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ARTIFICIAL INTELLIGENCE MACHINE LEARNING IN MARINE HYDRODYNAMICS

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ABSTRACT

Artificial Intelligence (AI) Support Vector Machine (SVM) learning algorithms have enjoyed rapid growth in recent years with applications in a wide range of disciplines often with impressive results. The present paper introduces this machine learning technology to the field of marine hydrodynamics for the study of complex potential and viscous flow problems. Examples considered include the forecasting of the seastate elevations and vessel responses using their past time records as “explanatory variables” or “features” and the development of a nonlinear model for the roll restoring, added moment of inertia and viscous damping using the vessel response kinematics from free decay tests as “features”. A key innovation of AI-SVM kernel algorithms is that the nonlinear dependence of the dependent variable on the “features” is embedded into the SVM kernel and its selection plays a key role in the performance of the algorithms. The kernel selection is discussed and its relation to the physics of the marine hydrodynamic flows considered in the present paper is addressed.

1 INTRODUCTION

SVM algorithms have their origins in statistical learning theory, functional analysis and convex optimization. Two standard applications of SVM involve classification and nonlinear regression of a dependent variable on a set of “features”. Regression is the pertinent application of SVM algorithms in

the present paper which considers the development of nonlinear models of complex marine hydrodynamic loads.

SVM algorithms represent the dependent variable as the linear superposition of a series of nonlinear basis functions which depend upon a set of explanatory variables or “features”. The mathematical form of the basis functions does not need to be made explicit, it is instead embedded into the form of the SVM kernel. In order to prevent the over-fitting of the input variables which may be corrupted by noise, SVM kernel algorithms minimize a cost function which includes an additive regulation term that penalizes the magnitude of the coefficients of the nonlinear basis function series. When the cost function is cast in the form of a Least-Squares quadratic penalty loss the popular LS-SVM algorithm is obtained. It leads to the solution of a linear system which may be carried out using standard matrix methods [1].

The selection of the kernel is essential for the successful performance of the SVM algorithms. The kernel encodes the covariance structure between the quantity being modeled and the features and is a positive definite function. This property brings to bear the tools of functional analysis and leads to the solution of a convex optimization problem which has a unique optimum. The positive definite Gaussian and polynomial kernels are popular choices and pertinent for the flow physics of the marine hydrodynamic flows studied in the present paper.

SVM kernels depend on a small number of hyper-parameters which are determined during the algorithm learning stage by a

cross-validation procedure. An additional hyper-parameter in the regularization term of the cost function is also determined by the same procedure. The Gaussian kernel hyper-parameter is its “scale” or standard deviation which encodes the degree to which neighboring input features interact. For small values of the scale the Gaussian models a weak correlation between the features. For large values of the scale the Gaussian kernel reduces to a “flat” function which implies a nonlinear polynomial-like dependence of the quantity under study upon the features. This dependence may include linear, quadratic, cubic or higher-order terms which follow from a Taylor series expansion of the Gaussian kernel. For complex fluid flows encountered in marine hydrodynamics the proper value of the scale is not a priori known and is “learned” by the SVM algorithm.

Often a large number of experimental samples is necessary for the training of the LS-SVM algorithm leading to the inversion of a large matrix. A key consideration is the numerical conditioning of the matrix equation to be inverted and robust algorithms must be developed. Positive definite kernels lead to matrices with positive eigenvalues which are easier to solve. An additional benefit of the Gaussian kernel is that its eigenvalues and eigenfunctions are known analytically. This permits the development of robust inversion algorithms even for large and ill-conditioned linear systems for a large number of samples necessary for the training of the LS-SVM algorithm for complex flows. These attributes of the Gaussian kernel have contributed to its widespread popularity.

In marine hydrodynamics a quadratic, cubic or higher-order nonlinear dependence of a load upon the flow or vessel kinematics are quite common. Examples include forces due to flow separation around bluff bodies and around bilge keels in the roll motion problem. Such nonlinear loads are usually modeled by Morison’s equation with inertia and drag coefficients determined empirically. In the multi-dimensional ship maneuvering problem the hydrodynamic derivatives are often modelled by including linear, quadratic and higher-order polynomial nonlinearities. For both types of problems may be treated by the LS-SVM algorithm using a Gaussian kernel trained against experiments. This leads to a unified nonlinear model which includes multi-dimensional polynomial representations obtained as a special case for small values of the scale of the Gaussian SVM kernel.

The LS-SVM treatment of the ship roll damping problem is carried out along the following lines. The availability is assumed of experimental measurements of the roll kinematics either from free-decay tests, forced oscillation experiments or the roll response record in regular or irregular waves. Invoking Newton’s law the hydrodynamic force time record may be derived from experiments as a function of the measured roll response kinematics defined as the “features”. The training of the SVM algorithm then leads to a nonlinear model of the hydrodynamic force as a function of the roll displacement,

velocity and acceleration including linear and nonlinear hydrostatic, potential flow and viscous separated flow effects.

Forecasting of seastate elevations and vessel responses is useful in a variety of contexts in the fields of seakeeping and ocean renewable energy. Such forecasts are valuable for the vessel navigation in severe seastates and the development of advanced algorithms for the control of offshore wind turbines and wave energy converters. The LS-SVM algorithm generates forecasts using past time records of the seastate elevation and vessel responses defined as “features”. Filtering of these records is not necessary, circumventing the undesirable phase shift that may result from the use of band-limiting filter transfer functions. Wave forecasts using the LS-SVM algorithm using the Gaussian kernel are found to perform consistently better relative to the advanced auto-regression algorithms that require filtering. Accurate forecasts of seastate elevation records and vessel responses based on towing tank data were generated 5-10 seconds into the future.

In the present paper the basic attributes of the LS-SVM algorithm are summarized. Its performance is then illustrated for the modeling of the nonlinear hydrodynamic forces in the roll motion problem from free decay tests and the forecasting of seastate elevations based on tank data.

2 SUPPORT VECTOR MACHINE ALGORITHMS

The present section reviews the basic attributes of the SVM algorithm establishing connections with the marine hydrodynamic flows studied in subsequent sections. Detailed presentations of the SVM algorithms may be found in [1] and [2].

2.1 Support Vector Machine Regression

Consider a physical quantity y dependent upon a set of k features cast in vector form $\mathbf{x} = (x^1, x^2, \dots, x^k)^T$. For example y may represent the seastate elevation at the current time step and $\mathbf{x} = (x^1, x^2, \dots, x^k)^T$ the record of the values of y over k past time steps. Alternatively y may represent the roll moment in a free decay test of a ship section obtained in terms of the roll kinematics by invoking Newton’s law. In his case the k features $\mathbf{x} = (x^1, x^2, \dots, x^k)^T$ are the contemporaneous values of the roll displacement, velocity and acceleration, i.e. $k=3$. If memory effects are important, past values of the roll kinematics must be included in the features. In this case each of the k features with $k=1, \dots, 3$, is a vector with dimension n , where n is the number of prior time steps over which the roll displacement, velocity and acceleration have been recorded. In the case of the wave elevation forecasting problem, the dependent variable is the current wave elevation and the scalar “features” are k past values of the wave elevation. In the case of the roll problem, the dependent variable is the roll hydrodynamic moment which

depends on k contemporaneous scalar “features” with $k=1$ being the displacement, $k=2$ the velocity and $k=3$ the acceleration. If memory effects are accounted for each of the k features with $k=1,2,3$ are vectors with dimension n , where n is the number of previous time steps over which their values has been recorded. The SVM algorithm can readily handle a large number k of scalar or vector features with a large vector dimension n . Moreover in order to train the SVM algorithm a sufficiently large number of “samples” N for each feature, scalar or vector, must be available. In the present context these are obtained from experimental measurements. The magnitude of k , n and N may be large and their relative size is not restricted. An extensive literature exists illustrating the development of SVM algorithms in a wide range of disciplines for very large of values of k , n and N depending on the application.

The SVM regression algorithm generates a nonlinear physical model for y in terms of the vector $\mathbf{x} = (x^1, x^2, \dots, x^k)^T$. The number of features k , scalar or vector, that are pertinent to include may be initially unknown and it is often appropriate to air on the side of caution and include more features than may be apparent by the flow physics. The SVM algorithm is often used in a subsequent stage to “prune” the features and reduce them to a compact subset in a parsimonious SVM model of acceptable accuracy.

The SVM nonlinear regression assumes the following functional dependence of y on \mathbf{x} :

$$y = \sum_{j=1}^M w_j \phi_j(\mathbf{x}) + b \quad (1)$$

The series expansion in (1) involves M unknown weights w_j and basis functions $\phi_j(\mathbf{x})$. The constant b is the bias or the mean value of the quantity being modeled and is also assumed unknown. The magnitude of M is a priori unknown and may be infinite. It does turn out that M does not need to be specified in most implementations of LS-SVM. The algorithm is also initially silent about the mathematical form of the basis functions and it turns out that the statement of their explicit form is not necessary. This is a key property of the SVM algorithm discussed later in this section.

Assuming that a sample of training data $\{(\mathbf{x}_i, y_i)\}_{i=1}^N$ is available the LS-SVM algorithm minimizes the following cost function:

$$\min_{w,e} R(w,e) = \frac{1}{2} \|w\|^2 + \frac{1}{2} \gamma \|e\|^2 \quad (2)$$

Where:

$$e_i = y_i - \sum_{j=1}^M w_j \phi_j(\mathbf{x}_i) - b, \quad i = 1, 2, \dots, N \quad (3)$$

$\|\cdot\|$ denotes the L_2 Euclidean norm, γ is the regularization parameter which controls the trade-off between the bias and variance of LS-SVM model and \mathbf{e} is the error vector, $\mathbf{e} = (e_1, e_2, \dots, e_N)^T$.

Eq. (2) and (3) form a standard optimization problem with equality constraints. The Lagrangian of this problem is:

$$L(w, b, e, \lambda) = R(w, e) - \sum_{i=1}^N \lambda_i (w^T \phi(\mathbf{x}_i) + b + e_i - y_i) \quad (4)$$

Where, λ_i are the Lagrange multipliers and a compact vector notation for the weights w_j and basis function $\phi_j(\mathbf{x}_i)$ has been adopted.

According to the Karush-Kuhn-Tucker Theorem, the conditions of optimality are:

$$\begin{aligned} \frac{\partial L}{\partial w} = 0 &\rightarrow w_j = \sum_{i=1}^N \lambda_i \phi_j(\mathbf{x}_i) \\ \frac{\partial L}{\partial b} = 0 &\rightarrow \sum_{i=1}^N \lambda_i = 0 \\ \frac{\partial L}{\partial e_i} = 0 &\rightarrow \lambda_i = \gamma e_i \\ \frac{\partial L}{\partial \lambda_i} = 0 &\rightarrow w^T \phi(\mathbf{x}_i) + b + e_i - y_i = 0 \end{aligned} \quad (5)$$

Cast Eq. (5) into a linear matrix equation:

$$\begin{bmatrix} 0 & \bar{\mathbf{1}}^T \\ \bar{\mathbf{1}} & \mathbf{K} + \gamma^{-1} \mathbf{I} \end{bmatrix} \begin{bmatrix} b \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{y} \end{bmatrix} \quad (6)$$

Where, $\bar{\mathbf{1}} = (1, 1, \dots, 1)^T$. \mathbf{I} is the identity matrix. $\mathbf{y} = (y_1, y_2, \dots, y_N)^T$. $\mathbf{K} = (k(\mathbf{x}_i, \mathbf{x}_j))_{i,j=1}^N$ is called the kernel matrix, and $k(\mathbf{x}_i, \mathbf{x}_j) = \phi^T(\mathbf{x}_i) \phi(\mathbf{x}_j)$. The length of the vector $\phi^T(\mathbf{x}_i)$ is M and the dimensions of the square kernel matrix \mathbf{K} are $N \times N$, where N is the size of the training sample.

It follows that using the LS-SVM regression model the quantity y can be expressed in the form:

$$y(\mathbf{x}) = \sum_{i=1}^N \lambda_i k(\mathbf{x}, \mathbf{x}_i) + b \quad (7)$$

From (6) and (7), it can be seen that neither the basis functions $\phi_j(\mathbf{x})$ nor their number M in (1) need to be specified explicitly. All LS-SVM requires is the inner product of $\phi_j(\mathbf{x})$, i.e., the kernel function $k(\mathbf{x}_i, \mathbf{x}_j)$. This property is known as the “kernel trick” and is a key attribute of the SVM machine learning algorithm.

Some widely used kernels are the linear, polynomial and Gaussian functions. In this study, the popular Gaussian kernel

$$k(\mathbf{x}, \mathbf{z}) = \exp(-\|\mathbf{x} - \mathbf{z}\|^2 / \sigma^2) \quad (8)$$

and the polynomial kernel

$$k(\mathbf{x}, \mathbf{z}) = (\mathbf{x}^T \mathbf{z} + t)^d \quad (9)$$

are used.

In (8), $\|\cdot\|$ denotes the 2-norm of a vector. σ is the “scale” that determines the width or variance of the Gaussian kernel. d is the degree of the polynomial kernel and t is its bias term. More generally, the value of d may be positive or negative, it does not need to be an integer, but its value and that of the bias must be such that the kernel (9) is positive definite ([2]).

Expression (7) provides an explicit nonlinear model for the dependent quantity $y(\mathbf{x})$. The summation in (7) is over the number of samples N used to train the SVM algorithm with the values of the sample features $\mathbf{x}_i, i=1, \dots, N$ which appear in the second argument of the kernel. The Lagrange multipliers λ_i are obtained from the solution of the linear system (6) and are known in the SVM literature as the “support vectors”.

The hyper-parameters (γ, σ) and (d, t) are calibrated to optimal values during the training and validation stages of the SVM nonlinear regression using a sufficiently large sample of features. As soon as the values of the hyper-parameters have been determined the nonlinear model (7) may be used either to generate forecasts or to represent complex hydrodynamic dominated loads dependent on the selected set of features.

2.2 Kernel Selection

The selection of the Gaussian kernel appears at first to be somewhat arbitrary. Moreover its connection to the set of basis functions $\phi_j(\mathbf{x})$ has not yet been made explicit. Assume that the physical quantity under study has a well-defined mean and that is otherwise oscillatory around its mean, a common occurrence in marine hydrodynamic applications dealing with signals that are deterministic or quasi-stationary and stochastic. In such cases appropriate basis functions would be a set of orthonormal functions in a multi-dimensional space with dimensions equal to the number of features.

The connection between the kernel and the basis functions in the SVM algorithm is established by Mercer’s theorem ([2]) which states that for a positive definite kernel:

$$\iint_{\mathbf{x}} k(\mathbf{x}, \mathbf{z}) \phi_j(\mathbf{z}) d(\mathbf{z}) = \mu_j \phi_j(\mathbf{x}) \quad (10)$$

$$k(\mathbf{x}, \mathbf{z}) = \sum_{j=1}^{\infty} \mu_j \phi_j(\mathbf{x}) \phi_j(\mathbf{z})$$

The solution of the first kind integral equation (10) is in principle not available in closed form nor is the a priori selection of the kernel evident. A reasonable selection of the basis functions capable to accurately describe the physical quantity under study according to (1) would a reasonable starting point. For such a basis function set the kernel would be the generating function as indicated by the second equation in (10). This would also require knowledge of the eigenvalues. Moreover the robust performance of the LS-SVM algorithm is a consequence of the positive definite kernel which guarantees a unique solution of the optimization problem (2). Within the LS-SVM algorithm the positive definiteness of the kernel matrix \mathbf{K} in (6) makes available robust algorithms for the inversion of large linear systems that arise when a large number of training samples is necessary.

For the Gaussian kernel the solution of (10) is available in closed form in any number of dimensions. The basis functions $\phi_j(\mathbf{x})$ are the generalized Hermit functions which are orthogonal over the entire real axis and are known to be a robust basis set for the representation of the wide range of sufficiently smooth functions. This is the case for the marine hydrodynamic applications considered in the present paper.

Consider the multi-dimensional Gaussian kernel assuming K un-correlated features. The explicit solution of (10) takes the form:

$$k(\mathbf{x}, \mathbf{z}) = \exp\left[-\varepsilon_1^2(x_1 - z_1)^2 - \varepsilon_2^2(x_2 - z_2)^2, \dots, -\varepsilon_K^2(x_K - z_K)^2\right]$$

$$= \sum_{k \in \mathbf{N}^K} \mu_k \phi_k(x) \phi_k(z) \quad (11)$$

Where, $\varepsilon_k^2 = 1/\sigma_k^2$, and σ_k refers to the constant determining the scale or variance of the k -th feature of Gaussian kernel (as in Eq. (8)). The cross-correlation of the features is assumed to vanish following a Principal Components Analysis or singular value decomposition of the covariance matrix of input feature dataset.

The eigenvalues and eigenfunctions in (11) are available in closed form:

$$\mu_k = \prod_{j=1}^K \mu_{k_j} = \prod_{j=1}^K \sqrt{\frac{\alpha_j^2}{\alpha_j^2 + \delta_j^2 + \varepsilon_j^2}} \left(\frac{\varepsilon_j^2}{\alpha_j^2 + \delta_j^2 + \varepsilon_j^2}\right)^{k_j - 1} \quad (12)$$

$$\phi_k(\mathbf{x}) = \prod_{j=1}^K \phi_{k_j}(x_j) = \prod_{j=1}^K \gamma_{k_j} \exp(-\delta_j^2 x_j^2) H_{k_j - 1}(\alpha_j \beta_j x_j) \quad (13)$$

Where, $H_n(\bullet)$ is the classical Hermite polynomial of degree n . α_j are the integral weights which are related to the global scale of the problem. ε_j are the scale parameters which are related to the local scale of the problem. $\delta_j, \beta_j, \gamma_{k_j}$ are auxiliary parameters defined in terms of α_j, ε_j . Refer to [3] for details on the derivation of (12) & (13).

This formulation of (12) and (13) allows us to select different shape parameters ε_j and different integral weights α_j for different space dimensions (i.e., \mathbf{K} may be an anisotropic kernel), or we may assume that they are all equal (i.e., \mathbf{K} is spherically isotropic).

The eigenvalues of the Gaussian kernel are seen in equation (12) to be positive therefore the matrix of the linear system (6) is positive definite. The basis functions $\phi_k(x)$ in (13) are the product of an exponential term and Hermite functions where both are dependent on the auxiliary parameters α_k which must be properly selected. While these parameters do not appear explicitly in the definition of the kernel they affect the condition number of the matrix in equation (6). They must be properly selected to determine the rank of the matrix \mathbf{K} and in order to develop a robust inversion algorithm for the inversion of large linear systems (6) that may be ill-conditioned. More details on the robust inversion of (6) are presented in [3].

The set of equations (11)-(13) underscore the popularity of the Gaussian kernel in LS-SVM applications. The reason is that the orthonormal Hermite functions are known to be a robust basis set for the approximation of a wide range functions on the entire real axis. These properties of the Gaussian kernel have led to the use of the LS-SVM algorithms in wide range of problems and underscore its popularity.

In a number of LS-SVM applications a polynomial kernel is used instead of the Gaussian. In the context of the marine hydrodynamics applications this is equivalent to replacing the Gaussian in the right-hand side of (7) by a polynomial of $(\mathbf{x}, \mathbf{x}_i)$ which may involve linear, quadratic, cubic and higher order terms. On closer inspection of (11) this is equivalent to expanding the Gaussian kernel into Taylor series for small values of the inverse scales ε_k^{-2} .

A polynomial representation of the physical quantity $y(\mathbf{x})$ would for example be justified when developing an LS-SVM model for a viscous load in terms of the ambient flow kinematics, the Morison drag formula being an example. Another example involves the representation of the hydrodynamic derivatives in the ship maneuvering problem by a high-order polynomial of the ship kinematics. It follows from the Taylor series expansion of (11) that the polynomial kernel with an integer power d is related to the Gaussian kernel for small values of ε_k^{-2} for some or all of the k features. Therefore

the use of the polynomial kernel may be unnecessary and emphasis must instead be placed upon the proper calibration of the parameters ε_k^{-2} for each of the k features depending of the physics of the flows under study. In a number of applications the same value of ε^2 for all features is selected simplifying the calibration process often with very satisfactory results. In marine hydrodynamics applications the selection of small values of ε_k^{-2} for some features may be appropriate but not for others, leading to a kernel that is a mixture of polynomial like factors for some features and exponential factors for others. These choices will be determined by the cross-validation procedure during the training of the LS-SVM algorithm.

3 SHIP ROLL HYDRODYNAMICS MODELLING VIA SVM REGRESSION

The hydrodynamic modelling of ship roll motions is of great interest and is significantly affected by various nonlinear effects. The LS-SVM regression algorithm is applied in this section to study the modelling of ship roll hydrodynamics. The study is based on free decay tests in a tank experiment of a barge with and without liquid cargo in spherical tanks. More detailed information about the tank tests is described in [4].

3.1 Free Decay Tests

For a free decay test of the ship rolling motion, the 1DOF equation of motion can be expressed as:

$$I\ddot{\xi} + F_h(\ddot{\xi}, \dot{\xi}, \xi) + K\xi = 0 \quad (14)$$

Where, I is the moment of inertia of the ship hull structure and K is the hydrostatic restoring coefficient. F_h denotes the hydrodynamic moment of the ship roll motion, which includes contributions from added mass, damping and nonlinear restoring effects. $\xi, \dot{\xi}, \ddot{\xi}$ are the ship roll displacement, velocity and acceleration, respectively.

From Eq. (13), the hydrodynamic force F_h in a free decay test can be derived from:

$$F_h(\ddot{\xi}, \dot{\xi}, \xi) = -(I\ddot{\xi}(t) + K\xi(t)) \quad (15)$$

The displacement $\xi(t)$ was directly measured in the experiments. The velocity and acceleration $\dot{\xi}, \ddot{\xi}$ are obtained from a finite difference approximation.

The free decay tests were conducted under three different conditions (Table 1). The sketch of the experimental set-ups [4] is shown in Figure 1.

TABLE 1. LOAD CONDITIONS OF THE FREE DECAY TESTS

Case NO.	Initial displacement (degrees)	Liquid or solid cargo	With or without bilge keels
1	10	Solid	No Bilge keels
2	10	Solid	Bilge keels
3	10	Liquid	Bilge keels

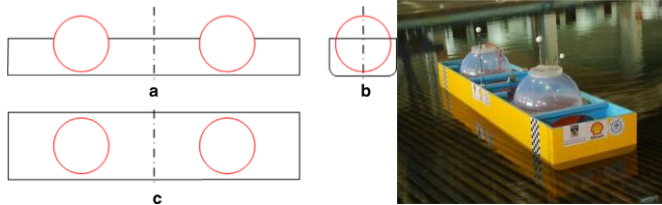


FIGURE 1. SKETCH OF EXPERIMENT SET-UPS [4]

3.2 SVM Regression Model

Clearly, the liquid cargo or the bilge keels would incite various different nonlinear flow effects and loads. The original time series of the free decay tests under the three loading conditions thus have different periods and decaying rates accordingly (Figure 2). When modelled using traditional nonlinear damping models as in [4], the nonlinear effects of the liquid cargo motion or bilge keels would be approximated using different linear and nonlinear damping coefficients. In the SVM regression model, the different flow effects would result in different optimized nonlinear kernel selections and hyper-parameter values.

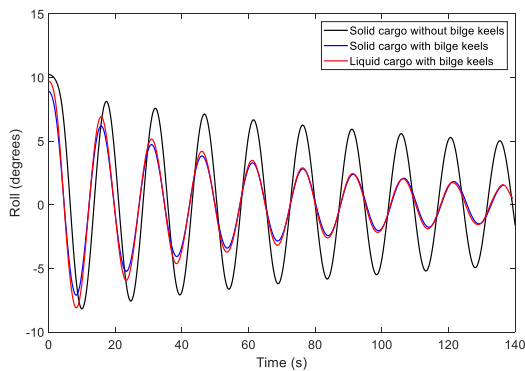


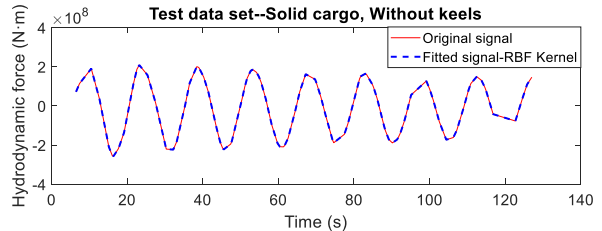
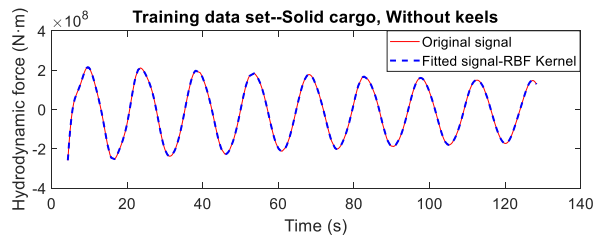
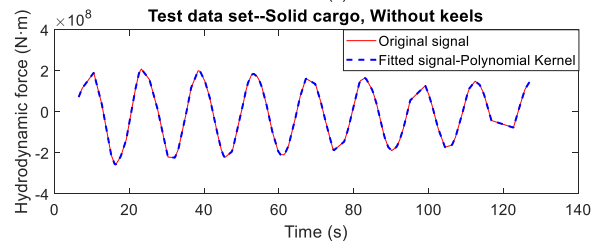
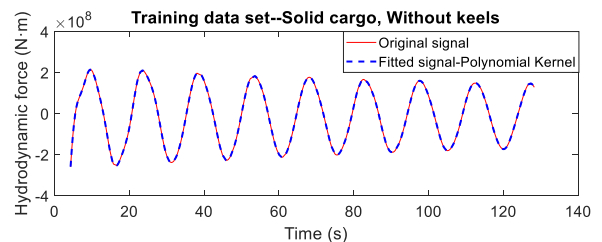
FIGURE 2. MEASURED TIME SERIES OF SHIP ROLL FREE DECAY TESTS

The total number of time samples used in each case is 400, among which a random selection of 300 samples are used to train the SVM hydrodynamic models and the rest 100 samples are used as test cases to validate the model.

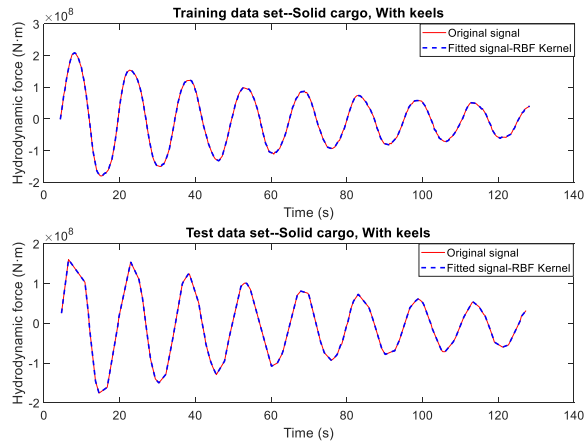
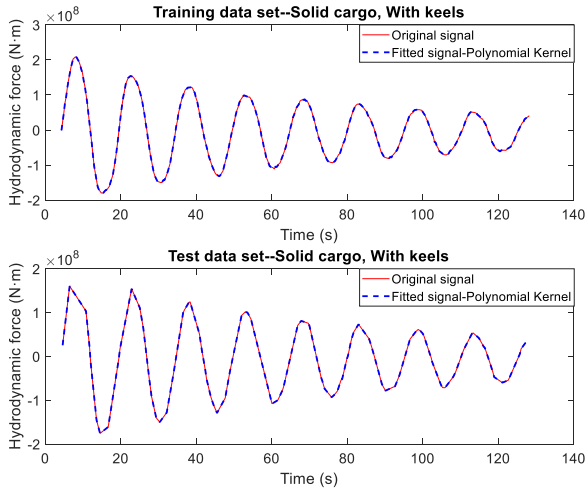
The ship roll displacement, velocity and acceleration are used as features (i.e., $\mathbf{x} = [\xi, \dot{\xi}, \ddot{\xi}]$) in the SVM algorithm. Both the polynomial kernel (Eq. 9) and Gaussian kernel (Eq. 8) have been tested. The hyper-parameters of the SVM regression model are optimized via a 10-fold cross-validation including the

regularization parameter γ , the Gaussian kernel width parameter σ or the power and bias d, t of the polynomial kernel. Intuitively the polynomial kernel implies that the hydrodynamic force is a high-order polynomial function of the ship roll displacement, velocity and acceleration and the Gaussian kernel implies a more general nonlinear dependence.

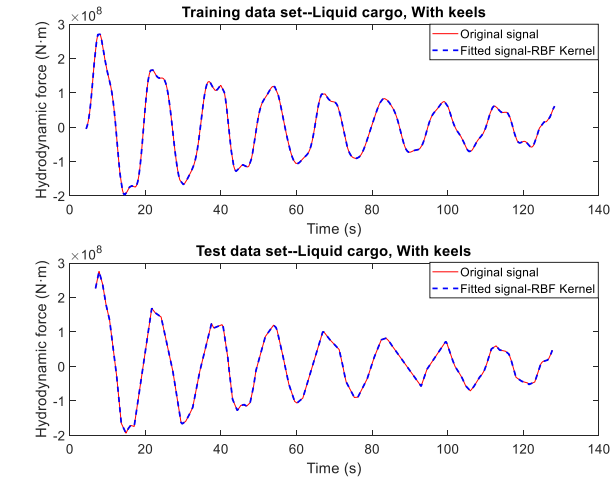
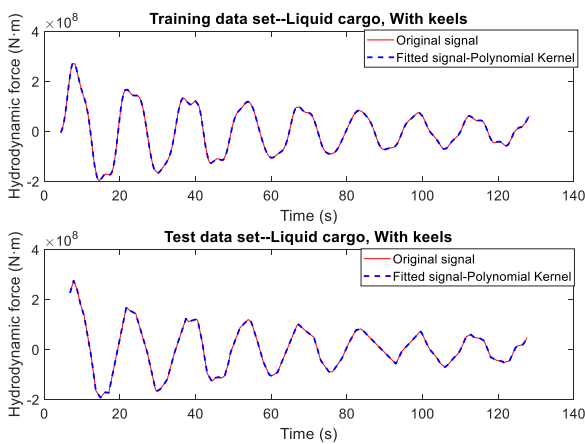
The results of the training and test data sets are shown in Figure 3 for each of the three cases. From these results, the SVM regression model can capture the nonlinear mapping relations between the ship roll kinematics and the corresponding hydrodynamic forces with appropriate training under all three scenarios. The power of the polynomial kernel optimized via cross-validation is 4 ~ 6.



(A) TRAINING AND TEST RESULTS OF CASE 1: SOLID CARGO, NO BILGE KEELS



(B) TRAINING AND TEST RESULTS OF CASE 2: SOLID CARGO, WITH BILGE KEELS



(C) TRAINING AND TEST RESULTS OF CASE 3: LIQUID CARGO, WITH BILGE KEELS

FIGURE 3. SVM REGRESSION RESULTS

The SVM regression models can be used to further study the complex flow physics of the hydrodynamic loads induced by bilge keels or the motion of the liquid cargo. From Eq. (7) and Eq. (15), the hydrodynamic force model learned by the SVM algorithm from separate free decay tests can be expressed as:

$$F_{hs}(\mathbf{x}) = \sum_i \alpha_i K_1(\mathbf{x}, \mathbf{x}_i) + b_1$$

$$F_{hl}(\mathbf{x}) = \sum_i \beta_i K_2(\mathbf{x}, \mathbf{x}_i) + b_2 \tag{16}$$

For example, F_{hs}, F_{hl} denote the hydrodynamic force models for the vessel with solid cargo and liquid cargo obtained from separate free decay tests. Analogous force models have been derived above for the vessel with solid cargo, without and with bilge keels from separate free decay tests. Equation (16) obtained from the training of the SVM algorithm for each free-decay test reveals a nonlinear dependence of the respective hydrodynamic forces on the features, namely the vessel displacement, velocity and acceleration of the feature samples used to train the algorithm. This dependence includes nonlinear hydrostatic effects, and viscous separated flow effects upon the vessel roll added-moment of inertia and damping mechanisms.

Equations (16) are nonlinear models of the hydrodynamic force expressed as functions of the current values of the vessel kinematics \mathbf{x} and the values \mathbf{x}_i of the N feature samples measured in the free decay test. Assume that the model (16) is valid in a more general setting where the current vessel kinematics \mathbf{x} corresponds to a forced oscillation experiment or the interaction of the vessel with ambient waves. In this setting \mathbf{x}_i are fixed at their values obtained from the controlled free decay tests and are constants of the models (16). The model (16) may be used to extract more information about the physics

of the individual force mechanisms associated with the flow around bilge keels and due to liquid cargo.

Taking the difference of the hydrodynamic forces derived from the two forced oscillation tests, one with solid and the second with liquid cargo, the contribution of the hydrodynamic forces due to the liquid cargo motion can be derived as:

$$\Delta F(\mathbf{x}) = \sum_i \{ \alpha_i K_1(\mathbf{x}, \mathbf{x}_i) - \beta_i K_2(\mathbf{x}, \mathbf{x}_i) \} + \Delta b \quad (17)$$

A similar derivation applies to study the hydrodynamic effects of bilge keels. The differential force model (17) may be further validated against independent experimental data and will be the subject of future studies.

4 SHORT-TERM WAVE ELEVATION FORECAST

The short-term forecast of wave elevations is a critical issue to various operational or control problems for ships, offshore platforms and ocean renewable energy systems [5].

The implementation of the LS-SVM regression for the prediction of wave elevations considers a one-step ahead prediction for a time series using the nonlinear autoregressive model first:

$$\eta_{t+1} = f(\eta_t, \eta_{t-1}, \dots, \eta_{t-d+1}) \quad (18)$$

Where, η_t denotes the sampled time series. d is the order of autoregressive model.

In the context of the LS-SVM, the training data $\{\mathbf{x}_i, y_i\}_{i=1}^{N_{training}}$ are formatted as:

$$\begin{aligned} \mathbf{x}_i &= [\eta_i, \eta_{i-1}, \dots, \eta_{i-d+1}] \\ y_i &= \eta_{i+1} \end{aligned} \quad (19)$$

Where, $N_{training}$ is the number of the data sets, or “samples”, used in the training process.

Denote the current time as t_c , then for the one-step ahead prediction, the input and output in Eq. (7) is:

$$\begin{aligned} \mathbf{x} &= [\eta_{t_c}, \eta_{t_c-1}, \dots, \eta_{t_c-d+1}] \\ y &= \eta_{t_c+1} \end{aligned} \quad (20)$$

To achieve multi-step ahead prediction, one only needs to repeat the one-step ahead prediction multiple times, substituting the output y_i in Eq. (17) as η_{i+k} in the training step and similarly y in Eq. (18) as η_{t_c+k} in the forecast ($k = 1, 2, \dots, N_{forecast}$).

Two wave records under different sea states measured in tank tests are used in this study to validate the forecast performance

of the SVM regression algorithm. The sampling rate of the wave records is 0.495 seconds and the forecast horizon is 5 seconds.

The Gaussian kernel is chosen for the forecast algorithm, and both hyper-parameters γ and σ are optimized through 10-fold cross-validation as well. The order of the autoregressive model d and the number of training samples $N_{training}$ are determined based on sensitivity studies to obtain the most consistent and robust results. The order of the autoregressive model corresponds to around 1~2 typical wave periods and the number of training samples is equivalent to about 50~60 wave periods.

The Root-Mean-Square (RMS) error of the forecasted signal is defined as:

$$RMS \text{ error} = \sqrt{\frac{1}{N} \sum_{k=1}^N |\eta_k - \tilde{\eta}_k|^2} \quad (21)$$

Where, $\tilde{\eta}_k$ is the forecasted wave elevation. η_k is the original wave elevation. To better evaluate the forecast performance, the RMS error is normalized using the significant wave height H_s .

Three 300-second segments of the original wave records are forecasted for the two sea states separately. The statistical results of the forecast error are summarized in Table 2. The overall RMS error of the entire forecasted signal of the three segments is summarized. Besides, the maximum RMS error for each five-second forecast horizon is listed as a measure of worst-case performance. Comparisons of the original and forecasted wave elevations of one segment are shown here to illustrate the forecast performance (Figure 5 and Figure 6).

TABLE 2. STATISTICAL RESULTS OF THE FORECAST ERROR

Sea state	Overall RMS Error/ H_s (%)	Maximum 5-second RMS Error / H_s (%)
Sea state 1: Hs=1.7m, Tp=8.7s	13.16	32.33
Sea state 2: Hs=4.5m, Tp=11.8s	12.74	32

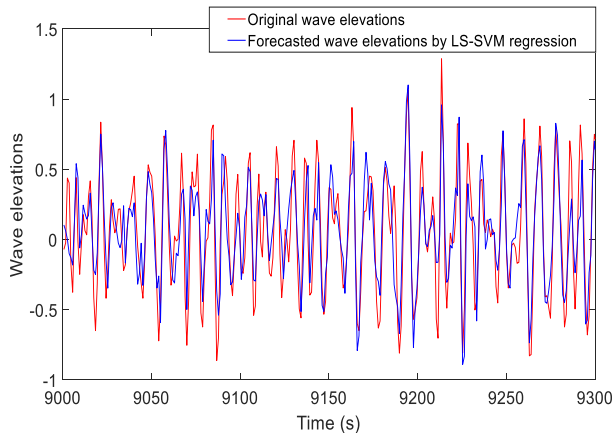


FIGURE 5 SEA STATE 1: RMS ERROR = 12.8% HS

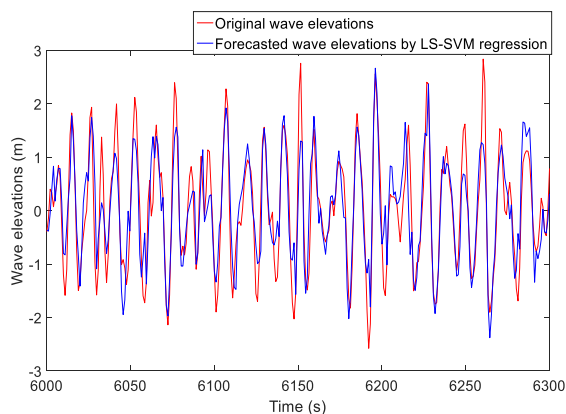


FIGURE 6. SEA STATE 2: RMS ERROR = 12% HS

All tested records are the original measured wave records without any filtering. The real-time measured wave elevations contain unknown noise. To forecast a time series, the challenge is to learn higher frequency components of the signal itself while canceling noise simultaneously. This can be interpreted as finding the optimal balance between overfitting the model during training in sample and achieving the best forecasts out of sample.

From the results shown in Table 2 and Figure 5~Figure 6, the LS-SVM regression method can forecast the real-time wave elevations 5 seconds into the future with good accuracy. Furthermore, its performance is consistent and robust regardless of the noise ratio or the band-width of the signal.

5 DISCUSSION AND CONCLUSIONS

In this study, the SVM regression algorithm is thoroughly reviewed. Two promising aspects of its applications are studied in this paper: to carry out the physical modelling of complex

marine hydrodynamic flow problems and to forecast real-time noisy signals in a seastate.

Through an appropriate training process combined with convex optimization schemes, the SVM regression method can produce nonlinear mapping relations from the “features” to “targets” with great accuracy for both the modelling (i.e., interpolation) and the forecast (i.e., extrapolation) problems. The kernelized method itself and the optimization on hyper-parameters through cross-validation have both enhanced its generalization capability.

As a result, it is able to model the ship roll hydrodynamics with different loading conditions and ship configurations. Moreover its performance on the forecast of real-time wave elevations is also consistent and robust with good accuracy.

The hydrodynamic modelling of the ship roll motions can be further applied and extended to the maneuvering or seakeeping problems in the presence of irregular waves. Under such scenarios, the real-time prediction of the wave elevations and the physical modelling of ship hydrodynamics can be combined to better predict or improve the ships’ maneuvering or seakeeping performance and will be the subject of future studies.

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