Type Synthesis Principle and Practice of Flexure Systems in the Framework of Screw Theory: Part II—Numerations and Synthesis of Complex Flexible Joints

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TYPE SYNTHESIS PRINCIPLE AND PRACTICE OF FLEXURE SYSTEMS IN THE FRAMEWORK OF SCREW THEORY
PART II: NUMERATIONS AND SYNTHESIS OF COMPLEX FLEXIBLE JOINTS

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ABSTRACT

In recent years, the increasing of application requirements call for development of a variety of high-performance (e.g. large-displacement, high-precision) flexible joints. In this paper we demonstrate how to use the proposed methodology for the type synthesis of flexure systems given in the companion paper to synthesize concepts for complex flexible joints. According to the joint characteristics other than other flexure systems, a basic design philosophy and a general type synthesis process for flexible joints are presented firstly. The numerations and type synthesis for four commonly used flexible joint types, i.e. flexible revolute joints (FRJs), flexible translational joints (FTJs), flexible universal joints (FUJs), and flexible spherical joints (FSJs) are investigated in detail. As a result, not only a variety of known flexible joints are systematically surveyed and classified, but also are some new flexible joints developed. The output of this process is the derivation of a multiple of flexible joint concepts that would then be modeled and optimized by existing modeling and analysis methods.

1 INTRODUCTION

Flexible joints are a type of precision-oriented compliant joints utilizing the inherent compliance of a material rather than restrain such deformation. These joints eliminate the presence of friction, backlash, and wear. Further benefits include up to sub-micron accuracy due to their continuous monolithic construction.

In the last decades, dozens of flexible joints have been invented and developed, and the detailed descriptions and comparisons in performance may refer to Smith [1, 2], Slocum [3], Howell [4], Lobontiu [5], Trease et al. [6], Tseytlin [7]. In terms of their topologies, all flexible joints can fall into two categories: primitive flexures and complex flexures [8]. The primitive flexure category includes small-length flexural pivots such as notch-type hinges [9], long flexible segments such as a wire straight or curved beam, a straight or curved blade, a tube or ring, a tape spring or angle spring, and a compliant bellow etc. (Fig. 1). Excluding notch-type primitives, other types can be acted as the large-deformation elements. In the mean time, these primitives can provide the corresponding compliant constraint reciprocal to its freedom according to Maxwell [10]. The complex flexure category, usually a combination of two or more flexure primitives, includes cross-axis flexural pivots [11], cartwheel flexures [12], split-tube flexural pivots [13], butterfly flexures [14], and Leaf-type Isosceles-trapezoidal Flexural Pivots (LIFPs) [15] etc. In term of motion pattern, however, flexures can be classified as flexible revolute joints, flexible translational joints and flexible compound joints according to Trease et al. [6]. Also, flexible joints can be divided into two categories with respect to their output stroke: large-displacement ones and small-displacement ones. In general cases, a large-displacement flexible joint always covers large-deformation flexure primitives. The advantages of complex flexures still include a large stroke, a high precision of rotation, or a high ratio of off-axis stiffness to axial stiffness to make up for part of drawbacks existing in the flexure primitives.

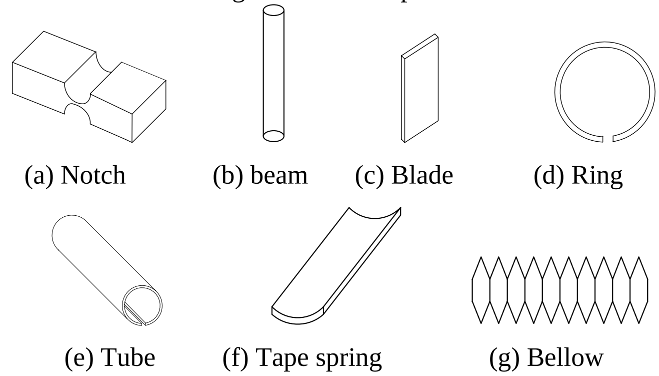
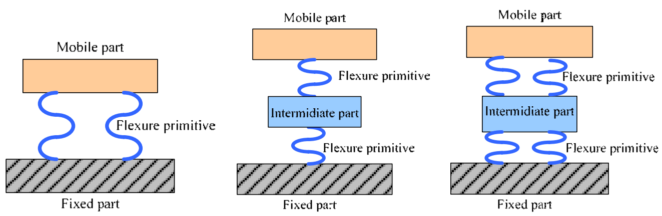


Fig. 1 Several flexure primitives

In recent years, the increasing of requirements call for a variety of *high-performance (e.g. large-displacement, high-precision)* flexible joints with various applications, including high-precision motion stages, aerospace devices, etc. Generally, three ways i.e. material, size and geometry may improve the output-stroke characteristics of a flexible joint. Pseudo-elastic flexure-hinges [16] and shape memory alloy wire actuated flexures [17] are both developed upon non-linear material, which may lead to some side effect on precision performance. Geometry of joints, however, is by far the most important factor in determining the output stroke of the system, as well as considering other performances such as accuracy, etc. For example, the large-displacement flexible joints in practical usages are always the complex ones by combination of flexure primitives in a serial, parallel or hybrid way (Fig. 2).



(a) Parallel (b) Serial (c) Hybrid

Fig. 2 Three types of flexible joints

Although there are also increasing designs for such complex flexible joints in the literatures, the joint types are still much fewer than those required. In this regards, a rigorous synthesis approach appears appealing. Lin [18] once applied the graph theory to the type synthesis of flexible translational joints. Unfortunately, the method seems not rigid because it can only lead to very limited types. In other words, the type synthesis for this class of flexible joints by aid of a systematic approach is to be explored. On the other hand, from a historical view, process of invention for most compliant joints is largely dependent on the experience and creativity of the designer, companied by dimensional synthesis, and usually leads to a specified design [6]. It is a common sense in rigid mechanism conceptual design field, however, that the type synthesis is always separated from dimensional synthesis.

Therefore, the objective of this research is to provide an effective means of realizing systematic type synthesis for high-performance complex flexible joints. The whole research is built upon the proposed approach given in the companion paper [19]. The purpose is to find as many complex flexible joint topologies as possible, without concerning the detailed size parameters and performance specifications of each joint.

The rest of this paper is organized as follows. Firstly, a basic design philosophy and a general type synthesis process for flexible joints are presented. After that, numerations and type synthesis of four commonly used flexible joint types, i.e. flexible revolute joints, flexible translational joints, flexible universal joints, and flexible spherical joints are investigated in detail.

2 TYPE SYNTHESIS PHILOSOPHY AND PROCESS

As a special flexure system other than a general case such as a flexure mechanism, a flexible joint should exhibit its particular geometry and criterion. The geometry of a flexible joint should be primarily simple and compact as well, which leads to as less intermediate parts as possible in the joint. Many flexure mechanisms may have the equivalent kinematics as a flexible joint does, but they generally can't be acted as flexible joints by themselves.

As mentioned in reference [6], in addition to the compactness of the geometry, a good and effective large-displacement flexible joint should have at least four most important criteria (*i*) a large range of motion, (*ii*) minimal parasitic motion, (*iii*) a large ratio of off-axis stiffness to axis-stiffness, and (*iv*) a reduced stress-concentration. The range of motion is determined largely by geometry of the joint, the parasitic motion can be improved by adding symmetry to the design of a joint. Leaf spring or plate as a flexure primitive generally has less stress-concentration than notched or wire flexure primitive. In all, the geometry arrangement of a flexible joint is particularly of importance for its performance. That is just the main task of the type synthesis.

In flexure geometry, when flexure primitives are connected in series, it prefers to add degrees of freedom (DOFs), and when flexure primitives are connected in parallel, it prefers to add degrees of constraints (DOCs), whereas a hybrid system with serial-parallel or parallel-serial structure may exhibit a compromised DOFs or DOCs. In the most cases, a flexible joint has at most three DOFs. Therefore, when synthesizing a single-DOF flexible joint, it is generally a serial or hybrid structural topology; on the contrary, a two-DOF or three-DOF flexible joint probably exhibits a parallel, serial, or hybrid topology.

Hence, based on the type synthesis principle for flexible joints investigated in the companion paper, and combined with the specialty of flexible joints, the whole type synthesis procedure is summarized as follows with a simple example.

Step 1. *Specify the desired freedom according to specifications of the flexible joint.*

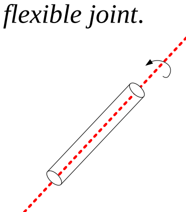


Fig. 3

Step 2. *Denote the specified freedom of the flexible joint using a visualized freedom space.*

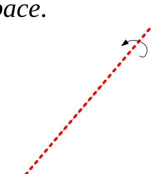


Fig. 4

Step 3. *Determine all possible reciprocal spaces representing the constraints of the joint, as shown in Table 3 in the companion paper.*

Step 4. *Select one appropriate reciprocal space type available for constructing a physical joint from the constraint space obtained in Step 3.*

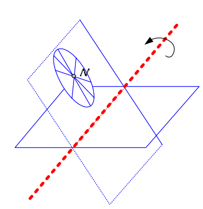


Fig. 5

Step 5. *Find the appropriate physical arrangement for each constraint space type obtained in Step 4.*

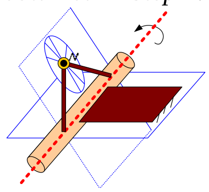


Fig. 6

Step 6: *Find an optimal joint profile with high performances in terms of performance requirements from the results in Step 5. Generally, a symmetrical redundant design is applied in this case.*

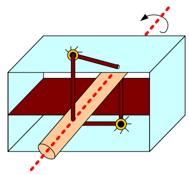


Fig. 7

Step 7. *Select another reciprocal space type available for constructing a physical joint and repeat the same thing as done from Step 4 to Step 6, until all available reciprocal space types are taken into consideration.*

From the next section on, we will make a systematic numeration and type synthesis of four types of flexible joints.

3 NUMERATIONS AND TYPE SYNTHESIS OF FLEXIBLE REVOLUTE JOINTS (FRJS)

A Flexible revolute joint, also called flexural pivot, flexure hinge, flexible bearing, or flexure, characterized by a high compliance with respect to the "in-plane" rotational degree of freedom (DOF) and high stiffness in all the other DOFs, is designed to generate pure rotational motion.

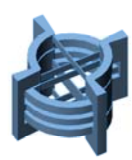
3.1 Regular FRJs

In the companion paper and last section in this paper, we have made an introductory type synthesis for FRJs by taking some special cases (e.g. $\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(N',\boldsymbol{n}')$) as example. In this section, we will continue this topic and give a comprehensive synthesis and numerations for this kind of joints.

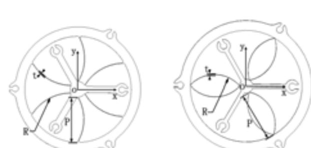
The detailed synthesis procedure is ignored, here only the results are numerated as follows.

(1). The known FRJs illustrated in Fig. 8 satisfy the case of $\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(N',\boldsymbol{n}')\cup\mathcal{L}(N'',\boldsymbol{n}'')\cup\cdots$.

(a) Three-plate[14]



(b) Multi-plate[29]

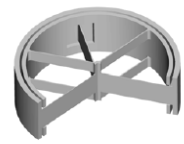


(c) Initiate curved six-plate [19]

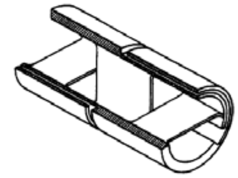
Fig. 8 $\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(N',\boldsymbol{n}')\cup\mathcal{L}(N'',\boldsymbol{n}'')\cup\cdots$

(2). The known FRJs illustrated in Fig. 9 satisfy the case of $\mathcal{F}_2(N_1,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(N_2,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{L}(N',\boldsymbol{n}')$.

(a)[12]



(b)



(c)

Fig. 9 $\mathcal{F}_2(N_1,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(N_2,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{L}(N',\boldsymbol{n}')$

(3). The known FRJs illustrated in Fig. 10 satisfy the case of $\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(N',\boldsymbol{u}',\boldsymbol{n}')\cup\mathcal{F}_2(N',\boldsymbol{u}',\boldsymbol{n}')\cup\cdots$.

Fig. 10 $\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(N',\boldsymbol{u}',\boldsymbol{n}')\cup\mathcal{F}_2(N',\boldsymbol{u}',\boldsymbol{n}')\cup\cdots$

(4). Torsion hinges

Torsion occurs in compliant members when out-of-plane load is applied to a flexure mechanism. Torsion also occurs in torsion bars that are used to obtain angular deflection using torsional compliant members.

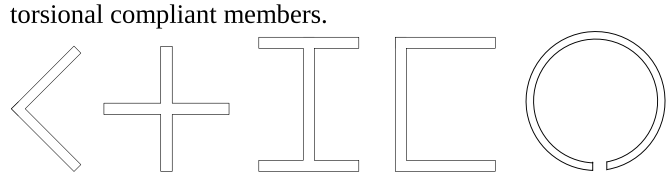
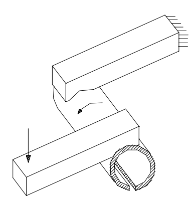
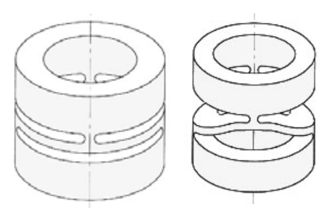


Fig. 11 Cross-section profiles

By using the synthesis procedure given in section 2, we may obtain different cross-section profiles of torsion hinges, as shown in Fig. 11. It is noted that not only rectangular beams can serve as torsion hinges, square, circular, and hollow cross-sections can also be used. An open shell is formed by cutting a slit lengthwise along a closed hollow beam, resulting in very lower in-axis compliance while maintaining the higher off-axis stiffness. Based on this concept, a typical torsion hinges called the split-tube joint was developed by Goldfarb [13]. It has the off-axis stiffness of a cylinder and very little torsional stiffness. Further, it has almost no center of rotation drift when the connected links are fixed along the line of center of rotation, shown in Fig. 12b



(a)



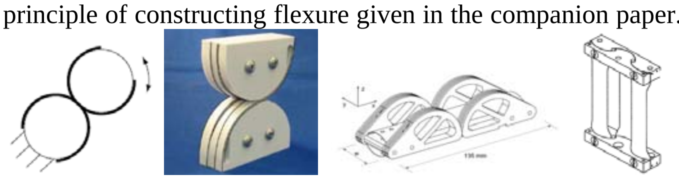
(b)

Fig. 12 Two torsion hinges

(5). Compliant rolling contact joints with tape springs

A compliant rolling contact joint [20, 21, 22] is composed of two surfaces rolling without slipping on each other, as illustrated in Fig. 13. Under such a classic configuration, the two surfaces are attached to each other by thin strips. These strips allow rolling contact, but prevent the surfaces from sliding or the two parts of the joint from separating. The strips can wrap over one or the other part of the joint. In order to ensure the rigidity in the direction normal to the plane of motion, the joint should be sufficiently wide and the load carried in that direction should be limited. This kind of joint is characterized by a very large rotation in desired directions but generally no fixed rotational axis.

In essence, compliant element arrangement of each compliant rolling contact joint should be also satisfied the principle of constructing flexure given in the companion paper.



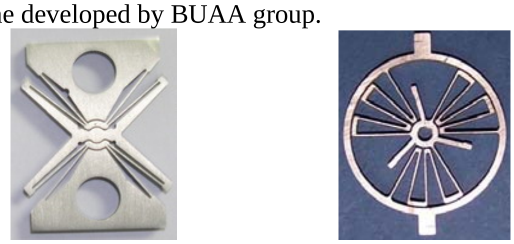
(a) (b) X_R-joint [20] (c) (d)

Fig. 13 Two torsion hinges

(6). Hybrid FRJs

Compared with pure parallel flexible joints, a hybrid flexible joint may have the merits of such as (*i*) exhibit comparatively large-displacement characteristics, (*ii*) cancel parasitic errors, etc.

In fact, the synthesis procedure for hybrid flexible joint has been investigated in the companion paper, as identical with that for hybrid flexure mechanisms. Two cases, as shown in Fig. 14, can be obtained by this method. One is the well-known butterfly-type joint developed by CSEM, the other is a new one developed by BUAA group.

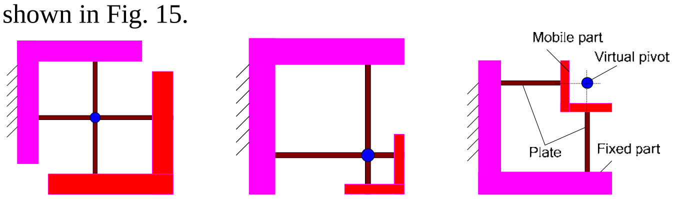


(a) Butterfly-type [14] (b)

Fig. 14 Two hybrid FRJs

3.2 Virtual-center-of-motion (VCM) FRJs

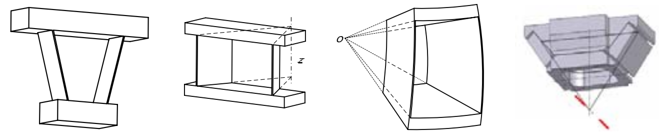
A virtual centre of motion (VCM) FRJ is a kind of flexible revolute joint that has a fixed rotary centre but the place around the centre is vacant, i.e. the virtual rotary centre, as shown in Fig. 15.



(a) Symmetry (b) Asymmetry (c) VCM structure

Fig. 15 Derivation process of a VCM FRJ

A typical VCM flexure is a leaf-type isosceles-trapezoidal flexural (LITF) pivot which is shown in Fig. 16a. The virtual center of the LITF pivot is located where the two leaf-type springs intersect to each other. There are other VCM FRJs shown in Fig. 16b-d. In Fig.16b, the planes of two leaf-type springs are intersected at axis *z*. When the bottom rigid part is fixed, the upper rigid part can rotate around this virtual axis *z*. In Fig.16c, the planes of two leaf-type springs and two rigid parts are intersected at point *o*. When the bottom rigid part is fixed, the upper rigid part can rotate around this virtual center *o*.



(a) (b) (c) (d) [23]

Fig. 16 VCM FRJs

Kyusojin and Sagawa [24] developed several VCM FRJs based on leaf springs. These joints generally have a good range of motion.

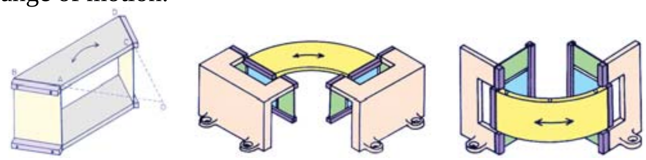


Fig. 17 VCM FRJs [24]

The VCM FRJs can be of great use for practical designs in which a pure rotation about a virtual pivot is required in the instance where the pivot point is to be vacant to enable the placement of specimens. Moreover, because of its virtual center, the VCM FRJs as a flexible building block (FBB) are very suitable for combination to create new complex FRJs, such as the Double-LITF pivot or butterfly pivot. In this case, the freedom space corresponding to any a VCM FRJ can be simple regarded as a one-dimension line. An illustration for large-displacement FRJs based on combinations of several LITF FBBs is shown in Fig. 18.

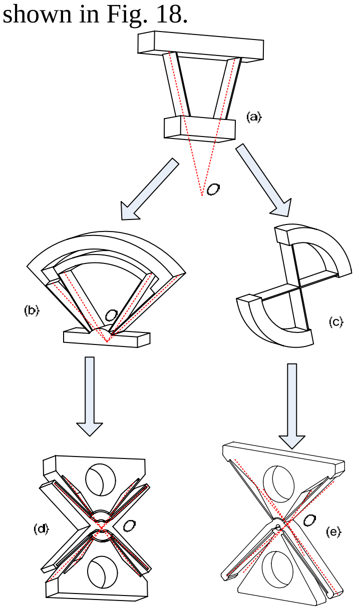


Fig. 18 Generating large-displacement FRJs based on LITF FBBs

3.3 Other kinds of FRJs

Some applications such as high-precision actuators or sensors require a monolithic FRJ with high mechanical advantage or displacement-amplification function. Under this background, compliant lever, chevron beam, the diamond, and the ellipse etc. are the main modules used to generate mechanical advantage or amplification of displacement. The diamond is a symmetrical implementation of two chevron beams. Elliptically-shaped amplification flexures, such as the one shown in Fig. 19, are generally designed with ratios of diameter to width 5 or less to avoid out-of-plane error motions. It is easily verified that these geometry also satisfy the topological requirement of a general FRJ based on their equivalent kinematic characteristics.

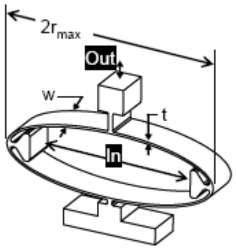


Fig. 19 Elliptically-shaped amplification flexure [25]

4 NUMERATIONS AND TYPE SYNTHESIS OF FLEXIBLE TRANSLATIONAL JOINTS (FTJS)

As a novel improvement of the leaf spring joint, the flexible translational joint is designed as a complex flexural device which may achieve a pure linear motion in precision applications to replace conventional rigid translation joints, such as proposed as sliders, rails, or linear bearings, etc.

In this section, we will make a comprehensive numeration and classified synthesis for known FTJs.

4.1 Set Operation on the Geometric Building Blocks

(1) Type 1

$$\begin{aligned}
&\mathcal{T}\cup\mathcal{U}(N,\boldsymbol{n})\\
&=\mathcal{T}_2(\boldsymbol{n})\cup\underline{\mathcal{P}(\boldsymbol{n})\cup\mathcal{R}(N,\boldsymbol{u})}\cup\mathcal{R}(N,\boldsymbol{v})\\
&=\mathcal{T}_2(\boldsymbol{n})\cup\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{R}(N,\boldsymbol{v})\\
&=\mathcal{T}_2(\boldsymbol{n})\cup\underline{\mathcal{R}(N,\boldsymbol{u})\cup\mathcal{R}(M,\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{v})}\\
&=\mathcal{T}_2(\boldsymbol{n})\cup\mathcal{L}(N,\boldsymbol{n})
\end{aligned}$$

$$\begin{aligned}
&\mathcal{T}_2(\boldsymbol{n})\cup\mathcal{L}(N,\boldsymbol{n})\\
&=\mathcal{P}(\boldsymbol{u})\cup\mathcal{P}(\boldsymbol{v})\cup\mathcal{R}(N,\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{v})\cup\mathcal{R}(M,\boldsymbol{w})\\
&=\underline{\mathcal{P}(\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{v})}\cup\underline{\mathcal{P}(\boldsymbol{v})\cup\mathcal{R}(N,\boldsymbol{u})}\cup\mathcal{R}(M,\boldsymbol{w})\\
&=\mathcal{F}_2(N,\boldsymbol{v},\boldsymbol{u})\cup\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{v})\cup\mathcal{R}(M,\boldsymbol{w})\\
&=\mathcal{R}(N,\boldsymbol{v})\cup\mathcal{R}(L,\boldsymbol{v})\cup\mathcal{R}(L,\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{u})\cup\mathcal{R}(M,\boldsymbol{w})\\
&=\underline{\mathcal{R}(N,\boldsymbol{v})\cup\mathcal{R}(N,\boldsymbol{u})\cup\mathcal{R}(M,\boldsymbol{w})}\cup\underline{\mathcal{R}(L,\boldsymbol{v})\cup\mathcal{R}(L,\boldsymbol{u})}\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{U}(L,\boldsymbol{n})
\end{aligned}$$

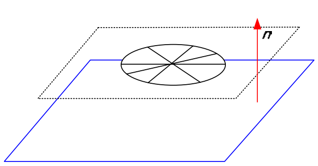


Fig. 20 $\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{U}(L,\boldsymbol{n})$

(2) Type 2

Consider existence of redundant constraints and the following equation.

$$\mathcal{F}(\boldsymbol{u})\cap\mathcal{F}(\boldsymbol{v})=\mathcal{P}(\boldsymbol{n})\quad(\boldsymbol{u}\times\boldsymbol{v}=\boldsymbol{n})$$

Thus we have

$$\begin{aligned}
&\mathcal{T}\cup\mathcal{U}(N,\boldsymbol{n})\\
&=\mathcal{T}_2(\boldsymbol{n})\cup\mathcal{L}(N,\boldsymbol{n})\\
&=\mathcal{F}_2(N,\boldsymbol{v},\boldsymbol{u})\cup\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{v})\cup\mathcal{R}(M,\boldsymbol{u})\\
&=\underline{\mathcal{R}(L,\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{u})\cup\mathcal{R}(M,\boldsymbol{u})}\cup\mathcal{R}(N,\boldsymbol{v})\cup\mathcal{R}(L,\boldsymbol{v})\cup\mathcal{P}(\boldsymbol{n})\\
&=\mathcal{F}(\boldsymbol{u})\cup\mathcal{F}(\boldsymbol{v})\quad(\boldsymbol{u}\times\boldsymbol{v}=\boldsymbol{n})
\end{aligned}$$

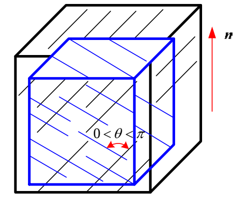


Fig. 21 $\mathcal{F}(\boldsymbol{u})\cup\mathcal{F}(\boldsymbol{v})$

(3) Type 3

Note that

$$\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(M,\boldsymbol{n})=\mathcal{P}(\boldsymbol{n})$$

Therefore, we have

$$\begin{aligned}
&\mathcal{T}\cup\mathcal{U}(N,\boldsymbol{n})\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{U}(M,\boldsymbol{n})\cup\mathcal{P}(\boldsymbol{n})\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{R}(M,\boldsymbol{v})\cup\underline{\mathcal{R}(M,\boldsymbol{u})\cup\mathcal{P}(\boldsymbol{n})}\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{R}(M,\boldsymbol{v})\cup\mathcal{F}_2(M,\boldsymbol{u},\boldsymbol{n})\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{R}(M,\boldsymbol{v})\cup\mathcal{R}(M,\boldsymbol{u})\cup\mathcal{R}(L,\boldsymbol{u})\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(M,\boldsymbol{n})
\end{aligned}$$

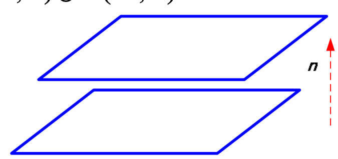


Fig. 22 $\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(M,\boldsymbol{n})$

(4) Type 4

In similar, we can derive that

$$\begin{aligned}
&\mathcal{T}\cup\mathcal{U}(N,\boldsymbol{n})\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(M,\boldsymbol{n})\\
&=\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(M,\boldsymbol{n})\cup\mathcal{L}(L,\boldsymbol{n})\cup\cdots
\end{aligned}$$

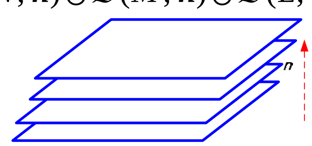


Fig. 23 $\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(M,\boldsymbol{n})\cup\mathcal{L}(L,\boldsymbol{n})\cup\cdots$

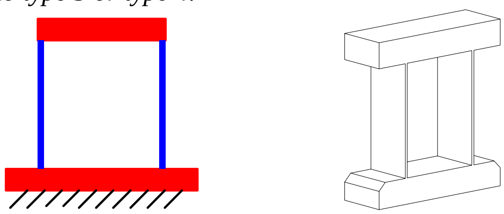
(5) Type 5

In similar, we can derive that

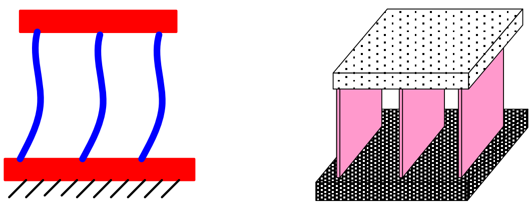
$$\begin{aligned}
&\mathcal{L}(M,\boldsymbol{n})\cup\mathcal{L}(N,\boldsymbol{n})\cup\mathcal{L}(L,\boldsymbol{n})\cup\cdots\\
&=\mathcal{F}_2(M,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(M,\boldsymbol{v},\boldsymbol{n})\cup\mathcal{F}_2(N,\boldsymbol{u},\boldsymbol{n})\\
&\quad\cup\mathcal{F}_2(N,\boldsymbol{v},\boldsymbol{n})\cup\mathcal{F}_2(L,\boldsymbol{u},\boldsymbol{n})\cup\mathcal{F}_2(L,\boldsymbol{v},\boldsymbol{n})
\end{aligned}$$

4.2 Numerations and Type Synthesis of FTJs

(1) A pure parallel configuration in which all flexure primitives are pure plates. In this case, the synthesized FTJs belong to type 3 or type 4.



(a) $\mathcal{L}(N,\mathbf{n})\cup\mathcal{L}(M,\mathbf{n})$

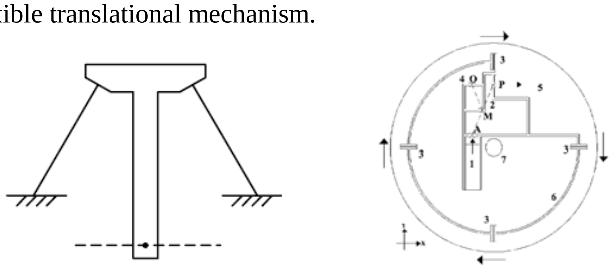


(b) $\mathcal{L}(N,\mathbf{n})\cup\mathcal{L}(M,\mathbf{n})\cup\mathcal{L}(L,\mathbf{n})$

Fig. 24

Most of the existing large-displacement FTJs are based on a parallel two-plate building block, as shown in Fig. 24a. Their flexibility is derived from leaf springs. The geometry is easy to be derived from type 3, which constrains the two rotational DOFs, while the parallelogram shape unites the remaining DOF together to create a curvilinear motion. In order to increase stiffness, more than two parallel plates are used to form a folded straight-beam or cured-beam structure, as shown in Fig. 24b.

Other kinds of parallel FTJs may come from its rigid counterpart, i.e. the instantaneous or approximate straight-line linkages, such as Roberts linkage, Scott-Russell linkage etc. In general cases, they are used as building blocks in a FTJ or a flexible translational mechanism.

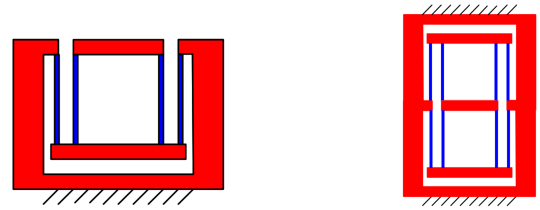


(a) Roberts linkage [26] (b) Scott-Russell linkage [27]

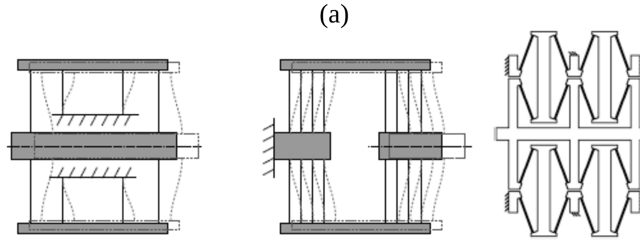
Fig. 25 Compliant linkages

(2) A hybrid configuration in which all flexure primitives are pure plates.

Most of currently existing hybrid FTJs are indeed compound joints by means of combinations of two and more linear or VCM building blocks, as shown in Fig. 26. They all deliver a larger range of straight-line motion. In fact, the philosophy of building block and the symmetric design of the configuration surely follow the exact constraint principle given in the companion paper. On the other hand, one can also easily derive all configurations shown in Fig. 26 by using the type procedure given in the companion paper.



(a)

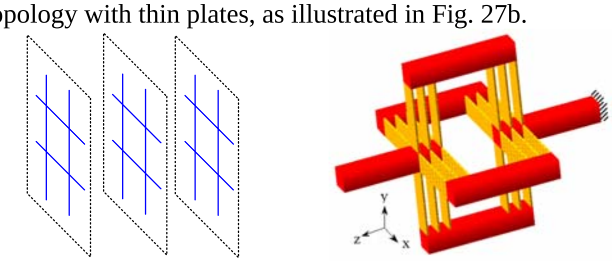


(b) [26]

Fig. 26 Hybrid FTJs

(3) *A planar (parallel or hybrid) configuration in which all flexure primitives are pure strips or bars. In this case, the synthesized FTJs belong to type 1, 3 or 5.*

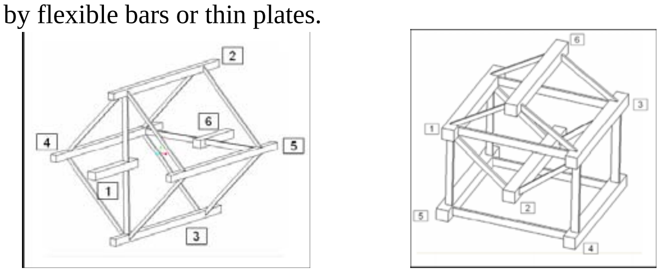
In order to increase the range of motion, multiple thin plates or strips (bars) can be used to design the FTJs, as done by the University of Michigan and shown in Fig 27. To specify it, According to the characteristics of constraint space (type 5, $\mathcal{F}_2(M,\mathbf{u},\mathbf{n})\cup\mathcal{F}_2(M,\mathbf{v},\mathbf{n})\cup\mathcal{F}_2(N,\mathbf{u},\mathbf{n})\cup\mathcal{F}_2(N,\mathbf{v},\mathbf{n})\cup\mathcal{F}_2(L,\mathbf{u},\mathbf{n})\cup\mathcal{F}_2(L,\mathbf{v},\mathbf{n})$), shown in Fig27a, it is easy to derive the joint topology using thin plates, For example, in order to increase the off-axis stiffness of the FTJ without decreasing the motion output, we can also apply this constraint space to construct a new joint topology with thin plates, as illustrated in Fig. 27b.



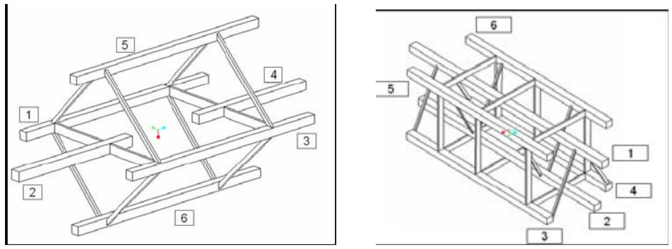
(a) Constraint space (b) Schematic

Fig. 27 FTJ designed by University of Michigan[6]

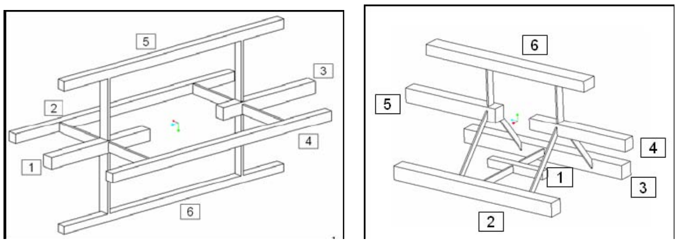
Using the same principle, we can also construct all six configurations proposed by Lin [18]. In every joint, one rigid link connects with the ground as fixed link, one rigid link acts as a mobile part, and four symmetric rigid links correspond to the intermediate part. In addition, all rigid links are connected by flexible bars or thin plates.



(a) Type 3 (b) Type 3



(c) Type 3 (d) Type 5 (type 4)



(e) Type 5 (f) Type 5 (type 1)

Fig. 28 Six FTJs [Lin, 2007[18]]

（4）*A new (pure parallel or hybrid) configuration in which flexure primitives are combinations of plates, strips or bars. In this case, the synthesized FTJs belong to type 3 or type 5.*

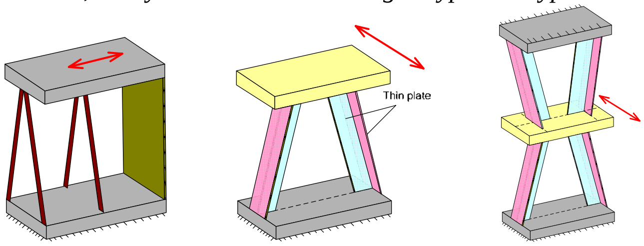


Fig. 29 Three novel FTJs

5 NUMERATIONS AND TYPE SYNTHESIS OF FLEXIBLE UNIVERSAL JOINTS (FUJS)

A flexible universal joint (FUJ), also called a flexible cardan joint, allows only two rotational DOFs, as does its rigid counterpart. In addition, two rotational axes should intersect into one point under an ideal circumstance (Fig. 30).

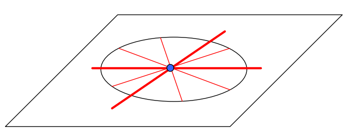
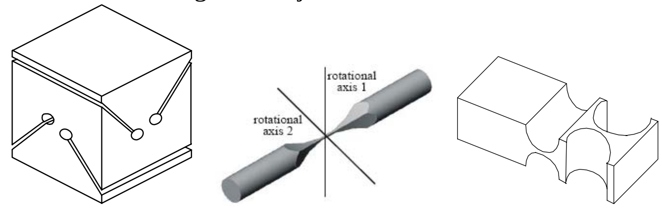
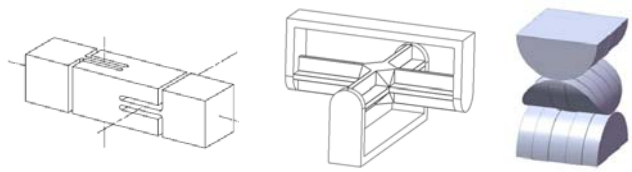


Fig. 30 Freedom space of FUJs

In theory, it is possible to construct a pure parallel FUJ using the method given in the companion paper, the resultant structure, however, is prone to be more complicate than its serial counterpart. Therefore, the flexible universal joint consists of a pair of flexures at 90º placed in a serial way in most cases. Figures 31a-f are showing possible geometries of this class of joints. The joint in Figure 31a has two notches which are perpendicular to each other. This allows rotations about two axes. The joint in Fig. 31b has a notch which is rotationally symmetrical, allowing arbitrary rotations and limited torsional movements. The joint in Fig. 31c has a notched flexure hinge with two DOF and non-intersecting axes of rotation. But these three joints shown in Fig. 31a-c can't be resorted to large-displacement category in a strictly sense. The two joints in Fig. 31d-f are all the large-displacement FUJs because of their leaf spring elements. The joint in Fig. 31e is created by concatenating two CR joints, whereas the joint in Fig. 31f is designed as concatenation of two flexible rolling contact joints.



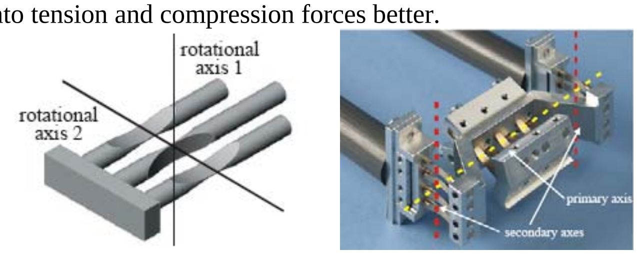
(a) (b) [16] (c)



(d) [31] (e) [6] (f) [21]

Fig. 31 FUJs with two flexures in series

Fig. 32a shows a combined FUJ with two DOF and intersecting axes of rotation, it is noted that the DOF of axis 2 in this joint is realized by a combination of two (or two more) flexures with one DOF. Figure 32b shows the combination of flexures. Due to parallel and angular arrangement of the hinges, the torsional moments are absorbed and transformed into tension and compression forces better.



(a) (b)

Fig. 32 Complex FUJs with two more flexures [16]

We can also construct a VCM FUJ, Just as does for the FRJ given in section 3. Fig. 33 shows such a derivation process of a new complex FUJ. In this example, several leaf-type springs can form a LITF pivot [16]; two LITF pivots can construct a Double-LITF pivot [20]; two Double-LITF pivots can further construct a flexible universal joint with a VCM.

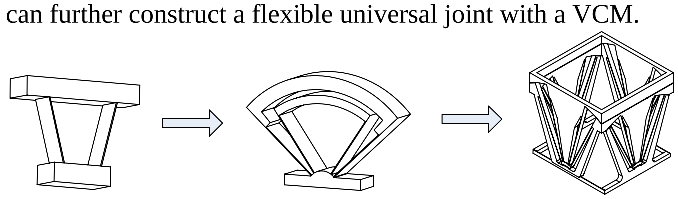
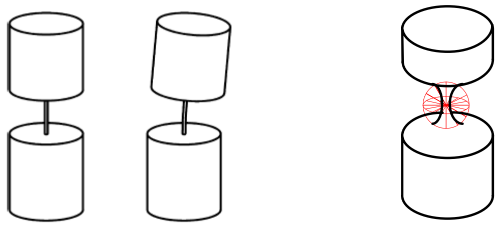


Fig. 33 Derivation of a complex FUJ

5 NUMERATIONS AND TYPE SYNTHESIS OF FLEXIBLE SPHERICAL JOINTS (FSJS)

A flexible spherical joint (FSJ) allows three rotational DOFs, as does its rigid counterpart. In general cases, three rotational axes should intersect into one point under an ideal circumstance. As shown in Fig. 34, the two FSJs with single flexure primitive can be obtained via compliance mapping.

In addition, there is a little difference between a FRJ or a FUJ and a FSJ in patterns, because of intermediated DOF, a FSJ may exhibit parallel, serial, or hybrid structure. Therefore, the numeration and type synthesis for this kind of joint will be made in terms of different cases.



(a) Cylindrical or steel wire (b) Notched-type

Fig. 34 FSJs with single flexure primitive

(1) Serial structure

Freedom-based method is applied to synthesize it.

Step 1: *Specify the desired freedom according to specifications of the flexure system*. For example, our task is the type synthesis of compliant spherical joints, which means that the flexure system should have three rotational axes with a common point.

Step 2: *Denote the specified freedom of the flexure system using a visualized freedom space*. For example, the freedom space of compliant spherical joints is a three-dimension line space $\mathcal{S}(N,\boldsymbol{u})$.

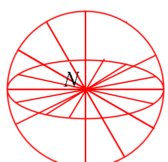
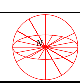
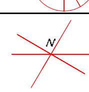
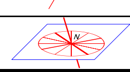


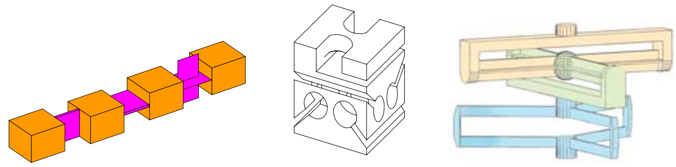
Fig. 35 Freedom space of a CSJ

Step 3: *Determine all possible subspaces by deposing the freedom space of the flexure system in terms of principle of the identical dimension and based on the approach given in section 2*. For example, the freedom space $\mathcal{S}(N,\boldsymbol{u})$ can exhibit several different subspaces, part of them are illustrated in Table 1.

Table 1 Subspaces of $\mathcal{S}(N)$

$\mathcal{S}(N)$	Illustrations
$\mathcal{S}(N)$	
$\mathcal{R}(N,\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{v})\cup\mathcal{R}(N,\boldsymbol{w})$	
$\mathcal{U}(N,\boldsymbol{u})\cup\mathcal{R}(N,\boldsymbol{w})$	

Step 4: *the approximate physical arrangement for each freedom subspace type obtained in Step 3*. For example, three physical configurations related to each freedom space type shown in Table 6 are illustrated in Fig. 36.



(a) RRR (b) UR (c) RU [6]

Fig. 36 Four FSJ types

(2) Parallel and hybrid structure

By using the method presented in the companion paper (Section 6), it is not difficult to find parallel and hybrid structures corresponding to FSJs. In fact, the comprehensive synthesis process for both parallel and hybrid FSJs has been investigated in Reference [28]. Hence the representatives of each type are illustrated in Fig. 37 and Fig. 38, respectively.

Fig. 37 Parallel FSJs [28]

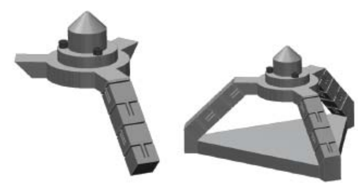


Fig. 38 A hybrid FSJ [28]

5 CONCLUSIONS

This paper makes a systematic numerations and synthesis of four types of commonly-used flexible joints with aid of the proposed methodology for the type synthesis of flexure systems given in Part I of the series of papers. As a result, not only a variety of known flexible joints are surveyed clearly and systematic classified, but also are some new types of high-performance complex flexible joints developed. What is more important, the philosophy and type synthesis approach presented in this paper may provide designers an effective tool to achieve a multiple of design concepts for new flexible joints in a simple but systematic way.

ACKNOWLEDGMENTS

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