DESIGN AND CONSTRUCTION OF AN ELECTRICALLY COMMUTATED A-C MOTOR WITH A FIXED TORQUE ANGLE

BY

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREES OF BACHELOR OF SCIENCE AND MASTER OF SCIENCE IN MECHANICAL ENGINEERING AT THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY - JULY 1974

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Submitted to the Department of Mechanical Engineering
July 10, 1974 in partial fulfillment of the requirements for
the degrees of Bachelor of Science and Master of Science in
Mechanical Engineering.

ABSTRACT

An electric motor which operates from a three-phase variable voltage power frequency A.C. source and has the control properties of an armature-controlled D.C. motor is designed, built and tested.

The torque angle is fixed (except for small oscillations) by the rectifying and commutating action of an SCR (Silicon Controlled Rectifier) circuit that connects the power lines to the stationary armature windings of the machine.

The SCR operation is controlled by trigger signals provided by a set of mechanical contacts that are switched by rotating carbon brushes in the shaft.

The motor provides full torque at speeds between zero and a fraction of the synchronous speed corresponding to the supply frequency. In operation, the speed of the motor adjusts linearly to the applied voltage with an in-phase current proportional to the shaft torque.

SCR performance is observed and analyzed at different motor speeds, armature currents and voltages. SCR failure under certain operating conditions is analyzed and suggestions are made to improve the design.


Title: Professor in Mechanical Engineering, MIT.
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ACKNOWLEDGEMENT

I want to express my gratitude to Professor J.L. Smith Jr., who has given me the opportunity of working as a research assistant in the Superconducting Generator Group in M.I.T. I also want to express my gratitude to him and to Dr. Tom A. Keim for their continuous interest and excellent advice that has guided my work and contributed to improving its quality.

The work reported in this thesis was done under the support of the Office of Naval Research, Contract N00014-67-A-0204-0068.

The suggestions made in different occasions by Professors Philip Thullen and J.I. Kirtley are much appreciated.

I also want to thank the technicians Mr. Carl Benner and Mr. Robert Gertsen for their help and advice in building the parts of our machine.

Finally I want to express my gratitude to Miss Ellen Kevorkian who has done an excellent job in typing the present document.

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Luis I. Cabezón G.
CHAPTER I

INTRODUCTION

I.1 Background and Objectives

The development of superconducting electric machines has motivated the development of various superconducting machines for application in systems which may be improved by the relative reduction of size and weight which are possible with these machines.

In ship propulsion applications, the advantages of superconducting machines apply not only in the generators but also in the propeller drive motors. This application requires good control characteristics similar to that achieved by a variety of D.C. motors. However, D.C. motors have some undesired characteristics, i.e. they require D.C. currents which cannot be generated as easily as A.C. currents, they have higher maintenance costs due to high current commutators, and they are limited in power and voltage by the commutator.

Thus, a superconducting A.C. motor with the control properties of a D.C. machine is highly desired. A first stage in the development of such a motor is the construction and testing of a normal conducting A.C. motor model in which the control properties are studied and developed. The present work is addressed to this problem.
I.2. Organization of the Thesis

In Chapter II, the concept to be used is explained in detail. As a first step the experimental motor is described in the context of a very quick introduction to D.C. motors and synchronous motors. The SCR commutating circuit, armature connections and idealized operation is explained in detail for the quasi-static rotation of the shaft. This chapter ends with an introduction of some important characteristics and properties of SCRs.

In Chapter III the design and construction of the actual motor is explained. It begins with the design of the SCR trigger circuit including some tests of the SCR performance with the proposed trigger circuit. The design and construction process of the signal commutator is explained in detail. This chapter ends with the description of the overall experimental installation, including pictures.

Chapter IV presents all the experimental results, beginning with the detailed description of armature magnetic flux, and verification of flux rotation caused by the commutation. SCR rectification process is also verified and compared with idealized rectification. Generated torques are observed next, including a brief explanation of reluctance torques. Also some undesired transient inductive effects during commutation are observed and eliminated. Section 5 of this chapter presents the results of motor operation under different conditions. SCR performance is
described in complete detail. Failure of these thyristors is observed under different circumstances. Causes for this malfunctioning are analyzed based on experimental information. The chapter ends with some proposed improvements and a preliminary study of some possible control actions over the transient response of the motor.
CHAPTER II

FIXING THE TORQUE ANGLE IN A THREE PHASE A.C. MOTOR. EXPLANATION OF THE CONCEPT

II.1. Preliminary Considerations

In a synchronous motor, the applied balanced three phase armature currents generate a magnetic field which rotates at synchronous speed. This speed is determined by the frequency of the armature current and the number of poles. In order to produce a steady electromagnetic torque, the magnetic fields of stator and rotor must be constant in amplitude and stationary with respect to one another. Thus, under normal operation, the synchronous motor runs at constant steady state speed and torque is determined by the currents and the angle between armature (stator) and rotor magnetic fields. It can be shown\(^1\) that for sinusoidal magnetic fields in the air gap, torque is given by the following alternative expressions:

\[
T = -\frac{P}{2} L_{sr} i_s r \sin \delta_{sr} \tag{2.1}
\]

\[
T = \frac{P}{2} \left( \mu_0 \Pi D l \right) F_s F_r \sin \delta_{sr} \tag{2.2}
\]

\[
T = \frac{P}{2} K F_s F_r \sin \delta_{sr} \tag{2.2a}
\]

\(^1\) *Electric Machinery*, Fitzgerald, Kingsley Kusko
in which:

\[ P = \# \text{ of poles} \]
\[ L_{sr} = \text{stator-rotor mutual inductance} \]
\[ i_s = \text{stator current} \]
\[ i_r = \text{rotor current} \]
\[ F_s = \text{peak value of stator m.m.f. wave} \]
\[ F_r = \text{peak value of rotor m.m.f. wave} \]
\[ \delta_{sr} = \text{electrical space phase angle between } F_s \& F_r \]
\[ D = \text{average diameter on air gap} \]
\[ l = \text{axial length} \]
\[ g = \text{air gap clearance} \]

There is no restriction that the m.m.f. wave or flux density wave need remain stationary in space. They may be stationary as in the case of a D.C. motor or they may be rotating as in the case of a synchronous machine.

It is a well-known fact that the rotor magnetic axis is kept very closely stationary in space in D.C. motors by means of commutators. This basic characteristic of this type of motor makes it possible to control the torque independently from the speed, i.e. for a fixed \( \delta_{sr} \) (since the field magnetic axis is also stationary in space), the torque depends only on armature and field currents, and the speed principally on the armature applied voltage.
In D.C. motors, a wide variety of operating characteristics can be obtained by selection of the method of excitation of the field windings. The method of excitation profoundly influences not only the steady state characteristics, but also the dynamic behavior of the machine. These are some of the outstanding characteristics of D.C. motors.

On the other hand, one of the disadvantages of these motors is the necessity of using power commutators to supply current to the rotating armature. Several complications arise from this fact which in general limits commutation capability of the device and also makes the mechanical design more complicated, hence more expensive.

We are not concerned here with the problem of studying in detail the complications and limitations of power commutators but instead we want to find an alternative way of fixing the torque angle $\delta_{sr}$ in the machine.

II.2. Proposed Method

A motor constructed by this proposed method would be classified in-between a typical D.C. and a synchronous motor.

In our motor, the field magnetic axis rotates at the shaft speed, and the armature magnetic axis is made to rotate at the same speed, with a nearly constant relative angle $\delta_{sr}$. Small oscillations of this angle will be seen to be due to geometric constraints, like the number of armature coils and their distribution.
Figure 2.1.a. Armature connection diagram for SCR-switched experimental motor.
Figure 2.1.b. Armature coils connection diagram. Coils have been laid out flat. There is a total of 42 coils. Points labeled 1 to 7 correspond to armature terminals.
Since the torque angle is fixed independently of the speed of the motor, torque is controlled by armature and field currents only. In the armature, counter e.m.f. is proportional to speed and since it should balance the applied voltage minus the resistance voltage drops, it is clear that the speed is determined by the applied armature voltage as it happens in D.C. motors. But as opposed to that type of machines, our motor is operated from a 60-cycle three phase power source and it does not have a power commutator.

The SCR (Silicon Controlled Rectifier) commutating circuit connects the power line to the stationary armature winding of the machine. Fig (2.1.a) shows a schematic diagram of armature terminals whose coils are shown in a simplified manner in the center. Each straight line crossing the center corresponds to a group of six coils whose resultant magnetic axis is perpendicular to it. Note that the axial direction of the rotor is a perpendicular line to the plane of the paper. Fig (2.1.b) shows armature coils and their connections in detail, laid out flat.

Successive groups of SCRs are triggered following the direction of rotation of the shaft so that for a given position of the rotor, a rectified current will flow between the corresponding armature terminals.

The triggering sequence is shown in Table 2.1. In the table, not all the stages are shown but it may be seen how to
TABLE 2.1

Triggering Sequence

<table>
<thead>
<tr>
<th>Stage Number</th>
<th>SCR Triggered</th>
<th>Current into Armature in Terminal</th>
<th>Current out of Armature in Terminal</th>
<th>Armature Magnetic Field Axis</th>
<th>Rotor (Field) Magnetic Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1A 1B 1C A5 B5 C5</td>
<td>5</td>
<td>1</td>
<td>0-12.85°</td>
<td>90→96</td>
</tr>
<tr>
<td>2</td>
<td>1A 1B 1C 2A 2B 2C A5 B5 C5</td>
<td>5</td>
<td>1-2</td>
<td>6.42°</td>
<td>96→97</td>
</tr>
<tr>
<td>3</td>
<td>2A 2B 2C A5 B5 C5</td>
<td>5</td>
<td>2</td>
<td>12.85°-25.7°</td>
<td>102.85+</td>
</tr>
<tr>
<td>4</td>
<td>2A 2B 2C A5 B5 C5 A6 B6 C6</td>
<td>5-6</td>
<td>2</td>
<td>19.27°</td>
<td>108.80+</td>
</tr>
<tr>
<td>5</td>
<td>2A 2B 2C A6 B6 C6</td>
<td>6</td>
<td>2</td>
<td>25.7°-38.55°</td>
<td>115.7+</td>
</tr>
<tr>
<td>6</td>
<td>2A 2B 2C 3A 3B 3C A6 B6 C6</td>
<td>6</td>
<td>2-3</td>
<td>32.12°</td>
<td>121.6+</td>
</tr>
<tr>
<td>7</td>
<td>3A 3B 3C A6 B6 C6</td>
<td>6</td>
<td>3</td>
<td>38.55°-51.4°</td>
<td>128.55+</td>
</tr>
<tr>
<td>8</td>
<td>3A 3B 3C A6 B6 C6 A7 B7 C7</td>
<td>6-7</td>
<td>3</td>
<td>44.97°</td>
<td>134.47+</td>
</tr>
</tbody>
</table>
Figure 2.2. SCR characteristics.
Figure 2.3.a. Stage #1 equivalent circuit. Figure 2.3.b. Stage #2 equivalent circuit.
Figure 2.4. Voltage waveforms at different points of stage #1 equivalent circuit assuming ideal behaviour of SCRs.
obtain the rest of them. There is a total of 28 stages.

It can be seen that stage #1 is equivalent to stage #5, stage #9,....etc.

When SCRs are not triggered, they behave nearly as an open circuit (within operating region) and as X diodes when they are triggered. Fig. (2.2) shows the V-I characteristic for a typical SCR.

For example, under normal operation, in stage #1, the system is equivalent to the diagram in Fig. (2.3a).

Assuming ideal behavior of the system components, the voltages across the load and SCRs would be as shown in Fig. (2.4) for resistive armature impedance. The direction of the rectified current is shown in Fig. (2.3).

When switching from stage #1 to stage #2 a new group of SCRs is triggered (i.e.: SCRs labeled 2A, 2B, 2C). Now there are two parallel paths for the current: 5→1 and 5→2. Transient effects during the commutation process will be seen in Chapter 4. Rectified currents through the load in stage #2 are similar to those in stage #1 and assuming that our components are ideal and identical we observe symmetry in the two paths in stage #2, see Fig. (2.3.b).

In reality, this idealized symmetry does not exist and departure from this idealization will affect the SCR operation. These effects will be observed experimentally.

Stages #2, 4, 8, 10, etc. can be considered as intermediate positions during commutation, since they correspond to the instant
when the signal commutator brush is touching two consecutive ring segments. (Operation of commutator will be seen in Chapter 3).

In stage #3 we again have a situation similar to stage #1 with the rectified current now flowing in the armature from terminals 2 to 5. The corresponding armature magnetic field axis has rotated $360^\circ/144^\circ$ with respect to the direction in stage #1.

In stage #4, a new group of "in-coming" SCRs are triggered, with rectified currents coming in at 5 and 6 and going out of armature at 2. The only important overall effect is to make the armature magnetic axis rotate $25.71^\circ$ ahead of the previous position of stage #3.

Stage #5 is identical to #1 except that we have rotated $25.7^\circ$ since then.

In a complete rotation of the shaft, there are 28 different positions of armature magnetic axis and since the field coils magnetic axis rotates smoothly along with the shaft, if we trigger the SCRs in the right moment, we can maintain $\delta_{rs}$ equal to $90^\circ \pm 12.85^\circ$.

In Chapter 3, we will see how the trigger signals to the SCRs are controlled by a set of stationary commutator and slip rings that are switched by rotating carbon brushes in the shaft. It is important to mention at this point, that this commutator works at a signal level only and therefore all the complications of power commutators are avoided.

The rest of this chapter will introduce the characteristics and properties of the SCRs in order to have the appropriate tools
to make the actual triggering circuit design as shown in Chapter III.

II.3. Characteristics and Properties of SCRs

As is shown in Fig. (2.2) SCRs behave nearly as ideal diodes when triggered and as open circuits otherwise. But this is not the whole story, these devices are very sensitive to numerous effects and departure from ideal behavior should be studied in detail either analytically or experimentally. An exhaustive analytical approach for the design may become extremely tedious, sophisticated and time consuming, therefore our design procedure is mostly experimental. Nevertheless, preliminary qualitative studies are done to achieve a basic intuitive understanding of the SCR operation.

II.3.a. Ratings

There are several factors that should be taken into consideration when designing an SCR circuit, specifically the power dissipation capabilities in the junction region of the SCR. There are mainly five components of dissipation:

1. Turn on switching
2. Conduction
3. Turn off commutation
4. Blocking
5. Triggering
On-state conduction losses are the major source of junction heating for normal duty cycles and power frequencies. However, for very steep (high di/dt) current waveforms or high operating frequencies turn-on switching losses may become the limiting consideration. Both, the on-state and reverse blocking losses are determined by integration of the appropriate E-I curves.

II.3.a.1. Current Ratings

Current ratings are selected according to the maximum allowable case temperature for the particular S.C.R., which is shown as a function of the average current rating in the specification sheets.

In the event of a type of overload or short circuit, the rated junction temperature can be exceeded for a brief instant, thereby allowing additional current rating. Ratings for this type of non-recurrent duty are given by the surge current and $I^2t$ rating curves.

Off-state blocking capability, $dv/dt$ and turn-off time, to name just a few device parameters, are not specified or guaranteed immediately following device operation in the non-recurrent current mode.

For irregularly shaped power pulses, an approximate method is used and the actual waveshape is converted into a rectangular form for which standard rating data can be found in specification sheets.
In addition to the current ratings, the di/dt ratings may become an important factor, for example, in cases where the rate of rise of anode current is very rapid compared to the spreading velocity of the turn-on process across the junctions, local "hot spot" heating will occur, due to high current density in those junction regions that have started to conduct. This is how turn-on switching dissipation in localized regions of the SCR may lead to an excessive temperature rise at a "hot spot" exceeding the device temperature rating.

II.3.a.2. Voltage Ratings

The peak off-state blocking voltage $V_{DRM}$ is given in the specification bulletin at maximum allowable junction temperatures (worst case) with a specified gate bias condition. The SCR will remain in the off-state if its peak off-state voltage rating is not exceeded and the gate is not triggered. However, a high rate of rise of off-state (anode-triggered-cathode) voltage may cause an SCR to switch into the "on" or low impedance conducting state. In practical applications, the maximum $dv/dt$ applied to a SCR can be limited by means of adder suppressor networks placed across a device's terminals.

II.3.a.3. Holding and Latching Currents

The SCR requires a certain minimum anode current to maintain it in the closed or conducting state. This minimum level is called holding current. Also, there is a minimum
principal current required to maintain the SCR in the on-state immediately after switching from the off-state to the on-state has occurred and the triggering signal has been removed, this current is the latching current.

This effect was seen in our circuit i.e. being in stage #1 (see Table 2.1) the gate signals were removed from SCRs labeled A5, B5 & L5 but it was found that still one of them failed to turn off. The reason was that due to the inductive-resistive type of load in armature coils, the current did not instantly go below the latching current value even though the voltage across the load was zero, hence the SCR that was conducting in that instant remained in the on-state.

II.3.b. Gate Triggering Characteristics

II.3.b.1. Effect of Load Impedance Over Triggering

In our particular application, we definitely need the simplest and most inexpensive type of gate trigger circuit, since we have 42 SCRs to trigger. A passive network that might provide a potential higher reliability would be desired.

In the previous sections, we have mentioned some considerations about the overall SCR ratings that should be taken into account in a preliminary SCR circuit design.

Similarly we will see now the most important triggering and gate characteristics looking forward to making an experimental type of design for the trigger circuit.
Figure 2.5. Effect of gate current $I_g$ over anode-cathode impedance of an SCR.
Proper triggering of the thyristor requires that the source of the trigger signal should apply adequate gate current and voltage, without exceeding the thyristor gate ratings, in accordance with the characteristics of the thyristor and the nature of its load and supply.

The criteria for triggering depends upon the nature of the external anode circuit impedance and the supply voltage, as well as the gate current. To visualize the process, let us look at Fig. (2.5).

$I_A$ is the short circuit load current, $V_L$ is the open circuit supply voltage. It is seen that the I-V characteristic of the SCR changes with gate current $I_G$.

For $I_G = 0$ the load line intersects the SCR I-V line at the stable point (1) and the device is in the off-stage. When the gate current is changed to $I_{G1}$, the load line becomes tangent at the unstable point (2) in which the negative resistance of the SCR is equal in magnitude to the external load resistance. The SCR then switches to the stable operating point (3) corresponding to the on-state, in this point, $I_G$ can be reduced to zero without altering conduction.

To turn off the device it is necessary to shift the load line to make it tangent to the I-V line of the SCR in the region where the curve bends upwards for the given $I_G$, the state would then be unstable and the SCR would switch to the off-state at the stable intersection of the two lines near the horizontal axis (i.e: points (1) (7) (8) & (9) for each case).
Figure 2.6. Gate equivalent circuit with anode disconnected.

Figure 2.7. Gate characteristics after triggering.
We can see that turning-on is made easier by increasing $I_G$, but obviously there is a maximum safe value for this current for each specific SCR.

Shifting the load line can be done either by increasing the load resistance or decreasing the applied voltage, holding current corresponds to the value of $I_{A\text{node}}$ just above the unstable tangent point, i.e. points (4)(5) or (6).

Trigger circuits should be designed to produce proper current flow between the gate and cathode terminals of the SCR. The gate to cathode impedance is then a determining factor in circuit design.

Manufacturers suggest different gate-cathode equivalent circuits for the conventional SCR, but they change radically according to the operating state of the device.

For example, when there is no anode current, i.e., when we want to examine the reverse gate bias or when we want to see the trigger circuits with anode disconnected, the gate equivalent circuit and gate cathode characteristic curve would be as shown in Fig. (2.6).

At triggering point $I_A$ is a function of $I_G$ and we will find that there is a maximum gate supply current ($I_{GT}$) required to trigger.

After the SCR has been triggered, gate impedance changes, it now behaves like a source having a voltage equal to the gate-cathode junction drop (at the existing anode current), and an internal impedance ($R_I$). Gate characteristics after triggering look like Fig. (2.7).
II.3.b.2. Effect of Gate-Cathode Trigger Circuit Impedance Over Operation of SCR

The relative effect of external gate-cathode resistance \( R_{GKe} \) is dependent upon the internal resistance \( R_L \) and \( R_S \). In general, \( R_{GKe} \) reduces sensitivity to triggering by thermally generated leakage currents, it slightly reduces the turn-off time, raises anode holding current and makes necessary to have higher anode current to initiate re-triggering.

A gate-cathode capacitance reduces sensitivity of SCR to \( dv/dt \) effects while maintaining high sensitivity to DC and low frequency gate signals. This is good when we want to eliminate the effects of high frequency noise, but it also will tend to retard the triggering process, action that can be detrimental if we want high \( (dI_A/dt) \).

Also, it can delay or even stop the turn-off process when gate signal is "removed" because it will provide a bias gate current when discharging.

Inductive reactance between gate and cathode reduces sensitivity to slowly changing anode current or gate source current, but maintains sensitivity to rapid changes. When the SCR anode current ceases, negative gate current will continue for a period of time decaying according to the \( L/R \) time constant. This current can reduce the turn-off time and also, faster rates of \( dv/dt \) can be applied. The inductive reactance also increases the holding current making turn-off easier to occur.
Positive gate current when we have a reverse anode-cathode applied voltage may increase the reverse (leakage) current, thus increasing the power dissipation in the device. Therefore this type of situation should be avoided if possible.

The gate to cathode voltage should never become more negative than is specified in the specification sheets. Serious consideration should be given to limit the gate power dissipation to within specified values, SCR gates are particularly sensitive and easy to damage.

Anode and gate circuits closely interact with each other and their interaction definitely affects triggering. These effects should be understood at least qualitatively before designing the trigger circuit, some of them are now mentioned:

i. Junction capacitance may couple high frequency signals from anode to gate, therefore affecting normal operation.

ii. During conduction, anode to gate voltage is almost zero, but during commutation from on-state to off-state, the gate goes through an intermediate phase which can result in a larger negative voltage appearing at the gate terminal. This negative transient at the gate can cause malfunction or damage in the external gate circuit. The trigger circuit should simultaneously supply adequate $I_G$ and its associated $V_{GK}$. 
CHAPTER III

DESIGN AND CONSTRUCTION

III.1. The SCR Trigger Circuit

III.1.a. Design Objectives

The trigger circuit should be designed to fulfill the following idealized requirements as closely as possible:

1. Upon closing a "signal switch" (that will constitute the triggering signal), the SCR should conduct whenever the anode-cathode voltage $V_{AK}$ becomes positive.

2. For negative $V_{AK}$ the device should not conduct, this should be independent of the "signal switch."

3. With "signal switch" off, the SCR should remain in the off state independently of $V_{AK}$.

It is seen from these requirements that the "signal switch" needs to be operated in coordination with the shaft position. Each SCR has a "signal switch" and it is closed whenever the angular position of the shaft is at a given value.

It is evident that a certain type of commutator should be used. A detailed description of this device will be done in Section (3.B).
Figure 3.1. Proposed SCR trigger circuit. $R_1 = 1\, \text{K} \Omega$, $R_2 = 10\, \text{K} \Omega$, $R_3 = 100\, \text{K} \Omega$, $R_4 = 200\, \Omega$. 
III.1.b. Proposed Trigger Circuit

The circuit in Fig (3.1) was set up and tested.

R₁ is placed following the suggestion of the manufacturer in the specification sheets. Its purpose is to guarantee that the SCR will block rated voltage over its rated operating temperature range.

The purpose of R₄ and R₃ is to produce a voltage divider type of effect and hence to provide an adequate gate-cathode voltage. R₃ and R₂ should be big resistances to limit the "signal switch" and gate currents. The diode protects the gate from reverse anode voltages and negative gate currents. R₄ is a variable resistance whose optimal value was determined experimentally.

Test #1

This circuit was tested for a sinusoidal input of the form: \( V_S = V_{S0} \sin \omega t \), with \( V_{S0} \) varying from zero to 300 volts, at 60 cycles/second, and for R₄ ranging from 0.2 KΩ to 27 KΩ. The SCR is equivalent to the GE type C106D1 rated for up to 400 volts and 4 amperes (RMS). More detailed characteristics are listed in specification sheets in Appendix.

For this trigger circuit, the values given in Table (3.1) were measured as follows: for a given value of R₄, \( V_{S0} \) was varied from 0 to 300 volts, the value for \( V_{S0} \) at which the SCR conducts is observed and the rest of the quantities were measured at maximum \( V_{S0} \).
It was observed that $V_{GT}$ was independent of $V_{So}$ maximum. The optimal value for $R_G$ was 0.2K, under which the SCR began to conduct when $V_{Ak} = 8$ volts. Since we had a pure resistive load $R_L$, as soon as $V_S$ became negative, $I_{Ak}$ became zero and the SCR was turned off. If the "signal switch" is opened when $V_{Ak}$ is positive the SCR continues to conduct until $I_{Ak}$ goes to zero when $V_{Ak}$ does so. In other words, although the gate "trigger signal" is removed, the SCR is not turned-off as far as it has a positive $I_{Ak}$ imposed by the anode-cathode-load circuit. This fact will constitute a small limitation in the overall performance of the motor.

In the case of an inductive load it will occur that the turn-off will be delayed to even after $V_{Ak}$ becomes negative.

<table>
<thead>
<tr>
<th>$V_{max}$</th>
<th>$V_{GT}$</th>
<th>$V_{BG}$ (for $V_{max}$)</th>
<th>$R_4$</th>
<th>$I_2$</th>
<th>$P_G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>.9</td>
<td>68</td>
<td>27KΩ</td>
<td>6.8mA</td>
<td>.0521W</td>
</tr>
<tr>
<td>300</td>
<td>.9</td>
<td>90</td>
<td>19KΩ</td>
<td>9.0mA</td>
<td>.073W</td>
</tr>
<tr>
<td>300</td>
<td>.9</td>
<td>110</td>
<td>14KΩ</td>
<td>11.mA</td>
<td>.09W</td>
</tr>
<tr>
<td>300</td>
<td>.9</td>
<td>150</td>
<td>8KΩ</td>
<td>15.mA</td>
<td>.127W</td>
</tr>
<tr>
<td>300</td>
<td>.9</td>
<td>190</td>
<td>4.4KΩ</td>
<td>19.mA</td>
<td>.163W</td>
</tr>
<tr>
<td>320</td>
<td>.9</td>
<td>200</td>
<td>4.4K</td>
<td>20.mA</td>
<td>.172W</td>
</tr>
<tr>
<td>300</td>
<td>.9</td>
<td>280</td>
<td>0.2K</td>
<td>28.mA</td>
<td>.244W</td>
</tr>
</tbody>
</table>

$P_G = V_{GT} \times I_G$
Test #2

An important practical fact is that different units of the same type of SCRs are not exactly alike due to manufacturing reasons.

The purpose of this experiment is to examine the triggering sensitivity of different SCRs as a function of $R_4$.

First, we proceeded to select 3 SCRs that proved to trigger at 3 different levels of $V_{AK}$ for the same triggering circuit.
(i.e. with $R_4 = 0.2 \text{K}\Omega$, SCR #1 triggered at $V_{AKT_0} = 9.5$ volts, SCR #2 at $V_{AKT_0} = 21$ volts and SCR #3 was turned on at $V_{AKT_0} = 29$ volts).

Next, we observed $V_{AK\text{trigger}}$ for each of them as a function of $R_4$. The results are shown in Table (3.2).

$V_{AK\text{trigg}}$ is a linear function of $R_4$ for each SCR, it is given by:

$$V_{AKT_j} = V_{AKT_0j} + m_j \times R_4 \quad (j=1,2,3)$$

$V_{AKT_j}$ is the anode-cathode voltage required to trigger SCR #j as a function of $R_4$ for our given trigger circuit (Fig. 3.1) in which:

$$m_1 = 0.85 \text{ volts/K}\Omega$$

$$m_2 = 2.1 \text{ volts/K}\Omega$$

$$m_3 = 3.1 \text{ volts/K}\Omega$$
Table 3.2 Results for Test #2

<table>
<thead>
<tr>
<th>$R_4$ (KΩ)</th>
<th>$V_{AKT1}$ (volts) (for SCR #1)</th>
<th>$V_{AKT2}$ (for SCR #2)</th>
<th>$V_{AKT3}$ (for SCR #3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>9.5 ($V_{AKT0}$)</td>
<td>21 ($V_{AKT0}$)</td>
<td>29 ($V_{AKT0}$)</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>22.5</td>
<td>32</td>
</tr>
<tr>
<td>2</td>
<td>11</td>
<td>24</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>30.5</td>
<td>44</td>
</tr>
<tr>
<td>10</td>
<td>18</td>
<td>42</td>
<td>60</td>
</tr>
</tbody>
</table>

Within the ranges of $R_4$ that we are interested in, $m_j$ can be approximated by the following linear equation:

$$m_j = \frac{V_{AKT0}}{10} \frac{\text{volts}}{\text{KΩ}}$$

Test #3

The objective of this test is to verify sensitivity and performance of the device for a step input $V_S$. In the previous tests, the applied voltages rose smoothly and the gate voltage varied accordingly until the SCR was triggered after which the gate voltage was not allowed to increase anymore, thus maintaining gate power dissipation at reasonable levels. In our application, "slowly" and smoothly rising voltages might not be always applied, and the gate might undergo undesirable high power transient in the instant before triggering.
Our test circuit is shown in Fig (3.1) but now our voltage source is a constant DC voltage $V_0$ applied at $t = 0$. $V_0$ ranged from 35 to 220 volts.

The circuit performed satisfactorily. The results are shown in Table 3.3.

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Initial Conditions (t=0-)</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Step input voltage $V_0$ with $35&lt;V_0&lt;220$</td>
<td>Off-state, &quot;signal switch&quot; off</td>
<td>Remains in off-state no damage to the system</td>
</tr>
<tr>
<td>2) Same as 1)</td>
<td>Off-state, &quot;signal switch&quot; on</td>
<td>SCR is triggered to the conducting state, no damage</td>
</tr>
<tr>
<td>3) Applied voltage is constant and &quot;signal switch&quot; turned on at (t=0+)</td>
<td>Off-state, &quot;signal switch&quot; is off, $V_S = V_0$</td>
<td>SCR is triggered and turned on, no damage</td>
</tr>
<tr>
<td>4) $V_S = V_0$ and &quot;signal switch&quot; is turned-off at (t=0+)</td>
<td>On-state, &quot;signal switch is on, $V_S = V_0$</td>
<td>Remains in on-state, no damage</td>
</tr>
<tr>
<td>5) $V_S = 0$</td>
<td>On-state, $V_S = V_0$, &quot;signal switch&quot; off</td>
<td>SCR is turned to the off-state, no damage</td>
</tr>
</tbody>
</table>
Figure 3.2. Stage #4 equivalent circuit. Switches represent brush action in signal commutator (they are in the "on" position in state 4).
The SCR circuit shown in Fig (2.1.a) was built, with each SCR having its own gate trigger circuit. Some small modifications were made and a more detailed diagram is shown in Fig. (3.2) in which only the SCRs that come into action when the motor is in stage #4 are shown. Resistors $R_S$ take care of limiting the armature current during transients in experimental tests. They are not part of the final design since they make the motor less efficient and alternative ways of limiting armature currents during transients may be used. (See Section IV.2).

The purpose of having the resister $R_K$ is explained in Section IV.4.

Resistors $R_3$ were eliminated for out-coming SCRs, (i.e. nA, nB, nC; n = 1,.....7) to eliminate a small gate signal that would exist due to a voltage divider effect between two phase lines.

Fig. (3.3) shows the armature equivalent circuit when the motor is in stage #2.

III.2 Design and Construction of Commutator

The commutator takes care of turning on and off the "signal switches" for the proper SCRs at the proper time.

It consists of a stationary set of rings and segmented rings held in the inside surface of an isolating tube (Part #1 in Fig. 3.5). Inside the tube rotates a co-axial cylinder rigidly connected to the shaft of the motor (Part #2 in Fig. 3.5). Radially oriented carbon brushes are held in the rotating member
Figure 3.3. Stage#2 equivalent circuit.
Figure 3.4. Developed view of rings and brushes. A, B and C refer to power line phases; A1, A2, etc. refer to the gate of the SCR between phase A and armature terminal 1, 2, .. etc., segments labeled 1 to 7 refer to armature terminals. The shadowed rectangles represent rotating brushes.
Figure 3.5. Commutator design. For simplicity only one brush is shown.
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and they short circuit successive combinations of rings and seg-
ments as they rotate.

The precise distribution of continuous and segmented rings
and the relative position of brushes are shown schematically in
Fig. (3.4) in which the cylindrical surface of contact is laid
out flat.

Fig. (3.5) shows orthographic projections of commutator
design.

We are faced now with the problem of constructing this
device. The main difficulty is to place the segmented rings in
the inside surface of Part #1 in an accurate way. It is not
possible with available tools here to glue a brass tube in the
inner surface of the holder (Part #1) and then cut from the in-
side the rings and the segments. The problem arises when we
want to make the axial cuts for the ring segments, there is a
drastic space limitation that makes it impossible to use the
milling machine inside the tube.

On the other hand, building the rings and the ring segments
outside and then gluing them accurately to the inner surface of
Part #1 is an almost impossible task without building a sophisti-
cated frame that would hold the segments conveniently from the
inside while they are being glued. This would be very time con-
suming.

We are also left with the problem of making the electric
connections from each ring and segment to the outside of hol-
der without damaging the brush contact surfact and epoxy resin
that is being used to hold rings and segments. Therefore the
Figure 3.6. Commutator construction method.
possibility of soldering the electric connections after the device has been built is not attractive. Also, this would be difficult to achieve (even if we did not have the danger of damaging the epoxy due to high temperatures) because of space limitations.

These difficulties were taken care of in our construction method. Please refer to Fig. (3.6).

Part #1 was built of micarta, it is a good isolating material, strong enough for our purposes and easily machined. The slots for electric connections were drilled and finished by hand.

Part #2 in Fig. (3.6) will constitute the rings and segments with their corresponding electric connections to the exterior. The axial cuts that separate each ring segment from the next as we rotate have already been done in the proper position with the milling machine. The dotted lines show the "future" shape of rings and segments. Electric connections are soldered in the right place and their length is a little longer than the final width of the outside tube holder.

The process now is conceptually straightforward. Part #2 is introduced into Part #1 and glued with epoxy resin that will fill in the slots for electric connections, therefore holding everything together for the operations that come next.

We now turn (in the lathe) carefully the inner diameter ($D_i$ in Fig. 3.5) that corresponds to the surface contact for carbon brushes. Then the cuts in the dotted lines are made from the inside and the rings and segments are identified. The final outside
Figure 3.7. Photograph of Commutator.
diameter of the holder is machined. The electric connections will appear at the surface. This operation should be done with special care since the material over which the cutting tool is working is not homogeneous. It was done at low speeds and taking layers of 0.003 inches thick at a time. No repercussions nor vibrations were observed when the cutting tool reached the copper wires embodied in the epoxy frame, thus insuring that no damage nor misalignments were produced on the rings and segments.

To prevent loosening of rings, the slots between them were filled with epoxy resin and finally the contact surface was carefully finished to a continuous smooth surface. A picture of the commutator is shown in Fig. (3.6.a)

III.3. Carbon Brushes Design

Our first design of cylindrical carbon brushes came out to be very inefficient for our purposes. Low contact resistance was never achieved smoothly along the surface of the ring, a condition that is very important for triggering the SCRs since contact resistance is in series with \( R_4 \) in the gate triggering circuit of Fig. (3.1).

The major difficulty lies in the fact that we had a curved contact surface and a small rotation of the brush around its axis would reduce drastically the surface contact between brush and ring.
It became absolutely necessary to design brushes that cannot rotate, therefore we decided to use square brushes even though holding them becomes more complicated. Making a square hole in the rotating member without going through it, is a difficult task, so we opted for introducing the standard square brush holder in the rotating member, holding the brush in the best possible way. The design is shown in Fig. (3.7.a).

The slot made in the surface of the square silver graphalloy brush is to avoid brush-epoxy contact in the region between two parallel rings. Silver graphalloy brushes are specifically manufactured for low noise, signal level contacts which makes them adequate for our application. The rings and segments are built of brass and according to the brush manufacturer the silver-graphalloy-brush combination performs well.

The design shown in Fig. (3.7.a) was verified to be not satisfactory. The difficulties were caused by the fact that parallel ring surfaces were not always at exactly the same level and this made the brush to be touching only one ring at a time in some points, hence the contact resistance was very high in those points. Fig. (3.8.a) shows the situation, exaggerating the proportions for better visualization.

Re-leveling the contact surfaces was not the best solution since we do not have any guarantee that after being used for a while they will become un-leveled again. This is very likely to occur because we are comparing segmented rings and continuous rings, therefore strains will be different in both cases.
We solved our problem by splitting the graphalloy brush in two and allowing each side to follow its contact surface independently. Fig. (3.7.b and 3.8.b) show the improved design.

The same spring is used for both brushes, it presses a copper square plate (shown in dark) which pushes both brushes allowing a small relative motion between them so that they may follow their own rings.

III.4. Experimental Installation

Our experimental motor is mechanically connected to a 3/4 H.P. DC motor with its field and armature separately excited. The DC motor is rated for 3450 rpm, 230 volts armature voltage, 3 amperes armature current and .25 amps maximum field current. It can be used as a drive to our experimental motor or as a generator providing a variable load by varying an external resistance connected to its armature terminals.

Our experimental motor was a 1 K watt synchronous machine, it has a two pole rotating field with capacity for a maximum field current of 2 amps. The armature coils have been entirely re-connected but basically they consist of 42 coils placed in 42 slots in the stationary armature core. The pictures in Fig. (3.9.a) and (3.9.b) show an overall view of the installation.
Figure 3.9.a. Photograph of Installation.
Figure 3.9.b. Photograph of Installation.
CHAPTER IV

EXPERIMENTAL MEASUREMENTS AND RESULTS

IV.1. Armature Connection Tests and Description of Armature Magnetic Flux.

Armature connections where made according to Figs. (2.1.a) & (2.1.b). Identification of coil terminals was not obvious since previous labeling was incomplete and direct physical verification of the coil to terminal connections would have required partial destruction of the coil set-up and insulation.

By a systematic and self-consistant method we made a good starting guess of the identification of the coil terminals. It became necessary however, to make an experimental verification of the armature magnetic flux.

The measurements were done as follows:

i. Isolate SCR circuit from armature, opening connections at points (1)(2)(3)(4)(5)(6) & (7) shown in Fig. (2.1.a).

ii. Set up a stroboscope with a photo electric pick up so as to generate a trigger signal synchronized with shaft rotation. This signal is fed to the oscilloscope.

iii. Apply a single phase AC voltage between armature terminals labeled 1 and 5.

iv. Measure with oscilloscope the induced voltage on field winding while turning the shaft slowly. The peak amplitude of this voltage is proportional to the rate of change
Figure 4.1. Relative position between an armature equivalent coil (stator) and the field equivalent coil (rotor) for an angle θ.

Figure 4.2. Voltage induced in field coil when a 60 cycle ac. voltage is applied to armature coils. The envelope describes the intensity of magnetic flux linkage λ as a function of angle θ.
of the magnetic flux linkage $\lambda$. Since the armature coils self inductance is closely independent of the angular position of rotor, and since we have a constant-frequency sinusoidal applied voltage, we will have a sinusoidal armature current and hence a magnetic flux whose amplitude varies sinusoidally with time.

Maximum induced-voltage amplitudes will occur when the magnetic axis of armature and field coils are parallel, i.e. when $\lambda$ mutual is maximum, $\theta = 0$, please refer to Fig. (4.1). This is seen more clearly as follows:

Assume that:

$$\lambda = \lambda_0 \cos \theta \sin \omega t$$

$$v = \frac{d\lambda}{dt} = w\lambda_0 \cos \theta \cos \omega t - \lambda_0 \sin \omega t \sin \theta \frac{d\theta}{dt}$$

in which $w$ is the line frequency. The rotor speed $\frac{d\theta}{dt}$ can be neglected and $v$ becomes:

$$v = w\lambda_0 \cos \theta \cos \omega t$$

In this particular case; $v_{\text{max}}$ occurs when $\lambda = \lambda_{\text{max}}$ therefore, $v_{\text{max}}$ is proportional to $\lambda_{\text{max}}$. Observation of $v_{\text{max}}$ as a function of $\theta$ tells us qualitatively how $\lambda_{\text{max}}$ varies with $\theta$.

When we have the correct armature connections we expect a nearly sinusoidal distribution of $\lambda_{\text{max}}$ as a function of $\theta$. 
Figure 4.3. Measured magnetic flux directions for each step in armature connections.

Figure 4.4. Typical anode to cathode voltage for an SCR during a triggering cycle at zero speed conditions.

Figure 4.5. Rectified armature current at zero speed versus time.
Finally we repeat step (iii) for all other possible combinations of "opposite" terminals. The photograph, Fig. (4.2), shows a typical $v = v(\theta, \omega)$ and the envelope corresponds to the shape of: $v_{\text{max}} = v_{\text{max}}(\theta)$. Fig. (4.3) shows the experimentally determined angles for maximum $\lambda$ for different pairs of terminals.

It can be seen that data from photographs like Fig. (4.2) does not show the sign of magnetic field. The directions shown in Fig. (4.3) were obtained by applying a "step" input at armature terminals and observing the sign of the "impulse" voltage induced in the field coils.

IV.2. Verification of Rectification Process

Having verified the correct armature interconnections and the shape of the magnetic-flux wave, the next step is to check how the SCRs are rectifying the 3-phase current applied to the armature terminals.

The circuit shown in Fig. (2.3.a) was isolated and two 25Ω power resistors $R_s$ were connected in series with armature impedance (as shown in Figs. 3.2 & 3.3) to limit the current to reasonable levels when 50-100 volts was applied at the terminals. When the motor is rotating under normal operation, resistors are not necessary since armature currents are limited by the back e.m.f; however, for starting and testing conditions, these resistors are desired.
It can be seen that they will reduce the efficiency of the motor and they can be removed provided that precautions are taken to prevent over-current damage when the motor is operating at very low speeds. (see Section IV.6.c).

The SCRs are triggered with mechanical switches, shown in Figs. (3.2 & 3.3).

The anode to cathode voltages $V_{AK}$ for each SCR are observed and a typical shape is shown in Fig. (4.4). It may be compared with the idealized $V_{AK}$ shape that is shown in Fig. (2.4), the positive peak observed in the actual voltage curve is due to the fact that the SCR has to wait until $V_{AK}$ reaches a certain level (in this particular case $V_{AKT} = 16$ volts) to be triggered. This delay effect also shows up in the rectified armature voltage, Fig. (4.5) as a departure from the ideal rectified voltage shown in Fig. (2.4).

IV.3. Transient Inductive Effect During Commutation

To examine switching effects that occur during commutation it became necessary to simulate the commutator action by means of mechanical switches that may be operated independently of shaft rotation.

In this way we had freedom to examine individually the different effects produced at each step in switching the armature connections as the rotor turns.
We can make the armature field axis rotate a whole revolution by operating these "signal switches" by hand. We can therefore identify the situation which might produce undesired effects.

This simulation of commutator action revealed a transition stage in which the brush shortcircuits an armature coil that was previously conducting. This is equivalent to suddenly shortcircuiting the secondary winding of an auto-transformer with the subsequent damage. Fig. (3.3) shows the situation explicitly.

In Stage #1, SRS labeled A5, B5, C5, lA, lB, lC are triggered, with armature current in the direction shown by the arrow. There is no current out of the #2 terminal. When we switch to state #2, the brush electrically connects together all the "signal switches" of the "outgoing" SCRs, by effectively connecting points S and S.1

The 2 K resistor labeled R_K is used to prevent shorting points 1 with 2 so that, armature coils are not short circuited by the brush during commutation. R_K does not dissipate appreciable power because it is in the signal level side of the circuit. Also it does not alter the trigger mechanism for the SCR as can be seen from the results of Test #2 in Chapter III.

IV.4 Static Torque

With the mechanical switch simulation described in the previous section we observed the static torque as a function of \( \theta \) for a given stage of armature connections. It was seen that the net torque was the superposition of a fundamental sinusoid and a
Figure 4.6. Static Torque. Net torque is the superposition of the two components shown above.
higher armonic sinusoid as shown qualitatively in Fig. (4.6).

It is shown (Appendix A) that the mechanical torque is given by:

$$T_{mech} = \frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta} + \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} \quad (4.1)$$

The higher harmonic component of torque corresponds to a reluctance torque which is caused by effect of the slots in the iron core over the magnetic field. In Equation (4.1) this effect shows up in the coefficients $\frac{\partial L_{12}}{\partial \theta}$, $\frac{\partial L_{11}}{\partial \theta}$ and $\frac{\partial L_{22}}{\partial \theta}$.

In the idealized case we have that the $L_{ij}$ are smooth sinusoidal functions of the angle $\theta$ but in our real apparatus this is not so. However, the higher armonic sinusoidal torque does not produce any net work per revolution and it was seen that under operation of the motor its vibrating effect is smoothed out by the inertia of the motor.

IV.5. Motor Performance, Description of SCR Operation and Discussion

The performance of the experimental motor was observed in several tests under different conditions. In every case the speed was clearly dependent on the AC peak voltage ($V_A$ peak) applied to the armature.
Figure 4.7. Speed versus armature peak applied voltage for no load conditions. $I_a = 0.025$ amperes.
Figure 4.8.a. Anode to cathode voltage $V_{AK}$ and gate to cathode voltage $V_{GK}$ versus time for a typical SCR.

Figure 4.8.b. $V_{R2}$ and $V_{GK}$ voltages versus time for an in-coming SCR.
Figure 4.8.c. $V_{R2}$ and $V_{GK}$ vs. time for an out-going SCR.

Figure 4.8.d. $V_{R2}$ and $V_{GK}$ vs. time for a different in-coming SCR.

Note: Data in Figures 4.8.a through 4.8.d was taken at zero speed conditions.
Fig. (4.7) shows RPM versus armature peak applied voltage. Under no load conditions, (the loading generator was disconnected at the shaft complying). The armature current was .025 amps. After 3 minutes of normal operation at 1200 rpm, two SCRs and two fuses burned out.

It was observed that the SCR performance was not regular for every cycle, but by increasing the load slightly on the motor shaft, the SCR performance became regular.

This can be explained by the fact that under no load conditions the back emf almost balances the applied voltage, (except for the small IR drop in armature). Thus the SCRs operate with a small forward voltage difference. As a result of small differences in trigger sensitivity, some of the SCR's will not conduct. Just enough of the SCR's will conduct to maintain the speed of the motor at the given steady state, and on the average just balancing back emf with the applied voltage.

These conditions are altered by increasing the load since armature current increases, speed decreases and therefore increasing the voltage across the SCRs when they are in the off state. As a result of this all SCRs come into action and they trigger regularly.

Fig. (4.8.a) shows the anode to cathode $V_{AK}$ and gate to cathode $V_{GK}$ of an SCR during operation at low speeds. Fig. (4.8.b and 4.8.c) shows the $R_2$ voltage together with $V_{GK}$. Gate trigger currents can be computed and they come out to be of the order of 0.1 ma.
Fig. (4.9) shows experimental data for the performance of the motor when the load is increased by connecting the load generator. Motor armature currents are close to 1 a. in this case. Data for two experiments are presented in this graph. Again, the first trial ended with the burning of 3 SCRs, and the second trial ended similarly. It was seen that no burning pattern exists and that there was no correlation between the SCRs that were damaged on the different occasions. Location of damaged SCRs seemed to occur at random.

Table (4.1) summarizes the data obtained in these two experiments.

<table>
<thead>
<tr>
<th>Peak Applied Armature Voltage</th>
<th>Motor Armature RMS Current (amps)</th>
<th>Open Circuit Generator Arm Voltage</th>
<th>Speed (rpm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>.73</td>
<td>11</td>
<td>190</td>
</tr>
<tr>
<td></td>
<td>.70</td>
<td>15</td>
<td>218</td>
</tr>
<tr>
<td>40</td>
<td>.80</td>
<td>21</td>
<td>345</td>
</tr>
<tr>
<td></td>
<td>.78</td>
<td>21</td>
<td>325</td>
</tr>
<tr>
<td>50</td>
<td>.84</td>
<td>30</td>
<td>477</td>
</tr>
<tr>
<td></td>
<td>.81</td>
<td>28</td>
<td>442</td>
</tr>
<tr>
<td>60</td>
<td>.90</td>
<td>35</td>
<td>570</td>
</tr>
<tr>
<td></td>
<td>.87</td>
<td>36</td>
<td>575</td>
</tr>
<tr>
<td>70</td>
<td>.95</td>
<td>42</td>
<td>680</td>
</tr>
<tr>
<td></td>
<td>.91</td>
<td>43</td>
<td>680</td>
</tr>
<tr>
<td>80</td>
<td>1.00</td>
<td>50</td>
<td>780</td>
</tr>
<tr>
<td></td>
<td>.98</td>
<td>52</td>
<td>825</td>
</tr>
</tbody>
</table>
Figure 4.9. Speed versus armature peak applied voltage and versus armature current (rms).
We can calculate the power output of the experimental motor knowing the generator characteristics. Fig. (4.10.a) shows speed versus armature voltage when the generator is operated as a motor without load. The field is excited with 240 volts DC. Fig. (4.10.b) shows armature current versus armature voltage and Fig. (4.10.c) shows armature resistance versus armature current for zero speed conditions.

Friction power dissipation is given by:

\[ P_f = I_a V_a - I_a^2 R_a \]  
(4.2)

Since \( I_a \) is small we can assume that hysteresis losses are small compared with friction losses.

Using the data presented in Fig. (4.10), we can obtain Table (4.2) by assuming that friction loss depends principally on speed.

The values of \( P_f \) are at the lowest range because the values of \( R_a \) are very conservative (high). In obtaining the data presented in Fig. (4.10.c) it was observed that \( R_a \) changes with the angular position of the shaft. The variation is attributed to voltage drops across the commutator. To be conservative the highest values of \( R_a \) was recorded. These values are within 20% accuracy.
Figure 4.10.a. No load Generator Characteristics.

Figure 4.10.b. No load Generator Characteristics

Figure 4.10.c. Armature resistance versus armature current for zero speed.
The mechanical power output for our experimental motor equals approximately the friction dissipation in the generator, therefore no electrical load is placed on the generator armature terminals.

The performance of the experimental motor is adequate so long as no SCR is damaged. From Fig. (4.9) and Table (4.2) it can be seen that $I_a$ (RMS) is proportional to torque and that speed is proportional to $V_a$ as it is expected.

If we compare Figs. (4.7 and 4.9) however, we can see that for a constant applied $V_a$, speed decreases proportionally to armature current. This behavior is expected because we have a non-negligible resistance in the armature. Since the back emf has to balance the applied voltage $V_a$ minus the $I_aR_a$ voltage drop, a higher current $I_a$ means lower back emf and hence lower

---

**TABLE 4.2**

Generator Friction Power Characteristics

<table>
<thead>
<tr>
<th>$P_f$ (Watts)</th>
<th>N (RPM)</th>
<th>Torque Watts</th>
<th>Watts rpm</th>
</tr>
</thead>
<tbody>
<tr>
<td>11.19</td>
<td>512</td>
<td>.022</td>
<td></td>
</tr>
<tr>
<td>21.49</td>
<td>780</td>
<td>.027</td>
<td></td>
</tr>
<tr>
<td>38.22</td>
<td>1080</td>
<td>.035</td>
<td></td>
</tr>
<tr>
<td>59.29</td>
<td>1320</td>
<td>.045</td>
<td></td>
</tr>
<tr>
<td>81.75</td>
<td>1650</td>
<td>.050</td>
<td></td>
</tr>
<tr>
<td>104.35</td>
<td>1920</td>
<td>.054</td>
<td></td>
</tr>
<tr>
<td>135.8</td>
<td>2180</td>
<td>.062</td>
<td></td>
</tr>
</tbody>
</table>
Burning of SCRs was clearly caused by sudden high currents since damage in the anode to cathode path of the actual device was observed, while no visible damage was seen at the gate of the burned SCR. There is a clear difference in the way the SCRs burn in this case as opposed to the type of damage observed in other past experiments were the gate was burned by excessive gate currents.

If we refer to Fig. (2.1.a), we can note that if an SCR is triggered by any means in the wrong moment we will make a short circuit between to supply lines with the subsequent destruction of more SCRs and the fuses.

Another reason for generating high currents in the system is the possible short circuit in armature coils due to poor insulation.

The state of armature coils insulation was verified with a megaohmmeter and, effectively, it was found that two coils were shorted to two adjacent laminations in the iron core of the stator. This might probably cause an intermittent short when the device is under thermal and mechanical effects during operation.

The poor insulated coils were disconnected together with other "good" ones that were also disconnected to maintain symmetry in the machine.

Also a 0.02 μf capacitor was connected in parallel to the 1K gate to cathode resistor of each SCR. Its purpose is to smooth out the possible dv/dt noise pick up at the gate that might possibly trigger the SCR at the wrong time. This capacitor
Figure 4.11.a. Steady state characteristics for experimental motor.
Figure 4.11.b. Peak (Δ) and RMS (○) armature currents versus speed.
will decrease the commutating capability of the SCR, but the
time constant RC equals 20 μsec which is still very small com-
pared with the period of the 60 HZ lime voltages.

A new run was made. Fig. (4.11.a) shows speed, N power input
$P_i$, and armature current $I_a$ (RMS) as a function of the peak
applied armature voltage $V_a$. Also, an estimated mechanical
power output for the motor is obtained based on an interpolation
from the data in Table (4.2), assuming that it is dissipated mainly
by friction in the generator. Fig. (4.11b) shows $I_a$ (RMS) and
$I_a$ (peak) versus RPM.

Figs. (4.12.a and 4.12.b) show the anode to cathode voltage
$V_{AK}$ and gate to cathode voltages $V_{GK}$ versus time of an SCR (in
particular, SCR labeled 1A in Fig. (2.1.a) when $V_{a\text{peak}} = 30$ volts
and 50 volts respectively. In the first case the motor speed is
190 rpm, it can be seen how the SCR conducts normally when it
receives the gate trigger signal. The effect of the back emf
can be appreciated better as we increase speed as can be seen
in Fig. (4.12.b) in which the speed is 520 rpm. The envelope
curve of $V_{AK}$ has the period corresponding to one revolution.
This becomes more evident when we see the voltage in a full
pitch search coil together with the anode-cathode voltage,
Fig. (4.15).

As speed increases, the effect of the back emf becomes more
important. It actually determines the intensity of the armature
current and the shape of the voltage $V_{AK}$. These two effects will
determine the behavior of the SCR.
Figure 4.12.a. $V_{Ak}$ and $V_{GK}$ voltages vs. time. $V_A = 30$ V. Speed = 190 RPM.

Figure 4.12.b. $V_A = 50$ V.; speed = 520 RPM.
Figure 4.13.a. $V_A = 60$ V.; speed = 690 RPM.

Figure 4.13.b. $V_A = 70$ V.; speed = 890 RPM.
Figs. (4.13.a and 4.13.b) show $V_{AK}$ for SCR labeled $V_2$ and the voltage $V_{RS}$ across the $11\Omega$ resistor in series with the armature windings. ($R_S$ in Fig. 3.3). $V_{RS}/11$ is therefore the current $I_{a1}$ coming out of the armature at point 1. (It is not the same as $I_a$ in Figs. (4.11.a and 4.11.b).

In Fig. (4.13.a), at $t = 26$ m. sec. SCR 1A starts to conduct, $I_{a1}$ raises from 0 to 1 amp in 2 m. sec. SCR 1A turns off at $t = 32$ m. sec. but we observe an increase in $I_{a1}$ at that time. This means that either 1B or 1C starts to conduct and the sudden increase in current is due to the fact that the voltage drive is now between the same back emf at point 1 and a different phase voltage B or C, whichever is the case.

The same phenomena of having another SCR (i.e. 1B or 1C) conducting after 1A has been turned off is also observed in Figs. (4.13, 4.14, 4.15.a, 4.16.a and 4.17), that correspond to operating conditions at different speeds and applied voltages $V_A$.

It is important to note how irregular the current $I_{a1}$ is, depending on what is the difference between the line voltage and back emf across the particular SCR that turned on at that instant.

In Fig. (4.17) we can observe peak anode currents of twice the typical peak amplitude for the same SCR during two different on-states. Under normal operation at higher speeds, we will have peak values of armature current that are considerably higher than the RMS values as can be seen in Fig. (4.11.b) in which after 600 RPM the difference between $I_a$ peak and $I_a$ RMS becomes more significant. This increase in irregularity of armature currents with speed may constitute an important speed limit factor of this experimental motor, since greater current-capacity SCRs are
Figure 4.14. $V_A = 80$ V.; speed = 1070 RPM.

Figure 4.15.a. $V_A = 90$ V.; speed = 1160 RPM.
Figure 4.15.b. $V_{AK}$ and search coil voltage $V_{sc}$ versus time. $V_A = 90$ V.; $\text{speed} = 1160$ RPM.

Figure 4.16.a. $V_A = 100$ V.; speed = 1340 RPM.
Figure 4.16.b. $V_{AK}$ and search coil voltage $V_{sc}$ versus time. $V_A = 100$ V.; speed = 1340 RPM.

Figure 4.17. Armature peak applied voltage $V_A = 110$ V. Speed = 1600 RPM approximately.
needed for higher speed operation. This is an important factor that should be considered when sizing the SCRs i.e. current capacity can be calculated directly from torque requirements but the values so obtained correspond to RMS values, the SCRs should be capable of taking twice as much when working at the normal steady state. Besides this, more current capacity should be added to take care of transient overloading. The run of Fig. (4.11) ended (a few seconds after the photograph shown in Fig. (4.17 was taken) with the burning of three SCRs and the fuses.

In a complex phenomena like this one, in which there are many effects influencing the operation of a device, it is not very reasonable to attribute a failure to only one effect, especially when they are all interconnected. We have obtained clear evidence of the following facts:

i. SCRs are damaged after a few minutes of normal operation. (In the last test, however, the SCRs were destroyed after 15 seconds of operation at approximately 1700 rpm and peak I_a values in the vicinity of 4 amps).

ii. Anode currents are very irregular for SCRs under normal steady state operation in which we can have peak values that might easily be two times the average values.

iii. Applied anode-cathode voltages for the SCRs contain high frequency ripple caused by the back emf.

We estimate that the cause of the SCR failure is the temporary overloading damage and/or the accidental triggering of the device. In both cases a short circuit is produced between line phases as
a consequence of the failure.

Both effects are influenced by the junction temperature. (Graph #7 in Appendix C shows the transient thermal impedance obtained from specification sheets of the GE type C106D1 SCRs that were used). It can be seen that two time constants are involved in the process. One, (the shorter one, i.e. 0.2 seconds) is associated with the junction to tab impedance, and the second one (the longer one, i.e. 15 seconds) is associated with the junction to ambient thermal impedance. It is reasonable to assume that maximum junction temperatures for a given amount of power dissipation will be reached after 10 or 20 seconds of operation.

Higher junction temperatures will make the SCR more sensitive to dv/dt effects and it will also reduce the current capability of the device.

From specification sheets we observe that for ambient temperature between 15°C to 25°C the allowable DC on-state currents range from 0.9 to 0.8 amps. respectively, for the SCRs that we have used. (See Graph #5, in specification sheets in Appendix).

Although we do not have a constant DC anode current for any SCR since they conduct only at some interval during the cycle, Fig. (4.17) shows us that the average (RMS) value of the anode current is close to one ampere which places us in the limit of the current capability of the SCR.

Simultaneously the device is working under a "noisy" $V_{AK}$ as can be seen in Fig. (4.16.b) and it is very likely to be trig-
gered when it should not (i.e. in absence of gate trigger signal). In this particular test (Fig. 4.11) either of the two effects would be the cause of the failure. In the no load test however, the armature current was low, therefore the failure was more likely to be the accidental triggering of an SCR.

IV.6. Suggestions for Improvements

IV.6.a. Current Capability

Current capability problems can be solved by increasing the heat transfer coefficient between tab and air (or using a better heat sink in general) and by proper selection of the SCRs.

Since it takes some time for the SCRs to fail, it may be concluded that the problem is related to the thermal impedance, between the junction and the ambient environment. The impedance can be reduced by a factor of 5 by using forced convection of air over the tab as opposed to free convection. An alternative approach would be to thermally connect all the tabs to a very good thermal conductor (copper) using a thin electric insulator as shown in Fig. (4.18).

The heat transfer coefficients for each case can be completed as follows:

1. For the electrical insulator layer:

\[ h_i = \frac{k}{\delta} \]  \hspace{1cm} (4.3)

\[ k = \text{thermal conductivity} \]
\[ \delta = \text{layer thickness} \]
Figure 4.18. SCR mounting with a metallic heat sink.
The minimum thickness $\delta_{\text{min}}$ is limited by the rupturing strength of the material $e \frac{\text{volts}}{\text{mil}} = 7.9 \times 10^{-5}$ inches.

The maximum $h_i$ is:

$$h_i_{\text{max}} = \frac{k}{\delta_{\text{min}}} = 38000 \frac{\text{Btu}}{\text{hr ft}^2 \cdot ^\circ\text{F}}$$

(4.4)

The value of the air heat transfer coefficient $h_a$ is between 5 and 50 (Btu/hr $^\circ$F ft$^2$) and a detailed calculation of its value will only tell us where in this region it is. But the conduction heat transfer is 3 orders of magnitude greater, therefore further calculations for $h_a$ are not relevant.

38000 is an optimistic value since we are assuming a perfect surface contact and a minimum insulator thickness. Even if we introduce a factor of 0.1 this solution is clearly superior to the forced convection one.

The ratio of the steady state thermal impedances for the tab as a function of the heat transfer coefficients $h$ is given by:

$$\frac{R_{Th1}}{R_{Th2}} = \frac{\tanh (h_2 C_1)}{\tanh (h_1 C_1)} \frac{h_2}{h_1}$$

(4.5)

$$C_1 = \frac{2(d + b)L}{k bd}$$

in which the fin effect at the tab is significant.
Dimensions are defined in Fig. (4.18).

This ratio indicates the improvements that we can achieve as far as power dissipation (or equivalently, as current capability) is concerned.

IV. 6.b. Accidental Triggering Prevention

Undesired high dv/dt are caused not only by back emf ripple but also by turn-off effect of other SCRs in the system.

To visualize this better, let us consider SCRs labeled Al and 1B. Assume that at t = 0(-) none of them are conducting and that at t = 0, Al receives its trigger signal and turns on in 1.2 micro-seconds. The incremental model of this system is shown in Fig. (4.19).

The impedance L is the output impedance of the variac used to supply the variable 3 phase "line" voltages. The phase to phase voltage can be approximated as a constant DC source over a short period of time (1.2 m sec). In our incremental model, we assume that the turn-on time of the SCR is a characteristic of the device depending on its internal structure, therefore it is not affected by an external anode-cathode impedance.

SCR Al turning-on is equivalent approximately to closing a switch, or, from the circuit point of view it is equivalent to applying a step-input voltage.
When using a pure resistor as the external anode-cathode impedance, the governing differential equation is the following for $t > 0$:

$$\frac{di}{dt} + i \frac{R_E}{L} = \frac{V_0}{L} \quad (4.6)$$

and:

at $t = 0$ (+)

$\quad i = 0$

$t = \infty$

$\quad i = \frac{V_0}{R_E}$

Solution of (4.6) yields:

$$i = \frac{V_0}{R_E}(1 - \exp\left(-\frac{R_E t}{L}\right)) \quad (4.7)$$

and

$$\left.\frac{dv_E}{dt}\right|_{t=0} = \frac{V_0 R_E}{L} \quad (4.8)$$

Preventing triggering requires that:

$$\frac{dv_E}{dt} < 10^7 \quad \text{[volts/sec]} \quad (4.9)$$
The output impedance of the variac used in these experiments is .0098H. If we consider phase-to-phase voltage of 200V, Condition (4.9) together with Equation (4.8) will give us:

\[ R_E < 475 \, \Omega \]  

(4.10)

As an alternative we can use a series R-C network as the protective external anode-cathode impedance. In this case, the governing differential equation is:

\[
\frac{d^2 v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{V_0}{LC} \]  

(4.11)

With boundary conditions:

at \( t = 0 \) \quad v_c = 0 \quad \text{and} \quad \frac{dv_c}{dt} = 0

\( t = \infty \) \quad v_c = V_0 \]  

(4.11.a)

Solving this equation (Appendix B) and using the definition that:

\[ v_E = v_R + v_C \]  

(4.12)

we obtain:

\[
\left. \frac{dv_E}{dt} \right|_{t=0} = -\frac{r_1^2 r_2 V_0}{r_2 - r_1} \frac{RC}{r_2 - r_1} + \frac{r_1 r_2^2 V_0}{r_2 - r_1} \frac{RC}{r_2 - r_1} \]  

(4.13)
in which \( r_1 \) and \( r_2 \) are defined in (B.8) and \( r_1 \neq r_2 \).

When \( r_1 = r_2' \),

\[
\left. \frac{dv}{dt} \right|_{t=0} = \frac{R^3 C}{4L^2} V_0
\]

(4.14)

The transient response time constants \( T_1 \) and \( T_2 \) are the inverse of \( r_1 \) and \( r_2 \) respectively and it can be seen that since they are dependent, increasing one implies reducing the other and vice-versa. It is desired to maximize the smaller one and we are going to look at the case when \( r_1 = r_2 = \frac{-R}{2L} \).

Combining Equations (4.9) and (4.14) we get:

\[
\frac{R^3 C}{4L^2} V_0 < 10^7
\]

(4.15)

If we assume that \( V_0 = 200V., C = .1 \mu F; \) and \( L = 0.0098H; \) we obtain for \( R \):

\[
R < 575 \Omega
\]

(4.16)

The values calculated in this section for the parameters of the (dv/dt) protection networks may provide a starting point for further and more detailed design of these circuits.
IV.6.c. Motor Operation, Armature Voltage Control

It was mentioned in Section IV.2 that it became necessary to introduce some current limiting resistors in series with the armature coils to prevent damage in the system during initial experimental tests.

Clearly the efficiency of the motor is therefore reduced and also the speed becomes more dependent on armature current. These resistors can be removed if we limit the armature current $I_a$ properly.

The fastest transients require the highest torques and therefore maximum armature currents. Steady state $I_a$ is given by the following expression:

$$I_a = \frac{V_a - E}{R_a} \quad (4.18)$$

in which:

$E = \text{back e.m.f.} = kw$

$w = \text{motor speed}$

In transients however, current levels may be much higher than the steady state values, depending on the dynamic characteristics of the power source, and the values of the different parameters in the system (inductances, inertias, etc.). Design and control actions will include constraining $I_a$ to safe limits and to provide reasonable transient responses.

If the open-loop transient response of the system is acceptable we can implement an on-off type of control which will come into action whenever an error signal becomes greater or equal than an error tolerance.
Figure 4.19. Incremental model for analyzing SCR turn-on dv/dt effects.
This type of control, however, does not have any action over the dynamic behavior of the system (i.e. does not alter transient time-constants nor prevent oscillatory behavior, etc). Therefore, it might become necessary to use more sophisticated feedback-control techniques to improve transient characteristics.
Analysis of Reluctance Torque

The reluctance torque depends on the geometry of the motor, i.e. it depends on the variation of the self-inductances of the different coils with the angular position of the rotor.

As a first approximation we assume for our analytic approach that we have a conservative system (i.e. energy is a state function). Energy balance yields:

\[ dW_{\text{electr}} = eidt = dW_{\text{fld}} + dW_{\text{mech}} \]  \hspace{1cm} (A.1)

or

\[ dW_{\text{fld}} = id\lambda - f_{\text{mech}} dx = dW_{\text{fld}}(\lambda, x) \]  \hspace{1cm} (A.2)

\((f_{\text{fld}} = f_{\text{mech}} \text{ for equilibrium})\)

but

\[ dW_{\text{fld}}(\lambda, x) = \frac{\partial W_{\text{fld}}}{\partial \lambda} d\lambda + \frac{\partial W_{\text{fld}}}{\partial x} dx \]  \hspace{1cm} (A.3)

\[ i = \frac{\partial W_{\text{fld}}(\lambda, x)}{\partial \lambda} \]  \hspace{1cm} (A.4)

\[ f_{\text{fld}} = f_{\text{mech}} = -\frac{\partial W_{\text{fld}}(\lambda, x)}{\partial x} \]  \hspace{1cm} (A.5)
If we use another set of variables (i.e. $i, x$) we can define coenergy as:

$$W'_{fld}(i, x) = i\lambda - W_{fld}(\lambda, x) \quad (A.6)$$

and similarly, we can prove that:

$$\lambda = \frac{\partial W'_{fld}(i, x)}{\partial i} \quad (A.7)$$

and

$$f_{mech} = \frac{\partial f_{fld}}{\partial x} = \frac{\partial W'_{fld}(i, x)}{\partial x} \quad (A.8)$$

The algebraic signs of equations (5) and (8) show that the field force acts in a direction to decrease the magnetic-field stored energy at constant flux or to increase the coenergy at constant current.

In the case of our motor, we have two independent sources of excitation i.e. magnetic field (rotor) excitation and armature (stator) excitation.

The field coils may be considered as a single inductance, but the armature coils for a given connection of armature terminals, is a complex net of interconnected inductances. Nevertheless, we will consider for simplicity an overall equivalent inductance and assume that there is no magnetic saturation in the iron coil.
With all these assumptions, we have that: $W_{\text{fld}}$ is a function of $\lambda_1$, $\lambda_2$, and $\theta$, the flux linkages of armature coils, field coils and angular position of rotor respectively, thus:

$$W_{\text{fld}} (\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 \, d\lambda_1 + \int_0^{\lambda_2} i_2 \, d\lambda_2$$

(A.9)

We can define for a linear system the relations between $\lambda$ and $i$ in terms of inductances:

$$\lambda_1 = L_{11} (\theta) i_1 + L_{12} (\theta) i_2$$

$$\lambda_2 = L_{21} (\theta) i_1 + L_{22} (\theta) i_2$$

(A.10)

in which:

$L_{11}$ = armature coils self inductance (function of $\theta$ because air gap is not uniform).

$L_{12}$ = armature coils and field coils mutual inductance (function of $\theta$).

$L_{21}$ = armature coils and field coils mutual inductance (function of $\theta$).

$L_{22}$ = field coil self inductance (function of $\theta$).

(a) Introducing (A.10) in (A.9) and integrating we obtain:

$$W_{\text{fld}} (\lambda_1, \lambda_2, \theta) = \frac{1}{2} \Gamma_{11} \lambda_1^2 + \Gamma_{12} \lambda_1 \lambda_2 + \frac{1}{2} \Gamma_{22} \lambda_2^2$$

where $\Gamma_{11}$, $\Gamma_{12}$, $\Gamma_{22}$ are functions of $L_{11}$, $L_{12}$, $L_{21}$ and $L_{22}$.

(A.11)
We can now compute $T_{\text{mech}}$ as follows:

$$T_{\text{mech}} = -\frac{\partial W_{\text{fld}}}{\partial \theta}(\lambda_1, \lambda_2, \theta) \quad (A.12)$$

$$T_{\text{mech}} = -\frac{1}{2} \lambda_1^2 \frac{\partial \Gamma_{11}}{\partial \theta} - \lambda_1 \lambda_2 \frac{\partial \Gamma_{12}}{\partial \theta} - \frac{1}{2} \lambda_2^2 \frac{\partial \Gamma_{22}}{\partial \theta} \quad (A.13)$$

Analogously, if we use the coenergy method, we find:

$$T_{\text{mech}} = \frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta} + \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta} \quad (A.14)$$

in which:

- $i_2$ = field current
- $i_1$ = armature current
- $x$ = displacement
- $W_{\text{fld}}$ = field energy
- $W_{\text{mech}}$ = mechanical work
- $f_{\text{fld}}$ = magnetic field force
- $f_{\text{mech}}$ = force in direction of displacement $x$
APPENDIX B

dv/dt Protection Circuits for SCRs

B.1 Pure Resistor Protection

In the incremental model shown in Fig. (4.19) a resistor \( R_E \) can be used as external anode-cathode impedance for the SCRs.

In this case the governing differential equation for \( t > 0 \) is the following:

\[
\frac{di}{dt} + i \frac{R_E}{L} = \frac{V_o}{L} \tag{B.1}
\]

with boundary conditions:

at \( t = 0 \) (+) \quad i = 0

t = \infty \quad i = \frac{V_o}{R_E}

Solution of (B.1) yields:

\[
i = \frac{V_o}{R_E} \left[ -\exp\left(\frac{-R_E}{L} t\right) + 1 \right] \quad t > 0 \tag{B.2}
\]

and:

\[
V_E = \left[ -\exp\left(\frac{-R_E}{L} t\right) + 1 \right] V_o \tag{B.3}
\]
\[ \frac{dv_E}{dt} = \frac{V_0 R_E}{L} \exp\left( -\frac{R_E}{L} t \right) \]  \hspace{1cm} (B.4)

and at \( t = 0 (+) \):

\[ \frac{dv_E}{dt} \bigg|_{t=0(+) = \frac{V_0}{L} \frac{R_E}{L} } \]  \hspace{1cm} (B.5)

### B.2 Resistor-Capacitor Protection

When we connect a series resistor-capacitor as the external anode-cathode impedance the differential equation valid for \( t > 0 \) is:

\[ \frac{d^2 v_c}{dt^2} + \frac{R}{L} \frac{dv_c}{dt} + \frac{v_c}{LC} = \frac{V_0}{LC} \]  \hspace{1cm} (B.6)

with boundary conditions:

at \( t = 0 (+) \) \hspace{1cm} v_c = 0 \hspace{1cm} \frac{dv_c}{dt} = 0

at \( t = \infty \) \hspace{1cm} v_c = V_0

Solution of (B.6) yields:

\[ v_c = V_0 - \frac{R^2 V_0}{r_2 - r_1} \exp(r_1 t) + \frac{r_1}{r_2 - r_1} V_0 \exp(r_2 t) \]  \hspace{1cm} (B.7)
in which \( r_1 \neq r_2 \) and

\[
\begin{align*}
\frac{1}{2} = & -\frac{R}{2L} + \frac{1}{2} \sqrt{\frac{R}{L}} - \frac{4}{LC} \\
\frac{1}{2} = & -\frac{R}{2L} - \frac{1}{2} \sqrt{\frac{R}{L}} - \frac{4}{LC}
\end{align*}
\] (B.8)

Defining \( v_E = v_c + v_R \) we obtain:

\[
\frac{dv_E}{dt} = \frac{RCV_0 r_1 r_2}{r_2 - r_1} r^2 e^{r_2 t} - r_1 e^{r_1 t}
\] (B.9)

\[
\frac{dv_E}{dt} \bigg|_{t=0} = \frac{RCV_0 r_1 r_2}{r_2 - r_1} r^2 - r_1
\] (B.10)

In the case that \( \frac{R^2}{L} = \frac{4}{C} \) (B.11)

The two characteristic values are equal, thus:

\[
r_1 = r_2 = -\frac{R}{2L}
\] (B.12)

and solution of (B.6) yields:

\[
v_c = -V_0 \exp\left(-\frac{R}{2L}t\right) - \frac{R}{2L} V_0 t \exp\left(-\frac{R}{2L}t\right) + V_0
\] (B.13)
and:

\[ \frac{dv_E}{dt} = v_0 \left[ \frac{R^4C}{8L^3} - \frac{R^2}{4L^2} \right] t \exp \left( -\frac{R}{2Lt} \right) + \]

\[ + v_0 \frac{R^3C}{4L^2} \exp \left( -\frac{R}{2L} \right) \quad (B.14) \]

due to:

\[ \frac{dv_E}{dt} \bigg|_{t=0} = \frac{R^3C}{4L^2} v_0 \quad (B.15) \]
APPENDIX C

Copies of the specification sheets for the GE, SCRs type C106 constitute this appendix. In our application the SCR type C106D1 was used.
<table>
<thead>
<tr>
<th>Type</th>
<th>Repetitive Peak Forward Blocking Voltage, ( V_{F XM} )</th>
<th>Working and Repetitive Peak Reverse Voltage, ( V_{R OM( rep)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>106Q1, C106Q2, C106Q3, C106Q4</td>
<td>15 Volts</td>
<td>15 Volts</td>
</tr>
<tr>
<td>106Y1, C106Y2, C106Y3, C106Y4</td>
<td>30 Volts</td>
<td>30 Volts</td>
</tr>
<tr>
<td>106F1, C106F2, C106F3, C106F4</td>
<td>50 Volts</td>
<td>50 Volts</td>
</tr>
<tr>
<td>106A1, C106A2, C106A3, C106A4</td>
<td>100 Volts</td>
<td>100 Volts</td>
</tr>
<tr>
<td>106B1, C106B2, C106B3, C106B4</td>
<td>200 Volts</td>
<td>200 Volts</td>
</tr>
<tr>
<td>106C1, C106C2, C106C3, C106C4</td>
<td>300 Volts</td>
<td>300 Volts</td>
</tr>
<tr>
<td>106D1, C106D2, C106D3, C106D4</td>
<td>400 Volts</td>
<td>400 Volts</td>
</tr>
</tbody>
</table>

- **Forward Current, On-State**: 4 Amperes
- **Rate of Rise of Forward Current (non-repetitive)**: 50 Amperes/Microsecond
- **Forward Current, On-State (repetitive)**: 75 Amperes
- **One Cycle Surge Forward Current, Non-Repetitive, \( I_{FM} \) (surge)**: 20 Amperes
- **Gate Power, \( P_{GM} \)**: 0.5 Watt
- **Gate Current, \( I_{GAV} \)**: 0.1 Watt
- **Reverse Gate Voltage, \( V_{GRM} \)**: 6 Volts
- **Gate Temperature, \( T_{G} \)**: -40°C to +150°C (For operation at 50°C)
- **Peak Temperature, \( T_{J} \)**: -40°C to +110°C
- **Rating applies for operation at 60 Hz, 75°C maximum tab (or anode) lead temperature, switching from 80 volts peak, sinusaloid current pulse width less than 1 microsecond.**

**CHARACTERISTICS**

<table>
<thead>
<tr>
<th>Test</th>
<th>Symbol</th>
<th>Min.</th>
<th>Typ.</th>
<th>Max.</th>
<th>Units</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reverse or Forward Blocking Current (All Types)</td>
<td>( I_{RX} ) or ( I_{FX} )</td>
<td>0.1</td>
<td>10</td>
<td>( \mu A )</td>
<td>( V_{RX} = V_{FX} ) Rated ( V_{ROM} ) (rep) Value, ( T_{L} = 25^\circ C, R_{G K} = 1000 \text{ Ohms} )</td>
<td></td>
</tr>
<tr>
<td>Gate Trigger Current</td>
<td>( I_{GT} )</td>
<td>10</td>
<td>100</td>
<td>( \mu A )</td>
<td>( V_{RX} = V_{FX} ) Rated ( V_{ROM} ) (rep) Value, ( T_{L} = 110^\circ C, R_{G K} = 1000 \text{ Ohms} )</td>
<td></td>
</tr>
<tr>
<td>Gate Trigger Voltage</td>
<td>( V_{GT} )</td>
<td>0.4</td>
<td>0.5</td>
<td>0.8</td>
<td>Volts/ DC</td>
<td>( T_{L} = 25^\circ C, V_{FX} = 6 \text{ Volts, } R_{L} = 100 \text{ Ohms, } R_{G K} = 1000 \text{ Ohms} )</td>
</tr>
<tr>
<td>Peak On-Voltage</td>
<td>( V_{F M} )</td>
<td>1.8</td>
<td>2.2</td>
<td>Volts</td>
<td>( T_{L} = 25^\circ C, I_{FM} = 4 \text{ Amperes Peak, Single Half Sine Wave Pulse, 2 Millisecond, Wide} )</td>
<td></td>
</tr>
<tr>
<td>Holding Current</td>
<td>( I_{H X} )</td>
<td>0.3</td>
<td>1.0</td>
<td>3.0</td>
<td>mA/ DC</td>
<td>( T_{L} = 25^\circ C, V_{FX} = 12 \text{ Volts, } R_{G K} = 1000 \text{ Ohms} )</td>
</tr>
<tr>
<td>Latching Current</td>
<td>( I_{L X} )</td>
<td>0.14</td>
<td>0.6</td>
<td>2.0</td>
<td>mA/ DC</td>
<td>( T_{L} = 25^\circ C, V_{FX} = 12 \text{ Volts, } R_{G K} = 1000 \text{ Ohms} )</td>
</tr>
<tr>
<td>Critical Rate of Rise of Forward Blocking Voltage</td>
<td>( dV/dt )</td>
<td>8</td>
<td>Volts/ Microsecond</td>
<td>( T_{L} = 110^\circ C, V_{FX} = \text{ Rated } V_{F XM} \text{ Value, } R_{G K} = 1000 \text{ Ohms} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turn On Time</td>
<td>( t_{d} + t_{r} )</td>
<td>1.2</td>
<td>Microseconds</td>
<td>( T_{L} = 25^\circ C, V_{FX} = \text{ Rated } V_{F XM} \text{ Value, } I_{FM} = 1 \text{ Ampere, Gate Pulse = 4 Volts, 300 Ohms, 5 Microseconds Wide} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Circuit Commutated Turn-Off Time | \( t_{off} \) | 40 | 100 | Microseconds | \( T_{L} = 110^\circ C, \text{ rectangular current waveform. Rate of rise of current } < 10 \text{ amps/second, Rate of reversal of current } < 5 \text{ amps/sec. } I_{FM} = 1 \text{ Amp, } 50 \text{ microsecond pulse} \). Repetition Rate = 50 pulses per second. \( V_{RX} = 15 \text{ Volts Minimum, } V_{F XM} = \text{ Rated. Rate of Rise Reapplied Forward Blocking Voltage } = 5 \text{ Volts/second, Gate Bias } = 0 \text{ Volts, 100 Ohms (during turn-off time interval).} \)
1. Maximum Forward Characteristics, On State

2. Maximum On-State Power Dissipation

3. Maximum Gate Trigger Current and Voltage Variation with Trigger Pulse Width
4. Maximum Allowable Temperatures for Half Sine Wave On-State Current

5. Maximum Allowable Temperatures for DC On-State Current

6. Maximum and Minimum Holding Current Variation with External Gate-to-Cathode Resistance

7. Maximum Transient Thermal Impedance

8. Maximum Allowable Non-Repetitive Peak Surge Forward Current

9. Turn-On Current Limit
BIBLIOGRAPHY

2. SCR Manual, General Electric
5. Heat, Mass and Momentum Transfer, by Warren M. Rohsenow and
Luis I. Cabezon Gil was born on October 10th, 1951 in Santiago, Chile. After three years of studies in the School of Engineering at the University of Chile, Santiago, he transferred in September 1971 to the Massachusetts Institute of Technology. He is a member of the Honors Course in the Department of Mechanical Engineering and of the Pi Tau Sigma honorary society.