Data set contents

Title: Data for *An optimal uncertainty principle in twelve dimensions via modular forms*

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List of files, types, and sizes:

File name	Type	Size
`roots.txt`	plain text	133.6 MB
`coefficients.txt`	plain text	133.6 MB

Notes:

This data set includes all the numerical data referred to in the paper *An optimal uncertainty principle in twelve dimensions via modular forms* by Cohn and Gonçalves (available on the arXiv with arXiv ID 1712.04438, at the URL `https://arXiv.org/abs/1712.04438`).

Specifically, the file `roots.txt` includes the roots $\rho_0, \dots, \rho_k$ for the numerically computed functions from Section 4. In the notation from this section, d is the dimension, s is the eigenvalue ± 1, and k is the number of forced double roots. In the file, a line such as

`s = 1, d = 12, k = 10:`

is followed by the roots $\rho_0, \dots, \rho_k$ corresponding to these parameters, with each root on a line by itself. Note that our normalization for the roots is the one from the beginning of Section 4, not Section 5.

The files contain data for the following parameter ranges:

s	d	k	s	d	k
1	1	1, 2, 5	−1	1	1–64
1	2	—	−1	2	1–64
1	3	1–27	−1	3	1–20, 26–31
1	4	1–35	−1	4–128	1–64
1	5–128	1–64			
1	28	1–128			

The file `coefficients.txt` includes the corresponding coefficients of the summation formulas from Section 5, normalized to work with the Laguerre eigenbasis rather than just polynomials (as is done in Section 5). Specifically, let $\psi_0, \psi_1, \dots$ be the Laguerre eigenbasis in $\mathbb{R}^d$, as in equation (3.3) from the paper, so that ψ_n is an eigenfunction with eigenvalue $(-1)^n$. Let V be the space spanned by the first $2k + 2$ of these functions that have eigenvalue s. In this framework, Proposition 5.1 amounts to constructing coefficients $a_0, \dots, a_k$ such that for all f in V,

$$a_0 f(\rho_0) + \cdots + a_k f(\rho_k) = s f(0).$$

The reason for this normalization is that it corresponds with Conjecture 2.5 in the limit as $k \to \infty$. The file `coefficients.txt` lists these coefficients in exactly the same format as `roots.txt` lists the roots.

Our numerical calculations are subject to floating-point error and have not been rigorously verified. We have reported all results to at least 100 decimal places, and we expect that all of the first hundred digits are accurate, aside from rounding the output. In most cases we have reported additional digits, to assist in verification and further exploration. We expect the additional digits are generally accurate, but we are a little less confident in them.