

A WAVELET MODEL OF SEISMIC NOISE

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A B S T R A C T

The work described in this thesis was carried out in order to examine the practical consequences arising from the assumption that a seismic trace may be treated as a finite interval of an infinitely long stationary time series. The "Predictive Decomposition Theorem" of Herman Wold states that a general type of stationary time series may be considered to represent the running summation in time of an infinite suite of wavelets, all of the stable shape or form b_s , but of random and uncorrelated arrival times and strengths. An artificial series satisfying the above requirements was generated, and subjected to a number of mathematical tests in order to ascertain some of its statistical and dynamic characteristics. Analyses of a similar nature could be performed on other types of time series that satisfy stationary requirements, such as those that meet the conditions of correlated decomposition.

A C K N O W L E D G E M E N T S

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The author has held a teaching assistantship in the Department of Geology and Geophysics during the fall term 1953/1954, and a research assistantship on the M.I.T. Geophysical Analysis Group (G.A.G.) since February, 1954.

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The major part of this thesis has been published as Section 6 of the M.I.T. Geophysical Analysis Group Report No. 9 (March 15, 1955). Most of the computations required for the work presented in the pages that follow were performed on the M.I.T. Whirlwind High Speed Digital Computer (courtesy of O.N.R.). The programs that have been used are briefly described in the appendix.

I. INTRODUCTION

During the past twenty-five years the seismic method has been used with ever increasing success for the location of sub-surface structural features that may bear oil. In spite of the effectiveness of the technique, however, a vast number of problems that have arisen since seismic prospecting has been used as an exploration tool remain yet to be solved. Progress has proceeded simultaneously along various lines. Our understanding of the nature of propagation of the disturbance caused by the explosion of a charge of dynamite has improved considerably. Field techniques have been refined to a degree where seismograph surveys have been successful in areas where negative results had been achieved several years before. Perhaps the greatest advances have been made in the design and construction of recording equipment. As a matter of fact, instrumentation is probably at a more advanced stage than our understanding of the information that it supplies the interpreter in the office.

Thus we often find that it is impossible to pick reflections on a seismic trace where the noise is of a nature that masks the signal, or reflection. The extraction of the signal from a noisy seismogram is obviously a problem of primary importance. Several alternate methods can be used towards its solution. One, which we may call the deterministic approach, involves writing down the differential

equations governing the behavior of a disturbance as it is propagated through the medium; and finding solutions that satisfy the appropriate boundary conditions; and another, which we may term the probabilistic approach, makes use of concepts from statistics and information theory. In the present investigation we are concerned with the statistical method of attack.

No matter what approach is used, we must begin by making a number of basic assumptions. In particular, we assume that a seismic pulse is propagated through the earth as a wavelet of a certain shape which is dependent on the absorption spectrum of the earth, and that a seismogram consists of a succession of overlapping wavelets which have reached the recording apparatus along various paths of refraction and reflection below the surface of the ground. This theory was first proposed by N. Ricker (1940); the wavelet has come to be known as the "Ricker Wavelet".

Robinson (1954) has postulated a direct connection between this "Wavelet Theory of Seismogram Structure" and the "Predictive Decomposition Theorem" of the statistician H. Wold. The theorem states in essence that a certain type of a stationary time series consists of a dynamical, non-statistical element, a wavelet shape b ; and a random element, a wavelet amplitude factor, ξ_t . In the work that follows, a stationary time series will be considered to be one whose integrated power spectrum is an absolutely continuous

function of frequency. A time series is said to be stationary if the probabilities associated with the series are not identified with any particular origin in time. Such a stationary process meets the requirements of the Predictive Decomposition Theorem of Wold.

The running summation in time of the wavelet, where the wavelet shape b is successively weighted by a different amplitude factor ξ_t , generates a stationary time series x_t . Robinson (1954) has shown that if a seismic trace is indeed describable by Ricker's theory, then it may be treated as a finite piece of an infinitely long stationary time series. In particular, the Ricker wavelet becomes the wavelet shape b in this case.

In order to examine these concepts experimentally, an artificial series satisfying the requirements of the Predictive Decomposition Theorem was generated.

The Ricker velocity type wavelet at infinity, $V(u|\infty)$, was selected as the wavelet shape b_s , while the random strengths, or amplitude factors, were drawn from a suitable Normal Distribution Function carefully tested for true randomness. Normalized autocorrelation functions, power spectra, and variances were computed for various sampling intervals of the generated series. These calculations were effected in order to secure a good picture of the statistical characteristics of the generated artificial time series. Provided that a seismic trace may indeed be

described as a short interval of a stationary time series, the investigation presented in the following pages may serve to enhance our knowledge of the statistical and mathematical behaviour of a true seismogram.

We thus hope to provide an artificial series satisfying the criteria of a stationary process. The series generated in this manner will be useful for future controlled experiments requiring the use of a truly stationary series sample.

II. THEORY

The Wavelet Theory of Seismogram Structure.

A seismic trace may be considered to represent the response of a system composed of the medium and the seismometers to the excitation, the detonation of a charge of dynamite. The wavelet theory of seismogram structure, proposed by N. Ricker in a series of papers (1940, 1941, 1943, 1944, 1945, 1949, 1953; a,b) holds that each trace of a seismic record may be considered to represent the result of the running addition along the time axis of a wavelet of constant shape but varying amplitudes and times of arrival. According to the laws of wave propagation in a homogeneous, isotropic medium, a disturbance remains unchanged in shape as it is transmitted through this medium. Under these ideal conditions an initial impulsive disturbance representing the explosion will be propagated in a completely unaltered

manner and received by the recording equipment in the form of an impulse. As this disturbance travels below the surface of the earth, it will suffer a certain number of reflections and refractions, but these will not alter the shape of the original pulse. It is assumed here that we are dealing with perfect plane reflectors, and homogeneous, elastic, and isotropic media of propagation. This idealized seismic trace will thus consist of a series of impulses of varying amplitudes and times of arrival, where each impulse represents a reflection or refraction of energy from the disturbance from a layer at depth, (see Figure 1). Alternatively, if it is assumed that the explosion can be mathematically represented by a doublet, then the resulting seismogram should consist of a series of doublets whose varying amplitudes and times of arrival are again a result of refractions and reflections at depth.

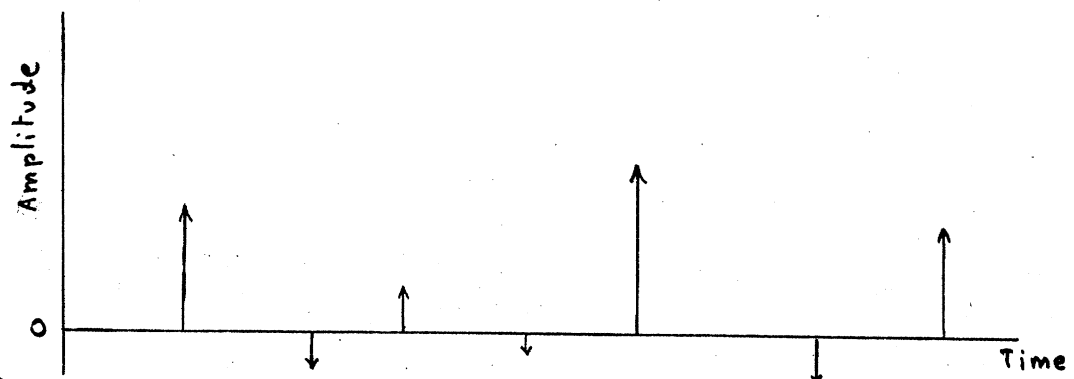


Figure 1. An ideal seismogram consists of a series of sharp spikes of varying amplitudes and times of arrival. Each spike represents the arrival of reflected or refracted energy propagated from the site of the explosion.

If the earth would satisfy the ideal elastic wave equations,

the process of picking reflections would be relatively simple, provided that the sub-surface geometry remains reasonably uncomplicated. It is of course an established fact that actual seismograms do not at all resemble the idealizations just described. This is a direct consequence of the fact that the earth is not a perfectly elastic and isotropic medium and does therefore not satisfy ideal elastic theory. However, the original assertion can be somewhat modified, and it may now be stated that the disturbance is propagated in the form of a wavelet whose shape is determined by the absorption spectrum of the earth for elastic waves. A seismic trace is now defined to consist of a running summation in time of a large number of these wavelets, whose shape is constant for a given disturbance but whose amplitudes and instants of arrival vary in time. These concepts may be worded in a somewhat different form as follows: a seismic trace consists of a dynamical element, the shape of the wavelet, and of two continually varying elements, the amplitudes and times of arrival of these same wavelets.

The Statistical Determination of Wavelets

One possible method of approach that might be employed in order to extract useful information from a seismogram is to establish a process that effects a separation of the dynamical element in a seismic trace, the wavelet shape, from the two continually varying elements,

the amplitudes and times of arrival of these wavelets. Let us thus consider the shape of a seismic wavelet to represent the response of the medium to the original excitation caused by the explosion. Let us assume also that the shape of this wavelet is physically stable, that is, that its representation in the form of a set of discrete ordinates is the solution of a stable difference equation (Robinson, 1954).

In order that the problem may be tested statistically, it must be assumed that it is not possible to predict the arrival times and amplitudes of one wavelet from knowledge of the arrival times and amplitudes of another wavelet. Using the notation of Robinson (1954), let us designate the strength, or amplitude of the wavelet arriving at time t by the symbol ξ_t . Thus ξ_t is a constant quantity for a given wavelet arriving at time t . Specifically, if the discrete set of ordinates that describe the shape of a wavelet arriving at a time t is called $(b_0, b_1, b_2, \dots, b_s)$, then the actual magnitude of this wavelet will be described by the ordinates $(\xi_t b_0, \xi_t b_1, \xi_t b_2, \dots, \xi_t b_s)$, (See Figure 2). ξ_t may be called the strength, or weighting factor of the wavelet b_s arriving at time t . From the foregoing discussion it becomes evident that the quantities ξ_t can be considered to represent the sharp impulsive disturbances propagated through the medium, whose response is given by the wavelet b_s which arrives at time t . Thus each wavelet forming part of

a seismic trace represents the response of the earth to an impulse that has reached the seismometer by a direct, a refracted, or a reflected path.

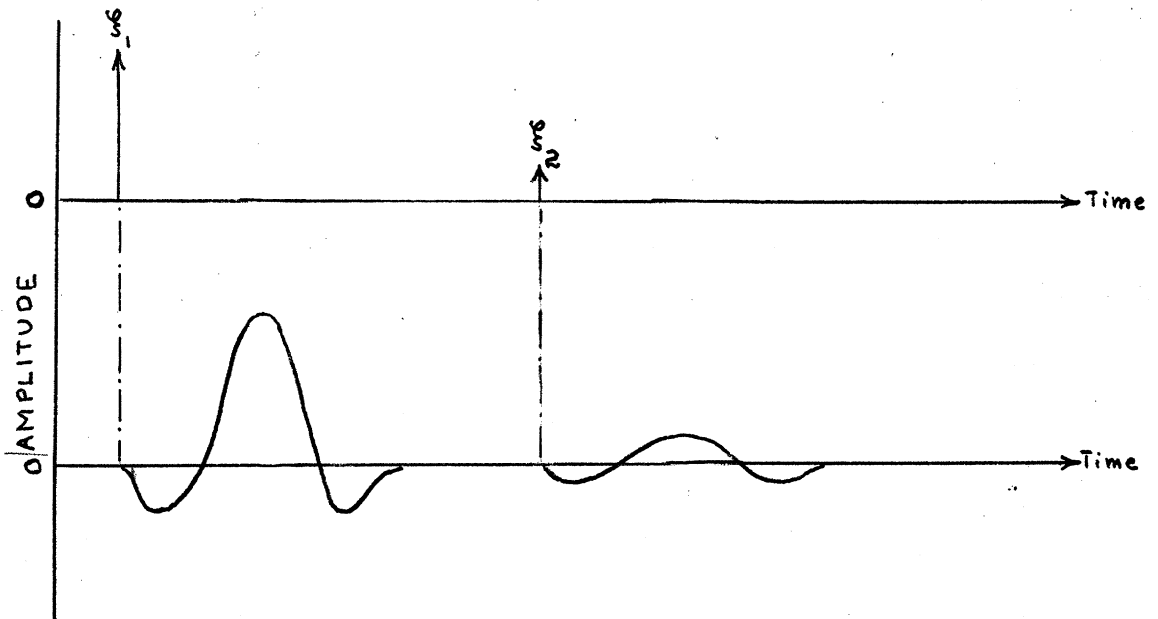


Figure 2. The amplitude of a wavelet arriving at time t is determined by the strength of the weighting factor .

For the purposes of the present investigation, the quantities ξ_t are assumed to be "random variables". Robinson (1954) explains that the use of the term "random variable ξ_t " does not mean that the values of these quantities are completely uncertain, since they are fixed by the geologic characteristics of the medium that the disturbance traverses. In spite of this apparently negative factor, it turns out that one cannot generally predict in advance the expected arrival time and strength of a seismic wavelet before the trace is actually recorded. Such incomplete information can therefore still be tested from the statistical point of view.

The sense in which the \mathcal{Y}_t are assumed to be random is that they are uncorrelated. This does not necessarily imply that they are independent; we merely assume that

$$E(\mathcal{Y}_t \mathcal{Y}_{t+\tau}) = 0 \quad \tau \neq 0 \quad (1)$$

where E is the expectation symbol.

We return again to a seismic trace x_t , which is the result of performing a running summation in time of a stable wavelet of shape b_s and random amplitudes \mathcal{Y}_t . Mathematically, this situation may be expressed as

$$x_t = \sum_{s=0}^{\infty} b_s \mathcal{Y}_{t-s} \quad -\infty < t < +\infty \quad (2)$$

if it be assumed that the above equation holds for all time between minus and plus infinity. It turns out (Robinson, 1954) that equation (2) is the mathematical statement of the Predictive Decomposition Theorem, due to the Swedish statistician Herman Wold, for a stationary time series with an absolutely continuous spectral distribution function. Consequently, if a seismic trace is indeed composed of wavelets of random, uncorrelated strengths and arrival times, then the trace may for the purposes of the present mathematical analysis be considered to be a stationary time series. It is next assumed that our seismic trace is of a length sufficient to warrant the application of concepts from time series analysis to its study. The validity of this assumption is discussed by Wadsworth et al (1953) and Smith, (1954).

The Structure of a Stationary Series

A stationary process possessing an absolutely continuous spectral distribution function may be adequately described by the Predictive Decomposition Theorem of Wold, (1938). The theorem states in essence that such a stationary time series is separable into two components—a dynamical, stable element b , and a random element ξ_t .

Let us assume a stationary process x_t , where $-\infty < t < +\infty$, which has an absolutely continuous spectral distribution. Then the process may be described by the decomposition

$$x_t = b_0 \xi_t + b_1 \xi_{t-1} + b_2 \xi_{t-2} + \dots + b_s \xi_{t-s} + \dots \quad (3)$$

or,

$$x_t = \sum_{s=0}^{\infty} b_s \xi_{t-s} \quad -\infty < t < \infty \quad (4)$$

In the above expression t is the discrete time parameter. The random elements are linear in $x_t, x_{t-1}, x_{t-2}, \dots$, have zero mean, $E(\xi_t) = 0$, and are mutually uncorrelated, $E(\xi_t \xi_{t+\tau}) = 0$ for $\tau \neq 0$ (Robinson, 1954).

The wavelet theory of seismogram structure, proposed by N. Ricker in a series of papers (1940, 1943, 1945) holds that each trace of a seismic record may be considered to represent the result of the running addition along the time axis of a wavelet of constant shape but varying amplitudes and times of arrival. If damping be neglected,

(which is a reasonable assumption to make, since damping effects become negligible in the region not in the immediate vicinity of the shotpoint), an initial impulsive disturbance representing the explosion will be propagated through the earth in the form of a wavelet whose shape is determined exclusively by the earth's absorption spectrum. The shape of this wavelet remains unaltered during its entire refraction or reflection path from the explosion site to the recording geophone. However, the strengths and times of arrival of each wavelet at the recording instrument are random and uncorrelated.

If we accept the assumptions stated above, it becomes possible to describe a seismic trace meeting Ricker's criteria by Wold's Predictive Decomposition Theorem. We may then assume that our seismic trace is a stationary process, and that it is therefore completely described by equation (4) in the limit.

In order to effect the separation of the random from the non-random, dynamic b_s 's, it becomes necessary to find a method to average out the random components ξ_t , so that the result of such an operation on the time series be the stable wavelet shape b . Once this wavelet shape has been found, it must next be removed from the trace; when this has been accomplished, there should remain the residual, random components ξ_t , which correspond to the prediction errors of Wadsworth et al (1953). For a more detailed

treatment of this problem, the reader is referred to Robinson, (1954).

Robinson shows that a purely random and uncorrelated process has a white-light power spectrum and autocorrelation coefficients which vanish except for the zeroth lag. This leads to the result that the autocorrelation of a stationary time series that satisfies predictive decomposition will approach the autocorrelation of the generating wavelet b_s in the limit. Such an autocorrelating process effectively averages out the random, non-dynamical element ξ_t .

The Fourier Transform of the autocorrelation function of the series x_t , $\phi_{xx}(\tau)$, is

$$\sum_{\tau=-\infty}^{\infty} \phi_{xx}(\tau) e^{-i\omega\tau} = \bar{\Phi}_x(\omega) \equiv \bar{\Phi}_b(\omega) \quad (5)$$

where $\bar{\Phi}_b(\omega)$ is the power spectrum of not only the time series x_t , but also of the wavelet b , again provided that the ξ_t 's be mutually uncorrelated random variables. The Fourier Transform of the wavelet b_s can be written as

$$B(\omega) = \sum_{t=0}^{\infty} b_s e^{-i\omega t} \quad (6)$$

and it can further be shown that

$$|B(\omega)|^2 = \bar{\Phi}_x(\omega) \quad (7)$$

that is, that the square of the absolute value of the wavelet amplitude spectrum is equal to the power spectrum

of the time series.

It is finally desirable to gain an idea of the rough measure of spread of the series ordinates x_t . For this purpose we introduce the concept of variance. Let X be a random variable assuming the values $x_1, x_2, x_3, \dots, x_k$ with corresponding probabilities $f(x_1), f(x_2), \dots, f(x_k)$. The mean value, or expectation of X is defined by

$$E(X) = \sum_{j=1}^k x_j f(x_j) = \mu \quad (8)$$

provided that this series converges absolutely.

The quantity

$$E(X^r) = \sum_{j=1}^k x_j^r f(x_j), \quad (9)$$

where r is an integer equal to or greater than zero is known as the r^{th} moment of X about the origin. In particular, if $r=2$, the resulting quantity $E(X^2)$ is called the second moment of X about the origin. Its existence is a sufficient condition for the existence of the mean,

$$\mu = E(x) \quad (10)$$

Introducing the quantity $X - \mu$, the deviation from the mean, we may write

$$E[(X - \mu)^2] = E(X^2) - \mu^2 = \text{Var}(X) \quad (11)$$

Either the first or the second member of (11) defines the variance of the variables X.

III.- DESCRIPTION OF EXPERIMENT

Choice of Wavelet

N. Ricker (1941) presents the theoretical derivation of the wavelet shapes that should be observable in a visco-elastic medium at various relative distances from the site of the dynamite detonation. He gives two families of such wavelets, one of which includes those that are recorded by a simple displacement type seismometer, while the second includes wavelets that a velocity type seismometer would record. These velocity type wavelets are of course merely the time derivatives of those of the displacement type. Ricker's notation,

$U(u|X)$ displacement type wavelet,

$V(u|X)$ velocity type wavelet,

where X is the relative distance from the explosion site, will be used here in a similar fashion. From Ricker's experimental work in the Pierre shale of Colorado it is evident that all wavelets $V(u|X)$ occur so near the shot-point that they can, if at all, only be recorded within the first few milliseconds time interval of a seismogram. It is therefore the Ricker wavelet "at infinity", $V(u|\infty)$ or $U(u|\infty)$, (which is symmetric), that is of interest here.

For this reason the wavelet selected for the generation

of our artificial seismogram was Ricker's $V(u|\infty)$, whose discrete numerical ordinate values are given in the appendix of his 1945 paper. The velocity type wavelet was chosen rather than the corresponding displacement type since most currently used seismometers are of the velocity class.

Generation of Stationary Series

It is recalled that the time series x_t composed of a stable wavelet shape b_s and random amplitude components ξ_t can be written as

$$x_t = \sum_{s=0}^{\infty} b_s \xi_{t-s} \quad -\infty < t < \infty \quad (12)$$

Since a seismogram is not recorded for an infinite length of time after the detonation of the charge, equation (12) is replaced by the expression

$$x_t = (x_0, x_1, \dots, x_T) = \sum_{s=0}^T b_s \xi_{t-s} \quad 0 < t < M \quad (13)$$

If the length of the resulting series is sufficiently great, the application of analysis that is only justified for equation (12) in a very strict sense can also be employed in the case of (13). In order to illustrate the summation procedure, the series arising from the running addition of three wavelets of stable shape b , where the wavelet b is displaced three times along the time axis, multiplied each time by a weighting factor ξ_t , and summed over overlapping portions of adjacent wavelets, is now examined.

The weighting factors γ_t were selected as entries 11,000 through 12,450 of Wold's (1948) random number deviates. These random deviates were constructed to fit a Normal Distribution Function and carefully tested for true randomness by the author of the tables. Both the wavelet ordinates and Wold's random numbers were scale-factored in order to permit their convenient use in a program for the M.I.T. Whirlwind High-Speed Digital Computer, where most of the calculations presented in this thesis were carried out.

The series computed in this manner and the generating wavelet are shown in Figure 19, and the ordinate values x_1 are presented in Table I. The generated series consists of 1407 discrete ordinates; the generating wavelet consists of 52 discrete ordinates. The series, composed of a dynamic element b_s and a random element γ_t , should thus conform to the requirements of the Predictive Decomposition Theorem of Wold. It now becomes necessary to subject this series to the mathematical analysis outlined previously in order to discover how well it actually does satisfy the requirements of this theorem.

Computation

In order to gain an idea of the spread of the magnitudes of the series ordinates x_1 , the running variances of the series for interval lengths of 5, 10, 20, 40, 80, and 160 ordinates were computed. These results are shown

graphically in Figure 17.

The normalized Wiener autocorrelation function is given by

$$\phi_{xx}(\tau) = \frac{\sum_{i=N}^{N+n-1-\tau} (x_i - \bar{x})(x_{i+\tau} - \bar{x})}{\sum_{i=N}^{N+n-1} (x_i - \bar{x})^2} \quad \tau = 0, 1, \dots, M \quad (17)$$

where

$$\bar{x} = \frac{1}{n} \sum_{i=N}^{N+n-1} x_i$$

Here x_1 is the value of an ordinate of the series to be autocorrelated, \bar{x} the mean of this series, N the index number of the first x_1 that is used in the process, n the number of x_1 's used, and M the highest lag desired. The function $\phi_{xx}(\tau)$ was computed for the generated series and random numbers \mathcal{V}_t over a variety of interval lengths and positions. The intervals are specified in Figures 5 through 10, where the corresponding graphs of $\phi_{xx}(\tau)$ are presented.

The power spectrum, or cosine transform

$$\Phi_x(\omega) = \frac{\phi_{xx}(0)}{2} + \sum_{\tau=1}^M \phi_{xx}(\tau) \cos \omega h \tau \quad (18)$$

where $\phi_{xx}(\tau)$ is the autocorrelation function defined in equation (17), ω the frequency in radians/sec., and

h the spacing of the series ordinates in milliseconds, was computed for the autocorrelations of Figures 5 through 10. These spectra are shown in Figures 11 through 13.

Smoothing of power spectra was effected by application of the Tukey-Hamming formulae, (See Ref.):

$$\begin{aligned}
 U_0 &= 0.54 L_0 + 0.46 L_1 \\
 U_1 &= 0.54 L_1 + 0.23 (L_0 + L_2) \\
 U_2 &= 0.54 L_2 + 0.23 (L_1 + L_3) \\
 &\dots\dots\dots \\
 U_{m-1} &= 0.54 L_{m-1} + 0.23 (L_{m-2} + L_m) \\
 U_m &= 0.54 L_m + 0.46 L_{m-1}
 \end{aligned}
 \tag{19}$$

where $L_0 \dots\dots\dots L_{m-1}$ are the unsmoothed, and $U_0 \dots\dots U_m$ the smoothed spectral line powers.

IV. EVALUATION

An examination of the autocorrelation functions $\phi_{xx}(\tau)$ as shown in Figures 5 through 10 reveals a considerable similarity of each $\phi_{xx}(\tau)$ with the autocorrelation function $\phi_{ww}(\tau)$ of the Ricker wavelet, $V(u|\infty)$. $\phi_{ww}(\tau)$ is shown at the top of each suite of autocorrelation curves for purposes of comparison. The shapes of the functions through approximately the twentieth lag ($\tau=20$) are almost identical within the limits of the approximation made here i.e., the assumption that a sufficiently long finite interval of a series generated by the method described previously can be treated as a stationary process. The autocorrelation functions invariably lose most of their power after the

twentieth lag. Figure 10 presents normalized Wiener autocorrelation curves of the random numbers y_t for five different sampling intervals.

In the ideal case the autocorrelation of a series of random numbers should be a spike at $\tau=0$, since all lags beyond the zeroth vanish. The computed curves approximately confirm this theoretical result. The deviations may be employed as a measure for the randomness of the numbers used for the generation of our series. It is seen that the "noise" beyond the first three or four lags is the least for the largest sampling intervals $i=0-800$ and $i=52-1152$; this result is to be expected since the agreement with theory should be best for the largest sampling interval taken.

The convolution of the autocorrelation function of the generating wavelet $V(u|\infty)$, $\phi_{ww}(\tau)$, with the autocorrelation function of the random number set for sampling interval I , $\phi_{y_I}(\tau)$ should approximate the autocorrelation $\phi_{x_I}(\tau)$ of the generated series x_I for the same sampling interval. In other words,

$$\phi_{x_I}(\tau) \approx \phi_{ww}(\tau) * \phi_{y_I}(\tau) \quad (20)$$

This result is clearly shown in the curves presented, and the rapid decay of power of the autocorrelation function for the various sampling intervals of the series x_I is due

to the spikelike shapes of the $\varphi_{\mathcal{Y}\mathcal{Y}_I}(\tau)$'s. We may look at this matter from a slightly different angle. The wavelet shape b_s used in the generation of the artificial series x_i consisted of 52 discrete ordinate values. Consequently, the autocorrelation function of any series generated from this particular b_s by our process should tend to zero after the 52nd. lag because all further contributions to $\varphi_{xxI}(\tau)$ beyond this point should be due to the random amplitude factors \mathcal{Y}_t , whose autocorrelation, $\varphi_{\mathcal{Y}\mathcal{Y}_I}(\tau)$, has been shown to approach a spike at the origin, ($\tau=0$).

The power spectra $\bar{\Phi}_x(\omega)$ were computed from all the autocorrelation functions previously calculated. The spectra shown in Figures 11 to 13 do not denote much uniformity individually, although they possess most of their power within the same general frequency range. Figure 20 shows the smoothed power spectrum of the generating Ricker wavelet, $V(u|\infty)$, compared to the average of smoothed spectra of the intervals $i=0-225$; $225-450$; and $450-675$. We see from Figure 20 that although individual spectra are not generally too similar to the generating wavelet spectrum in most cases, their average does show good agreement with the spectrum of the generating wavelet. In other words, the spectral estimates are confined to the expected frequency range. Oscillation about the true wavelet spectrum occurs randomly, so that the averaged spectrum shows better agreement with the wavelet spectrum than do the individual

spectra.

Figure 15 shows the proper spectra for the auto-correlation functions of various sampling lengths of the random numbers ξ_t . It is seen that the distribution of power is reasonably uniform at all frequency ranges, so that the theoretical result which predicts a white light spectrum for a suite of mutually uncorrelated random variables ξ_t is upheld within the limits of our approximation. A more constant spectrum for the entire frequency range would have been secured for greater sampling intervals than the ones that have been selected for the experiment described here.

The broken curve of Figure 14 presents the result of a multiplication of the computed unsmoothed power spectrum of the first $1100 \xi_t$ employed for the generation of the series by the unsmoothed power spectrum of the Ricker wavelet, $V(u | \infty)$. For purposes of comparison, the unsmoothed power spectrum computed for the corresponding first $1100 x_1$'s of the generated series is shown on the same graph. The similarity of these curves is evident, since both peak with reasonably similar amplitudes at $n=4, 6, \text{ and } 8$. In order to see why this result should be expected from theory, we may take the Fourier Transform of both sides of equation (20) so that

$$\Phi_{x_1}(\omega) \approx \Phi_w(\omega) \Phi_{\xi_1}(\omega) \quad (21)$$

where $\Phi_{x_I}(\omega)$ is the power spectrum of the series of interval I, $\Phi_w(\omega)$ the power spectrum of the wavelet $V(u|oo)$, and $\Phi_{\xi_I}(\omega)$ the power spectrum of the random numbers ξ_t used for the generation of the interval I of the series x_1 . Thus we have experimentally confirmed the theoretical result predicted by equation (21). Figure 17 presents variance curves computed from the generated series for sampling intervals of 5, 10, 20, 40, 80, and 160 x_1 's. Figure 18 presents variance curves computed for an actual noisy seismogram. A comparison of these curves with those computed for the artificial series reveals a similarity of general behaviour which suggests that our generated artificial stationary series will in fact simulate characteristics of a true seismogram.

Figure 16 presents a comparison between the spectral estimates on a "noisy" seismogram and on the stationary series generated in this thesis. The variability of these spectral estimates seems to be rather similar in both instances. This fact would serve to indicate that Predictive Decomposition is applicable to both situations in this case. It should be clearly emphasized, however, that such an analysis is only warranted when the seismic trace satisfies the conditions imposed by Predictive Decomposition; the technique must be suitably modified for other situations that might presumably arise. Thus a trace whose noise is not completely random, but which

rather involves a definitely non-random, deterministic element (such as correlated wavelet strengths) may be treated by correlative decomposition techniques. These are described in greater detail in M.I.T. G.A.G. Report No. 9 (March 15, 1955).

V.- CONCLUSIONS

We have generated an artificial series by a mechanism suggested by the "Predictive Decomposition Theorem" of H. Wold, and we have computed normalized Wiener autocorrelation functions, power spectra, and variances for various sampling intervals of this series. If we make the assumption that a seismic trace may be treated as a finite segment of a stationary time series which consists of a dynamic element b_s and a random element \mathcal{Y}_t , then the behavior of the autocorrelation functions, power spectra, and variances computed for our artificial case should be a reasonable indication of the corresponding behavior of these entities in an actual seismogram.

It should be emphasized that the work presented in this thesis is the result of an experiment performed on one series generated with the use of one particular wavelet shape b_s and a suite of random variables chosen from one particular distribution function. It does therefore not necessarily follow that the parameters that have been investigated here will show identical behavior

if the series is generated with other wavelet shapes and use is made of random numbers selected from a different distribution function.

The series generation procedure outlined in the preceding pages may be generalized to take into account not only random amplitudes of the wavelet b_s , but also random arrival times. While such random arrivals and strengths might well be drawn from different distribution functions, it is possible to devise one such distribution function from which both random amplitudes and arrival times of the wavelet b_s may be drawn. Let us divide our time axis into equal intervals of length h , and require that at each point $t_1 \dots t_s \dots t_n$ (Figure 3) there

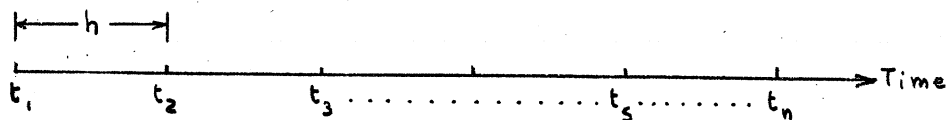


Figure 3. The time axis of a seismic trace is divided into equal intervals of length h .

exist a finite, none-zero probability that no wavelet arrive at time t_s —in other words, that the amplitude factor be of zero magnitude at time t_s .

Such conditions are satisfied by a distribution function illustrated in Figure 4, where a spike of infinitesimal width but finite area A is added to a normal distribution function at the origin. If the area under the normal curve is taken to be unity, then the probability

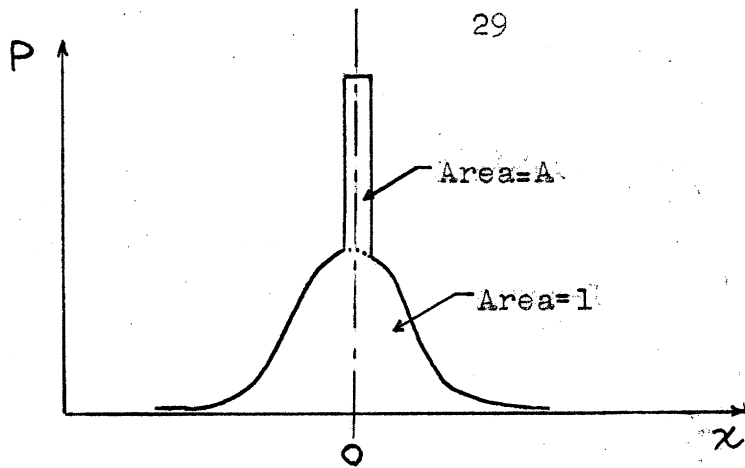


Figure 4. Distribution Function from which both the random amplitudes and arrival times of the wavelet may be drawn.

of zero wavelet amplitude at time t_s is given by

$$P(0, t_s) = \frac{A}{A+1} \quad (22)$$

A process of this nature may be shown to lead to a Poisson type distribution function in the arrival times.

Additional computations of this type should most certainly be carried out in the future, so that the results from such calculations can be extensively compared with autocorrelations, spectra, variances, and other statistical criteria found for actual seismograms. In particular, it might well be advisable to generalize the series generating procedure in the sense outlined above.

Such a series might perhaps be found to be an even more satisfactory tool in our understanding of the statistical characteristics of a seismic trace.

TABLE I
ORDINATES x_i OF GENERATED SERIES

i	X_i							
	-2966	-1055	-10917	-10569	-9870	-8400	-6888	-4904
	+4079	+3665	+955	+2422	+3405	+4127	+4538	+4082
	+2888	+3374	+3262	+3099	+2579	+2792	+2967	+3152
30	+698	-239	+3459	+3155	+3014	+2440	+2276	+1499
	-7007	-6700	-1504	-2426	-3677	-5241	-5797	-6596
	+4507	+5915	-6017	-4713	-3354	-1472	+277	+2409
	+4109	+2386	+6929	+8016	+8329	+7758	+7048	+5612
70	-3320	-2441	+452	-790	-2266	-3262	-3268	-3405
	-439	-586	-1949	-1245	-145	-346	-190	+181
	-2395	-2069	-1199	-1846	-1888	-2261	-2256	-2339
	+1254	+1695	-2019	-1503	-909	-710	-124	+425
	-1904	-2555	+1905	+2030	+1980	+1231	+642	-760
	-969	+219	-3486	-3957	-3980	-3599	-3020	-1795
110	+1643	+1630	+897	+1774	+2180	+2381	+2456	+1953
	+1643	+1785	+1455	+983	+1332	+1340	+1219	+1507
	+974	+904	+1787	+1661	+1695	+1467	+1266	+1221
	+123	+237	+698	+464	+407	+105	-49	-113
	+1000	+1043	+499	+1083	+1308	+1145	+1569	+1521
150	-1343	-1464	+642	+187	-43	-392	-801	-1028
	-2545	-2335	-1620	-1837	-2250	-2167	-2222	-2695
	-961	-909	-2187	-2015	-1731	-1314	-947	-1037
	-928	-140	-1196	-1598	-1737	-1725	-1900	-1630
	+4995	+4810	+673	+1528	+2598	+3519	+3941	+4460
190	-1772	-2989	+4327	+3852	+2776	+1864	+561	-955
	-1679	-509	-3747	-3931	-4092	-4029	-3496	-2557
	+2220	+2017	+447	+1284	+1862	+2169	+2360	+2575
	+127	+303	+1732	+1604	+970	+674	+545	+345
230	+3842	+4174	+691	+791	+1291	+1822	+2864	+3267
	-578	-1391	+4114	+3951	+3241	+2633	+1683	+515
	-4917	-5031	-2251	-3300	-3537	-4132	-4363	-4598
	-5858	-5238	-5345	-5415	-6111	-6120	-6290	-6142
	+4336	+5421	-4554	-3608	-2222	-638	+905	+2800
270	+2397	+1330	+6513	+6575	+6808	+6002	+5052	+3799
	+599	+1803	+50	-781	-1041	-1093	-1026	-143
	+7385	+6719	+3421	+4529	+5629	+6745	+7281	+7510
	-4765	-6158	+5482	+3932	+2635	+572	-1327	-3080
	-3388	-2156	-7082	-7509	-7429	-6789	-5851	-4744
	-1914	-2447	-1321	-449	-22	-147	-323	-965
310	-1403	-176	-3242	-3960	-4415	-4145	-3769	-2642
	+6901	+6512	+1606	+3184	+4662	+5549	+6498	+6855
	+385	-247	+5909	+5060	+4089	+3196	+2038	+1200
	-519	-603	-428	-484	-808	-650	-452	-284
350	-4130	-3974	-1199	-1888	-2198	-2997	-3266	-3843
	-548	+288	-3974	-3579	-3002	-2685	-1701	-1265
	+1696	+2088	+447	+823	+1236	+1405	+1293	+1393
	+3959	+4297	+2161	+2583	+3036	+3581	+4063	+4032
	+511	-37	+3824	+3578	+3106	+2442	+1831	+925
390	-1400	-1358	-462	-767	-740	-853	-622	-1157
	-4236	-3937	-2081	-2734	-2983	-3629	-3835	-3793
	+39	+1281	-3801	-3636	-3246	-2272	-1721	-1071
	+4574	+3833	+2349	+3378	+4374	+4783	+5354	+5276
	-5841	-6118	+2678	+1122	-624	-2472	-3622	-4787
430	-256	+879	-6097	-5718	-4957	-4056	-2696	-1534
	+4146	+4048	+2043	+2920	+3348	+4019	+4127	+4322
	+1698	+1252	+3652	+3612	+2962	+2533	+2079	+1800
	+244	-764	+1579	+1594	+1538	+1297	+1545	+1102
	-6333	-5680	-1608	-2950	-4003	-5342	-6270	-6143
	+4005	+4714	-4706	-3181	-1793	-85	+1467	+2949
			+5143	+5191	+5053	+4509	+4195	+3114

TABLE 1
ORDINATES x_1 OF GENERATED SERIES

i	X_i							
470	+2529	+1355	+6	-912	-1906	-2624	-3440	+3706
	-3608	-3187	-2834	-1984	-1071	-289	+438	+941
	+1400	+1213	+1121	+625	+191	-298	-829	-1091
	-1335	-1223	-1180	-1135	-992	-185	+78	+306
	+972	+1377	+1820	+2270	+2689	+3170	+3319	+3411
510	+3565	+3753	+3253	+2840	+2524	+1799	+925	+159
	-537	-1444	-2029	-3037	-3735	-4570	-4908	-5168
	-4921	-4451	-3759	-2327	-835	+380	+1792	+2955
	+3303	+3617	+3453	+3243	+2227	+1171	+675	-276
	-1076	-1726	-2159	-2750	-2982	-3309	-3295	-3217
550	-3477	-2828	-2407	-1821	-1256	+67	+758	+1831
	+2719	+3352	+4155	+4366	+4404	+4482	+4168	+3467
	+3404	+2399	+1642	+193	-65	-791	-1322	-1845
	-2542	-1934	-1847	-1801	-1286	-956	-764	-567
	-394	-871	-770	-945	-1250	-1166	-1140	-795
590	-666	-442	-79	-257	-249	-117	-370	-566
	-627	-898	-1136	-830	-1109	-706	-447	-439
	+446	+725	+1515	+1966	+2102	+2703	+2880	+2846
	+2895	+2138	+1900	+1625	+959	+293	-116	-7
	-768	-1075	-1298	-1347	-1696	-1810	-2164	-2337
630	-2246	-2630	-2107	-1889	-1293	-811	+16	+467
	+1021	+1271	+1316	+1267	+771	+365	+157	-390
	-626	-366	-835	-134	-190	+188	+351	+634
	+792	+508	+334	+351	+213	+62	+72	+101
	+803	+737	+1128	+1529	+1910	+1853	+1886	+1960
670	+1607	+1216	+1199	+749	+81	+90	-425	-735
	-986	-807	-899	-998	-637	-703	-215	-437
	-842	-1015	-1767	-2599	-3450	-4024	-4637	-5454
	-5518	-5278	-5215	-4537	-3372	-2434	-1053	+471
710	+1907	+3314	+4561	+5401	+6299	+7032	+7031	+7066
	+7212	+6898	+5540	+4788	+3444	+1927	+146	-1653
	-3530	-5210	-6368	-3172	-8496	-9243	-9061	-8177
	-7132	-5714	-3441	-929	+1246	+3700	+5903	+8046
	+936	+10217	+10518	+11110	+10592	+9572	+8643	+6852
	+5040	+3150	+751	-1417	-3485	-5581	-6898	-7858
750	-9123	-9364	-9127	-9217	-8371	-7052	-6058	-4792
	-3477	-2282	-630	+563	+1543	+2623	+3756	+4264
	+4822	+5063	+4751	+4556	+3938	+2889	+1626	+777
	+80	-907	-1584	-1901	-1798	-1493	-867	-24
	+659	+1846	+2744	+3122	+3279	+3243	+2550	+1477
790	+150	-1290	-3095	-4433	-6055	-7326	-8039	-8481
	-8468	-7956	-6741	-5504	-3641	-1784	+447	+2223
	+4309	+5910	+6865	+7763	+8120	+8029	+7672	+6869
	+5587	+4673	+3258	+1811	+462	-696	-1928	-3094
	-4128	-5082	-5820	-6110	-6197	-5852	-4841	-3334
830	-1674	+421	+2872	+4927	+7263	+8826	+10045	+10403
	+10160	+9038	+6995	+4886	+2097	-1510	-4040	-6665
	-9144	-10094	-10889	-10165	-8675	-6737	-4099	-926
	+1386	+3858	+6127	+7443	+8094	+7984	+7143	+6059
	+4148	+1910	-1	-2225	-3934	-5430	-6332	-7159
870	-6964	-6680	-5879	-5044	-3792	-2624	-1762	-583
	+26	+850	+1056	+1323	+1645	+1761	+1639	+1917
	+2245	+2636	+3094	+2915	+3376	+3243	+2644	+2558
	+2036	+1214	+778	-112	-999	-1454	-2816	-3421
	-4101	-4652	-4985	-5040	-5056	-4721	-3902	-2993
910	-1625	-574	+1169	+2465	+3900	+5112	+6019	+6662
	+6686	+6740	+6538	+5829	+4594	+3851	+2954	+1578
	+959	-263	-1086	-1433	-2435	-2781	-3010	-3302
	-3715	-4225	-4405	-4924	-5224	-4821	-4841	-4276
	-3592	-2505	-1234	+164	+1428	+2731	+3909	+4865

TABLE I
ORDINATES x_i OF GENERATED SERIES

i	X_i							
950	+5902	+6140	+6757	+6865	+6404	+6466	+5904	+4987
	+4366	+3385	+2350	+1278	-124	-1693	-2385	-3615
	-4843	-4972	-5545	-5494	-4800	-4147	-3769	-3052
	-2193	-1875	-1458	-1489	-1619	-1867	-2169	-2723
	-3058	-3209	-3431	-3066	-2757	-2583	-1789	-831
990	-55	+626	+1435	+2484	+2682	+3318	+3887	+4190
	+4327	+4377	+4449	+4270	+4037	+3495	+3176	+2920
	+2128	+1706	+1550	+732	-154	-585	-1325	-2297
	-2980	-3468	-4116	-4791	-4724	-4964	-4813	-4570
	-4352	-3349	-2729	-1935	-686	+491	+1594	+3074
1030	+4328	+5500	+6517	+7323	+7877	+7738	+7233	+5963
	+4641	+2340	+296	-1818	-4432	-6161	-7424	-8389
	-8698	-8378	-7616	-5717	-3946	-2035	+305	+2545
	+4151	+5599	+6896	+7108	+7448	+7335	+6380	+5577
	+4476	+2859	+1247	-349	-2204	-3391	-4389	-5431
1070	-5877	-5849	-5365	-4505	-3624	-2552	-979	+424
	+1776	+3061	+4015	+5044	+5329	+5273	+5231	+4337
	+3367	+2237	+1126	+112	-975	-2194	-3047	-3767
	-4099	-4366	-4185	-4085	-3490	-2534	-1990	-1466
	-934	-210	+83	+537	+573	+956	+1156	+1182
1110	+1049	+1176	+942	+1238	+1474	+1377	+1649	+1757
	+1778	+1560	+1177	+601	+98	-741	-1401	-1905
	-2034	-2252	-2207	-1700	-937	-432	+193	+880
	+1466	+1922	+1676	+1275	+994	-41	-1462	-2242
	-3548	-4755	-5231	-5906	-6063	-5809	-5044	-3806
1150	-2591	-861	+951	+2803	+4768	+6150	+7529	+8752
	+9512	+10219	+10460	+10727	+10215	+9761	+8930	+7585
	+5978	+4348	+2929	+548	-1224	-3071	-5191	-7049
	-8563	-10038	-10960	-12231	-12713	-12279	-12130	-11408
	-10361	-8583	-6809	-5358	-3146	-1018	+343	+2610
1190	+4045	+5119	+5959	+6500	+6901	+6693	+6155	+5818
	+4928	+4011	+3047	+2111	+1653	+934	+925	+754
	+1429	+2044	+2803	+3417	+3909	+4418	+4622	+4696
	+4202	+3702	+3263	+2476	+1606	+1066	+411	-407
	-960	-1909	-2501	-3578	-4476	-5099	-6086	-6400
1230	-6796	-6905	-6515	-5756	-4819	-3720	-2699	-1518
	-930	+175	+272	+320	+648	+355	+226	+485
	+421	+762	+1727	+2361	+3187	+3903	+5004	+5741
	+5970	+5887	+5870	+4995	+4249	+2944	+1362	-282
	-1922	-3540	-5084	-6254	-7155	-7404	-7553	-6729
1270	-5882	-4432	-2741	-1243	+305	+1818	+2631	+3523
	+3845	+4398	+4818	+4773	+5509	+5508	+6075	+6433
	+6489	+6327	+5971	+5243	+4366	+3092	+1395	-100
	-1765	-3268	-4750	-5815	-6590	-7055	-7539	-7259
	-7102	-6967	-6347	-5968	-5260	-4582	-3879	-3058
1310	-1961	-926	-148	+1040	+1523	+2012	+2304	+2690
	+2687	+2410	+2411	+2101	+2210	+1922	+1929	+2194
	+2250	+2687	+3508	+3797	+4689	+5115	+5568	+5811
	+5942	+5901	+5366	+4967	+4260	+3353	+1975	+1081
	-74	-1448	-2842	-4174	-4834	-5709	-6372	-6518
1350	-6340	-5929	-5723	-5143	-4212	-3885	-3284	-2763
	-2500	-1923	-1389	-489	+85	+1338	+2140	+3343
	+4672	+5296	+6304	+6468	+6382	+5757	+4856	+3768
	+2427	+855	-479	-1690	-2553	-3130	-3984	-4065
	-3932	-3590	-3355	-3108	-2075	-1051	-419	+353
1390	+1415	+2302	+3259	+3838	+3991	+4290	+3860	+3026
	+2491	+1515	-121	-1241	-2560	-3996	-4791	-5493
	-6041	-6382	-6400	-6199	-5583	-4736	-3951	-2997

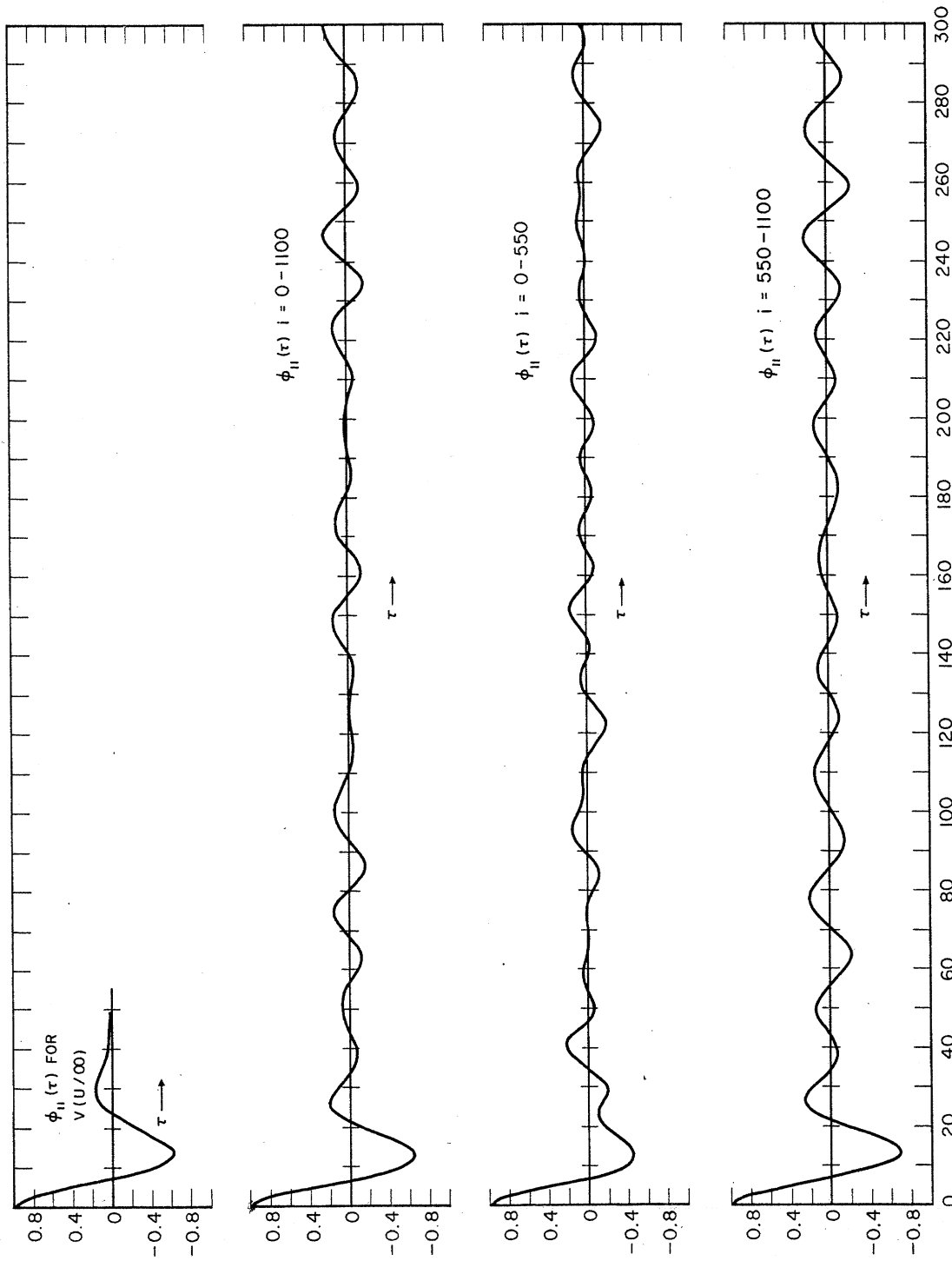


FIG. 5. NORMALIZED AUTOCORRELATION FUNCTIONS OF GENERATED SERIES SAMPLING LENGTHS: 1100 $\&$ 550 PTS. OF x_1 , INTERVALS AS SHOWN BY i RANGE

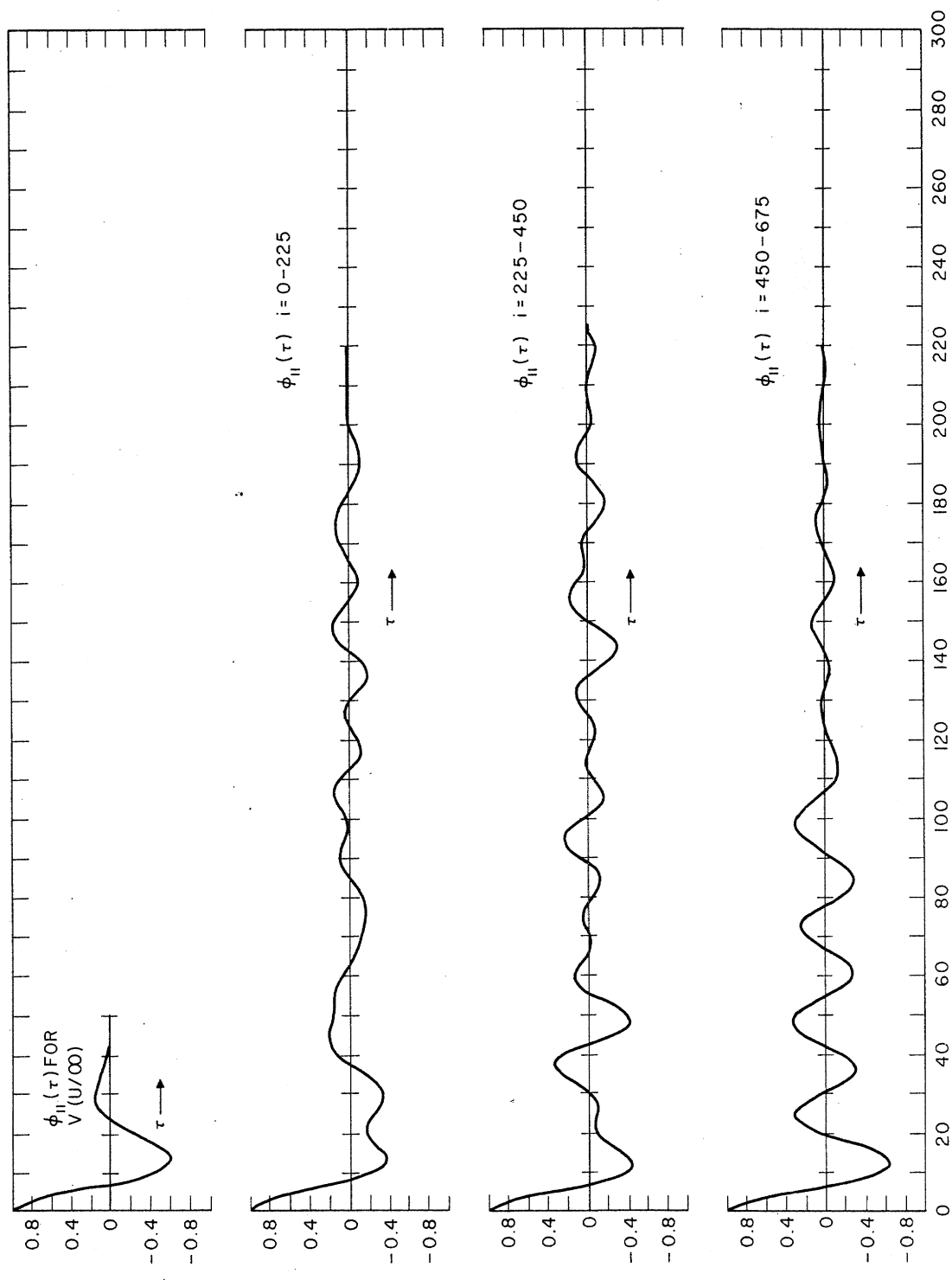


FIG. 6. NORMALIZED AUTOCORRELATION FUNCTION OF GENERATED SERIES SAMPLING LENGTH: 225 PTS OF x_i , INTERVALS AS SHOWN BY i RANGE

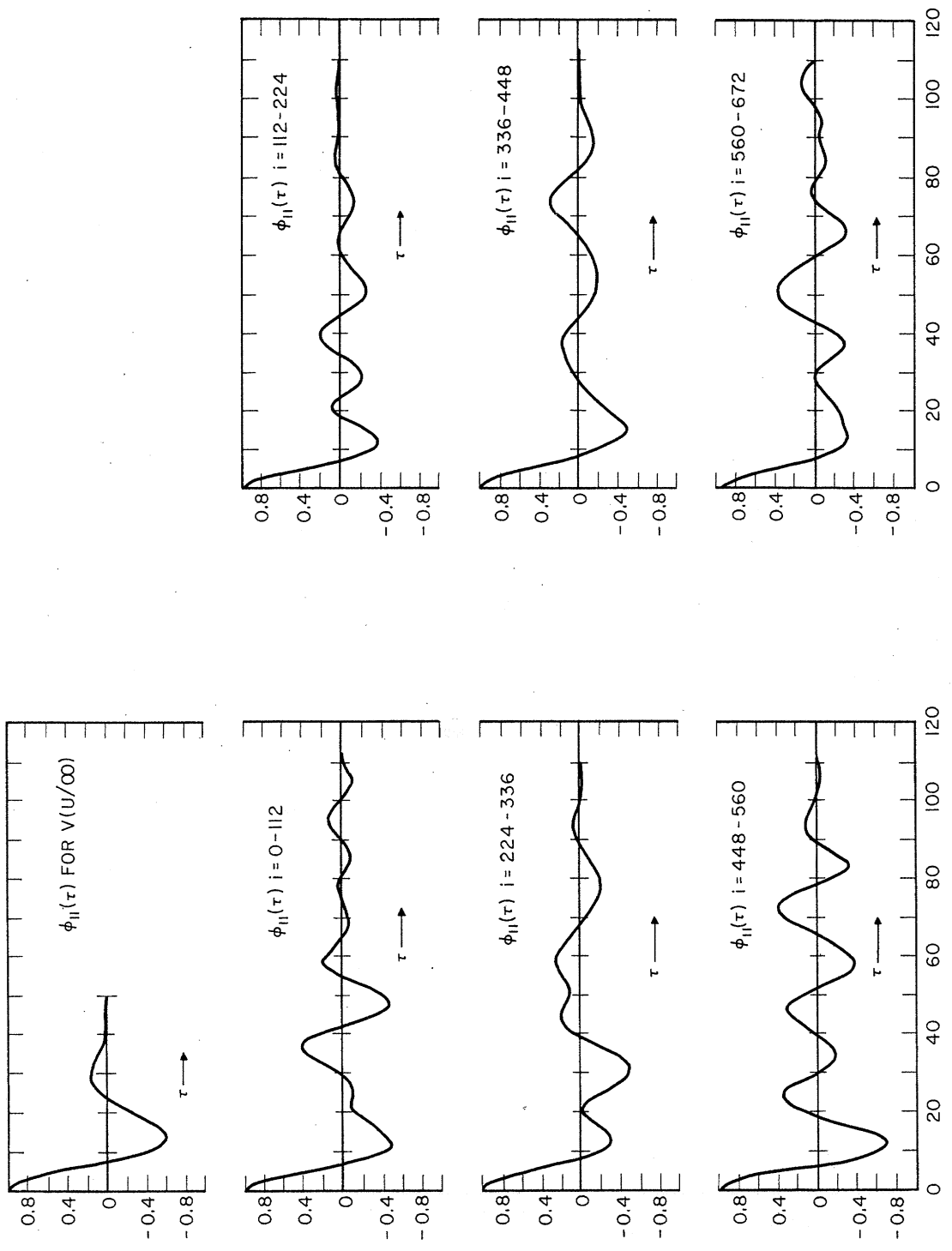


FIG. 7 NORMALIZED AUTOCORRELATION FUNCTIONS OF GENERATED SERIES
 SAMPLING LENGTH: 112 PTS. OF x_i INTERVALS AS SHOWN BY i RANGE

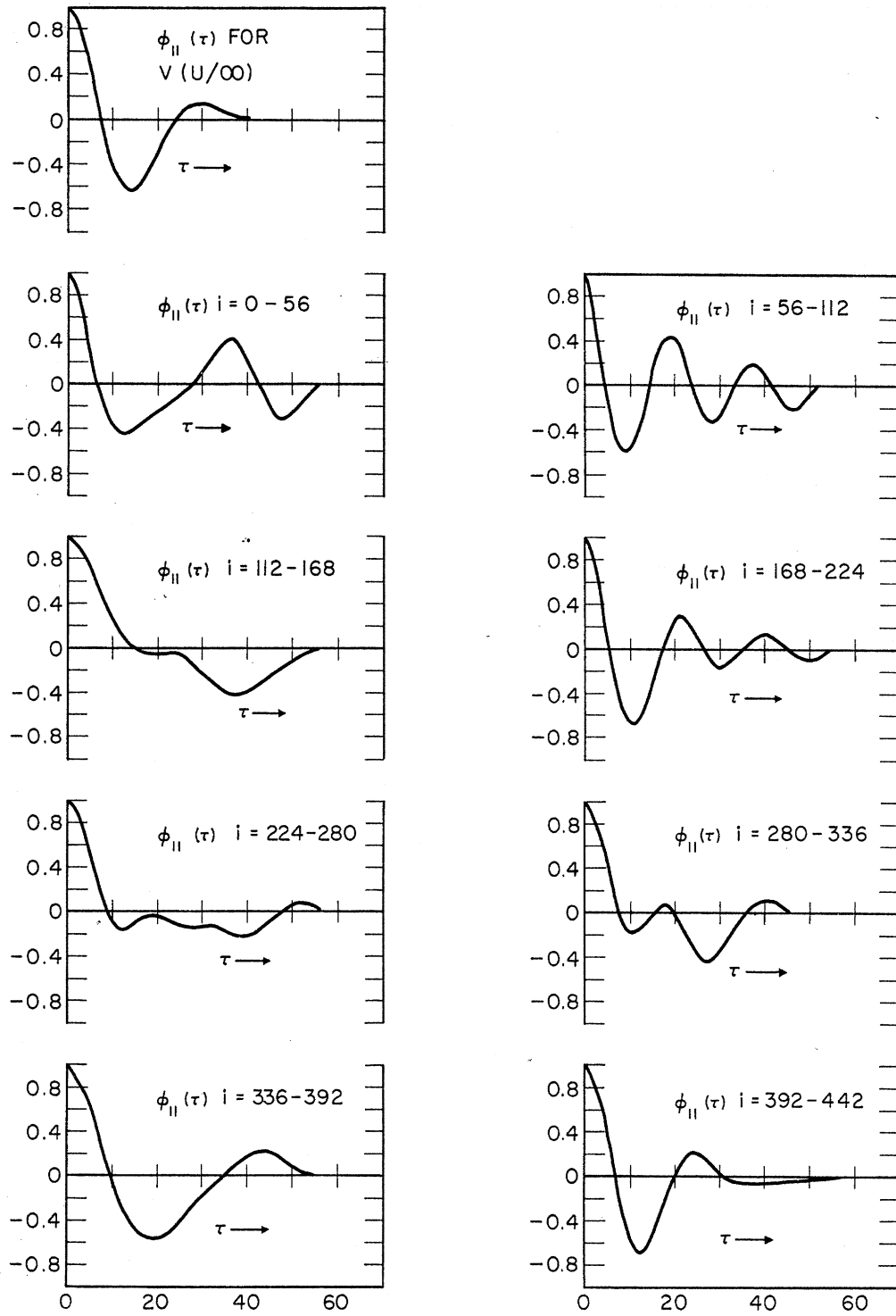


FIG. 8: NORMALIZED AUTOCORRELATION FUNCTIONS OF GENERATED SERIES SAMPLING LENGTH: 56 PTS OF x_i INTERVALS AS SHOWN BY i RANGE

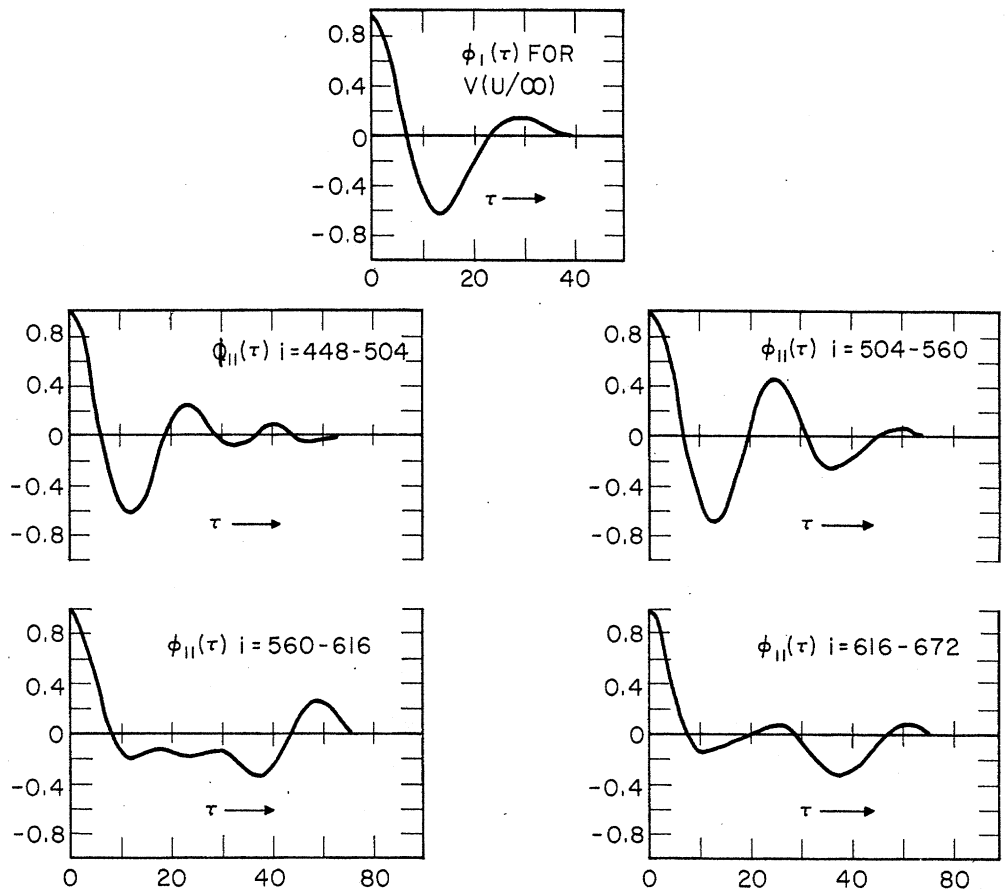


FIG. 9 NORMALIZED AUTOCORRELATION FUNCTIONS OF GENERATED SERIES SAMPLING LENGTH: 56 PTS OF x_i . INTERVALS AS SHOWN BY i RANGE

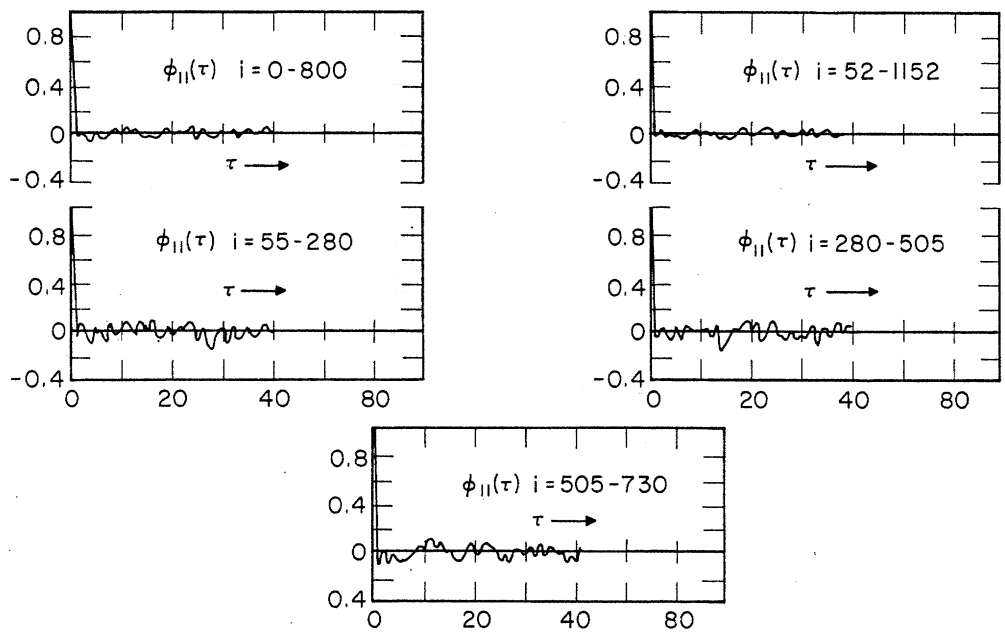


FIG. 10 NORMALIZED AUTOCORRELATION FUNCTIONS OF RANDOM NUMBERS USED FOR GENERATION OF SERIES INTERVALS AS SHOWN BY i RANGE

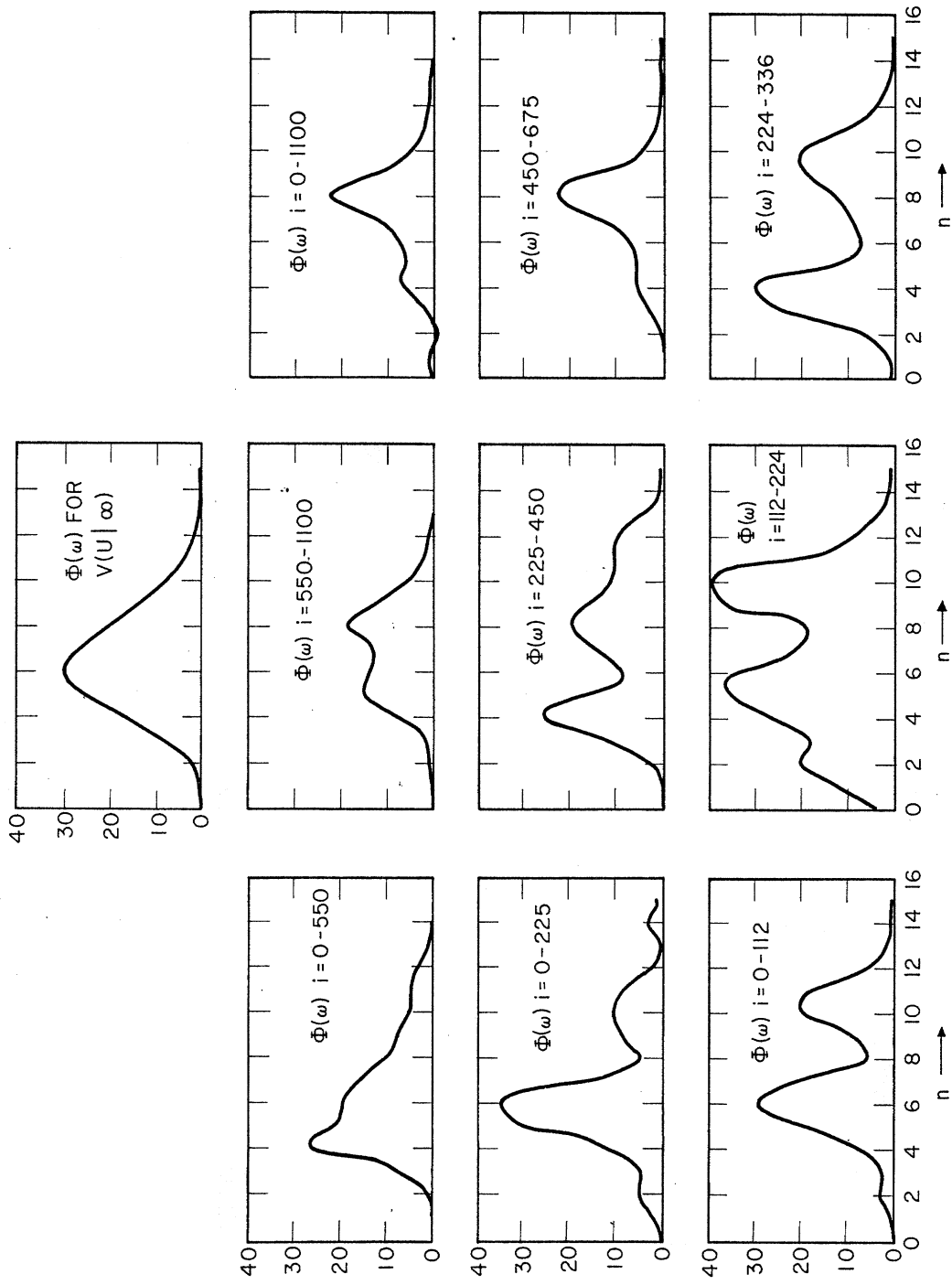


FIG. 11 SMOOTHED POWER SPECTRA OF GENERATED SERIES. SAMPLING LENGTHS AND INTERVALS AS SHOWN BY i RANGE.

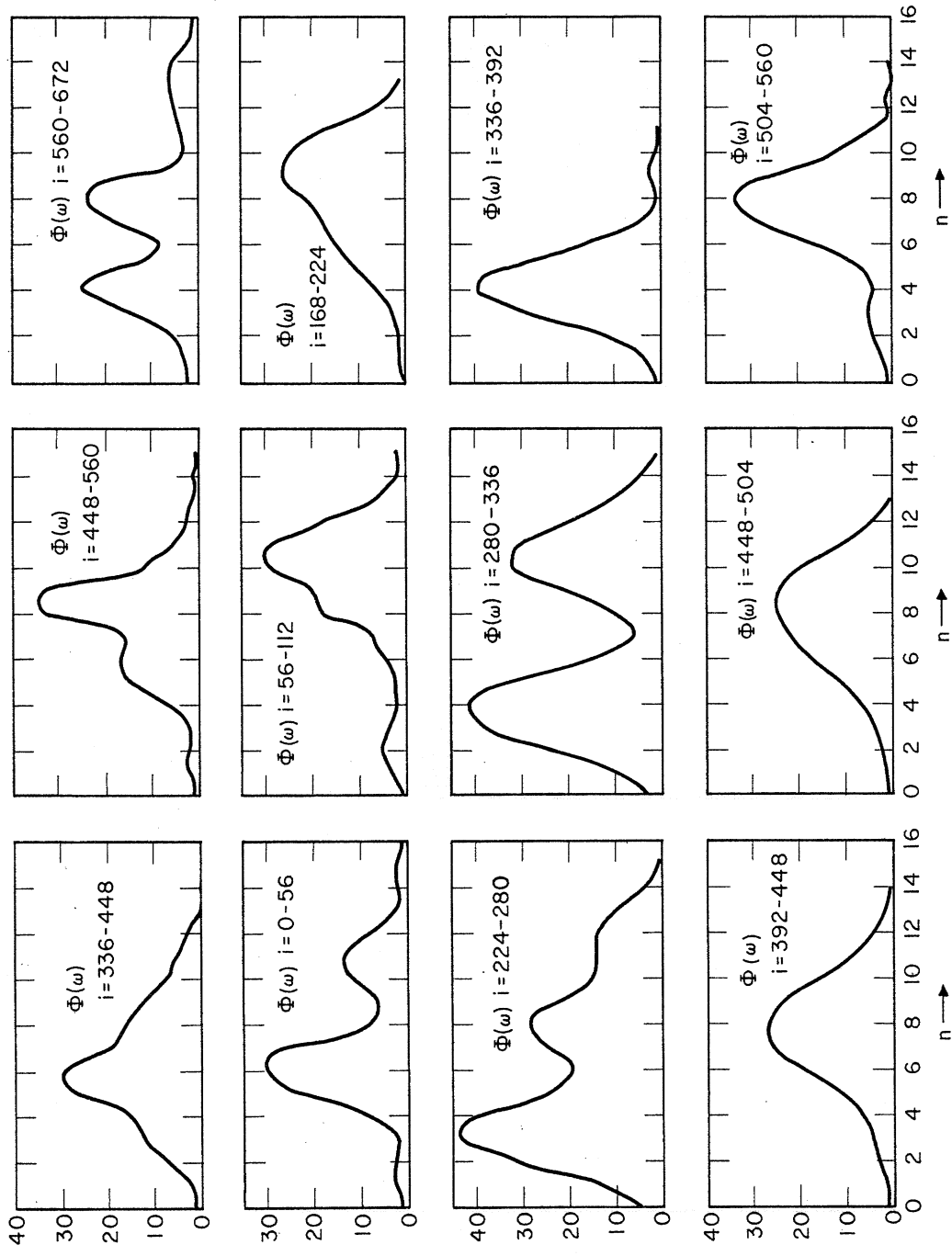


FIG. 12 SMOOTHED POWER SPECTRA OF GENERATED SERIES. SAMPLING LENGTHS AND INTERVALS AS SHOWN BY i RANGE.

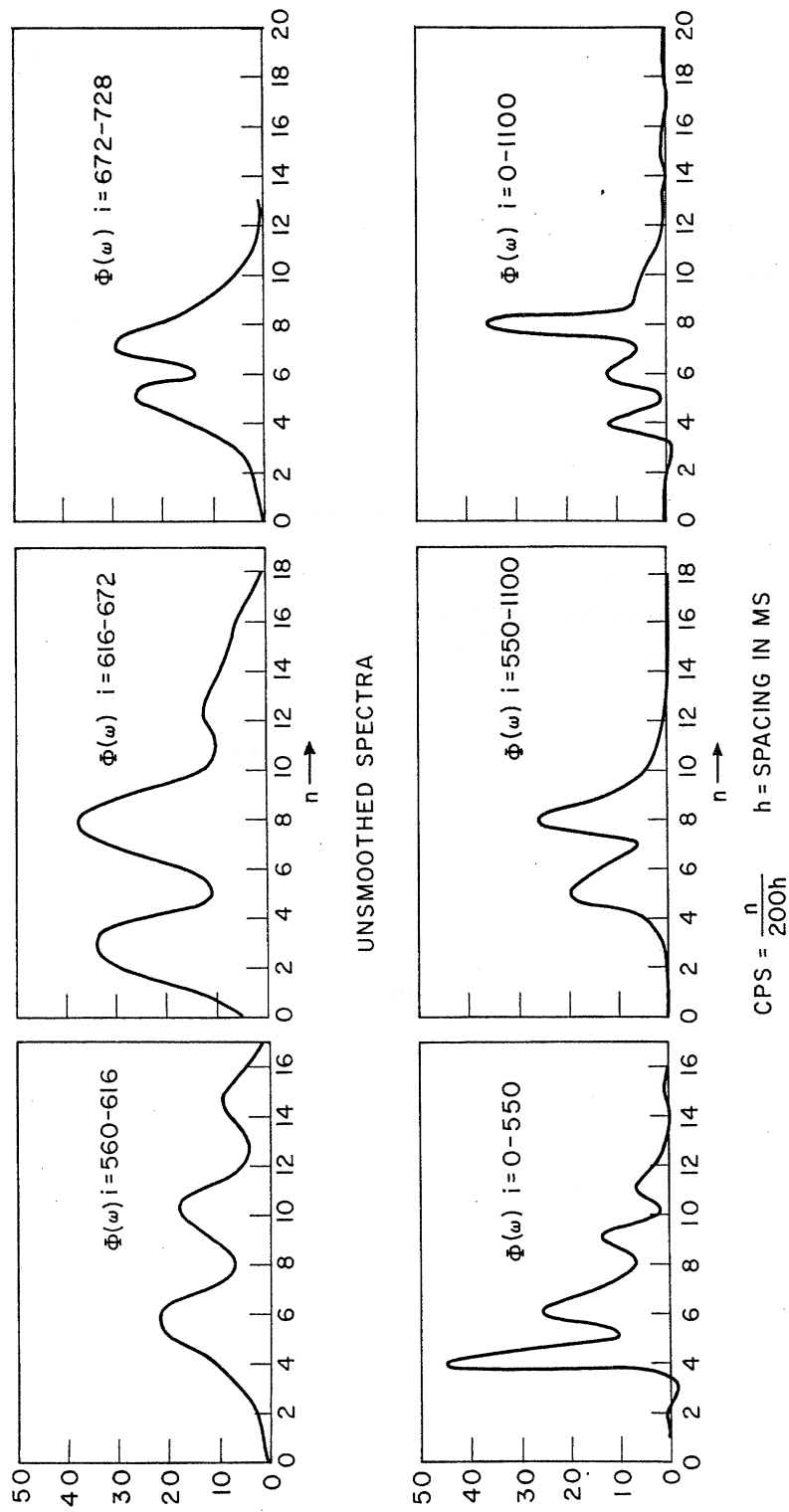


FIG. 13 SMOOTHED AND UNSMOOTHED SPECTRA OF GENERATED SERIES. SAMPLING LENGTHS AND INTERVALS AS SHOWN BY *i* RANGE.

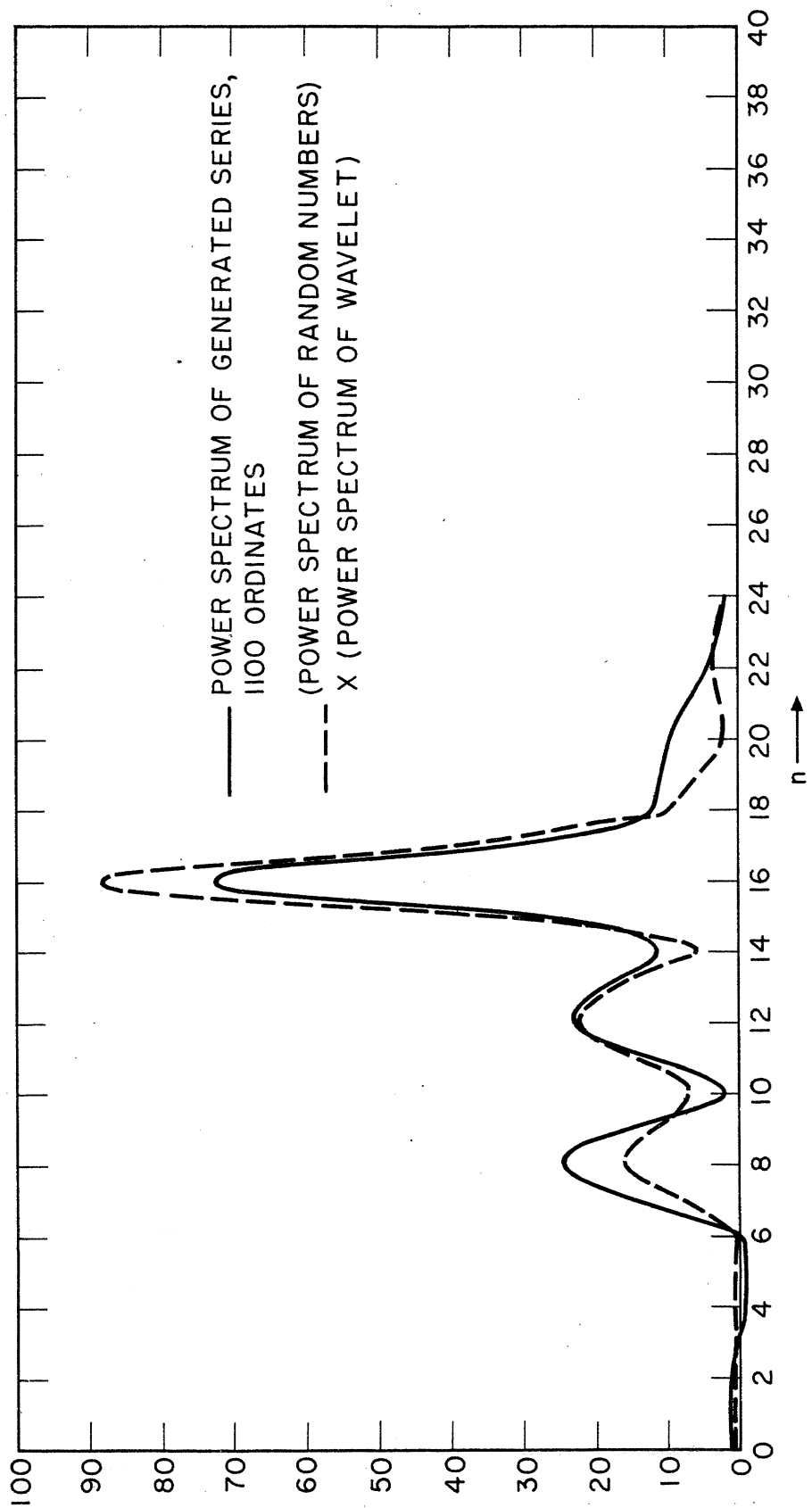


FIG. 14 OBSERVED SPECTRAL VARIATIONS AS DUE TO CORRELATION IN RANDOM NUMBER SERIES
 (SEE TEXT)

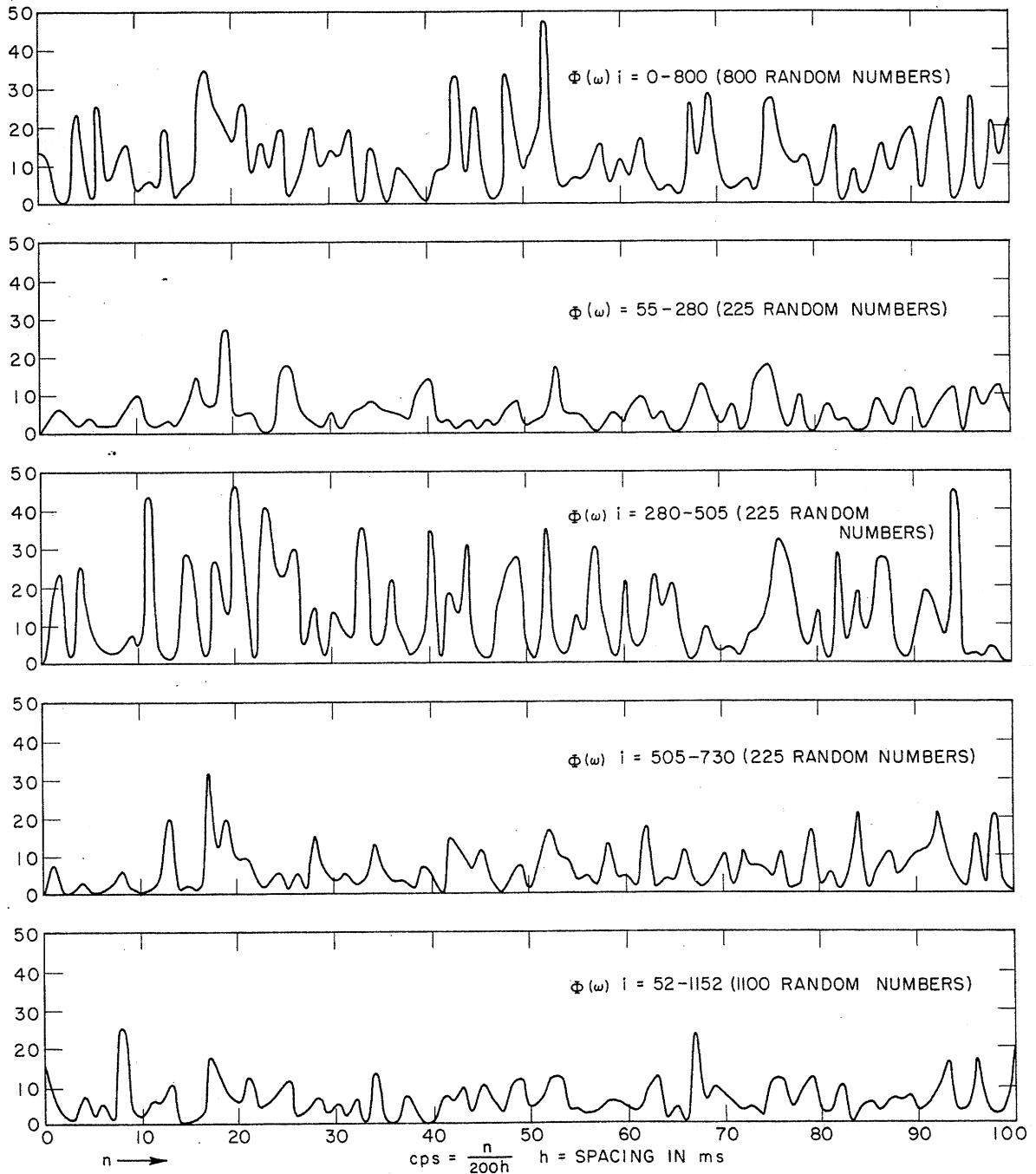


FIG. 15- POWER SPECTRA OF RANDOM NUMBERS USED FOR GENERATION OF SERIES.
 SAMPLING LENGTHS: 800, 225, AND 1100 ζ 'S AS SHOWN BY i RANGE.

RECORD 14.12

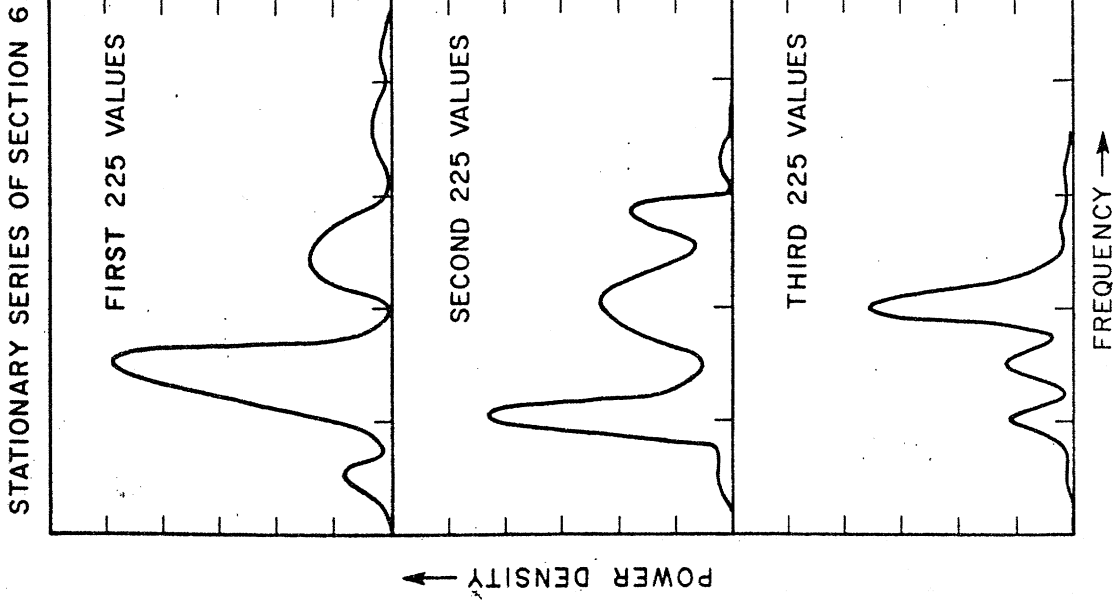
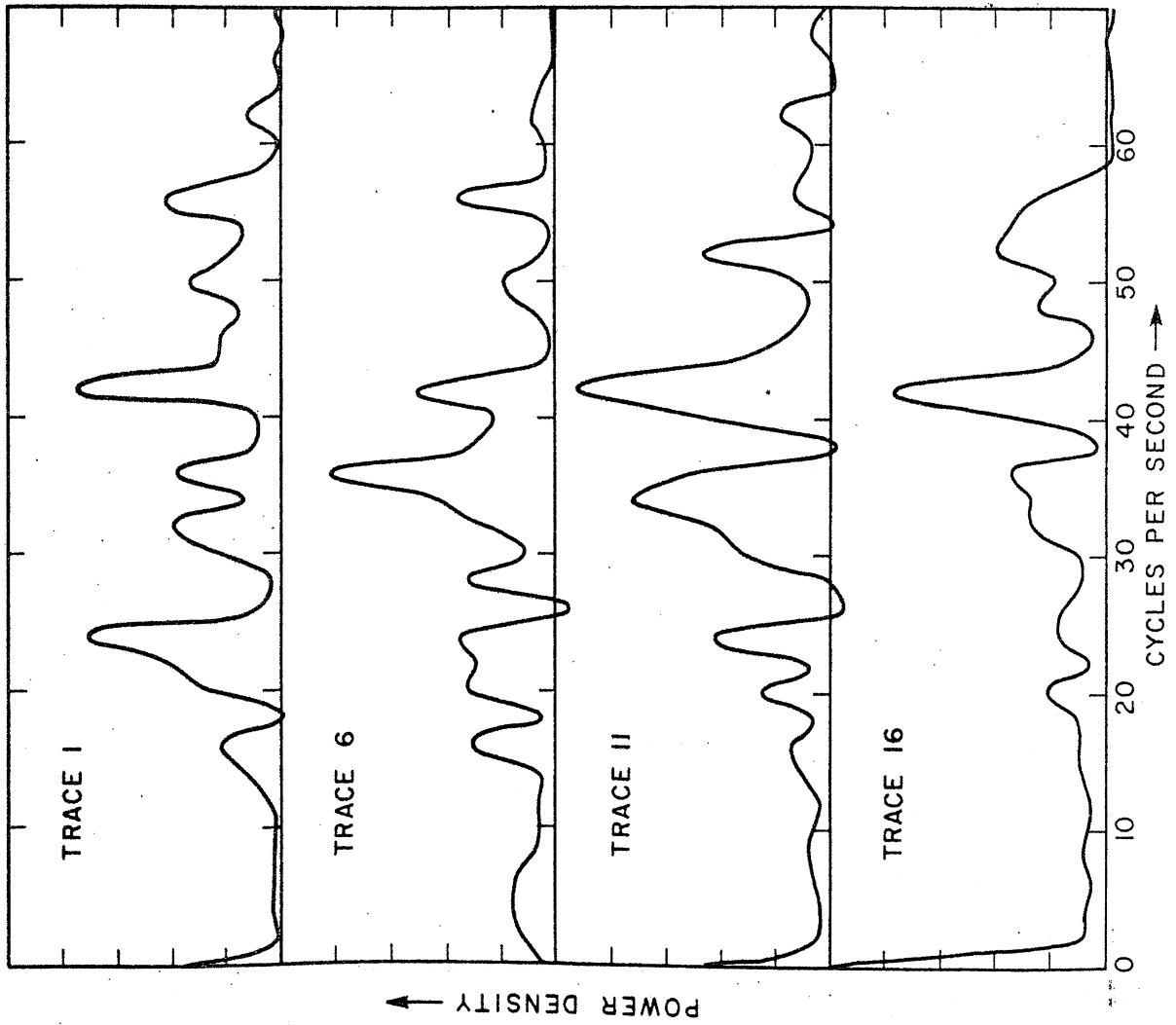


FIG. 16 SPECTRAL ESTIMATES ON A "NOISY" SEISMOGRAM AND ON A NON-DETERMINISTIC STATIONARY SERIES.

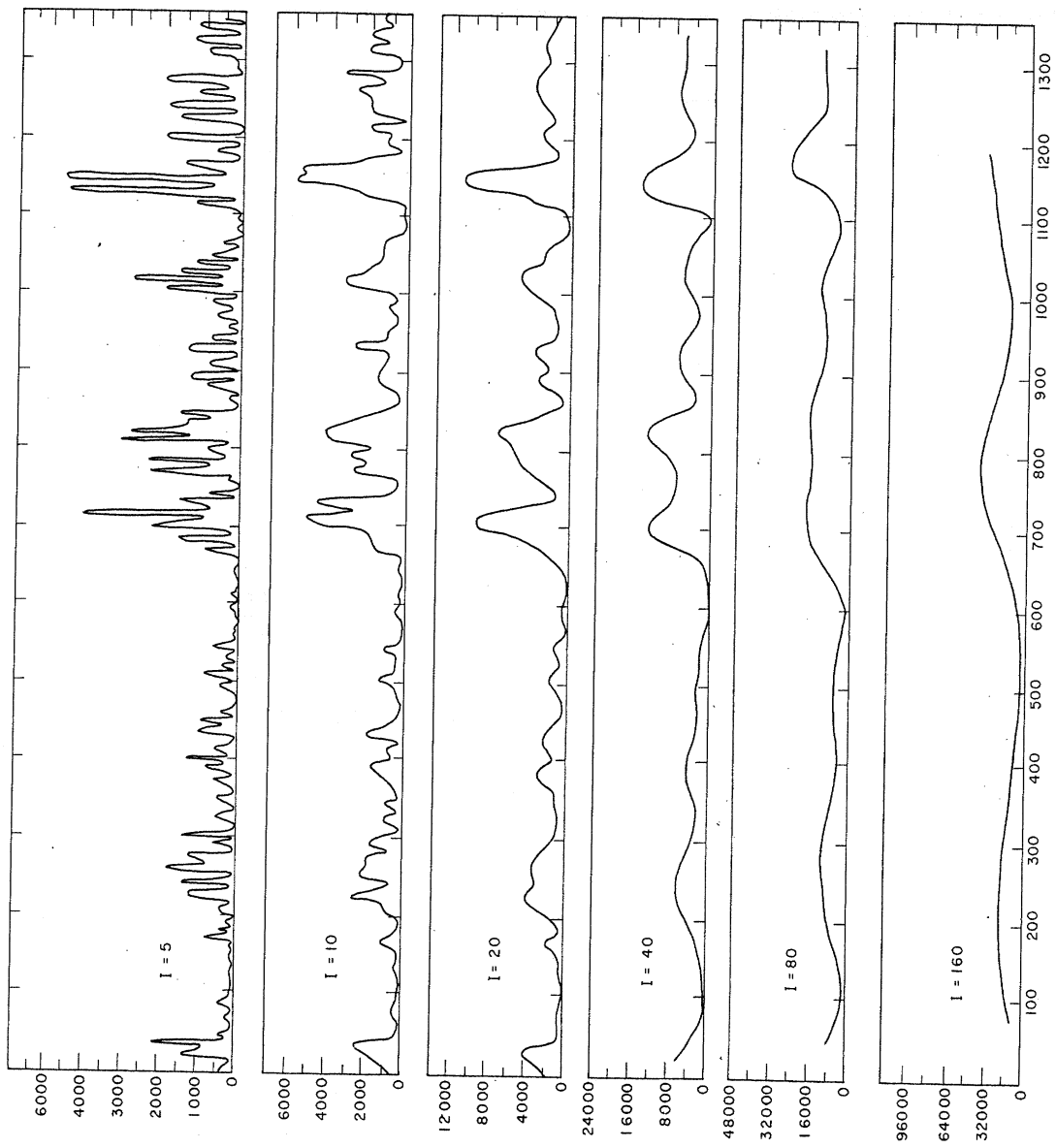


FIG. 17 RUNNING VARIANCES OF GENERATED SERIES. I = INTERVAL LENGTH

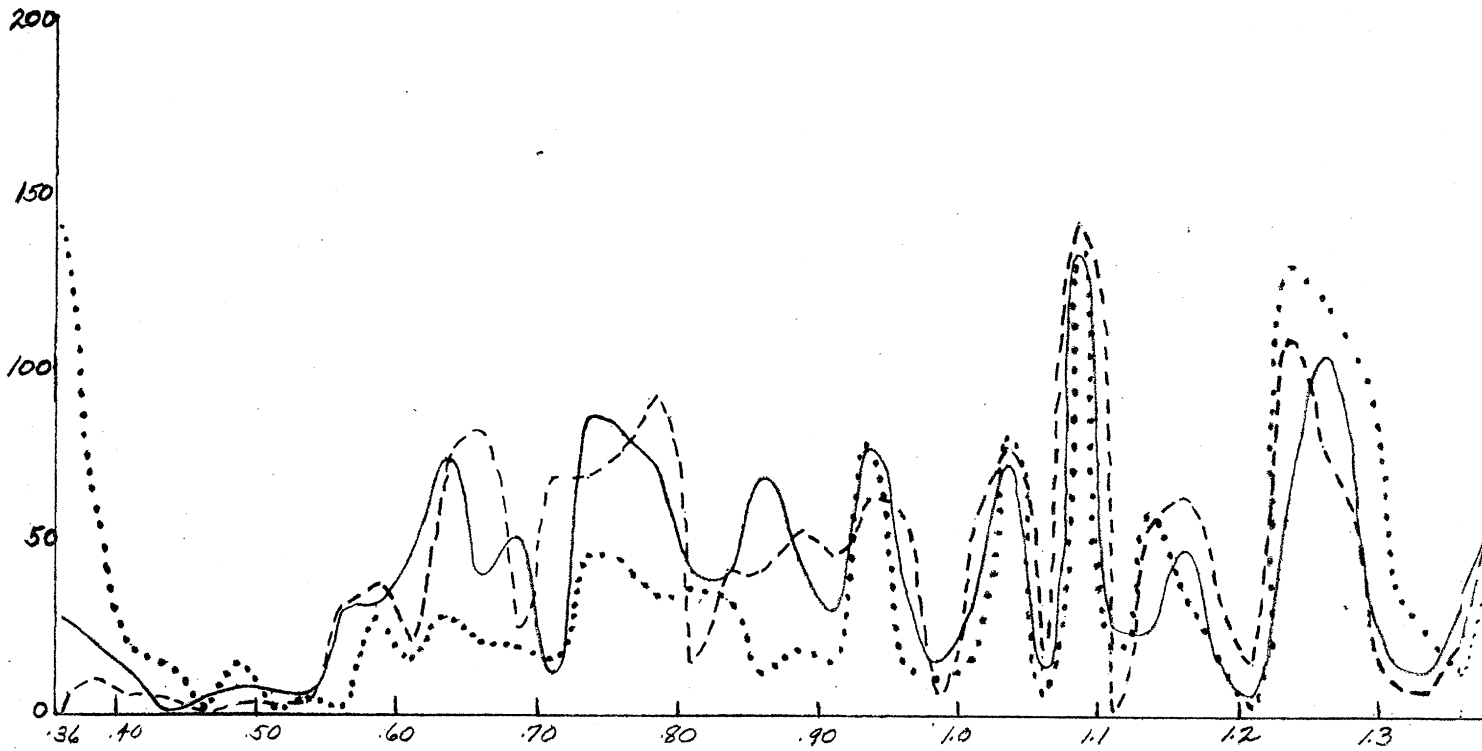
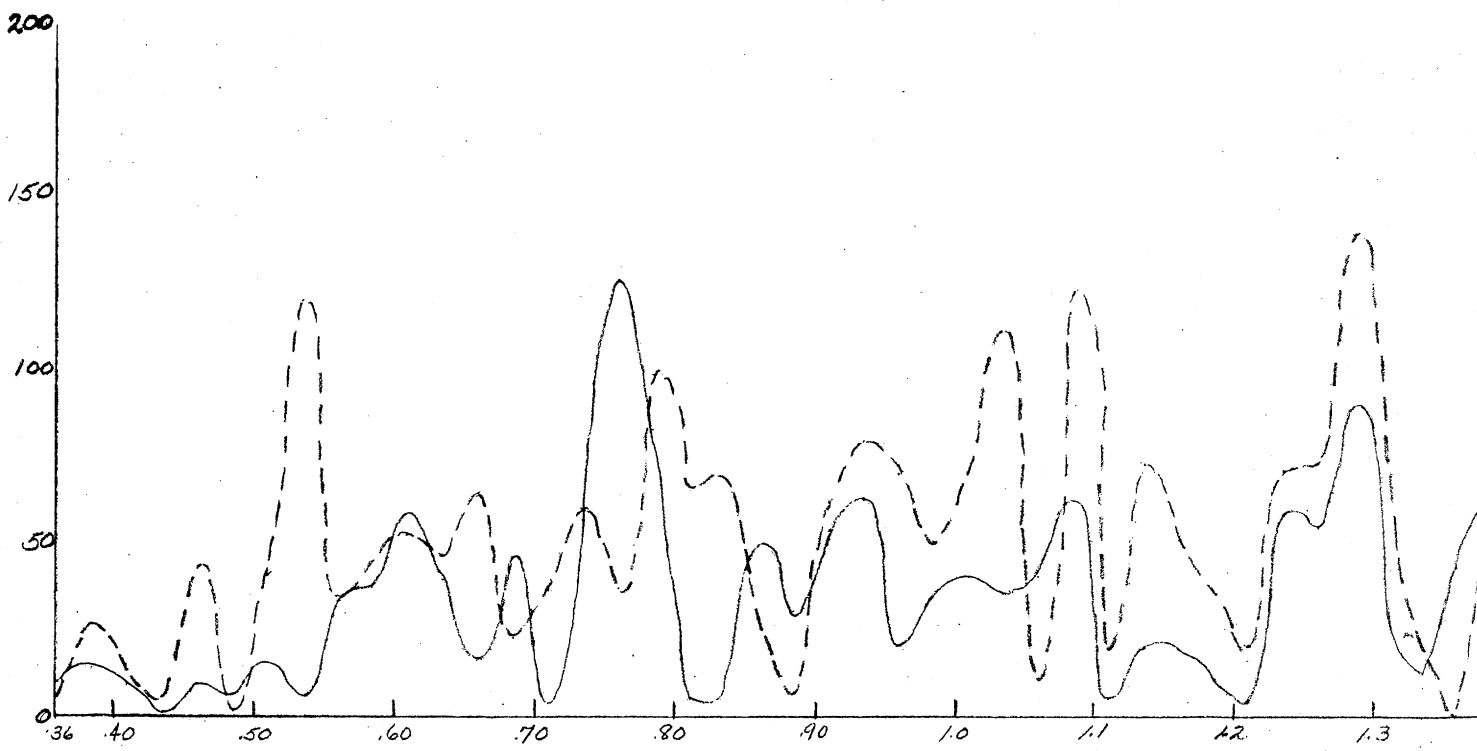


FIGURE 18.- VARIANCE CURVES OF AN ACTUAL "NOISY" SEISMOGRAM.-



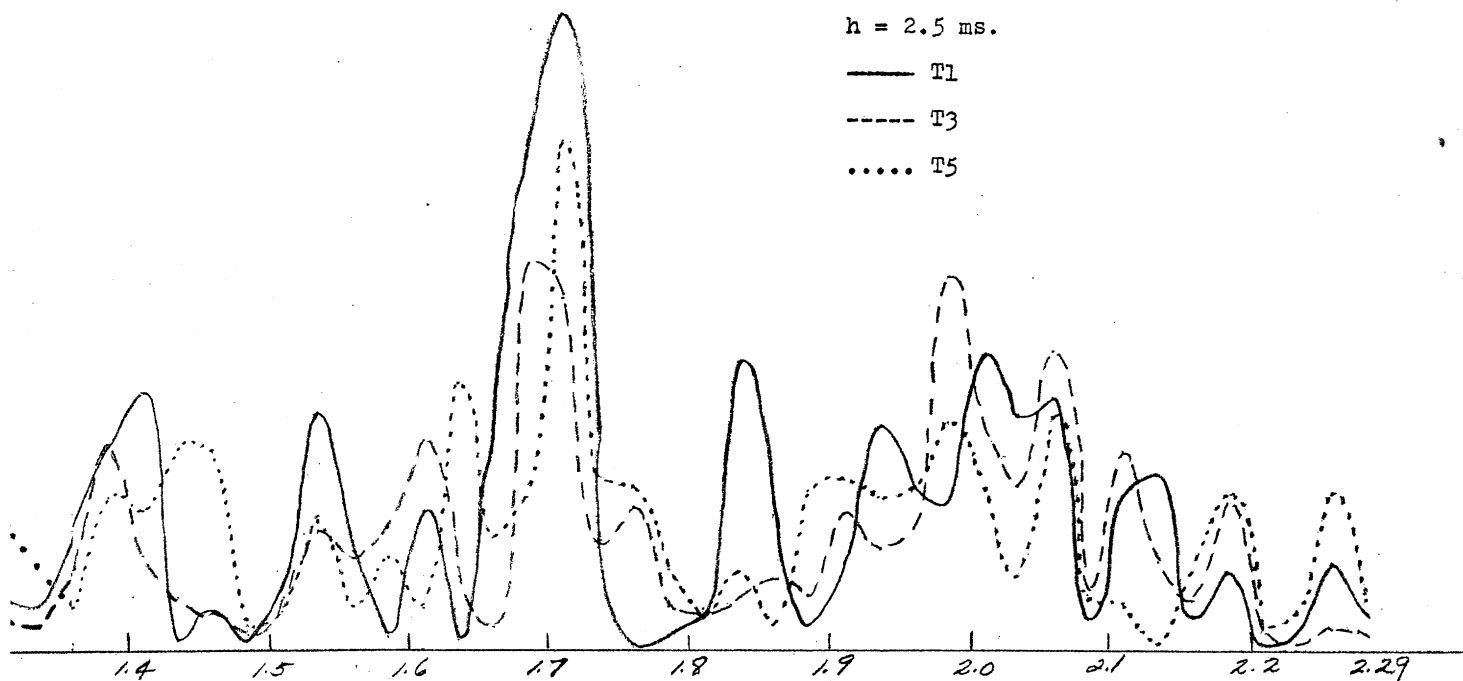
RECORD 12.4 VARIANCE CURVES (2p = 10)

h = 2.5 ms.

— T1

- - - T3

..... T5



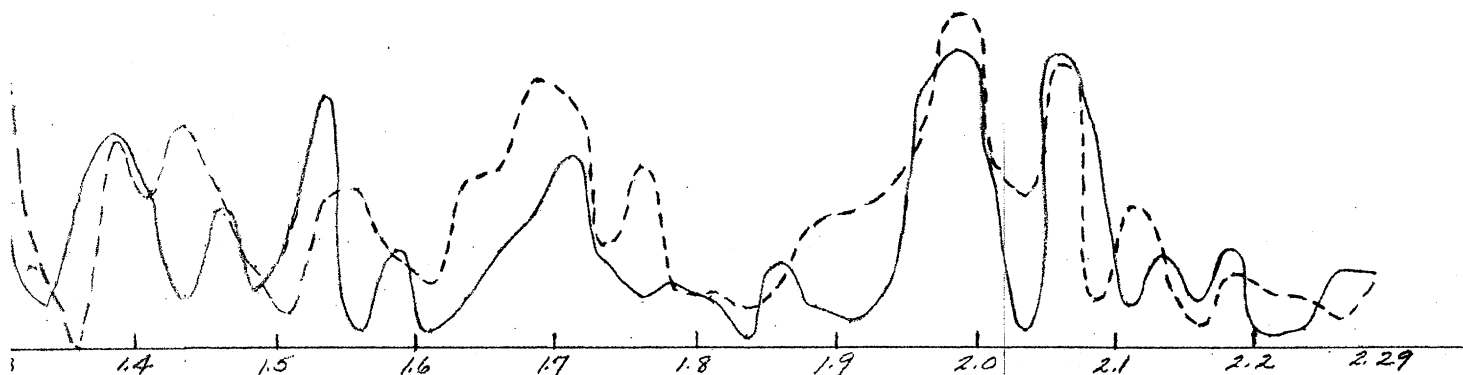
RECORD 12.4 VARIANCE CURVES (2p = 10)

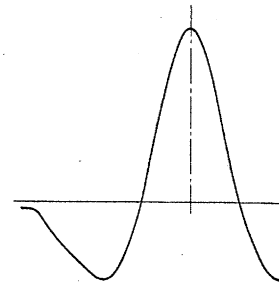
h = 2.5 ms.

— T2

- - - T4

..... T6





RICKER WAVELET

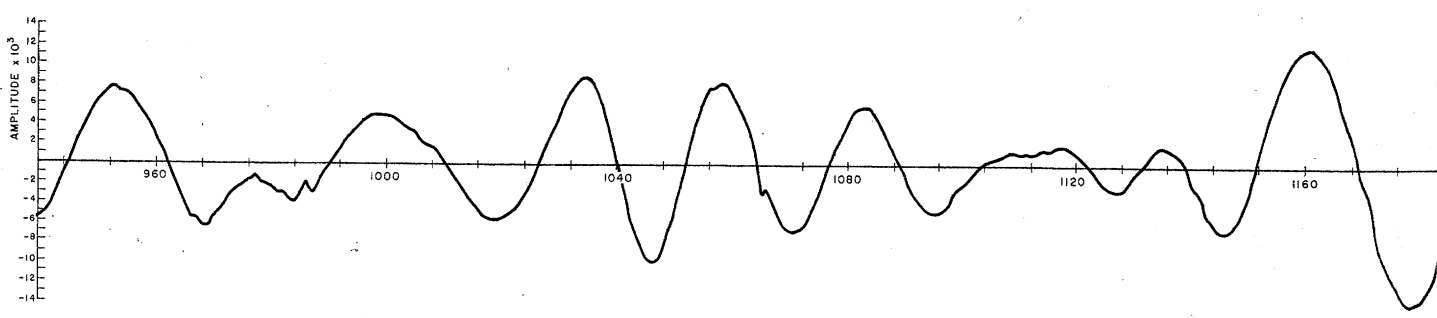
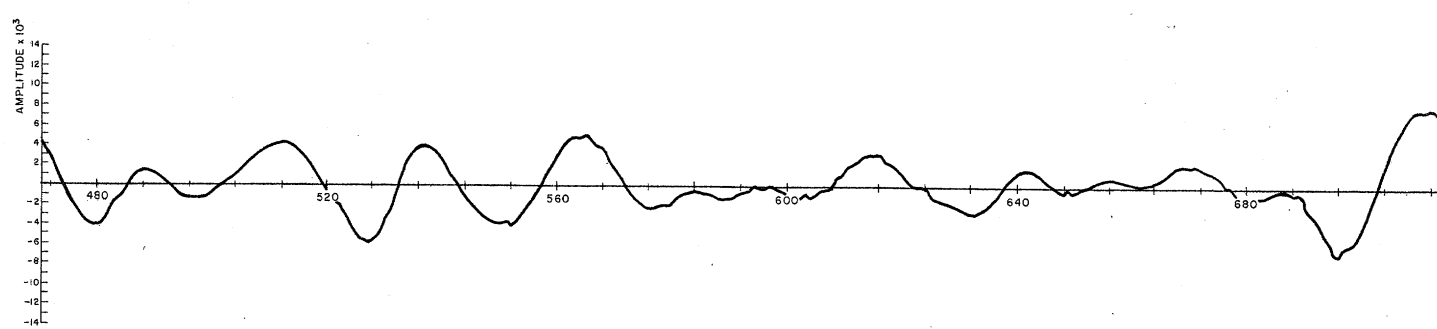
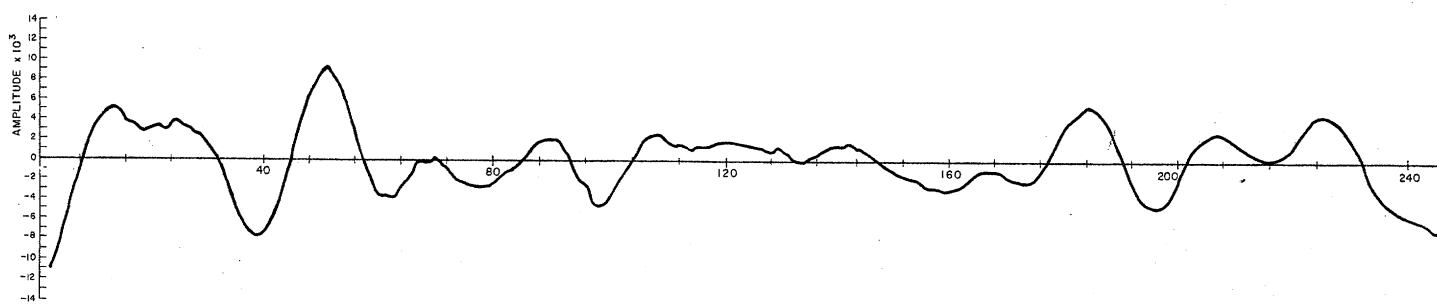
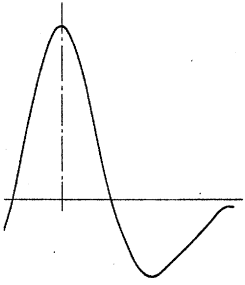
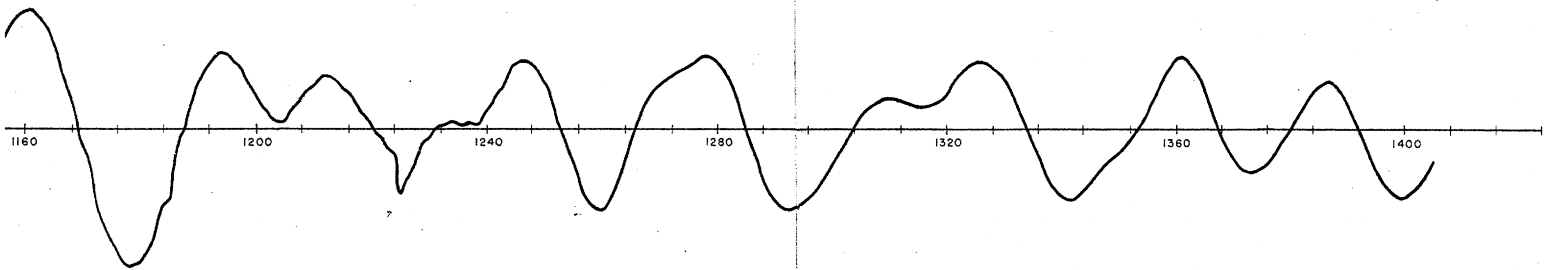
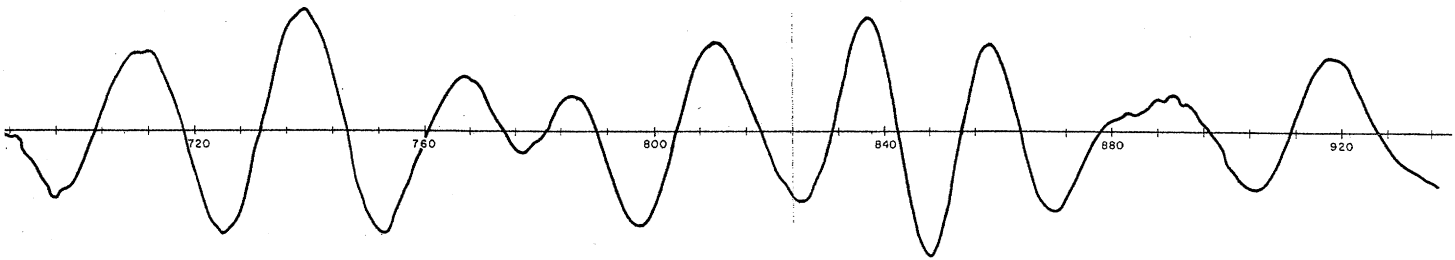
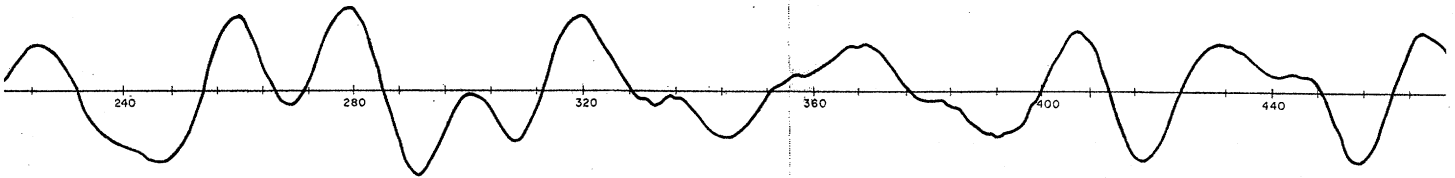


FIG. 19 GENERATE



AVELET $V(U|\infty)$



GENERATED SERIES

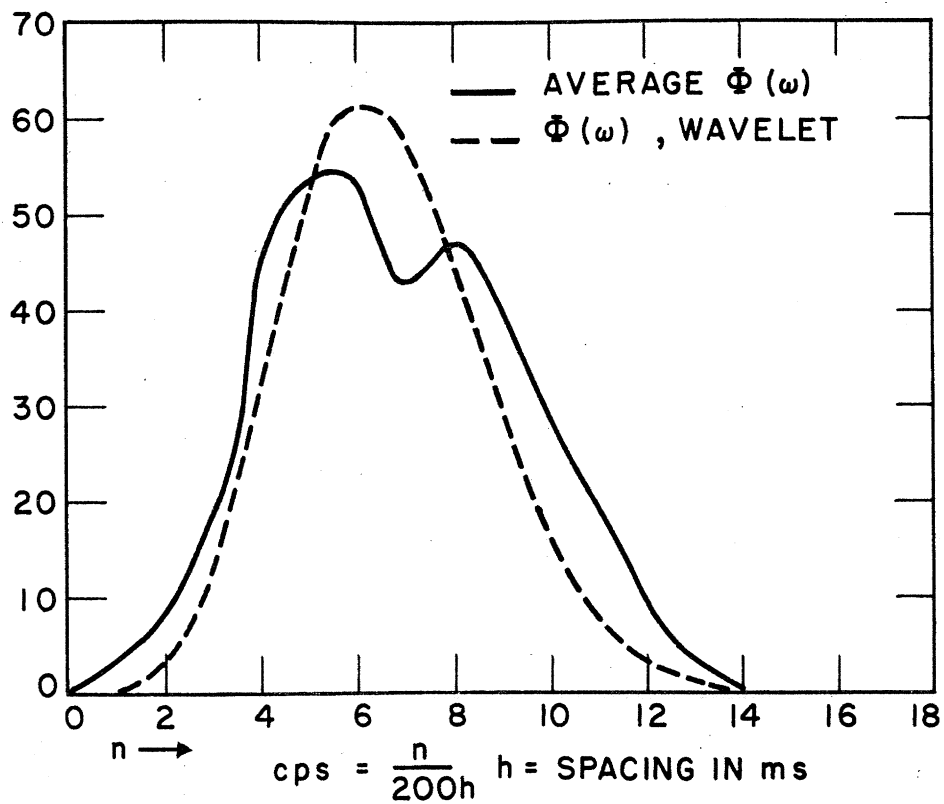


FIG. 20: - SMOOTHED SPECTRUM OF RICKER WAVELET
 COMPARED TO AVERAGE OF SMOOTHED
 SPECTRA OF INTERVALS $i = 0-225$; $225-$
 450 AND $450-675$

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Appendix

Brief Description of Program

Program	Input	Computation Performed
1) Wiener Autocorrelation and Spectrum (author: Prof. S.M.Simpson)	Series x_i , $i = 0, 1, \dots$ $N =$ index of first x_i to use $n =$ no. of x_i terms to use $M =$ highest lag desired	$x = \frac{1}{n} \sum_{i=1}^{N+n-1} x_i$ $\phi_{11}(\tau) = \frac{\sum_{i=1}^{N+n-1-\tau} (x_i - \bar{x})(x_{i+\tau} - \bar{x})}{\sum_{i=1}^{N+n-1} (x_i - \bar{x})^2}$
2) Generation of Stationary Series (author: S. Treitel)	Random Nos. Wavelet ordinates	$\tau = 0, 1, 2, \dots, M$ $x_t = \sum_{s=0}^n b_s^t t-s$
3) Variance (author: S. Treitel)	x_t series from (2) (2)	Variance of x_t at intervals of 5 ordinates
Output	1)	3)
$\phi_{11}(\tau)$ and $\Phi(\omega)$ or $\bar{\Phi}(\omega)$ only	x_t series	Variance values

T A B L E O F C O N T E N T S

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