

Essays in Financial Economics

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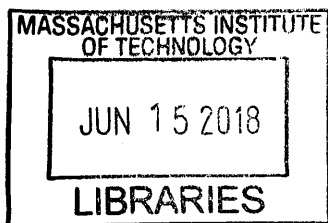
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Abstract

The essays in this thesis study issues in finance affecting large corporations, developing economies, and households. The common theme that connects these essays is a focus on how financial institutions, frictions, and policies affect the allocation of resources in the economy.

The first chapter explores a classic question in corporate finance: how valuable are restrictive debt covenants in reducing the agency costs of debt? I answer this question by exploiting the revealed preference decision to refinance fixed-coupon debt, which weighs observable interest rate savings against the unobservable costs of a change in restrictive debt covenants induced by refinancing. Plausibly exogenous variation in this trade-off reveals that firms require higher interest rate savings to refinance when it would add restrictive covenants and require much lower interest rate savings when refinancing sheds covenants. A high-yield bond's restrictive covenant package increases the value of speculative-grade firms by 2.4 percent on average.

Joint work with Ernest Liu in Chapter 2 provides a theory that explains how institutional weakness in credit markets can fail to stimulate development even when there is ample credit supply. We show that when borrowers lack credible mechanisms to commit not to borrow further from other lenders in the future, not only does the increasing availability of lenders raise the interest rate on loans and reduce the amount of funds that entrepreneurs can borrow, but perversely it is those entrepreneurs with more profitable investment opportunities that will end up raising fewer investments precisely because they have stronger desires to seek out additional lenders in the future. This effect further discourages entrepreneurs from initiating the most efficient and productive endeavors, generating persistent underdevelopment.

Chapter 3 explores the role of liquidity constraints in households' responses to fiscal stimulus programs. In joint work with Jonathan Parker, Brian Melzer, and Arcenis Rojas, this chapter evaluates the impact of the Car Allowance Rebate System (CARS) on vehicle purchases. We find that the liquidity provided by CARS amplified household responses to the economic subsidy. Liquidity provision was lower for the owners of clunkers encumbered by loans, since participation required loan repayment. Such households had a very low participation rate, which we attribute to liquidity constraints and distinguish from the effects of other indebtedness, household income, and the size of the program subsidy.

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Finally, I dedicate this dissertation to my loving wife Gail. She has made many sacrifices to support my education, including spending four years with me in this land of seemingly perpetual blizzards. Her pride and confidence in me has given me the strength to do my best in the pages that follow.

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Chapter 1

Corporate Refinancing, Covenants, and the Agency Cost of Debt

1.1 Introduction

A large theoretical literature demonstrates that the state-contingent allocation of control and cash flow rights is useful in preempting agency conflicts between creditors.¹ In debt contracts, this state-contingent allocation is achieved in large part through restrictive covenants (Smith and Warner, 1979). Empirically, studies of such restrictive covenants have largely focused on the ex-post impacts of covenant violations (Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009)). This leaves unanswered questions regarding the *ex-ante* efficiency gains achieved by using debt covenants to allocate control rights. In practice, how effective is this mechanism in ameliorating the agency costs of debt, or put differently, how much surplus is generated by the use of covenants? Given covenants are more likely to be used in the riskiest debt, how important are covenants in allowing value to be created from a high-leverage capital structure?

This paper provides quantitative answers to these questions. I develop a dynamic revealed preference framework that allows me to estimate the value of high-yield corporate bond covenants

¹Jensen and Meckling (1976) provides a foundation for the notion of inter-creditor agency costs. Aghion and Bolton (1992) and Zender (1991) show the value of debt in aligning incentives comes in part from the allocation of control rights.

by relating the observed timing of bond refinancings to changes in interest rates and to whether refinancing would impose or remove covenants from the firm's debt. I find that high-yield bond covenants add significant value. My baseline estimates suggest that they increase the total asset value of speculative-grade firms by approximately 2.4 percent. I also document substantial variation in the magnitude of the value added by these covenants: large firms and growth firms with speculative-grade capital structures benefit from restrictive covenants by as much as seven percent of enterprise value. Taken in concert with estimates in the literature of the net benefits of leverage, these estimates suggest restrictive debt covenants are essential in allowing high-leverage capital structures to add value to firms.

My methodology leverages a directly observable tradeoff in firms' decisions to refinance callable corporate bonds. This framework allows me to exploit time-series variation in risk-free interest rates and plausibly exogenous variation in how refinancing would change covenants to estimate the ex-ante surplus generated by the restrictive covenants in high-yield bond indentures. Like mortgage refinancing, the textbook consideration of fixed-coupon bond refinancing is to optimally exploit declines in borrowing costs. However, for firms that have gained or lost access to the investment grade bond market, refinancing also substantially alters the restrictive covenants imposed on the firm. I provide detailed evidence that only high-yield bonds contain covenants that substantially limit the control rights of the firm management, for example by preventing the firm from making equity distributions, raising additional debt, or undertaking new acquisitions. For firms that have been upgraded to investment-grade, which are referred to in capital markets as "rising stars," refinancing is an opportunity not only to exploit interest rate savings, but also to shed restrictive high-yield covenants, which may be inefficiently restricting the firm. For firms that have lost their investment-grade rating ("fallen angels"), refinancing to exploit declines in interest rates comes at the cost of adopting new limitations on managerial discretion.

A recent report by Moody's, a credit rating agency, highlights exactly this tradeoff:

"Investment-grade covenants typically do not restrict a company's ability to make dividends, buyback shares or incur unsecured debt—a fact that investors in investment-grade securities often overlook because of the issuers' strong credit profiles. But when investment-grade companies fall to speculative grade, the flexibility afforded by the covenants included in their bonds—which can have maturities of 20-40 years—can become a factor as these fallen angels seek to refinance." – Moody's (2016)

I begin my empirical analysis by documenting that, consistent with theory and anecdotal evidence, covenants are indeed an important factor in the decision to refinance callable corporate bonds. I do this by considering the refinancing decisions of firms that have experienced substantial credit rating downgrades or upgrades since issuance relative to decisions of firms that have not experienced such ratings changes but have the same potential interest rate savings from refinancing and the same current fundamentals. The only difference between these firms that is relevant for the decision to refinance an individual bond issue is how the debt covenants binding on the firm will change if the refinancing is undertaken.

For each bond in my sample I estimate the dynamically optimal refinancing strategy the issuing firm should follow in the absence of any covenant considerations. I find that this benchmark model of bond call policy is systematically biased when refinancing will significantly change the covenants binding on the firm. Firms that would face tighter covenants upon refinancing require larger declines in interest rates to initiate refinance than is implied by the model, relative to otherwise similar refinancing opportunities that do not involve changing covenants. Conversely, bonds which can *shed* covenants refinance too “early,” that is, they are called even though there is remaining time value in their option to wait for further declines in interest rates.

These refinancing patterns suggest that covenants have the effect on debt value hypothesized in the seminal work of Smith and Warner (1979): they limit firm actions to protect the value of debt. Fallen angel firms delay refinancing relative to always-junk firms because loose covenants allow shareholders to usurp wealth from debtholders (for example via asset substitution) and thus increase the cost of calling the bond relative to the opportunity cost of continuing to service it. Rising star firms refinance before the time value of their refinancing option has expired because restrictive covenants prevent these firms from taking profitable investment opportunities; refinancing eliminates these constraints.

The fact that firms are willing to sacrifice interest-rate savings to shed or avoid covenants reflects the fact that restrictive covenants materially limit control rights in a way that affects the distribution of value between debt and equity claims in a given notional capital structure. But it does not by itself identify the *total* value these covenants create or destroy, quantification of which is the central goal of my paper. I recover the surplus generated by restrictive covenants by considering both the

impact of covenant changes on the sensitivity of refinancing to changes in interest rates and the extent to which covenants increase the value of debt.

Why does the tradeoff between interest-rate savings and covenant considerations in refinance reveal the surplus generated by restrictive covenants? I show that the “delay” of fallen-angel firms to refinance to adopt covenants is an overhang problem.² Refinancing transfers significant value to existing bondholders unless interest-rate savings are large, precisely because loose covenants allow the firm to take actions that erode the value of debt claims relative to the pre-specified call price of the bond. The firm will only refinance if the surplus generated by adopting new restrictive covenants is at least as large as the net transfer to outstanding bondholders. Thus, combined with estimates of how covenants affect the value of corporate bonds, variation in interest rates and observed refinancing behavior identifies the surplus generated by fallen angel firms adopting restrictive debt covenants. A similar argument holds for firms that have attained investment-grade status: they could increase surplus by shedding high-yield bond covenants.

To exploit this insight, I structurally estimate a dynamic model of corporate bond refinancing that combines the intuition of the revealed preference tradeoff between interest rate savings and covenants with the dynamic considerations necessary to apply the model to the observed prices and refinancing decisions of real-world corporate bonds. In the model, covenants affect the stochastic process for the total enterprise value of the firm in a state-dependent fashion. This allows (but does not impose) restrictive covenants to affect the total value of the firm and the way firm value is distributed between equity and debt claims. For example, loose covenants could allow a highly levered firm to increase the variance of its profits at the expense of their mean, inefficiently transferring wealth from bondholders.

For a given parameterization of the model, I can solve the model for each bond in my sample at each time the bond is callable to generate predictions of the bond’s optimal refinancing policy and market price. I show that the parameters of the model related to covenants are identified by intuitive and directly observable variation in the data: the difference in refinancing policies and

²Because bonds are diffusely held, covenants are difficult to renegotiate, and the most effective way to modify them is to retire and replace debt issues. Indeed, this is one of the theories of why corporate debt is callable in the first place. If an investment opportunity were to arise for which excessive value would accrue to debt holders, debt overhang can be avoided by calling the original debt issue (Bodie and Taggart, 1978).

bond prices of issuers that stand to gain or shed covenants in refinancing relative to those of issuers whose covenants would be unchanged by refinancing. I structurally estimate these parameters in a maximum likelihood framework to reconcile the predictions of my model for bond prices and refinancing dates with what I actually observe in the data.

My estimation results imply that restrictive covenants increase the value of of speculative grade firms. The asset value of a typical speculative-grade firm would be around 2.4 percent lower if the firm had the same risky capital structure but its debt did not contain restrictive covenants. I take this figure as my headline quantification of the agency costs of debt solved by a standard speculative-grade package of restrictive covenants.

The model estimation also provides an assessment of how costly speculative grade covenants would be for an investment grade firm: the total asset value of such firms would only be around one percent lower if they were forced to abide by restrictive covenants. My model estimates are thus consistent with the notion that debt issued as investment-grade does not contain restrictive covenants because including them would reduce the joint surplus of equity and debt claimants by inefficiently limiting flexibility. However, the resulting reduction of value would be small compared to the value created by these same covenants for riskier firms. Together with the fact that investment-grade firms are willing to forgo substantial interest-rate savings to shed covenants, this implies the primary effect of covenants on investment-grade firms would be to change the distribution of value between equity and debt claimants, with little impact on total firm value.

I also explore heterogeneity in the value of high-yield restrictive covenants across my sample of firms. I find that there are substantial differences in the value of these covenants by firm size, industry, and growth opportunities. Large firms and firms with the highest growth opportunities gain as much as seven percent of asset value from restrictive covenants. The value of high-yield covenants for small speculative-grade firms is statistically indistinguishable from zero.

Finally, the effects that I find of restrictive debt covenants on risky firm asset value are quantitatively large. Most notably, they are on the same order of magnitude as estimates of the overall *net* value debt adds to the capital structure, accounting for the costs of financial distress. My contribution expands this in an important dimension: I show that the use of debt covenants in

risky capital structures is essential to solving agency problems that allow debt to generate positive value for the firm.

Relation to Prior Literature

My paper builds off of the large theoretical literature studying the design of securities in an incomplete contracting setting. Aghion and Bolton (1992) and Zender (1991) show that the allocation of control rights can improve the alignment of incentives between claimants. In these papers debt contracts, which allocate control rights in the event of default, emerge as optimal securities. Covenants can be understood in this context to operate through two mechanisms: increasing contractual completeness by restricting management control rights in a state-contingent manner and by inducing ex-post renegotiation.

The salience of these two mechanisms, and thus the design of covenants, depends on the cost of renegotiation. Corporate loans are narrowly monitored and controlled by individual or syndicate lenders, while bonds are publicly issued securities with disbursed ownership; therefore renegotiation is relatively more costly for bonds.³ It is thus not surprising that loan and bond covenants are substantially different. Violation of financial covenants in loans induces technical default, and thus these covenants serve as tripwires that induce renegotiation between firms and creditors.⁴ Bond covenants are instead “incurrence” based, and violation only restricts firms from taking certain actions that could reduce their ability to service debt. My paper studies bond covenants and thus speaks generally to the value of state-contingent control rights in general than specifically through renegotiation.⁵

³Further, The Trust Indenture Act of 1939 prevents issuers of public debt securities from modifying the terms of principal, interest, and maturity after issuance, which further limits the ability of bondholders and issuers to renegotiate debt structure.

⁴This idea dates back to Zinbarg (1975), which observes that the relatively restrictive covenants of loan agreements “provide a series of checkpoints that permit the lender to review proposed actions by the borrower with potential for substantially impairing the lender’s position.”

⁵Becker and Ivashina (2016) argue that the recent loosening of covenant strength in the leveraged loan market is due to the increasingly diffuse ownership structure of syndicated private debt. In fact, this market is becoming indistinguishable from the high yield bond market. The majority of the covenant loosening that has occurred recently in the leveraged loan market has indeed been the shift from maintenance covenants to “bond style” incurrence covenants, which is known in the industry as “covenant-lite.” Investment banks are increasingly arranging high-yield bond and leveraged loan financing from the same desk, further blurring the distinction between these two markets. Interestingly, the continued spread of covenant-lite terms in the private debt market, beyond just widely held institutional loan tranches, suggests that the importance of inducing state-contingent renegotiation may be declining.

Garleanu and Zwiebel (2009) study the optimal allocation of control rights in the context of asymmetric information about the magnitude of potential asset substitution. They show that when renegotiation costs are low, this information asymmetry results in optimal covenants that are tight and thus frequently violated, but also frequently relaxed upon violation. Murfin (2012) documents that bank lenders increase the tightness of these tripwires after experiencing defaults in their loan portfolios.

Chava and Roberts (2008), Roberts and Sufi (2009), and Nini, Smith, and Sufi (2009) empirically assess how firm activity responds to covenant violations. They show that covenant violations cause lenders demand concessions that materially restrict the activity of the firm. For example, Chava and Roberts (2008) uses a regression discontinuity design to show firm investment declines sharply around financial covenant violations. While this reveals that the state contingent allocation of control rights affects the operations of firms, it cannot quantify the value created by this mechanism.

Three papers have made progress in attempting such quantification. Bradley and Roberts (2015) use a two-equation regression methodology to estimate how much restrictive covenants lower the interest rate spreads of loan securities, accounting for the selection of riskier firms into more restrictive covenants. Reisel (2014) performs a similar analysis for bond covenants. They show that all else equal, the inclusion of covenants is in fact associated with lower interest rates. Thus, this work demonstrates that covenants address agency problems because they lower the cost of debt capital.

Finally, Matvos (2013) is the closest paper to mine because it goes beyond the estimation of the price effect of covenants to provide an assessment of the total value created by loan covenants. It identifies covenant surplus creation through the indifference condition that the firm's perceived cost of marginal covenant inclusion is exactly offset by the reduced price of debt associated with this marginal covenant inclusion.

My paper innovates on these papers in several dimensions. I approach the issue of identification from a new perspective: instead of trying to explicitly account for the selection into tight or loose covenants at issuance, I consider the information revealed by firms when they face an opportunity to change the covenants binding on their firm. I exploit the large discontinuity in covenant strength

across investment-grade and high-yield bonds to generate extensive margin variation in firms' ability to shed or avoid the restrictive incurrence covenants found in high-yield but not in investment-grade bonds. This allows me to obtain estimates of the total value created by high-yield bond covenant packages. The works cited above in contrast use intensive margin changes in covenant strength and thus estimate the marginal value of covenant strength (usually measured in terms of number of covenants). My paper is thus the first to credibly quantify the *total* value created by restrictive debt covenants.

My paper also innovates in how it uses structural estimation techniques to quantify important magnitudes in corporate finance. My approach attempts to address one of the criticisms of quantitative structural models in finance, raised by Welch (2012), that the identifying mechanisms of structural models in corporate finance should be more closely linked to observable data. The estimated parameters in my model are identified from “differences-in-differences” variation in the data that I show is directly revealing of how covenants change the distribution of firm value. The structural estimation is necessary *only* because the firm's decision to refinance is dynamic. This is similar in spirit to the exercise of Gornall and Strebulaev (2017), which directly applies an option pricing methodology to reveal the valuation of venture-backed firms implied by correctly considering the optionality structure of staged venture financing.

Also related to this paper is a literature that studies the endogenous choice of covenants and the innovation of new capital market instruments, including Nash, Netter, and Poulsen (2003), Billett, King, and Mauer (2007), Kahan and Yermack (1998), Asquith and Wizman (1990), and Beatty, Ramesh, and Weber (2002). King and Mauer (2000) also studies the call policy of nonconvertible corporate bonds and finds evidence that one of the determinants of calling a corporate bond is to remove restrictive covenants.

The rest of this paper is organized as follows. In Section 1.2, I introduce a static model to elucidate how refinancing decisions are influenced by inter-creditor conflicts and how the observed refinancing of firms reveals the magnitude of these conflicts. Section 3.4 describes the data I will use to explore this empirically. Section 1.4 presents evidence that firms' decisions to refinance debt are inconsistent with a benchmark model of refinancing in which covenants play no role. Instead, I show refinancing patterns support the hypothesis that firms are willing to pay to avoid new

restrictive covenants and to shed old ones. In Section 1.5, I outline a dynamic structural model of refinancing that explicitly incorporates firms' valuation of covenants in the refinancing decision, as motivated by the model in Section 1.2. Section 1.6 reports on the estimation of this model and what it reveals about the value of covenants in addressing inter-creditor agency conflicts. Section 3.7 concludes.

1.2 A Static Model of Covenants and Refinancing

I now introduce a stylized model that endogenizes the use of restrictive covenants. This model explains why they are only included in risky debt and illustrates the tradeoff that arises between the agency costs of debt and interest rate savings in firms' decisions to refinance callable bonds.

1.2.1 Model Setup

There are three periods $t \in \{1, 2, 3\}$. A firm has an investment opportunity and needs to issue debt K to finance the investment. The project is financed in period 1 by issuing a two period callable coupon bond, with coupon payment c_0 per dollar of face value due each period. In period 2 there is a shock to the risk-free rate and a shock to the fundamentals of the project. There is also at this time an opportunity to refinance the bond by calling it at par plus a call premium, CP per dollar of par, and issuing a new one period bond. The state of the project $s \in \{H, L\}$ is realized in period 3 and the firm takes action $a \in \{A, B\}$ after the state is realized. The capital market is competitive, and bonds are issued at a yield that ensures zero profit in expectation for bond investors.

The expected probability as of period t of state i being realized is p_t^i . The payoff of the project depends jointly on the realized state and chosen action, as shown in Table 1.1. $\{p_1^i\}$ is the distribution of the final state at period 1 and $\{p_2^i\}$ is the distribution of the final state as of period 2. The quantity q denotes the probability that project A is successful in state L —in this state the project returns zero with probability $(1 - q)$.

The payoff structure in Table 1.1 captures the idea that action A is uniformly good in state H

State	Action	Value	Equity Payout	Debt Payout
H	A	$X + \Delta$	$X + \Delta - D$	D
H	B	X	$X - D$	D
L	A	$(X + \Delta)w / \text{prob } q$	$q \times (X + \Delta - D)$	$q \times D$
L	B	X	$X - D$	D

Table 1.1: **States, Actions, and Payoffs in the Static Model**

but in state L the interests of equity and debt may diverge. This requires the following parameter assumption.

Assumption 1.2.1. (*Inter-Creditor Agency Conflict*)

$$(1 - q)X > q\Delta > (1 - q)(X - D)$$

Under Assumption 1.2.1, in state L shareholders prefer action A , but action B is value-maximizing. Again note that in state H it is optimal to take action A and there is no conflict between equity and debt holders.

1.2.2 Rating Dependent Optimality of Covenants

Covenants are imperfect and do not replicate complete contracting. I model this by assuming it is not possible to restrict the firm's action in a state-dependent manner. However, a covenant can be written that forces the issuer to always take action B . This gives rise to a tradeoff. By imposing this covenant, the firm solves its agency problem that would arise in state L , but at a cost of limiting its flexibility to make the value-maximizing decision in state H . This is intuitively realistic for many covenants in high-yield corporate bonds, for example those that restrict asset sales, mergers and acquisitions, and investment in non-core business lines. I define the total surplus generated by the covenant to be the difference in the expected project payout with and without the restrictive covenant in place. The total surplus generated by the covenant is given by:

$$S^{cov}(p) = Xp^L(1 - q) - \Delta(1 - p^L(1 - q)). \quad (1.1)$$

Conditional on debt being issued, either to fund the project initially or to refinance previous

debt, the covenant will be included in the new issue if and only if the covenant provides positive total surplus. This follows directly from the assumption that new debt is issued at a fair market price, which aligns equity holder's incentives with the maximization of total asset value. The above equation implies there is a threshold value of p^L above which any debt issuance will include covenants and below which no debt will include covenants.

Proposition 1.1. *Whenever debt is issued, it will contain the restrictive covenant if and only if $p^L > \bar{p}$, where $\bar{p} \equiv \Delta / [(1 - q)(X + \Delta)]$.*

This prediction is consistent with the fact highlighted in the Section 1.1 that riskier debt issues have strict covenants but safer debt does not. Section 1.4.1 examines this empirically in considerable detail.

1.2.3 Covenants and the Refinancing Decision

Now consider the model at period 2. The firm has legacy debt outstanding with payment due next period of $D = K(1 + c_0)$. The firm has the option to call this debt and replace it with a new debt issue. It does so by paying call price CP per unit of par value K to retire the old debt. The firm refinances if doing so will increase its equity value. This is a function of interest rate savings made possible by changes in the risk free rate and a change in whether or not the restrictive covenant will be imposed on the firm. There are four cases regarding a covenant change in refinance: maintaining the covenant, maintaining absence of the covenant, adopting the covenant, and shedding the covenant. In the model, these cases are characterized by if p^L has crossed \bar{p} from period 1 to period 2:

- | | |
|-----------------------------|------------------------------|
| 1. Always Junk | $\{p_1^L, p_2^L\} > \bar{p}$ |
| 2. Always Inv. Grade | $\{p_1^L, p_2^L\} < \bar{p}$ |
| 2. Fallen Angel | $p_1^L < \bar{p} < p_2^L$ |
| 4. Rising Star | $p_1^L > \bar{p} > p_2^L$ |

If a fallen angel firm decides to refinance, it must accept the restrictive covenant into its replacement debt issue, and it did not have such a covenant before. Similarly, a rising star firm, if it

chooses to refinance, is able to shed its restrictive covenant by issuing a replacement bond without the covenant. The other two categories of firms experience no change in their covenant status as a result of refinancing.

Proposition 1.2. *The refinancing decision of a firm can be characterized in terms of a threshold risk-free rate, below which the firm will choose to refinance and above which the firm will not refinance. The refinancing boundaries are given by the following expressions.*

$$\text{Always Junk Refi} \iff rf < \bar{r}^{AJ} = CP^{-1} (1 + c_0) K$$

$$\text{Always IG Refi} \iff rf < \bar{r}^{IG} = CP^{-1} (1 + c_0) K \left(1 - p_1^L (1 - q)\right)$$

$$\text{Fallen Angel Refi} \iff rf < \bar{r}^{FA} = b^{AJ} - \delta$$

$$\text{Rising Star Refi} \iff rf < \bar{r}^{RS} = b^{IG} + \delta$$

where

$$\delta \equiv CP^{-1} \left[\Delta \left(1 - p_1^L (1 - q)\right) - (X - K (1 + c_0)) p_1^L (1 - q) \right] > 0.$$

The proof of the proposition is a matter of straightforward algebra comparing the discounted expected value of refinancing against not refinancing for each of the four types of firms, and is shown in Appendix 1.A.3. The resulting expressions, however, are informative. For firms without changes in fundamental credit risk that imply a covenant change upon refinance, refinance is optimal if the risk-free rate is low so that paying the call premium today to retire the debt early is less expensive in present value terms than paying the principal and interest the following period. For firms with covenant changes, however, the expression also includes the firm's valuation of avoiding the restrictive covenant. This is captured by the expression for δ . Avoiding the covenant allows the firm to pick action A and possibly receive extra payment Δ , at the expense of increased probability of default. For firms that do not already have the covenant (fallen angels), the interest rate savings must be positive *net* of this lost benefit. For firms that have the covenant and are able to shed it (rising stars) the opposite is true, by refinancing the firm gains its valuation of being able to take action A and is willing to refinance even if interest savings are not very high.

These findings are summarized as empirically testable predictions of the model in the following

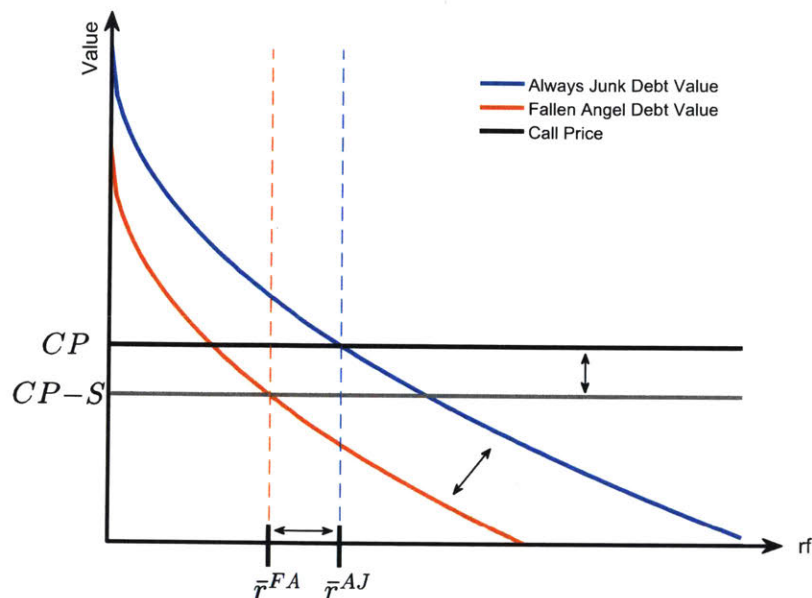
corollary.

Corollary 1.1. *All else held equal, the refinance boundary of a fallen angel firm is lower than an always junk firm. All else held equal, the refinance boundary of a rising star firm is higher than that of an always investment grade firm.*

It is important to understand the competing incentives of a firm to refinance into debt with the optimal covenant provision. Optimal covenant assignment increases total surplus, and assuming new debt issues are priced competitively, this puts the shareholders on the margin to maximize surplus *conditional on refinance*. However, *refinancing* the old debt may involve either a positive or negative transfer to existing debt holders. This is the net sum of two channels. On one hand, variation in the risk-free rate has changed the discounted present value of interest and principal payments relative to the fixed call price. On the other hand, the covenant changes the distribution of the project's returns, which differentially changes the value of debt and equity claims. This creates a wedge between the effective value of debt under the current covenant regime and the cost of refinancing the debt. For example, a risky firm without the protective covenant places a lower value on the debt claims it owes precisely because the lack of covenant allows the firm to undertake the risky activity and thus raise its probability of default. Thus, the failure of fallen angel firms to refinance can be thought of as classic debt overhang problem (Myers, 1977).

Figure 1.1 provides a graphical exposition of how the surplus generated by the restrictive covenant is identified in this framework. Imagine one could use refinancing decisions to observe the values of the refinancing boundaries \bar{r}^{FA} and \bar{r}^{AJ} as defined in Proposition 1.2, as well as the difference in the market value of the (risky) debt cash flows of fallen angel and always-junk firms with the same current value of p . The fact that the lack of the covenant reduces the value of debt implies that the fallen angel firms will only refinance for larger declines in the risk-free rate. However, the fact that conditional on refinance the firm would be able to capture the surplus S generated by the newly imposed covenant implies the firm will be willing to sacrifice some interest rate savings to refinance. This can be seen in Figure 1.1 by noting that when $r_f = \bar{r}^{FA}$, the value of the firm's outstanding debt claims is still below the call price CP . If the firm is indifferent to refinancing at this interest rate, it implies the surplus the firm can capture by accepting the restrictive covenant is exactly equal to this difference. The remainder of the paper builds on this insight to construct

Figure 1.1: Identification of Surplus Generated by the Restrictive Covenant



a framework from which data on real-world bond refinancings can be used to estimate the surplus generated by restrictive covenants.

1.3 Data

This paper empirically analyzes inter-creditor agency problems using firms' revealed preferences about when they should refinance debt. To do this I assemble data from several sources. First, I use FISD's Mergent Corporate Bond Securities Database to construct a sample of corporate bonds. This dataset, subsequently referred to as Mergent, contains detailed data on the issuance, outstanding amount, and ratings of a large sample of corporate bond issues. Mergent provides no formal documentation of their data collection process, but conversations with the company and analysis of the data reveal that the coverage of corporate bonds is near universal after 1993 and very good for older bonds, but even better for bonds still outstanding as of 1993. Where available, the dataset contains bond ratings issued by the three major credit agencies, including ratings assigned at issue and subsequent ratings revisions throughout the life of the bond. I augment these ratings data with additional historical firm and bond ratings from Standard and Poor's S&P RatingsXpress data package.

Using the Mergent data I construct a sample of bonds that meet the following criteria: the bonds are dollar denominated, issued by a corporation or utility, are not convertible, exchangeable, or puttable, pay a fixed coupon, are publicly placed, are non-callable or are continuously callable and lack “make-whole” protections.⁶ I also require that I be able to match each bond to an initial credit rating, that the bond have a face value of at least \$5 million, and for callable bonds that they have data on the call price schedule for the bond. This results in a final sample of 10,886 callable and 7,224 non-callable corporate bonds. Table A3 tracks the application of these screens sequentially to the sizes of the two samples.

Summary statistics for the sample of callable corporate bonds are displayed in Table 1.2. These bonds are issued by 3,293 distinct firms. The average size of a bond issue in the sample is \$164 million and the largest bond has a face value of \$4 billion. The median bond maturity is 10 years. Nearly 60 percent of the bonds in the sample are speculative grade at origination, with the median S&P equivalent credit rating at issuance being BB, one notch below the highest speculative-grade rating, BB+. Utility companies are associated with roughly 30 percent of the sample of bonds.

For this sample of callable bonds I collect data on the bonds issuance characteristics, the terms at which the bond can be called, the ratings history of the bond, and any subsequent action of the firm that affects the amount of outstanding principal remaining of an issue. Such actions include early repayments due to exchanges, calls, and puts, as well as repayments at scheduled maturity, defaults, and restructurings.

I also use Mergent’s Bond Covenant file to study the prevalence of various covenants in corporate bond issues. This file tracks the presence of the most common restrictive covenants in a large subset of the bond’s in the Mergent database. Covenant coverage is unfortunately fairly crude and for the majority of the sample is simply an indicator of if a given bond does or does not contain certain common debt covenants. There is a substantially more limited set of detailed notes on specific covenant provisions, but I only use these data to develop further my qualitative understanding of

⁶Make-whole protections are a relatively recent innovation in the optionality of corporate bonds. Bonds with make whole provisions are called at the maximum of the stated call price and a price computed by discounting all future coupon and principal payments using a discount rate specified as a fixed nominal spread above a benchmark Treasury rate. Thus, make-whole protected bonds have effectively variable call prices that increase as when interest rates fall. This has the effect of substantially reducing a bond’s optionality on risk free rates. Since my identification comes in part from exploiting variation in option value arising from changes in interest rates, I ignore these bonds.

the typical nature of these covenants.

I also use secondary market pricing data from TRACE, Moody's Default and Recovery Database, and from Bank of America Merrill Lynch's Corporate Bond Index constituents. I use these data sources to construct estimates of the replacement cost of debt of each callable bond in my sample, and to provide auxiliary evidence on the relationship between restrictive covenants and loss given default.

1.4 Supporting Evidence

In this section I provide suggestive evidence that firm's decision to refinance debt is not only a function of interest rate savings, but also of changes in restrictive covenants such a refinancing would entail. The analysis is comprised of two distinct parts. First, I describe the range of covenants that appear in corporate bond indentures, and document that only the high-yield bond market includes restrictive covenants. Second, I show that the refinancing behavior of firms reveals the patterns we should expect if bond covenants ameliorate agency costs of debt.

1.4.1 Covenant Strength in Investment Grade and High Yield Bonds

In this section I describe the restrictive covenants most commonly found in corporate debt. I limit this analysis to my sample of callable and non-callable bonds described in Section 3.4 for which I have data on bond covenants. This results in sample of 16,730 bonds, of which 3,211 are issued by utility companies. Figure 1.6 plots, by initial credit rating, the fraction of bonds issued by non-utility firms that contain a given type of covenant.⁷ First, we can see that the presence of negative pledge or stronger covenants protecting the seniority of the bond issue is uniform across bond ratings. However, limiting the analysis to covenants restricting debt that are *stronger* than a negative pledge covenant, we see that this fraction jumps precisely at the high yield boundary (BB+), and nearly all high yield bonds contain such a covenant. Other covenants, including restrictions on non debt payments, restrictions on asset sales, and restrictions on transactions with

⁷Utilities issuers often have regulated capital structures and different covenants. I report results for utility issuers in Figure 1.7.

affiliates, show significant discontinuity around the investment grade boundary. One exception is the takeover protective covenant, which typically gives bondholders a right to demand immediate repayment upon a change of control event. These covenants are designed to protect against debt dilution, for example in the event of a leveraged buyout. Asquith and Wizman (1990) study the ex-post value of this particular covenant in the event of leveraged buyouts. It is not surprising that takeover protection covenants are included in investment grade bond issues, as leveraged buyouts are conducted precisely to significantly modify the capital structure of a firm and often result in a substantial credit downgrade. In general, a defining feature of a high-yield covenant package is the inclusion of restrictive covenants that impose limits on future debt issuance and investment and payout policy.

These patterns may seem striking in the graphical form I have presented them. The best explanation is that the investment-grade and high-yield bond markets are quite segregated and different standards emerged in the two markets that reflect average differences in credit risk across these markets. Market participants I spoke with emphasized that the high-yield and investment-grade markets are indeed very separate, stressing that underwriting in these markets is deals with different sets of investors and firms. Despite these markets being distinct, there is ample evidence, anecdotally and empirically, that firms do gain and lose access to different subsets of capital markets as their credit ratings change. Graham and Harvey (2001) report that firms are conscious of their debt rating and its impact on their cost of capital.

These results show that there is a stark difference between the strength of covenants offered across high-yield and investment-grade corporate bonds in the cross section. However, do the covenants available for refinancings to specific firms whose debt is upgraded or downgrade also change this way? I further confirm that these differences remain within the bond issues of a given firm that crosses the investment-grade/high-yield boundary.

To do this, I start with my sample of called corporate bonds and, when possible, match them to the refunding bonds issued to replace them. My criterion for this match is that the firm issues the replacement bond within one year it repays the old bond issue. Figure 1.8 shows the distribution of timing gaps between my matches of called and refunding bond issues. In a textbook bond refunding a firm will simultaneously issue its new bond and announce it will call its old bond in thirty to

sixty days. Thus, a majority of the timing gaps are in the one month range. However, if the firm has the financial flexibility to temporarily increase its debt, it is also possible to prepay earlier than the new bond is financed; for example by temporarily drawing down a credit facility, or issuing new debt first and calling the bond months later.

In each case, I indicate if the called bond was in a different rating class at the time of refinancing than it was at origination, either upgraded to investment grade (a rising star bond) or downgraded to high yield (a fallen angel bond). In this matched sample of bonds I collect information from Mergent, when it exists, on the covenants contained in each of the original and replacement bonds.

Table 1.4 reports average differences between covenants in the called and refunding bonds by the rating change status of the called bond. In the first column we can see that when fallen angel bonds are refunded, their replacement bonds contain on average 3.5 more restrictive covenants. Similarly, when rising star bonds are refunded, the new issue sheds 4.1 covenants on average. Also worth noting is that high-yield bonds that are not downgraded shed covenants upon refinancing, further support of the hypothesis that one motivation for refinancing is to shed restrictive covenants when possible. Which covenants are the most likely to be added or dropped? The remaining columns of Table 1.4 are averages of differences in indicators for the presence of specific covenants between the called and refunding issue. For example, we see that nearly three quarters of called rising star bonds shed covenants that limit debt issuance, specify restricted payments, or limit transactions with affiliates. The most commonly adopted covenant in fallen angel refundings is a change in control protection, but the use of all other common covenants increases markedly as well.

Finally, I report similar analysis in regression form that controls for other sources of variation that could explain the difference between the quality of covenants of called and refunding bond issues. Most importantly, on average the refunding issues in my sample are originated much later in time than the original called bonds, so it is possible that time series variation in corporate bond covenant standards could be explaining these results. I show this is not the case. Table 1.5 displays regressions of the change in the number of covenants between the original issue and the refunding issue as a function of ratings upgrade and downgrade indicators and various fixed effects. All specifications include fixed effects for the called bond's S&P credit rating at the time of refunding, which means the coefficients on ratings changes can be interpreted as relative to

refundings that occurred at the same rating but that had not experienced a major upgrade or downgrade. For example, in the first column, the coefficient on the Rising Star indicator of -5.0 indicates that on average rising star refundings shed five more restrictive covenants than did refundings of originally investment grade bonds with the same current rating. The second column adds fixed effects for issuer industry and the quarterly date of origination of both the called and refunding bond. The fact that the coefficients magnitudes fall slightly indicates that indeed some of the differences in covenants are attributable to time series variation in average covenant quality. However, the magnitudes remain large and highly statistically significant. The third column shows these patterns broken down by broad industry category of issuer. The categories Energy, Mining and Transit issuer as well as Retail, Service, and Leisure experience the largest increases and decreases in binding covenants when refinancing after a significant ratings change. Consistent with the graphical analysis of covenant inclusion, changes in covenants around the investment-grade/high-yield boundary for utility issuers are much smaller and not statistically significant.

Overall, these results support the notion that capital markets supply loose covenant restrictions to highly rated firms and strong restrictive covenants to poorly rated firms, and that these patterns also hold *within firm* as firms cross the investment-grade/high-yield boundary.

1.4.2 Covenants and Debt Refinancing

In this section I test the second basic prediction of the model in Section 1.2—if covenants help to solve agency problems but are imperfect in doing so, firms' decisions to refinance callable debt should reflect not only interest rate savings but also their valuation of shedding or avoiding restrictive covenants. Testing this prediction is complicated by the dynamic nature of the firm's decision to refinance long-term debt in the presence of stochastic borrowing costs. In the single period model, firms decide to refinance by weighing the one-period interest cost savings of refinancing against the cost or benefit in changes to restrictive debt covenants. In reality, the interest rate savings is represented by the value of option embedded in callable debt to repay the bond ahead of maturity. I now introduce a simple statistical model of the refunding decision of long-maturity debt under stochastic interest rates that ignores covenant considerations. I then estimate this model and show that there are observed systematic deviations from the model-implied optimal refinancing policy

that exactly coincide with a role for changing covenants in the refinancing decision.

A Dynamic Refunding Model Ignoring Covenants

Consider a callable bond with maturity τ_m , continuous coupon process $c(t)$, principal normalized to 1, and call price schedule $CP(t)$. To reduce notation define $CP(\tau_m) = 1$ to capture the notion that at maturity the bond pays back its principal. Assume the bond is subject to exogenous default intensity $h(t)$. In the event of default at t , debt holders recover a fraction of market value of the bond $(1 - L(t))$. The price (per unit of principal) of this callable risky bond can be expressed as:

$$P(t) = \min_{\tau < \tau_m} \mathbb{E}_t^Q \left[\int_t^\tau e^{-\int_t^s R_m dm} c(s) ds + e^{-\int_t^\tau R_m dm} CP(\tau) \right]$$

$$R(t) = r^f(t) + h(t)L(t)$$

where $R(t)$ denotes the “default adjusted” short rate process, which can be used to price defaultable claims as if they were risk free, as shown by Duffie and Singleton (1999). I assume the default adjusted short rate process follows a single-factor diffusion process under the risk neutral measure.

$$dR_t = \mu(R_t) dt + \sigma(R_t) dZ_t^Q$$

The single-factor specification of the default-adjusted short rate process implies that the optimal call policy can be expressed as an *exercise boundary* $b(t)$ in the state variable $R(t)$:

$$\tau_c = \min t \text{ s.t. } b(t) > R(t)$$

where the exercise boundary is a function of time to maturity, the coupon and call schedule of the bond, and the parameterization of the stochastic process for the default adjusted short rate. Assuming no role of covenants in the refunding decision, the process for R embeds the aspects of the firm’s cost of capital that are important for the refunding decision. When the short rate falls, the firm assigns a higher valuation to the stream of interest and principal payments, relative to the fixed call price schedule at which the firm can retire the bond.

Figure 1.4 plots the model-implied optimal call boundary for a bond in the sample. This solution

is calculated using a discretization of the model to the monthly frequency. It is only optimal to call the bond if the firm's default adjusted short rate is below the blue line. The fact that the call boundary is upward sloping simply reflects the fact that the time value of waiting for interest rates to fall further is diminishing as remaining maturity decreases. The level of the call boundary is decreasing in interest rate volatility, consistent with standard intuition about the value of call options.

Hypothesis Specification

I now use this model to test if covenants play a role in the refinancing decision as described in Section 1.2. Consider two firms that are identical in every way except one has debt with loose covenants and the other has debt with strict covenants. Assume that if either firm were to refinance its debt, the new debt issue would contain the strict covenants. If firms are averse to adopting restrictive covenants, the firm that currently has loose covenants will not be as willing to refinance as the firm that already has covenants. However, the refunding model presented above has no way to account for the difference between these two firms and would deliver identical optimal refunding decisions for each firm. Thus, a simple and direct test of the hypothesis that covenants play a role in the refunding decision is to see if this model's prediction of refunding decisions of fallen angel and rising star bonds are systematically biased relative to the model's refinancing predictions of other bonds.

Specifically, I will consider the following statistical model specification to evaluate the naive bond refunding model:

$$\begin{aligned}
 Call_{it} &= \mathbf{1} \{b_{it}^* > R_{it}\}, \\
 b_{it}^* &= \hat{b}_{it} + \delta^{FA} FA_{it} + \delta^{RS} RS_{it} + \beta X_{it} + \epsilon_{it} \\
 \epsilon_{it} &\sim N(0, \sigma^2)
 \end{aligned} \tag{1.2}$$

Where $Call_{it}$ is an indicator of if bond i is called at time t , \hat{b}_{it} is the refinancing boundary for bond i at time t implied by the dynamic refinancing model, b_{it}^* is the true policy followed by the firm, and R_{it} is an estimate of the default-adjusted short rate of bond i at time t . The difference between

b_{it}^* and \hat{b}_{it} models the deviations of firm's *actual* refinancing decisions from this model. It accounts for linearly additive misspecification in both the model implied boundary and the estimated default adjusted short rate. If firms are willing to forgo interest rate savings to avoid restrictive covenants then we should expect $\delta^{FA} < 0$, or that the model is over-estimating the refinancing boundary of fallen angel firms. Similarly, if firms are willing to give up option value of future interest rate savings to *shed* restrictive covenants, we should expect $\delta^{RS} > 0$.

Estimation Strategy

To implement this test, I need to solve the optimal refinancing model for each bond and assemble an empirical estimate of each bond's default adjusted short rate at each time the bond is callable. This requires estimation of the risk-neutral dynamics of firms' default adjusted short rates and a procedure for mapping firm and bond observables into default adjusted short rates. I provide a detailed explanation of this procedure in Appendix 1.A.4, but summarize it briefly here.

I first assume the default adjusted short rate follows a single factor affine diffusion process under the risk-neutral measure, parameterized by θ . This allows the model to be solved for optimal exercise boundaries, now expressed as $b_{it}(\theta)$. I assume the same process for the default adjusted short rate R also prices non-callable corporate bonds, and that the price of risk is also linear in this process. This allows me to exploit the tools of affine term structure modeling to estimate θ by maximum likelihood, as outlined in Singleton (2001) and implemented in Duffee (2002).

I next construct estimates of default adjusted cost of capital R_{it} . I first obtain estimates of default adjusted costs of capital for my panel of non-callable bonds at each point in time I observe a price for the bond. I then estimate a flexible parametric relationship between these observable default adjusted short rates and observable characteristics at the firm-bond-time level, and project this relationship onto the callable bond sample to obtain short rate estimates for each callable bond at each time the bond is callable.

Results

Table 1.6 reports the results of the estimation of Equation 1.2 under various specifications of the vector of control variables X_{it} . The sample is comprised of all bond-months for which the bond is outstanding and currently callable. Consistent with a role of covenants in the refinancing decision, fallen angel firms delay their refinancing and rising star firms accelerate refinancing relative to firms for which ratings have not changed materially. Column 1 reports baseline estimates in which X_{it} contains controls for the origination ratings class of the bond and the number of months since the bond was issued. By controlling for current rating fixed effects, the coefficients on the fallen-angel and rising-star dummies can be interpreted as systematic deviations in observed refinancing behavior from the model-implied refinancing boundary relative to any systematic deviations of other bonds in the same current rating category that have not been upgraded or downgraded significantly since origination.

Column 2 adds current rating fixed effects, and column 3 further adds industry fixed effects. The fixed effects specifications imply that fallen angel firms, relative to always junk firms, on average wait until their default adjusted short rate is on average roughly 48 basis points below the model implied boundary. In contrast, rising star firms exercise when the default adjusted short rate is around 58 basis points above investment grade firms that would not be shedding covenants in a refinancing. The final column adds financial ratios from Compustat which may contain omitted variation about credit risk that is related to the firm's decision to refinance. The inclusion of these controls does not reduce the magnitude or statistical significance of the rising star and fallen angel coefficients.

I also explore how the individual restrictive covenants at stake in a refinancing decision affects the call exercise boundary. Table 1.7 repeats the specifications of Table 1.6 limited to the subset of bonds for which I am able to match data on individual covenants included in the bonds. The first column of Table 1.7 repeats the baseline fixed effects specification (column 3 of Table 1.6) on the subset of bonds for which covenant data is available. The second column adds the number of restrictive covenants and its interaction with rising star and fallen angel indicators. The more covenants included in a rising star bond, the more eager the now-investment grade firm is to

refinance it, holding fixed the interest rate incentive to refinance. In fact, variation in the number of covenants of in the current rising star indenture explains the entire early propensity of rising star firms to refinance earlier than comparable firms that have bonds issued as investment grade. This further supports the notion that it is exactly changes in covenants that are driving the differences in refinancing patterns observed here. The interaction of the fallen angel indicator and number of covenants is not precisely estimated. This is due to the fact that there is very little variation in the covenants of bonds issued as investment grade.

The third column of Table 1.7 consider the effects of the presence of specific covenants. Because there is so little variation in the covenants of fallen-angel bonds I report the interactions only for rising star firms' incentives to refinance. The possibility of shedding restricted payments, cross acceleration, and minimum net worth covenants seem to be particularly valuable to firms, though only the effect of the cross acceleration covenant is statistically significant. These types of covenants directly prevent shareholders from being able to extract value from the firm in bad states. It is thus not surprising that shareholder's revealed valuation of this restriction is higher than that of other covenants, which impose less direct limits on equity value maximization.

Robustness

The previous analysis ignored the fact that firms can have multiple bond issues outstanding and thus refinancing an individual debt issue may not change the set of restrictive covenants binding on the firm as a whole. Specifically, a fallen-angel firm may have little incentive to delay refinancing one bond if it has previously issued other speculative-grade debt that imposed covenants on the firm. On the other hand, some restrictive covenants act to protect the individual, and their absence in a fallen angle issue may provide the firm financial flexibility it would lose in a refinancing. To explore this channel I augment the previous analysis with interactions of the fallen angel indicator and variables that measure if the firm has other debt that was issued as speculative grade. Table 1.8 presents this analysis. The first column replicates column 3 of Table 1.6 and each additional column estimates a different parameterization of the presence on the balance sheet of debt likely to contain covenants. Column 2 shows that a fallen angel bond issued by a firm that has subsequently issued high yield debt significantly reduces the delay in refinancing relative to firms with no subsequent

high-yield debt issues. However, firms with outstanding high yield debt still delay refinancing. Column 3 considers a continuous measure, the fraction of the firm's outstanding debt that was originated as high-yield. The interpretation is that if essentially all of a firm's outstanding debt has restrictive covenants, there is very little incentive for the firm to delay refinancing its fallen-angel bonds. Finally, column 4 considers a "slope-intercept" parameterization by including interactions with both the existence of any high-yield issued debt and the fraction of this debt in the firm's capital structure. Tables A1 and A2 add the interaction of the fallen angel indicator and the fraction of the firm's debt issued as high-yield to specifications of Tables 1.6 and 1.7 and confirm the qualitative findings are unchanged. I interpret these findings as a powerful verification that my main results are entirely driven by covenant considerations.

I also conduct a placebo test to consider the possibility that these results are spurious or driven by an omitted mechanism. The model specifications in Tables 1.6 and 1.7 all (correctly) assume that crossing the investment-grade/high-yield boundary induces differences in the covenants of the outstanding and potential refunding bond issue. As a placebo check, I instead estimate the model assuming this difference in covenants occurs at various different credit ratings. Figure 1.10 reports the log-likelihood of these estimated models as a function of the hypothetical investment-grade/high-yield boundary. The fact that the log likelihood peaks exactly at the true boundary between the investment grade and high-yield market, beyond which covenants first are included in bonds, provides further evidence that this model is capturing a real relationship between covenants and the observed refinancing behavior of bond issuers. A related interpretation of this exercise is that it is using the model and observed refinancing data to estimate where in the distribution of credit ratings covenants on new bond issues change substantially, under the assumption that covenants are an important aspect of the refinancing decision. If potential covenant changes did not affect the refinancing decision we would not expect an estimation procedure using data on bond refinancings to reveal the true difference credit rating boundary between loose and restrictive covenants.

Unfortunately, the results presented in this section do not *quantify* how much covenants help reduce agency costs of debt or the magnitude of the costs covenants would impose on firms that do not need them. In order to do this, I now turn to estimating a dynamic model that explicitly

considers the role of restrictive covenants.

1.5 Dynamic Model of Refinancing and Covenants

In the previous section, I document that refinance interest rate boundaries are *lower* for fallen-angel firms, which face new restrictive covenants when refinancing, and boundaries are *higher* for rising-star firms, which can shed covenants by refunding debt, relative to a model that ignores covenants. This implies that loose covenants transfer value from debt to equity claimants. However, this analysis does not quantify the extent of this shift in surplus nor the surplus created or destroyed by the inclusion of restrictive covenants. As shown in the static model of Section 1.2, to answer these questions we need to understand both how restrictive covenants shift these exercise boundaries and affect the value of debt. This requires a model of refinancing that explicitly considers how covenants resolve agency conflicts in a dynamic setting. I now introduce such a model.

1.5.1 Model Primitives

The model is in discrete time and continues indefinitely until the firm defaults at random default time τ^D . The firm's only debt is a callable coupon bond with face value K which pays a per-period coupon c . The firm has assets in place that generate per-period revenue $A_t = A(X, \gamma)$ for $t < \tau^D$. X is a state variable capturing the distribution of per-period cash-flows and their dynamics and $\gamma \in \{0, 1\}$ is an indicator of if the firm is subject to restrictive debt covenants. There are N non-default states and one default state, and the firm can transition between non-default states and into the default state. I model the flow value of cash-flows to the non-defaulted firm as:

$$A(X_t, \gamma_t) = a(X_t) + f(X_t) \times \mathbf{1}\{\gamma_t = 0\}.$$

Here $a()$ and $f()$ are general functions of the state variable X . The firm only receives $f(X)$ if it is not subject to restrictive covenants. This term captures the state-dependent increase in asset returns allowed by not having restrictive covenants, conditional on not defaulting.

The covenant status of the firm also affects the firm's probability of default, because loose

covenants allow the firm to engage in projects with higher risk. The risk-neutral probability of defaulting in period t (conditional prior survival), $p_t^D = p^D(X, \gamma)$, is given by:

$$p^D(X_t, \gamma_t) = p(X_t) + s(X_t) \times \mathbf{1}\{\gamma = 0\}.$$

In the event of default, the firm permanently ceases debt payments and receives no future returns from assets. If the firm is not in default it pays per-period claims to debt-holders $d_t = d(\tau^m, c, \phi)$, where τ^m is the time to maturity of the firm's single debt issue, c is the per-period coupon, and $\phi \in \{0, 1\}$ is the firm's decision to call its outstanding debt. The required payment is either the coupon on its outstanding debt, the principal and final coupon at maturity, or the coupon and early prepayment of the debt if the firm exercises its option to call the bond. The debt service in each period is thus given by

$$d(\tau, c, a) = \begin{cases} cK & \phi = 0 \\ (1 + CP(\tau) + c)K & \phi = 1, \end{cases}$$

where the call premium due at maturity is always zero. In the case the outstanding bond is retired (either at maturity or due to the firm exercising its call option on the bond), the firm issues new debt with coupon $c'(r, x)$ set so that the new bond is offered at par and principal amount to finance the replacement cost of the previous bond.

The state variable capturing the firm's inherent risk X_t evolves as a Markov process with transition matrix Π specified under the risk neutral measure. Following the results of Proposition 1.1 and the empirical evidence in Section 1.4.1, the new debt contains covenants if and only if the firm's credit rating is below investment grade. The replacement bond has a maturity $\bar{\tau}^m(X_t)$ and call premium schedule $CP(\tau)$ that are set exogenously.

The risk-free short rate given by r_t and is assumed to evolve as an affine diffusion process under the risk-neutral measure. The firm is assumed to be incentivized to maximize the present value of equity claims. The only control variable of the firm in this model is the decision to refinance its debt each period. There are two reasons a firm would want to refinance: to lower its debt servicing costs

or to change the covenants imposed on the firm, which changes the distribution of asset returns, as well as the decomposition of expected asset returns between equity and debt. Given the primitives of the model the objective of the firm is to choose its refinancing to maximize the present value of equity. The equity value maximization problem can be written as:

$$V_t^E(r_t, X_t, \gamma, \tau^m) = \max_{\{\phi_s\}_{s>t}} \mathbb{E}_t^Q \left[\sum_{s=t}^{\tau^D} e^{-\int_t^s r_u du} [A(X_s, \gamma_s) - d(\tau_s^m, c_s, a_s)] \right] \quad (1.3)$$

s.t.

$$\tau_{s+1}^m = \begin{cases} (\tau_s^m - 1) & \phi_s = 0 \\ \bar{\tau}^m(p) & \phi_s = 1 \end{cases} \quad (1.4)$$

$$\gamma_{s+1} = \begin{cases} \gamma_s & \phi_s = 0 \\ \mathbf{1}(X_s \leq \bar{X}^{HY}) & \phi_s = 1 \end{cases} \quad (1.5)$$

$$c_{s+1} = \begin{cases} c_s & \phi_s = 0 \\ c'(r, p) & \phi_s = 1 \end{cases} \quad (1.6)$$

$$\tau_s = 0 \implies \phi_s = 1 \quad (1.7)$$

$$\Pr(\tau^D = s | \tau^D > s - 1) = p^D(X_t, \gamma_t). \quad (1.8)$$

Equation 1.3 specifies the state-dependent value of equity, and Equations 1.4-1.8 specify the constraints on the evolution of the endogenous state variables. Specifically, remaining maturity decreases until the bond is replaced, at which point it resets to the exogenously specified level. Covenants on debt do not change until debt is refinanced, in which case they are present only if the firm has a below investment-grade rating. To reduce notation, I specify that the bond is semantically refinanced ($\phi = 1$) upon maturity if it has not been refinanced already. Given an optimal refinancing strategy $\{\phi_t^*\}$, the value of the callable bond can be expressed as

$$P_t^D(r_t, X_t, \gamma, \tau^m) = \mathbb{E}_t^Q \left[\sum_{s=t}^{\bar{\tau}^m} e^{-\int_t^s r_u du} d(\tau_s^m, c, \phi_s^*) \right],$$

$$\bar{\tau}^m = \min(\tau^m, \tau^D).$$

It is also useful to develop an expression for the agency costs of debt solved by covenants in this model. A natural expression for this is simply the difference between actual firm asset value and the counterfactual asset value assuming it is never possible to use debt covenants. Denote this quantity as the asset value of covenants (AVC):

$$AVC(r_t, X_t) \equiv \mathbb{E}_t^Q \left[\sum_{s=t}^{\tau^D} e^{-\int_t^s r_u du} A(X_s, \gamma_t) \right] - \mathbb{E}_t^Q \left[\sum_{s=t}^{\tau_{\gamma=0}^D} e^{-\int_t^s r_u du} A(X_s, \gamma = 0) \right], \quad (1.9)$$

where $\tau_{\gamma=0}^D$ signifies that the random default time is given the firm never has restrictive covenants.

1.5.2 Model Discussion

The goal of this model is to capture the relationship between covenants, the distribution of asset returns, and optimal debt refinancing in a dynamic infinite horizon setting with as minimal complexity as possible. The model assumes a very simple parameterization of the distribution of asset returns: each period the firm either survives and generates revenue or it defaults permanently. The flow revenue and default probability are functions of two state variables: one capturing the inherent fundamentals of the firm and the other indicating if the firm is subject to restrictive covenants.

This model can be thought of as a generalization and multi-period extension to the static model presented in Section 1.2. In both models the distribution of firm cash-flows depend on the level of inherent risk in the firm and if the firm has restrictive debt covenants. The difference between these models is that here the periods are dynamically linked: by the fact that default affects all future cash-flows and because the firm's debt has a maturity longer than one period. These dynamics are essential to exploiting the variation in the data that credibly identifies the effect of covenants on the agency cost of debt.

It is important to note how the model is scaled: it is expressed in terms of \$1 of par value of debt. For every dollar of debt a firm has, the firm generates $A(X_t, \gamma)$ in cash-flow. Thus, the model objects' relationships with the state variable X_t capture the reduced-form relationship between earnings, leverage, maturity, and the probability of default. These features of the firm

are not modeled explicitly, but rather taken as given to model the refinancing decision of a firm.⁸ Further, note that the model does not explicitly distinguish between probability of default and loss given default. Instead, the state transition process and probability of default parameterization $p^D(\cdot)$ capture the stochastic process for the continuation value of servicing a bond. Further decomposition between probability of default and loss given default is neither identified nor relevant in this framework. Thus the estimated default probabilities should be interpreted as risk-neutral probabilities of default *as if* there is no recovery value of the bond in default.

My modeling choices necessitate a comparison to those more common in the dynamic capital structure literature, for example Strebulaev (2007), Hennessy and Whited (2005), DeAngelo, DeAngelo, and Whited (2011), and Morellec, Nikolov, and Schurhoff (2012).⁹ These models typically introduce various trade-offs of capital structure considerations in a dynamic setting and are calibrated or estimated to match empirical moments of firm capital structure, such as leverage ratios and their dynamics. The ability or inability of these models to fit aggregate moments is then used as support for or against their underlying economic mechanisms. In contrast, many (but not all) elements of my model are statistical and not structural in nature, and I use the model to estimate parameters that are reduced-form representations of objects of real world importance. Regarding model structure, I do not endogenize capital structure elements such as leverage, debt maturity, and their dynamics, or even consider a framework in which a certain capital structure is optimal. In theory the decision to perform a simple debt refunding is isolated from other aspects of firm capital structure. Thus, for the purpose of identifying how covenants shift the distribution of asset value between claimants, it is sufficient to treat the other decisions of the firm as exogenous and to let the exogenous state variables capture the implications of these decisions in reduced form. Further, I do not attempt to endogenize other aspects of capital structure because the variation I will exploit to identify my model only intuitively identifies how covenants change the distribution of asset returns.

⁸See Billett, King, and Mauer (2007) for a comprehensive overview of the empirical relationship between corporate leverage, maturity, and covenant usage.

⁹See Strebulaev and Whited (2012) for an excellent review of dynamic models and structure estimation in corporate finance.

1.5.3 Simplifying Assumptions

The above model can be solved numerically by value function iteration to a fixed point on the state variables (r, X, γ, τ) . Since the ultimate goal is to estimate key parameters of this model from data by applying it to each bond in my sample at each time the bond is callable, solving the model by fixed-point iteration on four state variables is computationally infeasible. Instead, I now introduce two assumptions that greatly reduce the complexity of solving the model but impose minimal loss of flexibility or realism.

Assumption 1.5.1. Any subsequent replacement bond issues after the retirement of the first bond are non-callable.

Assumption 1.5.2. The replacement bond issues are fairly priced. The firm and capital markets agree on the valuation of promised coupon and principal payments.

Assumption 1.5.3. The risk-free short rate evolves as a single-factor affine diffusion process.

Assumptions 1.5.1 and 1.5.2 together allow for the model to be solved by a combination of value function iteration and backward induction of smaller two-state variable problems. If replacement debt issues are priced fairly, then the cash-flow the firm receives from issuing the new bond is exactly equal to the firm's valuation of the principal and interest payments, and we can thus these terms cancel out beyond the initial refinancing. Also note that *some* assumption on the pricing of future debt needs to be made in order to estimate this forward looking dynamic model, and this particular assumption is both natural and convenient.

However, because covenants affect the equity value of the firm, Assumption 1.5.1 alone does not ensure the model can be solved by backward induction from maturity: we still need to know the continuation value of equity associated with the flexibility of covenants imposed in the future by all subsequent bond issues. If the replacement debt is callable, this involves pricing the option value of covenant flexibility afforded by the refinancing option embedded in all *future* bond issues. Specifying replacement debt as non-callable allows one to solve for the continuation value due to covenants in a straightforward fashion without explicitly modeling these dynamics. I do however, incorporate the realistic feature that speculative grade firms extend debt maturity in refinance while speculative grade firms do not alter maturity in refinance, as documented by Xu (2016).

Fortunately, the implications of this simplifying assumption are minor. Related to the intuition behind the results in Dunn and Spatt (2005), when debt can be refinanced more than once, there is a limited sensitivity of the contemporaneous refinancing decision to properties of subsequent refinancing options when the subsequent refinancing options are being priced into the replacement debt issue.

Assumption 1.5.3 is not strictly necessary, but drastically simplifies the model. By assuming the term structure of risk free rates is captured by a single factor, the firm's refinance decision can be expressed as an *exercise boundary* in the current level of the short rate for a given value of the other state variables (credit rating, remaining maturity, and covenant status). A more realistic model would allow a multi-factor term structure to match the term structure of interest rates, but in such a model optimal call policies would instead be an exercise *surface* in the interest rate factors and introduce additional state variables to the model. As mentioned previously, my model is already innovative in extending dynamic models of firm decisions to account for stochastic interest rates. Expanding dynamic corporate finance models to further capture the term structure of rates is an important direction for continued work in this area. For this paper, however, I assume the short rate itself is an affine diffusion process and evolves under the risk-neutral measure as a single factor Ornstein-Uhlenbeck process:

$$dr_t = \kappa (\bar{r} - r_t) dt + \sigma dZ_t^Q.$$

1.5.4 Solution

I now describe how the model is solved for each bond to determine the bond's optimal refinancing policy $\phi(\tau, r, X, \gamma)$ and price $P(\tau, r, X, \gamma)$ given the parameters of the model.

Given a parameterization, the first step in solving the model is to solve for the continuation value of assets after the initial refinancing. Given a solution value function, the optimal exercise policy is solved by backward induction from all states in which the bond matures, defaults, or is refinanced. This begins at the month of bond maturity, and proceeds backward each month to the first month the bond is callable. At each time step, the model produces functions for the value of the equity and debt, as well as the optimal refinancing policy as a function of the state variables.

For a given remaining maturity, covenant status, and credit rating, the optimal refinancing policy takes the form of an *exercise boundary* in the risk-free short rate:

$$b(\tau, X, \gamma) \equiv \max r \text{ st } \{a(\tau, r, X, \gamma) = 1\},$$

$$\phi(\tau, r, X, \gamma) = 1 \iff r \leq b(\tau, X, \gamma)$$

1.6 Structural Estimation of Covenant Value

1.6.1 Model Parameterization

I now specify the exact parameterization of the model of Section 1.5 that I will bring to the data. In the general model the state space and number of parameters are both potentially large. To simplify estimation and to ensure the model is identified, I introduce several parametric assumptions.

Empirically, I proxy for the state variable X with bond credit ratings. As shown in Table 1.14, covenant inclusion predicts credit spreads *within* credit rating, suggesting credit rating is an appropriate statistic for the firm's opportunity cost of capital. First, I assume the state variable X takes one of eight discrete values $X \in \{1, 2, \dots, 8\}$ where 1 is the highest rating. The transition matrix Π describes the evolution between the eight non-default states of X . I further assume that the firm can only transition into default from the worst state. Thus, the probability of default specification of the model is

$$p^D(X, \gamma) = \begin{cases} 0 & X \leq 7 \\ p + s \times \mathbf{1}\{\gamma = 1\} & X = 8 \end{cases}$$

Let \bar{X}^{HY} denote the cutoff credit rating at and below which firms receive covenants on new bond issues. I parameterize the relationship between covenants and non-default asset returns by assuming this value is only a function of if the firm's current credit rating is above or below this

credit rating cutoff:

$$f(X) = \begin{cases} f^B & X \leq \bar{X}^{HY} \\ f^G & X > \bar{X}^{HY} \end{cases}$$

Thus, firms generate additional asset return f^G when they are investment-grade and have no covenants, and f^B when they are speculative grade but have no covenants. I further assume that conditional on survival, the level of asset returns each period *without* debt covenants is a fixed constant:

$$a(X) = \bar{a}.$$

As previously specified, the risk free short rate is assumed to follow a single-factor Ornstein-Uhlenbeck process with parameters $(\bar{r}, \kappa, \sigma)$.

I have now described all unknown parameters in the model. Given properties of the callable coupon bond (its coupon, maturity, and call price schedule), the parameter vector θ completely describes the model:

$$\theta = (\bar{r}, \kappa, \sigma, \Pi, f^G, f^B, \bar{a}, p, s)$$

1.6.2 Calibrated Parameters

I calibrate the parameters of the model that are not intuitively identified by the model: those governing the dynamics of the exogenous state variables of the model, and the average asset returns per dollar of debt. For the benchmark specification the parameter \bar{a} is calibrated the sample median ratio of monthly *EBITDA* to long term debt, which is 0.025.

The risk free rate variable takes the form of a discretized Vasicek (1977) short rate process. I use 75 grid points spaced between 0 and 25 percentage points and fit the parameters to the monthly innovations of the three-month Treasury rate over the period 1970-2016. The estimated parameters (expressed in annualized units) are

$$\bar{r} = 0.035, \kappa = 0.12, \sigma^2 = 0.00028.$$

I calibrate the credit rating transition density using external data. The primary purpose of this is to keep the number of estimated parameters reasonable and ensure the estimated parameters are identified in the model. I map the eight levels of X in the model to observed S&P equivalent long term bond ratings according to the concordance in Table A4. There are four investment grade ratings and four speculative grade ratings. I use annual ratings transitions probabilities reported by S&P (1995) and estimate a generator matrix using the method described by Israel, Rosenthal, and Wei (2001) to create a monthly transition matrix. Table A5 reports the original data, estimated generator, and monthly transition probabilities.

1.6.3 Model Estimation

Given a calibration of $(\Pi, \bar{r}, \sigma, \kappa)$ we can now proceed to estimate (f^G, f^B, p, s) by matching the model implied bond prices and refinancing decisions to their observable counterparts. I estimate the above model using a maximum likelihood estimation strategy.

Recall that for given values of known and unknown parameters, and for a given realization of the state variables describing the firm's credit risk, the model solution is an exercise boundary for the risk free short rate. The firm should initiate the repayment of its bond only if the short rate is below this state contingent boundary. To formulate a likelihood function that identifies the parameters, I assume there is a source of unmeasured error in this exercise boundary that is not captured by the model. The true boundary is:

$$b^*(X_{it}, \tau, \theta) = b(X_{it}, \tau, \theta) + \epsilon_{it}^b.$$

Similarly, the true value of the callable bond i at time t is

$$P^*(r_t, X_{it}, \gamma, \tau^m) = P(r_t, X_{it}, \gamma, \tau^m) + \epsilon_{it}^p$$

Where the measurement error vector $(\epsilon_{it}^b, \epsilon_{it}^p)$ is assumed to be jointly normally distributed with mean zero and covariance matrix Σ_ϵ . Recall ϕ_{it} is an indicator of if bond i is actually refinanced

at time t . Then the model implies

$$\begin{aligned}\Pr(\phi_{it}^* = 1) &= \Pr(r_t - \epsilon_{it}^b < b_{it}(\theta)) \\ \Pr(\phi_{it}^* = 0) &= \Pr(r_t - \epsilon_{it}^b > b_{it}(\theta))\end{aligned}$$

Now consider the likelihood of the model implied bond price matching the data

$$\Pr(P_{it}^*) = \Pr(\epsilon_{it}^p = P_{it}^* - P_{it}(\theta))$$

Let ϕ_{it}^* be an indicator for if bond i is actually called at time t . And for brevity let $b_{it}(\theta)$ denote the model implied call boundary with state dependency implicit in the subscripts. The likelihood of an individual observation is:

$$\mathcal{L}_{it} = \Pr(\epsilon_{it}^b > r_t - b_{it}(\theta) \ \& \ \epsilon_{it}^p = P_{it}^* - P_{it}(\theta))^{\phi_{it}^*} \Pr(\epsilon_{it}^b < r_t - b_{it}(\theta) \ \& \ \epsilon_{it}^p = P_{it}^* - P_{it}(\theta))^{1-\phi_{it}^*}$$

Because the errors are assumed independent across observations we can write the log likelihood function of all the data as

$$\ell(\theta, \Sigma_\epsilon) = \sum_{i,t} \omega_{it} \log(\mathcal{L}_{it}) \tag{1.10}$$

The weights ω_i are constructed to ensure each bond receives the same influence in the likelihood function no matter how long the particular bond was callable in the data.

Likelihood evaluation is performed in a two step procedure. In the inner step, given a candidate value of θ the model produces policy functions of $b_{it}(\theta)$ and prices $P_{it}(\theta)$ that are not a function of Σ_ϵ .¹⁰ The likelihood function can thus be quickly maximized over Σ_ϵ in the inner step without recomputing the optimal bond policies, giving $\hat{\Sigma}_\epsilon(\theta)$. In the outer step, the likelihood function is simply maximized over parameters to be estimated in θ . Standard errors of the maximum likelihood estimates are calculated by evaluating the Hessian matrix of the likelihood function at the maximum likelihood estimates.

¹⁰Embedding structural errors into the nonlinear bond pricing and refinancing model would improve the model's ability to deal with selection, but comes at the cost of greatly increased computational complexity.

1.6.4 Identification of Estimated Parameters

The estimated parameters f^B , f^G , p and s are identified by reconciling observed values of debt and observed refunding decisions with their model-implied counterparts. The most intuitive source of variation comes from comparing the data to the model across the four types of refinancing classified by the covenant transition that a refinancing would induce: fallen-angel refincings *add* restrictive covenants, always-junk refincings *maintain* restrictive covenants, rising star refincings *remove* covenants, and always investment-grade refincings *avoid* covenants.

Figure 1.5 plots comparative statics of the model for a representative bond issue at a given point in time. The top left panel shows the optimal exercise boundary for refinancing in each of the four categories as a function of the parameter f^B . The refinancing boundary gap between always-junk and fallen-angel refincings is increasing in f^B , consistent with the reduced-form evidence in Section 1.4 that firms which will adopt covenants in a refinancing delay refinancing relative to firms that will maintain restrictive covenants in a refinancing.

Similarly, the top right figure shows that higher values of f^G increase the gap between the refinancing decision of rising star and always investment grade bonds: the restrictive covenants limit flexibility the more eager rising-star firms are to refinance and shed covenants for a given level of interest rate savings.

Increasing f^G also causes the optimal refinance boundaries of junk-rated bonds to decline, though in parallel to each other, and reflects a precautionary motive to locking in restrictive covenants when they would be harmful if the firm were to be upgraded to investment-grade. This sensitivity is a result of the simplifying assumption that replacement debt issues are non-callable and of a long maturity.

The middle panels of Figure 1.5 plot refinancing boundaries as a function of the parameters p and s . The bottom panels show the corresponding comparative statics for the price of the bond. Bond prices are averaged over all grid points of the state variable capturing the risk-free short rate. Higher values of p decrease call exercise boundaries for poorly-rated firms because a higher probability of default reduces the actual value of debt liabilities relative to the pre-specified call price. But unlike the effect of f^B on refinancing boundaries, the effect of p is the same for both junk-

rated debt with and without covenants. We can also see that the price of junk debt is decreasing in the probability of default parameter p . The parameter s captures the differential probability of default for firms without covenants. The fact that the refinancing boundary for fallen angel debt is increasing in s reflects the fact that as s increases and holding f^B fixed the surplus generated by covenants rises, and for a fixed price to refinance debt the firm is able to capture more of the surplus associated with covenants.

Again note that these figures describe the intuitive moments being matched by the model. These comparative statics are presented for specific bond coupon, remaining maturity, and call price schedule. Each observation used to estimate the model will generate a slightly different version of these figures. For example, for fixed unknown parameters, the gap between refinancing boundaries of fallen angel and always-junk bonds will be larger the longer the remaining maturity of the bond. These features vary in a non-linear way that reflects the optionality of the refinance decision. An alternative way to estimate the model would be to collapse this non-linearity and match aggregate moments of the model to the data, for example average refinancing boundary gaps or differential sensitivities of bond prices to interest rates between fallen angel and always-junk or between rising star and always investment-grade bonds. Instead, my maximum likelihood estimation strategy explicitly accounts for variation in these observed moments at the observation level.

1.6.5 Estimation Results

I begin by estimating the baseline model for my full sample of callable corporate bonds not issued by utility companies. As described in Section 3.4, this sample contains 4,185 callable corporate bonds for which I observe information on the key state variables of the model—the risk free rate and the bond’s credit rating—during the period in which the bond can be called. This sample comprises 289,794 bond-month observations, 23,233 of which I have data on the bond’s secondary market price that month.

The results of the estimation are displayed in Table 1.9. The parameter estimates are multiplied by a scaling factor of 100 to aide readability. Estimates of f^B and s loosely capture how covenants change in the distribution of asset returns for speculative-grade issuers. Firms in this category

without covenants have a higher per-period cash-flow return, but also a higher probability of default. The non-scaled estimate of f^B of 0.00105 is approximately four percent of the calibrated value of \bar{a} . The trade-off is that loose covenants increase the probability of default (conditional on being in the worst rating category) by s , which is estimated to be 64 basis points. The estimated value of f^G implies that investment-grade firms subject to debt covenants earn about eight percent lower asset returns than similar firms not burdened by covenants. The estimate of p of 356 basis points is the monthly risk-neutral implied probability of default (assuming zero recovery) of a firm in the worst credit rating *with* restrictive covenants. Again, the model estimation implies that this probability of default rises by 64 basis points for firm with comparable intrinsic risk but without debt covenants. This parameter is identified by the difference in prices of callable bonds with and without restrictive covenants, accounting for differences in optimal exercise policy of the bonds' embedded call options and how this exercise policy is affected by covenants.

These parameters should not be interpreted literally, but rather as what bond prices and firm decisions about refinancing callable bonds would imply if this were the true model of asset returns. The cash-flow process in the model is overly simplistic but provides a low-dimensional parameterization of how covenants change total surplus and the allocation of surplus between debt and equity for a given capital structure.

A better way to interpret the model estimates is to recast them as estimates of the agency costs of debt solved by covenants. To do this, I use the model to calculate the enterprise value of firms with the same state variable X_t but with and without restrictive covenants, as specified in Equation 1.9. I compare percent differences in value of investment grade and speculative grade firms with and without covenants, averaged across the remaining state variables of the model. These results are presented in Table 1.10. The main sample estimate reveals that the average speculative grade firm would be worth 2.4% less if it did not have debt covenants. This quantity reflects my model's estimate of the agency costs of debt solved by a typical high-yield covenants package. The 95% confidence interval of this estimate is [1.65, 3.14], implying the model quite widely rejects the neutral mutations hypothesis that covenants add positive but only quantitatively small value.

The model also reveals that imposing restrictive covenants on investment grade firms (which as an empirical regularity do not receive restrictive covenants in new debt issuance) would *decrease*

firm value by 1.3%. I also interpret this number as quantitatively large. My estimates of the value of covenants for high-yield and investment grade bonds thus together reconcile why strict covenants appear in only riskier debt issues. These covenants reduce agency costs for highly levered firms but inefficiently limit flexibility of firms not prone to agency conflicts. For low-risk firms, the benefits of typical high-yield covenants in reducing the firm's cost of capital are outweighed by the restrictions on investment activity they impose. For higher risk firms, the commitment induced by covenants *ex-ante* allows investors to demand a lower cost of capital that leverages the

It is useful to compare the estimate of the value of covenants to other estimates in the literature of the costs of financial distress and the value of debt. While my paper does not explicitly model the determinants of firm capital structure, its estimates are still useful for thinking about the cost benefit analysis in a standard trade-off theory. Korteweg (2010) and Van Binsbergen, Graham, and Yang (2010) estimate the net benefit of debt in firm capital structure to be 5.5% and 3.5% of enterprise value, respectively. Considered jointly with my estimate of the agency cost of covenants, this implies that in the absence of covenants the net value of debt would be significantly reduced. Assuming these figures are representative of the typical speculative-grade firm, my baseline estimates imply that 40 to 70 percent of the net benefits of debt are attributable to covenants. Thus, restrictive debt covenants are not only quantitatively important for firm asset value, but essential in allowing the typical firm from even being able to benefit from debt in its capital structure. Stated differently, these estimates together imply that the *existence* of risky high-leverage capital structures would not be possible without debt covenants.

1.6.6 Heterogeneity

The model estimated in Section 1.6.5 is sparsely parameterized. This raises the possibility that results may be biased due to selection and heterogeneity of the value of covenants in the sample. To partially address this, I estimate the model on various subsets of the sample of bond issues. The results are reported in Table 1.11. To streamline exposition I directly report in this table the implication of the model estimates for the value of strict covenants (as a function of firm enterprise value). First I report results for the subset of bonds for which I was able to match to issuer data in Compustat, which results in a slightly higher value of covenants for speculative

firms. Because Compustat matches are likely to be larger, this suggests that restrictive covenants are more valuable for larger firms. Indeed, I confirm this is true within the Compustat sample by splitting firms based on the median asset value of the issuer at bond issuance. The value of covenants for risky firms estimated in the large firm sample is 4.3% while the estimate from the smaller firm sample is estimated to be near zero and not statistically significant.

I also explore splitting the sample by the issuer's book-to-market ratio at origination. Interestingly, growth firms seem to benefit more from restrictive covenants than do value firms. This is consistent with Billett, King, and Mauer (2007), which finds that firms with growth opportunities take more bond covenants and that covenants help growth firms use leverage profitably. Again, my estimation contributes to this finding by quantifying the importance of debt covenants. My results suggest restrictive debt covenants create significant value relative to a similarly leveraged firm without them.

Finally, I split my sample into broad industry categories, based on the designations in the Mergent issuer data. Manufacturing firms, as well as firms in Retail, Services, and Leisure seem to benefit significantly from restrictive covenants relative to other firms. In contrast, media and communications issuers refinancing behavior and bond prices suggest there is little if any gain from restrictive covenants. Even the value of covenants to investment-grade issuers is estimated to be significantly more negative for media and communications issuers than firms in other industries.

1.6.7 Robustness

I also consider two alternative parameterizations of the structural model to investigate whether the chosen calibration is significantly influencing the results. First, instead of using the physical probability ratings transition density matrix from S&P, I consider a risk-neutral version derived in from Lando (2004, pg. 154). I chose to use the physical matrix instead because the methodologies for computing risk-neutral transition density matrices are relatively sensitive to assumptions and input data. Column 2 of Table 1.12 reports the estimates of the model with this alternative transition matrix. The parameters are relatively unchanged, most notably the estimated baseline probability of default and increase in probability of default due to lack of covenants decrease. The

estimated value of f^G also increases by approximately twenty percent. These changes result in somewhat larger magnitudes of the estimates of the value of covenants to investment grade and speculative grade firms. Next, I consider allowing more flexibility in the parameterization of a , the baseline ratio of firm cash flows to debt. Instead of calibrating this as a fixed parameter, I allow it to take a different value for each value of X . I calibrate these as the median ratio of monthly EBITDA to long term debt in each of the eight model-mapped credit ratings. This has an even smaller effect on the resulting estimation than the choice of transition matrix.

1.7 Conclusion

This paper is the first in the literature to structurally estimate a dynamic model to quantify the value of restrictive bond covenants. To do this, I abstract from other capital structure considerations of the firm and exploit an intuitive and clean trade-off between what I have shown are the two first order determinants of their decision to refinance debt: resulting changes in restrictive covenants imposed on the firm and the interest rate savings obtained through refinancing. When firms will face substantially tighter covenants, as proxied for by a potential refinancing of fallen-angel bonds, firms are willing to forgo substantial interest rate savings relative to firms refinancing debt originated as speculative grade into new speculative grade issues.

In my dynamic model this differential sensitivity to interest rate savings identifies the value of covenants in ameliorating agency costs of debt. The intuition behind this identification is that debt covenants are explicitly designed to protect the ability of firms to service debt when they are relatively close to default, at the expense of potentially preventing the firm from taking good investment opportunities. The absence of covenants close to default thus lowers recovery values and decreasing the firm's perceived value of its debt liabilities, which makes potential interest rate savings associated with refinancing less attractive. The more successful covenants are in reducing agency costs of debt, the lower the value of the debt of a firm without covenants in the same position as a firm with covenants, and the higher the difference is between the actual value of debt and the cost of refinancing the bond. To the extent that covenants would increase surplus, the higher the gap between the value of debt and the cost of repaying the debt, the less of this increase

in surplus would accrue to the equity claimants of the firm, and the less likely they are to refinance. This explains the observed gap in refinancing behavior between fallen angel and always junk bonds documented in Section 1.4. By instead holding the refinancing cost gap across firms with and without covenants fixed the correlation between the size of this gap and the differential propensity of firms with and without covenants to refinance into debt with covenants identifies the surplus that would be generated by covenants. A similar argument holds for the refinancing decision of firms able to shed covenants relative to that of firms that do not and will not have covenants after a refinancing.

I use a dynamic model that embeds the pricing of fixed income derivatives in a stochastic interest rate environment to collapse the nonlinear identifying variation described above into intuitive and interpretable quantities that are directly observable in the data—the decision to refinance debt and the market price of this debt. By explicitly considering how covenants affect the distribution of firm cash-flows in this model of optimal refinancing I am able to recover quantitative estimates of the costs and benefits of covenants.

My findings suggest that typical high-yield restrictive debt covenants add significant value to risky firms. In fact, because the value of covenants implied with my model is comparable to estimates in the literature of the net benefits of debt itself, I conclude that restrictive covenants are absolutely essential for allowing the high leverage capital structure to generate value. This finding is consistent with the rapid rise and collapse of the original junk bond market in the 1980s, in which many of these bonds lacked restrictive covenants but were part of highly leveraged capital structures.

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Figures and Tables

Figure 1.2: Corporate Bond Yields

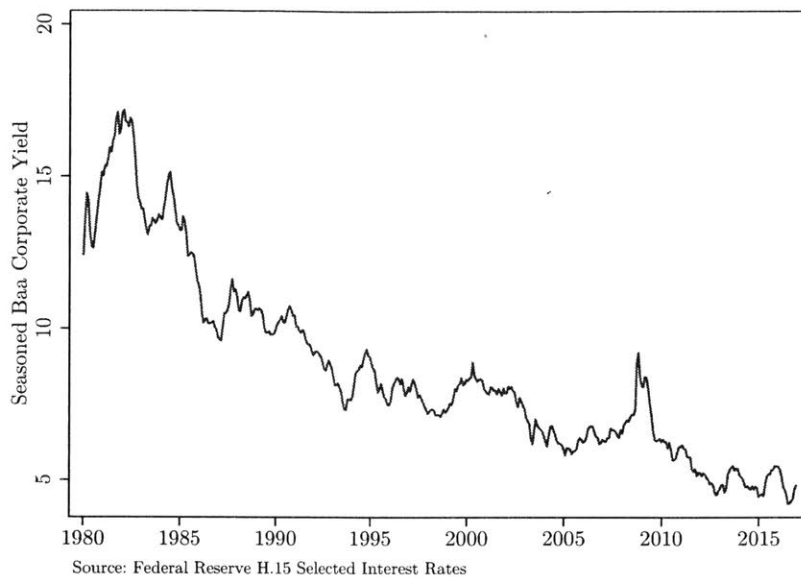
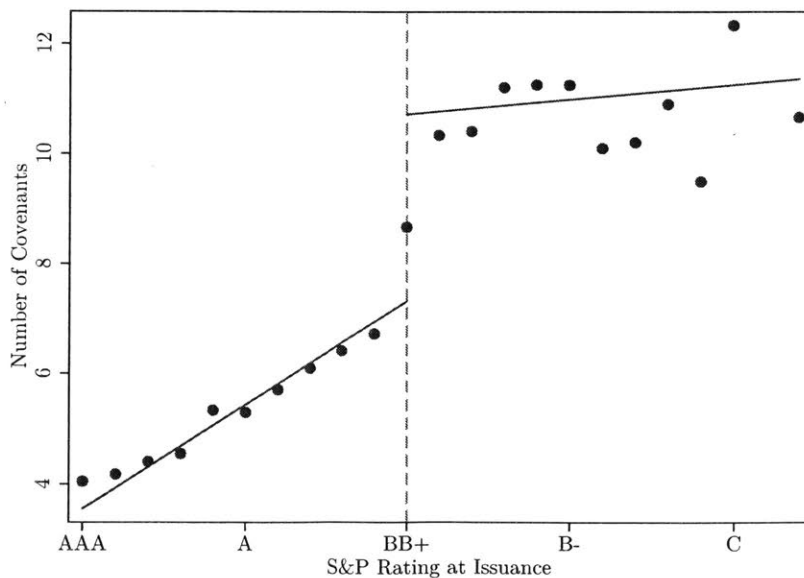


Figure 1.3: Corporate Bond Covenants and Credit Rating at Issuance



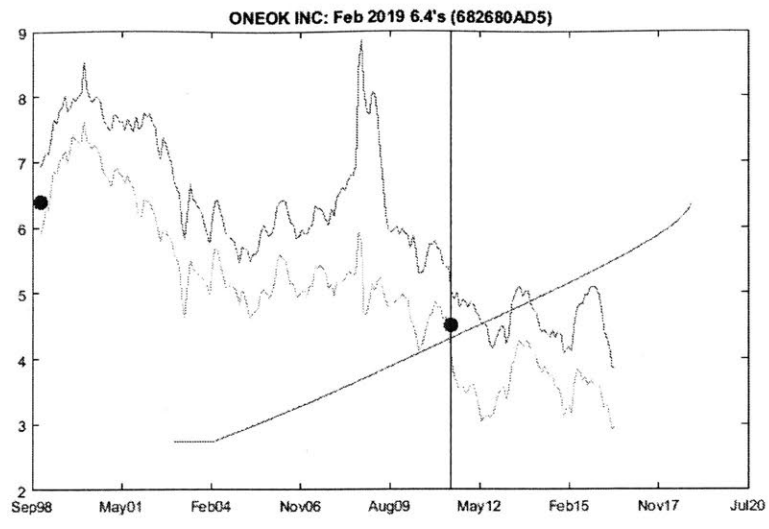
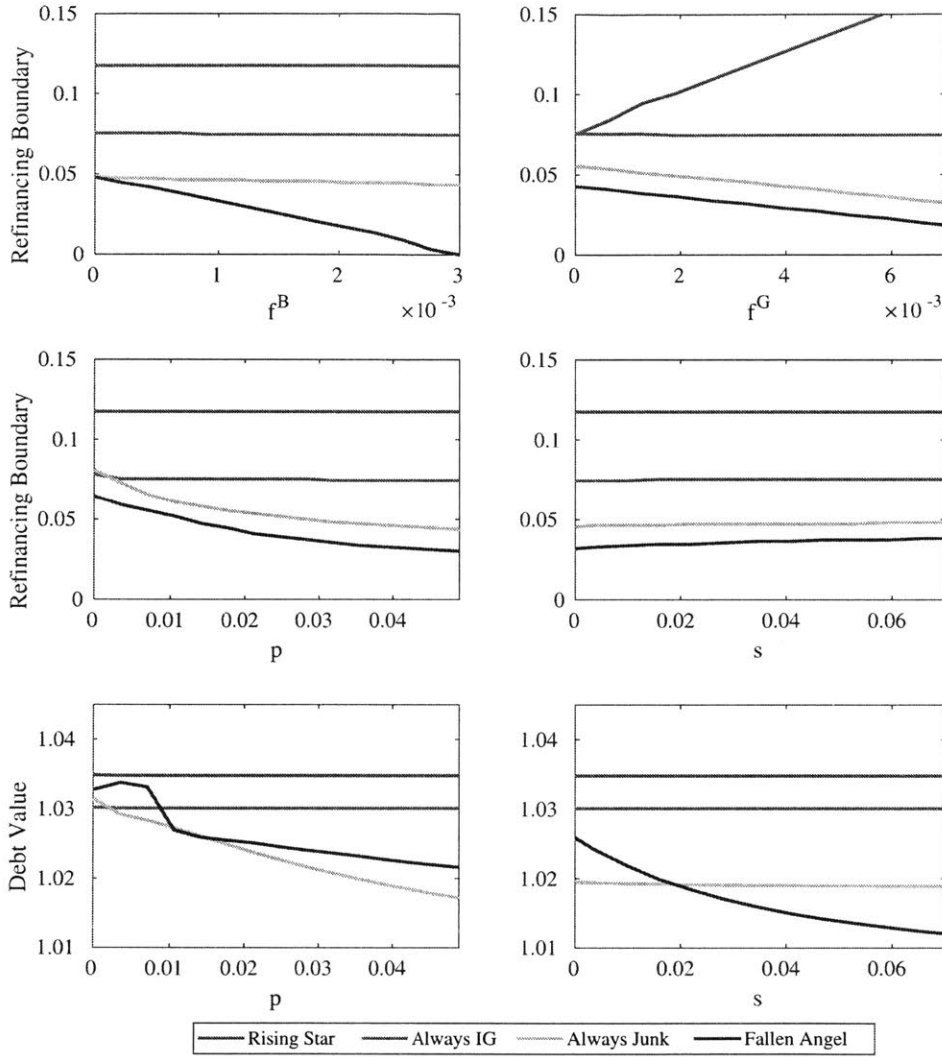


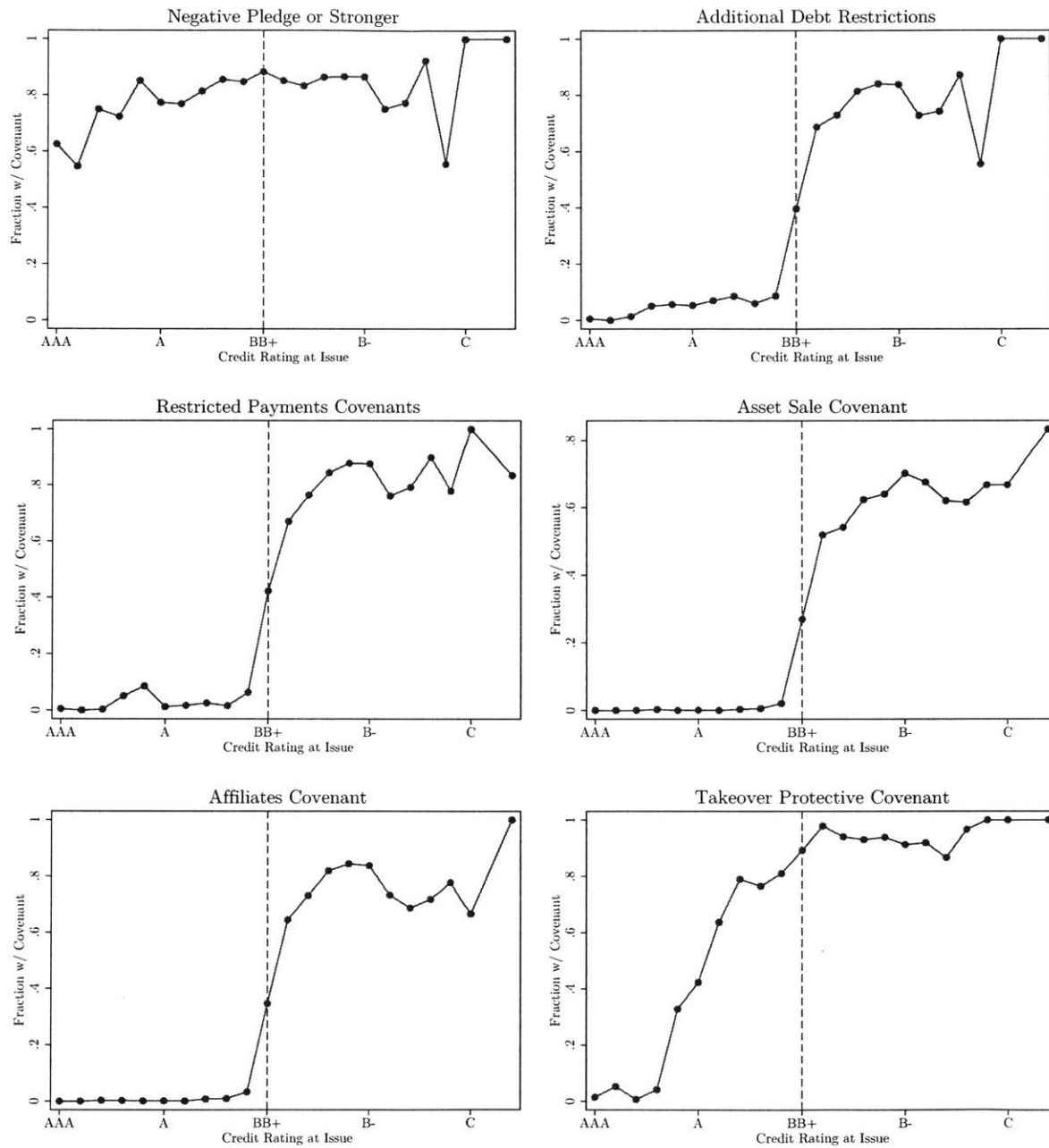
Figure 1.4: Example Call Exercise Boundary Solution

Figure 1.5: Identification of parameters f^B , f^G , p , and s



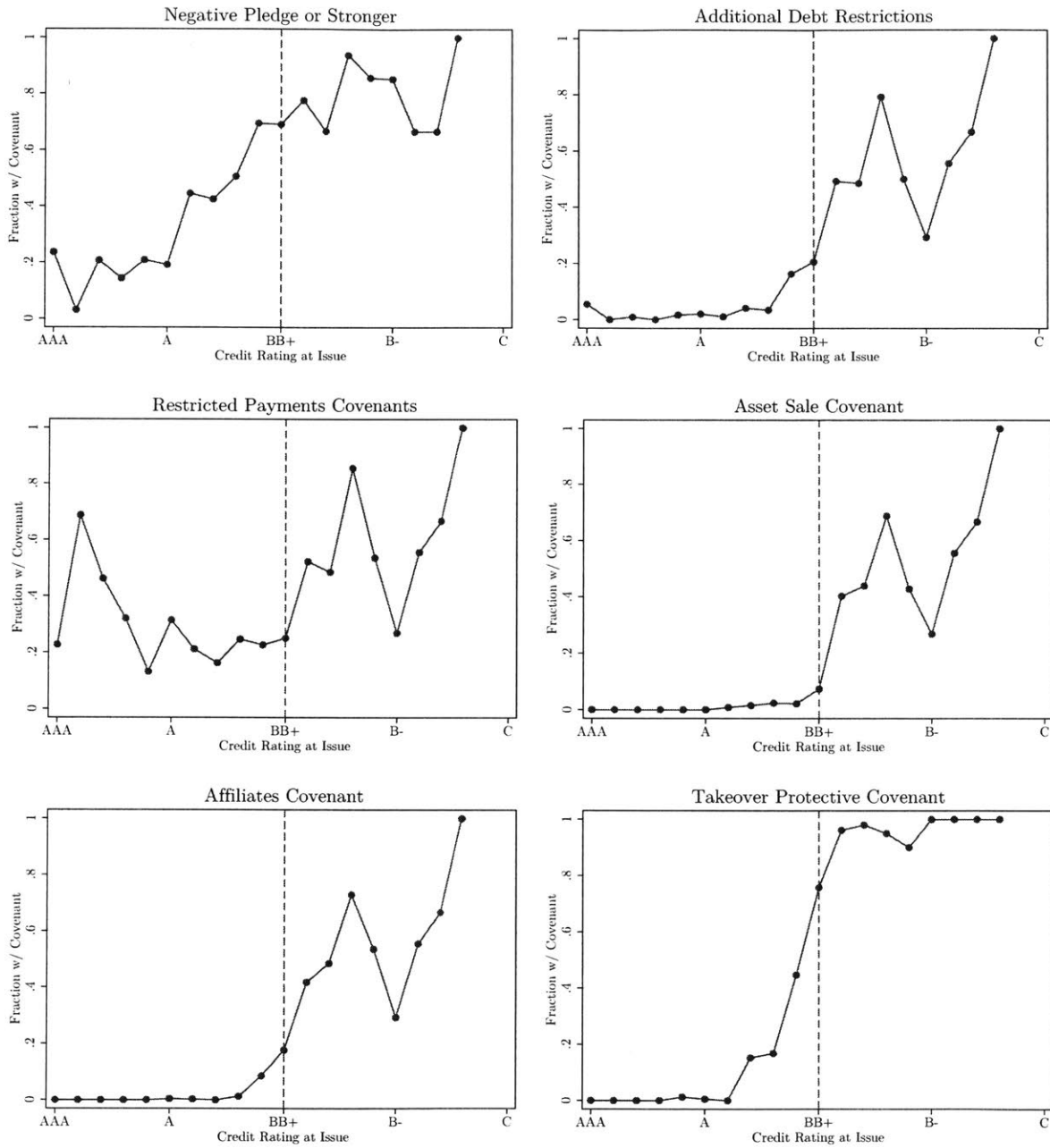
Notes: These figures show representative comparative statics of optimal exercise boundary debt value as a function of the parameters to be estimated. These comparative statics are for a hypothetical 9% coupon bond with 5 years until maturity and call price schedule: currently callable at 103.75%, after 1 year at 102.5%, after two years at 101.25%, and after three years at par. Rising Star and Always Investment-Grade bonds are AA rated and Always-Junk and Fallen Angel bonds are BB rated. Comparative statics of bond price are computed by averaging over all the bond price at all grid points of the risk-free short rate.

Figure 1.6: Covenant Quality at the Investment-Grade Boundary: Non-Utility Issuers



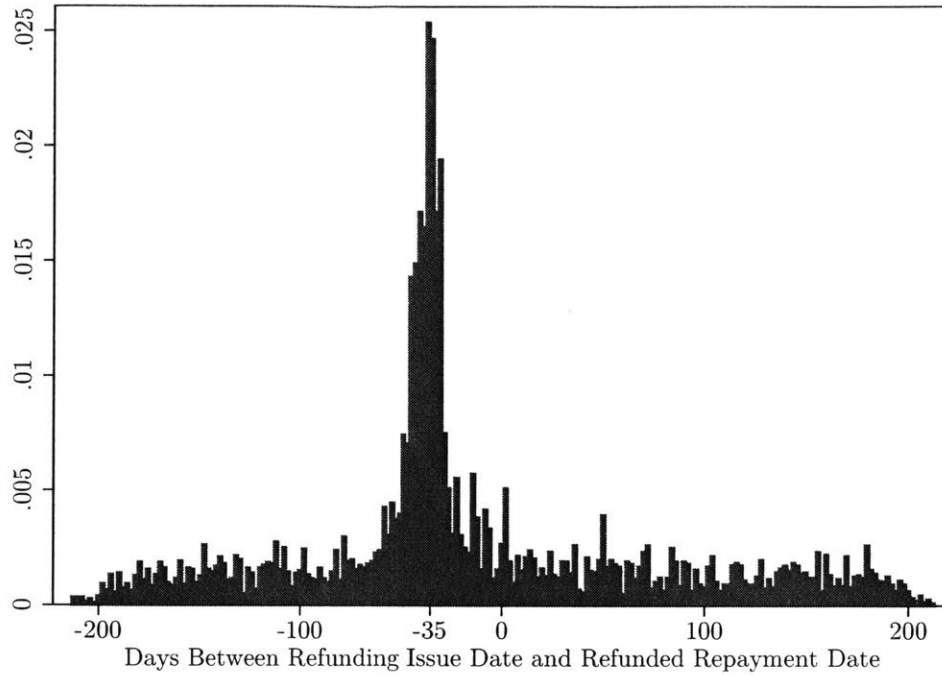
Notes: These figures show the fraction of bonds in the non-utility issuer covenant matched sample that contain a given type of covenant, sorted by S&P credit rating at bond issuance. BB+ is the highest S&P rating not considered to be investment grade.

Figure 1.7: Covenant Quality at the Investment-Grade Boundary: Utility Issuers



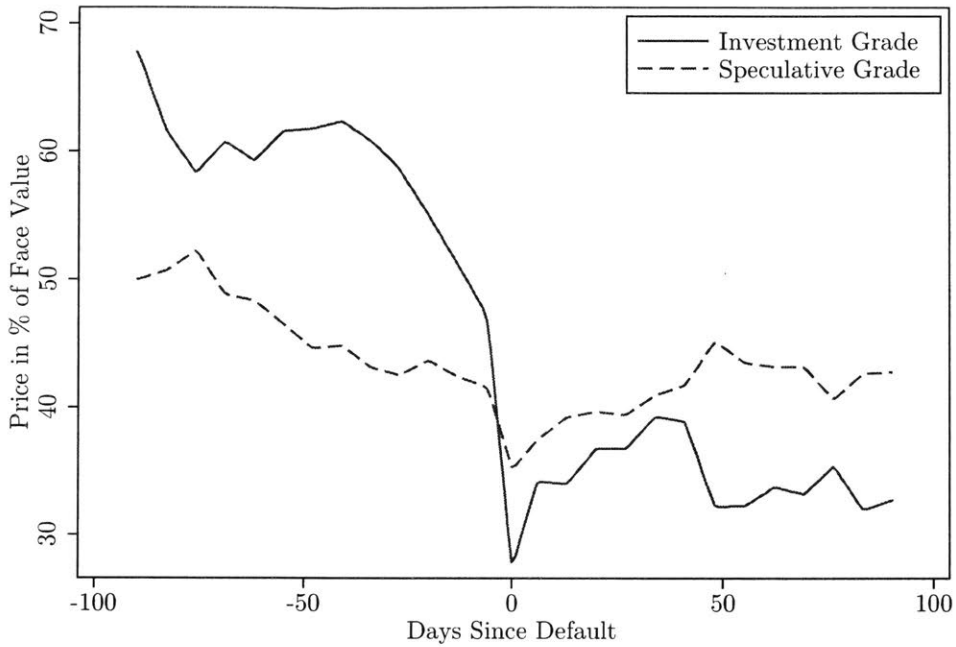
Notes: These figures shows the fraction of bonds in the utility issuer covenant matched sample that contain a given type of covenant, sorted by S&P credit rating at bond issuance. BB+ is the highest S&P rating not considered to be investment grade.

Figure 1.8: Histogram of Timing Delay Between Called and Refunding Bond Issues



Notes: This figure shows the distribution of the days between the issuance of a new bond and the repayment of a called bond issue. The standard refunding convention involves issuing a new bond simultaneously with the notification of the firm's intent to call the bond in 30 days, which is the typical minimum call notice period mandated in corporate bond indentures. I assemble matches by looking for all new bond issuances of a firm that fall within 200 days of the firm prepaying a callable bond. The significant mass around issuance of a new bond exactly 1 month before the early repayment of an outstanding bond suggests these transactions are textbook corporate bond refundings.

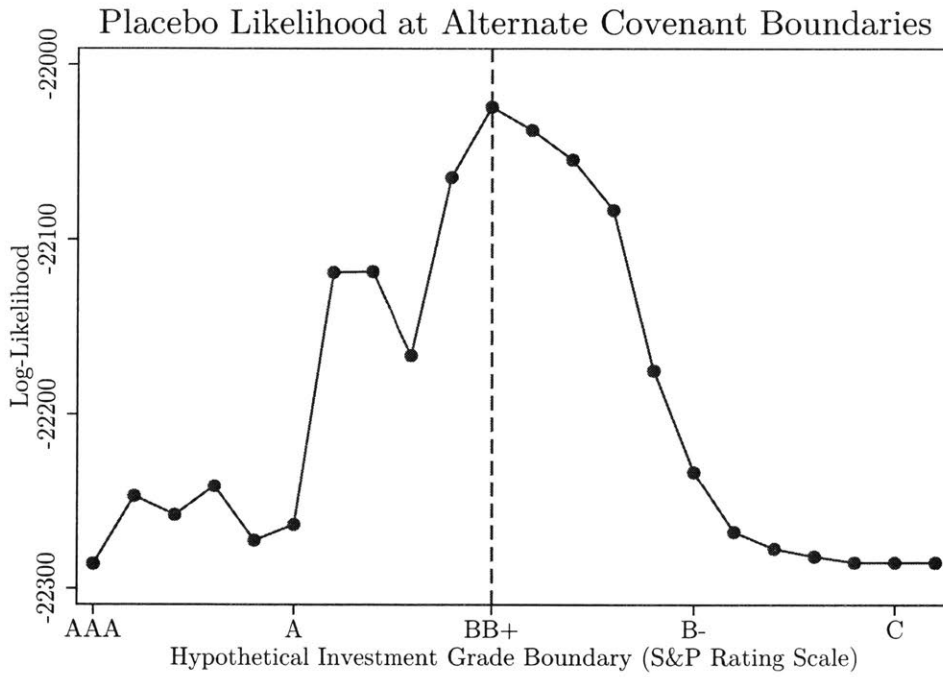
Figure 1.9: Trading Prices around Default by Credit Rating at Origination



Source: Jankowitsch, et. al (2014)

Notes: This figure combines the Investment Grade and Speculative Grade panels from Figure 1 of Jankowitsch, Nagler, and Subrahmanyam (2014). The plot uses TRACE transaction data to plot the average trading prices of defaulted bonds around the default event. The important takeaway here is that trading prices of defaulted investment grade bonds are lower than those of defaulted speculative grade bonds. See Jankowitsch, Nagler, and Subrahmanyam (2014) for more details about the construction of the data that leads to this figure.

Figure 1.10: **Placebo Test**



Notes: This figure plots the estimated log-likelihoods of versions of the model described in Equation 1.2 at different hypothetical cutoff values of the rating boundary between investment grade and high yield rating categories. The exact specification used includes credit rating and industry fixed effects. The fact that the likelihood peaks at exactly the true investment-grade / speculative-grade boundary is suggestive that there is not an omitted mechanism or interpretation behind the result that covenants affect corporate bond refinancing behavior.

Table 1.2: Callable Bond Sample Summary Statistics

	N/Mean	Median	Min	p25	p75	Max
Number of Bonds	10,886					
Number of Issuers	3,293					
Size (millions)	\$164	100	5	60	200	4,000
Tenor (years)	17.3	10	1	10	30	100
Investment Grade	41.3%	BB	C	B	A	AAA
Utility Issuer	29.6%					

Table 1.3: Corporate Bond Calls by Bond Status

Bond Status During Call Period	N	Frac Called	Call Price	$rf_{t_0} - rf_{t_{call}}$
Always Investment Grade	4,907	0.68	102.74	2.15%
Rising Star	245	0.78	103.68	0.91
<i>Difference</i>		-0.10	-0.94	-1.24
Always Speculative Grade	2,639	0.51	102.63	1.75
Fallen Angel	729	0.23	101.70	1.81
<i>Difference</i>		0.28	0.93	0.06

Notes: Bold indicates statistical significance at the 1% level.

Table 1.4: Covenant Differences Between Called and Refunding Bonds

	Average Difference Between Replacement and Called Bond Issue					
	Number of Covenants	Debt Restrictions	Restricted Payments	Asset Sales	Affiliates	Change of Control
Always HY	-1.76	-0.21	-0.23	-0.10	-0.20	-0.05
Always IG	1.19	-0.01	-0.00	0.20	-0.00	0.03
Fallen Angel	3.49	0.28	0.27	0.18	0.29	0.53
Rising Star	-4.06	-0.70	-0.74	-0.23	-0.74	-0.29
Total	-0.25	-0.12	-0.12	0.05	-0.11	-0.00

Notes: This table explores the relationship between measures of defaulted bond recovery values and the bond issue's credit rating at origination. Each specification controls for fixed effects of ratings 12 months before default, origination and default quarter, industry and seniority level of the bonds. IG at Origination is a dummy variable equal to one if the bond had an investment grade rating (and thus weak covenants) at origination. Debt More Senior is the percentage of debt in the firm's capital structure that is senior to this debt issue at the time of default. Dependent variables are nominal and discounted versions of two common recovery metrics: the value this issue receives in resolution and the bond's trading price after emergence from bankruptcy, both expressed as a percent of the issue's principal amount. Discounted measures are simply these values discounted back to the last date the bond made a cash payment or interest or principal. Standard errors are reported in parenthesis and clustered at the bond issuer level.

Table 1.5: **Covenant Differences Between Called and Refunding Bonds**

	Dependent Variable: Increase in Covenants on Refinancing		
	(1)	(2)	(3)
Fallen Angel	5.441*** (0.814)	4.532*** (0.739)	
Energy, Mining, Transit			7.738*** (2.052)
Media & Communications			4.377** (1.609)
Manufacturing			4.591*** (0.892)
Retail, Services, Leisure			7.781*** (2.057)
Utility			1.938 (1.100)
Rising Star	-5.015*** (0.666)	-5.053*** (0.593)	
Energy, Mining, Transit			-6.514*** (1.091)
Media & Communications			-3.253*** (0.731)
Manufacturing			-4.266*** (1.092)
Retail, Services, Leisure			-5.762*** (0.664)
Utility			-2.067 (2.258)
Observations	3,534	3,516	3,516

Notes: All specifications include fixed effects for the called bond's S&P credit rating at the time of the refunding. The second and third column add fixed effects for industry and quarters of issuance of original and refunding bonds. The third column separates the estimates by broad industry category. Standard errors are reported in parenthesis and clustered at the bond issuer level.

Table 1.6: Ratings Changes and Implied Shifts in Call Exercise Boundary

	Dependent Variable: Bond Called			
	(1)	(2)	(3)	(4)
Boundary Shift: Fallen Angel	-0.177*** (0.050)	-0.482*** (0.0643)	-0.471*** (0.0647)	-0.475*** (0.080)
Boundary Shift: Rising Star	0.604*** (0.076)	0.575*** (0.083)	0.564*** (0.082)	0.195 (0.104)
Debt/EBITDA				-0.005 (0.004)
Debt/Assets				0.212 (0.135)
Interest Coverage				0.029*** (0.007)
Price/Earnings				0.001** (0.0003)
Return on Assets				-0.066 (0.357)
Profit/Assets				-0.020 (0.152)
Observations	139,061	139,056	139,048	72,410
Bonds	3,615	3,615	3,614	2,083

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. The coefficients are estimates from the model of Equation 1.2. Each specification controls for the time elapsed from bond origination to the current month. Column (2) adds current credit rating fixed effects. Column (3) further adds industry fixed effects. Column (4) further adds controls for balance sheet characteristics of the issuing firm. Balance sheet variables are windsorised at the 1% level.

Table 1.7: Specific Covenants and Implied Shifts in Call Exercise Boundary

	Dependent Variable: Bond Called		
	(1)	(2)	(3)
Boundary Shift: Fallen Angel	-0.447*** (0.096)	-0.547** (0.182)	-0.549** (0.182)
# Num. Covenants		0.020 (0.038)	0.020 (0.038)
Boundary Shift: Rising Star	0.649*** (0.140)	-0.0151 (0.233)	-0.134 (0.298)
# Num. Covenants		0.071*** (0.021)	0.059 (0.079)
# Restricted Payments			0.070 (0.330)
# Restricted Subsidiaries			0.289 (0.533)
# Restricted Debt Issuance			-0.825 (0.560)
# Change in Control Put			-0.229 (0.335)
# Cross Acceleration			0.977* (0.426)
# Minimum Net Worth			0.274 (0.329)
# Asset Sale Restrictions			-0.035 (0.244)
Observations	43,521	43,521	43,521
Bonds	2,051	2,051	2,051

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. The coefficients are estimates from the model of Equation 1.2. Each specification controls for the time elapsed from bond origination to the current month, and current credit rating and industry fixed effects.

Table 1.8: Specific Covenants and Implied Shifts in Call Exercise Boundary

	Dependent Variable: Bond Called			
	(1)	(2)	(3)	(4)
Boundary Shift: Fallen Angel	-0.471*** (0.065)	-0.498*** (0.068)	-0.494*** (0.068)	-0.498*** (0.068)
# Any HY Debt		0.166 (0.138)		0.0620 (0.228)
# Frac Debt HY			0.374 (0.291)	0.274 (0.474)
Boundary Shift: Rising Star	0.564*** (0.082)	0.564*** (0.082)	0.564*** (0.082)	0.564*** (0.082)
Observations	139,048	139,048	139,048	139,048
Bonds	3,614	3,614	3,614	3,614

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. This specification uses data at the bond-month level. The coefficients are estimates from the model of Equation 1.2. Each specification controls for the time elapsed from bond origination to the current month, and current credit rating and industry fixed effects.

Table 1.9: **Baseline Structural Estimates**

<i>Non-Utilities Sample</i>	
f^B	0.105 (0.011)
f^G	0.200 (0.012)
p	3.575 (0.083)
s	0.637 (0.034)
N_{bonds}	4,185
N_{prices}	23,233
N_{bounds}	289,794

Notes: This table reports the baseline structural model estimates of Section 1.6. The coefficients and standard errors are reported after being multiplied by 100.

Table 1.10: Percent Change in Asset Value from Adding Restrictive Covenants

<i>Non-Utilities Sample</i>	
Speculative Grade Firms	2.39%
95% CI	(1.65, 3.15)
Investment Grade Firms	-1.33
95% CI	(-1.49, -1.18)
N_{bonds}	4,185
N_{prices}	23,233
N_{bounds}	289,794

Notes: This table reports the model-implied counterfactual estimates of how much covenants affect the value of assets by credit risk class. The exercise used the estimated model to calculate asset value under the status quo relative to two counterfactual scenarios: that either all firms or no firms are subject to typical high-yield covenant restrictions. The results estimate the value of speculative-grade firm assets would be 5.15% lower if they were not able to use covenants, and that investment grade firms would suffer a loss of asset value of approximately one percent if they were forced to adhere to strict covenants. 95% confidence intervals are calculated using the covariance matrix of the estimates reported in Table 1.9 and applying the delta method to the model-implied mapping between parameter estimates and the effect of covenants on asset values. See Section 1.6.5 for more details.

Table 1.11: Heterogeneity in the Value of Restrictive Covenants

	<i>Speculative Grade</i>	<i>Investment Grade</i>
Baseline Estimates	2.39% (1.65, 3.14)	-1.33 (-1.49, -1.18)
Compustat Sample	3.89 (2.56, 5.21)	-1.13 (-1.37, -0.88)
Large Firms (> \$2.5bn)	4.33 (2.92, 5.73)	-1.17 (-1.43, -0.90)
Small Firms (< \$2.5bn)	0.47 (-1.00, 1.94)	-0.78 (-1.05, -0.51)
Growth Firms ($BM < 0.65$)	6.44 (4.25, 8.63)	-0.95 (-1.13, -0.79)
Value Firms ($BM > 0.65$)	3.12 (1.95, 4.30)	-1.38 (-1.65, -1.10)
Manufacturing	7.17 (5.91, 8.44)	-0.73 (-0.86, -0.59)
Retail, Services, Leisure	6.87 (5.41, 8.34)	-0.38 (-0.68, -0.08)
Media	0.49 (-1.36, 2.33)	-2.07 (-2.36, -1.77)
Energy, Mining, Transit	3.55 (2.14, 4.96)	-1.15 (-1.29, 1.00)

Notes: This table reports the model-implied counterfactual estimates of how much covenants affect the value of assets by credit risk class. Each row represents a different subset of the data on which the model is separately estimated. 95% confidence intervals are calculated using the covariance matrix of the underlying parameter estimates and applying the delta method to the model-implied mapping between parameter estimates and the effect of covenants on asset values. See Section 1.6.5 for more details.

Table 1.12: Structural Estimates: Robustness

	<i>Baseline</i>	<i>Alt. Trans. Matrix</i>	<i>Flexible $a(X)$</i>
<i>Parameter Estimates</i>			
f^B	0.105 (0.011)	0.103 (0.011)	0.010 (0.022)
f^G	0.200 (0.012)	0.238 (0.011)	0.199 (0.016)
p	3.575 (0.083)	2.973 (0.061)	3.584 (0.087)
s	0.637 (0.034)	0.580 (0.101)	0.657 (0.038)
<i>Value of Covenants</i>			
HY	2.39% (1.65, 3.14)	4.20 (1.45, 6.94)	2.20% (-0.95, 5.35)
IG	-1.33 (-1.49,-1.18)	-1.91 (-2.06, -1.77)	-0.88 (-1.00, -0.76)
N_{bonds}	4,185	4,185	4,185
N_{prices}	23,233	23,233	23,233
N_{bounds}	289,794	289,794	289,794

Notes: This table reports the estimates of the structural model estimates of Section 1.6 to alternative assumptions. The first column replicates the baseline estimates of Table 1.9. The second column uses the calibrated risk-neutral transition matrix derived from the one shown in Table 6.3 of Lando (2004, pg. 154). Estimates in the third column allow the parameter a to vary with the risk state X , and these values are calibrated to match the sample average EBITDA to long term debt ratio of each rating category. The model coefficients and standard errors are multiplied by 100.

Table 1.13: Recovery Rates and Initial Credit Ratings

	(1) Settlement	(2) (discounted)	(3) Emergence Price	(4) (discounted)
IG at Origination	-14.08 (9.224)	-7.904 (7.041)	-19.68* (9.770)	-14.25* (7.092)
Debt More Senior	-0.0455 (0.110)	-0.0735 (0.0887)	-0.122 (0.101)	-0.160* (0.0782)
Observations	794	794	733	733
R^2	0.541	0.530	0.585	0.586

Notes: This table explores the relationship between four measures of defaulted bond recovery values and the bond issue's credit rating at origination and compliments the analysis of Figure 1.9. The data is from the Moody's Default and Recovery Database and matched to my sample of corporate bonds. The recovery measures are computed at restructuring settlement and emergence from bankruptcy. Results are also presented using these measures discounted back to the original default date. The results suggest that defaulted investment grade bonds have lower recovery values than defaulted speculative grade bonds, suggestive of the importance of restrictive covenants in protecting debt-holder value. Each specification controls for fixed effects of ratings 12 months before default, origination and default quarter, industry and seniority level of the bonds. IG at Origination is a dummy variable equal to one if the bond had an investment-grade rating (and thus weak covenants) at origination.

Table 1.14: Yield Spreads and Rating Change Status

	Dependent Variable: Secondary Market Yield Spread			
	(1)	(2)	(3: OLS)	(4: IV)
Fallen Angel	37.03 (25.33)	36.92 (24.93)		
Rising Star	-26.18* (11.03)	-25.25* (10.79)		
Number of Covenants			-4.974* (2.060)	-14.70** (5.468)
Observations	267,141	267,043	211,175	210,913
R^2	0.495	0.518	0.522	0.520

Notes: This table shows that credit ratings do not completely capture variation in priced credit risk due to loose covenants. If bond credit ratings fully account capture their risk including the risk associated with loose covenants, proxies for the inclusion of restrictive covenants in a bond should have no explanatory power for bond yields when controlling for credit ratings. Standard errors are clustered by bond issue and are reported in parenthesis. Column 4 instruments for number of covenants with indicators for fallen angel and rising star status. This instrumentation addresses the possibility that the number of covenants in a debt issue is correlated with current credit risk *within* current credit rating category. All specifications control for remaining maturity of the bond issue and quarter by industry and current credit rating fixed effects.

1.A Appendix

1.A.1 Additional Figures and Tables

Table A1: Ratings Changes and Implied Shifts in Call Exercise Boundary

	Dependent Variable: Bond Called			
	(1)	(2)	(3)	(4)
Boundary Shift: Fallen Angel	-0.300*** (0.0511)	-1.205*** (0.150)	-1.273*** (0.141)	-1.243*** (0.224)
# Frac Debt HY	1.069*** (0.277)	1.077*** (0.279)	1.048*** (0.285)	1.179*** (0.266)
Boundary Shift: Rising Star	0.438*** (0.0832)	1.210*** (0.139)	1.209*** (0.138)	0.947*** (0.229)
Debt/EBITDA				0.00439 (0.0104)
Debt/Assets				0.199 (0.174)
Interest Coverage				0.0342*** (0.00661)
Price/Earnings				0.000951** (0.000315)
RoA				-0.971* (0.409)
Profit/Assets				0.00709 (0.163)
Observations	307,360	307,353	307,345	189,996
Bonds	6,459	6,459	6,458	4,104

Notes: The coefficients are estimates from the model of Equation 1.2. Each specification controls for the time elapsed from bond origination to the current month. Column (2) adds current credit rating fixed effects. Column (3) further adds industry fixed effects. Column (4) further adds controls for balance sheet characteristics of the issuing firm. Balance sheet variables are windsorised at the 1% level.

Table A2: Specific Covenants and Implied Shifts in Call Exercise Boundary

	Dependent Variable: Bond Called		
	(1)	(2)	(3)
Boundary Shift: Fallen Angel	-1.630*** (0.167)	-1.358*** (0.239)	-1.321*** (0.241)
# Frac. Debt HY	0.847** (0.307)	0.883** (0.310)	0.936** (0.313)
# Num. Covenants		-0.0616 (0.0430)	-0.0808 (0.0452)
Boundary Shift: Rising Star	1.603*** (0.192)	1.195*** (0.278)	0.906** (0.333)
# Num. Covenants		0.0501* (0.0216)	0.0239 (0.0771)
# Restricted Payments			0.376 (0.345)
# Restricted Subsidiaries			0.0843 (0.524)
# Restricted Debt Issuance			-0.737 (0.525)
# Change in Control Put			-0.0856 (0.348)
# Cross Acceleration			1.001* (0.435)
# Minimum Net Worth			0.341 (0.363)
# Asset Sale Restrictions			-0.0511 (0.246)
Observations	63,244	63,244	63,244
Bonds	2,682	2,682	2,682

Notes: Standard errors are reported in parenthesis and clustered at the bond issuer level. This specification uses data at the bond-month level. The coefficients are estimates from the model of Equation 1.2. Each specification controls for the time elapsed from bond origination to the current month, and current credit rating and industry fixed effects.

1.A.2 Data Appendix

Sequential Screens	Callable	Non-Callable
Dollar Denominated US Corporate or Utility Bonds	30,509	13,960
Not Convertible, Exchangeable, or Puttable	27,648	12,252
Fixed Coupon	26,259	10,812
Public Placement	25,844	10,137
Continuously Callable	25,642	10,137
Matched initial credit ratings data	24,484	8,598
Face value at least \$5m	23,654	7,224
Have call price schedule data	16,507	n/a
No Make-Whole protections	10,886	n/a

Table A3: Number of bond issues meeting sequential screening requirements. Source data from Mergent database.

S&P Long Term Rating	Model Rating	Ratings Class
AAA	1	IG
AA+	2	IG
AA	2	IG
AA-	2	IG
A+	3	IG
A	3	IG
A-	3	IG
BBB+	4	IG
BBB	4	IG
BBB-	4	IG
BB+	5	HY
BB	5	HY
BB-	5	HY
B+	6	HY
B	6	HY
B-	6	HY
CCC+	7	HY
CCC	7	HY
CCC-	7	HY
CC	7	HY
C	7	HY
D	8	HY

Table A4: Concordance Table of Model Ratings and S&P Long Term Bond Ratings

Table A5: Ratings Transition Matrices

Panel A: Standard and Poor's One Year Transition Matrix

	1	2	3	4	5	6	7	8
1	90.79%	8.29%	0.72%	0.10%	0.10%	0.00%	0.00%	0.00%
2	0.10%	91.22%	7.85%	0.62%	0.10%	0.10%	0.00%	0.00%
3	0.92%	2.36%	90.04%	5.44%	0.72%	0.31%	0.10%	0.10%
4	0.00%	0.32%	5.94%	86.95%	5.30%	1.17%	0.12%	0.21%
5	0.00%	0.11%	0.66%	7.69%	80.55%	8.79%	0.99%	1.21%
6	0.00%	0.11%	0.23%	0.45%	6.47%	82.75%	4.09%	5.90%
7	0.23%	0.00%	0.23%	1.25%	2.28%	12.86%	60.64%	22.53%
8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Panel B: Generator Matrix

	1	2	3	4	5	6	7	8
1	-0.085	0.081	0.005	0.000	0.000	0.000	0.000	0.000
2	0.012	-0.093	0.078	0.001	0.002	0.000	0.000	0.000
3	0.001	0.028	-0.095	0.059	0.005	0.002	0.000	0.000
4	0.000	0.001	0.060	-0.132	0.063	0.007	0.001	0.000
5	0.000	0.001	0.003	0.058	-0.166	0.088	0.004	0.013
6	0.000	0.001	0.001	0.004	0.071	-0.200	0.029	0.094
7	0.000	0.005	0.005	0.010	0.030	0.078	-0.390	0.262
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Panel C: Monthly Transition Probabilities

	1	2	3	4	5	6	7	8
1	0.993	0.007	0.000	0.000	0.000	0.000	0.000	0.000
2	0.001	0.992	0.006	0.000	0.000	0.000	0.000	0.000
3	0.000	0.002	0.992	0.005	0.000	0.000	0.000	0.000
4	0.000	0.000	0.005	0.989	0.005	0.001	0.000	0.000
5	0.000	0.000	0.000	0.005	0.986	0.007	0.000	0.001
6	0.000	0.000	0.000	0.000	0.006	0.983	0.002	0.008
7	0.000	0.000	0.000	0.001	0.002	0.006	0.968	0.022
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000

1.A.3 Proofs

Proof of Proposition 1.1

Given any new debt is fairly priced by capital markets, shareholders have the incentive to maximize total asset value in choosing covenants. Thus, following Equation 1.1, new debt will contain the covenant if and only if $S(p^L) > 0$. Solving $S(\bar{p}) = 0$ gives the covenant boundary:

$$\bar{p} \equiv \Delta / [(1 - q)(X + \Delta)].$$

Proof of Proposition 1.2

The expressions for the refinancing boundaries are direct calculations of the difference between equity value if the firm does or does not refinance and solving for the equalizing interest rate. To show that rising star firms call at a higher boundary and fallen angel firms call at a lower boundary, it is sufficient to prove $\delta > 0$.

$$\delta = (1 + CP)^{-1} \left[\Delta (1 - p_1^L (1 - q)) - p_1^L (1 - q) (X - D) \right]$$

From Assumption 1.2.1, $q\Delta > (1 - q)(X - D)$. This and $\Delta > 0$ means we can write

$$\begin{aligned} \Delta (1 - p_1^L (1 - q)) &= \Delta (1 - p_1^L) + p_1^L \Delta q \\ &> \Delta (1 - p_1^L) + p_1^L (1 - q) (X - D) \\ &> p_1^L (1 - q) (X - D) \end{aligned}$$

which confirms $\delta > 0$.

1.A.4 Detailed Appendix for Section 1.4.2

Discretization and Solution to the Bond Pricing Model

To solve for optimal exercise boundaries in the “reduced form” section I discretize the bond pricing model to the solve by backward induction using dynamic programming methods from bond maturity. The discretization is to the monthly frequency. The interest rate process follows a Vasicek (1977) process discretized using the standard Euler method:

$$R_{t+\Delta} - R_t = \kappa^Q (\bar{R}^Q - R_t) \Delta + \sigma^Q \sqrt{\Delta} \times N^Q(0, 1)$$

Where $\Delta = \frac{1}{12}$ and thus the parameters are expressed in annualized terms. Denote the value of a callable riskless coupon bond with τ periods remaining to maturity, at which it pays one dollar of principal, as $V_\tau(R_t)$. For expositional simplicity, if the bond pays bi-annual coupon c^{ba} define the effective monthly coupon as $c = c^{ba}/6$. Assume the price per dollar of principal to call the bond. The optimal call policy solves the following recursive equations:

$$\begin{aligned} V_{\tau=0}(R_t) &= 1 + c \\ V_{\tau>0}(R_t) &= c + e^{-R_t \Delta} \times \min \left\{ CP_{\tau-1}, \mathbf{E}_t^Q [V_{\tau-1}(R_{t+\Delta})] \right\} \end{aligned}$$

I solve this with dynamic programming methods by discretizing the default-adjusted short rate process R_t into an evenly spaced grid of 150 points between 0 and 0.25.

Calibration of Default Adjusted Short Rate Process

I calibrate the risk-neutral default adjusted short rate process by to match the distribution of price innovations in a large sample of non-callable corporate bonds. First, I assume that the same default adjusted short rate process prices callable and non-callable bonds. Consider a defaultable τ -period zero coupon bond that pays one dollar upon maturity in the event of no default. Assuming the default adjusted short rate process follows a single factor affine diffusion process, its price can

be expressed as

$$P_\tau(R_t, \theta) = \exp(A(\tau, \theta) + B(\tau, \theta) R_t)$$

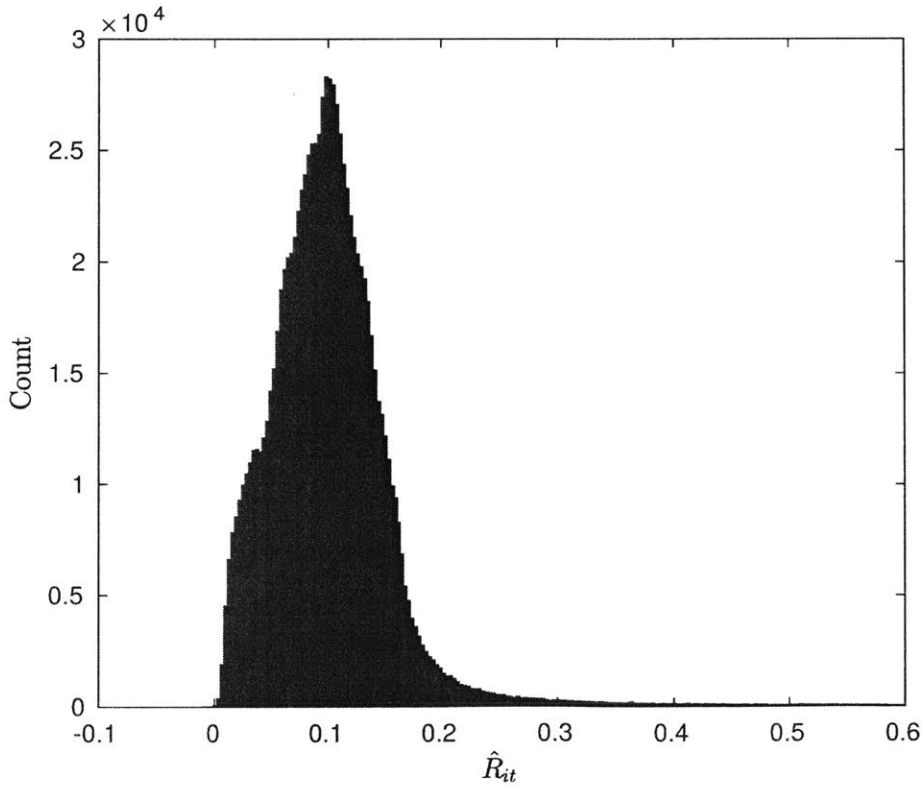
where θ parameterizes the default adjusted short rate process, and the coefficients A and B are well known as in Duffie and Kan (1996). A bond paying a coupon c every month thus has price given by

$$P_\tau^c(R_t, \theta) = \sum_{s=1}^{\tau} c \times P_s(R_t, \theta) + 1 \times P_\tau(R_t, \theta)$$

Since P_τ is monotone in R_t this equation can be solved uniquely for R_t given the current price of a coupon bond P_τ^c :

$$\hat{R}_t(\theta) = P^{-1}(P_\tau^c, \theta).$$

Figure 1.11: Histogram of \hat{R}_{it} in Non-Callable Pricing Sample



Thus, given a panel of corporate bond prices, assuming a single factor default adjusted short rate allows us to invert the pricing equation to obtain estimates of the current level of the short

rate process. Conceptually this state variable is a sufficient statistic about the term structure of interest rates to price the risky coupon bond. Of course, multi-factor models will do a better job capturing the true term structure of interest rates, but at the expense of significantly increasing the complexity of these methods. Duffee (2002) applies a multi-factor version of these methods to *indices* of corporate bond yields.

Given levels of the short rate implied by bond prices and a candidate θ we can express the likelihood of the bond price data as

$$\Pr(P_{i,t+1}|P_{i,t}) = \Pr(\hat{R}_{i,t+1}|\hat{R}_{i,t}) \left| \frac{\partial R_{i,t+1}}{\partial P_{i,t+1}} \right|.$$

Of course, we only observe prices under the physical measure, but have only specified the stochastic process for the default-adjusted short rate under the risk-neutral measure. Thus, I assume the price of risk in this economy satisfies $\eta_t = \lambda R_t$. This implies that the physical dynamics of the default adjusted short rate also follow a Vasicek process, where the mapping between risk neutral and physical parameters involves the price of risk coefficient λ :

$$\begin{aligned}\kappa &= \kappa^Q - \lambda\sigma^Q \\ \bar{R} &= \bar{R}^Q \frac{\kappa^Q}{\kappa} \\ \sigma &= \sigma^Q\end{aligned}$$

I then apply this maximum likelihood procedure to a large sample of monthly bond prices obtained from TRACE and Bank of America Merrill Lynch's bond index constituents data. I use the simplex method to find maximum likelihood estimates of the four parameters $\theta = (\kappa, \bar{R}, \sigma, \lambda)$. Figure 1.11 shows a histogram of the recovered values of the short-rate process given the maximum likelihood estimates $\hat{\theta}$. The maximum likelihood estimates are:

$$\hat{\kappa} = 0.0174, \quad \hat{\bar{R}} = 0.0774, \quad \hat{\sigma} = 0.0345, \quad \hat{\lambda} = -0.795$$

Hedonic Regression Model of Default Adjusted Cost of Capital

I now have maximum likelihood estimates of the stochastic process for the default-adjusted cost of capital and a way to solve the debt pricing model for optimal call policy. Optimal call policy is a boundary in the default adjusted cost of capital: bond i at time t should call the bond whenever its default adjusted replacement cost of capital is below the boundary $b_{it}(\theta)$. Thus to evaluate the refinancing decisions of callable bonds I need to obtain an estimate of the default adjusted cost of capital for every bond at every time the bond is callable.

I do this by inverting non-callable bond prices to obtain estimates of the default adjusted short rate and mapping these estimates onto the sample of callable bond months. To do this, I develop a hedonic regression model that estimates default adjusted cost of capital as a flexible function of a wide range of bond, firm, and time specific characteristics. I estimate this model on the non-callable bond \hat{R}_{it} obtained as described in the previous subsection using the maximum likelihood estimates of the parameters obtained there.

The model includes data on bonds' current credit rating, credit rating at issuance, industry, parent company credit rating, maturity-matched treasury yields, as well as time, rating by time, and industry by time fixed effects. Importantly, the model's inclusion of origination and current ratings captures, to the extent it exists in the data, any differences in the cost of capital of firms with different origination ratings but identical current ratings. This controls for the main source of potential bias in my methodology, that I am not capturing differences in the replacement cost of capital of firms between firms with dramatic ratings changes (that thus face difference covenants upon refinance) and firms that have not experienced such ratings changes.

The model is estimated on a sample of nearly 2 million bond-month observations, and contains thousands of parameters, most of which are the interacted fixed effects described above. The R^2 of the model is over 86%, suggesting that my calibrated single factor model of default adjusted short rates is able to capture a large majority of the variation in bond prices I then project these estimates onto the bond-month observations in my callable bond sample. To form estimates of the default adjusted cost of capital for each callable bond at each point in time.

Chapter 2

Growing Pains in Financial Development: Institutional Weakness and Investment Efficiency

2.1 Introduction

It is well understood that expanded access to finance can improve economic outcomes. However, this relationship does not seem to hold in the case of microfinance. While it has spread to over 100 countries and now serves more than 130 million borrowers (Microcredit Summit Campaign), there has been little systematic evidence that microfinance has a significant impact on poverty alleviation. Randomized control trials of microcredit expansions find little impact on borrower income and business size (Banerjee, Karlan, and Zinman, 2015). Perhaps more surprisingly, cash grant experiments show that, despite the existence of microfinance institutions, micro-entrepreneurs still face severe credit constraints as demonstrated by very high marginal returns to capital (de Mel, McKenzie, and Woodruff, 2008).

Why has microfinance failed to live up to its promise? Expanding access to credit is economically useful only if it improves the allocation of resources. In this paper we explore the relationship

* This chapter is joint work with Ernest Liu.

between the expansion of finance and allocative efficiency of investment, taking seriously two empirical regularities of the microfinance industry’s rapid expansion—an increasing number of lenders available in a given market, and the absence of credit market institutions that would allow borrowers to commit to exclusive contracting with individual lenders.¹ Our key insight is that if borrowers are unable to commit to a single lender *ex-ante*, then *ex-post* commitment induced by low marginal returns to further borrowing will be valuable in obtaining agreeable financing terms and facilitating investment. Our model formalizes this insight. We show that when borrowers cannot commit to exclusive contracts, better projects can receive less investment, borrowers may choose to forgo the most productive investments in favor of economically inferior alternatives, and that the severity of these distortions is increasing in the number of lenders available to borrowers. Together, these results outline a new explanation of why increased access to finance does not always improve aggregate outcomes.

In the model, an entrepreneur sequentially visits multiple lenders to obtain funds for a new investment opportunity. Crucially, the entrepreneur lacks the ability to commit to exclusive borrowing from a single lender and cannot write loan contracts that are contingent on the terms of contracts subsequently signed with other lenders. The entrepreneur faces an uncertain cost of default that is realized when loans come due, and defaults if the debt owed exceeds this cost. Thus, the more debt owed the less likely it is to be repaid. This gives rise to an externality between lenders. New lenders willingly provide additional investment that existing lenders would not, because new lenders do not internalize the decreased likelihood of repayment of *existing* debt when pricing new debt contracts. Rational lenders anticipate this additional borrowing and offer loan terms that compensate them for it, making multiple borrowing undesirable *ex-ante*, but without commitment unavoidable *ex-post*. Inability to commit to an exclusive lending relationship is thus a binding constraint and in equilibrium induces distortions in lending and investment outcomes.

The notion of commitment externalities in credit markets is not new to our paper. The economics of the commitment externality was analyzed in the seminal work of Arnott and Stiglitz (1991, 1993) in the insurance context and applied to credit markets by Bizer and DeMarzo (1992). Our

¹The pervasive “multiple-borrowing” phenomenon in microfinance due to lack of exclusive contracts has been well documented in several countries. See for example Faruquee and Khalily (2011), de Janvry, Sadoulet, McIntosh, Wydick, Luoto, Gordillo, and Schuetz (2003), and McIntosh, de Janvry, and Sadoulet (2005).

contribution is to show that the extent of the distortion induced by commitment problems is closely linked to the nature of the investment opportunity that needs to be financed. More productive projects receive *less* investment, inducing entrepreneurs to choose projects with lower returns and limited scope for growth because these are easier to finance. These problems are exacerbated by an increased availability of lenders, explaining why commitment externalities can substantially impede the ability of credit market expansion to improve the allocation of investment capital.

At the heart of these results is that lower marginal returns create *endogenous commitment power* not to borrow (much) from additional lenders—the benefits of marginal investment return are not as attractive relative to the cost of higher debt obligations. Since entrepreneurs cannot commit *ex-ante* to limit inefficient future borrowing, properties of their future marginal returns to investment that induce them to limit such borrowing *ex-post* are especially valuable. This induces a trade-off between an investment technology’s efficiency and concavity. Low marginal (and hence average) returns are bad for output, but declining marginal returns are good for incentives. When marginal returns are high, a borrower who lacks commitment not to continue borrowing may receive an amount of investment that could have been supported by a lower quantity of debt, if only the borrower could have committed to this level of borrowing *ex-ante*. Financial constraints are thus endogenously more severe for better projects when they also have sufficiently higher marginal returns.

We also show that commitment problems are not necessarily alleviated by relaxing financial constraints. Specifically, we consider how the forces we highlight interact with the ability of borrowers to credibly pledge to lenders a fraction of the investment capital they raise. We show that this in fact makes commitment problems *worse*. Pledgeability allows for leveraged borrowing, which on the one hand increases the total investment entrepreneurs can collect, but on the other hand makes *ex-post* commitment problems worse by effectively increasing marginal returns to risky borrowing. Under any level of partial-pledgeability, we show that better projects can always receive less investment than those with lower marginal returns.

Next, we show that not only can lack of commitment cause better projects to receive less investment, but it can also cause entrepreneurs to reject the most profitable projects entirely in favor of less efficient endeavors. When commitment problems are severe, constrained borrowers

choose business plans that have low prospects for expansion precisely because these are the easiest to finance. Those who instead choose better projects will receive low levels of funding at high interest rates.

The relationship we characterize between financial constraints and productivity generates a particularly stark form of misallocation: a negative correlation between the *level* of investment in a project and its productivity. This is consistent with a growing body of evidence from micro and experimental studies conducted in developing economies that productive firms may be especially credit constrained. McKenzie and Woodruff (2008) study microenterprise in Mexico and use a randomized experiment to estimate returns to investment capital. In addition to finding very high returns on average, they also find a positive relationship between the returns to investment capital of borrowers and the financial constraints they face, indicating that it is the projects with the best economic fundamentals that are most under-served. Similarly, Banerjee and Duflo (2014), using policy changes in credit access programs in India, find similar evidence of a negative selection effect. Their analysis of the relationship between loan growth and profit growth suggests that it is the least productive firms that are acquiring the most financing in credit. Our paper shows that such misallocation can be explained by commitment problems in credit markets.²

The setting of our model and focus on commitment problems is well grounded in the reality of microfinance around the globe, which as it expands is becoming increasingly associated with the narrative of *multiple borrowing crises* (e.g. see Faruquee and Khalily, 2011). The quintessential example of multiple borrowing is the spectacular boom and bust of the microfinance industry in the Indian state of Andhra Pradesh. A nascent industry in the 1990s, allegedly thousands of lenders entered the market to supply credit to nearly the entire state, much of it in the form of “overlapping” loans from many lenders to the same borrower.³ Borrowers accumulated large debt balances from many lenders, who individually had no way of observing or controlling a borrower’s total indebtedness. This ultimately proved unsustainable and culminated in a default crisis and near-collapse of the industry in 2010. Fear and realization of such multiple borrowing crisis have

²Misallocation induced by through frictions in microfinance lending has also been studied by Liu and Roth (2016), who show monopolistic lenders have incentives to generate debt traps via imposing contractual restrictions when future profits are nonpledgeable.

³At the peak in 2009, surveys estimated that 84% of rural villagers in Andhra Pradesh had loans from multiple lenders, including 58% borrowing from at least four lenders. See Johnson and Meka (2010), Taylor (2011) and references therein for a collection of facts and anecdotes about multiple borrowing in Andhra Pradesh.

also arisen around the world where microfinance has grown rapidly.⁴ In each of these cases, lenders, policymakers, and academics have all noted the importance the inability of borrowers to commit to an exclusive relationship with a single lender.

We also study optimal regulatory policy in the face of commitment problems and find that strongly mirrors India's regulatory response to the Andhra Pradesh crisis. In 2011, the Reserve Bank of India imposed new regulations on the banking industry and stated that they were in part meant to address multiple borrowing. A debt limit (USD 790) and interest rate cap (26%) were imposed.⁵ These policies work by explicitly limiting the scope of commitment externalities in lending markets. Of course, if borrowers could commit through contingent contracting, the externalities that arise between lenders would disappear and outcomes would improve.

The emergence of exactly this sort of contingent contracting in sophisticated financial markets thus further substantiates the idea that commitment distortions are important. In the syndicated loan market in the United States and Europe, widely employed "performance pricing" covenants allow interest rates to vary based on changes in observable firm characteristics that occur *after* the loan has been issued. Making interest rates increasing in a firm's total amount of debt, as the majority of these covenants dictate, internalizes the spillovers between lenders and restores the full-commitment lending outcomes, even when borrowers cannot explicitly agree to exclusive borrowing.⁶ Of course, implementing such contracts requires lenders can observe violations (for example through credit registries) and contract on them (for example through courts).

Theoretically, our model is based on the theme of common agency, which describes environments in which multiple principals with possibly conflicting interests act to influence the behavior of a single agent (Bernheim and Whinston (1986), Segal (1999)). Arnott and Stiglitz (1991, 1993) study common agency in insurance markets and recognize that additional insurance providers can impose externalities on each other through moral hazard of the buyer. More recently, Brunnermeier and Oehmke (2013) highlight that commitment externalities can endogenously shorten maturities

⁴Multiple borrowing crises have been a concern in Peru, Guatemala, Bolivia (de Janvry, Sadoulet, McIntosh, Wydick, Luoto, Gordillo, and Schuetz 2003), Uganda (McIntosh, de Janvry, and Sadoulet 2005), and Bangladesh (Faruqee and Khalily 2011).

⁵It was also stated that at least 75% of the total loans originated by any regulated MFI should be for the purpose of income generation. (Reserve Bank of India (2011))

⁶See Asquith, Beatty, and Weber (2005) for a detailed description of performance based pricing covenants.

in lending markets. Donaldson, Gromb, and Piacentino (2017) show that borrowers rely more on collateral when it is easier to take on new debt and dilute existing creditors. The closest paper to ours in this literature is Bizer and DeMarzo (1992). They study a model in which a borrower with a desire to smooth consumption between two periods can sequentially visit lenders to obtain loans backed by the borrower's stochastic next-period income, which is influenced by non-contractible effort choice. As in Arnott and Stiglitz (1991, 1993), moral hazard generates an externality that new lenders offer loans that harm previous lenders by decreasing their expected profits.⁷

Our main contribution to this literature is that we are the first to link commitment problems in credit markets to investment decisions. We show that commitment externalities cause better projects to receive lower levels of investment, and encourage entrepreneurs to favor projects with rapidly diminishing returns to scale, even if they generate lower returns for any given amount of investment than other available projects. These results highlight a general feature of credit markets with commitment externalities: better investment opportunities imply stronger desire to borrow, exacerbating the commitment externalities. These results cannot be derived from the model of Bizer and DeMarzo (1992) because in that model there is complete lack of commitment and equilibrium is reached only when the marginal return to borrowing is exactly one, preventing exploration of how commitment externalities affect equilibrium investment levels across projects with different profiles of marginal returns.

It is also important to emphasize that our paper relates to competition in credit markets only through competition's exacerbation of commitment externalities. We focus on the fact that easier access to additional lenders makes it harder to enforce exclusive borrowing and thus ex-post commitment mechanisms, such as selecting projects with limited returns to scale, are valuable. This channel is independent of the relationship between competition and market power in the context of imperfect credit markets, which has been studied in other papers. Of this work, the most related to our paper is Parlour and Rajan (2001), which shows that commitment problems prevent lender entry from competing away all lender profits. Also related is the theory of relationship lending developed by Petersen and Rajan (1994, 1995). This theory suggests that when borrowers cannot

⁷Other papers studying lack of commitment in credit markets include Boot and Thakor (1994), Bisin and Rampini (2006), Kahn and Mookherjee (1998), McAfee and Schwartz (1994), Parlour and Rajan (2001), and Petersen and Rajan (1995).

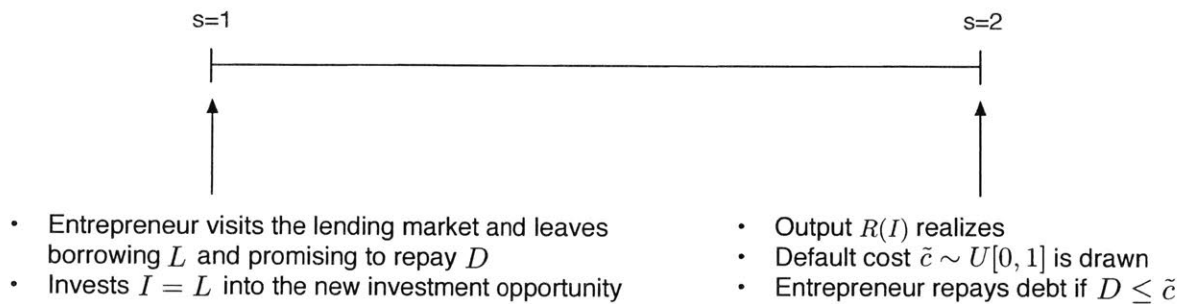


Figure 2.1: Timing of the Model

commit to long relationships with a single lender, limited market power of lenders erodes their ability to subsidize lending to early stage firms and recoup expenses later through rent extraction.

2.2 Model

Overview and Timing An entrepreneur attempts to finance a new investment opportunity with the constraints that it cannot commit to exclusive borrowing from a single lender and that there is limited enforcement of debt repayment, ie. the borrower only repays if the costs of default are high enough. The model has two stages ($s = 1, 2$) and contains two sets of agents: a single entrepreneur and an infinite sequence of potential lenders. All parties are risk neutral and have no discounting. The entrepreneur is endowed with a *variable-scale* investment opportunity that returns $R(I)$ deterministically at $s = 2$ for any $I \geq 0$ invested at $s = 1$. The returns to this investment opportunity are observable but not pledgeable (we relax this assumption in Section 2.4.3). After the projects returns are realized, the entrepreneur learns of its cost of defaulting on its debt, and only repays if the default cost exceeds the amount of debt owed. Specifically, we assume default costs \tilde{c} are drawn from a distribution $F_{\tilde{c}}$. Debt D is repaid if $\tilde{c} > D$, and we denote the probability of repayment as $p(D) \equiv 1 - F_{\tilde{c}}(D)$, which is of course a decreasing function of D . Figure 2.1 summarizes the timing of the model.

We model the entrepreneur raising capital from the lending market at $s = 1$ as an infinite horizon dynamic game of complete information in which the entrepreneur sequentially visits a potentially infinite number of lenders. All lenders are risk neutral and do not discount between $s = 1$ and

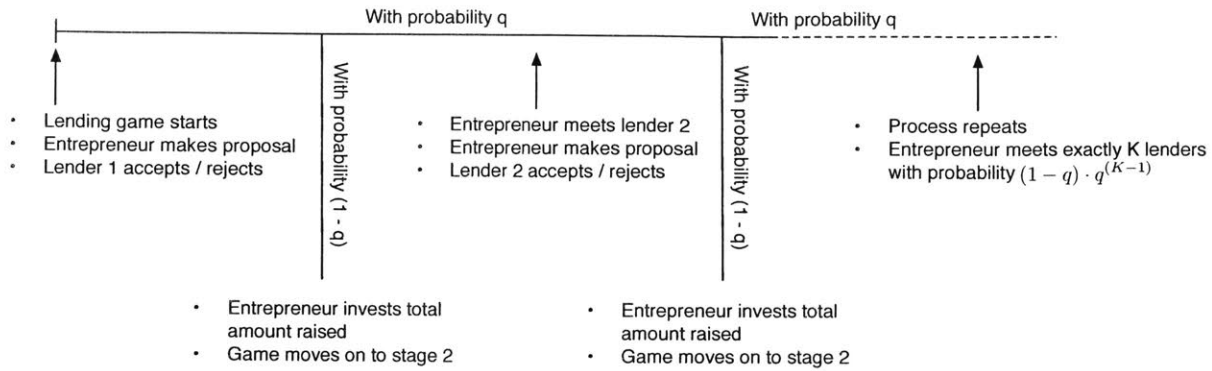


Figure 2.2: The Lending Market Game at $s = 1$

$s = 2$. Their opportunity cost of funds is the risk-free rate, which we normalize to one throughout the paper. Upon meeting a lender, the entrepreneur makes a take-it-or-leave-it offer for a simple debt contract (L_i, D_i) specifying the amount borrowed L_i and the promised repayment amount D_i . The lender, who can observe the history of the entrepreneur's borrowing from previous lenders, chooses whether to accept or reject this offer, and if accepted the funds are exchanged.

At this point, the entrepreneur loses access to the lending market with probability $1 - q$, with $q \in [0, 1)$. Otherwise, the entrepreneur meets a new lender and the process described above is repeated until eventually access to the lending market is lost or the last lender is visited. By assumption, no lender is visited more than once. After losing access to the lending market, the entrepreneur invests the aggregate financing it raised in the new project, and the model progresses to stage $s = 2$. Figure 2.2 provides a graphical exposition of the lending market game of stage $s = 1$.

A brief discussion of the important aspects of the model is in order. The crucial feature of the model is that the entrepreneur cannot credibly commit ex-ante to avoid borrowing from subsequent lenders it meets. More broadly, debt contracts cannot be contingent on the terms of other debt contracts written with subsequent lenders. This form of contractual incompleteness precisely reflects, in our view, important features of the institutional and contracting environments in which we think misallocation and suboptimal technology choice are most salient.

The assumption of sequential borrowing from a possibly large number of banks need not be taken

literally. What drives our results is that lack of commitment to an exclusive lending relationship leaves room for the externalities between lenders and perversely affects equilibrium outcomes. Our results also hold when the entrepreneur borrows simultaneously, rather than sequentially, from a finite number of lenders where loan terms cannot be made contingent on the other loans taken by the borrower. However, such a model is analytically less tractable and subject to the standard critique in the simultaneous contracting literature that the results are sensitive to the specification of agents' off-equilibrium beliefs (Segal and Whinston (2003)).

We formulate the endogenous choice of default via a random default cost that is realized ex-post. The formulation is similar to a random shock to an outside option, as in Aguiar, Amador, Hopenhayn, and Werning (2016), and reflects the fact that forces outside of the model generate ex-ante indeterminacy in the ex-post costs of defaulting on debt. Weak institutions are a salient driver of such indeterminacy; in the Andhra Pradesh default crisis, repayment rates plummeted from near-perfect to near-zero almost overnight when grandstanding local politicians urged borrowers to stop repaying their debts. In the appendix, we provide a microfoundation of the random default through moral hazard.

The parameter q exogenously limits the amount of commitment power borrowers can obtain in the lending market. The closer q is to zero, the less likely it is that contingencies arise in which early lenders can be exploited by further borrowing. There are several interpretations of this parameter. First, q can be thought of as inversely related to the difficulty or cost of subverting commitment, and reflects the quality of the contracting environment. As $q \rightarrow 0$ the contracting environment is able to perfectly enforce contingency in loan terms. When $q \rightarrow 1$ borrowers can costlessly find new lenders to provide marginal lending. A second view is that q reflects the composition of search frictions in the lending market and the limited time an entrepreneur has to raise money for an investment opportunity. Under this interpretation one would assume a complete inability to conduct contingent contracting or exclusive borrowing—the severity of the commitment problem is determined by the market structure of lending and time preference for funding.

To begin solving the model, we introduce two simplifying assumptions which aide greatly in the exposition of the model and in highlighting the underlying economic forces generating our results.

Assumption 2.2.1. (1) *The return on the new investment opportunity is linear and productive:*

$$R(I) = \alpha I \text{ and } \alpha > 1.$$

(2) *Default costs for the entrepreneur are uniformly distributed between zero and one.*

$$\tilde{c} \sim U[0, 1].$$

Linearity in returns to the investment opportunity means that the level of investment financing obtained from prior lenders is not directly relevant for the objective of the borrowers or lenders at any stage of the lending game, greatly simplifying the model. We restrict to *efficient* linear projects ($\alpha > 1$) to rule out trivial cases in which there is no demand for borrowing or investment. The uniform distribution of default costs between 0 and 1 normalizes the borrower's maximum debt capacity to one and generates simple expressions for properties of debt repayment. The probability of repayment is now simply $p(D) = 1 - D$ and the expected debt servicing cost (either repayment or strategic default) is $\mathbb{E}[\min(\tilde{c}, D)] = D - D^2/2$. In Section 2.4 we show our results are robust to allowing partial pledgeability of investment.

Illustrating the Commitment Problem First suppose the entrepreneur meets exactly one lender and makes a take-it-or-leave-it offer (D_1, L_1) , investing $I = L_1$ in the new investment opportunity. The lender accepts any loan offer that is weakly profitable in expectation. The full-commitment optimal lending contract solves:

$$\max_{D_1, I} \alpha I - \mathbb{E}[\min(D_1, \tilde{c})] \text{ s.t. } I \leq p(D_1) D_1.$$

The solution is characterized by the first-order condition:

$$\alpha \times [p(D_1) + p'(D_1) D_1] = p(D_1). \tag{2.1}$$

At the optimum, the costs and benefits of pledging an additional dollar of face value of debt must be equalized. The marginal cost is the increase in expected debt repayment costs associated with

the additional borrowing, which is simply the probability of actually repaying the marginal dollar pledged as shown on the right-hand-side of equation 2.1. The gain from pledging an additional dollar of debt is shown on the left-hand-side and is equal to the marginal return to investment (α) times the marginal investment that can be raised from the additional debt issuance. The term $p(D_1)$ represents the value of the marginal dollar of debt. Because extra repayment reduces the value of all debt claims, the value of *existing* debt D_1 falls when new debt is issued; this is reflected in the term $p'(D_1) D_1$.

To illustrate the source of the commitment problem, now suppose the borrower who has issued debt D_1 to the first lender gets to meet a second lender. The choice of debt promised to this second lender, d_2 , should satisfy the first-order condition

$$\alpha \times [p(D_1 + d_2) + p'(D_1 + d_2) \cdot d_2] = p(D_1 + d_2). \quad (2.2)$$

The critical difference between equations (2.1) and (2.2) lies in the second term in square brackets. Here, in pricing the marginal debt, the second lender only internalizes the change in the value of the debt it holds (d_2) due to its marginal debt issuance at the optimum, as opposed to the change in the total value of debt issued ($D_1 + d_2$). Thus the second lender imposes an externality on the first. Of course, in equilibrium lenders must anticipate all potential future borrowing and charge higher interest rates that compensate them for value they expect to lose. We now formally study the equilibrium of this model.

2.3 Equilibrium without Commitment

Our solution concept is Markov Perfect Equilibrium (MPE) in which the borrower's strategy is a function only of a single state variable D , the cumulative face value of debt the issued so far in the game, and the lender's strategy is a function of D as well as the current loan proposed by the borrower. Modeling borrowing as sequential, rather than simultaneous, allows us to avoid issues of the sensitivity of our results to off-equilibrium beliefs. As we show at the end of this section, the equilibrium we identify captures the limiting outcome of the unique subgame perfect equilibria of finite-lender versions of the model as the number of lenders grows large. We now provide a formal

definition of a Markov Perfect Equilibrium in the lending market game.

Definition 2.1. *A Markov Perfect Equilibrium (MPE) of the infinite lender lending market game is a set of borrower and lender strategies that are mutual best responses at every subgame when subject to the following constraints.*

- a) The entrepreneur's strategy when encountering any lender is a mapping from how much it has already pledged to repay, D , to a simple debt contract (L_i, D_i) .
- b) All lenders' strategies are represented by functions mapping from the state variable D and the loan contract proposed by the entrepreneur to the lender's decision to either accept or reject the proposal.

We characterize the equilibrium by first exploring the best responses of each player.

Lender Best Responses Lenders have rational expectations of future borrowing and thus only accept contracts that yield non-negative expected returns taking the strategies of the borrower and other lenders as given. A lender receiving a loan proposal must, given the state of the game, evaluate the expected profit from the loan taking into account the strategies of the borrower and future lenders and the likelihood that the borrower will be able to meet these lenders at all. Specifically, define $D' \equiv D + D_i$, and denote $\tilde{p}_i(D')$ to be lender i 's perceived probability that the entrepreneur will not default on a loan (L_i, D_i) to this lender, given the borrower has previously obtained total face value of debt D . Such a loan is weakly profitable in expectation if $L_i/D_i \leq \tilde{p}_i(D')$. Since lenders are risk neutral and do not discount stage 2 cash flows, lender i 's optimal strategy is to accept the loan offer if and only if it is weakly profitable. We focus on a symmetric equilibrium in which lenders' best response can be characterized by a function $\tilde{p}(\cdot)$ such that $\tilde{p}_i(\cdot) = \tilde{p}(\cdot)$ for all lenders i .

Borrower Best Response Conditional on meeting a lender, and given lender strategies represented by $\tilde{p}(\cdot)$, the borrower makes a loan offer that maximizes its expected continuation utility, taking into account the chances of being able to meet more lenders and obtaining further marginal borrowing. Solving for the entrepreneur's best response function thus involves a dynamic optimization problem. Taking lender pricing as given the entrepreneur forms strategies that maximize its

continuation utility at each value of the state variable D .

The borrower knows that lenders will accept any loan they expect to be profitable, so to maximize its own utility it will only offer loans that lenders expect to make exactly zero profits. If the entrepreneur has cumulative debt D upon meeting the lender and leaves the lender with cumulative debt D' , then the maximum amount of new investment the lender would provide is given by $\tilde{p}(D')[D' - D]$, the probability of repayment times the face value of new debt issuance. Thus, taking loan pricing as given the entrepreneur solves:

$$V(D) = \max_{D'} \alpha \tilde{p}(D')(D' - D) - (1 - q) \mathbb{E}[\min(D', \tilde{c})] + qV(D'). \quad (2.3)$$

Conditional on arriving at a new lender having already issued face value of debt D the entrepreneur optimally chooses D' , the new total face value of debt it will have issued after contracting with this lender. The first term on the right hand side of Equation 2.3 is the marginal payoff from the new investment opportunity associated with obtaining additional investment $\tilde{p}(D')(D' - D)$. With probability $1 - q$ the entrepreneur loses access to the lending market and either repays debt D' or defaults and pays the default cost \tilde{c} if it is lower than the cost of repayment. Finally, with probability q the entrepreneur does not lose access to the lending market and will receive continuation utility $V(D')$ from future borrowing.

Denote a policy function that solves the dynamic programming problem in Equation 2.3 by $g(D)$. Conditional on arriving to a new lender with aggregate face value of debt D , the borrower will leave with aggregate face value of debt $D' = g(D)$, having proposed a new additional loan $(\Delta D, \Delta L)$ with $\Delta D \equiv g(D) - D$ and $\Delta L \equiv \tilde{p}(g(D))(g(D) - D)$. On path, following this strategy generates a sequence of total aggregate face values of debt that have been accumulated up to a given lender, conditional on the lending market progressing that far: $\{g(0), g(g(0)), g^3(0), \dots\}$. The aggregate face value of debt obtained in the lending market is thus a random variable D^{agg} that realizes a particular value of this sequence depending on how many lenders the borrower is able to visit before the lending game ends.

Equilibrium Characterization Given the discussion of borrower and lender best responses above, it is clear that a MPE of the model is equivalent to a solution a fixed point dynamic programming problem. Given lender strategies (captured by $\tilde{p}(\cdot)$), the borrower's optimal strategy is represented by a policy function $g(\cdot)$ that solves the dynamic programming problem in Equation 2.3. Given the distribution of total aggregate face value of debt D^{agg} induced by $g(\cdot)$, each lender forms rational expectations over the probability the borrower will repay, denoted by $\tilde{p}(D') = \mathbb{E}[p(D^{agg})|D']$. A set of strategies forms a MPE if lender strategies given by $\tilde{p}(\cdot)$ are rational given $g(\cdot)$, and these borrower strategies are optimal given lender strategies embodied in $\tilde{p}(\cdot)$. We now summarize this characterization of an MPE in the following Proposition.

Proposition 2.1. *A symmetric Markov Perfect Equilibrium of the lending game is characterized by functions $\tilde{p}(\cdot)$ and $g(\cdot)$ that map from cumulative debt level $D \in [0, 1]$ to the interval $[0, 1]$ such that:*

- a) $g(\cdot)$ is the policy function in the solution to the dynamic programming problem in Equation 2.3 taking $\tilde{p}(\cdot)$ as given.
- b) Lenders' perceived expected repayment probabilities $\tilde{p}(\cdot)$ used to form accept/reject strategies are correct taking $g(\cdot)$ as given.

$$\tilde{p}(D) = \mathbb{E}[p(D^{agg})|D] = 1 - \mathbb{E}[D^{agg}|D]$$

Closed Form Solution We can solve for the unique linear symmetric MPE in closed form under Assumption 2.2.1. Notice that if $\tilde{p}(D)$ were linear then the dynamic programming problem would have a linear-quadratic form, so we look for an equilibrium where $\tilde{p}(D)$ and $g(D)$ are linear functions of D . The following lemma characterizes the form of the solution to this problem. The exact expressions for the closed form solution are provided in the Appendix.

Lemma 2.1. *For $\alpha > 1$ and $1 > q \geq 0$, There exists $\ell^*(\alpha, q) \geq 1$ and $b^*(\alpha, q) \geq 1$ such that the unique linear MPE takes the following form:*

$$\begin{aligned} \tilde{p}(D) &= (1 - D) \cdot \frac{1}{\ell^*} \\ 1 - g(D) &= (1 - D) \cdot \frac{1}{b^*} \end{aligned}$$

where ℓ^* and b^* respectively parameterize lender and borrower strategy in equilibrium.

There is an intuitive interpretation for both ℓ^* and b^* . First, if the borrower arrives to a lender with current debt D , its remaining debt capacity is $1 - D$.⁸ When leaving this lender the borrower will have pledged a total of $g(D)$ and thus the borrower will have remaining debt capacity $1 - g(D)$. Therefore $\frac{1}{b^*}$ is the fraction of current borrowing capacity that remains after visiting a lender. A higher b^* (or lower $\frac{1}{b^*}$) corresponds to more aggressive borrowing by the borrower; it will deplete its available debt capacity more rapidly. Given $g(D) > D$, in equilibrium the entrepreneur borrows a positive amount from each lender it gets to visit, no matter how much it has already borrowed.

Second, recall that with commitment, the repayment probability is exactly $\tilde{p}(D) = p(D) = 1 - D$, which corresponds to $\ell^* = 1$. Thus $\ell^* > 1$ corresponds to lower expected repayment probability and higher interest rates. In other words, when ℓ^* is high the lenders makes pricing decisions *as if* they expect the borrower to accumulate substantially more debt from future lenders, which deteriorates the value of the current lender's own claims, and set interest rates to reflect this.

Finally, there is also a static interpretation for the dynamic equilibrium. Taking lender's strategy parameterized by ℓ as given (which determines $\tilde{p}(D)$ and the interest rates), the entrepreneur chooses how much debt to issue when meeting each lender. This specifies a best response $b = B(\ell)$ for the entrepreneur, which can be thought of as representing a loan demand schedule. Since higher interest rates induce the borrower to take out debt less aggressively, the loan demand schedule is downward-sloping ($B'(\ell) < 0$). On the other hand, given the aggressiveness of entrepreneur's borrowing behavior parameterized by b , the lenders are able to determine the distribution of the face value of total borrowing. This maps into the distribution of the value of their own debt claims, and lenders set their decision rule such that they only accept loans they expect to be weakly profitable, and thus specifies a best response $\ell = L(b)$ as the loan supply schedule. The interest rates lenders have to charge in order to break-even increases as the entrepreneur takes out loans more aggressively, hence the loan supply schedule is upward-sloping ($L'(b) > 0$). The unique equilibrium is then the unique intersection of the "loan demand" equation $B(\ell)$ and the "loan supply" equation $L(b)$, which is depicted in Figure 2.3.

⁸If the borrower acquired additional debt of more than $1 - D$ it would default on all debt for sure. Recall default costs $\bar{c} \sim U[0, 1]$, so if the aggregate face value of debt equals or exceeds one the debt will never be repaid, as it will be less costly to default on it no matter what realization of default costs occur.

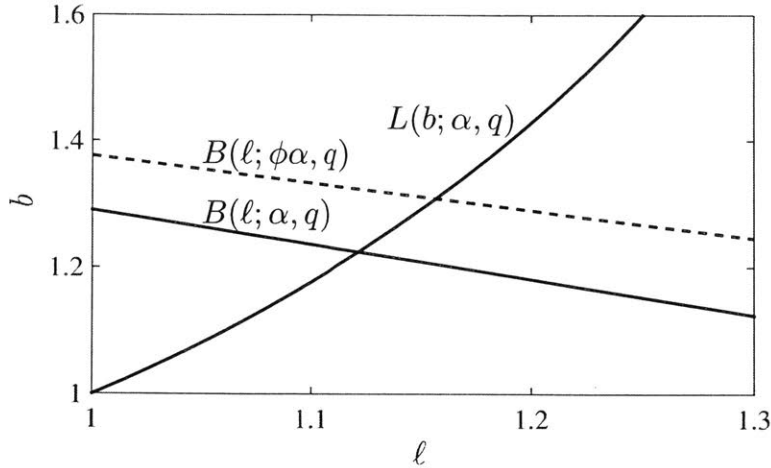


Figure 2.3: Static Representation of Lending Market Equilibrium. $\phi > 1$ indicates an increase in marginal returns to new investment. This shifts the borrower's loan aggressiveness best response curve upward but does not change the lender best response curve, which is only a function of q .

Equilibrium Choice We choose to focus on the unique linear symmetric MPE in our infinite lender model because it is closely related to the unique SPE of finite lender version of our game. Because the number of lenders in any real world credit market is finite, our infinite lender model and the linear MPE solution concept is merely an abstraction that comes with the benefits of algebraic tractability.

Formally, we define the finite N -lender game by modifying our infinite lending game as follows. After meeting the i -th lender, the borrower gets to meet $(i + 1)$ -th lender with probability q if and only if $i + 1 \leq N$, and with probability zero otherwise. That is, we truncate the game at a maximum of N lenders, while keeping the stochastic nature of the lending game unchanged for the first N lenders. Note that the finite N -lender game admits a unique SPE which can be solved by backward induction. The following proposition demonstrates that the unique linear symmetric MPE in the infinite lender game can be obtained as the limit of the sequence of the unique SPEs of the finite lender games as we take the number of lenders to infinity.

Proposition 2.2. *Fix i . In the finite N -lender game with $N > i$, as $N \rightarrow \infty$ the strategies at lender i in the unique SPE converge uniformly to the corresponding linear symmetric MPE strategy*

of the infinite-lender game.

2.4 Commitment and Investment

We now proceed to highlight the role of commitment in equilibrium investment outcomes. Because the outcome of the lending game depends on the stochastic number of lenders the borrower meets, we will focus our attention on *expected* outcomes, such as the expected face value of debt, the expected level of investment, the expected aggregate interest rate, and expected welfare. Due to the linear strategies in the game these concepts have simple closed form expressions. We first explore comparative statics of these outcomes to the model parameters q and α , which respectively correspond to the probability of meeting new lenders and the marginal return of the investment opportunities. We then show that allowing part of the raised investments to be pledgeable does not solve and in fact worsens the commitment problem.

2.4.1 More Lenders, Worse Allocations

How does the exogenous severity of the commitment problem q impact equilibrium outcomes? The expected number of lenders a borrower visits is $\frac{1}{1-q}$ which increases with q . Therefore a higher q can be seen as a worsening of the commitment problem. The following result echoes what has been experienced in the microfinance industry by a broad range of developing markets: the availability of lenders is associated not only with multiple borrowing, but also higher levels of debt, higher interest rates, and lower repayment rates.

Proposition 2.3. *The following results describe the equilibrium effects of increasing access to additional lenders, as parameterized by q , for any $\alpha > 1$ and $q > 0$:*

- a) *Expected aggregate face value of debt ($\mathbb{E}[D^{agg}]$) increases in q .*
- b) *Expected investment ($\mathbb{E}[I^{agg}]$) decreases in q .*
- c) *Expected probability of default ($\mathbb{E}[p(D^{agg})]$) increases in q .*
- d) *The ex-ante expected interest rate ($\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}]$) increases in q .*

e) Ex-ante welfare of the entrepreneur decreases in q , and in the limit as $q \rightarrow 1$ welfare converges to zero, the level that would be obtained if the entrepreneur does not have access to the lending market at all.

As q increases the expected face value of debt raised in equilibrium increases. This is the net effect of two forces. First, for higher q the borrower's demand aggressiveness curve shifts down, meaning the borrower issues a smaller fraction of its debt capacity at each round of financing for a given interest rate strategy. This is because higher q means it is more likely that the borrower will be able to exploit the externality future lenders impose and obtain favorable pricing on subsequent loans. In effect, when q is higher the borrower wants to smooth borrowing over multiple lenders to obtain better aggregate financing terms. From the lender perspective, for a given borrowing strategy a higher q increases lenders' expectations of debt dilution, causing them to raise interest rates. This further reduces borrower demand aggressiveness. However, the net reduction in aggressiveness of debt accumulation is dominated by the increasing likelihood that more loans will be realized.

In other words, while lenders respond to concerns about debt dilution by raising interest rates, without commitment this cannot prevent increased expected borrowing. As summarized in Proposition 2.3, increasing q also increases the effective interest rate and probability of default, while expected investment falls. The possibility of multiple borrowing makes it harder to use available debt capacity to fund new investment.

Entrepreneur welfare (and hence total surplus) is also declining in q . In fact, as $q \rightarrow 1$, the the commitment problem becomes so severe that while there is positive borrowing and investment, all surplus from investment is offset by costly default. The entrepreneur would be just as well off not investing at all. This result is related to the well-known Coase Conjecture in the industrial organization literature, that a durable good monopolist competing with itself inter-temporally is unable to obtain any monopoly rents as consumers become very patient.⁹ Both results illustrate the fact that dynamic commitment problems can completely unravel the ability of an agent to capture or generate surplus. In our model, welfare strictly decreases with q and at the limit of no commitment ($q = 1$) the welfare level is as if the entrepreneur does not have access to credit

⁹Specifically, the Coase Conjecture states that as consumers become extremely patient, a durable goods monopolist who cannot commit to future prices would have to sell the its good at competitive prices instantaneously and is unable to raise any profits. See Fudenberg and Tirole (Chapter 10, 1991).

markets at all.

The predictions of Proposition 2.3 bear similarity to what is being observed in microfinance markets facing increases in competition among lenders. Most directly, McIntosh, de Janvry, and Sadoulet (2005) find that increased competition between microfinance lenders in Uganda lead to increased debt accumulation and declining repayment rates of their borrowers. They cite conversations with local lenders that suggest that multiple borrowing is to blame:

“The chief executives of most of the [lending] institutions involved in this article were interviewed on the topic, and few were worried about competition insofar as it relates to the growth prospects of their institution. A common concern, however, was that, wherever two or more institutions are operating, many clients may be taking loans from several lenders simultaneously, or double-dipping...Nonetheless, they were unanimous in the opinion that this behavior does drive up default rates.”

The most striking aspect of this anecdote is not that multiple borrowing occurs in the face of competition, but that lenders are very concerned about its effects on repayment and that its incidence cannot be controlled. Such anecdotes are abundant in the narrative surrounding multiple borrowing in microfinance.¹⁰

2.4.2 Better Opportunities, Worse Commitment Problems

When returns to the investment opportunity α are higher the borrower has a greater incentive to exploit the externality associated with the commitment problem and take on more new financing at the expense of previous lenders. A central result of our paper is that for projects with higher returns, the effects of the commitment problem can be worsened to the extent that the equilibrium may involve lower levels of investment than would occur with less desirable projects.

To develop this result, first consider the effect of an increase in α on the equilibrium face value

¹⁰A set of case studies of microcredit markets in Peru, Guatemala, and Bolivia (de Janvry, Sadoulet, McIntosh, Wydick, Luoto, Gordillo, and Schuetz 2003) highlights the prevalence of multiple borrowing and high levels of debt that occurs following the rapid entry of lenders in each of these countries. Policy makers in Bangladesh are concerned that incidence of “overlapping borrowing” across multiple microfinance lenders is on the rise and incidence may be as high as 60% (Faruqee and Khalily 2011). A survey of microfinance usage in Andhra Pradesh analyzed by Johnson and Meka (2010) suggests that borrowers in Andhra Pradesh borrow from multiple lenders within the same month because they cannot obtain enough financing from individual lenders.

of debt. As shown graphically in Figure 2.3, this increases borrowing aggressiveness—the borrower shifts debt accumulation toward earlier lenders, increases the expected face value of debt. Why? At any point in the game the borrower faces the following tradeoff. Taking interest rate strategies as given, it weighs the benefits of borrowing more from the current lender for sure, or taking the gamble that it will be able to meet a new lender from which to borrow marginally at better interest rates that the previous lender would not have offered. When the returns to investment are higher this tradeoff tilts away from a less aggressive borrowing strategy of waiting to try and exploit the externality on the current lender. Of course, more aggressive borrowing means lenders have to charge higher interest rates in anticipation of higher aggregate borrowing. This result is summarized in Proposition 2.4.

Proposition 2.4. *Expected face value of debt $\mathbb{E}[D]$ and the effective interest rate $\mathbb{E}[D]/\mathbb{E}[I]$ are increasing in α , the returns to scale of the investment technology.*

Now we turn to the main result on the relationship between the equilibrium level of investment $\mathbb{E}[I]$ and the returns to scale of the new project .

Proposition 2.5. *Fix any $q \in (0, 1)$*

- a) *The equilibrium level of investment $\mathbb{E}[I]$ is non-monotone in α . In particular, there exists a cutoff $\bar{\alpha}(q)$ such that expected investment is increasing in α below this cutoff, and decreasing in α for $\alpha > \bar{\alpha}(q)$. Formally,*

$$\frac{d\mathbb{E}[I]}{d\alpha} \begin{cases} > 0 & \text{for } 1 \leq \alpha < \bar{\alpha}(q) \\ < 0 & \text{for } \alpha > \bar{\alpha}(q) \end{cases}$$

- b) *The stronger the commitment problem in the lending market, the lower is the cutoff level of marginal return $\bar{\alpha}(q)$ for decreasing investment:*

$$\frac{d\bar{\alpha}(q)}{dq} < 0$$

These results can be visualized in Figure 2.4, which plots equilibrium expected investment as a function of α for three different values of q . For all levels of q , the level of investment that the

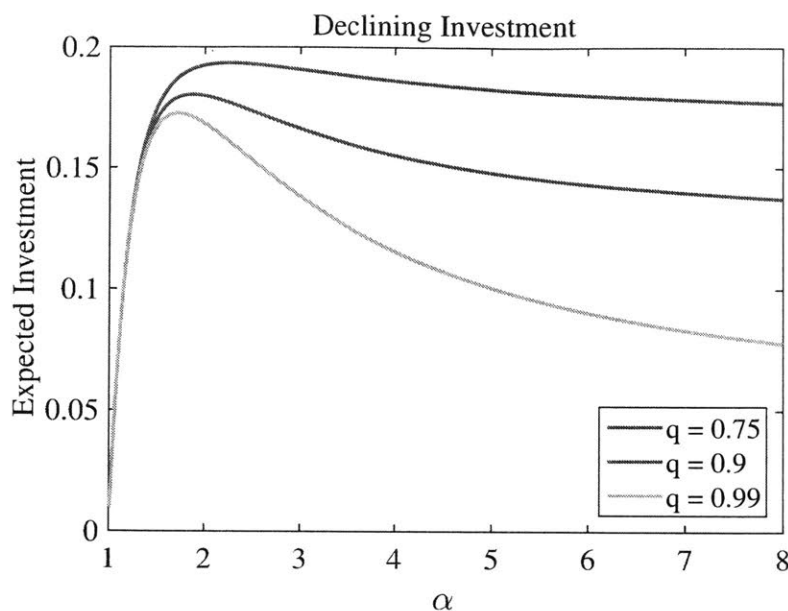


Figure 2.4: Expected Investment in the Dynamic Lending Game

entrepreneur gets to raise in expectation first increases in α and then decreases. Entrepreneurs with better opportunities could be facing tighter constraints. The second result of Proposition 2.5 shows that such endogenous misallocation of resources is more severe in markets where the commitment problem is worse: when q is higher the productivity α that maximizes equilibrium investment is lower.

The seemingly perverse outcome of lower investments in better opportunities arises because when commitment problems are present, it is possible that the equilibrium involves using available debt capacity inefficiently. To better understand the intuition behind this result, recall that the present value of promised repayments to creditors is affected by the face value of these claims in two directions. Holding repayment probability fixed, higher face value translates into higher present value. However, a higher face value of debt also reduces the probability of repayment. This gives rise to an inverted U-shaped (or “debt Laffer curve”) relationship: the value of debt is initially increasing in the amount of debt pledged, but for sufficiently high face values of debt, promising to repay an additional dollar of debt actually decreases the total amount of financing. Of course, facing an *individual* lender, no borrower would propose to borrow so much that it could receive more capital from that lender if it reduced its promised repayment. However, without commitment,

the entrepreneur will want to borrow from any future lenders it gets to meet. This means that lenders expect high future borrowing and charge high interest rates, and the equilibrium total amount of debt and investment could endogenously be on the wrong side of the *aggregate* debt Laffer curve. Proposition 2.5 shows that this happens precisely when marginal returns α are high. Entrepreneurs with the highest marginal returns have the strongest desire to borrow, hence the worst commitment problem. They in equilibrium end up promising such high levels of total repayment that they actually receive very little total investment.

These results are surprising and run against common intuition that more productive projects induce higher levels of investment despite the presence of financial constraints. In most classical theories of inefficient investment choice, investment occurs if and only if the net present value of a project exceeds a certain threshold, which can be above or below zero. Such models do not generate the stark patterns of resource misallocation that plagues developing countries. The commitment friction we explore in this paper however can generate these distortions. In fact, we show in Section 2.5 that this force is so strong that it can also distort the influence the choice of investment opportunity chosen by entrepreneurs.

2.4.3 Pledgeability Does Not Solve The Commitment Problem

The results presented thus far have focused on a simple borrowing environment in which investment capital raised through borrowing is not pledgeable. If the investment can be (partially) recouped by lenders in case of default, the entrepreneur can increase leverage and raise more investments for any given dollar of risky debt. Therefore, it seems reasonable to expect that pledgeability of investments can reduce inefficiency in the lending market. However, this intuition is incomplete. High leverage induced by partial pledgeability effectively raises marginal returns of issuing risky debt, thereby increases the desire to borrow and worsens the commitment problem. This insight can be illustrated through a simple extension in our model.

Specifically, suppose a fraction δ of the investment is pledgeable. Thus, in addition to promising a risky claim D_i , the borrower can credibly pledge to repay an amount $\delta \times L_i$ with certainty. Thus, if the borrower has already promised total risky claims D to previous lenders is issuing additional

risky debt D_i to lender i , the current lender's expected repayment amount is $\delta L_i + \bar{p}(D + D_i)D_i$. The borrower's value function can now be expressed as:

$$V(D) = \max_{D'} (\alpha - \delta) \frac{\bar{p}(D')(D' - D)}{1 - \delta} - (1 - q) \mathbb{E} [\min(D', \bar{c})] + qV(D'). \quad (2.4)$$

For any risky debt offering D_i , the lender is willing to supply $L_i \leq \frac{D_i \bar{p}(D + D_i)}{1 - \delta}$. The multiplier $\frac{1}{1 - \delta}$ can be interpreted as the leverage made possible by pledgeability. Because the borrower has to pay back δ fraction of the investment, the return on investment becomes $(\alpha - \delta) L_i$. Defining $\hat{\alpha} \equiv \frac{\alpha - \delta}{1 - \delta}$ one can see that Equation 2.4 is identical to Equation 2.3 with α replaced by $\hat{\alpha}$. This isomorphism to the baseline model allows us to establish the following claims.

Proposition 5'. *For any $q \in (0, 1)$ and $\delta \in [0, 1)$:*

- a) *The equilibrium level of investment $\mathbb{E}[I]$ is non-monotone in α . In particular, there exists a cutoff $\bar{\alpha}(q, \delta)$ such that expected investment is increasing in α below this cutoff, and decreasing in α for $\alpha > \bar{\alpha}(q, \delta)$.*
- b) *The cutoff level of marginal return $\bar{\alpha}(q, \delta)$ for decreasing investment is decreasing in q and δ :*

$$\frac{\partial \bar{\alpha}(q, \delta)}{\partial q} < 0, \quad \frac{\partial \bar{\alpha}(q, \delta)}{\partial \delta} < 0.$$

Proposition 5' verifies that the results of Proposition 2.5 hold in a model with partial pledgeability of investment capital. As long as $\delta < 1$, investment will be decreasing in marginal return α for sufficiently large α . For higher δ , the α that maximizes investment will be lower, reflecting that increased pledgeability actually makes commitment problems worse. This result might seem especially paradoxical because under full-pledgeability ($\delta = 1$), the investment opportunity can be self-financed and there is essentially no financial friction. To understand this, note that despite worse commitment externalities under high pledgeability, the level of investment always increases in pledgeability due to the canonical effect that each dollar of risky debt can be used to finance more investment. At δ close to one the commitment problem is severe but does not dominate the fact that nearly full pledgeability essentially removes financial frictions.

2.5 Endogenous Project Choice

The analysis in Section 2.4 demonstrates that high marginal returns dynamically exacerbate commitment problems. This suggests that the financing of projects with declining marginal returns in investment may be less distorted by lack of commitment. For projects with concave returns, obtaining sufficiently large amounts of capital from early lenders would lower the marginal returns to future borrowing, limiting borrowers' ex-post incentives to contract with additional lenders and therefore lowering ex-ante interest rates. Thus, concavity of investment returns can be desirable because it embeds endogenous commitment power. We now show that if entrepreneurs can select among a menu of projects to finance, lack of explicit commitment may distort this choice away from projects with the highest level of returns toward those with more concave returns.

We demonstrate the role of concavity in our model by studying a particularly tractable function form of returns to investment. Consider investment opportunities with linear returns up to a certain size of investment, but which deliver zero marginal return for any additional investment beyond that point. We denote these as “linear-flat” projects. Formally, a project is linear-flat if the return function can be parameterized by a slope parameter α and a cutoff parameter \bar{L} such that $R(I; \alpha, \bar{L}) = \alpha \min(I, \bar{L})$. The parameter \bar{L} can be interpreted as the *scale* of the investment opportunity because it imposes an effective ceiling on investment. The following proposition establishes that fixing any α , the entrepreneur's preference over the project's scale is maximized at some $\bar{L}^* < \infty$. In other words, more is not always better, and given the choice the entrepreneur could strictly prefer investment opportunities with smaller scales.

Proposition 6. *Fix $\alpha > 1$.*

- a) There exists a cutoff $\bar{L}^*(\alpha) < \infty$ that maximizes the entrepreneur's welfare. Specifically, welfare is strictly lower for projects with scale $\bar{L} > \bar{L}^*(\alpha)$.
- b) $\bar{L}^*(\alpha)$ is equal to the level of investment that would maximize welfare if the borrower could commit to exclusive contracting with a single lender.

The intuition behind Proposition 6 is simple. Consider the linear-flat project parametrized by scale $\bar{L}^*(\alpha)$. It is clear that without explicit commitment power, this project achieves the same

lending outcome as the linear project $R(I) = \alpha I$ would in the case of full commitment to a single lender. There are no marginal returns from borrowing beyond $\bar{L}^*(\alpha)$ in the linear-flat project, so the first lender that the entrepreneur visits would be willing to accept a loan at the same terms a borrower would propose if it could explicitly commit not to borrow from other lenders. Any equilibrium allocation of a linear flat project with $\bar{L} \neq \bar{L}^*(\alpha)$ is feasible in the full-commitment linear project model, but not optimal, and thus must generate lower surplus. In this example the extreme concavity of investment opportunity effectively solves the commitment problem. For project scales above $\bar{L}^*(\alpha)$, the commitment externality is present and the entrepreneur would benefit from reducing the scale of the investment such that loans could be obtained at lower interest rates.

As a corollary to Proposition 6, the entrepreneur may prefer an investment opportunity that is strictly dominated by another in terms of having strictly lower returns for any level of investment.¹¹ Thus, if the entrepreneur gets to choose ex-ante which investment projects to undertake, they may *endogenously* pursue an investment opportunity with lower returns, as long as it is sufficiently concave.

The results in this section highlights that in the presence of the commitment problem there is a powerful trade-off between the average and marginal returns of investment opportunities. Productivity is no longer the sole determinant of investment in economic activities. The commitment friction can be mitigated by undertaking instead in projects that have concave returns and thus embed some degree of commitment. Once these projects interior to the technology frontier are undertaken, they exhibit slower capital accumulation and growth potential.

2.6 Policy Implications

We now turn to studying simple regulatory policy tools that can improve outcomes in our model: limiting interest rates, imposing total borrowing limits, and limiting the number of lenders from

¹¹Consider for example a project with linear returns and no maximum scale, $R(I) = \alpha I$. Given α , we can solve for the $\alpha' < \alpha$ such that the linear-flat project parameterized by $(\alpha', \bar{L}^*(\alpha'))$ would provide the entrepreneur with the same welfare as the linear project. Then any linear-flat project $(\bar{\alpha}, \bar{L}^*(\bar{\alpha}))$ for $\bar{\alpha} \in (\alpha, \alpha')$ would generate a larger surplus than the linear project, even though the return to investment of the linear project is greater, for any level of investment, than the returns to these linear-flat projects.

which a borrower can obtain loans. Conventional arguments suggest that interest rate caps may be helpful in improving allocations when there is a *lack* of competition among lenders as they limit monopoly power. Yet in some scenarios, it seems to have been the *entry* of new lenders into markets and the resulting increase in competition that has driven regulators to consider usury regulations. The microfinance crisis in Andhra Pradesh was precipitated by the rapid entry of thousands of new microfinance lenders and characterized by over indebtedness of borrowers from multiple lenders. A report commissioned by the Reserve Bank of India to study the causes and potential regulatory responses to the Andhra Pradesh crisis states:

“It has been suggested that with the development of active competition between MFIs there has been a deluge of loan funds available to borrowers which has fueled excessive borrowing and the emergence of undesirable practices ... Finally, it is believed that in consequence of over-borrowing, default rates have been climbing in some locations but these have not been disclosed because of ever-greening and multiple lending.” Malegam (2011).

Despite highlighting a high degree of competition, the report proceeds to propose regulation that limits interest rates charged to borrowers. The following proposition illustrates that, through the lens of our model of multiple borrowing and commitment problems, this type of regulatory response is rational and welfare improving.

Proposition 7. *Adding an upper bound on interest rates generates the following results:*

- a) *For any increasing and concave investment opportunity $R(\cdot)$ with $R'(0) > 1$, there is an optimal interest rate cap $\bar{r}^{SL}(R)$ that induces the full commitment allocation with the borrower obtaining funding from a single lender. When $R(I) = \alpha I$, the optimal interest rate cap is $\bar{r}^{SL} \equiv 1 - \alpha^{-1}$.*
- b) *Any interest rate cap \bar{r} has an associated debt limit $\bar{D} = 1 - (1 + \bar{r})^{-1}$ that induces the same equilibrium.*
- c) *If $\bar{r} < \bar{r}^{SL}$ then single lender borrowing prevails but debt is inefficiently low and the interest rate cap is too restrictive. Welfare will be lower than the unregulated equilibrium (i.e. with no interest rate cap) if the interest rate cap is sufficiently low.*

d) If $\bar{r} > \bar{r}^{SL}$ then the interest rate cap is too loose. Imposing the cap increases expected investment and welfare while lowering expected debt and interest rates relative to the unregulated equilibrium. As $\bar{r} \rightarrow \infty$ the interest rate cap becomes irrelevant and outcomes converge to the unregulated equilibrium.

Interest rate caps have the potential to improve welfare because they embed commitment power. Recall that in the model the more debt the borrower has outstanding, the lower the probability of repayment, and thus the higher interest rates need to be on additional lending. Interest rate caps add commitment power because marginal borrowing at high enough levels of debt would need to violate the interest rate cap for these loans to break even and thus are never issued. Early lenders can then be assured that such future borrowing, which increases the anticipated probability of default, will not occur, and can provide initial loans at interest rates that are closer to those that would prevail if borrowers could fully commit to exclusive borrowing. This better aligns borrower incentives, reducing face values of debt and interest rates in equilibrium. Importantly, interest rate caps also increase investment and improve welfare as long as they are not too severe that they prevent productive investment.

Further, for a given investment opportunity the interest rate cap can be set to fully overcome the inefficiencies induced by lack of commitment. By setting the interest rate cap at exactly the interest rate that would prevail in the full-commitment equilibrium, the borrower can credibly raise exactly the full commitment level of debt, and at the commitment-level interest rate, restoring the full commitment outcomes. By proposing such a loan to the first lender, the lender is assured that any future borrowing would necessarily need to be at interest rates above the allowed limit, and can thus be sure that no such additional borrowing would take place. Thus this loan proposal is expected to earn zero profits for the lender and is always accepted. By definition this allocation maximizes the entrepreneur's welfare given lenders at least break even, so any rational lender strategy induces this loan proposal as a best response.

In our model a limit on the total face value of debt a borrower can obtain is mechanically equivalent to a particular interest rate cap, and the equivalent debt limit is increasing in the interest rate cap, so all the results above also apply to total debt limits. Both total borrowing limits and interest rate caps were adopted for microfinance loans in India in 2011 (Dr. D. Subbarao

(2011)). These policies are equivalent in the model because they both operate by shutting down contingencies of excessive debt accumulation: debt limits directly and interest rate limits indirectly through the fact that the lower bound on interest rates in any equilibrium is increasing in the total cumulative face value of debt. In the contingencies (potentially off-equilibrium) in which either the interest rate or debt limit is binding, the lender zero profit condition implies a unique relationship between interest rates and total face value of debt at this point: $p(\bar{D}) = (1 + \bar{r})^{-1}$, where \bar{D} is a borrowing limit and \bar{r} is the equivalent interest rate limit.

It is important to re-emphasize that in general interest rate caps (and total borrowing limits) have ambiguous implications for welfare, because caps that are too low can restrict productive investment. While there are always welfare improving interest rate limits for a given project, imposing a market-wide policy can have ambiguous effects on welfare if the investment opportunities in the economy are sufficiently heterogeneous. From a utilitarian perspective, however, “reasonable” interest rate caps can be quite beneficial if many projects in the economy could benefit from them. Further, taking into account the possibility of endogenous project choice, interest rate caps have the potential to “unlock” the best projects available that were previously infeasible due to the endogenous credit constraint induced by lack of commitment.

Finally, a surprising and controversial policy recommendation of Malegam (2011) was to limit borrowers to obtaining loans from at most two microfinance lenders. Limiting the number of lenders from which a borrower can obtain loans will mechanically increase commitment power by limiting the opportunities a borrower has not to commit. While this policy was not ultimately adopted, our model shows that in the face of commitment problems such a policy may actually be quite helpful.

2.7 Conclusion

This paper argues that commitment problems in lending markets can explain emerging empirical evidence that the rapid expansion of credit access can have perverse effects. When borrowers cannot commit to exclusive contracting, increasing the availability of lenders makes markets appear *less* competitive as interest rates rise and entrepreneur investment and welfare fall. More importantly, commitment problems can result in better projects receiving less investment than worse projects.

This force can be so severe that what look like good opportunities are passed over for inferior investment technology. Finally, we show how simple regulatory tools such as interest rate ceilings and debt limits can improve outcomes and ameliorate the misallocative forces we highlight.

The intuition for these result is that the externalities the lenders impose on each other when commitment or contingent contracting is not possible can prevent the borrower from being able to use pledgeable cash flows efficiently. When explicit commitment is impossible, there is value in any implicitly commitment mechanisms that attenuate the demand for further borrowing. Thus, the return profile of an investment opportunities itself is an important driver in the severity of commitment distortions.

Since commitment is less of a problem for projects with lower marginal returns, when given the choice entrepreneurs will endogenously choose investment opportunities that are everywhere less productive than other available opportunities, as long as they are sufficiently more concave. Thus our model provides a new micro-foundation for the idea that commitment problems in lending markets can induce substantial misallocation in capital investment and can explain observations both of low growth and of economic activity below the technological frontier. While we have augmented our study with a sample of the growing anecdotal evidence that multiple borrowing is problematic, there is much more to learn. Formally testing the empirical validity the mechanisms we highlight in explaining the failure of increased access to finance to significantly improve outcomes is an important topic for future research.

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2.A Appendix

Microfounding Random Default with Moral Hazard

In this section we provide an alternative microfoundation of the random default through limited pledgeability and moral hazard. Instead of assuming the entrepreneur borrows against his default cost \bar{c} as we do in the main text of the paper, we endow the entrepreneur with a pledgeable stochastic cashflows from assets-in-place that are realized at $s = 2$. The pledgeable cashflows realize one of two values: zero or one. The good payoff occurs with probability p , which is chosen by the entrepreneur at quadratic effort cost p^2 . Moral hazard arises because effort is chosen after cashflows are (partially) pledged to lenders. Since there are only two realizations of these cashflows and the entrepreneur has no other pledgeable wealth, it is without loss of generality that claims issued to lenders take the form of debt contracts: they are repaid in full when the good realization occurs and the entrepreneur defaults when cashflows are zero. With this in mind, assume the entrepreneur has issued a total face value of debt D in the first stage. At $s = 2$ the entrepreneur solves

$$p(D) \equiv \arg \max_p p[1 - D] - p^2$$

The entrepreneur expects the assets in place to pay out with probability p , and conditional on a positive payout the entrepreneur gets to keep $1 - D$ of the cashflows as the residual claim. The cost of choosing the probability of positive cashflows to be p is p^2 . Thus $p(D)$ denotes the solution to the entrepreneur's choice of effort at stage $s = 2$ conditional on having a total outstanding face value of debt D . The solution of entrepreneur's problem yields $p(D) = \frac{1-D}{2}$, and entrepreneur's expected payoff from the residual claim is $\frac{(1-D)^2}{4}$. The value function in (2.3) is modified to

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) - (1 - q) \frac{(1 - D)^2}{4} + qV(D')$$

and all of our results go through analogously.

Proof of Lemma 2.1

Borrower's problem can be formulated recursively as:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) + (1 - q) \left[\frac{(1 - D')^2}{2} - \frac{1}{2} \right] + qV(D')$$

where

$$\tilde{p}(D') \equiv \mathbb{E}[1 - D^{agg}|D']$$

We guess that borrower's policy function $g(\cdot)$ and lenders' loan pricing function $\tilde{p}(\cdot)$ both take a linear form and are each characterized by a single endogenous variable, b and ℓ , respectively:

$$\tilde{p}(D) = \ell^{-1} (1 - D)$$

$$1 - g(D) = b^{-1} (1 - D)$$

To solve for borrower's policy function, we proceed to take first order condition and use the envelope condition for borrower's problem. The first order condition is:

$$-\alpha \ell^{-1} (g(D) - D) + \alpha \ell^{-1} (1 - g(D)) - (1 - q) (1 - g(D)) + qV'(g(D)) = 0$$

and the envelope condition is:

$$V'(D) = -\alpha \ell^{-1} (1 - g(D))$$

Plugging the envelope condition into the first-order condition and after simplifying, we can express the Euler condition as a quadratic function of b^{-1} :

$$q\alpha \ell^{-1} b^{-2} + (1 - q - 2\alpha \ell^{-1}) b^{-1} + \alpha \ell^{-1} = 0$$

We thus solve for the endogenous parameter b^{-1} that governs the borrower's policy function as:¹²

¹²There are two roots to the quadratic equation, one of which leads to explosive debt accumulation. We choose the other, stable root.

$$b^{-1} = \frac{(2\alpha\ell^{-1} - (1-q)) - \sqrt{(1-q - 2\alpha\ell^{-1})^2 - 4q\alpha^2\ell^{-2}}}{2q\alpha\ell^{-1}} \quad (2.5)$$

To solve for the lender's loan pricing function, note

$$\begin{aligned} \tilde{p}(D) &= \mathbb{E}[1 - D^{agg}|D] \\ &= (1-q) \left[(1-D) + q(1-g(D)) + q^2(1-g(g(D))) + \dots \right] \\ &= (1-q) \left[(1-D) + qb^{-1}(1-D) + q^2b^{-2}(1-D) + \dots \right] \\ &= \frac{1-q}{1-qb^{-1}}(1-D) \end{aligned}$$

Hence

$$\ell^{-1} = \frac{1-q}{1-qb^{-1}} \quad (2.6)$$

Equation (2.5) characterizes b^{-1} as a decreasing function of ℓ^{-1} . On the other hand, equation (2.6) characterizes ℓ^{-1} as an increasing function of b^{-1} . The two equations therefore yields a unique solution (b^*, ℓ^*) for each $q \in [0, 1)$ and $\alpha \in (1, \infty)$. In particular, we have

$$(b^*)^{-1} = \frac{(2\alpha - 1 - \sqrt{4(1-q)(\alpha^2 - \alpha) + 1})}{2q(\alpha - 1)}$$

□

Lemma 2.2. *The best response functions have the following properties:*

$$\begin{aligned} \frac{\partial \ell(b; \alpha, q)}{\partial b} &\geq 0; & \frac{\partial \ell(b; \alpha, q)}{\partial \alpha} &= 0; \\ \frac{\partial b(\ell; \alpha, q)}{\partial \alpha} &\geq 0; & \frac{\partial b(\ell; \alpha, q)}{\partial \ell} &\leq 0; \end{aligned}$$

where the inequalities are strict for $q \in (0, 1)$ and $\alpha > 1$.

Proof. The results with respect to lender's best response immediately follow from equation (2.6).

We now with with equation (2.5) to derive the results respect to borrower's best response

function. Let $x \equiv 2\alpha\ell^{-1}$, we have

$$\begin{aligned} b^{-1} &= \frac{(x - (1 - q)) - \sqrt{(1 - q - x)^2 - qx^2}}{qx} \\ &= \frac{1}{q} - \frac{1 - q + \left((1 - q)(x - 1)^2 - q(1 - q) \right)^{\frac{1}{2}}}{qx} \end{aligned}$$

Let $\Delta \equiv \left((1 - q)(x - 1)^2 - q(1 - q) \right)$ and take derivative with respect to x , we have

$$\begin{aligned} \frac{\partial b^{-1}}{\partial x} &= - \frac{qx(1 - q)(x - 1) - q \left((1 - q)\Delta^{\frac{1}{2}} + (1 - q)(x - 1)^2 - q(1 - q) \right)}{(qx)^2 \Delta^{\frac{1}{2}}} \\ &= - \frac{q(1 - q)}{(qx)^2 \Delta^{\frac{1}{2}}} \left(x^2 - x - \Delta^{\frac{1}{2}} - (x - 1)^2 + q \right) \\ &= - \frac{q(1 - q)}{(qx)^2 \Delta^{\frac{1}{2}}} \left(x + q - 1 - \Delta^{\frac{1}{2}} \right) \end{aligned}$$

Since $\Delta^{\frac{1}{2}} = \sqrt{(x + q - 1)^2 - qx^2} \leq x + q - 1$, we have that

$$\frac{\partial b^{-1}}{\partial x} \leq 0$$

and the inequality is strict for $q \in (0, 1)$. Given the definition of x , we have that $\frac{\partial b(\ell; \alpha, q)}{\partial \alpha} \geq 0$ and $\frac{\partial b(\ell; \alpha, q)}{\partial \ell} \leq 0$. \square

Proof of Proposition 2.2

We begin with an outline of the proof. The proof will first show how to solve the finite-lender game by backwards induction, generating a recursive formulation for borrower and lender strategies. Next, we show that the fixed point of this recursion generates the strategies of the infinite-lender equilibrium defined in the main text. Finally, to demonstrate convergence, we show that this recursive formulation of strategies is characterized by a contraction mapping. This implies that, considering the strategies at a given lender, as the number of potential subsequent lenderse goes to infinity, the equilibrium strategies at this lender converge uniquely to the fixed point and thus to the strategies of the infinite-lender equilibrium.

Backward Induction in the Finite-Lender Game

Consider a finite version of the game with N lenders. For this proof, we abuse notation and index periods counting backwards from the end. Thus the last lender is indexed 1, and the first lender is indexed N . Therefore, after lender i there are at most $i - 1$ more lenders for the borrower to visit. Let D_i denote the amount of cumulative debt the borrower accumulates from meeting lenders N through $i + 1$, thus D_0 denotes the total amount of debt the borrower will accumulate if it gets to meet all N lenders. The probability of default from the last lender's perspective will be $\tilde{p}_1(D_0) = 1 - D_0$ if the lender accepts a proposal that brings the borrower's cumulative debt to D_0 . This defines the unique strategy of the final lender to accept only weakly profitable loans. Assume the borrower has any arbitrary face value of debt D_1 upon meeting the final lender. The borrower solves

$$V_1(D_1) \equiv \max_{D_0} \alpha (D_0 - D_1) (1 - D_0) + \left(\frac{D_0^2}{2} - D_0 \right)$$

and the solution is

$$1 - D_0 = (1 - D_1) \underbrace{\frac{\alpha}{2\alpha - 1}}_{\equiv B_1}$$

with borrower's maximized value function being

$$V_1(D_1) = (1 - D_1)^2 \underbrace{\left[\frac{2\alpha^3 - \alpha^2}{2(2\alpha - 1)^2} \right]}_{\equiv W_1} - \frac{1}{2}$$

We define

$$\begin{aligned} B_1 &\equiv \frac{\alpha}{2\alpha - 1} \\ L_1 &\equiv 1 \\ W_1 &\equiv \frac{2\alpha^3 - \alpha^2}{2(2\alpha - 1)^2} \end{aligned}$$

Thus the unique subgame perfect equilibrium strategies conditional on arriving to the last lender with some amount of debt D are:

$$\begin{aligned} 1 - g(D) &= B_1(1 - D) \\ \tilde{p}_1(D) &= L_1(1 - D) \end{aligned}$$

and any borrower considering leaving the second-to-last lender with a total face value of debt D realizes that continuation utility if it reaches the last lender is given by

$$V_1(D) = W_1(1 - D)^2 - \frac{1}{2}.$$

Thus we know that at lender $i = 1$ players use strategies linear in $1 - D$. Now we show by induction that all lenders use such linear strategies. Assume for some n that players at all stages $i < n$ use linear strategies and that the maximized value function at lender i is proportional to $(1 - D_{i+1})^2$. We will show that players at stage n also use linear strategies and that the maximized value function at lender n is proportional to $(1 - D_{n+1})^2$, and thus by induction prove that these claims do indeed hold for all $n \in \mathbb{N}$.

Now consider the subgame where the borrower meets lender n with cumulative debt D_n obtained from previous lenders. Since all future lenders and borrowers use linear strategies, we can compute lender n 's expected probability of repayment:

$$\begin{aligned} \tilde{p}_n(D_{n-1}) &= (1 - q)(1 - D_{n-1}) + q\tilde{p}_{n-1}(D_{n-2}) \\ &= (1 - q)(1 - D_{n-1}) + qL_{n-1}(1 - D_{n-2}) \\ &= (1 - q)(1 - D_{n-1}) + qL_{n-1}B_{n-1}(1 - D_{n-1}) \\ &= [(1 - q) + qB_{n-1}L_{n-1}](1 - D_{n-1}) \end{aligned}$$

where the first to second line follows from the assumption that lender $n - 1$ is using a linear strategy $\tilde{p}_{n-1}(D) = L_{n-1}(1 - D)$, and moving from the second to the third line relies on the assumption

that the borrower at $n - 1$ is using a linear strategy $1 - D_{n-2} = B_{n-1} (1 - D_{n-1})$. Thus we know that lender n follows the strategy given by

$$L_n = 1 - q + qB_{n-1}L_{n-1}$$

Next, under our inductive hypothesis we can write the borrower's problem visiting lender n as:

$$\begin{aligned} V_n(D_n) &= \max_{D_{n-1}} \alpha \tilde{p}_n(D_{n-1}) (D_{n-1} - D_n) + (1 - q) \left(\frac{D_{n-1}^2}{2} - D_{n-1} \right) + qV_{n-1}(D_{n-1}) \\ &= \max_{D_{n-1}} \left\{ \alpha (D_{n-1} - D_n) (1 - D_{n-1}) L_n + (1 - q) \left(\frac{D_{n-1}^2}{2} - D_{n-1} \right) \right. \\ &\quad \left. + q \left(W_{n-1} (1 - D_{n-1})^2 - \frac{1}{2} \right) \right\} \end{aligned}$$

Taking the first order condition and solving for D_{n-1} verifies that the borrower's strategy does indeed have the hypothesized linear form:

$$1 - D_{n-1} = (1 - D_n) \underbrace{\frac{\alpha L_n}{2\alpha L_n - (1 - q + 2qW_{n-1})}}_{\equiv B_n}$$

and maximized value function

$$V_n(D_n) = (1 - D_n)^2 \underbrace{\left(\alpha (1 - B_n) B_n L_n + (1 - q) \frac{B_n^2}{2} + qB_n^2 W_{n-1} \right)}_{\equiv W_n} - \frac{1}{2}$$

Thus the inductive proof is completed and all strategies satisfy the proposed form.

Recursive Formulation of Strategies

It is clear from above that the vector (B_n, L_n, W_n) is generated by a system of 3 difference equations:

$$\begin{aligned} L_n &= (1 - q) + qB_{n-1}L_{n-1} \\ B_n &= \frac{\alpha L_n}{2\alpha L_n - (1 - q + 2qW_{n-1})} \\ W_n &= \alpha(1 - B_n)B_n L_n + (1 - q)\frac{B_n^2}{2} + qB_n^2 W_{n-1} \end{aligned}$$

rearranging so each of (L_n, B_n, W_n) is a function of only the lagged variables:

$$\begin{aligned} L_n &= 1 - q + qB_{n-1}L_{n-1} \\ B_n &= \frac{\alpha(1 - q + qB_{n-1}L_{n-1})}{2\alpha(1 - q + qB_{n-1}L_{n-1}) - (1 - q + 2qW_{n-1})} \\ W_n &= \frac{\alpha^2(1 - q + qB_{n-1}L_{n-1})^2}{4\alpha(1 - q + qB_{n-1}L_{n-1}) - 2(1 - q + 2qW_{n-1})} \end{aligned}$$

where the last equation can be simplified to

$$W_n = \frac{\alpha}{2}B_n L_n.$$

Convergence to Infinite-Lender Strategies

Defining a new variable $x_n \equiv B_n L_n$, the set of difference equations above can be rewritten as

$$\begin{aligned} L_n &= (1 - q) + qx_{n-1} \\ B_n &= \frac{\alpha((1 - q) + qx_{n-1})}{2\alpha((1 - q) + qx_{n-1}) - (1 - q + \alpha qx_{n-1})} \\ W_n &= \frac{\alpha}{2}x_n \end{aligned}$$

Hence the sequence $\{x_n\}$ is defined by $x_1 \equiv B_1 L_1 = \frac{\alpha}{2\alpha-1}$ and a continuous function $f(\cdot)$ such that $x_n = f(x_{n-1})$, where

$$f(x) = \frac{\alpha((1-q) + qx)^2}{(1-q)(2\alpha-1) + \alpha qx}$$

First note that $f(x) > 0$ if $x > 0$, and since $x_1 > 0$, we have $x_n > 0$ for all n . It is also easily verified that $f(\cdot)$ has a unique fixed point x^* , which corresponds to the unique fixed point (B^*, L^*, W^*) of the system of difference equations above. Moreover, it is a matter of algebra to show that the fixed point coincides with our closed form solution of the infinite lender game, implying that $b^{*-1} = B^*$ and $\ell^{*-1} = L^*$.

We next show that $f(\cdot)$ defines a contraction mapping which, given the continuity of $f(\cdot)$, shows that $x_n \rightarrow x^*$. This in turn implies $(B_n \rightarrow b^{*-1}, L_n \rightarrow \ell^{*-1})$ as $n \rightarrow \infty$. In words, this means that as the number of potential future lenders in the game after lender n grows towards infinity, the unique subgame perfect strategies the players at stage n converge to the stationary strategies employed by all players in the infinite-lender game.

To show $f(\cdot)$ defines a contraction mapping, we show $\|f'(x)\| < 1$. Taking derivatives of f with respect to x , we have (after much simplification)

$$\begin{aligned} f'(x) &= q - \frac{q(\alpha-1)^2(1-q)^2}{(2\alpha+q-2\alpha q+\alpha qx-1)^2} \\ &= q \left(1 - \left(\frac{(\alpha-1)(1-q)}{(2\alpha-1)(1-q)+\alpha qx} \right)^2 \right) \end{aligned}$$

Since $x > 0$, $1 > q \geq 0$, $\alpha > 1$, we have

$$\begin{aligned} 0 &< \frac{(\alpha-1)(1-q)}{(2\alpha-1)(1-q)+\alpha qx} < 1 \\ 0 &< \frac{(\alpha-1)(1-q)}{(\alpha-1)(1-q)+\alpha(1-q)+\alpha qx} < 1 \end{aligned}$$

Hence

$$0 < f'(x) < 1$$

which shows that $f(\cdot)$ is a contraction mapping.

□

Proof of Proposition 2.3

Before proving the comparative static propositions, it will be useful to derive some expressions for equilibrium objects of interest in terms of parameters α and q . Let I^{agg} denote the aggregate investment and D^{agg} denote the aggregate debt that have been attained when the lending market game ends. In equilibrium, ex-ante, these are random variables with respect to the number of lenders the borrower will be able to visit.

Lemma 2.3. $\mathbb{E}[I^{agg}] = \mathbb{E}[p(D^{agg})D^{agg}]$

Proof. Let N denote the random number of lenders the borrower gets to visit before losing access to the lending market game. The random aggregate face value of debt and aggregate investment can be expressed as:

$$D^{agg} = \sum_{j=1}^{\infty} D_j \mathbf{1}(N \geq j)$$

$$I^{agg} = \sum_{j=1}^{\infty} I_j \mathbf{1}(N \geq j)$$

where D_j is the amount of debt given by the j -th lender. Similarly denote I_j to be the amount of investment capital provided by the j -th lender. Pick any $j > 0$, the zero-profit condition for his loan and investment size is:

$$\mathbb{E}[p(D^{agg}) | N \geq j] D_j \mathbf{1}(N \geq j) = I_j \mathbf{1}(N \geq j)$$

Taking expectation over N on both sides and applying the law of iterated expectation, we get:

$$\mathbb{E}[p(D^{agg}) D_j \mathbf{1}(N \geq j)] = \mathbb{E}[I_j \mathbf{1}(N \geq j)]$$

We next sum the previous equation over all lenders. By the linearity of the expectations operator,

we can bring the sum inside:

$$\mathbb{E} \left[p(D^{agg}) \sum_{j=1}^{\infty} D_j \mathbf{1}(N \geq j) \right] = \mathbb{E} \left[\sum_{j=1}^{\infty} I_j \mathbf{1}(N \geq j) \right]$$

Substituting in the definitions of D^{agg} and I^{agg} :

$$\mathbb{E} [p(D^{agg}) D^{agg}] = \mathbb{E} [I^{agg}]$$

□

Lemma 2.4. *We can express the expected debt and investment as functions of b^{-1} :*

$$\mathbb{E} [D^{agg}] = \frac{1 - b^{-1}}{1 - qb^{-1}}$$

$$\mathbb{E} [I^{agg}] = \frac{b^{-1} (1 - b^{-1}) (1 - q)}{(1 - b^{-1}q) (1 - b^{-2}q)}$$

Proof. Denote the expected aggregate debt upon leaving a given lender with cumulative debt D as $\mathbb{E} [D^{agg}|D]$. From lender's zero-profit condition, we have

$$\mathbb{E} [D^{agg}|D] = 1 - \tilde{p}(D)$$

The ex-ante expected aggregate debt $\mathbb{E} [D^{agg}]$ is simply the expected aggregate debt upon leaving a lender with zero outstanding debt, times $\frac{1}{q}$ (since the borrower meets the first lender with certainty, not probability q). Thus we have

$$\begin{aligned} \mathbb{E} [D^{agg}] &= \frac{1}{q} \mathbb{E} [D^{agg}|0] \\ &= \frac{1}{q} \left(1 - \frac{1 - q}{1 - qb^{-1}} \right) \\ &= \frac{1 - b^{-1}}{1 - qb^{-1}} \end{aligned}$$

To get the expression for expected investment:

$$\begin{aligned}
\mathbb{E}[I] &= \tilde{p}(g(0))g(0) + q\tilde{p}(g^2(0))\left[g^2(0) - g(0)\right] + q^2\tilde{p}(g^3(0))\left[g^3(0) - g^2(0)\right] + \dots \\
&= \ell^{-1}\left[(1-g(0))g(0) + q(1-g^2(0))\left[(1-g(0)) - (1-g^2(0))\right] + \dots\right] \\
&= \ell^{-1}\left[b^{-1}(1-b^{-1}) + qb^{-2}\left[b^{-1} - b^{-2}\right] + q^2b^{-3}\left[b^{-2} - b^{-3}\right] + \dots\right] \\
&= \ell^{-1}b^{-1}(1-b^{-1})\left[1 + qb^{-2} + q^2b^{-4} + \dots\right] \\
&= \ell^{-1}b^{-1}\frac{1-b^{-1}}{1-qb^{-2}} \\
&= \frac{b^{-1}(1-b^{-1})(1-q)}{(1-b^{-1}q)(1-qb^{-2})}
\end{aligned}$$

□

Lemma 2.5. Let $z \equiv \sqrt{4(1-q)(\alpha^2 - \alpha) + 1}$. The analytic solution of expected debt, investment, and welfare can be expressed as the following functions of parameters q and α :

$$\mathbb{E}[D^{agg}] = \frac{2\alpha - 1 - z}{2q\alpha}$$

$$\mathbb{E}[I^{agg}] = \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)}$$

$$V(0) = \frac{1 - 2\alpha(1 - q) - 2q + z}{4q}$$

Proof. These expressions can be obtained by substituting the analytic solution of b^* from lemma 1 into the expressions in lemma 2. □

Now continuing on, we can express equilibrium b^* as

$$(b^*)^{-1} = \frac{2\alpha - 1 - z}{2q(\alpha - 1)}$$

Also note that

$$\frac{\partial z}{\partial \alpha} = z^{-1}(1 - q)(4\alpha - 2)$$

$$\frac{\partial z}{\partial q} = -2z^{-1} (\alpha^2 - \alpha)$$

We now proceed to prove Proposition 2.3 claim by claim.

Claim 1. $\mathbb{E} [D^{agg}]$ is decreasing in q .

Proof. We first express $\mathbb{E} [D^{agg}]$ as a function of α , q , and z :

$$\begin{aligned} \mathbb{E} [D^{agg}] &= \frac{1 - \frac{2\alpha-1-z}{2q(\alpha-1)}}{1 - \frac{2\alpha-1-z}{2(\alpha-1)}} \\ &= \frac{2q(\alpha-1) - 2\alpha + 1 + z}{2q(\alpha-1) - 2q\alpha + q + qz} \\ &= \frac{(2q\alpha - 2q - 2\alpha + 1 + z)(z+1)}{q(z-1)(z+1)} \\ &= \frac{(2\alpha - 1 - z)(1-q)(\alpha-1)}{2\alpha q(1-q)(\alpha-1)} \\ &= \frac{2\alpha - 1 - z}{2\alpha q} \end{aligned}$$

Differentiating with respect to q , we get

$$\frac{d\mathbb{E} [D^{agg}]}{dq} = \frac{-2\alpha q \frac{dz}{dq} - (2\alpha - 1 - z) 2\alpha}{(2\alpha q)^2}$$

which implies

$$\begin{aligned} \text{sign} \left(\frac{\partial \mathbb{E} [D^{agg}]}{\partial q} \right) &= \text{sign} \left(2q(\alpha^2 - \alpha) - (2\alpha - 1 - z) z \right) \\ &= \text{sign} \left(2q(\alpha^2 - \alpha) + 4(1-q)(\alpha^2 - \alpha) + 1 - (2\alpha - 1) z \right) \\ &= \text{sign} \left((\alpha^2 - \alpha)(4 - 2q) + 1 - (2\alpha - 1) z \right) \end{aligned}$$

Let $RHS \equiv (\alpha^2 - \alpha)(4 - 2q) + 1 - (2\alpha - 1) z$. The remaining proof consists of three steps: 1) show $\frac{dRHS}{d\alpha} \geq 0$ for all $\alpha \geq 1$, $q \in [0, 1]$; 2) show $\frac{dRHS}{dq} \geq 0$ for all $\alpha \geq 1$, $q \in [0, 1]$, with equality holding only when $\alpha = 1$ or $q = 0$; 3) RHS evaluated at $\alpha = 1, q = 0$ is zero, concluding that

$RHS > 0$ for $\alpha > 1, q > 0$.

Step 1: show $\frac{dRHS}{d\alpha} > 0$ for all $\alpha \geq 1, q \in [0, 1]$. Differentiating RHS with respect to α , we have

$$\begin{aligned}
\frac{dRHS}{d\alpha} &= (2\alpha - 1)(4 - 2q) - 2z - (2\alpha - 1) \frac{\partial z}{\partial \alpha} \\
&= (2\alpha - 1) \left(4 - 2q - z^{-1}(1 - q)(4\alpha - 2) \right) - 2z \\
&> (2\alpha - 1) \left(4 - 2q - z^{-1}(1 - q)(4\alpha - 2) \right) \\
&\geq (2\alpha - 1) \left(4 - \max_{q \in [0, 1]} (2q) - \max_{q \in [0, 1]} z^{-1}(1 - q)(4\alpha - 2) \right) \\
&= (2\alpha - 1)(4 - 2 - 2) \\
&= 0
\end{aligned}$$

Step 2: show $\frac{dRHS}{dq} > 0$ for all $\alpha \geq 1, q \in [0, 1]$, with equality holding only when $\alpha = 1$ or $q = 0$.

Differentiating RHS with respect to q , we have

$$\begin{aligned}
\frac{dRHS}{dq} &= -2(\alpha^2 - \alpha) - (2\alpha - 1) \frac{\partial z}{\partial q} \\
&= (\alpha^2 - \alpha) \left(2z^{-1}(2\alpha - 1) - 2 \right) \\
&= \underbrace{2z^{-1}}_{>0} \underbrace{(\alpha^2 - \alpha)}_{\geq 0} (2\alpha - 1 - z)
\end{aligned}$$

The last term is non-negative and is zero only when $q = 0$. To see this, note

$$\begin{aligned}
z &= \sqrt{4(1 - q)(\alpha^2 - \alpha) + 1} \\
(\text{equal only if } q = 0) &\leq \sqrt{4\alpha^2 - 4\alpha + 1} \\
&= 2\alpha - 1
\end{aligned}$$

Hence we have $\frac{dRHS}{dq} > 0$.

Step 3: conclude the proof. Note

$$RHS|_{q=0, \alpha=1} = 0$$

Hence we have, for any $\alpha > 1$ and $q > 0$, $RHS > 0$. Thus $\frac{\partial \mathbb{E}[D^{agg}]}{\partial q} > 0$. □

Claim 2. $\mathbb{E}[I^{agg}]$ is decreasing in q .

Proof. Given the result in lemma (2.5), we first show that investment is decreasing in q if and only if the ex ante welfare for the borrower is decreasing in q . To see this, note

$$\begin{aligned} \mathbb{E}[I^{agg}] &= \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)} \\ &= \frac{2(\alpha - 1)}{\alpha(2\alpha - 1)} \cdot \left[\frac{(z + 1 - 2\alpha(1 - q) - 2q)}{4q} + \frac{1}{2} \right] \\ &= \frac{2(\alpha - 1)}{\alpha(2\alpha - 1)} \cdot \left[V(0) + \frac{1}{2} \right] \end{aligned}$$

To show $V(0)$ is decreasing in q , first note

$$\begin{aligned} V(0) &= \frac{1 - 2\alpha(1 - q) - 2q + z}{4q} \\ \frac{dV(0)}{dq} &= \frac{4q(2\alpha - 2 + z_q) - 4(1 + 2\alpha(q - 1) - 2q + z)}{16q^2} \\ &= \frac{1}{4q^2} [q(2\alpha - 2 + z_q) - 1 + 2\alpha(1 - q) + 2q - z] \\ &= \frac{1}{4q^2} \left[-2\alpha \left[(\alpha - 1) \left(\frac{q}{z} \right) - 1 \right] - (1 + z) \right] \end{aligned}$$

Define $\Omega \equiv -2\alpha \left[(\alpha - 1) \left(\frac{q}{z} \right) - 1 \right] - (1 + z)$, we then have $\text{sign} \left(\frac{dV(0)}{dq} \right) = \text{sign}(\Omega)$.

To compute the derivative of Ω with respect to q :

$$\begin{aligned} \frac{d\Omega}{dq} &= -2\alpha(\alpha - 1) \frac{z - qz_q}{z^2} - z_q \\ &= -\frac{1}{z^3} 4\alpha^2 (\alpha - 1)^2 q \leq 0 \end{aligned}$$

Therefore Ω is declining in q for all $\alpha > 1$. This means to check that $\frac{dV(0)}{dq} < 0$ it is sufficient to

check that $\Omega|_{q=0} \leq 0$.

$$\begin{aligned}\Omega(q=0) &= (2\alpha - 1) - z \\ &= 0\end{aligned}$$

Hence welfare is decreasing in q . □

Claim 3. Default probability is increasing in q .

Proof. Note $\mathbb{E}[Pr(\text{Default})] = \mathbb{E}[D^{agg}]$ and the claim follows directly from claim 1. □

Claim 3. $\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}]$ is increasing in q .

Proof. Follows directly from claims 1 and 2. □

Claim 4. $\lim_{q \rightarrow 1} V(0) = 0$.

Proof. We can write the ex-ante welfare as

$$V(0) = \frac{1 + 2\alpha(q - 1) - 2q + z}{4q}$$

Using the fact that $\lim_{q \rightarrow 1} z = 1$ and taking limit, the result is immediate. □

Proof of Proposition 2.4

Take $\alpha_1 > \alpha_2$. From lemma 2.2, we have $\ell(b, q, \alpha_1) = \ell(b, q, \alpha_2)$ and that $b(\ell, q, \alpha)$ is increasing in α . This means that the downward sloping borrower best response curve in Figure 2.3 shifts upwards as α increases, as the figure illustrates. Since the lender's best response slopes upwards ($\ell_b(b, q, \alpha) > 0$, also from Lemma 2.2), it follows that $b^*(q, \alpha_1) > b^*(q, \alpha_2)$ and $\ell^*(q, \alpha_1) > \ell^*(q, \alpha_2)$.

Next, from lemma (2.4) we have

$$\mathbb{E} [D^{agg}] = \frac{1 - b^{-1}}{1 - qb^{-1}}$$

$$\mathbb{E} [D^{agg}] / \mathbb{E} [I^{agg}] = \frac{1 - b^{-2}q}{b^{-1}(1 - q)}$$

both of which are increasing in b , which establishes the result that they are also both increasing in α .

□

Proof of Proposition 2.5

From Lemma (2.5) we have

$$\mathbb{E} [I^{agg}] = \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)}$$

Differentiating with respect to α , we get

$$\frac{d\mathbb{E} [I^{agg}]}{d\alpha} = \frac{q(2\alpha - 2\alpha^2 z - 10\alpha^2 + 8\alpha^3) - (z - 6\alpha + 4\alpha^2 z + 12\alpha^2 - 8\alpha^3 - 4\alpha z + 1)}{2\alpha^2 q(2\alpha - 1)^2 z}$$

From this expression, one can verify that for any given $q \in (0, 1)$, there exists a unique $\bar{\alpha}(q) \in (1, \infty)$ such that

$$\frac{d\mathbb{E} [I^{agg}]}{d\alpha} \begin{cases} < 0 & \text{for } 1 \leq \alpha < \bar{\alpha}(q) \\ > 0 & \text{for } \alpha > \bar{\alpha}(q) \end{cases}$$

and

$$\frac{d\bar{\alpha}(q)}{dq} < 0.$$

Proposition 2.5' follows from the fact that under partial-pledgeability, the cutoff level of marginal return can be written as $\bar{\alpha}(q, \delta) = \frac{\bar{\alpha}(q, 0) - \delta}{1 - \delta}$ where $\bar{\alpha}(q, 0)$ is the cutoff without pledgeability.

□

Proof of Proposition 5'

This follows immediately from Proposition 2.5 and from the fact that under partial pledgeability, the cutoff level of marginal return can be written as $\bar{\alpha}(q, \delta) = \frac{\bar{\alpha}(q, 0) - \delta}{1 - \delta}$ where $\bar{\alpha}(q, 0)$ is the cutoff without pledgeability.

□

Proof of Proposition 7

Proof of Claim 1. For an increasing and concave investment function $R(\cdot)$ with $R'(0) > 1$, the first-order condition in equation (2.1) that characterizes the single-lender equilibrium can be re-written as

$$R'(p(D^*)D^*) \times [p(D^*) + p'(D^*)D^*] = p(D^*)$$

If an interest rate cap were set to be $1 + \bar{r}^{SL} \equiv \frac{1}{p(D^*)}$, the borrower could propose to pledge D^* and raise $p(D^*)D^*$ from the very first lender. No future lender would be willing to provide additional investment to the borrower because doing so would require an interest rate higher than $1 + \bar{r}^{SL}$ to break even, but such a rate is prohibited by the interest rate cap. Hence the full commitment allocation can be achieved under \bar{r}^{SL} .

When $R(I) = \alpha I$, the first-order condition simplifies to

$$\alpha(1 - 2D^*) = 1 - D^*$$

hence $D^* = \frac{\alpha - 1}{2\alpha - 1}$. The optimal interest cap is thus

$$\begin{aligned} 1 + \bar{r}^{SL} &= \frac{1}{1 - D^*} \\ &= 1 - \frac{1}{\alpha} \end{aligned}$$

Claim 2 follows directly from the fact that there is an one-to-one relationship between risky debt issuance and the probability of repayment.

Proof of Claim 3. When $\bar{r} < \bar{r}^{SL}$, the interest rate cap is inefficiently low and the borrower can pledge less debt facevalue than he would have done under full commitment. The unique equilibrium under the interest rate cap would involve the borrower pledging $D = 1 - \frac{1}{1+\bar{r}}$ debt and raising $(1 - D)D$ investment from the very first lender. In the extreme case where $\bar{r} = 0$, the borrower would be unable to raise any investment from the lenders, achieving an even lower level of welfare than under the unregulated equilibrium.

Proof of Claim 4. When $\bar{r} > \bar{r}^{Sl}$, the full commitment allocation is unattainable as with probability q the borrower will meet the second lender and pledge a strictly positive amount of debt for any level of outstanding debt below one.

Next we show that for $\bar{r} < \infty$ the cap unambiguously improves expected investment and welfare while lowering expected debt and interest rate relative to the unregulated equilibrium. Using the techniques in the proof for proposition 2.2, for any game with finite lenders K we can find a sequence of aggregate debt $\{D_1^K, \dots, D_K^K\}$ where D_i^K corresponds to the aggregate debt level had the borrower reach lender i in a game with total lender K , where lender indices start backwards with the last lender being lender 1. Using a simple perturbation argument, we know that for K such that $D_1^K < \bar{D} \leq D_1^{K+1}$, the infinite lender game with debt cap \bar{D} would have a unique SPE where the borrower reaches the debt cap when borrowing from $(K + 1)$ -th lender. Furthermore, using the same recursive definition of lender and borrower strategies in equilibrium as we adopted in proposition 2.2, it is clear that borrower's ex-ante expected investment, aggregate debt level, and welfare with debt cap \bar{D} is in between the corresponding equilibrium quantities for the finite lender games with K and $K + 1$ lenders.

□

Chapter 3

Accelerator or Brake? Cash for Clunkers, Household Liquidity, and Aggregate Demand

3.1 Introduction

During the Great Recession, a range of fiscal policies were used to stimulate consumer demand, including temporary tax credits and price subsidies on durable goods. Temporary incentives for the purchase of durable goods, like temporary subsidies for capital investment by businesses, can in theory have large effects by altering the timing of purchases. But, the responses to such incentives are often found to be quite low.¹ A possible explanation is that intertemporal substitution is limited by financial constraints and a lack of liquidity. Agents may forgo substantial price subsidies if they lack the liquidity to make down payments, the debt capacity sufficient to secure loans, or the willingness to increase their leverage. And in the Great Recession, household were highly leveraged and lending standards had tightened.

* This chapter is joint work with Brian Melzer, Jonathan Parker, and Arcenis Rojas.

¹See, for example, Auerbach and Hassett (1992), Caballero, Engel, and Haltiwanger (1997), Desai and Goolsbee (2004), House and Shapiro (2008), Edgerton (2010), Zwick and Mahon (2016). Caballero (1993), Bar-Ilan and Blinder (1992), and Berger and Vavra (2015) study automobile purchase dynamics and Adda and Cooper (2000) studies previous vehicle scrappage subsidies in France.

We study the Car Allowance Rebate System (CARS) program to understand the importance of financial frictions for the impact and design of fiscal stimulus. Under CARS, colloquially known as “Cash for Clunkers,” the U.S. Government provided \$3,500 to \$4,500 rebates to consumers who traded in and scrapped old, fuel-inefficient automobiles and purchased new, more efficient ones during July and August of 2009. Transactions were submitted at roughly seven times the anticipated rate, and, despite Congress tripling available funding shortly after the program started, CARS ran out of money in just over a month. Because the rebates were paid at the time of the transaction, rather than as credits on households’ tax returns, they could be used as down payments for new vehicles. CARS rebates therefore provided not only a price subsidy, but also the liquidity to exploit it, unlike the State Energy-Efficient Rebate Program and (at times) the First-Time Homebuyer Tax Credit, both also used to stimulate durable purchases. Separating the liquidity feature of the program from that of the economic subsidy alone, we provide evidence that this aspect of CARS’ design—liquidity provision—was critical for the large response to the price subsidy. Further, we estimate the elasticity of new vehicle transactions to the CARS subsidy and so add to the literature quantifying the aggregate effect of the program (e.g., Mian and Sufi (2012) and Hoekstra, Puller, and West (2016)).

We use data from the Consumer Expenditure Interview Survey (CE) and a differences-in-differences approach to measure the causal impact of CARS. Passenger cars rated at 18 miles per gallon (MPG) or lower qualified for the CARS subsidy, while vehicles with efficiency of 19 MPG or higher did not. We use these fuel efficiency cut-offs to create a treatment group of vehicles eligible for CARS (“clunkers”) and a control group of similar, but ineligible, vehicles with fuel efficiency ratings above the cut off (“close-to-clunkers”). We identify the effect of CARS by comparing the rates at which treatment and control group vehicles are traded in for new vehicles during the program period and thereafter. We exclude from the estimation sample vehicles with fuel efficiency more than 6 MPG above or below the cut off and vehicles with estimated trade-in value above \$5,000, for which the CARS rebate provided no subsidy. Within the estimation sample the treatment and control groups have similar average vehicle value, vehicle age, and owner’s income.

We estimate that the CARS program raised the probability that a household with an eligible,

low-value vehicle exchanged its old vehicle for a new one by roughly one and a half percentage points, which represents roughly a quadrupling of the baseline probability. In dollar terms, CARS raised the average spending on new vehicle purchases by \$320 for each existing clunker eligible for trade-in under the program, consistent with our estimate of the increase in the probability of purchase and the average purchase price of about \$22,000. These results are robust to controlling for vehicle and household characteristics and to conducting the analysis at either the vehicle or the household level. As further validation of our model's identifying assumptions, we confirm that CARS had no effect in two placebo tests. Used vehicle purchases, which were ineligible for CARS, were similar between the treatment and control groups during the program period. New vehicle purchases were likewise similar between the treatment and control groups during the summer of 2008 when no program like CARS was in effect.

The rich vehicle- and household-level information in the CE allows us to measure differences in the program responses across households. We focus on three sources of heterogeneity: differences in the economic subsidy provided by CARS and differences in the liquidity and debt capacity of potential participants.

The participation in CARS increased with the economic subsidy it provided. Because the CARS program required scrapping the old vehicle, the true economic value of the subsidy was the face value of the credit minus the vehicle's trade-in value in the absence of the program. The CARS program increased the probability of purchasing or leasing a new vehicle by half a percentage point per \$1,000 of estimated economic subsidy. The program had roughly no effect for a household with an eligible vehicle that has a trade-in value greater than the maximum CARS subsidy of \$4,500. As compared with existing estimates of the average response, our measure of the program impact per dollar of economic subsidy is independent of the distribution of values of existing vehicles (which differ across samples used by previous studies).

The participation in CARS also increased with the liquidity it provided. Using new vehicle purchases observed during the same time period in the CE, we estimate that the higher rebate amount of \$4,500 (claimed by 71% of participants) exceeded the total down payment—cash plus trade-in value—on nearly 70% of purchases.² With no further down payment required, the vast

²We calculate 71% from the \$4,209 average CARS voucher reported in National Highway Traffic Safety Admin-

majority of households could thus participate in CARS irrespective of their liquid savings. For households that owned old vehicles with outstanding loans, however, the liquidity demands were much higher, as participation required immediate repayment of the prior loan. We find a substantially lower treatment effect of the program for this group of households, consistent with binding liquidity constraints limiting the response to the large price subsidy provided by CARS. In fact, the program had no rough effect at all on the purchases of households with outstanding loans on their old vehicles. This differential response remains after controlling for household income, liquid assets, and the size of the subsidy. The differential response is also specific to loans secured by the potential trade-in: we find no difference in program response for households with other outstanding loans, presumably because these debts are not due upon participation in CARS. As further evidence of liquidity constraints, we find economically significant but statistically weak evidence of lower CARS take-up among households in the bottom tercile of liquid assets (below \$400) compared to households in the middle tercile (between \$400 and \$6,000).

In contrast to our findings on liquidity constraints, we find no relationship between debt capacity and participation in CARS. To measure variation in debt capacity, we consider resources available to support future debt repayment: the household's income, alone and net of existing debt payments, and its housing equity. Despite apparently low debt capacity, households with income in the bottom tercile (less than \$24,000 per year after taxes), with high debt payment-to-income ratios (33% or higher), or with high mortgage leverage (loan-to-value ratio above 100%) still partake in the CARS program at close to the average rate in the full sample. One potential explanation for this finding is that the CARS rebate provided enough collateral coverage to substantially relax debt capacity constraints. Our statistical power in these subsample tests is low, however, so it may also be that debt capacity had significant effects that we are unable to detect within our sample.

We conclude by estimating the aggregate impacts of CARS on vehicle purchases and expenditures. Under the assumption that CARS had no impact on trade-ins of ineligible vehicles, we aggregate the predicted increases in individual purchases across the national distribution of clunkers. We find that CARS caused, in a partial-equilibrium sense, roughly 506,000 new purchases, relative to 680,000 vehicles traded in under the program. This estimate lies within, but at the upper end

istration (2009).

of, the range of aggregate impacts found in previous studies.³ With respect to the value of vehicles purchased, we estimate that CARS induced \$11 billion of new vehicle purchases in the third quarter of 2009 (\$44 billion at an annual rate) at a fiscal cost of \$2.85 billion. While CARS increased consumption demand with minimal government outlays and coincided with the end of the Great Recession, its effect on vehicle spending may have been short-lived. Our analysis does not reject the finding in Mian and Sufi (2012) that demand was drawn from purchases that would have occurred anyway over the subsequent seven months.

Our findings on household financial frictions are relevant to the design of stimulus programs. First, household liquidity constraints can meaningfully reduce a program's aggregate impact, especially during recessions when financial constraints are the most binding. Second, programs that bundle liquidity with subsidies, such as by disbursing rebates at purchase rather than as year-end tax credits or mail-in rebates, maximize take-up and are more equitable across households with varying amounts of liquid savings.⁴ These implications seem likely to hold more broadly because they held for CARS during a financial crisis in which lending standards had tightened dramatically.

3.2 Related literature

Our paper relates to the literatures on fiscal stimulus, household financial constraints, and purchases of durable goods. Studies of lump-sum stimulus programs also find an important role for household liquidity in causing spending, but for programs where payments naturally provide liquidity and do not depend on purchase behavior.⁵ Vehicle purchases in particular seem to follow from substantial increases in household liquidity, as caused by cash stimulus payments Parker, Souleles, Johnson, and McClelland (2013), minimum wage hikes Aaronson, Agarwal, and French (2012), and tax refunds Adams, Einav, and Levin (2009). The model in Rampini (2016) highlights the relevance of liquidity constraints for purchase of goods with high durability, such as new vehicles. Around the

³Previous studies by Council of Economic Advisors (2009), National Highway Traffic Safety Administration (2009), Li, Linn, and Spiller (2013), and Mian and Sufi (2012) find that CARS caused between 370,000 and 600,000 purchases in July and August of 2009.

⁴The First-Time Homebuyer Tax Credit and State Energy Efficient Appliance Rebate Program implemented around the same time as CARS were examples of programs that did not provide liquidity.

⁵See Johnson, Parker, and Souleles (2006), Agarwal, Liu, and Souleles (2007), Agarwal and Qian (2014), Broda and Parker (2014), and Di Maggio, Kermani, and Ramcharan (2014).

time of the CARS program, automobile purchases were also sensitive to credit supply Benmelech, Meisenzahl, and Ramcharan (2016). Finally, the analysis of the First-time Homebuyer Tax Credit by Berger, Turner, and Zwick (2016) suggests that take-up was amplified among households for which the credit relaxed down payment constraints (as proxied by FICO score).

With respect to existing research on the CARS program, our paper is unique in using nationally-representative, household-level data as well as the first to measure the roles of household financial constraints and the economic subsidy. Mian and Sufi (2012) was first to document the spending reversal following the CARS program. Comparing the rates of new vehicle registrations across cities that differed in their pre-program share of clunkers, the paper finds that CARS caused between 340,000 and 400,000 additional purchases by August 2009 but no difference in cumulative purchases by March 2010. We find a larger initial impact, a difference we analyze in Section 3.6.6. Unlike their study, we use microdata to study (and control for) the effect of household characteristics such as liquidity, and to more accurately assign vehicles to similar treatment and comparison groups. Despite these advantages, our data comprise a relatively small sample of households compared to their aggregated data on all households in each geographic area.

Hoekstra, Puller, and West (2016) uses the discontinuity in program eligibility at 18 MPG to identify the effect of CARS on total vehicle spending in Texas. The paper shows that CARS caused owners of just-eligible vehicles to purchase more fuel-efficient, but smaller and substantially less expensive vehicles. Their estimates imply that CARS ultimately reduced aggregate vehicle spending despite inducing an initial increase in spending and purchases at the time of the program. Since the response of owners of 18-MPG vehicles may not be representative of the response of all owners, we measure the average impact of the program using a wider range of fuel efficiencies.

3.3 The CARS program

3.3.1 Overview

The Car Allowance Rebate System (CARS) was designed to stimulate automobile sales and production, and to provide environmental benefits by reducing fuel consumption and pollution.

The program provided a \$3,500 or \$4,500 credit for trading in an old, fuel-inefficient vehicle and purchasing or leasing a new, more fuel-efficient vehicle. Cars that were traded in were scrapped by having the engine and drivetrain destroyed. Many countries have adopted similar scrappage programs, including Germany, France, the UK, Spain, South Korea, Japan, China, Italy, Portugal and the Netherlands.

In total, CARS provided \$2.85 billion of credits on nearly 680,000 transactions in July and August of 2009. Congress first considered the program in early 2009 and passed the authorizing legislation on June 24, 2009. The Department of Transportation established program rules one month later, and dealers began submitting transactions on July 27, 2009. Program participation exceeded expectations, with a flow of trade-ins seven times the expected rate National Highway Traffic Safety Administration (2009), and the initial funding of \$1 billion was exhausted in the first week. Congress responded by appropriating an additional \$2 billion that sustained the program through its ultimate end date of August 24, 2009, which was still more than two months ahead of the legal end of the program, November 1, 2009.

3.3.2 Program eligibility

In order to receive the CARS credit, a household had to trade in a qualifying vehicle and purchase or lease a new vehicle with sufficient improvement in fuel economy over the trade-in. Whether a trade-in vehicle qualified for the CARS credit also depended on its age, condition, and recent insurance and registration status. For fuel economy, passenger cars and small trucks (category 1 and some category 2 vehicles) qualified if they had a combined (city and highway) fuel economy of 18 miles per gallon or less. Large trucks (category 3 and some category 2 vehicles), for which the Department of Energy does not rate fuel economy, were screened instead on vehicle age, with model-years 2001 and earlier eligible for a credit. Regardless of vehicle type, all trade-ins had to be less than 25 years old (model-year 1984 or later) in order to qualify. Finally, all qualifying trade-ins had to be in drivable condition and continuously registered and insured to the current owner for the prior year.

Whether a new purchased or leased vehicle qualified for the CARS credit depended on its price,

its fuel economy, and the improvement in fuel economy between the trade-in and the new vehicle. New vehicles were ineligible if the manufacturer suggested retail price exceeded \$45,000. New passenger automobiles also had to have a combined fuel economy of 22 MPG or higher, and at least 4 MPG greater than the trade-in vehicle. New category 1 trucks were required to have fuel economy of at least 18 MPG and at least 2 MPG greater than the clunker. New category 2 trucks were required to get at least 15 MPG and 1 MPG more than the associated clunker. New category 3 trucks had no minimum MPG but could not be larger than the trade-in vehicle.

3.3.3 Program credit and economic subsidy

The credit on eligible transactions was either \$3,500 or \$4,500, with the larger credit granted for greater improvement in fuel economy between the trade-in and the new vehicle. For example, a customer purchasing a new passenger car received a credit of \$3,500 if the fuel economy improvement was between 4 and 9 MPG and received \$4,500 if the improvement was 10 MPG or more. Similar rules, but requiring smaller improvements in fuel efficiency, applied to each category of light truck. Table 3.1 summarizes the credit paid for each combination of new and trade-in vehicle. Credits were remitted directly to dealers, who were responsible for submitting the required documentation.

The economic subsidy provided by the CARS program was not the statutory \$3,500 or \$4,500 but instead was this amount less the value of the trade-in. That is, the program did not provide a fixed subsidy that could be received in addition to any private trade-in value. Rather, because trade-ins were scrapped, the CARS program effectively replaced the market value of the used car available outside of the program with the fixed CARS rebate. For example, for a CARS rebate of \$4,500, the true economic subsidy would be \$0 for a trade-in with value of \$4,500, whereas it would be \$3,500 for a trade-in worth \$1,000.

Finally, Busse, Knittel, Silva-Risso, and Zettelmeyer (2012) finds that CARS did not cause sellers to raise prices. With the incidence of the program entirely on consumers, there is no need to adjust for price changes in our measure of the economic subsidy.

3.4 Data

3.4.1 Data sources

We use data from four sources. We employ the Bureau of Labor Statistics' (BLS) Consumer Expenditure Survey (CE) for information on car purchases and trade-ins for a stratified random sample of US households. We merge this data with measures of vehicle fuel economy, trade-in values, and vehicle registrations from the Environmental Protection Agency (EPA), Edmunds, and R.L. Polk, respectively.

Our main data come from the CE Interview Survey, which tracks respondents' expenditures for one year through interviews every three months. The survey collects information on the make, model, and model-year of each household's vehicles when they enter the survey and in each subsequent interview. To preserve respondent confidentiality, the BLS suppresses the vehicle model in the public-use files but, following BLS protocols, we obtained access to confidential internal records that include the vehicle model. The CE provides detailed information on each vehicle purchase and disposal: the month of the transaction, the purchase or sale price, the type of vehicle (new or used), and whether it was purchased or leased. For purchases, the CE also reports the net purchase price as well as the value of the trade-in, if any. If the purchase was financed, the CE reports the amounts of the down payment and the loan. We use this information to measure: 1) the CARS eligibility of potential trade-ins; 2) the sale, trade-in or disposal of an existing vehicle; 3) the purchase or lease of an additional vehicle; 4) and the outstanding debt secured by a potential trade-in. We also use CE information on household demographics, income, assets, and debts.

We measure the fuel economy of CE vehicles using data from the EPA and R.L. Polk. The EPA rates the combined city-highway fuel economy by vehicle make, model, model-year, and the pertinent "model options" such as transmission type and drivetrain. R.L. Polk tracks vehicle registrations for each vehicle type. Since model options are not reported in the CE, we compute each vehicle's weighted average fuel economy, given its make, model, and model-year, weighing each model option by its share of nationwide registrations as of January 2009. We also calculate, for each vehicle, the share of registrations above or below the CARS MPG cutoff. For some records, the CE reports a vehicle that is not in the fuel economy file. For example, a household might

report having a 2005 Jeep Cherokee, though Jeep Cherokee was only made through 2004. For such instances, we use the MPG of the same model manufactured one year before or after the reported model-year if it exists. If no match exists within one model-year, we exclude the reported vehicle from our analysis since we cannot reliably estimate the vehicle's eligibility for CARS.

We measure the value of CE vehicles and the associated CARS subsidy using data from Edmunds.com. Edmunds calculates monthly estimates of trade-in value by make, model, and model year from actual transactions reported by car dealers. We use the estimated values in May 2009 for vehicles of average condition.

3.4.2 Validating the CE vehicle data: Trade-ins during CARS

Figure 3.1 provides validation that the CE data measure meaningful responses to the CARS program and that consumers are fairly accurate in timing their CARS-related purchases. Panel A shows the share of new vehicle purchases that are associated with vehicle trade-ins of exactly \$3,500 or \$4,500, the CARS credit amounts. In most months outside of the program period, very few respondents—roughly 5%—report trade-ins of such amounts. During the CARS program the share increases significantly to 22% in July 2009 and 39% in August 2009, the peak month of the program. In contrast, Panel B shows that the corresponding shares for purchases of used vehicles, which are clearly ineligible for CARS, are low and show no increase around the time of the CARS program.

Notably, the share of \$3,500 and \$4,500 trade-ins for new purchases remains elevated at 23% in September 2009 after the end of the program. This pattern of delayed program response may reflect the timing of vehicle delivery. An estimated 50,000 CARS transactions entailed September delivery despite the purchase occurring before the program's August 24th end date Krebs (2009). The timing in the CE is based on household reports of expenditures, and many consumers may have reported the delivery date rather than the purchase date. Another possibility is that the delayed response results from recall error, as households interviewed in the fall of 2009 recall their purchase as occurring in September as opposed to August. Such recall error does not appear to be too severe, however, since the proportion of \$3,500 and \$4,500 trade-ins returns to its normal low

level by October 2009.

3.5 Sample and methodology

We measure the effect of the CARS program on vehicle purchases by comparing a treatment group of eligible trade-in vehicles to a control group of similar but ineligible vehicles. For this comparison to identify the causal effects of the program, the rates of trade-in and new purchases in the treatment and comparison groups would have to be similar in the absence of the CARS program. Therefore, we construct a relatively homogeneous sample of vehicles, precisely allocate vehicles to treatment and comparison groups based on program eligibility at the vehicle level, and check the similarity of the characteristics of the treatment and control groups.

To construct the sample, we select vehicles owned by CE households as of June 2009. We exclude vehicles manufactured before 1985, since they were ineligible for CARS. We also exclude vehicles with average trade-in values above \$5,000, for which the CARS rebate likely provided no economic subsidy. Finally, we exclude vehicles of extreme fuel economy, for which more than 25% of registrations are below 12 MPG or above 25 MPG. The remaining sample includes vehicles of limited age, low value, and intermediate fuel economy.

Why not focus on a narrower range of MPG? One answer is sample size. But, more importantly, CARS linked both program eligibility and the size of the subsidy to the difference in fuel economy between the clunker and new vehicle. Conditional on having a clunker, the greater the fuel economy of the clunker, the more restricted was the set of new vehicles that qualified. It is reasonable to believe that households with high-MPG clunkers, who faced a limited choice of new cars that would be eligible for the subsidy, were less likely to participate than households with lower-MPG clunkers. In order to estimate the average effect of the program, we do not study responses only for clunkers immediately below the fuel efficiency cutoff. In a robustness exercise discussed in Section 3.6.5, we restrict the sample to a narrower range of 16 to 21 MPG.

Within our sample we measure CARS eligibility based on the vehicle's fuel economy and purchase date.⁶ We assign a vehicle of a given make, model, and model-year to the treatment group if at least

⁶Households report the purchase date of owned vehicles, but this information is sometimes missing. We do not

75% of its registrations have fuel economy of 18 MPG or lower and if it was purchased no later than July 2008. And we assign the vehicle to the control group if at least 75% of its registrations have fuel economy of 19 MPG or higher. We also assign vehicles to the control group if they satisfy the MPG restrictions of the treatment group but are ineligible for CARS because they were purchased after July 2008. We drop from the sample any vehicles that are not assigned by these rules—e.g. vehicles with equal shares of registrations above and below 18 MPG— because there is significant uncertainty over whether they belong in the treatment or control groups.

Table 3.2 displays summary statistics. Comparing the means of different characteristics, the treatment and control groups look quite similar. The control group consists of slightly newer vehicles that have a somewhat higher probability of having an outstanding loan, but with a slightly lower balance. Households that own vehicles in the treatment group have quite similar income to those that own vehicles in the control group. The unassigned vehicles, which are the majority, look quite different. They are younger, more fuel efficient, and (by construction) more valuable than vehicles assigned to the treatment and control groups.

Turning to our methodology, our main dependent variable is cumulative vehicle purchases associated with a potential trade-in. We measure program responses at the vehicle level, tracking if and when a vehicle is replaced by the purchase or lease of a new vehicle. In a robustness check we also measure new purchases or leases at the household level without conditioning on disposal of an existing vehicle. Because the CE does not explicitly link specific vehicle disposals to replacement vehicle purchases, we apply the following algorithm to match purchases and disposals. We first assume that a purchase or lease is associated with a vehicle disposal if it occurs in the same month as the disposal. If no contemporaneous disposal exists, we then assign the purchase to disposals within one month that are not otherwise assigned. We code the indicator variable $Transaction_{it}$ to be one if the household disposes of vehicle i within one month of the purchase or lease of a new car in month t . We also measure spending by taking the product of $Transaction_{it}$ and the gross price of the vehicle purchased. When there are multiple disposals that could be associated with a purchase, we divide the purchase equally among the disposals; when there are two purchases associated with a disposal, we include them both. We apply this procedure identically in the treatment

drop such households but instead assume that the vehicle is eligible. We may mis-categorize some cars as eligible when in fact they were ineligible due to being recently purchased.

and control groups. We then cumulate the purchases or leases (or spending) associated with each vehicle:

$$Transactions_{it} = \sum_{t = \text{July 2009}}^{\text{Month } T} Transaction_{it}. \quad (3.1)$$

We estimate a separate cross-sectional, vehicle-level regression for each month, T , July 2009 through April 2010:

$$Transactions_{iT} = \alpha_T + \beta_T Clunker_i + \sigma_T \mathbf{X}_i + \varepsilon_{iT}, \quad (3.2)$$

where $Clunker$ is an indicator variable for whether the vehicle is eligible for CARS. The regression coefficient β_T measures the cumulative difference—between June 2009 and month T —in the likelihood of purchase for a clunker relative to a close-to-clunker. α_T captures common variation in cumulative purchases in the treatment and control groups at each horizon. The additional control variables \mathbf{X} relax the assumption of parallel trends between the two groups by allowing for variation in the rate of purchases due to factors, such as household income and vehicle age, that may differ between the treatment and control groups. We vary \mathbf{X} to include survey features (interview number), household characteristics (income), and/or vehicle characteristics (vehicle age and value and MPG). Because σ_T differs across periods, the slope coefficients on these control variables are allowed to differ by month.

In the second part of our analysis, we add interaction terms to measure differential responses to CARS:

$$Transactions_{iT} = \alpha_T + \beta_T Clunker_i + \gamma_T \mathbf{Z}_i Clunker_i + \sigma_T \mathbf{X}_i + \varepsilon_{iT}. \quad (3.3)$$

In this model each β_T coefficient measures the cumulative difference in likelihood of trade-in for a clunker (relative to a close-to-clunker) conditional on the value of the variables in \mathbf{Z}_i equaling zero. And $\gamma_T \times \mathbf{Z}_i$ measures the differential change in the probability of purchase for vehicles with characteristic \mathbf{Z}_i . For example, to account for differential sensitivity to CARS based on the available subsidy, we estimate a model that includes an interaction between program eligibility and estimated trade-in value outside of the CARS program. That is, for this model with $\mathbf{Z}_i = Value_i$, the coefficient β_T measures the program response for the subset of vehicles with zero trade-in value,

which then receive a maximum subsidy equal to the CARS rebate. And $\gamma_T \times Value_i$ measures the change in the probability of purchase for vehicles with higher trade-in values. We also use these specifications to test the importance of liquidity and debt capacity for the response to the CARS program.

While we cannot directly test our identifying assumptions, in Section 3.6.5 we run two placebo tests to understand whether our treatment and control groups differ in observed ways. First, we check that there is no difference in purchases between our defined treatment and control groups when the CARS program is not run. Using 2008 data, we follow exactly the sample procedures to construct our sample and treatment and control groups, and run our analysis in exactly the same months when of course there was no CARS program. Second, we run our main analysis in the period of CARS with our identical treatment and control groups but with the dependent variable measuring purchases or leases of used vehicles. Both placebo analyses find no effects, and so support our assumption that absent CARS, our treatment and control groups would have behaved similarly.

What exactly does our methodology estimate? This approach estimates the response to having a vehicle that is eligible for CARS instead of a similar vehicle that is not eligible, in the world in which the CARS program was run. This has two implications. First, under the assumption that general equilibrium effects have the same average impact on households with similar vehicles that are eligible and ineligible for CARS, our estimate can be aggregated to reveal the partial-equilibrium impulse response of aggregate demand to the CARS program. Second, our estimates of the temporal dynamics of the program and of the heterogeneity in program impact across household and vehicle characteristics are both conditional on the aggregate outcomes we observe. For example, had the recession not ended when CARS was run, the effects of the CARS program may not have been rapidly reversed and the pattern of its impact across households and vehicles might have been different from what we find.

3.6 The impact of CARS on vehicle purchases

3.6.1 Average impact

Panel A of Table 3.3 reports estimates of the impact of the CARS program on new vehicle purchases and leases at several horizons and estimated on several sets of control variables based on Equation 3.2. The first column shows estimates without any control variables, reflecting the difference in cumulative purchase rate between the treated clunker vehicles and the otherwise similar close-to-clunker vehicles that were not eligible for the program. The main finding from the first column is that there is a statistically significant and substantial effect of the program, primarily during August 2009. Eligibility for the CARS program increased the rate at which households disposed of an existing vehicle for the purchase or lease of a new vehicle by 0.36% ($p < 0.10$) by the end of July, 1.22% ($p < 0.01$) by the end of August and by 1.43% ($p < 0.01$) by the end of September. During the same period, close-to-clunker vehicles had a disposal rate of 0.5% toward the purchase or lease of a new vehicle. The rate at which clunkers were traded in for a new purchase or lease therefore nearly quadrupled during the program period, from 0.5% to 1.9%.

Although the CARS program stopped accepting applications in August, we consider purchases made during September as part of the treatment effect of the program. Our reading of the CE questionnaire is that the reported purchase date could be interpreted as the delivery date, and many CARS purchases entailed September delivery. Further, the evidence on trade-in amounts shown in Figure 3.1 suggests that CARS purchases were indeed reported in September in the CE data.

The remaining columns of Panel A of Table 3.3 show that our estimate of the program response is robust to the inclusion of a variety of controls for survey structure (interview number and missing interviews), household characteristics (income), and vehicle characteristics (age and value, and then also fuel efficiency). The estimated impact rises as we increase the number of control variables, although the differences are not statistically significant. The only column with economically noticeably larger estimates is the last column which includes the control for fuel efficiency. Fuel efficiency is obviously highly correlated with the CARS program, and one can see that power substantially declines in the last column when we include fuel efficiency; standard errors increase by 50%.

Panel B of Table 3.3 shows the results from estimation of Equation 3.2 in which the dependent variable is the cumulative dollar amount of new vehicle transactions. According to the first column of Panel B, eligibility caused an average of \$263 of spending through August and \$327 through the end of September per eligible vehicle. The remaining columns of Panel B show that adding control variables tends to raise the estimate of dollar spending, but not statistically significantly. These dollar amounts are consistent with our estimates of the increase in the number of purchases and what we know about the average purchase price under CARS. Panel C reports estimates of unit prices of new car purchased caused by CARS that are implied by the estimates in Panels A and B. These figures are calculated by dividing the unconditional expenditure coefficients in Panel B by the incremental purchase probabilities in Panel A. According to the National Highway Traffic Safety Administration (2009b) report, the average vehicle purchased using the CARS program was \$22,450. Similarly, in the CE data, new vehicle purchases between July and September 2009 with trade-in value of \$3,500 or \$4,500 (plotted in Figure 3.1) have an average purchase price of \$22,283. These figures are both quite similar to the \$22,912 (Panel C, Column 1) implied by our estimates.

Our estimate of the average impact per eligible vehicle is not directly comparable to the estimates of other studies, which use different sample restrictions and rules for assigning program eligibility. We therefore defer such comparisons to Section 3.6.6, where we estimate the program's aggregate impact.

3.6.2 Economic subsidy

How responsive were households to the program's economic subsidy? In addition to providing a parameter useful for the design of similar programs, the answer to this question provides an estimate of CARS on the behavior of a household facing a given subsidy, rather than an estimate that is intermediated by sample selection and the distribution of subsidies within any given sample.

Table 3.4 displays estimates from Equation 3.3 with an interaction between *Clunker* and the trade-in value of the vehicle. The economic value of the CARS subsidy is the rebate of \$4,500 (or, less common, \$3,500 for a smaller increase in fuel efficiency) less the trade-in value of the vehicle. The first two columns of Panel A show that an eligible vehicle of no value has a two and half percent

chance of being traded in under CARS (first row), an effect roughly double the baseline effect in Table 3.3. The second row shows that each additional \$1,000 in estimated trade-in value reduces the probability of purchase under CARS by around half a percentage point, so that an eligible make, model, and model-year vehicle worth \$4,500 is estimated to be no more likely to be traded in during CARS than an equivalent ineligible vehicle. These results are summarized graphically in Figure 3.2.

The last two columns in Panel A of Table 3.4 display the results of the same regressions with cumulative value of new vehicles as the dependent variable. Looking at the last column, for each \$1,000 of used vehicle value, the average value of spending on new vehicles was \$74 lower. A worthless eligible vehicle generated \$553 in expected new vehicle transactions, and implied a unit purchase value of \$21,265. An eligible vehicle worth \$4,500 still generated an expected \$220 in new vehicle purchases or leases.

Is it reasonable to believe that CARS caused increased an rate of purchase for vehicles worth on average \$4,500? It is. There is actually a distribution of trade-in values associated with any make, model, and model year. Since presumably the least valuable vehicles within any model year are the most likely to be traded in under CARS, the average trade-in value may be an overestimate, particularly for vehicles that are marginal, around the \$4,500 value. Thus, vehicles traded in under CARS that are of a make, model, model-year that are worth \$4,500 on average are actually worth less and so receive some subsidy for participation in CARS.⁷

To investigate this point further, Panel B of Table 3.4 reports the results of expanding our sample to include vehicles of more valuable make, model, and model years and allowing for a nonlinear effect of the CARS subsidy on purchasing and spending. In this approach, if these more valuable vehicles are unaffected by CARS, the regression coefficients will capture this. The first row of Panel B shows that there is only an economically small and statistically insignificant effect of CARS on vehicles of make, model, and model year worth on average between \$5,000 and \$6,500.

⁷It also possible that some people were not aware of the trade-in value of their vehicle so that some vehicles worth more than \$4,500 were traded-in in error. In this case, we would expect that dealers would not trade in the vehicle under CARS, but simply pay the customer \$4,500 for the vehicle worth more. In our data, since we do not distinguish these cases, such instances would be included in our measure and be a true effect of the CARS program (although potentially an effect that might not survive repeated CARS-type policies). Such a possibility is consistent with the household responses to the employee-pricing-for-everyone sales event of the summer of 2006 which lead to enormous increases in vehicle sales at prices slightly higher than the previous months Busse, Simester, and Zettelmeyer (2010).

The coefficients on the indicator variables for different values have the expected pattern, so that the less valuable an eligible vehicle is the more likely it is to be traded in and the more spending it causes in expectation.

3.6.3 Household financial constraints

Households who wanted to participate in CARS either had to have sufficient liquid wealth to purchase the new vehicle they wanted outright, or they had to have enough liquidity to make a required down payment, sufficient income in excess of debt payments, and a sufficiently high credit score to be approved for financing or a lease. Roughly 80% of new vehicle purchases are financed, so that for the vast majority of households, the ability to buy a vehicle at any time, including as part of CARS, hinges critically on being approved for a lease or financing and having the liquidity to make the down payment. Typically, debt capacity limits households' ability to purchase new vehicles, and the impact of CARS may have also been reduced for households with low income, high debt payments, and high mortgage leverage. However, we hypothesize that the CARS program was massively oversubscribed because the subsidy provided immediate liquidity that could be used for a down payment. Accordingly, the large response to the program was potentially not only due to liquid households changing the timing of their purchases in response to a temporary price subsidy, but also due to illiquid households who responded to the liquidity provided by the program and would not otherwise have been able to purchase a vehicle in the summer of 2009.

In this section, we use the CE survey data to estimate the impacts of household liquidity and debt capacity on participation in CARS. We begin by studying how program participation is affected by two different measures of liquidity.

First, the CE Survey contains information on household indebtedness that allows us to measure differences in liquidity required to participate in the CARS that are distinct from the size of the economic subsidy provided by the program. The CE collects outstanding vehicle debt balances, by month and by vehicle, for the entire survey period. The survey also collects outstanding unsecured debt balances as of the first and final CE interviews. By observing outstanding vehicle debt, we are able to measure differences in liquidity required to participate in the CARS program. For

a would-be participant with a loan secured by its clunker, the liquidity provided by CARS was reduced by the amount of the outstanding loan because the household trading in its vehicle would have to repay the outstanding loan before using any CARS funds as a down payment. Within our regression sample, 5.7% and 7.2% have outstanding debt on their eligible and ineligible vehicles, respectively, and 41.2% of households have outstanding unsecured debt.

Second, the CE Survey contains a measure of household liquid assets—checking and savings account balances—as of the final interview. This asset information, however, is missing for a non-trivial share of households and contains significant measurement error: respondents only report balances if they reach the final interview and they do not report balances contemporaneous with the vehicle purchase decision that we analyze. We divide households into terciles: illiquid households have less than \$300 in liquid assets, low liquidity households have between \$300 and \$4,500 in liquid assets, and liquid households have more than \$4,500.

We find that CARS participation was significantly reduced for households with existing vehicles secured by outstanding loans. As shown in the first column of Table 3.5, the *Clunker* coefficient of 2.33 ($p < 0.01$) indicates that owners of eligible vehicles unencumbered by outstanding loans increased purchases at substantial rates during the CARS program period. The interaction coefficient of -2.80 ($p < 0.01$), meanwhile, shows that CARS had a much smaller impact on the probability of an old vehicle encumbered by a loan being traded in to purchase new vehicle. In fact, the point estimates in this first column suggest that there was essentially no response by households with outstanding vehicle loans because CARS provided them only an economic subsidy but insufficient liquidity. To be clear, we would not claim that the program induced purchases *only* through the liquidity it provided. Among households without vehicle loans, the program had a large effect, likely because it induced purchases purely through the subsidy for some households and through the combination of the subsidy and liquidity for other households.

One might be concerned that households with loans against their old vehicles are different than households that own their vehicles outright. First, households with encumbered vehicles might simply not purchase new vehicles. Figure 3.3 shows that this is not the case by displaying the trade-in dynamics of clunker and close-to-clunker vehicles that are either securing loans or are owned by households with unsecured debt such as credit card balances. Figure 3.3.B shows that

among vehicles in the control group, those with outstanding loans are traded in for new vehicles at the same rate as vehicles owned by households with unsecured debt. Thus, vehicles used to secure loans are in fact used in the purchase of new vehicles. Figure 3.3.A shows that among vehicles eligible for CARS, only vehicles associated with unsecured debt are traded in under the CARS program. Clunkers encumbered with debt—and thus unable to benefit from the liquidity provision of CARS—do not respond to the program. This pattern suggests that households that have borrowed against their vehicles can accumulate down payments and so do buy new cars at rates not unlike those of owners of other old vehicles. But such households, on short notice, may not have been able to come up with the down payment needed to take advantage of the large and unexpected economic subsidy provided by CARS.

A more specific version of this hypothesis is that the presence of debt is reducing household participation in CARS rather than the presence of debt secured by the clunker *per se*. We offer two sets of results to evaluate this alternative to our interpretation. First, we find that CARS take-up is not reduced for households with unsecured loans. The second column of Table 3.5 shows that the muted program response is not due to the existence of debt in general or to the fact that households with debt are somehow different (e.g. have lower incomes, or they just do not buy new vehicles). The presence of debt that does not secure a vehicle does not mute program participation; the interaction coefficient of 0.27 is small and statistically insignificant. Second, we show that the estimated decline in CARS participation for encumbered vehicles is robust to controlling for measures of the household's debt capacity and mortgage leverage (see Appendix Table A3). These findings are consistent with a difference in program response due to the liquidity requirement for encumbered clunkers rather than a general difference in indebtedness or borrowing capacity.

The remaining columns of Table 3.5 control directly for other factors possibly correlated with having a loan secured by the clunker and that might affect participation in CARS, such as income, the value of the clunker, and existing liquidity. For example, poorer households may be more likely to have vehicle loans and also may be generally less likely to purchase new vehicles. Alternatively, the effect of an existing loan secured by the vehicle may be due to a correlation between existing auto debt and vehicle value and thus effective subsidy. In the third column, we show that these factors

are not driving the result; we include interactions with income and vehicle value and continue to find a distinct decline in purchases associated with encumbered vehicles. In the final two columns, we consider whether there is an important role for a loan on a vehicle after controlling for liquidity directly, which we measure as the amount in checking and savings accounts. The presence of a loan secured by the potential trade-in continues to have a large negative effect on participation in the CARS program even after controlling for liquid wealth and other covariates. These effects are also robust to a number of further checks and placebo tests described in Section 3.6.5.

The difference in response to CARS across households with different levels of liquid assets provides further evidence, albeit statistically weak evidence, about the role of liquidity in program participation. The fourth and fifth columns of Table 3.5 show that we unfortunately do not have much power to measure these differences, at least in part due to mis-measurement and missing asset data. However, point estimates suggest that households with very little liquidity, less than \$300, and households with lots of liquidity, more than \$4,500, were both less likely to take up CARS. Households with high levels of liquid wealth respond less to CARS (although not statistically significant), consistent with their having liquidity without CARS and therefore benefitting less from the liquidity provided by participation in the CARS program. The behavior of households with low liquid wealth is less consistent with an important role for liquidity (although also statistically weak), but could be explained by their having insufficient liquidity or access to credit to take advantage of CARS.

Having established that liquidity provision was crucial to the CARS program uptake, we now turn to studying the role of debt capacity. Did the ability of households to qualify for loans based on payment-to-income requirements affect response to the program? From the income and balance sheet information reported in the CE, we construct a debt payment-to-income ratio (the sum of mortgage and vehicle debt payments as a fraction of income) and a mortgage leverage ratio (the total mortgage balance as a fraction of the estimated home value). We then test whether the response to CARS varies with income and indebtedness. Households with more income and home equity have more capacity to repay or secure additional borrowing. If the supply of credit were constraining CARS participation, one would therefore expect limited participation by households with low incomes, high payments-to-income ratios and high mortgage leverage.

Table 3.6 presents the results of our analyses. First, as shown in the first column, the baseline effect of the CARS program remains and there is little statistical or economic difference in the impact of CARS across income terciles. Households in the bottom tercile of the sample, with annual after-tax income below \$24,000, still respond strongly to the program. Their purchases increase by a statistically significant 1.76% when they own a clunker rather than a close-to-clunker.⁸ Second, we measure each household's capacity to take on additional monthly payments before reaching a payment-to-income ratio of 1/3 or greater. As shown in the second column of Table 3.6, CARS participation is actually highest for households with the least debt capacity, although this effect is again statistically weak. While it may be a statistical fluke, it does not appear that high existing debt levels constrained participation. Finally, we examine the impact of existing mortgage debt. The last two columns show that both homeowners without mortgages and those with loan-to-value ratios in excess of 50% were more likely to participate in CARS, although as before this result is statistically insignificant. In the final specification, when we sum the coefficients on *Clunker* and its interaction with the indicator for negative home equity, we see that homeowners increased purchases by 1.66% even when they lacked equity to support further mortgage borrowing. In each of these models, we lack the statistical power to draw strong conclusions on how responses varied with debt capacity within our sample. Nevertheless, we find strong CARS participation among the various subsets of households for which debt capacity is most limited, which suggests that debt capacity did not substantially constrain CARS participation.

Why did debt capacity play so limited a role? One possible reason is that, given a large enough down payment, lenders were willing to finance purchases based on collateral value of the new vehicle. Automobiles provide solid collateral that is easier to repossess and re-sell than a home for example, and the CARS subsidy alone provides a 20% down payment at the average new car price of vehicles purchased using CARS.

However, an important issue to consider is whether measurement error in debt payments and income prevent us from identifying borrowing constraints. Our measure of debt payment-to-income is similar to the measure that Johnson and Li (2010) analyzes using both the CE survey and the Survey of Consumer Finances (SCF). That study presents two relevant findings. First, households in

⁸This effect is calculated by adding the 1.97 coefficient on *Clunker* with the -0.21 coefficient on the *Clunker*-Income Bottom Tercile interaction.

the SCF are substantially more likely to be turned down for credit when they have a high payment-to-income ratio, which indicates that survey-based payment-to-income ratios can effectively measure borrowing constraints. Second, the study validates, to some extent, the quality of the CE liability data by showing that the distribution of debt payment-to-income is remarkably similar in the SCF and CE samples. So, while we cannot rule out the possibility that measurement error is obscuring a debt capacity constraint in CARS participation, the evidence from prior work suggests that the CE is measuring real variation in debt capacity.

3.6.4 Intertemporal substitution

Having analyzed the contemporaneous impact of the CARS program, we now turn to analyzing the longer run response after the program ended. Figure 3.4.A and Appendix Table A1 display expanded versions of the results reported in Panel A of Table 3.3. Figure 3.4.A plots the difference in the cumulative share of trade-ins for new vehicles between the treatment group of eligible vehicles and the similar group of ineligible vehicles from the a July 2009 to April 2010 (Figure 3.4.B plots each of these series separately.) The figures reveal two points about the dynamics of the response to CARS.

The first finding is that the increase in purchases caused by CARS lasted for a few months—the effect of CARS was not immediately reversed by lower sales in the few months that followed the program. Our point estimates suggest that the effect of CARS on new vehicles transactions actually continued to rise until November, although this rise is statistically and economically small. We do not treat this increase as part of the main effect of the CARS program and suspect it is due to statistical measurement error.

The second main finding in Figure 3.4 is that following November 2009, there is a rapid reversal in the differential cumulative purchases between households with clunkers and those with close-to-clunkers. The effect of the CARS program reverses quickly, so that by January 2010, there is no longer a statistically significant effect of CARS. By March 2010, the point estimate of the effect of CARS on new vehicle transactions is estimated to be zero, and by April it is slightly negative.

While this result confirms the intertemporal substitution documented in both Mian and Sufi

(2012) and Hockstra, Puller, and West (2016), our evidence is comparatively statistically weak. The shortcoming of the CE data is that households enter the survey on a rolling monthly basis and remain for only 12 months. As a result, roughly 1/12th of the households from the June 2009 cohort exit the survey in each month, leading to greater statistical uncertainty at longer horizons. Indeed, Figure 3.4.B shows that much of the spending reversal that we find relates to a decline in cumulative purchases within the treatment group, which is impossible in a fixed cohort but arises in our sample due to the small sample and the attrition of treatment group members that happened to purchase new vehicles.

3.6.5 Placebo tests and robustness

We cannot directly test our identifying assumption that vehicle purchases would have been similar in the treatment and control groups in the absence of CARS. However, we can test for evidence of bias in the average response and in the differential responses by subsidy amount and liquidity provision. First, we check whether there was any differential purchase or lease of new vehicles in June of 2009, before the program started which might indicate bias in our estimated program impact. Second, we check whether there is any difference in purchases between similarly defined treatment and control groups over the same months of 2008, a year in which the CARS program was not run. Third, we check whether there is any difference in transactions during the CARS period for used vehicles that would not have qualified for the CARS subsidy.

One might have been concerned that the large response of CARS-eligible vehicles was in part due to households delaying purchase from preceding months to take advantage of the CARS rebate. We re-estimate Equation 3.2 with the dependent variable measuring purchases during June 2009, the month before the program. We find an economically small and statistically insignificant effect of the CARS in June of 0.21 with a standard error of 0.25 (reported in Table A2 in the Appendix). The fact that we estimate a positive rather than negative coefficient suggests (somewhat surprisingly) that households did not significantly delay purchases prior to CARS.

Second, using 2008 data, we follow identical sample procedures to construct treatment and control groups, and run our analysis in exactly the same months of 2008, during which the CARS

rebate was not available. Figure 3.5 plots the coefficients starting in July 2008 and is completely analogous to Figure 3.4.A which starts in July 2009. The treatment and control groups purchase or lease new cars at similar rates in July, August, and September of 2008. Although this difference is statistically weak, if anything, the control group has a slightly higher rate of purchase. Panel A of Table 3.7 shows that we no difference in purchases through September of 2008 when we include the full set of control variables.

Third, we estimate Equation 3.2 but with the dependent variable replaced by cumulative purchases or leases of used vehicles during the CARS period. Panel B of Table 3.7 shows that there is no evidence of any difference in purchases of used vehicles caused by CARS: all coefficients are less than one standard error distance from zero. Thus our procedure does not appear to be picking up any differences between treatment and control group in the propensity to purchase vehicles in general that would apply to both new and used vehicles. In sum, these placebo tests do not reject the validity of our identifying assumptions and the resulting estimates of the impact of the CARS program.

Our results are also robust to a number of alternative reasonable assumptions about the sample and the dependent variable, as we show in Appendix Table A2. One might be concerned, for example, that our results are driven by vehicles at the very low and high ends of MPG in our sample, which may be quite different. When we reduce the sample size by narrowing the range of fuel efficiency in our sample to 16 to 21 MPG, we find a slightly larger program response of 1.74% through September, but we lose statistical power, as the standard error more than doubles to 0.97. Another concern is that our sample restriction to make, model and model-years with less than \$5,000 trade-in value omits some vehicles that are worth much less than the average for their make, model and model year and for which CARS may have an effect. In an expanded sample of vehicles with average trade-in value less than or equal to \$6,500, we find an estimated effect of CARS of a 1.61% ($p < 0.01$) increased probability of purchase. We also find similar results when we conduct our analysis at the household level rather than the vehicle level, an alternative assumption that reduces the effective variation in the data slightly. A third concern is that CARS causes some people to scrap an old vehicle to make a purchase that they otherwise would have made while continuing to hold on to their old vehicle. Such behavior could bias upward our measured effect

of CARS on purchases. For such people taking advantage of CARS, we would count the purchase because of the associated trade-in, while for such people with ineligible vehicles, we would not count their purchase because it would not be associated with a trade-in. To investigate this possibility, we replace our dependent variable with the cumulative purchases or leases of new all new vehicles rather than just those associated with the disposal of a vehicle by a household. Instead of a lower effect of CARS however, we find a slightly larger measured effect of 1.62% ($p < 0.05$). In sum, our results are generally robust to reasonable alternatives.

Finally, we provide further evidence that our findings on the importance of liquidity provision by CARS are not instead driven by other factors. As discussed in the previous section, we obtain our main liquidity findings by comparing the response to CARS of potential trade-in vehicles with and without outstanding loans. If the difference in trade-in rates between cars with and without outstanding loans were driven by something other than the differential liquidity provided to these groups by the CARS program, then these differences should also appear in our placebo analyses using purchases in 2008 and purchases of used vehicles. Including the interaction with outstanding loan in each placebo test, the presence of a loan securing a vehicle has a statistically insignificant (and if anything positive) effect on participation in pseudo-CARS (see Table 3.7). We also address the concern that our estimates of treatment effect heterogeneity with respect to liquidity may actually be capturing treatment heterogeneity in other dimensions. As we show in Table A3 of the Appendix, our findings are robust to allowing for the response to CARS to also vary with income, existing payment-to-income ratio, and mortgage loan-to-home value terciles. Finally, we continue to find a strong negative effect of having a loan secured by a vehicle when we control for a broader measure of wealth rather than just liquid wealth.

In summary, our estimates of the response to the CARS program appear to be well identified and not driven by several possible biases or sources of mis-measurement. We now turn to using these estimates to study the aggregate effects of the program.

3.6.6 Partial-equilibrium impact on aggregate demand for vehicles

We use our household-level estimates to draw inferences about the aggregate impact of the CARS program on the number and dollar value of vehicle purchases. Below, we describe this calculation and compare our estimate of aggregate impact to the estimates of prior studies.

First, we estimate the number of CARS-eligible vehicles in the U.S. Assuming vehicle ownership is unrelated to CE data being missing, we use the CE sample weights scaled up for missing data to estimate that there were 35,423,323 of CARS-eligible vehicles with value less than \$5,000. We also calculate an alternative measure using the Polk data on registrations merged with vehicle values from Edmunds. This calculation yields a similar number: 38,737,677 such vehicles.

Second, we multiply the number of CARS-eligible vehicles by the number of purchases per CARS-eligible vehicle estimated in Section 3.6.1. According to the first column of Table 3.3, the CARS program raised the probability of purchase by 1.43% over the three month period from June to September 2009. To calculate the number of vehicle purchases at the time of the program caused by the CARS, we multiply the percentage increase in purchases per CARS-eligible vehicle by the total number of CARS-eligible vehicles, which implies that the CARS program directly caused an additional 506,553 purchases or leases of new vehicles between July and September 2009 based on our CE estimate. The corresponding number from the Polk-Edmunds total is 553,949.

Third and finally, we calculate the impact on aggregate demand using the average reported purchase price in the CE data for new vehicle purchases between July and September 2009 with trade-in value of \$3,500 or \$4,500. This average purchase price of \$22,283 is very close to the National Highway Traffic Safety Administration (2009b) report of the average MSRP of vehicles purchased using the CARS program, which is \$22,453. These numbers imply that the CARS program raised demand by \$11 billion in incremental purchases or leases (506,553 purchases x \$22,283 per purchase) according to our CE-based estimate or \$12 billion according to our Polk-Edmunds based estimate. According to the National Highway Traffic Safety Administration (2009), just under half of the vehicles purchased were produced domestically, and vehicles purchased that were produced domestically were slightly more expensive than those that were imported.

Our baseline estimates therefore imply that the CARS program increased demand (meaning a partial-equilibrium, accounting estimate) for durable goods by \$11 billion in the third quarter of 2009, or by \$44 billion at an annual rate. In terms of the expenditure accounts, roughly half of this was an increase in demand for imported vehicles, and potentially some of the demand was met through reduced inventory investment (of imported and domestically-produced goods), so that the impact on production was almost surely less than the full \$44 billion (again in a partial-equilibrium, accounting sense). Due to inventory reduction, the accounting effect on national income is likely larger than the production-side effect and may be closer (at least contemporaneously) to the complete \$44 billion. To put these numbers in perspective, GDP increased by \$43.7 billion in the third quarter of 2009, coinciding with the end of the recession (the NBER dates the trough as June). Real GDP had fallen \$200 billion per quarter in the two worst quarters of the recession—the last quarter of 2008 and the first quarter of 2009—and it fell by \$43.5 billion in the second quarter immediately before CARS.

Our estimate of the aggregate impact is within the range of estimates reported in prior studies, but at the high end of the range. Based on transactions in other periods and the prevalence of CARS-eligible vehicles, Council of Economic Advisors (2009) estimates that 240,000 of the purchases made under the CARS program would have occurred anyway, so that CARS caused 440,000 additional purchases.⁹ Based on a survey of households that participated in CARS, National Highway Traffic Safety Administration (2009) estimates that CARS caused an additional 600,000 purchases. Mian and Sufi (2012) estimates that CARS caused between 340,000 and 400,000 new purchases. However, their analysis may underestimate the program's impact. They assume CARS caused no purchases in cities with a bottom-decile share of clunkers, despite the fact that these cities still had 5.8 clunkers per 2004 purchase (compared to a city average of 9.9).¹⁰ Li, Linn, and Spiller (2013) consider the experience of Canada as a counterfactual to the United States and estimate 370,000 incremental purchases due to CARS.

⁹At the time of the program, however, the economy was just emerging from the Great Recession, so coincident changes to incomes, wealth, and uncertainty could also be responsible for deviations from the estimated path of sales.

¹⁰If one assumed that CARS had an effect on purchases in these bottom-decile cities, this would raise the estimated aggregate effect by a factor of $9.9/(9.9 - 5.8)$, to 893,000.

3.7 Conclusion and discussion

This paper uses household expenditure data to evaluate the CARS stimulus program and to investigate whether financial frictions dampened the response to the program. To identify the program impact, we compare purchases by owners of eligible vehicles to purchases by owners of ineligible vehicles with similar value but fuel economy above the CARS program cutoff. We also use information on households' assets and liabilities, unique to our evaluation of CARS, to understand whether take-up varied with liquidity, debt capacity, and the size of the program subsidy.

Our estimates of the average effect of the CARS program lie within the range of previous estimates. We provide new evidence that take-up increased with the size of the economic subsidy, which was the official credit less the value of the trade-in. In aggregate, we find that during the period of the program, purchases using CARS-eligible vehicles doubled relative to the comparison group, generating roughly \$11 billion in additional (partial-equilibrium) demand from a Federal outlay of only \$2.9 billion. However, consistent with theory and previous research, this large effect was due to short-term intertemporal substitution in response to the temporary price subsidy: although we have limited power, our point estimates suggest that cumulative (partial-equilibrium) auto sales were unaffected by the program seven months after its initiation.

Our evidence is consistent with the hypothesis that the response to CARS was significantly amplified by the liquidity it provided. Since roughly 80% of new vehicle purchases are financed, a household's ability to borrow was critical for its participation in CARS. By offering a large credit available at the time of sale, CARS provided liquidity that could be used to meet the down payment requirement typical of a new vehicle loan. For households with preexisting loans on their potential trade-in vehicles, however, participation required further liquidity to immediately repay the loan on the scrapped vehicle. Consistent with binding liquidity constraints, we show that program participation decreased significantly for these households, even when controlling for any differences in their income, liquid assets, existing car value and the economic subsidy offered by trading it in, and baseline propensity to purchase new vehicles. In contrast, while statistical power is limited, we find no measurable differences in take-up for households with unsecured outstanding debts, nor any evidence that household responses were constrained by debt capacity. Households with modest

income, high debt payment-to-income ratios, and high mortgage leverage all show strong responses to the program. By making possible a large down payment, it is possible that CARS facilitated loans to risky borrowers on the strength of the collateral rather than the borrowers' ability to repay. We conclude that household liquidity significantly constrained participation in the CARS program.

Our findings offer lessons for the design of similar programs. Our findings suggest that responses to such programs are larger if subsidies are timed so that they can contribute to down payments and alleviate liquidity constraints, rather than being given as tax incentives to be paid at later dates. We would also expect significantly lower responses if subsidies were insufficient to contribute a substantial portion of the typical down payment. While significantly larger subsidies would draw in more households, we would expect the per dollar responses to be lower as additional funds beyond typical down payment amounts would only have the subsidy benefit and not also a liquidity benefit.

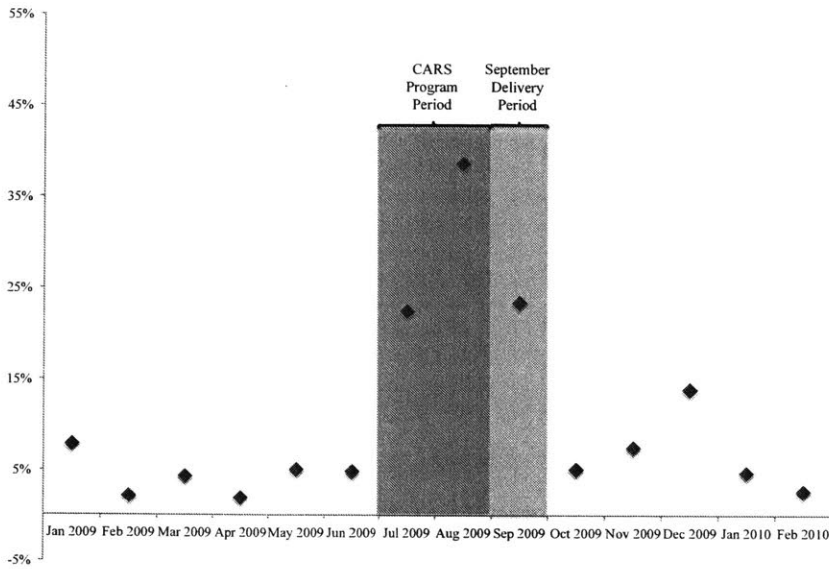
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Panel A: New Vehicle Purchases



Panel B: Used Vehicle Purchases

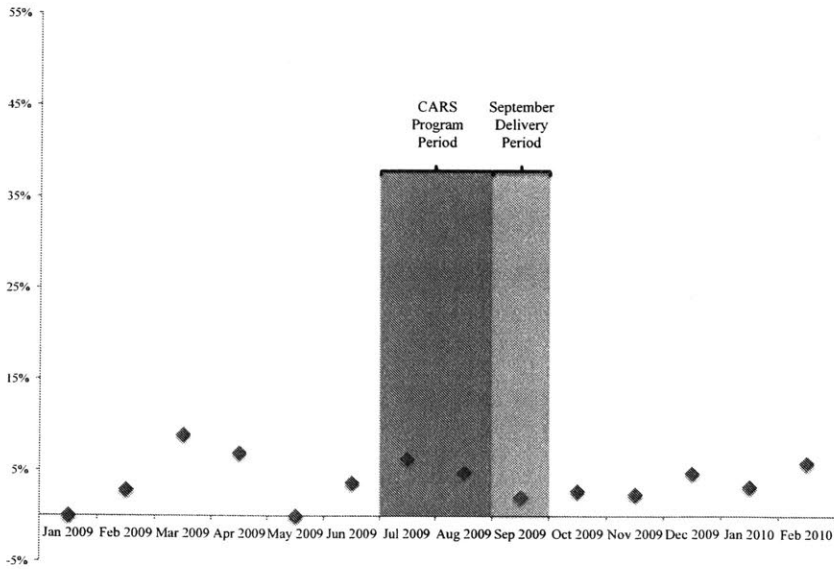
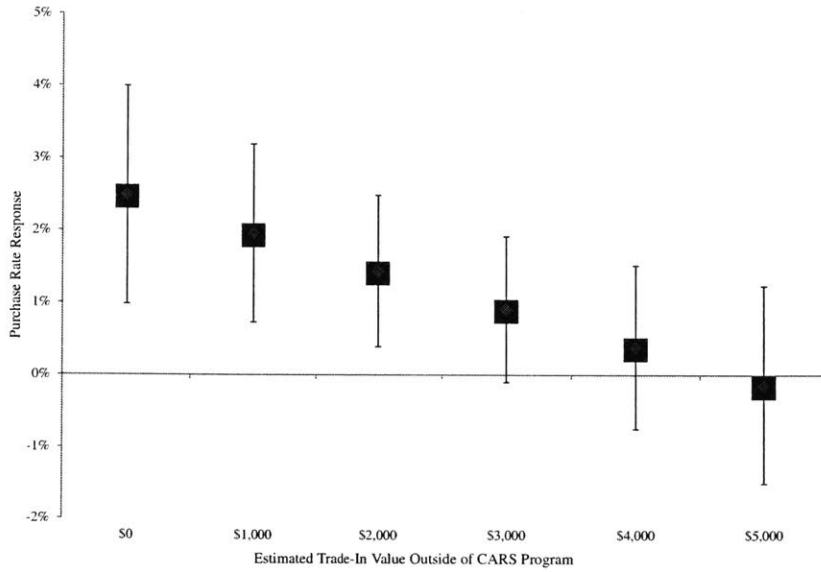


Figure 3.1: **Proportion of Trade-ins with Value of \$3,500 or \$4,500.** These figures plot the proportion of trade-ins with value of \$3,500 or \$4,500 on new (Panel A) and used vehicle (Panel B) purchases between January 2009 and February 2010. The x-axis corresponds to the month of the purchase. The sample is constructed from CE survey responses between 2009 through 2013, and includes transactions that occurred during the respondents' participation in the survey and transactions that were reported retrospectively in interviews between 2010 and the first quarter of 2013.

Panel A: CARS Response: Purchase Rate



Panel B: CARS Response: Dollars

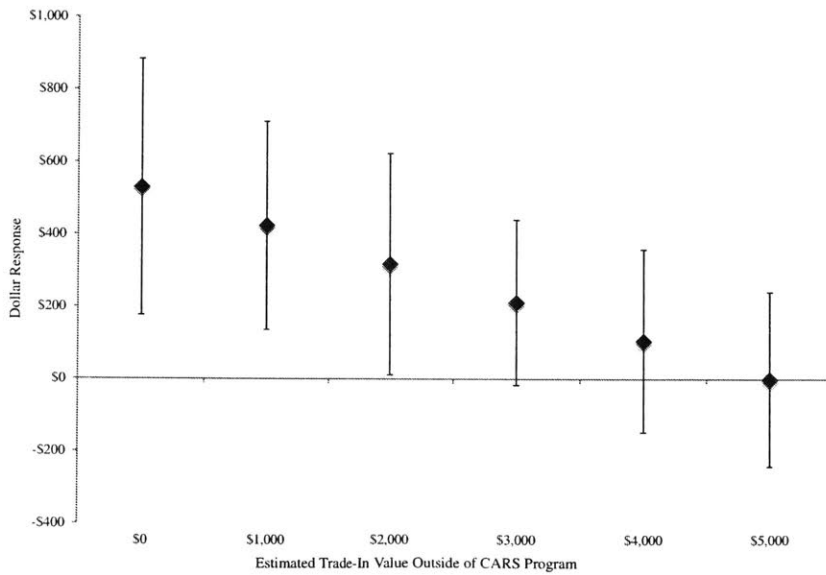
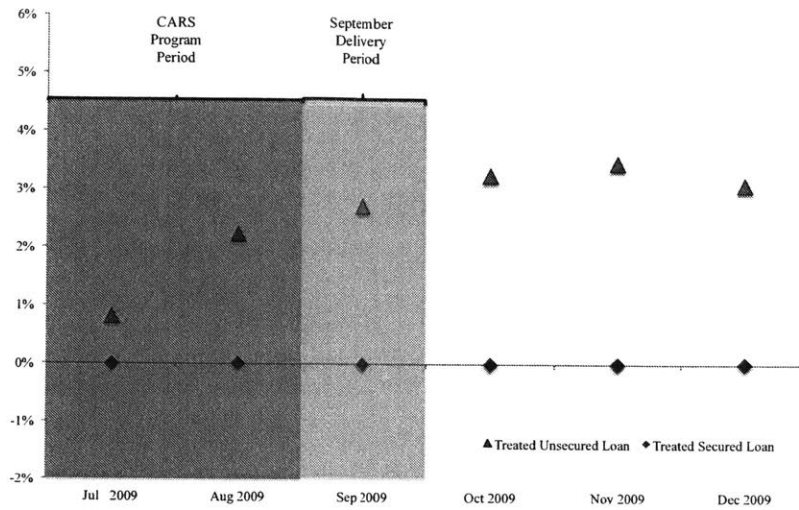


Figure 3.2: **CARS Response by Trade-in Value.** This figure plots the CARS response for eligible vehicles of different trade-in values. The y-axis is change in the rate of new vehicle purchases or leases associated with CARS. The point estimates and confidence intervals are calculated from the model reported in the first column of Table 3.4, Panel A.

Panel A: Treatment Group Cumulative Purchase Rate with Vehicle Loan or Unsecured Loan



Panel B: Control Group Cumulative Purchase Rate with Vehicle Loan or Unsecured Loan

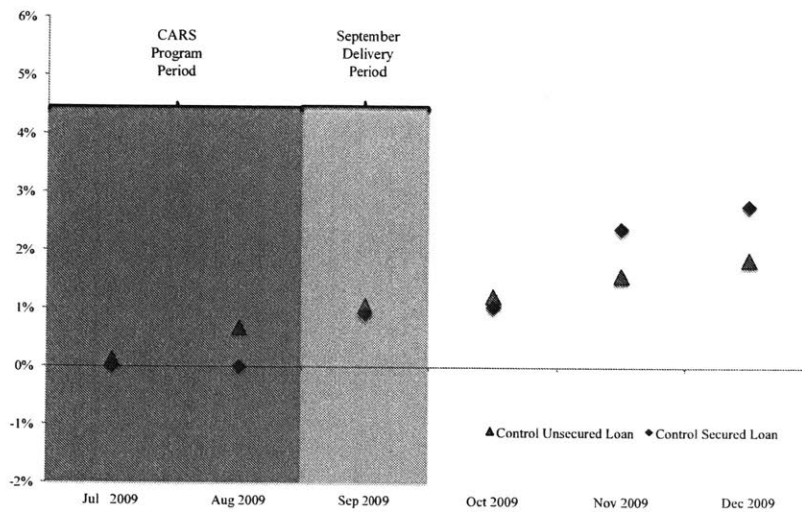
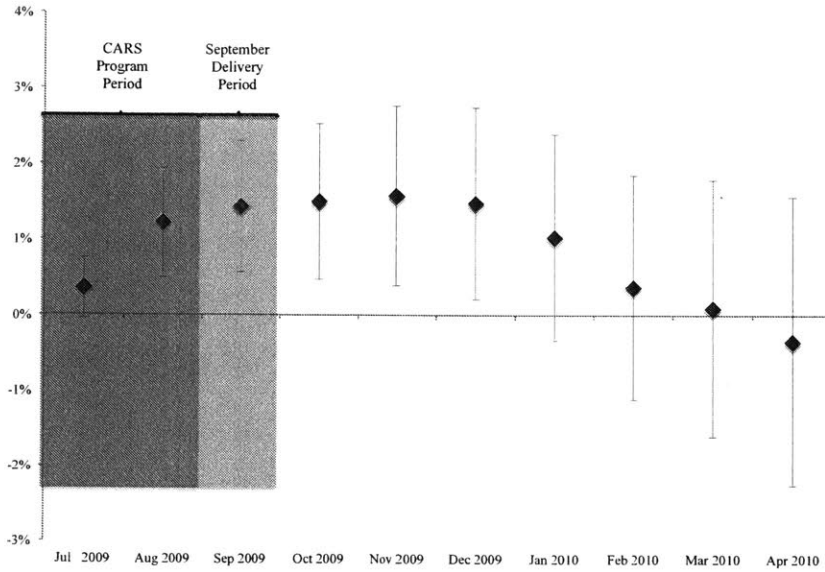


Figure 3.3: **CARS Response and Outstanding Debt.** This figure plots the cumulative rate of new vehicle purchases or leases in four subgroups of the main sample, defined by CARS eligibility (“treated” or “control”) and household indebtedness. Within each panel, we report cumulative purchases or leases separately for vehicles encumbered by a loan and vehicles owned by households with outstanding unsecured loans. To obtain these estimates we include interactions of the *Clunker* indicator with indicators of secured and unsecured debt balances.

Panel A: Cumulative Difference between Treatment and Control Groups



Panel B: Cumulative Purchases in Treatment and Control Groups

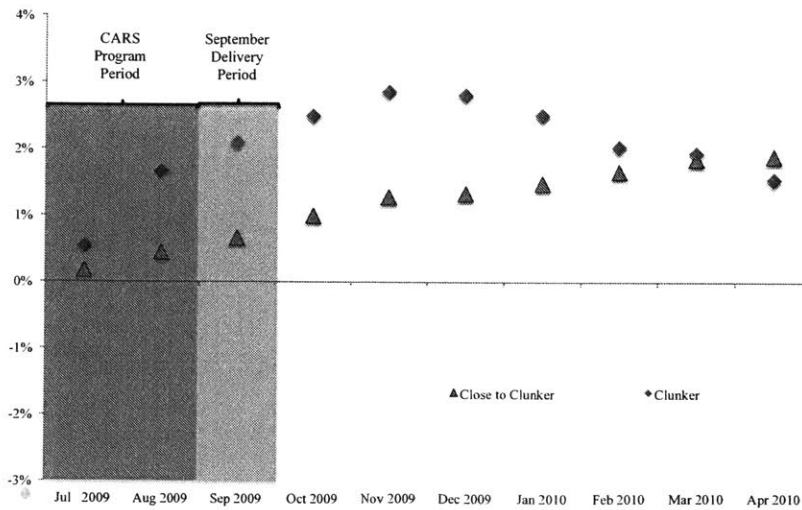


Figure 3.4: **Cumulative Impact of CARS on Rate of New Vehicle Purchases.** Panel A plots the cumulative difference (since June 2009) in the rate of new vehicle purchases for CARS-eligible vehicles compared to similar ineligible vehicles. The lines indicate the 95% confidence intervals, computed with clustering at the household level. Panel B plots the cumulative rate of purchases for eligible and ineligible vehicles separately.

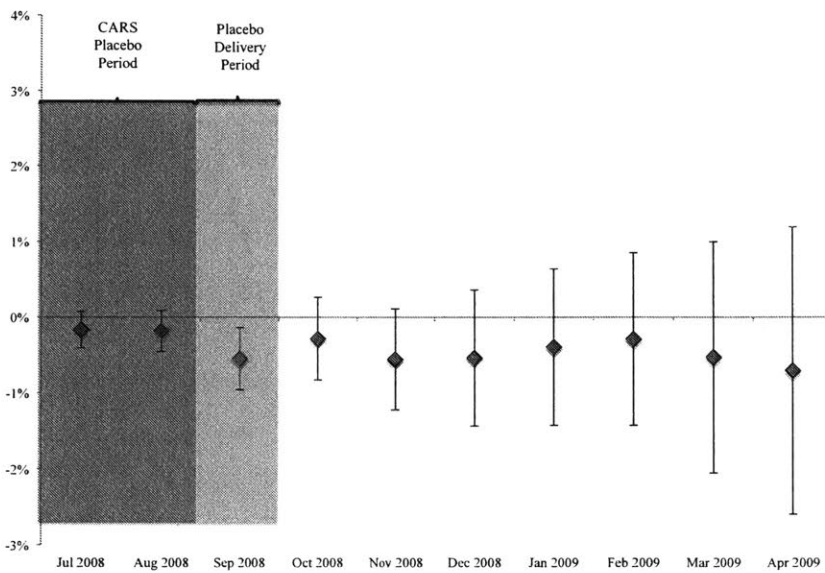


Figure 3.5: **Placebo Test: Cumulative Change in Purchases during 2008.** This figure plots the full set of *Clunker* coefficients and 95% confidence intervals for the model reported in Table 3.7, Panel A. For each month between July 2008 and April 2009, the coefficient measures the cumulative difference (since July 2008) in the rate of new vehicle purchases associated with hypothetically-eligible vehicles compared to similar ineligible vehicles.

Table 3.1: CARS eligibility requirements and rebate amounts

NEW VEHICLE TYPE	NEW VEHICLE FUEL ECONOMY	TRADE-IN VEHICLE TYPE	DIFFERENCE IN MPG, NEW VS. TRADE-IN	REBATE AMOUNT
Passenger Automobile: <ul style="list-style-type: none"> All passenger cars. 	At least 22 MPG	Passenger car, Category 1 or 2 truck with MPG 18 or less	4-9 MPG	\$3,500
			10 MPG or more	\$4,500
Category 1 Truck: <ul style="list-style-type: none"> All SUVs w/GVWR $\leq 10,000$ lbs. Pickups w/GVWR $< 8,500$ lbs. and wheelbase ≤ 115 in. Passenger vans and cargo vans w/GVWR $< 8,500$ lbs. and wheelbase ≤ 124 in. 	At least 18 MPG	Passenger car, Category 1 or 2 truck with MPG 18 or less	2-5 MPG	\$3,500
			5 MPG or more	\$4,500
Category 2 Truck: <ul style="list-style-type: none"> Pickups w/GVWR $\leq 8,500$ lbs. and wheelbase > 115 in. Passenger vans and cargo vans w/GVWR $\leq 8,500$ lbs. and wheelbase > 124 in. 	At least 15 MPG	Category 2 truck with MPG 18 or less	1 MPG	\$3,500
			2 MPG or more	\$4,500
		Category 3 truck	NA	\$3,500
Category 3 Truck: <ul style="list-style-type: none"> Trucks w/GVWR 8,500–10,000 lbs. that is either large cargo van or pickup trucks w/cargo bed > 72 in. 	NA	Category 3 truck	NA. New vehicle must be no larger than trade-in	\$3,500

Source: National Highway Traffic Safety Administration (2009). Fuel economy requirements are based on EPA's combined city/highway ratings. To be eligible, a trade-in vehicle must have a fuel economy rating of 18 MPG or less. Category 3 trucks do not have EPA fuel economy ratings.

Table 3.2: Summary Statistics, Stratified by *Clunker* Status

Sample characteristics	Subsample: <i>Classified</i>		<i>Unclassified</i>			
	<i>Clunker</i>	<i>Close-to-Clunker</i>	<i>Outside Window</i>	<i>Uncertain</i>	<i>Too Valuable</i>	<i>Missing Data</i>
Number of vehicles	1,676	2,265	563	467	4,504	1,160
Number of households	1,480	2,014	538	452	3,390	967
Sample mean						
Vehicle age (years)	13.1	11.8	12.4	14.2	4.4	15.3
Vehicle value (\$ thousands)	2.1	2.1	1.6	1.6	11.4	0.6
Vehicle fuel economy (MPG)	15.6	21.0	27	19	20	
Vehicle loan outstanding (indicator)	5.7%	7.2%	5.0%	5.9%	45.0%	13.1%
Vehicle loan balance, if > 0 (\$ thousands)	5.0	4.7	2.8	4.0	11.9	12.2
Household income (\$ thousands)	51.7	50.8	49	49	71	49
Financial assets (\$ thousands)	50.5	46.0	42	58	105	55
Unsecured loan outstanding (indicator)	41.5%	41.1%	45.9%	45.0%	47.0%	36.2%
Unsecured loan balance, if > 0 (\$ thousands)	9.7	9.1	15.5	11.6	11.5	10.4
Mortgage loan-to-value ratio, if homeowner	36.8%	34.9%	39.2%	36.7%	39.7%	33.3%
Negative home equity (indicator)	8.5%	6.0%	8.6%	6.6%	7.9%	7.9%
Sample median						
Monthly debt-payment-to-income ratio	6.7%	5.0%	3.9%	5.2%	11.9%	6.6%
July - September 2009 Vehicle Purchases						
Mean Purchase Price (\$ thousands)	24.7	26.8	25.0	23.6	27.9	33.5
conditional on trade-in	23.8	25.7	27.2	21.8	28.6	25.8
\$3,500 or \$4,500 trade-in	21.8	13.5		16.1	21.8	11.8

Notes: This table reports descriptive statistics for the Consumer Expenditure Survey (CE). The main regression sample includes *Clunker* and *Close-to-Clunker* vehicles with estimated trade-in value of \$5,000 or less that were owned as of June 2009. A *Clunker* is a vehicle purchased prior to July 2008 for which at least 75% of 2009 registrations in the same make-model-year had fuel economy between 12 MPG and 18 MPG. A *Close-to-Clunker* is a vehicle purchased before July 2008 for which at least 75% of registrations are between 19 MPG and 25 MPG or a vehicle purchased after July 2008 for which 75% of registrations are between 12 MPG and 18 MPG. Unclassified vehicles are, by column: those with either fuel economy above 25 MPG or purchase date between July 2008 and June 2009 (“Outside Window”); those for which more than 25% of the vehicles are eligible but at least 25% ineligible based on fuel economy (“Uncertain”); those with average trade-in value above \$5,000 (“Too Valuable”); and those with insufficient CE data reported to classify (“Missing Data”).

Table 3.3: The Impact of CARS on New Vehicle Purchases and Leases

Panel A: Cumulative rate of new vehicles purchased or leased since July 2009

<i>Estimated impact during CARS</i>	Sample Size	No Controls	All Controls Except MPG	All Controls
Clunker (through Jul 2009)	3,941	0.36 (0.20)	0.37 (0.20)	0.54 (0.32)
Clunker (through Aug 2009)	3,548	1.22 (0.36)	1.29 (0.37)	1.43 (0.52)
Clunker (through Sep 2009)	3,162	1.43 (0.43)	1.56 (0.45)	1.72 (0.63)

Panel B: Cumulative expenditure on new vehicles purchased or leased since July 2009

<i>Estimated impact during CARS</i>	Sample Size	No Controls	All Controls Except MPG	All Controls
Clunker (through Jul 2009)	3,941	64.1 (43.0)	69.9 (43.4)	99.9 (71.4)
Clunker (through Aug 2009)	3,548	263.0 (86.4)	286.2 (88.0)	315.8 (120.7)
Clunker (through Sep 2009)	3,162	326.5 (106.9)	368.0 (111.3)	403.7 (151.2)

Panel C: Implied unit purchase prices associated with CARS transactions

<i>Estimated impact during CARS</i>	Sample Size	No Controls	All Controls Except MPG	All Controls
Clunker (through Jul 2009)	3,941	17,784	19,024	18,470
Clunker (through Aug 2009)	3,548	21,523	22,161	22,146
Clunker (through Sep 2009)	3,162	22,912	23,632	23,475

Notes: This table reports analysis of the rate and value of new vehicle purchases and leases during the CARS program and thereafter. The Clunker coefficients in Panel A are multiplied by 100 and measure the percentage point difference in cumulative vehicle purchases (since July 2009) associated with CARS-eligible trade-in vehicles compared to similar but ineligible trade-ins. Panel B reports specifications that measure the total dollar expenditure in the Clunkergroup. The regression sample includes Clunker and Close-to-Clunker vehicles, as defined in Table 3.2, with estimated trade-in value of \$5,000 or less that were owned as of June 2009. Panel C reports the vehicle purchase prices that took place because of CARS that are implied by the coefficient estimates in Panels A and B. The column headings indicate the control variables, which include vehicle age, trade-in value and fuel economy (MPG), household after-tax income, the number of CE interviews ever completed by the household, and the total number of CE interviews missed to date. Vehicle age is the the number of months since the January of the vehicle model-year. Vehicle trade-in value and fuel economy are averaged across drivetrain configurations of the make, model, and model-year. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table 3.4: Trade-in Value and the CARS Program Response

Panel A: Baseline sample of vehicles with trade-in value less than or equal to \$5,500				
Dependent variable:	Cumulative purchases and leases of new vehicles		Cumulative dollars spent on new vehicles	
Sample period:	Through Aug 2009	Through Sep 2009	Through Aug 2009	Through Sep 2009
Clunker	2.49%	2.60%	\$530.1	\$552.9
	(0.75)	(0.85)	(176.8)	(199.6)
Clunker × Value (in \$ thousands)	-0.53	-0.44	-106.0	-74.1
	(0.21)	(0.26)	(47.5)	(63.8)
Observations	3,548	3,162	3,548	3,162
Clunker effect @ \$1500 Value	1.70	1.94	370.9	441.8
	(0.56)	(0.66)	(130.7)	(155.1)
Panel B: Sample of vehicles with trade-in value less than or equal to \$6,500				
Dependent variable:	Cumulative purchases and leases of new vehicles		Cumulative dollars spent on new vehicles	
Sample period:	Through Aug 2009	Through Sep 2009	Through Aug 2009	Through Sep 2009
Clunker	0.46	0.38	166.0	164.8
	(0.65)	(0.77)	(203.2)	(234.8)
Clunker × (Value < \$1000)	1.50	1.51	294.2	288.2
	(0.82)	(0.95)	(235.6)	(266.0)
Clunker × (< \$1000 < Value < \$2500)	1.04	1.79	119.0	289.4
	(0.88)	(1.10)	(248.0)	(306.7)
Clunker × (< \$2500 < Value < \$5000)	0.48	1.06	70.0	217.4
	(0.71)	(0.90)	(213.9)	(260.6)
Observations	4,197	3,744	4,197	3,744

Notes: This table reports estimates from regressions of the number or value of new vehicle purchases on an indicator for CARS eligibility (*Clunker*) and its interaction with vehicle trade-in value. Panel A uses the same sample as Table 3.3 but includes an interaction with the vehicle's average trade-in value. Panel B uses an expanded sample—vehicles with trade-in value up to \$6,500—and includes interactions of *Clunker* with indicators for various ranges of trade-in value. The excluded group is vehicles with trade-in value between \$5,000 and \$6,500. Each regression includes the full set of controls described in Table 3.3. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table 3.5: Household Liquidity and the CARS Program Response

Dependent variable:	Number of new vehicles purchased or leased between July and September 2009				
Clunker	1.79 (0.52)	1.63 (0.59)	2.85 (0.96)	2.12 (0.92)	2.24 (1.47)
Clunker X Outstanding loan on vehicle	-2.70 (1.05)		-2.30 (1.03)		-2.02 (1.24)
Clunker × Outstanding unsecured loan		0.04 (1.03)			
Clunker × Assets bottom tercile ($< \$300$)				-1.67 (1.07)	-1.19 (1.18)
Clunker × Assets middle tercile (omitted) (\$300 – \$4,500)					
Clunker × Assets upper tercile ($> \$4,500$)				-0.72 (1.41)	-0.73 (1.60)
Clunker × Income			0.00 (0.01)		0.02 (0.02)
Clunker × Value			-0.40 (0.30)		-0.49 (0.32)
Observations	2,673	2,722	2,673	2,106	1,821

Notes: This table reports estimates from regressions of the number of new vehicle purchases on an indicator for CARS program-eligibility (*Clunker*) and its interaction with household financial variables. These variables include indicators for whether the potential trade-in is encumbered by an outstanding loan, whether the household has an outstanding unsecured loan, the tercile of the household's liquid assets, and the household's after-tax income. The model includes a control for each financial variable when it is interacted with *Clunker*. Coefficients are multiplied by 100 to reflect purchase rates in percentage points. The model also includes the full set of control variables used in Table 3.3. Assets are measured as the sum of reported checking and savings account balances. The regression sample is the same as in Table 3.3, subject to the further requirement that the financial variables included in the specification are non-missing in the CE. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table 3.6: Household Debt Capacity and CARS Program Response

Dependent variable:	Number of new vehicles purchase or leased between July and September 2009			
Clunker	1.97 (0.85)	1.32 (0.80)	1.23 (1.32)	2.12 (0.99)
Clunker × Income bottom tercile ($< \$24,000$)	-0.21 (0.91)			
Clunker × Income middle tercile (omitted) ($\$24,000 - \$57,500$)				
Clunker × Income upper tercile ($> \$57,500$)	-0.58 (1.15)			
Clunker × (PTI already $> 1/3$)		1.87 (1.23)		
Clunker × (PTI $> 1/3$ with $\$0 < \text{payment} < \500)				
Clunker × (PTI $> 1/3$ only with payment $> \$500$)		0.80 (0.93)		
Clunker × Mortgage LTV bottom tercile ($LTV = 0$)			1.04 (1.37)	
Clunker × Mortgage LTV middle tercile (omitted) ($0 < LTV < 0.5$)				
Clunker × Mortgage LTV upper tercile ($LTV > 0.5$)			1.25 (1.58)	
Clunker × Negative home equity (indicator)				-0.47 (1.39)
Observations	3,162	2,722	2,010	2,010

Notes: This table reports estimates from regressions of the number of new vehicle purchases on an indicator for CARS program-eligibility (*Clunker*) and its interaction with household financial variables. These variables include indicators for ranges of household income, debt-payment-to income ratio, mortgage loan-to value ratio, and an indicator for negative home equity. The model includes a control for each financial variable when it is interacted with *Clunker*. Coefficients are multiplied by 100 to reflect purchase rates in percentage points. The model also includes the full set of control variables used in Table 3.3. The payment-to-income ratio (PTI) indicators consider whether a household's PTI would be above 1/3 after including a hypothetical new debt payment of various sizes. The regression sample is the same as in Table 3.3, subject to the further requirement that the financial variables included in the specification are non-missing in the CE. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table 3.7: Placebo Tests of CARS Program Response

Panel A: Analysis of Placebo Period, 2008			
Dependent Variable:	Number of new vehicles purchased or leased between July and September 2008		
Clunker	-0.36 (0.36)	-0.18 (0.45)	-0.03 (0.53)
Clunker × Value		-0.09 (0.19)	-0.02 (0.21)
Clunker × Outstanding loan on vehicle			-0.31 (0.31)
Clunker × Income			0.00 (0.00)
Observations	3,003	3,003	2,500
Panel B: Analysis of Placebo Outcome, Used Vehicle Purchases			
Dependent Variable:	Number of used vehicles purchased between July and September 2009		
Clunker	0.58 (0.52)	0.66 (0.83)	1.40 (0.99)
Clunker × Value		-0.04 (0.26)	-0.06 (0.30)
Clunker × Outstanding loan on vehicle			1.05 (1.64)
Clunker × Income			-0.01 (0.01)
Observations	3,162	3,162	2,673

Notes: This table reports regression estimates from two placebo tests. Panel A reports analysis of CARS as if it had run in July and August of 2008. Following an identical procedure to our main analysis, we classify vehicles owned as of June 2008 according to their hypothetical eligibility for CARS and estimate the cumulative new vehicle purchases or leases associated with eligible vehicles compared to ineligible vehicles. Panel B reports analysis of used vehicle purchases, which were not eligible for the CARS rebate. Repeating the main analysis of Table 3.3 Panel A, we estimate the impact of CARS on used vehicle purchases. Coefficients are multiplied by 100 to reflect purchase rates in percentage points. Each model includes the full set of control variables listed in Table 3.3. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table A1: The Cumulative Impact of CARS on the Likelihood of New Vehicle Purchases and Leases through April 2010

Dependent variable: Number of new vehicles purchased or leased between July 2009 and month:										
Panel A: Cumulative Program Impact - No Controls										
	Jul 2009	Aug 2009	Sep 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	Mar 2010	Apr 2010
Clunker	0.36 (0.20)	1.22 (0.36)	1.43 (0.43)	1.50 (0.51)	1.58 (0.60)	1.48 (0.64)	1.03 (0.68)	0.37 (0.74)	0.09 (0.85)	-0.35 (0.95)
Observations	3,941	3,548	3,162	2,737	2,401	2,072	1,720	1,371	1,061	749
R^2	0.001	0.004	0.004	0.003	0.003	0.003	0.001	0.000	0.000	0.000
Panel B: Cumulative Program Impact - All Controls except MPG										
	Jul 2009	Aug 2009	Sep 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	Mar 2010	Apr 2010
Clunker	0.37 (0.20)	1.29 (0.37)	1.56 (0.45)	1.67 (0.53)	1.65 (0.61)	1.52 (0.65)	1.07 (0.71)	0.34 (0.76)	0.10 (0.86)	-0.29 (0.96)
Observations	3,941	3,548	3,162	2,737	2,401	2,072	1,720	1,371	1,061	749
R^2	0.007	0.008	0.011	0.010	0.010	0.009	0.006	0.005	0.005	0.004
Panel C: Cumulative Program Impact - All Controls										
	Jul 2009	Aug 2009	Sep 2009	Oct 2009	Nov 2009	Dec 2009	Jan 2010	Feb 2010	Mar 2010	Apr 2010
Clunker	0.54 (0.32)	1.43 (0.52)	1.72 (0.63)	2.03 (0.74)	2.48 (0.87)	2.57 (0.96)	1.85 (0.97)	0.84 (1.06)	0.73 (1.20)	0.14 (1.55)
Observations	3,941	3,548	3,162	2,737	2,401	2,072	1,720	1,371	1,061	749
R^2	0.007	0.008	0.011	0.010	0.011	0.010	0.007	0.005	0.006	0.004

Notes: This table expands on Table 3.3, reporting cumulative estimates of the impact of the CARS program at each monthly horizon from July 2009 to April 2010. Coefficients are multiplied by 100 to reflect purchase rates in percentage points. The panel headings indicate the control variables, which include vehicle age, trade-in value, and fuel economy, household after-tax income, the number of CE interviews ever completed by the household, and the total number of CE interviews missed to date. Vehicle age is the the number of months since the January of the vehicle model-year. Vehicle trade-in value and fuel economy are averaged across drivetrain configurations of the make, model, and model-year. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table A2: Measuring the Impact of CARS with Alternative Assumptions

Dependent Variable:	Cumulative purchases or leases of new vehicles					
Specification:	Program Anticipation? Purchase in June 2009	Narrow Fuel Economy Range: 16-21 MPG	Wide Trade-in Value Range: < \$6500	Household-level Purchases with Disposal	Purchases with or without Disposal	State Fixed Effects
Clunker (during Jun 2009)	0.21 (0.25)					
<i>Estimated impact during CARS</i>						
Clunker (through Jul 2009)		0.73 (0.47)	0.46 (0.29)	0.52 (0.33)	0.54 (0.36)	0.62 (0.35)
Clunker (through Aug 2009)		1.45 (0.77)	1.31 (0.46)	1.35 (0.57)	0.96 (0.57)	1.05 (0.58)
Clunker (through Sep 2009)		1.74 (0.97)	1.61 (0.56)	1.19 (0.69)	1.62 (0.69)	1.80 (0.70)

Notes: This table presents analysis of the CARS program under alternative assumptions. Coefficients are multiplied by 100 to reflect purchase rates in percentage points. The first model examines program anticipation, measuring the difference in the number of purchases or leases in the month before the program (June 2009). The second and third models use different rules for sample construction: a narrow range of fuel economy (16 to 21 MPG) or a wider range of trade-in value (less than \$6,500). The final two models use measures of eligibility and purchases at the household level. In these specifications, *Clunker* is an indicator for whether the household owns any program-eligible vehicle. In the fourth model, the dependent variable is the total number of new vehicle purchases or leases by the household that coincide with disposal of a *Clunker* or *Close-to-Clunker*. In the fifth column, the dependent variable is the total number of new vehicle purchases or leases irrespective of vehicle disposal. The final model includes state fixed effects. Each model includes the full set of control variables: vehicle age, trade-in value and fuel economy (MPG), household after-tax income, the number of CE interviews ever completed by the household, and the total number of CE interviews missed to date. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.

Table A3: Household Liquidity and CARS Program Response, Controlling for Debt Capacity

Dependent variable:	Number of new vehicles purchased or leased between July and September 2009			
Clunker	2.81 (1.06)	1.50 (0.84)	1.27 (1.37)	2.15 (1.02)
Clunker × Outstanding loan on vehicle	-2.93 (1.12)	-2.76 (1.07)	-1.65 (0.77)	-1.64 (0.63)
Clunker × Income bottom tercile ($< \$24,000$)	-0.48 (1.09)			
Clunker × Income middle tercile (omitted) ($\$24,000 - \$57,500$)				
Clunker × Income upper tercile ($> \$57,500$)	-1.00 (1.32)			
Clunker × (PTI already $> 1/3$)		1.87 (1.24)		
Clunker × (PTI $> 1/3$ with $\$0 < \text{payment} < \500)				
Clunker × (PTI $> 1/3$ only with payment $> \$500$)		0.79 (0.94)		
Clunker × Mortgage LTV bottom tercile (LTV = 0)			1.00 (1.40)	
Clunker × Mortgage LTV middle tercile (omitted) ($0 < \text{LTV} < 0.5$)				
Clunker × Mortgage LTV upper tercile (LTV > 0.5)			1.25 (1.60)	
Clunker × Negative home equity (indicator)				-0.35 (1.41)
Observations	2,673	2,673	1,983	1,983

Notes: This table repeats the analysis of Table 3.5 but with additional controls for *Clunker* interacted with household debt capacity. Coefficients are multiplied by 100 to reflect purchase rates in percentage points. The standard errors, which are reported in parentheses, are calculated with observations clustered by household.