Essays on Economic Geography and Networks

by

Yuhei Miyauchi

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Abstract

This thesis consists of three chapters that analyze how the networks of firms, people, and locations shape socio-economic activities.

The first chapter analyzes the role of supplier to buyer matching in the firm-to-firm trade as a source of geographic concentration of economic activities. Using a panel of firm-to-firm trade data covering over a million Japanese firms, I first provide evidence that the new supplier matching rate upon unexpected supplier bankruptcies increases in locations and industries when there are more alternative suppliers selling in the buyer’s location, while this rate remains stable in the presence of other buyers looking for a match. I then estimate a new structural trade model that incorporates dynamic firm-to-firm matching across space in a standard Melitz model and concludes that this agglomeration mechanism drives a large part of spatial inequality of firm density and real wages in Japan.

The second chapter (co-authored with Gabriel Kreindler) investigates how people's mobility patterns are associated with urban spatial economic activities. We use cell phone transaction data to extract commuting flows at a fine spatially and temporarily scale, and use a model to empirically associate commuting flows with spatial economic activity distributions in Bangladesh and Sri Lanka. We validate our predicted measures of economic activities with a government survey and show several applications to provide a proof of concept of our approach.

The third chapter develops an econometric framework to estimate structural parameters underlying a network formation model. I show that the set of equilibria is a complete lattice under certain conditions, and extend this characterization to an econometric framework based on the moment inequality model. I then apply this method to a student friendship network formation in the U.S.

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Chapter 1

Matching and Agglomeration: Theory and Evidence from Japanese Firm-to-Firm Trade

1.1 Introduction

Economic activities are geographically concentrated. In the case of Japan, 50% of the firms are concentrated in only 3% of the usable area, as observed in Figure 1.A1. There is no shortage of theories that explain why agglomeration may occur.\(^1\) However, there is much less consensus about the empirical and quantitative relevance of the various mechanisms that the literature proposes.

In this paper, I focus on one such mechanism for agglomeration: firms find input suppliers more easily in denser areas. Although this is one of the most classical ideas dating back to Marshall (1890), empirical evidence is limited beyond a cross-sectional correlation (Holmes (1999)). In this paper, I first provide new reduced-form evidence of this agglomeration mechanism. Based on the reduced-form evidence, I develop a new structural trade model to quantify the importance of this mechanism in explaining the spatial distribution of economic activities, as well as understanding

\(^1\)Duranton and Puga (2004); Rosenthal and Strange (2004) and Head and Mayer (2004) provide a review.
its distinctive policy implications specific to this agglomeration mechanism.

The first part of this paper uses a panel of firm-to-firm trade data for over one million Japanese firms to provide reduced-form evidence of increasing returns in firm-to-firm matching. These data allow me to estimate the matching rate with new suppliers upon a supplier loss. If the matching rate improves in the presence of more alternative suppliers, but does not decrease as much by the presence of other buyers, there is evidence of increasing returns in matching. To ensure the exogeneity of a supplier loss, I select plausibly unexpected bank ruptcies of suppliers – accidental reasons (CEO death, natural disaster, etc.,) and spillovers from other bankruptcies – that are reported in the data set, and use them as instruments for a supplier loss.

The results are summarized as follows. First, I find that the new supplier matching rate increases in the density of alternative suppliers transporting to sell to the buyer’s location (i.e., the number of suppliers that already serve other firms in the buyer’s location). The magnitude is large: while a loss of one supplier leads to 0.16 new supplier matching per year within the same input sector on average, the matching rate is as high as 0.24 at the 75th percentile and as low as 0.08 at the 25th percentile of the density of alternative suppliers. This heterogeneity of new supplier matching rate is robust by just using within-location-and-buyer-industry variation, i.e., by comparing firms in the same location and industry that loses a supplier in different input industries with a varying supplier density. This alleviates the concern that firm’s selective entry drives the results; i.e., firms which enter in urban areas may be better at supplier matching than those in rural areas.

Second, I find that among various definitions of supplier density, what directly matters for the supplier matching rate is the density of alternative suppliers transporting to sell to the buyer’s location as defined earlier, and once I condition on this, the density of suppliers established and producing in the buyer’s location does not matter. In other words, the first type of supplier density is a sufficient statistic that governs the supplier matching rate. This finding is important, as a typical model

---

2The data set has been used by several previous papers, including Nakajima, Saito, and Uesugi (2012); Bernard, Moxnes, and Saito (2015); Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016), and Furusawa, Inui, Ito, and Tang (2017).
in the literature used for spatial policy analysis assumes that agglomeration benefit arises from the latter, often without specifying agglomeration mechanisms.\(^3\) I will investigate the differential policy implications more concretely with a structural model in the latter part of the paper.

Third, I find that conditional on the density of suppliers, the density of other buyers looking for a match does not affect the matching rate. This implies that buyers do not crowd out each other for matching with a supplier. This is in a stark contrast to firm-to-worker matching in the labor market context, where the presence of unemployed workers is often found to decrease other unemployed workers’ reemployment rate.\(^4\) These differences are intuitive. In the context of firm-to-firm matching, suppliers can simultaneously serve multiple buyers without inducing crowding out among buyers, while in the labor market, a vacant job can be filled by only one unemployed worker, necessarily creating crowding out.

Taking all the evidence together, there is evidence of increasing returns in matching: the presence of more suppliers and more buyers improves the firm-to-firm matching rate. Additional findings confirm that such firm-to-firm matching is important for firm production: I find that a supplier bankruptcy leads to a lower sales growth and a higher exit probability of buyer-side firms through an imperfect supplier recovery, and firms do not cope with these supplier bankruptcies by substituting inputs from other existing suppliers.

The second part of the paper develops a structural model to quantify the importance of increasing returns in matching as a source of agglomeration. The model extends a multi-location multi-sector Melitz model (Melitz (2003)) to incorporate dynamic firm-to-firm matching in input trade across space. As in a standard Melitz model, potential producers enter in each location by paying a fixed cost and draw an idiosyncratic productivity; upon the realization of this productivity draw, they make a decision to sell into various locations by paying a fixed marketing cost. In

\(^3\)See, for example, Allen and Arkolakis (2014); Kline and Moretti (2014a); Monte, Redding, and Rossi-Hansberg (2015); Ahlfeldt, Redding, Sturm, and Wolf (2015); Faber and Gaubert (2016); Nagy (2017). Redding and Rossi-Hansberg (2016) provide a survey.

\(^4\)See Petrongolo and Pissarides (2001) for a survey on this literature.
addition to these standard assumptions, firms require inputs for production, which they can source from matched suppliers. The matching rate with a supplier increases in the number of suppliers selling in the location but it is unaffected by the number of input buyers in the location; this assumption is in line with the empirical findings of increasing returns in matching in the first part of the paper.

The model exhibits agglomeration through circular causation between the measure of input sellers and downstream market size. In a location with more input sellers, input buyers enjoy a higher supplier matching rate and hence a cost advantage, i.e., a "forward linkage". This, in turn, creates a larger market for suppliers and encourages more supplier to sell in the location, i.e., a "backward linkage." The key parameter that governs this circular causation is the elasticity of the supplier matching rate with respect to the number of potential suppliers. I estimate this parameter to match the reduced-form estimates from the first part of the paper.

The first takeaway from the structural estimation is the quantification exercise to understand how important the increasing returns to scale in firm-to-firm matching is in explaining the equilibrium spatial distribution of economic activities. To ask this question, I simulate a counterfactual equilibrium by hypothetically shutting down the increasing returns to scale in matching, i.e., assuming that the elasticity of the supplier matching rate with respect to the supplier density is 0, unlike the estimates of 0.36 from the structural estimation. I find that, under this counterfactual world, the standard deviation of firm density across space would be 7% smaller and that of real wages would be 16% smaller. The remaining geographic variations are induced by the exogenous population distribution and the location and sector-specific productivity heterogeneity, including natural advantages. These results imply that a non-negligible part of the geographic concentration of firms can be explained by increasing returns in firm-to-firm matching.  

The second takeaway from the structural estimation is the policy implications. In

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5To understand the magnitude, note that Ellison and Glaeser (1999) claim that about 20% of the spatial variation in firm density can be explained by natural advantages (i.e., exogenous location and sector productivity) by correlating the firm density and a proxy for natural advantages. The magnitude at which increasing returns to scale in matching explains the spatial distribution of firm density is quantitatively comparable to their number.
particular, I study policies that are targeted to promote one of the most economically lagged areas in Japan; Hokkaido area (Figure 1.A1). Large economic disparity of economic activities is an important concern in Japan just as in many countries in the world, and policy makers are facing challenges to promote economically lagged areas. Two prominent examples of such policies are firm subsidy and transportation infrastructure improvement, and I analyze these policies through the lens of my estimated model.

I first analyze firm subsidy. Here, I consider two distinct types of firm subsidy: subsidies for input suppliers to sell in Hokkaido, and subsidies to produce in Hokkaido. Note that the former subsidies can go to firms regardless of their production location, as long as they make input sales in Hokkaido. In reality, the former subsidies are commonly implemented in the form of trade exhibitions or business matching events, and the latter in the form of tax exemptions or subsidies for new business establishment. I find that first type of subsidies is much more effective than the latter to improve the economic welfare of Hokkaido for the same dollar spent. The intuition is simple: because the agglomeration benefit from this mechanism arises from the density of input suppliers selling in the location, the subsidies should be directly targeted to this margin. This result has a strong implication on the recent discussions of place-based policies. Given the past lack of spatial models of agglomeration through firm-to-firm matching in input trade, the literature has only discussed the second type of subsidy, and the first type of subsidy has not attracted an attention despite the prevalence of such policies through business matching events.6

Second, I analyze the impacts of transportation infrastructure that improves the access between Hokkaido and the main island of Japan. One may expect that such policy would benefit Hokkaido in a similar way as the subsidies for input sales. In fact, a new bullet train connected Hokkaido and the main island Japan (Tohoku area) in 2016 largely aiming for improving economic welfare in Hokkaido. Surprisingly, I find that the reduction of transportation costs initially decreases and then increases the

6See Glaeser and Gottlieb (2008); Kline and Moretti (2014b); Neumark and Simpson (2015) for a review of recent discussions on place-based policies.
economic welfare in Hokkaido. The intuition comes from the two counter-forces from transportation infrastructure improvement: On one hand, the reduction of transportation costs benefits firms in Hokkaido through the reduction of unit cost of input goods. On the other hand, reducing transportation costs harms firms by exposing them for more competition. While the first force is initially weak because the matching probability is close to 0, as the transportation cost decreases, more input suppliers sell in Hokkaido, which increases the supplier matching rate and exponentially increases the benefit from a marginal improvement of transportation infrastructure.

Agglomeration is a core issue intersecting in urban economics, economic geography, and international trade, and this paper contributes to these strands of the literature. First, this paper is directly related to the literature of the microfoundation of the agglomeration from increasing returns to scale in matching. In terms of empirics, the closest evidence is limited to a cross-sectional correlation between a fraction of purchased inputs per firm and spatial firm density in the United States (Holmes (1999)). In terms of theory, there are papers that embed increasing returns to scale in matching as a source of agglomeration (i.e., Diamond (1982); Helsley and Strange (1990)), but these models do not incorporate geography and cross-locational matching, which matters policy implications by differentiating the density of sellers and producers as mentioned earlier.

Second, this paper contributes to the literature of economic geography. There is a recent wave of quantitative spatial economic models to incorporate realistic geography in theoretical models of the New Economic Geography (NEG) literature\(^7\) as surveyed in Redding and Rossi-Hansberg (2016). This paper's contribution is to explicitly model a particular micro-foundation of agglomeration and study its distinct policy implications.

Third, this paper is related to several sub-fields of international trade. First, this paper is related to the literature of firm sourcing behavior, with particular emphasis on geographic proximity (Antràs, Fort, and Tintelnot (2014); Bernard et al. (2015); Krugman (1991); Krugman and Venables (1995); Fujita, Krugman, and Venables (1999) for the theoretical literature of the New Economic Geography.

\(^7\)See Krugman (1991); Krugman and Venables (1995); Fujita, Krugman, and Venables (1999) for the theoretical literature of the New Economic Geography.
Bernard, Moxnes, and Helene Ulltveit-Moe (2016); Blaum, Lelarge, and Peters (2016); Furusawa et al. (2017)). Second, this paper is related to the literature on firm-to-firm trade network formation (Oberfield (2013); Eaton, Kortum, and Kramarz (2016b); Lim (2016); Tintelnot, Kikkawa, Mogstad, and Dhyne (2017)). Third, it is related to the literature that studies search and matching frictions in trade relationships (Allen (2014); Startz (2016); Eaton, David Jinkins, Tybout, and Xu (2016a); Krolikowski and McCallum (2017); Brancaccio, Kalouptsidi, and Papageorgiou (2017)).

The rest of the paper is organized as follows. Section 1.2 provides the description of the main data set used in this paper, a panel of firm-to-firm trade of over a million Japanese firms. Section 1.3 provides reduced-form evidence of increasing returns in firm-to-firm matching. Section 2.3 develops a structural trade model to understand the implications of increasing returns in matching for agglomeration patterns. Section 1.5 structurally estimates the key parameters of the model and presents the procedures for computing counterfactual equilibrium. Section 1.6 presents three counterfactual simulations to illustrate the quantitative implication of the agglomeration forces. Section 2.7 concludes.

1.2 Data and Descriptive Facts

In this section, I briefly describe this paper's main data set, a panel of firm-to-firm trade between over one million Japanese firms. I also document a set of descriptive facts that motivates the empirical exercise in Section 1.3 and the structural model in Section 2.3.

1.2.1 Firm-to-Firm Trade Data in Japan

The main data set utilized in this paper comes from a major credit reporting agency, Tokyo Shoko Research (TSR). The data is based on face-to-face and phone interviews, as well as public resources such as financial statements, corporate registrations, and public relations documents. The panel covers three waves, 2006, 2011, and 2014, and it contains basic firm-level characteristics as well as the precise locations of firm
The most important feature of the data set is that it contains dynamic transitions of supplier-buyer relationships. In each period and for each firm, the data reports up to 24 suppliers and buyers for each firm. The information is collected in annual interviews by field surveyors of TSR by asking whether the relationship from previous years continues to exist or whether a new relationship has been initiated. Figure 1.B1 reports that, while the upper-bound of 24 suppliers is not binding for most firms, there are a non-negligible number of cases with more than 24 suppliers once I include the number of supplier-buyer relationships reported by supplier-side firms (but not by buyer-side firms). Following other recent papers that utilize the same data set (e.g., Carvalho et al. (2016); Bernard et al. (2016)), I define a supplier-buyer relationship to exist if either the supplier-side or the buyer-side firm reports a relationship in each period.

Another important feature of the data set is that it reports the main reasons for bankruptcy in each case. This information is also collected through the interviews by TSR. Table 1.A1 reports the list of reasons recorded in this data set. In Section 1.3, I make use of this information as an instrument when estimating matching rate with new suppliers upon a supplier loss.

1.2.2 Descriptive Patterns of Firm-to-Firm Matching

This section provides a brief overview of the descriptive patterns of the data that motivates the empirical exercise and the structural model in the subsequent sections.

First, an extensive margin of firm-to-firm trade is geographically concentrated, with non-negligible long-distance trade. As is already documented in Nakajima, Saito, and Uesugi (2013) and Bernard et al. (2016), who use the same data set, and as is replicated in Figure 1.A2, the median of the geodesic distances between buyers and suppliers across all three years is about 25 km, which is much

---

For the empirical analysis of this paper, I define the location of the firm by its headquarter location. I show robustness by restricting the samples to firms with few establishments to minimize the mismeasurement of firm locations.
smaller than that of all possible pairs of firms in Japan (464 km). At the same time, the 75th percentile of the geodesic distances of firm-to-firm trade is about 250 km, suggesting that firms are not constrained to trade within locations. These observations are naturally modeled by market penetration decisions à la Melitz (2003) as pursued in Section 2.3; firms decide their production locations, and based on the production locations, firms engage in input sales in various locations under geographic frictions.

**Second, firms in denser areas have more suppliers.** Figure 1.A3 shows that it is true, and Table 1.B1 further confirms that this correlation is robust to various controls, including industry fixed effects and employment size fixed effects. Together with the strong cross-sectional correlation between number of suppliers and revenue productivity per worker, the results are at least consistent with increasing returns in matching; in denser areas where there are more potential supplier and buyers, the probability that downstream firms match with suppliers for each input sector is high. At the same time, the patterns may be driven by endogenous firm entry resulting from unobserved input demand; i.e., firms who unobservably demand more input goods for production may selectively enter in denser locations.

**Third, firms in denser areas have a higher rate of matching with new suppliers, while the separation rate with existing suppliers does not decrease with firm density.** From a dynamic point of view, the cross-sectional correlation between the number of suppliers and firm density can be driven either by a higher rate of matching with suppliers or a lower rate of separation with existing suppliers in denser areas. Figure 1.A4 shows that the former is increasing in firm density while the latter is flatter (with a U-shape), indicating that the new matching rate is the key driving force of cross-sectional correlation between the number of

---

9 A similar finding has been documented in the literature, i.e., externally sourced input share of firms is positively correlated with firm density in the United States (Holmes (1999)).

10 Figure 1.B2 provides a raw correlation; Table 1.B2 shows that the correlation is not driven by various controls. A similar cross-sectional correlation has been documented by Bernard et al. (2016).

11Firms have suppliers in various input sectors. On average firms have 4.6 suppliers across all three years, and they have suppliers in 2.3 two-digit input sectors and 3.1 four-digit input sectors (Panel (A) of Table 1.A2). There is also a wide dispersion in the number of suppliers and input sectors across firms, as shown in Figure 1.B3.

12To see this point more formally, note that the following accounting relationship holds at the
suppliers per firm and firm density.

In sum, the descriptive patterns are consistent with agglomeration benefits from increasing returns in firm-to-firm matching; in denser areas there are more buyers and suppliers, giving a better chance of matching with a supplier for each input sector. At the same time, the patterns may be driven by endogenous firm entry resulting from unobserved demand for supplier matching. Testing increasing returns in matching requires, to a first order, identifying a case where firms in different locations are equally in search of suppliers. Section 1.3 explores this point by studying the matching rate with new suppliers upon an exogenous loss of a supplier.

### 1.3 Reduced-Form Evidence of Increasing Returns in Firm-to-Firm Matching

This section provides reduced-form evidence of increasing returns in firm-to-firm matching. Section 1.3.1 discusses the main empirical strategy; estimating the new supplier matching rate upon a supplier loss instrumented by unexpected supplier bankruptcies. Section 1.3.2 presents the main results, and Section 1.3.3 shows additional results on other outcome variables to confirm the importance of matching frictions in this context.

**firm-level:***

\[
\text{Number of Suppliers}_{t+1} = \text{Number of Suppliers}_t \times (1 - \text{Probability of Supplier Separation}_t) \\
+ \text{Number of New Suppliers}_t.
\]

If a firm has a steady-state number of suppliers, i.e., \( \text{Number of Suppliers}_{t+1} = \text{Number of Suppliers}_t \), then it follows that

\[
\text{Number of Suppliers}_t = \frac{\text{Number of New Suppliers}_t}{\text{Probability of Supplier Separation}_t}, \quad (1.1)
\]

giving the variation in the number of suppliers per firm is decomposed into number of new suppliers and probability of supplier separation.
1.3.1 Empirical Strategy

While no formal test of increasing returns in matching has been conducted in the context of firm-to-firm matching in input trade, that of the labor market has been studied extensively. As reviewed in Petrongolo and Pissarides (2001), early literature uses aggregate relationship between the number of unemployed workers and the job vacancies,\(^{13}\) while more recent literature uses micro data and estimates reemployment probabilities as a function of the number of job vacancies and unemployed workers in their local areas. For example, Petrongolo (2001) finds that while more job vacancies increase the reemployment rates, more unemployed workers crowd out the reemployment rates at a similar magnitude, supporting the constant returns in matching.\(^{14}\)

Testing increasing returns in firm-to-firm matching follows the same idea. I estimate the matching rate with new suppliers upon an exogenous supplier loss. If the matching rate improves in the presence of more alternative suppliers, but does not decrease as much by the presence of other buyers, there is evidence of increasing returns in matching. Exogeneity of a supplier loss is important; if the separation is initiated by the buyer-side firm, it may not be in need and search of alternative suppliers, which biases the matching rate estimates. For this purpose, I instrument a supplier loss by unexpected supplier bankruptcies – accidental reasons (CEO death, natural disaster, etc..) and spillovers from other bankruptcies – from the reasons of bankruptcies reported in the data.\(^{15}\) The validity of these instruments is further confirmed by testing the absence of pre-trends in the outcome variables.

\(^{13}\)See, for example, Blanchard and Diamond (1989b,a) for an early attempt to estimate matching functions using aggregate data.

\(^{14}\)Other papers that study reemployment probability as a relationship with local market conditions include Bleakley and Lin (2012) and Macaluso (2016). In a similar vein, Jäger (2016) estimates the implication of an unexpected worker death on new hires and demonstrates how it depends on the presence of alternative workers with similar skill sets.

\(^{15}\)Table 1.A1 lists the full reasons of the bankruptcies. “Spillover from other bankruptcies” are those caused by management difficulties due to chain reactions such as business partners, subsidiary companies, related bankruptcies, voluntary liquidation, etc. “Accidental reasons” include those with unanticipated accidental problems such as the death of representatives, flood disaster, fire, earthquake, traffic accident, fraud, theft, embezzlement, etc. For “spillovers from other bankruptcies,” I omit the cases in which the buyer-side firms go bankrupt before the suppliers, thereby avoiding cases in which the “spillover” comes from the buyer-side firms in focus.
To proxy for the density of alternative suppliers, I count the number of firms for each input industry that supply to some firms in the buyer's location in the baseline period (2006), which I call "locally-selling suppliers." Reflecting the nonnegligible presence of long-distance trade as reported in Section 1.2.2, the proxy includes firms which are established anywhere and does not restrict to local suppliers; in fact, I show that the matching rate is increasing in this proxy, but once it is conditioned, the simple density of suppliers physically established and producing in the buyer's location does not matter. It should be also stressed that the proxy provides a location and input sector level variation. Such variation allows me to address the concern that firms in a particular location and sector are good at supplier matching in nature, by controlling for the buyer's location and industry specific heterogeneous effects. Figure 1.A5 provides visual illustrations of this proxy for two industries, the forestry industry and the steel industry; while the forestry industry has higher supplier density in the northern part of Japan, the steel industry has more suppliers in the south.

In turn, I examine various proxies for the density of buyers due to the lack of theoretical guidance on which buyer-side firms are competing each other for supplier matching. The proxies considered include the number of firms in buyer's industry and location, the number of firms in buyer's industry and location facing supplier separation in the same input sector, and the number of firms in buyers industry in any locations.

1.3.2 Main Results

This section presents the evidence of increasing returns in firm-to-firm matching. Section 1.3.2.1 argues that the new supplier matching rate upon a supplier loss increases in locations and industries when there are more alternative suppliers transporting to sell in the buyer's location. Section 1.3.2.2 argues that this particular notion of supplier density is a sufficient statistic for new supplier matching rate; I show that once I condition on it, density of suppliers or firms which are established and producing in the buyer's location does not matter for the matching rate. Lastly, Section 1.3.2.3 shows that conditional on the density of suppliers, the density of buyers does not
mater for the matching rate.

1.3.2.1 Matching Rate Increases with the Density of Suppliers

I first show that the new supplier matching rate upon an exogenous separation is higher with more potential suppliers. To show this, I run the following regression:

\[
\text{NewSupplier}_{fkt} = \beta_1 \text{Separation}_{fkt} + \beta_2 \text{Separation}_{fkt} \times \log \text{LocallySellingSuppliers}_{loc(f),k} \\
+ \delta_{loc(f),ind(f),k,t} + \varphi_t(\text{BaselineSupplier}_{fkt}) + \epsilon_{fkt},
\]

where \( f \) is the firm, \( k \) is the two-digit input sector of the JSIC industry classification, \( t \) is year (2006 and 2011). \( \text{NewSupplier}_{fkt} \) is the number of new suppliers that firm \( f \) matches per year from \( t \) to the next period (2006 to 2011 or 2011 to 2014), and \( \text{Separation}_{fkt} \) is the number of suppliers that firm \( f \) loses per year between \( t \) and the next period, including supplier exits and the dissolution of supplier-buyer relationships. \( \text{LocallySellingSuppliers}_{loc(f),k} \) is the proxy for the density of alternative suppliers transporting to sell as discussed in Section 1.3.1; the number of firms in input sector \( k \) that supply to any firms in firm \( f \)'s headquarter location \( loc(f) \) (defined by 0.5 degree grid cell) at the baseline year (2006). \( \delta_{loc(f),ind(f),k,t} \) is the fixed effects at the level of the location and sector of firm \( f \), input sector \( k \), and year \( t \), to make sure that the new supplier matching rate is identified as a comparison between firms in the same location and industry with a same-input-sector supplier with and without a supplier loss. \( \varphi_t(\text{BaselineSupplier}_{fkt}) \) are the flexible controls for \( \text{BaselineSupplier}_{fkt} \), included as fixed effects, to control for the fact that firms with more baseline suppliers mechanically face a higher rate of supplier separation. \( \text{Separation}_{fkt} \) and its interaction with \( \log \text{LocallySellingSuppliers}_{loc(f),k} \) are instrumented by \( \text{UnexpectedSupplierBankruptcy}_{fkt} \), the number of unexpected supplier

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16The industry classification follows the Japan Standard Industrial Classification (JSIC). There are 98 two-digit sectors and 1248 four-digit sectors in JSIC classification. JSIC roughly corresponds to International Standard Industrial Classification (ISIC), although they do not uniquely match. For more detail on JSIC, see http://www.soumu.go.jp/english/dgpp_ss/seido/sangyo/index.htm. Table 1.B5 reports the robustness checks for different levels of industry classification.

17Note that the fixed effects \( \delta_{loc(f),ind(f),k,t} \) saturate \( \text{LocallySellingSuppliers}_{loc(f),k} \), so \( \beta_2 \) is identified only off of the interaction, not by the baseline effects of \( \text{LocallySellingSuppliers}_{loc(f),k} \).
bankruptcies that firm \( f \) faces in input sector \( k \) per year from \( t \) to the next period, and its interaction with \( \log \text{LocallySellingSuppliers}_{loc(f),k} \). As reported in Table 1.B3, the first stage is strongly significant and is not driven by the pre-trends.\( ^{18} \) Standard errors are clustered at the firm level, and the regressions are weighted to equalize the weight at the firm and year level. To avoid the case that the results are solely driven by extremely large firms, I take out firms whose baseline number of suppliers is above the 99th percentile for each year.

Table 1.A3 reports the baseline results. Column (1) starts with the specification with just the average impacts of a supplier loss. The point estimate indicates that a loss of one supplier per year leads to 0.16 more new supplier matching per year in the same two-digit input industry. The fact that it is significantly above 0 implies that firms recover suppliers; the fact that it is significantly below 1 implies that the recovery is far from perfect.\( ^{19} \)

Columns (2) of Table 1.A3 reports the results with heterogeneous impacts with respect to the number of potential suppliers.\( ^{20} \) The inter-quartile range of the log locally-selling suppliers is 2.6, implying that the impact of supplier separation increases by \( 0.06 \times 2.6 = 0.156 \) by going from the 25th percentile to 75th percentile of the log number of potential suppliers. The magnitude is large relative to the average impact of supplier separation; it implies that the yearly recovery rate ranges from 24\% to 8\% by going from the 75th percentile to 25th percentile of the number of potential suppliers.

The rest of the columns show that the positive heterogeneous effects with respect to the locally-selling suppliers are robust. One may worry that these differential matching rates arise because firms in denser locations are better at matching with

\( ^{18} \) The reason why the first stage coefficient is less than 1 is because control firms lose suppliers at some rate at the same time as treated firms face a supplier bankruptcy.

\( ^{19} \) It should be noted that the estimates of the average effect reported in these tables are underestimated if there is misclassification of input industries. To see this, Table 1.B4 reports that the re-matching rate is higher than 0.16 when one takes the dependent variable to be the new supplier matching rate in all input sectors. However, the average rematching rate is still significantly below 1, indicating that the supplier rematching is imperfect.

\( ^{20} \) The log density of locally-selling suppliers is normalized to be mean 0, hence the coefficients on the number of separated suppliers roughly reflect the average impacts of the supplier separation. The same is true for other interactions with the supplier separation.
new suppliers by nature and not because they face more potential suppliers. To deal with these concerns, Column (3) includes the interaction between unexpected supplier bankruptcies (instruments) and firm f’s location and year fixed effects. The regression now utilize the within-location variation; two firms in the same location face an unexpected supplier bankruptcy in two different input industries. The results show that the heterogeneous effects with respect to the locally-selling suppliers are robust and still significant. Column (4) further controls for the heterogeneous effects at the level of prefecture and buyer’s industry for each year. The heterogeneous effects with respect to the number of potential suppliers are now identified by the within-location-and-buyer-industry variation, further ensuring that the results are not driven by the selective entry of firms. Furthermore, Column (5) shows that the differential matching rates are not driven by the firm sizes by controlling for the heterogeneous effects with respect to firm’s baseline employment size.

Lastly, I argue that the results are not driven by the correlated shocks over the supply chain, i.e., firms which face unexpected supplier bankruptcy are on a differential trend for supplier matching patterns. For example, if the underlying reason of the supplier bankruptcy is a natural disaster, it is plausible that the natural disaster also hits their buyers directly. To deal with this concern, Column (6) shows that the results are robust to controlling for the firm and year fixed effects to use only the within-firm-across-input-industry variation; i.e., the new supplier matching rate is identified as a comparison within a firm across different input sectors with and without a supplier loss. In addition, Columns (7) and (8) show that firms which face unexpected supplier bankruptcies from 2011 to 2014 do not show differentially higher rates of supplier matching from 2006 to 2011, and this lack of pre-trends hold for the heterogeneous effects. Hence, the results are not driven by the pre-trends.

**Other robustness results.** Table 1.B5 discusses additional robustness results.

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21 Because these added fixed effects saturate the number of unexpected supplier bankruptcy that is used for supplier separation, the average effects of supplier separation are not identified and omitted from the table.

22 Region is a geographic administrative unit in Japan, with 8 regions in total. I take regions, rather than the 0.5 degree grid cells, to control for the heterogeneous effects at the level of region, buyer industry and year due to the lack of power by using the latter geographic unit.
In (A) endogeneity of entry, I further deal with the concern related to the endogeneity of supplier density through endogenous firm entry. Of particular concern is a possibility that firms have heterogeneous comparative advantage in supplier matching rate in different input industries, and such unobserved comparative advantage is correlated with the composition of supplier densities across different input industries. To resolve these concerns, I remove the samples where the input sector is the primary sector for the buyer-side firms (Column 1 and 2), as well as only taking buyer-side firms whose CEO’s birth location is the same as the current firm headquarter location. These are the cases where the input sector of concern is not the primary driving force of the entry decision of buyer-side firms.

The rest of the robustness concerns are addressed as follows. In (B) endogeneity of supplier bankruptcy, I show that the results are robust by taking each reason of bankruptcy one-by-one (Columns 4 and 5), as well as controlling for the baseline solvency score of the suppliers (Column 6). (C) heterogeneity of input sectors makes sure whether a particular combination of supplier and buyer industry do not drive the results, by controlling input coefficients of the IO matrix, which proxies the importance of each input industry (Column 7), by controlling the number of suppliers all over Japan (Column 8), and by dividing by manufacturing and non-manufacturing sectors (Columns 9 and 10). The results are further robust to geographic and industry definitions, as well as various sample definitions (Panels D, E, and F)\(^2\)\(^3\).

1.3.2.2 Density of Locally-Selling Suppliers is a Sufficient Statistics of the New Supplier Matching Rate

While the results from Section 1.3.2.1 confirms that new supplier matching rate upon a supplier loss is higher if the density of locally-selling supplier is higher, it remains to be investigated whether this particular notion of supplier density is the most relevant

\(^{23}\)Another potential concern is the external validity; unexpected supplier bankruptcies are not representative of the supplier separation. To address this concern, Table 1.B6 shows the results by using supplier exits for any reasons as an instrument for supplier separation, rather than unexpected supplier bankruptcies. Although the presence of pre-trends makes it hard to draw precise causal estimates, the qualitative patterns are consistent with the results using unexpected supplier bankruptcies as an instrument; firms match more with new suppliers upon separation, and they do so more in the presence of more suppliers.
margin of supplier density among various definitions of supplier density.

To investigate this point, Table 1.A4 scrutinizes what definition of supplier density is actually driving the heterogeneous matching rate. In Column (1) and (2), instead of the density of locally-selling suppliers, I study the heterogeneous rematching rate with respect to the density of firms in any industry whose headquarters are located in the buyer-side firm’s location (“Locally-Established Firms”), as well as the density of suppliers in each input industry established in buyer’s location (“Locally-Established Suppliers”). The results show that, while the rematching rate is higher if these proxies are higher (Columns 1 and 2), they lose significance once I include the interaction of supplier separation and the density of locally-selling suppliers, while that of the “Locally-Selling Suppliers” remains significantly positive (in Columns 3 and 4). This indicates that “Locally-Selling Suppliers” is a sufficient statistics that governs the supplier rematching rate. As mentioned earlier, this finding is important, as a typical model in the literature assumes that agglomeration benefit arises from the latter, often without specifying agglomeration mechanisms. I will investigate the differential policy implications more concretely with a structural model in the latter part of the paper.

While this result provides important policy implications by changing the margin from which agglomeration benefit arises, the results themselves are intuitive: Firms can match with suppliers at a distance, and therefore the relevant margin of supplier density should take such possibility of long-distance matching into account. In fact, in Table 1.B7, I show that more than half of the newly matched suppliers are outside 0.5 degree longitude times latitude grid cells, and these long-distance matches are what drives the patterns of heterogeneous effects of Table 1.A4.
1.3.2.3 Matching Rate Does Not Decrease with the Density of Buyers

In this section, I test whether the matching rate decreases with the density of other buyers. The specification is as follows:

\[
NewSupplier_{fkt} = \beta_1 \text{Separation}_{fkt} + \beta_2 \text{Separation}_{fkt} \times \log \text{LocallySellingSuppliers}_{loc(f),k} \\
+ \beta_3 \text{Separation}_{fkt} \times \log \text{Buyers}_{loc(f),ind(f)} \\
+ \delta_{loc(f),ind(f),k,t} + \varphi_t(\text{BaselineSupplier}_{fkt}) + \epsilon_{fkt},
\]

(1.3)

where \( \text{Buyers}_{loc(f),ind(f)} \) are various proxies of the number of buyers. One would expect \( \beta_3 < 0 \) if downstream firms crowd out each other for supplier matching. As in regression (1.2), \( \text{Separation}_{fkt} \) and all of its interactions are instrumented by \( \text{UnexpectedSupplierBankruptcy}_{fkt} \) and its relevant interactions.

The results reported in Table 1.A5 indicate that there is no evidence of crowding out by other buyers. Using the number of firms in the same two-digit industry in firm \( f \)'s location (0.5 degree grid cells) as a proxy for the number of buyers (Column 1), as well as those in the same four-digit industries (Column 2), the coefficients \( \beta_3 \) are precisely estimated and close to 0, while \( \beta_2 \) remains significantly positive. In Column (3), I show that the results remain unchanged by taking the number of firms in firm \( f \)'s industry that also faced an unexpected supplier bankruptcy in the same input sector, exploiting the plausibly exogenous variation in the number of other buyers searching for suppliers. Column (4) shows that the results are also unchanged by counting the buyers in any industry, rather than restricting to those within firm \( f \)'s industry, as long as they face unexpected supplier bankruptcy in the same input sector \( k \). Furthermore, Columns (5) and (6) demonstrate that these results are unchanged when I count firms that faced supplier separation in input sector \( k \), not only those with unexpected supplier bankruptcies. Lastly, one may worry that buyers may be under competition with nonlocal buyers for matching with suppliers. Columns (7) and (8) show that the results are unchanged by defining the buyers as firms in firm \( f \)'s industry located anywhere that also faced an unexpected supplier bankruptcy in the same input sector.
The findings of no crowding out is in a stark contrast to worker-to-firm matching in the labor market context, where the presence of unemployed workers is often found to decrease other unemployed workers' reemployment rate.\(^{24}\) The differences come from the fact that, in firm-to-firm matching, suppliers can simultaneously serve multiple buyers without inducing crowding out among buyers; in the labor market, a vacant job can be filled by only one unemployed worker, necessarily creating crowding out. In other words, the fact that firms can share suppliers limits crowding-out by other buyers.\(^{25}\) In the model, the differences can be expressed that suppliers can simultaneously serve multiple buyers (i.e., \textit{many-to-one} matching), unlike the \textit{one-to-one} matching between a vacant job and an unemployed worker.

1.3.3 Additional Results

This section briefly presents additional results that confirm the importance of matching frictions. Section 1.3.3.1 demonstrates that a supplier loss leads to lower sales growth and a higher exit probability. Section 1.3.3.2 illustrates that there is no evidence that firms cope with a loss of supplier by substituting inputs from other existing suppliers.

1.3.3.1 Sales Growth and Exit Upon Unexpected Supplier Bankruptcy

In this section, I investigate the impacts of unexpected supplier bankruptcy on sales growth and exit. The specification is as follows:

\[
\Delta Y_{ft} = \beta_1 \text{UnexpBankruptcy}_{fkt} + \beta_2 \text{UnexpBankruptcy}_{fkt} \times \log \text{LocallySellingSuppliers}_{loc(f),k} \\
+ \delta_{loc(f),ind(f),t} + \varphi_{t}(\text{BaselineSupplier}_{fkt}) + \epsilon_{fkt}.
\]

(1.4)

The results are presented in the reduced form rather than the IV form, mainly because the supplier separation cannot be defined when firm \(f\) exits. Compared to

\(^{24}\)See Petrongolo and Pissarides (2001) for a survey on this literature.

\(^{25}\)In this sense, the increasing returns in matching documented here is related to "sharing," in addition to "matching," among the three classifications of agglomeration mechanisms as introduced in Duranton and Puga (2004).
regression (1.2), the outcome variables are defined at the firm and year level, but not at the input sector $k$ level. Kaido and Miyauchi (in progress) show that even if the outcome variables do not depend on $k$, above regression consistently estimates average and heterogeneous treatment effects.

Table 1.A6 reports the results. Columns (1) and (2) show the impacts on sales growth, where it is defined as the arc-elasticity (Davis and Haltiwanger (1992)), i.e.,

$$
\Delta Y_{ft} = \frac{Sales_{ft}}{\frac{1}{2} \{Sales_{ft} + Sales_{ft+1}\}}
$$

where $Sales_{ft}$ is defined to be 0 if the firm exits in $t + 1$. This measure is bounded between 2 and -2, avoids outliers with a large increase in sales from skewing the distribution of percentage changes, and allows inclusion of firms with no sales (i.e., exit). The results show that, while a supplier bankruptcy leads to 3% reduction of sales per year on average relative to control groups, this reduction is mitigated in the presence of more potential suppliers, despite insignificantly. Columns (3) to (6) decompose these impacts in terms of firm $f$'s exit and sales growth conditional on survival. While the average effects are driven more by exit margins (Columns 3 and 5), the heterogeneous effects are driven more by the sales growth conditional on survival (Columns 4 and 6). Lastly, Columns (7) and (8) show that these results are not driven by the pre-trends.26

1.3.3.2 Evidence of Lack of Substitution by Other Existing Suppliers

While a new supplier matching is one way that firms cope with a supplier loss, they can potentially deal with the shock by sourcing more from other existing suppliers. I show that, in this context, there is no such evidence of substitution by other existing suppliers.

26While the lack of significance in heterogeneous effects on sales growth and exits are plausibly driven by the noise, another possibility is that the heterogeneity in the importance of matched suppliers offsets the benefit of improved supplier matching rates. More concretely, if suppliers' productivity is increasing in the proxy of the number of potential suppliers, a loss of these suppliers is more damaging to buyers, which offsets the benefit of a higher recovery rate of suppliers. I show that the structural model in this paper can rationalize this pattern, and I provide suggestive evidence that this is potentially an explanation of the empirical results.
To test this, I run the following specification:

\[
\Delta Y_{fkt} = \beta_1 UnexpBankruptcy_{fkt} + \beta_2 UnexpBankruptcy_{fkt} \times \log LocallySellingSuppliers_{loc(f),k} \\
+ \delta_{loc(f),ind(f),k,t} + \varphi_t(BaselineSupplier_{fkt}) + \epsilon_{fkt}.
\] (1.5)

The regression specification is the same as regression (1.4), while now the outcome variables \( \Delta Y_{fkt} \) are various outcomes of other existing suppliers of firm \( f \) in input sector \( k \) at year \( t \). Samples of the regressions are those with more than one baseline suppliers within the input sector.

Table 1.A7 reports the results. Columns (1) and (2) show the impacts on the probability of separation with other existing suppliers. If firms substitute inputs from other existing suppliers, one would expect that the retention rates of other existing suppliers are higher with unexpected supplier bankruptcies. The results show that there is no such evidence. Columns (3) and (4) show that unexpected supplier bankruptcy actually reduces the sales growth of other existing suppliers on average. The results are in fact the opposite of the prediction under the substitution by other existing suppliers; rather, the results imply that other suppliers are complements of the bankrupting suppliers. This evidence of lack of substitution implies that the shock from a supplier loss cannot be mitigated by the presence of other existing suppliers, confirming the importance of new supplier matching.

### 1.4 Model of Firm-to-Firm Matching and Agglomeration

This section develops a new structural model building on the reduced-form evidence in Section 1.3. The model extends a multi-location multi-sector Melitz model (Melitz (2003)) to incorporate dynamic firm-to-firm matching in input trade. As in a standard Melitz model, potential producers enter in each location by paying a fixed cost and draw an idiosyncratic productivity; upon the realization of this productivity draw, they make a decision to sell into various locations by paying a fixed marketing cost.
In addition to these standard assumptions, firms use input goods for production, which they can source from matched suppliers. The matching rate with a supplier increases with the number of suppliers selling in the location, but it is unaffected by the number of buyers at the location; this assumption is in line with the empirical findings of increasing returns in matching in Section 1.3.

1.4.1 Model Set-up

Space is partitioned into a discrete number of locations, denoted by $i, j, n \in N$. Each location is endowed with $L_i$ measure of workers who consume final goods. In the baseline, I assume workers are immobile, while I relax this assumption in Appendix 1.D. Time is continuous and denoted by $t$. In this paper, I only consider a steady-state equilibrium in which aggregate variables (e.g., wages, number of entrants) are constant. Only firm-level variables like supplier matching status vary by $t$. Without a risk of confusion, the subscript $t$ is omitted from the aggregate variables.

There is a continuum of potential entrants in each location and sector, where sector is denoted by $k, m \in K$. I assume that firms do not change production location over their life-cycles. All firms produce both final goods, consumed by final goods consumers, and input goods, used for production by other firms. In this sense, each firm can be simultaneously a buyer and a supplier in input trade. Input trade is only possible when two firms match as a supplier and a buyer. I assume that each buyer-side firm can be matched with at most one supplier in each input sector at a time, though suppliers can be matched with multiple buyers simultaneously.

1.4.1.1 Technology

Each firm can produce both final goods and input goods with the Cobb-Douglas production function. I assume that the unit cost for both final goods and input goods by firm $\omega$ in location $i$ in sector $m$ is written as follows:

$$ c_{\omega t} = \frac{1}{\varphi_{\omega} \lambda_{i, m}} \omega_{i, m}^{\gamma_{i, m}} \prod_{k \in K} P_{\omega t, k}^{\gamma_{k, m}}, $$  

(1.6)
where \( \varphi_\omega \) is the exogenous productivity of firm \( \omega \), \( A_{i,m} \) is the exogenous location-sector level productivity, which can be interpreted as natural advantages or other agglomeration forces, \( \gamma_{L,m} \) is the labor share in production for sector \( m \), \( w_i \) is the wage in \( \omega \)'s production location \( i \), \( \gamma_{k,m} \) is the input share of sector \( k \)'s input goods in sector \( m \)'s production, and \( p_{\omega t,k} \) is the unit cost of input goods that firm \( \omega \) has access to in period \( t \). I assume that production function is constant returns to scale, i.e., 
\[
\gamma_{L,m} + \sum_k \gamma_{k,m} = 1 \quad \text{for all } m \in K.
\]

There are two possible ways to source input goods: match with a supplier for customized input goods or purchase in the generalized input goods market. If firm \( \omega \) with production location \( i \) is matched with supplier \( v \) with production location \( n \) in sector \( k \), firm \( \omega \) can source customized inputs with unit cost at \( c_{vt} \psi \tau_{ni,k}^I \), where \( c_{vt} \) is the unit cost of production for firm \( v \), \( \psi \) is the constant mark-up ratio charged by supplier \( v \) (\( \psi \geq 1 \)), and \( \tau_{ni,k}^I \) is iceberg trade cost of input goods (\( \tau_{ni,k}^I \geq 1 \)). The iceberg trade cost captures the combination of shipment cost, transaction cost, and other sources of geographic frictions. If firm \( \omega \) is not matched with a supplier, the firm can source input goods from the generalized input goods market, where these inputs are produced by perfectly competitive fringe suppliers with linear production technology with labor, charging the unit cost of \( \chi_{i,k} w_i \).\(^{27}\) Taken together, the cost of input goods is written as
\[
p_{\omega t,k} = \begin{cases} 
\min \{ c_{vt} \psi \tau_{ni,k}^I, \chi_{i,k} w_i \} & \text{if matched with supplier } v \text{ producing in } n, \\
\chi_{i,k} w_i & \text{otherwise.}
\end{cases}
\quad (1.7)
\]

I consider an equilibrium where \( \chi_{i,k} \) is sufficiently high so that whenever a firm is matched with a supplier, it uses the input goods from the matched supplier rather than sourcing from a generalized input goods market.

\(^{27}\) Alternatively, one can interpret these generalized input goods as in-house production of input goods by using labor.
1.4.1.2 Final Goods Demand and Market Structure

As in a standard Melitz model, for firms in sector \( k \) to make final goods sales in location \( j \), they have to pay a fixed marketing cost at a flow rate \( f_{j,k}^F \) regardless of their production location \( i \). For shipping final goods from production location \( n \) to \( j \), the firm incurs an iceberg trade cost \( \tau_{n,j,k}^F \). Each seller provides a differentiated variety in a monopolistically competitive manner. Representative final goods consumers have a standard CES utility function:

\[
U = \prod_{k \in K} \left( \int_{\omega \in \Omega_{i,k}} q_k(\omega) \frac{\sigma - 1}{\sigma} d\omega \right)^{\frac{\sigma}{\sigma - 1} \alpha_k},
\]

where \( q_k(\omega) \) is the consumption of the goods produced by firm \( \omega \), \( \alpha_k \) is the consumption share of sector \( k \) final goods, \( \sigma > 1 \) is the elasticity of substitution, and \( \Omega_{i,k} \) is the set of varieties available for final goods consumers in location \( i \).

1.4.1.3 Input Goods Demand, Matching, and Market Structure

For firms in sector \( k \) to make input goods sales in location \( j \), they have to pay a fixed marketing cost at flow rate \( f_{j,k}^I \) regardless of their production location \( i \). I assume that these decisions are independent of seller entry for final goods sales described in the previous subsection. All producers in location \( j \) are potential input buyers. I refer to firms in sector \( k \) which pay a fixed marketing cost for input goods sales in location \( j \) as "input sellers" in location \( j \), and its measure by \( S_{j,k}^I \). I refer to firms producing in location \( j \) in sector \( k \) as "input buyers" in location \( j \), and its measure by \( B_{j,k} \). Due to matching frictions, input sellers can be only stochastically matched with input buyers.

In each period \( t \), firm \( \omega \) will randomly match with an input seller in location \( i \) at the Poisson rate \( M_{i,km} (S_{i,k}^I) \). Following the reduced-form results in Section 1.3, I assume that the rate that input buyers match with input sellers is increasing in the number of input sellers \( S_{i,k}^I \), but it does not depend on the number of input buyers, i.e., other buyers do not crowd out matching. From the perspective of an input buyer,
they match with a supplier at a Poisson rate $M_{i,km} \left( S_{i,k}^I \right) / S_{i,k}^I$. I also assume that the matching rate with suppliers in input sector $k$ is independent across all input sectors. Once the relationship is formed, the relationship continues until it is exogenously destroyed at the Poisson rate $\beta_{i,km}$.

For tractability purposes, I make several additional assumptions, largely in order to simplify the firm-level life-cycle dynamics. First, I assume that firm $\omega$ makes input goods sales decision in each period based only on its instantaneous unit cost $c_{\omega t}$. This implies that firms’ input goods sales decisions do not depend on the exogenous productivity of a firm $\varphi_\omega$ as well as the current supplier matching status conditional on the unit cost $c_{\omega t}$.$^{28}$ Second, I assume that, if a supplier stops input sales in location $j$ or if the supplier dies (explained in Section 1.4.1.4), the buyers of these exiting suppliers instantaneously recover a supplier from the pool of suppliers which newly start to sell in location $j$ in the steady state. Similarly, I assume that, if an input buyer dies, the suppliers of exiting buyers instantly match with input buyers which newly enter in location $j$. These two assumptions essentially shut down the firm-level life-cycle dynamics, and firms instantly reach to the steady-state matching with suppliers and buyers when it is born.

1.4.1.4 Entry and Exit

Firms in location $i$ in sector $m$ exogenously dies at rate $\xi_{i,m}$. I consider a steady state where the same flow rate of firms enter so that the number of firms, or equivalently input buyers $B_{i,m}$, is constant over time. Potential entrant $\omega$ pays a fixed cost $C_{i,m}$ and draws a productivity $\varphi_\omega$ from a Pareto distribution, where its cumulative distribution function is written as $F(\varphi_\omega) = 1 - \left( \frac{\varphi}{\varphi_\omega} \right)^\theta$, where $\varphi$ and $\theta$ are parameters, and $\varphi_\omega \geq \varphi$.

When I analyze the equilibrium, I consider the limit where $\varphi \to 0$. The motivation of this approximation is in line with the assumption that $\varphi$ is sufficiently small in a standard Melitz model; this is to ensure that not all firms make sales (or “export,” in the context of international trade) and the set of sellers is endogenously determined.

$^{28}$Another interpretation of this assumption is that prior to the firm entry and drawing its productivity $\varphi_\omega$, each firm determines $c^t_{j,k}$ above which all firms pays a fixed marketing cost to make input sales in location $j$. 

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1.4.1.5 Total Expenditure and Trade Balance

Aggregate final goods sales from location $i$ is written as

$$X_{i,k}^{F} = \sum_{j \in N} \alpha_{k} w_{j} L_{j} \pi_{ij,k}^{F},$$

(1.9)

where $\pi_{ij,k}^{F}$ is location $j$'s final goods expenditure share in sector $k$ of goods from location $i$, and $\alpha_{k}$ is the final goods consumption share of final goods in sector $k$. $\pi_{ij,k}^{F}$ are endogenously determined in the equilibrium and are derived in the next section.

Aggregate input goods sales from location $i$ is written as

$$X_{i,k}^{I} = \sum_{m \in K} \sum_{j \in N} \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^{F} + X_{j,m}^{I} \right) \pi_{ij,k}^{I},$$

(1.10)

where $\Psi_{j,km}$ is the share of final goods sales in location $j$ and sector $m$ sold by firms that are matched with a supplier in input sector $k$, and $\pi_{ij,k}^{I}$ is location $j$'s input goods expenditure share in sector $k$ of goods from location $i$.

I assume a steady state where each location $i$ has trade deficit $D_{i}$. Trade balancing condition equates the aggregate sales from location $i$ with the final and input goods purchases in location $i$ net of trade surplus, i.e.,

$$\sum_{k \in K} X_{i,k}^{F} + \sum_{k \in K} X_{i,k}^{I} = w_{i} L_{i} + \sum_{k,m \in K} \gamma_{k,m} \Psi_{i,km} \left( X_{i,m}^{F} + X_{i,m}^{I} \right) + D_{i}.$$

(1.11)

1.4.2 Steady-State Equilibrium

In this section, I define and characterize the steady-state equilibrium.

1.4.2.1 Definition of Equilibrium

To define the equilibrium, I first define the measure of firms in location $i$ in sector $m$ whose unit cost of input goods is below $c$ by $H_{i,m}(c)$. From equations (1.6), (1.7),

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and the Pareto distribution of $\varphi_\omega$,

$$H_{i,m}(c) = \int_{p_1, \ldots, p_N} B_{i,m} \left( 1 - F \left( \frac{1}{cA_{i,m}} \sum_{k \in K} p_k \gamma_{k,m} \right) \right) \prod_{k \in K} dG_{i,k}^I(p_k) \quad (1.12)$$

where $G_{i,k}^I(\cdot)$ is the distribution of unit cost of input goods $k$. Since $G_{i,k}^I(\cdot)$ depends on the unit cost of input suppliers selling in location $i$, it in turn depends on $\{H_{j,k}(\cdot)\}$ in all locations and sectors. This constitutes a fixed point problem of unit cost distributions $\{H_{j,k}(\cdot)\}$.

Using the unit cost distributions of producers $\{H_{i,m}(\cdot)\}$ as defined as above, the equilibrium is defined as follows.

**Definition 1.** The steady-state equilibrium is characterized by unit cost distributions of firms producing in $i$ $\{H_{i,k}(\cdot)\}$, the measure of firms which makes input and final goods sales $\{S^I_{i,k}, S^F_{i,k}\}$, firm entry $\{B_{i,k}\}$ and wages $\{w_i\}$ which satisfy:

1. Stationary stationary distributions of unit costs $\{H_{i,k}(\cdot)\}$ are determined by (1.12).

2. Firms optimality conditions are satisfied.

   (a) In each period, firm $\omega$, given its unit cost $\{c_{\omega,t}\}$, determines to make final goods and input goods sales in each sales location $j$ if and only if the expected flow of profit is greater than 0. Per-period expected profits are determined as described in Sections 1.4.1.2 and 1.4.1.3. In other words, zero-profit conditions of marginal sellers are satisfied.

   (b) The free entry conditions of potential entrants equate the discounted sum of expected profit with the fixed cost, as described in Section 1.4.1.4.

3. Goods and labor market clears. The conditions come down to:

$^{29}$More precisely, by noting that this depends on the probability that a firm is matched with a supplier, as well as the distribution of unit cost of suppliers, $G_{i,k}^I(c)$ is defined as

$$G_{i,k}^I(c) = \left\{ 1 - \Lambda_{i,k,m} (S^I_{i,k}) \right\} 1[c \leq \chi_{i,k} w_i] + \Lambda_{i,k,m} (S^I_{i,k}) \frac{\sum H_{i,m}^I(c/\tau_{i,k}^I)}{\sum H_{i,m}^I(c_{i,m}/\tau_{i,k}^I)} 1[c \leq c_{i,m}^I].$$

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(a) Total expenditure condition of final goods (1.9) and input goods (1.10).

(b) Trade balancing conditions (1.11).

The equilibrium definition highlights two important distinctions from a standard Melitz model. First, unit cost distributions \( \{H_{i,k}(\cdot)\} \) not only depends on exogenous productivity \( \{A_{i,k}\} \) and wages \( \{w_i\} \), but also on the steady-state probability of matching with a supplier, as well as the unit cost distributions of matched suppliers. Second, there are input goods sales market, where the set of input sellers are determined by zero-profit condition, in addition to the final goods sellers as in a standard Melitz model.

### 1.4.2.2 Characterizing Equilibrium

To further characterize the equilibrium, I make use of an approximation where \( \varphi \) is sufficiently small. It is also a common underlying assumption in a standard Melitz model to ensure that the set of sellers (or "exporters") is endogenously determined. To see this, if \( \varphi \) is large, all firms may find it beneficial to pay a fixed marketing cost to make sales. Here, to ensure that it is true however low the unit cost of the matched suppliers, I explicitly take an approximation where \( \varphi \) is close to 0. Below, I excerpts the main equilibrium characterization, while the detailed algebraic derivation is left in the appendix.

#### Unit Cost Distribution of Production

First, I characterize \( H_{i,m}(c) \), the measure of producers whose unit cost of input goods is below \( c \) in location \( i \) in sector \( m \). Appendix 1.C.1.1 shows that

\[
H_{i,m}(c) = (1 - o(\varphi)) \varphi^\theta B_{i,m} A_{i,m}^\theta w_i^{-\theta} \prod_{k \in K} \Omega_{i,km} (S_{i,k}^l, \tau_{i,k}^l) c^\theta,
\]
where $o(\varphi)$ is the term that goes to 0 as $\varphi \to 0$. $\Omega_{i,km}(S_{i,k}^I, c_{i,k}^I)$ represents the cost advantage from supplier matching, i.e.,

$$\Omega_{i,km}(S_{i,k}^I, c_{i,k}^I) = 1 - \Lambda_{i,km}(S_{i,k}^I) + \Lambda_{i,km}(S_{i,k}^I) \nu_{i,km}(c_{i,k}^I),$$

(1.13)

where $\Lambda_{i,km}(S_{i,k}^I) \equiv \frac{M_{i,km}(S_{i,k}^I)}{M_{i,km}(S_{i,k}^I) + \rho_{i,km}}$ is the steady state probability of matching with a supplier, and $\nu_{i,km}(c_{i,k}^I) \equiv \frac{1}{1 - \gamma_{k,m}} (c_{i,k}^I / w_{i,k}^T)^{-\gamma_{k,m} \theta}$ governs the average cost advantage upon being matched with a supplier.

Gravity Equations

The previous argument shows that $H_{i,m}(\cdot)$ is approximated by an inverse of Pareto distribution if $\varphi$ is small. This implies that, just like a standard Melitz model with a Pareto productivity distribution (e.g., Chaney (2008)), the expenditure share of final goods follow a gravity equation

$$\pi_{i,m}^F = \frac{B_{i,k} A_{n,m}^\theta w_i^{-\theta} (\tau_{i,m}^F)^\theta \Pi_{k \in K} \Omega_{i,km}(S_{i,k}^I, c_{i,k}^I)}{\sum_{v' \in N} B_{v',m} A_{v',m}^\theta w_{v'}^{-\theta} (\tau_{v',m}^F)^\theta \Pi_{k \in K} \Omega_{v',km}(S_{v',k}^I, c_{v',k}^I)},$$

(1.14)

as well as the expenditure input goods also follow a gravity equation,

$$\pi_{i,j,m}^I = \frac{B_{i,k} A_{n,m}^\theta w_i^{-\theta} (\tau_{i,j,m}^I)^\theta \Pi_{k \in K} \Omega_{i,km}(S_{i,k}^I, c_{i,k}^I)}{\sum_{v' \in N} B_{v',m} A_{v',m}^\theta w_{v'}^{-\theta} (\tau_{v',j,m}^I)^\theta \Pi_{k \in K} \Omega_{v',km}(S_{v',k}^I, c_{v',k}^I)}.$$

(1.15)

Furthermore, $\{\pi_{i,m}^F, \pi_{i,j,m}^I\}$ also correspond to the extensive margin of sellers, i.e. the proportion of firms that sell from location $i$ to location $j$ out of all sellers to location $j$.

Zero Profit Conditions of Input Goods Sellers

Zero-profit conditions of input goods sellers yield the number of input sellers $S_{j,k}^I$ and input seller entry cut-off of unit cost $c_{j,k}^I$. Appendix 1.C.1.3 shows that $S_{j,k}^I$ is solved
as

$$S_{j,k}^I = \frac{1}{w_j f_{j,k}^I} \frac{\psi - 1}{\psi} \sum_{m \in K} \{(1 - \gamma_{k,m}) \gamma_{k,m} \Psi_{j,k,m} (X_{j,m}^F + X_{j,m}^I)\},$$  \hspace{1cm} (1.16)

where

$$\Psi_{j,k,m} = \frac{\Lambda_{j,k,m}(S_{j,k}^I)}{1 - \Lambda_{j,k,m}(S_{j,k}^I)}$$

is the share of final goods sales in location $j$ and sector $m$ sold by firms that are matched with a supplier in input sector $k$ (also appearing in total expenditure condition in Equation 1.10).\textsuperscript{30} Given the number of sellers $S_{j}^I$, the entry cut-off of sellers is derived as

$$c_{j,k}^I = \left(\frac{S_{j,k}^I}{\sum_{i' \in N} B_{i',k} \omega_{i',m} \omega_i^{-\theta} \pi_{i,j,m}^F (\gamma_{l,i,m}^F)^{\theta} \prod_{l \in K} \Omega_{l,k,m} (S_{l,j,k}^I, c_{l,j,k}^I)}\right)^{1/\theta}.$$  \hspace{1cm} (1.17)

**Free Entry Conditions**

The expected profit of firms upon an entry depends both on the final goods profits and input goods profits. Appendix 1.C.1.4 shows that the free entry conditions yield the number of firms, equivalently as input buyers, as

$$B_{i,k} = \frac{1}{\xi_{i,k} C_{i,k} \omega_i} \sum_{j \in J} \left\{ \pi_{F,j,k}^F \sigma - 1 \over \sigma \alpha_k w_j L_j + \psi - 1 \over \psi \pi_{ij,k}^F \sum_{m \in K} \gamma_{k,m} \Psi_{j,k,m} (X_{j,m}^F + X_{j,m}^I) \right\}. \hspace{1cm} (1.18)$$

Together, the equilibrium is characterized as follows:

**Proposition 1.** Under sufficiently small $\varphi$, the steady-state equilibrium is characterized by $\{S_{j,k}^I, B_{i,k}, \psi, X_{j,k}^F, c_{j,k}^I, \pi_{F,j,k}^F, \pi_{ij,k}^F\}$, which satisfy total expenditure conditions (1.9, 1.10), trade balancing conditions (1.11), gravity equations for input goods (1.15) and final goods (1.14), zero profit conditions (1.16), entry cut-off of input goods sellers (1.17) and free-entry conditions (1.18).

\textsuperscript{30}The intuition of the expression of $\Psi_{j,k,m}$ is as follows: Firms that are matched with a supplier in input sector $k$ enjoy a cost advantage by $\nu_{j,k,m}(c_{j,k}^I)$ over unmatched firms. Hence, the sales share of firms with a supplier is expanded from $\Lambda_{j,k,m}(S_{j,k}^I)$, the steady state probability of matching with a supplier in input sector $k$, by the factor of $\nu_{j,k,m}(c_{j,k}^I)$. $\Psi_{j,k,m}$ enters in zero-profit conditions, because the aggregate input sales is written as $\gamma_{k,m} \Psi_{j,k,m} (X_{j,m}^F + X_{j,m}^I)$.
1.4.3 Matching and Agglomeration in the Model

In this subsection, I briefly discuss the main agglomeration forces of the model: circular causation between the input seller entry $S^f_{i,k}$ and the final goods sales of input buyers $X^F_{i,m}$. For the sake of the discussion of this section, I will ignore the endogeneity of $w_i$ and the number of producers $B_{i,m}$, which constitute a further general equilibrium loop as in the standard Melitz model.

Final goods sales of input buyers in location $i$ in sector $m$, $X^F_{i,m}$, is determined by the total expenditure condition for final goods (Equation 1.9). By combining the expenditure share from the gravity equation (1.15), I have the relationship corresponding to a “forward linkage”:

$$X^F_{i,m} = \sum_{j \in N} \alpha_j w_j L_j \frac{B_{i,m} \gamma_{i,m} w_i^{-\gamma_{i,m}} (\tau^F_{i,j,m})^\theta \Pi_{k \in K} \Omega_{i,k,m} (S^f_{i,k}, \tau^f_{i,k})}{\sum_{j' \in N} B'_{i',m} \gamma'_{i',m} w_i^{-\gamma'_{i',m}} (\tau^F'_{i',j,m})^\theta \Pi_{k' \in K} \Omega_{i',k,m} (S^f_{i',k}, \tau^f_{i',k})}.$$

The intuition of these equations is simple: If there are more sellers $S^f_{i,k}$, producers have a higher chance of matching with a supplier, which gives a cost advantage to producers in location $i$ (high $\Omega_{i,k,m} (S^f_{i,k}, \tau^f_{i,k})$) and hence increases aggregate final goods sales $X^F_{i,m}$. The number of sellers, in turn, is again derived by the zero profit condition (1.16), reproduced here:

$$S^f_{i,k} = \frac{1}{w_i \Psi_{i,k}} \left\{ \left( 1 - \gamma_{i,m} \right) \gamma_{i,m} \Psi_{i,k} (X^F_{i,m} + X^F_{i,m}) \right\}.$$

It shows that $S^f_{i,k}$ is increasing in the final goods sales of input buyers, $X^F_{i,m}$. This corresponds to a “backward linkage.” The “forward linkage” and “backward linkage” constitute a positive feedback loop, reinforcing each other to create a force toward agglomeration.

In this circular causation, the elasticity of supplier matching rate with respect to the number of input sellers, $\lambda$, serve a crucial role. To see this, recall that

$$\Omega_{i,k} (S^f_{i,k}, \tau^f_{i,k}) = 1 - \left\{ \nu_{i,k,m} (\tau^f_{i,k}) - 1 \right\} \times M_{i,k,m} (S^f_{i,k}) / \left\{ M_{i,k,m} (S^f_{i,k}) + \rho_{i,k,m} \right\}$$

by plugging in Equation (1.13) in the definition of $\Omega_{i,k,m}$. Hence, the sensitivity of $\Omega_{i,k,m}$ with respect to $S^f_{i,k}$ is crucially governed by the parameter $\lambda$, the elasticity of the supplier
matching rate with respect to the number of potential suppliers. In Section 1.5, I estimate $\lambda$ to replicate the reduced-form estimates in Section 1.3.

While related, the circular causation through vertical linkages presented here is somewhat distinct from the theoretical models developed by Krugman and Venables (1995) and Venables (1996). In their models, there are no matching frictions, and firms have access to all producers all across the world. There, the circular causation arises from the production location decision of suppliers and from downstream market size. Here, on the other hand, the circular causation arises from supplier market penetration decision à la Melitz (2003) and from downstream market size. The distinction leads to crucial differences in policy implications, as illustrated in the subsidies for input sales in Section 1.6.2.1.

Just like Krugman and Venables (1995), Venables (1996), and other models of economic geography with agglomeration forces, the circular causation may potentially lead to multiple equilibria. While the presence of multiple equilibria does not cause issues in parameter estimation because the estimation procedure only utilizes the partial equilibrium relationships of the model, it complicates the counterfactual equilibrium simulation. In this paper, I compute counterfactual equilibrium that follow the same bifurcation path as in the observed equilibrium. Section 1.5 discuss these points further.

1.5 Equilibrium Computation and Model Estimation

This section discusses the procedure for obtaining counterfactual equilibrium and parameter estimation and calibration.

1.5.1 Computation of Counterfactual Equilibrium

To compute the counterfactual equilibrium, I follow the approach of "hat algebra" initiated by Dekle, Eaton, and Kortum (2008). This procedure allows me to limit the set of parameters and baseline variables required to compute the counterfactual equilibrium.
To make the computation simple, I assume that the matching rate is an exponential function of the measure of input sellers, i.e., 
\[ M_{i,k,m} \left( S_{i,k}^f \right) = \eta_{i,k,m} \times \left( S_{i,k}^f \right)^\lambda, \]
where \( \{\eta_{i,k,m}\} \) and \( \lambda \) are parameters. The following proposition describes the conditions of counterfactual equilibrium under two policy counterfactual simulations: Subsidies for input sales and production (Section 1.6.2.1), and the change in transportation cost (Section 1.6.2.2).

**Proposition 2.** Given baseline equilibrium variables \( \{B_i, w_i, X_{i,k,0}^F, \pi_{i,j,k}^F, \psi_{i,j,k}, \Lambda_{i,k,m}, \nu_{i,k,m}, \Psi_{j,k,m}\} \), parameters \( \{\lambda, \psi, \theta, \{\gamma_{L,k}, \gamma_{k,m}, \alpha_k\}\} \), counterfactual subsidies for input sales \( \{T_{i,k}^S\} \), subsidies for production \( \{T_{i,k}^B\} \), and changes in transportation costs \( \{\hat{\tau}_{i,j,k}^F, \hat{\tau}_{i,j,k}^I\} \), the counterfactual equilibrium is computed by solving the following set of equations with respect to \( \{\hat{S}_{i,k}^f, \hat{B}_{i,k,0}, \hat{w}_{i}, \hat{X}_{i,k,0}^F, \hat{X}_{i,k,0}^I, \hat{\pi}_{i,j,k}^F, \hat{\pi}_{i,j,k}^I\} :\)

(i) **Gravity equation of input goods**

\[
\hat{\pi}_{i,j,m}^F = \frac{\hat{B}_{i,m,w_{i}^{-\theta}} \left( \hat{\tau}_{i,j,m}^I \right)^\theta \prod_{k \in K} \hat{\Omega}_{i,k,m}}{\sum_{i' \in N} \hat{\pi}_{i',j,m} \hat{B}_{i',m,w_{i'}^{-\theta}} \left( \hat{\tau}_{i',j,m}^I \right)^\theta \prod_{k \in K} \hat{\Omega}_{i',k,m}},
\]

and that of final goods

\[
\hat{\pi}_{i,j,m}^F = \frac{\hat{B}_{i,m,w_{i}^{-\theta}} \left( \hat{\tau}_{i,j,m}^F \right)^\theta \prod_{k \in K} \hat{\Omega}_{i,k,m}}{\sum_{i' \in N} \hat{\pi}_{i',j,m} \hat{B}_{i',m,w_{i'}^{-\theta}} \left( \hat{\tau}_{i',j,m}^F \right)^\theta \prod_{k \in K} \hat{\Omega}_{i',k,m}}.
\]

(ii) **Zero-profit conditions of input goods sellers**

\[
\hat{S}_{j,k}^F = 1 \sum_{m \in K} \left( 1 - \gamma_{k,m} \right) \gamma_{k,m} \psi_{j,k,m} \hat{\psi}_{j,k,m} \left( X_{j,m}^F \hat{X}_{j,m}^F + X_{j,m}^I \hat{X}_{j,m}^I \right) + \frac{\psi_{j,k,m} T_{i,k}^S}{\psi_{j,k,m} T_{i,k}^B}.
\]

(iii) **Free entry conditions**

\[
\hat{B}_{i,k} = \frac{1}{\hat{w}_{i}} \sum_{j \in N} \left\{ \frac{\hat{\pi}_{i,j,k}^F \hat{\pi}_{i,j,k}^F \psi_{j,k,m}^2}{\psi_{j,k,m}^2} \alpha_k w_j \hat{w}_{j} L_j + \frac{\psi_{j,k,m} T_{i,k}^F}{\psi_{j,k,m} T_{i,k}^B} \right\} + \frac{\psi_{j,k,m} T_{i,k}^B}{\psi_{j,k,m} T_{i,k}^F}.
\]

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(iv) cost advantages from supplier matching

\[ \hat{\Omega}_{i,km} = \frac{1 - \Lambda_{i,km} \hat{\Lambda}_{i,km} + \Lambda_{i,km} \nu_{i,km} \hat{\nu}_{i,km}}{1 - \Lambda_{i,km} + \Lambda_{i,km} \nu_{i,km}} \]

where steady state supplier matching probability is \( \hat{\Lambda}_{i,km} = \frac{1}{1 + \Lambda_{i,km} + \Lambda_{i,km} \times \left( \frac{\hat{S}_{i,k}}{\hat{S}_{i,k}} \right)^{-\gamma_{km}}} \)

and cost advantage upon a match is \( \hat{\nu}_{i,km} = \left( \frac{\hat{S}_{i,k}}{\hat{S}_{i,k}} \right) \left( \sum_{\prime \in N} \hat{\pi}_{i,k} \hat{B}_{i,k} \hat{\hat{w}}_{i,k}^{-\theta} \hat{\hat{\pi}}_{i,k} \hat{\hat{\pi}}_{i,k} \right) \prod_{i \in K} \hat{\hat{\Omega}}_{i,ik} \).

(v) trade balancing condition (no change in trade surplus)

\[ 1 = \frac{\sum_{k \in K} X_{i,k}^F \hat{X}_{i,k}^F + \sum_{k \in K} X_{i,k}^I \hat{X}_{i,k}^I - w_i \hat{w}_i L_i - \sum_{k,m \in K} \gamma_{k,m} \Psi_{i,km} \hat{\Psi}_{i,km} \left( X_{i,m}^F \hat{X}_{i,m}^F + X_{i,m}^I \hat{X}_{i,m}^I \right)}{\sum_{k \in K} X_{i,k}^F + \sum_{k \in K} X_{i,k}^I - w_i L_i - \sum_{k,m \in K} \gamma_{k,m} \Psi_{i,km} \left( X_{i,m}^F + X_{i,m}^I \right)} \]

where input goods sales is \( X_{i,k}^F \hat{X}_{i,k}^F = \sum_{m \in K} \sum_{j \in N} \gamma_{k,m} \Psi_{j,km} \hat{\Psi}_{j,km} \left( X_{i,m}^F \hat{X}_{i,m}^F + X_{i,m}^I \hat{X}_{i,m}^I \right) \hat{\pi}_{ij,k} \hat{\hat{\pi}}_{ij,k} \).

Proposition 2 states that computing an equilibrium does not require all the parameters in the model; rather, a subset of parameters \( \{\lambda, \psi, \{\gamma_{L,m}, \gamma_{k,m}, \alpha_k\}, \theta, \sigma\} \) and baseline variables \( \{B_{i,k}, w_i, X_{i,k}^F, \hat{\pi}_{ij,k}, \hat{\hat{\pi}}_{ij,k}, \Lambda_{i,km}, \nu_{i,km}, \Psi_{i,km}\} \) are sufficient. These parameters and baseline variables are either estimated, calibrated, or directly obtained from the data. Next subsection is devoted to the discussion.

As noted in Section 1.4.3, there may be multiple equilibria, i.e., there may be multiple set of endogenous variables that solve the system of equations in Proposition 2. For the counterfactual simulations provided later, I obtain the counterfactual equilibrium by gradually changing the subsidies \( \{T_{i,k}^S, T_{i,k}^B\} \) or transportation cost \( \{\hat{\pi}_{ij,k}, \hat{\hat{\pi}}_{ij,k}\} \); in this sense, the obtained counterfactual equilibrium follows the same bifurcation paths as in the observed equilibrium. This is an intuitive equilibrium selection rule under the counterfactual policies, particularly when the policies are at a relatively small scale and are unlikely to induce switching to equilibria on a different bifurcation path.

In the counterfactual simulation exercise, I mostly focus on the change in the firm density \( B_{j,k} \) and the real wages \( \frac{w_j}{P_j} \), where \( P_j \) is the consumer price index. The following proposition is useful for characterizing the changes in real wages in each location.
Proposition 3. The change in real wages are derived as
\[
\left( \frac{w_i}{P_i} \right) = \prod_{m \in K} \left( \frac{\hat{\pi}_{i,m}^F}{\hat{B}_{i,m} \hat{\psi}_{i,m}^{\gamma_{L,m}} (\hat{\tau}_{i,m}^F)^{\nu_{i,k,m}} \prod_{k \in K} \hat{\Omega}_{i,k,m}} \right)^{\alpha_{m}}.
\]

Proof. See Appendix 1.C.2. □

1.5.2 Parameter Estimation and Calibration

As explained in the previous section, the knowledge of \{\lambda, \psi, \{\gamma_{L,m}, \gamma_{k,m}, \alpha_k\}, \theta, \sigma\} and baseline variables \{B_{i,k}, w_i, X_{i,k}^F, \pi_{i,j,k}^F, \pi_{i,j,k}^{F_i}, \Lambda_{i,k,m}, \nu_{i,k,m}, \Psi_{i,k,m}\} are sufficient for computing counterfactual equilibrium. Below, I discuss the estimation and calibration of these parameters and variables in turn.

1.5.2.1 Elasticity of Supplier Matching Rate \( \lambda \)

As discussed in Section 1.4.3, the elasticity of supplier matching rate \( \lambda \) is a particularly important parameter that governs the degree of agglomeration. Below, I illustrate the estimation procedure of \( \lambda \) to match the reduced-form estimates presented in Section 1.3.

In the model introduced in Section 2.3, input buyers in location \( i \) in sector \( m \) who lost a supplier in sector \( k \) matches with a new supplier at Poisson rate \( \eta_{i,k,m} \times (S_{i,k}^I)^\lambda \). This implies that the probability that the firm matches with an alternative supplier after one year is approximated by \( 1 - \exp(-\eta_{i,k,m} \times (S_{i,k}^I)^\lambda) \). This object is directly mapped to the data to estimate \( \lambda \), i.e.,

\[
\hat{\lambda} = \min_{\lambda} || \left\{ 1 - \exp(-\eta_{i,k,m} \times (S_{i,k}^I)^\lambda) \right\} - \left\{ \hat{\beta}_1 + \hat{\beta}_2 \times \log \text{LocallySellingSuppliers}_{i,k} \right\} ||^2
\]

where \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) are the reduced-form estimates of Equation (1.2), where we choose \( \hat{\beta}_1 = 0.160 \) and \( \hat{\beta}_2 = 0.066 \) from Column (2) of Table 1.A3. The estimation results suggest the value of \( \lambda = 0.36 \), implying that 1 percent increase of \( S_{i,k}^I \) leads to 0.36 percent higher rate of supplier matching.\(^{31}\)

\(^{31}\)In order to estimate \( \lambda \), further parametric assumptions are required for \( \eta_{i,k,m} \) and \( S_{i,k}^I \). By
It should be noted that the multiplicity of equilibria, as discussed in Section 1.4.3, does not cause issues in the estimation of parameter \( \lambda \). This is because the estimation of the parameter is conducted by just using the partial equilibrium relationship of a model without using an entire equilibrium structure. The same argument applies to markup ratio of input sales \( \psi \), and other baseline parameters explained below.

1.5.2.2 Cost Advantages upon Supplier Matching \( \{ \nu_{i,km} \} \)

Another key set of parameters are the production benefit of supplier matching, \( \{ \nu_{j,km} \} \). \( \{ \nu_{i,km} \} \) themselves are not exogenous parameters in the model, and it is partly determined by the productivity of costly generalized input goods that firms use if not matched with a supplier \( \{ \chi_{j,k} \} \). However, since \( \{ \nu_{j,km} \} \) are easier to match with data and also because they are sufficient statistics to conduct counterfactual simulation, I directly calibrate \( \{ \nu_{j,km} \} \) in this paper.

To calibrate \( \{ \nu_{j,km} \} \), I use the relationship that \( \nu_{j,km} = \frac{\Lambda_{j,km}/\Psi_{j,km}}{1-\Lambda_{j,km}+\Lambda_{j,km}/\Psi_{j,km}} \) where \( \Lambda_{j,km} \) and \( \Psi_{j,km} \) are directly obtained from the data. This relationship is derived from equation (1.20). Intuitively, \( \nu_{j,km} \) is identified from the sales premium of matching with a supplier.

1.5.2.3 Other Parameters and Baseline Variables

Table 1.A8 provides the remaining list of estimated and calibrated parameters and variables required for computing counterfactual equilibrium following Proposition 2. In terms of parameters, I calibrate \( \{ \alpha_k, \gamma_{k,m} \} \) from the input-output matrix and \( \sigma, \theta \) from the literature (Broda and Weinstein (2006); Head and Mayer (2014)). \( \psi \) are assumed to be equal to \( \sigma/(\sigma - 1) \), implying that the mark-up for final goods sales is the same as that of the input goods sales. For the remaining baseline variables required for the counterfactual simulation, while some variables are directly obtained from the data \( \{ B_{i,k}, w_i, \Lambda_i, \chi_{i,k}, \psi_{i,k} \} \), others are obtained indirectly from the model noting the relationship

\[
\eta_{i,km} \left( \frac{\psi_{i,k}}{\Gamma_i} \right)^\frac{1}{\psi_{i,k}} \frac{1}{w_j} \sum_{m \in K} \left\{ \frac{\theta + \gamma_{m-1}}{\theta - 1} - \gamma_{m,m-1} \psi_{i,k} \chi_{i,k} \right\} \right]^{\psi_{i,k}}
\]

I parametrize that \( \eta_{i,km} \left( \frac{\psi_{i,k}}{\Gamma_i} \right)^\frac{1}{\psi_{i,k}} \sim \log N(\mu_{i,km}, \sigma_{i,km}^2) \) and \( \mu_{km} \sim N(\bar{\mu}_i, \bar{\sigma}_i^2) \).
1.6 Results of Counterfactual Simulations

This section presents the results from counterfactual simulations. Section 1.6.1 illustrates how important the increasing returns to scale in matching is in explaining the equilibrium spatial distribution of economic activities. Section 1.6.2 analyzes two important policies to economically lagged areas; firm subsidies and transportation infrastructure.

1.6.1 How Much Does Increasing Returns in Matching Explain Geographic Concentration of Economic Activity in Japan?

To understand how important the increasing returns to scale in matching in explaining the equilibrium spatial distribution of economic activities, I hypothetically shut down the increasing returns to scale in matching, i.e., compute the equilibrium under $\lambda = 0$ rather than the estimated value of $\lambda = 0.36$, and study how the equilibrium changes. More specifically, I assume that the supplier matching rate is $M_{i,k,m}(\hat{S}_{i,k}^l) = \eta_{i,k,m} \times (\bar{S}_{i,k}^l)^{\lambda}$, where $\bar{S}_{i,k}^l \equiv \frac{1}{N} \sum_{j \in N} S_{i,k}^l$, i.e., the average number of input sellers in sector $k$ in the baseline equilibrium. These matching rates are assumed to be unchanged under the counterfactual equilibrium and they do not depend on $\hat{S}_{i,k}^l$. Hence, the equilibrium is computed following Proposition 2, except that $\hat{S}_{j,k}^l$ are exogenously specified as $\hat{S}_{j,k}^l = \bar{S}_{j,k}^l / \bar{S}_{j,k}^l$ rather than zero profit conditions.

Figure 1.A6 shows that counterfactually assuming that $\lambda = 0$ significantly reduces the variance of the firm density and real wages across space. Quantitatively, the standard deviation of firm density decreases to 7% of the baseline, while the variance of real wages decreases by 16%. The remaining geographic variations are induced by the exogenous population distribution, $L_i$, and the heterogeneity of productivity across locations and sectors, $A_{i,m}$, which include natural advantages. The results
confirm that a significant part of geographic inequality in firm density and real wages are attributed to the increasing returns in matching.

To interpret the magnitude, note that Ellison and Glaeser (1999) document that about 20% of the variation of firm density across cities in the United States is explained by the proxies of natural advantages. Compared to this number, the extent to which the increasing returns to scale in matching explains the spatial dispersion of economic activities is smaller. It should be noted, however, that this magnitude becomes larger if one incorporates other agglomeration forces or labor mobility. To see this, note that while shutting down increasing returns to scale in matching, a big city (e.g., Tokyo) becomes less dense, which reduces other agglomeration forces in Tokyo (e.g., knowledge spillovers), which makes Tokyo less dense.

1.6.2 Policy Implications to Promote Economically Lagged Areas

In this section, I analyze two policies targeted to promote economically lagged areas. Throughout the world, policies targeting the improvements of disadvantaged locations are common and are often refereed as place-based policies (Kline and Moretti (2014b); Neumark and Simpson (2015)). Here, I study implications of such place-based policies in one of the most economically disadvantaged areas in Japan: Hokkaido (Figure 1.A1). It is one of the most economically lagged areas in Japan; an illustrating event is the bankruptcy of Yubari municipality in 1996 due to the lack of tax revenue. Partly aimed for improving the economic conditions of Hokkaido area, a new bullet train connected Hokkaido island and the northern part of the main island Japan (i.e., Tohoku area) in 2016, and is planned to fully extend over the next 10 to 20 years.

The policies studied in this section are two-folds. First, Section 1.6.2.1 analyzes the implication of different type of firm subsidies. Second, Section 1.6.2.2 analyzes the change of iceberg trade cost for input goods between Hokkaido and Tohoku area, motivated by the new bullet train.
1.6.2.1 Are Input Sales Subsidies More Effective than Production Subsidies?

In this subsection, I analyze firm subsidies. In particular, I compare two types of firm subsidies: subsidies for input suppliers to sell in Hokkaido, and subsidy to produce in Hokkaido. The former are given to suppliers conditional on serving the location as input sellers regardless of their production location, and the latter are given to suppliers who produce in these locations. In the model, these are translated as $T_{i,k}^S$ and $T_{i,k}^P$ in Proposition 2. In reality, the former subsidies are commonly implemented in the form of trade exhibitions or business matching events, and the latter in the form of tax exemptions or subsidies for new business establishment.

Figure 1.A7 presents the results. While firm density and real wages increase as a function of total dollar spent for both types of subsidies, subsidy for input sales is much more effective than production subsidy for the same dollar spent. The intuition of these results is simple: Since the agglomeration benefit arises from the density of input suppliers selling in each location, one should directly target this margin. It should be noted, however, that this implication does not hold if the supplier matching elasticity $\lambda$ is not sufficiently large. To see this, if $\lambda \approx 0$, there is no impact on firm density and welfare by subsidizing input sellers. Hence, this results come from the fact that the estimated increasing returns to scale in matching is quantitatively large.

1.6.2.2 What Are the Welfare Implications of Transportation Infrastructure?

In this subsection, I analyze the impacts of change of iceberg trade cost for input goods between Hokkaido and Tohoku area. Figure 1.A8 shows the results. The x-axis represents the inverse of the change of iceberg trade cost for input goods between Hokkaido and Tohoku areas (i.e., $1/\tau_{Hokkaido,Tohoku,k}$ for all $k$). For firm density, improving the transportation infrastructure actually reduces the firm density. As for

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32 These implications are unchanged even if one considers the impacts of these subsidies in other locations. As shown in Figure 1.B4, the impacts of subsidies in Hokkaido on locations outside Hokkaido are much smaller in magnitude.
the real wages in Hokkaido, it exhibits a non-monotonic pattern: it initially decreases and then increases.

The intuition is as follows. There are two counter-forces as a response to the change of input goods trade cost. For one thing, reduction of input goods trade cost increases the unit cost of suppliers selling in Hokkaido, which benefits firms in Hokkaido. On one hand, the reduction of transportation costs benefits firms in Hokkaido through the reduction of unit cost of input goods. On the other hand, reducing transportation costs harms firms by exposing them for more competition. If the latter force is stronger, it reduces the incentive for potential entrants to enter in Hokkaido and reduces firm density in Hokkaido, which results in the welfare reduction. This intuition also explains why there are non-monotonic pattern of real wages. While the first force is initially weak because the matching probability is close to 0, as the transportation cost decreases, more input suppliers sell in Hokkaido, which increases the supplier matching rate and exponentially increases the benefit from a marginal improvement of transportation infrastructure.

1.7 Conclusion

This paper investigates the importance of increasing returns in firm-to-firm matching in input trade as a source of agglomeration. I first provide reduced-form evidence of increasing returns to scale in firm-to-firm matching. Using unexpected supplier bankruptcies as an instrument, I show that the new supplier matching rate upon a supplier loss increases in locations and industries when there are more alternative suppliers selling in the buyer's location, while this rate remains stable in the presence of other downstream firms. Based on these findings, I build a structural trade model to quantify the importance of increasing returns in firm-to-firm matching for the geographic concentration patterns of economic activity in Japan. The structural model highlights distinct policy implications compared to a typical model in the literature that assumes agglomeration benefit arises from local firm density producing in a location. In particular, I find that subsidies for input suppliers to sell in the target
location is much more effective than subsidies to produce in these locations. I also find that transportation infrastructure development initially harms and then improves the welfare of the target location.

There are several important directions for future work. First, further understanding of why matching frictions exist and how the policies can reduce such frictions is important. To understand this, Daisuke Miyakawa and I are partnering with Tokyo Shoko Research to conduct a randomized control trial to provide information about potential transaction partners. Such an RCT will reveal how information frictions interact with geographic space and how reducing such frictions improves firm production. Second, dynamic implications of matching frictions and increasing returns in firm-to-firm matching for regional business cycles and long-term growth should be studied both theoretically and empirically. Lastly, other micro-foundations of agglomeration should be studied empirically, theoretically and quantitatively. By no means does this paper claim that increasing returns in firm-to-firm matching is the only source of agglomeration. Reflecting Marshall (1890), labor market pooling and knowledge spillovers may be equally important forces that drive agglomeration. With this respect, a general key message of this paper is that both theoretical and empirical understandings of different microfoundations of agglomeration are important for proper understanding of policy consequences, and further research is awaited.
Appendices

1.A Main Tables and Figures

Figure 1.A1: Geographic Distribution of Firms in Japan

(A) Map of Japan

(B) Cumulative Distribution of Firms by Local Firm Density

Note: Local firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km).
Figure 1.A2: Cumulative Distribution of Geodesic Distances between Supplier and Buyers

Note: The graph shows the cumulative distributions of geodesic distance between supplier and buyer's headquarter locations for the years of 2006, 2011, and 2014.
Figure 1.A3: Number of Suppliers per Firm and Local Firm Density

Note: Samples are firms in the data set from 2006. Firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km).
Figure 1.A4: New Supplier Matching Rate and Separation Rate by Local Firm Density

Note: “New Supplier Matching Rate” is the number of suppliers that firms gain per year (averaged across 2006 to 11 and 2011 to 14), and “Separation Rate” is the probability that firms lose suppliers per year, including both the exits of suppliers and dissolution of trading relationships.
Figure 1.A5: Examples of Geographic Variation of the Density of Locally-Selling Suppliers

(A) Forestry Industry

(B) Steel Industry

Note: The density of locally-selling suppliers is the density the number of firms for each sector that supply to any firms whose headquarter is located in each location (defined by 0.5 degree grid cell) at the baseline year (2006). See Section 1.3 for more discussion.
Figure 1.A6: Counterfactual Simulation: Shutting Off Increasing Returns in Matching

*Note:* The graphs show the results from the counterfactual simulations in Section 1.6.1. Each graph shows the distribution of log firm density and log real wages in the data (in dotted blue lines) and that under the counterfactual equilibrium with \( \lambda = 0 \) (in solid red lines). Panel (B) shows the same objects for the real wages.
Figure 1.A7: Counterfactual Simulation: Subsidies for Input Sales and Production in Hokkaido

Note: The graphs show the results from the counterfactual simulations in Section 1.6.2.1. In each graph, blue circle dots represent the impacts of input sales subsidies and red diamond dots represent the impacts of production subsidies. X-axis represents the total subsidies given as a fraction of total input goods expenditure in Hokkaido area in the baseline ($X^I_{Hokkaido}$).
Figure 1.A8: Counterfactual Simulation: Transportation Infrastructure

Note: The graphs show the results from the counterfactual simulations in Section 1.6.2.2. X-axis represents the inverse of the change of iceberg trade cost for input goods between Hokkaido and Tohoku areas (i.e., $1/\tau_{Hokkaido,Tohoku,k}$ for all $k$).
Table 1.A1: Main Reasons of Bankruptcies

<table>
<thead>
<tr>
<th>Reasons of Bankruptcies</th>
<th>Number of Bankruptcies 06-14</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>reasons used for IV</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>spillover from other bankruptcy</td>
<td>2,310</td>
<td>5.8%</td>
</tr>
<tr>
<td>accidental reasons</td>
<td>615</td>
<td>1.5%</td>
</tr>
<tr>
<td><strong>reasons NOT used for IV</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non-performing sales</td>
<td>26,073</td>
<td>65.4%</td>
</tr>
<tr>
<td>accumulation of debt</td>
<td>5,614</td>
<td>14.1%</td>
</tr>
<tr>
<td>insufficient capital</td>
<td>2,791</td>
<td>7.0%</td>
</tr>
<tr>
<td>management failure</td>
<td>1,311</td>
<td>3.3%</td>
</tr>
<tr>
<td>overinvestment in capital</td>
<td>438</td>
<td>1.1%</td>
</tr>
<tr>
<td>difficulty in collecting account receivable</td>
<td>249</td>
<td>0.6%</td>
</tr>
<tr>
<td>non-project related failure</td>
<td>232</td>
<td>0.6%</td>
</tr>
<tr>
<td>issuance of accommodation debt</td>
<td>155</td>
<td>0.4%</td>
</tr>
<tr>
<td>over-accumulation of inventory</td>
<td>51</td>
<td>0.1%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>39,839</strong></td>
<td><strong>100.0%</strong></td>
</tr>
</tbody>
</table>

*Note:* The frequency of the bankruptcies assigned to each main reason of bankruptcies is reported. "Spillover from other bankruptcy" and "accidental reasons" are the two categories of bankruptcies used for an instrument for a supplier loss in Section 1.3. "Spillover from other bankruptcies" are those caused by management difficulties due to chain reactions such as business partners, subsidiary companies, related bankruptcies, voluntary liquidation, etc. "Accidental reasons" include those with unanticipated accidental problems such as the death of representatives, flood disaster, fire, earthquake, traffic accident, fraud, theft, embezzlement, etc.
Table 1.A2: Summary Statistics of Japanese Firm-to-Firm Matching in Input Trade

(A) Cross-Sectional Patterns

<table>
<thead>
<tr>
<th></th>
<th>2006</th>
<th>2011</th>
<th>2014</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Number of Suppliers per Firm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>4.65</td>
<td>4.63</td>
<td>4.62</td>
</tr>
<tr>
<td>median</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5 percentile</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>95 percentile</td>
<td>12</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>99 percentile</td>
<td>33</td>
<td>35</td>
<td>37</td>
</tr>
<tr>
<td>(ii) Number of 2-digit Input Sectors with a Supplier per Firm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>2.39</td>
<td>2.34</td>
<td>2.33</td>
</tr>
<tr>
<td>median</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(iii) Number of 4-digit Input Sectors with a Supplier per Firm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>3.16</td>
<td>3.11</td>
<td>3.10</td>
</tr>
<tr>
<td>median</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>(iv) Number of Buyers per Firm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mean</td>
<td>4.64</td>
<td>4.66</td>
<td>4.65</td>
</tr>
<tr>
<td>median</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5 percentile</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>95 percentile</td>
<td>13</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>99 percentile</td>
<td>42</td>
<td>40</td>
<td>41</td>
</tr>
<tr>
<td>(v) Number of Firms</td>
<td>695,418</td>
<td>1,030,965</td>
<td>1,095,617</td>
</tr>
</tbody>
</table>

(B) Dynamic Patterns

<table>
<thead>
<tr>
<th></th>
<th>2006 to 11</th>
<th>2011 to 14</th>
<th>per-year average</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Probability of Exit</td>
<td>0.099</td>
<td>0.053</td>
<td>0.019</td>
</tr>
<tr>
<td>(ii) Change of Suppliers (Conditional on Survival)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proportion of Exiting Suppliers</td>
<td>0.10</td>
<td>0.05</td>
<td>0.018</td>
</tr>
<tr>
<td>Proportion of Lost Supplier Relationships</td>
<td>0.37</td>
<td>0.24</td>
<td>0.077</td>
</tr>
<tr>
<td>No. of New Suppliers / No. of Baseline Suppliers</td>
<td>0.54</td>
<td>0.24</td>
<td>0.094</td>
</tr>
<tr>
<td>(iii) Relocation Probability of Firm Headquarters</td>
<td>0.014</td>
<td>0.008</td>
<td>0.003</td>
</tr>
</tbody>
</table>

*Note:* Proportion of lost suppliers in Row (ii) of Panel (B) includes both the cases of supplier exits and dissolution of trading relationships in the specified time interval. Relocation of firm headquarters in Row (iii) of Panel (B) is defined by the relocation outside the grid cell of 0.5 degree latitude by 0.5 degree longitude. The two-digit and four-digit sectors follow the Japan Standard Industrial Classification (JSIC) with 98 two-digit sectors and 1248 four-digit sectors in JSIC classification. For more detail on JSIC, see [http://www.soumu.go.jp/english/dgpp_ss/seido/sangyo/index.htm](http://www.soumu.go.jp/english/dgpp_ss/seido/sangyo/index.htm)
### Table 1.A3: Evidence of New Supplier Matching Rate Increasing in Density of Locally-Selling Suppliers

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
<th>Pretrend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.156***</td>
<td>0.160***</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.054)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.066**</td>
<td>0.104***</td>
</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.040)</td>
</tr>
<tr>
<td>Unexp. S. Bankruptcy x Location x Year FE</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Unexp. S. B. x Buyer Industry x Pref. x Year FE</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Unexp. S. B. x log Employment Size</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Firm x Year FE</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

*Note:* Regression specification follows Equation (1.2). The regression is run at the firm, input sector, and year level. The outcome variable is the number of new suppliers per year, and "Separated Suppliers" indicates the number of lost suppliers per year. "Locally-Selling Suppliers" are defined as the number of firms in each input sector that supply to any firms in the same location (0.5 degree grid cells) in the baseline period (2006). "Separated Suppliers" and its interactions are instrumented by "Unexpected Supplier Bankruptcy" and its relevant interactions. All regressions control for the buyer's location, buyer's sector, input sector and year fixed effects, as well as the fixed effects for the baseline number of suppliers. "log Locally-Selling Suppliers" are normalized to be mean 0, and the inter-quartile range is 2.6. Prefecture (Pref.) is a geographic administrative unit in Japan, with 47 prefectures in total. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 1.A4: Evidence that Density of *Locally-Selling* Suppliers is a Sufficient Statistics for New Supplier Matching Rate

<table>
<thead>
<tr>
<th>New Suppliers</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separated Suppliers</td>
<td>0.151***</td>
<td>0.131***</td>
<td>0.124**</td>
<td>0.129***</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.050)</td>
<td>(0.055)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td></td>
<td>0.077*</td>
<td>0.085*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.040)</td>
<td>(0.049)</td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Established Firms</td>
<td>0.051*</td>
<td>-0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.045)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Established Suppliers</td>
<td>0.047**</td>
<td>-0.018</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.034)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations: 2,288,772 2,288,772 2,288,772 2,288,772

*Note:* Regression specification follows Equation (1.3). See the footnote of Table 1.A3 for the definitions of samples, outcome variables and other controls. "Locally-Selling Suppliers" are defined as the number of firms in each input sector that supply to any firms in the same location (0.5 degree grid cells) in the baseline period (2006). "Locally-Established Firms" are the number of firms in *any sectors* whose headquarters are located in the buyer-side firm's location, and "Locally-Established Suppliers" are the number of firms in *each input sector* whose headquarters are located in the buyer-side firm's location.
Table 1.A5: Evidence of New Supplier Matching Rate Not Decreasing in Buyers

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Supplier Separation</td>
<td>0.179***</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
</tr>
<tr>
<td>Sep. S. x log Potential Suppliers</td>
<td>0.091***</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (Same 2-digit Industry)</td>
<td>-0.036</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (Same 4-digit Industry)</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (Same 2-digit Industry) with Unexp. S. Bankruptcy</td>
<td>0.012</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (all) with Unexp. S. Bankruptcy</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.046)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (Same 2-digit Industry) with Supplier Separation</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (all) with Supplier Separation</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers Anywhere (Same 2-digit Industry) with Unexp. S. Bankruptcy</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers Anywhere (all) with Unexp. S. Bankruptcy</td>
<td>0.035*</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
</tr>
</tbody>
</table>

Observations: 2,288,793 2,288,793 2,288,793 2,288,793 2,288,793 2,288,793 2,288,793 2,288,793

Note: Regression specification follows Equation (1.3). See the footnote of Table 1.A3 for the definitions of samples, outcome variables, other controls, and the variable "Locally-Selling Suppliers". "log Buyers (Same 2-digit Industry)" indicates the number of firms in the same 2-digit industry in firm \( j \)'s location. "log Buyers (Same 2-digit Industry) with Unexp. S. Bankruptcy" indicates the number of firms in the same 2-digit industry in \( j \)'s location that faced unexpected supplier bankruptcy in the same 2-digit input sector, while "log Buyers (Same 2-digit Industry) with Supplier Separation" follows the same definition except that firms are counted if they faced any supplier separation (including supplier exits and dissolution of trade relationships) in the same 2-digit sector. "log Buyers (Same 2-digit Industry) Anywhere with Unexp. S. Bankruptcy" follows the same definition as "log Buyers (Same 2-digit Industry) with Unexp. S. Bankruptcy," except firms in any locations are counted as long as they face unexpected supplier bankruptcy in the same input sector. **p<0.05; ***p<0.01.
Table 1.A6: Impacts of Unexpected Supplier Bankruptcy on Sales Growth and Exit

<table>
<thead>
<tr>
<th></th>
<th>Sales Growth</th>
<th>Exit</th>
<th>Sales Growth (Surviving)</th>
<th>Sales Growth (Pretrend)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Unexpected Supplier Bankruptcy</td>
<td>$-0.031^*$</td>
<td>$-0.036^*$</td>
<td>$0.007^*$</td>
<td>$0.007^*$</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.019)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Unexp. S. Bankruptcy x log Locally-Selling Suppliers</td>
<td>0.013</td>
<td>$-0.0005$</td>
<td>0.011</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>Observations</td>
<td>2,496,861</td>
<td>2,496,861</td>
<td>2,496,861</td>
<td>2,309,804</td>
</tr>
</tbody>
</table>

Note: The regression specification follows Equation (1.4). See the footnote of Table 1.A3 for the definitions of samples, outcome variables, other controls, and the variable “Potential Suppliers”. Sales growth is defined by the arc-elasticity (Davis and Haltiwanger (1992)). Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 1.A7: Impacts of Unexpected Supplier Bankruptcy on Other Existing Suppliers

<table>
<thead>
<tr>
<th></th>
<th>Pr[Other Supplier Separation]</th>
<th>Other Supplier Sales Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Unexpected Supplier Bankruptcy</td>
<td>0.002</td>
<td>−0.002</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Unexp. S. Bankruptcy x log Locally-Selling Suppliers</td>
<td>0.007</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
</tbody>
</table>

Observations 870,764 870,764 865,615 865,615

Note: The regression specification follows Equation (1.5). See the footnote of Table 1.A3 for the definitions of samples, outcome variables, other controls, and the variable “Potential Suppliers”. Sales growth is defined by the arc-elasticity (Davis and Haltiwanger (1992)). Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 1.A8: Estimated and Calibrated Parameters

<table>
<thead>
<tr>
<th>Estimated and Calibrated Parameters</th>
<th>value</th>
<th>source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of supplier matching rate $\lambda$</td>
<td>0.36</td>
<td>Section 1.5.2.1</td>
</tr>
<tr>
<td>Cost advantage upon supplier matching $\nu_{i,km}$</td>
<td></td>
<td>Section 1.5.2.2</td>
</tr>
<tr>
<td>Input share in production $\gamma_{L,m}, \gamma_{km}$</td>
<td></td>
<td>IO table</td>
</tr>
<tr>
<td>Final goods expenditure share $\alpha_k$</td>
<td></td>
<td>IO table</td>
</tr>
<tr>
<td>Productivity distribution shape parameter $\theta$</td>
<td>5</td>
<td>Head and Mayer (2013)</td>
</tr>
<tr>
<td>Elasticity of substitution $\sigma$</td>
<td>5</td>
<td>Broda and Weinstein (2006)</td>
</tr>
<tr>
<td>Mark-up for input sales $\psi$</td>
<td>1.25</td>
<td>Assume same mark-up as final goods, i.e., $\psi = \sigma/(\sigma - 1)$</td>
</tr>
</tbody>
</table>

Baseline Variables

- Number of producers $B_{i,k}$
- Wages $w_i$
- Steady state supplier matching probability $\Lambda_{i,km}$
- Market share of firms with a supplier $\Psi_{i,km}$
- Expenditure share of input goods from $i$ in $j$ $\pi_{ij,k}^I$
- Expenditure share of final goods from in $j$ $\pi_{ij,k}^F$
- Final goods sales $X_{i,k}^F$
- Input goods sales $X_{i,k}^I$

Expenditure share of input goods from $i$ in $j$ $\pi_{ij,k}^I$ was smoothed with the gravity equation; See Appendix 1.E

Expenditure share of final goods from in $j$ $\pi_{ij,k}^F$ was assumed $\pi_{ij,k}^F = \pi_{ij,k}^I$ in the baseline

Final goods sales $X_{i,k}^F$ were used with the total expenditure condition (Equation 1.9)

Input goods sales $X_{i,k}^I$ were used with the total expenditure condition (Equation 1.10)
1.B Additional Figures and Tables

Figure 1.B1: CDF of Number of Suppliers

Note: Cumulative distribution of the number of suppliers under two definitions. “Only reported by buyer-side firm” defines the existence of a link if only buyer-side firms report. “Reported by either side” defines the existence of a link if either party reports that the relationship exists.
Figure 1.B2: Per-Worker Revenue Productivity and Number of Suppliers per Firm

Note: Samples are based on the data from 2006. The graph shows the average log sales per worker for each number of suppliers, censored at 100.
Figure 1.B3: Distribution of Suppliers and Input Sectors per Firm

Note: Samples are based on the data from 2006. The graph shows the CDF of the number of suppliers and number of input sectors with a supplier at the firm level, censored at 20.
Figure 1.B4: Counterfactual Simulation: Subsidies for Input Sales and Production outside Hokkaido

Note: The graphs show the results from the counterfactual simulations in Section 1.6.2.1. In each graph, blue circle dots represent the impacts of input sales subsidies and red diamond dots represent the impacts of production subsidies. X-axis represents the total subsidies given as a fraction of total input goods expenditure in Hokkaido area in the baseline ($X^I_{Hokkaido}$).
Figure 1.B5: Additional Figures for Counterfactual Simulation of Transportation Infrastructure

Note: The graphs show the results from the counterfactual simulations in Section 1.6.2.2. X-axis represents the inverse of the change of iceberg trade cost for input goods between Hokkaido and Tohoku areas (i.e., 1/\tilde{\tau}_{Hokkaido,Tohoku,k} for all k).
Table 1.B1: Number of Suppliers per Firm and Firm Density

<table>
<thead>
<tr>
<th></th>
<th>No. of Suppliers</th>
<th>log No. of Suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>log Firm Density</td>
<td>0.784***</td>
<td>0.956***</td>
</tr>
<tr>
<td></td>
<td>(0.193)</td>
<td>(0.218)</td>
</tr>
</tbody>
</table>

Sample

- 4-digit Industry FE: X X X X X
- Employment Size FE: X X X X X
- Prefecture FE: X X X X X

Observations: 696,454 696,453 670,740 670,740 591,312 670,740

R²: 0.002 0.034 0.928 0.929 0.895 0.447

Note: Samples are firms in the data set from 2006. Firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km). Prefecture is a geographic administrative unit in Japan, with 47 prefectures in total. Standard errors are clustered at the grid cell level. *p<0.1; **p<0.05; ***p<0.01.
Table 1.B2: Revenue Productivity and Number of Suppliers per Firm

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log (Supplier Number + 1)</td>
<td>0.373***</td>
<td>0.305***</td>
<td>0.340***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>4-digit Industry FE</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Employment Size FE</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Observations</td>
<td>670,092</td>
<td>670,091</td>
<td>670,091</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.107</td>
<td>0.313</td>
<td>0.324</td>
</tr>
</tbody>
</table>

*Note:* Samples are firms in the data set from 2006. Firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km). Robust standard errors are reported. *p<0.1; **p<0.05; ***p<0.01.
Table 1.B3: First Stages of New Supplier Matching Regression

<table>
<thead>
<tr>
<th>Supplier Separation</th>
<th>Pretrend</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Unexpected Supplier Bankruptcy</td>
<td>0.864***</td>
<td>0.861***</td>
<td>0.028</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Unexp. Sup. Bankruptcy x log Potential Suppliers</td>
<td>0.007</td>
<td>-0.011</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.011)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Observations 2,288,793 2,288,793 989,957 989,957
R² 0.367 0.367 0.243 0.243

Note: The first stage regression of Equation (1.3) are reported. See the footnote of Table 1.A3 for the definitions of samples, outcome variables, other controls, and variable definitions. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 1.B4: New Supplier Matching in All Input Sectors

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separated Suppliers</td>
<td>0.668***</td>
<td>0.551***</td>
</tr>
<tr>
<td></td>
<td>(0.188)</td>
<td>(0.173)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.272***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.097)</td>
</tr>
<tr>
<td>Observation</td>
<td>2.2M</td>
<td>2.2M</td>
</tr>
</tbody>
</table>

*Note:* Regression specification follows Equation (1.2), except that the dependent variables are the new supplier matching rate in any sectors, not only in each input sector. See the footnote of Table 1.A3 for the definitions of samples, outcome variables, other controls, and variable definitions. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 1.B5: Additional Robustness on New Supplier Matching Regression

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A) Endogeneity of Entry</td>
<td>(B) Endogeneity of Supplier Bankruptcy</td>
<td>(C) Heterogeneity of Input Sectors</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td></td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td></td>
<td>(7)</td>
<td>(8)</td>
<td>(9)</td>
</tr>
<tr>
<td></td>
<td>(10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.070 0.082* 0.122** 0.359** 0.157*** 0.160*** 0.198*** 0.116*** 0.111** 0.206***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.046) (0.048) (0.060) (0.156) (0.060) (0.056) (0.057) (0.055) (0.044) (0.055)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.056** 0.048** 0.054** 0.100* 0.080*** 0.066** 0.087*** 0.097*** 0.042** 0.058**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.022) (0.023) (0.025) (0.058) (0.028) (0.026) (0.026) (0.032) (0.019) (0.026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observation</td>
<td>1.7M 1.2M 1.6M 2.2M 2.2M 2.2M 2.2M 2.2M 2.2M 2.2M 0.6M 1.6M</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample</td>
<td>Excl. top 1 Excl. top 3 Local Only Only bankruptcy Ctrl. Ctrl. Input Ctrl. Input Sector Manufacturing Nonmanufacturing</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

|                | New Suppliers                                      |                |                |
|                | (D) Robustness to Geographic Definition            | (E) Robustness to Industry Definition | (F) Other Misc. Robustness |
|                | (1)                                                 | (2)            | (3)            |
|                | (4)                                                 | (5)            | (6)            |
|                | (7)                                                 | (8)            | (9)            |
|                | (10)                                                |                |                |
| Separated Suppliers | 0.169*** 0.149** 0.145** 0.154*** 0.096* 0.066 0.102* 0.292*** 0.209*** 0.110** 0.128** |                |                |
|                | (0.040) (0.062) (0.069) (0.050) (0.054) (0.071) (0.067) (0.073) (0.048) (0.047) (0.059) |                |                |
| Sep. S. x log Locally-Selling Suppliers | 0.045** 0.048 0.037 0.065*** 0.043 0.050* 0.066* 0.067** 0.085*** 0.055** 0.110*** |                |                |
|                | (0.021) (0.029) (0.027) (0.024) (0.028) (0.030) (0.035) (0.026) (0.026) (0.022) (0.022) |                |                |
| Observation    | 2.0M 1.7M 2.2M 2.2M 2.2M 2.2M 2.2M 1.2M 2.2M 2.3M 2.4M |                |                |
| Sample         | Excl. Excl. 0.1 Degree 1 Degree 3 digit 4 digit Locally-Selling Only Relationship Only Incl. |                |                |
|                | Tokyo Est.>5 Grid Cells Grid Cells Sector Sector Suppliers within Same 2-digit Buyer Sector |                |                |

Note: The specification follows that of Table 1.A3. Column (1) controls for the interaction between unexpected supplier bankruptcy and the input coefficients of the IO matrix. In Column (2), the ranking of industry pair is based on the baseline probability that firms in buyer industries match with suppliers in each input sector. Columns (4) and (5) restrict the instruments for separated suppliers to be unexpected supplier bankruptcy due to accidental reasons and due to other bankruptcy spillovers, respectively. Column (8) and (9) indicate the regressions run at the three digits and four digits industry classification levels, respectively.
Table 1.B6: New Supplier Matching Regression with All Supplier Exits as an Instrument

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
<th>Pretrend</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
<td>(6)</td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.125***</td>
<td>0.127***</td>
<td>-0.038***</td>
<td>-0.039***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.053***</td>
<td>0.062***</td>
<td></td>
<td>0.031***</td>
<td>0.027***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Firm x Year x Supplier Exit FE</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>989,957</td>
<td>989,957</td>
<td>989,957</td>
</tr>
</tbody>
</table>

Note: The specification follows that of Table 1.A3, except that the instruments are supplier exits for any reasons, not unexpected supplier bankruptcies as in Table 1.A3.
Table 1.B7: New Supplier Matching Within and Outside Locations

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Local)</td>
<td>(Nonlocal)</td>
<td>(Local)</td>
<td>(Nonlocal)</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Supplier Separation</td>
<td>0.063*</td>
<td>0.093**</td>
<td>0.020</td>
<td>0.151***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>-0.041</td>
<td>0.126***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Established Suppliers</td>
<td>0.077***</td>
<td>-0.095***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.027)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
</tr>
</tbody>
</table>

Note: The specification follows that of Table 1.A3. “Local” indicates the number of new suppliers within the same 0.5 degree grids, and “Nonlocal” indicates that outside the grids.
Table 1.B8: Correlation between $\nu_{i,km}$ and Proxy of Potential Suppliers in Reduced-Form Regression

<table>
<thead>
<tr>
<th></th>
<th>log $\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>log Locally-Selling Suppliers</td>
<td>0.029***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Sector FE</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Buyer Sector FE</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Buyer Location FE</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>Observations</td>
<td>89,739</td>
<td>89,739</td>
</tr>
</tbody>
</table>

*Note: See Appendix 1.5.2.2 for the calibration of $\nu_{i,km}$, i.e., the cost advantage upon supplier matching in the structural model, and Section 1.3 for the definition of the proxy for the number of potential suppliers in the reduced-form exercise. *p<0.1; **p<0.05; ***p<0.01.
1.C  Proofs and Derivations

1.C.1  Proof of Proposition 1

This section provides a proof of Proposition 1, following in the order of Section 1.4.2 by filling the detailed algebraic derivations.

1.C.1.1 Unit Cost Distribution

The unit cost of firm \( \omega \) in production location \( i \) in sector \( m \) is reproduced from equation (1.6) as

\[
\begin{align*}
\omega_{i,m} &= \frac{1}{\varphi_{\omega} A_{i,m}} w_i^{\gamma_{i,m}} \prod_{k \in K} P_{i,m}^{\gamma_{k,m}},
\end{align*}
\]

where

\[
\begin{align*}
P_{i,m} &= \chi_{i,k} w_i \quad \text{w.p. } 1 - \Lambda_{i,km} (S_{i,k}^I) \\
P_{i,m} &\sim G_{i,k}^I (\cdot) \quad \text{w.p. } \Lambda_{i,km} (S_{i,k}^I)
\end{align*}
\]

hold in the steady state, where \( G_{i,k}^I (\cdot) \) is the probability distribution of unit cost of input goods in location \( i \) in sector \( k \). From the argument in Section 1.4.2, \( G_{i,k}^I (\cdot) \) is approximated by the inverse of the Pareto distribution with upper bound \( \bar{c}_{i,k}^I \), where \( \bar{c}_{i,k}^I \) is the entry cutoff of input goods sellers in sector \( k \) in location \( i \). The measure of
firms whose unit cost is below $c$ is derived as

$$
H_{i,m}(c) = \int_{p_1, \ldots, p_N} B_{i,m} \left( \frac{1}{cA_{i,m} w_i^{\gamma L,m} \prod_{k \in K} p_k^{\gamma_k m}} \right)^{-\theta} \left\{ \begin{array}{l}
\frac{1}{cA_{i,m} w_i^{\gamma L,m} \prod_{k \in K} p_k^{\gamma_k m}} \geq \varphi \\
\prod_{k \in K} dG_{i,k}^l(p_k)
\end{array} \right\}
= \int_{p_1, \ldots, p_N} B_{i,m} \left( \frac{1}{cA_{i,m} w_i^{\gamma L,m} \prod_{k \in K} p_k^{\gamma_k m}} \right)^{-\theta} (1 - o(\varphi)) \prod_{k \in K} dG_{i,k}^l(p_k)
= (1 - o(\varphi)) \varphi^{\theta} B_{i,m} A_{i,m}^{\theta} w_i^{-\theta \gamma L,m} c^\theta
\times \prod_{k \in K} \left\{ (1 - \Lambda_{i,km} (S_{i,k}^l)) + \Lambda_{i,km} (S_{i,k}^l) \int_{p_k} \left( \frac{p_k}{w_i X_{i,k}} \right)^{-\theta \gamma_{k,m}} dG_{i,k}^l(p_k) \right\}
= (1 - o(\varphi)) \varphi^{\theta} B_{i,m} A_{i,m}^{\theta} w_i^{-\theta \gamma L,m} c^\theta
\times \prod_{k \in K} \left\{ (1 - \Lambda_{i,km} (S_{i,k}^l)) + \Lambda_{i,km} (S_{i,k}^l) \frac{1}{1 - \gamma_{k,m}} \left( \frac{\bar{c}_{i,k}}{w_i X_{i,k}} \right)^{-\theta \gamma_{k,m}} \right\}
\equiv (1 - o(\varphi)) \varphi^{\theta} B_{i,m} A_{i,m}^{\theta} w_i^{-\theta \gamma L,m} \prod_{k \in K} \Omega_{i,km} (S_{i,k}^l, \bar{c}_{i,k}^l) c^\theta.
$$

$\Omega_{i,km} (S_{i,k}^l, \bar{c}_{i,k}^l)$ represents the cost advantage from supplier matching, i.e.,

$$
\Omega_{i,km} (S_{i,k}^l, \bar{c}_{i,k}^l) = 1 - \Lambda_{i,km} (S_{i,k}^l) + \Lambda_{i,km} (S_{i,k}^l) \nu_{i,km} (\bar{c}_{i,k}^l),
$$

where $\Lambda_{i,km} (S_{i,k}^l) \equiv \frac{M_{i,km}(S_{i,k}^l)}{M_{i,km}(S_{i,k}^l) + p_{k,i,m}}$ is the steady state probability of matching with a supplier, and $\nu_{i,km} (\bar{c}_{i,k}^l) \equiv \frac{1}{1 - \gamma_{k,m}} \left( \frac{\bar{c}_{i,k}^l}{w_i X_{i,k}} \right)^{-\gamma_{k,m}}$ governs the average cost advantage upon being matched with a supplier.

### 1.C.1.2 Zero Profit Condition of Final Goods Sellers

As in a standard Melitz model, zero profit conditions of marginal final goods sellers determine the entry cutoff unit cost of final goods sellers as well as the number of final goods sellers. Note that the aggregate final goods sales in sector $k$ in location $j$ is $\alpha_k w_j L_j$, and $\frac{1}{\sigma - 1}$ fraction of this goes to final goods seller's profit. Furthermore,
the measure of input goods suppliers whose unit cost is below $c$ is approximated by
\[
\sum_{i' \in N} B_{i',m} \theta \frac{w_{i'}}{u_{i'} \cdot \omega_{i'} \cdot \omega_{i'}} \left( \tau_{i',m}^{F} \right)^{\theta} \prod_{k \in K} \Omega_{i,k,m} \left( S_{i,k}^{I} \right) \equiv \Gamma_{j,k}^{F}. \]
From these relationships, I solve for the cut-off value of unit cost $\bar{c}_{j,k}^{F}$:

\[
\frac{1}{\sigma - 1} \alpha_k w_j L_j = \int_0^{\bar{c}_{j,k}^{F}} \left\{ \Pi_{j,k,m}(c) + f_{j,k}^{F} w_j \right\} d \Gamma_{j,k}^{F} \theta c^{\theta - 1} dc
\]

\[
= \int_0^{\bar{c}_{j,k}^{F}} \left\{ \Pi_{j,k,m}(\bar{c}_{j,k}^{F}) + f_{j,k}^{F} w_j \right\} \left( \frac{\bar{c}_{j,k}^{F}}{c} \right)^{(1-\theta)} \Gamma_{j,k}^{F} \theta c^{\theta - 1} dc
\]

\[
= \left\{ \Pi_{j,k,m}(\bar{c}_{j,k}^{F}) + f_{j,k}^{F} w_j \right\} \frac{\theta}{\theta + \sigma - 1} \Gamma_{j,k}^{F} \left( \bar{c}_{j,k}^{F} \right)^{\theta}
\]

\[
= f_{j,k}^{F} w_j \frac{\theta}{\theta + \sigma - 1} \Gamma_{j,k}^{F} \left( \bar{c}_{j,k}^{F} \right)^{\theta},
\]

where the last equation comes from the fact that $\Pi_{j,k,m}(\bar{c}_{j,k}^{F}) = 0$. This yields the final goods seller entry cutoff as

\[
\bar{c}_{j,k}^{F} = \left( \frac{\theta + \sigma - 1}{\theta(\sigma - 1)} \frac{\alpha_k L_j}{f_{j,k}^{F}} \right)^{1/\theta},
\]

and the measure of final goods sellers, $S_{j,k}^{F} \equiv \Gamma_{j,k}^{F} \left( \bar{c}_{j,k}^{F} \right)^{\theta}$, is

\[
S_{j,k}^{F} = \frac{\theta + \sigma - 1}{\theta(\sigma - 1)} \frac{\alpha_k L_j}{f_{j,k}^{F}}.
\]

1.C.1.3 Zero Profit Condition of Input Goods Sellers

Zero-profit conditions of input goods sellers yield the number of input sellers $S_{j,k}^{I}$ and input seller entry cut-off of unit cost $\bar{c}_{j,k}^{I}$. To see this, first observe that aggregate profit from input sales in location $j$ by sector $k$ to sector $m$ is $\psi_{j,k,m} \Psi_{j,k,m} \left( X_{j,m}^{F} + X_{j,m}^{I} \right)$, where $\Psi_{j,k,m}$ is the share of final goods sales in location $j$ and sector $m$ sold by firms with a supplier in input sector $k$, out of total sales by firms in location $j$ and sector $m$ (introduced in Equation (1.10)). I first derive $\Psi_{j,k,m}$ as a function of $\nu_{i,k,m} \left( \bar{c}_{i,k}^{I} \right)$ and $\Lambda_{i,k,m} \left( S_{i,k}^{I} \right)$. To see this, note that conditional on being matched, the probability of
making a sale is \( \nu_{i,km}(\bar{c}_{i,j,k}) \) times the unmatched one. This yields

\[
\Psi_{j,km}(S^f_{j,k}, \bar{c}^f_{j,k}) = \frac{\Lambda_{j,km}(S^f_{j,k}) \nu_{j,km}(\bar{c}^f_{j,k})}{1 - \Lambda_{j,km}(S^f_{j,k}) + \Lambda_{j,km}(S^f_{j,k}) \nu_{j,km}(\bar{c}^f_{j,k})}.
\] (1.20)

Now, the ratio of the profit from input goods of two firms with unit cost \( c \) and \( c' \) is written as \( \frac{\Pi^f_{j,km}(c')}{\Pi^f_{j,km}(c)} = (\frac{c'}{c})^\gamma_{k,m} \theta \), by noting that the unit cost differences translates to downstream final goods sales by the factor of \( \gamma_{k,m} \). Furthermore, the measure of input goods suppliers whose unit cost is below \( c \) is approximated by \( \sum_{i' \in N} B_{i',m} \ell_{i',m} \nu_{i',m}(\tau_{i',j,m})^\theta c^\theta \prod_{k \in K} \Omega_{i,km}(S^f_{i,k}, \bar{c}^f_{i,k}) \equiv \Gamma^f_{j,k} c^\theta \). From these relationships, I solve for the cut-off value of unit cost \( \bar{c}^f_{j,k} \):

\[
\psi - 1 - \gamma_{k,m} \Psi_{j,km}(X^F_{j,m} + X^I_{j,m}) = \int_0^{\bar{c}^f_{j,k}} \Pi^f_{j,km}(c) d\Gamma^f_{j,k} c^\theta = \int_0^{\bar{c}^f_{j,k}} \Pi^f_{j,km}(\bar{c}^f_{j,k}) \left( \frac{\bar{c}^f_{j,k}}{c} \right) \gamma_{k,m} \theta \Gamma^f_{j,k} c^\theta - 1 dc
\]

\[
= \Pi^f_{j,km}(\bar{c}^f_{j,k}) \frac{1}{1 - \gamma_{k,m}} \Gamma^f_{j,k} \bar{c}^f_{j,k}^\theta
\]

\[
= \Pi^f_{j,km}(\bar{c}^f_{j,k}) \frac{1}{1 - \gamma_{k,m}} S^f_{j,km}.
\]

By noting that the zero profit condition is derived as \( f_j^* w_j = \sum_{m \in K} \Pi^f_{j,km}(\bar{c}^f_{j,k}) \), I have

\[
S^f_{j,k} = \frac{1}{w_j f^*_{j,k}} \frac{\psi - 1}{\psi} \sum_{m \in K} \{(1 - \gamma_{k,m}) \gamma_{k,m} \Psi_{j,km}(X^F_{j,m} + X^I_{j,m})\}.
\]

The entry cut-off of unit cost is derived as

\[
\bar{c}^f_{j,k} = \left( \frac{S^f_{j,k}}{\sum_{i' \in N} B_{i',k} \left( \tau^I_{i,j,m} \right)^\theta \ell_{i',m} \nu_{i',m}(\tau^f_{i',j,m})^\theta \prod_{k \in K} \Omega_{i,km}(S^f_{i',k}, \bar{c}^f_{i',k})} \right)^{1/\theta}.
\]
1.C.1.4 Free Entry Conditions

Total input goods profit of firms in location $j$, net of the fixed marketing cost, is written as

$$\sum_{m \in K} \int_{0}^{\Gamma_{j,k,m}(c)} \left( \Pi_{j,k,m}(c) - w_{j} f_{i,k} \right) d\Gamma_{j,k,m} \theta = \sum_{m \in K} \frac{\psi - 1}{\psi} \gamma_{k,m} \Psi_{j,k,m} \left( X_{j,m} + X_{j,m}^{l} \right) \left\{ \gamma_{k,m} + 1 \right\} \frac{1}{\psi} \gamma_{k,m} \Psi_{j,k,m} \left( X_{j,m} + X_{j,m}^{l} \right)$$

$$= \sum_{m \in K} \frac{\psi - 1}{\psi} \gamma_{k,m} \Psi_{j,k,m} \left( X_{j,m} + X_{j,m}^{l} \right)$$

Total final goods profit of firms in location $j$, net of fixed marketing cost, is similarly derived as

$$\int_{0}^{\Gamma_{j,k,m}(c)} \left( \Pi_{j,k,m}(c) - w_{j} f_{i,k} \right) d\Gamma_{j,k,m} \theta = \frac{\sigma - 1}{\sigma \theta} \alpha_{k} w_{j} L_{j}.$$

Hence, the free entry condition to enter in location $i$ is written as

$$B_{i,k} = \frac{1}{\xi_{i,k} C_{i,k} w_{i}} \sum_{j \in N} \left\{ \frac{\sigma - 1}{\sigma \theta} \alpha_{k} w_{j} L_{j} + \frac{\psi - 1}{\psi} \sum_{m \in K} \gamma_{k,m} \Psi_{j,k,m} \left( X_{j,m}^{F} + X_{j,m}^{l} \right) \right\},$$

where $\xi_{k}$ is the bankruptcy rate, and $C_{i,k} w_{i}$ is the fixed entry cost.

1.C.2 Proof of Proposition 3

From the definition of CES consumer utility and pareto distribution of unit cost of final goods sellers, the consumer price index in location $i$ is written as

$$P_{i} \propto \prod_{m \in K} \left( S_{i,m}^{F} \right)^{a_{m}} \left( c_{j,m} \right)^{a_{m}}$$

$$= \prod_{m \in K} \left( S_{i,m}^{F} \right)^{a_{m}} \left( \sum_{l} \frac{S_{i,m}^{F}}{\tau_{i,l,m}^{\theta} 0 L_{m}} \right)^{a_{m}} \theta \left( \tau_{i,m}^{F} \right)^{\theta} \Pi_{k \in K} \Omega_{i,k}$$

$$= \prod_{m \in K} \left( S_{i,k}^{F} \right)^{a_{m}(1 - \sigma + \theta)} \left( \frac{\pi_{i,m}^{F}}{\frac{\theta - \gamma_{i,l,m}^{\theta}}{\pi_{i,m}^{F}} 0 L_{m}} \right)^{a_{m}} \theta \left( \tau_{i,m}^{F} \right)^{\theta} \Pi_{k \in K} \Omega_{i,k}$$

Note that under the Pareto distribution $S_{i,k}^{F}$ only depends on $L_{i}$, hence the values
are unchanged under the counterfactual simulations. Therefore, the change of real wages is written as

$$\left(\frac{\bar{w}_i}{\hat{P}_i}\right) = \prod_{m \in K} \left(\frac{\hat{n}_{i,m}^{F,*}}{\hat{B}_{i,m}^{*}w_i^{1-\beta}L_i^{\beta}} \left(\frac{\bar{\pi}_i^{F,*}}{\hat{\pi}_{i,m}^{F}}\right)^{\beta} \prod_{k \in K} \hat{\Omega}_{i,km} \right)^{\alpha_m \beta}$$

1.D Model Extension to Incorporating Labor Mobility

To incorporate labor mobility, I make the following additional assumption. I assume that workers also consume housing goods in addition to final goods, with Cobb-Douglas utility with share $\beta$. In addition, each worker has heterogeneous preferences for locations, $\epsilon = \{\epsilon_1, \ldots, \epsilon_N\}$. Together, the utility of a worker that draws preference shock $\epsilon$ is written as

$$U_i(\epsilon) = A_i \frac{w_i}{P_i^{1-\beta}R_i^\beta} \epsilon_i,$$

where $A_i$ is the exogenous amenity level of the locations and $R_i$ is the rent in location $i$. I assume that housing supply in each location is fixed to 1 at each location. From the land market clearing condition, the rent is determined as $R_i = w_i L_i$ hence the utility function $U_i(\epsilon) = A_i \left(\frac{w_i}{P_i}\right)^{1-\beta} (L_i)^{-\beta} \epsilon_i$. Assuming that $\epsilon_i$ is drawn from Fréchet distribution with scale parameter $\nu$ independently for each worker and location, and normalizing the total population $L = \sum_i L_i = 1$, I have free labor mobility conditions:

$$L_i = \frac{A_i^\nu \left(\frac{w_i}{P_i}\right)^{(1-\beta)\nu} (L_i)^{-\beta \nu}}{\sum_{i'} A_{i'}^\nu \left(\frac{w_{i'}}{P_{i'}}\right)^{(1-\beta)\nu} (L_i)^{-\beta \nu}}. \quad (1.21)$$

The equilibrium with free labor mobility is simply characterized by just adding free labor mobility conditions in the characterization of Proposition 1 and including $L_i$ as an additional endogenous variables.

**Proposition 4.** Under sufficiently small $\varphi$, the steady-state equilibrium with free labor mobility is characterized by $\{S_{i,k}, B_{i,k}, w_i, X_{i,k}^{F}, \bar{\epsilon}_{j,k}, \pi_{i,k}^{F}, \pi_{i,j,k}^{F}, L_i\}$ that satisfy total
expenditure conditions (1.9, 1.10), trade balancing condition (1.11), gravity equations for input goods (1.15) and final goods (1.14), zero profit conditions (1.16), entry cut-off of input goods sellers (1.17), free-entry conditions (1.18), and free labor mobility (1.21).
1.E Calibration of Expenditure Share of Input Goods

By noting that $\pi_{ij,k}$ also corresponds to the extensive margin of trade, empirical frequency of the probability of sourcing input goods from location $i$ in sector $k$, $\pi_{ij,k}$, can be in principle directly obtained from the data set by counting the fraction of sector $k$ suppliers producing in location $j$ from location $i$. However, this measure is noisy due to the sparseness of the data and involves many zeros. To deal with this, I first estimate the parametrized gravity equation (1.15) to obtain the smoothed predictor of $\pi_{ij,k}$. To implement this, I estimate the following model

$$\hat{\pi}_{ij,k} = \delta_{i,k} \pi_{ij,k}^{\theta} \overline{\sum_{i' \in N} \delta_{i',k} \pi_{ij,k}^{\theta}}$$

where $I$ parametrize $\pi_{ij,k}^{\theta} = (D_{ij})^{\kappa}$ and $D_{ij}$ is the straight line distance between $i$ and $j$. $\delta_{i,k}$ is the location fixed effects, defined as $\delta_{i,k} = B_i \omega_{i,k} w_i^{-\alpha}$. The model is estimated by a Poisson regression as suggested by Santos Silva and Tenreyro (2006). I use the predictor of the model

$$\hat{\pi}_{ij,k} = \hat{\delta}_{i,k} \hat{\pi}_{ij,k}^{\theta} \overline{\sum_{i' \in N} \hat{\delta}_{i',k} \hat{\pi}_{ij,k}^{\theta}}$$

for the baseline $\pi_{ij,k}$. 

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Chapter 2

Billions of Calls Away from Home: Measuring Commuting and Productivity inside Cities with Cell Phone Records

2.1 Introduction

Measures of urban economic activity at fine temporal and spatial scale are important yet scarce. Such data is useful for researchers and policy-makers to understand how cities respond to shocks such as floods or industry-specific demand shocks, as well as to urban policies such as transportation infrastructure improvements, or to monitor informal economic activity not covered by tax records. Traditionally, fine grained economic data is only available infrequently and often with long delays, such as from population and economic censuses. These issues are especially salient in large cities in developing countries, which are growing fast (commonly by 30-40% per decade\(^1\)) and thus experiencing high urban environment stress, yet which are least covered by conventional data sources.

\(^1\)Author calculation based on (UN DESA 2016).
In this paper we measure economic activity indirectly, based on urban commuting flows. The logic of our approach is simple. A core function of cities is to connect workers and jobs. While many factors enter into workplace choice decisions, areas with high labor productivity should disproportionately attract commuting workers, keeping distance and worker home locations fixed. This suggests a revealed preference approach to measure high economic activity areas based on the pattern of observed commuting flows.

We formalize this intuition using an urban economic model of commuting flows derived from individual utility maximizing behavior. The model relates aggregate bilateral commuting flows and commuting time costs, which appear in the data, and wages, or workplace productivity levels, which we seek to estimate. The relationship takes the form of an unconstrained gravity equation on log commuting flows, with origin and destination fixed effects. Inverting this relationship to derive wages amounts to estimating the gravity equation and recovering the destination fixed effects.

To implement this approach, we use two data sets of commuting flows extracted from cell phone transaction data in Sri Lanka and in Bangladesh. Together, they hold information on almost half a billion days with commuting data. We first confirm that our measure indeed captures commuting flows, by comparing it with a commuting survey from Dhaka. We find that the two measures line up well, even when controlling for travel time, meaning that commuting flows from cell phone data pick up subtle route-specific variations in commuting flows. The advantage over conventional data sources is the much higher sample size, as well as very fine geographic resolution at the level of cell phone towers, and daily time variation.

To validate our model-based approach of inferring relative productivity, we compare model-predicted income and income from the transportation survey in Dhaka. The model prediction is derived purely from commuting flows and the matrix of commuting travel times. We find a strong correlation with self-reported income from the survey, at the origin-destination pair level. The model-predicted income measure depends on how much commuting time and idiosyncratic shocks affect productivity (in addition to affecting utility). We show how these parameters – as well as the Fret
shape parameter – can be directly estimated using the survey income data, and we find that distance is a pure utility cost, while idiosyncratic shocks contribute partially to productivity.

We show how the constructed income and economic activity measures can be used in practice with two applications that roughly map on the space and time dimension of the data. We first compare the city profiles of Dhaka and Colombo in terms of population and average incomes at the residential and employment locations level, and find that Colombo has a distinctly more concentrated Central Business District in terms of employment population and income. In the second application, we estimate the economic impact of hartals, a form of strike action intended to disrupt transportation that is common in Bangladesh (UNDP 2005). We find that on hartal days, people travel less along both extensive and intensive margins, and this effect is biased towards high-income commuting links. Using the model estimated on non-hartal days, and assuming commuters receive the income of the destination, we account for a 4.6% decrease in output on hartal days relative to usual workdays (95% CI of 0.9 to 8.6% decrease). For reference, Fridays (the main free day in Bangladesh) are associated with a 11% decrease in output using this measure.

Our project has two main contributions. We explore a model linking commuting and destination location productivities, and show that the model predictions are generally satisfied. Secondly, we show that the model and model output are useful in practice for measuring spatial and temporal changes in economic activity.

This project contributes to several strands of literature. Our proof of concept method is applicable to many data-scarce large cities in developing countries, and it should be of interest to researchers and urban planners alike. Similar to nighttime lights data from satellites (Henderson, Storeygard, and Weil, 2012), we show how big data can be used to infer economic activity over space and time. Our focus is on distinguishing differences within cities, and using the model we can compute how income “moves” across the city, specifically to compute residential income, and income of commuters on a specific commuting link. This is a contribution over existing purely statistical measures of economic activity, such as nighttime lights, which do
not easily allow making this distinction. The topic of how economic productivity and commuting costs interact to determine urban structure is fundamental in urban economics models (Alonso, 1960; Mills, 1967; Muth, 1968). Here, we use a new generation of models inspired from the trade literature, designed to better match the real data (Ahlfeldt et al., 2015). Finally, we contribute to a recent, growing and diverse literature that uses CDR data to measure human mobility and economic activity (Calabrese, Di Lorenzo, Liu, and Ratti, 2011; Wang, Hunter, Bayen, Schechtner, and González, 2012; Csáji, Browet, Traag, Delvenne, Huens, Van Dooren, Smoreda, and Blondel, 2013; Iqbal, Choudhury, Wang, and González, 2014).

Increasingly, new data sources on mobility are becoming available, such as public transport ticketing data in digital format, electricity metering data, cell phone transaction data, passively collected smartphone app location data, etc. This data is usually highly multi-dimensional, so it is not \textit{a priori} obvious how to relate it to economic activity. One approach uses statistical learning techniques to relate mobility or the underlying data to economic indicators (Blumenstock, Cadamuro, and On, 2015). This approach focuses on predictive power, yet is agnostic about any theoretical relations in the data.

The paper is organized as follows. Section 2.2 describes the cell phone and commuting data, and compares survey-based commuting flows with those derived from cell phone data. Section 2.3 sets up and analyzes the model, and section 2.4 reports the main validation results. Sections 2.5 and 2.6 report the results from the the two applications, and section 2.7 concludes.

### 2.2 Cell-Phone Data and Commuting Flows

In this section we describe the cell phone data and the procedure to extract commuting flows. We perform a validation exercises based on a transportation survey from Dhaka.

\footnote{The ideas in these models, especially the relation between discrete choice and the gravity equation, have been explored previously (Anas, 1983).}
2.2.1 Data Sources

**Cell phone transaction data and commuting flows.** We use call detail record (CDR) data from multiple operators in Sri Lanka and Bangladesh to compute detailed commuting matrices.\(^3\) CDR data includes an observation for each transaction, such as making or receiving a voice call, sending or receiving a text message, or initiating a GPRS internet connection. Each observation has a timestamp, the participant user identifiers, and their locations at the cell tower level. Towers are unevenly distributed in space; they are denser in urban and developed areas. We focus on the greater metropolitan areas around the capital cities of Colombo and Dhaka. The data covers a little over a year in Sri Lanka and four months in Bangladesh, and it is anonymized at the telephone number level, which allows us to observe all transactions associated with a given user throughout the study period.

We infer within-day movement by observing a user connect to different cell towers during the day. On a given day, we define a user's *origin* as the location of the first transaction between 5am to 10am, and the user's *destination* as the location of the last transaction between 10am and 3pm.\(^4\) By definition, a user has at most one commuting trip per day. If the origin and destination correspond to the same cell tower, we consider that the user was *stationary* that day, and if they are different we consider that the user made a commuting trip that day. If transaction data is missing in either time interval, commuting behavior is not defined for that user and day. Table 2.E1 shows that commuting data (either stationary or trips) is available for 16% and 29% of the theoretical maximum if we observed each user on every day in the sample in Dhaka and Colombo, respectively. In theory, incomplete coverage

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\(^3\)The data for Bangladesh is prepared by the Asian Development Bank for the project (A-8074REG: “Applying Remote Sensing Technology in River Basin Management”), a joint initiative between ADB and the University of Tokyo.

\(^4\)In addition to the validation discussed below, where we compare commuting flows computed using this definition to commuting flows from a separate transportation survey in Dhaka, we have also experimented defining the *home* and *work* locations as the most popular destinations of a user in certain time intervals, computed using the data for the whole period for that user. The commuting flows computed using the two methods are strongly correlated, even after controlling for distance (not reported).

\(^5\)We focus on the morning commute as other types of trips (e.g., shopping) are more likely in the evening (Frank and Murtha, 2010).
can lead to biased findings if travel behavior and calling behavior are correlated. For example, people may be more likely to make calls when traveling, which would lead us to measure more travel than in reality. In the second application in Section 2.6, we use a Heckman selection model to control for fewer calls during hartal days, as this may influence observed travel behavior; we find very similar results.

In each city, we aggregate trips over all non-holiday weekdays to obtain an origin-destination (OD) matrix of commuting flows between every pair of cell towers. For analysis, we restrict the sample to trips between tower pairs that are not very close or very far away. Specifically, we exclude stationary (within tower) flows, which account for roughly half of all flows, because they may partly capture non-commuting behavior from individuals not in the labor market. We also exclude commuting flows between nearby tower pairs, as we are concerned that they may partly reflect stationary users who randomly connect to different nearby towers, instead of real travel.

Figure 2.A2 shows one cut of the data, plotting in each city the spatial distribution of total commuting inflows for each cell phone tower. Inflows are a measure of workplace population, as they measure where people are during the day. In both cities, inflows are highest around the center of the city, they decay with distance, yet other local centers are also visible. In particular, several concentrated centers are visible in Dhaka.

Google Maps distance data. We obtain typical driving travel times and road distances between pairs of cell towers using the Google Maps API. For a given pair of cell towers, we query the Google Maps Distance Matrix API for the typical driving travel time on a typical weekday with 8am departure time. Because of the large number of bilateral pairs (on the order of \( \sim 10^6 \)), in each city we obtain Google data for 90,000 randomly selected pairs of towers in the sample, and predict the travel time and road distance for the remaining pairs of towers. The prediction for an origin-destination pair \((i, j)\) is based on measured travel times for pairs whose origins are close to \(i\) and whose destinations are close to \(j\).\(^6\)

\(^6\)See Appendix C.2 for more detail about the exact prediction procedure. To assess the predictive power, for each pair with original Google Maps data we compare the original travel time and the predicted travel time using all data except for the pair itself. The \(R^2\) is 0.979 and 0.962 in Sri Lanka.
Household commuting survey. In Dhaka, we use survey data from the Dhaka Urban Transport Network Development Study or DHUTS (JICA 2010). This survey interviewed individuals in randomly selected households, in each of 72 “commuting zones” defined in Dhaka. The sample size that we use for analysis is 13,905 individuals.

2.2.2 Commuting Data Validation

We now explore quantitatively how commuting flows derived from cell phone data in Dhaka relate to flows from the DHUTS commuting survey. Previous work shows that origin-destination commuting flows derived from cell phone data correlate well with commuting and origin population measured from transportation surveys or census data (Calabrese et al., 2011; Iqbal et al., 2014; Wang et al., 2012). However, a simple correlation between two flow measures may in principle be due to strong intermediary factors, such as distance.

Figure 2.A1 shows how log average commuting flows decreases with travel time in the two data sources. To construct this figure, we first aggregate the cell phone commuting data up to the level of commuting zones defined in the DHUTS survey. The sample consists of 7,676 pairs of distinct commuting zones, with a total of 7,903 trips in the DHUTS survey and \( \sim 6 \cdot 10^6 \) trips in the cell phone data. For each pair of commuting zones, we estimate the average travel time by summing over all cell phone tower pairs included in the two commuting zones, weighted by commuting flow. We then divide log travel time into 100 bins and compute the log of mean flow for all pairs of distinct commuting zones with log travel time in that bin.7 Figure 2.A1 plots the resulting relationship, as well as point-wise bootstrapped 95% confidence intervals clustered at the origin commuting zone level. The decay with distance is virtually identical throughout the range of distance. For small distances, the cell phone data tends to pick up slightly higher commuting compared to the survey data.

and Bangladesh, respectively, indicating that the prediction performance is good.

7The DHUTS data is sparse, so it is important to take the average of the flow for pairs in a certain distance bin before taking logs. Appendix Figure 2.E1 shows that without this adjustment, there is considerable bias in the slope with respect to distance due to the large number of pairs with zero flows. Commuting flows from cell phone data do not have this problem due to their much higher coverage, and the adjustment does not make any notable difference.
This may be due to bouncing between towers, short non-commuting trips, or due to underreporting in the survey data.

Table 2.E2 shows regression results with the same data, including specifications that control for origin and destination commuting zone fixed effects, and log travel time. Throughout, commuting flows derived from cell phone data are a strong predictor of survey-based commuting flows. That cell phone data detects real variations in commuting flows controlling for these factors implies that it contains rich information about how people move around the city for work.

It is not obvious \textit{a priori} how to link commuting flows – which are defined at the location pair level – and location-specific productivity and income. We now introduce a simple theoretical model that clarifies the relationships that we expect to see in the data.

### 2.3 Theoretical Framework

In this section, we set up a simple urban economic model that we will use to interpret the commuting data. Commuters decide their work location taking into account wages at different potential work locations, commuting costs, as well as destination-specific idiosyncratic utility shock. This discrete choice model implies that log bilateral commuting flows follow an unconstrained gravity equation, with destination fixed effects capturing log wages. The basic intuition of the model is to assign higher wages to a destination location that attracts more workers net of how far workers live and commuting costs. We use this relationship and the commuting flow data to \textit{back out} the distribution of wages across locations. The model also allows us to compute several important measures, such as the average income at a given location or for commuters on a specific route. We will then compare these predicted income measures with survey-collected income data.

The model presented here is a partial equilibrium version of the model developed by Ahlfeldt et al. (2015), which is in turn inspired by models in international trade (Eaton and Kortum, 2002). We assume that competitive forces lead wages to re-
flect productivity. We are otherwise agnostic about how rents and firm production decisions are determined in general equilibrium. As we will show, this approach is sufficient for our purpose of inferring wages and income from commuting flows using equilibrium relationships. Hereinafter, we describe our model, followed by the empirical estimation procedure.

2.3.1 Model Setup

Space is partitioned into a finite set of locations $L$, which may serve as both residential locations and work locations. In our application, these correspond to Voronoi cells around cell phone towers. Each worker supplies one unit of labor inelastically. A worker $\omega$ who lives at residential location (or origin) $i$ can choose to work at any work location (or destination) $j \in L$ that offers employment. The utility if she chooses destination $j$ is:

$$U_{ij\omega} = \frac{W_j Z_{ij\omega}}{D_{ij}}$$  \hspace{1cm} (2.1)

$W_j$ is the wage offered at location $j$ (all firms at location $j$ offer the same wage), $D_{ij}$ is the travel time between $i$ and $j$, and $Z_{ij\omega}$ is an idiosyncratic utility shock that is i.i.d. following the Fret distribution, with scale parameter $T$ and shape parameter $\epsilon$. Standard results on the Fret distribution (reviewed in Appendix 2.C) imply that $U_{ij\omega}$ is also a Fret-distributed random variable, with shape $\epsilon$ and scale $T_{ij} = TW_j D_{ij}^{\epsilon T}$. Each worker chooses the work location $j$ where $U_{ij\omega}$ is maximized. This implies that the probability that a worker residing in $i$ commutes to $j$ is given by:

$$\pi_{j|i} = \frac{(W_j / D_{ij})^\epsilon}{\sum_k(W_k / D_{ik})^\epsilon}$$  \hspace{1cm} (2.2)

In the absence of random shocks, all workers from a given location would choose the same work location. The $Z_{ij\omega}$ shocks lead to a non-degenerate distribution of work location choices, and a higher variance of $Z_{ij\omega}$ decreases the relative importance of distance and wage.

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8It should be noted, however, that if one is interested in how different policies will change prices and commuting flows, a complete general equilibrium framework is needed.
Equation (2.2) describes a gravity equation for commuting probabilities. Taking logs, and denoting log quantities by lowercase letters:

$$\log(\pi_{j|i}) = \epsilon \log(W_j) - \epsilon \tau \log(D_{ij}) - \log \left( \sum_s \left( \frac{W_s}{D_{is}} \right)^\epsilon \right)$$

$$= \epsilon w_j - \epsilon \tau d_{ij} - \log \left( \sum_s \exp (\epsilon w_s - \epsilon \tau d_{is}) \right)$$ (2.3)

We estimate this equation through the following empirical gravity model:

$$\log(\pi_{j|i}) = \psi_j + \beta \log(D_{ij}) + \mu_i + \epsilon_{ij}$$ (2.4)

where $\mu_i$ is an origin fixed effect, $\psi_j$ is a destination fixed effect, and $\epsilon_{ij}$ accounts for measurement error. Gravity equations have been widely used to model international trade (Anderson, 1979), transportation (Erlander and Stewart, 1990) and commuting behavior (Duran-fernandez and Santos, 2014; Sohn, 2005). Applications in transportation usually specify a constrained gravity model that is guaranteed to match inflows, outflows, or both, and are used to estimate the effect of distance. Here, we are primarily interested in the destination fixed effects, so the model is unconstrained. Equation (2.3) does, however, imply a constraint between the origin fixed effects and the other quantities. For computational reasons, we choose to estimate (2.4) unconstrained and check the relationship post estimation. In practice, the R-squared in the regression of $\hat{\mu}_i$ on $\log \left( \sum_s \exp \left( \hat{\psi}_s + \hat{\beta} d_{is} \right) \right)$ is 0.64, suggesting that the bias due to unconstrained gravity estimation is not problematic.

### 2.3.2 Model-Predicted Wages and Income

The gravity equation identifies relative wages at all employment locations. Indeed, the destination fixed effects directly estimate

$$\hat{\psi}_j = \epsilon w_j$$ (2.5)

Notice that the Fret scale parameter $\epsilon$ also enters in the destination fixed effect.
We explain below how we estimate this parameter using survey data on income.9

We next derive average income at a given residential location, and for workers commuting between a given pair of residential and work locations. Equation (2.1) shows that distance and idiosyncratic shocks affect utility. In order to compute average worker income, we must take a stand on how the two variables affect productivity and thus labor income. Our approach is to derive a general formula for income and let the data speak as to the role of shocks and distance in explaining income.

Assume income is given by $Y_{ij\omega}(\alpha_z, \alpha_d) = W_jZ_{ij\omega}^{\alpha_z}D_{ij}^{-\alpha_d}$, where $\alpha_z \in [0, 1]$ controls how much the shocks $z_{ij\omega}$ are productive, and $\alpha_d \in [0, 1]$ controls how much distance $d_{ij}$ is a productive cost. At the extreme where $\alpha_z = \alpha_d = 0$, these variables affect utility but not productivity; when $\alpha_z = \alpha_d = 1$, the variables affect utility and income equally. Taking logs, we express income as a convex combination of four extreme cases:

$$y_{ij\omega}(\alpha_z, \alpha_d) = \alpha_z (\alpha_d \cdot y_{ij\omega}(1, 1) + (1 - \alpha_d) y_{ij\omega}(1, 0)) + (1 - \alpha_z) (\alpha_d \cdot y_{ij\omega}(0, 1) + (1 - \alpha_d) y_{ij\omega}(0, 0))$$

Expected income in these four cases is given by the following formulas:

$$y_{ij\omega}(0, 0) = w_j$$
$$y_{ij\omega}(0, 1) = w_j - \tau d_{ij}$$
$$y_{ij\omega}(1, 1) = \frac{1}{\epsilon} \log \left( \sum_s \exp (\epsilon w_j - \epsilon \tau d_{ij}) \right) - \frac{K}{\epsilon} \text{ for some absolute constant } K$$
$$y_{ij\omega}(1, 0) = y_{ij\omega}(1, 1) + \tau d_{ij}$$

When neither shocks nor distance are productive, income is simply the destination wage, and thus it is constant regardless of commuting origin and does not vary between individuals. In the second case, when distance imposes a productivity cost, income is origin-destination specific but still does not vary between individuals. When the shocks $z_{ij\omega}$ are productive as in the third and fourth cases, log income for a worker

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9Faced with a similar situation, Ahlfeldt et al. (2015) calibrate $\epsilon$ to match the wage standard deviation from survey data across the city.
commuting between $i$ and $j$ depends on the distribution of the shock conditional on destination $j$ being chosen. By virtue of the Fret distribution, the conditional distribution $y_{ij|j} \in \text{arg max}_s U_{i\omega}$ is also Fret with the same shape parameter $\epsilon$ and scale $T_i = \sum_s T_{is} = \sum_s W_s^* D_{is}^{-\epsilon \tau}$. In particular, this distribution only depends on the origin $i$ and thus expected log income is the same for all destinations $j$. Detailed derivations can be found in Appendix 2.C.

We next show how we can use data on income to learn how distance and idiosyncratic shocks affect productivity, and thus identify the parameters $\alpha_z$ and $\alpha_d$.

### 2.3.3 Taking the Model to Data

**Estimating Wages and Income.** We estimate the gravity equation (2.4) using the matrix of commuting flows derived from the cell phone data. Equipped with estimated fixed effects and the coefficient on log travel time, we can compute log wages at each location (up to the factor $\epsilon$), as well as the income measures described in the previous section.

In order to let the data speak about which income measures provides a better fit, we will estimate the parameters $\alpha_z$ and $\alpha_d$. The procedure described below has the added advantage of providing an estimate of the shape parameter $\epsilon$. We begin by comparing the income measures with self-reported income from a transportation survey. The survey data also contains residential and work locations, so we can compare the two measures at the origin and destination level. Next, note that plugging in the four extreme values into equation (2.6) simplifies to

$$y_{ij\omega} (\alpha_z, \alpha_d) = \alpha_z y_{ij\omega} (1, 0) + (1 - \alpha_z) w_j - \alpha_d \tau d_{ij}$$  

(2.7)

We estimate this equation using OLS with survey income as outcome and model predicted measures on the right-hand side:

$$y_{ij\omega}^S = \rho_1 y_{ij}^{1,0} + \rho_2 \hat{y}_j + \rho_3 d_{ij} + \varepsilon_{ij\omega}^S$$  

(2.8)

where $y_{ij\omega}^S$ is survey-based income of commuter $\omega$ who lives at $i$ and works at $j$. 

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\[ \hat{y}_{ij}^{1.0} = \log \left( \sum_s \exp \left( \hat{\psi}_s - \hat{\beta} d_{ij} \right) \right) + \beta d_{ij} \]

is the estimated counterpart of \( \mathbb{E}y_{ij}\omega (1, 0) \) without the \( \epsilon \) factor, \( \hat{\psi}_j \) is the estimated destination fixed effect at destination \( j \), and \( d_{ij} \) is log distance. The asymptotic relationship between the regression coefficients and equation (2.7) is

\[ \rho_1 = \frac{\alpha_z}{\epsilon}, \quad \rho_2 = \frac{1 - \alpha_z}{\epsilon}, \quad \text{and} \quad \rho_3 = -\frac{\alpha_d}{\epsilon}. \]

We invert this system of equations to obtain estimates for the productivity shares of shocks and distance, and the Fret shape parameter:

\[ \hat{\alpha}_z = \frac{\hat{\rho}_1}{\hat{\rho}_1 + \hat{\rho}_2}, \quad \hat{\alpha}_d = -\frac{\hat{\rho}_3}{\hat{\rho}_1 + \hat{\rho}_2}, \quad \text{and} \quad \hat{\epsilon} = \frac{1}{\hat{\rho}_1 + \hat{\rho}_2}. \]  

**Mapping Model Locations to the Data**. An important aspect of taking the model to the data is choosing how to represent residential and work locations in the data. Specifically, we need to choose an aggregation level for our data, and to take a stand on the geographic level where the shocks from the model are realized. Previous work in urban economics uses fine urban administrative levels for one or both purposes (Ahlfeldt et al., 2015), yet at its most granular geographic level our cell phone data contains cell phone tower locations. If predictions on wages, income, output, depend on how geography is defined, this could mean that the model is not a reliable tool for empirical work. In fact, we show that the model has a general (approximate) invariance property for the level of geographic aggregation of both origin and destination locations. We summarize the main results here, and relegate detailed derivations to Appendix (2.C).

At the origin level, the model is approximately invariant with respect to the origin aggregation level, because the basic discrete choice problem is individual specific. At the destination level, the aggregation level affects the interpretation of wages \( W_j \) in a straightforward way. Assume that location \( j \) is in fact composed of several sublocations \( \{k_1, k_2, ..., k_{N_j}\} \), and we estimate the model at the higher level \( (j) \) and ignore the sub-locations. The wage we obtain, \( W_j = \left( \sum_{\ell=1}^{N_j} W_{k_{ij}} \right)^{1/\epsilon} \), represents a C.E.S. aggregate with elasticity \( \epsilon \) of the true underlying wages at all sub-locations within \( j \).
In particular, this implies a simple adjustment for the destination fixed effect \( \psi_j = \epsilon w_j \) estimated using the gravity model. Assume that the "real" underlying wage is constant and denoted by \( W_j^R \) within each location \( j \), then the C.E.S. relationship becomes \( W_j = N_j^{1/t} W_j^R \), or in logs the underlying wage is given by \( w_j^R = w_j - \frac{1}{t} \log (N_j) \). In terms of estimated quantities, this becomes \( \hat{\psi}_j^R = \hat{\psi}_j - \log (N_j) \). The underlying destination fixed effect \( \hat{\psi}_j^R \) is obtained from the fixed effect \( \hat{\psi}_j \), estimated ignoring sub-locations, minus an adjustment factor equal to the log of the number of true underlying locations where shocks are realized, \( N_j \).

For most of the analysis we map locations \( i \) and \( j \) from the model to cell phone towers in the data. (When comparing with survey data, we aggregate up to survey commuting zones.) We also assume that shocks \( Z_{ijw} \) are drawn for each area in Euclidian geometry, and the true wage \( W_j^R \) is approximately constant within cell phone towers. Following the argument above, we adjust each estimated destination fixed effect downward by \( \log (K_j) \) where \( K_j \) is the area of cell tower \( j \).

### 2.4 Results

We now turn to estimating the gravity equation with commuting data from Colombo and Dhaka. We then validate the model's ability to predict income by comparing model-predicted income measures with self-reported income data collected in the DHUTS transportation survey in Dhaka.

#### 2.4.1 Gravity Estimation to Recover Destination Wages

In this section, we estimate equation (2.4) using the cell phone commuting flows and the Google Maps travel time distance measure. Our goal is to recover the destination fixed effects, which are a measure of workplace productivity (or, equivalently, wages). We estimate the gravity equation using OLS and a procedure to account for two-way fixed effects at the level of origin and destination. In our analysis sample, we exclude tower pairs further than 50 kilometers away, and those with travel time less than 180 seconds (which account for 0.1% of the tower pairs within 50 km).
Table 2.B1 reports the results. As expected from Figure 2.A1, the travel time between the origin and destination is strongly and negatively associated with the commuting flow. The specification implied by the model is shown in columns (1) and (3) for Dhaka and Colombo, respectively. The coefficients for the two cities are very similar, at $-1.64$ and $-1.75$. This is surprising, as these cities vary considerably in terms of their level of development and population. In columns (2) and (4) we omit the destination fixed effects (which in the model capture variation in productivity levels). The coefficient drops significantly in Dhaka, but less so in Colombo. This suggests that wages are distributed spatially differently in the two cities, a theme that we explore in more details in our first application in section 2.5. In short, these results are consistent with a “flatter” distribution of productivity in Dhaka.

2.4.2 Validating Model-Predicted Income using Survey Income Data

We now move on to the main validation exercise of our approach. Figure 2.A3 shows the simple correlation between our preferred model-predicted income measure and earnings at the workplace or destination level (panel A) and at the residential or origin level (panel B). The size of each scatter point indicates the relative number of commuters at that location. In both cases, we observe a strong correlation between the survey data set and model predictions.

Table 2.B2 compares survey income to four model-predicted income measures, and selects the best fit measure, implementing the procedure described in Section 2.3.3. Columns (1) through (4) regress individual-level income from the DHUTS survey on the four model-predicted measures, separately. Measures where distance is productive ($y_{ijw} (0, 1)$ and $y_{ijw} (1, 1)$) are either not correlated with income, or if anything negatively correlated. However, the two measures where distance is a pure utility shock are robustly correlated with income.

In column (5) we estimate equation (2.8) in order to obtain the estimates for the productivity levels for distance and shocks ($\alpha_d$ and $\alpha_z$) and the Frechet distribution
shape parameter $\varepsilon$. The transformed results are shown in Panel B, with standard errors computed using the Delta method. Columns (1) shows the unconstrained regression, where we obtain a negative point estimate for the productivity of distance. The result is not significantly different from zero, so in column (2) we run the same regression constraining $\alpha_d = 0$. We obtain an intermediate value for the productivity of shocks $\alpha_s = 0.56$ and a value of the shape parameter $\varepsilon = 6.4$ that is in line with estimates in other urban contexts (Ahlfeldt et al., 2015).

2.5 Urban Economic Structure in Colombo and Dhaka

In this section we use the commuting data and model-predicted income data to explore the spatial distribution of residential and work population, as well as average incomes at these locations. A canonical way to think about cities is given by the monocentric model (Mills, 1967), where commuters from the entire city commute to the Central Business District (CBD) in the center on the city. While in the model all business activity happens exactly at the city center, the model is used as a general way to think about business activity that is much more concentrated than residential population. In reality, cities depart from this structure to various extents, and in some cases have a polycentric structure, with multiple regional centers attracting commuters.

Figure 2.A4 analyzes these patterns for Colombo and Dhaka. In Panel A, we look at the cumulative population as we go away from the CBD. We compute this separately for tower residential population (the number of commutes originating at that tower) and employed population (the number of commutes ending at that tower location). In general, the two measures track each other well, suggesting that business activity is also very highly distributed across the city, consistent with evidence from the U.S. (Wheaton, 2004). In Dhaka, the employed population is only slightly more concentrated than residents (the line for employed is slightly to the left of that for residents), while in Colombo this effect is much more pronounced. This indicates that employment population is more concentrated in Colombo.

In Panel B, we plot the average model-predicted income as we move away from the
CBD, calculated separately for residents and for employees. Surprisingly, we find that residents in Colombo and Dhaka have a very similar spatial distribution of income as a function of distance to the city center. Moreover, in Dhaka there is virtually no difference between residential and work locations, again emphasizing the polycentric, mixed nature of this metropolis. In Colombo, we observe a starkly more concentrated income at the destination level, consistent with a monocentric structure for Colombo.

2.6 The economic costs of Hartal days

We now use the data in Bangladesh to estimate the economic costs of hartals. Hartals are a form of political strike action that involve a partial shutdown of urban transportation and businesses. They are common in South Asia, and especially in Dhaka (UNDP 2005). On hartal days, which are typically announced several days in advance, groups of people in the streets enforce the transportation shutdown, especially on major roads and in certain locations of the city.

Hartals have the potential to seriously disrupt economic activity – indeed, this is their declared goal. At the same time, they may be concentrated only in certain areas of the city, and may or may not affect high economic activity commuting routes. In addition, commuters may adapt to hartals, for example by substituting their usual trips with shorter but at least somewhat productive trips. The economic cost of hartal is thus an empirical question; our data and model are uniquely suited to analyze this question.

We find that people travel less and shorter distances on hartal days, and trips with high predicted income are disproportionately affected. We conclude with an estimation that income is 5% lower on hartal days compared to working days. To benchmark this effect, we show that relative to working days, income is 11% lower on Fridays, which are the free day in Bangladesh.

Table 2.B3 shows reduced form results comparing extensive and intensive margin behavior on hartal and non-hartal days. The sample is a 5% random sample of all commuters in our data that have at least one hartal and one non-hartal day. Panels
A, B and C report the results for the extensive margin of making any trip, total trip duration and total trip distance. The first three columns use different empirical specifications, including commuter fixed effects in column (1), plain OLS in column (2), and a two-step Heckman selection procedure in column (3). In the last column, in panels B and C, we condition on making a trip, that is we drop zeros. For comparison, we also control for travel behavior on Fridays (the free day of the week in Bangladesh), Saturdays and other holidays.

We find a large, robust negative effect on travel on hartal days. Hartal days have 2 percentage points lower travel probability compared to regular weekdays. This effect is roughly a third of the effect of a typical Friday (5.4 percentage points). However, the results in Panels B and C suggest that different types of trips are affected on hartal days. Indeed, commuting trips on hartal days have roughly 10% smaller trip duration and distance, an effect similar or larger than for Fridays.

Figure 2.A5 shows the change in travel behavior on hartal versus working days, as a function of trip predicted income. For each pair of origin and destination cell phone tower locations, we compute the predicted income (preferred specification) based on the gravity equation estimated on non-hartal days. To construct this graph, for each bin in the trip predicted income distribution, we run a regression of making a trip that falls in that bin on a given day, on commuter fixed effects, and a hartal dummy. The top panel plots the coefficients on "hartal" as well as point-wise 95% confidence intervals clustered at the trip origin level. The bottom panel shows the mean values on control days and hartal days.

The results show that there is a statistically significant decrease in trips at the high end of the income distribution, and the size of the effect is moderate in size (around 10% at its highest). Interestingly, there is also a slightly but statistically significant increase in trips with low predicted income. This suggests that commuters avoid certain high-income destinations during hartals, and re-route to other (likely closer) destinations.

Our final point is an accounting exercise, where we integrate the impact on travel over the distribution log income in Figure 2.A5 to obtain a total fraction of income
lost on hartal days due to commuters not traveling to their usual destinations. This exercise makes several assumptions, notably that the income is additive across days, and that commuters who change their destination on hartal days nevertheless gain the income of the chosen destination. We find that income is 4.6% lower on hartal days, with a 95% confidence interval of 0.9 to 8.6%. To benchmark this effect, the impact on Fridays is approximately 11%. In other words, hartals have a statistically and economically significant impact on economic activity.

2.7 Conclusion

In this paper we used commuting flows derived from cell phone transaction data to infer productivity variations at a fine geographic scale within cities. We use a urban economic model of workplace choice to derive a gravity equation that allows us to “back out” productivity levels that rationalize observed commuting flows. We show empirically that cell phone commuting flows pick up fine variations in commuting controlling for distance, origin and destination, while offering much finer and frequent coverage. We validate the model-predicted income using self-reported survey income, and allow the data to inform the productivity effects of distance and idiosyncratic shocks.

The method we introduce can be used to study the impact on commuting and economic activity of various natural shocks and policies. Among other applications, we believe this method can also be extended to study the relationship between commuting and economic activity, disaggregated by skill level and industry sector (at the origin and destination levels, respectively). Given high levels of inequality and occupational heterogeneity in large cities in developing countries, this seems a particularly important extension. Similarly, this framework can be used to study commuting behavior at different wealth levels, using individual-level wealth predictions from the cell phone data itself (Blumenstock et al., 2015). The data and framework are uniquely suited to study the extent and causes of “wasteful” or “excess” commuting (Hamilton and Röell, 1982; White, 1988). This method can also be used by or in collabora-
tion with policy-makers to study discrepancies between official accounts of economic activity, such as tax records, and our measure, which also includes informal activity.
Appendices

2.A Figures

Figure 2.A1: Comparison of Commuting Flows from Survey Data and Cell Phone Data

Notes. This figure shows the relationship between commuting flows and log commuting travel time (in seconds) at the route level in Dhaka, using two different data sets: the DHUTS transportation survey (red, solid line) and commuting flows constructed using cell phone data (blue, dashed line). Commuting flows from the cell phone data were aggregated at the larger commuting zone level defined in the DHUTS survey. The sample consists of 7,676 pairs of distinct commuting zones, with a total of 7,903 commuters in the DHUTS survey and $\sim 6 \cdot 10^6$ commuters in the cell phone data. (Commuting zone pairs below the 1st and over the 99th percentiles of the log distance distribution are not included.) To adjust for pairs with zero flow (where log is not defined), for each of 100 log travel time bins, we first take the average commuting flow, and then take logs. (Figure 2.E1 shows results using average log flow.) Pointwise bootstrapped 95% confidence intervals clustered at the origin commuting zone level are shown in gray.
Figure 2.A2: The Distribution of Commuting Arrivals in Colombo and Dhaka

(A) Colombo, Sri Lanka

(B) Dhaka, Bangladesh

Notes. These figures show the distribution of inflows (total commuting arrivals) by cell phone tower Voronoi cell in Colombo, Sri Lanka, and Dhaka, Bangladesh.

Figure 2.A3: Correlation Between Self-reported Survey Income and Model-predicted Income using Cell Phone Data

(A) Workplace level

(B) Residential level

Notes. These figures show the correlation between income reported in the DHUTS survey on the Y axis, and optimal model-predicted income on the X axis. Panel A shows results aggregated at the (DHUTS commuting zone) destination (workplace) level, and Panel B at the origin (residential) level. The size of the scatter point indicates the relative number of commuters at that location (total in-flows in Panel A and total out-flows in panel B).
Figure 2.A4: Application (1) The Urban Structures of Colombo and Dhaka

(A) Cumulative Employment and Number of Residents, by Distance from CBD

(B) Model Predicted Income, for Residents and Employees, by Distance from CBD

Notes. These figures plot the distribution of population and mean income at the residential and employment location in Colombo, Sri Lanka and Dhaka, Bangladesh. To construct it, towers are first ordered by distance to the CBD, which is Colombo Fort in Colombo (red, thick lines) and Motijheel in Dhaka (blue, thin lines). Panel A plots the cumulative population with respect to distance to the CBD. Dashed lines indicate residential population (at the origin level) and solid lines indicate employment population (at the destination level). In panel B, origin-destination link specific model-predicted log income (using the optimal weights, see Section 2.3.3 for details) is averaged at the origin level (dashed lines) and at the destination level (solid lines), in each case using commuting flows as weights. The figure plots a local linear regression as a function of distance to the CBD. Gray lines around the destination-level plots indicate pointwise 95% confidence intervals.
Figure 2.A5: Application (2) Impact of Hartal on Travel by Trip Predicted Income

Notes. This figure shows the change in travel behavior on hartal versus working days, as a function of trip predicted income. For each pair of origin and destination cell phone tower locations, we compute the predicted income (preferred specification) based on the gravity equation estimated on non-hartal days. To construct this graph, for each bin in the trip predicted income distribution, we run a regression of making a trip that falls in that bin on a given day, on commuter fixed effects, and a hartal dummy. The top panel plots the coefficients on "hartal" as well as point-wise 95% confidence intervals clustered at the trip origin level. The bottom panel shows the mean values on control days and hartal days.
## Tables

Table 2.B1: Commuting Flows and Travel Time (Gravity Equation)

<table>
<thead>
<tr>
<th>City</th>
<th>Dhaka</th>
<th>Dhaka</th>
<th>Colombo</th>
<th>Colombo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Origin FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Destination FE</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1,541,912</td>
<td>1,541,912</td>
<td>1,169,267</td>
<td>1,169,267</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td>0.618</td>
<td>0.395</td>
<td>0.754</td>
<td>0.474</td>
</tr>
</tbody>
</table>

### Dependent variable:

<table>
<thead>
<tr>
<th>log Commuting Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>(2)</td>
</tr>
<tr>
<td>(3)</td>
</tr>
<tr>
<td>(4)</td>
</tr>
<tr>
<td><strong>log Travel Time</strong></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

Notes. This table reports estimates of the gravity equation (2.4). The outcome data is commuting flows from cell phone data between pairs of cell phone tower locations, aggregated over all weekdays in the data. For each individual in the data, the origin of their commuting trip on a given day is defined as the first location (tower) between 5 am and 10 am, and the destination as the last location between 10 am and 3 pm (see Section 2.2 for more details). Tower pairs less than 180 seconds away (including same tower pairs) and pairs at more than 50 km are dropped. Travel time between each pair of towers is measured from Google Maps. Each column reports the coefficient on travel time from an OLS regression that accounts for two-way (origin and destination) fixed effects, except in columns (2) and (4) where only origin fixed effects are included. *p<0.1; **p<0.05; ***p<0.01.
Table 2.B2: Validation of Income Measure

Panel A Correlation of Reported Income and Predicted Income

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pred log Income (( \alpha_z = 1, \alpha_d = 1 ))</td>
<td>0.056</td>
<td>(0.097)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pred log Income (( \alpha_z = 0, \alpha_d = 1 ))</td>
<td></td>
<td></td>
<td>-0.045**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pred log Income (( \alpha_z = 1, \alpha_d = 0 ))</td>
<td></td>
<td></td>
<td>0.101***</td>
<td>0.092</td>
<td>(0.023)</td>
<td>(0.118)</td>
</tr>
<tr>
<td>Pred log Income (( \alpha_z = 0, \alpha_d = 0 ))</td>
<td></td>
<td></td>
<td>0.128***</td>
<td>0.077***</td>
<td>(0.025)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>log Travel Time</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.016</td>
<td>(0.169)</td>
</tr>
<tr>
<td>Pred log Income with Estimated ( \alpha_z, \alpha_d, \epsilon )</td>
<td></td>
<td></td>
<td></td>
<td>1.000***</td>
<td></td>
<td>(0.171)</td>
</tr>
</tbody>
</table>

| Observations     | 11,785    | 11,785    | 11,785    | 11,785    | 11,785    | 11,785    |
| Adjusted R^2     | 0.0002    | 0.006     | 0.029     | 0.019     | 0.035     | 0.035     |

Panel B Estimated Structural Parameters

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Parameter ( \epsilon )</td>
<td>6.95</td>
<td>6.41***</td>
</tr>
<tr>
<td></td>
<td>(5.30)</td>
<td>(0.96)</td>
</tr>
<tr>
<td>Shock Productivity ( \alpha_z )</td>
<td>0.51***</td>
<td>0.56***</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>Distance Productivity ( \alpha_d )</td>
<td>-0.09</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>(4.64)</td>
<td></td>
</tr>
<tr>
<td>Constraint ( \alpha_d = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>11,785</td>
<td>11,785</td>
</tr>
</tbody>
</table>

Notes. This table reports results regression results from estimating equation 2.8. In Panel A, the dependent variable is income as reported in the DHUTS survey, at the individual level. The first four columns correlate survey income with the four model-predicted measures of income, based on the extreme assumptions on how shocks and distance affect productivity (\( \alpha_z \) and \( \alpha_d \)) respectively. Column (5) implements equation 2.8. Column (6) uses the estimated parameters from Column (5) to construct our preferred measure of income. Panel B inverts the coefficients in Column (4) to recover \( \alpha_z, \alpha_d \) and \( \epsilon \).
Table 2.B3: Impact on Hartal on Probability of Travel, Duration and Distance

<table>
<thead>
<tr>
<th>Specification</th>
<th>(1) Commuter FE</th>
<th>(2) OLS</th>
<th>(3) Heckit</th>
<th>(4) Commuter FE</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Makes trip</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hartal day</td>
<td>-0.021*** (0.0045)</td>
<td>-0.027*** (0.0050)</td>
<td>-0.027*** (0.0050)</td>
<td></td>
</tr>
<tr>
<td>Friday</td>
<td>-0.054*** (0.0057)</td>
<td>-0.069*** (0.0083)</td>
<td>-0.069*** (0.0083)</td>
<td></td>
</tr>
<tr>
<td>Saturday, Holiday FE</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Control Mean</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>$5.1 \cdot 10^6$</td>
<td>$5.1 \cdot 10^6$</td>
<td>$24.8 \cdot 10^6$</td>
<td></td>
</tr>
</tbody>
</table>

| **Panel B. Trip duration (minutes)** | | | | |
| Hartal day | -0.89*** (0.19) | -1.05*** (0.21) | -1.28*** (0.019) | -0.86*** (0.19) |
| Friday | -0.50* (0.23) | -0.98*** (0.27) | -1.01*** (0.022) | 1.10*** (0.22) |
| Sample: only trips | X | | | |
| Saturday, Holiday FE | X | X | X | X |
| Control Mean | 9.64 | 9.64 | 9.64 | 17.3 |
| Observations | $5.1 \cdot 10^6$ | $5.1 \cdot 10^6$ | $24.8 \cdot 10^6$ | $2.7 \cdot 10^6$ |

| **Panel C. Trip distance (kilometers)** | | | | |
| Hartal day | -0.31*** (0.068) | -0.35*** (0.074) | -0.42*** (0.0071) | -0.32*** (0.072) |
| Friday | -0.073 (0.080) | -0.22* (0.095) | -0.23*** (0.0081) | 0.50*** (0.085) |
| Sample: only trips | X | | | |
| Saturday, Holiday FE | X | X | X | X |
| Control Mean | 3.04 | 3.04 | 3.04 | 5.45 |
| Observations | $5.1 \cdot 10^6$ | $5.1 \cdot 10^6$ | $24.8 \cdot 10^6$ | $2.7 \cdot 10^6$ |

Notes: This table reports regression results in Dhaka of the impact of hartal days on the extensive margin of travel (panel A), trip duration (panel B) and trip distance (panel C). The sample is a 5% random sample of all users with at least one hartal day with commuting data and at least one non-hartal day with commuting data. Column (1) includes commuter fixed effects, columns (2) is plain OLS, and columns (3) implements two-step Heckit on the full rectangular sample (all days and user combinations) where the selection variable is whether we observe commuting for a given user on a given day. In panels B and C, column (4) restricts the sample to days with trips; in other words it provides results on the intensive margin. Standard errors are clustered at the date level in columns (1), (2) and (4).
2.C Additional Model Derivations

Properties of the Fret Distribution. We review some basic properties of the Fret distribution.

The cumulative distribution function of a Fret random variable with scale parameter \( T \) and shape parameter \( \epsilon \) is \( F(z) = \exp(-Tz^{-\epsilon}) \).

Consider a sequence of independent Fret random variables \( z_k \) with scale \( T_k \) and the same shape \( \epsilon \), for \( k = \{1, \ldots, K\} \). The probability that the maximum is achieved by the \( j \)th variable, with \( j \in \{1, \ldots, K\} \), is given by \( \Pr(j \in \arg\max_k z_k) = \exp(T_j) / (\sum_k \exp(T_k)) \).

The class of Fret random variables is closed with respect to the max operator. The random variable \( \max_k z_k \) is Fret distributed with scale \( T = \sum_k T_k \) and shape \( \epsilon \). Moreover, the conditional maxima, namely \( z_j | j \in \arg\max_k z_k \), have exactly the same distribution as the unconditional maximum.

The mean of a Fret distributed variable is \( \langle z \rangle = T^{1/\epsilon} \Gamma \left( 1 - \frac{1}{\epsilon} \right) \) where \( \Gamma(\cdot) \) is the gamma function. It follows that \( \ln \langle z \rangle = \frac{\ln(T)}{\epsilon} + \ln \left( 1 - \frac{1}{\epsilon} \right) \). Also, for some absolute constant \( K \) we have \( \ln \langle z \rangle = \frac{\ln(T)}{\epsilon} - \frac{K}{\epsilon} \).

Derivation of Income Measures.

Standard errors for \( \hat{\alpha}_z \), \( \hat{\alpha}_d \) and \( \hat{\epsilon} \) are derived using the Delta method. Differentiating the equations in (2.9) with respect to \( \rho \)'s yields:

\[
\nabla \begin{pmatrix}
\epsilon \\
\alpha_z \\
\alpha_d \\
\end{pmatrix} = \begin{pmatrix}
-\frac{1}{(\rho_1 + \rho_2)^2} & -\frac{1}{(\rho_1 + \rho_2)^2} & 0 \\
\frac{\rho_2}{(\rho_1 + \rho_2)^2} & -\frac{-\rho_1}{(\rho_1 + \rho_2)^2} & 0 \\
\frac{\rho_2}{(\rho_1 + \rho_2)^2} & \frac{\rho_3}{(\rho_1 + \rho_2)^2} & -\frac{-1}{(\rho_1 + \rho_3)^2} \\
\end{pmatrix}
\]

and thus

\[
\text{Var} \begin{pmatrix}
\hat{\epsilon} \\
\hat{\alpha}_z \\
\hat{\alpha}_d \\
\end{pmatrix} = \nabla^T \Sigma \nabla
\]

where \( \Sigma \) is the variance covariance matrix of the estimator \((\rho_1, \rho_2, \rho_3)\).
2.D Additional Data Details

Geographic Information and Census Population. We use population counts from Sri Lanka’s 2011 census, at the Grama Niladhari (GN) level, the lowest administrative level in Sri Lanka. There are 14,021 GN’s in Sri Lanka in our data. We obtained geographic administrative GN boundaries from the Survey Department of Sri Lanka, which we combine with spatial data on cell towers. For each cell tower, we interpolate the population based on the information from the census. Specifically, we assume that population is uniformly distributed within each GN. We partition every cell tower’s Voronoi cell into subareas corresponding to different GNs, and calculate the population of each subarea, based on its land surface relative to the entire GN it belongs to. Our estimate of the cell tower’s population is obtained by summing over all subareas.
2.E Additional Figures and Tables

Figure 2.E1: Comparison of Commuting Flows from Survey Data and Cell Phone Data

Notes. This figure replicates Figure 2.A1 including plots that do not adjust for origin-destination pairs with zero flows. The short dashed lines plot the local linear regression of log flow on log travel time for survey data (red, short dash dot line) and for cell phone data (blue, short dash line).
Table 2.E1: Cell Phone Data Coverage at the User-Day Level

<table>
<thead>
<tr>
<th></th>
<th>Dhaka, Bangladesh</th>
<th>Colombo, Sri Lanka</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Users in sample</td>
<td>$5.3 \cdot 10^6$</td>
<td>$3.0 \cdot 10^6$</td>
</tr>
<tr>
<td>(2) Days in sample</td>
<td>122</td>
<td>395</td>
</tr>
<tr>
<td>(3) All user-days possible = (1) x (2)</td>
<td>$6.5 \cdot 10^8$</td>
<td>$1.2 \cdot 10^9$</td>
</tr>
<tr>
<td>(4) User-days with data</td>
<td>$2.9 \cdot 10^8$</td>
<td></td>
</tr>
<tr>
<td>(5) User-days with data (5-10am)</td>
<td>$1.5 \cdot 10^8$</td>
<td></td>
</tr>
<tr>
<td>(6) User-days with data (10am-3pm)</td>
<td>$2.4 \cdot 10^8$</td>
<td></td>
</tr>
<tr>
<td>(7) User-days with data (5-10am and 10am-3pm)</td>
<td>$1.0 \cdot 10^8$</td>
<td>$3.4 \cdot 10^8$</td>
</tr>
<tr>
<td>(8) Coverage rate = (7)/(3)</td>
<td>16.1%</td>
<td>28.8%</td>
</tr>
</tbody>
</table>

Notes: This table shows descriptive statistics on data coverage in the two data sets. The first row indicates the number of unique users (who appear at least once in the data set). The second row shows the total number of calendar dates with data. The third row is the product of the previous two, which is the theoretical upper bound of user-day combinations that could appear in the data. (Note that in practice some users only start using a cell phone partway through the period, so this is an overestimate.) Rows 4-6 describe the actual number of user-days in the Bangladesh data under different restrictions. The seventh row shows the number of user-days for which we have at least one location between 5 am and 10 am, and at least one location between 10 am and 3 pm – this corresponds to the data necessary to define commuting behavior for that user and that day.
Table 2.E2: Comparison of Commuting Flows from Survey Data and Cell Phone Data

<table>
<thead>
<tr>
<th></th>
<th>Log flow survey data (DHUTS)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Log flow cell phone data</td>
<td>0.29***</td>
<td>0.50***</td>
<td>0.18***</td>
<td>0.58***</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>(0.018)</td>
<td>(0.038)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>Log travel time</td>
<td>-0.48***</td>
<td>0.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.097)</td>
<td>(0.13)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Origin and destination fixed effects</th>
<th>Yes</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>1859</td>
<td>1857</td>
</tr>
<tr>
<td>R2</td>
<td>0.24</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Notes: This table shows the relationship between commuting flows from two different data sets: the DHUTS transportation survey (red, solid line) and commuting flows constructed using cell phone data (blue, dashed line), in Dhaka.
Chapter 3

Structural Estimation of Network Formation under Nonnegative Externality

3.1 Introduction

Since the seminal paper of Jackson and Wolinsky (1996), the theory of the economics of networks has been extensively studied and applied to numerous fields.\textsuperscript{1} Many theories and applications that consider network formation, including Jackson and Wolinsky (1996), use the equilibrium concept known as "pairwise stability." This equilibrium concept basically requires that no pairs have an incentive to deviate from the current network structure. The concept is appealing because of its simplicity and intuition and has been applied to many fields, such as collaborative research and development networks (Goyal and Joshi, 2003), free-trade networks (Goyal, Joshi, and Trade, 2006), and risk-sharing networks (Kranton and Bramoullé, 2007).

However, few papers have conducted the structural estimation of network formation models using this equilibrium concept. One of the reasons for this is a lack of characterization. Neither the existence nor the uniqueness of the equilibrium is en-

\textsuperscript{1}See Jackson (2008) for a survey.
sured in general. Although the issue of existence in a generic case is examined by Jackson and Watts (2002) where they characterize sufficient conditions for the existence of the equilibrium based on the concept of "improving paths," their conditions are difficult to check, so their approach is not directly applicable upon estimation. The lack of an econometric framework for pairwise stable network formation has been an obstacle to the development of the field.

This is not merely a theoretical concern. For example, in the context of friendship network formation, economists and sociologists have extensively studied the degree of "racial homophily" in a friendship network, i.e. the degree to which people of the same race are likely to be friends with each other (Lazarsfeld and Merton, 1954). This parameter is of particular importance when we consider racial integration policies. For example, Moody (2001) conducts a regression of whether a pair of people are friends with each other on a dummy variable that indicates whether the pair are of the same race. He concludes that even racially integrated schools may still exhibit racial segregation in friendships if the degree of racial homophily is high. However, it is also pointed out in the literature that there exists a tendency that a pair with a mutual friend are more likely to be friends, i.e. clustering (Simmel, 1908). Then, omitting the term of existence of mutual friend in the regression as in Moody (2001) is likely to overestimate the coefficient on the same-race dummy.2

To deal with the stated difficulty associated with the lack of existence and characterization of equilibria, we take an approach that restricts the form of utility functions. By assuming that there is no negative externality, i.e. the incentive to form a link weakly increases as the number of links increases, we show that the set of equilibria is a nonempty complete lattice. The concept of nonnegative externalities is not new; it is the same as "strategic complementarity and convexity in own links" studied in Hellmann (2013). Hellmann (2013) showed the existence of an equilibrium using the approach of Jackson and Watts (2002). However, we adopt a different approach for the proof. We characterize the equilibrium as a fixed point of a certain mapping and apply Tarski’s fixed-point theorem (Tarski, 1955). The value added of this approach

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2The intuition behind this bias is stated in more detail in Section 3.4.4.
is that we can show that the set of equilibria is a complete lattice, in addition to existence. This implies that there is a unique maximum equilibrium and minimum equilibrium under a suitably defined partial order, and all equilibria lie in between them. This characterization is essentially an application of Topkis (1979), Milgrom and Roberts (1990) and Vives (1990), although in the context of network formation, no paper thus far has proposed a characterization of the equilibrium with this approach.

Using the equilibrium characterization above, we provide a framework for structural estimation. The basic approach is to use moments that maintain the partial ordering. Then, the moments of the observed network exist in between that of the minimum and the maximum equilibria. In this way, we can construct a moment-inequality model without assuming any equilibrium selection rule other than that of pairwise stability. This approach of utilizing Tarski's fixed-point theorem for structural estimation is conducted in Jia (2008), Nishida (2015), Uetake and Watanabe (2013b) and Uetake and Watanabe (2013a) in different contexts. We review the relationship between these papers in Section 3.2.

We then apply our framework to friendship networks from the National Longitudinal Study of Adolescent Health (Add Health). The dataset contains data on U.S. students in grades 7-12 from a nationally representative sample of roughly 130 private and public schools during the years 1994-1995.

Our empirical objective is to obtain reliable estimates of the preference toward racial homophily after controlling for the preference toward clustering. We find that omitting the variable of clustering overestimates the degree of racial homophily (in a sense that larger values of the parameters are more likely to be rejected and smaller values of the parameters are more likely to be accepted). This finding has an important implication for racial integration policy, such as class assignment in schools.

The rest of the paper is organized as follows. Section 3.2 summarizes the related literature. Section 3.3 describes the model and the theoretical characterization of the equilibria. Section 3.4 introduces an econometric model and explains how we construct moment inequalities. Section 3.5 provides the results of Monte-Carlo sim-
ulation. Section 3.6 explains the application of our method to the student friendship networks. Section 3.7 presents our conclusion.

3.2 Related Literature

3.2.1 Structural Estimation of Network Formation Models

There are several recent papers that attempt to structurally estimate network formation models with different approaches. One strand of the literature takes the approach of exploring an entirely different equilibrium concept than that here. Christakis, Fowler, Imbens, and Kalyanaraman (2010) models network formation as a sequential process where in each period a single, randomly-selected pair of agents has the opportunity to form a link, and Mele (2016) considers a stationary distribution of such a process. Chandrasekhar and Jackson (2014) investigates the approach based on random graph models, which have been developed outside economics (i.e. Kolaczyk (2009)).

There are also several recent papers that employ the same equilibrium concept of pairwise stability as this paper, such as Sheng (2014), de Paula, Richards-Shubik, and Tamer (2015), and Leung (2016). Sheng (2014) conducts moment inequality inference based on the concept of “subnetworks.” de Paula et al. (2015) consider a new model set-up where similarly defined “subnetworks” carry over enough information to characterize the identified set of the parameters, and provide a method to compute the identified set. Leung (2016) takes a completely different approach and develops a weak law of large numbers for sparse networks. In this paper, we impose different assumptions and take an entirely different approach for inference.

3.2.2 Structural Estimation of Supermodular Games

The method developed in this paper is one application of the growing literature on the structural estimation of supermodular games. On the theoretical side, the characterization of the equilibrium in this paper is essentially an application of Topkis (1979),
Milgrom and Roberts (1990) and Vives (1990), although in the context of network formation, no paper we know of has proposed a characterization of the equilibrium with an approach similar to ours.

Empirically, many papers have utilized the characterization of equilibria of supermodular games for structural estimation. Most notably, Jia (2008) considers the entry game between Walmart and Kmart. Nishida (2015) extends Jia (2008)'s result to the situation where each brand can choose multiple branches in the Japanese convenience-store industry. Both of these papers assume extremal equilibria of the lattice as an equilibrium selection rule, and they obtain two different estimates corresponding to each equilibrium. On the other hand, Uetake and Watanabe (2013b) illustrates how the lattice equilibrium structure of a noncooperative supermodular game can be translated to moment inequalities.

The methodology in this paper of utilizing Tarski's fixed-point theorem for structural estimation should be viewed as an extension of Jia (2008), Nishida (2015) and Uetake and Watanabe (2013b). How we translate lattice equilibrium structure to moment inequalities closely follows Uetake and Watanabe (2013b), though we consider a more general construction of moment inequalities.\(^3\)

### 3.2.3 Structural Estimation of Two-sided Matching Models

There have been substantial developments in the literature of structural estimation of two-sided matching. Unlike network formation models where any pairs can potentially form a link, they consider a situation where markets are divided into two sides and matches are only formed in between them. In this subsection, we find it useful to mention some of the similarities and distinctions that exist in the literature of estimation of matching and networks.

First of all, there are some similarities in the equilibrium concepts. In two-sided matching models with non-transferable utility, a standard equilibrium concept is stable matching, where no pairs of individuals have an incentive to deviate from the current match to form another match, and no individuals have an incentive to unilat-
eraly sever the match. On the other hand, in the pairwise stable network formation considered in this paper, the only requirement is that no pairs of individuals have an incentive to change the linking status taking all other links unchanged. Therefore, the equilibrium conditions are generally stronger in stable matching. Furthermore, in many cases, matching models impose stronger assumptions on preferences. In particular, they often assume away direct externality, i.e. incentive of an agent to form a match does not depend on other pairs’ decision. This implies that the moment inequality method based on necessary conditions of pairwise stable network developed in this paper can be directly applied to many-to-many two-sided matching models if the externality is nonnegative.4

It has been shown that stable matching in a two-sided matching model can be characterized by a complete lattice under suitable conditions (Adachi (2000), Hatfield and Milgrom (2005)). However, the space where the mapping is applied is different; it is usually constructed on the space of “opportunity sets,” i.e. the hypothetically offered set of contracts. In our case, the mapping takes place in the space of “realized contracts” (or realized links). In general, extending the characterization of stable matching to network formation models is not straightforward, mainly due to direct payoff externalities which are often not allowed in the two-sided matching models.

In terms of the structural estimation of two-sided matching with nontransferable utility,5 Agarwal and Diamond (2016) points out the difficulty in point identification.6 Hence, some papers utilize a partial-identification approach. For example, Uetake and Watanabe (2012) and Uetake and Watanabe (2013a) extend the lattice structure of stable matching to moment inequalities for structural estimation along the lines of

4The argument also clarifies that the approach is not directly applicable to the case of one-to-one or many-to-one matching models, as they implicitly impose negative externality through capacity constraint, i.e. a match may make the involved agents unable to form any other matches.

5For the case of transferable utility, see, for example, Fox (2016), Fox (2010) and Graham (2011). We do not discuss it here in detail because the case of nontransferable utility is more relevant to our paper.

6This difficulty in identification does not come solely from the multiplicity of equilibria, but also from the nature of simultaneous equations. In particular, in one-to-one or many-to-one matching, a decision of a match may make the involved agents unable to form any other matches, creating the highly simultaneous nature of the equations estimated. The problem of simultaneity also arises in the context of network formation models through more general forms of externality.
Section 3.2.2.

3.3 Theory

In this section, we explore our theoretical characterization of equilibrium. We first describe our model and define an equilibrium concept, which can be characterized as a fixed point of a certain mapping. Then we show that the set of equilibria is a nonempty complete lattice under the condition of nonnegative externality (Proposition 5). This result is essentially an application of Topkis (1979), Milgrom and Roberts (1990) and Vives (1990). Since this characterization is new in the context of network formation but extensively investigated in other contexts, we also discuss the relationship to other contexts.

3.3.1 The Model

We introduce a generic model of network formation. The set of agents is denoted by \( N = \{1, \ldots, n\} \). The network is defined as follows. A complete network is defined by \( g^N = \{(i, j) : i, j \in N\} \). Thus, \( g^N \) is a set of all possible pairs within \( N \). The set of possible networks is defined by \( \mathcal{G} \equiv \{g : g \subseteq g^N\} \). If there is a link between \( i, j \in N \), \( (i, j) \in g \). We also define \( y_{ij} = 1 \) if \( (i, j) \in g \) and \( y_{ij} = 0 \) if \( (i, j) \notin g \). We also denote \( y = [y_{1,2}, y_{1,3}, \ldots, y_{n-1,n}]' \in \mathbb{I}^{n(n-1)/2} \), where \( \mathbb{I} = \{0, 1\} \). Thus, \( y \) is a \( n(n-1)/2 \) binary vector that specifies whether there exists a link for all possible pairs.

Next, we define a utility function. Agent \( i \)'s utility is defined by \( U_i(g) \). At this point, agents’ utility freely depends on the structure of the network. In our empirical application, we impose a particular parametric assumption.

Next, we define marginal utility. The marginal utility of a link with \( j \) for \( i \) is defined as follows:

\[
MU_{i,j}(g) = \begin{cases} 
U_i(g) - U_i(g \setminus (i,j)) & \text{if } (i,j) \in g, \\
U_i(g \cup (i,j)) - U_i(g) & \text{if } (i,j) \notin g.
\end{cases}
\] (3.1)
Thus, $MU_{i,j}(g)$ is the marginal utility of $i$ from a link between $i$ and $j$ with other links fixed. Since the mapping from $G$ to $\mathbb{P}^{n(n-1)/2}$ is a bijection, with an abuse of notation, we write $U_i(y) = U_i(g)$ and $MU_{i,j}(y) = MU_{i,j}(g)$.

Next, we introduce the definition of the equilibrium.

**Definition 2 (Pairwise Stability).** A network $g \in G$ is pairwise stable if no link will be cut by a single player, and no two players want to form a link:

1. $\forall (i, j) \in g$, $U_i(g) \geq U_i(g \setminus (i, j))$ and $U_j(g) \geq U_j(g \setminus (i, j))$.

2. $\forall (i, j) \notin g$, $U_i(g \cup (i, j)) > U_i(g) \Rightarrow U_j(g \cup (i, j)) < U_j(g)$.

Note that the definition of pairwise stability is quite standard in the literature initiated by Jackson and Wolinsky (1996). Also, we do not allow transfers between agents.

It is straightforward that Definition 2 is equivalently rewritten as a fixed point of the following mapping $F(\cdot) : \mathbb{P}^{n(n-1)/2} \mapsto \mathbb{P}^{n(n-1)/2}$.

\[
F(y) = \begin{bmatrix}
1[MU_{1,2}(y) \geq 0]1[MU_{2,1}(y) \geq 0] \\
1[MU_{1,3}(y) \geq 0]1[MU_{3,1}(y) \geq 0] \\
\vdots \\
1[MU_{n-1,n}(y) \geq 0]1[MU_{n,n-1}(y) \geq 0]
\end{bmatrix}.
\] (3.2)

**3.3.2 Existence and Characterization of the equilibria**

Now, we introduce the key assumption on the utility function, which we refer to as "nonnegative externality."

**Definition 3 (nonnegative externality).** The set of utility functions $\{U_i(y)\}_{i=1}^n$ satisfies nonnegative externality if $MU_{i,j}(y)$ is weakly increasing in $y$ for all $i$ and $j$.

Intuitively, nonnegative externality requires that if links within the network increase, the incentive to form a new link weakly increases for all pairs. This property
is equivalent to "convexity in own links and the strategic complements property" introduced by Hellmann (2013).\footnote{Since the assumption of nonnegative externality plays a crucial role in our framework, it would be useful to test this assumption directly. In the case of continuous games, Echenique and Komunjer (2009) provides a way to test complementarity. It may be possible to extend their result to the case of discrete games, including network formation models. We leave this work for future research.}

Under this property, we show that there always exists a pairwise stable equilibrium, which is shown in the following theorem:

**Proposition 5** (existence and characterization of the equilibria). *Suppose the set of utility functions \( \{U_i(y)\}_{i=1}^{n} \) satisfies nonnegative externality. Then there exists a pairwise stable equilibrium. Moreover, the set of equilibria is a nonempty complete lattice, where partial order \( \succeq \) in \( \mathbb{R}^{n(n-1)/2} \) is defined as follows: \( y \succeq y' \) if \( y_k \geq y'_k \) \( \forall k = 1, \ldots, n(n-1)/2 \), where \( y_k \) is the \( k \)-th component of \( y \).

*Proof.* From the assumption that \( M_{ij}(y) \) is weakly increasing in \( y \) for all \( i \) and \( j \), \( F(\cdot) \) is also weakly increasing because it is a weakly increasing function of a weakly increasing function. Also, \( \mathbb{R}^{n(n-1)/2} \) is a complete lattice with respect to \( \succeq \). Consequently, Tarski’s fixed point theorem (cf. Tarski (1955)) can be applied, and the statement is proved. \( \square \)

The fact that the set of equilibria becomes a complete lattice implies that there always exist maximum and minimum equilibria. Note that the definition of partial order in the statement of Proposition 5 can be rephrased as the set inclusion of \( g \). In other words, the set of links of any equilibrium includes that of the minimum equilibrium and is included by that of the maximum equilibrium.

Several comments are in order here. Firstly, existence itself is shown by Hellmann (2013), but he does not characterize the equilibrium as a complete lattice, which is the value added of our approach.

Secondly, this theorem is essentially an application of Topkis (1979), Milgrom and Roberts (1990) and Vives (1990), although in the context of network formation, no paper thus far has proposed a characterization of the equilibrium using this approach. As is already stated in Section 3.2, the same characterization of the equilibrium is
used in structural estimation of noncooperative games with strategic complementar-
ities (Jia (2008), Nishida (2015) and Uetake and Watanabe (2013b)), and two-sided
matching (Uetake and Watanabe (2012) and Uetake and Watanabe (2013a)). In the
former case, the mapping $F(\cdot)$ corresponds to a best-response correspondence, and
in the latter case, the mapping $F(\cdot)$ corresponds to the opportunity sets, i.e. the
hypothetically offered set of contracts.

3.3.3 Discussions on the Applicability of Nonnegative Extern-
ality

Although nonnegative externality gives a nice characterization, it is at the same time
restrictive and not suitable in many applications. For example, the connections game
introduced by Jackson and Wolinsky (1996) does not satisfy this property. In this
subsection, we discuss the validity of the assumptions in and out of the friendship
network which we take as an empirical example.

Whether friendship networks satisfy nonnegative externality is not clear. On
the one hand, the presence of positive externality is pointed out in the context of
friendship networks. Starting from Simmel (1908), many papers provide empirical
evidence of a clustering feature, i.e. a friend of a friend is likely to be a friend.
However, our nonnegative externality assumption requires an absence of negative
externality rather than the presence of positive externality. Negative externality can
be present in many different scenarios: For example, if the utility of friendship is an
increasing function of the time spent with each friend, the marginal utility decreases
as the person herself and/or the potential partner has more friends, as is theoretically
illustrated in Goyal and Joshi (2006). Even more simply, if the marginal cost of
forming additional friend is increasing, the assumption is not satisfied. This is a valid
core of our empirical result and alternative approaches should be developed that
do not require this assumption.

Departing from the friendship network, one example where the nonnegative ex-
ternality assumption holds is a collusive network where linked firms commit not to
enter each other’s territory, as considered in Belleflamme and Bloch (2004).\(^8\) They assume that the profit function is convex in the number of entrants in the market, i.e. the decline in profit (measured as a positive number) is a decreasing function of the number of competitors, which implies nonnegative externality. They also provide primitive assumptions of the nature of oligopolistic competition that support their assumption.

### 3.4 Econometric Model

In this section, we introduce an econometric model and propose a general procedure for constructing moment inequalities. We also provide an example of the construction of moment inequalities in the context of our application of friendship networks.

#### 3.4.1 Construction of Moment Inequalities in Generic Case

Here we introduce an econometric model. We assume that the utility function is parametrized by a finite dimensional parameter vector \( \theta \in \Theta \subset \mathbb{R}^D \) and denote \( \theta_0 \) as the true parameter vector. We also assume that we observe independent networks \( s = 1, \ldots, S \). Each network \( s \) is associated with an observable random vector \( X_s \) and an unobserved random vector \( E_s \). An example of \( X_s \) and \( E_s \) which applies to many applications is the set of observed characteristics of all agents in network \( s \) and the set of unobserved preference shocks, respectively. We assume that \( X_s \) and \( E_s \) are independent. The utility function is denoted by \( U_i(g_s; \theta, X_s, E_s) \), where \( g_s \) is the set of realized links in network \( s \). We also assume that \( U_i(g_s; \theta, X_s, E_s) \) satisfies the definition of nonnegative externality for all \( \theta \in \Theta \) almost surely.

As is shown in Proposition 5, we can characterize the maximum and minimum equilibria given \( (\theta, X_s, E_s) \). We denote them \( \bar{g}(\theta, X_s, E_s) \) and \( g(\theta, X_s, E_s) \) respectively. Also, we denote \( G(\theta, X_s, E_s) \) as the set of equilibria sustained by \( (\theta, X_s, E_s) \). Since the realized network is unique, we assume that there is a true equilibrium selection

\(^8\)Belleflamme and Bloch (2004) considers pairwise strong Nash equilibrium, a refinement of pairwise stable equilibrium.
mechanism for each \((\theta, X_s, E_s)\). We denote it by \(g^*_s(\theta, X_s, E_s)\). Note that by definition \(g^*_s(\theta, X_s, E_s) \in G(\theta, X_s, E_s)\). Note that this is just notation; we do not assume any equilibrium selection rule. In fact, the true equilibrium selection rule can depend on \(s\) and can be non-stationary.

Now we define the moment functions. The moments are constructed based on \(L\) sets of statistics that can depend on \(g_s\) and \(X_s\). Define the function \(h_l(\cdot, \cdot) : G \times X_s \rightarrow \mathbb{R}\) for \(l = 1, \ldots, L\) where \(G\) and \(X_s\) are the spaces of \(g_s\) and \(X_s\), respectively. Our choice of the moment functions takes the following form:

\[
m^*_l(\theta, X_s, g_s) = E[h_l(g^*_s(\theta, X_s, E_s), X_s)|X_s] - h_l(g_s, X_s).
\] (3.3)

In other words, the moment functions are the differences between the realized statistics and the conditional mean of the statistics with respect to \(X_s\).

Since \(g^*_s(\theta, X_s, E_s)\) corresponds to the true equilibrium selection rule, \(E[m^*_l(\theta_0, X_s, g_s)] = E[h_l(g^*_s(\theta_0, X_s, E_s), X_s)] - E[h_l(g_s, X_s)] = 0\). We also denote that \(\overline{m}_l(\theta, X_s, g_s) = E[h_l(\tilde{g}(\theta, X_s, E_s), X_s)|X_s] - h_l(g_s, X_s)\) and \(\underline{m}_l(\theta, X_s, g_s) = E[h_l(\tilde{g}(\theta, X_s, E_s), X_s)|X_s] - h_l(g_s, X_s)\).

Next we introduce a necessary property for \(h_l(\cdot, \cdot)\) to construct moment inequalities. Essentially, the property asserts that the moment functions succeed the partial order of equilibria as defined in Proposition 5.

**Property 1** (monotonicity of moment statistics). \(\forall j, m, \theta, \forall \tilde{g}_s \in G(\theta, X_s, E_s), h_l(\tilde{g}(\theta, X_s, E_s), X_s) \geq h_l(\tilde{g}_s, X_s) \geq h_l(g(\theta, X_s, E_s), X_s)\) almost surely.\(^9\)

From Property 1, it is easy to show that

\[
E[\overline{m}_l(\theta, X_s, g_s)] \geq E[m^*_l(\theta, X_s, g_s)] \geq E[\underline{m}_l(\theta, X_s, g_s)].
\] (3.4)

Together with the fact that \(E[m^*_l(\theta_0, X_s, g_s)] = 0\) (by taking the unconditional

---

\(^9\)It should be noted that it is sufficient to assume that the inequality in Property 1 holds for \(g_s^*(\theta, X_s, E_s)\), but not for all \(\tilde{g}_s \in G(\theta, X_s, E_s)\), to construct moment inequalities. However, it is usually the case that we have no information on the equilibrium selection rule a priori. Therefore it is natural to choose \(h_l(\cdot, \cdot)\) based on the criteria in Property 1.
expectation of equation (3.3)), we have the following 2L set of moment inequalities:

\[
\begin{align*}
E[\bar{m}_l(\theta_0, X_s, g_s)] & \geq 0, \\
E[m_l(\theta_0, X_s, g_s)] & \leq 0.
\end{align*}
\tag{3.5}
\]

It is usually the case that parameters that satisfy moment inequalities (3.5) turn out to be a set, not a point. We denote this set \( \Theta_I \). Note that \( \Theta_I \) is not the sharpest bound implied by the model because we only utilize a necessary condition to characterize \( \Theta_I \). Therefore a suitable choice of informative moments should be considered carefully for each application. An example of this type of argument is described in the next subsection.\(^{10}\)

Several comments are in order. First, in relationship to the previous literature on structural estimation of supermodular games, Jia (2008) and Nishida (2015) assume two extremal equilibrium selection rules and obtain two different estimators by assuming either \( E[\bar{m}_l(\theta_0, X_s, g_s)] = 0 \) or \( E[m_l(\theta_0, X_s, g_s)] = 0 \). Uetake and Watanabe (2013b) does not assume equilibrium selection rules and proposes to use particular moments that satisfy Property 1 (namely, the summation of the strategies and the utilities of all players in the market in a noncooperative supermodular game).

Secondly, \( m_l(\theta, X_s, g_s) \) and \( \bar{m}_l(\theta, X_s, g_s) \) are not solved analytically in our case. However, we can construct these moments by simulation. More specifically, we define simulated moments as follows:

\[
\hat{m}_l(\theta, X_s, g_s, \{E_s^{(r)}\}_r) = \frac{1}{R} \sum_{r=1}^{R} h_l(\theta, X_s, g_s, \{E_s^{(r)}\}_r) - h_l(g_s, X_s),
\tag{3.6}
\]

where \( E_s^{(r)} \) is the \( r \)-th number of simulated error terms. \( \hat{m}_l(\theta, X_s, g_s, \{E_s^{(r)}\}_r) \) is defined similarly. 3.A describes the more detailed procedure as well as the computational algorithm of \( g(\theta, X_s, E_s^{(r)}) \) and \( \bar{g}(\theta, X_s, E_s^{(r)}) \).

Thirdly, the considered asymptotics here is in the direction that the number of

\(^{10}\)There is recent literature that characterizes the sharpest bound of the identified set of partially identified models; see Beresteanu, Molchanov, and Molinari (2011) or Galichon and Henry (2011). It is possible to apply their idea to our framework as well. We leave this work for future research.
networks goes to infinity. In some cases, however, we can obtain information on only one network. Although extending our method to such a case is beyond the scope of this paper, there are several potential directions one could go. One direction is to take a Bayesian approach, as Christakis et al. (2010) or Mele (2016) do. Another potential idea is to take the approach of spatial statistics of weak dependence as in Leung (2016). We leave this work to extend our result to the case with a single network for future research.

3.4.2 Parametrization in the Friendship Network Application

In this subsection, we illustrate the construction of moment inequalities in our friendship network application.

We assume that the utility function takes the following functional form:

$$U_{i,s}(g_s; \theta, X_s, E_s) = \sum_{j:(i,j) \in g_s} MU_{ij,s}(g_s; \theta, X_s, E_s),$$

$$= \sum_{j:(i,j) \in g_s} \beta_0 + 1[x_{i,s}^1 = x_{j,s}^1] \beta_1 + 1[x_{i,s}^2 = x_{j,s}^2] \beta_2 + M_s(i,j) \gamma + \epsilon_{ij,s},$$

where $\theta = [\beta_0, \beta_1, \beta_2, \gamma]'$, $x_{i,s}^1$ denotes $i$'s gender, $x_{i,s}^2$ denotes $i$'s race, $M_s(i,j)$ denotes the dummy variable that takes 1 if $i$ and $j$ have a mutual friend, i.e. $\exists k \neq i, j$ s.t. $(i,k), (j,k) \in g_s$, and $\epsilon_{ij,s}$ is an unobserved preference shock of $i$ toward $j$. Note that the summation is taken for each direct friend $j$, i.e. $(i,j) \in g_s$. Parameters $\beta_1$ and $\beta_2$ represent the degree of preference toward the same gender and the same race, respectively. In the empirical application, we consider different racial homophily for different races, so $\beta_2$ is a vector. $\gamma$ represents the degree of preference toward clustering. In order to ensure the presence of nonnegative externality, we assume $\gamma \geq 0$, i.e., there is only a nonnegative effect of having a mutual friend. We assume $\epsilon_{ij,s}$ is i.i.d. over $i$, $j$ and $s$, and it follows the standard normal distribution. In the general setup explained in the previous subsection, $X_s = [x_{1,s}, x_{2,s}, \ldots, x_{n_s,s}]$ and $E_s = [\epsilon_{12,s}, \ldots, \epsilon_{n_s(n_s-1),s}]$, where $n_s$ is the number of agents in market $s$. Note that we allow the number of agents in the network $n_s$ to depend on $s$ as a random variable.
We assume \((X_s, E_s)\) are i.i.d. over \(s\).

Although the empirical results using the above model are more of an illustration of our approach, we still need to make some comments on the empirical specification. First, one may argue that the nonnegative externality property may not hold when applied to friendship network formation. For example, if the utility of friendship is an increasing function of the time spent with each friend, the marginal utility decreases as the person herself and/or the potential partner has more friends. We assume away such possibility. Absence of negative externality is a crucial assumption that we have to impose, and alternative approaches should be developed that do not require this assumption.

Secondly, other extensions of the model can be considered. For example, the friendship network may exhibit a clustering feature because of unobserved heterogeneity, i.e. individuals who are close by in some unobservable attributes to an econometrician is likely to be a friend. This is pointed out by Badev (2013), for example. Not taking into account such unobserved sorting effect may bias racial homophily terms and clustering terms. We leave this investigation for future research.

3.4.3 Moment Choice in the Friendship Network Application

In this subsection, we explain the choice of \(h_l(\cdot, \cdot)\) in our application. Although the choice of moment functions should be tailored to each application, some of them can be directly extended to other applications. Table 3.C1 lists all the moments used in the empirical section. Although there is no explicit correspondence between moments and parameter dimension, we explain the intuition behind why each moment has some identifying information about the identified set.

(a) proportion of pairs with a link has the identifying information on the constant term, \(\beta_0\). (b) proportion of pairs with a link between the same \(x_i^p, s\) for \(p = 1, 2\), and (c) proportion of pairs with a link between different \(x_i^p, s\) for \(p = 1\), contains information on the degree of homophily with respect to gender and race, i.e. \(\beta_1\) and \(\beta_2\).\(^{11}\) We also included (d) interaction of (a)-(c) and the dummy variable that takes 1 if the

\(^{11}\)We excluded (c) for \(p = 2\) in our application because this moment has high correlation with (a).
number of nodes is less than 30. This is motivated by the observation that networks with fewer agents should be more informative about $\beta_1$ and $\beta_2$.\textsuperscript{12}

Other moments (e) to (h) contain information on the clustering term, $\gamma$. There is one technical caveat here: For (e) to (h), the denominator potentially takes the value of 0, in which case we assume $h_l(\cdot)$ to be 1, rather than 0. This normalization is used to obtain a tighter identified set for certain parameter configurations. To see this, consider the following example. Suppose that $\gamma$ is a large positive value, while $\beta_0$, $\beta_1$, and $\beta_2$ are large negative values. In such a case, the minimum network is the empty network ($\emptyset$) and the maximum network is the complete network ($g^N$). If we define $h_l(\emptyset) \equiv 0$, the bound has no empirical content because $0 = h_l(\emptyset) \leq h_l(g) \leq 1 = h_l(g^N)$ for any observed network $g$. On the other hand, if we define $h_l(\emptyset) \equiv 1$, we obtain a much tighter bound, and the exclusion of such parameter configurations is possible.\textsuperscript{13}

The nontrivial part is whether this definition satisfies Property 1, which we prove in 3.B.\textsuperscript{14}

It should also be noted that there are other network-level statistics that one can use for other applications, such as the number of stars or weighted average of network centrality. In this particular application, we do not use them because these moments are not particularly informative about our model or parameters. Of course, if the underlying model predicts some pattern on these statistics particularly, one may consider these statistics as well, after checking that Property 1 is satisfied.

\textsuperscript{12}To see the intuition, consider an extreme case with only two agents. Then, the mutual friend effect is never relevant, and we can purely identify $\beta_1$ and $\beta_2$.

\textsuperscript{13}We can of course use both definitions of the moments for inference. However, in practice, the correlation of these two moments is too high, which hampers the finite sample performance of Andrews and Soares (2010). Hence, we have decided to use only the moments defined in Table 1.

\textsuperscript{14}Another related note is that we do not use the conventional clustering coefficient, i.e. $\{\sum_{j \neq i, k \neq j, k \neq i} y_{ij,s} y_{jk,s} y_{ik,s}\}/\{\sum_{j \neq i, k \neq j, k \neq i} y_{ij,s} y_{jk,s} y_{ik,s}\}$ for one of the moments. This is because the conventional definition of clustering coefficient does not satisfy Property 1. To see this, consider a case where there are four nodes (A, B, C, D), and the minimum equilibrium is \{(A, B)\}, while the maximum equilibrium is \{(A, B), (B, C), (B, D), (C, D)\}. In this case, $h_l(\cdot, \cdot)$ takes 1 for the minimum equilibrium and $\frac{1}{3}$ for the maximum equilibrium.
3.4.4 Why Classical Bivariate Regression is Biased

To illustrate the usefulness of our method, consider the following bivariate regression:

\[
y_{ij,s} = 1\{\beta_0 + 1[x_{i,s} = x_{j,s}]\beta_1 + 1[x_{i,s} = x_{i,s}]\beta_2 + M_s(i,j)\gamma + \epsilon_{ij,s} > 0\} \\
\times 1\{\beta_0 + 1[x_{j,s} = x_{i,s}]\beta_1 + 1[x_{j,s} = x_{i,s}]\beta_2 + M_s(j,i)\gamma + \epsilon_{ji,s} > 0\}
\]

(3.7)

If \(\epsilon_{ij}\) follows a standard normal distribution, the regression model looks similar to the Partially observable bivariate probit model (Poirier, 1980). However, naively running this regression involves several problems. Firstly, in the presence of multiple equilibria, the above regression is incomplete (Tamer, 2003) and special attention should be paid to identification. Secondly, there are endogenous variables \(M_s(i,j)\) in the regression model (3.7). Since \(M_s(i,j)\) depends on other pairs’ \(y_{ij,s}\), the variables are endogenous. In other words, the model is a particular example of nonlinear simultaneous equation model.

If there is no clustering (i.e. \(M_s(i,j)\)), then a simple bivariate probit estimate as above is consistent.\(^{15}\) Because of the simplicity of the bivariate probit, sometimes people have made this assumption. However, this leads to an omitted-variable-bias problem. Namely, if the preference toward clustering is not controlled in the estimating equation, one is likely to overestimate the preference toward racial homophily. To illustrate why, consider three students of the same race, A, B and C. Suppose that they are all friends with each other, i.e., they form a “triangle”. If one does not take into account the preference toward clustering in the utility function, one is

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\(^{15}\)One important comment here is that the identification issue may arise even without an externality term or multiple equilibria in the partially observable bivariate probit model with exogenous regressors, as illustrated in Poirier (1980). Poirier (1980)’s results suggest that point identification is attained if the utility indices are symmetric for both sides of potential partners without an externality term. On the other hand, if they are not symmetric, one may not attain point identification even without an externality term nor multiple equilibria. To see this, suppose that there are only two explanatory variables: one for the preference of men toward women, and the other of women toward men. These two parameters are not separately identified, because exchanging the two coefficients will yield completely same data generation process. Even if one takes an approach of partial identification, this point must be considered when constructing a model and deciding what moments to use.
attributing all of the links (A-B, B-C, C-A) to a preference toward racial homophily, even though C and A may have become friends due to the preference toward clustering, i.e. conditional on the existence of mutual friend B. Hence, it is expected to overestimate preference toward racial homophily if preference toward clustering is not controlled. With the estimator proposed in this paper, clustering can be handled correctly. Furthermore, the estimates in the following section show that clustering is indeed present.

In the presence of externality term $M_s(i,j)$, the model becomes a particular example of nonlinear simultaneous equation models, whose discussion at least date back to Heckman (1978). He assumed away the possibility of multiple equilibria and discussed the identification in two simultaneous equations models. The argument centers around the derivation of the reduced form of the model. In the case of the network formation models where the equations interact in more complicated manner and it is difficult to derive reduced form\textsuperscript{16}, showing point identification is even more challenging. In the presence of multiple equilibria, the problem is even more complicated. Tamer (2003) elegantly showed an point identification condition even under multiple equilibria in two-by-two entry game. How this result is extended in more complicated simultaneous equations models such as the network formation models is yet to be investigated.\textsuperscript{17}

\section{Monte Carlo Simulation}

In this section, we present the results from the Monte Carlo experiment. We choose the specification to be as close as the empirical application in Section 3.6. Throughout the exercise, we assume 4 dimensions of parameter space, which resembles the specification where we assume homogeneous parameters for Whites and Blacks. The number of agents and the distribution of the covariates (gender and race) are set to

\textsuperscript{16}To see this, consider how to write down $y_{ij,s}$ as a function of only exogenous regressors. Since $M_s(i,j)$ depends on $x_{k,s}$ for $k \neq i,j$, the reduced form of equation (3.7) depends on $x_{k,s}$ for all $k$ in the network $s$. Thus, it is almost impossible to estimate the reduced form, even if we ignore the problem of multiplicity of equilibria. It follows that the reduced-form approach does not work here.

\textsuperscript{17}Similar difficulty in identification arises in matching models, as is discussed in Section 3.2.3.
match the empirical proportion in the data (see Table 3.C3).

The true parameter value of the Monte Carlo exercise is set as follows: for the parameters $\beta_0$, $\beta_1$, and $\beta_2$, we take the midpoint of the minimum and maximum values of the confidence set in the empirical application reported in column 2 of Table 3.C4 for each parameter, while for $\gamma$ we take the minimum value of the confidence set. As we will show below, the test has relatively low power in the direction in which $\gamma$ increases than it decreases. In other words, the true parameter value of $\gamma$ tends to locate closer to the minimum boundary of the confidence interval of $\gamma$ than its maximum. Hence, this parameter choice is meant to better resemble the true data generating process. The choice of true parameter value comes down to $\beta_0 = -1.4099, \beta_1 = 0.1873, \beta_2 = 0.2406, \gamma = 0.8850$. In terms of true equilibrium selection rules, we assume four different types of equilibrium selection: (1) minimum equilibrium, (2) maximum equilibrium, (3) mixture of minimum and maximum equilibria (i.e. a half of the networks are assumed to choose maximum equilibrium and the rest are assumed to choose minimum equilibrium), and (4) equilibrium with initial random networks. For (4), we first draw a network with $\frac{1}{2}$ probability of forming a link independently across all pairs. Using this network as a initial state, we run the algorithm specified in Corollary 1.18 The number of independent networks are 150 (149 in the application of friendship networks). The number of Monte Carlo simulation draws is 1000.

Table 3.C2 shows the empirical coverage probability of the true parameter value. First, for (1) and (2), with a sample size of 150, the empirical size of the test is somewhat below the nominal size of the test (95%) but within a reasonable range. Under (3) and (4), the test is much more conservative. This is natural in light of the observation that the moment inequalities (3.5) are satisfied with a slack at the true parameter value for (3) and (4), while for (1) and (2) half of them are satisfied without a slack. To see this, consider the minimum equilibrium. Among the moment inequalities in (3.5), $E[m_4(\theta_0, X_s, g_s)] \leq 0$ hold without slack while $E[\overline{m}_t(\theta_0, X_s, g_s)] \geq 0$ hold with slack. For mixture and equilibria with initial random networks all the

\footnote{Note that if initial state is not empty or complete network, the algorithm is not guaranteed to converge. If the algorithm does not converge, we take the network after 1000 iteration as the realized networks.}
inequalities hold with some slack, hence it is more difficult to reject the null.

Figure 3.C1 summarizes the empirical power of the test. Each graph indicates the results under different true equilibrium selection rules. Each graph reports the rejection probability when we vary one parameter value fixing the other parameter values as the true parameter values. For example, the line "beta0" reports the rejection probability of the null at the true $\beta_1, \beta_2$ and $\gamma$, but at different $\beta_0$ by the amount specified in the horizontal axis. The actual calculation of the power is conducted at every 0.01 interval in $[-0.05, 0.05]$, and 0.1 interval in $[-1.0, -0.1]$ and $[0.1, 3.0]$.

First, focus on (1) minimum equilibrium selection. Several points are noteworthy. First, the power function exhibits a nonlinear pattern. As each dimension of the parameter values decreases from the true values, the rejection probability decreases to even less than 0.05 for a while, and then suddenly increases to 1. On the other hand, the rejection probability shoots up for a small increase of parameter values from the true values. This is because the identified set is not singleton and the true parameter value is on its boundary. As is explained above, under minimum equilibrium selection $E[m_t(\theta_0, X_o, g_o)] \leq 0$ hold without slack while $E[m_t(\theta_0, X_o, g_o)] \geq 0$ hold with slack. Hence, as the null parameter value increases the former moments are immediately violated, while the decrease of null parameter value keeps the moment inequalities to hold.

For (2) maximum equilibrium selection, we confirm the opposite patterns: strong power in the direction of decreasing parameters and weaker power in the opposite direction. One important comment is that the rejection probability stays low in the direction of increasing $\gamma$. One potential explanation for this is that the identified set covers large values of $\gamma$. This is consistent with the observation that the increase of $\gamma$ increases the possibility of multiple equilibria (i.e. the equilibrium is unique when $\gamma = 0$), which widens the moment bounds under such parameter configurations.

For (3) mixture of minimum and maximum equilibria, the test has intermediate power in both direction of parameter changes, which is consistent with the fact that under the equilibrium selection rule the true parameter value is strictly within the identified set. (4) equilibrium with initial random networks exhibits similar patterns
as (2) maximum equilibrium. This is because under the parameter values considered in the Monte Carlo experiment, the equilibrium network is very sparse, and setting the initial network with $\frac{1}{2}$ probability of links for the algorithm in Corollary 1 results in an almost similar equilibrium selection as the maximum equilibrium.

### 3.6 Application: Friendship Networks

#### 3.6.1 Data

We apply our methodology to the unique dataset of the National Longitudinal Study of Adolescent Health (Add Health). The Add Health database was designed to study the impact of social environment (i.e. friends, family, neighborhood and school) on adolescents' behavior in the U.S. It includes data on students in grades 7-12 from a nationally representative sample of roughly 130 private and public schools during the years 1994-1995, and contains four successive waves. Every pupil attending the sampled schools on the interview day of the first wave was asked to complete a brief questionnaire (in-school data) containing questions on respondents' demographic and behavioral characteristics, education, family background and friendships. In this paper, we use only the first wave of the survey, which collect friendship nominations from all the samples. In total, the in-school data contains information on 90118 participants.\(^{19}\)

The data has been applied in other papers on structural estimation of network formation (Christakis et al. (2010) and Mele (2016)) and in the literature on peer effect (Bramoullé, Djebari, and Fortin (2009) and Calvó-Armengol, Patachini, Zenou, O-armengol, Calvó-Armengol, Patachini, and Zenou (2009)). It is also extensively studied outside economics, such as in the field of sociology (Moody (2001), for example). Thus we believe that the data is a good choice to illustrate our approach.

Our empirical objective is to obtain reliable estimates of the preference toward racial homophily after controlling for the preference toward clustering by avoiding

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\(^{19}\)The Add Health website (http://www.cpc.unc.edu/projects/addhealth) describes surveys and data in more detail.
the problem discussed in Section 3.4.4. From the perspective of racial integration policy, the utility parameter associated with the preference toward racial homophily is usually of interest.

For characteristics vector \( x_i \), we take gender and race (Black and White). Some students answered different races (Hispanics, Asian, native American, and others), but since these samples are relatively small, we only focus on Black and White.\(^{20}\)

For the friendship links, we use data based on friends nominations. In the in-school questionnaire, students were asked to identify their best friends up to five males and five females.\(^{21}\) We assume that a friendship network is undirected, and hence we assume that a pair of students have friendship links if both students nominated the other as their friend. This specification follows Christakis et al. (2010), but not Mele (2016), as is explained and discussed in the introduction.

To define networks, the networks were divided by schools and grades. It may be controversial to treat different grades within the same school, or even different schools, as independent networks. However, to obtain a sufficient number of observation of networks to conduct asymptotic analysis, such treatment is necessary.\(^{22}\) Furthermore, we only take networks with size of 10 to 100 to enable computation. We take relatively small networks as our samples because of computational reasons, although it is not impossible to use larger networks.

Consequently, our final data set contains 149 networks with 7671 students. In the next subsection, we introduce the descriptive statistics of our sample.

\(^{20}\)In fact, Monte Carlo simulation not reported in this paper suggests that including racial category with small proportion with skewed distribution (0 agents in many schools) results in low performance of the test. This is because the variance of some moments (in particular (b), (c) and (h) in Table 3.C1) become close to 0, which violates one of the regularity conditions in Andrews and Soares (2010). See Page 124, (2.2, vii) of Andrews and Soares (2010) for the condition.

\(^{21}\)As is noted in Moody (2001), of all students, 3% nominated ten in-school friends, 23% of all students nominated five in-school male friends, and 25% of all students named five in-school female friends. This censoring potentially leads to a bias of the estimate. We leave the methodological question on how to deal with censoring, as well as the empirical question on how much the estimates are biased due to the censoring, for future research. In this paper, we do not impose such a cap when we simulate equilibria in the estimation procedure.

\(^{22}\)We believe that this treatment is not a big problem in our analysis, because fewer than 5% of all the links are across grades and with mutual friends. In other words, very few observations are formed across grades conditional on the existence of mutual friends, indicating the dependence across networks within grades which are created through links across grades is not large.
3.6.2 Descriptive Statistics

Table 3.C3 presents the descriptive statistics of the observed networks. The first five rows show the composition of the agents in each network. We observe that there is great heterogeneity over networks even after restricting the number of students to be between 10 and 100. For example, although the average proportion of Blacks among all students is 16.02%, there is a network in which it reaches 100% and in which there are no Blacks.

The last six rows show the network related statistics. The average degree, which means the average number of links each agent has, shows that students have on average about 1.5 friends. The fact that the minimum average degree is 0 implies that in some networks, there are no friendship links. There is also a large heterogeneity in the average degree across networks: The 10 percentile is 0.5 and 90 percentile is 2.5 (not reported in the table).

The next four rows show the degree of gender or racial homophily. One can infer from the table that there is strong racial homophily. For example, since the proportion of Whites is 0.7220, if links are formed completely randomly, the expected proportion of links between Whites is \((0.7220)^2 = 0.5213\). This is much less than is the observed proportion of links between Whites, which is 0.6784. The last row shows the clustering coefficient.\(^{23}\) The clustering coefficient of 0.1568 shows that there are many more triangles than the case if links are formed completely randomly, as is pointed out in the previous literature on social networks.\(^{24}\)

To illustrate the clustering and racial homophily more visually, we draw a network of one of the samples (school ID 121, grade 8). All the nodes that are connected to at least one node are shown. Pink, white, black nodes represent Hispanics, Whites, and Blacks, respectively. We observe the same phenomena as we have seen with Table 3.C3: clustering and racial homophily.

\(^{23}\)The clustering coefficient here follows the conventional definition rather than the modified one we defined in Section 3.4. For the definition, refer to Section 3.4.

\(^{24}\)See Jackson (2008), for example.
3.6.3 Estimation Results

Table 3.C4 reports our main empirical results. Column 1 reports the estimation results of our full model, and column 2 reports the estimation results restricting the coefficients of the dummies for both Whites and both Blacks to be the same. In both specifications, the dummy variable for the same gender is significantly positive. For racial preferences, the coefficient of the dummy for both Whites is significantly positive, while the dummy for both Blacks is not in column 1. However, if we restrict the two coefficients to be the same in column 2, the variable is jointly significant. These findings indicate the importance of racial homophily even after controlling for the mutual friend effect. Lastly, $\gamma$ is significantly different from 0 in both specifications, and the coefficient is large compared to the other coefficients. These findings suggest the importance of the parameter in forming a friendship link.

To understand the potential bias of gender and racial homophily terms without the mutual friendship effects $M_s(i, j)$, in column 3 we show the estimates under the restriction that $\gamma = 0$. The estimates and the confidence set under this specification is obtained by the simulated method of moments (McFadden (1989) and Pakes and Pollard (1989)), because in this case the moment inequalities come down to equalities. We find that the lower and upper bounds of the confidence set of $\beta_1$ and $\beta_2$ are smaller in column 1 than in column 3 in most cases. The only exception is the lower bound of the coefficient of Dummy (1: both Whites), but these two lower bounds are very close. A particularly striking fact is that the coefficient of Dummy (1: both Blacks) is significantly positive in column 3, while it is not in column 1. This confirms that preference toward gender and racial homophily is likely to be "overestimated" without controlling for preference toward clustering; i.e. larger values of the parameters are more likely to be rejected and smaller values of the parameters are more likely to be accepted in column 1 than in column 3.\(^{26}\)

\(^{25}\)It may not be accurate to use the phrase "overestimated," because it may be the case that the true parameters could be covered by both confidence sets. However, for the sake of brevity, we use this phrase to refer to the existence of this observation.

\(^{26}\)It should be also noted that there is a mechanical positive effect of introducing an additional explanatory variable on the absolute values of the coefficients. This is because the coefficients are normalized by the variance of the error term as is true in any discrete choice models. Including an
Another potential bias in the estimates is the model misspecification across different network size. The model considered here assumes that the preference shocks are drawn independently across all pairs. This implies that as the number of agents in the network increases, the average number of friends for each person should increase. In reality, it may well be that the set of agents who each agent considers as potential friends remain relatively constant even if the number of agents in the network increases. If this is true, and if there is correlation between the number of agents and the average characteristics of agents in the network (e.g. number of people of particular race), then the misspecification directly biases $\beta_1$ or $\beta_2$. To deal with the concern, in column 4 we report the empirical results by adding an additional term of the number of students in each network in the utility specification. The coefficient of the number of students is not significantly different from 0, and the confidence bounds of other parameters are close between column 1 and column 4. These findings alleviate the concern to some extent.

3.7 Conclusion

This paper develops a framework with which we can structurally estimate pairwise stable network formation with nonnegative externality. We first characterize the equilibrium by a fixed point of a certain mapping. By applying Tarski's fixed point theorem, we show that the set of pairwise stable equilibria with nonnegative externality is a nonempty complete lattice. This equilibrium characterization is then extended to an econometric framework for structural estimation. We apply our methodology to the friendship network of students in the United States, using data from Add-Health. We estimate the preference toward racial homophily and the preference toward clustering at the same time. We find that the preference toward racial homophily is overestimated if we do not take into account the preference toward clustering. Although our approach is only applicable to the case with nonnegative externality, we additional explanatory variable decreases the variance of the error term, which in turn increases the absolute values of the coefficients. Hence, the result here shows that the "overestimation" bias is strong enough to exceed the opposite mechanical effect.
believe that the paper is a step forward in the literature on social networks.
Appendices

3.A Procedure to obtain the confidence set

3.A.1 Algorithms to compute the maximum and minimum equilibria

We first propose a simple algorithm to compute the maximum and minimum equilibria. As is stated in the main section of the paper, the same algorithm is used for different applications of supermodular games (Jia (2008), Nishida (2015) and Uetake and Watanabe (2013b)). We state the algorithm as a corollary to Proposition 5.

Corollary 1 (algorithm for computing extremal equilibria). Suppose the set of utility functions \{U_i(y)\}_{i=1}^{n} satisfies nonnegative externality. Let \(y^0\) be the initial state. Define \(y^1(y^0) = F(y^0)\), \(y^2(y^0) = F(F(y^0))\), and so forth. Then, the iteration starting from 0 and 1 always converges within finite time, where 0 = \([0, \ldots, 0]\)' and 1 = \([1, \ldots, 1]\)'). Furthermore, the converging networks correspond to the maximum and minimum equilibria.

Proof. First, we show that the iteration starting from 0 is always weakly increasing. \(y^1(0) \geq 0\) is obvious. Suppose the claim holds until \(k\)-th iteration, i.e. for all \(0 < l \leq k\), \(y^l(0) \geq y^{l-1}(0)\). Then, \(y^{k+1}(0) \geq y^k(0)\) must hold. To see this, suppose the contrary, i.e. there exists \(i, j\) such that \(y^{k+1}_{ij}(0) = 0\) and \(y^{k}_{ij}(0) = 1\) where \(y^{k}_{ij}(0)\) is the \((i, j)\) component of \(y^k(0)\). Then, there must exist \(l\) within \(0 < l \leq k\) such that \(y^{l}_{ij}(0) = \cdots = y^{l-1}_{ij}(0) = 0\) and \(y^{l}_{ij}(0) = \cdots = y^{k}_{ij}(0) = 1\). Since we have assumed

\(^{27}\)Echenique (2005) also provides a related constructed proof for Tarski's fixed point theorem.
\( y^k(0) \geq y'(0), \) together with the weakly increasing assumption of \( F(\cdot), \) it must be that \( y^{k+1}(0) = F(y^k(0)) \geq y'(0) = F(y^{i-1}(0)). \) This contradicts \( y^{i+1}_y(0) = 0. \)

Given that the iteration is always weakly increasing, it is obvious that the algorithm converges within finite time, as the space of \( y \) is finite.

Next, we show that \( y^*(0) \) is the minimum equilibrium. Take any \( \tilde{y} \) such that \( \tilde{y} \neq 0. \) Note that by definition of \( 0, \tilde{y} \geq 0. \) Then, for any \( k, \) \( y^k(\tilde{y}) \geq y^k(0) \) since \( F(\cdot) \) is weakly increasing. By taking sufficiently large \( k \geq 2^{n(n-1)/2}, \) \( y^*(\tilde{y}) \geq y^*(0). \)

Note that any fixed point of \( F(\cdot) \) is included in \( y^*(\tilde{y}) \) by assigning \( y^*(\tilde{y}) \) as the initial point. Therefore, \( y^*(0) \) is the minimum equilibrium.

That the iteration starting from \( 1 \) converges within finite steps to the maximum equilibrium can be shown similarly.

Using the result of Corollary 1, we can compute the minimum and maximum equilibria within finite time. Of course converging within finite time is not an informative bound about the speed of convergence, and in general the speed is determined on a case by case basis. Also, this algorithm is analogous to the improving path defined by Jackson and Watts (2002), although their concept of "improving path" is not a simultaneous revision of links as in this paper, but rather a case where one link is updated at one iteration. But the more important difference between Jackson and Watts (2002) and our paper is again that Jackson and Watts (2002) does not provide a characterization of the equilibrium by complete lattice, and they provide entirely different sufficient conditions for existence.

### 3.A.2 Steps to obtain the confidence set of parameters

We describe a more detailed description of the procedure to obtain the confidence set reported in Table 3.C4 and Section 3.5. The notation follows the main part of the paper.

Step 1: Draw \( R \) set of error terms for all pairs of agents and for all networks. The \( r \)-th error term is denoted by \( E^{(r)} = \{E_1^{(r)}, \ldots, E_i^{(r)}, \ldots, E_s^{(r)}\}, \) where \( E_s^{(r)} = \)
Step 2: For each parameter value \( \theta \), simulate the maximum and minimum equilibrium following Corollary 1 for each \( E^{(r)} \) for \( r = 1, \ldots, R \) and for each \( s = 1, \ldots, S \). Denote them by \( \bar{g}(\theta, X_s, E_s^{(r)}) \) and \( \underline{g}(\theta, X_s, E_s^{(r)}) \), correspondingly.

Step 3: Obtain the simulated moment by

\[
\bar{m}_l(\theta, X_s, g_s, \{E_s^{(r)}\}_r) = \frac{1}{R} \sum_{r=1}^{R} h_l(\bar{g}(\theta, X_s, E_s^{(r)}), X_s) - h_l(g_s, X_s), \quad (3.8)
\]

\[
\underline{m}_l(\theta, X_s, g_s, \{E_s^{(r)}\}_r) = \frac{1}{R} \sum_{r=1}^{R} h_l(\underline{g}(\theta, X_s, E_s^{(r)}), X_s) - h_l(g_s, X_s), \quad (3.9)
\]

Step 4: By noting that \( \theta_0 \in \Theta_I \) satisfies

\[
E[\bar{m}_l(\theta_0, X_s, g_s, \{E_s^{(r)}\}_r)] \geq 0,
\]

\[
E[\underline{m}_l(\theta_0, X_s, g_s, \{E_s^{(r)}\}_r)] \leq 0, \quad (3.10)
\]

we compute the test statistics \( T(\theta) \) and the estimated critical value \( \hat{c}(\theta, 1 - \alpha) \) for the null hypothesis that \( \theta \in \Theta_I \), where \( \alpha \) is the nominal level of significance, following Andrews and Soares (2010). Note that \( T(\theta) \leq \hat{c}(\theta, 1 - \alpha) \) implies that \( \theta \in \Theta_I \) is not rejected at the significance level \( \alpha \). We construct the critical value \( \hat{c}(\theta, 1 - \alpha) \) by a nonparametric bootstrap by redrawing \( \{X_s, g_s, \{E_s^{(r)}\}_r\}_s \). Since \( \{X_s, g_s\}_s \) and \( \{\{E_s^{(r)}\}_r\}_s \) are independent by assumption, we can redraw them independently.

Step 5: Obtain the maximum and minimum parameter values of each dimension of the parameter set among the parameter values that do not reject the null hypothesis.
For example, to obtain the minimum parameter value of dimension $j$,

$$
\min_{\theta} \theta^j \\
\text{s.t. } T(\theta) \leq \hat{c}(\theta, 1 - \alpha)
$$

Note that this is a constrained optimization problem, so a standard constrained optimization package can be used.

At this juncture, it is necessary to make several comments on the computational procedure and the use of Andrews and Soares (2010). Firstly, in Step 5, we essentially obtain the rectangle that covers the confidence set. Thus the reported interval is too conservative in some sense. However, there are 5 dimensions of parameters in our application, and showing the shape of the confidence set in high-dimensional space is neither possible for expository reasons and also computationally burdensome because this requires a grid search method in the entire parameter space. In this paper we then follow the procedure in Step 5.

Secondly, the use of Andrews and Soares (2010) is more computationally costly than other methods of inference of moment inequality models, such as Pakes, Porter, Ho, and Ishii (2015). The computational burden of Andrews and Soares (2010) arises from the fact that the critical value $\hat{c}(\theta, 1 - \alpha)$ is recomputed at each parameter value $\theta$ by removing moment conditions which are unlikely to be binding. Although this process is computationally burdensome, this procedure provides much less conservative estimates of the confidence set. For this reason we follow the procedure in Andrews and Soares (2010) in this paper.\(^{28}\)

Thirdly, we use simulated moments because moments are not obtained in a closed form. Although Andrews and Soares (2010) do not explicitly consider the case with simulated moments, the nonparametric bootstrap of redrawing $\{X_s, g_s, \{E_s^{(r)}\}_r\}_{s}$

\(^{28}\)There is also a conceptually different type of inference method which constructs the confidence set of the identified set $\Theta_I$, not that of the true parameter $\theta_0$, such as Chernozhukov, Hong, and Tamer (2007) or Beresteanu and Molinari (2008). See Imbens and Manski (2004) the difference between the two approaches.
could be seen as a special case of what they consider. To see this, note that the simulated statistics \( h_l(g(\theta, X_s, E_s^{(r)})) \) and \( h_l(g(\theta, X_s, E_s^{(r)})) \) enter linearly in the simulated moments. This implies that there is no simulation bias, i.e.

\[
E[\tilde{m}_l(\theta, X_s, g_s, \{E_s^{(r)}\}_r)] = E\left[ \frac{1}{R} \sum_{r=1}^{R} h_l(g(\theta, X_s, E_s^{(r)}), X_s) - h_l(g_s, X_s) \right],
\]

\[
= E[h_l(g(\theta, X_s, E_s), X_s) - h_l(g_s, X_s)],
\]

\[
= E[\tilde{m}_l(\theta, X_s, g_s)].
\]

This is a special case of McFadden (1989) and Pakes and Pollard (1989), where a consistent estimator is obtained with simulated method of moments with fixed simulation draws \( R \). In such a case, although the asymptotic distribution of the test statistics based on \( \tilde{m}_l(\theta, X_s, g_s) \) and \( \tilde{m}_l(\theta, X_s, g_s) \) are different, the nonparametric bootstrap that resamples \( \{X_s, g_s, \{E_s^{(r)}\}_r\}_r \) yields a consistent critical value of the latter.

3.B Proof that moments in Table 3.C1 satisfy Property 1

In this subsection of the appendix, we show that Property 1 is satisfied for \( h_l(\cdot, \cdot) \) in Table 3.C1.

Given the partial order defined in Proposition 5, it is trivial that Property 1 is satisfied for (a) the proportion of pairs with a link, (b) the proportion of pairs with a link between same \( x_{i,s}^p \) for \( p = 1, 2 \), (c) the proportion of pairs with a link between different \( x_{i,s}^p \) for \( p = 1, 2 \) and (d) the interaction of (a)-(c) and the dummy variable that takes 1 if the number of nodes is less than 30, because they are monotonic to the set inclusion of links.

For the rest of the moments, we first note that if \((i, j) \in \tilde{g}_s\) for some \( \tilde{g}_s \in G(\theta, X_s, E_s) \) and \((i, j) \not\in \tilde{g}(\theta, X_s, E_s)\), \((i, j)\) must be included in the triangle in \( \tilde{g}_s \), i.e., \( \exists k \) s.t. \((i, k), (j, k) \in \tilde{g}_s\). This is because if we suppose the contrary, we must
observe \((i, j) \in g(\theta, X_s, E_s)\) under the parametrization of the utility function. Using this result, it is immediate that the choice of \(h_l(\cdot, \cdot)\) of (e) to (h) satisfies Property 1, because whenever the denominator increases depending on different \(g_s \in G(\theta, X_s, E_s)\), the numerator also increases by the same amount. It follows that (e) to (h) also satisfy Property 1.
3.C Figures and Tables

Table 3.C1: Choice of Statistics $h_i(\cdot, \cdot)$ in the Friendship Network Application

<table>
<thead>
<tr>
<th>Choice of Statistics</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) proportion of pairs with a link</td>
<td>$(\sum_{i,j} y_{ij,s})/(n_s(n_s - 1)/2)$</td>
</tr>
<tr>
<td>(b) proportion of pairs with a link between same $x_{i,s}$ for $p = 1, 2$</td>
<td>$(\sum_{i,j} y_{ij,s} 1[x_{i,s} = x_{j,s}^p])/(\sum_{i,j} 1[x_{i,s} = x_{j,s}^p] - 1)/2)</td>
</tr>
<tr>
<td>(c) proportion of pairs with a link between different $x_{i,s}$ for $p = 1$</td>
<td>$(\sum_{i,j} y_{ij,s} 1[x_{i,s} \neq x_{j,s}^p])/(\sum_{i,j} 1[x_{i,s} \neq x_{j,s}^p] - 1)/2)</td>
</tr>
<tr>
<td>(d) interaction of (a)-(c) and the dummy variable that takes 1 if the number of nodes were less than 30</td>
<td></td>
</tr>
<tr>
<td>(e) proportion of links belonging to triangles out of all links</td>
<td>$\sum_{i,j} 1[\exists k \text{ s.t. } i,j,k \text{ and } y_{ik,s} = y_{ik,s} = y_{jk,s} = 1] / \sum_{i,j} y_{ij,s} = 0, \ 1 \text{ otherwise}$</td>
</tr>
<tr>
<td>(f) proportion of nodes belonging to triangles out of all nodes with at least one link</td>
<td>$\sum_{i} 1[\exists j \text{ s.t. } y_{ij,s} = 1] / \sum_{i,j} y_{ij,s} = 0, \ 1 \text{ otherwise}$</td>
</tr>
<tr>
<td>(g) proportion of nodes belonging to triangles out of all nodes with at least two links</td>
<td>$\sum_{i} 1[\exists j, k \text{ s.t. } y_{ij,s} = y_{ik,s} = y_{jk,s} = 1] / \sum_{i} 1[\exists j \text{ s.t. } y_{ij,s} = y_{ik,s} = y_{jk,s} = 1] = 0, \ 1 \text{ otherwise}$</td>
</tr>
<tr>
<td>(h) proportion of links between same $x_{i,s}$ belonging to triangles out of all links for $p = 1, 2$</td>
<td>$\sum_{i,j} 1[\exists k \text{ s.t. } i,j,k \text{ and } y_{ik,s} = y_{ik,s} = y_{jk,s} = 1] 1[x_{i,s}^p = x_{j,s}^p] / \sum_{i,j} y_{ij,s} = 0, \ 1 \text{ otherwise}$</td>
</tr>
</tbody>
</table>
Table 3.C2: Monte Carlo Results: Size

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>(1) maximum</th>
<th>(2) minimum</th>
<th>(3) mixture</th>
<th>(4) initial random networks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>93.6%</td>
<td>91.4%</td>
<td>100.0%</td>
<td>96.2%</td>
</tr>
</tbody>
</table>

Note: The empirical size of the test is reported. The nominal size of the test is 95%, and the test is based on Andrews and Soares (2010). The number of agents and the distribution of the covariates (gender and race) are set to match the empirical proportion in the data (see Table 3.C3). The true parameter value is set at $\beta_0 = -1.4099$, $\beta_1 = 0.1873$, $\beta_2 = 0.2406$, $\gamma = 0.8850$. Each column indicates different true equilibrium selection rules. (3) mixture means a half of the networks are assumed to choose maximum equilibrium and the rest are assumed to choose minimum equilibrium. For (4) initial random networks, we first draw a network with $\frac{1}{2}$ probability of forming a link independently across all pairs, and then we run the algorithm specified in Corollary 1 for 1000 iterations. The number of independent networks are 150 and the number of Monte Carlo simulation draws is 1000.
<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>51.4832</td>
<td>30.1728</td>
<td>100.000</td>
<td>10.0000</td>
</tr>
<tr>
<td>Proportion of Male</td>
<td>0.4853</td>
<td>0.1029</td>
<td>0.8000</td>
<td>0.1818</td>
</tr>
<tr>
<td>Proportion of Whites</td>
<td>0.7220</td>
<td>0.2789</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>Proportion of Blacks</td>
<td>0.1602</td>
<td>0.2471</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>Average Degree</td>
<td>1.4868</td>
<td>0.7589</td>
<td>4.0000</td>
<td>0</td>
</tr>
<tr>
<td>Proportion of links between Males</td>
<td>0.3112</td>
<td>0.1786</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>Proportion of links between Whites</td>
<td>0.6784</td>
<td>0.3179</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>Proportion of links between Blacks</td>
<td>0.1115</td>
<td>0.2327</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>Clustering Coefficient</td>
<td>0.1568</td>
<td>0.1402</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>Sample Size</td>
<td></td>
<td></td>
<td></td>
<td>149</td>
</tr>
</tbody>
</table>
Table 3.C4: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_0$: Const.</td>
<td>[-1.7836, -1.3176]</td>
<td>[-1.5710, -1.2765]</td>
<td>[-1.9131, -1.5498]</td>
<td>[-1.8113, -1.3255]</td>
</tr>
<tr>
<td>$\beta_1$: Dummy (1: same gender)</td>
<td>[0.0891, 0.3277]</td>
<td>[0.1444, 0.2301]</td>
<td>[0.2563, 0.3589]</td>
<td>[0.0873, 0.2849]</td>
</tr>
<tr>
<td>$\beta_2$: Dummy (1: both Whites) Dummy (1: both Blacks)</td>
<td>[0.1655, 0.4327]</td>
<td>[0.1557, 0.6039]</td>
<td>[-0.0495, 0.4580]</td>
<td>[-0.0712, 0.4376]</td>
</tr>
<tr>
<td>$\gamma$: Dummy (1: exists mutual friends)</td>
<td>[0.7534, 1.3686]</td>
<td>[0.8850, 1.1328]</td>
<td>[0.7781, 1.2020]</td>
<td>[-0.174, 0.0487]</td>
</tr>
</tbody>
</table>

Number of Students / 100 | 149

Note: The rectangle that covers the 95%-confidence set is reported. The confidence set is computed following Andrews and Soares (2010) with the S1 test statistic and 1000 bootstrapped samples for columns 1, 2 and 4. To obtain the simulated moments, we draw 100 sets of simulated error terms. For column 3, we obtained estimates and from simulated method of moments following McFadden (1989) and Pakes and Pollard (1989).
Figure 3.C1: Monte Carlo Results: Power

(1) True Equilibrium Selection: Minimum Equilibrium

(2) True Equilibrium Selection: Maximum Equilibrium

(3) True Equilibrium Selection: Mixture of Minimum and Maximum Equilibrium

(4) True Equilibrium Selection: Equilibrium with Random Initial Networks

Note: The empirical power of the test at different null parameter values is reported. The nominal size of the test is 95%, and the test is based on Andrews and Soares (2010). The true parameter value is set at \( \beta_0 = -1.4099, \beta_1 = 0.1873, \beta_2 = 0.2406, \gamma = 0.8850 \). See the footnote of Table 3.C2 for more details about the test. Each graph reports the rejection probability when we vary one parameter value fixing the other parameter values as the true parameter values. For example, the line "beta0" reports the rejection probability of the null at the true \( \beta_1, \beta_2 \) and \( \gamma \), but at different \( \beta_0 \) by the amount specified in the horizontal axis. Each graph reports the rejection probability when we vary one parameter value fixing the other parameter values as the true parameter values. For example, the line "beta0" reports the rejection probability of the null at the true \( \beta_1, \beta_2 \) and \( \gamma \), but at different \( \beta_0 \) by the amount specified in the horizontal axis. The actual calculation of the power is conducted at every 0.01 interval in \([-0.05, 0.05]\), and 0.1 interval in \([-1.0, -0.1]\) and \([0.1, 3.0]\).
Figure 3.C2: Graph of the network of school 121, grade 8

Note: All the nodes connected to at least one node in grade 8 of school ID 121 are shown. Pink, white, black nodes represent Hispanics, Whites, and Blacks, respectively. The picture is drawn using the software Cytoscape. For more about the software, see http://www.cytoscape.org/.
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