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Mismatched Front and Back Gratings for Optimum Light Trapping in Thin Crystalline Silicon Solar Cells

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ABSTRACT

The implementation of a front and back grating in thin photovoltaic cells is a promising approach towards improving light trapping. A simple design rule was developed using the least common multiple (LCM) of the front and back grating periods. From this design rule, several optimal period combinations can be found, providing greater design flexibility for absorbers of indirect band gap materials. Using numerical simulations, the photo-generated current (J_{ph}) for a 10- μm -thick crystalline silicon absorber was predicted to be as high as 38 mA/cm^2 , which is 11.74% higher than that of a single front grating ($J_{ph}=34 \text{ mA}/\text{cm}^2$).

KEYWORDS : Light trapping, double grating, crystalline silicon, solar cell, thin absorber

1. Introduction

Solar photovoltaics (PV) technology has advanced considerably in recent years with current state-of-the-art single junction PV cells exhibiting efficiencies approaching the Shockley-Queisser limit.¹ The need to reduce costs have spurred the development of thin-film PV devices. Conventional bulk crystalline silicon (c-Si) PV cells typically have device thicknesses of 140-180 μm and require a 300- μm -thick wafer to produce one cell due to kerf losses associated with Si boules dicing. Including kerf losses, raw material costs have been shown to contribute over 30% of total module cost in conventional PV cells.²⁻⁴ To produce thin c-Si cells without excessive material waste, new manufacturing technologies have been recently developed including silicon epitaxial growth and direct wafering.⁵⁻⁸ Despite the cost benefits, the combination of an indirect band gap and thinness of the absorber make thin c-Si cells a poor absorber of near-infrared light, leading to significant photocurrent losses. Therefore, light trapping is paramount for thin c-Si PV cells to approach the Shockley-Queisser limit.

In order to enhance light-trapping performance, a myriad of structures using surface plasmons,⁹⁻¹¹ photonic crystals,¹²⁻¹⁴ or diffraction gratings¹²⁻⁴⁰ were proposed. In particular, diffraction gratings incorporated onto the front of PV cells have shown great potential by coupling incident light into guided modes in the thin film. Additionally, recent studies have shown that triangular, pyramidal, or conical gratings exhibit higher absorption due to the impedance match from air to silicon.^{15-22,28,30,39} To improve light trapping even further, the inclusion of a back grating structure has been studied as a means to better scatter near-infrared light into guided resonances.^{12-14,17,27-28,33} By combining a front and back grating, recent studies have demonstrated enhancement to light trapping that outperforms absorbers that are textured only on one side.²⁷⁻³⁹ Further improvements for these double grating structures have included the

introduction of a phase shift through misalignment of two gratings and the use of blazed gratings to break symmetry.^{30,38-40} To optimize the double grating structure, a general design strategy was proposed in which the front grating is treated as an anti-reflection layer to couple incident light while the back grating diffracts light into guided modes to trap it within the thin film.^{28,33} The periodicities of both the front and back grating are therefore crucial to the overall performance. However, as these two gratings serve different purposes, they were typically optimized separately in prior studies. For instance, the optimization of the front grating requires striking a balance between larger in-coupling of incident sunlight and decreasing leakage of the guided modes.^{24-26,28,33} The period of the back grating is then tuned to further improve light trapping. However, such an optimization can converge to a local maximum, which is determined by the choice of the front grating. To avoid this potential pitfall and simultaneously reduce computational costs of optimization, we propose a more general optimization approach that relies on tuning the periods of two gratings relative to each other, resulting in a simple rule that can be used to design double grating structures to achieve higher absorption in indirect band gap materials.

In this work, we focus on developing an optimized double grating light-trapping structure for a 10- μm -thick c-Si PV cell. Using numerical simulations to solve Maxwell equations, we demonstrate that when a mismatch is introduced between the periods of two gratings, the back grating will diffract incident light into guided modes that couple weakly to the diffraction orders of the front grating. This minimizes the number of incoming solar photons escaping solar cells without getting absorbed, thus improving photocurrent and cell conversion efficiency. By varying the degree of mismatch between the front and back gratings, the highest absorption occurs when the least common multiple (LCM) of the periods between the two gratings is large.

This approach offers more design flexibility and *yields multiple near-optimum designs*, which can be chosen based on the material and fabrication processes. Interestingly, since our approach reveals that the mismatch between two gratings is the most important parameter, optimum designs can have back gratings with either larger or smaller periods than the front grating.

2. Principle

A simple one dimensional (1D) grating based on triangular grooves, in Fig. 1(a), is chosen in this study to act as a 2D analog of inverted nanopylramids or nanocones, which was shown in previous studies to exhibit good light-trapping performance.^{15-22,27-28} By using this grating design, it is possible to apply well-established grating theory to better understand the underlying mechanisms and find a design rule to govern light trapping.

In general, gratings are characterized by their dispersion of light into discrete diffraction orders. These orders are determined by adding a characteristic grating momentum, which depends on the period, to the parallel momentum of incoming light,⁴⁰

$$\vec{k}_{2||} = \vec{k}_{1||} + \vec{G}_m \quad (1)$$

where $k_{1||}$ and $k_{2||}$ are parallel wavevectors of the incident and diffracted light respectively, and \vec{G}_m is grating momentum. The subscript \parallel corresponds to the in-plane direction and m is the index for the diffraction order. For the 1D grating in Fig. 1(a), the momentum equation can be simplified as

$$n_2 \frac{2\pi}{\lambda} \sin\beta_m = n_1 \frac{2\pi}{\lambda} \sin\theta + m \frac{2\pi}{W} \quad (2)$$

where λ is wavelength of light, θ is incident angle, β_m is the diffraction angle for order m , W is the period of the grating, and n_1 and n_2 are refractive indices of air and silicon, respectively. Each diffraction order corresponds to a particular β_m and depends on θ , W , λ , m , and n_2 . A drawing of each grating order is shown in Fig. 1(a).

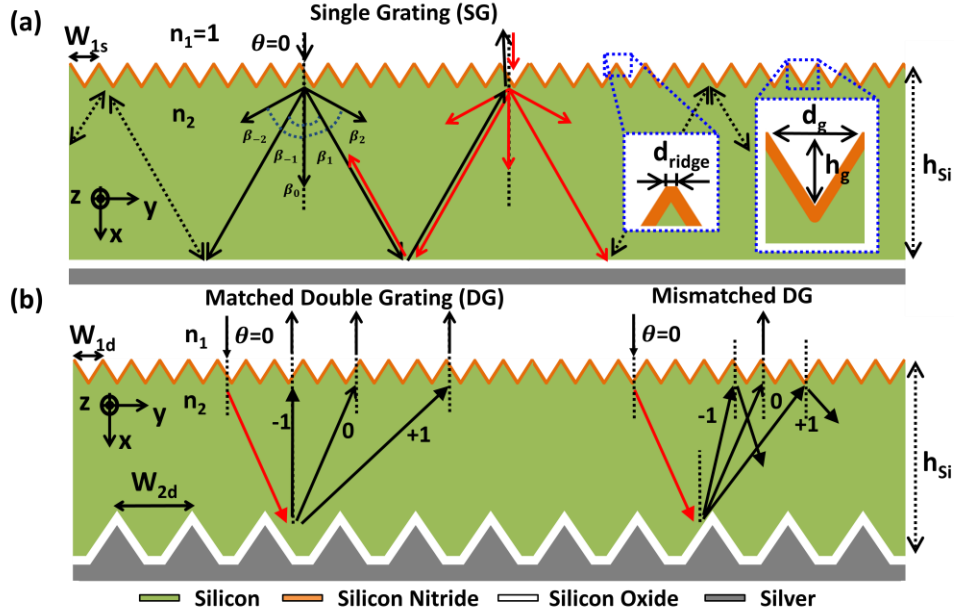


FIG. 1. A schematic of (a) the single grating (SG) case and (b) the double grating case illustrating the design of thin c-Si PV cells. In SG cases, the black and red arrows indicate the different diffraction light is coupled into c-Si by the front grating for the case where $W_{1s} = 700$ nm and $\lambda = 1000$ nm. In DG cases, two grating periods are considered and denoted as W_{1d} and W_{2d} for the front and back gratings respectively. The black and red arrows correspond to the diffraction orders at the wavelength of 1000 nm. Two cases are considered: the matched DG case where $W_{1d} = W_{2d} = 700$ nm and mismatched DG case where $W_{1d} = 700$ nm and $W_{2d} = 3100$ nm. The red arrows represent light at normal incidence diffracted into the 1st diffraction order ($m = +1$) by the front grating. The black arrows represent light diffracted by the back grating. Only the first three diffraction orders ($m = -5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5$) for this grating structure.

By using a grating to diffract incident light, the addition of the grating momentum along the film plane direction will lead to an increase in the optical path length of light in PV cells as waves propagate more parallel to the film. In addition, for thin c-Si cells, which support guided modes, the diffraction of light also provides greater opportunities to couple into these modes over a broad wavelength range, resulting in improvements in absorption.

In order to further improve light trapping, a second grating is introduced on the back of the PV cell with a period of W_{2d} , as shown in Fig. 1(b). The purpose of the back grating is to add

additional momentum to the diffracted light such that light reflected by the back metal reflector cannot couple back to air through the same diffraction channel. However, it is crucial to note that the addition of a second grating by itself may not lead to better light trapping as improvements will depend on the difference in grating periods, W_{1d} and W_{2d} . For example, if $W_{1d}=W_{2d}=700$ nm, which corresponds to the optimal grating period in the front single grating (SG) case,⁴¹ the front and back gratings support the same diffraction modes. In this matched double grating (DG) case, the dispersion of the front and back gratings as well as the waveguide modes within the c-Si PV cell are shown in Fig. 2(a). When incident light diffracts into the PV cell by the front grating, it can initially couple into waveguide modes. However, once light reaches the back metal reflector, it is diffracted into the same diffraction order as the front grating, as indicated by the overlap in the dispersion of both grating orders in Fig. 2(a), and it can thus couple back into air. As in the front SG case, unless light is fully absorbed within two passes through the PV cell, these leaky guided modes will allow light to couple back into air.

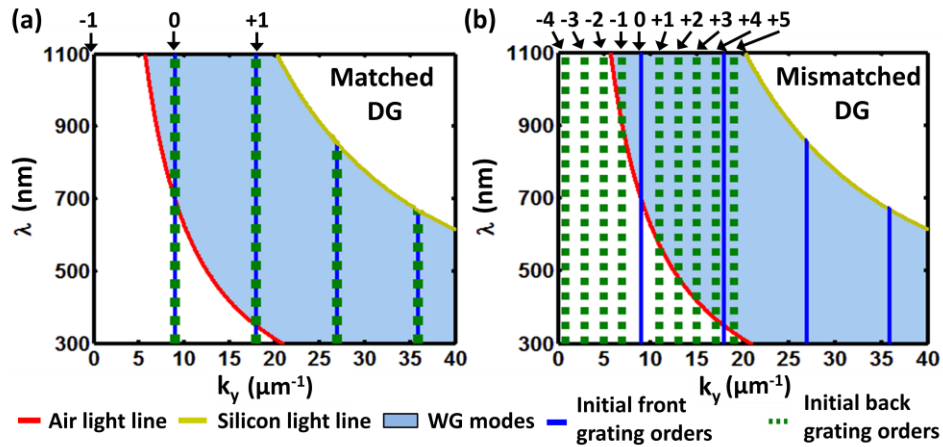


FIG. 2. Waveguide dispersion and diffraction orders are shown in (a) for the matched DG case and (b) for the mismatched DG case. The light line for air is shown in red and light line for c-Si is shown in yellow. The thickness of c-Si is much larger than wavelengths of incident light, thus the density of waveguide modes (WG) is high and their dispersion appears as a continuous light blue region.⁴¹ Diffraction orders of the front and back gratings are shown by the blue vertical lines and dashed green vertical lines, respectively. For simplicity, the diffraction orders shown

correspond to light initially incident on the front and back gratings since subsequent reflections within the c-Si film will result in higher diffraction orders. Furthermore, the diffraction orders of the back grating shown correspond to light incident on the back grating that was diffracted into the $m = +1$ order of the front grating.

On the other hand, if the periods of both gratings are not matched, light diffracted by the back grating can avoid coupling into diffraction orders of the front grating. For this mismatched DG case, if $W_{1d} = 700$ nm and $W_{2d} = 3100$ nm, which is a relatively strong mismatch, the dispersion of the grating orders will no longer overlap as shown in Fig. 2(b). As a result, incident light can strongly couple into the guided modes within the PV cell without coupling back into free space. Although it is possible for light to couple back into the front grating orders after multiple diffractions, the effective optical path length can be significantly improved resulting in greater light trapping.

3. Results and discussion

3.1 Numerical proofs for the proposed principle

To assess the light-trapping performance of this grating structure, *numerical finite element electromagnetic simulations (Wave-Optic Module in COMSOL Multiphysics®)* were performed to prove the proposed principle by simulating the SG, matched DG, and mismatched DG cases. As shown in Fig. 1, the system under consideration consists of a thin c-Si absorber with a thickness, h_{Si} , of 10 μ m. The triangular grating is used on both sides of PV cells with a height (h_g) to width (d_g) ratio of 0.707, which corresponds to the aspect ratio when monocrystalline {100} silicon is etched with an anisotropic etchant such as potassium hydroxide. A constant ridge width, d_{ridge} , of 50 nm, is included in all grating design to emulate realistic grating structures fabricated in previous studies.¹⁸⁻¹⁹ The total period, W_{1s} , thus equals $W_{1s} = d_g + d_{ridge}$. In addition, a 100-nm-thick silicon nitride antireflection layer is placed on the front of PV cells

to reduce reflection losses. On the backside, a 200-nm-thick silicon dioxide passivation layer and a 300-nm-thick silver back reflector are also included. In these simulations, Floquet boundary conditions are used to simulate an infinitely wide structure. The optical constants of materials were obtained from literatures.⁴¹ For all grating structures, light is assumed to be normally incident and unpolarized, which was calculated by taking the average of the transverse magnetic and transverse electric polarizations.⁴¹

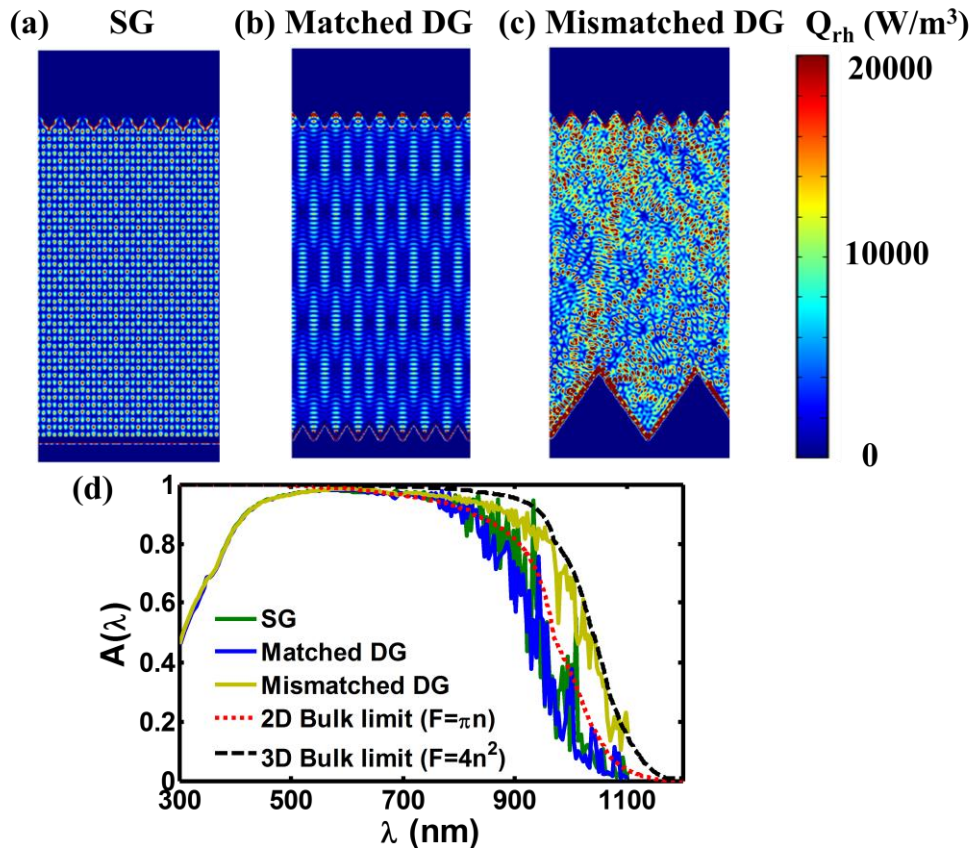


FIG. 3. The spatial distribution of the absorbed intensity, Q_{rh} , for (a) the SG case where $W_{1s} = 700$ nm, (b) the matched DG case where $W_{1d} = W_{2d} = 700$ nm, and (c) the mismatched DG case where $W_{1d} = 700$ nm and $W_{2d} = 3100$ nm. These Q_{rh} are shown at wavelength, λ , of 1000 nm and a integration bandwidth of 4nm. These pictures are cut, and pictures with their full simulation unit cell are shown in supplementary material.⁴¹ (d) A comparison of absorbance spectra for the SG, matched DG, and mismatched DG cases, as well as 2D and 3D bulk limits.

In Fig. 3, the spatial distribution of absorbed intensity, Q_{th} , at a wavelength of 1000 nm is shown to visually compare the effects of three cases, and the interference patterns between them are markedly different. In the front SG and matched DG cases, a well-defined interference pattern can be observed, which indicates that light diffracted by the front and back gratings are within phase of each other due to the overlap in diffraction orders. In contrast, in the mismatched DG case, Q_{th} is more spatially randomized with higher values compared to the other cases. This suggests greater absorption occurs in the mismatched DG case. In addition, absorptance spectra in c-Si for three cases are shown in Fig. 3(d). It can be observed that, in near-IR wavelengths ranged from 800 nm to 1100 nm, the spectral absorptance for the front SG case and the matched DG case are similar, which confirms the earlier intuition that the inclusion of a back grating with a period equal to the front grating does not improve light trapping. For the mismatched DG case, the spectral absorptance clearly exceeds the previous two cases.

3.2 Least common multiple (LCM) rule

Based on the spectral absorptance, the photo-generated current (J_{ph}), which is a more relevant parameter to evaluate the light-trapping performance of PV cells, is calculated using the following expression, $J_{ph} = \int_0^{\lambda_g} A(\lambda) I_{AM1.5G}(\lambda) \frac{q\lambda}{hc} d\lambda$, where A is spectral absorptance, $I_{AM1.5G}$ is intensity of AM1.5G solar spectrum, λ_g is the corresponding wavelength of bandgap, q is the elementary charge, h is the Planck constant, and c is the speed of light in vacuum.

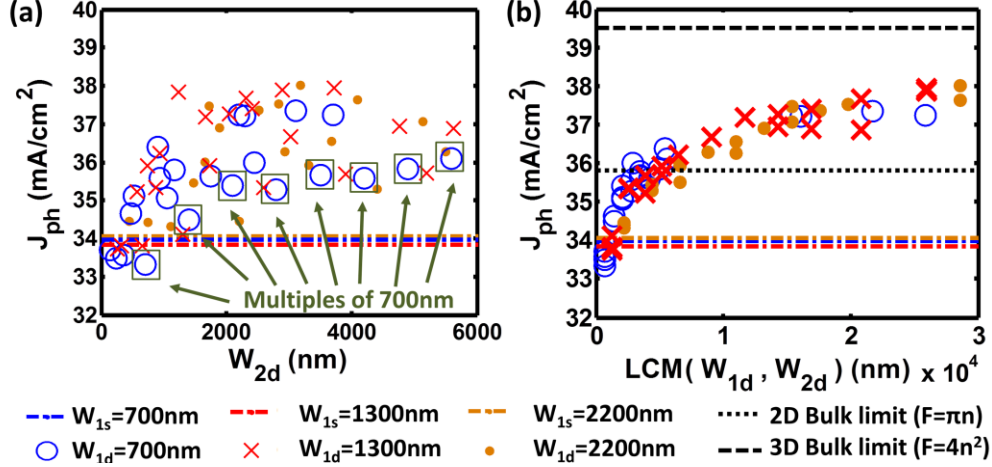


FIG. 4. Double grating (DG) results. Simulations were performed at normal incidence ($\theta=0^\circ$) and $h_{Si}=10\mu\text{m}$. (a) The photo-generated current (J_{ph}) as a function of the back grating period (W_{2d}) assuming front grating periods of $W_{1s} = 700$ nm, 1300 nm, and 2200 nm. (b) J_{ph} as a function of the least common multiple, $LCM(W_{1d}, W_{2d})$. In both (a) and (b), single grating cases with front grating periods of $W_{1s} = 700$ nm, 1300 nm, and 2200 nm and 2D and 3D bulk limits are shown for comparison.

Using the expression, J_{ph} is plotted as a function of the back grating period (W_{2d}) assuming the front grating period is $W_{1d} = 700$ nm, 1300 nm, and 2200 nm, as shown in Fig. 4(a) at normal incidence. The results for the front SG case assuming $W_{1s} = 700$ nm, 1300 nm, and 2200 nm are plotted for comparison.⁴¹ As shown, J_{ph} exhibits an overall increase with W_{2d} , but with an oscillatory behavior that exhibits local minima at multiples of W_{1d} . This suggests that, even with a large mismatch in the front and back grating periods, light can still efficiently couple into diffraction orders of the front grating if W_{2d} is a higher multiple of W_{1d} . For example, in Fig. 4(a), the case of $W_{1d} = 700$ nm has lower J_{ph} while W_{2d} is a multiple of W_{1d} .

To show the impact of this effect, J_{ph} is plotted as a function of the least common multiple (LCM) of W_{1d} and W_{2d} in Fig. 4(b). As shown, J_{ph} now exhibits a clear trend of monotonically exponential saturation. This is due to absorption occurring as an exponential decay in accordance to the Beer-Lambert law.⁴² To show this dependence, J_{ph} can be expressed in terms of the optical

path length of light, ℓ , as follows, $J_{ph} = \int_0^{\lambda_g} I_\lambda (1 - e^{-\alpha_\lambda \ell_\lambda}) \frac{q\lambda}{hc} d\lambda$, where I_λ is spectral intensity of light upon entering the absorber, and α_λ is the absorption coefficient. From this expression, J_{ph} will saturate when ℓ_λ is sufficiently large, which coincides with the trend shown in Fig. 4(b). Based on these results, the LCM can be used to determine the optimal mismatch between the front and back grating periods, thus providing a means to maximize light trapping in double grating systems. Based on the various cases in Fig. 4(b), the highest J_{ph} found at *normal incidence* is 38.02 mA/cm² corresponding to $W_{1d} = 2200$ nm, $W_{2d} = 3177.78$ nm ($W_{2d} = \frac{13}{9} W_{1d}$), and a LCM = 28600 nm. This value is nearly 4 mA/cm², or 11.76%, higher than the optimal front SG case ($J_{ph} \approx 34$ mA/cm²)⁴¹ and exceeds the 2D bulk limit ($F=\pi n$). However, it does not exceed the 3D bulk limit, which is also known as the Yablonovitch or Lambertian limit ($F=4n^2$), where absorptance spectra are given by $A(\lambda) = \alpha_\lambda h_{Si} / (\alpha_\lambda h_{Si} + 1/F)$ with F the enhancement factor.^{24-26,28,43} To properly compare our results with the bulk limits, angular dependence of these structures was also investigated in section 3.4. Additionally, it should be noted that an increase in the grating period will lead to greater removal of the absorbing medium and modification of guided modes from the planar thin film. Thus, it is possible to achieve different J_{ph} and deviate from the exponential saturation curve for the same LCM. For example, if $W_{1d} = 2200$ nm, $J_{ph} = 37.64$ mA/cm² when $W_{2d} = 4085.7$ nm ($W_{2d} = \frac{13}{7} W_{1d}$) and $J_{ph} = 38.02$ mA/cm² when $W_{2d} = 3177.8$ nm ($W_{2d} = \frac{13}{9} W_{1d}$), despite the fact that both cases exhibit an LCM equal to 28,600 nm.

3.3 Dependency on crystalline silicon film thickness

To understand the effect of the crystalline silicon film thickness (h_{Si}) for various least common multiple front and back gratings, additional simulations were performed for $h_{Si} = 2\mu\text{m}$, $5\mu\text{m}$, and $10\mu\text{m}$. The photogenerated current (J_{ph}) is plotted as a function of the least common

multiple parameter in Fig. 5. The front period is fixed to be $W_{1d} = 700\text{nm}$ for all three cases. At these three thicknesses, the photogenerated currents generally increase with their least common multiple following an exponential saturation trend. For the $2\text{-}\mu\text{m}$ -thick case, the J_{ph} decreases at a very large least common multiple due to excess removal of material from the silicon absorber leading to poor absorption. The single grating case with a front grating period of $W_{1s} = 700\text{ nm}$, the 2D bulk limit ($F=\pi n$), and the 3D bulk limit ($F=4n^2$) are also shown for comparison.

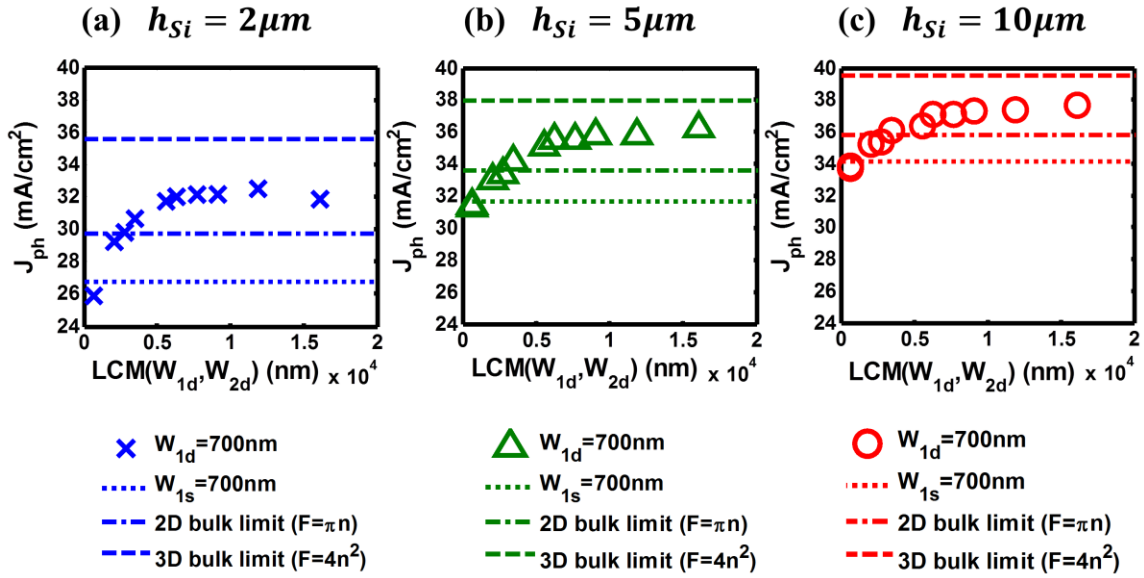


FIG. 5. Double grating (DG) results for various thickness of crystalline silicon absorber (h_{Si}). Simulations were performed at normal incidence ($\theta=0^\circ$) and $W_{1d}=700\text{nm}$. The photo-generated current (J_{ph}) as a function of the least common multiple, $LCM(W_{1d}, W_{2d})$ are shown with $h_{Si} = 2\text{ }\mu\text{m}$ in (a), $h_{Si} = 5\text{ }\mu\text{m}$ in (b), and $h_{Si} = 10\text{ }\mu\text{m}$ in (c). In (a), (b), and (c), the single grating cases with front grating periods of $W_{1s} = 700\text{ nm}$, 2D bulk limit ($F=\pi n$), and 3D bulk limit ($F=4n^2$) are shown for comparison.

3.4 Angular dependence

To compare the most optimal DG structure based on the LCM design rule ($J_{ph} = 38.02$ mA/cm², $W_{1d} = 2200$ nm, $W_{2d} = 3177.78$ nm ($W_{2d} = \frac{13}{9} W_{1d}$), and LCM = 28,600 nm.) to the 2D bulk limit,²⁴⁻²⁶ further simulations were performed to evaluate the absorptance for incident angles ranging from 0° to 90° as plotted in Fig. 6a. As shown, the SG, matched DG, and mismatched DG cases all exhibit a similar angular dependence, due to the same top grating structure, with a minor decrease in absorptance up to an angle of incidence about 50 degrees after which the absorptance decreases significantly. This sudden decrease is due to light at large angles of incidence observing the grating structure as a planar surface. Based on the aspect ratio of the triangular grating in our simulation, this transition occurs at approximately 54.73°. As a result, the angle-averaged absorptance is lower than at normal incidence, as shown in Fig. 6b. However, for the mismatched DG case, the angle-averaged absorptance can still achieve a significantly higher absorptance from 965 nm to 1100 nm compared to the SG and the matched DG cases. In fact, within these wavelengths, the angle-averaged mismatched DG case also exceeds the 2D bulk limit. To understand the reason, it is important to recall that the bulk limit was derived assuming the photon density of state (DOS) for a bulk medium. For a thin film, the DOS can deviate dramatically since the guided modes supported in the film exhibit a flat dispersion near the cut-off frequency of the mode.^{24,44-46} This can result in a photon DOS that exceeds the DOS of a bulk medium, thus enabling a thin PV cell to potentially exceed the ray optics or bulk limit.

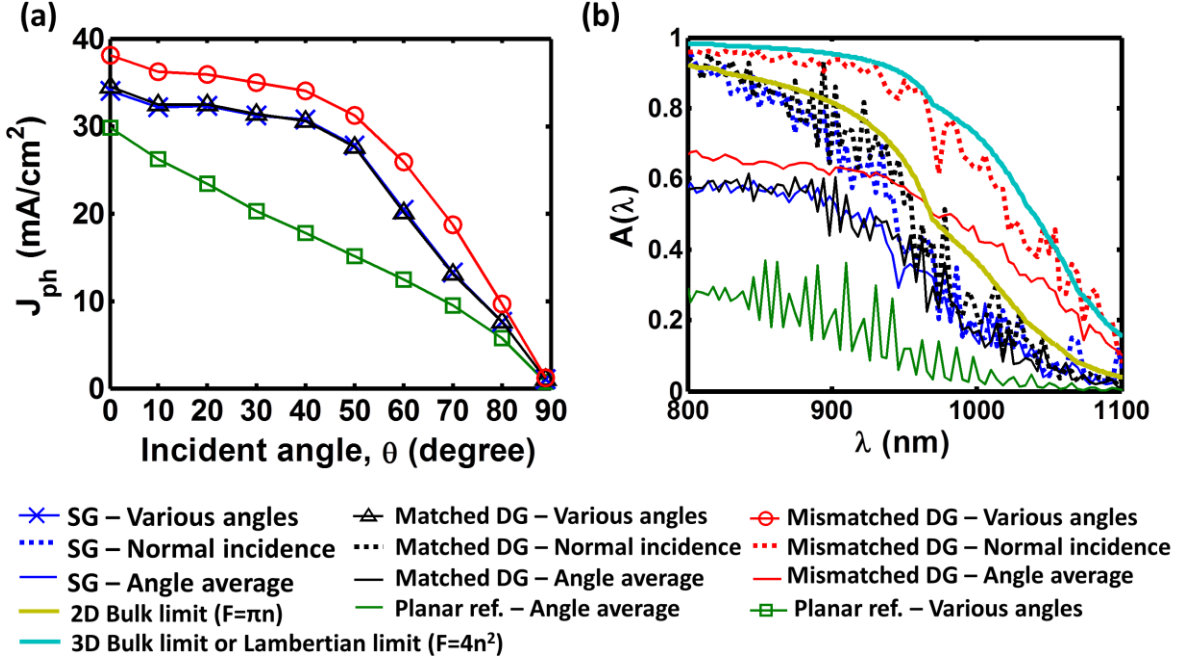


FIG. 6. (a) The angular dependence of J_{ph} for each case at various angles from 0° to 90° were computed for the SG case ($W_{1s} = 2200$ nm), matched DG case ($W_{1d} = W_{2d} = 2200$ nm), and mismatched DG case ($W_{1d} = 2200$ nm and $W_{2d} = 3177.78$ nm). (b) The angle-averaged absorptances of three cases were in comparison with the 2D bulk limit and 3D bulk limit (Lambertian limit).

3.5 Generalization of the LCM rule

This design methodology is also not restricted to only PV, but can be applied to any materials in different applications such as photodetectors. As an example, the LCM design rule was also applied to a 3- μ m-thick germanium (Ge) film, which is a common material used for near-IR photodetectors. In general, Ge weakly absorbs light in the 1.5 to 1.65 μ m wavelength range. By applying the same grating structure to this film, we show that the spectral absorptance can again be dramatically improved for an optimally mismatched DG structure in Fig. 7.

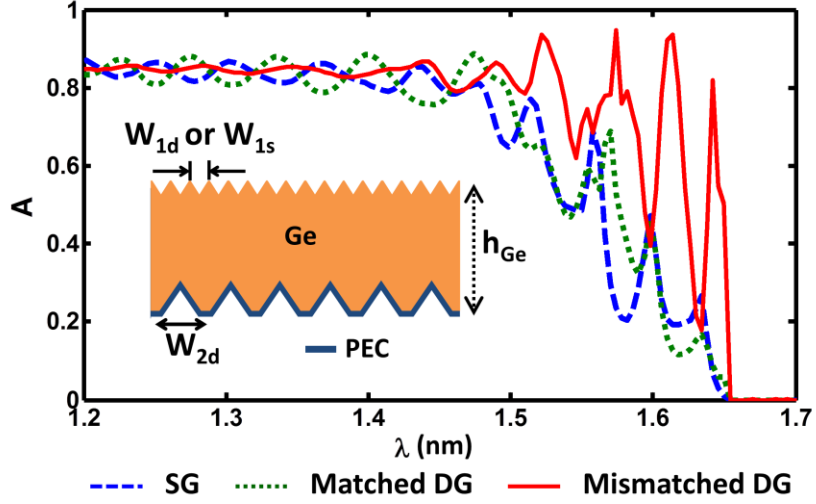


FIG. 7. The LCM design rule is general and can be applied to any material. As a demonstration, the LCM design rule is applied to a thin Ge film with a thickness of $h_{\text{Ge}} = 3 \mu\text{m}$ (inset). The absorptance spectra is shown for the SG case ($W_{1s} = 700 \text{ nm}$), the matched DG case ($W_{1d} = W_{2d} = 700 \text{ nm}$), and the mismatched DG case ($W_{1d} = 700 \text{ nm}$ and $W_{2d} = 1100 \text{ nm}$). PEC is the boundary condition of perfect electric conductor.

Another important point is that the LCM rule can be directly applied to 3D PV cells using 2D gratings. The improvement to light trapping that we propose is purely dependent on the ratio of the front and back grating periods. From Eq. (1), 2D gratings would provide lattice momentum along two orthogonal directions. This means that 2D gratings can be viewed as a combination of two 1D gratings. Thus, it is expected that the light-trapping performance for 3D PV cells using 2D grating structures on the front and back sides will also be significantly improved. One should be noted that the shape of the gratings such as a rectangle and trigon, the proposed least common multiple rule will also yield optimized optical designs. Different symmetries of the gratings (both in one and two dimensions) can serve as additional fitting parameters to produce even more optimal thin-film PV cell designs.

3.6 Limitation of the LCM rule

Direct implementation of the LCM rule yields a large LCM value for a double-grating design with two similar periods on the front and back gratings. However, there is a limitation

using LCM rule in this situation, which stems from the finite angle width ($\Delta\beta_m$) of the discrete grating channels. When the periods of the front and back gratings are very close to each other, the diffraction orders of the front grating will overlap with the diffraction orders of the back grating, causing the leakage of light and a breakdown of the LCM rule. The finite angle width of a grating channel can be calculated using Eq. (2). For example, a case with $W_{1s} = 700$ nm, $\lambda = 1000$ nm, $n_2 = 3.588$, and $m = -1$, and the $\Delta\beta_{-1} = (\beta_{-1})^+ - (\beta_{-1})^- = 42.60^\circ - 23.46^\circ = 19.14^\circ$.

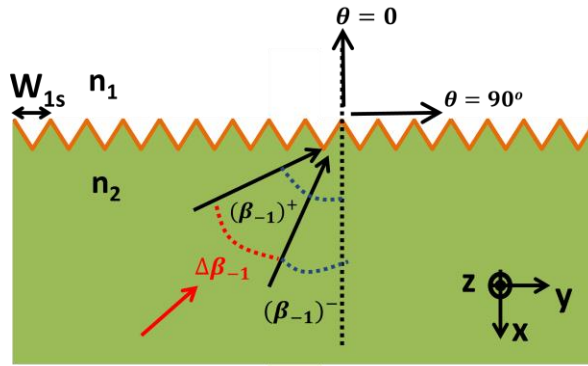


FIG. 8. The angle width ($\Delta\beta_m$) of the grating channel. The finite angle width of the grating channel will cause to the breakdown of the least common multiple rule when periods of the front and back gratings are too close to each other.

4. Conclusion

To summarize, light trapping and absorption in thin c-Si PV cells can be dramatically improved by using a front and back grating with mismatched periods. To facilitate the design of a double grating structure, we propose the least common multiple (LCM) design rule, which provides a simple method to determine the optimal mismatch between periods of two gratings in order to increase light absorption. To illustrate this rule, we optimized the front and back grating periods for a thin 10- μ m-thick c-Si PV cell and show that it yields several optimal designs, which deviate from previous studies that predict the existence of a single optimal design for a double grating structure.^{28,30} For example, a double grating structure designed with this rule is

predicted to increase the photo-generated current for a 10- μm -thick c-Si PV cell from ~ 34.0 mA/cm² for a single front grating to 38.02 mA/cm² for a double-grating case at normal incidence. Even when averaged over the incident angle from 0° to 90°, the optimal mismatched DG case can still achieve an absorptance that exceeds the fundamental 2D bulk limit in the wavelength range from 965 nm to 1100 nm. This design rule is general and can be applied to any indirect band gap material such as germanium, which is commonly used in near-infrared photodetectors. Given the simplicity of the proposed design strategy and its compatibility with standard manufacturing techniques, the mismatched double grating design can enable significant improvements to device performance as well as other optoelectronic devices such as photodetectors.

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- [41] Supplemental material. **Section I** shows absorption profile of 10- μm -thick c-Si and absorption depths of c-Si. **Section II** shows waveguide modes in crystalline silicon. **Sections III** shows optical constants of materials for simulated solar cells. **Section IV** shows absorptance of transverse magnetic and transverse electric polarized light. **Section V** shows profiles of absorbed intensity, electric fields, and magnetic fields for various light-trapping cases. **Section VI** shows all combination of W_{1d} and W_{2d} in Fig. 4.
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