATES AND HYPOTHESIS TESTS– NUMBER OF MARKET-MARKET TRADES, ALL STOCKS.

This table presents in Panel A abbreviated coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NMMT_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \varepsilon_t$$

Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. Estimates of the coefficients δ_1 to δ_{12} are not presented. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. ℓ is the log-likelihood. * indicates that the coefficient is significant at the 5% level; ** at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = ... = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	0.551*	-0.038	-0.288*	1.060*	-1.282*	-0.008	0.376*	0.654*	1.235*
	(0.063)	(0.047)	(0.119)	(0.062)	(0.131)	(0.085)	(0.068)	(0.056)	(0.173)
β_1	0.346*	0.599*	-0.130	0.235*	0.197*	0.204*	0.407*	0.226*	0.587*
	(0.075)	(0.035)	(0.130)	(0.062)	(0.072)	(0.067)	(0.076)	(0.053)	(0.061)
β_2	0.167**	0.362*	0.419*	0.308*	-0.018	0.063	0.489*	0.222*	0.521*
	(0.087)	(0.096)	(0.081)	(0.059)	(0.118)	(0.072)	(0.076)	(0.056)	(0.079)
β_3	0.555*	0.295*	0.224*	0.704*	0.489*	0.831*	0.354*	0.669*	0.312*
	(0.123)	(0.043)	(0.066)	(0.105)	(0.228)	(0.146)	(0.082)	(0.120)	(0.137)
β_4	0.414*	0.122*	1.231*	0.378*	0.535*	0.661*	0.774*	0.665*	0.373*
	(0.109)	(0.045)	(0.156)	(0.089)	(0.199)	(0.136)	(0.129)	(0.094)	(0.145)
β_5	-0.402*	0.571*	1.100*	0.046	0.679*	0.537*	0.587*	-0.035	-0.531*
	(0.186)	(0.148)	(0.134)	(0.155)	(0.129)	(0.122)	(0.120)	(0.095)	(0.163)
β_6	0.740*	0.574*	0.727*	0.314	0.361	-0.026	0.538*	0.356**	-2.273*
	(0.218)	(0.135)	(0.151)	(0.200)	(0.421)	(0.287)	(0.156)	(0.186)	(0.900)
β_7	0.818*	0.159*	-1.864*	0.898*	0.384*	0.518*	0.137	0.302*	-0.853*
	(0.095)	(0.076)	(0.583)	(0.093)	(0.046)	(0.063)	(0.297)	(0.058)	(0.094)
β_8	-1.008*	0.633	0.705*	-2.594*	-0.311	-2.529*	0.738*	-1.108*	-1.331*
	(0.204)	(0.408)	(0.058)	(0.236)	(0.423)	(0.397)	(0.075)	(0.201)	(0.495)
β_9	0.554*	0.528*	-1.877*	0.469*	1.002*	0.443*	-2.094*	0.027*	1.314*
	(0.048)	(0.068)	(0.347)	(0.050)	(0.334)	(0.079)	(0.285)	(0.010)	(0.127)
γ	0.442*	-0.434*	0.146**	0.598*	-1.129*	-0.330*	-0.040	-0.136*	0.222*
	(0.034)	(0.074)	(0.079)	(0.033)	(0.113)	(0.052)	(0.063)	(0.044)	(0.046)
ℓ	-0.209	-0.505	-0.867	0.979	-0.810	-0.839	-0.431	-0.306	-0.125

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	158.100	225.099	190.678	285.800	88.386	131.290	213.382	102.449	474.963
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	10.751	23.395	18.879	15.807	2.986	5.681	34.755	13.913	79.582
p-value	(0.005)	(0.000)	(0.000)	(0.000)	(0.225)	(0.058)	(0.000)	(0.001)	(0.000)
H_2	29.090	38.570	76.145	53.745	13.736	29.666	74.564	51.857	94.740
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.008)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	3.794	6.323	22.655	0.632	11.347	8.837	6.951	2.530	23.369
p-value	(0.150)	(0.042)	(0.000)	(0.729)	(0.003)	(0.012)	(0.031)	(0.282)	(0.000)
H_4	92.963	128.989	77.404	166.295	44.790	63.751	67.613	30.355	227.376
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 10 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	0.599 (0.635)	1.170• (0.098)	-0.125• (0.055)	-0.891• (0.137)	0.351• (0.057)	0.605• (0.081)	1.205• (0.062)	-0.120 (0.078)	-0.536• (0.112)
β_1	0.360• (0.049)	0.292• (0.056)	-0.437• (0.114)	0.122 (0.100)	0.351• (0.077)	0.458• (0.061)	0.093 (0.059)	0.419• (0.093)	0.303• (0.122)
β_2	0.179• (0.061)	0.294• (0.047)	0.058 (0.061)	0.144 (0.108)	0.157•• (0.086)	0.323• (0.067)	0.194• (0.053)	0.152 (0.114)	0.465• (0.123)
β_3	0.427• (0.111)	0.396• (0.132)	-0.040 (0.067)	1.348• (0.189)	1.037• (0.124)	0.547• (0.121)	0.541• (0.127)	0.832• (0.148)	0.614• (0.181)
β_4	0.530• (0.123)	0.603• (0.097)	0.705• (0.198)	1.123• (0.173)	0.575• (0.123)	0.257• (0.118)	0.460• (0.106)	0.765• (0.132)	0.505• (0.159)
β_5	-0.129 (0.112)	0.030 (0.210)	0.692• (0.141)	0.503• (0.157)	-0.580• (0.138)	0.251• (0.131)	-0.298• (0.130)	0.315•• (0.171)	0.599• (0.228)
β_6	0.032 (0.168)	-1.301• (0.451)	0.605• (0.090)	0.207 (0.563)	0.570• (0.228)	-0.257 (0.328)	0.304•• (0.182)	0.906• (0.247)	-0.025 (0.469)
β_7	0.380• (0.056)	-0.873• (0.092)	-0.341 (0.412)	0.424• (0.063)	1.159• (0.057)	0.206• (0.096)	0.772• (0.086)	1.269• (0.089)	0.281•• (0.172)
β_8	-2.173• (0.207)	-2.969• (0.580)	0.328• (0.040)	-1.101• (0.490)	-1.433• (0.133)	-1.719• (0.414)	-2.410• (0.276)	-1.046• (0.231)	-1.610• (0.656)
β_9	0.538• (0.083)	1.321• (0.093)	-2.077• (0.319)	1.141• (0.165)	0.143• (0.057)	0.603• (0.115)	0.539• (0.056)	0.022• (0.010)	1.169• (0.256)
γ	0.142• (0.040)	0.334• (0.044)	-0.137• (0.096)	-0.329• (0.069)	0.233• (0.038)	-0.191• (0.045)	1.205• (0.029)	0.247• (0.037)	-0.476• (0.063)
l	-0.265	0.246	-0.832	-0.781	-0.567	-0.359	1.522	-0.666	-0.864

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0 p-value	294.630 (0.000)	581.519 (0.000)	66.351 (0.000)	130.603 (0.000)	222.570 (0.000)	140.802 (0.000)	232.502 (0.000)	154.791 (0.000)	83.830 (0.000)
H_1 p-value	32.807 (0.000)	35.358 (0.000)	55.189 (0.000)	0.628 (0.731)	9.484 (0.009)	34.096 (0.000)	8.213 (0.016)	8.845 (0.012)	5.042 (0.080)
H_2 p-value	58.674 (0.000)	61.246 (0.000)	47.128 (0.000)	43.325 (0.000)	52.481 (0.000)	56.195 (0.000)	33.485 (0.000)	49.280 (0.000)	20.800 (0.000)
H_3 p-value	1.893 (0.388)	7.577 (0.023)	40.927 (0.000)	4.395 (0.111)	10.749 (0.005)	1.894 (0.388)	1.895 (0.388)	3.159 (0.206)	4.412 (0.110)
H_4 p-value	182.330 (0.000)	316.331 (0.000)	30.385 (0.000)	64.046 (0.000)	147.326 (0.000)	43.567 (0.000)	162.373 (0.000)	88.452 (0.000)	27.733 (0.000)

TABLE 11: MODEL ESTIMATES AND HYPOTHESIS TESTS-- NUMBER OF BLOCK TRADES, ALL STOCKS.

This table presents in Panel A abbreviated coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NBLOCK_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \delta_1 T1_t + \delta_2 T2_t + \delta_3 T3_t + \delta_4 T4_t + \delta_5 T5_t + \delta_6 T6_t + \delta_7 T7_t + \delta_8 T8_t + \delta_9 T9_t + \delta_{10} T10_t + \delta_{11} T11_t + \delta_{12} T12_t + \varepsilon_t$$

Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. Estimates of the coefficients δ_1 to δ_{12} are not presented. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. l is the log-likelihood. * indicates that the coefficient is significant at the 5% level; ** at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = ... = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	-0.898*	-1.351*	-2.332*	-0.967*	-2.046*	-2.339*	-1.719*	-1.443*	-0.286
	(0.112)	(0.139)	(0.514)	(0.111)	(0.496)	(0.177)	(0.147)	(0.116)	(0.184)
β_1	0.390*	0.201*	0.663*	0.301*	-0.085	0.052	-0.026	0.177	0.522*
	(0.125)	(0.078)	(0.194)	(0.105)	(0.207)	(0.187)	(0.162)	(0.139)	(0.102)
β_2	0.158	0.003	-0.363	0.187**	0.107	-0.204	0.128	0.164	-0.022
	(0.127)	(0.074)	(0.302)	(0.111)	(0.165)	(0.155)	(0.139)	(0.137)	(0.133)
β_3	1.213*	0.588*	0.519	0.744*	-0.071	1.109*	1.007*	1.103*	0.671*
	(0.221)	(0.261)	(0.401)	(0.189)	(0.554)	(0.345)	(0.273)	(0.246)	(0.244)
β_4	0.598*	0.901*	0.165	0.578*	-0.081	1.157*	0.898*	0.921*	0.637*
	(0.176)	(0.202)	(0.422)	(0.174)	(0.500)	(0.272)	(0.192)	(0.195)	(0.212)
β_5	-0.811*	0.074	0.273	0.284	0.495	1.003*	1.509*	-0.653*	-0.708*
	(0.334)	(0.127)	(0.394)	(0.267)	(0.357)	(0.273)	(0.326)	(0.242)	(0.335)
β_6	-0.187	0.522	-0.021	0.348	-0.016	0.236	-0.262	0.609*	-2.988*
	(0.414)	(0.549)	(2.515)	(0.323)	(2.416)	(0.656)	(0.504)	(0.287)	(0.842)
β_7	-1.724*	-1.073*	-0.067	-0.655*	-0.648*	-0.479*	-1.427*	-0.320*	-1.980*
	(0.221)	(0.156)	(0.167)	(0.186)	(0.138)	(0.165)	(0.241)	(0.139)	(0.210)
β_8	2.596*	2.475*	0.951	1.283*	0.852	4.310*	2.784*	2.472*	3.798*
	(0.213)	(0.223)	(0.822)	(0.253)	(1.140)	(0.573)	(0.433)	(0.246)	(0.726)
β_9	0.657*	0.656*	0.660*	0.615*	0.374	0.759*	0.303*	0.022	1.668*
	(0.059)	(0.064)	(0.313)	(0.074)	(1.127)	(0.141)	(0.088)	(0.018)	(0.152)
γ	-1.095*	-0.789*	-1.509*	-1.104*	-1.595*	-1.666*	-1.089*	-1.016*	-0.900*
	(0.115)	(0.093)	(0.167)	(0.096)	(0.229)	(0.178)	(0.108)	(0.106)	(0.102)
l	-0.698	-0.594	-0.330	-0.806	-0.238	-0.432	-0.549	-0.593	-0.617

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	159.365	118.872	20.138	100.536	28.666	65.645	81.515	80.947	260.219
p-value	(0.000)	(0.000)	(0.017)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	6.324	4.426	3.147	11.381	17.916	1.262	1.264	1.265	20.843
p-value	(0.042)	(0.109)	(0.207)	(0.003)	(0.000)	(0.532)	(0.532)	(0.531)	(0.000)
H_2	27.193	13.278	13.845	22.763	17.319	10.730	11.374	16.442	31.580
p-value	(0.000)	(0.010)	(0.008)	(0.000)	(0.002)	(0.030)	(0.023)	(0.002)	(0.000)
H_3	3.794	1.265	0.629	1.265	4.180	5.681	24.012	3.794	14.527
p-value	(0.150)	(0.531)	(0.730)	(0.531)	(0.124)	(0.058)	(0.000)	(0.150)	(0.001)
H_4	122.053	92.316	14.474	61.965	18.513	46.078	61.294	56.916	143.373
p-value	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 11 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	-1.223*	-0.358**	-0.470*	-2.057*	-1.306*	-1.275*	-0.311*	-1.662*	-2.391*
	(0.160)	(0.200)	(0.071)	(0.285)	(0.145)	(0.172)	(0.106)	(0.199)	(0.174)
β_1	0.302*	0.277*	-1.639*	0.183	0.149	0.607*	0.239*	0.542*	0.111
	(0.079)	(0.085)	(0.242)	(0.218)	(0.175)	(0.153)	(0.075)	(0.198)	(0.266)
β_2	-0.026	0.152	0.107	0.155	-0.085	0.301**	0.192*	0.420*	-0.149
	(0.136)	(0.110)	(0.129)	(0.238)	(0.158)	(0.159)	(0.072)	(0.197)	(0.252)
β_3	0.324	0.153	-0.203	0.592	0.880*	-0.054	0.396*	0.609*	0.690*
	(0.266)	(0.265)	(0.153)	(0.447)	(0.251)	(0.296)	(0.190)	(0.283)	(0.335)
β_4	0.438**	0.379**	0.068	-1.101**	0.772*	0.260	0.144	0.420**	0.730*
	(0.239)	(0.202)	(0.439)	(0.610)	(0.238)	(0.238)	(0.164)	(0.255)	(0.257)
β_5	-0.226**	-0.562**	0.059	0.810*	-0.054	0.069	-0.706*	0.281	0.564
	(0.253)	(0.307)	(0.420)	(0.325)	(0.259)	(0.321)	(0.197)	(0.328)	(0.457)
β_6	-0.323	-1.498	0.161	-1.784**	0.177	-0.672	-0.046	0.458	0.334
	(0.640)	(0.985)	(0.209)	(1.044)	(0.503)	(0.684)	(0.388)	(0.820)	(0.561)
β_7	-0.415*	-2.478*	-0.179	-1.085*	-1.222*	-1.448*	-0.684*	-1.272*	-2.351*
	(0.132)	(0.225)	(0.890)	(0.191)	(0.164)	(0.266)	(0.160)	(0.226)	(0.387)
β_8	1.207*	5.120*	-0.798*	5.665*	1.227*	5.997*	1.248*	1.688*	9.764*
	(0.149)	(0.725)	(0.110)	(0.668)	(0.139)	(0.669)	(0.270)	(0.229)	(0.692)
β_9	0.843*	1.963*	0.589	1.285*	0.228*	1.036*	0.769*	0.050*	2.792*
	(0.104)	(0.119)	(0.449)	(0.222)	(0.063)	(0.179)	(0.070)	(0.012)	(0.331)
γ	-0.841*	-0.486*	-0.024	-1.891*	-1.772*	-1.162*	-0.649*	-1.373*	-1.358*
	(0.084)	(0.083)	(0.206)	(0.254)	(0.157)	(0.114)	(0.072)	(0.128)	(0.153)
l	-0.608	-0.621	-0.342	-0.290	-0.568	-0.548	-0.815	-0.520	-0.419

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0	92.742	347.270	62.630	81.627	61.965	80.819	180.063	48.649	107.781
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	4.416	8.840	1.860	0.628	0.632	11.365	10.741	1.264	0.630
p-value	(0.110)	(0.012)	(0.394)	(0.731)	(0.729)	(0.003)	(0.005)	(0.532)	(0.730)
H_2	9.463	11.365	1.860	1.884	13.911	11.997	13.900	5.054	5.673
p-value	(0.051)	(0.023)	(0.761)	(0.757)	(0.008)	(0.017)	(0.008)	(0.282)	(0.225)
H_3	0.631	5.683	0.620	3.767	1.265	3.788	5.686	1.264	1.891
p-value	(0.729)	(0.058)	(0.733)	(0.152)	(0.531)	(0.150)	(0.058)	(0.532)	(0.389)
H_4	76.339	260.137	52.709	72.836	51.849	71.348	144.050	36.644	103.369
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 12: MODEL ESTIMATES AND HYPOTHESIS TESTS-- NUMBER OF TRADES, FIRST HOUR, ALL STOCKS.

This table presents in Panel A coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NTRADE_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \varepsilon_t.$$

This model is estimated on observations between 9:30 AM and 10:30 AM. Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. NOBS is the number of observations. l is the log-likelihood. * indicates that the coefficient is significant at the 5% level; ** at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = \ldots = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	2.066*	1.686*	0.868*	2.361*	0.337*	1.242*	1.438*	1.537*	1.906*
	(0.060)	(0.091)	(0.108)	(0.055)	(0.084)	(0.060)	(0.083)	(0.051)	(0.175)
β_1	0.057	0.168*	0.371*	-0.060	0.283*	0.098	0.228*	-0.023	0.511*
	(0.082)	(0.056)	(0.125)	(0.076)	(0.099)	(0.085)	(0.088)	(0.082)	(0.094)
β_2	-0.020	0.091	0.127	0.079	0.064	-0.086	0.169*	-0.012	0.254*
	(0.112)	(0.062)	(0.213)	(0.074)	(0.105)	(0.101)	(0.084)	(0.082)	(0.102)
β_3	0.335**	0.389**	0.650*	0.455*	0.412	0.735*	0.274	0.406*	0.438*
	(0.195)	(0.229)	(0.297)	(0.144)	(0.286)	(0.284)	(0.216)	(0.161)	(0.210)
β_4	0.454*	0.185	0.530*	0.302*	0.361*	0.638*	0.345*	0.479*	0.077
	(0.108)	(0.148)	(0.233)	(0.119)	(0.154)	(0.172)	(0.146)	(0.086)	(0.276)
β_5	0.118	0.245*	0.401	0.351	0.056	-0.062	0.645*	0.037	-0.491
	(0.277)	(0.109)	(0.252)	(0.218)	(0.173)	(0.204)	(0.251)	(0.175)	(0.312)
β_6	0.198	0.783**	0.078	0.183	0.996*	0.650*	0.519	0.931*	-0.426
	(0.251)	(0.438)	(0.447)	(0.241)	(0.262)	(0.257)	(0.383)	(0.127)	(0.894)
β_7	-0.956*	-0.396	-0.324*	-0.348**	-0.370*	-0.651*	-0.716*	-0.365*	-1.808*
	(0.436)	(0.256)	(0.155)	(0.205)	(0.069)	(0.150)	(0.343)	(0.148)	(0.233)
β_8	0.504	0.210	-0.455	0.013	0.464	0.664	-0.132	0.554	-0.105
	(0.588)	(1.140)	(0.709)	(0.590)	(0.736)	(0.743)	(1.097)	(0.421)	(2.387)
β_9	0.458*	0.405*	-0.079	0.324*	0.184	0.367*	0.316*	0.222*	0.905*
	(0.052)	(0.108)	(0.306)	(0.057)	(0.824)	(0.108)	(0.078)	(0.092)	(0.191)
γ	0.322*	0.954*	-0.347*	0.106	-2.223*	-0.226**	0.244*	-0.405*	0.490*
	(0.099)	(0.098)	(0.129)	(0.103)	(0.711)	(0.122)	(0.111)	(0.143)	(0.114)
NOBS	758	757	756	757	757	756	755	758	758
l	12.717	9.096	0.069	20.822	-0.860	1.882	4.423	5.056	6.451

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	70.646	45.420	22.680	49.811	38.229	55.944	54.587	55.182	146.294
p-value	(0.000)	(0.000)	(0.007)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	0.379	3.709	3.704	1.363	3.709	2.873	2.869	0.455	26.530
p-value	(0.827)	(0.157)	(0.157)	(0.506)	(0.157)	(0.238)	(0.238)	(0.797)	(0.000)
H_2	5.761	1.287	6.955	4.391	5.980	11.038	5.738	8.035	32.367
p-value	(0.218)	(0.864)	(0.138)	(0.356)	(0.201)	(0.026)	(0.220)	(0.090)	(0.000)
H_3	1.895	7.721	1.285	3.331	5.072	5.216	5.889	15.842	0.834
p-value	(0.388)	(0.021)	(0.526)	(0.189)	(0.079)	(0.074)	(0.053)	(0.000)	(0.659)
H_4	50.331	15.367	4.914	25.889	13.702	16.103	19.026	11.673	76.785
p-value	(0.000)	(0.002)	(0.178)	(0.000)	(0.003)	(0.001)	(0.000)	(0.009)	(0.000)

TABLE 12 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	1.525*	1.816*	1.075*	0.582*	1.622*	1.604*	2.459*	1.608*	1.049*
	(0.070)	(0.084)	(0.097)	(0.089)	(0.054)	(0.063)	(0.066)	(0.065)	(0.074)
β_1	0.233*	0.367*	0.062	0.150	0.020	0.125	0.026	0.151	0.169
	(0.095)	(0.065)	(0.160)	(0.150)	(0.099)	(0.098)	(0.057)	(0.111)	(0.165)
β_2	0.064	0.305*	0.102	0.274**	-0.065	0.246*	-0.018	-0.078	0.129
	(0.095)	(0.083)	(0.142)	(0.152)	(0.086)	(0.104)	(0.049)	(0.165)	(0.153)
β_3	0.365**	0.374**	0.956*	0.319	0.541*	0.533*	0.523*	0.526*	0.377
	(0.211)	(0.224)	(0.293)	(0.336)	(0.207)	(0.195)	(0.159)	(0.189)	(0.295)
β_4	0.489*	0.328*	0.637*	0.272	0.541*	0.367*	0.345*	0.544*	0.550*
	(0.134)	(0.135)	(0.162)	(0.190)	(0.153)	(0.122)	(0.102)	(0.134)	(0.204)
β_5	0.229	-0.406	-0.019	0.173	0.331**	0.192	0.280	0.307	0.421
	(0.214)	(0.274)	(0.171)	(0.271)	(0.173)	(0.218)	(0.205)	(0.262)	(0.334)
β_6	-0.103	-0.136	0.151	0.803*	0.284	0.315	0.351*	-0.032	0.295
	(0.299)	(0.331)	(0.344)	(0.390)	(0.199)	(0.204)	(0.173)	(0.255)	(0.312)
β_7	0.034	-1.776*	-0.498*	-0.281*	-0.211**	-0.524*	-0.559*	-0.572*	-0.760*
	(0.124)	(0.316)	(0.077)	(0.098)	(0.129)	(0.251)	(0.259)	(0.162)	(0.379)
β_8	-0.340	0.236	0.313	1.451	-0.048	0.730	0.901**	-0.010	0.367
	(0.355)	(2.262)	(0.440)	(0.901)	(0.261)	(1.158)	(0.506)	(0.346)	(1.785)
β_9	0.609*	1.388*	0.472*	1.257*	-0.033	0.567*	0.609*	0.100	1.462*
	(0.090)	(0.150)	(0.111)	(0.379)	(0.115)	(0.175)	(0.065)	(0.116)	(0.340)
γ	0.184	1.077*	0.082	-0.454*	-0.878*	0.055	0.927*	-0.567*	0.197**
	(0.118)	(0.093)	(0.135)	(0.151)	(0.209)	(0.107)	(0.098)	(0.180)	(0.121)
NOBS	748	753	756	759	758	754	753	755	758
l	4.702	9.451	0.499	-0.155	4.502	5.487	32.663	3.361	1.345

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0	46.301	155.570	50.274	33.851	21.679	37.549	103.161	23.329	38.203
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.010)	(0.000)	(0.000)	(0.005)	(0.000)
H_1	2.768	21.235	0.756	3.567	0.076	5.278	0.226	1.585	0.152
p-value	(0.251)	(0.000)	(0.685)	(0.168)	(0.963)	(0.071)	(0.893)	(0.453)	(0.927)
H_2	8.078	29.216	7.938	6.831	5.458	8.671	0.979	6.266	3.108
p-value	(0.089)	(0.000)	(0.094)	(0.145)	(0.243)	(0.070)	(0.913)	(0.180)	(0.540)
H_3	0.224	1.882	0.302	3.264	1.895	0.528	1.506	0.679	1.895
p-value	(0.894)	(0.390)	(0.860)	(0.196)	(0.388)	(0.768)	(0.471)	(0.712)	(0.388)
H_4	23.786	99.773	32.508	13.283	1.743	10.179	90.887	8.833	15.842
p-value	(0.000)	(0.000)	(0.000)	(0.004)	(0.627)	(0.017)	(0.006)	(0.032)	(0.001)

TABLE 13: MODEL ESTIMATES AND HYPOTHESIS TESTS-- NUMBER OF TRADES, MIDDAY, ALL STOCKS.

This table presents in Panel A coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NTRADE_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \varepsilon_t$$

This model is estimated on observations between 10:30 AM and 2:59 PM. Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. NOBS is the number of observations. ℓ is the log-likelihood. • indicates that the coefficient is significant at the 5% level; •• at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = ... = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. F-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	1.732•	1.250•	0.448•	2.073•	0.252•	0.663•	0.985•	1.195•	2.306•
	(0.057)	(0.058)	(0.089)	(0.053)	(0.095)	(0.053)	(0.088)	(0.055)	(0.143)
β_1	0.102••	0.112•	0.306•	0.047	0.009	0.129•	0.319•	-0.012	0.316•
	(0.060)	(0.037)	(0.076)	(0.040)	(0.058)	(0.059)	(0.062)	(0.048)	(0.063)
β_2	0.091	0.065•	0.018	0.123•	-0.090	0.085	0.295•	0.035	0.289•
	(0.061)	(0.033)	(0.067)	(0.042)	(0.067)	(0.060)	(0.093)	(0.049)	(0.079)
β_3	0.802•	0.911•	1.747•	0.801•	1.330•	1.301•	0.908•	0.921•	0.795•
	(0.088)	(0.107)	(0.136)	(0.077)	(0.162)	(0.116)	(0.117)	(0.102)	(0.123)
β_4	0.722•	0.902•	1.558•	0.586•	1.360•	1.194•	0.896•	0.870•	0.735•
	(0.083)	(0.111)	(0.127)	(0.069)	(0.144)	(0.107)	(0.104)	(0.099)	(0.120)
β_5	0.139	0.345•	0.462•	0.290•	0.889•	0.511•	0.540•	0.363•	-0.359•
	(0.114)	(0.046)	(0.147)	(0.085)	(0.098)	(0.096)	(0.133)	(0.072)	(0.142)
β_6	0.409	1.311•	0.034	0.332	0.048	1.155•	1.036•	0.931•	-4.022•
	(0.299)	(0.286)	(0.446)	(0.259)	(0.462)	(0.229)	(0.456)	(0.252)	(0.756)
β_7	-0.897•	-0.425•	-0.255•	-0.370•	-0.271•	-0.459•	-0.558•	-0.361•	-1.626•
	(0.072)	(0.047)	(0.055)	(0.054)	(0.037)	(0.054)	(0.072)	(0.046)	(0.077)
β_8	0.497•	0.212	-0.394	0.018	0.147	0.849•	-0.098	0.513•	-0.450
	(0.102)	(0.139)	(0.254)	(0.086)	(0.254)	(0.234)	(0.174)	[illegible]	(0.343)
β_9	0.362•	0.304•	0.348•	0.211•	0.117	0.201•	0.145•	-0.011	[illegible]
	(0.035)	(0.043)	(0.163)	(0.041)	(0.287)	(0.081)	(0.057)	(0.016)	(0.122)
γ	0.012	0.401•	-0.764•	0.072••	-1.657•	-0.529•	0.003	-0.383•	0.210•
	(0.046)	(0.041)	(0.076)	(0.042)	(0.165)	(0.063)	(0.050)	(0.054)	(0.048)
NOBS	4554	4554	4551	4554	4432	4554	4554	4554	4554
ℓ	6.066	4.093	-0.599	12.101	-0.911	0.236	1.364	2.120	2.363

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	293.733	294.188	197.058	248.193	163.984	231.343	244.094	144.362	672.170
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	3.188	10.019	6.826	10.019	1.773	1.822	26.869	1.366	28.235
p-value	(0.203)	(0.007)	(0.033)	(0.007)	(0.412)	(0.402)	(0.000)	(0.505)	(0.000)
H_2	66.488	68.310	128.338	81.972	60.275	93.812	79.240	66.944	71.953
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_3	1.366	26.869	4.551	1.822	42.990	21.404	10.019	14.573	33.244
p-value	(0.505)	(0.000)	(0.103)	(0.402)	(0.000)	(0.000)	(0.007)	(0.001)	(0.000)
H_4	194.456	118.859	21.845	98.822	29.251	51.916	64.211	35.977	415.325
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

TABLE 13 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	1.322• (0.070)	1.414• (0.094)	0.880• (0.074)	0.387• (0.129)	1.375• (0.062)	1.284• (0.062)	2.220• (0.061)	1.312• (0.082)	0.491• (0.102)
β_1	0.292• (0.038)	0.146•• (0.086)	-0.064 (0.049)	-0.098 (0.077)	0.095•• (0.055)	0.282• (0.072)	0.071 (0.057)	0.015 (0.074)	0.098 (0.119)
β_2	0.096•• (0.051)	0.129•• (0.073)	-0.150• (0.049)	0.085 (0.098)	0.018 (0.052)	0.232• (0.063)	0.051 (0.037)	0.037 (0.073)	0.119 (0.120)
β_3	0.803• (0.104)	0.990• (0.136)	0.938• (0.149)	1.429• (0.162)	0.865 (0.103)	0.823• (0.120)	0.629• (0.094)	0.859• (0.113)	1.506• (0.161)
β_4	0.822• (0.105)	0.920• (0.120)	1.116• (0.124)	1.134• (0.148)	0.762• (0.092)	0.687• (0.116)	0.532• (0.086)	0.911• (0.099)	1.428• (0.143)
β_5	0.462• (0.096)	-0.031 (0.200)	0.513• (0.067)	0.554• (0.114)	0.388• (0.079)	0.243• (0.106)	0.294• (0.085)	0.296• (0.110)	0.551• (0.184)
β_6	-0.049 (0.346)	0.073 (0.492)	0.007 (0.341)	0.397 (0.664)	0.404 (0.300)	0.540•• (0.289)	0.410 (0.307)	0.156 (0.412)	0.694 (0.506)
β_7	-0.438• (0.047)	-1.483• (0.081)	-0.524• (0.031)	-0.444• (0.050)	-0.279• (0.045)	-0.705• (0.081)	-0.648• (0.057)	-0.422• (0.064)	-0.727• (0.134)
β_8	-0.061 (0.075)	0.237 (0.387)	0.332• (0.116)	0.828• (0.267)	-0.030 (0.053)	0.732• (0.270)	0.833• (0.078)	0.040 (0.098)	0.371 (0.503)
β_9	0.311• (0.050)	1.382• (0.119)	0.061• (0.011)	0.641• (0.132)	0.011 (0.017)	0.417• (0.117)	0.514• (0.048)	-0.001 (0.004)	1.272• (0.231)
γ	-0.159• (0.052)	0.612• (0.047)	-0.456• (0.071)	-0.800• (0.086)	-0.811• (0.071)	-0.069 (0.051)	0.773• (0.042)	-0.509• (0.061)	-0.095•• (0.055)
NOBS	4552	4554	4515	4554	4554	4552	4554	4554	758
l	1.861	2.883	-0.395	-0.693	2.484	2.366	19.018	1.542	-0.209

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0 p-value	267.202 (0.000)	652.588 (0.000)	261.419 (0.000)	190.813 (0.000)	144.362 (0.000)	205.295 (0.000)	423.522 (0.000)	107.474 (0.000)	228.155 (0.000)
H_1 p-value	30.043 (0.000)	6.831 (0.033)	4.515 (0.105)	3.188 (0.203)	2.277 (0.320)	20.029 (0.000)	4.554 (0.103)	0.911 (0.634)	1.366 (0.505)
H_2 p-value	70.101 (0.000)	55.559 (0.000)	48.762 (0.000)	66.033 (0.000)	73.319 (0.000)	69.190 (0.000)	42.352 (0.000)	69.676 (0.000)	103.831 (0.000)
H_3 p-value	10.014 (0.007)	0.911 (0.634)	24.833 (0.000)	12.296 (0.002)	10.474 (0.005)	1.366 (0.505)	1.822 (0.402)	2.732 (0.255)	5.465 (0.065)
H_4 p-value	84.212 (0.000)	440.372 (0.000)	153.059 (0.000)	73.775 (0.000)	25.047 (0.000)	78.294 (0.000)	347.470 (0.000)	22.315 (0.000)	49.639 (0.000)

TABLE 14: MODEL ESTIMATES AND HYPOTHESIS TESTS– NUMBER OF TRADES, FINAL HOUR, ALL STOCKS.

This table presents in Panel A coefficients and standard errors from negative binomial maximum likelihood estimation for the following equation:

$$NTRADE_t = \beta_0 + \beta_1 PTRET_{t-1} + \beta_2 NTRET_{t-1} + \beta_3 PMRET_{t-1} + \beta_4 NMRET_{t-1} + \beta_5 TVOL_{t-1} + \beta_6 MVOL_{t-1} + \beta_7 SPREAD_{t-1} + \beta_8 DEPTH_{t-1} + \beta_9 VOLUME_{t-1} + \varepsilon_t$$

This model is estimated on observations between 3:00 PM and 4:00 PM. Stock ticker symbols are given in Table 1; see Tables 2 and 3 for variable descriptions. γ is a scalar dispersion parameter. Hetroscedasticity robust standard errors are reported in the parenthesis. NOBS is the number of observations. ℓ is the log-likelihood. • indicates that the coefficient is significant at the 5% level; •• at the 10% level. Panel B presents likelihood ratio test statistics and p-values from model exclusion tests over different sets of parameters. We test the following five hypotheses: H_0: $\beta_1 = \ldots = \beta_9 = 0$, H_1: $\beta_1 = \beta_2 = 0$, H_2: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$, H_3: $\beta_5 = \beta_6 = 0$, H_4: $\beta_7 = \beta_8 = \beta_9 = 0$. In each case, the two-sided alternative hypothesis is that the given coefficients are not equal to zero. All test statistics are distributed $\chi^2(k)$, where k is the number of restrictions. P-values are reported in parenthesis.

Panel A: Model Estimates.

Parameter	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
β_0	1.94[illegible]	1.515•	0.799•	2.268•	0.575•	0.921•	1.201•	1.342•	1.981•
	(0.090)	(0.103)	(0.135)	(0.085)	(0.240)	(0.117)	(0.103)	(0.105)	(0.161)
β_1	-0.005	0.187•	0.221•	0.087	0.074	0.046	0.212••	0.159•	0.261•
	(0.100)	(0.070)	(0.121)	(0.074)	(0.118)	(0.109)	(0.120)	(0.081)	(0.112)
β_2	-0.028	0.111•	0.113	0.043	0.033	0.174••	0.147	0.171••	0.280•
	(0.083)	(0.038)	(0.148)	(0.067)	(0.107)	(0.093)	(0.120)	(0.089)	(0.100)
β_3	0.659•	0.704•	0.983•	0.746•	0.823•	0.883•	0.720•	0.685•	0.137
	(0.126)	(0.152)	(0.173)	(0.113)	(0.246)	(0.159)	(0.168)	(0.137)	(0.178)
β_4	0.601•	0.579•	0.823•	0.568•	0.595•	0.722•	0.669•	0.753•	0.293••
	(0.088)	(0.098)	(0.179)	(0.096)	(0.188)	(0.162)	(0.164)	(0.119)	(0.153)
β_5	0.102•	0.299•	0.324•	0.347	0.256	0.450•	0.566•	0.207	-0.347
	(0.222)	(0.099)	(0.226)	(0.214)	(0.171)	(0.195)	(0.223)	(0.152)	(0.261)
β_6	0.434	1.334•	0.108	0.365	-0.125	1.121•	1.038•	0.847••	-1.015
	(0.441)	(0.443)	(0.675)	(0.415)	(1.139)	(0.544)	(0.501)	(0.485)	(0.860)
β_7	-0.865•	-0.639•	-0.265•	-0.333•	-0.304•	-0.409•	-0.412•	-0.242•	-1.472•
	(0.163)	(0.135)	(0.107)	(0.118)	(0.082)	(0.111)	(0.145)	(0.108)	(0.162)
β_8	0.511•	0.139	-0.360	0.022	1.086••	0.608	-0.023	0.534••	-0.081
	(0.239)	(0.262)	(0.581)	(0.184)	(0.639)	(0.399)	(0.385)	(0.284)	(0.998)
β_9	0.390•	0.505•	-0.084	0.230•	0.499	0.076	0.224	0.013•	1.327•
	(0.076)	(0.175)	(0.147)	(0.063)	(0.535)	(0.186)	(0.201)	(0.006)	(0.232)
γ	0.310•	0.789•	0.022	0.444•	-0.164	-0.009	0.320•	-0.191••	0.590•
	(0.086)	(0.085)	(0.099)	(0.085)	(0.148)	(0.100)	(0.092)	(0.110)	(0.085)
NOBS	1012	1012	986	1012	783	1002	1010	1012	1004
ℓ	9.956	8.346	0.164	19.914	-0.625	1.404	3.478	4.033	4.759

Panel B: Model Hypothesis Tests.

Hypothesis	BA	BC	BCC	BMY	FLE	GNN	JCP	JPM	KMB
H_0	78.430	99.783	48.117	77.519	23.725	65.531	70.195	48.272	124.797
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	0.607	5.566	2.268	1.417	0.078	1.603	4.141	2.732	6.325
p-value	(0.738)	(0.062)	(0.322)	(0.492)	(0.962)	(0.449)	(0.126)	(0.255)	(0.042)
H_2	18.418	25.300	25.340	27.931	8.691	30.060	23.634	35.521	13.353
p-value	(0.001)	(0.000)	(0.000)	(0.000)	(0.069)	(0.000)	(0.000)	(0.000)	(0.010)
H_3	0.911	8.400	2.564	0.202	0.861	4.008	5.454	1.113	6.325
p-value	(0.634)	(0.015)	(0.278)	(0.904)	(0.650)	(0.135)	(0.065)	(0.573)	(0.042)
H_4	51.207	37.545	3.845	17.204	8.848	6.814	9.696	3.744	80.521
p-value	(0.000)	(0.000)	(0.279)	(0.001)	(0.031)	(0.078)	(0.021)	(0.290)	(0.000)

TABLE 14 (continued).

Panel A: Model Estimates.

Parameter	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
β_0	1.476*	1.752*	1.117*	0.759*	1.606*	1.470*	2.412*	1.519*	0.670*
	(0.108)	(0.113)	(0.180)	(0.166)	(0.103)	(0.098)	(0.088)	(0.100)	(0.142)
β_1	0.200*	0.220*	0.072	-0.139	-0.007	0.447*	0.075	0.110	-0.047
	(0.093)	(0.086)	(0.094)	(0.121)	(0.083)	(0.100)	(0.068)	(0.123)	(0.234)
β_2	-0.149	0.198*	-0.115	-0.139	0.050	0.183**	0.085	0.016	0.234**
	(0.105)	(0.080)	(0.082)	(0.135)	(0.085)	(0.099)	(0.057)	(0.135)	(0.138)
β_3	0.738*	0.165	0.472*	0.978*	0.702*	0.678*	0.539*	0.471*	0.822*
	(0.146)	(0.188)	(0.237)	(0.207)	(0.135)	(0.138)	(0.115)	(0.124)	(0.216)
β_4	0.645*	0.478*	0.793*	1.154*	0.543*	0.700*	0.529*	0.548*	0.753*
	(0.134)	(0.137)	(0.133)	(0.185)	(0.104)	(0.105)	(0.092)	(0.140)	(0.188)
β_5	0.475*	-0.046	0.373*	0.671*	0.392*	0.038	0.267	0.394*	0.151
	(0.199)	(0.254)	(0.136)	(0.220)	(0.166)	(0.226)	(0.198)	(0.186)	(0.339)
β_6	-0.035	-0.002	0.038	-0.347	0.421	0.591	0.463	0.167	1.664*
	(0.538)	(0.545)	(0.895)	(0.762)	(0.501)	(0.455)	(0.408)	(0.460)	(0.696)
β_7	-0.280*	-1.246*	-0.469*	-0.547*	-0.210*	-0.733*	-0.642*	-0.388*	-0.749*
	(0.100)	(0.216)	(0.070)	(0.101)	(0.112)	(0.196)	(0.116)	(0.140)	(0.310)
β_8	0.087	0.280	0.202	2.304*	-0.099	0.718	0.827*	0.184	0.288
	(0.151)	(0.974)	(0.279)	(0.594)	(0.131)	(0.690)	(0.152)	(0.225)	(1.167)
β_9	0.352*	0.948*	0.149	0.154	0.102	0.384*	0.352*	0.335*	0.903*
	(0.082)	(0.198)	(0.124)	(0.265)	(0.082)	(0.141)	(0.097)	(0.138)	(0.366)
γ	-0.092	0.949*	0.139	0.031	-0.346*	0.232*	1.003*	-0.133	0.483*
	(0.099)	(0.079)	(0.110)	(0.106)	(0.113)	(0.085)	(0.073)	(0.105)	(0.094)
NOBS	1009	1007	930	966	1011	1008	1011	1009	993
l	3.482	5.511	0.336	-0.217	5.201	4.542	27.454	3.610	0.904

Panel B: Model Hypothesis Tests.

Hypothesis	MA	MHP	NCB	RAD	SBC	UNP	UPJ	USW	VO
H_0	59.027	82.272	51.522	48.686	44.282	71.568	85.328	37.535	55.211
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
H_1	5.751	2.316	1.767	0.773	0.404	9.173	1.415	0.706	1.390
p-value	(0.056)	(0.314)	(0.413)	(0.679)	(0.817)	(0.010)	(0.493)	(0.702)	(0.499)
H_2	25.326	9.365	12.369	20.479	17.591	35.986	15.367	14.025	20.654
p-value	(0.000)	(0.053)	(0.015)	(0.000)	(0.001)	(0.000)	(0.004)	(0.007)	(0.000)
H_3	4.440	0.101	3.348	4.540	2.022	0.907	1.314	1.917	1.986
p-value	(0.109)	(0.951)	(0.187)	(0.103)	(0.364)	(0.635)	(0.518)	(0.383)	(0.370)
H_4	14.025	51.860	25.761	18.451	6.167	17.136	53.785	8.576	8.639
p-value	(0.003)	(0.000)	(0.000)	(0.0[illegible]0)	(0.104)	(0.001)	(0.000)	(0.035)	(0.034)

PAPER 2:

COMPARATIVE FINANCIAL MARKET PERFORMANCE UNDER DIFFERENT DEGREES OF CENTRALIZATION.

0. Introduction.

The generic notion of a marketplace is based on assembling traders in a single trading arena. Yet markets can be centralized in varying degrees, depending on the number of marketplaces and on the ability of traders in one marketplace to transact with traders in other marketplaces. This paper examines how the degree of a market's centralization affects its operational characteristics, and provides a framework for evaluating the consequences of market consolidation. We view a market mechanism as a transformation process that accepts orders from individual traders as "inputs," and generates transactions (usually specified by price and quantity) as "outputs." We say that the market mechanism is *consolidated* if all the order data are available when this transformation takes place; e.g., when all orders are channeled to a central trading post. We say that the market is *fragmented* when orders are decomposed into a number of disjoint subsets, and the transformation is applied to each subset separately; e.g., when an asset is traded in a number of secluded locations. In this context, market fragmentation is the result of incomplete information (about all available orders), that could result in the absence of potentially mutually beneficial trades and loss of the associated gains from trade.[1] In contrast, a consolidated market takes all

[1] Note here that "fragmentation" does not necessarily imply "price independence." Further, geographically-dispersed markets may well be consolidated (e.g., if they use a consolidated "order book").

the available order data into account in determining market outcomes.

An analysis of the centralization issue calls for balancing the costs of incomplete order information against the costs of consolidation. Consolidated market-mechanisms require all orders to be called to a central location, thus leading to substantial communication requirements. According to Garbade (1982, Ch. 20), "market fragmentation will exist whenever transactors are unable to communicate with each other quickly and cheaply. The greater costs of communication, and the longer lags in communication, the more extensive will be the fragmentation of the market, and the more likely it will be that transactions will take place away from the best available prices." Thus, market design requires an analytic framework for studying the effects of market centralization on market performance and by the effects of market liquidity as a determinant of asset returns. The interest in such a framework is enhanced by the trend towards automation of the exchange process, which calls for an explicit analysis of the costs and benefits of alternative exchange procedures.

This paper analyzes two alternative models of market organization used in the market micro-structure literature, which represent two approaches to order execution in securities exchanges.[2] The first market mechanism is the *clearing house*, which accumulates orders and intersects the resulting demand and supply schedules periodically, yielding a single market-clearing price that applies to all orders. The second mechanism is the *dealer market*, in which dealers quote bid and ask prices and stand ready to buy

[2] For example, see Whitcomb (1985).

and sell at these prices.[3] We analyze the polar cases of a consolidated market versus a fragmented market under both organizations, yielding four alternative models. For the clearing house organization, we compare the fully consolidated alternative to the fully fragmented one. Then we study the operation of a monopoly-dealer (specialist) market and compare it to an inter-dealer market, thus examining the effects of competition among dealers on the performance of a dealer market.

The issues studied in this paper arise in a number of securities markets in the United States and around the world.[4] The over-the-counter (OTC) market operates as an inter-dealer market that is partially fragmented; our results bear on the effects of inter-dealer competition on the performance of this market. The debate over in-house execution of orders by brokerage houses (internalization) highlights another aspect of the centralization issue. Internalization is a special type of fragmentation occurring when a broker-dealer executes orders in-house before they reach the exchange floor. Both the New York and American Stock Exchanges have imposed restrictions on off-board trading, citing the evils of fragmentation in response to the SEC's attempts to repeal these restrictions. In the OTC market, brokers/dealers often internally execute orders that could not be executed in the inter-dealer market. Another example is provided by securities that are multiply listed on the New York and regional exchanges. These markets are connected by a communications

[3] Amihud and Mendelson (1980) also model market-making in a dealer market to determine the effect on market liquidity.
[4] For further discussion of the public policy nature of these issues, see Stoll (1993).

system that allows dealers to examine the best available quotes and may thus be viewed as an inter-dealer market.

The operation of fragmented markets has been examined previously by a number of authors.[5] Garbade and Silber (1976, 1978, 1979) studied various phenomena of price dispersion in geographically separated markets, considering the role of transaction costs and information asymmetries as determinants of price dispersion. Cohen, Maier, Schwartz, and Whitcomb (1982) discussed the reasons for and possible implications of internalization by brokerage firms when prices are discrete and, hence, the time component of the price-time execution priority becomes important. They demonstrated that under a fragmented market implementation in which brokerage houses assume the dealership function, the performance of the market as a whole will deteriorate.[6] Chowdhry and Nanda (1991) focuses on how an informed trader can exploit his private information when there are multiple locations that simultaneously trade an asset. They examine strategies that market makers may use to deter informed trading across markets. However, their paper does not consider competition between dealers for order flow and it does not address the consequences of market consolidation. Madhaven (1992) compares price formation under a quote-driven mechanism and an order-driven mechanism using various measures of market performance. While Madhaven models trading as a game between strategic traders with rational expectations, our research abstracts from these informational issues in order to consider the centralization issue.

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[5] A comprehensive survey of the literature on this subject is provided in Stoll (1992).
[6] Stoll (1982), in discussing their paper, questions their conclusions.

Finally, Biais (1993) compares centralized and fragmented markets with an emphasis on the nature of interdealer behavior and competition. His primary measure of market performance in contrasting market structures is the bid-ask spread.

There have also been a number of related empirical studies. Hamilton (1979) studied empirically the impact of the NASDAQ system on bid-ask spreads, demonstrating that the introduction of NASDAQ has reduced the spread somewhat, and almost eliminated the edge of the NYSE for comparable securities. Branch and Freed (1977) demonstrated that inter-dealer competition tends to reduce the NYSE spread, and Benston and Hagerman (1974) found that inter-dealer competition reduces the spreads of OTC stocks. More recently, Neal (1992) compares empirical bid-ask spreads for equity options traded on both monopoly dealer markets and interdealer markets. He finds that specialist markets provide more liquidity relative to interdealer markets, which is consistent with the results in this paper.

In Section 1, we present the basic models of the dealer market and the clearing house. We study four models of distinct market mechanisms in Section 2. Section 3 studies the implications of the organization of the market on its performance. In Section 4, we study the robustness of our assumptions for our results. Concluding remarks are offered in Section 5.

1. The Market Structure.

1.1. The Demand/Supply Structure.

Our model of traders' demand and supply follows conventional assumptions made in the market microstructure literature.[7] Each trader is either a potential seller, offering for sale one unit of the traded asset, or a potential buyer, bidding on one unit of the asset. Traders' reservation prices are independent and identically distributed (i.i.d.) over a finite interval, that is assumed (without loss of generality) to be the unit interval.[8] The market is cleared at fixed time intervals of length T.

The market demand and supply are obtained by aggregation of the individual schedules that depict stochastic processes in *price* space, assumed to be independent Poisson processes. The positively-sloped supply function rises in random price increments denoted by τ_j^S (= the interval between the (j - 1)st and the jth reservation price). Similarly, the demand function is downward sloping with random price steps τ_j^D. We assume that τ_j^S and τ_j^D are exponential i.i.d. random variables with mean $1/\lambda$, implying that the demand and supply schedules are independent Poisson

[7] Numerous examples of this setup exist. See Garman (1976), Amihud and Mendelson (1980), Ho and Stoll (1981), Mildenstein and Schleef (1983), or Chen and Jain (1992). Garman (1976) argues: "The Poisson assumption is justified in the same fashion that we normally rationalize the distribution of call arrivals to a telephone switchboard, automobile arrivals at a car wash, or customer arrivals at a bank. Loeve (1963, p. 317) gives the necessary conditions for Poisson convergence, but these are rather abstract in nature. For our purposes there are reasonably mild sufficient conditions: (1) that there be a large number of market agents; (2) that agents act independently in selecting the timing of their orders; (3) that no small subset of agents dominate overall order generation; and (4) that no agent can generate an infinite number of orders in a finite period of time." (p. 259-260)

[8] Since there is no information of a fundamental nature in this model, our results pertaining to the price process should be interpreted as being a relative price rather than an absolute price. In other words, the prices produced in one model must be considered in relation to the prices produced in another model rather than to some fundamental basis for pricing the asset.

processes in price space. The Poisson rate λ, that represents the expected number of buy (sell) orders per unit price, is called the *order intensity*.

The demand and supply processes can be summarized by $N_D(p)$, the number of buy orders with reservation prices greater than or equal to $(1 - p)$; and $N_S(p)$, the number of sell orders with prices less than or equal to p. Formally, these are the *renewal functions*[9]

$$N_D(p) = \max\{m \mid \sum_{j=1}^{m} \tau_j^D \le p\}, \tag{1.1}$$

and

$$N_S(p) = \max\{m \mid \sum_{j=1}^{m} \tau_j^S \ge p\}. \tag{1.2}$$

Both $N_D(p)$ and $N_S(p)$ are Poisson processes whose first two moments are given by

$$E[N_S(p)] = E[N_D(p)] = \lambda p, \tag{1.3}$$

and

$$E[N_S(p)]^2 = E[N_D(p)]^2 = \lambda^2 p^2 + \lambda p. \tag{1.4}$$

We next introduce the market mechanisms operating in the above environment.

1.2. The Clearing House.

The clearing house is a model of a periodic call market. The clearing procedure, applied every T units of time, is a sealed-bid double auction: aggregation of the individual buy and sell orders generates the classical demand and supply schedules of the market, that intersect at a market-clearing price. The expected quantity exchanged in a clearing house with order intensity λ is given by

[9] A sequence is called a *renewal process* provided that the arrival times are i.i.d. non-negative random variables. For a discussion of the properties of these models, see Parzen (1962, Ch. 5) or Ross (1970, Ch. 3).

$$E[Q|\lambda] = \lambda/2 - 1/4(1 - e^{-2\lambda}). \qquad (1.5)$$

Increasing the order intensity increases the trading volume, as might be expected.[10] The execution price variance in the clearing house is

$$\mathrm{Var}(P|\lambda) = \frac{1}{4\lambda}\left[1 - \frac{1 - e^{-2\lambda}}{2\lambda}\right]. \qquad (1.6)$$

The price variance is a decreasing and convex function of the order intensity, λ: increasing λ increases the "depth" of the market, thus reducing the price fluctuations. As the order intensity λ tends to infinity, the execution price variance tends to zero.

1.3. The Monopoly-Dealer (Specialist) Market.

In this model, exchange is performed through a dealer who has a monopoly on all trading. At the beginning of the inter-clearing interval T, the dealer (specialist) announces a bid price β and ask price α; at clearing time he buys the quantity offered to him at β and sells the quantity desired at α.

The cost and revenue structure faced by the dealer is as follows. If he quotes bid price β and ask price α, the quantity supplied to him is given by the random variable $N_S(\beta)$, and the quantity demanded is $N_D(1 - \alpha)$. If the dealer's initial inventory position is θ, his final inventory position will be

[10] For a given trading volume Q, the combined (supply and demand) process is obtained by counting every second renewal in a Poisson process with inter-arrival intervals $\tau_1^D, \tau_1^S, \tau_2^D, \tau_2^S, \tau_3^D, \tau_3^S, \ldots$ Hence, we have

$$P\{Q = k\} = e^{-\lambda}\frac{(\lambda)^{2k}}{(2k)!} + e^{-\lambda}\frac{(\lambda)^{2k+1}}{(2k+1)!}.$$

The expected trading volume is given by the expectation of the renewal function of the combined process.

$$I = \theta + N_S(\beta) - N_D(1 - \alpha). \qquad (1.7)$$

We assume that the dealer values his final inventory I at ν per unit; in addition, he incurs a cost h(I) whenever his final position I is nonzero. This cost is an increasing function of the deviation from a zero position, and is assumed to have the quadratic form $h(I) = k \cdot I^2$. For convenience, we assume $0 \leq k \leq 1/2$ and $|\theta| \leq \lambda/2$.[11]

The dealer, acting as a monopoly, sets the bid and ask prices (β and α) so as to maximize his expected overall profit, given by

$$\pi(\beta, \alpha) = E[\alpha \cdot N_D(1 - \alpha) - \beta \cdot N_S(\beta)] + \nu \cdot I - E[h(I)], \qquad (1.8)$$

where the final inventory position I is given in equation (1.7). Using equations (1.3) and (1.4) to evaluate the expectation, we obtain

$$\pi(\beta, \alpha) = [\nu\theta - k\theta^2] - \lambda(1 + k\lambda)\beta^2 + \lambda(\nu - k - 2\theta k)\beta$$
$$- \lambda(1 + k\lambda)(1 - \alpha)^2 + \lambda(1 - \nu - k + 2\theta k)(1 - \alpha)$$
$$+ 2k\lambda^2\beta(1 - \alpha). \qquad (1.9)$$

The first order conditions for maximizing equation (1.9) give the solution

$$\beta^* = 1/4(1 - 2k) - \left[\frac{4\theta k + 1 - 2\nu}{4(1 + 2k\lambda)}\right], \qquad (1.10)$$

$$\alpha^* = 1 - 1/4(1 - 2k) - \left[\frac{4\theta k + 1 - 2\nu}{4(1 + 2k\lambda)}\right]. \qquad (1.11)$$

Note that both the bid price β^* and the ask price α^* are decreasing functions of the specialists's inventory position. The bid-ask spread,

$$\alpha^* - \beta^* = 1 - 1/2(1 - 2k), \qquad (1.12)$$

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[11] That is, inventory holding cost is nonnegative, the cost of holding one unit of inventory is bounded by the price at the intersection of the expected demand and supply schedules, and the dealer's inventory position is bounded by the corresponding quantity. Amihud and Mendelson (1980) were among the first researchers to use this approach.

is an increasing function of the inventory holding cost parameter. The expected net change in the dealer's inventory as a result of trading with the public, $\lambda(\beta^* + \alpha^* - 1)$, is a decreasing function of his starting inventory, reflecting the dealer's desire to balance his position.

2. The Models.

We now turn to the issue of market centralization. Consider a market that consists of N submarkets indexed by i = 1, 2, ..., N. Each submarket i follows the demand/supply specification of Section 1.1: traders submit orders at their local submarket, and each submarket demand and supply process is a Poisson process in price space, characterized by order intensity λ_i. When these markets are consolidated, the order intensity becomes $\lambda = \sum_{i=1}^{N} \lambda_i$.[12] We analyze the consequences of executing orders under each of the following market mechanisms: fragmented clearing houses (FC), a consolidated clearing house (CC), a monopoly dealer market (MD), and an interdealer market (ID).

The mode of operation of the *fragmented clearing houses* (FC) is simple: intersect the demand and supply schedules constructed in each submarket separately, without allowing arbitrage between them.[13] The *consolidated clearing house* (CC) has a single clearing house for the entire

[12] This statement is true because the superposition of independent Poisson processes with rates λ_i generates a Poisson process with rate $\lambda = \sum_{i=1}^{N} \lambda_i$. See Ross (1970, Ch. 2).

[13] This market structure corresponds to Alfred Marshall's (1949) notion of "secluded markets in which all direct competition from afar is shut out," and represents a polar form of fragmentation.

market. The demand and supply schedules generated by all orders are intersected every T units of time to determine the quantity traded and the market clearing price. The analysis of Section 1.2 applies to each of the fragmented clearing houses (with order intensity λ_i) as well as to the consolidated clearing house (with order intensity λ).

The *monopoly dealer* market (MD) is operated by a specialist that announces his bid and ask prices, β and α, buys the quantity offered at β and sells the quantity offered at α. The dealer operates as in Section 1.3 and faces the consolidated order intensity $\lambda = \sum_{i=1}^{N} \lambda_i$.

The *interdealer market* (ID) consists of N competing dealers, one in each submarket. Each dealer i sets a bid price β_i and an ask price α_i. The order flow is attracted to the highest bid $B = \max_{1 \leq i \leq N} \beta_i$ and to the lowest ask price $A = \min_{1 \leq i \leq N} \alpha_i$. That is, trading is carried out at the inside bid and ask quotes. We assume that dealers face the revenue and cost structures specified in Section 1.3., and differ in their initial inventory positions θ_i. If dealer i quotes the bid-ask prices (β_i, α_i), he will attract no buy orders if $\beta_i < B$, and no sell orders if $\alpha_i > A$. Consequently, a dealer will increase his bid price β_i above other dealers' bids as long as buying is profitable to him, and will reduce his ask price α_i below the competing dealers' offers as long as selling is profitable to him. A dealer is called *neutral* if he is neither a buyer nor a seller, i.e., when his quotes are inferior to the market quotes. A neutral dealer's expected profit (given initial inventory position θ) is $\nu\theta - k\theta^2$.

The differences between dealers are due to the variation in their initial inventory positions θ_i. Intuitively, a dealer with a long position will offer a lower price than a dealer with a short position in order to avoid the inventory holding costs. Similarly, a dealer with a short position will increase his bid to avoid the costs of a short closing position. Thus, each dealer will have a different reservation price, depending on his initial inventory position.

We define the *reservation bid price* (ask, respectively) of a dealer with inventory position θ, $\beta^*(\theta)$ ($\alpha^*(\theta)$, respectively), as the bid (ask) price that drives his expected profit to its neutral level ($\nu\theta - k\theta^2$) when he faces the entire market supply (demand, respectively). We define the *profit-maximizing bid* (ask, respectively) as the bid (ask) price that maximizes his buying (selling) profits, assuming he faces the entire market supply (demand, respectively). The dealer with the highest reservation bid (say $\beta^*(\theta_i)$) can outbid all other dealers. If $\beta^*(\theta_j)$ is the second-highest reservation bid, it is sufficient for dealer i to bid slightly above $\beta^*(\theta_j)$ to attract the entire market supply. Thus, the market bid B will be given by the maximum of the highest profit-maximizing bid and the second-highest reservation bid. Similarly, the market ask A will be given by the minimum of the lowest profit-maximizing ask and the second-lowest reservation ask.

Denoting the ordered inventory position by $\theta_{(1)}$, $\theta_{(2)}$, ..., $\theta_{(N)}$, the dealer with the lowest position ($\theta_{(1)}$) will be the highest bidder, and his bid will be the maximum of the reservation bid of the dealer with position ($\theta_{(2)}$) and his own profit-maximizing bid. Similarly, the dealer with the highest inventory position ($\theta_{(N)}$) will quote the market ask price, equal to the minimum of the reservation ask price of the dealer with inventory position ($\theta_{(N-1)}$) and his own profit-maximizing ask. Thus, unlike the case of the monopoly dealer market, a dealer will be either a buyer or a seller in a given clearing, but not both.

Next, we derive $\alpha^*(\theta)$, $a^*(\theta)$, $\beta^*(\theta)$, and $b^*(\theta)$ as functions of the dealer's inventory position. The profit function of a dealer with initial inventory θ who captures the market supply is given by

$$\pi_B(\beta) = -\lambda\beta^2 + \nu \cdot E[I] - k \cdot E[I^2], \tag{2.1}$$

where I is his closing inventory position. If X denotes the quantity bought by the dealer (a Poisson variate with mean $\lambda\beta$), we have $I = \theta + X$. Hence,

$$\pi_B(\beta) = -\lambda\beta^2 + \nu \cdot E[\theta + X] - k \cdot E[(\theta + X)^2]$$
$$= [\nu\theta - k\theta^2] + \lambda\beta[\nu - 2k\theta - k - \beta(1 + k\lambda)]. \tag{2.2}$$

Thus, $\pi_B(\beta)$ is a quadratic function of β. If the dealer is neutral, his profit is simply $[\nu\theta - k\theta^2]$. The reservation bid at which a dealer is indifferent between buying and not buying is given by

$$\beta^*(\theta) = \frac{\nu - 2k\theta - k}{1 + k\lambda}. \tag{2.3}$$

The profit-maximizing bid of a dealer with inventory position θ is

$$b^*(\theta) = \beta^*(\theta)/2. \tag{2.4}$$

Thus, the equilibrium market bid B^* will be given by

$$B^* = \max\left\{\frac{\nu - 2k\theta_{(2)} - k}{1 + k\lambda}, \frac{\nu - 2k\theta_{(1)} - k}{2(1 + k\lambda)}\right\}. \tag{2.5}$$

Similarly, the profit function of a dealer who captures the market as a seller

$$\pi_A(\alpha) = \lambda\alpha(1 - \alpha) + \nu \cdot E[I] - k \cdot E[I^2]$$
$$= [\nu\theta - k\theta^2] + \lambda(1 - \alpha)[1 - \nu + 2k\theta - k - (1 - \alpha)(1 + k\lambda)], \tag{2.6}$$

implying that his reservation ask price is

$$\alpha^*(\theta) = 1 - \frac{\nu - 2k\theta - k}{1 + k\lambda}. \qquad (2.7)$$

The corresponding profit-maximizing ask is

$$a^*(\theta) = 1 - (1 - \alpha^*(\theta))/2. \qquad (2.8)$$

Thus, the market ask price is given by

$$A^* = \max\left\{1 - \frac{\nu - 2k\theta_{(N-1)} - k}{1 + k\lambda},\ 1 - \frac{\nu - 2k\theta_{(N)} - k}{2(1 + k\lambda)}\right\}. \qquad (2.9)$$

With these models in hand, we are now ready for a comparative study of the performance of different market regimes.

3. Comparative Market Performance.

In this section, we study the performance of the market under four alternative regimes. We distinguish between the various models using a superscript over the variable being studied: the superscript FC identifies the fragmented clearing houses with order intensities $\lambda_1, \lambda_2, \ldots, \lambda_N$; CC identifies the consolidated clearing house with order intensity $\lambda = \sum_{i=1}^{N} \lambda_i$; MD identifies the monopolistic dealer market, and ID is the interdealer market.[14] Our comparative study examines four measures of market performance: the expected quantity traded, the price variance faced by an individual trader, the quality of price signals provided by the exchange process, and the expected gains from trade. Finally, we examine the implementation costs associated with each of the alternative regimes.

3.1. Expected Quantity Traded.

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[14] In models MD and ID, we take $\nu = 1/2$, i.e., the price at the intersection of the expected demand and expected supply functions.

The quantity traded in a market is a direct measure of its performance: higher quantities are usually associated with greater "depth" and higher liquidity. Since the expected quantity traded in the consolidated clearing is

$$E[Q^{CC}] = E[Q|\lambda] = \lambda/2 - 1/4(1 - e^{-2\lambda}), \quad (3.1)$$

and in the fragmented clearing houses

$$E[Q^{FC}] = \sum_{i=1}^{N} E[Q|\lambda_i] = \lambda/2 - 1/4\sum_{i=1}^{N}(1 - e^{-2\lambda_i}), \quad (3.2)$$

we always have

$$E[Q^{CC}] > E[Q^{FC}], \quad (3.3)$$

i.e., consolidation increases the expected quantity traded in a clearing house.

Next, we consider the expected quantities traded in the dealer-based markets. These quantities are obtained by substituting the bid and ask prices into the market demand and supply functions. When $k = 0$ (i.e., there are no inventory costs), the expected quantities bought and sold in the monopoly dealer market are $\lambda/4$, whereas they are twice as large ($\lambda/2$) for the interdealer market. For moderate values of θ, the expected quantities bought and sold in the monopoly dealer market are lower than $E[Q^{CC}]$, but may be either higher or lower than $E[Q^{FC}]$. The expected quantities exchanged in the interdealer market may be higher or lower than the expected quantity traded in the consolidated clearing house.

3.2. Price Variance—Individual Trader.

Next we consider the effects of market fragmentation on the price variance faced by a trader in a single market. Let P_i^{FC} and P_i^{CC} denote the price in submarket i under the respective regimes. Then, for all $\lambda_1, \lambda_2, \ldots, \lambda_N > 0$ and $N > 1$, fragmentation increases the price variance faced by a trader,

$$\mathrm{Var}(P_i^{CC}) < \mathrm{Var}(P_i^{FC}). \tag{3.4}$$

Clearly, the price variance is an increasing function and concave function of N. A comparison to the dealer market regimes is difficult due to the fact that point estimates are to be compared to interval estimates.

3.3. Quality of the Relative Prices.

The process of exchange is important not only for traders, but also for market observers who do not directly participate in the trading process. Various consumption, investment, and production decisions require the knowledge of relative prices, and there are real costs associated with errors in the estimation of these prices. Thus, noisy price signals produced by the exchange process inflict costs on the economy as a whole.

It is tempting to conclude from the results of the previous section that (in the case of the clearing house) fragmentation reduces the quality of price signals. There is no reason to presume, however, that decision makers use in their assessments the outcomes of any single submarket. Rather, they can obtain better estimates by processing the information on the outcomes of trading in each fragmented market. As all markets clear synchronously, an observer of the outcomes can compute the volume-weighted average (or, more precisely, the intensity-weighted average) of the prices obtained in each of the separate submarkets.

Consider the fragmented market regime, and let the weighted-average price, AP^{FC}, be defined by

$$AP^{FC} = \sum_{i=1}^{N} (\lambda_i/\lambda)\cdot P_i^{FC}. \qquad (3.5)$$

The following theorem (proofs are provided in Appendix A) states that the variance of the price obtained by averaging across markets is smaller than the price variance of the consolidated clearing house.

Theorem 1. *For all* $N > 1$, *and* $\lambda_1, \lambda_2, \ldots, \lambda_N > 0$,

$$\mathrm{Var}(AP^{FC}) < \mathrm{Var}(P^{CC}). \qquad (3.6)$$

Theorem 1 demonstrates that fragmentation actually improves the quality of price signals, reducing the weighted-average price variance. To study this effect in detail, we consider the symmetric case where the consolidated intensity λ is obtained by aggregating N equivalent submarkets with $\lambda_i = \lambda/N$. In this case, we obtain

$$\mathrm{Var}(AP^{FC}) = \frac{1}{4\lambda}\cdot\left[1 - \frac{1 - e^{-2\lambda/N}}{2\lambda/N}\right]. \qquad (3.7)$$

It is now easy to verify that equation (3.7) is a decreasing function of N —i.e., fragmentation reduces the weighted-average price variance. One way to interpret this result is to view the effect of fragmentation as the net outcome of two opposite effects: first, the order intensity in each submarket declines as the system is fragmented; as demonstrated above, increasing "thinness" increases the price variance in each separate fragment. On the other hand, since AP^{FC} is an average of the fragmented market prices, there is an effect of price diversification across submarkets, that tends to reduce the overall variance. Theorem 1 then states that the "diversification" effect dominates the "thinness" effect. This follows from the fact that even though the price variance within each submarket is an increasing function of N, this function is concave. Thus, doubling the number of fragments N increases the price variance by less than a factor of 2. On the other hand, the diversification effect is linear in 1/N: doubling N (while maintaining each submarket price variance the same) reduces the weighted-average price variance by a factor of 2. Since

$$Var(AP^{FC}) = 1/N \cdot Var(P_i^{FC}), \qquad (3.8)$$

the net effect of increasing N is that the "diversification" factor 1/N dominates, and the weighted-average price variance is a decreasing function of N.

3.4. Gains from Trade.

A common summary-measure of market performance is the magnitude of gains from trade accruing to market participants. These gains represent a monetary value that traders should be willing to pay to have the exchange in operation. In this subsection, we assess the effects of the market mechanism on this surplus measure.[15]

We first evaluate gains from trade under the clearing house organization. These gains are represented by the area between the demand schedule, the supply schedule, and the price axis. Suppose that the

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[15] The analysis in this section considers only the *gross* surplus. We could further model implementation costs by considering fixed-costs of trading, variable costs of trading, order processing costs, and the costs of clearing shares.

execution price is P, and that a trader j has placed a sell or buy order with limit price p_j. Trader j's gain from trade is $|P - p_j|$ if his order is executed, and zero otherwise. The aggregate measure of traders' gains is thus given by

$$G = \sum_{sells} (P - p_j)^+ + \sum_{buys} (p_j - P)^+, \qquad (3.9)$$

where $x^+ = \max\{x, 0\}$ denotes the positive part of x.

The following theorem provides the expected gains from trade in a clearing house.

<u>Theorem</u> <u>2</u>. *The expected gains from trade in a clearing house are given by*

$$E[G|\lambda] = \frac{1}{4}\lambda - \frac{1}{4} + \frac{1}{8\lambda}(1 - e^{-2\lambda}). \qquad (3.10)$$

Expression (3.11) for the expected market surplus can be written as

$$E[G|\lambda] = \frac{1}{4}\lambda - \lambda \cdot Var(P). \qquad (3.11)$$

The first expression is the "deterministic" traders' surplus corresponding to the expected-demand and expected-supply functions, and the second represents the expected surplus loss due to uncertainty.[16] That is, an increase in the price variance is directly translated into an expected surplus loss per order: the fluctuations of P have an adverse effect on the aggregate of traders gains in an expected-value sense.

Comparing the gains from trade in the consolidated and fragmented clearing houses, we have from Theorem 2:

$$E[G^{CC}] = \frac{1}{4}\lambda - \frac{1}{4} + \frac{1}{8\lambda}(1 - e^{-2\lambda}), \qquad (3.12)$$

---

[16] Note that the expected loss per order is $1/2 \cdot Var(P)$.

and

$$E[G^{FC}] = \frac{1}{4}\lambda - \frac{1}{4}N + \sum_{i=1}^{N} \frac{1}{8\lambda_i}(1 - e^{-2\lambda_i}). \tag{3.13}$$

It is straightforward to verify that

$$E[G^{CC}] > E[G^{FC}], \tag{3.14}$$

i.e., fragmentation reduces the expected gains from trade.

Next consider the gains from trade in the monopoly dealer market. These gains consist of three parts: the dealer's profits, the surplus of public buyers (reflecting the difference between buyers' reservation prices and the ask price), and the surplus of public sellers (resulting from the difference between sellers' reservation prices and the bid price). The gains from trade for the dealer market are given by the following theorem.

<u>Theorem</u> <u>3</u>.

$$E[G^{MD}] = \nu\theta - k\theta^2 + \frac{1}{8}\lambda(1 - 2k)^2 + \frac{2\lambda\theta^2k^2}{1 + 2k\lambda} + \frac{1}{2}\lambda(\beta^*)^2 + \frac{1}{2}\lambda(1 - \alpha^*)^2, \tag{3.15}$$

where β^* *and* α^* *are given by equations* (1.10) *and* (1.11), *respectively.*

Finally, consider the interdealer market. Here, the market surplus is the sum of (i) the expected profit of the dealer quoting the bid price, $\pi_B(B^*)$ (where $\pi_B(\cdot)$ is given by equation (2.2), and B^* —by equation (2.4); (ii) the expected profit of the dealer quoting the ask price $\pi_A(A^*)$ (given by equations (2.5) and (2.7)); (iii) the expected surplus of public sellers, given by

$$E[G_S] = 1/2\lambda(B^*)^2; \tag{3.16}$$

and (iv) the expected surplus of public buyers,

$$E[G_B] = 1/2\lambda(1 - A^*)^2 \qquad (3.17)$$

(the latter two follow as in Theorem 3). Thus, it is straightforward to compute the expected gains from trade. The overall surplus of the interdealer market depends on the dealers' inventory positions, but will typically be between the surplus of the monopoly dealer market and that of the consolidated clearing house.

3.5. Implementation Costs.

As already pointed out, the surplus measure studied in the previous subsection is incomplete since it does not take into account the costs of the exchange process, and, in particular, the processing and communication costs.[17] The latter component, i.e., the cost associated with the communication infrastructure of the exchange, is of special interest in the centralization context.

We consider a market consisting of N identical submarkets with total order intensity λ, and classify the exchange implementation costs (E) into four components: (i) fixed costs denoted by F per exchange; F depends on the market mechanism;[18] (ii) variable (volume dependent) data-communication costs, assumed proportional to the expected number of orders communicated from the submarkets (we denote the cost per order communicated by γ, and assume that γ is independent of the market mechanism); (iii) order-processing costs; we assume that the processing cost per order is a

[17] The analysis of the market mechanisms involving dealers who actively take a position already accounted for the associated costs.

[18] F also includes fixed data-communication costs that are independent of the order volume (and are necessary to support the market mechanism).

constant f, depending on the market mechanism; and (iv) the cost of clearing shares that were actually exchanged—we assume that the marginal cost per share is a constant e, and that the clearing cost is independent of the market mechanism.[19]

Consider first the mechanisms based on the clearing house procedure. Denote the corresponding fixed cost by F^C (per clearing house) and the order-handling cost by f^C (per order). In the consolidated clearing house, each buy and sell order must be communicated to the central exchange, hence the expected data-communication cost is $2\gamma\lambda$. In the fragmented clearing houses, orders are executed locally, hence,

$$E^{CC} = F^C + 2\gamma\cdot\lambda + f^C\cdot\lambda + e\cdot E[Q^{CC}], \tag{3.18}$$

and

$$E^{FC} = N\cdot F^C + f^C\cdot\lambda + e\cdot E[Q^{FC}]. \tag{3.19}$$

As seen in equations (3.18) and (3.19), an important benefit of fragmentation is the reduction in order-communication requirements. This is achieved at the cost of a deterioration in market performance (as demonstrated above) as well as incurring the fixed cost F^C in each of the submarkets rather than once.

Next consider the dealer-based market mechanisms. Denoting the fixed[20] and order handling costs of the monopoly-dealer market regime by F^{MD} and f^{MD}, respectively, we have

$$E^{MD} = F^{MD} + (\gamma + 1/2e)\cdot\lambda(\beta^* + 1 - \alpha^*) + \lambda f^{MD}, \tag{3.20}$$

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[19] Alternative assumptions are also possible; they will not change the essence of our results.

[20] Recall that we included the cost of disseminating quotations, which is independent of the order volume, in F^D.

where $\lambda(\beta^* + 1 - \alpha^*)$ is the sum of the expected quantities bought and sold. Note that the order-communication volume has been reduced from $2\gamma\lambda$ in the consolidated clearing house to $\gamma\lambda(\beta^* + 1 - \alpha^*)$ under the monopolistic dealer market. This is because, unlike the consolidated clearing house, which requires transmission of all orders, here, sell orders at prices that are higher than the bid or buy orders at prices lower than the ask need not be communicated, since they will not be executed. Thus, by quoting bid and ask prices, the dealer provides useful information that enables a reduction in the order-communication volume. This, however, is achieved at the cost of disseminating quotations (included in F^{MD}) as well as the surplus loss due to the dealer's monopoly.

Finally, consider the interdealer market. Let F^{ID} denote the fixed cost per dealer, and let f^{ID} denote the order-handling cost. As in the monopoly dealer market, only orders that will be executed need to be communicated. The resulting order-communication volume, $\lambda(B^* + 1 - A^*)$, is greater than that of the monopoly dealer market, but this is because the trading volume increases. This order-communication volume is substantially below the corresponding volume in the consolidated clearing house, despite the fact that the trading volume is not substantially lower: the availability of open quotations enables transactors to know in advance whether their orders will be executed. As a result, there is no need to communicate orders that would not be executed against the inside market quotes. Thus, for the interdealer market,

$$E^{ID} = N \cdot F^{ID} + (\gamma + 1/2e)\lambda(B^* + 1 - A^*) + \lambda f^{ID} \qquad (3.21)$$

Ranking the alternative regimes by increasing order-communication costs, we have FC (the lowest), then MD, then ID, and, finally, CC. While the dissemination of quotations by the dealers in regimes MD and ID are costly, they enable a substantial reduction in the order-communication costs: since traders become aware of the current state of the market, it becomes unnecessary to submit orders that would not qualify for execution. The price leadership provided by the dealer based quotations is an important service that increases the efficiency of the exchange process. As we have shown, interdealer competition substantially reduces the costs charged by dealers for providing these services.

4. Model Robustness to Distributional Assumptions.

In order to make our model tractable, we employ assumptions and operating characteristics which only approximately describe reality. This section explores the robustness of our findings to several of the key assumptions we need in order to solve the model.[21]

Researchers discussing the behavior of securities markets commonly identify two types of investors: *liquidity traders* that lack information and merely want to convert securities into cash or vice-versa; and *informed traders* who transact with special information. Our study focused only on the former type. A trader motivated by liquidity will either buy or sell the asset, depending on his current liquidity preferences. As first described by Bagehot (1971), the statistical nature of this ensemble of traders creates unsystematic and independent shocks to market clearing so

[21] We provide further discussion of the implications of our distributional assumptions in Appendix B.

that "market price is virtually unaffected by these errors." (p. 14) By studying this population of traders, characterized by random demand and supply schedules, we are able to provide quantitative and qualitative insights into the effect of the market mechanism on the performance of a market.

The assumed independence between the demand process and the supply process is consistent with the nature of liquidity-motivated transactions: sell orders are generated by transactors who are willing to convert the traded asset into cash, while buy orders originate from a different population of traders who are willing to convert cash into the traded asset. The Poisson assumption for the arrival of traders to the marketplace and to demand transaction services is appropriate when there is a large number of potential traders who act independently in the placement of orders and are statistically interchangeable. This interpretation enables a complete analysis of market performance. The key to the analysis lies in the application of concepts from the study of stochastic processes and renewal theory. Both the independence and Poisson assumptions have been made in numerous studies of market microstructure[22] and are entirely appropriate in the study of arrival processes such as the one we set out to model in this paper.

An alternative approach would be to model the aggregation of information in market prices using a richer population of traders. Kyle (1985, 1989) developed models of trade in financial markets with both informed and uninformed participants engaging in either imperfect or

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[22] See the references and discussion in Footnote 7.

strategic competition. In this class of models, researchers typically must specify the distribution of information among traders, their endowments, utility functions, and risk aversion in order to discuss an analytically tractable model of trade. Kyle analyzes a market in which many trades are batched together and processed at a price that is calculated to give zero expected profits to the market maker given the total order flow. In Kyle's model, prices are not posted but instead are determined after traders submit orders and the size of the total order imbalance is observed.[23]

With normally distributed information increments, Kyle (1985) finds that the variance of the price process is a *decreasing linear function* of the order intensity within a periodically cleared call market. Admati and Pfleiderer (1988) use Kyle's framework to consider the relationship between price variability and order intensity. They argue that the variance of price changes is *constant* across time periods and should be independent of the amount of information in the market and the total order flow.[24] They explain that this result occurs because trading patterns arise endogenously as a result of the strategic interaction of liquidity traders and informed traders. Intuitively, informed traders act to keep price variability constant so that liquidity traders cannot deduce their information.

The implications of the Kyle and Admati and Pfleiderer models contrast with the results developed in Sections 1 and 3 which arise from the fact that the variance of the price process is a *decreasing and convex function*

[23] Admati and Pfleiderer (1989) extend this model to include a more realistic trading mechanism and find that intraday and day-of-the-week mean asset return effects are possible.

[24] Examine, in particular, Admati and Pfleiderer (1988), Proposition 3, p. 19, and for the diverse information case, p. 25.

of order intensity. If we use Kyle-style model of trade in financial markets to study market performance, then the results in Theorem 1 as well as our other results that rely on the decreasing and convex relationship between price variability and order intensity property no longer obtain without additional assumptions. However, we would argue that neither model is a complete depiction of reality and our model has a simpler, more plausible description of both the arrival of liquidity traders and the manner in which the trading mechanism affects market performance.

5. Conclusions.

This paper studies trade-offs between market consolidation and fragmentation, using a number of models of market organization and a number of measures of market performance. The classical comparison is between a consolidated clearing house and fragmented clearing houses. Our results demonstrate that, for the clearing house market regime, fragmentation reduces the expected quantity traded, increases the price variance faced by individual traders, and reduces the expected gains from trade. Furthermore, we demonstrate that fragmentation may improve the quality of market price signals when they are aggregated to yield the cross-market weighted average price variance. This variance is lower in the fragmented regime than in the consolidated regime, since the price "diversification" effect across submarkets dominates the "thinness" effect within submarkets. Finally, we study the performance of dealer-based market regimes, demonstrating how interdealer competition affects our measures of market performance.

We do not purport to suggest that certain market organization is superior or "optimal." The diversity of exchange mechanisms that prevail around the world as well as across assets reflects the dependence of the appropriate market design on specific circumstances and on factors that are probably not captured by the stylized facts of the market microstructure literature. It is hoped, however, that these models can contribute to our understanding of the basic trade-offs and to the definition and evaluation of performance measures, in addition to the provision of an analytic foundation that can serve as the basis for more detailed and intricate analyses.

APPENDIX A

Proof of Theorem 1. It is easy to verify that the function

$$g(x) = x - 1 + e^{-x} \tag{A.1}$$

is strictly convex, and $g(0) = 0$. Thus, for all x_1, $x_2 > 0$,

$$g(x_1 + x_2) > g(x_1) + g(x_2); \tag{A.2}$$

substituting $x_i = 2\lambda_i$ $(i = 1, 2)$ and rearranging terms, we obtain

$$2(\lambda_1 + \lambda_2) - 1 + e^{-2(\lambda_1+\lambda_2)} >$$
$$(2\lambda_1 - 1 + e^{-2\lambda_1}) + (2\lambda_2 - 1 + e^{-2\lambda_2}) \tag{A.3}$$

that is equivalent to

$$\frac{1}{4(\lambda_1 + \lambda_2)} \cdot \left[1 - \frac{1 - e^{-2(\lambda_1+\lambda_2)}}{2(\lambda_1 + \lambda_2)}\right] > \left[\frac{\lambda_1}{\lambda_1 + \lambda_2}\right]^2 \cdot \frac{1}{4\lambda_1} \cdot \left[1 - \frac{1 - e^{-2\lambda_1}}{2\lambda_1}\right]$$
$$+ \left[\frac{\lambda_1}{\lambda_1 + \lambda_2}\right]^2 \cdot \frac{1}{4\lambda_2} \cdot \left[1 - \frac{1 - e^{-2\lambda_2}}{2\lambda_2}\right]. \tag{A.4}$$

However, in light of equation (1.6), this inequality implies the theorem for $N = 2$; the result for $N > 2$ now follows easily by induction on N. ∎

Proof of Theorem 2. The analysis of traders' surplus utilizes the variables

$$C_m = \sum_{j=1}^{m} (\tau_j^D + \tau_j^S); \quad m = 1, 2, 3, \ldots \tag{A.5}$$

where C_1, C_2, C_3,... are the renewal epochs of a renewal process generated by the variables $\{\tau_j^D + \tau_j^S\}_{j=1}^{\infty}$, that are Erlang variates possessing scale parameter λ and shape parameter 2.[25]

The gains from trade are given by

$$G = \sum_{n=1}^{Q}\left(1 - \sum_{j=1}^{n}\tau_j^S - \sum_{j=1}^{n}\tau_j^D\right) = \sum_{n=1}^{Q}(1 - C_n) = Q - \sum_{n=1}^{Q}C_n, \tag{A.6}$$

and the expected gains are thus

$$E[G] = E[Q] - E\left(\sum_{n=1}^{Q}C_n\right). \tag{A.7}$$

Now, let M denote the number of renewals of the Poisson process with interarrival intervals $\tau_1^D, \tau_1^S, \tau_2^D, \tau_2^S, \ldots,$ in the interval [0, 1]. Conditioning on the value of M, we distinguish between two cases:

(i) M = 2m. Here,

$$E\left(\sum_{n=1}^{Q}C_n | M = 2m\right) = \sum_{n=1}^{m}E[C_n | M = 2m]. \tag{A.8}$$

Now, given M = 2m, the nth renewal epoch of the Poisson process is distributed as the nth order statistic from a sample of (2m) i.i.d. variates.[26] That is, C_n is distributed as the (2n)th order statistic of such a sample, and

$$\sum_{n=1}^{m}E[C_n | M = 2m] = \frac{m(m + 1)}{2m + 1} = \frac{1}{2}m + \frac{m}{2(2m + 1)}. \tag{A.9}$$

(ii) M = 2m + 1. Here,

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[25] An Erlang-n distribution is commonly used to describe the successive arrival times at a fixed boundary. See Parzen (1962), Ch. 5.

[26] See Parzen (1962), Ch. 5, or Ross (1970), Ch. 2).

$$E\left[\sum_{n=1}^{Q} C_n \mid M = 2m + 1\right] = \frac{1}{2}m. \tag{A.10}$$

The theorem follows by combining parts (i) and (ii) and summing over n. ■

Proof of Theorem 3. First, the dealer's profits are given by

$$\pi(\beta^*, \alpha^*) = [\nu\theta - k\theta^2] + \frac{1}{8}\lambda(1 - 2k)^2 + \frac{2\lambda\theta^2k^2}{1 + 2k\lambda} \tag{A.11}$$

obtained by substituting equations (1.10) and (1.11) in to equation (1.9) and rearranging terms. The rest of the gains accrue to public transactors. Letting G_S denote the surplus of public sellers, we have

$$G_S = \sum_{n=1}^{N_S(\beta^*)} \left(\beta^* - \sum_{j=1}^{n} \tau_j^S\right). \tag{A.12}$$

To evaluate $E[G_S]$, assume first that $N_S(\beta^*) = m$ is given. Given $N_S(\beta^*) = m$, the renewal epochs $\sum_{j=1}^{n} \tau_j^S$ are distributed as m order statistics from a uniform distribution over $[0, \beta^*]$, hence,

$$E[G_S \mid N_S(\beta^*) = m] = m \cdot (\beta^*/2). \tag{A.13}$$

But since $N_S(\beta^*)$ follows a Poisson distribution with expected value $\lambda\beta^*$ (recall section 2.1), we have

$$E[G_S] = 1/2\lambda(\beta^*)^2. \tag{A.14}$$

A similar argument regarding the surplus of public buyers completes the proof. ■

APPENDIX B

In this section we address the issue of how the distributional assumptions in the model can be altered. In particular, we examine whether the normal distribution can be substituted for the stochastic process hypothesized. We address this issue in two ways: first, we examine whether a normal distribution can be used to describe a renewal process, and second, we study the implications of replacing the Poisson price process with a Brownian Motion price process.

B.1. The Renewal Process.

By definition, a *renewal process* is a sequence of nonnegative, discrete, independent and identically distributed random variables. The model in Section 1.1 employs a renewal process in the form of a Poisson process to model the expected number of buy (sell) orders per unit price or the order intensity. This counting process has the property that the times between successive events are independent and identically distributed exponential random variables.

There are four reasons why we cannot simply substitute a normal distribution for the Poisson distribution in this model. First, the normal distribution is continuous whereas the expected number of orders per unit time must be discrete. For example, our model cannot handle the case in which 3 buyers attempt to transact with 3.5 sellers. Second, the normal distribution takes on values over the entire real line. We cannot model 3 buyers attempting to transact with -3.5 sellers. This example is ill-defined as a renewal process. Third, the normal distribution lacks the property that it be increasing in time. In order to solve the model, we need the property that each successive interarrival time be greater than or equal to the previous interarrival time. The normal distribution does not yield this property. Finally, the normal distribution cannot serve as a renewal process because it takes on values between negative infinity and positive infinity. In other words, under a normally distributed renewal process (if one existed), it would be theoretically possible that an infinite number of renewals can occur in a finite amount of time. This possibility is explicitly ruled out in renewal theory since the distribution of the number of events can only be obtained if and only if they occur in a finite length of time. A normally distributed model will not rule this case out.

In conclusion, we cannot simply insert a normal distribution when we model aggregate order flow since that distribution is neither discrete, increasing in time, non-negative, or finite. Only a renewal stochastic process of the type developed in Section 1.1 can adequately model order intensity. Of course, it is not strictly necessary to replace the distributional assumption surrounding the renewal process. One could simply assume a different price price. We explore this possibility next.

B.2. Developing an Alternative Model of the Price Process.

In Section 1.1, we assume that the market demand and supply are obtained by the aggregation of individual schedules that depict stochastic processes in price space, assumed to be independent Poisson processes. As discussed in Duffie (1988, p. 136), "A real-valued stochastic process N is a *Possion process* on (Ω, F, P) with parameter $\lambda > 0$ provided: (a) for any $0 \leq s < t \leq \infty$, $N_t - N_s$ is a random variable with Poisson distribution having expected value (and variance) $\lambda(t - s)$." Furthermore, Duffie develops this stochastic process as an acceptable model of price dynamics under uncertainty and as an abstract model of the revelation of information through time. Our model uses this basic class of stochastic process to model the price process. One implication of this modeling decision is that we observe in equation (1.6) that the execution price variance is a decreasing and convex function of the order intensity, λ.

An alternative model of price dynamics under uncertainty is to use a real-valued stochastic process B defined to be a *Standard Brownian Motion* on (Ω, F, P) provided: "(a) for any given $0 \leq s < t \leq \infty$, $B_t - B_s$ is a normally distributed random variable with expected value equal to zero and variance equal to t - s." (Duffie, 1988, p. 137) In this case for a given trading interval, the price variance is a constant function, independent of the order intensity.

Within this framework, we contend that altering the call market structure yields little in terms of substantive comparative statics. First, the nature of the model is unchanged by having a single consolidated clearing house or a fragmented clearing house. This is true because the variance of a sum of uncorrelated random variables is equal to the sum of the variances of each random variable. Hence, having a single consolidated clearing house or a fragmented clearing house would make no difference to any of our measures of market performance. Operationally, the market performance under both types of market structure would be equivalent.

Furthermore, the model is unaltered by replacing one monopoly dealer with multiple competitive dealers. We observed that the expected net change in a single dealer's inventory as a result of trading with the public is a decreasing function of his starting inventory (reflecting the dealer's desire to balance his position) and a decreasing function of the order intensity of his market. If we replaced the Poisson assumption with the normal model assumption, then the single dealer's inventory change would be a function of his starting inventory only, and be independent of the order flow observed in the market yielding results analogous to those observed in the consolidated and fragmented clearing house case explored above. When there are multiple dealers, their demands aggregate to the demand of a single dealer in the normal price process case. This result demonstrates that our findings are not robust to a change in the distributional assumption.

In conclusion, the degree of market centralization has no effect on market performance under the normal model assumption because of the inherent linearity in that model. The price variance across a number of clearing house markets aggregates to the same value as the price variance in a single market. Finally, the addition of multiple dealers does not change market performance in the Standard Brownian Motion process case since dealers would continue to base their quotes on their inventory positions, independent of the market's order intensity. However, as discussed above, we contend that a Standard Brownian Motion process is not an appropriate distribution with which to study the problem we address.

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PAPER 3:

INFORMATION AGGREGATION, BUBBLES, AND HERD TRADING.

0. Introduction.

The possibility that asset prices might deviate from intrinsic values based on market fundamentals, because of 'speculative bubbles' or 'fads,' has long intrigued economists. Prices might drift away from intrinsic values because social forces create fads or fashions in asset markets, as in markets for cars, food, houses, and entertainment. Many of the oldest studies of the business cycle focused on asset price instability, and posited that some collective 'mania' occasionally caused investors to bid up asset prices to unsustainable levels, eventually but inevitably ending in a 'panic' as prices crashed. Keynes (1936) in his famous 'beauty contest' metaphor pointed out that asset traders who attempt to profit from short-run price fluctuations must rationally compare price with their expectations of others' expectations, rather than their own estimates of fundamental values, so perhaps investor behavior is not entirely irrational.[1]

Our principle goal in this paper is to provide a model of asset trading that is consistent with semi-strong efficient markets and trader rationality, but leaves room for at least moderate 'bubbles' in asset

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[1] Several modern writers have related this Keynesian theme to the multiplicity of rational expectations equilibria in many models. For example, Azariadis (1981) suggests that bubbles should be though of as instantaneous transitions, triggered by extraneous events, between different 'self-fulfilling prophecy' (or 'sunspot') equilibria. The perfect coordination of expectations (and actions) in this approach, however, appears quite at odds with the turmoil and confusion generally associated with historical bubbles.

prices arising from imperfect aggregation of private information. To provide a context for our ideas, we begin with a brief review of previous approaches to bubbles, or discrepancies between an asset's market price and its fundamental value.

Perhaps the most popular approach to modeling bubbles in recent years is based on the observation (originally due to Hahn (1966)) that no-intertemporal-arbitrage conditions do not yield a unique price path in most perfect foresight or Rational Expectations Equilibrium asset-market models. Typically, one has a convergent saddle-path identified as the 'fundamental,' as well as other price paths that diverge from the saddle path at an exponential rate. One cannot always eliminate such divergent paths by transversality conditions, particularly in stochastic versions of the model. For instance, Blanchard and Watson (1983) gives examples of such bubbles with random lifetimes that grow at a known exponential rate (the discount rate plus a risk premium) until they burst. Researchers have rarely explicitly discussed the information conditions in models of this type, but it is hard to avoid the interpretation that once the bubble has started, everyone knows that price is above fundamental value and therefore the game has an expected negative sum for current and future transactions in the asset market, as in a Ponzi scheme or chain letter. Tirole (1982) points out that such bubbles are impossible in Rational Expectations Equilibrium once the negative sum aspect is common knowledge, except for special cases in which participants are able to pass the losses on forever to later entrants. Nevertheless, this 'exponential rational bubbles' approach has generated numerous articles.

An alternative rational expectations view of bubbles posits an incomplete information game between ordinary speculators, better informed speculators, and liquidity-motivated transactors. In this class of models, one can demonstrate that in perfect Bayesian-Nash equilibrium the fundamental value (based on the aggregate information) can differ with positive probability from the transacted price. The better-informed speculators (a non-negligible fraction of the market) must be able to form an effective cartel for such a bubble to arise. Such an 'information-monopoly bubble' reminds one of some popular accounts of the Hunt brothers' silver market activities in the late 1970s. For example, Summers and his coauthors (e.g., DeLong *et. al.* (1990)) introduced an approach in which irrational traders create excess volatility that rational traders cannot eliminate by arbitrage. Indeed, rational speculation may actually reinforce discrepancies between the fundamental value and price created by irrational 'noise' or 'positive feedback' traders.

Our own model emphasizes heterogeneous beliefs, but we do not assume behaviorally distinct types of traders. In the spirit of Hirshleifer's (1989) critique of Tirole, we study a dynamic market in which traders receive heterogeneous private information ('news'), trade, observe the price, and receive more 'news,' trade again, etc., over many periods. Our focus is the extent to which price aggregates the diverse news. Under general specifications regarding expectation formations, we show that aggregation is imperfect and that substantial discrepancies can arise between price and fundamental value.

We take care to model the market institution through which traders' decisions yield transactions and observed prices, since this institution largely determines the extent and timing of the public information conveyed by prices. We employ a simplified version of the Clearinghouse institution (sometime referred to in the literature as a *call market* or a *sealed bid-offer auction*) because it is widely used in practice[2], analytically tractable, and bears some resemblance to the Walrasian institution that is usually employed in theoretical discussions (but almost never used in practice).[3]

In other respects, we keep our model as rudimentary as possible. To distinguish our model clearly from its predecessors, we employ the following simplifications: (1) traders pursue buy-and-hold strategies, eliminating 'beauty contests' bubbles; (2) traders neglect the possibility that they may affect prices, eliminating 'information-monopoly' bubbles; and (3) traders are otherwise rational wealth-maximizers, eliminating the most obvious 'lemming bubbles.'

A final modeling choice deserves brief discussion. It is convenient to avoid specifying risk preferences because they do not play a central role in our view of information aggregation. However, a risk-neutral agent will want to take an arbitrarily large position if he perceives even a

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[2] See Schwartz (1988) for a discussion of worldwide trading systems.

[3] In our view, the Walrasian auctioneer institution is inappropriate for serious modeling of information aggregation despite its widespread use for this purpose. For example, in his classic rational expectations equilibrium model Grossman (1976) assumes that traders are able to observe equilibrium prices before submitting their demand schedules to a Walrasian auctioneer, a logistical impossibility. (Also see the critical comments of Allan Kraus following the Grossman article.) Here, we work with an explicit, feasible market institution and seek updating rules that can be applied even outside of equilibrium.

small price discrepancy, so difficulties arise when agents' perceptions differ. We avoid the problem by imposing arbitrary limits on position sizes—a choice that we feel (given bankruptcy costs, impediments to short-selling, etc.) does not stray too far from current practice.

In the next section we introduce the market structure and present a parametric example to build intuition and sharpen the issues. We begin the following section with a rather general formal specification of private information and Bayesian updating processes, and then state several analytical results.

We derive a formula for the optimal trader action and investigate its dependence on past prices in the first two Propositions. Next, we characterize the market clearing price and obtain relations between this price, the fundamental, and the unobservable 'true value' of the asset. The rest of our results concern the dynamic behavior of bubbles: we find that they can be 'self-feeding' and lead to 'herd trading' in that once started, small positive (or negative) bubbles tend to grow (Proposition 6) but that they eventually are self-correcting in that massive positive (or negative) bubbles tend to shrink (Proposition 5). We also discuss the comparative statics of the information arrival process, and argue that, other things equal, bubbles tend to be larger when information is 'lumpier,' or less frequent but more precise. For example, since news is likely to be lumpier in this sense for a smaller company than for a large diversified company, this result suggests that a small company mutual fund has unusual profit opportunities to buy at the bottom of a negative bubble and to sell at the top of a positive bubble (and also greater risks of doing the opposite). Of course, it is central to our analysis that no trader really knows whether there is a bubble, but each trader must make his own estimate.

The final section summarizes our results and discusses the empirical implications and the relevant empirical literature. The Appendix contains the proofs and some calculations.

1. The Asset Market: An Example.

At some given time T in the future, each of M indivisible shares in some venture will pay off \$X.[4] We assume here that it is common knowledge that X is an exponentially distributed random variable with mean $1/\lambda$. The true value $\bar{\lambda}$ of λ is unknown.

N risk neutral traders may exchange their shares for cash in a simple Clearinghouse market (the rules are specified below) at times $t = 1, 2, \ldots, T-1$. We assume that traders pursue buy-and-hold strategies oriented to period T wealth, and that they neglect the possibility that they can influence prices.[5] We also assume a constant discount rate r and exogenous limits on traders' position sizes; for simplicity we set $r = 0$ and set the lower position limit at 0 (no short sales) and the upper limit

[4] Approximate counterparts of such 'ventures' in contemporary financial markets include European call options, takeovers in which the firm is to be liquidated at a specified date, and the conversion of a closed end mutual fund to an open end fund at a specified date. Theorists' 'state-contingent claims' also fall into this class. See Glosten and Milgrom (1984) for a tradition-based justification of assets similar to our 'venture.'

[5] Satterthwaite and Williams (1989) theoretically demonstrate that the ability to influence price in a Clearinghouse market is negligible even with only a few traders.

at 1. For present purposes, we make the convenient assumption there are $N = 2M - 1$ traders with M shareholders and $M - 1$ non-shareholders.

Our simplified Clearinghouse works as follows. At the end of each period t, each trader privately submits a single 'limit order' $\nu_{it} \geq 0$, sometimes referred to as a 'bid.' For a shareholding trader i, this order is an offer to sell his share at any price exceeding ν_{it}.[6] Thus one obtains the supply curve $S_t(p)$ by arranging shareholders' orders in ascending order and the demand curve $D_t(p)$ by arranging non-shareholders' orders in descending order. The clearing price p_{t+1} is set at the (highest) intersection of D_t and S_t, by the rule:

$$p_{t+1} = \max\{p: S_t(p) \leq D_t(p)\}, \tag{1.1}$$

and is announced (if any transactions occurred) at the beginning of period $t + 1$. The corresponding orders are executed at the same time, so (apart from ties) the M highest bidders are the shareholders at time $t + 1$ and the clearing price is always the Mth highest (here the median) bid.

The focus of our analysis is the information flow. We assume that in each period each participant costlessly receives, with known probability $\pi > 0$, some private information ('news'), denoted z_{it}. In our parametric example, news is an independent realization from a Gamma distribution with mean $\bar{a}/\bar{b}$ and variance $\bar{a}/\bar{b}^2$, where $\bar{a} > 1$ is a known positive constant and $\bar{b} = \overline{\lambda a}$. This distribution is convenient because its population mean is

$$\bar{a}/(\overline{\lambda a}) = 1/\bar{\lambda} = E(X|\bar{\lambda}) = \bar{X}, \tag{1.2}$$

[6] The requirement that all traders submit limit orders is not restrictive. A seller can in effect submit a market order by specifying $\nu = 0$, and can effectively withdraw from the market by specifying ν at an absurdly high level; and analogously for buyers.

the 'true' expected payoff, and its variance is proportional to the known parameter $\bar{a}$.

We directly compute the impact of such news using the basic properties of the Gamma distribution.[7] Suppose that the (conjugate) Gamma distribution $g(\lambda | a, b)$ with $a > 1$ and $b > 0$ fully summarizes traders' prior beliefs regarding payoffs. Then a news message $z = z_{it}$ induces posterior beliefs which are also Gamma with parameters $a + \bar{a}$ and $b + \bar{a}z$, i.e., the posterior distribution is $g(\lambda | a + \bar{a}, b + \bar{a}z)$. It follows that the prior expected payoff is

$$\bar{x}^0 = b/(a - 1), \tag{1.3}$$

and the posterior expected payoff is

$$\bar{x} = (b + \bar{a}z)/(a + \bar{a} - 1). \tag{1.4}$$

We also must consider the impact of publicly observed transactions prices p_t. Since neither these prices nor their first differences Δp_t generally have a Gamma distribution, there is no tractable formula for the precise impact of such public information. However, it is reasonable to suppose that the main impact of an observed price change is a proportional change in the expected payoff, and that the effect on the precision parameter (a) is negligible. To extend the present example, we therefore assume that beliefs summarized in the Gamma distribution $g(\lambda | a, b)$ are modified to $g(\lambda | a, b + c\Delta p)$ after participants observe the price change Δp, for some $c \geq 0$. We can justify this formula as a first-order approximation in Δp, with the parameter c depending (inversely) on the amount of

[7] For example, see DeGroot (1970).

exogenous 'noise' (or perhaps liquidity) trading in the market.[8] The cumulative impact of public information $\{p_0, p_1, \ldots, p_t\}$ on beliefs in this formulation is therefore a shift in the b parameter of

$$\sum_{s=1}^{t} c\Delta p_s = c(p_t - p_0). \tag{1.5}$$

Next, we analyze trader behavior and market outcomes. Suppose a trader (index i is suppressed in the next two paragraphs) enters period t with beliefs $g(\lambda | a_{t-1}, b_{t-1})$, observes the p_t (generated from period (t - 1) orders) and receives news z_t. Thus, his new beliefs are $g(\lambda | a_t, b_t)$, where $a_t = a_{t-1} + \bar{a}$ and $b_t = b_{t-1} + \bar{a}z_t + c(p_t - p_{t-1})$. At first one might suppose that our buy-and-hold assumption leads to an optimal order price of

$$\nu_t = \bar{x}_t = E(X | a_t, b_t) = b_t/(a_t - 1), \tag{1.6}$$

but further reflection discloses a variant of the 'winner's curse' problem[9]: generally, a price change will be required for the order to be

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[8] Although c can also depend on the time period and/or the current parameter values a and b, e.g., in Rational Expectations Equilibrium, we will treat it as a constant for simplicity in this section. Assumption (A3) in Section 2 below is much less restrictive.

We could explicitly model 'noise traders' whose activity is driven by exogenous liquidity shocks. For instance, a trader may sell because he has an unanticipated need for cash. Therefore, his sale does not necessarily imply that he received new unfavorable information, and there is a 'surplus' available for other traders. Absent such noise, only an individual who believed he had superior information would attempt to transact, and therefore, transaction would not occur in equilibrium, as Tirole (1982) has pointed out. We were not able to obtain any further insights by explicitly considering liquidity shocks and so we omit them in the present model to keep our notation and statement of results less cumbersome. However, liquidity shocks remain implicit in the present model as an explanation of traders' willingness to trade and as the underlying determinant of the parameter c, or more generally, of the price response sensitivity η specified in assumption (A3) below.

[9] Milgrom and Weber (1982), among others, discusses the 'winner's curse' in the context of first-price, sealed-bid, common-value auctions. Welch (1992) develops a model of learning and herd behavior in the context of the

executed. If a trader does not condition his order on this prospective price change, then he often will pay more (or receive less) for the asset than he is willing *ex post*.

Therefore, a prospective buyer (a non-shareholding trader) optimally chooses his bid ν_t by solving

$$\max_{\nu \geq 0} \mathrm{E}(X - p_{t+1} | [p_{t+1} \geq \nu], a_t, b_t, p_t)\Pr[p_{t+1} \geq \nu], \tag{1.7}$$

that maximizes the expected gain $(X - p_{t+1})$ conditioned on the event $[p_{t+1} \geq \nu]$ that the bid will be executed and conditioned on current information (a_t, b_t, p_t). The first order-condition for this problem is

$$\nu = \mathrm{E}(X | a, b, \Delta p_{t+1} = \nu - p_t) = (b + c(\nu - p_t))/(a - 1), \tag{1.8}$$

and it follows (restoring subscripts) that the optimal bid is

$$\nu_{it} = (b_{it} - cp_t)/(a_{it} - c - 1). \tag{1.9}$$

A similar calculation for a prospective seller (a shareholding trader) yields the same formula. Note that

$$\nu_{it} \gtrless \bar{x}_{it} = b_{it}/(a_{it} - 1) \text{ as } \bar{x}_{it} \gtrless p_t \text{ for } c > 0, \tag{1.10}$$

as a result of Lemma 1 of the Appendix. In particular, bids and expectations have the same ordering over traders $i = 1, \ldots, N$.

By the Clearinghouse rules, the price p_{t+1} will be the median bid $\nu_{(M)t}$, and traders with higher bids will be shareholders at $t + 1$ (having purchased or retained their shares) while those with lower bids will be non-shareholders (having sold or failed to purchase shares). The expectations $\bar{x}^0_{it+1}$ at the beginning of period $t + 1$ (with p_{t+1} having been announced) will have the same ordering with shareholders more optimistic

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market for IPOs that relies a winner's curse argument. Copeland and Galai (1984) and Glosten and Milgrom (1985) analyze a similar phenomenon in the context of specialist markets.

(higher $\bar{x}^0$) than non-shareholders, but as news arrives during the period, the ordering of expectations may be altered and transactions (announced at the beginning of period t + 2) can result.

We can usefully compare the Clearinghouse-generated price p_{t+1} to the fundamental value, f_{t+1}, the expected payoff conditioned on all information received by market participants up to time t. If traders receive news messages $\{z_1, \ldots, z_n\}$ in periods $\{1, \ldots, t\}$, then

$$f_{t+1} = (1/n)\sum_{j=1}^{n} z_j, \tag{1.11}$$

the *sample mean* of news.

Simulation Experiment. We ran a series of Monte Carlo simulations of this parametric example to compare price to fundamental. A typical simulation appears in Figure 1, Panel A; the 'true value' of the expected payoff $\bar{X} = 1/\bar{\lambda}$ is normalized to 1.00, and traders' time 0 priors for λ all have $a_{i0} = 2$ and $b_{i0} = 1$ so they are unbiased. There are $M = 5$ shares and $N = 9$ traders, each with price-sensitivity $c = 0.5$. News arrives with probability $\pi = 0.5$ and has precision parameter $\bar{a} = 1$. As one can see, the fundamental initially jumps to over 1.60 at $t = 3$, but appears to converge to the 'true value' 1.00 by the end of the simulation at $t = 20$. The price appears to take longer to settle down and appears to be weakly correlated with the fundamental.

Perhaps the first question one might wish to ask is whether (or to what extent) the market price p_t is more volatile than the fundamental value f_t. An answer is difficult to gleam from a time graph since in our simple example both converge to the 'true value' $\bar{X}$. Table 1 addresses the question by computing the ratio EV_t of the (cross sectional) variance of p_t

to the variance of f_t in each time period over 1000 Monte Carlo simulations. In the first column, parameters are the same as in Figure 1 except that the market is about twice as large (N = 19 traders, M = 10 shares) and so is the price sensitivity at c = 1.0. In the other columns, the probability π of receiving news is reduced as indicated, with compensating increases in news precision. The EV_t statistics seem to rise[10] as news becomes 'lumpier' (i.e., less frequent but more precise), and that (except in the first few periods) the price seems considerably (typically 2-3 times) more volatile than the fundamental.

A very interesting set of questions arise concerning the co-movements of price and fundamental. Defining the *bubble* as the difference between the two, $b_t = p_t - f_t$, one might ask whether prices merely follow a random walk or, more interestingly, whether bubbles tend to feed on themselves[11] and/or tend to burst.[12] For reasons to be explained in the next section, we investigate these possibilities by replacing c in the bid formula (but not in the updating formula) by dc where the 'hypothetical discount factor' d is between 0 and 1.

The simulation displayed in Panel B of Figure 1 employs the same parameters as that in Figure 1 except that d = 0.5 rather than 1.0. One can imagine that the positive bubble $(p_t - f_t)$ at t = 3 feeds on itself

[10] The main exception is for t = 17-20 when $\pi = 0.2$. The corresponding price variances happened to be low in this sample of 1000 runs, depressing EV_t. An explanation of the consistently low EV_t for small t is that for a small sample of news messages with a long-tailed distribution (Gamma), the median is less sensitive to outliers than the mean.
[11] For example, one empirical implication would be that returns exhibit a higher positive autocorrelation during these periods.
[12] In this case, larger $|b_t|$ implies a higher probability that $|b_{t+1}| < |b_t|$.

before collapsing at $t = 8$, overshooting slightly, reversing itself and then feeding on itself again during periods 10-19.[13]

These simulations are meant only to be suggestive. In the next section, we present a more formal analysis of excess volatility, overshooting, etc., in the context of a much more general information environment.

2. The Formal Model.

Notation and Assumptions. Our main interests are in the processes of belief adjustment and information aggregation. To model these processes in a general but tractable manner, we minimize the complexity of other parts of the model. Thus, we suppose that M indivisible shares are trades in a simple Clearinghouse market by $N > M$ risk-neutral traders. Each share (a maximum of one per customer) pays a liquidating dividend of X at a known time T. Subject to the single share constraint, traders buy and sell at times $t = 1, 2, \ldots, T$ to maximize current expectation of final wealth, $E_t(X - p_t)$ for a buyer and $E_t(p_t - X)$ for a seller.[14]

The payoff X is uncertain and its distribution is not known, but traders receive information that improves their estimates of the

[13] Of course, note that the underlying trend for $b_t \to 0$, since both p_t and f_t converge to the 'true value.'

[14] This assumption rules out more complex intertemporal strategies such as buying now at a price above expected value in hopes of selling later (to a 'greater fool') at an even higher price. We are happy to rule out such strategies, which underlie the main 'rational bubbles' models as well as many irrational bubbles stories, in the interest of simplicity and (more importantly) to emphasize that imperfect information aggregation by itself can lead to bubbles.

distribution. We begin to formalize the belief adjustment process by assuming there is an indexed family of possible distributions $F(X;\theta)$ for X, and we express beliefs in terms of the index θ. In the previous section, for example, we assumed that F was the family of exponential distributions with index $\theta = \lambda$. Two other possibilities are that F is binomial[15] and the index θ is the probability p that $X = 1$, or that $\ln(X)$ is Normal with index vector $\theta = (\mu, \sigma)$. For our general model, it is harmless to assume that θ is a point in a subset of Θ of R^n, but we may easily accommodate even more general specifications.

We take the very general view that a trader's belief is represented by a point γ that identifies a particular conjugate distribution $H(\theta | \gamma)$ over the set of possible payoff distributions $F(X;\theta)$. The point γ lies in some vector space, possibly infinite dimensional. To allow for the possibility of complete ignorance, we postulate a belief γ^0 such that $H(\theta | \gamma^0)$ is diffuse.[16] The previous section used the example $\gamma = (a, b)$ in the conjugate Gamma distribution for the exponential parameter. In this case, complete ignorance is represented by $\gamma^0 = \lim_{b \to \infty}(2, b)$, which says that the exponential parameter λ has the (improper) uniform distribution on $[0, \infty)$. We summarize this discussion in the following general assumption.

(A0) *There is some family of distributions* $H(\theta | \gamma)$, γ *in some convex open subset* Γ *of a topological vector space, such that at any time* t *any trader i's beliefs regarding the venture's payoff can be summarized by* $H(\theta | \gamma_{it})$ *for some* $\gamma_{it} \in \Gamma$. *There is some element* $\gamma^0 \in \Gamma$ *such that* $H(\theta | \gamma^0)$ *is diffuse.*

Traders' beliefs change over time largely in response to private news.

[15] So $X = 0$ or 1, as in an Arrow security.
[16] See DeGroot (1970), p. 190.

We assume for simplicity that each trader i receives a non-trivial private news message z_{it} in period t with probability $\pi > 0$, and no message (sometimes conventionally denoted $z_{it} = 0$) with probability $1 - \pi$. We have in mind the view that news could include such diverse events as 'a brokerage issues a research report' or 'a company announces its current earnings and dividend.' We let S denote the set of possible news messages. For many purposes, we require no further structure on S, but sometimes we assume that z_{it} is independently drawn from a specific distribution $G(z|\bar{\theta}, \bar{a})$ on S, where $\bar{\theta}$ is the true but unknown value of θ in the payoff distribution and $\bar{a}$ is the precision of G.[17]

We permit traders' beliefs to differ because of different prior beliefs, or, more importantly, because of different private information. In the interest of parsimony we assume that beliefs do not diverge because of idiosyncratic information processing. That is, we assume that all traders use the same updating function ψ when responding to private news z and to (publicly observed) price changes Δp. Formally,

(A1) *Each trader updates his beliefs by means of the same continuous updating function* ψ: $\Gamma \times S \times R \rightarrow \Gamma$, *so* $\gamma_{it+1} = \psi(\gamma_{it}, z_{it}, \Delta p_{t+1})$.

In the previous section, we used an updating function that we now express as

$$\psi((a, b), z, y) = (a + \bar{a}, b + \bar{a}z + cy), \qquad (2.1)$$

where y equals Δp_{t+1}. We may derive simple formulae of this type for special conjugate families of distributions G and H, but in general ψ has

[17] We define the precision as 1/variance.

no closed form and Γ is infinite dimensional.[18] In Rational Expectations Equilibrium (REE) models ψ is shown to exist by fixed point argument. In our model, we do not exclude the possibility that traders use REE updating procedures, but we prefer not to impose the very strong assumptions underlying REE: that traders fully understand the market environment, have unlimited computational powers, know (as common knowledge) that the same is true for other traders, etc. We feel more comfortable assuming that traders' expectations are unbiased and consistent, but not necessarily minimum variance.

To formalize this view, we need some further notation. Let ψ^* denote the extension of ψ to updating over several periods, defined inductively by

$$\psi^*(\gamma;\ z_1;\ y_1) = \psi(\gamma,\ z_1,\ y_1) \tag{2.2}$$

and

$$\psi^*(\gamma;\ z_1,\ldots,z_{m+1};\ y_1,\ldots,y_{n+1}) = \psi(\psi^*(\gamma;\ z_1,\ldots,z_m;\ y_1,\ldots,y_n),\ z_{m+1},\ y_{n+1}). \tag{2.3}$$

Slightly abusing notation, we abbreviate $\psi^*(\gamma;\ z_1,\ \ldots,\ z_{m+1};\ 0)$ as $\psi^*(z_1,\ \ldots,\ z_{m+1})$ and abbreviate $\psi^*(\gamma;\ 0;\ y_1,\ \ldots,\ y_{n+1})$ as $\psi^*(y_1,\ \ldots,\ y_{n+1})$ when γ is understood.

Let $\bar{\theta}$ denote the *unknown index value for the true payoff distribution*, so with perfect (but unobtainable) information the payoff expectation would be

$$\bar{X} = E(X;\bar{\theta}) \equiv \int xF(dx;\bar{\theta}). \tag{2.4}$$

We denote *trader i's current payoff expectation* by

$$\bar{x}_{it} = E(x;\gamma_{it}) \equiv \iint xF(dx;\theta)H(d\theta \mid \gamma_{it}). \tag{2.5}$$

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[18] See DeGroot (1970), Chapter 9.

The *expectation function* ϕ used by traders is induced by the belief updating function ψ, so

$$\phi(\gamma, z, y) = E(X | \psi(\gamma, z, y)), \tag{2.6}$$

and we denote its multiperiod extension by ϕ^*. In particular, the *sample expectation* for the observation z is denoted by $\phi^*(z) \equiv \phi^*(\gamma^0, z, 0)$; recall that γ^0 indicates a diffuse prior.

Our main rationality assumption is formalized as follows:

(A2) *News messages are interpreted so as to provide consistent and unbiased information regarding the true expected payoff* $\bar{X} = E(X;\bar{\theta})$. *That is:*

(A) $\bar{X} = E\phi^*(z)$ *(so news interpretation is unbiased);*

(B) *For each* $\gamma \in \Gamma$, *the posterior expectation* $\phi(\gamma, z, 0)$ *is strictly increasing in the sample expectation* $\phi^*(z)$, *and lies between* $\phi^*(z)$ *and the prior expectation* $\phi(\gamma, 0, 0)$ *(so news is informative); and*

(C) *For any fixed* $\gamma \in \Gamma$, *the expectation* $\phi^*(z_1, \ldots, z_n) \to \bar{X}$ *with probability* 1 *as* $n \to \infty$ *(so news is consistent).*

Now consider how public information (Δp) affects expectations. In the example of Section 2, we assume that the expectation function is

$$\phi((a_0, b_0), z, y) = (b + cy)/(a - 1), \tag{2.7}$$

where $a = a_0 + \bar{a}$ and $b = b_0 + \bar{a}z$. In this case, the *sensitivity of expectations to public information* is

$$\partial\phi/\partial y \equiv \eta = c/(a - 1) > 0. \tag{2.8}$$

Note that $\eta < 1$ as long as $c < a - 1 = a_p + n\bar{a}$, that is, as long as the number n of messages received is sufficiently large, given the initial ($t = 0$) precision a_p and the news precision $\bar{a}$. For the general model, we allow complicated, possibly non-linear and time dependent, responses of expectations to price changes. We assume only that the responses are non-negative and not overly sensitive. Information aggregation can be short-circuited if traders allow their own private information to be outweighed by public information.[19] To rule out 'cascade' or 'lemming' behavior, we assume $\eta < 1$.[20] Our formal assumption, then, is

(A3) *The expectations function* ϕ *is continuously differentiable and, for some* $\delta > 0$, *satisfies* $0 \leq \eta \leq 1 - \delta$ *for all* γ, z, *and* y.

Assumptions (A0)-(A3) involve some mild technical conditions.[21] For some purposes we use other mild technical conditions regarding boundedness and onto-ness. Specifically,

(A4) (A) *There is some* $B > 0$ *such that* $E(X \mid \gamma) < B$ *for all* $\gamma \in \Gamma$. *(So payoff expectations are uniformly bounded), and*

(B) *For each* $\gamma \in \Gamma$ *there is some* $z \in S$ *such that* $E(X \mid \gamma) = \phi^*(z)$. *(So all parameter values* γ *give expectations that conceivably could be news-justified).*

Main Results. Now that all elements of the model are in place, we define the *fundamental* as the sample expectation based on all news previously received by all traders, or as $f_{t+1} = \phi^*(z_{11},\ldots,z_{it},\ldots,z_{Nt})$. The bubble at time t is the difference between the current price and fundamental, $b_t = p_t - f_t$. We begin our analysis with a derivation of the optimal bid, taking into account the 'winner's curse' problem discussed is the previous section.

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[19] See Bikchandani, Hirshleifer, and Welsh (1992).
[20] When $\eta \geq 1$, we can have unstable situations in which a price rise of 1 cent causes expectations to rise by, say, 2 cents, causing a 2 cent price rise, in turn causing expectations to rise by 4 cents, etc.,... .
[21] The differentiability assumptions probably are the strongest, and we could replace them with less restrictive but more cumbersome Lipshitz continuity conditions.

Proposition 1. *Under assumptions* (A0)-(A3), *the equation* $y = \phi(\gamma_{it}, 0, y - p_t)$ *has a unique solution* y^* *for each* $p_t > 0$ *and* $\gamma_{it} \in \Gamma$. *If* p_t *is the most recently announced price and* γ_{it} *describes trader* i's *current beliefs regarding the asset payoff, then his optimal bid is* $\nu_{it} = y^*$.

The proof of this and other propositions appears in the Appendix. We summarize the main idea as follows: to avoid the winner's curse, the optimal bid differs from the current expectation $\bar{x}_{it}$ to the extent that the current expectation differs from the current price.

Before turning to price determination, we examine in more detail how the optimal bid depends on the updating process ψ. In the context of i.i.d. private news z_{it}, it is natural to assume that $\psi^*(z_{i1},\ldots,z_{it})$ is a symmetric function. Hence, the order in which these messages arrive is irrelevant. A corresponding (but more problematic) symmetric property for the public information embodied in price changes is *price path invariance* (PPI). We say ψ is (or has) PPI if $\psi^*(\gamma;\, z_1,\ldots,z_t;\, y_0,\ldots,y_t)$ depends on the price changes $(y_0,\ldots,y_t)$ only through the sum $\sum_{\tau=0}^{t} y_\tau$. That is, the order in which price changes occur is irrelevant, and only their net effect matters. Thus, the current price p_t (with the time 0 price p_0) is a sufficient statistic for the sequence of price changes under PPI since $p_t - p_0 = \Sigma \Delta p_\tau$, as is the case in many REE models.

An important alternative to PPI is that traders respond less strongly to prospective than to actual price changes.[22] To formalize this idea, let $\hat{\phi}$ be the expectation function obtained from ϕ by replacing its last

[22] There is considerable evidence for such behavior in related economic contexts e.g. Arrow (1981), Cox, Smith, and Walker (1983), and Grether (1978); specifically we are suggesting that traders may not fully adjust for the winner's curse, as documented by Kagel and Levin (1986).

argument Δp_{t+1} by a hypothetical price change that traders have not (yet) observed. If $0 \le \hat{\eta} < \eta$ and PPI holds for observed price changes, then we say that traders *discount hypothetical price* changes, or DHP holds.[23]

Proposition 2. *Assume that assumptions* (A0)-(A3) *hold. Under PPI, a trader's optimal bid* ν_{it} *depends only on private news* $\{z_{1t},\ldots,z_{it}\}$ *and is independent of observed prices* $\{p_1,\ldots,p_t\}$. *Under DHP, the optimal bid* ν_{it} *is an increasing function of the most recent price change* Δp_t, *centeris paribus.*

In the DHP case, we note that $\eta > 0$ so that (absent contrary news) $\Delta\bar{x}_{it} \gtrless 0$ and $\Delta\nu_{it} \gtrless 0$ as $\Delta p_t \gtrless 0$; that is, expectations and bids respond to price changes in the same direction. This property seems natural to us, but we discovered that it does not hold under PPI. Indeed, Proposition 2 shows that under PPI optimal bids will not respond at all to price changes, even when expectations respond quite strongly. The main idea is that $y^* - p_0$ is a sufficient statistic under PPI for all price changes in the equation that defines ν_{it}, but y^* is independent of observed prices. In retrospect, this suggests that under PPI, traders care only about the prospective transaction price, which does not really depend on previous price changes.

We characterize the prices that emerge from our Clearinghouse market, in terms of traders' prior and posterior expectations, as $\bar{x}^0_{it+1} = \phi(\gamma_{it}, 0, \Delta p_{t+1})$ and $\bar{x}_{it+1} = \phi(\gamma_{it}, z_{it}, \Delta p_{t+1}) = \phi(\gamma_{it+1}, 0, 0)$. We let H denote the set of shareholders, $\bar{H}$ denote the set of non-

[23] Unlike the 'positive feedback traders' of DeLong *et. al.* (1990), our DHP traders do not blindly extrapolate price trends. They merely weight the evidence regarding the expected final payoff contained in Δp against other evidence from history and their own private news. Indeed, our DHP traders may well understate the inferences that can be drawn from Δp.

shareholders, and #S denote the number of elements in the set S.

Proposition 3. *Under the rules of the simple Clearinghouse market, we have* $p_{t+1} = \nu_{(M)t}$ *for* $t = 0, 1, \ldots, T\text{-}2$. *Given optimal bidding under assumptions* (A0)-(A3), *shareholders' posterior expectations are no lower than those of non-shareholders,* $H_{t+1} \subset \{i: \bar{x}^0_{it+1} \geq p_{t+1}\}$ *and* $\bar{H}_{t+1} \subset \{i: \bar{x}^0_{it+1} \leq p_{t+1}\}$. *Under strict PPI for* ψ, *we have* $p_{t+1} = \bar{x}^0_{(M)t+1}$, *while* p_{t+1} *is between* p_t *and* $\bar{x}^0_{(M)t+1}$ *under DHP.*

Proposition 3 allows us to investigate price behavior. Recall $\bar{X} = E(X|\bar{\theta})$ is the 'true value' of the asset. Our next result tells us that the fundamental f_t and price p_t are both consistent estimators of the asset's true value $\bar{X}$, and are essentially unbiased (any bias is inherited from biased initial priors). Therefore, bubbles are zero on average and have variance that becomes arbitrarily small when sufficient information accumulates.

Proposition 4. *Under assumptions* (A0)-(A4), *both* p_t *and* f_t *converge to* $\bar{X}$ *with probability* 1 *as* $t \to \infty$. *If the prior distributions for* θ *are unbiased (if* $E(X|\gamma_{i0}) = \bar{X}$ *for* $i = 1, \ldots, N$), *then the unconditional expectations of* p_t *and* f_t *are both* $\bar{X}$ *for all* $t = 0, 1, 2, \ldots$.

In view of this proposition, bubbles are significant only when $Var(p_t)$ far exceeds $Var(f_t)$, or p_t is much less efficient than f_t. It turns out that this relative efficiency depends on the nature of the private news arrival process, which we characterize largely by the precision $\bar{a} = 1/Var(z)$ of a news message and the probability π of its arrival during a trading day. The *intensity of the news process*, $I = \bar{a}\pi$, is the average rate at which a trader becomes (privately) informed. One might be tempted to consider the comparative statics of news intensity, but a moment's reflection reveals that greater intensity is equivalent merely to more frequent market clearings—one just redefines the time scale to obtain unit intensity. Therefore, the interesting comparative statics concern lumpiness, the extent to which news arrival is rare and decisive versus frequent but ambiguous. We say that a news process (π, a) is *lumpier* than another ($\bar{\pi}$, $\bar{a}$) with the same intensity if $\pi < \bar{\pi}$ (so $a > \bar{a}$). The limiting cases are $\pi = 1$, in which case traders are always equally well informed, and $\pi \to 0$, in which case a few traders could be far better informed than the general population.

We obtain the following approximation (see the Appendix) for the excess variance ratio:

$$EV_t \equiv \frac{Var(p_t)}{Var(f_t)} \cong k \cdot \left[\frac{1 - (1 - \pi)^{t+1}}{1 - (1 - \pi)^{Nt+1}}\right], \tag{2.9}$$

where k depends on η, e.g. $k \approx 1.57$ for $\eta = 0$. Since the RHS is decreasing in π, we conclude that lumpier news tends to decrease the efficiency of p_t relative to f_t as an estimator of $\bar{X}$, so bubbles become increasingly important. Indeed, by L'Hospital's rule, the RHS $\to \infty$ as $\pi \to 0$, so bubbles tend to dominate price movements in the case of extremely lumpy news (extreme asymmetries in traders' private information). The intuition is that p_t is essentially a median of traders' expectations of $\bar{X}$ while f_t is an appropriately weighted mean, which is much more efficient when the (information-received) weights are very unequal.

We wish to analyze whether bubbles are self-generating and/or have some built-in tendency to burst. We now proceed to consider these matters more formally.

Proposition 5. *Assume PPI holds. Then* $E(\Delta p_{t+1} | p_t, \bar{\theta})$ *is a decreasing function of* p_t *whose sign agrees with that of* $\bar{X} - p_t$.

The essential idea here is that, when p_t is above the asset's true value $\bar{X}$, unbiased private news is more likely to lower expectations and bids of shareholders and less likely to raise expectations and bids of non-shareholders, thus increasing the probability of negative price changes. Similarly, positive (and larger) price changes are more likely to the extent that p_t is below $\bar{X}$. Hence, we have mean reversion tendencies.

The same tendencies are also at work under DHP, and actually another mechanism reinforces them. In view of the relative efficiency of f_t (and Lemma 5.1 in the Appendix) one has a positive correlation of bubbles with $(p_t - \bar{X})$. When $b_t > 0$, non-shareholders tend to be 'better informed' and less responsive to news than shareholders. Since price decreases arise from lower bids by shareholders and the absence of higher bids by non-shareholders, this differential in news-responsiveness (which requires $\pi < 1$) will tend to lead to price decreases. Similarly, price increases are more likely when $b_t < 0$. In the interest of brevity, we do not attempt to formalize this point.

Perhaps the most interesting problem is to determine when bubbles are *self-feeding*, or lead to *herd trading*, which we define as the property that $E(\Delta b_{t+1} | \Delta p_t)$ is an increasing function of Δp_t.

Proposition 6. *Suppose that news arrival is not certain* ($\pi < 1$) *and DHP holds. Then bubbles are self-feeding when the asset's fundamental value* f_t *is sufficiently near its true value* $\bar{X}$.

Next, we elaborate on the mechanism driving this result. Figure 2 shows the most likely events (called Events ± 1) that produce a transaction: a near-median trader responds more or less strongly to public information than does the median trader, and hence transacts with him. Given our assumption of a common updating function ψ, it must be the case that our less responsive trader was better informed.[24] Hence, one could say that self-feeding bubbles arise in our model as less informed expectations (or bids) overtake better informed expectations (bids). In this case, trade becomes self-generating with traders contagiously following the actions of other participants.

Clearly this mechanism operates more strongly when π is small, so both self-feeding bubbles as well as excess variance are more important when news is lumpy. On the other hand, PPI makes $\Delta\nu_{it+1}$ independent of Δp_t by Proposition 2, so evidently bubbles are not contagious in the PPI case.

3. Discussion.

Our main argument may be summarized as follows. Even if the asset price is not a sufficient statistic for aggregate information, traders nonetheless will generally find it informative, and find it in their interest to respond to it. Under these circumstances, prices will not *a priori* be biased away from the fundamental value (which is based on the aggregate of all current information), but will generally exhibit higher variance than the strong efficient markets hypothesis would suggest. In addition, bubbles, or discrepancies between prices and fundamentals, can be

[24] For example, an informed trader may have a larger number of news messages.

self-feeding, in the sense that an increase (or decrease) tends to provoke a further increase (or decrease). Bubbles of the sort that we examine are eventually self-limiting in that the probability of reversal increases with the bubble's magnitude. The last two features imply overshooting: when a positive (or negative) bubble disappears, its momentum tends to produce a negative (or positive) bubble. These market inefficiencies are not arbitragable by market participants, who have access only to observed prices and their own private information, not aggregate information.

We developed this argument in terms of a theoretical asset market model, with a parametric illustration and simulations, featuring news (dispersed private information arrival) and Bayesian traders oriented to long-term asset value. The model suggests that prices will be excessively volatile (relative to fundamental values) under the following conditions: (1) news is of high quality relative to prior information, so it induces large revisions in asset value estimates; (2) news arrives infrequently, so individual asset value estimates remain uncertain over much of the life of the asset, and (3) there is not so much background noise (e.g. liquidity shocks) that participants regard price changes as uninformative. In our model, self-feeding (and overshooting) arise if (4) traders underweight prospective price changes relative to actual price changes. Thus, the empirical implications concern the relation between news arrival (and news processing) and certain types of price behavior.

Although difficult to identify by econometric analysis of historical data, many participants and analysts on contemporary asset markets have suggested the existence of excess price volatility, self-feeding, and overshooting. For example, with respect to excess price volatility, the 'variance bounds' literature attempted to demonstrate that U.S. stock market prices had higher variance than could be accounted for by rationally valued underlying dividend or earnings streams.[25] The present consensus appears to be that these empirical tests are inconclusive because the theoretical models they employ rely too heavily on the exogeneity and stationarity of the relevant time series (e.g. dividend or earnings streams).[26] A more direct test requires data on the information employed by investors, but of course such data are not normally available to the econometrician. Roll (1984) studied perhaps the most assessable case: he argued persuasively that, over the period he considers, the only information relevant to short-term fluctuations in orange juice futures prices is the predicted temperature in central Florida, yet 'surprises' in the latter variable explain only a small fraction of the observed daily price variability.[27]

There is also a set of recent articles that seek empirical tests of the exponential rational bubbles literature. For example, Meese (1986), employing a specification test suggested by West (1988), finds that monthly foreign exchange rate data reject a joint hypothesis of no bubbles and a

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[25] For instance, see Shiller (1981) or LeRoy and Porter (1981). These authors base the variance bound on the fact that conditioning on a larger information set can only increase the conditional variance. This fact has no bite in our model because our asset price is the expectation of a particular trader (the median) whose identity changes over time, and our traders have heterogeneous, non-nested information sets.

[26] See Marsh and Merton (1986) or Kleidon (1984).

[27] He also points out that price variance across weekends should be three times as large as across weekdays under the efficient markets hypothesis, but it is actually only about 1.5 times as large.

stable driving process for a monetary exchange rate model. He carefully qualifies his tentative conclusion that bubbles are present. On the other hand, Hamilton and Whiteman (1986) argues strongly that bubbles are impossible to detect econometrically when market participants respond to variables not observed by the econometrician, and that previous empirical detection of bubbles (and excess volatility) was invalid.

A different empirical approach involves laboratory experiments with asset markets in which researchers control and observe each trader's information. Smith, Suchanek, and Williams (1988) reports massive positive (but non-exponential) bubbles in a double auction asset market even when the news arrival process is trivial. It remains to be seen whether such phenomena persist under conditions more closely resembling those of our model, but we regard the experimental approach as a promising empirical technique for studying information aggregation and asset price dynamics.

Our economy as presented in this paper involves a single payoff, extreme indivisibility, and no exogenous public information, and therefore we cannot directly apply this model to most contemporary asset markets. However, intuition and some preliminary analysis suggest that its main conclusions survive considerable generalization. For example, most important securities have a payoff stream that extends over time. The cost of modeling such securities is more complex notation and calculations. Preliminary work with models of this sort reveals no major new insights, but does point up the convenient fact that the expected present value of an infinite-lived payoff stream need not have a variance that decreases over time, so excess volatility can be measured directly. Relaxing the indivisibility and risk neutrality assumptions blurs the distinction between the median and mean, but some preliminary work indicates that (except under some special parametric specifications) price is still not a sufficient statistic for all private information and our main argument remains valid. Exogenous public information is not usually transparent. When asset market participants differ in their evaluations of public news, then we can regard this as independent sample information.[28] Perhaps one could model the impact of transparent exogenous public information in the same manner as the impact of a price change, but this remains a matter for further investigation.[29]

We are not the first to argue that asset prices do not fully aggregate information available to market participants.[30] As we see it, our main contributions are to show how excess variance due to imperfect information aggregation is related to asset price bubbles that are self-feeding, overshoot, etc., and to show how these phenomena arise from the underlying information conditions and market structure. Once bubbles of our sort become prevalent, some traders can find it worthwhile to pursue short-run technical strategies, in which 'beauty contest' bubbles become plausible.[31]

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[28] In Bayesian terms, participants may have different priors as well as different likelihood functions for the asset value implications of news, so even weather forecasts may present an information aggregation problem for the orange juice futures market.

[29] Froot and Obstfeld (1991) develops the notion of intrinsic bubbles that rely on exogenous public information in the form of aggregate dividends. This rational bubbles approach is unrealistic since the economy must be stuck on a path along which price-dividend ratios approach infinity in the limit.

[30] See Grossman (1976), Froot, Scharfstein, and Stein (1992), and especially Figlewski (1982) for an excess variance argument.

[31] It is precisely this insight that DeLong *et. al.* (1990) exploit.

Thus our theory of bubbles does not replace the older theories surveyed in the introduction, but rather suggests conditions under which these theories may become germane.

APPENDIX

Lemma 1. Let a, b, c, d be positive numbers, then $\frac{a}{b} \gtrless \frac{a+c}{b+d} \gtrless \frac{c}{d}$.

Proof. Observe that

$$\frac{a+c}{b+d} = \left(\frac{b}{b+d}\right)\frac{a}{b} + \left(\frac{b}{b+d}\right)\frac{c}{d}$$

$$= \alpha \frac{a}{b} + (1 - \alpha)\frac{c}{d} \text{ for } 0 < \alpha = \frac{b}{b+d} < 1,$$

and the conclusions follow immediately. ∎

Proof of Proposition 1: Since X is non-negative,

$$E(X \mid \psi(\gamma_{it}, 0, y - p_t)) = \phi(\gamma_{it}, 0, y - p_t) \geq 0,$$

for y = 0 and all $p_t \geq 0$ and $\gamma_{it} \in \Gamma$. By (A3) we have

$$\phi(\gamma_{it}, 0, y - p_t) < y,$$

for y sufficiently large. Hence, (employing (A1) and (A3)) the function

$$h(y) = \phi(\gamma_{it}, 0, y - p_t) - y$$

is continuous, decreasing, positive at y = 0, and negative for y sufficiently large. Therefore, by the intermediate value theorem, h(y) has a unique root y^*, as required. Note that $h(y) \geq 0$ iff $y \leq y^*$.

The optimization problem for a non-shareholder is

$$\max_{v \geq 0} E\{(X - p_{t+1})I[p_{t+1} \leq \nu] \mid \gamma_{it}, [p_{t+1} \leq \nu]\},$$

where I[e] = 1 if the event e occurs and is 0 otherwise. Thus the maximand is

$$\int_0^{\nu}\{E(X \mid \psi(\gamma_{it}, 0, y - p_t)) - y\}dF^*(y) = \int_0^{\nu} h(p_{t+1})dF^*(p_{t+1}),$$

where F^* is the (subjective) distribution of p_{t+1}. Clearly, whatever F^*

might be, the maximum is achieved by integrating over $\{y: h(y) \geq 0\}$, so $\nu = y^*$ is optimal.

The optimization problem for a shareholder i is

$$\max_{v \geq 0} E\{(p_{t+1} - X)I[p_{t+1} \geq \nu] \mid \gamma_{it}, [p_{t+1} \geq \nu]\}.$$

A similar analysis also leads to the same conclusion in this case. ∎

Proof of Proposition 2. For arbitrary γ_{it} (suppressed), we know by (A3) that the following function is differentiable:

$$h_t(y, \rho) = \phi^*(\Delta p_1, \ldots, \Delta p_{t-1}, \rho, y - (p_{t-1} + \rho)) - y,$$

where $\rho = \Delta p_t$ is the most recently observed price change. By Proposition 1, the optimal bid ν_{it} is $y^* = h_t^{-1}(0, \rho)$, the inverse function taken with respect to the first (y) argument of h. Under PPI, h_t is independent of ρ, so y^* is also independent of $\rho = \Delta p_t$. By induction, it is also independent of Δp_τ, $\tau = 1, 2, \ldots, T\text{-}1$ as well so the first part of the proposition follows. Under DHP, we see that h_t is increasing in ρ since the last argument $y - (p_{t+1} + \rho)$ of ϕ^* is hypothetical but the previous argument ρ is not. Consequently, y^* is also increasing in $\rho = \Delta p_{t+1}$. ∎

Proof of Proposition 3. Assume that no two ν_{it}'s coincide. Recall that traders are assumed not to attempt to exercise monopoly power and that

$$S_t(p) = \#\{i \in H_t: \nu_{it} \leq p\},$$

and

$$D_t(p) = \#\{i \in \overline{H}_t: \nu_{it} \geq p\}$$

are the supply and demand functions. For $p > \nu_{(l)t}$, one clearly has $S_t(p) = M$ and $D_t(p) = 0$. As p decreases from $p^+ = \nu_{it} + \varepsilon$ to $p^- = \nu_{it} - \varepsilon$, we have either $S_t(p^-) = S_t(p^+) - 1$ (when $i \in H_t$) or $D_t(p^-) = D_t(p^+) + 1$ (when $i \in \bar{H}_t$). Consequently, $S(p) > D(p)$ iff $p > \nu_{(M)t}$. It then follows from the definitions that $p_{t+1} = \nu_{(M)t}$, and $H_{t+1} = \{i: \nu_{it} \geq p_{t+1}\}$ while $\bar{H}_{t+1} = \{i: \nu_{it} < p_{t+1}\}$. Now

$$\bar{x}^0_{it+1} = \phi(\gamma_{it}, 0, \Delta p_{t+1}) > \nu_{it} \text{ iff } p_{t+1} > y^* = \nu_{it},$$

by (A3) so we replace ν_{it} by $\bar{x}^0_{it+1}$ in the previous sentence. By the same argument, $\nu_{it} = p_{t+1}$ implies $\bar{x}_{it+1} = \nu_{it}$, so we conclude that $p_{t+1} = \bar{x}_{(M)t+1}$ under PPI. The weaker statements in the proposition then follow from assuming only DHP and allowing for the possibility of two or more traders bidding $\nu_{(M)t}$. ■

<u>Proof</u> <u>of</u> <u>Proposition</u> <u>4</u>. Let n_{it} = the number of messages received by trader i up to time t, and let $n_t = \sum_{i=1}^{n} n_{it}$. Since $n_t \to \infty$ with probability 1 as $t \to \infty$, it follows from (A2) and the definition of f_t that f_t is unbiased and consistent. Suppose now that $\eta = 0$, the prices are uninformative. Since by (A4), an unbiased prior γ_{i0} can be regarded as $\gamma(z_{i0})$ for some realization z_{i0} of the news process and since $n_{it} \to \infty$, the same argument shows that $\bar{x}_{it}$ is also unbiased and consistent for each i, so by Proposition 3, we obtain the same conclusion for p_t. When $\eta > 0$, the argument is more delicate; essentially we must show that the news $\{z_{i1},\ldots,z_{it}\}$ eventually dominates the effect of price changes. We first note that

$$p_t \leq \max\{\bar{x}_{it}: i = 1,\ldots,n\} \leq \sup\{E(X|\gamma): \gamma \in \Gamma\} \leq B,$$

by the inequalities from Proposition 3, the definition of $\bar{x}$, and (A4) respectively. Consequently,

$$\sum_{\tau \leq t} \Delta p_\tau = p_t - p_0 \leq B,$$

so the bound $\eta \leq 1 - \delta$ from (A3) then yields a uniform bound on $E(X \mid \psi^*(\Delta p_1, \ldots, \Delta p_t))$. Now (A4) implies that $E(X \mid \gamma(z_0))$ exceeds this bound with positive probability, but (A2) assures us that even in this event we have

$$E(X \mid \gamma(z_0, z_1, \ldots, z_n)) \to E(X \mid \bar{\theta}) \text{ as } t \to \infty \text{ with probability } 1,$$

for $(z_1, \ldots, z_n)$ the actual news received by a trader i. Hence, by Propositions 1 and 3 and the Dominated Convergence Theorem, we must have

$$E(X \mid \gamma_{it}) \to E(X \mid \bar{\theta}) \text{ for each i},$$

establishing the consistency of each $\bar{x}_{it}$ and hence p_t. Finally, for unbiasedness, note that $\bar{x}_{i0}$, p_0, and p_1 are unbiased by the hypothesis on γ_{i0}. Under the inductive assumption, that

$$Ep_{t-1} = Ep_t = E(X \mid \bar{\theta}) = E\bar{x}_{it},$$

for all i, we note that $E\bar{x}_{it}$ is a convex combination of the latter variable and

$$E(E(X \mid \gamma_{it}(z_{it+1})) = E(X \mid \bar{\theta}),$$

since $E\Delta p_t = 0$, so $E\bar{x}_{it+1} = E(X \mid \bar{\theta})$ also. Consequently, $Ep_{t+1} = E(X \mid \bar{\theta})$. ∎

<u>Derivation of Equation (2.9)</u>. We now analyze the dependence of the bubble index,

$$EV_t \equiv Var(p_t)/Var(f_t),$$

on the parameters describing the news process. Assume that news z_{it} is i.i.d. $G(z \mid \bar{a}, \bar{\theta})$, where $a = Var^{-1}(z)$ is the precision of a news message, and π is its probability of arrival. Thus the *intensity of the news process*, defined as $I = \bar{a}\pi$, summarizes the rate at which an individual becomes informed. By redefining the time scale t, we can (apart from the effect of more frequent market clearings) normalize on a given news intensity I_0. Recall that a news process (π, a) is *lumpier* than a process ($\bar{\pi}$, $\bar{a}$) with the same intensity ($I_0 = a\pi = \overline{\pi a}$) if $\pi < \bar{\pi}$. The limiting cases are $\pi = 1$, in which case all traders are always equally well informed, and $\pi \rightarrow 0$, in which case news is very lumpy and typically a few traders are far better informed than others.

It is convenient in this analysis to assume that prior beliefs at $t = 0$ arise from a preliminary message $z_{0i} \sim$ i.i.d. $G(z|a, \bar{\theta})$. Now traders' expectations at time t are described by $S^t = \{\bar{x}_{it}: i = 1,\ldots,N\}$; for the case $\eta = 0$ (prices regarded as uninformative), S_t is an i.i.d. random sample from a distribution G_t^* derived from G and the binomial distribution with parameters π and t, as noted below. We employ the decomposition

$$EV_t = (Var(p_t)/Var(S_t))(Var(S_t)/Var(f_t)),$$

where $Var(S_t)$ is the variance of S^t (the sample mean), and analyze the two factors separately.

We explicitly analyze the case that $\eta = 0$. Let n_{it} be the number of actual (not preliminary) news messages received by trader i by the end of period t; clearly it has the binomial distribution so

$$Pr[n_{it} = n] = \binom{t}{n}\pi^n(1 - \pi)^{t-n}.$$

The conditional variance of $\bar{x}_{it}$ is

$$Var(\bar{x}_{it}|n_{it}) = a^{-1}(n_{it} + 1)^{-1};$$

if follows that the unconditional variance is

$$\mathrm{Var}(\bar{x}_{it}) = \frac{1}{a}\,\mathrm{E}\left(\frac{1}{n_{it}+1}\,|\,\pi,\ t\right).$$

But

$$\mathrm{E}\left(\frac{1}{n+1}\,|\,\pi,\ t\right) = \sum_{n=0}^{t} \frac{1}{n+1}\,\frac{t!\pi^n(1-\pi)^{t-n}}{n!(t-n)!} = \sum_{k=1}^{t+1} \frac{(t+1)!\pi^k(1-\pi)^{t+1-k}}{k!(t+1-k)!(t+1)\pi}$$

$$= (1 - \Pr[k=0])\cdot\frac{1}{(t+1)\pi} = \frac{1-(1-\pi)^{t+1}}{(t+1)\pi}.$$

Consequently,

$$\mathrm{Var}(S_t) = \frac{1}{N}\,\mathrm{Var}(\bar{x}_{it}) = \frac{1}{Na}\,\mathrm{E}\left(\frac{1}{n+1}\,|\,\pi,\ t\right) = \frac{1-(1-\pi)^{t+1}}{Na(t+1)\pi}.$$

Now $\mathrm{Var}(f_t)$ can be computed in an analogous fashion. For $n_t = \Sigma n_{it}$, the total number of messages incorporated in f_t (counting, as we should in this context, the preliminary messages) is $n_t + N$, so the conditional variance of f_t is $a^{-1}(n_t + N)^{-1}$ and the unconditional variance is

$$\mathrm{Var}(f_t) = \frac{1}{a}\,\mathrm{E}\left(\frac{1}{n_t+N}\,|\,\pi,\ Nt\right) \cong \frac{1}{a}\,\frac{\pi t + N^{-1}}{\pi t + 1}\,\mathrm{E}\left(\frac{1}{n_t+1}\,|\,\pi,\ Nt\right)$$

$$= \frac{(\pi t + N^{-1}(1-(1-\pi)^{Nt+1})}{(\pi t + 1)(Nt+1)\pi a}.$$

We employed the approximation

$$\frac{1}{n_t+N} = \frac{1}{n_t+1}\,\frac{n_t/N + 1/N}{n_t/N + 1} \cong \frac{1}{n_t+1}\,\frac{\pi t + 1/N}{\pi t + 1},$$

valid for t sufficiently large that $\pi \cong n_t/Nt$. It follows that the second factor in the decomposition of EV_t is

$$\mathrm{Var}(S_t)/\mathrm{Var}(f_t) \cong \frac{(Nt+1)}{N(t+1)}\,\frac{(\pi t+1)}{(\pi t + 1/N)}\,\frac{(1-(1-\pi)^{t+1}}{(1-(1-\pi)^{Nt+1})}$$

$$\cong \frac{1-(1-\pi)^{t+1}}{1-(1-\pi)^{Nt+1}}.$$

The last formula clearly shows that the second factor is decreasing in

π (increasing in the lumpiness of news), is equal to 1 for $\pi = 1$, and (by L'Hospital's rule) approaches ∞ as $\pi \rightarrow 0$. These conclusions may also be confirmed more laboriously from the exact formulae. The result may be familiar from statistical theory: $Var(S_t)$ corresponds to an OLS estimator and $Var(f_t)$ corresponds to a GLS estimator employing the known heterosckasticity of the 'sample' S^t.

As for the first factor $Var(p_t)/Var(S_t)$, Proposition 2 implies that we are comparing the efficiency of the sample median (or more generally, the M^{th} order statistic) to that of the sample mean. It is known that this ratio depends on the shape of the underlying distribution G_t^* but is rather insensitive to its location or scale, and therefore is, to a first approximation, independent of t, π and a. Specifically, Fisz (1963, p. 383) shows that if N is reasonably large, then $Var(p_t)$ is approximately $(M/N)(1 - M/N)N^{-1}g_t^{-2}(\bar{x})$, where g_t is the density of G_t^*, assumed continuous at the M/N quantile $\bar{x}$. By (A2), $\bar{x}$ is the mean of G_t^*, and the Central Limit Theorem suggests that $g_t(\bar{x}) \cong (2\pi\sigma_t^2)^{-1/2}$, where here $\pi \cong 3.14$ and σ_t^2 is the variance of G_t^*. Hence, for the case $M \cong N/2$, we have

$$Var(p_t) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{N}\cdot 2\pi\sigma_t^2 = \frac{\pi}{2}\cdot\frac{\sigma_t^2}{N}.$$

Since in the present case $Var(S_t) = \sigma_t^2/N$, we obtain $Var(p_t)/Var(S_t) \cong 1.57$ as an approximation valid for reasonably large N and πt.

We do not attempt an explicit analysis of the case $\eta > 0$ (informative news), but note that η should depend on news intensity rather than news lumpiness. Indeed, ϕ is probably chosen to minimize $Var(\bar{x}_{it})$; when this is successful (and 'noise' does not overwhelm 'signal'), $Var(S_t)$ and

$Var(p_t)$ should both be reduced relative to the $\eta = 0$ case. On the other hand, S_t is positively correlated when $\eta > 0$, tending to increase $Var(S_t)$ and $Var(p_t)$. On balance, the first factor of EV_t appears little affected by η and the second factor appears to exhibit the same qualitative features (e.g., it $\rightarrow 1$ as $\pi \rightarrow 1$, and it $\rightarrow \infty$ as $\pi \rightarrow 0$) for any η.

We conclude that given the news process defined above, EV_t is an increasing function of news lumpiness. For N and πt large, we have

$$EV_t \cong k \cdot \left\{ \frac{(1 - (1 - \pi)^{t+1})}{(1 - (1 - \pi)^{Nt+1})} \right\},$$

where k depends on η and the news distribution G. For η small, $k \cong 1.57$. In particular, $EV_t \rightarrow \infty$ as $\pi \rightarrow 0$, or as news becomes extremely lumpy.

<u>Lemma 5.1</u>. $E(\Delta f_{t+1} | f_t)$ is a decreasing linear function of f_t whose sign agrees with that of $\bar{X} - f_t$.

<u>Proof</u>: Note that

$$f_{t+1} = \phi^*(z_{11},\ldots,z_{m}; z_{1t+1},\ldots,z_{m+1}) = (1 - a)f_t + a\phi^*(z_{1t+1},\ldots,z_{m+1}),$$

since the first equality restates the definition of f_t and the second holds by (A2(B)) and induction, for some $a \in (0,1)$. Consequently

$$\Delta f_{t+1} = -af_t + a\phi^*(z_{1t+1},\ldots,z_{m+1}).$$

Now by (A2(A)),

$$E\phi^*(z_{1t+1},\ldots,z_{m+1}) = \bar{X}.$$

Since z_{it+1} is independent of f_t, we conclude that

$$E(\Delta f_t | f_t) = a(\bar{X} - f_t). \quad \blacksquare$$

Lemma 5.2. Assume PPI holds. $E(\Delta\nu_{it+1} | \bar{x}_{it}, p_t)$ is a decreasing function of $\bar{x}_{it}$ whose sign agrees with that of $\phi^*(z_{it}) - \bar{x}_{it}$. Given $\bar{x}_{it}$, it is independent of p_t.

Proof: For fixed γ_{it} and p_t, we see that ν_{it} is predetermined and ν_{it+1} depends only on $z = z_{it}$. More specifically, let $\hat{\gamma}(y) = \psi(\gamma_{it}, 0, y - p_t)$ and define $h(y, z) = \phi(\hat{\gamma}(y), z, 0) - y$. Thus, $h(\nu_{it}, 0) = 0$ and $h(\nu_{it+1}, z) = 0$ by Proposition 1, PPI, and Proposition 2. Applying the implicit function theorem to h and noting assumption (A2(B)) we see that $\Delta\nu_{it+1} - \nu_{it}$ is a strictly increasing and sign-preserving function of $\phi^*(z) - \bar{x}_{it}$. Since the response to z is independent of previous news, we conclude (upon taking expectation over all news paths that yield the given γ_{it} and p_t) that $E(\Delta\nu_{it+1} | \bar{x}_{it}, p_t)$ inherits the same properties. The independence from p_t follows from Proposition 2. ■

Proof of Proposition 5. Observe:

$$E(\Delta\nu_{it+1} | p_t) = E[E(\nu_{it+1} | \bar{x}_{it}, p_t)],$$

since $\bar{x}_{it}$ is monotone increasing in p_t, we conclude from Lemma 5.2 that $E(\Delta\nu_{it+1} | p_t)$ is decreasing in p_t. Now $\Delta p_{t+1} = \Delta\nu_{it+1}$ if [i = (M) at t and t + 1] ('Event 0'), and is an increasing function of $\Delta\nu_{it+1}$ for [i = (M ± k) at t and i = (M) at t + 1] ('Event ±k'). Hence, in any event, $E(\Delta p_{t+1} | p_t)$ is decreasing in p_t. Assume for the moment that event 0 occurs, $\Delta p_{t+1} = \Delta\nu_{(M)t+1}$. By Proposition 3, we have $\nu_{(M)t} = \bar{x}^0_{(M)t} = p_t$ and $\nu_{(M)t+1} = \bar{x}^0_{(M)t+1} = p_{t+1}$. Thus,

$$E(\Delta p_{t+1} | p_t, \text{event } 0) = E((\Delta\nu_{(M)t} | \bar{x}^0_{(M)t}, p_t) | \bar{x}_{(M)t}) = p_t \gtrless 0,$$

as

$$\phi^*(z_t) - \bar{x}_{(M)t} \gtrless 0,$$

by Lemma 5.2 applied to the $\bar{x}^0$'s. Pairing events ±k and making a similar argument, the conclusion follows. ■

Proof of Proposition 6. Suppose that no trader receives news during period t and the observed price change was $\Delta p_t > 0$. As a consequence of Proposition 2, we then have $\Delta \nu_{it+1} > 0$ for all traders i. Consider the following events:

(0) $[(M)_t = (M)_{t+1}]$, the same trader made the M^{th} highest bid in period t + 1 as in period t;

(±k) $[(M \pm k)_t = (M)_{t+1}]$, the trader who made the $(M \pm k)^{th}$ highest bid in period t makes the M^{th} highest bid in period $t + 1$, $0 < k < M$.

All these events have positive probability since $\pi < 1$. Events ±k produce transactions and so involve an announced price change, while with positive probability event 0 does not involve a transaction and hence no announced price change. Since $\Delta \nu_{it+1} > 0$ for all i implies $\Delta p_{t+1} \geq 0$ in any event, we see that under present assumptions Δp_{t+1} is a non-negative random variable with positive expectation. Moreover, since the magnitude of $\Delta \nu_{it+1}$ as well as the probabilities of events (±k) are increasing in Δp_t, we conclude that $E(\Delta p_{t+1} | \Delta p_t)$ is positive and increasing in Δp_t. Likewise, if $\Delta p_t < 0$, we find that $E(\Delta p_{t+1} | \Delta p_t)$ is negative and decreasing in $|p_t|$. Consequently, $E(\Delta p_{t+1} | \Delta p_t)$ is a monotone and sign-preserving function of Δp_t. Since $\Delta f_{t+1} = 0$ when no news arrives, we conclude that $E(\Delta b_{t+1} | \Delta p_t)$

has the same property.

Consider now the possibility of private news arrival when $f_t = \overline{X}$. It is not hard to see that

$$E(\overline{x}_{it+1} - \overline{x}^0_{it+1}) = 0,$$

for $i = (M)_{t+1}$ in this case so $E(\Delta p_{t+1} | \Delta p_t)$ is still an increasing, sign-preserving function of Δp_t, even though Δp_{t+1} can differ in sign from Δp_t with positive probability. Again, $E(\Delta f_{t+1}) = 0$ —see Lemma 5.1 above— so $E(\Delta b_{t+1} | \Delta p_t)$ remains monotone and sign-preserving in Δp_t. Finally, a continuity argument shows that $E(\Delta b_{t+1} | \Delta p_t)$ is monotone (though not necessarily sign-preserving) when $|f_t - \overline{X}|$ is small. ■

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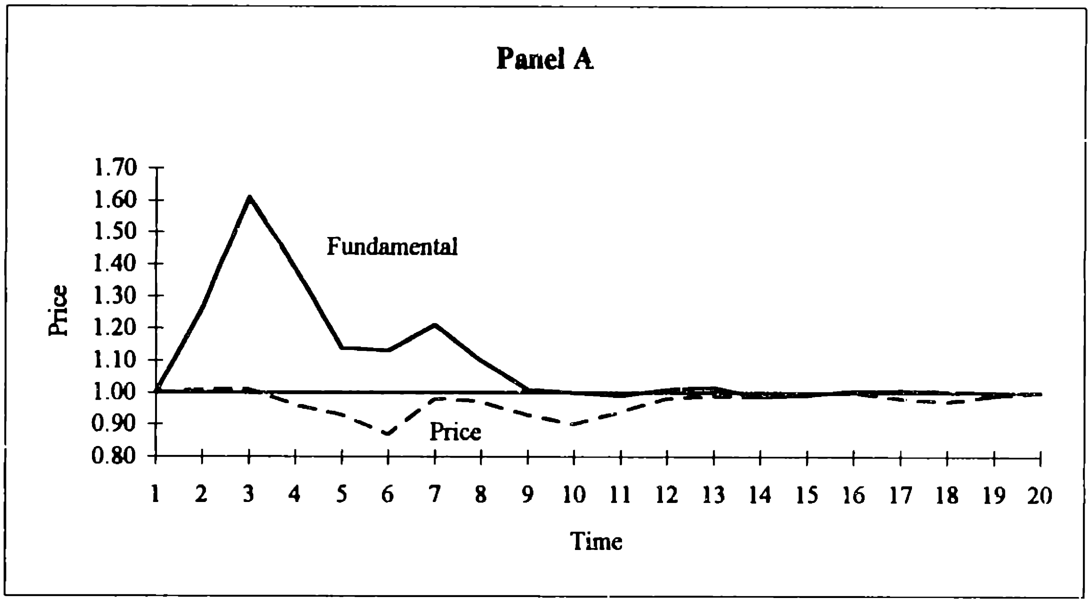
FIGURE 1: SAMPLE MODEL SIMULATIONS.



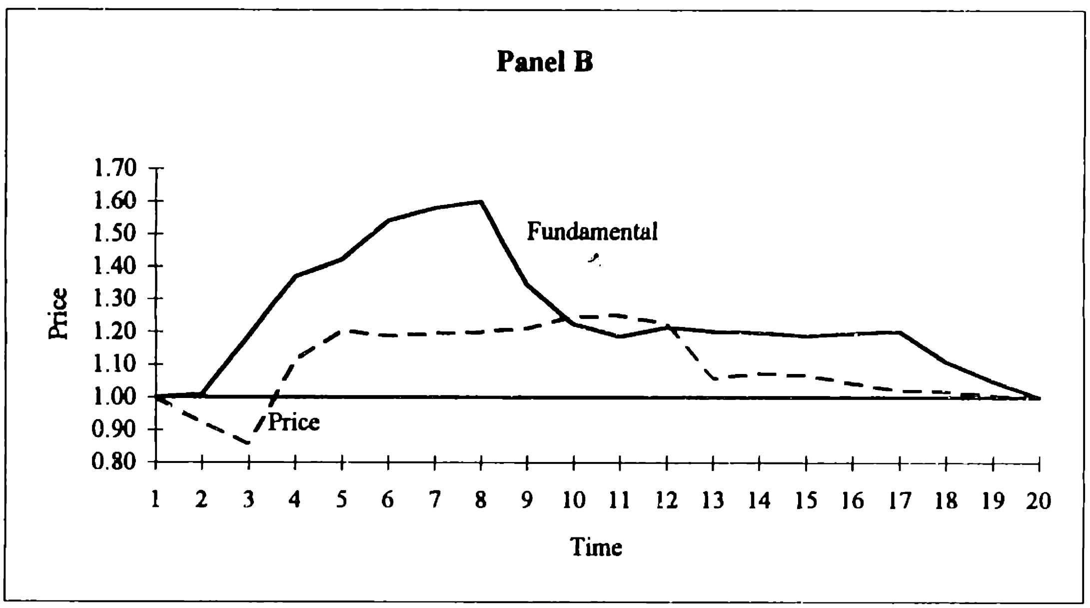
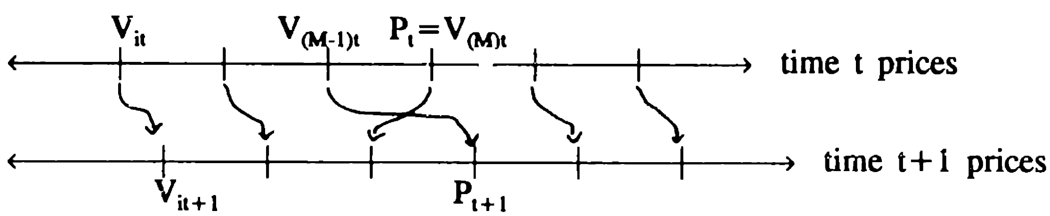


FIGURE 2: PRICE TRANSITION EVENTS.

Event -1:



Event +1:

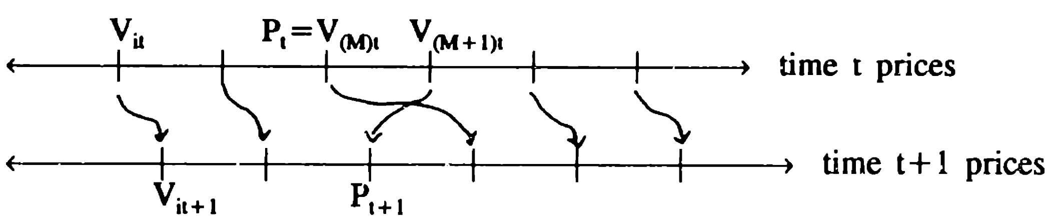


TABLE 1: EXCESS VARIANCE RATIOS, $EV_t = Var(p_t)/Var(f_t)$.

t	$\pi = 1.0$	$\pi = 0.4$	$\pi = 0.2$	$\pi = 0.1$
1	0.00	0.00	0.00	0.00
2	0.20	0.13	0.02	0.00
3	0.48	0.61	0.35	0.07
4	0.83	1.09	0.81	0.35
5	1.35	1.30	1.63	1.58
6	1.32	1.68	2.08	1.81
7	1.17	1.48	2.61	2.72
8	1.77	1.51	3.05	2.05
9	1.69	2.05	2.45	2.55
10	1.60	1.89	2.26	2.70
11	1.45	1.83	2.31	2.87
12	1.45	1.99	2.32	2.41
13	1.48	1.86	2.25	3.07
14	1.74	2.29	1.93	3.16
15	1.50	2.11	2.68	3.06
16	1.46	1.47	2.13	3.81
17	1.43	1.73	1.43	4.55
18	1.57	1.97	1.69	3.79
19	1.54	1.92	1.72	3.54
20	1.43	2.04	1.21	3.15

Notes: $\bar{a} = 1/\pi$. Other parameters are constant at N = 19, M = 10, $a_0 = 2.0$, $b_0 = 1.0$, c = 1.0, HDF = 1.0, and $\bar{\lambda} = 1.0$. Each column of estimates is based on 1000 Monte Carlo simulations, across which $Var(p_t)$ and $Var(f_t)$ were computed for each t.