AFTERSHOCKS IN ENGINEERING SEISMIC RISK ANALYSIS

by

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ABSTRACT

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Submitted to the Department of Civil Engineering on May 11, 1973 in partial fulfillment of the requirements for the degree of Master of Science.

This study deals with the role of aftershocks in seismic risk analysis. The purpose of risk analysis is to assess the probability that the maximum seismic intensity experienced at a structural site in a lifetime of $\underline{T}$ years will exceed $\underline{a}$ units. It has been noted in several historical events that certain sites located at some distance from the mainshock epicenter have experienced more severe shaking during the temporally and spatially distributed aftershock sequence than during the larger mainshock itself; the cause is apparently the closer proximity of a particular aftershock to the site. This study evaluates this "additional aftershock risk". The analysis of both main- and aftershocks accounts for uncertainty in the times, locations and magnitudes of earthquakes as well as uncertainty in the attenuation "laws" (correlations). Mainshock occurrences are governed by a homogeneous Poisson process in time whereas the temporal characteristics are represented as a non-homogeneous Poisson process triggered by a mainshock and with parameters dependent on the mainshock magnitude. Aftershocks are assumed to occur at random spatially in a region whose location and extent depend upon the (random) mainshock location and size.

Analytical expressions for the risk are given for two cases ~~of spatial assumptions: mainshocks and~~ aftershocks occur on a "line" fault; and mainshocks occur on a line, but aftershocks may occur in a surrounding areal region. Because the analytical expressions are complicated and difficult to evaluate, a method is presented to determine an upper bound on the risk from both main- and aftershocks. Numerical results (upper bounds) are given for the simplest, first case and compared with a method that accounts

only for mainshocks and a method that treats all earthquakes as mainshocks. The preliminary conclusions are that the contribution of aftershocks to the seismic risk are in general small for large examined seismic intensities at the site (say, 0.1g or more), especially when only mainshocks of moderate to small magnitudes (7.0 or smaller) are anticipated in the future. However, it might be significant in regions where mainshocks are usually followed by a large number of aftershocks. In any case, risk analysis of both main- and aftershocks is complicated and the simple method, which accounts only for mainshocks, yields risk estimates adequate (but underestimated) for engineering purposes.

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TABLE OF CONTENTS

TABLE OF CONTENTS (Continued) Page

TABLE OF CONTENTS (Continued)

LIST OF FIGURES

LIST OF FIGURES (Continued) Page

LIST OF SYMBOLS

p_y^T : Probability, that the maximum peak seismic intensity at the site exceeds y in T years

y : Seismic intensity (e.g., maximum peak ground acceleration)

T : Time periods in years

{A} : Event, that at least one mainshock in T years causes a maximum peak seismic intensity at the site in excess of y

{B} : Event, that at least one aftershock in T years causes a maximum peak seismic intensity at the site in excess of y

{C} : Event, that at least one mainshock occurs in T years

$E[nm_y^T]$: Total expected number of mainshocks of interest (i.e., causing a seismic intensity at the site in excess of y) in T years in an earthquake source

$E[nm^T]$: Total expected number of mainshocks of magnitudes in excess of m_o^m in T years

$E[na_y^T]$: Total expected number of aftershocks of interest in T years in an earthquake source

$\overline{p}_m$: Probability that, given a mainshock occurs in a "point" source, the seismic intensity at the site will exceed y

$\overline{p}_a$: Probability that, given an aftershock occurs in a "point" source, the seismic intensity at the site will exceed y

R : Distance from a "point" source to the site

M, m : Magnitude

M_m, m_m : Mainshock magnitude

m_1 : Upper bound on mainshock magnitude

LIST OF SYMBOLS (Continued)

m_o^m	:	Lower bound on mainshock magnitude
m_o^a	:	Lower bound on aftershock magnitude
L	:	Length of fault line
$\Delta\ell$	:	Length of a (linear) "point" source
Δf	:	Area of a (areal) "point" source
$\Delta\ell_i$	:	Length of intervals on fault line
E	:	Energy of an earthquake
β	:	Parameter of the magnitude-energy law
$\overline{\beta}$, b	:	Parameter of the magnitude-frequency law
ν_m	:	Average mean rate of occurrences of mainshocks (per year)
ν_a	:	Average mean rate of occurrences of aftershocks (per year)
c, p, r	:	Parameters of the modified Omori law
t, t'	:	Time (in years)
b_1, b_2, b_3	:	Parameters of the attenuation "law"
ε	:	"Error" term in the attenuation "law"
σ	:	Standard deviation of $\ln \varepsilon$
D	:	Linear dimension of potential aftershock area
F	:	Area of potential aftershock occurrences
γ', δ', γ, δ	:	Parameters of the spatial distribution law for aftershocks
$\Phi^*(\cdot)$	:	Complementary cumulative distribution function of the standardized Gaussian distribution
$\Gamma(\cdot)$	:	Gamma function

CHAPTER 1

INTRODUCTION

The purpose of seismic risk analysis as defined in this study is to assess the probability p_y^T that the maximum peak seismic intensity (e.g., maximum peak ground acceleration) experienced at a structural site will exceed y units (e.g., 0.2g) in a time period T. Seismic risk analysis has been studied by Cornell[1,2], by Esteva[3] and others[4,6] and applied[5,6,7] for several years. Reported analyses account for uncertainty in the times, magnitudes and locations of earthquake events in potential earthquake sources, as well as for uncertainty in the attenuation "laws" (correlations) which estimate the seismic intensity at a site as a function of the distance from the earthquake location to the site. In several historical events it has clearly been noted and often been presumed that certain localized sites located at some distance from the mainshock epicenter have experienced a larger seismic intensity (e.g., larger maximum peak ground acceleration) during the temporally and spatially distributed aftershock sequence that usually follows a mainshock than during the larger mainshock itself.[16,17,18,19] (These observations which are related to seismic intensities such as peak ground acceleration, are entirely different from the frequent observations that structures at certain sites have experienced more severe damage during the aftershock sequence, because their structural resistances have been weakened during the mainshock.

However, these two phenomenons might sometimes be interrelated and hard to separate.) The underlying reasons for the potentially larger intensities during the aftershock sequences are the closer proximity of a particular aftershock to the site and possible differences in the attenuation "laws" for different locations of the epicenters, which in both cases can occur because aftershocks are in general scattered around the mainshock epicenters (see Figures 1-1 and 1-2). These reasons are therefore not related to any particular structure. This study is aimed at incorporating these observations into seismic risk analysis, at evaluating the "additional risk" and at determining under what conditions it might prove important (e.g., be of the same order as the risk due to mainshocks alone).

In reported seismic risk analyses, the differences between main- and aftershock sequences and the influence on the seismic risk have either been neglected or accounted for with simplistic assumptions. On one side, it has been proposed(6,7), that for the purpose of seismic risk analysis it is appropriate to treat all earthquakes as equivalent events and therefore not to distinguish between main- and aftershocks ("equivalent event" model). Because the locations of aftershock epicenters are not independent of the locations of the causative mainshock epicenters, but generally occur only in a limited region around it, this approach will always yield too conservative risk estimates. On the other hand, it has been proposed(5) that mainshocks alone contribute to the seismic risk and in establishing the rate of occurrence of earthquakes only (past) mainshocks have to be considered ("mainshock" model). This is only true if it is assumed that all aftershocks originate at the same location as the mainshock. Because this strong dependence does not hold, the risk obtained under this assumption will always be too low, particularly for sites close to the potential earthquake sources. Since, in general, 50% or more of all earthquakes with magnitudes of engineering interest can be classified as aftershocks[(5)], the risks will be approximately half or less of the ones obtained under the former approach (the approximation being valid for small risks (<0.1)). The "real" value of the risk will lie somewhere in between these bounds. A comparison of approximate results of a seismic risk analysis based upon the model proposed in this paper and based upon the two traditional models will be shown in an example in Chapter 4.

In this analysis, the temporal characteristics of aftershocks are represented as a non-homogeneous Poisson process in time, triggered by a mainshock and with parameters dependent on the (random) mainshock magnitude. These and other functional relationships and assumptions will be described in Chapter 2. In Chapter 3 the analytical models are derived, first, for a case where it is assumed that both the main- and the aftershock epicenters lie on the same (fault) line and, second, for a case where mainshocks are assumed to occur on a (fault) line but aftershocks in an areal region around it. Because the analytical expressions of these models are very complicated and difficult to evaluate, a method to calculate an upper bound on the seismic risk of main- and aftershocks will be presented. The corresponding expressions are much easier to evaluate and are still a significant improvement over the present upper bound ("equivalent event" model). Results of this method will be shown in the example in Chapter 4 for the case where both the main- and the aftershocks lie on the same (fault) line.

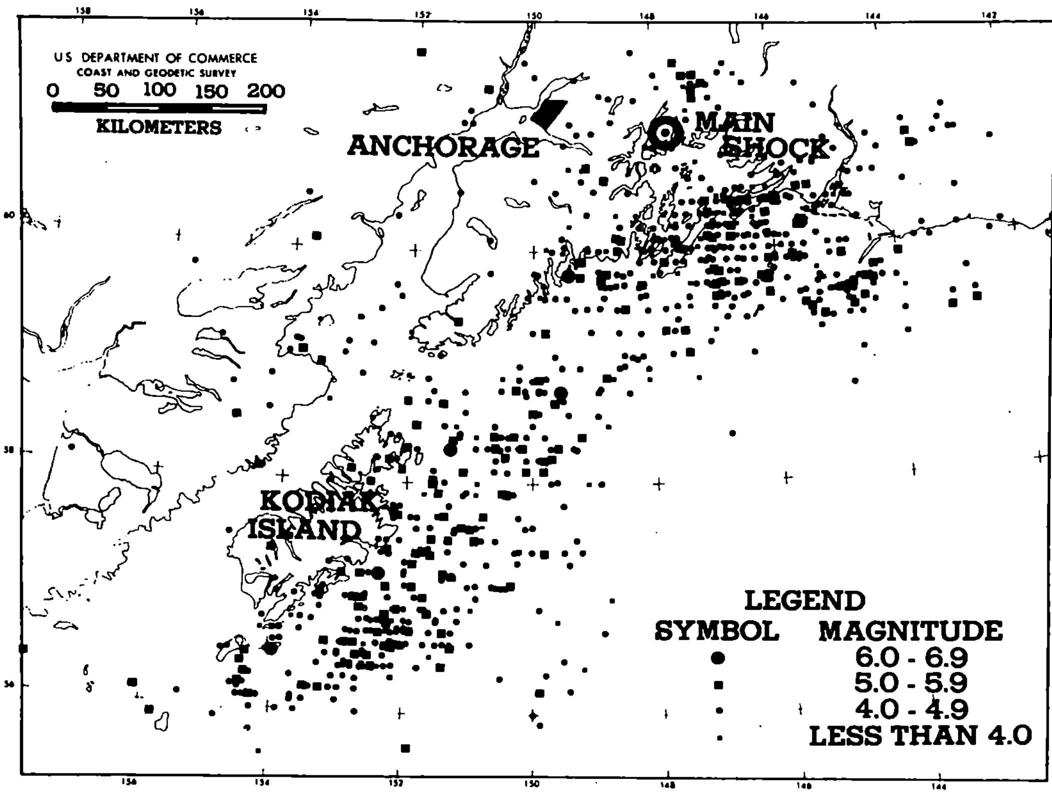

Figure 1-1 Spatial Distribution of Aftershock of the Prince William Sound, Alaska Earthquake of 1964. (Mainshock magnitude 8.3.)

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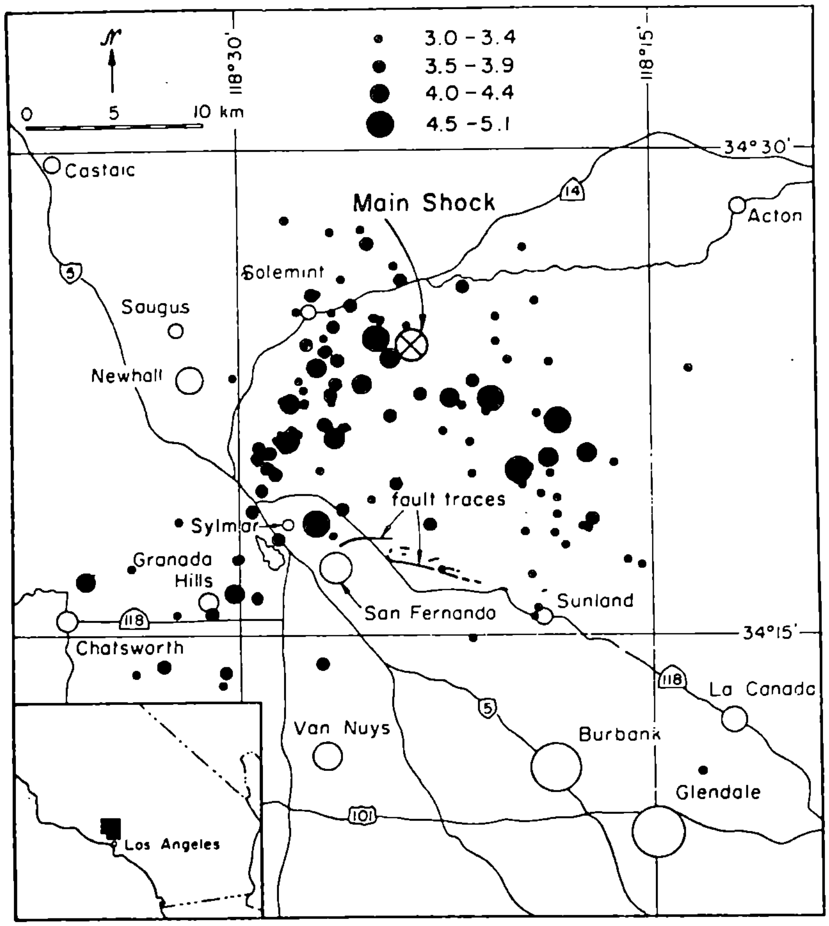

Figure 1-2 Spatial Distribution of the San Fernando, California, Earthquake, February 9, 1971. (Mainshock magnitude 6.6.)

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CHAPTER 2

ASSUMPTIONS AND PARAMETERS OF THE SEISMIC RISK MODEL

2.1 Temporal Characteristics of Main- and Aftershock Sequences

For the purpose of this study it is assumed that all earthquakes can be classified as either main- or aftershocks, although this assumption does not always hold. When the seismic activity in an area surrounding the epicenter of a major earthquake is considerably increased after its occurrence, all shocks originating in this activity are considered to be aftershocks of the major earthquake (mainshock). Utsu[9,10] and others[15] have established more sophisticated criteria and methods to distinguish between main- and aftershocks, based upon the time history as well as the spatial distribution of earthquake sequences. Many seismologists (Omori, Utsu[9,10], Aki[11], Vere Jones[12,13] and others) have studied and modeled earthquake sequences in time. In particular, it was found that generally the temporal characteristics of earthquake sequences cannot be described by a single Poisson (memoryless) process, but rather by more general renewal or Markov processes. However, when aftershocks are excluded from observed earthquake sequences, a Poisson process for the mainshock occurrences in time seems to be a reasonable assumption[5,13,15], especially for large shocks. For engineering purposes the Poisson assumption has been considered adequate for numerous reasons. Furthermore, even though it is not fully satisfactory, earthquake occurrences can be described in a general way by a trigger model, where the conditional probability of a shock (aftershock) occurring at a time t after a trigger event (mainshock) is proportional to a decay function $\nu(t)$.[13] In this study, the time aspects of earthquake sequences are described by such a trigger model where the temporal characteristics of the aftershocks are represented as a non-homogeneous Poisson process in time, triggered by a mainshock and with parameters such as the decay function $\nu(t)$, dependent on the mainshock magnitude. The occurrences of mainshocks, the triggering events, in time are themselves governed by a homogeneous Poisson process in time (see Figure 2-1).

2.1.1 Parameters of the Mainshock Occurrence Model

The homogeneous Poisson process for the occurrence of mainshocks is fully determined by the constant (time invariant) average mean rate of occurrence, ν_m, of mainshocks. The mean rate gives the average number of mainshocks with magnitudes larger that a lower bound m_0^m per time interval (e.g., per year). Usually it is derived from the earthquake history of the particular earthquake source.* Since for the calculation of ν_m only mainshocks should be considered, the separation of past earthquakes into main- and aftershocks is a necessary prerequisite.

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* An earthquake source is defined as a geographical region of limited size, in which future earthquakes can potentially occur. (e.g., fault line). The occurrence of an earthquake (i.e., its epicenter) is associated with a single point in that source.

2.1.2 Parameters of the Aftershock Occurrence Model: Modified Omori Law

The non-homogeneous Poisson process, which has been assumed for the occurrence of aftershocks during the time after a triggering mainshock, is determined by the frequency function $\nu_a(t)$, which gives the expected number of aftershocks to occur at time t as a function of the time t, elapsed since the occurrence of the main-shock. By the modified Omori law(9,10), the number of aftershocks per day with energy exceeding E_ℓ (i.e., with magnitude above a certain level) is given by

$$\nu_a(t) = \frac{A}{(t+c)^p} \tag{1}$$

in which t is the elapsed time in days since the mainshock, and c and p are regional constants. The parameter A is a constant for any aftershock sequence and given by the formula(9)

$$A = \left\{\frac{(\beta-b)\Gamma(\beta/b)c\,E_a}{b\,E_\ell}\right\}^{b/\beta} \tag{2}$$

where E_ℓ = lower limit of shock energies considered, corresponding to a lower limit on the aftershock magnitudes m_0^a.

$\Gamma(\cdot)$ = gamma function

β = coefficient in the energy-magnitude relation for earthquakes $\log E = \alpha + \beta M$ where E is the energy in a shock of magnitude M (see Section 2.3).

b = coefficient in the Gutenberg-Richter formula (magnitude-frequency law) $\log n(M) = a - bM$, where $n(M)$ is the number of shocks of magnitude M or larger (see Section 2.2).

E_a = total energy of aftershocks in an aftershock sequence, which can be related to the energy of the mainshock E_0 by $E_a = r\, E_0$.

The values of the parameters c, p and r can vary considerably according to observed aftershock sequences. In Japanese aftershock sequences(10), p most frequently fell into the range between 1.0 and 1.5, c was usually smaller than 2 days and r varied between 0.02 and 1.0. No significant correlations have been found between the parameters themselves or with the mainshock magnitude. The obvious conclusion would be to treat these parameters as independent random variables in the observed ranges. However, the expected number of aftershocks in a particular earthquake source in a time period T is extremely sensitive to the values of these parameters, especially to the value of p. Because in this analysis the expected total number of aftershocks is more important than the exact form of the frequency decay function, it is suggested that, when possible, c, p and r are adjusted in order to reflect the past aftershock history of a particular earthquake source. Analytical expressions for the expected number of aftershocks are given in Appendix A. For reasons to be given later, the value p = 1.0 is not permitted in this analysis.

2.2 Magnitude-Frequency Law

The formula most widely used for representing the frequency of occurrence of earthquakes as a function of magnitude is the Gutenberg-Richter formula.*

$$\log_{10} n(M) = a - bM \tag{3}$$

The parameter b or its base 10 counterpart $\bar{\beta} = b \cdot \ln 10 = 2.302 \cdot b$ is important in calculating the probability that given an earthquake occurs, its magnitude, M, will be of a certain size. It has been found[10] that for both the main- and aftershock sequences the Gutenberg-Richter formula applies individually, but the b-values are not necessarily the same for both sequences. However, it seems reasonable to assume in this analysis that the two b-values are equal and in addition, time invariant. A typical value for Southern California is b = 0.86 [5,10] for magnitudes between 3.0 and 8.0. In the following analysis, the magnitude-frequency law is truncated at an upper bound on the magnitudes. The upper bound implies that it is not reasonable to assume the possibility of infinitely large magnitudes, but that with each source an upper

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* Based upon observed data, Shlien and Toksöz[14] have recently proposed a quadratic form of the magnitude-frequency law:

$$\log n(M) = a_0 + b_1 M - b_2 M^2$$

Using this quadratic form, rather than the linear Gutenberg-Richter formula, the expressions for the seismic risk due to mainshocks have been derived by Merz and Cornell[8]. These results can easily be incorporated into the following seismic risk analysis for main- and aftershocks, but they significantly complicate the mathematical expressions.

bound on the magnitudes of mainshocks can be associated and that aftershock magnitudes cannot exceed a certain level in any particular sequence. The lower bound either gives the magnitude below which earthquakes are not of engineering importance, or serves as reference magnitude to establish the rate of occurrence (e.g., the rate of occurrence of mainshocks ν_m has to be calculated for mainshocks equal to or larger than a selected lower bound on the mainshock magnitude). If the examination of the seismic risk at a site requires the consideration of magnitudes below the lower bound, the analytical model accounts automatically for the necessary adjustments in the rate of occurrence.*

The magnitude-frequency law used in this analysis takes, therefore, the form of

$$\log_{10} n(M) = \begin{cases} a(m_o^m) - b(M - m_o^m) & m_o^m \leq M \leq m_1 \\ 0 & M > m_1 \end{cases} \quad (4)$$

for mainshocks sequences, where m_1 denotes the upper bound and m_o^m the lower bound on the mainshock magnitudes. For an aftershock sequence, the upper bound on the magnitudes is assumed to be the (random) magnitude m_m of the triggering mainshock. Thus m_1 in Eq. (4) has to be replaced by m_m. This assumption has proved valid

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* For a more detailed discussion of the meaning and the implications of the lower bound on the magnitudes in seismic risk analysis, see Merz and Cornell(8).

independent of the way of classifying main- and aftershocks.[10] For a reason to be explained in Section 3.2.2.2.1, the lower bound on the aftershock magnitudes m_o^a has to be smaller than m_o^m in this analysis.

2.3 Magnitude-Energy Law

A relationship between the energy released in an earthquake and its magnitude is required in the parameter A in Eq. (2) of the modified Omori law. It is generally accepted that the energy E of an earthquake is related to the magnitude M by the following formula

$$\log_{10} E = \alpha + \beta M \qquad (5)$$

Several values have been reported for the constants α and β. In this analysis only β is of importance, since only the energy of earthquakes relative to a lower limit is needed. Gutenberg[23] gives a value of $\beta = 1.5$.

2.4 Spatial Distribution of Earthquakes

2.4.1 Spatial Distribution of Mainshocks

For a seismic risk analysis it is necessary to have designated potential earthquake sources, i.e., regions in which earthquakes and, specifically, mainshocks are expected to occur in the future. These sources can be lines (e.g., tectonic faults) or areas, and are usually determined from the earthquake history and from geological considerations of a region. The likelihood of an earthquake varies often from location to location. In the following analysis, however, it is assumed that mainshocks are equally likely to occur anywhere in a designated earthquake source. If there are indications against the equal-likelihood model, it is relatively easy to assign other relative values to each of the many portions into which a source has to be divided in this analysis and over which the equally likely assumption is reasonable.

2.4.2 Spatial Distribution of Aftershocks

It has been recognized for a long time that aftershocks of a particular mainshock do not originate at the same location as the mainshock itself, but are scattered in a surrounding region of a limited size. As the magnitude of the mainshock is increased, the size of the aftershock region is also increased. Utsu[9,10] and others have extensively investigated the correlation between the size F of the aftershock region and the mainshock magnitude M_m, and found that F and M_m can be connected by the following formula

$$\log_{10} F = \gamma + \delta M_m \qquad (6)$$

The shapes of the areas have been found to be approximately elliptic. The values of the parameters γ and δ, found in the literature, vary from author to author, and depend often on the way aftershocks are separated from mainshocks. For Japanese earthquakes, Utsu[9,10] reports a value of γ = 1.02 and δ = 4.01, for F in square kilometers and magnitudes between 5.5 and 8.5. By approximating the elliptical area by a circle, the linear dimension (diameter) D of the area can be written as

$$\log_{10} D = \gamma' + \delta' M_m \qquad (7)$$

For Utsu's values for γ and δ , γ' = -1.8 and δ' = 0.5 for D in kilometers.

In the following analysis the relationships (6) and (7) will be used with a single set of parameter values for the entire range of mainshock magnitudes considered. For lower bounds on the mainshock magnitudes below 5.5, this assumption neglects that Eq. (6) gives considerably smaller areas for small magnitudes than actually observed. However, it is possible to account for different sets of parameter values for different ranges of magnitudes, but it complicates the analysis significantly. Furthermore it is assumed that it is equally likely for an aftershock to occur anywhere in the determined area. If data or judgement should rule against the equal-likelihood assumption and in favor of other relative values, they can be included in the analysis, but again, it will complicate the mathematical expressions.

2.5 Attenuation Law

The parameters discussed so far describe the model for the occurrence of main- and aftershocks in time and space. Since the engineer's interest lies in the seismic intensity at particular sites with various distances to the earthquake sources, it is necessary to project the effects of a distant earthquake to the site. The function of the attenuation "law" (correlation) is to estimate the seismic intensity at a site (e.g., maximum peak ground acceleration) as a function of the event magnitude M and the distance R from the site to the location of the earthquake (epicenter or hypocenter). Kanai[22], Esteva and Rosenblueth[20] have recommended the particular form

$$y = b_1 e^{b_2 M} R^{-b_3} \tag{8}$$

for peak ground acceleration (y = A), peak ground velocity (y = V) and peak ground displacement (y = D). The latter authors suggest that the constants $\{b_1, b_2, b_3\}$ be {2000, 0.8, 2.}, {16, 1.0, 1.7} and {7, 1.2, 1.6} for A, V and D respectively in Southern California with A, V and D in units of centimeters and seconds and R in kilometers. There have been discussions in recent years about the accuracy of the above formula, especially for short distances R (25 km and less). It is important to note that the expressions for the seismic risk at a site remain valid, whether R stands for epicentral or focal distance or any function thereof, as long as the above formula is basically maintained. For instance, Esteva's suggestion[21] to express the distance in the formula as (R + 25), where R is the physical distance, can therefore easily be incorporated. It is, however, extremely difficult to account for the proposed magnitude dependency of the parameters b_1, b_2 and b_3 in Eq. (8) for small distances.

Because Eq. (8) is, in fact, only a crude correlation with important scatter of observed data around the predicted values, an "error" term ε is added to the former equation. Thus,

$$y = b_1 e^{b_2 M} R^{-b_3} \varepsilon \qquad (9)$$

Esteva[3] has found that ln ε is approximately normally distributed with mean zero and standard deviation σ (usually of the order 0.5 to 1.0). Since the seismic risk is obtained by analyzing many small portions in which an earthquake source has to be divided, it is also possible to account for a variation of the values of σ with the distance R.

In regions where the seismic history is available only in terms of some intensity measure such as Modified Mercalli Intensity, it is necessary for this model to translate the intensity measures into magnitudes, in order to make the different relationships (attenuation law, Modified Omori law, spatial distribution law) compatible in the dimensions. Gutenberg[23] has proposed relationships between MM-Intensities and magnitudes.

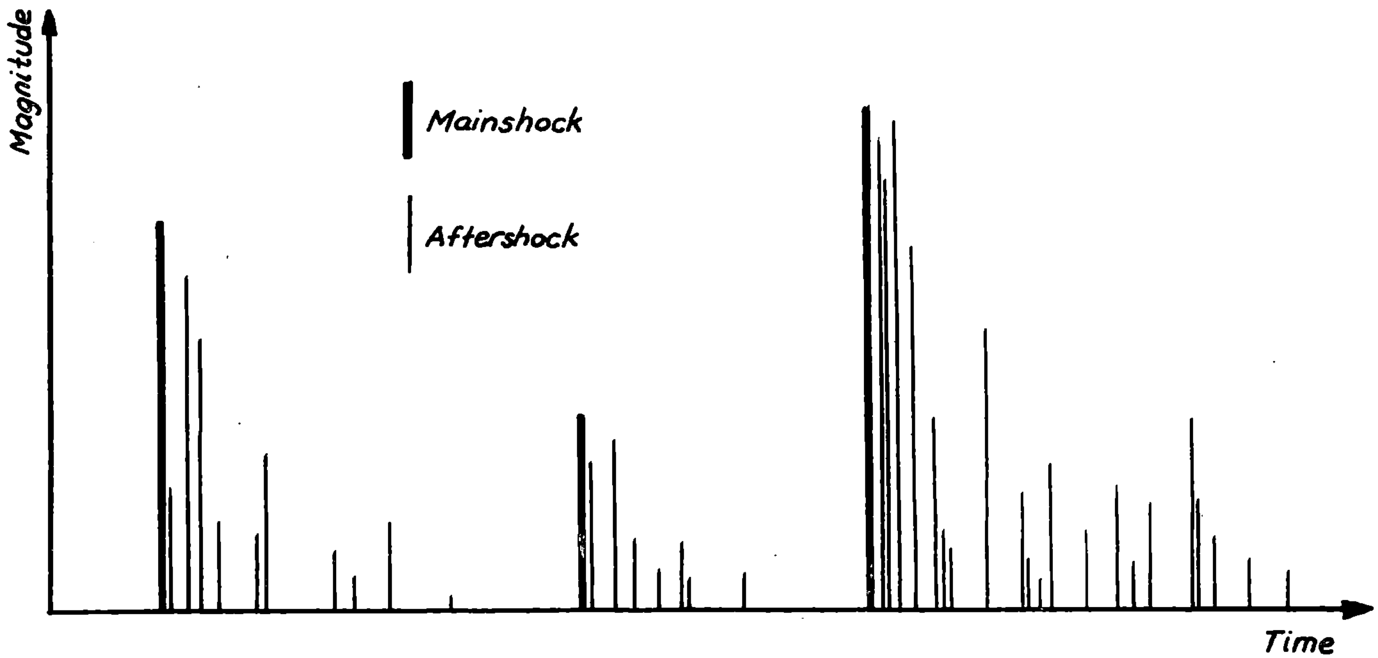

Figure 2-1 Example of Main- and Aftershock Sequences in Time

CHAPTER 3

DERIVATION OF THE ANALYTICAL MODELS

3.1 General Formulas

The analysis of the seismic risk at a structural site involves basically two steps: first, an analysis of the seismic intensity at the site, given an earthquake (main- or aftershock) occurs; and second, an analysis of the random multiple occurrences of both main- and aftershocks. Combining these two probabilistic analyses yields the desired probability p_y^T that at least one mainshock or at least one aftershock occurs in a designated earthquake source* in a specified time period T and produces a seismic intensity at the site above a certain level y (e.g., maximum peak ground acceleration > 0.2g). If the event {A} is defined as: "At least one mainshock occurs during T which produces $y_{site} > y$," and event {B} as: "At least one aftershock occurs during T which produces $y_{site} > y$," the desired probability p_y^T can be expressed as the probability that event {A} or {B} or both {A} and {B} occur

$$p_y^T = P[A \cup B] \qquad (10)$$

which can be expanded to

$$p_y^T = P[A] + P[B] - P[A \cap B] \qquad (11)$$

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* See footnote on p. 19 for the definition of an earthquake source.

Because event {B} can only occur if at least one mainshock occurs during T, it may be written as

$$P[B] = P[B \cap C] \qquad (12)$$

where event {C} is defined as: "At least one mainshock occurs during T." (Note that the event C contains the event A.) By expanding event {C} , P[B] takes the form of

$$P[B] = P[B \cap C] = \sum_{n=1}^{\infty} P\left[B \cap \begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur during } T\end{pmatrix}\right]$$
$$= \sum_{n=1}^{\infty} P\left[B / \begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur during } T\end{pmatrix}\right] \cdot P\left[\begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur during } T\end{pmatrix}\right] \qquad (13)$$

Analogously, $P[A \cap B]$ can be expressed as

$$P[A \cap B] = \sum_{n=1}^{\infty} P\left[B \cap \begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur in } T \text{ which all} \\ \text{cause } y_{site} > y\end{pmatrix}\right]$$
$$= \sum_{n=1}^{\infty} P\left[B / \begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur in } T \text{ which all} \\ \text{cause } y_{site} > y\end{pmatrix}\right] P\left[\begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur in } T \text{ which all} \\ \text{cause } y_{site} > y\end{pmatrix}\right] \qquad (14)$$

Mainshocks are assumed to occur in time according to a homogeneous Poisson process. The probability P[A] can therefore be written as

$$P[A] = 1 - \exp\{-E[n m_y^T]\} \qquad (15)$$

where $E[nm_y^T]$ denotes the expected number of mainshocks in T which produce a seismic intensity at the site in excess of y ($\exp\{-E[nm_y^T]\}$ is the probability that no such mainshock occurs in T). Thus

$$P_y^T = 1-\exp\{-E[nm_y^T]\} + \sum_{n=1}^{\infty} P[B/(\text{exactly } n \text{ mainshocks occur during } T)] \cdot P[(\text{exactly } n \text{ mainshocks occur during } T)]$$
$$- \sum_{n=1}^{\infty} P[B/(\text{exactly } n \text{ mainshocks occur in } T \text{ which all cause } y_{site} > y)] \cdot P[(\text{exactly } n \text{ mainshocks occur in } T \text{ which all cause } y_{site} > y)] \qquad (16)$$

Because mainshocks, and therefore also the individual aftershock sequences, are assumed to be independent events, the probability $P[B/(\text{exactly } n \text{ mainshocks occur during } T)]$ can be expressed as a function of the probability $P[B/(\text{exactly one mainshock occurs during } T)]$, namely

$$P[B/(\text{exactly } n \text{ mainshocks occur during } T)] = 1-(1-P[B/(\text{exactly one mainshock occurs during } T)])^n \qquad (17)$$

where $(1-P[B/(\text{exactly one mainshock occurs during } T)])^n$ is the probability that no aftershock of the n (independent) aftershock sequences produces $y_{site} > y$. Analogously, the probability $P[B/(\text{exactly } n \text{ mainshocks occur in } T \text{ which all cause } y_{site} > y)]$ can be written as a function of the probability $P[B/(\text{exactly one mainshock occurs in } T \text{ which causes } y_{site} > y)]$

$$P[B|(\text{exactly } n \text{ mainshocks occur in } T \text{ which all cause } y_{site} > y)] = 1 - (1 - P[B|(\text{exactly one mainshock occurs in } T \text{ which causes } y_{site} > y)])^n \quad (18)$$

The probability that exactly n mainshocks occur during the time period T is given by (Poisson process)

$$\frac{(E[nm^T])^n \, e^{-E[nm^T]}}{n!} \quad (19)$$

where $E[nm^T]$ denotes the expected number of mainshocks in T. Analogously, the probability that exactly n mainshocks occur in T and cause $y_{site} > y$, is given by

$$\frac{(E[nm_y^T])^n \, e^{-E[nm_y^T]}}{n!} \quad (20)$$

By multiplying Eq. (17) by Eq. (19), respectively Eq. (18) by Eq. (20), and carrying out the summations, the results for P[B] and P[A ∩ B] are

$$P[B] = \sum_{n=1}^{\infty} P[B|(\text{exactly } n \text{ mainshocks occur during } T)] \cdot P[(\text{exactly } n \text{ mainshocks occur during } T)]$$

$$= 1 - \exp\{-E[nm^T] \, P[B|(\text{exactly one mainshock occurs during } T)]\} \quad (21)$$

$$P[A \cap B] = \sum_{n=1}^{\infty} P\left[B \middle/ \begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur in } T \text{ which all} \\ \text{cause } y_{site} > y\end{pmatrix}\right] P\left[\begin{pmatrix}\text{exactly } n \text{ mainshocks} \\ \text{occur in } T \text{ which all} \\ \text{cause } y_{site} > y\end{pmatrix}\right]$$

$$= 1 - \exp\left\{-E[nm_y^T] \cdot P\left[B \middle/ \begin{pmatrix}\text{exactly one mainshock} \\ \text{occurs in } T \text{ which} \\ \text{causes } y_{site} > y\end{pmatrix}\right]\right\} \tag{22}$$

Thus

$$p_y^T = 1 - \exp\{-E[nm_y^T]\} - \exp\left\{-E[nm^T] \cdot P\left[B \middle/ \begin{pmatrix}\text{exactly one mainshock} \\ \text{occurs during } T\end{pmatrix}\right]\right\}$$

$$+ \exp\left\{-E[nm_y^T] P\left[B \middle/ \begin{pmatrix}\text{exactly one mainshock} \\ \text{occurs in } T \text{ which} \\ \text{causes } y_{site} > y\end{pmatrix}\right]\right\} \tag{23}$$

In order to keep the analytical expressions for the various terms tractable, it is necessary to break an earthquake source up into many individual "point" sources, for which fixed distances to the site can be assumed. These point-by-point calculations have the additional advantage that different parameter values for the functional relationships described in the previous chapter can be used for different portions of an earthquake source. Criteria for the choice of the size of these "point" sources will be given below.

In the following, the analytical expressions for p_y^T will be derived for a "line-line" occurrence model (Section 3.2) and a "line-area" model (Section 3.3). It can easily be seen that the numerical evaluation of these expressions is extremely complicated, even for automated computation. However, it is comparatively easy to derive and evaluate the analytical expressions for an upper bound on the value of p_y^T. In this chapter, the expressions will be given for both the "exact" risk p_y^T and for the upper bound on p_y^T. Unfortunately, this upper bound can in some cases, especially for small seismic intensities examined, be a very conservative estimate of the "exact" value of p_y^T. Nevertheless, it is a significant improvement over the present estimates of the contribution of aftershocks to the seismic risk, and still shows the influence of the most important parameters. This will be demonstrated in an example in Chapter 4.

3.2 Seismic Risk Analysis of a "Line-Line" Occurrence Model

The "line-line" occurrence model is defined by the assumption that both the mainshock and the aftershock epicenters lie on the same fault line, i.e., that an earthquake source can be reduced to a line. This assumption clearly contradicts the observed two-dimensional scattering of aftershock epicenters around the mainshock epicenters, that was discussed in Section 2.4.2. However, it is the simplest model that retains the spatial distribution of aftershocks and it simplifies the analytical expressions, especially those for the estimate of an upper bound on the risk. A typical fault-site configuration of this model and a typical mainshock with its aftershocks are shown in Figure 3-1. As mentioned above, this "line source" will be represented in this analysis as a set of many "point" sources (see Figure 3-2).

3.2.1 Special Assumptions

With the exception of the spatial distribution law for aftershock epicenters (Eq. (6), Section 2.4.2), all the assumptions and parameter relationships of the previous chapter can be used without modifications in this model. Because of the assumption that the aftershock epicenters lie all on the fault line, rather than being scattered in an areal region around the mainshock epicenters, the spatial distribution law has to be written in a different form. Eq. (7) in Section 2.4.2 gives the linear dimension D (diameter) of the aftershock area as a function of the mainshock magnitude. It seems reasonable to use this relationship of the form

$$\log_{10} D = \gamma' + \delta' M_m \qquad (24)$$

as the spatial distribution law in the "line-line" model. In addition, it is assumed that in general (exceptions will be given below), the linear aftershock region D extends D/2 on both sides of the mainshock epicenter. This implies that a mainshock with magnitude M_m can potentially produce an aftershock in a "point" source on the fault if it occurs within a region of $D(M_m)/2$ on either side of the "point" source.

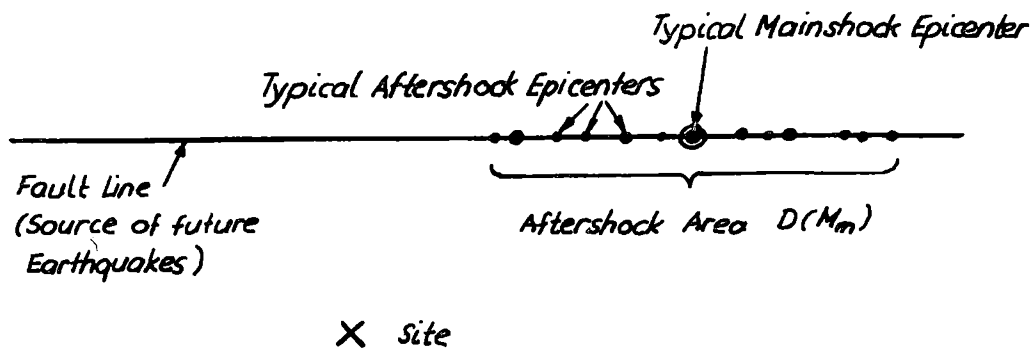

Figure 3-1 Typical Fault-Site Configuration with a Typical Mainshock and its Aftershocks ("Line-Line" Model)

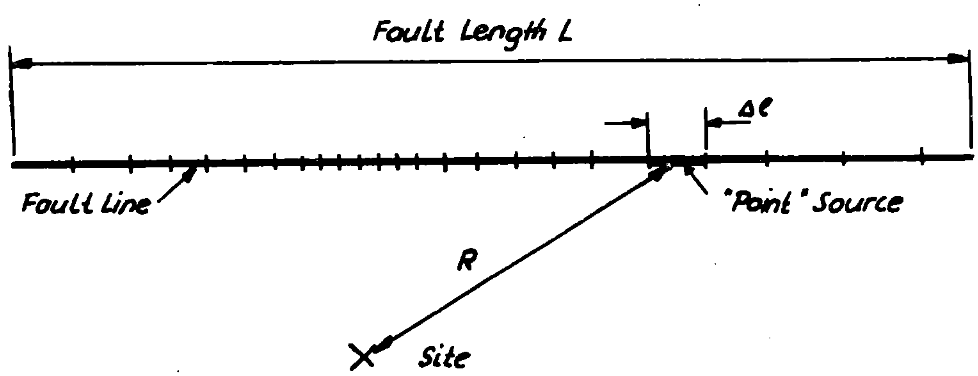

Figure 3-2 Point-by-Point Analysis of the Seismic Risk ("Line-Line" Model)

3.2.2 Derivation of the Analytical Expressions

3.2.2.1 Mainshock Analysis: Calculation of P[A]

Given a mainshock occurs in a "point" source (see Figure 3-2) with distance R to the site, the probability $\bar{p}_m$ that the seismic intensity at the site will exceed y is given by (Cornell[1], formula (13))*

$$P[Y_{site} > y] = \bar{p}_m = (1 - k_{m_1})\,\phi^*(z/\sigma) + k_{m_1}\,\phi^*(z/\sigma - \bar{\beta}\sigma/b_2)\; e^{\bar{\beta}^2\sigma^2/2b_2^2}\, e^{\bar{\beta} m_o^m}\, R^{-\bar{\beta} b_3/b_2}\,(y/b_1)^{-\bar{\beta}/b_2} \tag{25}$$

where $\bar{\beta} = b \cdot \ln 10$ = constant of the magnitude frequency law (Section 2.2)

b_1, b_2, b_3, σ are the constants in the attenuation law (Section 2.5)

$$k_{m_1} = \left(1 - e^{-\bar{\beta}(m_1 - m_o^m)}\right)^{-1} \tag{26}$$

m_1, m_o^m are an upper and lower bound on the mainshock magnitude (Section 2.2)

---

* A formula for the same probability using a quadratic instead of a linear magnitude-frequency law, as proposed by Shlien and Toksöz[14], has been derived by Merz and Cornell[8]. In addition, it is shown there that it is possible to use a partly linear, partly quadratic magnitude-frequency law.

$$z = \ln y - \ln (b_1 e^{b_2 m_1} R^{-b_3}) \tag{27}$$

$\Phi^*(\cdot)$ = complementary cumulative distribution function of the standardized Gaussian distribution.

If ν_m denotes the mean number of mainshock events per year with magnitudes larger than a lower bound m_o^m occurring on the entire fault of length L, then the expected number of events in T years $E[nm^T]$ is given by

$$E[nm^T] = \nu_m T \tag{28}$$

If it is equally likely for a mainshock to occur anywhere on the fault line, the expected number of events with magnitudes larger than m_o^m occurring in the "point" source of length $\Delta\ell$ during T years is given by

$$\nu_m T \frac{\Delta\ell}{L} = E[nm^T] \frac{\Delta\ell}{L} \tag{29}$$

Therefore, the expected number of mainshocks in $\Delta\ell$ during T years producing a seismic intensity at the site in excess of y is

$$E[nm^T] \frac{\Delta\ell}{L} \bar{p}_m \tag{30}$$

Thus, the total expected number of mainshocks $E[nm_y^T]$ on the fault during T years, which produce $y_{site} > y$, can be written as

$$E[nm_y^T] = E[nm^T] \sum_{\substack{\text{all } \Delta\ell \\ \text{on fault}}} \frac{\Delta\ell}{L} \bar{p}_m \qquad (31)$$

And, finally,

$$P[A] = 1 - \exp\left\{-\nu_m T \sum_{\substack{\text{all } \Delta\ell \\ \text{on fault}}} \frac{\Delta\ell}{L} \bar{p}_m\right\} \qquad (32)$$

3.2.2.2 Aftershock Analyses

3.2.2.2.1 Calculation of P[B]

In order to give the expression for P[B], the probability $P[B/(\text{exactly one mainshock occurs during } T)]$ has to be calculated (see Eq. (21)). Given the mainshock in question occurs at time t' to t' + dt' during T in a particular "point" source of $\Delta\ell$ and has a magnitude between m_m and $m_m + dm_m$, the total expected number of aftershocks, E[na], with magnitudes larger than a lower bound m_o^a during t' and T (T and t' in years) is given through the modified Omori law (Section 2.1.2) as

$$E[na] = \int_{t'}^{T} \nu_a(365(t''-t'))365\,dt'' = \int_{t'}^{T} \frac{A}{(365(t''-t')+c)^p} 365\,dt''$$

$$= \frac{A}{1-p}\left[(365(T-t')+c)^{1-p} - c^{1-p}\right] \quad 0 \leq t \leq T \qquad (33)$$

where A is given by Eq. (2) in Section 2.1.2. The factor 365 is the average number of days per year and is necessary because the modified Omori law gives the aftershock frequency per <u>day</u>. Clearly, Eq. (33) is valid only for $p \neq 1$.

The size of the aftershock region $D(m_m)$ is given by the spatial distribution law (Eq. (24)). If this region is divided into intervals of lengths $\Delta\ell_i$, for which fixed distances to the site can be assumed, the expected number of aftershocks in any particular interval is, with the equally likely assumption for the location of aftershocks

$$E[na]\frac{\Delta\ell_i}{L} \tag{34}$$

From these aftershocks of magnitudes above m_0^a, only a fraction will produce a seismic intensity at the site in excess of y, namely

$$E[na]\frac{\Delta\ell_i}{D(m_m)}\bar{p}_a^i \tag{35}$$

where $\bar{p}_a^i$ is the probability that, given an aftershock occurs in $\Delta\ell_i$, the seismic intensity at the site will exceed y. The probability $\bar{p}_a^i$ is analogous to Eq. (25) given by

$$P[y_{site} > y] = \bar{p}_a^i = (1-k_{m_m})\Phi^*(\bar{z}/\sigma) + k_{m_m}\Phi^*(\bar{z}/\sigma - \bar{\beta}\sigma/b_2)\, e^{\bar{\beta}^2\sigma^2/2b_2^2} e^{\bar{\beta}m_0^a} R^{-\bar{\beta}b_3/b_2} (y/b_1)^{-\bar{\beta}/b_2} \tag{36}$$

where m_m is the upper bound on the aftershock magnitude which is equal to the magnitude of the causative mainshock (Section 2.2)

m_o^a is the lower bound on the aftershock magnitude ($m_a^o < m_o^m$ for reasons to be given later)

$$k_{m_m} = (1 - e^{-\bar{\beta}(m_m - m_o^m)})^{-1} \qquad (37)$$

$$\bar{\bar{z}} = \ln y - \ln (b_1 e^{b_2 m_m} R^{-b_3}) \qquad (38)$$

and the other parameters are equal to the ones described in Eq. (25).

Analogous to the step from Eq. (30) to Eq. (31) in the mainshock analysis, the total expected number of aftershocks during t' to T which produce $y_{site} > y$, due to a mainshock of magnitude m_m at time t' in the "point" source $\Delta\ell$, can be written as

$$E[na] \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m) \\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)} \bar{p}_a^i \qquad (39)$$

Thus, the probability that at least one aftershock occurs during t' to T and causes $y_{site} > y$, given a mainshock of magnitude m_m occurs at time t' in "point" source $\Delta\ell$, is (Poisson process)

$$1 - \exp\left\{-E[na] \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m) \\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)} \bar{P}_a^i \right\} \tag{40}$$

The probability that the mainshock occurs in the "point" source $\Delta\ell$ is given by $\Delta\ell/L$ (see Eq. 29). Because, given one occurs, it is equally likely for a mainshock to occur at any time during T, the probability that it occurs during t' to $t' + dt'$ is simply dt'/T. The truncated magnitude-frequency law for mainshocks (Eq. (4), Section 2.3) implies a probability density function over the magnitudes of

$$f_{M_m}(m_m) = \begin{cases} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_0^m)} & m_0^m \leq m_m \leq m_1 \\ 0 & m_m > m_1 \end{cases} \tag{41}$$

where

$$\bar{\beta} = b \ln 10 \tag{42}$$

$$k_{m_1} = \left(1 - e^{-\bar{\beta}(m_1 - m_0^m)}\right)^{-1} \tag{43}$$

Now, the probability that at least one aftershock occurs during t' to T and causes $y_{site} > y$, due to a mainshock at t' to $t' + dt'$ in a "point" source of length $\Delta\ell$ and of magnitude between m_m and $m_m + dm_m$, can be given by

$$[1-\exp\{-E[na]\sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m)\\ \text{around } \Delta\ell}}\frac{\Delta\ell_i}{D(m_m)}\bar{p}_a^i\}]\frac{\Delta\ell}{L}\frac{dt'}{T}k_{m_1}\bar{\beta}e^{-\bar{\beta}(m_m-m_o^m)}dm_m \tag{44}$$

In order to account for all possible times t', and all possible mainshock magnitudes m_m as well as all possible locations $\Delta\ell$, Eq. (44) has to be integrated over t' from t' = 0 to t' = T and over m_m from $m_m = m_o^m$ to $m_m = m_1$ and finally summed over all $\Delta\ell$ on the fault line. Thus,

$$P[B/(\text{exactly one mainshock occurs during } T)] = \sum_{\substack{\text{all } \Delta\ell \text{ on}\\ \text{fault}}}\int_0^T\int_{m_o^m}^{m_1}\frac{\Delta\ell}{L}\frac{1}{T}k_{m_1}\bar{\beta}$$

$$e^{-\bar{\beta}(m_m-m_o^m)}[1-\exp\{-\frac{A}{1-p}[(365(T-t')+c)^{1-p}-c^{1-p}]\sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m)\\ \text{around } \Delta\ell}}\frac{\Delta\ell_i}{D(m_m)}\bar{p}_a^i\}]dm_m\,dt' \tag{45}$$

where k_{m_1} is given by Eq. (26)

$\bar{p}_a^i$ is given by Eq. (36)

A is given by Eq. (2)

$D(m_m)$ is given by Eq. (24)

The expression for P[B] is therefore (see Eq. (21) and (28))

$$P[B] = 1 - \exp\left\{-E[nm^T]\,P[B/(\text{exactly one mainshock occurs during } T)]\right\}$$

$$= 1 - \exp\left\{-\sum_{\substack{\text{all } \Delta\ell \text{ on}\\ \text{fault}}} \int_0^T \int_{m_o^m}^{m_1} \frac{\Delta\ell}{L} \nu_m k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_o^m)}\right.$$

$$\left[1 - \exp\left\{-\frac{A}{1-p}\left[(365(T-t')+c)^{1-p}\right.\right.\right.$$

$$\left.\left.\left.\left. - c^{1-p}\right] \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m)\\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)} \bar{p}_a^i \right\}\right] dm_m\, dt' \right\} \quad (46)$$

The integration over the mainshock magnitudes from m_o^m to m_1 implies that only aftershock sequences of mainshocks with magnitudes between these limits are considered. However, the lower limit m_o^m can be chosen arbitrarily, as long as at the same time the condition $m_o^a < m_o^m$ is satisfied. The necessary adjustments in the other parameters are carried out automatically.* The reason

* If the lower bound is changed from m_o^m to $m_o^{m'}$, on one hand the mean rate of occurrence ν_m has to be changed by a factor

$$e^{\bar{\beta}(m_o^m - m_o^{m'})}$$

and on the other hand, the probability density function over the magnitudes changes by a factor of

$$e^{-\bar{\beta}(m_o^m - m_o^{m'})}$$

Thus, the two factors cancel. See Merz and Cornell.[8]

for the condition $m_o^a < m_o^m$ is that Eq. (36) for $\overline{p}_a^i$ and the underlying parameter relationships are not valid for the case $m_o^a = m_o^m$.* However, the difference between m_o^m and m_o^a can be small, say 0.1 magnitudes.

A special situation arises for "point" sources located near the end of the fault line where the distance from the "point" source to the end of the fault line is smaller than $D(m_m)/2$, where $D(m_m)$ is the potential aftershock region associated with a mainshock of magnitude m_m. In this case, the assumptions that all aftershock epicenters lie on the fault line and that the region of potential aftershock epicenters extends $D(m_m)/2$ on either side of the mainshock epicenter are no longer compatible. This problem emerges essentially from the problem of defining the beginning or the end of a fault-line, and, at this time, there is no way of solving it completely and satisfactorily. In any case, either one of the two assumptions has to be given up. For instance, it is possible to maintain the size of the aftershock region but allow near the end of the fault-line that the mainshock epicenters are no longer the center of the region. This approach implies a concentration of aftershocks in the end regions of a fault-line. In cases where the site lies close to one of the defined endpoints of the fault and the concentration of the aftershocks near the end-

---

* For $m_o^a = m_o^m$ the factor k_{m_m} (Eq. (37)) in the cumulative distribution function of the aftershock magnitudes goes to infinity. The reason is that in this case the magnitude-frequency law of the form $\log_{10} n = a - bm^a$ is not valid, but is reduced to a single point.

points is believed to be wrong, it is suggested that the equally likely assumption for the location of the mainshocks along the fault (Section 2.4.1) be abolished and the end of a fault line be defined by a diminishing probability of mainshock occurrences in this region. With this method, a fault line can theoretically be extended to infinity and Eq. (46) for P[B] can be applied without changes.

Under the equally likely assumption for the occurrence of mainshocks on the fault, it is a necessary condition that the length L of a fault line is equal or larger than the linear dimension of the aftershock region for the largest possible mainshock (magnitude m_1)

$$L \geqslant D(m_1) = e^{\ln 10(\gamma' + \delta' m_1)} \qquad (47)$$

This condition reflects the statement by seismologists that the length of a fault is correlated with the maximum size of the earthquake magnitudes.

3.2.2.2.2 Calculation of P[A ∩ B]

In order to give the expression for P[A ∩ B], the probability $P[B/(\substack{\text{exactly one mainshock occurs} \\ \text{in } T \text{ and causes } y_{site} > y})]$ has to be calculated. The knowledge that the mainshock in question caused $y_{site} > y$ changes only the probability density function (p.d.f.) over the mainshock magnitudes in each of the individual "point" sources $\Delta\ell$, because the larger the distance R from the mainshock location to the site, the more likely it is now that a mainshock of a larger magnitude occurred. In contrast to Sections 3.2.2.1 and 3.2.2.2.1, where the same p.d.f. over the mainshock magnitude (Eq. (41)) has been applied to all the "point" sources, now, each of these "point" sources has its individual p.d.f. Thus, once the expression for the new p.d.f.'s has been found, the expression for

$$P\left[B / \left(\begin{smallmatrix}\text{exactly one mainshock occurs} \\ \text{in } T \text{ and causes } y_{site} > y\end{smallmatrix}\right)\right]$$

is obtained from $P\left[B / \left(\begin{smallmatrix}\text{exactly one mainshock} \\ \text{occurs during } T\end{smallmatrix}\right)\right]$ (Eq. (45))

by simply replacing the p.d.f. over the mainshock magnitudes.

Given the mainshock which produced $y_{site} > y$ occurs in a particular "point" source with distance R to the site, the probability that its magnitude lay between m_m and $m_m + dm_m$ can be written with Bayes' Theorem as

$$P[m_m \leq M_m \leq m_m + dm_m / y_{site} > y] = \frac{P[y_{site} > y / (m_m \leq M_m \leq m_m + dm_m)]}{P[y_{site} > y]} P[m_m \leq M_m \leq m_m + dm_m] \quad (48)$$

where

$$P[m_m \leq M_m \leq m_m + dm_m] = k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_0^m)} dm_m$$

(see Eq. (41)). (49)

$$P[y_{site} > y] = \bar{p}_m$$

(see Eq. (25)) (50)

With the attenuation law (Eq. (9)), it can be shown that

$$P[y_{site} > y / (m_m \leq M_m \leq m_m + dm_m)] = P[\ln \varepsilon \geq (\ln y - \ln b_1 e^{b_2 m_m} R^{-b_3})] = \Phi^*(z'/\sigma) \qquad (51)$$

where R = distance from "point" source to the site

$$z' = \ln y - \ln(b_1 e^{b_2 m_m} R^{-b_3}) \qquad (52)$$

$\Phi^*(\cdot)$ = complementary cumulative distribution function of the standardized Gaussian distribution

Thus, the new p.d.f. over the mainshock magnitude in a "point" source with distance R to the site can be written as

$$\frac{\Phi^*(z'/\sigma)}{\bar{p}_m} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_0^m)} \quad m_0^m \leq m_m \leq m_1 \qquad (53)$$

Replacing the "old" p.d.f. (Eq. (41)) in Eq. (45) by the "new" p.d.f. (Eq. (53)) yields

$$P[B / \left(\begin{smallmatrix}\text{exactly one mainshock occurs}\\ \text{in } T \text{ and causes } y_{site} > y\end{smallmatrix}\right)] = \sum_{\substack{\text{all } \Delta\ell \text{ on}\\ \text{fault}}} \int_0^T \int_{m_0^m}^{m_1} \frac{\Delta\ell}{L} \frac{1}{T} \frac{\Phi^*(z'/\sigma)}{\bar{p}_m} k_{m_1} e^{-\bar{\beta}(m_m - m_0^m)} \Big[1 - \exp\Big\{-\frac{A}{1-p}[(365(T-t') + c)^{1-p} - c^{1-p}] \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m)\\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)} \bar{p}_a^i\Big\}\Big] dm_m \, dt' \qquad (54)$$

where z' is given by Eq. (52)

$\bar{p}_m$ is given by Eq. (25)

k_{m_1} is given by Eq. (26)

A is given by Eq. (2)

$D(m_m)$ is given by Eq. (24)

$\bar{p}_a^i$ is given by Eq. (36)

The expression for $P[A \cap B]$ is therefore (see Eq. (22) and (31))

$$P[A\cap B] = 1 - \exp\left\{-E[nm_y^T]\, P\left[B \,\middle|\, \begin{pmatrix}\text{exactly one mainshock} \\ \text{occurs in } T \text{ and causes} \\ y_{site} > y\end{pmatrix}\right]\right\}$$

$$= 1 - \exp\Bigg\{-\Big[\sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \frac{\Delta\ell}{L}\bar{p}_m\Big] \cdot \sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \int_0^T \int_{m_0^m}^{m_1} \frac{\Delta\ell}{L}\nu_m \frac{\Phi^*(z'/\sigma)}{\bar{p}_m} k_{m_1} e^{-\bar{\beta}(m_m - m_0^m)} \Big[1 - \exp\Big\{-\frac{A}{1-p}\big[(365(T-t') + c)^{1-p} - c^{1-p}\big] \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m) \\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)}\bar{p}_a^i\Big\}\Big]\, dm_m\, dt'\Bigg\} \tag{55}$$

The same remarks with respect to "point" sources located near the end of the fault line and to the condition $m_0^m > m_0^a$, and with respect to the maximum magnitude and the length of the fault line, made at the end of Section 3.2.2.2.1 apply here as well.

3.2.2.3 <u>Main- and Aftershock Analysis of the "Line-Line" Model</u>

3.2.2.3.1 Exact Solution for p_y^T

With the expressions for P[A] (Eq. (32)), P[B] (Eq. (46))

and P[A ∩ B] (Eq. (55)), the desired probability p_y^T, that at least one mainshock or at least one aftershock occurs in T years and causes a seismic intensity at the site in excess of y, can now be given as

$$
\begin{aligned}
p_y^T &= P[A] + P[B] - P[A \cap B] \\
&= 1 - \exp\Big\{-\nu_m T \sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \frac{\Delta\ell}{L} \bar{p}_m\Big\} \\
&\quad - \exp\Big\{-\sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \int_0^T \int_{m_o^m}^{m_1} \frac{\Delta\ell}{L} \nu_m k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_o^m)} \\
&\qquad \Big[1 - \exp\Big\{-\frac{A}{1-p}\big[(365(T-t')+c)^{1-p} - c^{1-p}\big] \\
&\qquad \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m) \\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)} \bar{p}_a^i\Big\}\Big]\, dm_m\, dt'\Big\} \\
&\quad + \exp\Big\{-\Big[\sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \frac{\Delta\ell}{L} \bar{p}_m\Big] \sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \int_0^T \int_{m_o^m}^{m_1} \frac{\Delta\ell}{L} \nu_m \frac{\Phi^*(z/\sigma)}{\bar{p}_m} \\
&\qquad k_{m_1} e^{-\bar{\beta}(m_m - m_o^m)} \Big[1 - \exp\Big\{-\frac{A}{1-p}\big[(365(T-t') \\
&\qquad + c)^{1-p} - c^{1-p}\big] \sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m) \\ \text{around } \Delta\ell}} \frac{\Delta\ell_i}{D(m_m)} \bar{p}_a^i\Big\}\Big]\, dm_m\, dt'\Big\}
\end{aligned}
\qquad (56)
$$

where $\bar{p}_m$ is given by Eq. (25)

$\bar{p}_a$ is given by Eq. (36)

k_{m_1} is given by Eq. (26)

A is given by Eq. (2)

$D(m_m)$ is given by Eq. (24)

z' is given by Eq. (38)

$\Phi^*(\cdot)$ = complementary cumulative distribution function of the standardized Gaussian distribution

Eq. (56) gives the probability that at least one main- or after-shock produces a maximum peak seismic intensity in excess of y in T years. For small risks (say $p_y^T < 0.1$) an "average annual" probability p_y^1 can be obtained by dividing p_y^T by T

$$p_y^1 \approx \frac{p_y^T}{T} \tag{57}$$

So far, nothing has been said about criteria for the choice of the size $\Delta\ell$ of the individual "point" sources. Unlike the "line-area" model to follow, the lengths $\Delta\ell$ are governed only by the assumption of fixed distances to the site. If, for instance, the parameter b_3 in the attenuation "law" (Eq. (8), Section 2.5) is 2.0, a variation of the distance R by 5% will change the risks by approximately 10%. A reasonable rule for the determination of $\Delta\ell$ is that the distance should not vary more than 1% when going from one endpoint of the "point" source to the other, for "point" sources near the site, and not more than 2 to 5% for more distant "point" sources.

Because the double integral over the time and mainshock epicenters can neither be separated nor solved analytically, it has to be evaluated numerically. In addition, the values of P[B] and $P[A \cap B]$ can be very close in many cases, which requires precision in evaluating the terms. The numerical evaluation of Eq. (56) for p_y^T is therefore very complicated and time consuming. In the following section a method to calculate an upper bound on p_y^T will be presented.

3.2.2.3.2 Calculation of an Upper Bound on p_y^T

Compared with the computations necessary to obtain the "exact" value of p_y^T from Eq. (56) in the previous section, it is relatively easy to calculate an upper bound on p_y^T, which is still a significant improvement over the present upper bound ("equivalent event" model, see Chapter 1).

An upper bound on p_y^T is obtained if in the equation of p_y^T (Eq. (11), Section 3.1)

$$p_y^T = P[A] + P[B] - P[A \cap B]$$

the term $P[A \cap B]$ is omitted and if P[B] (Eq. (46)) is approximated by

$$P[B] = 1 - \exp\{-E[n a_y^T]\} \tag{58}$$

where $E[na_y^T]$ is the total expected number of aftershocks in T years which produce a seismic intensity at the site in excess of y. Details about this approximation are given at the end of this section. With Eq. (58) it is assumed that aftershocks are marginally governed by a Poisson process. Especially for small values of y, this is a very bad assumption, because the aftershocks cannot be considered as totally independent of each other at these low levels of y. For higher values of y, however, the Poisson assumption becomes more reasonable. As it will be shown in the following, this approximation greatly simplifies the analytical expressions and their numerical evaluation.

The expected number $E[na_y^T]$ can be written as (see also Eq. (19), (20), (21))

$$E[na_y^T] = \sum_{n=1}^{\infty} \frac{(E[nm^T])^n e^{-E[nm^T]}}{n!} \cdot n\, E[na_y^T \mid (\text{exactly one mainshock occurs during } T)]$$

or

$$E[na_y^T] = E[nm^T]\, E[na_y^T \mid (\text{exactly one mainshock occurs during } T)] \qquad (59)$$

where $E[nm^T]$ is defined as the total expected number of mainshocks with magnitudes above m_o^m and given by Eq. (28) (Section 3.2.2.1) as

$$E[nm^T] = \nu_m T \qquad (60)$$

Given a mainshock of magnitude between m_m and $m_m + dm_m$ occurs at time t' to t' + dt' ($0 \leq t' \leq T$), the expected number of aftershocks

E[na] with magnitudes in excess of m_o^a is given by Eq. (33) (Section 3.2.2.2.1) as

$$E[na] = \frac{A}{1-p}\left[(365(T-t')+c)^{1-p} - c^{1-p}\right] \qquad (61)$$

The probability that the mainshock occurs "at t" and has "a magnitude m_m" can be taken from Eq. (44) (Section 3.2.2.2.1) as

$$\frac{dt'}{T} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_o^m)} dm_m \qquad (62)$$

As mentioned at the end of Section 3.2.1, a mainshock of magnitude m_m can potentially produce aftershocks in a particular "point" source of length $\Delta\ell$, only if it occurs within a region of $D(m_m)/2$ on either side of the "point" source. Thus, the probability $p_{\Delta\ell}$ that, given an aftershock occurs, it lies in a particular "point" source can be expressed as the probability that the mainshock occurs within $D(m_m)$ around the "point" source times the probability that the aftershock, which can occur anywhere in the region $D(m_m)$, occurs in the "point" source $\Delta\ell$. With the equally likely assumption on the location of mainshocks and aftershocks, $p_{\Delta\ell}$ can be written as

$$p_{\Delta\ell} = \frac{D(m_m)}{L} \cdot \frac{\Delta\ell}{D(m_m)} = \frac{\Delta\ell}{L} \qquad (63)$$

Given that an aftershock occurs in a particular "point" source, the probability $\bar{p}_a$ that it will produce a seismic intensity at the site in excess of y is given by Eq. (36) (Section 3.2.2.1) as

$$\bar{p}_a = (1-k_{m_m})\,\Phi^*(\bar{z}/\sigma) + k_{m_m}\,\Phi^*(\bar{z}/\sigma - \bar{\beta}\sigma/b_2)\; e^{\bar{\beta}^2\sigma^2/2b_2^2}\, e^{\bar{\beta}m_o^a}\, R^{-\bar{\beta}b_3/b_2}\,(y/b_1)^{-\bar{\beta}/b_2} \qquad (64)$$

Thus, the expected number of aftershocks during t' to T in a particular "point" source, that produce $y_{site} > y$, due to a mainshock of "magnitude m_m" at time t', is the product of Eqs. (61) through (64)

$$E[na]\, \frac{\Delta \ell}{L}\, \bar{p}_a\, \frac{dt'}{T}\, k_{m_1}\, \bar{\beta}\, e^{-\bar{\beta}(m_m - m_o^m)}\, dm_m \qquad (65)$$

In order to get the total expected number of aftershocks which cause $y_{site} > y$, given a mainshock occurs in T, Eq. (65) has to be integrated over t' (from t' = 0 to t' = T) and over m_m (from m_o^m * to m_1) and summed over all "point sources on the fault-line

$$E[na_y^T \,|\, (\text{exactly one mainshock occurs during } T)] = \sum_{\text{all } \Delta\ell \text{ on fault}} \frac{\Delta \ell}{L}\, \frac{1}{T}\, k_{m_1}\, \bar{\beta}$$

$$\int_0^T \int_{m_o^m}^{m_1} \bar{p}_a\, e^{-\bar{\beta}(m_m - m_o^m)}\, \frac{A}{1-p}\, [[365(T-t')+c]^{1-p} - c^{1-p}]\, dm_m\, dt' \qquad (66)$$

The integration over the time t' can be carried out, yielding

---

* See comments on page 45.

$$E[na_y^T/(\text{exactly one mainshock occurs during } T)] = \frac{k_{m_1}\bar{\beta}}{LT(1-p)}$$

$$\left[\frac{(365T+c)^{2-p}-c^{2-p}}{365(2-p)} - Tc^{1-p}\right]\int_{m_o^m}^{m_1} \bar{p}_a A e^{-\bar{\beta}(m_m-m_o^m)} dm_m \qquad (67)$$

After Eq. (2) in Section 2.1.2

$$A = \left\{\frac{(\beta-b)\Gamma(\beta/b)\, c\cdot E_a}{b E_\ell}\right\}^{b/\beta}$$

By using the relationships

$$\bar{\beta} = b \ln 10$$
$$E_a = r E_o = r e^{\alpha \ln 10 + m_m \beta \ln 10}$$
$$E_\ell = e^{\alpha \ln 10 + m_o^a \beta \ln 10}$$

from Chapter 2, the expression of A can be transformed into

$$A = \left\{\frac{(\beta \ln 10 - \bar{\beta})\Gamma(\beta \ln 10/\bar{\beta})\, c r}{\bar{\beta}}\right\}^{\bar{\beta}/\beta \ln 10} e^{\bar{\beta}(m_m - m_o^a)}$$

$$= \bar{A} e^{\bar{\beta}(m_m - m_o^a)} \qquad (68)$$

The integral over the magnitude in Eq. (67) can now be written as

$$\int_{m_o^m}^{m_1} \bar{p}_a A e^{-\bar{\beta}(m_m-m_o^m)} dm_m = \bar{A} e^{\bar{\beta}(m_o^m - m_o^a)} \int_{m_o^m}^{m_1} \bar{p}_a \, dm_m \qquad (69)$$

Using Eq. (64) for $\bar{p}_a$ and Eq. (60) for $E[nm^T]$, $E[na_y^T]$ is given by

$$E[na_y^T] = \sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}} \nu_m \frac{\Delta\ell}{L} k_{m_1} \bar{\beta} \frac{\bar{A}}{1-\rho} \left[\frac{(365T+c)^{2-\rho} - c^{2-\rho}}{365(2-\rho)} - Tc^{1-\rho} \right] e^{\bar{\beta}(m_0^m - m_0^a)} \left\{ \int_{m_0^m}^{m_1} (1-k_{m_m}) \Phi^*(\bar{z}/\sigma) dm_m + e^{\bar{\beta}^2\sigma^2/2b_2^2} e^{\bar{\beta} m_0^a} R^{-\bar{\beta} b_3/b_2} (y/b_1)^{-\bar{\beta}/b_2} \int_{m_0^m}^{m_1} k_{m_m} \Phi^*(\bar{z}/\sigma - \bar{\beta}\sigma/b_2) dm_m \right\} \tag{70}$$

where $\bar{A}$ is given by Eq. (68)

k_{m_1} is given by Eq. (26)

k_{m_m} is given by Eq. (37)

$\bar{z}$ is given by Eq. (38)

$\Gamma(\cdot)$ = Gamma Function

$\Phi^*(\cdot)$ = complementary cumulative distribution function of the standardized Gaussian distribution

This expression is relatively easy to evaluate, and the corresponding computer program is available.

The upper bound on p_y^T can now be expressed as the sum of P[A] given by Eq. (32) plus the estimate of P[B] given by Eq. (58)

$$\underset{\sim}{p_y^T} = 2 - \exp\{-E[nm_y^T]\} - \exp\{-E[na_y^T]\} \tag{71}$$

where $E[nm_y^T]$ is given by Eq. (31) (Section 3.2.2.1) and $E[na_y^T]$

by Eq. (70).

A comparison of $(1 - \exp\{-E[na_y^T]\}$ with the exact expression of P[B] (Eq. (46), Section 3.2.2.2.1) reveals that in the calculation of $\underset{\sim}{p}_y^T$, the following term in Eq. (46)

$$1-\exp\left\{-\frac{A}{1-p}[\ldots\ldots\ldots]\sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m)\\ \text{around } \Delta\ell}}\frac{\Delta\ell_i}{D(m_m)}\bar{p}_a^{\,i}\right\}$$

has been approximated by

$$\left\{-\frac{A}{1-p}[\ldots\ldots\ldots]\sum_{\substack{\text{all } \Delta\ell_i \text{ in } D(m_m)\\ \text{around } \Delta\ell}}\frac{\Delta\ell_i}{D(m_m)}\bar{p}_a^{\,i}\right\}$$

and the two summations $\sum_{\substack{\text{all } \Delta\ell \text{ on}\\ \text{fault}}}(\,.\,)$ and $\sum_{\substack{\text{all } \Delta\ell_i\\ \text{in } D(m_m)}}(\,.\,)$ have been interchanged.

It can now clearly be seen from the approximation that for small values of y (i.e., large values of $\bar{p}_a^i$), the upper bound method yields very conservative estimates of the contribution of aftershocks to the seismic risk. However, it will be shown in the example of Chapter 4 that for values of y of engineering interest (say, maximum peak ground accelerations > 0.1g) the upper bound p_y^T is considerably smaller than the present estimate of an upper bound ("equivalent event" model). Unfortunately, it is not possible to assess the degree of over-estimation implicit in $\underset{\sim}{p}_y^T$ without actually calculating the exact value of p_y^T.

3.3 Seismic Risk Analysis of a "Line-Area" Occurrence Model

The "line-area" occurrence model is defined by the assumptions that the mainshock epicenters lie on a fault line, whereas the aftershock epicenters are scattered in a limited area around the mainshock epicenters. The model is clearly more realistic than the "line-line" model, but, on the other hand, the evaluations of the analytical expressions become considerably more complicated and time consuming, even for the calculation of an upper bound on the risk. For the latter reason, this model has not yet been applied to an example.

A typical fault-site configuration of the "line-area" model with a typical mainshock and its aftershocks is shown in Figure 3-3. In the analysis, the fault line will be represented as a set of many (linear) "point" sources, the total potential aftershock area as a set of many (areal) "point" sources, for all of which fixed distances to the site and in the case of areal "point" sources also fixed perpendicular distances to the fault line can be calculated (see Figure 3-4).

3.3.1 Special Assumptions

Basically, all assumptions and parameter relationships which were discussed in Chapter 2 can be used in this model. However, more detailed assumptions are necessary for the spatial distribution law of aftershocks. In order to avoid undue geometrical complications while still capturing the basic characteristics of the areal model, the aftershock area is approximated by a circle with a

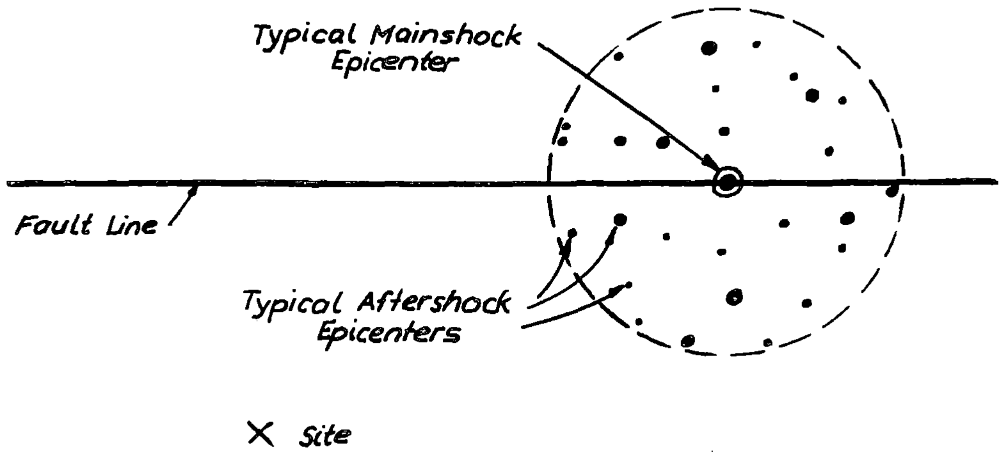

Figure 3-3 Typical Fault-Site Configuration with Typical Mainshock and its Aftershocks ("Line-Area" Model)

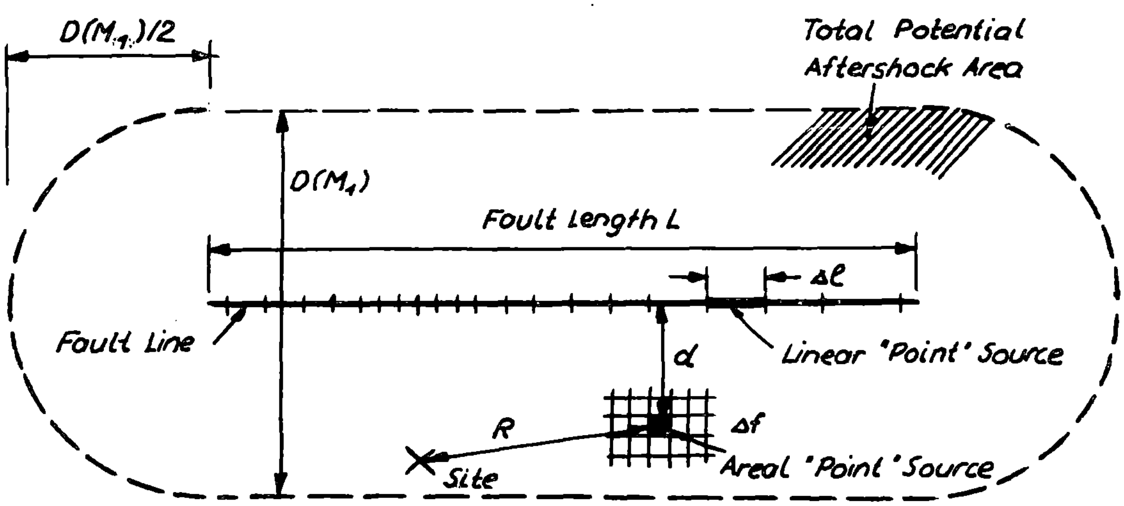

Figure 3-4 Point-by-Point Analysis of the Seismic Risk ("Line-Area" Model)

mainshock epicenter as its center and the diameter D related to the mainshock magnitude M_m by the following formula (see Section 2.4.2)

$$\log_{10} D = \gamma' + \delta' M_m \tag{72}$$

Therefore, with $F = \pi/4D^2$, the relation between the aftershock area $F(m_m)$ and the mainshock magnitude M_m can be written as

$$\log_{10} F(M_m) = \gamma + \delta M_m \tag{73}$$

It is furthermore assumed that aftershocks of a particular mainshock are equally likely to occur anywhere within the determined area F.

3.3.2 Derivation of the Analytical Expressions

3.3.2.1 Main- and Aftershock Analyses of the "Line-Area" Model

3.3.2.1.1 Exact Solution for p_y^T

The only difference between the "line-line" and the "line-area" model is the spatial distribution of aftershocks. The analytical expressions for the probability p_y^T, that during T years at least one mainshock occurs on the fault and produces $y_{site} > y$ or at least one aftershock occurs in the total potential aftershock area (see Figure 3-4) and produces $y_{site} > y$, can therefore easily be derived from the analytical expressions for p_y^T of the "line-line" model.

Given a mainshock of "magnitude m_m" occurred at "time t'" in one of the (linear) "point" sources, in which the fault line has been divided, the probability that at least one aftershock occurs during t' to T in any one of the (areal) "point" sources by which the aftershock region $F(m_m)$ is approximated, and causes $y_{site} > y$, is analogous to Eq. (40) (Section 3.2.2.2.1)

$$1-\exp\left\{-E[na]\sum_{\substack{\text{all } \Delta f_i \text{ in } F(m_m) \\ \text{around } \Delta\ell}} \frac{\Delta f_i}{F(m_m)}\,\bar{p}_a^{\,i}\right\} \qquad (74)$$

where Δf_i denotes the area of the (areal) "point" source i, E[na] the expected number of aftershocks during t' to T due to a mainshock of "magnitude m_m at time t'" (see Eq. (33)), and $\bar{p}_a^i$ the probability that, given an aftershock occurs in "point" source i, it will cause $y_{site} > y$ (see Eq. (36)). The analytical expression for p_y^T is now obtained by simply replacing Eq. (40) in the final expression (Eq. (56)) of the "line-line" model by Eq. (74). Thus

$$p_y^T = P[A]+P[B]-P[A\cap B]$$

$$= 1-\exp\left\{-\nu_m T\sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}}\frac{\Delta\ell}{L}\bar{p}_m\right\}$$

$$-\exp\left\{-\sum_{\substack{\text{all } \Delta\ell \text{ on} \\ \text{fault}}}\int_0^T\int_{m_0^m}^{m_1}\frac{\Delta\ell}{L}\nu_m k_{m_1}\beta e^{-\beta(m_m-m_0^m)}\left[1-\right.\right.$$

$$\exp\left\{-\frac{A}{1-p}\left[[365(T-t')+c]^{1-p}-c^{1-p}\right]\right.$$

$$\sum_{\substack{\text{all } \Delta f_i \text{ in } F(m_m)\\ \text{around } \Delta\ell}} \frac{\Delta f_i}{F(m_m)} \bar{p}_a^i\}\Big] dm_m dt'\Big\}$$

$$+\exp\Big\{-\Big[\sum_{\substack{\text{all } \Delta\ell \text{ on}\\ \text{fault}}} \frac{\Delta\ell}{L}\bar{p}_m\Big] \sum_{\substack{\text{all } \Delta\ell \text{ on}\\ \text{fault}}} \int_0^T\int_{m_0^m}^{m_1} \frac{\Delta\ell}{L} \nu_m \frac{\Phi^*(z'/\sigma)}{\bar{p}_m} k_{m_1}$$

$$e^{-\beta(m_m - m_0^m)}\Big[1-\exp\Big\{-\frac{A}{1-p}\Big[(365(T-t')+c)^{1-p} -$$

$$c^{1-p}\Big] \sum_{\text{all } \Delta f_i \text{ in } F(m_m)} \frac{\Delta f_i}{F(m_m)} \bar{p}_a^i\Big\}\Big] dm_m dt'\Big\} \qquad (75)$$

where $\bar{p}_m$ is given by Eq. (25), Section 3.2.2.1

$\bar{p}_a$ is given by Eq. (36), Section 3.2.2.2.1

k_{m_1} is given by Eq. (26), Section 3.2.2.1

A is given by Eq. (2) , Section 2.1.2

$F(m_m)$ is given by Eq. (73)

z' is given by Eq. (38), Section 3.2.2.2.2

$\Phi^*(\cdot)$ = complementary cumulative distribution function of the standardized Gaussian distribution

For small risks (say, $p_y^T < 0.1$) an "average" annual probability p_y^1 can be obtained by dividing p_y^T by T

$$p_y^1 \approx \frac{p_y^T}{T} \qquad (76)$$

As was the case in the "line-line" model (see Section 3.2.2.2.1 following Eq. (46)), the expression for p_y^T (Eq. (75)) is valid only for $m_0^m > m_0^a$. The remarks with respect to the implications of m_0^m as the lower limit of the integral over the mainshock magnitudes apply here as well.

The problem with "point" sources near the defined endpoints of a fault-line does not come up in this particular analysis of the "line-area" model, because it has been assumed that mainshocks occur only on the fault and aftershocks anywhere in the surrounding region (i.e., also in a region beyond the endpoint of a fault (see Figure 3-4)). However, if it is believed that the assumption is unreasonable, the same problem as discussed at the end of Section 3.2.2.2.1 arises and the remarks made there apply to this model as well.

In choosing the size of a particular (areal) "point" source Δf, which can vary from location to location, two conditions have to be satisfied. On one hand, the size should be small enough that a fixed distance to the site can be assumed. As proposed in the "line-line" model, the distances from the points inside a "point" source should not vary more than 1% for close "point" sources and not more that 2 - 5% for more distant "point" sources. On the other hand, the size should be such that a fixed perpendicular distance from the "point" source to the fault line can be assumed. This criterion insures that the error in approximating the circular aftershock area by sets of (areal) "point" sources is small. A reasonable rule is that the perpendicular distance from points inside a particular "point" source should not vary more than 5% for "point" sources close to the fault line and not more than 5 to 10% for more distant ones.

The criteria set up in the "line-line" model for the choice of the length's $\Delta\ell$ of the (linear) "point" sources, which represent the fault line, can be used without changes in the "line-area" model.

3.3.2.1.2 Calculation of an Upper Bound on p_y^T

As in the "line-line" model, an upper bound on p_y^T, which is easier to calculate than the exact value of p_y^T, is obtained, if in the equation of p_y^T (Eq. (11), Section 3.1)

$$p_y^T = P[A] + P[B] - P[A \cap B]$$

the term $P[A \cap B]$ is omitted and if $P[B]$ respectively $(1 - P[B])$ (second term in Eq. (75)) is approximated by

$$P[B] = 1 - \exp\{-E[na_y^T]\}$$

$E[na_y^T]$ is the total expected number of aftershocks in T years which produce $y_{site} > y$. This approximation implies the assumption that aftershocks are marginally governed by a Poisson process. As mentioned in the "line-line" model, this assumption does not hold, especially for small values of y. However, it simplifies the analytical expression and their numerical evaluation significantly.

The expected number $E[na_y^T]$ can be written as

$$E[na_y^T] = E[nm^T]\, E[na_y^T / (\text{exactly one mainshock occurs during } T)]$$

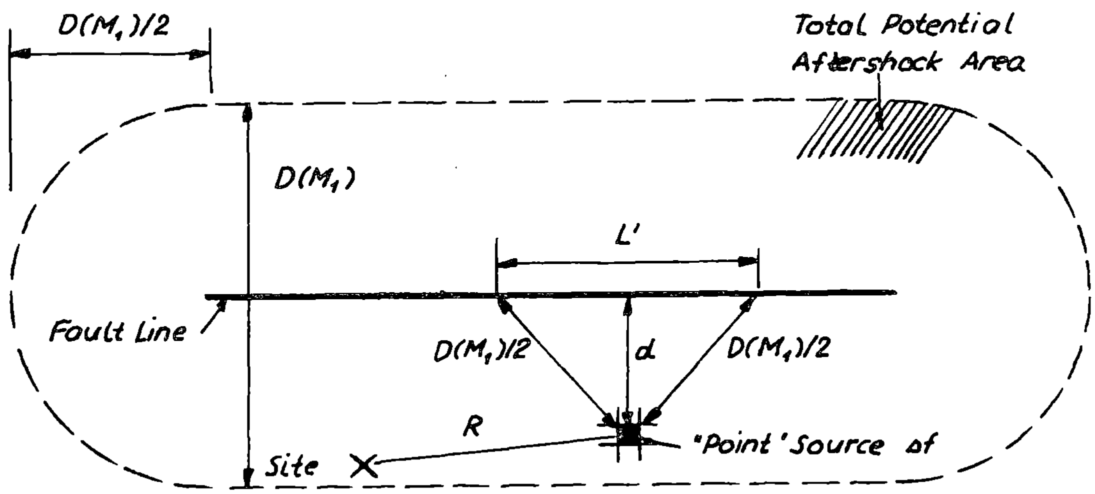

Figure 3-5 Point-by-Point Calculation of $E[na_y^T]$

with $E[nm^T]$, the total expected number of mainshocks with magnitudes above m_o^m, given by Eq. (28) (Section 3.1.2.1) as

$$E[nm^T] = \nu_m T$$

The total area of potential aftershock locations has to be divided (as was done in Section 3.3.2.1.1) into smaller areas Δf, which can be treated as "point" sources with fixed distances to the site and the fault line. With an upper bound m_1 on the mainshock magnitude, the maximum extent of the aftershock area is determined by the relation

$$\log_{10} D_1 = \gamma' + \delta' m_1$$

as shown in Figure 3-5.

In order to induce aftershocks in a particular "point" source, a mainshock of magnitude m_m has to occur in a limited region L' on the fault line, where L' is a function of m_m and the perpendicular distance d from the source to the fault line (see Figure 3-5). With the spatial distribution law, L' is given by

$$L' = 2\sqrt{\exp\{2 \ln 10 (\gamma' + \delta' m_m)\} - d^2} \qquad (77)$$

Clearly only mainshocks of magnitudes in excess of m_m'

$$m_m' = \frac{\log_{10} d - \gamma'}{\delta'} \qquad (78)$$

can induce aftershocks in this particular "point" source.

Given a mainshock of "magnitude m_m" ($m_m > m_m'$) occurs "at time t'" ($0 \leq t' \leq T$) with its epicenter in $L'(d,m_m)$, the expected number of aftershocks that occur during t' to T in the "point" source Δf_i in question and produce $y_{site} > y$, can be written as

$$E[na] \frac{\Delta f_i}{F(m_m)} \bar{p}_a^i \tag{79}$$

where the expected number of aftershocks, E[na], with magnitudes in excess of m_a^o is given by Eq. (33) (Section 3.2.2.2.1), the aftershock area $F(m_m)$ by Eq. (73), and $\bar{p}_a^i$, the probability that, given an aftershock occurs in Δf_i, it will produce $y_{site} > y$, by Eq. (36) (Section 3.2.2.2.1). The probability that the mainshock is of "magnitude m_m" ($m_m > m_m'$) and occurs "at time t'" in $L'(d,m_m)$ is given by

$$\frac{dt'}{T} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_o^m)} dm_m \frac{L'}{L} \quad m_m' \leq m_m \leq m_1 \tag{80}$$

Thus, the expected number of aftershocks in "point" source Δf_i during t' to T, that produce $y_{site} > y$, due to a mainshock of "magnitude m_m" at "time t'", is the product of Eq. (79) and Eq. (80).

$$E[na] \frac{\Delta f_i}{F(m_m)} \bar{p}_a^i \frac{dt'}{T} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_o^m)} dm_m \frac{L'}{L} \tag{81}$$

In order to get the total expected number of aftershocks which occur in the total potential aftershock area and produce

$y_{site} > y$, given a mainshock occurs in T, Eq. (81) has to be integrated over t' (from t' = 0 to t' = T) and over m_m (from $m_m = m_m'(d)$* to $m_m = m_1$) and summed over all "point" sources of the total potential aftershock area, yielding

$$E[na_y^T|(\text{exactly one mainshock occurs during } T)] = \sum_{\substack{\text{all } \Delta f_i \text{ in total} \\ \text{aftershock area}}} \frac{\Delta f_i}{L} \frac{1}{T} k_{m_1} \bar{\beta} \int_0^T \int_{m_o^m}^{m_1} \frac{L'}{F(m_m)} e^{-\bar{\beta}(m_m - m_o^m)} \bar{p}_a^i \frac{A}{1-p} [(365(T-t') + c)^{1-p} - c^{1-p}] \, dm_m \, dt' \tag{82}$$

By carrying out the integration over t', replacing A by Eq. (68) and by using Eq. (28) for $E[nm^T]$, the final expression for $E[na_y^T]$ can be written as

$$E[na_y^T] = \sum_{\substack{\text{all } \Delta f_i \text{ in total} \\ \text{aftershock area}}} \frac{\nu_m}{L} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_o^m - m_o^a)} \Delta f_i \frac{\bar{A}}{1-p} \left[\frac{(365T+c)^{2-p} - c^{2-p}}{365(2-p)} - Tc^{1-p}\right] \int_{m_m'}^{m_1} \frac{L'}{F(m_m)} \bar{p}_a^i \, dm_m \tag{83}$$

---

* For reasons given in Section 3.2.2.2.1, the lower limit of the integral has to be larger than m_o^a, the lower bound on the aftershock magnitudes. For small values of d, where $m_m' \leq m_o^a$, either the lower limit of the integral can be set to m_o^m (if only mainshock magnitudes between m_o^m and m_1 want to be considered) or m_o^a can be lowered until it satisfies the above condition.

where $\bar{p}_a^i$ is given by Eq. (36)

L' is given by Eq. (77)

$F(m_m)$ is given by Eq. (73)

$\bar{A}$ is given by Eq. (68)

k_{m_1} is given by Eq. (26)

m_m' is given by Eq. (78) (see also footnote on page 70)

$m_o^m > m_o^a$ (See Section 3.2.2.2.1 for explanation)

This expression is not easy to evaluate but significantly easier than the expressions of Eq. (75).

The upper bound on p_y^T can now be expressed as

$$\underset{\sim}{p_y^T} = 2 - \exp\{-E[nm_y^T]\} - \exp\{-E[na_y^T]\} \qquad (84)$$

where $E[nm_y^T]$ is given by Eq. (31) (Section 3.1.2.1) and $E[na_y^T]$ by Eq. (83).

A comparison of $\exp\{-E[na_y^T]\}$ with the exact solution of $(1 - P[B])$ (Eq. (75), second term) reveals that in the calculation of p_y^T the following term in $(1 - P[B])$

$$1 - \exp\left\{-\frac{A}{1-p}[\dots\dots\dots\dots] \sum_{\substack{\text{all } \Delta f_i \text{ in} \\ F(m_m)}} \frac{\Delta f_i}{F(m_m)} \bar{p}_a^i\right\}$$

has been approximated by

$$\left\{-\frac{A}{1-p}[\dots\dots\dots\dots] \sum_{\substack{\text{all } \Delta f_i \text{ in} \\ F(m_m)}} \frac{\Delta f_i}{F(m_m)} \bar{p}_a^i\right\}$$

and the summations and the integral over the mainshock magnitudes have been rearranged. As in the "line-line" model, the approximation yields very conservative estimates for small values of y. Again, it is not possible to assess the degree of over-estimation without calculating the exact value of p_y^T.

When using the expression for an upper bound on p_y^T, (areal) "point" sources near the defined endpoints of the fault-line have to be treated specially. If the region L' (Eq. (77), see also Figure 3-5) on the fault, where the mainshocks have to occur in order to be able to induce aftershocks in an (areal) "point" source, extends beyond the defined fault-line, L' in Eq. (83) for $p_{\tilde{y}}^T$ has to be replaced by the portion of L' that lies on the fault-line. The same is true should L' for a particular "point" source be larger than the total fault length L.

CHAPTER 4

APPLICATION OF THE "LINE-LINE" MODEL

In this chapter the "line-line" model will be applied to an example and numerical results of a seismic risk analysis using the upper bound method outlined in Section 3.2.2.3.2 will be presented and discussed. As it has been pointed out several times, it is not possible to estimate how conservative the upper bound on p_y^T is without actually calculating the exact value of p_y^T. Despite this limitation, the results are useful and provide new insight and understanding of the influence of aftershocks on the seismic risk in various situations. However, it has also to be kept in mind that the "line-line" mdoel is a crude simplification of the spatial occurrence of earthquakes and that the results cannot yet be generalized with great confidence. This can probably only be done after an application of the more realistic "line-area" model (Section 3.3).

The numerical results of the upper bound on p_y^T will be compared with the results of a seismic risk analysis described in Cornell[(2)] of the "equivalent event" model (the aftershocks are treated as mainshocks) and the "mainshock" model (only mainshocks are considered). This comparison will be done for a set of dif-ferent upper bounds on the mainshock magnitude and a fixed fault-site distance, and for a set of different fault-site distances and a fixed upper bound on the mainshock magnitude, as well as for different ratios of the ("average") expected number of aftershocks ν_a to the expected number of mainshocks ν_m per year, with the sum of ν_a and ν_m kept constant.

All computations have been done on the IBM-370 computer of the M.I.T. Information Processing Center.

4.1 Parameter Values of the Example

The fault-site configuration of the following example of the "line-line" model is shown in Figure 4-1. The site at which the seismic risk will be estimated is located at a distance d in a perpendicular direction from the midpoint of the 200 kilometer-long fault-line.

The seismic risk will be calculated for nine different cases: four cases with a distance d of 30 kilometers, but each with a different upper bound on the mainshock magnitude, 6.0, 7.0, 8.0 and 9.0; two cases with an upper bound on the mainshock magnitude of 9.0, but each with a different distance d, 10 and 50 kilometers. For these six cases it has been assumed that 50%* of all earthquakes with magnitudes above ~4.5 that occurred in the past on the fault-line, can be classified as aftershocks. Therefore, the expected number of aftershocks with magnitudes above 4.4 (lower bound on aftershock magnitude) in a certain time period T

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* This number, as well as the value of ν_m (for mainshock with magnitudes above 4.5), can be considered as reasonable and typical values for faults in Southern California (see Kallberg[5], p. 24, Sources 9 and 10).

(here 10 years) should be approximately equal to the expected number of mainshocks in excess of 4.5 (lower bound on mainshock magnitude; note that $m_o^m > m_o^a$ as required), which was set to 0.5 (i.e., annual mean rate of mainshocks > 4.5 $\nu_m = 0.05$). This condition was met by adjusting the values of c and p in the modified Omori law (see Section 3.1.2) with a method explained in Appendix A. For a value of r = 0.07* (r is the ratio of the total energy of all aftershocks larger than 4.4 to the energy of the causative mainshock) the following values for c and p have been calculated (c = 1.0 = constant):

	Upper bound on mainshock magnitude			
	6.0	7.0	8.0	9.0
c	1.0	1.0	1.0	1.0
p	1.64	2.025	2.435	2.845

For the last three cases, an upper bound on the mainshock magnitude of 9.0 and a distance d of 30 kilometers was assumed, but each with a different ratio of ν_a/ν_m, 0.43, 2.33, and 9.0 The corresponding c and p values have been calculated in the same way as in the other six cases:

* Median value of 42 observed aftershock sequences in Japan (Utsu[9]).

	Ratio ν_a/ν_m		
	0.43	2.33	9.0
c	1.0	1.0	1.0
p	5.3	1.79	1.025

A typical value of b (magnitude frequency law) for Southern California is b = 0.86, thus $\bar{\beta} = b \ln 10 = 2.0$. The parameter β in the magnitude-energy law is assumed to be 1.5 (Gutenberg[23]).

Utsu's[9,10] relationship between the linear dimension D of the potential aftershock area and the mainshock magnitude M_m of the form

$$\log_{10} D = -1.8 + 0.5 M_m$$

for D in kilometers, is used as spatial distribution law.

The attenuation "law" in this example takes the following form (Esteva[21])

$$y = \begin{cases} 1280\, e^{0.8M} R^{-2.0}\, \varepsilon & \text{for } R > 30 \text{ kilometers} \\ 1.42\, e^{0.8M} & \text{for } R \leq 30 \text{ kilometers} \end{cases}$$

where y is the maximum peak ground acceleration in cm/s², M the magnitude of the earthquake and R the distance from the epicenter to the site (in kilometers). The "error" term ε is lognormally distributed with a mean of ln ε of zero and a standard deviation σ of ln ε of 0.01. This small value has been chosen in order to avoid masking effects by σ and because the case $\sigma = 0$ requires special and more complicated treatment.

4.2 Numerical Results

In this section, the upper bounds on the seismic risk of the "main- and aftershock" model (see Section 3.2.2.3.2) will be presented and compared with the "exact" results of a seismic risk analysis of the "mainshock" and the "equivalent event" model. In all three models it was assumed that the epicenters of future earthquakes will lie on the fault line.

The results of the "mainshock" model, obtained with a seismic risk analysis described in Cornell[2] (corresponding to term P[A] in Eq. (32), Section 3.2.2.1) are plotted in Figure 4-2. For each of the six cases with $\nu_a/\nu_m = 1$ described in Section 4.2, the probability that the maximum peak ground acceleration at the site will exceed a units in any year, is given as a function of a. Since the risks are small (less than 0.1) the corresponding probabilities of the cases with different ratios, $x = \nu_a/\nu_m$, are obtained by multiplying the results of Figure 4-2 by $2/(1 + x)$. Because $(\nu_a + \nu_m)$ is constant in all cases, the risks from the "equivalent event" model are twice the ones shown in Figure 4-2 (valid also for small risks). Truncating the magnitudes at an upper bound leads to an effective upper bound on the maximum peak ground acceleration at the site a^u, which is a function of the closest distance from the site to the fault, of the upper bound on the mainshock magnitude and (however, to a far lesser degree) of the standard deviation, σ, of $\ln \varepsilon$. For $\sigma \neq 0$, infinitely large values of $\underline{a}$ are theoretically possible, but the risks fall off very rapidly when approaching the effective upper bound $\underline{a}^u$.

A comparison of risk estimates of the "main- and aftershock" model with the "equivalent event" and "mainshock" model for the nine different cases examined are shown in Figures 4-3 to 4-6. In Figure 4-3 the (annual) risks of the three models are plotted as a function of maximum peak ground acceleration $\underline{a}$ for the case with upper bound on the mainshock magnitudes = 9.0, distance d = 30 kilometers and $\nu_a/\nu_m = 1$. In the other figures the ratio of the ("average annual") risk from the "main- and aftershock" model (i.e., upper bound on the exact risk) to the risk of the "equivalent event" model is plotted as a function of the ratio of examined ground acceleration $\underline{a}$ to the effective upper bound on the ground acceleration $\underline{a}^u$. Figure 4-4 shows the influence of a variation of the upper bounds on the mainshock magnitudes for the cases with constant distance d = 30 kilometers and constant ratio $\nu_a/\nu_m = 1$ (i.e., risk from "mainshock" model is approximately 50% of risk from "equivalent event" model). Figure 4-5 shows the influence of a variation of the perpendicular distance d from the site to the fault line for the cases with a constant upper bound on the mainshock magnitude $m_1 = 9.0$ and a constant ratio of $\nu_a/\nu_m = 1.0$. The dashed lines in both figures correspond to a fixed level of examined ground acceleration (i.e., 10% of g).

Finally, in Figure 4-6, the results of a variation of the ratio $x = \nu_a/\nu_m$ are shown for the case with upper bound on the mainshock magnitude $m_1 = 9.0$ and distance $d = 30$ kilometers. Because $(\nu_a + \nu_m)$ was kept constant, the 100% line corresponds to the "equivalent event" model for all values of x. The actual risk values for the "equivalent event" model can be taken from Figure 4-3 and the risks for the "mainshock" model are obtained by multiplying these values by $1/(1 + x)$, as shown by the horizontal lines in Figure 4-6.

4.3 Discussion of the Results

Throughout this chapter it has to be kept in mind that, first, the risk values for the "main- and aftershock" model are only an upper bound on the "true" values, and, second, that the "line-line" model is based upon a simplistic assumption about the spatial distribution of aftershock epicenters. The observations and conclusions drawn from the results of this example can therefore not necessarily be generalized. However, some of the more general conclusions are judged not to be restricted to the above limitations of this example.

In all the examined cases, it can clearly be observed that the influence of aftershocks on the seismic risk is largest for small ground accelerations examined, and rapidly decreases when the examined ground acceleration approaches the effective upper bound on the ground acceleration at the site (see Figures 4-4 to 4-6). In any case, the "equivalent event" model significantly overestimates the risks, especially for larger ground accelerations, whereas the "mainshock" model underestimates them. In the view that the risks of the "main- and aftershock" model are upper bounds, it seems that the "mainshock" model represents more closely the aftershock influence, particularly for ground accelerations of engineering interest (in general, 10% of g or more).

When looking at a fixed ground acceleration (say, 10% of g) as indicated by the dashed lines in Figure 4-4, it can be noticed that the contribution of aftershocks to the seismic risk decreases with decreasing upper bounds m_1 on the mainshock magnitude. For instance, the aftershock risk at 10% of g drops from 26% of the total risk from the "equivalent event" model of an upper bound $m_1 = 9.0$ to 16% of an upper bound $m_1 = 7.0$ (see Figure 4-4). This implies indirectly that the aftershock risk is associated mainly with mainshocks of large magnitudes, which cause a considerable spread of aftershock epicenter, thus permitting considerably shorter distances to the site. The decrease is believed to be more significant for the exact risks of the "main- and aftershock" model, because the maximum possible aftershock region $D(m_1)$ (see equation for $D(m_m)$ in Section 3.1) for small upper bounds on the mainshock magnitude m_1 (say, $m_1 = 6.0$) is not very much different from the length $\Delta\ell$ of a "point" source (for which a fixed distance to the site was assumed). In these cases, the aftershock risk has to be nearly zero (equal to zero if $D(m_1) \leq \Delta\ell$).

As shown in Figure 4-5, a variation of the perpendicular distance d from the site to the fault-line does not significantly change the relative contributions of the aftershocks to the seismic risk. Clearly a variation of d does change the absolute risk values and the effective upper bound on the ground acceleration at the site (see Figure 4-2).

It can be expected that a variation of the "average" expected number of aftershocks ν_a to the expected number of mainshocks ν_m, given the sum ν_a and ν_m remains constant, greatly influences the contributions of aftershocks to the seismic risk. If an earthquake source (fault-line) is characterized by a small ratio of $x = \nu_a/\nu_m$ (i.e., mainshocks are usually followed by a relatively small number of aftershocks), the influence of aftershocks remains small. For the case of x = 0.43 in Figure 4-6, aftershocks contribute less than 20% (for a > 10% of g) to the risk from the "mainshock" model. The "equivalent event" model, on the other hand, overestimates the risk by 20 to 45%. However, it must be emphasized that such a variation of the risk can be easily obtained also by changing parameter values in the "mainshock" model. This aspect will be discussed in the following chapter. For large values of x (x = 9.0, see Figure 4-6) the "mainshock" model underestimates the risks by less than 400% (for a > 10% of g), whereas the "equivalent event" model overestimates the risks by 300 to 900% (relative to the "mainshock" model). Thus, for increasing values of x, the influence of the aftershocks becomes more and more important and neither of the simple models estimates the risk adequately.

Summarizing the discussions of the results, it can be said that the most significant parameters which influence the relative contribution of aftershocks are the upper bound on the mainshock magnitude and the ratio x of the "average" expected number of aftershocks ν_a to the expected number of mainshocks ν_m. For low upper bounds and/or a small (say, less than 1.0) value of x, the "mainshock" model yields, in this example, results closer to the "true" results. For high upper bounds and especially for large values of x, neither of the two models yields accurate results (up to 900% difference for x = 9.0, a value that has been observed in several earthquake regions in Japan[15]). However, for ground accelerations of engineering interest, the "mainshock" model always comes closer to the "true" results.

Other parameters, such as d, affect the absolute risk levels but not significantly the relative contributions of the aftershocks to the seismic risk. It is anticipated that a more realistic two-dimensional spatial assumption for aftershock locations in future analytical models will demonstrate further influences of parameters on the contribution of aftershocks.

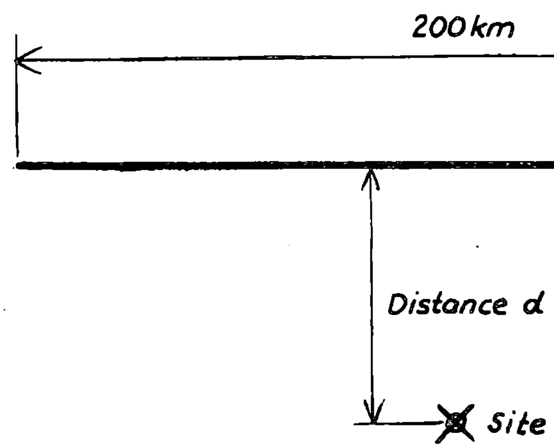

Figure 4-1 Fault-Site Configuration of the Example

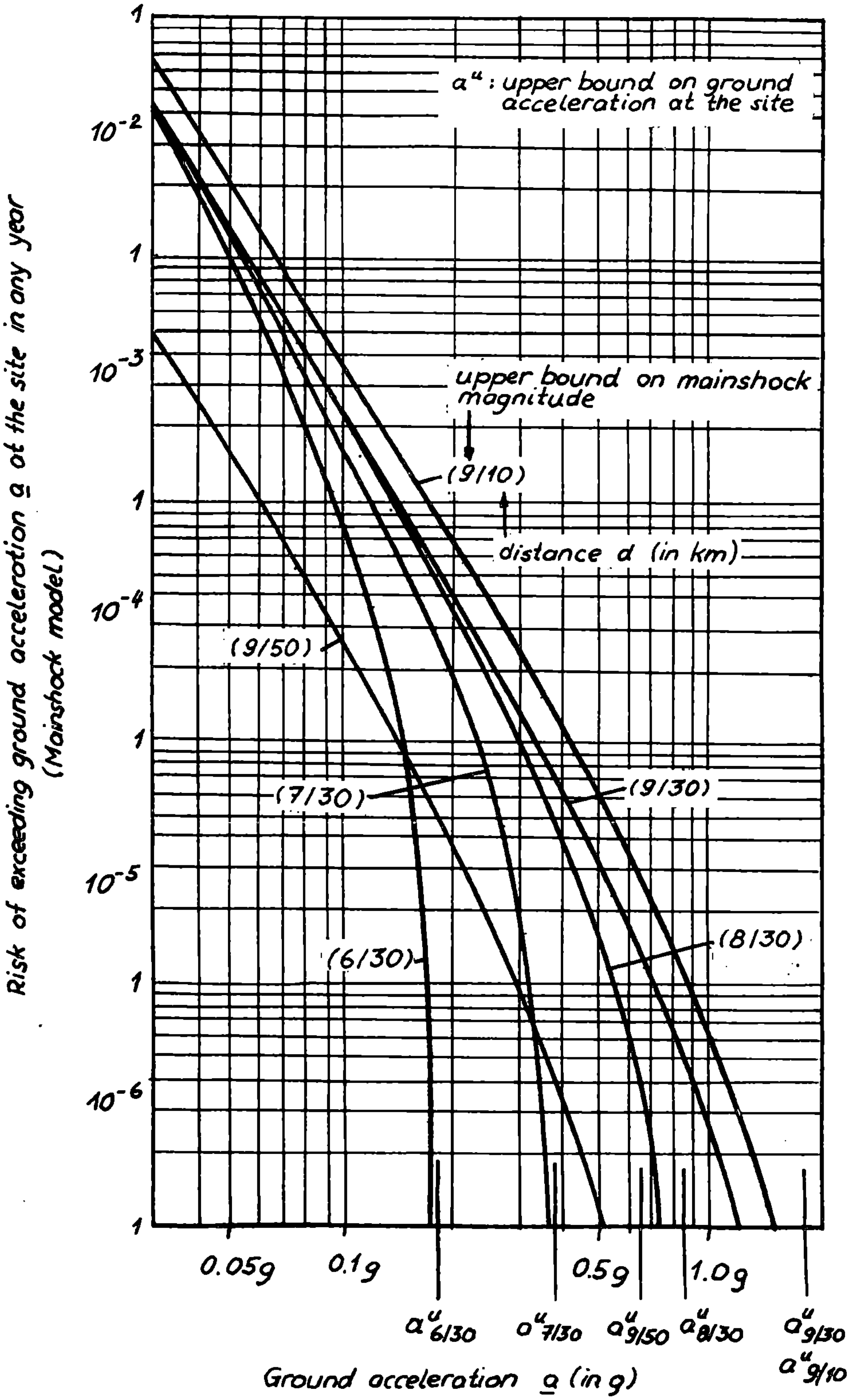

Figure 4-2 Seismic Risk for the "Mainshock" Model

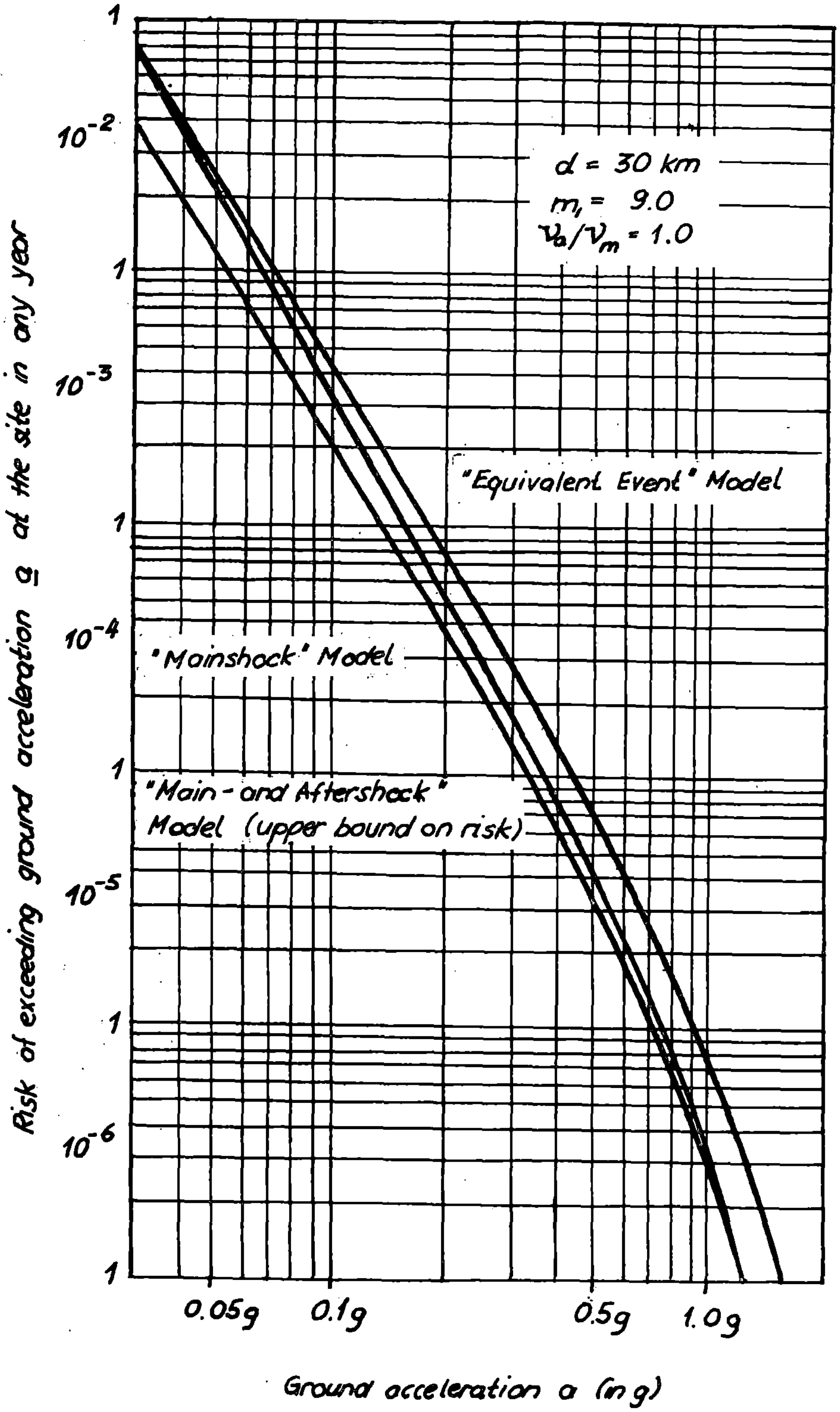

Figure 4-3 Comparison of the "Main- and Aftershock" Model with "Equivalent Event" and "Mainshock" Model for the case (m_1 = 9.0, d = 30 km, ν_a/ν_m = 1.0)

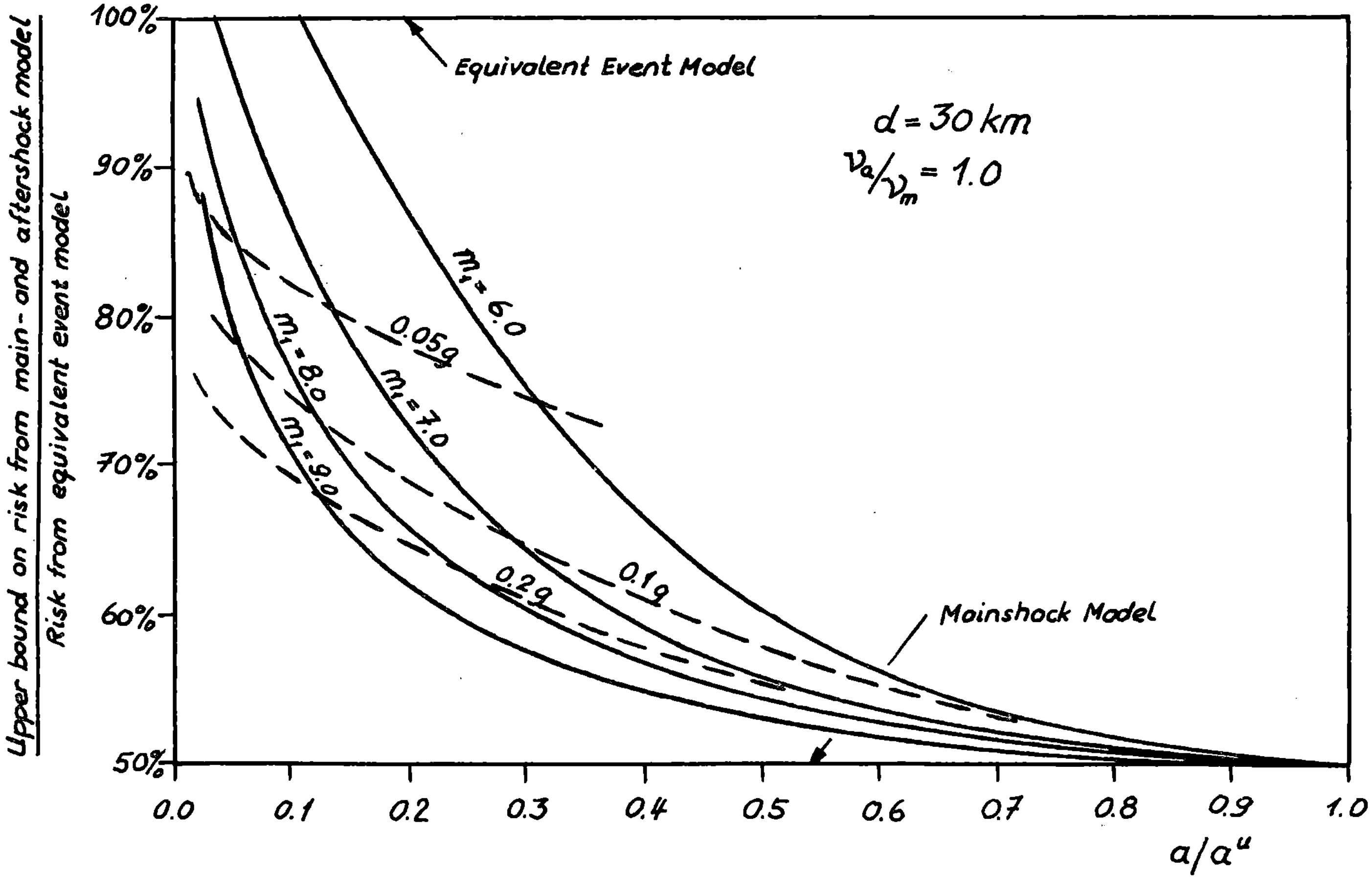

Figure 4-4 Comparison of "Main- and Aftershock" Model with "Equivalent Event" Model and "Mainshock" Model: Variation of Upper Bound on Mainshock Magnitude

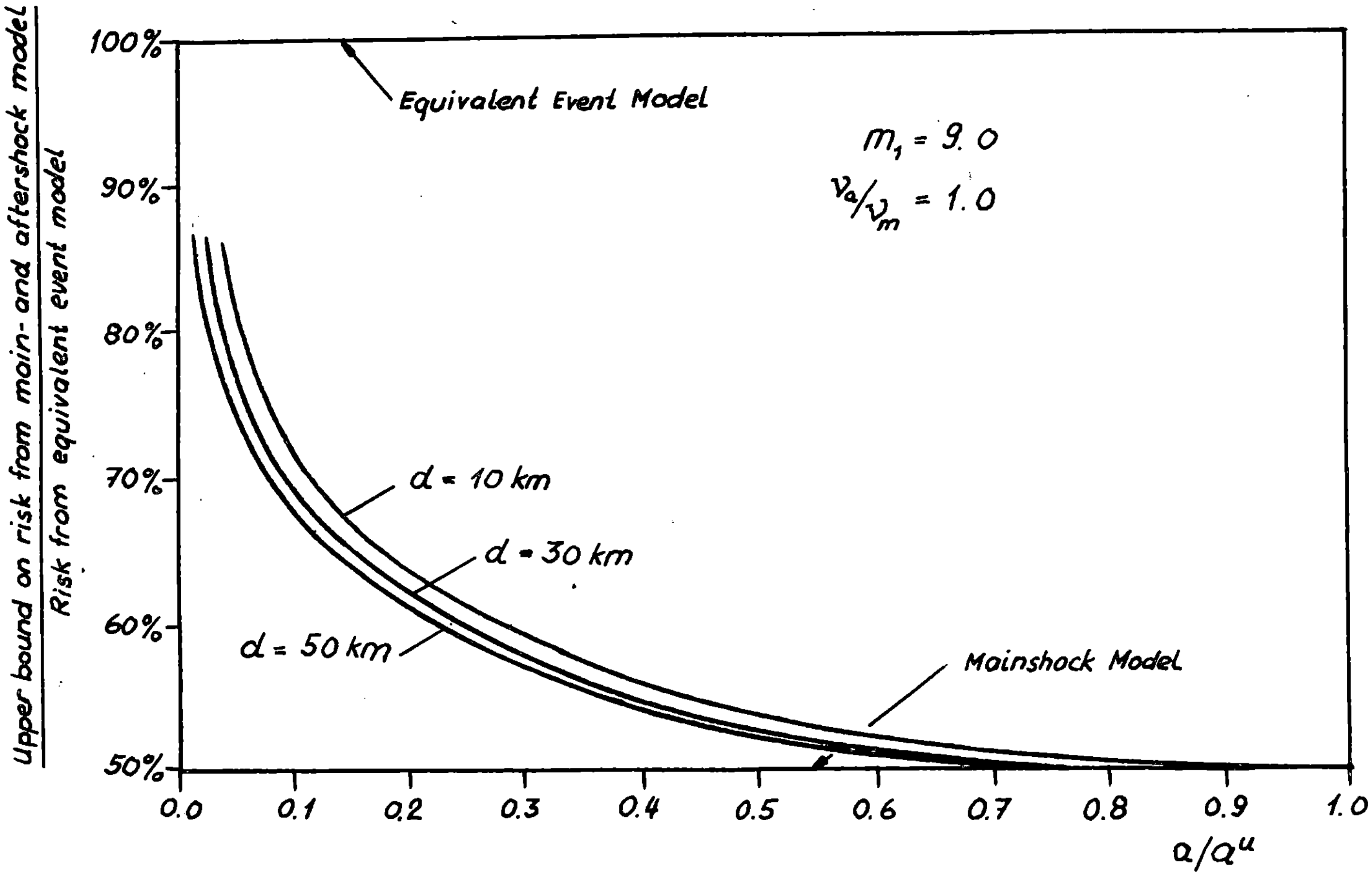

Figure 4-5 Comparison of "Main- and Aftershock" Model with "Equivalent Event" and "Mainshock" Model: Variation of Distance

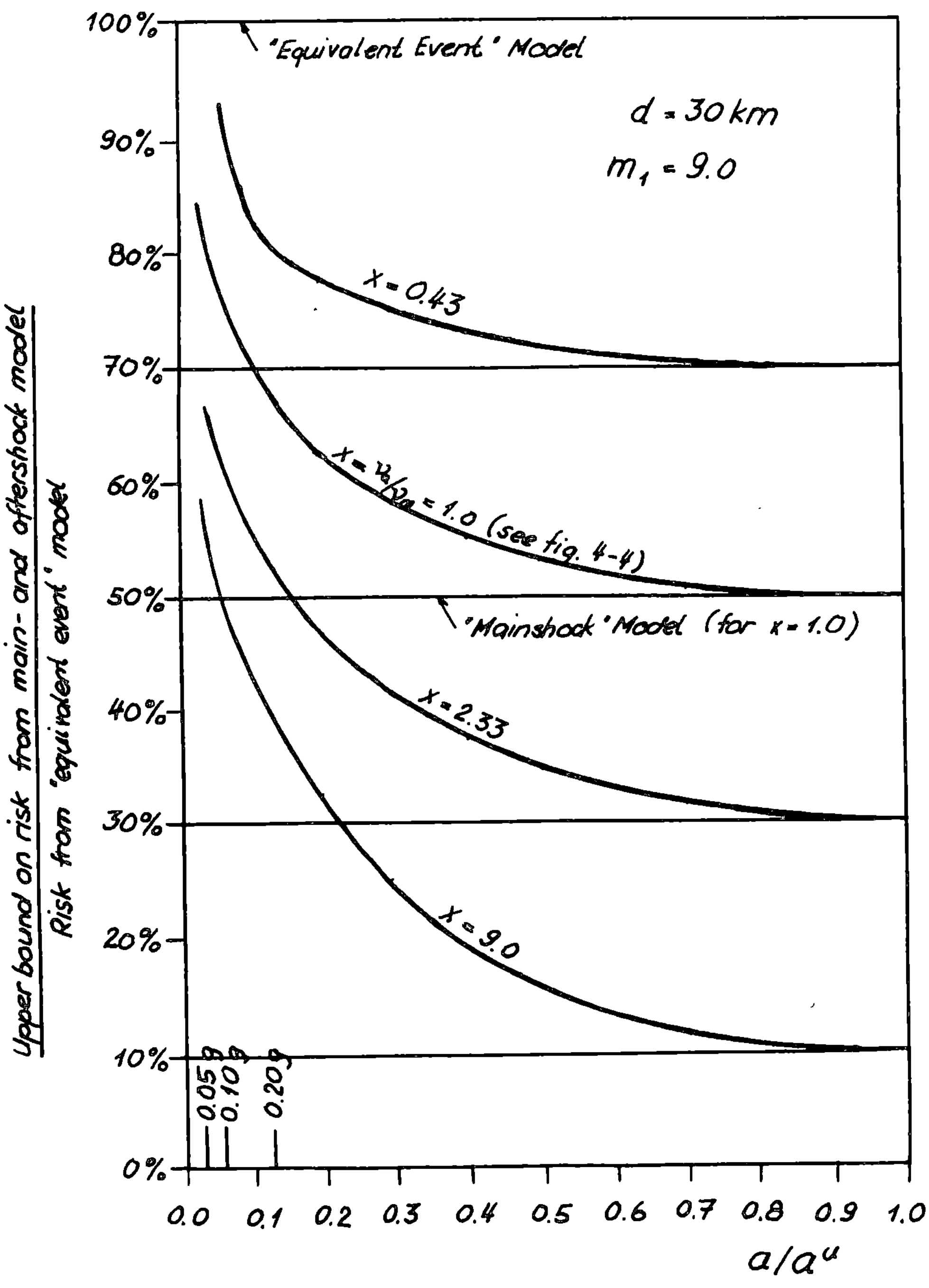

Figure 4-6 Comparison of "Main- and Aftershock" Model with "Equivalent Event" and "Mainshock" Model: Variation of $x = \nu_a/\nu_m$, where $\nu_a = \nu_m$ = Constant

CHAPTER 5

CONCLUSIONS

Seismic risk analysis for main- and aftershocks involves many random variables, which make the analytical expressions and the numerical calculations extremely complicated and the results difficult to anticipate. It is therefore somewhat ambitious to draw conclusions from the experience gained in the development of the analytical expressions for the two models presented in this study, and from the approximate (upper bound) results of examples of the simpler of the two models.

Seismic risk analysis for main- and aftershocks is mathematically much more expensive to carry out than the two other models ("equivalent event" and "mainshock" model), especially when the more realistic "line-area" model is used. However, it is relatively easy to get an upper bound on the risk of the "main- and aftershock" model. The question is, under what circumstances is it worth the effort to gain more "accuracy" in determining the seismic risk at a site. It has to be kept in mind that the presented seismic risk models are based on many assumptions (such as the truncated exponential distribution of the magnitudes, or the assumption that a zone of recent past activity is equally likely to be the source of the next earthquake than a previously active zone which has been relatively quiet for some time), which are simplifications of the actually observed behavior. Furthermore, the values and forms of many parameters and parameter relationships (such as the attenuation "law" or the modified Omori law) are often only "best estimates" and can vary considerably from earthquake to earthquake. It has been found in various applications of seismic risk analysis that the results are relatively insensitive to some of them, but others can easily change the risk values by a factor of 2 (for small ground accelerations, say around 10% of g) to 10 (for large ground accelerations, say 50% of g or more). It is also known that a small, but close earthquake (i.e., a typical situation of aftershocks) produces a different response spectrum for structures than large but more distant earthquakes, a fact which has not yet been included in seismic risk analysis. This critical appraisal of seismic risk analysis is not for the purpose of reducing its value, but of explaining that the results should always be interpreted as "best estimates" with a possible spread around them. Because of the multitude of different parameters, whose values are known with various degrees of certainty, it is difficult to quantify this spread, but for all models a variation of the seismic risk between 50% and 200% of the "best estimate" value should be anticipated.

Five fairly general conclusions can be drawn from the present study of seismic risk analysis for main- and aftershocks:

(1) The relative contribution of aftershocks to the total seismic risk decreases with increasing values of examined seismic intensities at the site.

(2) The relative contribution of aftershocks to the total seismic risk increases with increasing upper bounds on the mainshock magnitudes, which implies that the aftershock risk is mainly associated with large but rare mainshocks which cause a large number of aftershocks and a considerable spread of aftershock epicenters and thus allow considerably shorter distances of certain aftershocks to the site.

(3) The relative contribution of aftershocks to the total seismic risk increases significantly with an increasing ratio of the "average" expected number of aftershocks to the expected number of mainshocks, independently of the conclusions (1) and (2). Thus, the aftershock risk can be an important part of the total seismic risk in regions where mainshocks are usually followed by a large number of aftershocks of significant magnitudes (say 4.0 or larger).

(4) The "equivalent event" model always yields conservative risk estimates, especially for large examined seismic intensities and small upper bounds on the mainshock magnitudes. The "mainshock" model underestimates the risk, but with the exception of small examined seismic intensities, usually comes closer to the "true" results than the "equivalent event" model.

(5) A seismic risk analysis for both main- and aftershocks is complicated and expensive, particularly when exact results are wanted and for models which try to simulate the actual spatial distribution of main- and aftershocks.

With these conclusions and with the previous remarks, it can be stated at the present time that it is, in general, not worth the effort to use a seismic risk model for both main- and aftershocks in regions where the aftershock activity is small and/or only mainshocks of small magnitudes are anticipated in the future, especially when the interest lies in the larger seismic intensities at the site (e.g., ground accelerations above 10% of g). In these cases, a seismic risk analysis with the "mainshock" model is appropriate and gives risk estimates adequate for engineering purposes. However, in regions with a large aftershock activity, it may well be of value to use the more complicated seismic risk analysis for both main- and aftershocks, or at least the method to obtain an upper bound on the "true" risk values. In these cases, the contribution of aftershocks can easily exceed the possible variation in the risk due to uncertainty in the parameter values. Furthermore, it can, in general, be said that if no seismic risk analysis for both main- and aftershocks is available, the "mainshock" model gives "better" (underestimated, however) risk estimates than the "equivalent event" model, particularly for seismic intensities of engineering interest.

It has to be emphasized that these conclusions are still based upon a relatively small experience and are therefore to be used with caution. Only the application of more realistic models such as the "line-area" model, and calculations of the exact (instead of an upper bound) risk can improve and sharpen the conclusions of this study.

As mentioned earlier, future work in this area has to concentrate on the application of the "line-area" model, in order to verify or change the conclusions drawn mainly from a "line-line" model. These findings might indicate whether it is necessary to apply the more complicated formulas for the exact risk values. At the same time, it is necessary to collect data on aftershock sequences (e.g., parameters of the Omori law and the spatial distribution law) in specific earthquake areas such as California. Finally, this work could be extended to the most general "area-area" model, if it should prove important to do so.

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APPENDIX A

CALCULATION OF AN "AVERAGE" EXPECTED NUMBER OF AFTERSHOCKS PER YEAR IN A PARTICULAR EARTHQUAKE SOURCE

As mentioned in Chapter 1, seismologists have established methods and criteria to classify earthquakes as main- or aftershocks. With such an information on the seismic history of an earthquake source, it is possible to determine an average mean number of mainshocks, ν_m, and of aftershocks, ν_a, per year. The values of ν_m and ν_a can change from earthquake source to earthquake source. The values found in the literature for the parameters c and p (and r) in the modified Omori law (Section 2.1.2) which determine most significantly the value of ν_a are very often average values from many earthquake sources together (earthquake region, such as Southern California, for instance). Depending on the location of a structural site, the seismic risk is often determined primarily by nearby earthquake sources only. With an expression for an "average" expected number of aftershocks per year in a particular source, the values of c and p (and eventually r) can be adjusted in order to reflect the past seismic history of this source.

The expected number of aftershocks of magnitudes in excess of m_o^a during t' to T, given a mainshock of magnitude between m_m and $m_m + dm_m$ occurred at time t' to t' + dt', is given by Eq. (33) (Section 3.2.2.2.1).

$$\frac{A}{1-p}\left[(365(T-t')+c)^{1-p}-c^{1-p}\right] \tag{A-1}$$

In order to get a finite number of aftershocks for $T \to \infty$, the value of p has to be larger than 1.

The probability that the mainshock occurs at t' to t' + dt' and has a magnitude between m_m and $m_m + dm_m$ can be written as

$$\frac{dt'}{T} k_{m_1} \bar{\beta} e^{-\bar{\beta}(m_m - m_o^m)} dm_m \tag{A-2}$$

The total expected number of aftershocks in T years, given a mainshock occurs during T years, is obtained by multiplying Eq. (A-1) by Eq. (A-2) and by double integration over all possible mainshock magnitudes from m_o^m to m_1 and all possible times t' from 0 to T, yielding

$$\frac{1}{T}\bar{\beta}\frac{\bar{A}}{1-p} e^{\bar{\beta}(m_o^m - m_o^a)}\left[\frac{(365T+c)^{2-p}-c^{2-p}}{365(2-p)} - Tc^{1-p}\right](m_1 - m_o^m) \tag{A-3}$$

Thus, ν_a is obtained by multiplying Eq. (A-3) by the average mean number of mainshocks ν_m per year.

$$\nu_a = \nu_m \frac{1}{T}\bar{\beta}\frac{\bar{A}}{1-p} e^{\bar{\beta}(m_o^m - m_o^a)}\left[\frac{(365T+c)^{2-p}-c^{2-p}}{365(2-p)} - Tc^{1-p}\right](m_1 - m_o^m) \tag{A-4}$$

For explanation of the parameters and symbols, see Chapters 2 and 3.

If, from the earthquake history of a particular earthquake source, the values of ν_m and ν_a are known, the parameters c and p (and eventually r) can be determined such that Eq. (A-3) is satisfied. Because ν_a is a nonlinear function of T, ν_a should be determined for "large" values of T (say, 10 to 50 years).