The Traveling Salesman Problem with Multiple Drones:
An Optimization Model for Last-Mile Delivery

by

Justin J. Yoon

Bachelor of Science, Economics
United States Military Academy at West Point, 2013

SUBMITTED TO THE DEPARTMENT OF SUPPLY CHAIN MANAGEMENT
IN PARTIAL FULLFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN SUPPLY CHAIN MANAGEMENT
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2018

Signature redacted

Signature of Author ....

Justin J. Yoon
Department of Supply Chain Management
May 11, 2018

Signature redacted

Certified by ....

Dr. Mohammed Moshref-Javadi
Postdoctoral Associate, Megacity Logistics Lab
Thesis Supervisor

Signature redacted

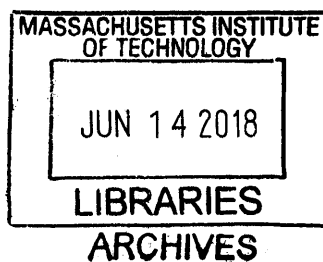Certified by ....

Dr. Matthias Winkenbach
Director, Megacity Logistics Lab
Thesis Supervisor

Signature redacted

Accepted by ....

Dr. Yossi Sheffi
Director, Center for Transportation and Logistics
Elisha Gray II Professor of Engineering Systems
Professor, Civil and Environmental Engineering

[This page intentionally left blank.]

The Traveling Salesman Problem with Multiple Drones:
An Optimization Model for Last-Mile Delivery

by

Justin J. Yoon

Submitted to the Department of Supply Chain Management
on May 11, 2018 in Partial Fulfillment of the
Requirements for the Degree of Master of Engineering in
Supply Chain Management

ABSTRACT

With the increasingly competitive landscape of e-commerce and omni-channel delivery execution, the last mile has emerged as a critical source of opportunity for cost efficiency. Unmanned aerial vehicles (UAVs) have historically been utilized for military applications, but they are quickly gaining traction as a viable option for driving improvements in commercial last-mile operations. Although extensive literature currently exists on vehicle routing problems, research integrating drones as a supplement to these routing problems is scarce. This thesis explores the feasibility of deploying drones to the last mile, by modeling the cost of serving customers with one truck and multiple drones in the context of the traveling salesman problem. The model is constructed with mixed integer linear programming (MILP) optimization and assessed with a sensitivity analysis of several key parameters. We find significant median cost savings over TSP of 30 percent in the base case, and that these effects on savings can diminish to a median 4 percent in the worst case while surging up to 55 percent in the best case.

Thesis Supervisor: Dr. Mohammad Moshref-Javadi
Title: Postdoctoral Associate, MIT Megacity Logistics Lab

Thesis Supervisor: Dr. Matthias Winkenbach
Title: Director, MIT Megacity Logistics Lab

Acknowledgements

Many individuals have helped me during my time at MIT and throughout the course of my research. I would like to extend my specific words of gratitude to the following individuals:

My classmate, Eugene Sohn - For serving as a partner with whom to embrace and conquer the unknown. Specifically, for encouraging me to pursue this thesis as well as for accompanying me towards acquiring personal office space in hostile territory.

My classmate, Darryl Yau - For opening my eyes towards a perpetual journey of formatting and presentation perfection, and more importantly for procuring the paper upon which this thesis is printed on.

My advisor, Dr. Mohammad Moshref-Javadi - For providing me with steady guidance and light as I traversed an otherwise intimidating journey through the ambiguity of research. You instilled in me a sense of confidence in my path on a day-to-day basis and encouraged me to raise my own expectations with every step. Although the deadlines were persistent and there was never enough time to run the models we truly wanted to, we made it.

My advisor, Dr. Matthias Winkenbach - For entrusting me with the freedom to pursue my research in a way that I found personally inspiring and intrinsically valuable. Your approachability as a mentor allowed me to explore difficult questions about my own career and ultimate life goals, and I will be forever grateful to have conducted academic research under your leadership.

My dad, Daniel - For instilling in me a hunger to pursue academic and professional excellence from my very first years of life. You are the reason I pursued a path towards MIT, and I only hope to somehow repay you in the short time we have left for giving me the inspiration to be a better person in the most formative years of my life.

My mom, Julie - For being the rock to our family as Edmund, Andrew, and I took major risks in pursuit of better lives and somehow kept afloat amidst the grave uncertainties of the real world. You have always been there for us to lean on in difficult times, and your faith has never failed to keep us on the right path. Although not everything goes as planned, you have always reminded us that everything happens for a reason. Now I realize more than ever - you were right.

My brother, Edmund - For always stepping up to the plate and being the leader our family needs. You have always taken the first step into the unknown so that Andrew and I may look up to you and follow with faith. Being the first born son is a huge burden to take on, and I are extremely grateful that it is you that has embraced the responsibility and that you bear it with pride.

My brother, Andrew - For always pushing the boundaries of achievement in our family and instilling a deep sense of pride in each of us with every single step that you take in life. Although you could be flying Black hawks.

My wife, Sylvia - For being the steadfast rock to our young family and for inspiring me to be a better husband and father every day. I could not fathom life without you and your calming presence, and am extremely thankful that I get to explore the rest of life with you by my side. You are my perpetual source of inspiration and drive.

My daughter, Tippi - For breathing life into my sense of purpose. Everything I do is for you, and I am certain that I will never experience more pride than when I watch you navigate your way through life. I hope and wish that you eventually find your purpose and pursue whatever you find to be meaningful, and that you know in your heart that nothing will ever shake my support for you as your father.

Biographical Note

Justin Yoon was born in Garden Grove, California and has lived in Southern California, New York, Georgia, and Massachusetts. Justin graduated from the United States Military Academy at West Point with a Bachelor of Science in Economics. Upon graduation from college, he attended and completed the US Army Flight School in Fort Rucker, Alabama, where he was certified as a UH-60M Blackhawk helicopter pilot and completed the Survival Evasion Resistance and Escape (SERE) school.

Prior to matriculating at MIT, Justin was a regional learning and development manager for operations at Amazon, where he spent most of his time training new management teams to rapidly launch fulfillment centers in the southeast region of North America. After graduating from MIT, Justin will join Tesla as a global supply manager on the team responsible for the procurement of capital equipment (CAPEX) in Fremont, California.

Contents

List of Figures

List of Tables

1 Introduction

1.1 Market Scope

In 2016, the cost of the global parcel delivery market amounted to approximately 82 billion US dollars, with 40 percent being generated in China, Germany, and the United States (Joerss et al., 2016). Although already large, this market faced significant growth in recent years and shows no signs of slowing down. Relatively mature markets, such as the US and Germany, experienced 7 to 10 percent growth in the last few years, which means that this volume is poised to double over the next decade (Joerss et al., 2016).

Given the vast and growing size of the parcel delivery market, it is crucial for companies to identify pivotal opportunities to gain the competitive edge. The rapid expansion and establishment of e-commerce by companies like Amazon are primary drivers and leading indicators for this growth. In 2015, consumers worldwide spent 1.7 trillion dollars on e-commerce; this amount is projected to double to 3.5 trillion by 2019 (Lindner, 2015). This increased demand on online retail consumption has shifted the market landscape from one that has been previously B2B-dominated to one in which B2C drives over 50 percent of global volume. To capitalize on this shift, it is important for companies to understand that over half of global parcel delivery costs are primarily incurred in the last mile (Joerss et al., 2016). This sculpts a developing environment in which the companies that establish the most cost efficient last mile logistics operations will dominate the competition.

1.2 The Role of Drones in Last-Mile Delivery

Drone delivery as a method to solve for the last mile was first publicly proposed by Jeff Bezos, the CEO and founder of Amazon, in an interview conducted in CBS's *60 Minutes* (Rose, 2013). Amazon was quickly followed by Google (Madrigal, 2014) and DHL in 2014 (Hern, 2014), Dominoes in 2016 (Reid, 2016), and UPS in 2017. As Figure 1.1 shows, Amazon has already patented several ideas and developed drones in anticipation of establishing dominance
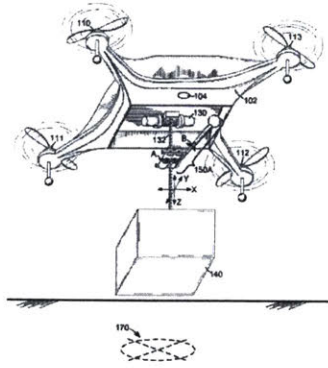
(a) Adjustable arm patent, 2017.
(source: geekwire.com)
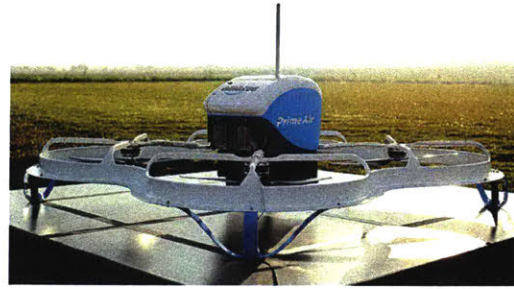
(b) Amazon test drone.
(source: techcrunch.com)

Figure 1.1: Amazon Prime Air investing heavily in drone delivery.

in the field. Last mile solutions with drones have since emerged as a front runner in solving for this particular challenge, due to the cost-saving nature of its labor and fuel efficient design. Furthermore, the increasing congestion and density of urban populations leaves trucks with the challenge of fighting through traffic and idling more often - a challenge from which drones are wholly exempt. From the delivery company's perspective, the opportunity for significant operational cost savings itself is enough to justify the pursuit of drone implementation.

1.2.1 Social Responsibility

Societal incentives for integrating drone delivery into industry provide an additional justification that goes beyond the immediate marginal cost-cutting advantages listed above. Companies today are facing increasing pressures to commit to sustainable practices and exhibit responsible supply chain behavior (Bateman, 2015). This has led them to re-evaluate their strategies and respond to consumer preferences by publicly adopting sustainable methods such as those that reduce carbon emissions. According to Stolaroff et al. (2018), the proper application of drone-based delivery could reduce greenhouse gas emissions and energy use in the freight sector by up to 50 percent. This is due in part by the significantly lower energy consumption rate of electric powered drones, which consumes approximately 0.097 megajoules per mile (MJ/mi) compared to the 11.748 MJ/mi of a typical diesel truck used

by companies like UPS (Stolaroff et al., 2018). With drones integrated into the last mile, the resulting reduction in carbon footprint per package delivered could lead to large cuts in global transportation-related greenhouse gas emissions - which truck transport is responsible for 24 percent of today (Stolaroff et al., 2018).

1.2.2 Customer Preferences

Today's consumers have a powerful inclination towards speedy delivery. In a survey conducted by McKinsey in 2016, 20 to 25 percent of consumers indicated that they would pay significant premiums to receive their packages within the same day (Joerss et al., 2016). Increasingly complex customer expectations are also pushing retailers to adapt to meet these varied customer demands with innovation in delivery (Lee et al., 2016). To overcome this challenge, many retailers offer several options, with same-day delivery often making it into the mix. A key advantage offered by drones in this competitive landscape is the speed at which they can navigate through open airspace as opposed to congested traffic on the roads. Furthermore, they are able to bypass obstacles such as water or unpaved rural areas to significantly cut delivery times (Lee et al., 2016). With this newfound capability, Amazon is exploring offering 30-minute delivery options with drones for lightweight products that weigh up to 5 pounds - a market segment that accounts for over 86 percent of Amazon's total offerings (Gross, 2013). These factors, combined with an increasing consumer appetite for speed, have urged e-commerce players to double down on increasing their offerings and delivering better fulfillment strategies, including ones with drones.

1.2.3 Government Regulation

Although the benefits of using drones for package delivery appear relatively clear, government regulation has largely lagged behind (Figure 1.2). In the United States, the Federal Aviation Administration (FAA) has been resistant to the development and testing of commercial drones within its jurisdiction since adding regulations in 2015 (Heater, 2017). Although

technically permitted by the US to test, commercial drone operators are required to maintain line-of-sight, operate vehicles under 400 feet above ground level (AGL), register each vehicle, face a limited selection of drone models, and are restricted from flying in many population-based locations (Romm, 2017). For this reason, the companies that have led research and deployment of drones for commercial purposes have largely developed their technologies overseas. Amazon kicked off its research in the United Kingdom (Glaser, 2016) while Google has based its "Project Wing" operations out of Australia (Madrigal, 2014), and Dominoes delivered its first pizza by drone in New Zealand (Reid, 2016). However, recent developments prove optimistic for the US; the White House signed a bill in 2017 overturning the FAA's previous regulation and citing a direction of loosened legislation towards commercial drone operation (Heater, 2017). Since then, new licenses have been issued by the FAA and the agency is estimating that the number of drone operators will exceed that of private pilots with 450,000 by 2022 (Pasztor, 2018) as depicted in Figure 1.3. Although testing has already somewhat kicked off in international locations, the United States is now opening up its
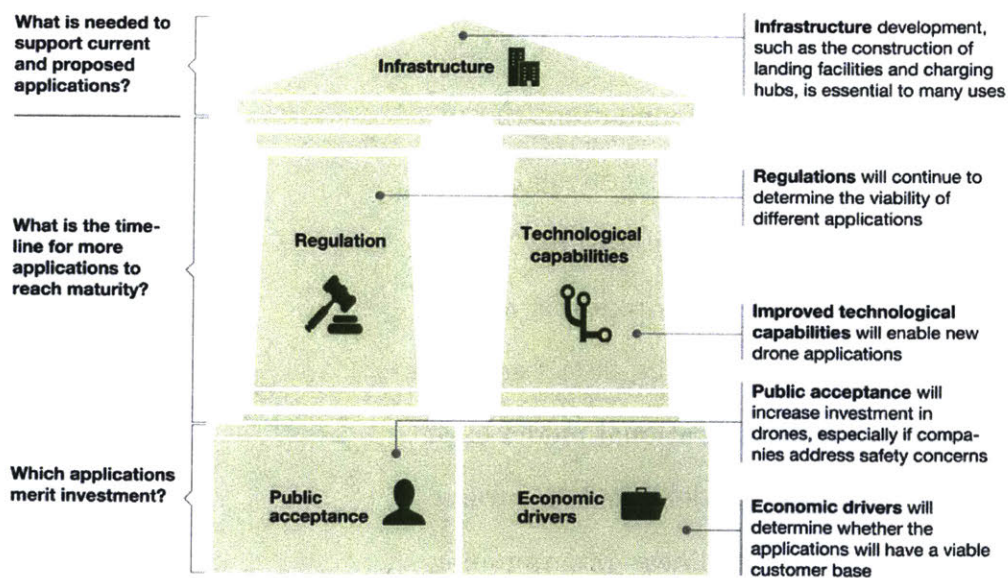
Figure 1.2: Five factors for UAV growth.
(source: McKinsey)

11

jurisdiction to serve as a testing ground for drones.

1.2.4 Outlook and Future Prospects

With these recent developments, the integration of drones into last mile delivery is quickly becoming imminent and in need of a robust foundation of research for operational implementation. The drone is favorable over a truck for its ability to maintain high speeds in the face of congested roads, but it is not without its downsides. The feasible boundary in which drone-based operations can occur are bound by several physical constraints. Two crucial limitations that guide the framing of this problem are the drone's limited payload capacity of approximately 5 pounds and range of approximately 10 miles, assuming a speed of 50 miles per hour (Gross, 2013).

To overcome this constraint and combine drone capabilities with longer range truck capabilities, Murray and Chu (2015) proposed a model in which a single drone is attached to a truck and dispatched enroute to serve customers. This approach has gained traction and allows the benefits of both vehicles to be utilized effectively. On this foundation, the research community has introduced several angles and other approaches to the standing literature.

Following the introduction of drones as a possible solution to the last mile in 2013, several studies exploring a feasible operational model have emerged. The primary model used to frame the problem is mixed integer linear programming (MILP), with the objective being to minimize the truck's arrival time back to the origin depot. A

COMMERCIAL DRONES DEPLOYMENT FORECAST
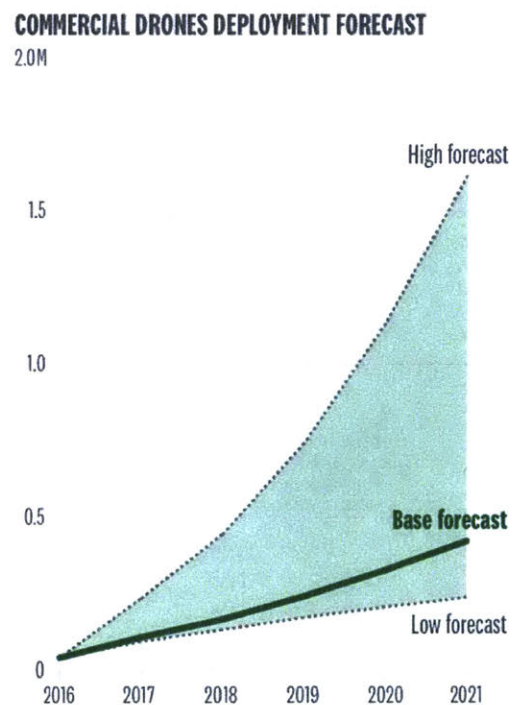
Figure 1.3: FAA commercial drone deployment forecast.
(source: hbr.org)

(a) Delivery driver loading package.
(source: pressroom.ups.com)

(b) Drone launch testing from truck.
(source: ups.com)

Figure 1.4: Real-world drone testing by UPS.
(source: ups.com)

drone, attached to the truck at launch (See Figure 1.4), is then dispatched along the truck's route to deliver a package before rendezvousing for a package replenishment and re-launch.

1.3 Thesis Plan

In this thesis, we first formulate a mathematical model and define its various components. The objective function is to minimize cost, which is composed of the fixed and variable costs of deploying multiple drones as well as the truck's operating fuel and labor costs. This is done to capture the tradeoff in costs between choosing to serve each customer in the network with either a drone or the truck. The model allows for multiple drones to be deployed, which captures the additional potential upside of dispatching more than one drone per truck tour. This configuration permits the model to explore a larger solution space and provide deeper insight into deploying the synergistic technology as a whole. Finally, we identify and conduct a sensitivity analysis on several key parameters that are subject to fluctuate or improve over time: drone speed and endurance, the number of drones made available, truck speed, and customer geographic area. As we change these parameters, we measure the effect on cost savings over TSP with truck only as well as the effect on number of drones used. The results are used to determine whether drones as a last-mile delivery solution are feasible or favorable for real-world application.

2   Literature Review

This literature review is organized into two sections: Vehicle Routing 2.1 reviews classic literature on vehicle routing problems (VRP), which is a generalization of the traveling salesman problem (TSP). Drone Vehicle Routing 2.2 focuses on papers that have integrated drones into a routing problem.

2.1   Vehicle Routing Problem

Classic VRP research with one truck serves as the logical starting point to establish a foundational understanding for the underlying methodology of drone delivery with truck. VRP was first introduced by Dantzig and Ramser (1959) as a generalization of the older and fundamentally applied TSP. TSP can be described as a problem in which a driver must visit a set of n cities exactly once and travel back to the origin point with the objective of minimizing either distance or time (Lin, 1965). While TSP is defined as a single tour with a single vehicle, VRP builds upon this preliminary framework and incorporates additional complexities such as multiple vehicles and origin points. Many researchers over the years have proposed a wide range of exact and approximate approaches to find solutions for VRP and TSP; therefore, the academic environment for these problems has consequently developed into a robust foundation on which we are able to build upon to establish a preliminary understanding of vehicle routing with drones.

2.1.1   Exact Methods

Laporte (1992) discusses the challenges and methods of solving vehicle routing problems with mixed integer linear programming. In his paper, VRP is loosely defined as designing optimal delivery or collection routes from one or several depots to several customers or cities, subject to a series of constraints. The exact approach starts by defining the objective as minimizing total cost or travel time. The decision variables determine which nodes the truck should visit

to optimize based on the objective. He goes on to identify several constraints that are crucial to properly frame the problem: capacity constraints on vehicles or depots, total time or cost restrictions, and precedence relations that ensure sequential ordering of city visits. Toth and Vigo (2002) suggest that exact methods can be classified into three parts: direct tree search methods, dynamic programming, and three integer linear programming algorithms. These methods are typically used to solve small instances due to constraints on computing capability.

2.1.2 Heuristics and Alternative Solving Methods

Although exact algorithms for vehicle routing are precise in their solutions, they cannot be scaled to real-world size problems due to the NP-hard nature of TSP and VRP. The growth in number of possible combinations of routes per node is exponential and quickly reaches over 100 billion with only 15 customer nodes, which equates to equally large demands on computing power and time. Thus heuristics are introduced to provide reasonably good solutions in a significantly lower amount of time.

Junger and Thienel (1994) propose the "branch and cut" method for VRP, which is loosely based on the "branch and bound" method. The quality of solutions that result from an approximate method such as this is assessed by comparing them to exact solutions and never being worse than a predetermined fixed fraction of the optimal solution. Although not always yielding the optimal solution itself, these approximate approaches can perform exceptionally well in terms of run-time and applicability to large scale problems. The "branch and cut" method iterates and creates shorter tours from an initial solution by branching into subproblems and establishing increasingly better lower bounds on the length of the optimal tour. Each iteration shrinks the feasible space in which a solution can be found for each branch until the list of subproblems that can be fathomed is empty. Throughout the process, each subproblem establishes its own local lower bound. This lower bound is compared to the global lower bound of all iterations and replaced by the global lower bound if the global

is more restrictive. Junger and Thienel (1994) find that the "branch and cut" method consistently yields reasonably good solutions while effectively applying more stringent lower bound guarantees.

Simulated Annealing (SA) is a heuristic that applies the properties of cooling metal towards determining solution exploration in the context of complex combinatorial problems like TSP (Kirkpatrick et al., 1983). The system starts at a high temperature, where particles are highly fluid, and cools at a constant rate to an eventual equilibrium in which particles have settled and established a more solid state. In the initial high temperature fluid state, lower quality solutions are allowed to be explored. As the system cools, the tolerance for exploring deviations from the global optimum decreases until the system is fully cooled and a single solution is ultimately identified as the best solution (See Figure 2.1). This eventual settling of the solution exploration process prevents the heuristic from getting stuck in a local optimum by committing to a solution too early in the process. By starting with a
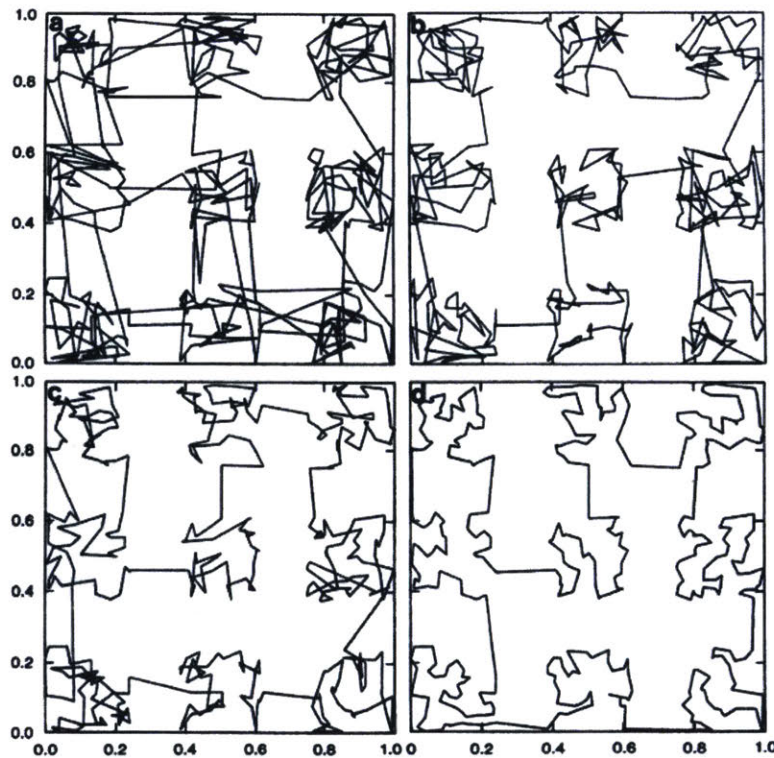
Figure 2.1: Simulated Annealing. Results at four temperatures for a clustered 400-city traveling salesman problem. (source: Kirkpatrick et al. (1983))

16

relaxed exploration of global solutions, SA as a heuristic yields solutions relatively close to optimal.

More recently, a class of nature-inspired optimization approaches to solving complex problems such as TSP have emerged. Swarm intelligence (SI) based heuristics mimic the behavior of social creatures such as colonies of ants, flocks of birds, and schools of fish that are individually unintelligent but driven by population-based self organization to yield efficient results (Fister Jr. et al., 2013). The ant colony optimization (ACO) heuristic is used by Yang et al. (2008) to solve for TSP, which emulates the process in which ants forage for food and leave pheromones to attract others towards a desirable path. Ants, or solutions, are generated at random, and the use of pheromones is represented by assigning a higher probability bias towards favorable paths. After many iterations, ACO yields consistently efficient population-based solutions that perform better than local heuristics such as 2-opt algorithm (Yang et al., 2008).

2.2 Drone Routing Problem

While a significant amount of literature exists for TSP and VRP, the introduction of drones into these problems have inspired the development of a dedicated branch of literature. Although the approach is fundamentally identical to classic VRP, drone-based routing problems must address additional complexities such as coordinated simultaneous movements, unique vehicle constraints, and new transactional costs.

One approach to this problem involves deploying drones as "sidekicks" to trucks by launching them from the trucks at the depot or customer nodes, which Murray and Chu (2015) call the Flying Sidekick TSP (FSTSP). Their approach to drone-based last mile delivery includes the use of mixed integer linear programming (MILP) to minimize the final arrival time of the truck and drone back at the origin point, while traveling in tandem to serve customers 2.2.

Due to the NP-Hard nature of classic TSP problems, FSTSP inherently faces the same

challenge in terms of scaling to larger practical customer sets using a purely mathematical MILP approach. With drones integrated into the model, Gurobi requires exponential amounts of time to solve small instances of even 10 customers. Thus Murray and Chu (2015) formulate a route and re-assign heuristic to generate efficient results, which serves to capture decision trade-offs between drone usage and truck usage to serve each customer in order to yield overall time savings. They do this by defining a list of drone-eligible customers, generating a solution for the truck-only TSP, and then finally reassigning the route by inserting drones and assessing time savings. This last step is iterated several times until no more savings or improvements can be achieved.

Agatz et al. (2015) takes a similar approach in terms of deploying a delivery truck in collaboration with a drone to solve TSP, but they refer to the approach as TSP with Drone (TSP-D). Instead of using a route and re-assign algorithm, they expand their MILP upon larger customer sets by utilizing several fast route first - cluster second heuristics based on local search techniques. This involves constructing an initial truck-only tour then reassigning solutions with greedy or exact partitioning algorithms. They find that the resulting objective

(a) Truck only   (b) Truck and drone
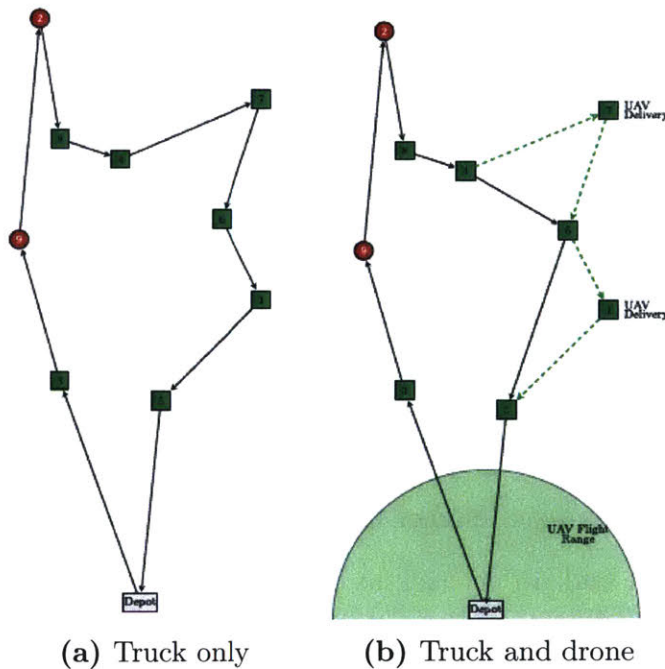
Figure 2.2: Traditional TSP compared to FSTSP with drone. Murray and Chu (2015)

18

distance traveled is about 35 percent less with drones than without.

Another variant of TSP-D is proposed as VRPD by Poikonen et al. (2017). They propose the approach of starting with a close to optimal solution, then improving it by modifying them to maintain VRPD feasibility and applying local optimization techniques. A second approach that they take is starting with a relatively good solution then inserting drones to minimize completion time of the tour.

In an expansion of the previously proposed models with truck and drone, Carlsson and Song (2017) suggest that the savings yielded from the drone sidekick model equates to the square root of the ratio of the speed of the truck and the drone. This mathematical relationship is proposed as a potential method to significantly cut down solve times. A heuristic is used to assess this theory and produces compelling results, but the lack of using global optima is an opportunity for future research.

Further expansion on the FSTSP and TSP-D is conducted by Minh Ha et al. (2018) by applying two separate heuristics and optimizing the MILP by minimizing cost. In their min-cost TSP-D model, the variable cost incurred for the truck is distinguished between traveling cost and waiting cost for when it must rendezvous with the drone. The heuristics used for their approach are the greedy randomized adaptive search procedure (GRASP) and TSP-LS, which yield an average 30 percent savings when optimizing cost compared to an average 20 percent savings when optimizing time.

In another approach to solving large-scale practical networks for the truck and drone tandem TSP, Luo et al. (2017) apply and test two heuristics that they call H1 and H2. On customer sets scaling up to 200 nodes, they set the objective to minimize total routing time. H1 involves creating an initial solution using targets (customers) then splitting them to reassign. H2 involves starting with a solution using feasible rendezvous nodes, then reassigning. While both approaches yield similar objective function results, H2 takes significantly less time as more customer nodes are introduced into the system (Figure 2.3).

Genetic algorithm (GA) and k-means clustering are classic heuristic approaches that
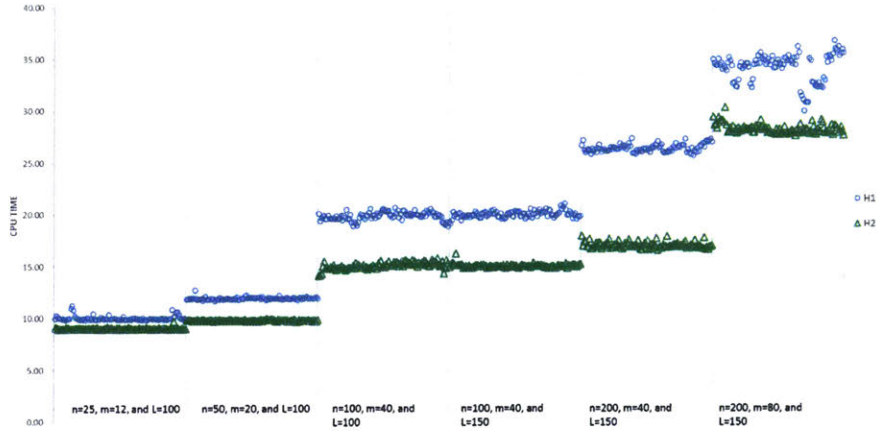
(a) H1 vs H2: Solve times across problem size
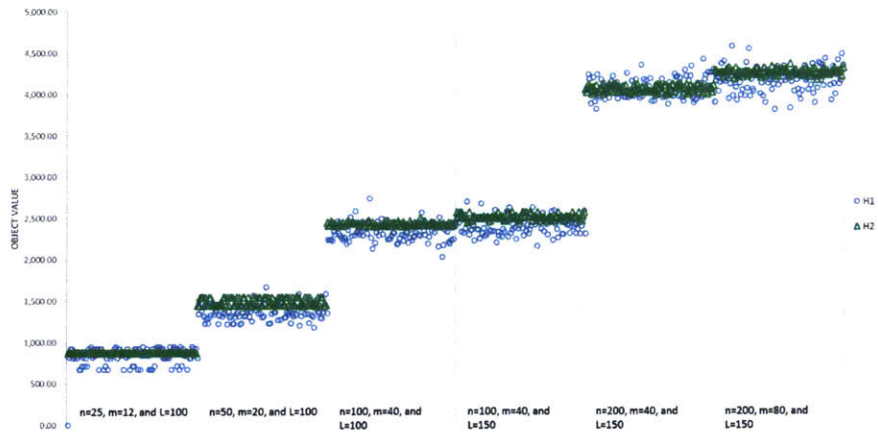
(b) H1 vs H2: Objective values across problem size

Figure 2.3: Comparing H1 and H2 heuristic results. Luo et al. (2017)

have been used extensively on TSP. In order to optimize the TSP with drone, Ferrandez et al. (2016) start with an initial solution, then execute mutations and crossovers to mix the potential solutions. They then accept the best solutions to move forward to the next iteration. K-mean clustering involves identifying features by which to split customers, and then minimizing the mean deviation across clusters. Findings suggest that the use of drones in tandem with trucks are only beneficial when the drone is at least twice the speed of the truck and when at least two drones are used.

Similarly, Chang and Lee (2018) use k-mean clustering and nonlinear programming to assign shift-weights to areas around the depot. A truck then travels through these determined areas and dispatches drones to serve customers in the respective zones. The combined method

proves effective as they yield lower service times compared to pure TSP without k-means clustering.

Genetic algorithm is also used by Savuran and Karakaya (2015), but they focus on maximizing target coverage by the drone's range. The genetic operators in this case are local search methods that serve to achieve this objective. They find improvements over classic GA of 11 to 21 percent and 75 to 260 percent over other heuristics.

Though not directly applicable to commercial drone delivery to customers, emergency medical response is another notable field in which the viability of drone utilization is being researched to cut delivery times and mitigate risk. Advancements in this field are likely to be highly correlated with growth in the commercial last mile sector. Because traditional air transport through fixed or rotary aircraft is a high-risk operation, drones can replenish hospitals or deliver crucial medical supplies directly to remote locations without gambling the lives of crew or the patients (Thiels et al., 2014). Although further research for feasibility in different use-cases is required, the potential for high impact is undeniable. This unique application outlines the versatility of problems that can be solved or improved by introducing drones into many systems, even beyond TSP for last-mile delivery.

3 Methodology

3.1 Overview

This section first introduces and describes the mathematical model then describes the selection of key parameters and the design of test instances relative to real-world applications. The method of solving is described by outlining the establishment of the optimality gap. The foundation for the model is based on mixed integer linear programming (MILP), with the objective of minimizing total cost. The outcome is optimized by determining the binary decisions for each node to be visited by truck or drone as well as the order of visitation. A sensitivity analysis is then conducted in order to determine how trade offs are made as key parameters are adjusted.

3.2 The Traveling Salesman Problem with Multiple Drones

3.2.1 Model Notation

This section first briefly explains the notation used to define the mathematical model. For mixed integer linear programming (MILP), indexes are set to independently track truck and drone movements according to Manhattan distance instead of Euclidean. Sets are defined to constrain feasible nodes from which the truck or drone can depart from, arrive to, or serve a customer at. One set defines the number of drones that are available to be deployed per truck tour. The largest set is N, which contains all nodes and starts and ends with the depot $(c0, c+1)$. Variables are set to define moving relationships between the inputs and the model. Parameters are established to scale the magnitude of each variable's overall effect on the objective of the model and ultimately to assess model sensitivity.

In order to formulate the problem and frame the environment, the following notation is defined in Table 3.1:

22

Table 3.1: Mathematical Model Notation.

Indexes	
h, i, j, k, l, m, o:	Represents Node of Network, Total $c+1$
n:	Represents Deployed Drones
Sets	
N:	$\{0,1,...,c+1\}$: Set of all nodes in problem
N_0:	$\{0,1,...,c\}$: Set of all nodes that can be departed from
N_+:	$\{1,2,...,c+1\}$: Set of all nodes that can be arrived to
C:	$\{1,2,...,c\}$: Set of all customers
D:	$\{1,...,n\}$: Set of available drones for deployment
Parameters	
min_D:	Drone Endurance Time (min)
mph_D:	Drone Speed (mph)
mph_T:	Truck Speed (mph)
s_L:	Drone Launch Setup Time (min)
s_R:	Drone Retrieval Time (min)
A:	Customer Grid Area (mi^2)
C_F:	Variable Operating Cost for Truck Fuel (USD/min)
C_L:	Variable Operating Cost for Truck Labor (USD/min)
C_E:	Variable Operating Cost for Drone Electricity (USD/min)
F_D:	Fixed Cost of Deploying Unique Drone per Tour (USD)
M:	Linking Constraint
Variables	
τ:	Travel Time
t:	Arrival Time
x:	Binary, Customer Served by Truck
y:	Binary, Customer Served by Drone
z:	Binary, Drone Deployed
p:	Binary, Tour Order Sequencing
u:	Binary, Sub-tour Elimination

3.2.2 Assumptions

The following operating conditions are assumed:

- Only one truck can be used per tour, while several drones can be dispatched in the same tour. Only one tour can be run at a time.

- Both the drone and truck are set to travel Manhattan distance instead of Euclidean as a method to emulate movement on road networks to avoid obstacles (Figure 3.1).

23
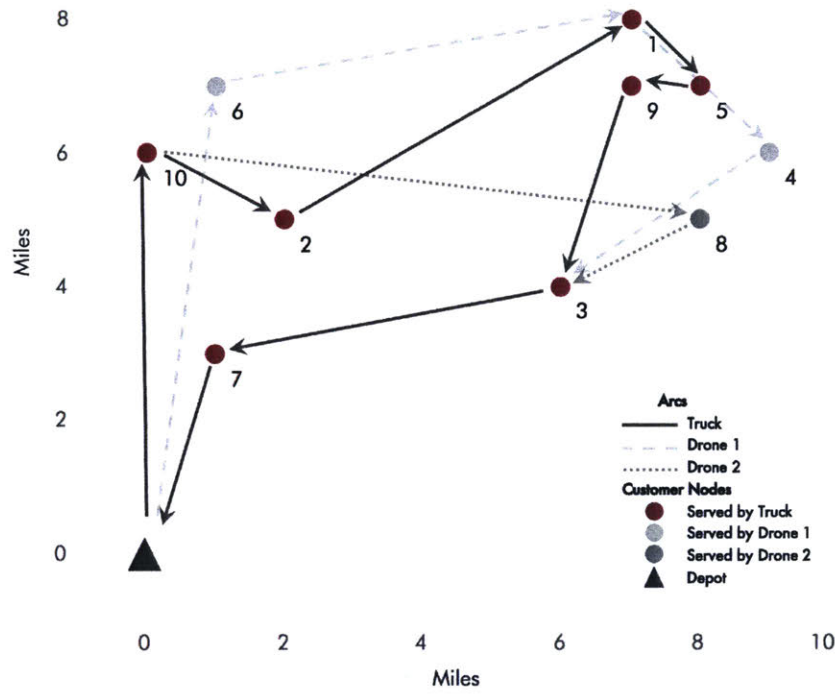
(a) Truck and Drones traveling Euclidean Distance
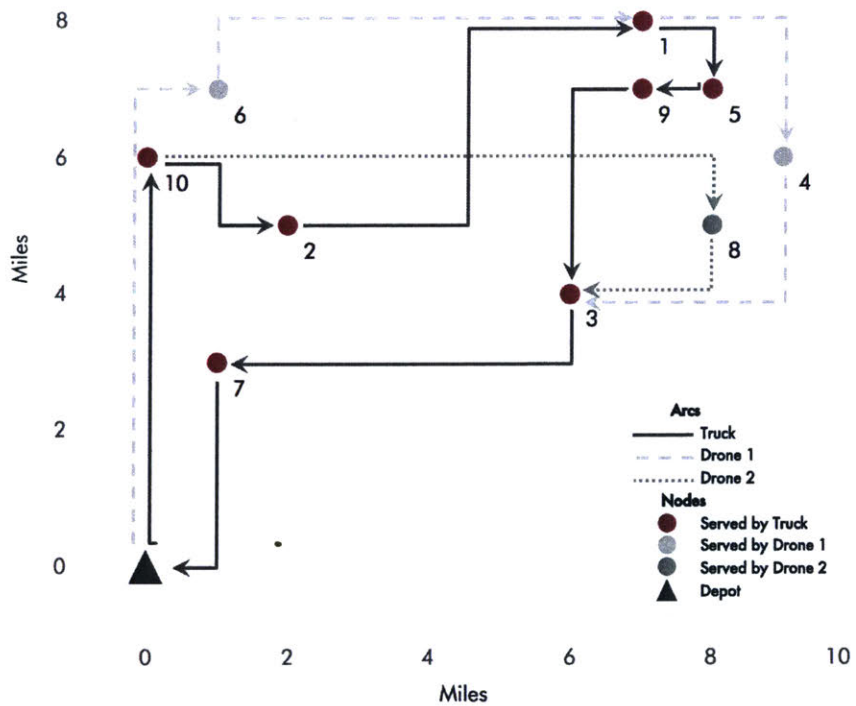
(b) Truck and Drones traveling Manhattan Distance

Figure 3.1: Comparing Euclidean versus Manhattan Distance with TSPMD.

- The drone may only visit one customer node per dispatch from the truck, while the truck may visit more than one customer before rendezvousing with the drone.

- Drone travel is limited by the flight-time limit, or endurance min_D.

- If a drone rendezvous with a truck at node k, the drone can then be re-launched from node k. However, after this point, the drone or truck cannot return to node k.

- Drone launch and rendezvous can only occur at the depot or customer node. This transaction cannot occur en route between nodes.

- Once the drone visits the depot again at any point after the truck has initiated the tour, the drone is removed from the tour.

- Drones are considered fully autonomous, which means that their operation does not incur any additional labor costs.

3.2.3 Mathematical Model

The Traveling Salesman Problem with Multiple Drone (TSP-MD):

$$MinCost = t_{c+1}(C_F + C_L) + \sum_{i \in N_0} \sum_{j \in N} \sum_{k \in N_+} \sum_{n \in D} y_{ijkn}(\tau'_{ij} + \tau'_{jk}) * C_E + \sum_{n \in D} z_n * F_D \qquad (1)$$

$$\sum_{i \in N_+} x_{i0} = 0 \quad \forall j \in N \qquad (2)$$

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} + \sum_{\substack{i \in N_0 \\ i \neq j}} \sum_{\substack{k \in N_+ \\ \langle i,j,k \rangle \in P}} \sum_{n \in D} y_{ijkn} = 1 \quad \forall j \in C \qquad (3)$$

$$\sum_{j \in N_+} x_{0j} = 1 \qquad (4)$$

$$\sum_{i \in N_0} x_{i,c+1} = 1 \qquad (5)$$

$$u_i - u_j + 1 \leq (c+2)(1 - x_{ij}) \quad \forall i \in C, j \in \{N_+ : j \neq i\} \qquad (6)$$

25

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{ij} = \sum_{\substack{k \in N_+ \\ k \neq j}} x_{jk} \quad \forall j \in C \tag{7}$$

$$\sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i,j,k \rangle \in P}} y_{ijkn} \leq 1 \quad \forall i \in N_0, n \in D \tag{8}$$

$$\sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in c \\ \langle i,j,k \rangle \in P}} y_{ijkn} \leq 1 \quad \forall k \in N_+, n \in D \tag{9}$$

$$2y_{ijkn} \leq \sum_{\substack{h \in N_0 \\ h \neq i}} x_{hi} + \sum_{\substack{l \in C \\ l \neq k}} x_{lk} \quad \forall i \in C, j \in \{C : j \neq i\}, k \in \{N_+ : \langle i,j,k \rangle \in P\}, n \in D \tag{10}$$

$$y_{0jkn} \leq \sum_{\substack{h \in N_0 \\ h \neq k}} x_{hk} \quad \forall j \in C, k \in \{N_+ : \langle 0,j,k \rangle \in P\}, n \in D \tag{11}$$

$$u_k - u_i \geq 1 - (c+2)(1 - \sum_{\substack{j \in C \\ \langle i,j,k \rangle \in P}} y_{ijkn}) \quad \forall i \in C, k \in \{N_+ : k \neq i\}, n \in D \tag{12}$$

$$t'_i \geq t_i - M(1 - \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i,j,k \rangle \in P}} y_{ijkn}) \quad \forall i \in C, n \in D \tag{13}$$

$$t'_i \leq t_i + M(1 - \sum_{\substack{j \in C \\ j \neq i}} \sum_{\substack{k \in N_+ \\ \langle i,j,k \rangle \in P}} y_{ijkn}) \quad \forall i \in C, n \in D \tag{14}$$

$$t'_i \geq t_k - M(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle i,j,k \rangle \in P}} y_{ijkn}) \quad \forall k \in N_+, n \in D \tag{15}$$

$$t'_k \leq t_k + M(1 - \sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle i,j,k \rangle \in P}} y_{ijkn}) \quad \forall k \in N_+, n \in D \tag{16}$$

$$t_k \leq t_h + \tau_{hk} + s_L(\sum_{\substack{l \in C \\ l \neq k}} \sum_{\substack{m \in N_+ \\ \langle k,l,m \rangle \in P}} y_{klmn}) + s_R(\sum_{\substack{i \in N_0 \\ i \neq k}} \sum_{\substack{j \in C \\ \langle i,j,k \rangle P}} y_{ijkn}) - M(1 - x_{hk})$$
$$\forall h \in N_0, k \in \{N_+ : k \neq h\}, n \in D \tag{17}$$

$$t'_j \geq t'_i + \tau'_{ij} - M(1 - \sum_{\substack{k \in N_+ \\ \langle i,j,k \rangle \in P}} y_{ijkn}) \quad \forall j \in C', i \in \{N_0 : k \neq j\}, n \in D \tag{18}$$

26

$$t'_k \geq t'_j + \tau'_{jk} + s_R - M(1 - \sum_{\substack{i \in N_0 \\ \langle i,j,k \rangle \in P}} y_{ijkn} \quad \forall j \in C', k \in \{N_+ : k \neq j\}, n \in D \tag{19}$$

$$t'_k - (t'_j - \tau'_{ij}) \leq min_D + M(1 - y_{ijkn})$$
$$\forall k \in N_+, j \in \{C : j \neq k\}, i \in \{N_0 : \langle i,j,k \rangle \in P\}, n \in D \tag{20}$$

$$u_i - u_j \geq 1 - (c+2)p_{ij} \quad \forall i \in C, j \in \{C : j \neq i\} \tag{21}$$

$$u_i - u_j \leq -1 + (c+2)(1 - p_{ij}) \quad \forall i \in C, j \in \{C : j \neq i\} \tag{22}$$

$$p_{ij} + p_{ji} = 1 \quad \forall i \in C, j \in \{C : j \neq i\} \tag{23}$$

$$t'_l \geq t'_k - M(3 - \sum_{\substack{j \in C \\ \langle i,j,k \rangle \in P \\ j \neq l}} y_{ijkn} - \sum_{\substack{m \in C \\ m \neq i \\ m \neq k \\ m \neq l}} \sum_{\substack{o \in N_+ \\ \langle l,m,o \rangle \in P \\ o \neq i \\ o \neq k}} y_{lmon} - p_{il})$$
$$\forall i \in N_0, k \in \{N_+ : k \neq i\}, l \in \{C : l \neq i, l \neq k\}, n \in D \tag{24}$$

$$M * z_n \geq \sum_{i \in N_0} \sum_{j \in C} \sum_{k \in N_+} y_{ijkn} \quad \forall n \in D \tag{25}$$

$$t_0 = 0 \tag{26}$$

$$t'_0 = 0 \tag{27}$$

$$p_{0j} = 1 \quad \forall j \in C \tag{28}$$

$$x_{ij} \in \{0,1\} \quad \forall i \in N_0, j \in \{N_+ : \langle i,j,k \rangle \in P\} \tag{29}$$

$$y_{ijkn} \in \{0,1\} \quad \forall i \in N_0, j \in \{C : j \neq i\}, k \in \{N_+ : \langle i,j,k \rangle \in P\}, n \in D \tag{30}$$

$$1 \leq u_i \leq c + 2 \quad \forall i \in N_+ \tag{31}$$

$$t_i \geq 0 \quad \forall i \in N \tag{32}$$

$$t'_i \geq 0 \quad \forall i \in N \tag{33}$$

27

$$p_{ij} \in \{0, 1\} \, \forall i \in N_0, j \in \{C : j \neq i\} \tag{34}$$

3.2.4 Model Description

The objective function (1) is to minimize total cost, which considers operational costs per minute spent as well as the fixed cost associated with introducing each additional drone. The truck variable costs account for fuel and labor expenses per minute, while the drone variable costs account for electricity expenses per minute.

Constraint (2) prevents the truck from returning to the origin depot.

Constraint (3) ensures that all customer nodes are visited exactly once, by either a drone or a truck.

Constraints (4) and (5) ensure that the truck departs and returns to the depot exactly one time.

Constraints (6) and (31) eliminate sub-tours from the model by ensuring that for the truck and drones, node i will be visited before node k. This is accomplished by implementing the routing variable, u_i, which tracks the sequence in which nodes are visited.

Constraint (7) synchronizes the truck's departure node with its next movement's departure node.

Constraints (8) and (9) ensure that each drone can only be launched up to one time from each node and that it can rendezvous only up to one time from each node, respectively.

Constraint (10) ensures that if a drone is launched at node i and arrives to node k, the truck must be assigned to both nodes. Similarly, constraint (11) ensures that if a drone is launched from the origin depot and arrives at node k, then the truck must be assigned to node k. Furthermore, constraint (12) ensures sequential integrity so that if a drone departs node i and arrives to node k, the truck must visit i before it visits k.

Constraints (13) and (14) coordinate the time at which the truck and a drone are launched from node i. Similarly, constraints (15) and (16) coordinate the time at which the truck and a drone arrive to node k. A single drone may not be launched multiple times from

the same node, but these constraints allow for multiple drones to be launched or arrived simultaneously.

Constraint (17) accounts for the truck's travel time from node h to k when considering its effective arrival time at node k (t_k). Furthermore, it accounts for the additional time accrued from drone launch setup (s_L) if a drone launches from node k when calculating t_k. Similarly, the constraint will account for drone retreival time (s_R) when a drone rendezvous at node k.

Constraint (18) accounts for a drone's travel time from node i to j when considering its effective arrival time at node j. This constraint does not consider s_L becuase the drone arrival time will be synchronized with that of the truck (constraints 13 and 14), which already accounts for s_L.

Similarly, constraint (19) accounts for a drone's travel time from node j to k when considering its effective arrival time to node k. However, in this case the retrieval time of the drone s_R must be considered due to the possibility that the truck may arrive to node k prior to the drone's arrival.

Constraint (20) incorporates the drone's flight endurance parameter. This constraint only applies when a drone travels from nodes i to j to k and ensures that the sum of the two legs are less than or equal to e. M is a big enough number that must exceed the final arrival time of the model and serves as the linking constraint.

Constraint (21) through (23) ensures that the proper values of $p_i j$ for the truck are determined so that sequential integrity is upheld. Furthermore, it disregards the movements of the drone and ensures ordering within the truck path only.

Constraint (24) ensures that the drone's arrival time to future nodes does not precede that of a prior visit.

Constraints (28) to (33) specify the decision variables. x_{ij}, y_{ijkn}, and p_{ij} are defined as binary and can only equal 0 or 1. The arrival times t_i and t'_i are ensured to be positive with non-negative constraints (32) and (33).

3.3 Baseline Parameters

We selected the following baseline parameters to establish a practical scenario in terms of average historical costs and current available technologies. Unless otherwise stated in the sensitivity analysis, all parameters default to the baseline values listed below (Table 3.2):

3.3.1 Truck Variable Costs

Labor cost C_L is 25.10 US dollars per hour (BLS, 2018), which is the national average hourly wage for an express delivery service driver and converts to 0.418 US dollars per minute. The burn rate is 10.2 miles per gallon for a standard diesel delivery truck at city speeds (Stolaroff et al., 2018). The average speed of travel mph_T is 25 miles per hour (Corps, 2009) due to the prevalence of stop-and-go traffic within densely populated areas. The fuel cost is 3.04 US dollars per gallon based on 2018 average diesel costs in the United States (EIA, 2018b), which converts to 0.124 dollars per minute. Considering all of the above listed truck variable costs, the total operating cost is base-lined at 0.485 US dollars per minute.

3.3.2 Drone Variable Costs

The cost of electricity is 0.12 US dollars per kWh (EIA, 2018a), as this was the national average cost in the United States as of 2018. We used the standard energy conversion rate of 1 Joule per 3.6 x 10^{-6} kWh to convert the burn rate into Joules. Thus the resulting value of 96,560 Joules per mile (Stolaroff et al., 2018) is used to represent cost of drone usage over

Table 3.2: **Baseline Costs.** Fixed and variable costs with baseline parameters: drone speed 35 miles per hour, drone endurance 30 minutes, and truck speed 25 miles per hour.

Truck Variable Costs	
Labor Cost	0.124 $USD/minute$
Fuel Cost	0.418 $USD/minute$
Drone Variable Costs	
Electricity Cost	0.002 $USD/minute$
Drone Fixed Cost	
Deployment Cost	1.136 USD

time. The average speed mph_D is 35 miles per hour, which accounts for stopping and turning to follow the road network and avoid obstacles. The endurance min_D is base-lined at 30 minutes per drone. Considering the above listed drone variable costs, the total operating cost is base-lined at 0.002 US dollars per minute.

3.3.3 Drone Fixed Cost

The fixed cost F_D for the drone is computed by taking the acquisition cost of a drone and spreading it over a specified payback period of 2 years. This is the time span allotted by the FAA after which a drone certificate of authorization expires (FAA, 2016). We assume 1,500 US dollars for drone acquisition as well as 365 available days of usage per year. The drone is assumed to be used twice per day and down for scheduled maintenance for 5 percent of the year. All factors considered, we base-lined to 730 total uses of the drone per year. A single use is defined as deployment within a single delivery truck tour. We use an annuity applied per use and discounted at 10 percent annually to determine the fixed cost per iteration. The fixed cost F_D per drone deployment is 1.13 US dollars.

3.4 Test Instances

3.4.1 Geographic Representation

Test instances were generated to simulate a typical city TSP route in which a single delivery driver's tour would be contained to a sub-section of a densely populated city. Ten customers were randomly plotted on a 100 square mile region (10 mile x 10 mile), with the distribution center always located on the bottom left corner as depicted in Figure 3.2. This considers that the distribution center is likely located outside of the city where real estate prices are relatively cheaper, and that the truck would be traveling from this external point into the city. Because the truck would be used exclusively to make the first movement from the depot to the concentrated delivery area, we excluded this line haul movement from all calculations and focused on the relevant trade-offs that would be made within the area in which drones
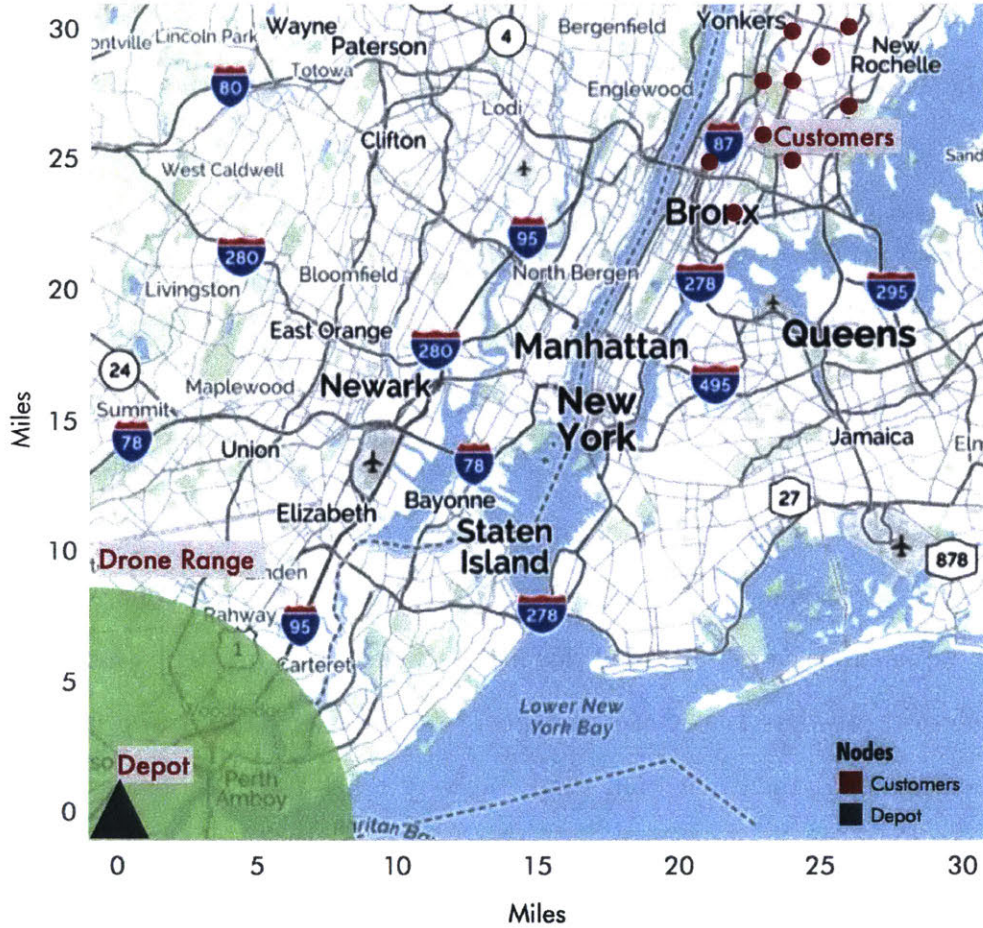
Figure 3.2: Sample Customer Set in NYC. A depiction of a depot in NYC relative to the geographic location of customers in the city. Although the customers are out of reach from drones at the depot, a delivery truck can serve to bring them closer within deployable range.

are available for deployment. To represent movement within a city environment in which many obstacles exist on the fringes of the road network, both the truck and drone are set to travel on the Manhattan metric. Thus instead of moving directly from point to point as a Euclidean space would dictate, the drone and truck are set to travel along roads in a squared-off manner. A total of 170 test scenarios were generated, each containing 10 customers distributed across the aforementioned 100 square foot region.

3.4.2 Optimality Gap

Due to the NP-hard nature of TSP, solve times for TSP-MD consequently exceed several hours for 10-customer problems and grow exponentially with each additional node added.
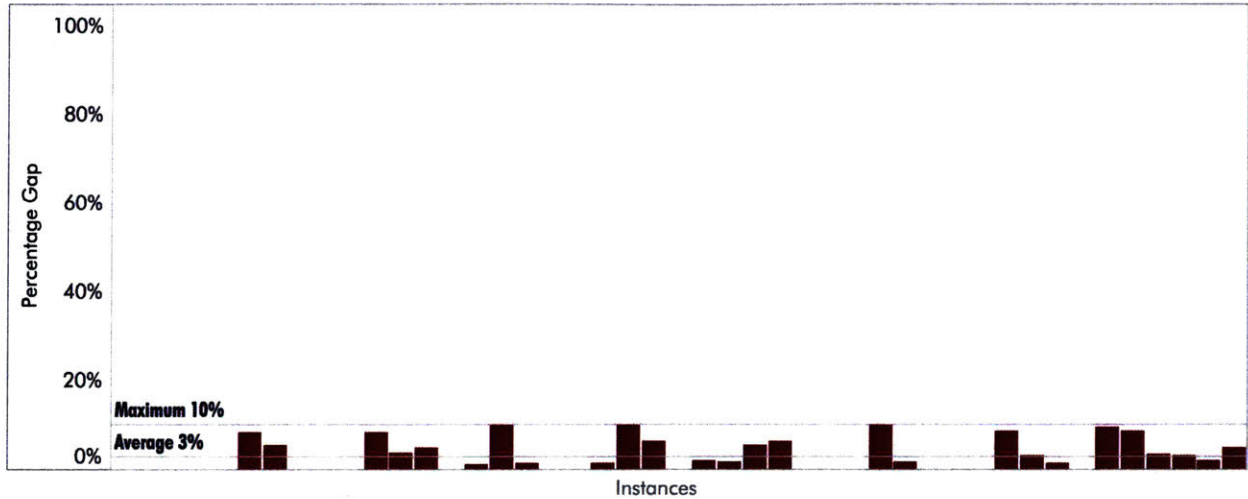
Figure 3.3: Optimality Gap. This gap is determined by comparing the total cost savings of a TSP-MD tour in a 10-customer network with that of a 9-customer network. The former is limited to a solve time of 30 minutes while the latter is solved to optimality. The resulting difference, or gap, depicts a worst-case scenario that defines an upper limit to the true optimality gap of the 10-customer solve.

Although heuristic solutions are often applied to solve such problems at quicker speeds and good enough accuracy, we decided instead to streamline our analysis with the mathematical model by limiting to 30 minute solve times. Five resulting instance solutions per scenario were then compared to those of 9 customer node problems which were solved to optimality in order to determine a worst-case gap. These 9 customer node problems were generated by taking the respective 10 customer node instance and removing a single customer node in order to keep the potential gap minimal. We found the resulting optimality gap across instances to be relatively small (Figure 3.3), with an average gap of 3 percent and an overall maximum observation of 10 percent.

3.5 Sensitivity Analysis

Given that future technology and costs are bound to fluctuate from the baseline established in section 3.3, we conducted a sensitivity analysis to identify threshold circumstances in which the applicability of drones in TSP are significantly affected. Each test instance was then solved with the MILP mathematical formulation described in 3.2 for the varying set of

circumstances.

3.5.1 Key Parameter Selection

In order to truly understand the effect of drones on last-mile delivery, a wide range of scenarios must be explored. Our analysis starts with the establishment of the base case, which we determined to be the following: two available drones that travel at an average 35 miles per hour with 30 minute endurance, an average truck speed of 25 miles per hour, and a grid area of 100 square miles. These values are kept constant as each parameter is fluctuated to represent the use cases depicted in Table 3.3. We determined these particular parameters to be key due to the following reasons:

3.5.2 Drone Speed

Although commercial drones are legally limited to fly up to 100 miles per hour (Feist, 2018) and rotary vehicles are technically capable of flying up to 60 miles per hour (Thiels et al., 2014) under delivery conditions, true average travel speeds are likely to be much lower in

Table 3.3: Key Parameters for Sensitivity Analysis.

Parameter of Interest	available drones D_n	endurance (min_D)	drone speed (mph_D)	truck speed (mph_T)	grid area (A)
Speed/Endurance	2	20 30 40	25 35 45	25	100
Available Drones	1 2 3	30	35	25	100
Truck Speed	2	30	35	5 10 15 20 25	100
Grid Area	2	30	35	25	100 225 400

practice. Delivery drones would be able to sustain a more stable and high-endurance flight when traveling at speeds between 25 and 45 miles per hour.

3.5.3 Drone Endurance

The drone's endurance is largely dependent on existing battery technology as well as the rate of energy consumption which is primarily determined by the speed of travel. Gross (2013) suggests that drones are capable of staying in the air for 30 minutes, but many factors can bring this value down to something closer to 20 minutes while emerging technological advancements could potentially bring it up to 40.

3.5.4 Number of Drones Available

With varying availability of drones in the model, the usage and utilization is expected to vary. In real-life scenarios, the upper bound to this parameter is likely determined with a trade off between capacity of packages in the delivery truck and added efficiency with drones. We determine 2 available drones to be a reasonable base case scenario.

3.5.5 Truck Speed

In highly populated urban locations, average truck speeds can range from 10 miles per hour to 30 miles per hour (Gorzelany, 2017). In order to capture the dynamic effect on cost savings as truck speed varies, we set a baseline of 25 miles per hour and explore speeds down to 5 miles per hour in increments of 5.

3.5.6 Geographic Customer Grid Area

Although the premise of this study is focused on dense urban areas, the effects of serving wider geographic locations are worth exploring. Outside of a city, a single delivery truck may be responsible for a tour covering up to 15 or 20 square miles of customer area. This extends the case to potentially apply towards suburban and rural situations.

4 Computations and Results

This section presents our findings from computational iterations of the model proposed in section 3.2. All algorithms were implemented in Python 2.7.5 and conducted on an Apple Macbook Pro with a 3.1 GHz Intel Core i5 processor and 8 GB RAM running macOS Sierra in 64-bit mode. Mixed integer linear programming (MILP) models were solved with Gurobi version 8.0.0. As mentioned previously in Section 3.5, the following sensitivity analysis will assess tradeoffs between the key parameters: drone speed and endurance, number of drones available, truck speed, and geographic customer area.

4.1 Drone Speed and Endurance

When varying drone speed and endurance simultaneously, drone speed immediately emerges as the more significant parameter in terms of percent savings gained over TSP. A 10 mile
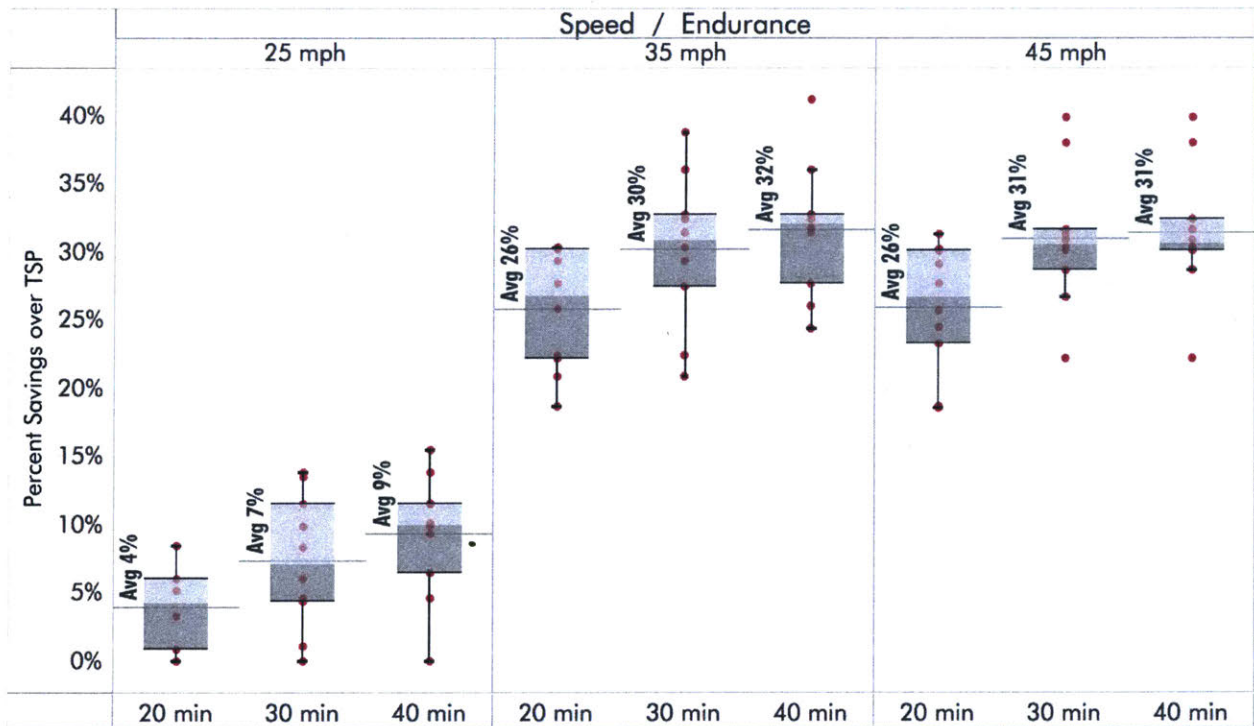

Figure 4.1: Percent cost savings over TSP across drone speed and endurance. The y-axis measures savings earned when drones are deployed compared to the same TSP tour with truck only.

36

per hour increase from 25 to 35 yields an improvement of over 20 percent in cost savings while an increase in 10 minutes of endurance leads to an improvement in the range of only 2 to 4 percent of average savings (Figure 4.1).

Although this upward trend driven by drone speed appears to lose its impact when the speed moves from 35 miles per hour to 45, Figure 4.2 suggests that this is explained by a constraint on the number of drones available for deployment. The maximum number of drones available is reached for all cases at 35 miles per hour drone speed, and this maximum is maintained as the speed moves to 45 miles per hour. While the utilization per drone does
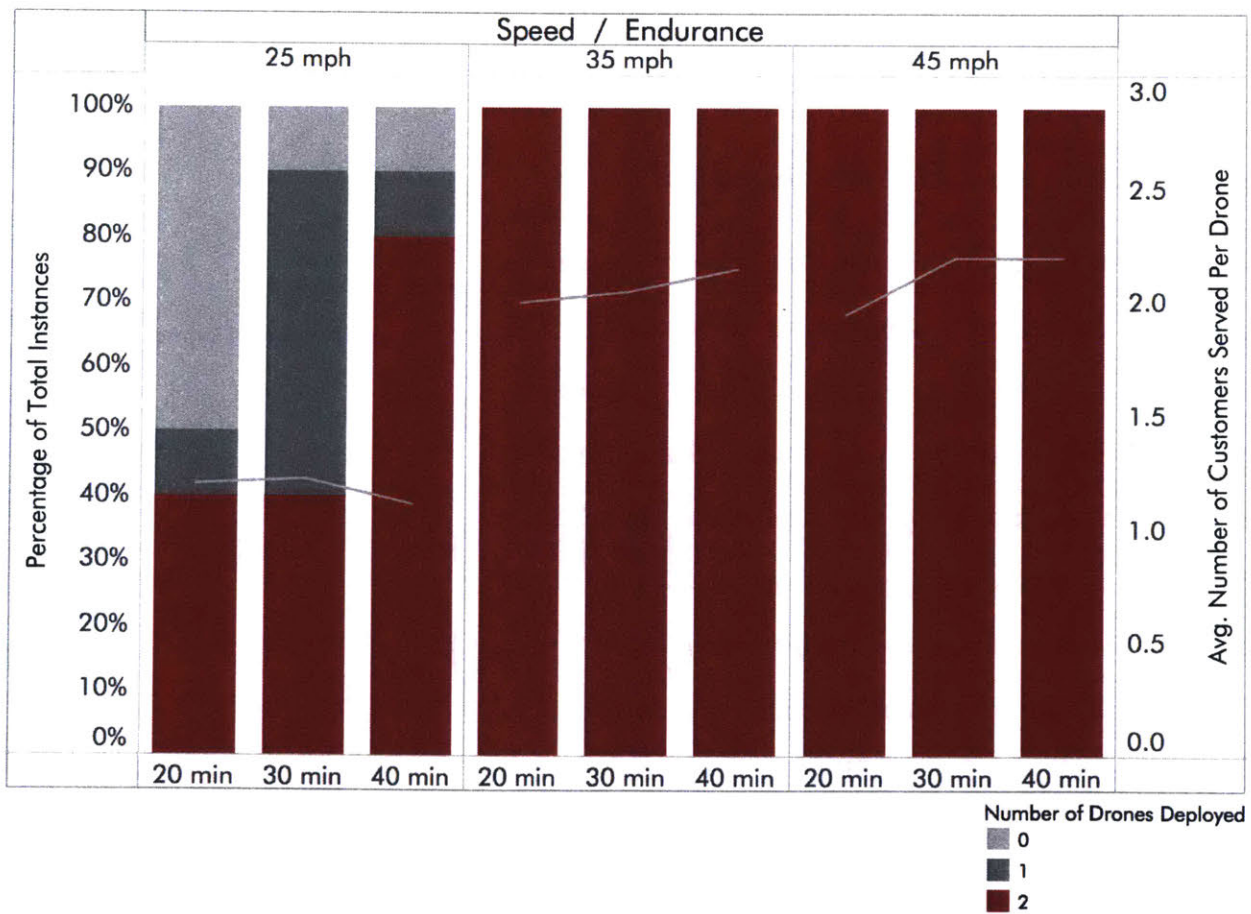
Figure 4.2: Drone usage across drone speed and endurance. The left axis corresponds with the stacked bars, which measures the frequency that a particular number of drones is deployed per instance as a percentage of all instances in the same use case. The right axis corresponds with the lines, which measures each drone in the instance's utilization in terms of average number of customers served per drone.

increase slightly as depicted as average number of customers served per drone by the line graph in Figure 4.2, the limit is reached for this metric as well at 30 minutes of endurance and drone speed of 45 miles per hour.

Another interesting trend exists when drones are set to travel at 25 miles per hour and the endurance changes from 30 minutes to 40 minutes. The drone utilization decreases with this parameter change, which suggests that the costs savings earned from increasing the number of drones deployed is traded off for slightly less utilization per drone. While number of times in which 2 drones is deployed increases from 40 percent to 80 percent as drone endurance increases from 30 minutes to 40 minutes, drone utilization drops by approximately 0.2 customers per drone.
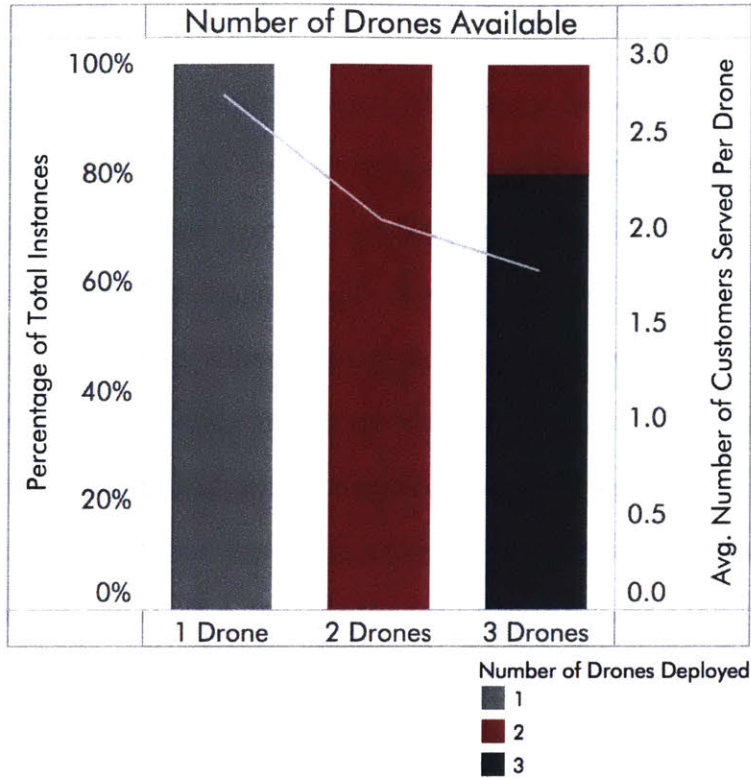
4.2 Number of Drones Available

As the number of drones available for deployment is increased, the percent savings over TSP intuitively follows the same trend. As Figure 4.3 shows, increasing availability from 1 drone to 2 drones yields an over 10 percent increase in savings over TSP. However, the relative gain is significantly reduced to only 3 percent as the third drone is introduced into the model.

Figure 4.4 shows a sharp decline in utilization per drone across all scenarios, falling by approximately one customer per deployed drone. When two drones were made available instead of only one drone (Figure 4.5), this lower per-drone utilization is offset by the increased amount of work that could be completed with the added drone throughout the network. However, when the model moved to 3 available drones, there were not
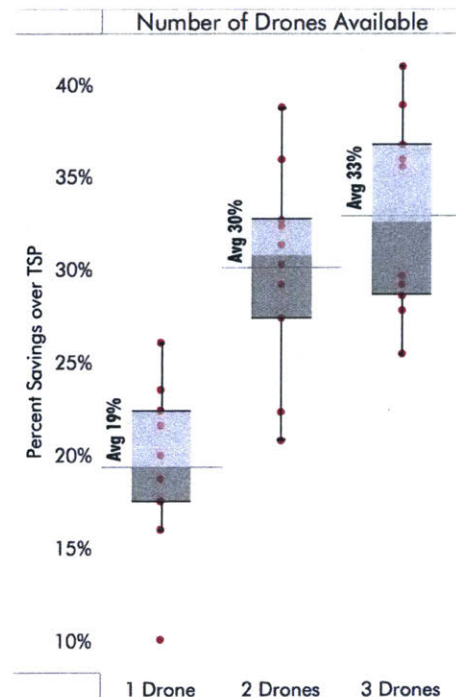

Figure 4.3: Percent savings over TSP across number of drones made available.

38

Figure 4.4: Drone usage over TSP across number of drones made available. The left axis corresponds with the stacked bars, which measures the frequency that a particular number of drones is deployed per instance as a percentage of all instances in the same use case. The right axis corresponds with the lines, which measures each drone in the instance's utilization in terms of average number of customers served per drone.

enough customers to serve with the newfound capac-

ity. Because there are only 10 customer nodes in the network and 7 serviceable by drones, the three-drone instances were constrained by the lack of density of customers.
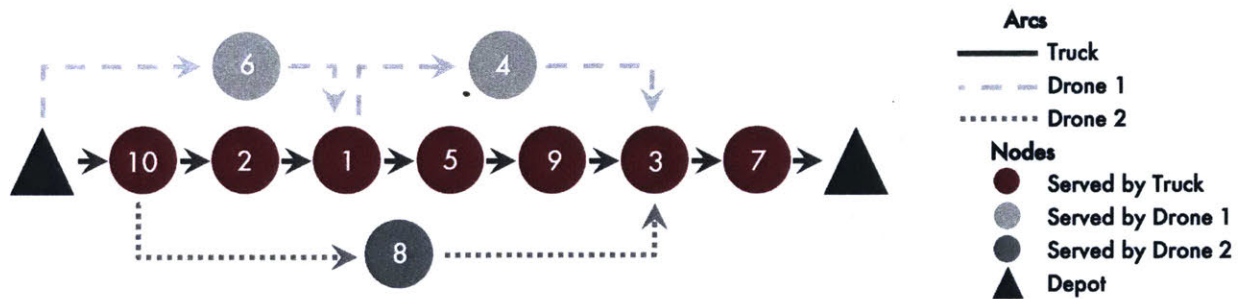

Figure 4.5: Sample test instance where two drones are deployed.

39

4.3 Truck Speed

Changes in truck speed showed a relatively uninterrupted and consistent trend upward as the average truck speed decreased in increments of five. The cost savings over TSP increases by approximately 10 percent as truck speed moves from 25 to 20, and by approximately 7 percent as truck speed moves from 10 to 5. The savings improvements from the baseline start relatively large and slowly diminish as truck speed is marginally decreased.

The steady trend can be explained by the significant disparity in the savings trade off between deploying a drone to serve a customer when truck speeds are relatively low. Although the number of drones deployed stayed constant at two per instance, the utilization per drone increased steadily as the truck speed decreased (Figure 4.7). When the average truck speed reaches its minimum of 5 miles per hour, the average number of customers served by each drone reaches its maximum of approximately 3. This observation represents the highest
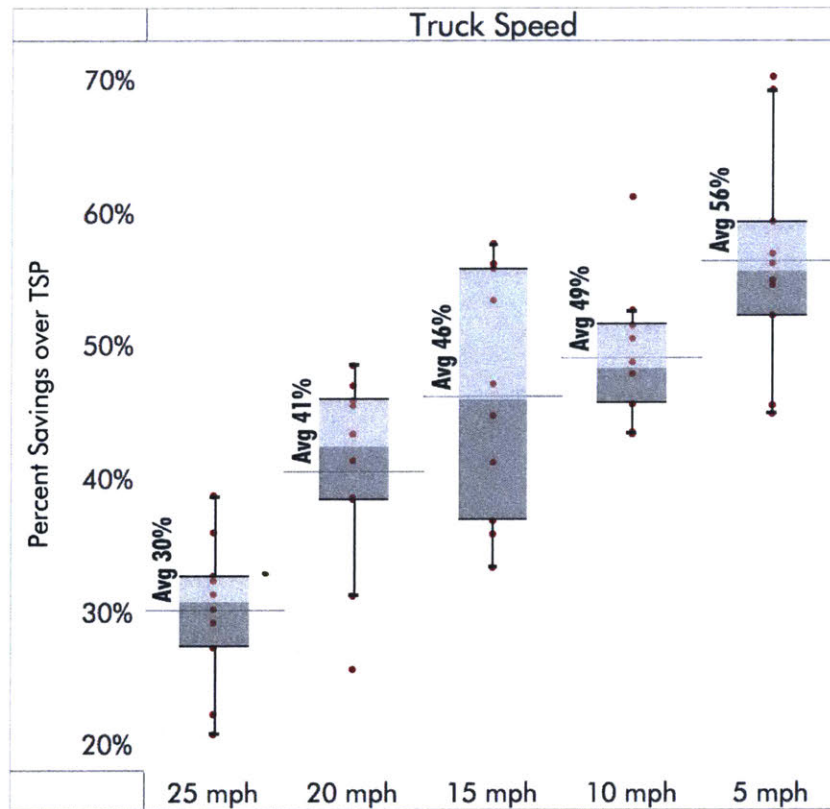
Figure 4.6: Percent cost savings over TSP across truck speeds. The y-axis measures savings earned when drones are deployed compared to the same TSP tour with truck only.

40
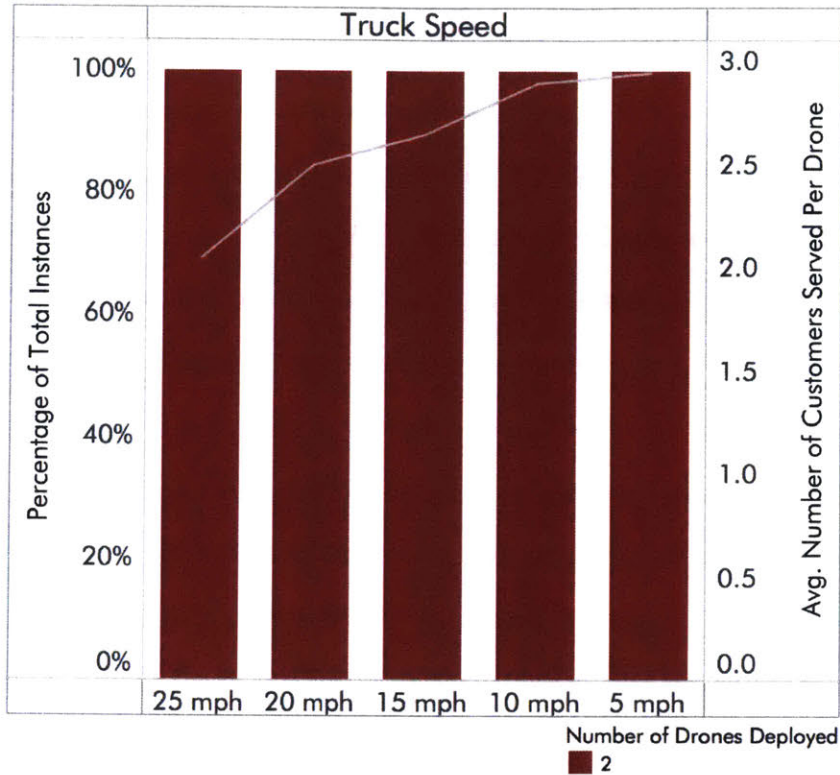
Figure 4.7: Drone usage across truck speeds. The left axis corresponds with the stacked bars, which measures the frequency that a particular number of drones is deployed per instance as a percentage of all instances in the same use case. The right axis corresponds with the lines, which measures each drone in the instance's utilization in terms of average number of customers served per drone.

average drone utilization attained throughout the entirety of this study.

The significant trade off in savings gains between serving customers with a slow moving truck compared to serving them with a drone is apparent when the cost of labor is considered. Labor cost is an important parameter to consider because we assumed in Section 3.2.2 that the drones are autonomous and therefore incur no labor costs to operate. The trade off between using a truck versus a drone to serve customers in this case is compelling enough to overcome the majority of other factors that constrains other parameters, such as limited availability of drones and low density of customers.

4.4 Geographic Customer Grid Area

While the base case parameters intend to simulate drone operation in a dense and geographically tight urban location, the effects of the same application in larger areas representative of suburban populations is highly relevant. Although the cost savings are clear in the 100 square foot use case, the government regulations mentioned in Section 1.2.3 may prove to be more difficult to overcome in highly populated city areas. Therefore the exploration of larger geographic customer areas representative of rural and suburban populations is worthwhile.

We find that increases in the area in which customers are geographically dispersed result in a negative trend in terms of percent savings over TSP as show in Figure 4.8. The effect was similar in magnitude and inverse to the impact derived from changes in drone endurance, as
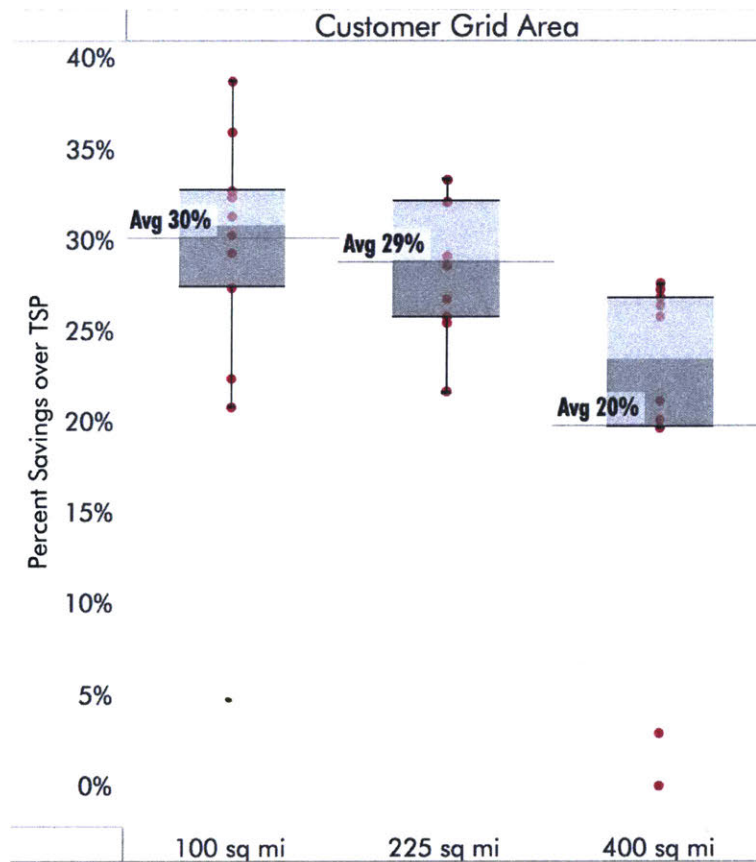

Figure 4.8: Percent cost savings over TSP across customer grid areas. The y-axis measures savings earned when drones are deployed compared to the same TSP tour with truck only.

changes in distance can be similarly represented as limited range from customer to customer. In terms of drone usage, we find that the maximum number of available drones is utilized in the 100 and 225 square mile instances (Figure 4.9).

However, in the 400 square mile case, the number of times that two drones are deployed decreased by 30 percent to 70 percent while each drone experiences a slight increase of 0.2 more customers on average. For 10 percent of observations in the 400 square mile customer grid, zero drones are deployed in the tour. This sharp drop off in drone utilization as shown in Figure 4.9 can be attributed to insufficient range relative to the size of the grid; the drone is constrained to 17.5 miles of total travel - less than 50 percent of the grid - at the baseline speed of 35 miles per hour and endurance of 30 minutes. This can also be translated to
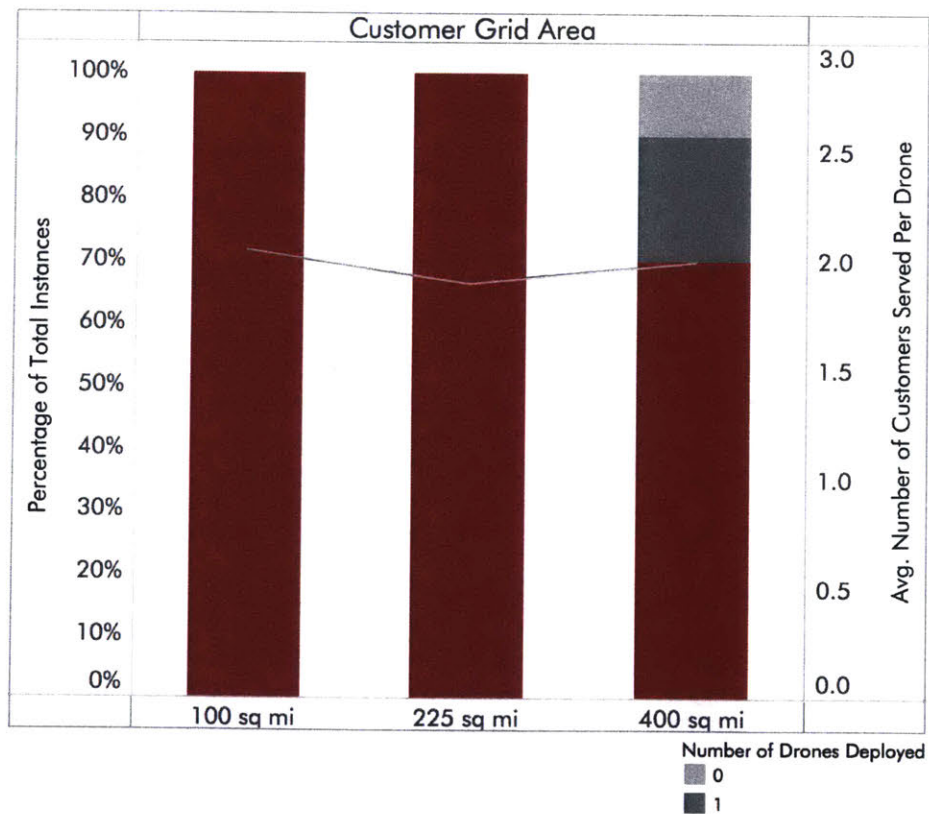
Figure 4.9: Drone usage across customer grid areas. The left axis corresponds with the stacked bars, which measures the frequency that a particular number of drones is deployed per instance as a percentage of all instances in the same use case. The right axis corresponds with the lines, which measures each drone in the instance's utilization in terms of average number of customers served per drone.

43

a range of only 8.75 miles if a drone is expected to dispatch from and return to the same origin point. The effect of this range constraint is further amplified by the flight path of the drones, which is restricted to over road networks as modeled with Manhattan distance (Section 3.2.2). Although this same range restriction is not fully realized in the case of city applications where the relative grid area can be 100 square miles or less, the negative effects of the constraint are high in magnitude and quickly noticeable as the geographic space scales in size.

4.5 Discussion

Our findings show that the cost savings for TSP with drones is significant across several potential scenarios. In the realistic case of integrating a tour with the base case determined in Section 3.3, we computed a median cost savings of 31 percent over TSP with truck only. As the endurance improves, we find that this savings increases marginally due to the relatively sufficient range in a restricted 10 mile x 10 mile grid. However, the impacts from speed increases were significant - yielding up to 10 percent cost savings improvements per 10 miles per hour increase. We found that increasing the number of drones available shows promise in terms of improving savings, but the benefits can only be fully realized as the number of customers increases due to limitations on utilization per drone when there are not enough customers to serve. Average truck speed had the largest effect drone use, as the savings surged by over 25 percent from 30 to 55 as truck speed moved from 25 to 5. When we increased the geographic grid area of customers, we found that drone usage in terms number of drones used dropped sharply due to limited relative range and capability of the base case drone.

Overall, the effect of drones is positive and relatively significant in all tested scenarios. Although there are clear use cases that have emerged as an ideal starting point to initiate testing or real-life application with the objective of highest possible gain, all scenarios generated considerable savings over TSP with truck only.

5 Conclusion

Last-mile delivery stands to face significant cost reductions and operational improvements with the integration of drones. We conducted a sensitivity analysis which varied parameters such as number of drones available for deployment, drone speed and endurance, truck speed, and geographic customer area - all which supported the notion that the benefits heavily outweigh the costs across the board when it comes to drone delivery. Our findings show that under a wide range of potential scenarios, the use of drones as a supplement to delivery trucks in the traveling salesman problem yields a minimum 4 percent savings over TSP with truck only. The most negative impact on savings comes from low endurance and speed of the drone as well as large geographic customer areas. However, these savings depict the results of the worst case of the scenarios that we tested. In more realistic scenarios, we find a median 30 percent savings over TSP and up to 55 percent savings in the best case.

We find that the largest positive impact came from diminishing average truck speed, which improved cost savings over TSP by an average 4 to 10 percent with each 5 mile per hour decrease in assumed average truck speed. However, although savings improve as truck speed is modeled to be lower, the inclusion of a truck in the model is still crucial. In all scenarios, the truck serves essentially as a moving depot that brings drones into deployable range of otherwise inaccessible customers.

5.1 Future Research

There are several notable topics that would be interesting to explore in terms of impact to drone usage in TSP. One is the trade off between truck capacity and number of drones available per tour. Optimization of this decision can potentially define an industry standard on how any drones that each delivery truck should carry. Another interesting topic of future research is the feasibility of each drone being able to carry more than one package and serve more than one customer at a time. Once this is determined to be feasible, the upside to

drone development and usage in the last-mile can be explosive. Finally, future optimizations and models should work to include the capability of drones to launch and rendezvous with the truck at a point other than a network node. Because the drones are assumed to be autonomous in the first place, this operational function is feasible within the boundaries of technological capabilities today.

Although the mathematical model defined in Section 3.2.3 can solve simple problems in polynomial time, the NP-hard nature of the traveling salesman problem causes it to become exponentially complex with each additional node. Thus complex sets exceeding approximately 10 nodes require extensive solving time and ultimately make the math model impractical for real industry use.

To solve this complication, several heuristics stand out as potential candidates to explore. Swarm intelligence (SI) is a class of heuristics inspired by the naturally occurring behavior of collective organisms such as schools of fish, flocks of birds, colonies of ants, or swarms of bees (Karaboga and Basturk, 2006). Because these creatures behave in strict accordance with others in their group, the resulting population-based approach is significantly less likely to get stuck in local optimums. Genetic algorithm is another effective heuristic that has been applied by Ferrandez et al. (2016) and Savuran and Karakaya (2015) on this specific problem, but still faces immense opportunity in terms of unexplored approaches.

Regardless of the method in which future research unfolds in terms of exploring the opportunities associated with using drones in the last mile, disruption is imminent. With a recent and rapidly growing demand for better last-mile solutions and technological advancements exceeding commercial applications, very little stands in the way of drone-based delivery becoming a commonality. While our findings show specific use-cases in which drones help yield immense cost savings over TSP with truck only, the overall benefits observed across a wide range of residual scenarios are still compelling enough to attract significant investment into research and growth of the field.

References

Agatz, N., Bouman, P., and Schmidt, M. (2015). Optimization approaches for the traveling salesman problem with drone. *SSRN Electronic Journal*, pages 1–42.

Bateman, A. H. (2015). Innovation strategies: Tracking the value of traceability. *Supply Chain Management Review*.

BLS (2018). Occupational employment and wages, may 2017, 53-3033 light truck or delivery services drivers. Technical report, U.S. Bureau of Labor Statistics. https://www.bls.gov/oes/current/oes533033.htm.

Carlsson, J. G. and Song, S. (2017). Coordinated logistics with a truck and a drone. *Management Science*.

Chang, Y. S. and Lee, H. J. (2018). Optimal delivery routing with wider drone-delivery areas along a shorter truck-route. *Elsevier*, 104:307–317.

Corps, I. M. (2009). How fast is your city? http://infinitemonkeycorps.net/projects/cityspeed/.

Dantzig, G. B. and Ramser, J. J. (1959). The truck dispatching problem. *Management Science*, 6:80–91.

EIA (2018a). Electric power monthly. Technical report, U.S. Energy Information Administration. https://www.eia.gov/electricity/monthly/index.php.

EIA (2018b). Weekly retail gasoline and diesel prices. Technical report, U.S. Energy Information Administration. https://www.eia.gov/petroleum/.

FAA (2016). Faa form 7711-uas coa. https://www.faa.gov/uas/.

Feist, J. (2018). How fast can a drone fly? - how to fly, the science of flight. https://www.dronerush.com/how-fast-can-a-drone-fly-science-of-flight-10953/.

Ferrandez, S. M., Harbison, T., Weber, T., Sturges, R., and Rich, R. (2016). Optimization of a truck-drone in tandem delivery network using k-mean and genetic algorithm. *Journal of Industrial Engineering and Management*, 9:374–388.

Fister Jr., I., Yang, X.-S., Fister, I., Brest, J., and Fister, D. (2013). A brief review of nature-inspired algorithms for optimization. *Elektrotehniski Vestnik*, 80:1–7.

Glaser, A. (2016). Why amazon is testing drone delivery in the u.k. — and not in the u.s. https://www.recode.net/2016/12/14/13955818/amazon-drone-delivery-uk-us-faa-testing.

Gorzelany, J. (2017). The world's best and worst cities for drivers. https://www.forbes.com/sites/jimgorzelany/2017/09/27/the-worlds-best-and-worst-cities-for-drivers.

Gross, D. (2013). Amazon's drone delivery: How would it work? https://www.cnn.com/2013/12/02/tech/innovation/amazon-drones-questions/index.html.

Heater, B. (2017). Trump signs bill reinstating the faa's drone registration requirement. https://techcrunch.com/2017/12/12/trump-signs-bill-reinstating-the-faas-drone-registration-requirement/.

Hern, A. (2014). Dhl launches first commercial drone 'parcelcopter' delivery service. https://www.theguardian.com/technology/2014/sep/25/german-dhl-launches-first-commercial-drone-delivery-service.

Joerss, M., Schroder, J., Neuhaus, F., Klink, C., and Mann, F. (2016). Parcel delivery: The future of last mile.

Junger, M. and Thienel, S. (1994). Provably good solutions for the traveling salesman problem. *ZOR - Mathematical Methods of Operations Research*, 40:183–217.

Karaboga, D. and Basturk, B. (2006). A powerful and efficient algorithm for numerical function optimization: artificial bee colony (abc). *J Glob Optim*, 39:459–471.

Kirkpatrick, S., C. D. Gelatt, J., and Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, 220:671–680.

Laporte, G. (1992). The vehicle routing problem: an overview of exact and approximate algorithms. *European journal of operational research*, 59:345–358.

Lee, H. L., Chen, Y., Gillai, B., and Rammohan, S. (2016). Technological disruption and innovation in the last-mile. *Stanford value chain innovation initiative.*

Lin, S. (1965). Computer solutions of the traveling salesman problem. *The Bell System Technical Journal*, pages 2245–2269.

Lindner, M. (2015). Global e-commerce sales set to grow 25 percent in 2015. https://www.digitalcommerce360.com/2015/07/29/global-e-commerce-set-grow-25-2015/.

Luo, Z., Liu, Z., and Shi, J. (2017). A two-echelon cooperated routing problem for a ground vehicle and its carried unmanned aerial vehicle. *Sensors*, pages 1–17.

Madrigal, A. C. (2014). Inside google's secret drone-delivery program. https://www.theatlantic.com/technology/archive/2014/08/inside-googles-secret-drone-delivery-program/379306/.

Minh Ha, Q., Deville, Y., Pham, Q. D., and Ha, M. H. (2018). On the min-cost traveling salesman problem with drone. *Transportation Researh Part C: Emerging Technologies*, 86:597–621.

Murray, C. C. and Chu, A. G. (2015). The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery. *Transportation Research Part C: Emerging Technologies*, 54:86–109.

Pasztor, A. (2018). Faa projects fourfold increase in commercial drones by 2022. https://www.wsj.com/articles/faa-projects-fourfold-increase-in-commercial-drones-by-2022-1521407110.

Poikonen, S., Wang, X., and Golden, B. (2017). The vehicle routing problem with drones: Extended models and connections. *Networks: An international journal*, 70:1:34–43.

Reid, D. (2016). Domino's delivers world's first ever pizza by drone. https://www.cnbc.com/2016/11/16/dominos-has-delivered-the-worlds-first-ever-pizza-by-drone-to-a-new-zealand-couple.html.

Romm, T. (2017). A new trump policy could let amazon and google test more drones in u.s. cities. https://www.recode.net/2017/10/25/16542940/amazon-google-drones-us-government-trump.

Rose, C. (2013). Amazon's jeff bezos looks to the future. https://www.cbsnews.com/news/amazons-jeff-bezos-looks-to-the-future/.

Savuran, H. and Karakaya, M. (2015). Efficient route planning for an unmanned air vehicle deployed on a moving carrier. *Springer.*

Stolaroff, J. K., Samaras, C., O'Neill, E. R., Lubers, A., Mitchell, A. S., and Ceperley, D. (2018). Energy use and life cycle greenhouse gas emissions of drones for commercial package delivery. *Nature Communications 9*, 409.

Thiels, C. A., Ado, J. M., Zietlow, S. P., and Jenkins, D. H. (2014). Use of unmanned aerial vehicles for medical product transport. *Air Medical Journal*, 34:2:104–108.

Toth, P. and Vigo, D. (2002). *The Vehicle Routing Problem*. Society for Industrial and Applied Mathematics, 3600 University City Science Center, Philadelphia, PA 19104.

Yang, J., Shi, X., Marchese, M., and Liang, Y. (2008). *Progress in Natural Science*, 18:1417–1422.