A Comparative Study: The Dynamic Behavior of Tall Buildings with Diagrid and Hexagrid Structural Systems Subjected to Seismic Loads

by

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Submitted to the Department of Civil and Environmental Engineering on May 11, 2018 in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Civil and Environmental Engineering

ABSTRACT

Most advancements in achieving new structural heights for tall buildings are not possible without the innovation in structural systems. One type of tubular shell structural system, diagrids, like the one used in Hearst Tower, have gained much popularity in high-rise constructions over the past decade due to its high efficiency by resolving both gravity and lateral loads with the same elements. A new iteration of such tubular shell system named hexagrid was examined in this paper. The aim of the study is to evaluate the dynamic response and behavior of such systems when they are subjected to seismic loads, and to compare their efficiency to the better understood diagrid system. Three 60story and three 36-story models using diagrid and hexagrid exoskeleton systems designed to control the governing wind drift requirements were constructed in commercial software ETABS. For each height, one benchmark diagrid structure and two hexagrid structures using horizontal and vertical configurations were modeled. ASCE 7-10 Code based linear dynamic Modal Response Spectrum Analyses and modal analyses were then carried out for two locations. San Francisco and San Diego, to study the seismic performance based on the dynamic response and modal properties. The results from hexagrid architype models were compared against those of benchmark diagrid models to study the difference in dynamic behavior and relative efficiency. The analysis results showed similar mode shapes across different systems, which was attributed to the similar geometry and load-resisting mechanism of tubular shell structures. However, neither of the hexagrid configurations are as stiff as the diagrid system, resulting in larger seismic-induced lateral displacements and acceleration. An efficiency analysis shows, from the perspective of structural weight, that neither one of the two hexagrid configurations are as efficient as diagrids in controlling lateral drifts, but vertical hexagrids are comparable to diagrids in controlling lateral acceleration. It was also concluded that the studied vertical hexagrid configuration is stiffer and more efficient than the studied horizontal hexagrid configuration. The results of the study could be used by design professionals, architects and structural engineers alike, to make a more informed decision in system selection.

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Table of Contents

List of Figures	8
List of Tables	10
1. Introduction	11
2. An Overview of the Structural Systems for Tall Buildings	14
2.1 Diagrids	16
2.2 Hexagrids	17
3. Critical Literature Review: Seismic Performance and Optimization of Diagrid and Hexagid Structural Systems	20
3.1 Optimal Diagonal Angles and Patterns	20
3.1.1 Diagrid Structural System	20
3.1.2 Hexagrid Structural System	24
3.3 Seismic Performance & Design Factors	26
4. Model definition	29
4.1 Overall Geometry	29
4.2 Material	33
4.3 Selected Diagrid Structural System	34
4.4 Hexagrid Structural System	35
4.4.1 Horizontal Hexagrid	35
4.4.2 Vertical Hexagrid	35
4.5 Typical Member Sizes	36
5. Analysis and Design Methodology	37
5.1 Code-Based Structural Analysis and Design	37
5.2 Seismic Analysis Procedure	37
5.2.1 Procedure Selection	37

5.2.2 Overview of Modal Response Spectrum Analysis and Special Requirements Base	ed on
ASCE 7-10 §12.9	
5.2.3 Modal and Directional Combination	
5.3 Site Location and Parameters	
6. Results and Analysis	43
6.1 Comparative Study 1 – 60 Story	43
6.1.1 Modal Properties	44
6.1.2 Drift and Acceleration	46
6.1.3 Structural Weight, Embodied Carbon and Efficiency	51
6.2. Comparative Study 2 – 36 Story	53
6.2.1 Modal Properties	54
6.2.2 Drift and Acceleration	55
6.2.3 Structural Weight, Embodied Carbon and Efficiency	
7. Conclusion	62
8. Future Potential Research Areas	65
Bibliography	66

List of Figures

Figure 1: Diagrid (left) and Vertical Hexagrid (right) Structural System (Moon et al. 2007;
Mashhadiali and Keyroddin, 2012a)11
Figure 2: Poly International Plaza, SOM12
Figure 3: Acceleration Zoning Map of China (Gao, 2003)12
Figure 4: Timeline of World's Tallest Buildings and Increase in Height from Previous Record
(Kayvani, 2018)14
Figure 5: Structural System for Different Height by F.R. Khan (Gu, 2015)15
Figure 6: Elevation View of Traditional Braced Tube and Diagrid (Moon et al., 2007)15
Figure 7: Upper Row from Left to Right: IBM Building (Boake, 2013), Swiss Re Tower, Heast Tower;
Lower Row from Left to Right: Lotte Super Tower, Guangzhou West Tower (Sun, 2015).16
Figure 8: Pushover Curves of Diagrid Structures with Different Diagonal Angles (Kim and Lee,
2012)
Figure 9: Hexagon Shape in Natural Structure: Balsa and Cork Structural Patterns (Montuori et al.,
2015)
Figure 10: Pushover Curve of Hexagrid Structural System (Mashhadiali and Keyroddin, 2012)18
Figure 11: Sinosteel Building and Nanotower Conceptual Rendering (Montuori et al., 2015)19
Figure 12: Al Bahar Tower Hexagonal Frame (Arup, 2013)19
Figure 13: Module Definition20
Figure 14: Angle between the Diagonal Members and the Horizontal Plane
Figure 15: Top Story Displacement vs. Diagonal Angle of Different Building Heights (Moon et al.,
2007)22
Figure 16: Aspect Ratio Definition22
Figure 17: Weight in Steel Diagrid Structures as a Function of Diagonal Angles (Kim and Lee, 2012)
Figure 18: Selected Performance Indicators (Story Drift, Inter-story Drift, Steel Consumption)
Indicating 4-story Diagrid Modules as the Optimal Size (Isaac and Ipe, 2017)
Figure 19: Efficiency Comparison by Montuori et al. (2015)24
Figure 20: Effect on Horizontal Dilation of Diaphragm Restraint (a) No Restraint (b) Restraints at
Every 9th Floor (c) Restraints at Every Floor) (Montuori et al., 2015)
Figure 21: Module Node Constraints Comparison: (a) Single-story (b) Double-story (Montuori et al.,
2015)
Figure 22: Definition of Seismic Parameters (FEMA, 2003)27

Figure 23: Recommended R Value for Steel Diagrids (Baker et al., 2010)	27
Figure 24: Geometric Definition of Six Models	32
Figure 25: 3D Rendered View of Six Models, Showing the Braced Frame Core in 60-Story M	lodels
and Gravity Only Columns in 36-Story Models	33
Figure 26: Typical Angle of Diagrid Diagonal Members	34
Figure 27: Horizontal Hexagrid Module Definition	35
Figure 28: Vertical Hexagrid Module Definition	36
Figure 29: Table of Permitted Analytical Procedures as Listed in ASCE 7-10 Chapter 12	37
Figure 30: Sample Design Response Spectrum Used for Equivalent Lateral Force Analysis a	and Modal
Response Spectrum Analysis (ASCE, 2010)	
Figure 31: First 10 seconds San Francisco Design Response Spectrum with Spectral Accele	ration on
Y-axis, Period on X-axis	40
Figure 32: First 10 seconds San Diego Design Response Spectrum with Spectral Acceleration	on on Y-
axis, Period on X-axis	41
Figure 33: Ground Shaking Intensity Map FEMA E-74 (FEMA, 2012)	42
Figure 34: First Five Modes of 60 Story Models: Diagrid, Horizontal Hexagrid, and Vertical	Hexagrid
from Top to Bottom	45
Figure 35: RSA Story Displacement vs. Floor Plots of 60 Story Models	49
Figure 36: Shear vs. Flexural Deformation Mode (Hiraish, 1983)	50
Figure 37: Deflected Shapes of 60-story Diagrid, Horizontal Hexagrid, Vertical Hexagird Sh	owing
Flexural (Diagrids on the Left) and Shear Dominated Deflected Shapes (Two Hex	agrids on
the Right)	51
Figure 38: Steel Mass and Seismic Performance Correlation for 60 Story Models	53
Figure 39: First Five Modes of 36 Story Models: Diagrid, Horizontal Hexagrid, and Vertical	Hexagrid
from Top to Bottom	55
Figure 40: Story Displacement vs. Floor Number Plots of 36 Story Models	58
Figure 41: Steel Mass and Seismic Performance Correlation for 36 Story Models	60
Figure 42: Complex Connections Commonly Present in Diagrid Structures (Left: Swiss Re	Гower,
Right: Hearst Tower) (Boake, 2016)	63

List of Tables

Table 1: Seismic Design Factors for Steel Diagrid Structures (Baker et al., 2010)	28
Table 2: Seismic Design Factors for Steel Diagrid Structures (Sadeghi and Rofooei, 2018)	
Table 3: Typical Member Sizes	
Table 4: ASCE 7-10 §12.9 Calibration Requirements	
Table 5: Seismic Parameters of Two Sites	40
Table 6: Values Required to Construct Design Response Spectra	40
Table 7: Modal Mass Participating Ratio	43
Table 8: MRSA to ELF Base Shear Ratio	43
Table 9: Fundamental Periods and Fundamental Natural Frequency of 60 Story Models	45
Table 10: Top Story Drift of 60-Story Models	46
Table 11: Top Story Acceleration of 60-Story Models Under Seismic Loading	51
Table 12: Steel Mass and Embodied Carbon of 60 Story Models	52
Table 13: Material Efficiency of Different Systems of 60 Story Models	52
Table 14: Fundamental Periods and Fundamental Natural Frequency of 36 Story Models	55
Table 15: Top Story Drift of 36-Story Models	55
Table 16: Top Story Acceleration of 36-Story Models Under Seismic Loading	59
Table 17: Steel Mass and Embodied Carbon of 36 Story Models	60
Table 18: Material Efficiency of Different Systems of 36 Story Models	60
Table 19: Efficiency Study: Stiffened 36 Story Vertical Hexagrid Results Comparison	61

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1. Introduction

From the Great Pyramid of ancient Egypt to the modern Jeddah Tower in Saudi Arabia set to exceed 1000 meters, human ambition to reach new heights has never ceased. The astonishing progress would not be possible without the innovations and development of more efficient structural systems for different heights. Diagrids, i.e., diagonalized grids, for example, are among the most innovative and efficient structural systems for mid to high-rise buildings that have emerged in the recent decades (Boake, 2013). The essence of structural engineering is to design structures, by the means of choosing geometric form and material, to resist the anticipated vertical and lateral loads on a structure. Conventionally, such loads have been resolved by either frames with specially-detailed rigid connections that can resist loads in both directions by compression and bending action, or by separately resolving the lateral loads using more efficient elements such as braced frames, which deal with lateral forces through elements in compression, tension, or both. In diagrid structures, an innovative approach is taken by combining the advantages of the two traditional methods by eliminating most of perimeter columns and resolving both the gravitational and lateral forces through tension and compression actions of the diagonal members, as illustrated in Figure 1(a) (Moon et al., 2007). First proposed as a structural system in published literature in 2012 by Mashhadiali and Kheyroddin, Hexagrid system is the new iteration of the exoskeleton tubular structural system. It features modules composed of hexagon-shaped structural elements on the perimeter of the structure to resolve lateral and gravitational forces in a similar manner to diagrids. The load path diagram is illustrated on the right side of Figure 1(b).



Figure 1: Diagrid (left) and Vertical Hexagrid (right) Structural System (Moon et al. 2007; Mashhadiali and Keyroddin, 2012a)

There has also been recognized research work by Moon et al. (2007), Boake (2013), and Kim and Lee (2013) on the design, optimization and comparative studies of diagrids, as they relate to other traditional frame structural systems. Although there are few existing constructed diagrid structures in high seismic zones at this time, it is still essential to investigate the dynamic response behavior and performance of any new structural systems for applications in those regions. One example of the use of diagrids in relatively high seismic zone is the Poly International Plaza in Beijing, as shown in Figure 2. The seismic zoning map is shown in Figure 3, indicating high peak acceleration near Beijing (Gao, 2003).



Figure 2: Poly International Plaza, SOM



Figure 3: Acceleration Zoning Map of China (Gao, 2003)

At this point, the seismic behavior and performance of diagrids are relatively well-understood through analytical and experimental work carried out by leading design firms like Skidmore, Owings & Merrill (Baker et al., 2010) and researchers in the academia (Sun, 2015; Kim and Lee, 2013; Kim and Lee, 2012). However, there has been less work devoted the understanding of seismic behavior and performance of the new iteration in tubular exoskeleton structural system: hexagrids.

This paper aims at shedding some light on the seismic performance and systematic exploration of the dynamic behavior of tall buildings using different configurations of the hexagrid structural system, with the purpose of investigating the relative efficiency and performance of the newly proposed structural system under strong seismic load. The paper will start with an overview of the structural system development for tall buildings with a focus on exoskeleton tubular structures: diagrids and hexagrids in Section 2. A state-of-the-art literature review on the pattern optimization and performance of the two systems will be presented in Section 3. In section 4, the analytical models used for this study will be defined. Then, an introduction to the analysis procedure and criteria to be used will be presented in Section 5. With the analysis complete, comparative studies of the structures using hexagrid system to comparable benchmark diagrid structures of different height and aspect ratio will be carried out in Section 6. Section 7 and 8 will conclude the study with design implications and potential future research areas.

2. An Overview of the Structural Systems for Tall Buildings

The immense economic and technological advancement due to the second Industrial Revolution, especially the emergence of the first vertical transportation hoist designed by Elisha Otis in 1852, enabled the development of modern high-rise buildings in the two major cities: Chicago and New York City (Kayvani, 2014). Structural heights of tall buildings have since been pushed higher and higher. Such vertical growth would not be possible without the innovations in structural systems along the way.

Figure 4 shows the rapid height growth of the tallest building in the world in a just little over one century. Some of the step-ups are a direct result of the use of an innovative structural system. For example, the first ever modern skyscraper—Home Insurance Building—in Chicago was made possible by the application of structural steel framing in place of thick and heavy masonry bearing wall system. (Kayvani, 2014) The Sears Tower finished in 1974 that held the tallest building title for over two decades owes its glory to the Bundled Tube system, which was a major shift from the traditional tubular system and the even older portal frame system used in tall buildings like the Empire State Building in New York City. The current tallest building, Burj Khalifa (initially named Burj Dubai), brought a dramatic 60% increase in height compared to Taipei 101 thanks to its Buttressed Core structural system capable of resisting all the expected loads and meeting serviceability requirements on the 828-meter tower (Baker and Pawlikowski, 2012).



Figure 4: Timeline of World's Tallest Buildings and Increase in Height from Previous Record (Kayvani, 2018)

One of the greatest structural engineers and the pioneer in innovating structural systems for skyscrapers, especially tubular structures, Fazlur Rahman Khan, put together diagrams illustrating the most efficient structural system for various heights with steel and reinforced concrete in the 1970s, as shown in Figure 5:



Figure 5: Structural System for Different Height by F.R. Khan (Gu, 2015)

Categorization of these systems is just like any other engineering problem: there is no one unique solution. Different methods were proposed and used in previous research on structural systems, like the one done by Mir M. Ali in which he categorized systems based on the location (interior or exterior) of the main load bearing system. A previous study carried out by Gu in 2015 categorized structural systems into six main types: Rigid Frame, Core and Outrigger, Framed Tube, Trussed Tube, Tube in Tube, and Bundled System. She gave definition of these systems and introduced the main advantages. The systems of particular interest to this paper—diagrid and hexagrid—are considered as iterations of trussed tube system. An evevation view showing the defining characteristics of braced tube system and diagrid system is shown in Figure 6.



Figure 6: Elevation View of Traditional Braced Tube and Diagrid (Moon et al., 2007)

2.1 Diagrids

Like mentioned in the previous section, diagrids are considered, by some, as a type of trussed tube structural system. It was defined and discussed in more detail in the introduction section. Similar to braced tube used in John Hancock Tower in Chicago, it was found to be efficient for structures up to 100 story tall (Gu, 2015).

The first real world application of diagrids was designed by Leslie E. Robertson Associates Consulting Structural Engineers in the IBM Building in Pittsburgh back in 1965. Many more diagrids mid to highrise buildings have been designed and constructed since then, especially in the early 2000s, such as the Swiss Re Tower in London and the Hearst Tower in midtown Manhattan, New York City. The height of these earlier buildings does not represent the full potential of the diagrid structural system. Some other designs in recent years like the Lotte Super Tower proposed by Skidmore, Owings and Merrill and Guangzhou West Tower utilized the diagrid system to reach super tall status defined by Council on Tall Buildings and Urban Habitat, standing at 440 meters and 556 meters (Sun, 2015).



Figure 7: Upper Row from Left to Right: IBM Building (Boake, 2013), Swiss Re Tower, Heast Tower; Lower Row from Left to Right: Lotte Super Tower, Guangzhou West Tower (Sun, 2015) Given its popularity, it is not surprising that there has been much research on the diagrid structural system with focus on its angle optimization, wind-induced behavior, and progressive collapse behavior.

However, there is one major drawback in this system. Kim and Lee (2012) were among the first to investigate the seismic performance of diagrid system. They found out that, with nonlinear static and dynamic analysis, buildings with such system do not have high ductility, as shown in the pushover curve in Figure 8. All models failed at below 1% interstory drift, exhibiting unwanted brittle behavior for seismic applications (Kim and Lee, 2012). This is due to the inherent load-resolving mechanism, compression and tension of diagonal members, neither of which action would behave in a ductile manner.



Figure 8: Pushover Curves of Diagrid Structures with Different Diagonal Angles (Kim and Lee, 2012)

2.2 Hexagrids

Hexagon and hexagon-like shapes are very common in natural structures and materials at both miniscule and larger scales. Montuori et al. (2015) discussed the presence of such shapes in nature in their studies on the mechanistical properties of different hexagrid structure configurations. For example, in culler solids like balsa and cork, hexagon-shaped cells efficiently populate the space to form the material, as illustrated in Figure 9.



Figure 9: Hexagon Shape in Natural Structure: Balsa and Cork Structural Patterns (Montuori et al., 2015)

At a larger scale, the most obvious hexagon natural structure is beehive. The hexagon shape of honeycomb is believed by Roman scholar Marcus Terentius Varro to be the one that holds the largest amount of honey with least amount of wax required to build. Mathematician Thomas Hales formulated mathematical proof of "honeycomb Conjuncture", stating that one can divide a surface into equal regions with least perimeter length with hexagonal grids. This highly efficient structural pattern from nature has inspired material engineers to create light artificial composite material to possess high compression and shear strength (Montuori et al., 2015). Structural engineers and researchers are also interested in incorporating this naturally efficient pattern in load resisting system. As discussed in the introduction, Mashhadiali and Kheyroddin (2012a, 2012b) first proposed, in the form of a published literature, the use of such system on high-rise buildings, praising it as an innovative and efficient tubular structural system that is also architecturally pleasing. Aside from studying wind-induced behavior, they also carried out nonlinear static and dynamic analyses on three archetype models constructed in commercial software SAP2000. The linear pushover curve similar to those constructed by Kim and Lee (2012) in Figure 8 are shown in Figure 10 below. It was determined that hexagrid system has higher ductility and better inherent energy dissipation potential when compared to diagrid system (Mashhadiali and Keyroddin, 2012b).



Figure 10: Pushover Curve of Hexagrid Structural System (Mashhadiali and Keyroddin, 2012) In general, high ductility is highly desirable for seismic application, for that the high ductility can help achieve life safety goal set forth in building codes by giving ample time and warning to occupants. This one specific property might make hexagrid system a more desirable choice than diagrid structural system in high seismic zones.

At the time of writing this paper, there is no finished structure that utilizes hexagrid structural system alone as the sole lateral and vertical force resisting system yet. However, there has been conceptual designs such as the Sinosteel building and the Nanotower discussed by Montuori et al. (2015) in their study, which are shown in Figure 11.



Figure 11: Sinosteel Building and Nanotower Conceptual Rendering (Montuori et al., 2015) The Al Bahar Tower engineered by Arup in Abu Dhabi incorporated hexagrid frame in the concrete core system tower. However, the hexagrid frame only provides about 10% of the overall lateral stiffness to the finished building (Lee and Kim, 2017). Figure 12 shows the finished project, which has architectural finishing but still shows the hexagonal frame pattern on the façade (Arup, 2013).



Figure 12: Al Bahar Tower Hexagonal Frame (Arup, 2013)

All this information presents an opportunity for research to compare other seismic performance parameters such as drift and acceleration, as well as structural efficiency to those of similarly configured diagrid system, which will be the focus of this study.

3. Critical Literature Review: Seismic Performance and Optimization of Diagrid and Hexagid Structural Systems

Recently, significant efforts (Boake, 2013; Kim and Lee, 2013; Kim and Lee, 2012; Moon et al., 2007) had been put into investigating and optimizing the performance of diagrid structures of different modules. Hexagrids have also been studied (Mashhadiali and Keyroddin, 2012; 2013; Mathews et al., 2016) to some extent in the past few years. To make the comparison valid, it is essential that equivalent and also the best performing modules are used in the analysis of the building models. For exoskeleton tubular structures like diagrids and hexagrids, modules are defined as the same geometric section that repeats itself throughout the height and perimeter of the building. For example, one module is highlighted for each of the structural models in Figure 13.



Figure 13: Module Definition

Thus, state-of-the-art literature review related to the optimal patterns and seismic performance is conducted and the important findings are presented in the following sections.

3.1 Optimal Diagonal Angles and Patterns

3.1.1 Diagrid Structural System

For diagrid structures, it is commonly accepted that the angle between the diagonal members and the horizontal plane is the one of most important parameters that controls the efficiency of the system, noted as angle α in Figure 14.



Figure 14: Angle between the Diagonal Members and the Horizontal Plane

Moon et al. (2007) studied and published a paper about the mechanical characteristics and angle optimization of diagrid structures of various heights using several numerical models and an iterative design process to establish guidelines for design professionals. The fundamental mechanics, including shear and bending stiffness of diagrid modules, was studied to establish formulas for initial member sizing and optimal angle α . The important graphs and results are shown in Figure 15 below. The graphs represent the resulting horizontal displacements at the top of the diagrid structures of 3 different heights, 60, 42, and 20 Stories, with different diagonal angle α .



(b) 42 Story Diagrid Models



(c) 20 Story Diagrid Models

Figure 15: Top Story Displacement vs. Diagonal Angle of Different Building Heights (Moon et al., 2007)

Moon et al. (2007) concluded with the statement that the range of 65 to 75 degrees is the optimal one for diagrid structures of 60 stories and an aspect-ratio of 7. Aspect ratio is simply defined as the ratio between the height and the base width of the structure, h/b, defined in Figure 16.



Figure 16: Aspect Ratio Definition

This range becomes lower for lower structures with a smaller aspect ratio. Namely, it is 55 to 65 degrees for a 42-story structure with an aspect ratio of around 5 (Moon et al., 2007). These results

highly agree with research conducted by Kim and Lee (2012), who studied the seismic behavior of multiple 36-story diagrid structures with a fixed aspect ratio of 3.6 and various diagonal angles using nonlinear static and dynamic analyses. They found out that brace angles between 60-70 degrees will result in the highest efficiency for resisting vertical and lateral loads. The important figure illustrating the findings regarding efficiency as related to diagonal angles is listed in Figure 17. Isaac and Ipe (2017) arrived at similar conclusions in their studies as well.



Figure 17: Weight in Steel Diagrid Structures as a Function of Diagonal Angles (Kim and Lee, 2012) In addition, Isaac and Ipe (2017), in their comparative study work of Diagrid, Hexagrid and Octagrid systems, also found the optimal module sizes (the height of each "Diamond" shape) to maximize efficiency at resisting dynamic lateral loads. They concluded that the Diagrid building with 4-story modules and 67.38-degree diagonal angle exhibits the most desirable behavior among 2-12 story modules studied, resulting in greater lateral stiffness and low structural weight. Some key graphs are shown in Figure 18.



Figure 18: Selected Performance Indicators (Story Drift, Inter-story Drift, Steel Consumption) Indicating 4-story Diagrid Modules as the Optimal Size (Isaac and Ipe, 2017)

3.1.2 Hexagrid Structural System

For hexagrid structures, similar studies were carried out by multiple groups varying the module orientation, sizes, and diagonal angles. Among those studies, the work by Montuori et al. (2015) and Mathews et al. (2016) provided insights on maximizing the efficiency of hexagrid structural system. The following figure is obtained from the paper by Montuori et al. (2015), and the formula used to define efficiency is:

$$Efficiency = \frac{1}{\rho_v * \rho * b}$$

Where ρ_v is visual density, ρ is structural weight density, and b is the thickness of the exoskeleton tube. Each of the perimeters are defined differently for each system. It is discussed in more detail in Section 9 of Montuori et al. (2015). In Figure 19, the efficiency of each model is plotted, where H_H_30 means 30 story horizontal hexagrids; V_H means vertical hexagrids; and D means diagrids.



Figure 19: Efficiency Comparison by Montuori et al. (2015)

In terms of diagonal angles, Montuori et al. (2015) pointed out that, using preliminary parametric analyses and complete building design analyses, for horizontal hexagrids and vertical diagrids, diagonal angles of 60 degrees and 40-50 degrees respectively, are the most efficient in terms of a combined performance considering module stiffness, structural weight and visual density, which is a very significant architectural concern as it directly affects the amount of daylight and feel of openness of a building.

Regarding module height, they presented that, for horizontal hexagrids (here two parallel sides of the hexagon are horizontal), the edge constraints imposed by the rigid diaphragm at each floor can significantly increase the axial rigidity of hexagrid tubes by constraining the lateral dilatation of the structure, as illustrated in the Figure 20 below (Montuori et al., 2015).



Figure 20: Effect on Horizontal Dilation of Diaphragm Restraint (a) No Restraint (b) Restraints at Every 9th Floor (c) Restraints at Every Floor) (Montuori et al., 2015)

For unit cell (module) whose height equals to the inter-story height (Figure 21(a)), only partial horizontal movement of the nodes are possible when the structure is subjected to axial load. However, for modules of double-story height (Figure 21(b)), all the nodes land on diaphragms and such undesired movements can be fully blocked. Thus, double-story height is more advantageous and efficient in resisting axial load and controlling both vertical and lateral displacements of structural members.



(a)





The findings about module height were in line with what Mathews et al. (2016) found in their research on the seismic performance of hexagrid systems. The group studied three patterns of hexagrids proposed by previous researchers and used a total of 15 different configurations by varying the module geometry (width and height), concluding that: vertical hexagrids have the highest stiffness among others; and that for horizontal hexagrids, double-story height modules perform better (stiffer) than the 4-story modules.

The literature review related to optimal modules for the two structural systems provide significant foundation and baseline information for the comparative study to be conducted in this paper, especially for the model set-up and definition.

3.3 Seismic Performance & Design Factors

The focus of this paper is on the systematical study of dynamic response behaviors of diagrids and hexagrids under moderate to high seismic load. Thus, it is essential to obtain and apply the correct seismic design factors—Response Modification Factor (R), Over-strength Factor (Ω_0), and Displacement Amplification Factor (C_d). The definitions of these factors are illustrated in Figure 22 (FEMA, 2003).



Figure 22: Definition of Seismic Parameters (FEMA, 2003)

Baker et al. (2010) from one of the world's leading structural engineering firm with long and distinguished tradition in the design of tall buildings Skidmore, Owings & Merrill conducted research on these factors and published a paper proposing a methodology to determine the design factors for steel diagrids based on ATC-63 project/FEMA P695 approach in 2010. It is a probability-based approach to achieve the collapse prevention goal set forth by ASCE 7 by studying 300 different archetype models with varying aspect ratios, diagonal angles, seismic design categories, and gravity load intensity and subjecting the aforementioned archetype models to nonlinear static analyses. Iterations were used for the convergence of the R-factor, which are shown in Figure 23.



Figure 23: Recommended R Value for Steel Diagrids (Baker et al., 2010)

The analysis results for the design factors recommended for the design of steel diagrid structures are summarized in the Table 1.

Seismic Design Factor	R	Ω_0
Value	3.64	1.5

Table 1: Seismic Design Factors for Steel Diagrid Structures (Baker et al., 2010)

However, the industry and academia has not come to a consensus on these numbers to this day. Sadeghi and Rofooei just published a paper on March 23rd, 2018 (One month before this paper was written) questioning the validity of the number found by Baker et al. Sadeghi and Rofooei (2018) conducted similar numerical studies using FEMA P695 approach, concluding that R of 3.64 is unconservative for seismic design category (SDC) of D and above and suggesting that different R factors to be used based on the diagonal angle. R factors proposed by Sadeghi and Rofooei (2018) are listed in Table 2.

Table 2: Seismic Design Factors for Steel Diagrid Structures (Sadeghi and Rofooei, 2018)

Diagonal Angle	71.5	63.4	45
Response Modification	3	2.5	1.5
Factor R			

As for the hexagrid structural system, there has been little work done on establishing appropriate seismic design factors. Researchers conducting studies on hexagrid systems have used different response modification factors to obtain the seismic design forces and conducting published comparative studies, which are scarce at this time. The R values range from 3 to 6 (Mashhadiali and Keyroddin, 2012; Mathews et al., 2016; Darbandsari and Abdadi, 2017).

Since there is no consensus on this subject matter for either Diagrid or Hexagrid structural systems, for the purpose of this study, the Seismic Design Factors R and Ω_0 will be set at 3 and 1.5, respectively, for models of both structural systems. This is to control the variables in seismic design forces by using the same Design Response Spectrum for both models in order to compare their behavior and performance under the same expected earthquake intensity.

4. Model definition

Geometry has been proven to have a tremendous effect on the efficiency of a structural system for building structures. Two heights were chosen for the scope of this study to examine the seismic behavior of the new hexagrid system: a 60-story at a height over 200 meters (656 ft) and a 36-story that stands about 130 meters tall (427 ft). Two hexagrids structure models were constructed for each height with different patterns (Horizontal and Vertical), totaling four hexagrids models. Additionally, two diagrid structures of the same heights as the hexagrid models were also constructed as the benchmark structure to carry out the comparative study. This section is devoted to describing, in detail, the various geometric aspects of the models constructed.

4.1 Overall Geometry

Like discussed in Section 3, aspect ratio is one of the most important factors that influences the behavior and efficiency of a building (Sun, 2015; Kim and Lee, 2012). For example, the lateral deformation of short and stocky structures tends to be shear governed while a tall and slender building to behave more like a bending beam.

The base for the 60-story building was chosen to be a square of 42 meters (138 ft) in width while the 36-story building has a base width of 36 meters (118 ft). These choices were made based on typical real buildings at similar height and previous work of other researchers (Sun, 2015; Mathews et al., 2016; Darbandsari and Abdadi, 2017).

The story height was set at 3.6 meters (11.8 ft) for both building heights, making the 60-story building stand at 216-meter tall (709 ft) and the 36-story at 129.6 meters (425 ft). Thus, two different aspect ratios were set at 5.14 and 3.6. Typical floor and elevation views of the 6 models are shown in Figure 24. 3D rendered views are listed in Figure 25. Module geometry will be discussed in more detail in Section 4.3 (for diagrids) and 4.4 (for hexagrids).



(a) Plan (3-m constant Grid Spacing, Typical Unless Noted Otherwise), Elevation, and 3D views of 60-story Diagrid Model



(b) Plan (2.08-m and 3.625-m Alternating Grid Spacing), Elevation, and 3D views of 60-story Horizontal Hexagrid Model



(c) Plan, Elevation, and 3D views of 60-story Vertical Hexagrid Model



(d) Plan, Elevation, and 3D views of 36-story Diagrid Model



(e) Plan (2.08-m and 2.77-m Alternating Grid Spacing), Elevation, and 3D views of 36-story Horizontal Hexagrid Model



(f) Plan, Elevation, and 3D views of 36-story Vertical Hexagrid Model

Figure 24: Geometric Definition of Six Models



Figure 25: 3D Rendered View of Six Models, Showing the Braced Frame Core in 60-Story Models and Gravity Only Columns in 36-Story Models

4.2 Material

Structural steel was the only material assumed for all 6 models in order to control the variables as different material will drastically change the dynamic behaviors of a tall building as shown by Sun (2015) in his study of steel versus concrete diagrids.

4.3 Selected Diagrid Structural System

The primary focus of this paper is on the new iteration of tubular structural system: Hexagrids. Two diagrids are selected and modeled to be considered as the benchmark structures to which the hexagrid system will be compared. Thus, it is essential to select an optimal configuration for the modules to be used in the models.

As discussed in detail in Section 3.1.1, diagonal angles of 60-70 degrees are represented as the general consensus on the optimal range for structures of interested height and aspect ratio. Based on the previous work of Moon et al. (2015), Kim and Lee (2012), as well as Mathews et al. (2016), 67.4 degree was chosen to be the diagonal angles to be used on both the 36- and the 60-story benchmark structures, as shown in Figure 26. The 4-story module, which has the highest stiffness and relatively low structural weight among 5 other module heights, was chosen based on previous research from Mathew et al. (2016) discussed in more detail in Section 3.1.1.



Figure 26: Typical Angle of Diagrid Diagonal Members

It shall also be noted that all 60 story models have an identical braced steel frame structural core dimensioned at 18 meters by 18 meters located at the geometric center of the floor plan to provide additional lateral stiffness. The core is connected to the diagrid or hexagrid perimeter structure with transfer beams at each floor. It is also understood that such core is not needed for tubular structures in an ideal situation because of the high stiffness inherent with exoskeleton tubular structures like these. Nonetheless, this choice was made based on previous research (Sun, 2015), architectural needs for vertical transportation, as well as common construction practice for buildings of similar height and structural system. The inclusion of the core also allows the reduction of the member sizes and an increase in redundancy by using a dual system (Sun, 2015).

Structural core is not included in any of the 36-store models, however. Additional gravity columns are added in the interior to reduce the beam spans and to share gravity loads with the tubular shell structure.

It is important to note that such cores and columns are kept constant (same location and size) for all analytical models to reduce their effect to the minimum. Transfer beams and gravity beam locations are not matched 100% between the diagrid and hexagrid models due to the node location difference in each system, although great effort was made to keep those elements as similar as possible to minimize the effect on the overall systematic assessment of the dynamic behavior of the building models.

4.4 Hexagrid Structural System

Two types of hexagrid modules are studied and compared to diagrids to have a more comprehensive understanding of this innovative system. One is horizontal and the other is vertical. Horizontal hexagrid is defined as those structures having the two parallel sides of the hexagon are parallel to a horizontal plane, shown in Figure 27, whereas vertical hexagrid is defined as having two parallel sides of the hexagon perpendicular to the horizontal plane, shown in Figure 28.

Similar to Diagrid structures, the behavior of hexagrids is also highly related to the module shape, especially the size and diagonal angle. This section is devoted to defining the module geometry.

4.4.1 Horizontal Hexagrid

The module used to model horizontal hexagrids in this study has double-story height and a diagonal angle of 60 degrees, as illustrated in Figure 27.



Figure 27: Horizontal Hexagrid Module Definition

This module selection is an informed decision based on previous studies by Montuori et al. (2015) discussed in detail in Section 3.1.2.

4.4.2 Vertical Hexagrid

In a similar fashion as the horizontal hexagrids, the module of the vertical hexagrids is selected. The diagonal angle is

$$\theta = \arctan\left(\frac{story\ height}{half\ hexagrid\ width}\right) = \arctan\left(\frac{3.6\ meter}{3\ meter}\right) = 50.19^\circ$$

This angle is right at the upper limit of the recommended 40-50 degrees range by Montuori et al. (2015). The 4-story module height is selected based on the work of Mashhadiali and Kheyroddin (2012), and Mathews et al. (2016) hexagrid systems. A typical vertical hexagrid module is shown in Figure 28.



Figure 28: Vertical Hexagrid Module Definition

4.5 Typical Member Sizes

Some typical member sizes for the models are shown in Table 3 below. Beams and girders are selected from readily available wide flange sections, while members of the grid shell structures are custom square HSS sections. Member sizes are controlled across different models to minimize the effect caused by different sections on the system behavior.

Model	Typical Beam	Typical Girder	Typical Grid Shell at
			Dase (mm)
60 Story All	W24x94 (B1)	W27x102;(G1)	HSS550x75
		W27x129 (G2)	
36 Story Diagrid and	W21x83 (B2),	W24x117; (G3)	HSS450x25
Vertical Hexagrid	W24x94 (B1)	W27x129 (G2)	
36 Story Horizontal	W24x94 (B1)	W27x102 (G1)	HSS450x35
Hexagrid			

Table 3: Typica	Member Sizes
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*Note: Typical beams and girders are indicated by name B1, B2, G1, G2, and G3 in Figure 24.

5. Analysis and Design Methodology

5.1 Code-Based Structural Analysis and Design

The objective of this paper is to study the dynamic response behavior and the relative efficiency of the innovative hexagrid structural system considering two different building heights and aspect ratios, and to compare the results to those of the more well-understood diagrid structural system to inform future design professional of the potential of the new system under seismic load. The six (6) analytical building models described in Section 4 are subjected to the design loads including Dead, Live, Partition, Wind, and Seismic set forth in ASCE 7-10, Minimum Design Loads for Buildings and Other Structures (ASCE, 2010), at two different locations. Structural members are sized based on a generally accepted stiffness criterion of horizontal displacement of the top story in tall buildings: 1) H/500 for wind load; 2) 2% drift under seismic load.

The following sub-sections will discuss the code-based seismic analytical procedures used in this study in more detail.

5.2 Seismic Analysis Procedure

5.2.1 Procedure Selection

As mandated by ASCE 7-10 Section 12.6, the structural analysis of the analytical models shall consist of one of the permitted procedures outlined in table 12.6-1. This is shown in Figure 29. The structures to be studies fall into the category of "all other structures". Thus, Modal Response Spectrum Analysis or Seismic Response History Procedures are allowed. For this study, Modal Response Spectrum Analysis (RSA) will be used. A brief overview of this analytical procedure is provided in the next subsection.

Seismic Design Category	Structural Characteristics	Equivalent Lateral Force Analysis, Section 12.8 ^e	Modal Response Spectrum Analysis, Section 12.9 st	Seismic Response History Procedures Chapter 16*
B, C	All structures	Р	Р	Р
D, E, F	Risk Category I or II buildings not exceeding 2 stories above the base	Р	Р	Р
	Structures of light frame construction	Р	Р	Р
	Structures with no structural irregularities and not exceeding 160 ft in structural height	Р	Р	Р
	Structures exceeding 160 ft in structural height with no structural irregularities and with $T < 3.5T_c$	Р	Р	Р
	Structures not exceeding 160 ft in structural height and having only horizontal irregularities of Type 2, 3, 4, or 5 in Table 12.3-1 or vertical irregularities of Type 4, 5a, or 5b in Table 12.3-2	Р	р	Р
	All other structures	NP	Р	Р



5.2.2 Overview of Modal Response Spectrum Analysis and Special Requirements Based on ASCE 7-10 §12.9

The dynamic behavior of any system can be studied if its equation of motion is written out. For a freevibrating single Degree of Freedom dynamic system of mass m, stiffness k and damping c, the equation of motion can be expressed as:

$$m\ddot{x} + c\dot{x} + kx = 0$$

A building structure can be generally idealized as a N-degree-of-freedom system whose equation of motion can be written in a similar form with scalars m, c, and k being replaced as three $N \times N$ matrices, [M], [C], and [K]. The most important dynamic attributes of such building structures, natural periods and their corresponding vibrating modes, can be extracted from the equation of motion by solving the following matrix eigenvalue and eigenvector problem:

$$[K]{x} - \omega^2[M]{x} = {0}$$

Modal Response Spectrum Analysis is a linear dynamic analysis procedure used to predict the dynamic behavior of a structure subjected to a response spectrum by modal superposition. In general, the first few natural modes of a structure are extracted and subjected to the response spectrum to obtain the response of each mode and then the responses are combined numerically to provide the prediction of the overall response. The Design Response Spectra are to be divided by R/I_e to obtain the specific spectra to be used for a specific system to account for the post-elastic strength of the system, i.e., ductility. A sample Design Response Spectrum is shown in Figure 30.



Figure 30: Sample Design Response Spectrum Used for Equivalent Lateral Force Analysis and Modal Response Spectrum Analysis (ASCE, 2010)

Different jurisdictions have different special requirements for the calibration of the design base shear and Modal Mass Participation Ratio. In this case, the code states that the number of modes shall be selected that the total Modal Mass Participation Ratio shall be at least 90% of the seismic weight in each orthogonal directions. Even though Equivalent Lateral Force Analysis is not allowed for the studied buildings, it is required in ASCE 7-10 §12.9.4 that the resulting base shear from Modal Response Spectrum Analysis (Vt) to be at least 85% of the base shear found by Equivalent Lateral Force Procedure (V). If not, the response spectrum shall be scaled to achieve the 85% ratio to obtain member forces as well as drift. The requirements are summarized in Table 4:

Table 4: ASCE 7-10 §12.9 Calibration Requirements

Parameter	Modal Mass Participation Ratio	MRSA Base Shear Vt
Requirement by ASCE 7- §12.9	$0 \geq 90\%*W$	≥85%*V

5.2.3 Modal and Directional Combination

It shall be noted that, for this study, results from different modes are combined using Complete Quadratic Combination (CQC) because of the expected closely spaced modes. Member forces resulted from the different application directions are also combined using Complete Quadratic Combination (CQC) method.

5.3 Site Location and Parameters

The building models are to be set at two different locations, both of which with high magnitude ground motion, namely, San Francisco and San Diego in California, United States of America. San Francisco was chosen because of a previous study conducted by Sun (2015) about the seismic performance of diagrid structures with different materials. It is desired that the analysis results of the 60-story benchmark diagrid structure have similar results to validate the analysis and design procedures.

The ground motion parameters (S_s, S₁, T_L) are obtained from ASCE 7-10 §22, and soil profile were obtained from United States Geological Survey (USGS) "Bay Area Soil Type and Shocking Hazard Map" from Sun's work (Sun, 2015). The site class is set as Class B for both locations to control the ground motion variation for comparison studies. These variables are summarized in Table 5.

Table 5: Seismic Parameters of Two S	Siles
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Location	Ss	S ₁	TL
San Francisco	2.00 g	0.95 g	12 sec.
San Diego	1.22 g	0.471 g	8 sec.

ASCE 7-10 Design Response Spectra for the two locations are then constructed. The critical values needed to construct the Design Response Spectrum as defined in ASCE 7-10 §11.4.5 are listed in the Table 6:

Table 6: Values Required to Construct Design Response Spectra

Critical Values	San Francisco	San Diego	
S _{DS}	1.33 g	0.815 g	
S _{D1}	0.63 g	0.314 g	
$T0 = 0.2 \times \frac{S_{D1}}{S_{DS}}$	0.095 sec.	0.077 sec.	
$TS = \frac{S_{D1}}{S_{DS}}$	0.470 sec.	0.385 sec.	
TL	12 sec.	8 sec.	

The Design Response Spectra for these two sites are then constructed as shown in Figure 31 and Figure 32:



Figure 31: First 10 seconds San Francisco Design Response Spectrum with Spectral Acceleration on Y-axis, Period on X-axis



Figure 32: First 10 seconds San Diego Design Response Spectrum with Spectral Acceleration on Yaxis, Period on X-axis

It is noted that, as the S_1 value at the San Francisco site is greater than the limit of 0.6g, extremely high base shears were obtained because of the lower bound limit set forth by equation 12.8-6 in ASCE 7-10 §12.8.1, which reads:

"for structures located where S_1 is equal to or greater than 0.6g, C_s shall not be less than:

$$C_s = \frac{0.5S_1}{(\frac{R}{I_e})}$$

In this case, this will result in an extremely high C_s value as follows

$$C_s = \frac{0.5S_1}{(\frac{R}{I_e})} = 0.158$$

In the practice industry, such a high C_s value will warrant a Performance Based Design (PBD), as a building will never see such high base shear due to seismic because of its long period, which is beyond the scope of this study. That said, Code Based Response Spectrum Design was applied on the building models for this location to compare the results to those found by Sun (2015) to validate the analysis and design approach.

San Diego site was selected based on a lower S_1 value (0.6g), which leads to the design of the buildings out of the lower-bound C_s requirement to achieve a more realistic seismic design force and drift levels

compared to the San Francisco Site. A ground shaking intensity map from FEMA (2012) document E-74 is shown in Figure 33.



Figure 33: Ground Shaking Intensity Map FEMA E-74 (FEMA, 2012)

Comparative studies were carried out after an iterative design process consisting of models being analyzed, members designed, model calibrated, and reanalyzed. Specifically, the seismic analysis and design were based on the Response Spectra shown above. Different seismic performance aspects were examined and discussed in detail in the following sections.

6. Results and Analysis

A total of six structural models constructed in ETABS Version 16.2 Ultimate (CSI, 2016) with the same loading criteria were analyzed at two locations of different seismic parameters, resulting in 12 sets of analysis results. The results will be discussed in detail in two comparative studies to be presented in this section.

As detailed in the previous Section, Modal Response Spectrum Analyses were used for all the models and calibrated to ASCE 7 §12.9 requirement. The final Modal Mass Participating Ratios and Base Shear Ratios of all the models are documented in Tables 7 and 8.

Model	X-Direction	Y-Direction
60 Story Diagrid	96.78%	96.82%
60 Story Horizontal Hexagrid	95.22%	96.16%
60 Story Vertical Hexagrid	95.22%	96.16%
36 Story Diagrid	95.70%	96.88%
36 Story Horizontal Hexagrid	95.11%	96.24%
36 Story Vertical Hexagrid	94.47%	94.49%

Table 7: Modal Mass Participating Ratio

Table 8: MRSA to ELF Base Shear Ratio

Model	San Francisco		San Diego	
	X-Direction	Y-Direction	X-Direction	Y-Direction
60 Story Diagrid	85.00%	85.04%	85.00%	85.04%
60 Story	85.00%	85.84%	85.00%	86.14%
Horizontal				
Hexagrid				
60 Story Vertical	87.72%	85.00%	87.80%	85.00%
Hexagrid				
36 Story Diagrid	85.00%	85.30%	85.59%	86.01%
36 Story	85.01%	85.95%	85.00%	86.21%
Horizontal				
Hexagrid			4	
36 Story Vertical	88.23%	85.00%	88.26%	85.00%
Hexagrid				

6.1 Comparative Study 1 – 60 Story

To study the important dynamic response characteristics of these models, linear dynamic analyses were carried out on these structures. To calculate the seismic weight, all the models are subjected to:

- self-weight of the structural steel and 180 mm (7 inch) thick slabs (normal weight concrete) at each floor;
- an additional superimposed dead load of 1.675 kN/m² (35 psf)
- Partition load of 0.48 kN/m² (10 psf) required per ASCE 7-10 §12.7.2.

6.1.1 Modal Properties

Modal properties, being the basis of Modal Response Spectrum Analysis, are undoubtfully the most important properties when it comes to understanding the seismic response behavior of a structure. Mode shapes and natural periods can give important imformation on the overall stiffness and performance of a structure under dynamic lateral loads.

Even though a structure can have millions of degrees of freedom, it is the first couple of modes that participate the most in the combined response of a building. The first five mode shapes for each of the 60-story structures are shown in Figure 34 below.





Figure 34: First Five Modes of 60 Story Models: Diagrid, Horizontal Hexagrid, and Vertical Hexagrid from Top to Bottom

All three models exhibit similar first five mode shapes: the first two are the translational mode in the two minor axes of the building; the third mode is torsional mode; and the last two are the second-degree translational mode in the direction of the minor axis. This is due to the similarities in the overall geometry. All three systems—diagrids, horizontal hexagrid, and vertical hexagrid—are tubular structures that resemble a cantilever truss system, which will result in translational governed deformation, as can be observed from the fundamental mode. Also, the triangulated diagonal members in diagrids, and the diagonal members for both hexagrid system configurations, provides torsional stiffness through axial action, which is more efficient than the more traditional frame structure such as core-and-outrigger system (Sun, 2015). The difference in stiffness between hexagrid system and the more traditional diagrid system is not very pronounced in terms of mode shapes obtained for the 60-story models.

Next, fundamental periods and the corresponding natural frequencies are compared. The fundamental periods of these structures are listed in Table 9.

Structural System	Fundamental Period	Natural Frequency (Hz)
Diagrid	3.389	0.295
Horizontal Hexagrid	6.530	0.153
Vertical Hexagrid	4.986	0.201

Table 9: Fundamenta	l Periods and Fundamenta	l Natural Frequence	cy of 60 Story	Models
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The natural frequency and fundamental period are good indicators of the overall lateral stiffness of the structure. They can be defined written as follows for a simple vibrating system:

$$\omega = \frac{1}{T} = \sqrt{\frac{k}{m}}$$

where ω is the natural frequency, T is the fundamental period, k is the stiffness, and m is the mass.

Like discussed in detail in Section 4, all the 60 story models are constructed with an identical braced frame core to provide additional stiffness. Thus, the core should not affect the validity of the stiffness comparison. For all 60-story models, the members of the tubular system are sized to control static wind deflection to less than H/500 and are the same in all three systems to control the variables in the comparative study. As shown by the results, diagrid structural system has the lowest fundamental period, indicating a high stiffness compared to hexagrid models. This result is expected due to the high inherent stiffness of triangular shapes prevalent in a diagrid structural system.

Comparing the two configurations of hexagrid systems, it can be observed that the 4-story tall vertical hexagrid module has a higher lateral stiffness than the horizontal hexagrid system. The differences in these two different configurations will be examined in more depth in the following sections in terms of stiffness and weight efficiency.

6.1.2 Drift and Acceleration

Top story drift is recorded for all models under seismic loads using Modal Response Spectrum Analysis discussed in detail in Section 5. It has a strong correlation with the stiffness characteristics of a structure and is a very critical design parameter for any building. The top story drift of the three 60-story models under the two selected ASCE Response Spectral acceleration input as well as code prescribed static wind load is summarized in Table 10.

Structural System	San Francisco ASCE 7-10	San Diego ASCE 7-10	ASCE 7-10 Static
	RSA	RSA	Wind
Diagrid	514.6 mm	113.2 mm	118.9 mm
Horizontal Hexagrid	2019.2 mm	455.8 mm	401.7 mm
Vertical Hexagrid	1227.5 mm	276.1 mm	251.0 mm

The max story displacement at each floor based on seismic analyses performed for the San Francisco and San Diego location design parameters is plotted for these systems and presented in Figure 35.



(a) Diagrid Drift Profile, SF on the Left, SD on the Right



(b) Horizontal Hexagrid Drift Profile, SF on the Left, SD on the Right



(c) Vertical Hexagrid Drift Profile, SF on the Left, SD on the Right



Maximum Story Displacement (SF RSA) - 60 Story Models

(d) Three Systems on the Same Plot, San Francisco RSA



Maximum Story Displacement (SD RSA) - 60 Story Models

(e) Displacements of Three Systems on the Same Plot, San Diego RSA Figure 35: RSA Story Displacement vs. Floor Plots of 60 Story Models

The difference in drift can be clearly observed in both wind and seismic responses shown in Table 10. With the San Francisco location Spectrum, the drift level difference under seismic load is more dramatic than that with San Diego Spectrum. This is due to the fact that: 1. Higher S_s and S_1 values were used at the San Francisco location; 2: C_s value used to calculate the Equivalent Lateral Force procedure base shear is controlled by the lower-bound limit described in Section 5.3, and thus it is not possible to take advantage of the long period of both of the Hexagrid structures to obtain a lower spectral acceleration.

Nonetheless, the benchmark diagrid structure underwent the least amount of drift across the board under different loading criteria despite being loaded with higher spectral acceleration due to its low period compared to the less stiff hexagrid structures, putting it at the first place for controlling lateral drift because of its high stiffness. It is also noted that the structural weight of the different tubular system is different, and this aspect will be examined in more detail in the next subsection.

Besides stiffness, the overall deformation mode may have an impact on the amount of drift. Different types of deflected shapes are shown in Figure 36. It is observed that, under the prescribed seismic load in both directions simultaneously, the diagrid system, exhibits more flexural-like deflection while the hexagrid systems show a more translational (shear) deformation mode. This can be observed from the drift profile plots in Figure 35 and the amplified deflection shape in Figure 37, especially in the lower story range, where the curve is more linear for the hexagrid system but more nonlinear for the diagrid system.



Figure 36: Shear vs. Flexural Deformation Mode (Hiraish, 1983)



Figure 37: Deflected Shapes of 60-story Diagrid, Horizontal Hexagrid, Vertical Hexagird Showing Flexural (Diagrids on the Left) and Shear Dominated Deflected Shapes (Two Hexagrids on the Right)

The acceleration of the diaphragm is also recorded and listed in Table 11. It is another important parameter for measuring the motion perceived by the occupants in the building during a seismic dynamic event.

Structural System	San Francisco ASCE 7-10	San Diego ASCE 7-10
	RSA	RSA
Diagrid	0.528 g	0.127 g
Horizontal Hexagrid	0.622 g	0.145 g
Vertical Hexagrid	0.610 g	0.144 g

Table 11: Top Story Acceleration of 60-Story Models Under Seismic Loading

The top story lateral acceleration of the different systems is much closer across the 60 story structures than the drift values. The diagrid system has a slight advantage over the hexagrid system in terms of controlling the seismic lateral acceleration.

6.1.3 Structural Weight, Embodied Carbon and Efficiency

Sustainability is becoming an increasingly significant parameter in the design and construction industry. In this section, the material weight and the amount of embodied carbon of the three systems are compared and are normalized with respect to the provided stiffness to provide additional insight on the efficiency of the studied systems. Table 12 summarizes the steel mass needed and the embodied carbon for each system to achieve the performance described in the previous subsections.

Structural System	Steel Mass	Embodied Carbon (0. 482 kgCarbon/kg) (Hammond and Jones, 2008)
Diagrid	14657 Ton	7064 Ton
Horizontal Hexagrid	12634 Ton	5167 Ton
Vertical Hexagrid	10720 Ton	6090 Ton

Table 12: Steel Mass and Embodied Carbon of 60 Story Models

As noted in Table 12, the diagrid, despite its high stiffness, costs the most in terms of material. It is understood that in general there shall a positive correlation between the mass of structure to the stiffness of it. However, to better understand these numbers and their implication to the efficiency of the corresponding structural system in a simple way, they are normalized to unitless artificial weight efficiency terms with respect to the two key performance indicators: seismic drift and acceleration. This number shall be higher for lower mass and lower drift and acceleration. The stiffness efficiency term is simply defined as:

$$\eta_{\Delta} = \frac{10^7}{Mass (ton) \times Drift (mm)} \times (ton \times mm)$$

The higher the number, the more mass efficient the system is to achieve the desired stiffness. Similarly, the acceleration efficiency is defined as:

$$\eta_{a} = \frac{10^{8}}{Mass (ton) \times acceleration (mm/s^{2})} \times (ton \times mm/s^{2})$$

The results are shown in Table 13.

ctural System	η_{Δ}	η_a

Table 13: Material Efficiency of Different Systems of 60 Story Models

Structural System	η_{Δ}	η_a
Diagrid	1.325	1.317
Horizontal Hexagrid	0.392	1.296
Vertical Hexagrid	0.760	1.559

The correlations between weight and displacement/acceleration can be observed from Figure 38, where bottom left corner of the graphs indicates high efficiency and top right corner indicates low efficiency.









It is clear that, despite being the heaviest system (given the same section properties), the traditional diagrid system is the most efficient in controlling the seismic-induced drift. However, when it comes to acceleration control, the diagrid has little to no advantage compared to hexagrid structural system.

6.2. Comparative Study 2 – 36 Story

Same analyses were carried out for the 36-story structural models. Key assumptions and the loading criteria remain the same as the 60 story models, which is discussed in more detail at the beginning of the previous section. It shall be noted that there is no structural core included the 36 story

structures, i.e., the tubular structure provides the lateral stiffness alone. Because of this, larger members are used in the less stiff horizontal hexagrid system to control the static wind drift.

6.2.1 Modal Properties

The first five eigen modes of the 36-story buildings are shown in Figure 39. Similar patterns are found: the first two are translational in the two minor axis; the 3rd mode is the torsional; the 4th and 5th are the 2nd translational mode in the two minor axis.





Figure 39: First Five Modes of 36 Story Models: Diagrid, Horizontal Hexagrid, and Vertical Hexagrid from Top to Bottom

Structural System	Fundamental Period	Natural Frequency (Hz)	
Diagrid	2.678	0.373	
Horizontal Hexagrid	4.816	0.208	
Vertical Hexagrid	4.311	0.232	

Table 14: Fundamental Periods and Fundamental Natural Frequency of 36 Story Models

Table 14 shows the fundamental periods and natural frequencies of 36 story models. Without the structural core that provides additional lateral stiffness, the diagrid system remains to have the lowest fundamental period. The horizontal configuration of the hexagrid system, despite being stiffened by increasing member sizes, still has the longest fundamental period, indicating an inherent low stiffness from the geometric configuration.

6.2.2 Drift and Acceleration

Table 15: Top Story Drift of 36-Story Models

Structural System	San Francisco ASCE 7-10	San Diego ASCE 7-10	ASCE 7-10 Static
	RSA	RSA	Wind
Diagrid	353.7	88.1	92.1
Horizontal Hexagrid	1163	293.0	279.3 (H/460)
Vertical Hexagrid	953	240.1	218.9

The drift profile plots for these 36-story models are shown in Figure 40.



(a) Diagrid Drift Profile, SF on the Left, SD on the Right



(b) Horizontal Hexagrid Drift Profile, SF on the Left, SD on the Right



(c) Vertical hexagrid Drift Profile, SF on the Left, SD on the Right



Maximum Story Displacement (SF RSA) - 36 Story Models

(d) Three Systems on the Same Plot, San Francisco RSA



(e) Displacement of Three Systems on the Same Plot, San Diego RSA Figure 40: Story Displacement vs. Floor Number Plots of 36 Story Models

The overall deformation profiles show similarities to those of the 60 story models. The nonlinearity is more pronounced for the diagrid system model at the lower levels compared to both configuration of the hexagrid system.

The diagrid structural system has the samllest drift magnitude, followed by the vertical hexagrid, then the horizontal hexagrid.

Structural System	San Francisco ASCE 7-10	San Diego ASCE 7-10
	RSA	RSA
Diagrid	0.461 g	0.125 g
Horizontal Hexagrid	0.515 g	0.134 g
Vertical Hexagrid	0.557 g	0.142 g

Table 16: Top Story Acceleration of 36-Story Models Under Seismic Loading

Again, similar results were found for the 36-story models with regards to controlling seismic-induced motion. Without considering the structural weight, the diagrid system is the best at controlling the diaphragm acceleration. However, the advantage is very slim and shall be examined in the next subsection, where the amount of material needed to achieve such performance is factored in.

6.2.3 Structural Weight, Embodied Carbon and Efficiency

The steel mass of the 36 story models are recorded in Table 17 below. The embodied carbon and material efficiency for drift and acceleration control were calculated using the methodology outlined in detail in section 6.1.3. It shall be noted that one of the normalization factors is modified due to the reduction in drift and structural weight to keep the magnitude consistent with the 60 story models. The drift control efficiency factor is now found by:

$$\eta_{\Delta} = \frac{10^{6}}{Mass (ton) \times Drift (mm)} \times (ton \times mm)$$

The results are shown in Table 18. Again, similarity to 60 story structures are found in terms of the overall ranking of the system. However, it was also noted that, without the help of the structural core, the drift efficiency difference between the diagrids and horizontal hexagrids are more prevalent. This further confirmed that the more traditional diagrid system is more efficient in being the sole lateral force resisting system in terms of drift control.

In terms of efficiency for acceleration control, the vertical hexagrid system exceeded diagrid system by a slight margin. The horizontal hexagrid configuration appears to be an even less favorable option when the structural weight of the system is taken into consideration. Correlations between steel mass and seismic performance indicators (displacement and acceleration) are plotted in Figure 41. Similarly, bottom left corner indicates higher efficiency.

Structural System	Steel Mass	Embodied Carbon (0. 482 kgCarbon/kg) (Hammond and Jones, 2008)
Diagrid	2350	1133
Horizontal Hexagrid	2968	1431
Vertical Hexagrid	1687	813

Table 17: Steel Mass and Embodied Carbon of 36 Story Models

Table 18: Material Efficiency of Different Systems of 36 Story Models

Structural System	η_{Δ}	η_a
Diagrid	1.202	0.941
Horizontal Hexagrid	0.290	0.668
Vertical Hexagrid	0.622	1.086

Steel Mass vs. Top Story Displacement Correlation for 36 Story Models

Steel Mass vs. Top Story Accelerataion Correlation for 36 Story Models



Figure 41: Steel Mass and Seismic Performance Correlation for 36 Story Models

To further investigate the weight efficiency, the 36-story vertical hexagrid structure was stiffened by increasing the grid member size. The criterion was to match the wind deflection to that of the 36

hexagrid structure. The model was reanalyzed with the new larger member sizes, which resulted in higher steel mass. The results are summarized in the Table 19.

Model	Weight	Wind	ASCE 7-10 SF	ASCE 7-10 SD
	(ton)/% of	Displacement	Displacement	Displacement
	Diagrid	(mm)/% of	(mm)/% of Diagrid	(mm)/% of Diagrid
		Diagrid		
36 Story Diagrid	2350/100%	92.1/100%	353.7/100%	88.1/100%
36 Story Original	1687/72%	218.9/238%	953/269%	240.1/272%
Vertical Hexagrid				
36 Story Stiffened	4663/198%	106.7/116%	685/194%	172/195%
Vertical Hexagrid				

Table 19: Efficiency Study: Stiffened 36 Story Vertical Hexagrid Results Comparison

To reduce the static wind deflection of the 36-story vertical hexagrid structure by 50% to 116% of that of the same height diagrid structure, about 176% increase in steel mass is needed. This indicates a rather low stiffness sensitivity to grid member size, which means that increasing member size does not have a very significant impact on increasing lateral stiffness. It is more so for the earthquake loads, which is calculated based on the weight of the structure. As member size increases, the mass as well as seismic induced lateral force and base shear would also increase. This results only in about a 30% reduction in seismic displacement for both locations at a cost of 176% weight increase, indicating a very low seismic stiffness sensitivity to member size increase.

7. Conclusion

With carefully controlled variables, the dynamic seismic response behavior of 6 archetype analytical models of two different heights with diagrid, horizontal hexagrid, and vertical hexagrid tubular structural systems are studied using linear dynamic analysis (Modal Response Spectrum Analysis) with the same response modification factor R of 3 in commercial software ETABS Ultimate Version 16.2 (CSI, 2016), while subjected to various ground shaking intensity levels and selected pertinent minimum loading criteria prescribed by American Society of Civil Engineers document ASCE Standard 7-10: Minimum Design Loads for Buildings and Other Structures. The performance results of the hexagrid models are compared to those of the more traditional diagrid system to explore the seismic performance potential and efficiency of the innovative hexagrid structural system.

It was found that the three tubular systems behave similarly for 36 and 60 story structures overall due to their resemblance to a cantilever truss. The three systems at two different heights exhibit similar first five mode shapes. The fundamental periods, however, are very different, indicating different stiffness the systems can provide. The benchmark diagrid system showed lowest fundamental period and highest stiffness because of the inherently stiff nature of the triangulated geometry formed by the diagonal member. Because of this, the diagrid structures also showed greater capacity to control the dynamic seismic and static wind-induced drift and lateral acceleration.

The efficiency of these systems is also compared by normalizing the drift and acceleration values with respect to the steel mass. Based on the results of the analysis of all the models at two different heights, it can be concluded that diagrid structural system, despite being the heaviest system, is the most efficient among the three systems at controlling the drift. The sensitivity study conducted on the 36-story vertical hexagrid structure showed that a very large increase in structural weight caused by increasing grid shell member size only has small impact on reducing the seismic displacements, further reaffirming the superior efficiency of diagrids compared to hexagrids. However, the efficiency difference in acceleration control is much smaller. Three systems showed similar efficiency for the 60 story models, while diagrid and vertical hexagrid systems have better efficiency for the 36 story models, leaving the horizontal hexagrid system behind. The vertical hexagrid system showed the most efficient in controlling the seismic induced acceleration at both heights among the three systems. It is essential to understand that the comparative study of the seismic performance is conducted assuming the same response modification factor for all three systems. The higher ductility shown by previous research (Kim and Lee, 2012; Mashhadiali, and Kheyroddin, 2012a, 2012b) of

hexagrid systems compared to diagrid systems can potentially lead to a higher response modification factor, making it more suitable for seismic applications.

Looking at the hexagrid structural systems by themselves, the vertical configuration as defined in Section 4.4 showed better performance across the board and higher efficiency for both heights studied in this paper than the horizontal configuration defined in Section 4.3. Thus, it is deemed to be the configuration in the hexagrid system family.

Besides the structural performance, one other important thing to consider is the constructability. Nodes, the point at which structural members join in a three-dimensional space, are one of the most critical links to help achieve the desired structure performance and are also significant concerns for diagrid structures (Boake, 2016) in terms of design complexities and construction costs. It is noted that every node on the diagrid system involves multiple slanted members, and there are 43 nodes on one side of each 4-story module. Two examples of these complex nodes from the Swiss Re Tower and the Hearst Tower are shown in Figure 42 (Boake, 2016). However, there are only 33 nodes on one side of each 4-story vertical hexagrid modules, 21 of which are typical perpendicular beam-column connections that builders and designers are familiar with. This difference in construction complexion will add up fast considering the height and size of the tall buildings in which such systems are used on. This might or might not make the vertical hexagrid more favorable options for building owners, architects, designers, and contractors.



Figure 42: Complex Connections Commonly Present in Diagrid Structures (Left: Swiss Re Tower, Right: Hearst Tower) (Boake, 2016)

Overall, the vertical hexagrid system seems to be a viable alternative to diagrids that can provide a unique aesthetics, sufficient lateral and vertical stability and stiffness, a higher ductility, a potentially lower seismic design force, as well as a simpler design and construction procedure.

8. Future Potential Research Areas

There are some key potential future research areas to better understand the hexagrid structural system.

- 1. Use parametric studies to evaluate proper seismic design factors (response modification factor (R), over-strength Factor (Ω_0), and displacement Amplification Factor (C_d)).
 - a. These parameters are the basis for the seismic design of any structural system.
 Previous research has shown about double the ductility in hexagrid structural system than diagrid system (Kim and Lee, 2012; Mashhadiali, and Kheyroddin, 2012a, 2012b), which is likely to lead to a higher response modification factor. Using the more accurate factors will result in more economical designs because of the potentially reduced force levels, making hexagrid system a more favorable system than diagrid system for seismic applications.
- 2. Carry out nonlinear static analysis on parametric models to find the ductility and verify with physical push-over experiments.
 - a. Ductility is of great importance for seismic applications. It may also help with codifying the innovative tubular system.
- 3. Conduct research on the progressive collapse behavior.
 - a. Progressive collapse gained its fair share of attention since the turn of the century due to tragic events. It shall be studied for all the systems that may be subjected to attacks, especially those used in high-rise constructions, to safe guard the safety of the general public and the building owner.
- 4. Study materiality (structural steel, reinforced concrete, and composite like concrete-filled steel) of hexagrid and its influence in the system behavior.
 - a. As previous study as shown (Gu, 2015), there is the trend to switch to composite constructions for tall buildings in the past decade. Construction materials have been proven (Sun, 2015) to have great impacts on the performance and dynamic behavior of a structural system. Thus, it is essential to understand the effects of using different materials for this innovative hexagrid system.

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