Permeability Estimation from Field Data

by

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ABSTRACT

Strong correlations exist between core measured permeability values and the phase velocity and attenuation of the tube wave obtained from full waveform acoustic logs. A number of authors have applied the Biot theory to the borehole wave propagation problem in order to explain the observed correlations with reasonably good success. In this paper we present two methods of estimating the absolute in-situ permeability from acoustic log data. Comparisons between measured tube wave velocity and attenuation and model predictions indicate that the Rosenbaum formulation of the Biot model can explain most of the tube wave attenuation data. Based on these findings, an inverse problem is formulated to estimate in-situ permeability from tube wave attenuation measurements using the Biot-Rosenbaum model. Resulting permeability estimates from two field data sets are in agreement with smoothed core permeability measurements. A second estimation method is based on tube wave velocity measurements. By taking the difference between the measured tube wave travel time and the calculated elastic travel time, a new measure is obtained which is referred to as the $\Delta\Delta T$ value. A cross plot of logarithmic core permeability values versus $\Delta\Delta T$ values for data from two different lithologies gives an excellent linear trend for the permeability range of 0.1 to 2000 millidarcies.

INTRODUCTION

Williams et al. (1984) and Zemanek et al. (1985) have demonstrated that significant correlations exist between core measured permeability and the velocity and attenuation of the tube wave arrival on full waveform acoustic logs. In order to explain the observed tube wave behavior, and try to estimate the in-situ permeability from these measurements, a number of authors have utilized the Biot (1956a,b) model of wave propagation in a porous and permeable medium which was first applied to the borehole geometry by Rosenbaum (1974) (Schmitt, 1985; Hsui et al., 1985; Cheng et al., 1987; Burns and Cheng, 1986). White (1983) developed a low frequency approximation to this problem which is in general agreement with the Biot theory results. In this paper, two methods of obtaining permeability estimates from tube wave measurements are presented. One method is based on Biot theory. The Biot-Rosenbaum model predictions are compared to field data and an inverse problem is formulated to estimate permeability, shear wave Q, and pore fluid compressibility. The second estimation method is empirical. In this method, the measured tube wave phase velocity is corrected for elastic parameter variations. The difference between the measured tube wave velocity and the predicted elastic velocity is assumed to be due to permeability effects. Data is presented from two lithologic sequences that support this assumption.

FORWARD MODELLING - THE BIOT-ROSENBAUM MODEL

Overview of the Model

Biot (1956a,b) developed a theoretical model of a two phase medium which can be used to describe wave propagation in a porous and permeable formation. The model treats the medium as a solid elastic matrix containing a compressible viscous fluid. Biot defines average stresses on the solid and fluid phases of the medium, and strains in terms of the displacements of the skeleton and fluid. Equations of motion are derived from the Lagrange equations which yield two coupled differential equations in terms of displacements in the solid and fluid phases. These equations are then separated into equations in terms of dilatation and rotation only. Dissipation is proportional to the relative motion between the solid frame and the viscous pore fluid. The dissipation, therefore, is controlled by the ease with which the pore fluid moves through the solid skeleton of the medium. As such, the permeability of the medium and the viscosity of the pore fluid enter the formulation through this dissipation term. At low frequencies the fluid flow is assumed to be laminar and to follow Darcy's law. In order to extend his formulation to high frequencies (non-laminar flow), Biot (1956b) introduced a complex correction factor to the dissipation function by deriving the friction and viscous forces due to the oscillation of cylindrical tubes containing a viscous fluid. He extended this factor to other pore shapes and tortuosity values (referred to as the structural factor), but found that the solutions are relatively insensitive to most reasonable variations.

The coupled equations result in two dilatational waves and one rotational wave which propagate in the porous medium. The dilatational wave of the first kind is the normal P wave and is related to the in-phase motion of the solid and liquid phases. The dilatational wave of the second kind, or slow P wave, is a diffusive type wave which is related to an out of phase motion of the two phases (Biot, 1956a). All three body waves are dispersive and dissipative due to the viscous pore fluid motion. Lab measurements on a synthetic porous material have substantiated the existence and behavior of these three body waves (Plona, 1980).

Rosenbaum (1974) applied the Biot model to the borehole geometry and generated synthetic full waveform acoustic logs. Schmitt (1986b, 1986c) has generated synthetic logs in porous formations in both open and cased hole geometries, and has studied the effect of pore shapes on the arrivals. In this paper, only the simple open borehole geometry will be considered. For such a geometry, there are four boundary conditions which must be satisfied at the borehole wall:

i) continuity of radial displacement

ii) continuity of radial stress

iii) vanishing of axial stress

iv) continuity of fluid pressure (between the borehole and pore fluids)

If the borehole wall is completely sealed, by steel casing for example, the fluid pressure continuity condition is no longer applicable, and the new boundary condition would be that skeleton and pore fluid displacements be equal at the borehole wall. The period equation is derived by satisfying the four boundary conditions at the borehole wall. The result is a four by four determinant whose elements are given in Rosenbaum (1974).

The Rosenbaum (1974) formulation will be used to model the behavior of the borehole guided waves in a simple open hole geometry and porous and permeable formation. Following Rosenbaum, the formation structural factor is taken as $\sqrt{8}$ and the mass coupling coefficient is set at 3 for all calculations in this chapter. These values correspond to typical rocks (White, 1984), and represent a pore structure consisting of an orthogonal orientation of cylindrical tubes. Test calculations have been carried out with other values of these factors and the results indicate that the tube wave behavior is only slightly affected by the choice of values. The other parameters needed to model the porous formation using this model are:

- borehole fluid velocity and density

- pore fluid velocity, density, and viscosity

- dry P and S velocities of the formation

- grain material density and bulk modulus

- formation porosity and permeability

A final parameter in the Rosenbaum formulation is the acoustic pressure impedence factor κ. When κ is zero, the borehole fluid pressure and pore fluid pressures are equal. Rosenbaum referred to this situation as the 'open' hole case. As κ increases, the pressure communication between the two fluids decreases until in the limit as κ goes to infinity there is no pressure communication. This is referred to as the 'sealed' case by Rosenbaum and he used this as a model of the effects of an impermeable mudcake on the borehole wall. The actual effect of a thin mudcake layer on the borehole wall is unknown; however it seems likely that such a layer would still allow pressure communication between the two fluid systems.

One further note on the model concerns the pore fluid viscosity. The permeability and pore fluid viscosity appear in the Biot model as a ratio, that is, the conductivity or pore fluid mobility is the key factor.

Applications to Field Data

Williams et al. (1984) presented data from three boreholes which compared the Stoneley wave velocity and attenuation to smoothed core permeability measurements. All available geophysical measurements and core data from these boreholes have been made available for analysis. It should be emphasized that core permeability measurements can vary from actual in-situ values (as measured by a packer test, for example) by as much as two orders of magnitude (Brace, 1977). Inadequate sampling, damaged cores, and the presence of highly permeable fractures in the formation all contribute to such variations. The results presented here should be viewed in this light.

Two data sets will be examined in this paper. The first is from a borehole that penetrated a limestone-dolomite lithologic section, and the second is from a borehole that penetrated a very highly permeable sandstone unit in a sand-shale sequence. In both of these data sets, information on several necessary model parameters was not available. These parameters had to be estimated in order to generate theoretical results, and the methods used to make such estimates will be explicitly addressed when appropriate.

The first data set corresponds to the limestone-dolomite lithology. The data set for this example consists of: the full waveform acoustic log, the shear wave acoustic log, density log, caliper log (borehole radius measurement), and core measured porosity and permeability values at one foot increments in depth. Plots of the Stoneley wave travel time and energy ratio together with permeability values were published by Williams et al. (1984). For the purposes of comparing the Biot-Rosenbaum model to the Stoneley wave data, however, these curves are inadequate. In Figure 1 the core measured permeability data (with a five foot running average applied) are plotted against the peak frequency amplitude ratio and phase velocity of the Stoneley wave for this data set. The peak frequency of the Stoneley wave in this example is between 5500 and 6000 Hz. The amplitude ratio is simply the spectral ratio of the near and far receivers, and the measured phase velocity is obtained from phase spectrum differences (Aki and Richards, 1980). The phase unwrapping algorithm used is identical to that used by Willis (1983). The correlation between these measured values and the measured permeability data is excellent (the Stoneley wave curves have been shifted by + 3.05 m (10 feet) in depth relative to the core data to give the best correlation).

In order to test the applicability of the model to actual field data, the predicted phase velocity and amplitude ratios can be compared to the measured values. A number of parameters which are needed by the model, however, are unavailable for this first data set. These parameters are: the borehole fluid velocity, the pore fluid velocity, density, and viscosity, the bulk modulus of the matrix material, and the fluid and formation Q values. Each of these parameters has been estimated based on given information. The pore fluid present in the porous zones of the dolomite section is reported to consist of a mixture of oil and water. However, it is most likely that the pore fluid near the borehole wall consists of the borehole fluid filtrate. The borehole fluid, which is at pressures slightly greater than the pore fluid pressure, invades the formation leaving its solid particles on the borehole wall as a mudcake layer. The pore fluid velocity and density, then, should be related to the borehole fluid properties. In this borehole, the borehole fluid density is given as 1.2 gm/cc, and the velocity is estimated to be 1525 m/sec (5000 ft/sec) (the velocity estimate is based on comparisons between the measured and predicted Stoneley wave velocities in the non-permeable zone). Based on these values, the pore fluid velocity is estimated to be 1490 m/sec and the density is set at 1.0 gm/cc. The viscosity of the pore fluid is estimated to be 0.1 cP, which is the approximate viscosity of water at the reported bottom hole temperature of $132^{o}C$ ($270^{o}F$). The bulk modulus of the matrix material is assumed to be the value for calcium carbonate (77.9 GPa). Inversion of the Stoneley and pseudo-Rayleigh wave spectral ratios in the non-permeable zones of the section resulted in a fluid Q value of about 40 and a formation shear wave Q value of about 70 (Burns and Cheng, 1987). The formation compressional wave Q value, which has a negligible effect on the Stoneley wave attenuation, is set at 100 for the calculations. All modelling parameters are given in Table 1.

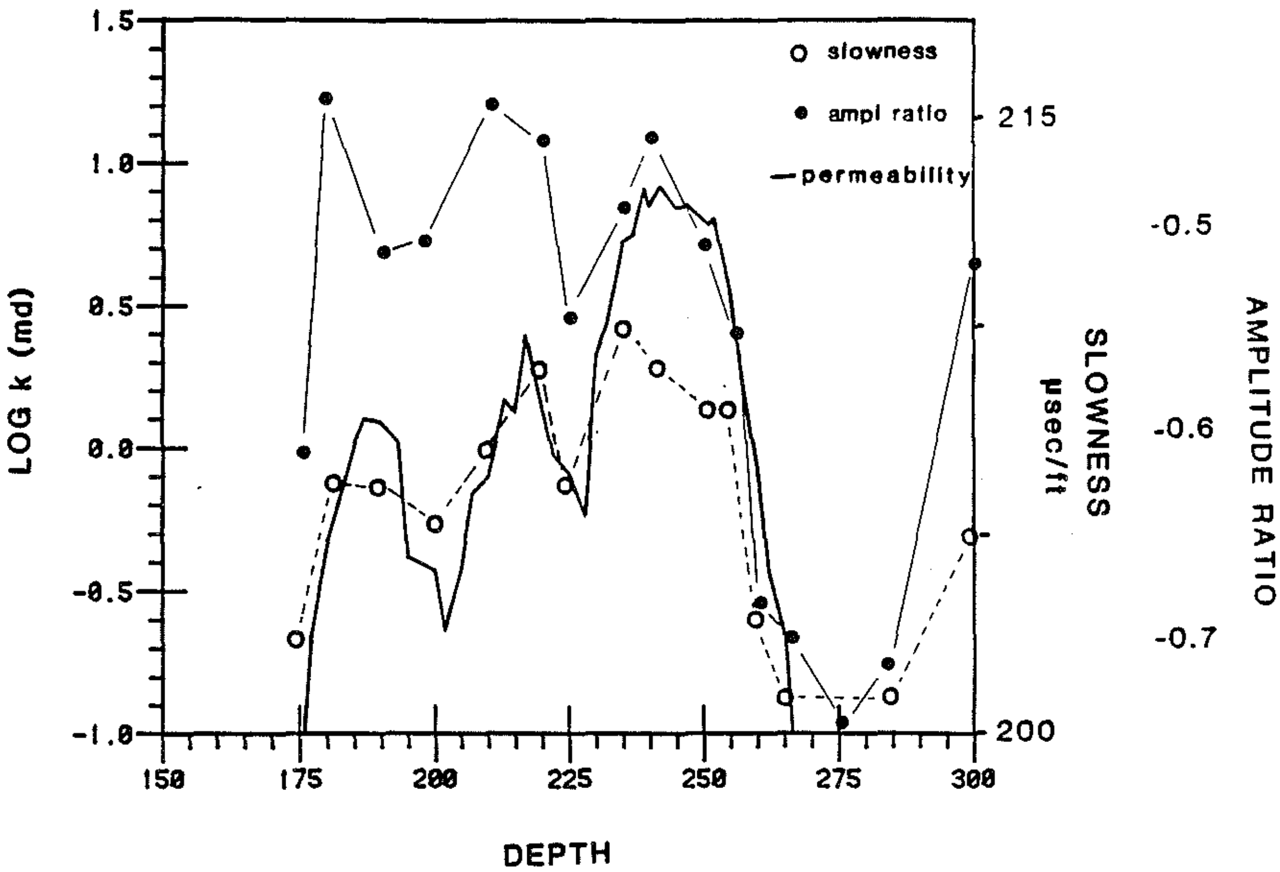

Figure 1: Measured Stoneley wave phase velocity and amplitude ratio (peak frequency) plotted against core measured permeability (smoothed) for the limestone example.

Table 1: Biot model parameters used in forward modelling and inversion

DEPTH	V_p (m/sec)	V_s (m/sec)	ρ_{matrix} (g/cm^3)	ϕ	Q_p	Q_f	V_{bfluid} (m/sec)	ρ_{bfluid} (g/cm^3)	η (cP)
Limestone:									
5165	6100	3430	2.71	0.03	100	40	1525	1.2	0.1
5170	5550	3600	2.71	0.08	100	40	1525	1.2	0.1
5180	5400	3430	2.71	0.10	100	40	1525	1.2	0.1
5200	5290	3570	2.71	0.05	100	40	1525	1.2	0.1
5210	5190	3470	2.71	0.05	100	40	1525	1.2	0.1
5213	5190	3420	2.71	0.15	100	40	1525	1.2	0.1
5225	5050	3260	2.71	0.15	100	40	1525	1.2	0.1
5230	5050	3360	2.71	0.14	100	40	1525	1.2	0.1
5239	5600	3360	2.71	0.09	100	40	1525	1.2	0.1
5250	6100	3460	2.71	0.09	100	40	1525	1.2	0.1
5275	6100	3560	2.71	0.02	100	40	1525	1.2	0.1
5290	5600	3460	2.71	0.13	100	40	1525	1.2	0.1
Sandstone:									
2000	3200	2000	2.65	0.35	100	200	1475	1.05	1.0
2010	3200	1935	2.65	0.35	100	200	1475	1.05	1.0
2020	3240	1850	2.65	0.35	100	200	1475	1.05	1.0
2035	3450	1725	2.65	0.35	100	200	1475	1.05	1.0
2040	3520	1765	2.65	0.35	100	200	1475	1.05	1.0
2050	3200	1775	2.65	0.35	100	200	1475	1.05	1.0
2070	3420	1935	2.65	0.35	100	200	1475	1.05	1.0
2090	3330	1850	2.65	0.35	100	200	1475	1.05	1.0

The model results over the depth range of 1570 - 1616 m (5150 - 5300 feet) are compared to the measured phase velocity and amplitude ratio values in Figure 2. The model amplitude ratio predictions are in reasonably good agreement with the data, both qualitatively and quantitatively. The predicted phase velocity values, however, show almost no variation with permeability variations, while the measured phase velocities are quite sensitive to these variations.

The forward modelling results for this data set indicate that the Biot-Rosenbaum model can explain the Stoneley wave amplitude variations fairly well, but cannot adequately explain the velocity variations. The model results, however, are also very dependent on a number of poorly constrained parameters. In light of these observations, it may be possible to use the Biot-Rosenbaum model to predict absolute permeability values using the Stoneley wave amplitude information. The Stoneley wave velocity information, on the other hand, may provide a simple measure of the relative changes in permeability. One way to take advantage of this is to correct the measured phase velocity values for elastic property variations. Velocity measurements are more accurate than the amplitude measurements, and corrections for changes in the formation properties and borehole radius are relatively easy to obtain. With this in mind, predicted elastic Stoneley wave phase velocity values have been calculated for the limestone-dolomite data set by using the elastic period equation and all available velocity, density, and borehole information. Figure 3 is a plot of the difference between the measured and predicted elastic phase velocities versus the core measured permeability. The plot uses the travel time difference (slowness) which will be referred to as the $\Delta\Delta T$ value. The agreement between the core permeability values and the $\Delta\Delta T$ values is very good. Although the differences in travel time for these low permeability values are fairly small, the variations appear to be a good indicator of the relative permeability changes. Further discussion of this measure will be treated in a moment.

The second data set is from a shallow sand-shale sequence. A sand unit was penetrated in the depth interval of 610 - 640 m (2000 - 2100 feet) from which core permeability measurements have been obtained. The permeability values in this unit range between 0 and 3000 md. The data set for this example consists of: the full waveform acoustic log data, shear wave acoustic log data, electric logs, and core measured permeability values. Figure 4 shows the Stoneley wave peak frequency (2500-3500 Hz) amplitude ratios and phase velocity values measured from the full waveform acoustic logs plotted against the core permeability values (five foot running average) for this data set. No depth shift has been performed in this figure since the core values had already been corrected to the full waveform log depths. As in the previous example, the agreement between the Stoneley wave measurements and the permeability variations is very good. In order to generate model predictions, a number of parameters must be estimated. In addition to the pore fluid and borehole fluid properties, the matrix bulk modulus, and the fluid and formation Q values, the porosity and borehole radius values must also be estimated. The borehole was drilled with a 0.13 m (5") diameter drill bit; however, the low formation

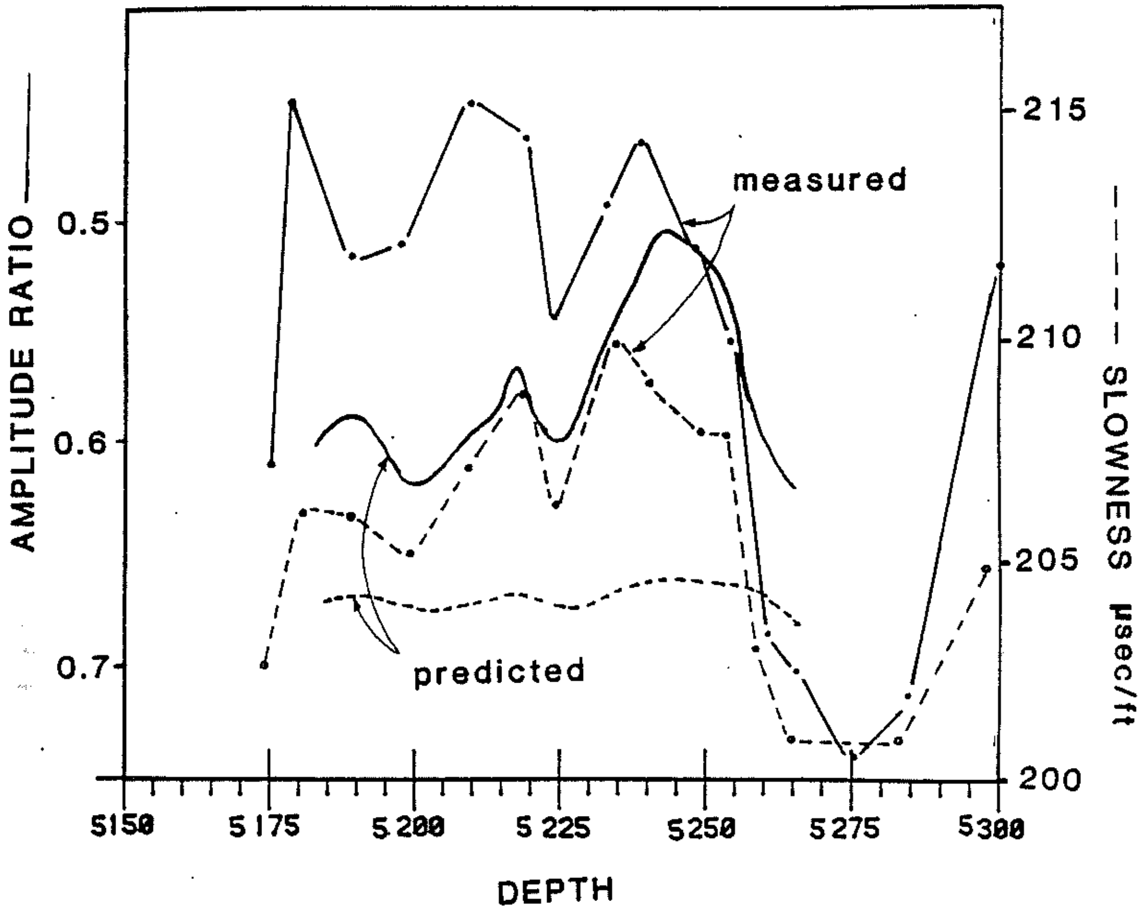

Figure 2: Measured Stoneley wave phase velocity and amplitude ratio (peak frequency) plotted against predicted values from the Biot-Rosenbaum model for the limestone example.

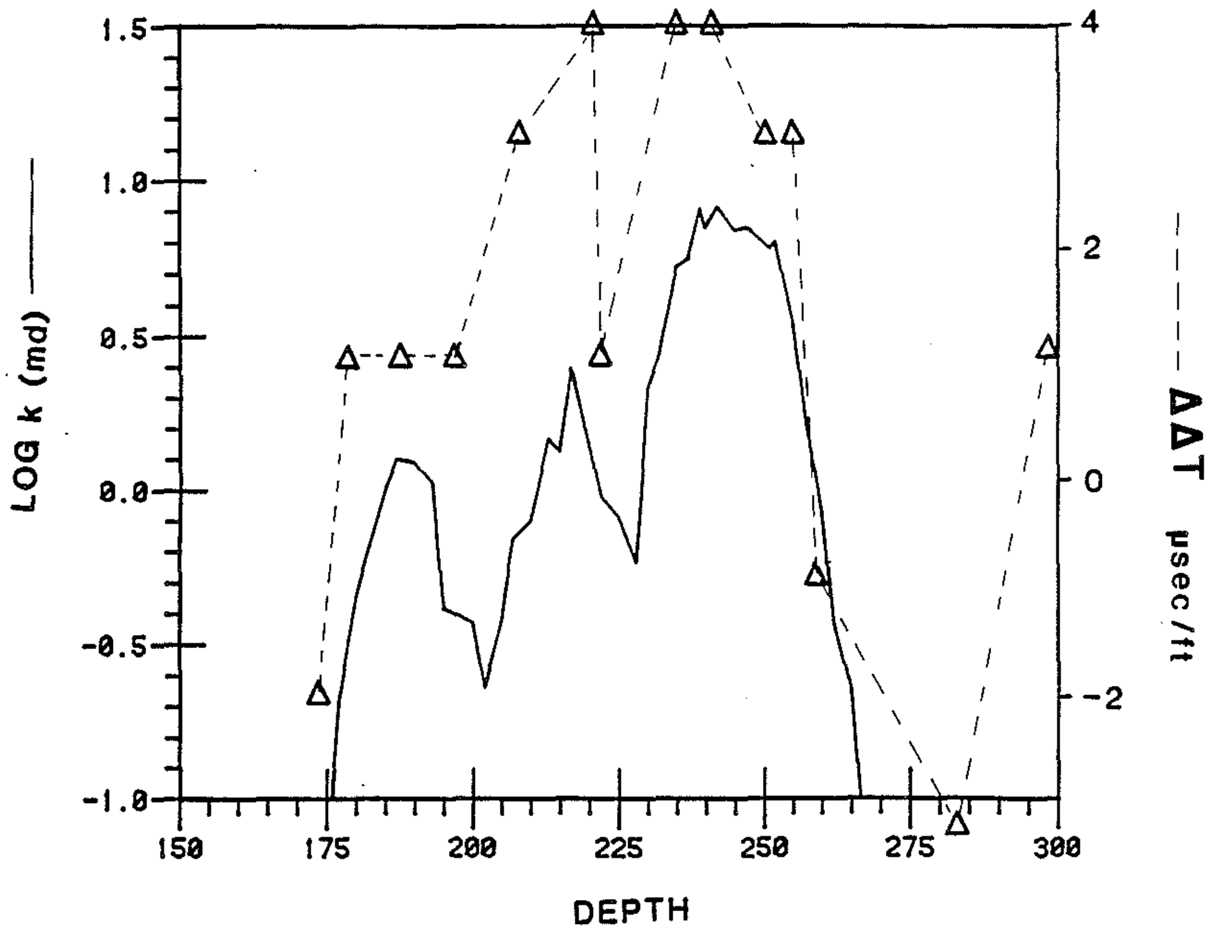

Figure 3: Comparison of the difference between the measured slowness and the predicted elastic slowness ($\Delta\Delta T$) and the core measured permeability values for the limestone example.

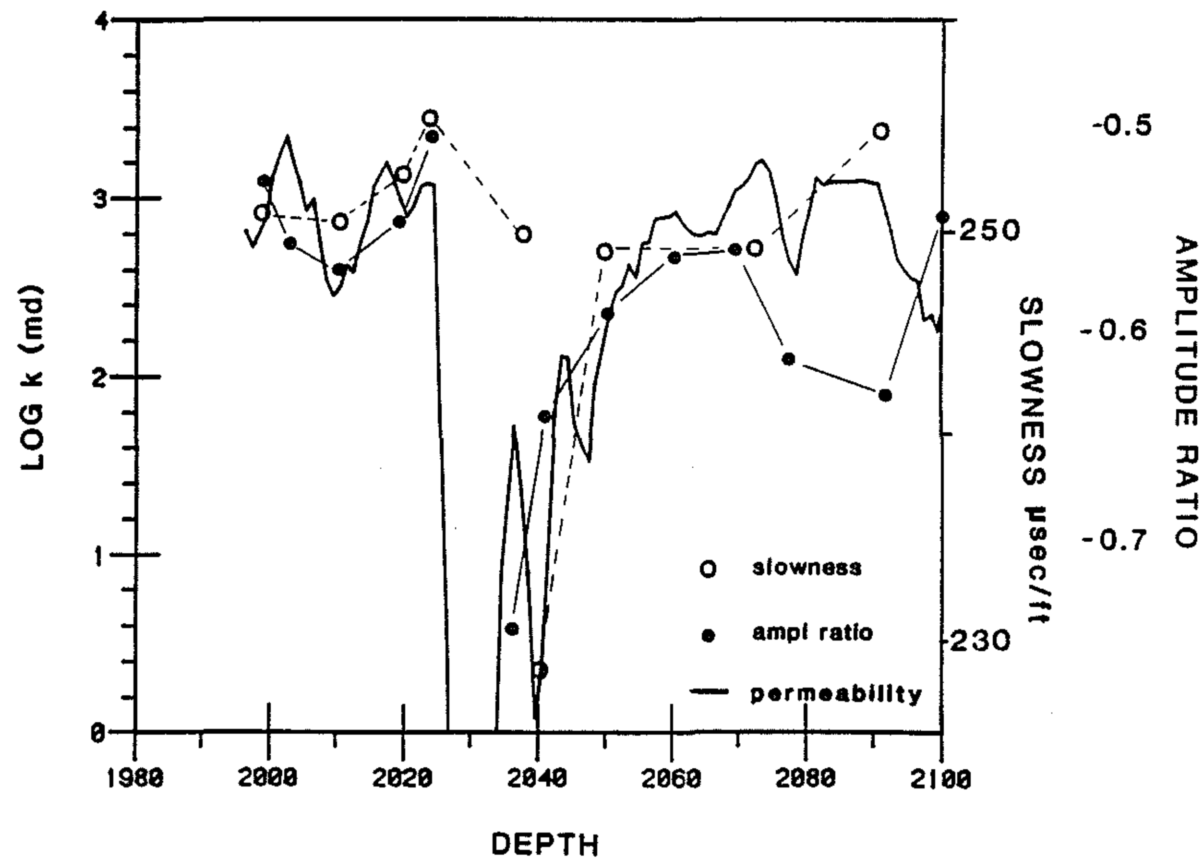

Figure 4: Measured Stoneley wave phase velocity and amplitude ratio (peak frequency) plotted against core measured permeability (smoothed) for the sand/shale example.

velocities and high permeability values suggest that the radius may be quite variable due to washouts in the borehole wall. In order to get reasonable agreement between the measured amplitude ratio data and the model calculations, a borehole diameter of 0.152 m (6") has been assumed (0.076 m (3") radius). The borehole fluid is reported to consist of water only, although it is likely that the fluid contains some suspended clay and silt from the soft shales encountered during the drilling process. The fluid velocity and density are assumed to be 1475 m/sec and 1.05gm/cc respectively. The pore fluid is assumed to be identical to the borehole fluid. The pore fluid viscocity is set at 1.0 cP (water at 20^oC). The sandstone porosity is set at 35% based on rough estimates from the time average equation. The density is assumed to be 2.16 gm/cc. The sand is assumed to be entirely quartz in composition with a grain density of 2.65 gm/cc and a bulk modulus of 37.9 GPa. The fluid and formation shear wave Q values are set at 200 and 50 respectively, based on the inversion results of Burns and Cheng (1987). The formation Q_p is assumed to be 100. Again, all parameters are summarized in Table 1.

The calculated Stoneley wave amplitude ratio and phase velocity values are compared to the measured data in Figure 5. As in the previous example, the amplitude ratio predictions and measured values are in good agreement both qualitatively and quantitatively, while the predicted phase velocity values, in contrast to the measured values, are very insensitive to permeability variations. In order to get the agreement shown in this figure, the acoustic coupling parameter (κ) has also been adjusted. The Rosenbaum model includes an acoustic coupling parameter which allows the amount of pressure communication between the borehole and pore fluid systems to be adjusted to account for possible mudcake effects. In the first data set example, κ was set equal to zero (0) representing complete communication. In this example, however, the predicted amplitude ratio values were much lower than the measured values with κ set equal to 0 (e.g. - measured values of about 0.55 compared to predicted values of 0.15 to 0.25). The predicted values could not be sufficiently increased by adjusting the other model parameters (such as viscosity, porosity, velocity, or Q values) within reasonable limits. The values shown in Figure 5 were generated with a κ value of 20. Since κ is an impedence factor which relates the fluid flow velocity across the borehole wall to the acoustic pressure difference between the two fluids, as κ increases from 0 the borehole fluid pressure coupling to the pore fluid is damped out. This coupling reduction forces the relative displacement between the formation matrix and pore fluid towards zero at the borehole wall as κ gets large, resulting in reduced attenuation due to viscous losses. It is difficult to relate a κ value of 20 to any physical parameter, but it is reasonable to assume that a mudcake may have built up on the borehole wall adajent to these highly permeable sandstones. No mudcake factor was needed in the limestone-dolomite example because the clay content of the borehole fluid may have been much less, and the low permeability values in that section would result in the formation of a much thinner mudcake.

The $\Delta\Delta T$ curve for the sandstone example is shown in Figure 6. The values are

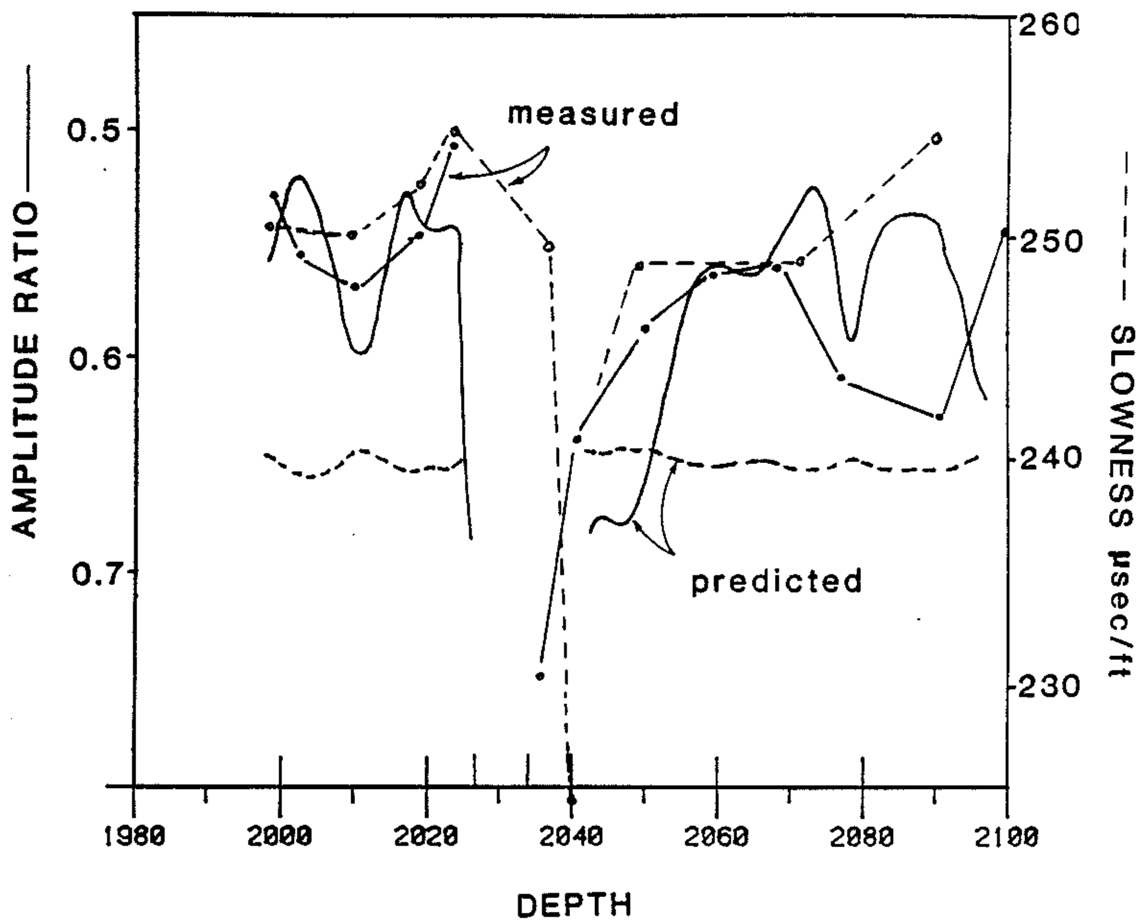

Figure 5: Measured Stoneley wave phase velocity and amplitude ratio (peak frequency) plotted against predicted values from the Biot-Rosenbaum model for the sand/shale example.

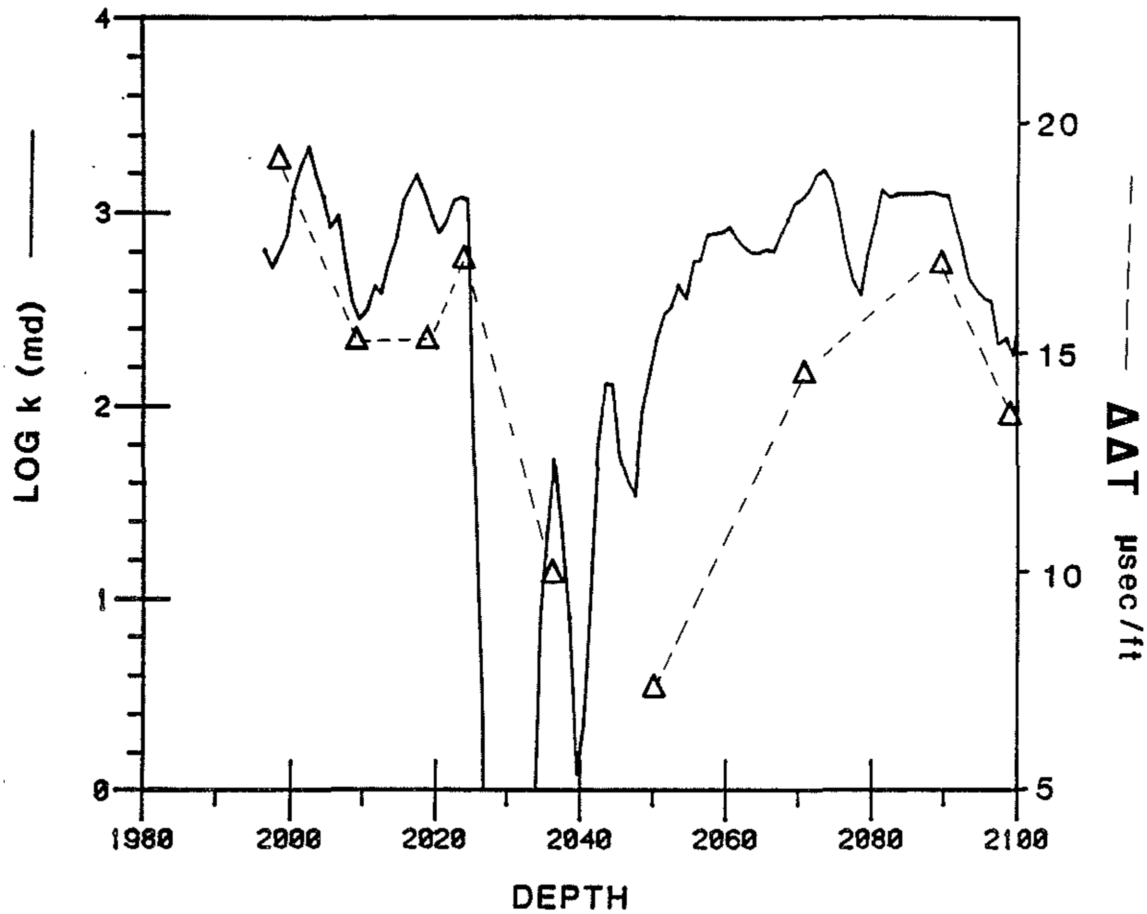

Figure 6: Comparison of the difference between the measured slowness and the predicted elastic slowness ($\Delta\Delta T$) and the core measured permeability values for the sand/shale example.

much larger than in the limestone example reflecting the much higher permeability values. The correlation between this curve and the permeability values is again very good, supporting its use as a relative permeability indicator. It is also interesting to note that the $\Delta\Delta T$ values for this example vary from 5 to 20 microseconds for a permeability range of roughly 10 to 2000 md. Comparison of these values with those obtained for the limestone example indicates that the values are approximately the same for zones of similar permeability (values of 4 microseconds for permeability values of about 10md). Although this may be coincidental, perhaps this measure will allow comparisons of data from different boreholes to be made. Figure 7 shows a plot of the $\Delta\Delta T$ values versus permeability for both data sets which supports this possibility. Although more data points are needed to define a true linear trend, the data from these two boreholes indicate that such a trend may exist.

The results obtained for the two field data examples used in this section indicate that the Biot-Rosenbaum model can explain the measured Stoneley wave attenuation with a selection of reasonable input parameters; however the model has difficulty in explaining the measured phase velocity variations. It appears that the attenuation mechanism modelled by Biot theory adequately represents the physics of the problem, but something is missing in the description of the Stoneley wave dispersion in porous media. One possible problem is the presence and behavior of a mudcake layer along the borehole wall. The Stoneley wave is very sensitive to layer properties adjacent to the borehole wall, and the presence of a very low velocity, low density mudcake may be affecting the measured Stoneley wave velocity. It is also possible that the pressure coupling behavior of a mudcake region is not adequately represented by the κ term in the Rosenbaum formulation. A second possible problem is the presence of a radial velocity gradient in the formation. Such a gradient could be caused by drilling damage or fluid invasion, and the model used here does not account for this situation.

INVERSE PROBLEM

Formulation

The forward modelling results of the previous section indicate that the Biot- Rosenbaum model predictions of the Stoneley wave amplitude ratios are in good agreement with measured data in permeable formations. There are several shortcomings of the model, however. First, the predicted phase velocity variations show much less sensitivity to permeability variations than the data and, second, the model requires knowledge about a number of parameters which are often not well constrained. In spite of these shortcomings, however, the model provides a very promising means of estimating in-situ permeability variations. In this section an inverse problem based on the

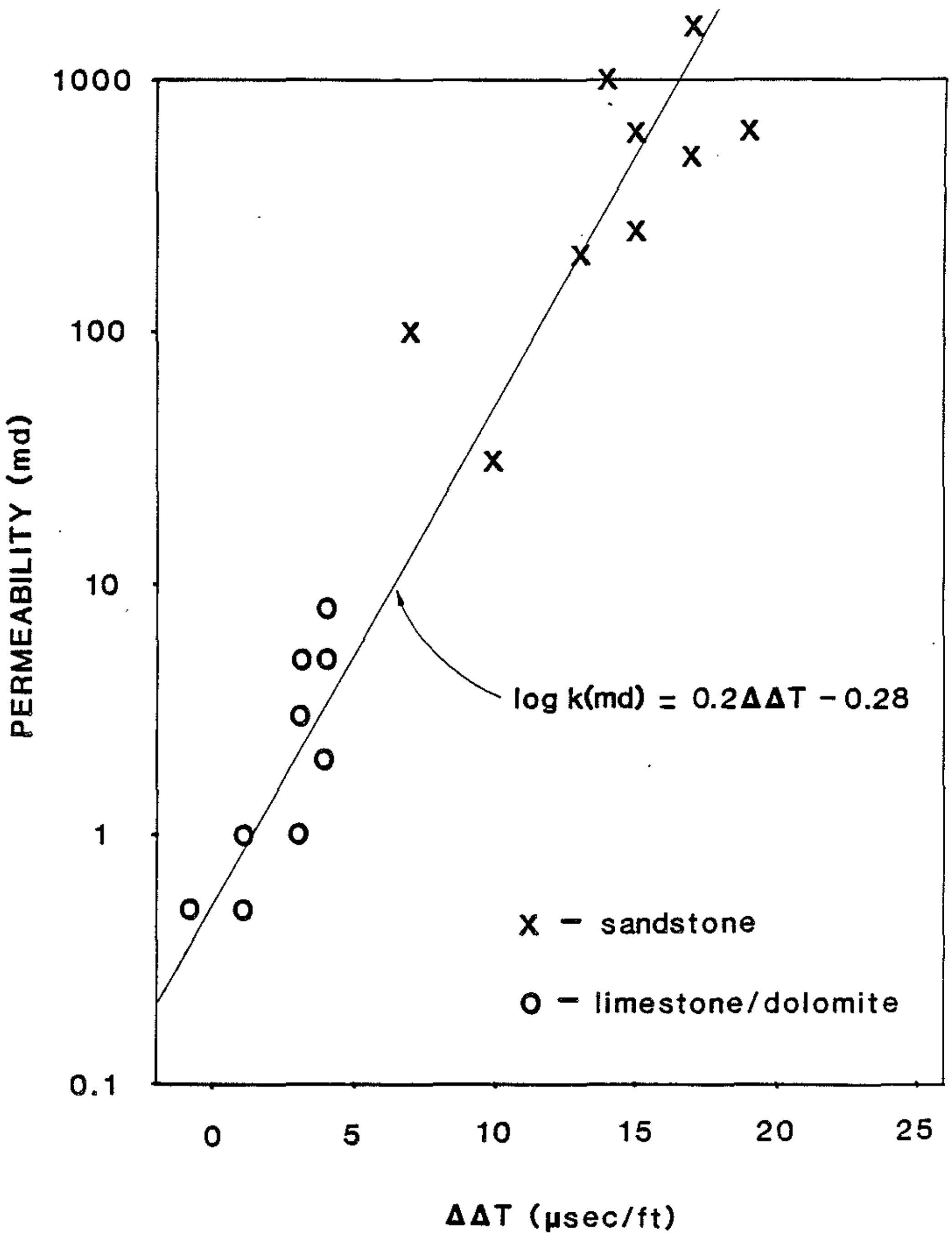

Figure 7: Plot of the difference between the measured slowness and the predicted elastic slowness ($\Delta\Delta T$) against the core measured permeability values for both the limestone/dolomite and sand/shale examples.

Biot-Rosenbaum model is formulated to see how well the model can predict in-situ permeability variations from Stoneley wave amplitude variations. The problem is formulated as a linearized damped least squares problem. Modifications of this model to correctly account for mudcake effects are currently being developed. Such modifications may help to explain the phase velocity variations seen in the data, and should result in better agreement between the predicted and measured Stoneley wave behavior. The development of this inverse method, in addition to providing estimates of permeability from the existing model, provides a framework for monitoring the utility of model modifications to the permeability estimation problem.

Although the permeability parameter is of primary interest, several other parameters which affect the Stoneley wave attenuation also must be estimated. Specifically, the formation shear wave Q value, the fluid Q value, and the pore fluid compressibility will all affect the measured attenuation. The parameters that will be estimated, therefore, will be: the formation permeability, the formation shear wave Q value, and the pore fluid density (pore fluid velocity is held constant). All other parameters are assumed to be known. Certainly the inverse problem can be set up to estimate all other parameters in the model, but most of the other parameters (such as porosity, matrix parameters, formation velocities and density) are usually fairly well constrained by other borehole measurements. Borehole fluid velocity and density are very important parameters in the model, but they are not estimated in the inverse problem because the values must remain constant throughout the borehole, and the values can be estimated from drilling information or simple forward modelling of Stoneley velocities in non-permeable sections. The fluid Q value is also very important, but it must also remain constant and its value can be estimated by the inversion procedure of Burns and Cheng (1987) in the non-permeable zones of the borehole. The pore fluid viscosity is assumed to be known as well. Since the model only treats the fluid mobility (ratio of permeability and viscosity), the viscosity cannot be estimated along with the permeability. The viscosity is usually chosen to correspond to water viscosity at the temperature of the depth of interest. The pore fluid velocity is also assumed to be known.

Using a Taylor series expansion of the model response to the parameters of interest and neglecting second and higher order terms, the inverse problem takes the form:

$$\mathbf{G}\,\Delta\mathbf{x} = \Delta\mathbf{b} \tag{1}$$

where:

$\mathbf{G}$ = Jacobian of the model response with respect to parameters
$\Delta\mathbf{x}$ = vector containing changes in model parameters

$\Delta \mathbf{b}$	= vector containing changes in data

For the permeability estimation problem, the parameter vector is given by:

$$\Delta \mathbf{x}^{\mathbf{T}} = [\Delta 10/Q_\beta \ \Delta log_{10}k \ \Delta \rho_{pf}/\rho_{bf}] \tag{2}$$

where:

k	= permeability in millidarcies
ρ_{pf}	= pore fluid density (gm/cc)
ρ_{bf}	= borehole fluid density (gm/cc)

The variations in scaling of each parameter ensure that all values are of the same order. The data vector is given in general by:

$$\Delta \mathbf{b}^{\mathbf{T}} = [c_{St}(f_i)/v_f ... A_{St}(f_i) ... A_{pR}(f_j) ...] \tag{3}$$

where:

$c_{St}(f_i)$	= Stoneley wave phase velocity (freq: i=1,n)
$A_{St}(f_i)$	= Stoneley wave spectral ratios (freq: i=1,n)
$A_{pR}(f_j)$	= pseudo-Rayleigh spectral ratios (freq: j=1,m)
v_f	= borehole fluid velocity

The pseudo-Rayleigh wave amplitude ratios can be included to help constrain the formation Q_β value, although it should be kept in mind that these values are quite susceptible to noise near the cutoff frequency. Stoneley wave phase velocity data will be used in a synthetic example, but will not be used in any real data applications. Because the problem is non-linear, a number of iterations are necessary to achieve convergence. Equation (1) is solved by a damped least squares method (IMSL routine ZXSSQ). The Jacobian is calculated via forward differences and the damping factor is adjusted at each iteration to maintain convergence. In order to help constrain the estimation procedure, penalty functions are defined for the formation Q_β and pore fluid density parameters (none is imposed on the permeability parameter). The penalty function for Q_β takes the form:

$$cqs = max\,[0,\ |1/Q_\beta\ -\ 1/qspnlty|\,] \tag{4}$$

where $qspnlty$ is an input value corresponding to the minimum Q_β value allowed. The pore fluid density penalty functions are:

$$\begin{aligned} cpf_1 &= max\,[0,\ (\rho_{pf}\ -\ \rho_{max}\,)] \\ cpf_1 &= max\,[0,\ (\rho_{min}\ -\ \rho_{pf}\,)] \end{aligned} \tag{5}$$

where:

ρ_{min} = minimum pore fluid density
ρ_{max} = maximum pore fluid density

These functions can be used to force the procedure to use permeability variations as the primary means of matching the model response to the data. The penalty functions also insure that the parameter values do not vary too far from what are considered reasonable values.

The model used in the inverse problem consists of the Biot-Rosenbaum period equation which is solved for Stoneley wave roots at the frequencies of interest. Measured amplitude ratio values are then subtracted from the calculated values to define the errors. The starting model is input and should be fairly close to the actual solution to obtain convergence.

RESULTS AND DISCUSSION

Synthetic Example

The inversion routine is first tested on synthetic acoustic log data. Figure 8 shows two synthetic traces (at offsets of 4.25m and 5.25m) generated by the discrete wavenumber summation method for a simple open borehole surrounded by a Biot porous formation. The porosity and permeability for this example are 19% and 1000 md. The pore fluid and borehole fluid are modelled as water with velocity of 1500m/sec and density of 1.0gm/cc. The synthetics were generated with the fluid and formation Q values set at 10000. Measured Stoneley wave phase velocity and spectral ratios and pseudo-Rayleigh wave spectral ratios were used as input data for the inversion. Using a starting model of: log k (md) = 2.0, and pore fluid density (normalized) = 0.8 (Q_s was not used in the inversion due to its extremely high value), the final results for this example are:

$$\begin{aligned} log\, k &= 2.97 + -0.014 \\ \rho_{pf} &= 0.94 + -0.05 \end{aligned} \tag{6}$$

No constraints on the pore fluid density were imposed on this example. Convergence was attained in 11 iterations. A convenient measure of the ability of the model to fit the data is the residual energy reduction factor (RER) which is calculated by (Beydoun, 1985):

$$RER = 100\left(1 - \frac{E}{E_0}\right) \tag{7}$$

where:

E = sum of squared errors at a given iteration
E_0 = sum of squared errors at the starting model

In this synthetic example, the RER at the final solution is 99.9.

The inversion results for the synthetic data indicate that the problem is well resolved in the parameter of interest, namely permeability. However, the data were generated using the Biot model on which the inversion procedure is based, and the data were also noise free so the success of this application is not too surprising. Next, the routine is applied to actual field data. The two data sets used in the previous section will be inverted.

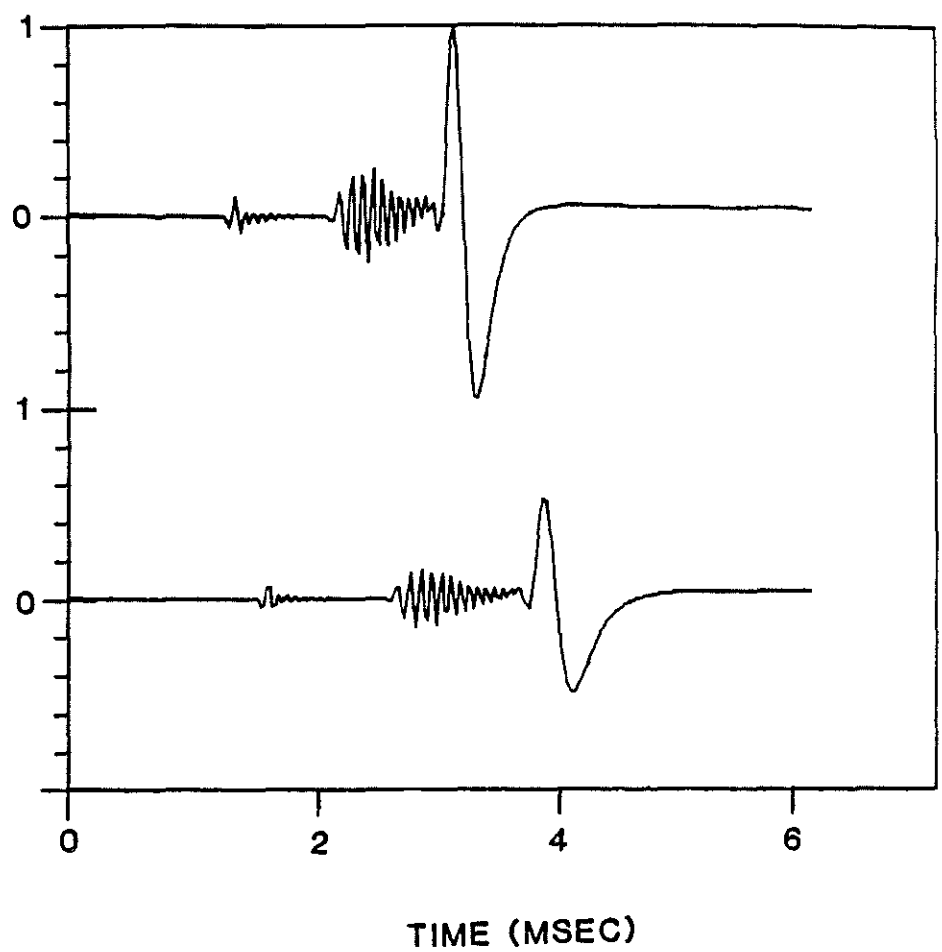

Figure 8: Synthetic full waveform acoustic logs generated for a Biot porous formation. The offsets are 4.25 and 5.25 m. The borehole radius is 0.07m and the source is a 6.5kHz Ricker wavelet.

Table 2: Limestone/dolomite estimated permeability values from inversion routine

Depth	log k (md)
5165	0.238
5170	1.336
5180	1.176
5200	1.965
5213	0.888
5225	1.087
5230	1.172
5239	0.939
5250	0.837
5275	-1.12
5290	0.963

Field Data Examples

The first example corresponds to the limestone-dolomite data set. The input data at each depth consists of Stoneley wave spectral ratio values at a number of frequencies. The pseudo-Rayleigh wave spectral ratios are not included as input data because of noise contamination problems (Burns and Cheng, 1987). Without the pseudo-Rayleigh wave data, the formation Q_β estimates will be poorly resolved since the Stoneley wave attenuation is insensitive to this parameter in fast formations such as this limestone sequence. The inversion is carried out for the three parameters of interest (permeability, Q_β, and pore fluid density). The minimum Q_β value constraint is set at 70 based on the results of the estimates obtained in Burns and Cheng (1987). The density of the pore fluid, which is assumed to be water, is constrained to lie between 0.8 and 1.2 times the borehole fluid density. The pore fluid viscosity is set at 0.1 cP and the acoustic pressure impedence factor (κ) is set equal to zero. All other parameters are given in Table 1. The inversion results are given in Table 2, and the permeability estimates are plotted against the smoothed measured values in Figure 9.

The predicted values are in very good agreement with the measured values in this example. Although the estimated values do not fully reflect the permeability variations in the interval, the absolute values and general structure of the permeability changes is well represented. The permeability parameter is well resolved in this example while the Q_β parameter resolution is very poor, as is the pore fluid density value.

The second field data example to be treated is the sandstone example. The model parameters used in this example correspond to the parameters used in the forward

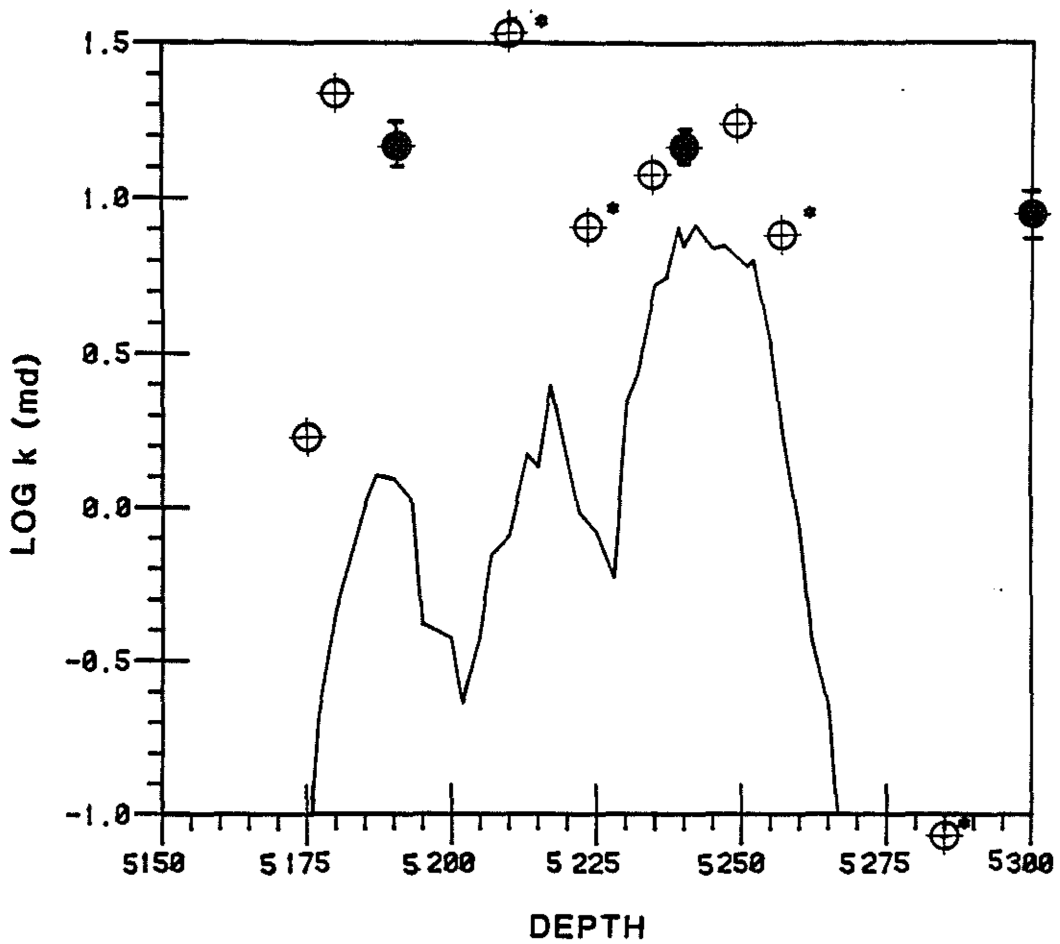

Figure 9: Inversion results for the limestone example. The circles are the predicted permeability results. The open circles represent fair to poor resolution (< 0.5), filled circles represent results with good resolution (> 0.75), asterisks indicate convergence not attained.

Table 3: Sand/shale estimated permeability values from inversion routine

Depth	log k (md)
2000	2.42
2010	2.35
2020	2.59
2035	1.59
2040	2.15
2050	1.75
2070	2.18
2090	2.16

modelling and are given in Table 1. In this case the acoustic impedence factor (κ) is set at twenty (20) as discussed in the forward modelling section. The Q_s estimates are constrained to be greater than 50 and the pore fluid density, as in the last example, is constrained to lie between 0.8 and 1.2 of the borehole fluid value. The resulting parameter estimates are given in Table 3, and the permeability estimates are plotted against the smoothed measured values in Figure 10.

The estimated permeability values are again in very good agreement with the measured values. The general variations are well represented, although the inversion results tend to underestimate the permeability by between a factor of 2 to as much as a factor of 10. The estimates are particularly low for the zone between 2050 and 2100 feet in depth. The porosity, borehole radius, and pore fluid characteristics were assumed to be constant for the entire depth range in this example (except for the depths of 2035 and 2040 feet which coincide with a more shaly zone). It is certainly possible that some or all of these parameters are different in this lower sand unit (2050'- 2100'), which may explain the low estimates. The permeability parameter resolution is quite good in several of the runs, while the other two parameters are poorly resolved.

CONCLUSIONS

The Stoneley wave phase velocity and attenuation provide a very promising means of estimating in-situ permeability variations. By taking the difference between the measured and predicted elastic slownesses, a good relative permeability measure ($\Delta\Delta T$) is obtained. The Biot-Rosenbaum model can explain some of the measured Stoneley wave variations in permeable formations. The model is in good agreement with the data for Stoneley wave attenuation variations, but does not adequately reflect the Stoneley

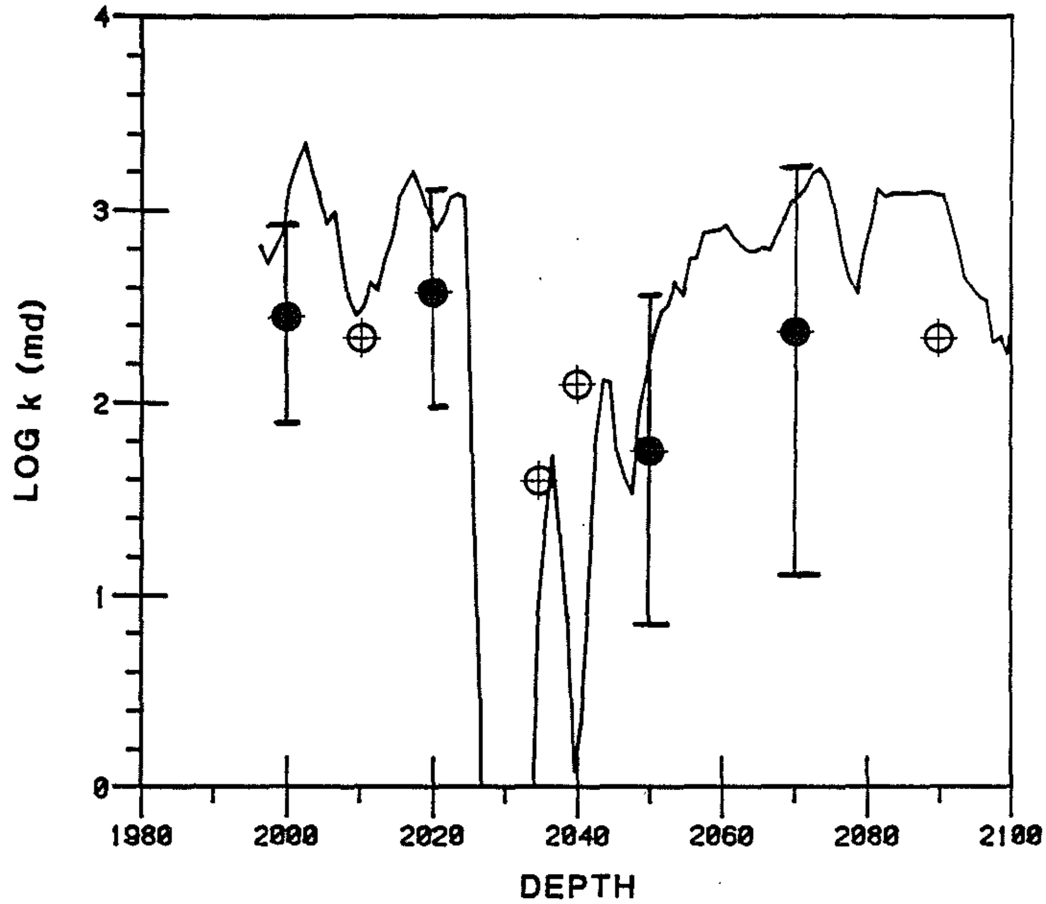

Figure 10: Inversion results for the sand/shale example. The circles are the predicted permeability results. The open circles represent fair to poor resolution (< 0.5), filled circles represent results with good resolution (> 0.65).

wave phase velocity variations that are seen in the data. The presence of a mudcake layer which inhibits the pressure communication between the borehole and pore fluid systems can explain some of the discrepancies which are seen between the model and the data, but further work is needed to understand the mudcake effect more fully. An inverse problem, based on the Biot-Rosenbaum model, provided estimates of the in-situ permeability from measured Stoneley wave spectral ratios. The predicted permeability variations obtained from this inversion are in very good agreement with the measured values in the two field data sets studied.

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