Using K-Means Clustering to Create Cost and Demand Functions that Decrease Excess Inventory and Better Manage Inventory in Defense

by

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ABSTRACT

Excess inventory is prevalent in both the armed forces and defense companies; it takes up space and resources that could be used elsewhere. This thesis proposes a method to reduce the excess inventory and associated costs, while maintaining instant part availability, despite design changes which alter the number of parts required. A single period model extension was created based on K-means clustering of the parts according to lead-time and cost. These groupings provided the backbone of the cost functions created in the thesis. A predictive demand function was also created so that the design change's alterations to demand would be captured. The cost function was optimized using the predicted demand, to find an optimal order quantity that met the demand requirements and was the lowest cost option. Together these single period model function extensions allowed for a 31 percent decrease in excess inventory and 34 percent decrease in total cost.

Due to the nature of this report the companies' names have been removed, and the data naming conventions were altered so as to protect the nature of the parts.

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1. Introduction

Inventory reduction strategies exist in most industries, and there are many strategies to pick from, except when it comes to defense. Defense procurement does not fall into any one of the already predetermined categories such as Just-In-Time, economic order quantity, reorder point, periodic review, etc. Instead, defense inventory procurement straddles multiple categories, sampling from each. Typical inventory models and strategies rely on the ability to dual source or choose from a range of suppliers, use the market to set prices, and have longer production runs. However, defense inventory items are primarily high-value, complex, non-interchangeable, and critical, as there are few suppliers. Furthermore, demand for the end product (large equipment) can be relatively small; five, twelve, or eighteen units are built, which does not give much room for the economies of scale that common inventory models rely on. Moreover, defense is primarily government funded, and as such, it is focused on maximizing the ability to respond and build, and not always maximizing profits.

The initial catalyst for reviewing inventory and procurement practices in defense stemmed from the global decrease in defense spending prior to 2017, which both Deloitte and PwC noted in their annual Aerospace & Defense (A&D) reports (Deloitte, 2016; PwC, 2016). Both the armed forces and the tier one contractors experienced pressure to reduce costs, inventory, overhead, and material spend due to the decrease in government defense spending. Increased pressure, in turn, led to a need to find savings and cut costs; it is often easier to reduce inventory than headcount.

With inventory and procurement policy reviews in defense underway globally, multiple reports on excess inventory in defense from the United States, Canada, and the United Kingdom emerged. Each report noted a lack of process in procuring parts, a lack of control in selling off excess or obsolete parts, and every report highlighted the hundreds of thousands or hundreds of millions of dollars in inventory that is not attributed to a project, or that was procured without being previously approved or signed off (NAO, 2012), (GAO, 2012), (CAF, 2015). Of note: bids were often cost-plus in nature, which allowed for substantial cost over-runs, (Wang, 2013). Given the above reasons behind excess inventory, there was a pressing need to find inventory strategies that were specific to defense. This thesis offers one such strategy: using k-means clustering to group parts to create cluster specific demand and cost functions that would allow for better inventory management.

The thesis is structured into six chapters. Chapter 1 introduces the thesis. Chapter 2 reviews the literature on excess inventory in the defense industry and the current excess inventory problem, including the costs, and proposed solutions. Chapter 2 also includes a specific case study based on the author's experience with a Tier 1 A&D contractor. Chapter 3 discusses the methodology used to create new inventory management solutions, including clustering techniques, clusters, and the creation of a both a cost and a demand function. Chapter 4 provides the results of the created solutions. Chapter 5 discusses the results and offers a recommendation regarding how the defense industry can better manage inventory. Chapter 6 concludes the report.

1.1 A History of defense:

Knowledge of the history of defense is useful to understand how the problem of excess inventory came about. The US defense industry was once a leader in both revenue and profit, but since the end of the Cold War, global defense budgets have been shrinking (PwC, 2017) and mergers between specialized defense companies have become common, leading to an increase of inventory reserves.

Prior to and during the Cold War, global defense spending was estimated at twenty percent of Global Gross Domestic Product (GDP) (Harrison, M. 2003; Dobson, 2005). Defense firms had soft budgets, which allowed them to focus on building rather than on procurement processes or inventory management. Before the end of the Cold War, most inventory held by defense companies had the ability to be modified, reused, or recalibrated, keeping inventory levels relatively stable with low obsolescence and marginal cost associated for the upgrades. An ample amount of inventory was not uncommon, nor was it feared because write-downs were rare, costs could be pushed downstream onto the customer, and inventory could be reused (Dobson, 2005).

Post-Cold War, this was no longer the case. The increased pace of technological change (Drezner, 2009), coupled with declining defense spending by governments, caused defense firms to compete over a smaller pool of money, with increased technological requirements (Deloitte, 2014). This decrease in spending rippled through the defense market, leading to a decrease in the number of A&D firms (Drezner, 2009). Many could no longer compete and were bought by the strongest remaining firms (Deloitte, 2017).

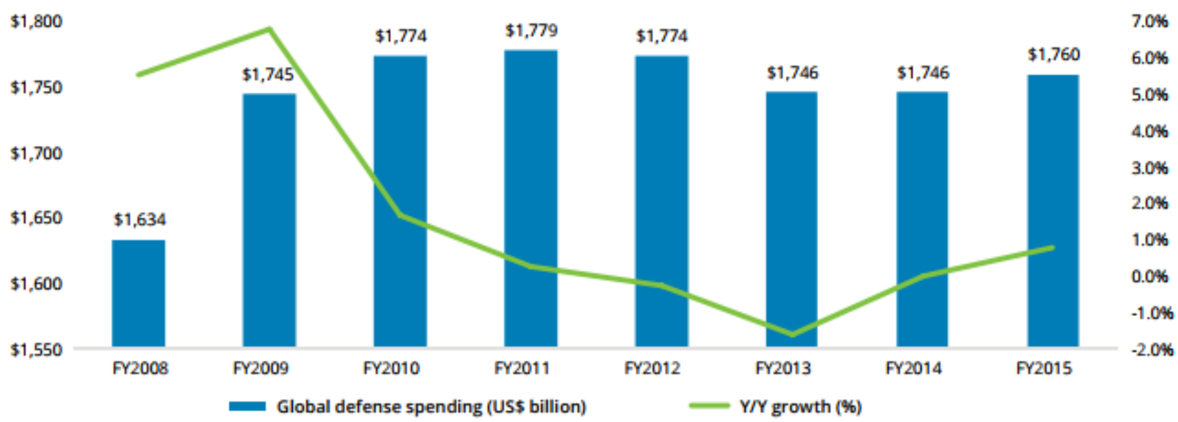


Figure 1: Comparison of defense company growth to global defense spending (PwC, 2016)

The ever-increasing ramp up of technological advancements resulted in increased inventory levels and obsolescence. The technology and the complexity of parts (for all military equipment) was increasing substantially (Drezner, 2009), pushing up the cost of the goods and products, and customers were routinely expecting more for less (Deloitte GA&D, 2017). The UK has seen a steady rise in defense inventory, which is attributed to new types of equipment being brought in without older equipment being disposed of (NAO, 2012). Factors to be considered when disposing of inventory include storage costs, potential long-term demand, potential re-purchase costs, and life expectancy of the item. Forecasting demand and the parts required is not easy. Many A&D companies retain inventory indefinitely, compared with the finite period a consumer manufacturing company would use (ibid). Coyle et al. (2016) state that defense inventory is costly; it requires storage space and service costs to ensure it does not expire. Moreover, holding inventory also poses the risk that the technology will change and become obsolete, as well as the associated carrying costs (GAO, 2001).

Furthermore, while military purchasing appears to be similar to that of a private entrepreneur, the associated cost differences are tremendous; if equipment is not available in wartime, a nation could be in danger, whereas a private citizen can find a different manufacturer or store with ease (Masuda, 1977). Therefore, militaries and their direct contractors order extra inventory to have on hand, in the event of an emergency, which includes spare parts for later in the life cycle of the equipment. Spare parts are yet another inventory challenge, as they are often far more expensive to order once construction is finished (Masuda, 1977). Each tier one contractor and each unit in the military orders their own spares, using their own calculations.

As previously mentioned, further to decreasing the funds, governments wanted to know the breakdown of the fund usage. Attention turned directly to the armed forces in several countries (Canada, United States of America, and the United Kingdom), most notably when the US Government Audit Office (GAO) reported an excess inventory level of 9.4 billion dollars between the Army and Navy in 2001 after years of extensive research (GAO, 2001). Great Britain's Ministry of Defense (MOD), which falls under the National Audit Office (NAO) also published a report stating that its defense parts also represented billions in excess inventory (NAO, 2011). Immediately, inventory reduction strategies and inventory best practices were put forward (GAO, 2012), (NAO, 2011). However, according to the reports, none of the proposed strategies reduced inventory significantly enough (GAO, 2015). Clearly, inventories of defense goods cannot be managed using the same strategies employed by private companies. Most commercial and private companies create and manage their own internal inventory systems to cover operating changes and lead-times. The models used by these manufacturers are not working in defense. Therefore, a need for models specific to defense procurement and inventory management has arisen. While a number of government reports stated the need for change, only one report outlined a solution: an inventory reduction strategy that would retire or sell unneeded parts. A follow-up report two years later found that the strategy did not work, and had been too vague (GOA, 2015).

1.2 Case Study

The company from which the data was obtained was awarded a government contract in the late 2000's – they were tasked with designing and building six different classes of equipment for the Armed Forces. Each piece of equipment can take approximately two years to build. As the size of each piece of equipment is large, it is difficult to make many at a time. The design phase is often not completed prior to the beginning of the build phase.

Inventory management strategies included Vendor Managed Inventory (VMI) though this was not controlled from the consumption side, and only monitored from the replenishment side. The company's largest issue with VMI was having the wrong items in VMI – expensive or highly customized items that were purchased before a design was finalized often ended up requiring write downs, as the vendor was not able to make changes to their order fast enough, this removed then power from the company as they were left to handle the costly mistakes from the VMI. While VMI for common off the shelf (COTS) was better managed, items are ordered, and when they arrived were placed in the part warehouse in specific bins and are used by the tradespeople. No reconciliation is performed, there is no sign out of the inventory – it is an open bin system. When a particular bin looks low, it is replenished until it looks full. The company is looking for a better system to manage the ordering of expensive and customized parts.

A further break down in the supply chain occurs when there is a design change created by engineering that is not passed along to procurement. For instance, a new design requires a new pipe width, but the procurement team is unaware and thus orders the previous width. This pipe is then unable to be used and is either discarded at a loss or reworked at a cost. It was found through interviews that ordered parts are often unable to be returned, as they were a customized part with complex specifications. This part is then rendered obsolete and is stored on site, taking up valuable space and adding to the excess inventory.

When parts arrive on site, they are given a part number that is related to the project that the part will belong to; this however, prevents other projects from being able to use the part, should they need it first. Specialized, made to order and non-COTS goods are managed on a project basis, rather than by company, resulting in duplicate orders or multiple small batch orders (one order per project), rather than one large order, which would generate increased buying power. The break down in the supply chain visibility exacerbates the excess inventory problem.

As with most A&D companies, at this company there is both the defense side of the business and the commercial side (Wang, 2013). However, defense COTS parts are stored in conjunction with commercial COTS parts. To add to the confusion, parts for the defense side are ordered and tracked using a separate ERP system from the commercial side.

1.3 Current Procurement Process

There is a break down in the lead-time on parts; of the 48,000 different parts ordered, over 5300 of them (13.61% of items ordered) have lead-times listed as 999, which is the system default. Therefore, no lead-time is known. Knowledge of lead-time is crucial as it prevents parts from arriving after they are needed. The most common known lead-time at this company is 30 days, which accounts for 18,818 or 47.91 % of items ordered. This suggests that the absolute minimum for a frozen period should be 30 days. There are currently situations where parts are being added to work orders that have a required finish date of months previous according to a data analyst at Company A (R. Smith, personal communication, February 7-8, 2018).

To further complicate matters, a single class of equipment can have over 48,000 unique parts ordered from over 270 suppliers from fifteen different countries; however, this does not include any of the parts made in-house. These parts are categorized into Long Lead Items (LLI), common off the shelf (COTS), and a variety of other categories, each with its own unique set of requirements and its own buyer. Excess inventory is often a result of last minute design changes, which can create a significant change in demand for parts, some of which have already been ordered due to their long lead-time. Further compounding the issue are the unpredictable lead-times and quality issues, as well as the fear that the highly customized parts will not arrive on time.

While the buyers order what is requested, those placing the orders do not have visibility into what design changes have been made or are going to be made and how that will affect the order amount required. Furthermore, emerging technology changes has increased complexity and has resulted in an increase in the number of rejected parts, as well as an increase in number of parts ordered; these parts cannot be reworked or reused later. Thus, with all the last-minute requirement changes to the parts, companies are left with substantial excess inventory, which not only takes up space and creates issues for finding the correct parts in a timely manner, but also drains resources, as some of the parts require maintenance even when not in use.

This thesis will cover the gap that has left the defense industry drowning in excess inventory by proposing an inventory management model specific to defense procurement.

2. Literature Review

Excess inventory occurs in most industries that have highly customized, secretive, and time sensitive inventory, for example, high-end fashion, pharmaceuticals, and luxury vehicles (AT Kearney, 2014) (Peltz et al., 2015). Most of these manufacturing companies are able to fix excess inventory in a fairly simple manner: decrease lead-time, increase sales, produce less, push off production until the item has been ordered, etc. This is not the case in A&D firms because the solutions surrounding excess inventory are far more complicated: costs are rarely passed down to the end user, the ability to rework a part is difficult, and suppliers hold more power because there is little to no substitution available (Raemekers et al., 1970). Furthermore, Yoho et al (2013) wrote a report for the International Journal of Physical Distribution and Logistics Management stating that while scholarly articles exist on topics in defense, they are largely published in specialist military journals, though other industries could benefit from the knowledge, specifically humanitarian logistics. He further mentions that there is a need for more researchers and articles on Defense logistics.

Excess inventory in defense stems from supply uncertainty, ever-changing technology, and the lack of a strategy for discarding items. This excess inventory matters because it reduces a firm's available cash and leads to a decrease in profit, which drives consolidation among defense and aerospace firms, thus decreasing the talent pool and the ability for a quick ramp up in wartime (PwC, 2016).

In most manufacturing facilities, excess parts can be modified, returned, or reused on a different project. However, in defense, these excess parts are often too complex or labour intensive to rework or require such significant alterations that the time and money required to rework them would be better spent buying or making new ones (Cameron, 2011). Accordingly, the general inability to reuse parts at a later time, coupled with a small but lengthy production run over a finite period of time, is best suited to a single period model, other times called a News Vendor Problem (NVP). The classic NVP problem is as it sounds: a vendor selling newspapers must anticipate the demand without ordering too many, as the newspapers are useless at the end of the day and will be thrown out. Porteus (2008) states more specifically that this problem occurs when the demand is unknown; there is a finite time to use the goods, while the consequences of having too much or too little are generally known.

2.1 Previous Suggestions

The inability to accurately predict demand is a leading cause of excess inventory in nearly every industry (Erwin, 2015). In defense, this problem is further aggravated due to the requirement of spare parts. These parts will be used later on in the equipment's life, although the exact number needed is unknown at the time, when they are least expensive to purchase. Replacement items, though necessary, can cost considerably more later on in the life cycle of the equipment than if they had been ordered during manufacturing (Pearson, 1994). Not only are replacements expensive to procure later on, but the ability to make them, as well as the required lead-time, can render equipment useless until the replacement part has arrived. Therefore, it is ideal to purchase the spare parts at the same time as the original parts needed for the manufacturing (Pearson, 1994).

2.2 Inventory Strategies

To avoid the excess inventory plaguing defense companies and the armed forces, the UK's NAO suggests that defense contractors, and the Ministry of Defence itself, should have inventory strategies, which are currently non-existent (NAO, 2012). When PwC interviewed executives in various industries, only sixty-three percent of A&D executives said they had a well-defined inventory strategy (PwC, 2016). The reason for the low percentage, compared to the seventy-eight percent in other industries, is that most industries are profit driven, whereas defense, which is largely government funded, is less profit focused and more results focused. A quick Google search for 'inventory model' will bring up models which focus on maximizing profit, rather than availability or response. Defense requires part and product availability and is less concerned with cost (Peltz, 2013).

One inventory management strategy is to decrease complexity, of the number of parts or the parts' usage (Mayer, 2014), as a way of mitigating inventory overages. However, commercial aerospace reaped more benefits than A&D firms when this strategy was implemented. This was due to the economies of scale that aerospace could attain. Commercial aircraft carriers buy a fleet, upwards of one hundred units, whereas in defense, the unit number is closer to twenty or thirty. Furthermore, a commercial aircraft requires less technology than a fighter jet.

Another new inventory management strategy is additive manufacturing, which was once used for toys and art, but is now moving into health care, food, and A&D. As the quality of 3-D printing production increases it could be a viable solution to the inventory issues any industry or company is experiencing (Cotteleer et al., 2014). However, the precision and size of parts required for military operations are not feasible via additive manufacturing as of this moment. PwC's recent runway-to-growth study stated that the maintenance, repair, and overhaul (MRO) market could save 3.4 billion dollars annually in materials and logistics alone through using additive manufacturing. The downfall is the comprehensive training, increased quality control, large potential for counterfeit parts, and IT system overhauls, all of which come with a steep price tag (Techbrief Media, 2016).

2.3 Model Research

Other research suggests improved supplier relationships as a way to mitigate excess inventory. Minner (2003) asserts that competition between suppliers gives increased buying power to the purchaser, which would allow for the ability to return product or push for a better price and push for a more equal partnership. However, there is a lack of competition in the defense industry (due to strict rules and regulations), which, according to Porter's 5 Forces (Porter, 1979), shifts the power to the supplier, and prevents a working relationship between the purchaser and the supplier. Moreover, defense companies have a relatively small number of suppliers they can choose from, due to geographic, nationality, or clearance restrictions, thus limiting the pool further and giving more control to the suppliers.

Just-In-Time (JIT), created in the 1970's by Toyota, is an inventory management strategy that reduces waste (excess parts) by providing parts right before the process required them (*The*

Economist, 2009). While JIT is a suitable and well-used strategy for auto manufacturing, the defense industry is largely unable to predict lead-time as precisely as JIT requires, and few (if any) substitutes are available for some parts. Reducing lead-time has also been suggested as an inventory management strategy (Silver, Pyke & Peterson, 1998). However, the complexity of the parts required in defense, and the lack of substitutes, rarely allow for a lead-time reduction. Additionally, (R, S) policies otherwise known as order-up-to policies generally have two constraints: the review interval and the order up to replenishment periods that are expected to minimize costs, which can reduce excess. The model uses infinite production period and does not account for obsolete or decaying inventory (Tarim and Kingsman, 2005). Later in the equipment lifetime, if these parts are no longer available, they can prevent equipment maintenance, or repairs from happening. Therefore, with especially complex and expensive parts, a single period model is the most promising inventory management strategy.

Additionally, inventory for A&D firms is primarily a critical buy; custom designed specialty items with a limited number of suppliers. Though traditional ways of moving a critical buy item to a strategic one will not work in A&D, neither will grouping inventory into traditional A, B, and C categories, nor any model that depends on a service level of less than one hundred percent. Why is defense so different that the most widely used inventory models do not work? In almost every industry it is acceptable to have stock-outs or planned backorders, or both. Typical service levels are agreed on and sit between eighty-five to ninety-eight percent depending on the industry. For example, AT Kearney published a report on the pharmaceutical industry stating that the service level average was 94.5% (AT Kearney, 2014), which is below the one hundred percent needed in defense. This is because the cost of carrying enough inventory is so great, it outweighs the cost of stocking out (King, 2011).

Studies exist on inventory management including procurement, sourcing, policies, and the creation of models that minimize inventory and maximize profit. However, there is a distinct lack of studies on inventory, procurement, and models for defense. Single period models are best for defense due to the nature of the non-infinite production line and the severe restrictions on suppliers, but mostly due to the fact that stock-out and excess costs are well known.

2.4 Single Period Model Research

Previous efforts and literature have used single period models in defense, with an end focus on maximizing profit (through decreasing inventory, decreasing lead-time), or minimizing cost (discounts, Vendor Managed Inventory or VMI), though none have included shortage costs which are squared, decaying inventory, with a requirement for spare parts. Squared shortage costs were first used by Padgett (1994) as both her and Masuda (1977) stated that to not have a part on hand was far more of a problem in defense than to have extra. For example, a destroyer or submarine cannot surface during wartime to pick up extra parts, they must be readily available and on hand. Thus, I also used a squared shortage cost. Defense parts include those that deteriorate over time and then must be retired for example, fire suppressant fluid, therefore I included the element of decay in my model. Furthermore, spare parts are a necessity for defense, as Masuda (1977) states, there is not always the ability to find a substitute and so spare parts are required to be on hand. This report aims to fill that gap by providing a single period model which not only minimizes cost, but also maximizes the on-hand part availability (also called demand coverage), while accounting for deteriorating inventory, and a squared stock-out cost. Defense companies require increased part availability, as a stop in production can be fateful to a company or a nation (Masuda, 1977). Furthermore, due to changes in design, the demand for parts is adjusted often over the equipment build, which triggers defense companies to order a higher number of parts to account for these changes.

The literature on single period models is plentiful, although these models are not always appropriate for A&D, given their simplistic use of demand coverage based on “simple expressions for overage and underage costs” (Minner 2003, pg. 269). In the literature, single period models are often simplified into single period supply lead-time, instantaneous emergency supply, and limit the number of suppliers. Baranakin (1961) created a single period model with T periods, focused on delivery-lag, which is not as important in defense as other industries. A more notable model consists of utilizing multiple suppliers due to the varying lead-times between them (Minner, 2003). This approach is not always possible in defense, as there are restrictions on which companies can be used to supply. An extension to the single period model is Khouja’s (1999), which highlights different states of information about demand. However, Khouja’s model relies on subjective judgement and a fuzzy set theory, which is too complicated for use in defense.

While the simplistic nature of the NVP model does not work well in defense – it assumes the product being demanded is simple, that demand is random and unknown, ignores product decay and that the cost of a shortage or stock-out is linear and relatively low – it does provide a good framework; ordering enough to cover for the whole project, or mission at one time, using the cost of excess and shortage. Therefore, we require an extension to this model; one that sets a high value on a stock-out or non-linear relationship, while maximizing demand coverage and minimizing cost, which includes product decay, and potential discounts. In order to create a better fitting model for defense companies that will decrease overall inventory but cover demand, I looked at how inventory was currently being ordered and managed.

2.5 Single Period Model Extensions

Extensions to the single period models specific to defense exist, though there is minimal literature, likely due to the nature of the topic – the defense industry is highly secretive which limits access to the types and kinds of data needed for research and reports, only since 2011 has the topic of excess inventory become popular, largely due to the research done by the GAO, NOA, and CAF. Three such extensions are proposed by Padgett (1994), Masuda (1977), and Badiru (2009). Padgett utilizes single period models to ensure that demand is covered with the use of quadratic relationships, rather than the traditional linear ones. This creates a large surplus of inventory, which is the problem I am trying to avoid (Padgett, 1994). Additionally, her model was more compatible with large quantity ordering: more than a thousand units. Therefore, I used Padgett's quadratic cost model as inspiration but created my own model that incorporated more variables and additional constraints. Masuda (1977) focuses on single period models when ordering for spare parts: should the spares be ordered with the original order, or later on. He too, discusses a large stock-out penalty, which I have used in my model. Badiru (2009)'s single period model relies on time-dependent demand during mission critical time and is used primarily for contingency operations. Furthermore, he uses a Poisson distribution (common for sporadic demand) for this demand, which is appropriate for that situation, given the unknown nature of contingency. His model is specific to contingency planning, rather than the entire mission.

My model is to be utilized prior to the mission and can be applied to manufacturing aircraft carriers, other vessels, and large equipment. Lastly, Badiru's models are complex, and specific to few situations, whereas I have created a model that is applicable to general situations and can easily have constraints added or removed.

2.6 Findings and the Gap

There is a need for a model that better fits the defense industry's requirement for lower overall inventory. Large vessels such as aircraft carriers, submarines, destroyers, and other combat ships are made in small batches every twenty or so years. The batch size creates a difficult situation where scalability is difficult to achieve, and the number of spare parts required should be purchased or built when the original vessel is being built. If spares are to be bought later when they are needed, the costs increase substantially, as does the lead-time (Masuda 1977). This issue falls into the category of an NVP. Most inventory models, including the NVP, maintain a linear relationship between a cost of a stock-out and potential demand. The traditional NVP model looks at the cost of excess, the cost of being short, and the potential demand. It then sets the optimal ordering quantity Q* to cover only enough demand that would maximize the profit (Khouja, 1999).

Graves and Willems (2003) suggest that safety stock placement could be broken down into stages, with each stage having its own policy. They use an Economic Order Quantity (EOQ) model, as their product was one that would be continuously made, where I am focused on single period, as all items need to be ordered within a window of time for the original build, and then to be available as spare parts later on. Rather than create stages, I created clusters of like inventory. I obtained data from a defense manufacturing company and I included cost, lead-time, lead-time variability (delta), part numbers, descriptions, and the number of parts ordered.

While these studies highlight excess inventory, their results do not actually reduce excess inventory; Padgett's model created an optimal order quantity that doubled the demand (Padgett, 1994), and Masuda (1977) focused on spare parts. While spares are necessary in defense, clean energy, and humanitarian relief, not all excess inventory can be used later as spare parts. There is a need for a model that is neither profit maximizing nor cost minimizing, where instead the focus is on increased part availability without excess. Industries in which a model like this would be useful include defense, humanitarian, clean energy, and government funded initiatives. This model fills the need for a function that takes into account the importance of having product immediately available, with some contingency, while minimizing costs as much as possible. My model is a single period model extension that includes; multi-phase lead-time and production time, minimal inventory holding costs, and an extremely high service level, that aims to minimize costs rather than maximize profit. A minimal holding cost was used in this model as parts purchased as per the design and when they arrive on site are either used right away or within a week. Furthermore, the parts are paid for by the client (in this case the Government) and so the loss of capital is minimal, as the company is reimbursed in a reasonable time frame, while reimbursement may not always be the case, that assumption was made in this model. Moreover, as defense is a national interest, suppliers are limited to the same nationality or an ally so as to preserve and protect the national interests (Badiru, 2009).

As noted in the 2011 GAO report, a high service level is required as the planning of missions depends on having the exact number of each required part and piece of equipment. This model will also be appropriate for situations where profit is minimal and not maximized, build time or deployment is long, and funding comes largely from the government or donors.

3. Methodology:

As the Literature review highlights, few models exist for our situation and as such, models were created to fill the gap. Based on the clusters a cost function was created that took into account holding costs, shortage penalties, and a need for spares. A demand function was created based on the changes to demand in the different phases. These models worked to decrease costs and excess inventory, as seen in sections three and four. Though specifications for the company's situations were added, they did not change the model.

3.1 Data Analysis

Data was obtained from a defense company, with their permission. It was cleansed for any null values such as missing costs and negative calculated lead-times. Once these lines were removed 40,875 unique rows remained. Data was adjusted so all values were positive and then standardized using both a z-score and minimum-maximum normalization technique. I assumed that the data was normally distributed as the Central Limit Theorem states that for a large n (equal to or greater than 30) data tends to be normally distributed. The z-score results were then compared to the minimum-maximum distribution, as I knew both the minimum value and the maximum value. I used both normalization techniques to find the normalization that best suited the data. In the end, the z-score was used as it created more defined clusters and segregation between data points, allowing for cleaner clusters. Once the data was normalized, the cost, lead-time, and delta were then adjusted so that the normalized lead-time value for 0, would be 0, and the normalized delta value for 0 would be 0.

3.2 Clustering Technique

The unsupervised clustering grouped inventory based on features such as lead-time, cost, product family type, and product number. These clusters were used to generate class-based procurement strategies. Class-based procurement strategies are useful as there is the ability to be a blanket strategy for a specific group with similar attributes. This current company groups by ordering strategy which has led to excess parts. While K-means clustering was applied for the case in this thesis, other types of clustering can be used, as long as the cluster includes lead-time and cost, the models mentioned in section 3.4 will apply.

The data was graphed in Tableau and then grouped via k-means clustering. K-means clustering "is a method commonly used to automatically partition a data set into *k* groups" (Wagstaff et al., pg. 577). In using this type of clustering, the goal is to sort the parts into specific groups or clusters with similar attributes that the human eye would not have acutely picked up on. Tableau utilizes the Calinski-Harabasz (Tableau) criterion to assess cluster quality:

Equation 1

$$\frac{SS_B}{SS_W} \times \frac{(N-k)}{(k-1)}$$

SS_B is the overall between-cluster variance and SS_W is the overall within-cluster variance, k is the number of clusters, and N is the number of observations. The clusters were then used to help create purchasing policies and algorithms.

Delta, cost, and lead-time were all variables in the clustering, as K-means clustering uses distance to measure points, descriptive data was unable to be used – part family, type etc. as it is not able to be given a distance or ranked, therefore the only factors that could be considered in the clustering were cost, lead-time, and the delta. While it would have been preferable to have a clustering technique that included descriptive attributes along with the cost and lead-time values when the cluster results were reviewed it was found that each cluster had specific part types which were rarely present in other clusters.

The current method used by the company groups parts solely by their description, this lead to substantial excess inventory. This report used a different type of clustering which would allow for the creation and testing of a new procurement policy, one that would decrease excess inventory.

Figure 2 shows the results of the k-means clustering for Delta vs Cost, where colour represents the cluster. The delta is the difference between when the item was scheduled to arrive and when it actually arrived. The clustered data was then downloaded and analyzed using single factor analysis of variance (ANOVA), and confidence levels. As shown in Figure 2 the colors are the clusters, and the shape is the part description. The data had over 40,000 points, and the clustering was returned with no null values. Unfortunately, the level of confidence was low, and the clusters were extremely uneven in size – the largest cluster comprised ninety-five percent of the data points. Therefore, the Cost vs Delta was not used, as the models generated would not have been accurate or reliable.

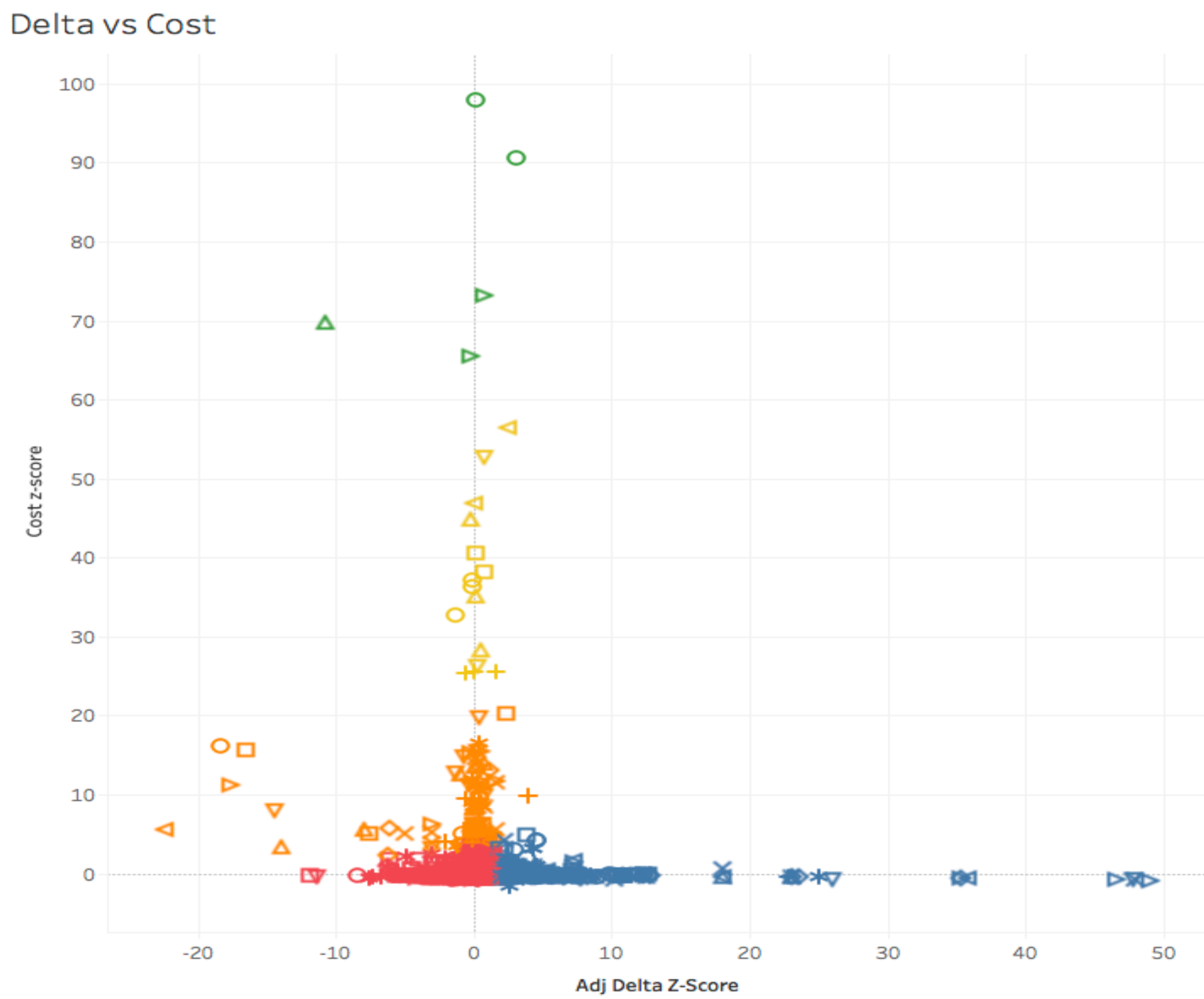


Figure 2 Delta versus Cost graphed. Shapes represent the part family, colors represent the cluster.

As a result, Cost vs Lead-Time data was placed into Tableau, as shown in Figure 3:

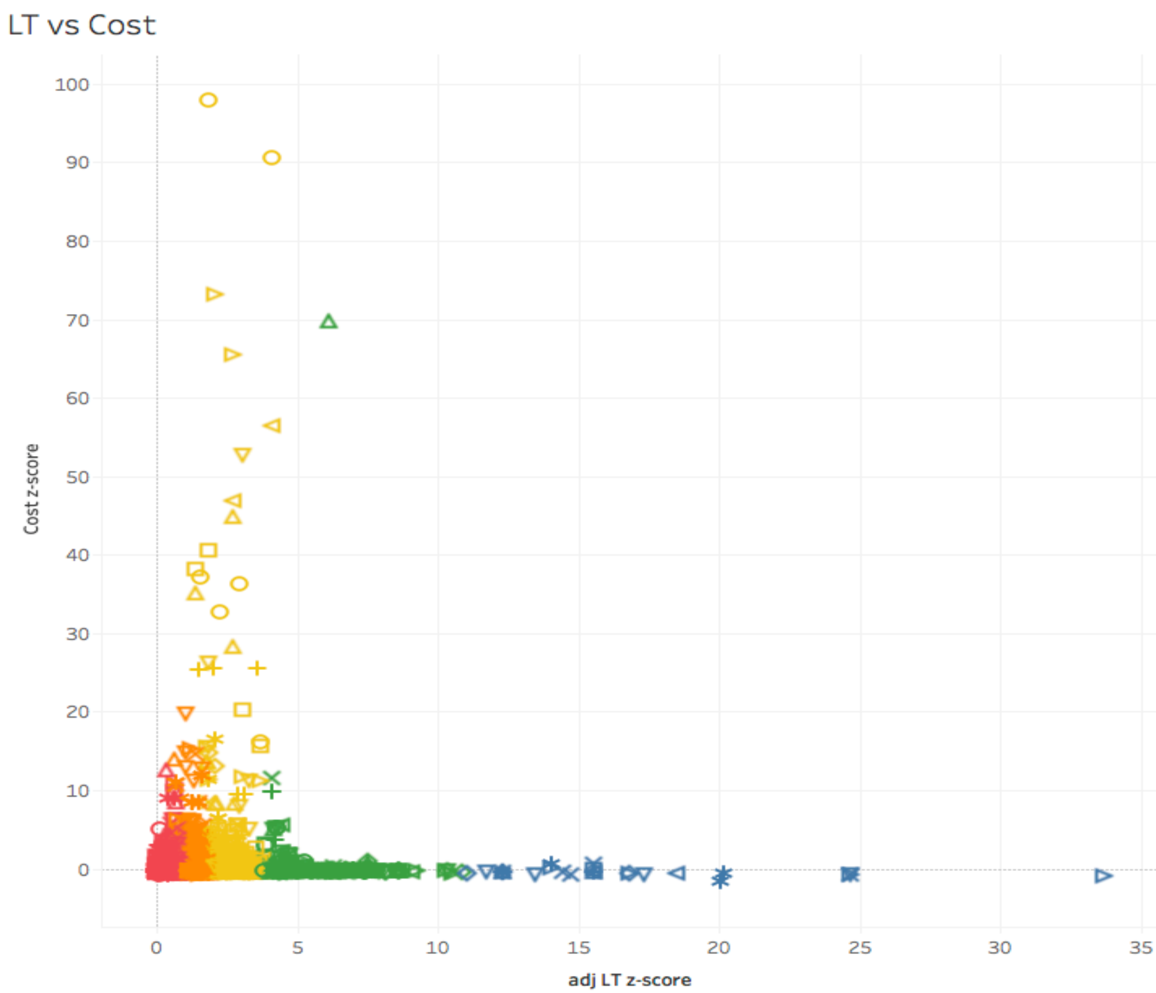


Figure 3 Lead Time versus Cost, colours are clusters, and shape is part type

As shown in Figure 3, this approach yields a much more balanced cluster approach, with the largest cluster holding ~20,000 data points, and the second largest holding ~14,000, followed by ~6,000, ~1,000 and ~30. The more even clusters will allow each model for each cluster to be more accurate.

- Cluster one: Red, short lead-time, within one to thirty days, pricing is inexpensive to mid-range.

- Cluster two: Orange, lead-time of 160 days, prices varies from inexpensive to moderately expensive.
- Cluster three: Yellow, lead-time between 281 days, price is inexpensive to the most expensive parts.
- Cluster four: Green, lead-time of 566 days, pricing is generally inexpensive.
- Cluster five: Blue, lead-time of 1,874 days, and price is inexpensive.

The clusters created by Tableau are similar to the part groups used by the company, though they are not identical; furthermore, the procurement strategies will differ greatly, which will allow for better inventory management when the new models are used.

The company previously clustered the parts into categories and families that were based on how the item would be ordered and its lead-time as shown in Table 1.

CAT	DESCRIPTION	CONTRACT
A	Single System Integration (SSI)	Full Development
	Complex LLI	Full Development
B	LLI with Design %	10-15% Development
C1	COTS LLI	Zero Development
C2	MSA	Framework Supply Contract
C3	PO	Purchase Order
D	On Site Supply	Supply & Install

Product Family	Product Family Description
FAB	Fabricated Hull Specific part
NS	Non-Spec Purchased Material
PS	Pipe Spool
SPEC	Major Spec Equipment
RM	Raw Material
WP	Workpackage
OS	Outfit Steel
NEST	Nest (Steel Plate or Profile)
SPECL	Major Spec Equipment Loose Part
CBL	Cable, Electrical
SA	Subassembly
DUMMY	Dummy Parts Never Ordered

Table 1 Current grouping method used by the company

This category management allowed for substantial excess, as each type of ordering had its own strategy, primarily defined by the purchasers themselves. The company has no over-arching inventory management strategy, and as such, each department and category manager created their own.

3.3 Demand Distribution

Due to design changes, either from the customer changing the requirements, or a new technology emerging, demand for each part can vary over the life time of the design and build. What makes ordering parts even more difficult is that often the design phases and build phases overlap, such that when a design change occurs, it can affect the number of parts already ordered. For this thesis we will assume five build phases and five design phases. While not every defense company will have five of each, it was a large enough number of phases to test the models, while still being computationally friendly.

This model assumes that each design phase will take place, though there is no requirement for a specific number of phases. All that is required is knowledge of an average change in demand per phase, and probability that the change will occur. While my model did not have a probability for a decrease in demand, one can easily be added.

Each cluster's part descriptions were reviewed, and equations were created based on the type of parts, the lead time of the cluster, the cost of the cluster, and the correlation of the cluster. Equations for each cluster were created, which maximized the demand coverage [quantity of parts required during the build (which can change due to changes in the design), as well as during the lifetime of use] and minimized the overall cost, using supply as the variable. Though demand is assumed to be known, in cluster one, demand is dependent on design changes, where events $B_1, B_2, \ldots$ form a partition of a set Ω, $\sum_{i=1}^{\infty} B_i = \Omega$, then for any event A, we have Bayes formula of total causes (Brémaud 2017)

Equation 2

$$P(A) = \sum_{i=1}^{\infty} P(A|B_i)P(B_i)$$

Through discussions with management, it was found that there is a range of probability, in which, an item that was not ordered would have an increase in demand due to the design change. It was also found that some items that had already been ordered would require additional parts due to changes in design. However, a probability was not calculated, as enough data could not be found on the range of changes; for this reason, it was left out of this thesis.

Given the data regarding changes in demand between phases, a downward sliding scale was used on the estimated increase in demand, as only overall averages for increases in demand were measured, and not on a by part or design phase basis. Generally, changes in early design or planning phases are much more significant than the later phases, largely due to all of the unknowns at the beginning of the design or plan; however, as decisions are made, the ability to change decreases. Table 2, the probability of increased demand and the estimate of the demand increase is particular to the company in this thesis, though they worked with other data it would be beneficial to find averages for the industry or specific company when used.

Phase	Probability item not ordered yet will have an increase in demand	Estimated increase in demand
1	75%	25%
2	70%	20%
3	65%	15%
4	60%	12%
5	55%	10%

Table 2 Percentage change of demand during a phase

In this particular case there was an assumption that each design phase would happen, so a uniform distribution was applied. Though the model has the ability to handle a probability for a decrease in demand, due to conversations and data from the company, in this case there is an assumption that demand will not decrease; it will either increase or remain the same. Per the assumption, we can now use a simplified function to find the average expected increase. Given that the item is going to be ordered in design phase three, and used in design phase four, we select the probability of change for phase four and multiply it by the percent increase for phase four. There is no change to the demand if the design change does not affect the demand; therefore, that part of the equation would be zero and is ignored.

Equation 3

$$P(D_3|D_4) = 0.60 \times 0.12 = 7.2\%$$

Therefore, there is a 7.2 percent increase in demand during phases 3 to 4. For items that span multiple stages, the average expected values for the increases are added together such that a part ordered in design phase three but used in design phase five would be

Equation 4

$$P(D_3|D_5) = (0.60 \times 0.1) + (0.55 \times 0.10) = 12.7\%$$

While the percent increases and changes in the above model were tailored to this company, these values provided similar results when used on another set of data - decreasing their excess inventory on average by twenty four percent. The same assumptions applied: five phases, increased probability of demand change in each phase, and no decrease in demand.

3.4 Equations

Equations were created based on the result of the clusters. While these equations inputs were specific to the case and the company's data, they can be adapted as the inputs are based on company data. To test their robustness, the equations were tested on another set of data from the Armed Forces. The equations provided improved demand accuracy, which reduced costs. Though assumptions were applied which included: cluster creation using cost and or lead-time, knowledge of the phase the item will be used in and the phase the item will be ordered in, the average demand change during the phases is known and the probability of a demand change in each phase is known. Clusters which require spares know the general number of spares required, a penalty cost is set between ten and thirty dollars an item, and the holding cost is not the same as the salvage value. The holding cost in this model is the financial burden the excess parts take, multiplied by the cost of the item multiplied by the length of time the item is on site and unused.

The general cost function is:

Equation 5

$$Total\ Cost = \begin{cases} c_iD_i + \delta(D_i - S_i)^2 - (1+\pi_p)c_iD_i + \theta(X_i) + c_i\varphi;\ \pi_p > 1,\ D_i \geq S_i \\ c_iD_i + c_ih(T_{EP} - T_U)(S_i - D_i) - g(S_i - D_i) + \varepsilon\left(\frac{1}{T_{EP} - T_U}\right);\ T_{EP} > T_U,\ S_i > D_i \end{cases}$$

where

$c = cost\ of\ the\ unit$

$\delta = penalty\ for\ being\ short$

$\alpha = srink\ (items\ that\ are\ lost, stolen, or\ break)$

$\pi_p = probability\ value\ of\ a\ design\ change\ leading\ to\ demand\ change, p\ is\ the\ phase$

$D_i = Demand\ for\ item\ i\ in\ cluster\ j$

$S_i = Supply\ for\ item\ i\ in\ cluster\ j$

$h = cost\ to\ have\ the\ inventory\ on\ site$

$g = salvage\ value$

$\varphi = number\ of\ spares, calculated\ by\ ](lifetime\ years\ of\ equipment + 15\ years)/(lifetime\ years\ of\ part)]$

$\theta = cost\ of\ reworking\ the\ items$

$T = life\ time\ of\ item\ from\ use\ (U) to\ the\ end\ of\ the\ project\ (EP)$

$\varepsilon = \ cost\ of\ decaying\ items$

and

$c_i D_i$ The total cost of items ordered; the cost of the item, multiplied by the demand of the item – this is a common term in all cost equations, as it is the total cost of the parts before the holding, order, salvage and other terms are multiplied in.

$c_i h(T_{EP} - T_U)(S_i - D_i)$ The holding cost ch (cost of the space the item is taking up) is multiplied by number of months the excess will sit on site. As inventory is ordered to arrive right before it is used, and the suppliers are not paid until the inventory arrives, the holding cost is only attached to the excess, and the length of time the excess is held for.

$g(S_i - D_i)$ The salvage value only applies to the inventory that was ordered in excess, this is why it is a max function. Salvage values decrease the cost of over ordering, as the parts have some value and can either be reworked, sold, or used at a later time.

$\delta(D_i - S_i)^2$ The shortage cost is squared and multiplied by the units short. A squared short cost is rarely used, as it would not maximize profitability, however, in defense, or other government projects, the goal is not to maximize profit, but to complete the work in a timely manner.

$(1 + \pi_p)c_i D_i$ The phase change is π_p and is calculated using the demand function's probability forecast

$\varepsilon\left(\frac{1}{T_e - T_a}\right)$ The cost of replacing the decaying items, given when they arrive minus when they are used. The inverse is used as these parts decay.

$c_i\varphi$ This is the cost of the part multiplied by the number of spare parts required.

The cost function was different depending on the cluster's attribute; cluster one includes parts that can be used in other projects, so it included a salvage value, whereas cluster three does not have parts which would have a salvage value, so a salvage value was not included.

Chapter 4 (Results) includes tables and graphs which compare the previous cost function to the newly created cost functions provided in sections 3.4.1 - 3.4.5.

3.4.1 Cluster One

Cluster one includes 20,400 parts, using twenty percent of the total spend. The items in this cluster are primarily compromised of COTS, such as elbow joins, gaskets, nuts, bolts, etc which can be used on other equipment or other projects so there is a salvage value attached to them.

While cluster one includes a probability function for demand change, this is only included in the demand function and not the cost, as the order quantities are so large and the cost function includes a salvage value for any excess, therefore using the phase in the cost function would only further encourage additional excess.

The ordering model for items in cluster one is below and only includes the parts of the cost function that are relevant to the items in the cluster.

Equation 6

$$Total\ Cost = \begin{cases} c_i D_i + \delta(D_i - S_i)^2;\ D_i \geq S_i \\ c_i D_i + c_i h(T_{EP} - T_U)(S_i - D_i) - g(S_i - D_i);\ T_{EP} > T_U,\ S_i > D_i \end{cases}$$

The predictive demand function will generate a new demand amount that takes into account the number of phases the part order will straddle: the phase the part is needed in and when it has to be ordered according to the lead-time it has. The predicted demand number is then placed into the cost function, which provides a cost associated to that demand.

Equation 7

$$\alpha \sum (1 + \pi_p)(D_i)$$

Demand is an expected value based on the lead-time and the phases in which the lead-time over lays.

α corresponds to the amount of shrink this cluster experiences. These parts are primarily small and inexpensive allowing them to easily be misplaced. For cluster one, α is set at 7 percent as per conversations with the management team in warehousing felt this was representative of the shrink.

$1 + \pi_p D_i$ Is the design multiplier; the p represents the phases between when the item is needed and when it must be ordered, as this number is an additive percentage it must be added to 1 to create the increase in demand.

3.4.2 Cluster Two

Cluster two has 13,700 parts and accounts for forty-six percent of the total cost, including a large portion of customized parts. Some of the customized parts can be reworked; however, it costs both time and money to rework them. The penalty for being short is squared as the items in this cluster will hold up production if they do not arrive on time or arrive damaged. Lead-time plays more of a role in this equation (compared to the other clusters), as the average lead-time is 157 days, and we are including a probability that the cost of buying the spares now will be less expensive than buying them later. Also introduced is φ, which accounts for the number of spares required for that item.

Equation 8

$$Total\ Cost = \begin{cases} c_i D_i + \delta(D_i - S_i)^2 - \left(1 + \pi_p\right) c_i D_i + c_i \varphi;\ \pi_p > 1,\ D_i \geq S_i \\ c_i D_i + c_i h(T_{EP} - T_U)(S_i - D_i);\ T_{EP} > T_U,\ S_i > D_i \end{cases}$$

$$\sum (1 + \pi_p)(D_i) + \varphi$$

3.4.3 Cluster Three

Cluster three holds 5,900 parts and makes up thirty-three percent of the total cost – these items are critical to the production. Should these items arrive late, production will stop. Some of these items can be reworked on site. Therefore, it is essential to include a large penalty for stocking out, and cover demand over the lead-time and its variability, as well as the probability of stocking out.

$$Total\ Cost = \begin{cases} c_iD_i + \delta(D_i - S_i)^2 - (1+\pi_p)c_i(1.07)D_i + \theta(X_i) + c_i\varphi;\ \pi_p > 1,\ D_i \geq S_i \\ c_iD_i + c_ih(T_{EP} - T_U)(S_i - D_i);\ T_{EP} > T_U,\ S_i > D_i \end{cases}$$

$$\sum(1+\pi_p)(D_i) + \varphi$$

3.4.4 Cluster Four

Cluster four is comprised of 700 parts, worth 2.6% of total spend; however, these items can increase spend quickly as they have a shelf life and will deteriorate while both in or out of service. A cost is included for replacing them and will get larger as the time to expiration draws nearer.

Equation 10

$$Total\ Cost = \begin{cases} c_iD_i + \delta(D_i - S_i)^2 - (1+\pi_p)c_iD_i + c_i\varphi;\ \pi_p > 1,\ D_i \geq S_i \\ c_iD_i + c_ih(T_{EP} - T_U)(S_i - D_i) + \varepsilon\left(\frac{1}{T_{EP} - T_U}\right);\ T_{EP} > T_U,\ S_i > D_i \end{cases}$$

$$\sum(1+\pi_p)(D_i) + \varphi$$

3.4.5 Cluster Five

Cluster five holds twenty-eight items, for less than one percent of total spend, but the lead-time is in years, not months. Parts included in this cluster are specialized paint and glue, as well as large pieces of steel. This inventory is primarily ordered prior to the first phase and required in the first phase (the steel) or not until the last phase (paint and glue). Glue and paint have expiry dates and thus decay if ordered and arrive too soon. It is inventory that must arrive on time and in perfect condition or the production will halt.

Equation 2

$$Total\ Cost = \begin{cases} c_i D_i + \delta(D_i - S_i)^2 - (1+\pi_p)c_i D_i;\ \pi_p > 1,\ D_i \geq S_i \\ c_i D_i + c_i h(T_{EP} - T_U)(S_i - D_i) + \varepsilon\left(\frac{1}{T_{EP} - T_U}\right);\ T_{EP} > T_U,\ S_i > D_i \end{cases}$$

$$\sum (1+\pi_p)\,(D_i)$$

3.5 Testing

The above models were trialed in excel using Solver. The model was optimized to find the optimal ordering quantity using the predicted demand and the minimized cost function. New ordering strategies were created using the model results.

The model assumes that each stage is a finite length of time, demand has variation, though is normally distributed, that a large cost is incurred for a stock-out, and that there are five phases for both design and build. Their probability of change has been estimated based on the data and conversations with the company, though when applied to other data provided.

Once the equations were created, the next step was to test them. Excel Solver was used for the optimized cost function, and a macro was used for the demand function. The new cost functions were compared to the previous cost function. Cost testing included determining the optimal quantity that minimized costs, further to that, I manually adjusted the purchased amount to differ, and recorded the results. The costs were compared and either charted or graphed. The cost model also included a shortage cost that was non-linear due to the high service level requirement, in defense parts need to be on hand and available immediately (DODIG, 2016).

The new demand model was set up in excel, where the initial order quantity is used and then multiplied by the probability of demand increase per phase the item covers, this results in a predicted demand value for when the item will be used. The new predicted demand quantity was compared to the actual amount ordered, and in every case the predicted demand was less than the actual amount ordered. The predicted demand amount was then placed into the cost function and the optimized supply quantity was found, in some instances it was less than the predicted demand and, in some instances, it was more, the reason for this was the size of the phase change; the larger the phase changes the larger the optimized supply quantity.

The quantity provided by the new demand model was then placed into the cost function, where an optimized supply quantity that minimized cost was found. For smaller phase changes the optimized supply quantity matched the predicted demand, however, once the phase change was larger than 23 percent the optimized supply quantity was larger than the demand. While this will lead to excess demand, the optimized supply quantity is still below the previous demand function's quantity, thereby still decreasing the excess.

4. RESULTS

After the company's data was tested, new data from the DOD (Inspector General) was tested, which yielded similar results, and, in the case of the demand function, better results. These models will be used to set new procurement policies. The demand model will be used to find the future demand for when the part will be used, that quantity will be placed into the demand part of the cost function, the purchased amount is the changing variable, and the cost is minimized.

The function was optimized using Solver in excel, this provided the optimal purchase quantity that minimizes the costs. The optimized supply quantity was similar to the predicted demand function, though began to increase at a faster rate for phase changes above 23 percent, though 80 percent of phase change probability is below 23 percent.

4.1 Cost Function

The cost functions were adapted to suit the clusters in this particular case; not all clusters require spares, have decaying parts, or a salvage value. Therefore, each cluster can pull from the general. Accordingly, each cluster's cost function is related to the type of parts, the general cost function includes all possible types of inventory and can be used with success, as seen in the cluster break down.

Clusters one to five decreased the cost of purchasing the parts when they were compared to the previous cost function. The cluster results are described below in Section 4.2.

4.2 Cluster Results

After running the data through Tableau, five clusters of parts were created. Table 3 gives the breakdown of the clusters:

Cluster	Color	Number of Parts	Total Cost	Average Cost	Average LT in Days	Max LT	Min LT	Average Delta in Days
1	Red	20,412	$5.9M	$290.5	34	96	0	4.4
2	Orange	13,772	$14.0M	$1,019.2	158	220	70	24.7
3	Yellow	5,875	$10.0M	$1,706.9	282	460	156	35.3
4	Green	703	$807.9K	$1,150.9	566	1,220	424	91.0
5	Blue	28	$28.3K	$1,048.6	1,875	3,787	1,242	1,029.3

Table 3 Overview of all clusters

Each cluster's cost and demand function were derived from its properties. In the cost function, the shortage cost included an exponent. As previously discussed in the Literature Review it is far more expensive in defense to not have the part available then to have excess parts. The cost function used the quantity demanded derived from the demand function and then was optimized in excel using Solver, the changing variable was the purchase (supplied) amount and the cost was minimized. The minimized cost was compared to the previous model's cost, the new cost function had lower costs - a sample of the differences can be found in Table 4. Combing these two models provided an optimal order quantity that also provided the least cost. This was true in the generalized model as well as the case study specific models.

4.2.1 Cluster One

After testing the model and comparing it to the previous cost function, I found that the cluster's cost increased both when there was a shortage of parts, as well as when there was a significant abundance. Historical data was used to test the cost model. In the case of a short on product, the cost would be far greater than the traditional model; in the event of too much inventory, the cost was also greater than the traditional cost model. This is a significant difference from the previous cost model as it was linear and thus the cost increased with an increase in parts or decreased with a decrease in parts. Equation 6 creates a valley and optimizes the quantity at the lowest cost.

4.2.2 Cluster Two

Cluster 2 included phase changes; this occurs when parts are needed for example in phase 5 but due to their long lead time, must be ordered in phase 3. The demand model will create a predicted future demand and this demand will be used in the cost model, which will then be optimized, and a suggested supply quantity that minimizes the costs will be found. It was found that with smaller phase changes the optimal ordering quantity was less than the predicted demand quantity. However, as the phase change amount grew, so did the optimized supply quantity result in excess inventory. Table 4 highlights a single part's experience with the changing phases. An interesting note is that for this particular part the small phase change (phase 5) creates an optimized supply quantity that is less than the predicted demand. While this did not hold for the remainder of the parts tested – they either optimized to match the predicted demand or generated a single unit of excess – the reason for this was likely the part cost, which was higher than the average part cost for this cluster.

Predicted Demand	Optimized Supply	Difference	Phases	Phase Probability
11	10	-1	5	5%
11	12	1	4 & 5	12%
11	12	1	4	7.20%
11	12	1	3	9.75%
11	12	1	2	14%
12	14	2	1	18.75%
13	16	3	1 & 2	32.75%
10	10	0	No Phase	0%

Table 4 A sample that compares the predicted demand quantity on part A to the optimized purchased quantity that minimizes the cost

When Equation 8 was graphed using phase changes between phases 4 and 5, (highlighted in purple), and phase 1 (highlighted in gold), Figure 6 below, selected 12 units as the optimized quantity to minimize the total cost for phase 4 and 5, however, it selected 14 units for phase 1. While this creates an excess number of units, it is because the model is set to force an integer answer, as 11.2 parts cannot be ordered. As such the quantity is rounded up to the nearest integer - in this case 12 and 14.

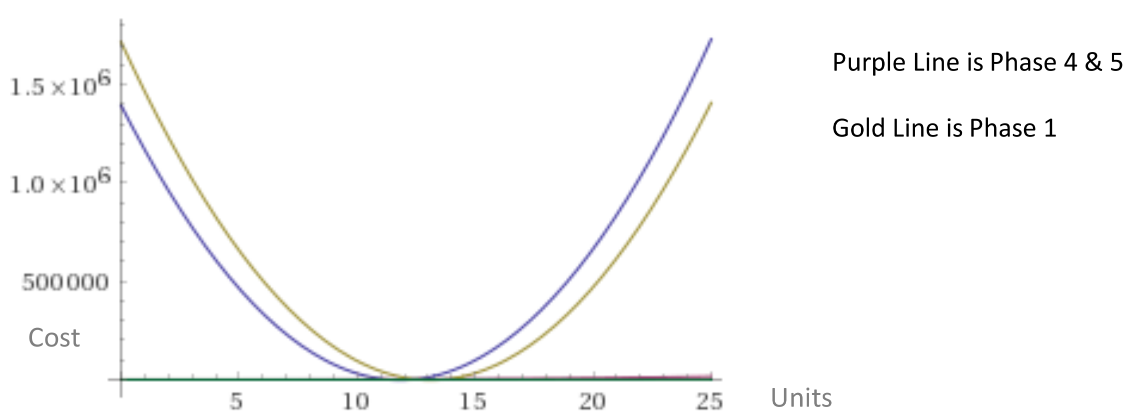


Figure 4 Graph of the equation 8 from Table 4, phases 4 & 5 (purple) vs phase 1 (gold) were used, Cost vs Number of units to order.

Also, of note is the holding cost has a significant impact on the optimized supply quantity. The lower the holding cost, the higher the optimized supply number – this observation is not uncommon as the less costly it is to hold an item, the more likely it is to hold them; especially as the shortage cost is substantial. The disadvantage is that the function is highly sensitive to the holding cost; therefore, it is crucial to select a holding cost of between 0.10 and 0.13, (10 percent and 13 percent respectively). Table 4 has a holding cost of 12% per year, when this was changed to 13% the optimized supply decreased for the earlier phases and matched the predicted demand.

4.2.3 Cluster Three

As with cluster 2, the phase change affects the optimal supply quantity; as the probability of the phase change grows, the optimized supply quantity also grows. See Figure 7 below, highlights the changes in optimal order quantities when the phase changes - Table 5 provides the exact quantity that minimizes the cost. Equation 9 is graphed below, the green parabola has the largest phase change, and the light purple, the lowest phase change.

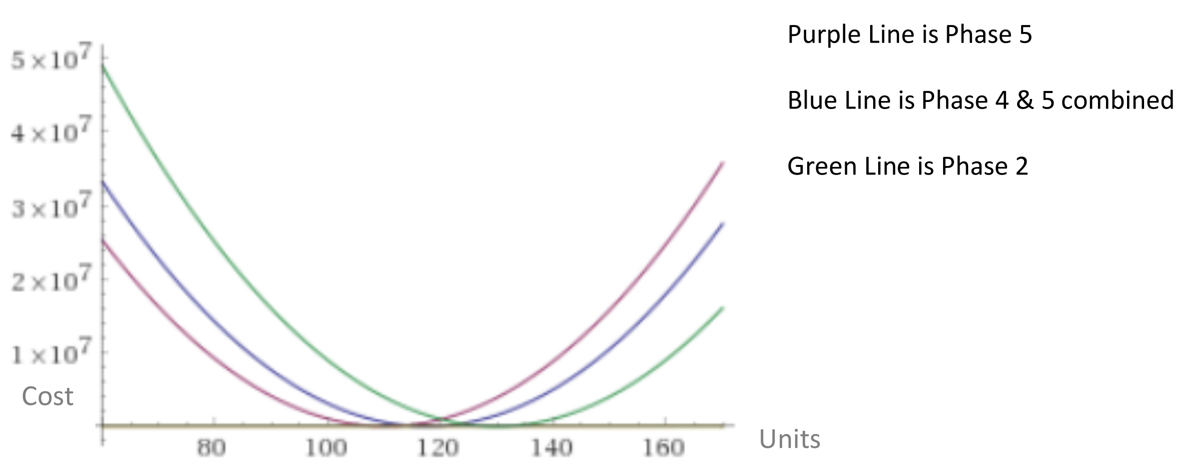


Figure 5 Equation 9 graphed with an example of a single part's optimal supply quantity over different phases, see Table 5 for the details

The amount of excess in the optimized supply in cluster 3 was far higher than that of cluster 2's. It is expected that this is largely attributed to the fact that cluster 3 includes spares, and as such, most excess has the ability to be used as a spare at a later date. The other factor is that the original order quantities are substantially larger than that of cluster 2, so while the percentages remain the same, the physical number of parts appears larger. Table 5 has the detailed highlighted quantities for Figure 5:

Predicted Demand	Optimized Supply	Difference	Phases	Phase Probability
108	110	2	5	5%
115	125	10	4 & 5	12%
110	115	5	4	7.20%
113	121	8	3	9.75%
117	130	13	2	14%
122	141	19	1	18.75%
137	178	41	1 & 2	32.75%
105	105	0	No Phase	0%

Table 5 A sample that compares a single part's predicted demand to optimized supply over different phases.

As with cluster 2 the holding cost has a significant impact on the optimized supply quantity in that the higher the holding cost the smaller the difference between the predicted demand and the supply. Therefore, a holding cost of between 0.10 and 0.13 (10 and 13 percent) is optimal. Table 5 and the adjoining graphs in Figure 7 used a holding cost of 12 percent.

For small order quantities, much like in cluster 2, there is little variation between the predicted demand and the optimized supply quantity, this is regardless of the cost of the item. Therefore, the larger the order quantity the larger the difference between the predicted demand, and the optimized supply.

While the previous total cost equation decreased as the number of parts decreased, the new model as seen in Figure 7 will increase the cost when too few items are supplied. The new cost function is quadratic, which will have a global minimum where the cost is minimized. The penalty for a short is not sensitive but is recommended to be set between 30 and 100 dollars per part, with a higher penalty used when the quantity is low and under 20 units.

4.2.4 Cluster Four

The cluster saw even larger gaps form between the phases, as seen in Equation 10 in Figure 6 below.

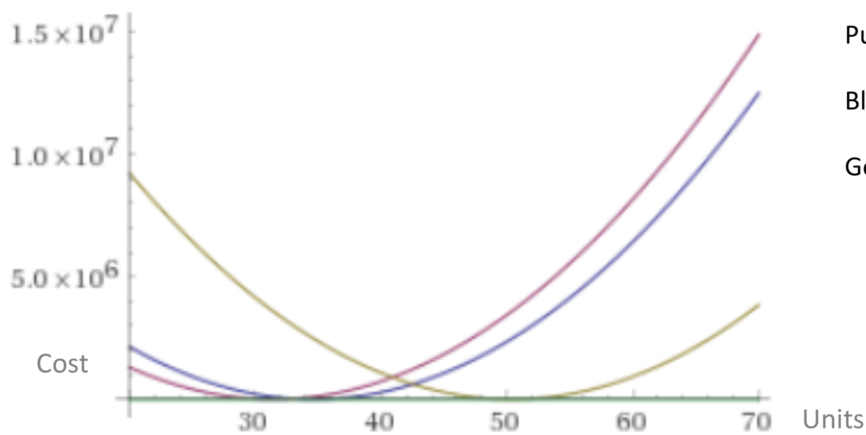


Figure 6 Equation 10 graphed with an example of a single part's optimal supply quantity over different phases see Table 6 for the details

Predicted Demand	Optimized Supply	Difference	Phases	Phase Probability
30	30	0	5	5%
32	32	0	4 & 5	12%
31	31	0	4	7.20%
32	32	0	3	9.75%
33	33	0	2	14%
34	35	1	1	18.75%
38	50	12	1 & 2	32.75%
30	30	0	No Phase	0%

Table 6 A sample that compares a single part's predicted demand to optimized supply over different phases.

Again, the cost function is a quadratic with a global minimum which minimizes costs, again of note is that as the probability of a change in phase increases, so does the optimized supply quantity. Cluster 4 also includes parts that decay or expire (fire suppressant material and chemicals). These also contribute to the larger gap between the predicted demand and the optimized supply quantity. However, unlike clusters 2 and 3, the holding cost in cluster 4 did not create large changes in the optimized supply quantity when it was decreased. Instead the quantity remained the same, regardless of using a holding cost being between 0.05 and 0.12.

4.2.5 Cluster Five

Cluster five reacted the same way as cluster 4; optimized supply increased far faster with the phase changes and regardless of the holding cost used the optimized quantity remained the same. Cluster 5 is made up of specialized consumables such as paint and glue, as well as steel plates; there were only 30 items in this cluster. The extreme difference in parts and the small number of items (though similar lead-times and cost), as well that only some parts in the cluster decay are the causes of the large variation between phases as seen in the graph of Equation 11 in Figure 7 below and in Table 7:

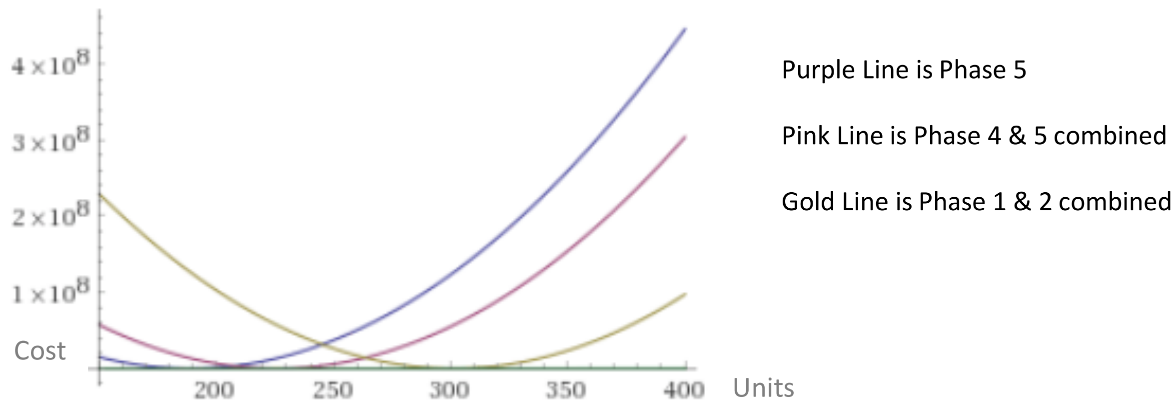


Figure 7 Equation 11 graphed with an example of a single part's optimal supply quantity over different phases, see Table 7 for the details

See the table 7 below.

Predicted Demand	Optimized Supply	Difference	Phases	Phase Probability
180	180	0	5	5%
192	193	1	4 & 5	12%
183	183	0	4	7.20%
188	190	2	3	9.75%
195	203	8	2	14%
203	241	38	1	18.75%
227	301	74	1 & 2	32.75%
171	171	0	No Phase	0%

Table 7 A sample that compares a single parts' predicted demand to optimized supply quantity

As the above table shows, the results of the optimized supply quantity were closely aligned with the predicted demand, though the difference grew as the phase probability increased.

4.3 Demand Function

The previous demand function did not use a critical ratio, and instead ordered based on all of the potential demand, as the service level was expected to be high. The problem with this method of finding demand is that when a design change occurred, the increased requirement would sometimes be added to the original demand and sent to the procurement team. The procurement team would then order the new demand, despite having already logged an original order for the original demand. However, given the single period function, a previous order could not always be added to if it had already been submitted; therefore, another supplier would need to be found to make the parts. As a result, this lead to the potential for duplicating the amount of parts and costs. This type of ordering is reactionary. In contrast, this demand function is proactive and takes into account the design phases and potential changes, and places those into the initial order. This prevents potential duplicate ordering. Table 8 is a sample of the part demand where instances of duplicate demand have been found, see the old demand column and compare it to the actual need;

Part	Original demand	New Demand	Old Demand	Optimized Supply	Actual Need
G	3,864	4,142	7,728	4,143	3,909
H	924	975	1,036	1,023	932
I	1,071	1,253	2,142	1,253	1,156
J	566	852	849	701	657

Table 8 Sample part demand using Equation 7, and adjoining demand functions from Equations 8-11 are compared to the original and actual and optimized quantity.

In Table 8, Part G's initial demand was 3,864, but after the design changes and the additional order using the previous method it rose to 7,728. Whereas the new model puts in an amount of 4,142, and the optimized supply was 4,143. While 4,142 parts initially appears to be larger and inaccurate, it is closer to the needed demand of 3,909, as is the optimized supply number. Each cluster was tested and similar results were found; the new demand function created less excess. Generally, the optimized supply quantity matched or was larger than the predicted demand. Although in some situations the optimized supply number was lower than the predicted demand although this only happened with small probably of demand changes such as in Phase 5.

The new demand function over all of the clusters decreased the excess inventory as compared to the previous demand function. Figure 8 highlights the average percentage decrease from a sample of each cluster. Cluster 1 has the lowest decrease in parts, which aligns with the type of parts in cluster 1; COTS, joints, screws, nuts etc. and they are used in mass quantities. The smaller decrease in cluster 3 also aligns with the attributes of that cluster; the parts have spare requirements, which were not captured in the original demand function, but are now. Cluster 5's smaller decrease is likely due to the smaller nature of the cluster in general, as well as the type of parts; liquids which makes it harder to decrease numbers, as a 1-gallon decrease is not possible due to paint coming in 4-gallon bottles.

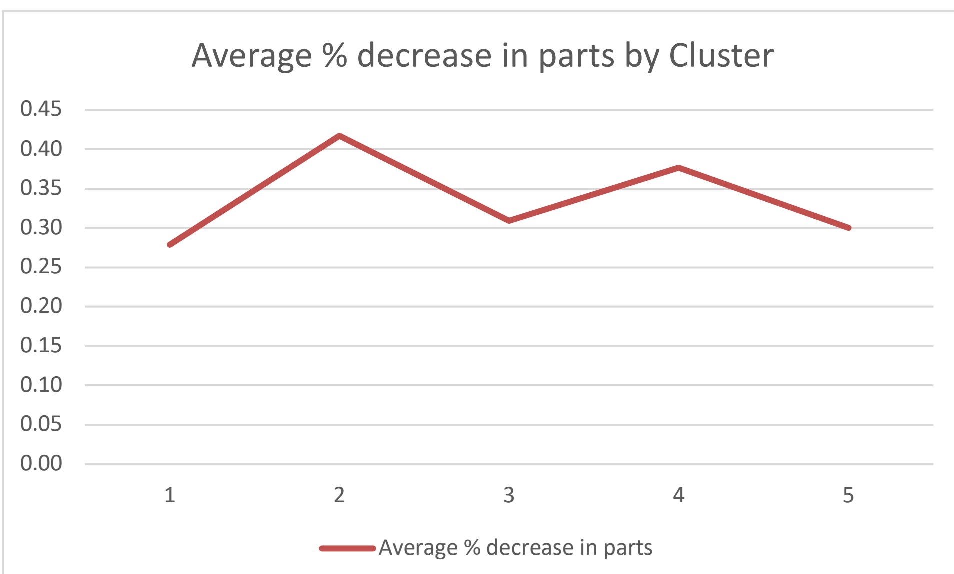


Figure 8 Demand equation 7 and the demand function from equation 8-11 highlight the decrease in number of parts when the new demand function is used

While the model still creates some excess, it is minimal compared to the original (old) demand function. The reasons for the difference are because the new model uses the probability increases over the phases and adds that to the original demand, whereas the previous (old) process would place a hold with the supplier for the original amount of demand. Thus, when the design change triggered an increase, the increase was sometimes added to the original amount, which created a new demand number, and that new amount was then added to the previous demand, which created a duplicate

More interesting still, was the cost savings between the original model and the new model. Figure 8 is a sample of the total cost comparison of the items ordered with their original demand compared to the total cost of the items ordered with the new demand function. This is solely using the demand function and not the cost function’s optimized supply quantity.

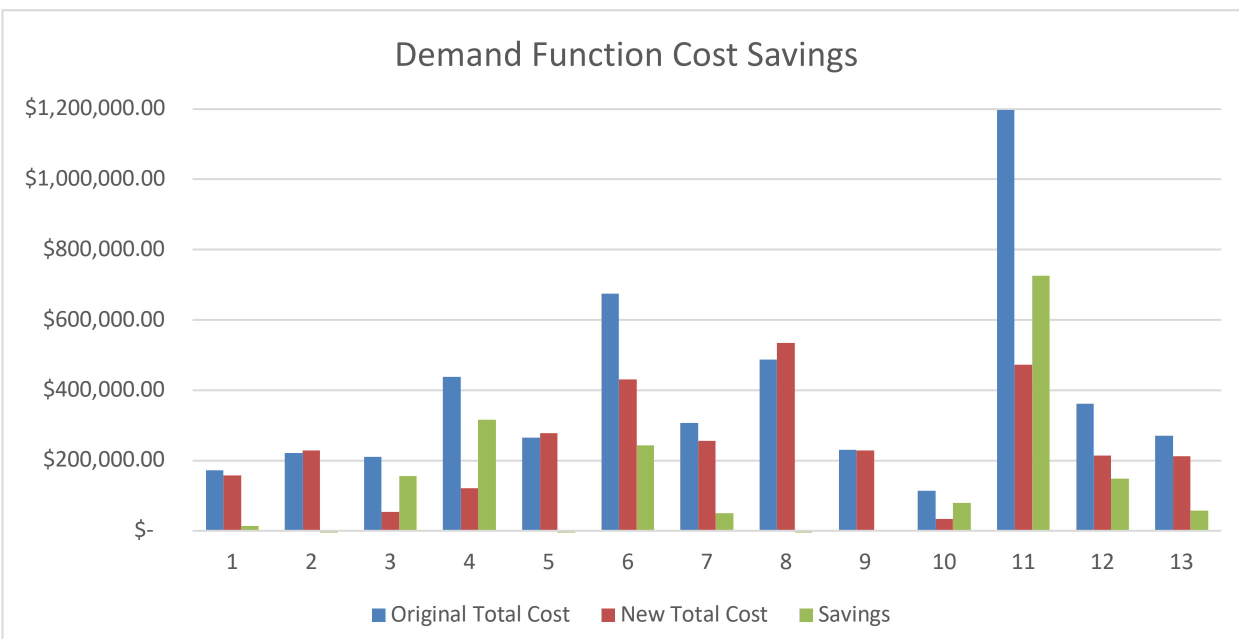
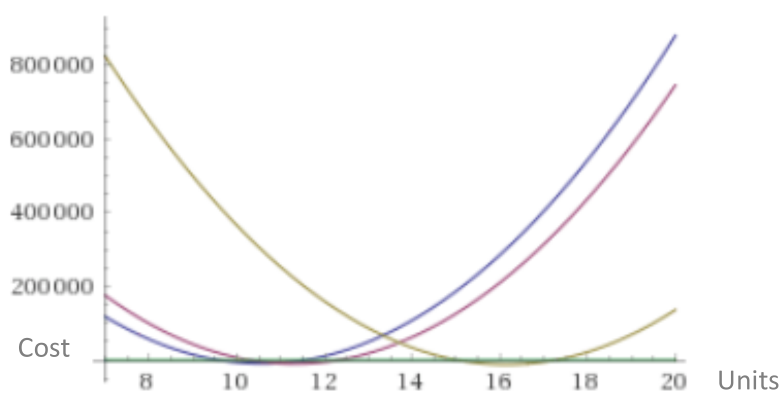


Figure 8 Sample comparison between total cost for items ordered with the old demand function and the new demand functions in Equations 7-11.

There were times when my model generated more demand than the previous model needed – these are noted in Figure 8's sample above, although this was rare and likely due to the small order quantity. On average, the new demand function decreased excess inventory parts by 30 percent, and the associated cost by approximately 51 percent; the large difference between part decrease and cost decrease was due to the fact that substantial excess was tied to inventory with a cost above $100,000 per unit.

The generalized model, when tested on a different set of data, also decreased demand - number of parts decreased by 31 percent, and total cost decreased by 34 percent. As there was no knowledge of the phases for this data, each part was run through each potential phase scenario and the average was taken from those results. Below is a single part's reaction to the phase changes.



Purple Line is Phase 5

Pink Line is Phase 4 & 5 combined

Gold Line is Phase 1 & 2 combined

Figure 8 Equation 5 graphed with a sample of a single part's optimal supply quantity over different phases, see Table 9 for the details

Below is a sample of a single part's reaction to the phase changes (industry data was used):

Predicted Demand	Optimized Supply	Difference	Phases	Phase Probability
11	10	-1	5	5%
11	11	0	4 & 5	12%
11	12	1	4	7.20%
11	11	0	3	9.75%
11	12	1	2	14%
12	13	1	1	18.75%
13	16	3	1 & 2	32.75%
10	10	0	No Phase	0%

Table 9 Sample from industry data of how the optimized supply function compares to the predicted demand.

The industry data reacted in a similar manner as the company data: the optimized supply remained close to the predicted demand until the phase change probability grew over 23 percent, or any of phase 1 and 2 or 2 and 3 combined.

Therefore, the new ordering policy should use the predictive demand function first, which will include using the lead time and phase the part is required in to find the probability increases during each demand phase. The quantity found from the demand function will then be put into the appropriate cluster's cost function and optimized to ensure that the quantity the demand function found, is also the lowest in cost. The quantity found in the cost function will then be placed as an order when the part lead-time has been reached.

5. DISCUSSION

The new clustering allows for better attribute alignment, parts with similar costs and lead times are treated similarly whereas previously, parts were grouped haphazardly based on the supplier. K-means is a tool used often, although it depends on number values to decide the cluster. The downside of k-means is that it is unable to take into account characteristics or traits that are not value based; therefore, it cannot cluster on names, or identification numbers.

Fortunately, there are different ways to cluster than k-means, such as manually by what the part does, or when it will be needed, hierarchical, K-nearest neighbours, etc. These were not explored in this thesis due to time constraints. However, it would be beneficial to explore them at a later time.

The demand function and the cost function are not required to be used together, although it is useful to find the demand first using the model and insert the calculated demand into the new cost function. The cost function optimizes the quantity at the lowest cost and the optimized supply quantity does not always equal the predicted demand, however this provides useful insight into what quantity should be ordered given the phases. Even though the demand function created often provides a number that is greater than the actual requirement, it is still less than the original demand function used by the company. Moreover, the cost function can be adjusted to use a different demand quantity, and then optimized to find the lowest cost supply quantity; the function provides options most of which decrease spend and excess inventory. That said, the created demand model had the highest accuracy in clusters two and three, which is ideal, given that the company commits 78 percent of their spend to those two clusters.

Another note is that further work needs to be performed regarding changes to demand that happen after the parts have been ordered. This scenario was left out of this thesis as data was largely unavailable, and the data that was available was scarce and with large differences. Moreover, the extent of the increases had not been tracked, nor the frequency at which they happened. That left too much “guess work,” so it was not a viable option to add to this study. If the data were collected, it would be beneficial to further explore a demand function that looked at both future demand changes prior to the order, as well as the probability of demand changes after the order had been placed.

Despite the cost model's sensitivity to changes in holding costs, and or design changes, they were and are able to scale effectively, and reduce long-term costs, especially when paired with the predictive demand function. Further testing should be completed on cluster five – in future tests it would be beneficial to break cluster five into two different parts. The physical difference of the parts is large and while this cluster worked during my model, it may not hold in future tests. The cost model was tested further with non-company data and provided similar results; however, clusters were not used, as that information was unavailable. Instead, the general cost function was used, and it yielded similar results to the clustered results.

It would be beneficial to have clustering based on when the parts were required, rather than the lead-time, or cost or part family. A time frame would better help shape the demand function, and then could be removed from the cost side, pushing the model to an even simpler version of the current one more like a traditional single period model.

The demand function is particularly important as it takes into account a number of factors that normal NVP do not: decay, spares, and future design changes. The demand function can easily be altered to include phases where the design changes require a decrease in demand for a certain part, although more testing would need to be done to ensure that the model acts as expected.

Although there were constraints on the models, the results based on the company data and other data demonstrated improvements in both cost and inventory reduction. Many improvements still can be made in this space: my models are just the beginning of what I foresee becoming a larger field of study.

6. Conclusion

The primary driver of this thesis was the lack of inventory management models for a situation where profit is not the focus. Although the models created can be further refined, they provide a much-needed start to filling a problematic gap. While profit and costs drive most businesses, there are situations when a different driver is the focus; as such, models need to be available for those situations. Those situations include government-funded projects for defense, humanitarian relief, disaster recovery, and other large-scale infrastructure projects, where a timely response is key. It is still important to minimize costs, but equally important to be able to respond in situations: a submarine cannot surface because it is missing a part; it must be prepared. Therefore, these models will help decrease the excess inventory that is plaguing the defense industry, while not hindering the availability of inventory.

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