



Room 14-0551
77 Massachusetts Avenue
Cambridge, MA 02139
Ph: 617.253.5668 Fax: 617.253.1690
Email: docs@mit.edu
<http://libraries.mit.edu/docs>

DISCLAIMER OF QUALITY

Due to the condition of the original material, there are unavoidable flaws in this reproduction. We have made every effort possible to provide you with the best copy available. If you are dissatisfied with this product and find it unusable, please contact Document Services as soon as possible.

Thank you.

Due to the poor quality of the original document, there is some spotting or background shading in this document.

INVESTIGATION ON THE INLET DESIGN
OF THE REGENERATIVE PUMP

by

RENE BICARD

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
MAY 1957

Signature of Author _____

Department of Mechanical Engineering, May 20, 1957

Certified by _____

Thesis Supervisor _____

Accepted by _____

Chairman, Departmental Committee on Theses

ABSTRACT

INVESTIGATION ON THE INLET DESIGN
OF THE REGENERATIVE PUMP

by

RENE BICARD

Submitted to the Department of Mechanical Engineering on May 20, 1957 in partial fulfillment of the requirements for the degree of Bachelor of Science.

This investigation involves the design of a new inlet for a regenerative pump with a semi-torus shape channel. The design of the inlet is based on a proposed model of the fluid mechanism in the inlet region.

The results obtained were not as satisfactory as was expected, however some improvement was achieved in increasing the length of the linear region of the channel and therefore increasing the head across the same. This small improvement shows, however, that the line of analysis followed in the model is in the right direction, and a further development of the model is encouraged.

Thesis Supervisor: W.A. Wilson

Title: Professor of Mechanical Engineering

TABLE OF CONTENTS

Abstract	i
List of illustrations	ii
List of symbols	iii
Foreward	1
1. Introduction	2
2. The Circulatory Flow Theory	4
3. The Inlet Problem	5
4. The Inlet Design	7
5. Description of Apparatus and Test Procedure	13
6. Discussion of Results	15
7. Conclusions and Recommendations	19

Illustrations

Appendix

Bibliography

LIST OF ILLUSTRATIONS

Figure No.

- 1 Schematic Diagram of Experimental Unit
- 2 Schematic Diagram of New Design Inlet
- 3 Basic Control Volume Assumed at Inlet
- 4 Pressure Variation within Experimental Unit Before New Inlet Design
- 5 Pressure Variation within Experimental Unit With New Inlet Design
- 6 Performance Curves of Experimental Unit Before New Inlet Design
- 7 Performance Curves of Experimental Unit With New Inlet Design

LIST OF SYMBOLS

g	acceleration of gravity
H	head in ft. of fluid pumped
k_c	circulatory flow coefficient
P	Pressure
Q	volume rate of flow
L/D_n	length to hydraulic diameter ratio
u	internal energy in absence of
U	wheel speed
V	velocity
W	work input

DIMENSIONS

r	radius
r_g	radius to centroid of cross-section of channel
r_o	position of mean stream line leaving impeller passage
r_i	position of mean stream line entering impeller passage
b_o	radial length of circulatory flow area leaving impeller
b_i	radial length of circulatory flow area entering impeller
A	cross-sectional area of channel
D	diameter to cg of channel

GREEK SYMBOLS

α	V_{ti} / U_i
Δ	V_{to} / U_o slip factor

β	inlet angle
ρ	density
τ	torque
ω	angular velocity
θ	angular measure from inlet, radius
θ_p	linear region of pump, radius
φ	$\frac{Q}{Q_s}$ ratio of actual flow rate to solid body rotation of channel area
$\frac{d\psi}{d\theta}$	$\frac{gH}{\omega^2 D^2} \frac{1}{\theta_p}$ pressure coefficient
$\frac{dTP}{d\theta}$	$\frac{g\tau}{\rho \omega^2 D^5} \frac{1}{\theta_p}$ power coefficient
η	$\frac{\rho Q H}{W}$ efficiency

SUBSCRIPTS

a	average throughout linear region
c	circulatory flow
i	at mean stream line entering impeller passage
o	at mean stream line leaving impeller passage
s	solid body rotation
t	tangential direction
x	at inlet region

FOREWORD

The present investigation is a continuation of a research program initiated at M.I.T. in 1952 under a grant-in-aid established by the Worthington Corporation for the purpose of rationalizing the nature of the regenerative pumping process with a view to achieving improved performance.

This thesis is concerned with the design of the inlet port of a circular channel regenerative pump. The pump tested was originally designed by Mr. D.P. Dewitt, and except for the inlet section other things in the unit were left the same.

The initial step in the design of the inlet is to predict the kinematics of the fluid at the said section. The analysis is based on the circulatory flow theory, which was originally developed by W.A. Wilson, M.A. Santalo, and J.A. Oelrich, and later by D.P. Dewitt for the particular pump geometry of the present experimental unit. On this basis an inlet was designed, built and tested.

The author hereby wishes to acknowledge the support received from Worthington Corporation for this project, and is indebted to Professors W.A. Wilson and M.A. Santalo, and Mr. D.P. Dewitt for their help and valuable advice which contributed greatly to this investigation.

INTRODUCTION

A regenerative pump is a hydrodynamic unit, sometimes referred to in the literature as a periphery pump, friction pump, turbulence pump, etc. A particular feature of this pump is that it develops high heads at low specific speeds and with a single rotor. As compared to other units its efficiency is not impressive, but it becomes acceptable when compared to units of similar specific speeds. These characteristics of a regenerative pump have made it particularly attractive for lubrication, filtering and booster systems, where high heads at low flows are required with as much unit simplicity as possible.

The present investigation is concerned with the inlet effects of a given unit, and the design of an inlet port which will improve the performance characteristics of the pump.

A schematic diagram of the unit used in this investigation is shown in Fig. 1. It has a torus-shaped chamber, half of which is in the casing and is called the open channel. The other half is in the impeller, and similarly it is called impeller channel. The impeller channel has cast into it a large number of straight radical blades. At one section of the open channel there is an obstruction called the stripper, at each end of which there are located the inlet and outlet ports of the unit. The fluid passes from the inlet into the

channel and while it circulates along its periphery, it also circulates in and out of the impeller blades, i.e. if supposing a cross sectional view of the channel the fluid would be entering at the inside end of the blade (with respect to the axis of rotation of the impeller), and leaving at the outside end. The mean stream line along the periphery of the channel will then have an approximate helical path, and it is from this interval "multistaging" or "regeneration" which takes place that the unit owes its name. It should also be recalled that the object of the stripper mentioned above is to provide an obstruction so that all fluid does not continue to circulate in the channel but some of it is pumped out through the outlet port.

In the following section the theory of operation for the pump in question is discussed.

2. THE CIRCULATORY FLOW THEORY

The circulatory flow theory is the basis for the analysis done in this thesis. The said theory represents the latest development in the study of the fluid dynamics mechanism of regenerative pumps, and was proposed by Wilson, Santalo and Oelrich.^{1*} In this investigation however, their original expressions are not used directly because they are derived for a particular pump geometry which differs from the one here in question. Instead the expressions to be used are those which were later developed by D.P. Dewitt,^{2*} the designer of the present experimental unit. These expressions however, follow the same methods of reasoning as in the original development.

A resume of the circulatory flow theory as derived by Dewitt for this particular test unit is shown in the appendix.

*Superscript refers to references found in the Bibliography.

3. THE INLET PROBLEM

Previous experimental work on regenerative pumps shows that the pressure distribution within the pump, i.e. in the channel, not including the inlet and outlet ducts, is marked by five characteristic regions as follows:

- a) Inlet region: A pressure drop occurs in this section of the channel which is opposite to the outlet port.
- b) Acceleration region: It is a transition region where circulatory flow is not fully developed and the pressure increases at an increasing rate.
- c) Linear region: The circulatory flow is here fully developed, and the rate of pressure increase is constant.
- d) Acceleration region: This is again a transition region at which circulatory flow begins to degenerate near the exit and pressure reaches a maximum.
- e) Outlet region: A pressure drop takes place as the fluid leaves the channel.

These regions are indicated on Fig. 4, which shows the pressure distribution at various flow rates, for the experimental unit, before any special inlet design had been made. The inlet which the unit had at the time had been an intuitive design and no special investigation was undertaken for that purpose.

The regions mentioned above are not only true for the present test unit, but they are marked in various degrees in other regenerative pumps which have been previously built.

From considerations on the pressure distribution discussed above, it has been suggested that if an inlet were designed so that the pressure drop in the inlet region would be eliminated, and the acceleration region be reduced and at the same time moved so as to overlap the inlet region, then the head across the pump would be increased because the linear region would start earlier, meaning that each of the curves in Fig. 4 would move upwards and to the left. And, correspondingly, if the head across the pump is increased, its performance will also be improved.

From the previous analysis it was considered worthy to undertake the investigation leading to a new inlet design of the succeeding sections.

4. THE INLET DESIGN

As it was said in the previous section, the purpose to be achieved with a new inlet design is to increase the length of the linear region by reducing as much as possible the acceleration region, and also to reduce any entrance losses which may occur in the inlet region. This new inlet will then bring about an increase in head across the pump, and consequently an increase in overall pump performance will follow.

4.1 THE BASIC INLET MODEL

The first step in the design of the new inlet was to produce a model of the flow in the inlet region which would match the circulatory flow in the channel so as to avoid any energy losses other than friction losses which may take place at the inlet section. In other words the flow is to be assumed to follow continuous stream lines in its transition from the inlet duct to the linear region of the channel. If this flow pattern can be achieved then we would have the pressure losses in the inlet region which originally were due to turbulence effects eliminated, and at the same time the acceleration region would be superposed over the inlet region.

The proposed basic model can be seen on Figure 3, and it was drawn under the following assumptions and considerations:

- a) The inlet region is defined as $R; \theta_x$ where $\theta = 0$ at the tip of the stripper and $\theta = \theta_x$ is the end of

the inlet at a point on a plane between the open channel and the tip of the blades.

- b) The linear region is assumed to start right at the end of the inlet region, i.e. at $\theta = \theta_x$.

Therefore at this point we have:

$$(V_i)_{\theta=\theta_x} = (V_i)_c = \frac{(Q_c)_c}{R_i B_i}$$

and

$$(v_{ti})_{\theta=\theta_x} = (\alpha)_c U_i$$

or

$$(\alpha)_{\theta=\theta_x} = (\alpha)_c$$

- c) At the beginning of the inlet region or at we have that the fluid in the impeller is undergoing solid body rotation, and therefore at this point we have:

$$(V_i)_{\theta=0} = 0$$

and

$$(v_{ti})_{\theta=0} = (\alpha)_{\theta=0} U_i = U_i$$

or

$$(\alpha)_{\theta=0} = 1$$

- d) A linear velocity distribution is assumed over the inlet region. In the tangential direction, as well as in the circulatory direction, so that from $\theta = 0$ to $\theta = \theta_x$

V_i	changes linearly	from 0	to	$(V_i)_c$
V_t	"	"	"	U_i to $(\alpha)_c U_i$
α	"	"	"	1 to $(\alpha)_c$

- e) If the relations in d) are true, then the absolute velocity at any one point in the inlet is

$$V = \sqrt{V_t^2 + V_i^2}$$

and the average absolute velocity across the inlet

section is:

$$(V)_{AVE} = \sqrt{(V_t)_{x-AVE}^2 + (V_i)_{x-AVE}^2}$$

- f) If continuity is to be satisfied, we have that through-flow across the pump is equal to the the average absolute velocity across the inlet times the cross-sectional area of the inlet, therefore:

$$Q = (B_i R_i \theta_x) (V)_{AVE}$$

- g) If friction losses are considered small in the inlet region, and are therefore neglected, then the pressure difference from $\theta = 0$ to $\theta = \theta_x$ is given by

$$\Delta P \Big|_{\theta=0}^{\theta=\theta_x} = \int_{\theta=0}^{\theta=\theta_x} \frac{\rho Q_c}{g R_c A} [\alpha U_o R_o - \alpha U_i R_i] d\theta$$

or

$$= \int_{\theta=0}^{\theta=\theta_x} \frac{\rho V_i B_i R_i}{g R_c A} [\alpha U_o R_o - \alpha U_i R_i] d\theta$$

which are developed in the Appendix.

4.2 THE INLET ANGLE

On the basis of the model discussed above, the following conclusions may be drawn:

- a) At point $\theta = 0$ the circulatory velocity $V_i = 0$ and the tangential velocity $V_{t1} = U_i$; therefore it is obvious that at this point the wall of the inlet duct should be in the plane of the tip of the blades and perpendicular to the axis

of rotation of the impeller. This accounts for the fact that the stripper ends in the form of a wedge in the new design inlet shown in Figure 2.

b) At point $\theta = \theta_x$, the total absolute velocity is

$$V = \sqrt{(V_i)_c^2 + [(\alpha)_c U_i]^2}$$

and it is at an angle

$$\beta = \tan^{-1} \left(\frac{(\alpha)_c U_i}{(V_i)_c} \right)$$

with respect to the axial direction of the impeller.

This angle is the inlet angle shown in Figure 2.

4.3 THE INLET SIZE

The radial dimension B_i (see Figures 1, 2, and 3) of the inlet section is arrived at from circulatory flow considerations. As defined by Dewitt² B_i is the radial length of circulatory flow area entering impeller, therefore if the flow across the inlet is to correspond with the circulatory flow, then B_i is the logical dimension.

In the tangential direction, the dimension $(R_i \theta_x)$ is determined for a given volume rate of flow which is chosen as the design point for the pump. This is a consequence of the discussion of section 4.1 f), where the following relation was derived:

$$Q = B_i (R_i \theta_x) (V)_{AVE}$$

4.3 OTHER INLET CONSIDERATIONS

A great deal of thought went into the author's decision about what to do with that section of open channel in the inlet region which is not occupied by the inlet orifice or opening. This is the upper section of the channel where the fluid leaves the impeller. Finally it was arrived at the conclusion that if from continuity at any section of the channel, the circulatory velocity distribution, had to be similar in entering the blades, as well as leaving the blades. (This is shown graphically in Figure 3 .) Therefore if this were the case, the section of channel in question should have a shape corresponding velocity distribution so as to follow the stream line pattern of the flow. Upon this line reasoning the "s" shaped wedge filling shown in Figure 2 was arrived at. The length of this wedge is the same as that of the inlet region from the fact that the circulatory velocity increases from zero to the maximum achieved when there is fully developed circulatory flow, and according to the model, this occurs at the end of the inlet region.(i.e. at $\theta = \theta_x$)

4.5 CALCULATIONS FOR EXPERIMENTAL INLET

Since the dimensions of the inlet are a function of mainly the flow and the speed of the impeller, the following design conditions were established:

$$(\varphi)_{\text{DESIGN}} = \left(\frac{Q}{Q_s} \right)_{\text{DESIGN}} = 0.7$$

$$(N)_{\text{DESIGN}} = 1000 \text{ RPM}$$

$$\alpha = 0.820$$

$$(\alpha)_{C-DESIGN} = 0.46$$

$(\phi)_{DES} = 0.7$ the ratio of actual flow rate to solid body rotation in open channel was picked because from previous experience it has been found to be the condition for which the pump has a relatively good efficiency. $(N)_{des} = 1000$ RPM was also picked because it has been found to be within the general range of operation for this experimental pump.

$\alpha = 0.820$ was found experimentally by Dewitt² for this particular pump in operation.

$(\alpha)_{C-DES} = 0.46$ is calculated in the Appendix (A-2.2) from the relation

$$\phi = \frac{1}{2} \left[\frac{R_o}{R_c} \alpha + \frac{R_i}{R_c} \alpha \right]$$

A detailed calculation of the experimental inlet is shown in the Appendix (A-2). The calculated dimensions are:

$$\text{Inlet angle } \beta = 33.2^\circ$$

$$\text{Length of inlet region } R_i \theta_x = 1.35 \text{ inches}$$

$$\text{or } \theta_x = 0.442 \text{ radians.}$$

5. DESCRIPTION OF APPARATUS AND TEST PROCEDURE

5.1 DESCRIPTION OF PUMP UNIT

The details of the unit used are shown in Figure 1.

The essential elements include:

1. Impeller with 40 radial teeth. $D = 7.50''$
2. Casing with a half-torus channel, 2" dia.
3. Support plate for casing, serving also as back cover.
4. Four radial seals with a running clearance of 0.008.
5. Axial inlet made of cast plaster of Paris according to dimensions given in Appendix (A-2). A separate view of the inlet is shown in Figure 2
6. Stripper or septum region made of cast plaster of Paris.
7. Axial outlet port made of cast plaster of Paris
8. Clearance between impeller face and casing $0.004 \pm 1''$

5.2 INSTRUMENTATION

The instrumentation installed on the unit included:

1. Venturi tube with inclined manometer at inlet.
2. Static pressure taps connected to a water manometer.
Tap locations are shown in Figure 1
3. Cradled dynamometer (electric) rated at 2.62 in.-lb. torque. Sliding weight on a calibrated arm measured the torque.
4. Commercial strobatac to measure speed of rotation.

5.3 TESTING PROCEDURE

Air was used as the pumping fluid. Tests were made at a constant speed of 1000 RPM which is the speed for which the inlet was designed, and over the whole range of $\psi = Q/Q_s$. Check runs were also made for points in the vicinity of the inlet design conditions.

In obtaining the experimental data, the following procedure was followed:

1. The through-flow was measured by a Venturi meter, and was controlled by a gate valve located at the exit. For very high flow rates, above the design point, the use of a vacuum cleaner had to be used, in order to "help" the fluid to overcome the friction losses in the piping outside the channel.
2. Pressure variation in the channel were measured by water manometers connected to the taps.
3. The torque measurement was by means of a dynamometer with calibrated arm and sliding weight. The dynamometer was adjusted so that it measured absolute torque.
4. Speed measurements were made with a commercial strobatac, and the control of the speed was by means of a Variac.

6. DISCUSSION OF RESULTS

6.1 PRESSURE VARIATION WITHIN THE PUMP

The experimental results obtained for the pump unit after the new inlet design was put in are shown in Figure 5. And for comparison Figure 4 shows the results obtained before the new inlet was put in. It should be noticed however, that in Figure 5 the pump speed is 1000 RPM, and in Figure 4, 990 RPM, and since it has been found that little change in speed makes quite a difference in the pressure variation for a given flow rate, precaution should be taken in making a comparison, so that only a qualitative analysis can be made.

In Figure 5, the heavy line represents the pressure variation at the flow rate for which the new inlet was designed. The lighter lines are flow rates other than design. The pressure (in inches of water) is plotted versus the distance along the channel (in degrees). Location of taps is also shown. Tap 1, according to the new inlet design, is located at the end of the inlet region, i.e. at $\theta = \theta_z = 24.2^\circ$. Therefore according to the proposed model, at this point the circulatory flow should be fully developed and consequently the pressure rise should be constant from this point on along the channel. This implies that starting at tap 1 the slope of the pressure distribution line should be constant. This means that if the proposed model were to be in agreement with the actual flow pattern in the inlet region, the pressure distribution along

along the channel for points after tap 1, should be a straight line for the flow rate for which the inlet was designed.

A look at the heavy line on Figure 5, which corresponds to inlet design conditions, shows that the constant pressure rise characteristic of the fully developed circulatory flow starts at tap 3, and between taps 1 and 3 there is an acceleration region in which the rate of pressure rise is increasing. From this experimental fact, it can be said that the proposed model is not quite in agreement with the actual conditions of the pump.

It is important to notice however that some improvement was achieved with the new inlet design because the curvature of the pressure distribution line in the acceleration region between taps 1 and 3 is only very slight. And if by extrapolating the straight line for the pressure rise in the linear region to tap 1 it is found that pressure loss due to this acceleration region is about 0.1 inches of water which is only about 7% of the total pressure rise across the linear region between taps 1 and 10.

With regard to the predicted pressure drop across the inlet region, it was not possible to check directly the predicted value calculated in the Appendix (A-2.4), since no method could be devised for measuring the pressure at the beginning of the inlet region ($\theta = 0$), however by extrapolation of the pressure distribution line for design conditions it can be said that the predicted value of $(\Delta P)_x = 0.036$ inches

of water is within narrow limits a good approximation.

Comparison of the results in Figures 4 and 5 shows that even though the improvement due to the new inlet design has not been very large, some slight reduction in the length of the acceleration region was achieved. And this is not only true for the new inlet design conditions, but in general for the whole range of flows.

6.2 PERFORMANCE CONSIDERATIONS

In figures 4 and 5, are shown the performance curves for the linear region of the channel. Figure 4 represents the results obtained before the new inlet design was put in, and Figure 5 is for the new inlet design. In both cases the linear region has been taken between taps 1 and 10.

In other words to correlate the results obtained before and after the inlet was put in which were not taken under the same conditions of operation, the results in Figure 4 and 5 are plotted in a dimensionless form. This is permissible since the regenerative pump has been shown to follow the laws of similitude¹. The dimensionless parameters used are:

$$\begin{array}{ll}
 \text{flow coefficient} & \varphi = \frac{Q}{R_c A w} = \frac{Q}{Q_s} \\
 \text{head coefficient} & \frac{d\psi}{d\theta} = \frac{g H}{\omega^2 D^2} \frac{1}{\theta_p} \\
 \text{power coefficient} & \frac{dTP}{d\theta} = \frac{g \dot{E}}{\rho \omega^2 D^5} \frac{1}{\theta_p} \\
 \text{efficiency} & \eta = \frac{\rho Q H}{W}
 \end{array}$$

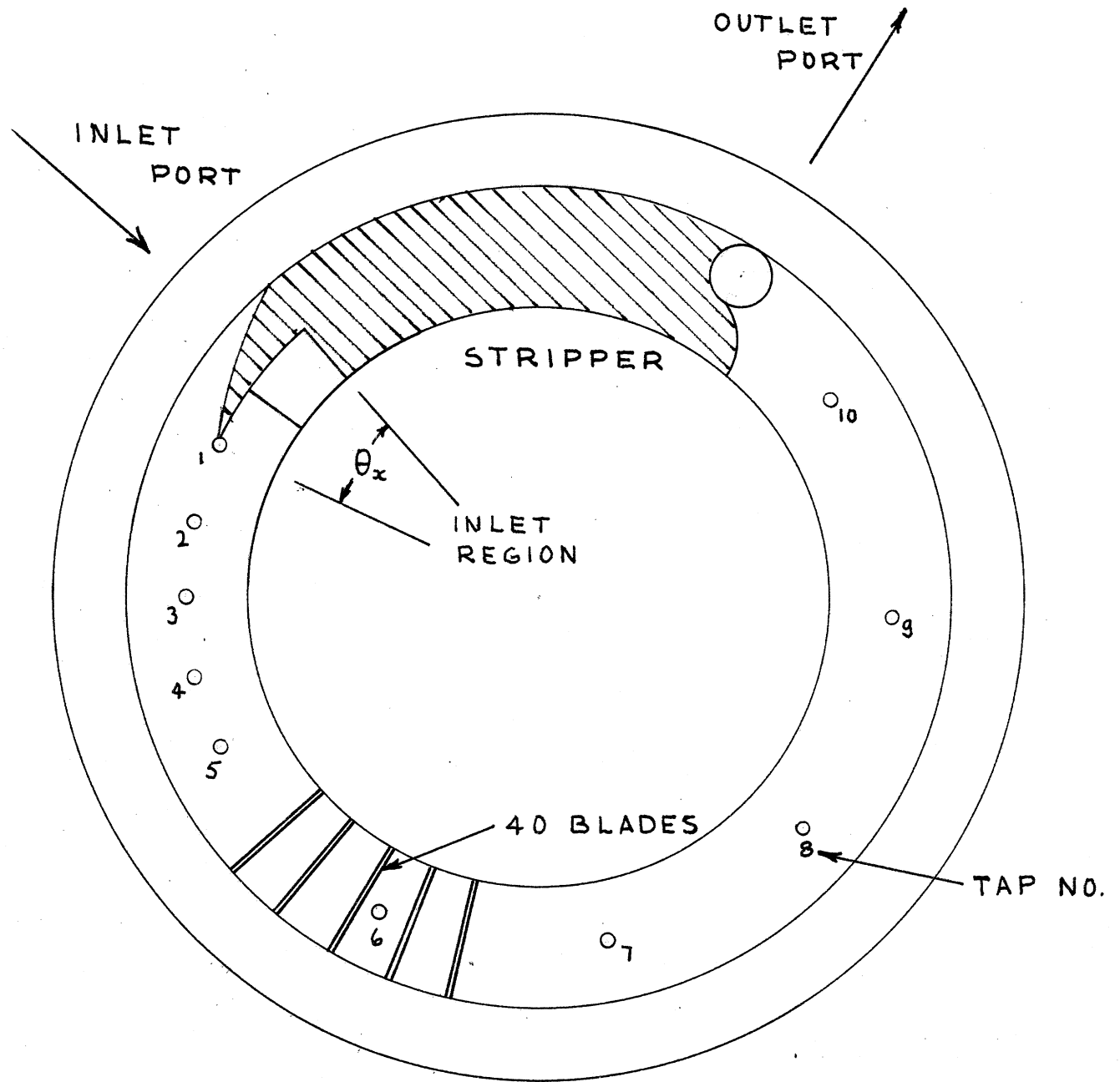
Comparison of the results in Figures 4 and 5, shows that the head coefficient is higher for any given flow coefficient,

with the new inlet design. This fact could account for the fact that the linear region considered includes part of the acceleration region, and therefore if the acceleration region was reduced with the new inlet it is reasonable to expect that the head across the channel has been increased, and therefore the head coefficient.

With regard to the power and efficiency curves, the higher power and lower efficiency obtained with the new inlet design cannot be very well accounted for. The author's opinion is that this large discrepancy is certainly not due to the changes induced by the new inlet design, but rather due to errors induced by the torque measuring device. The fact is that the dynamometer used for this purpose underwent a series of changes in the period between which the two sets of data were obtained.

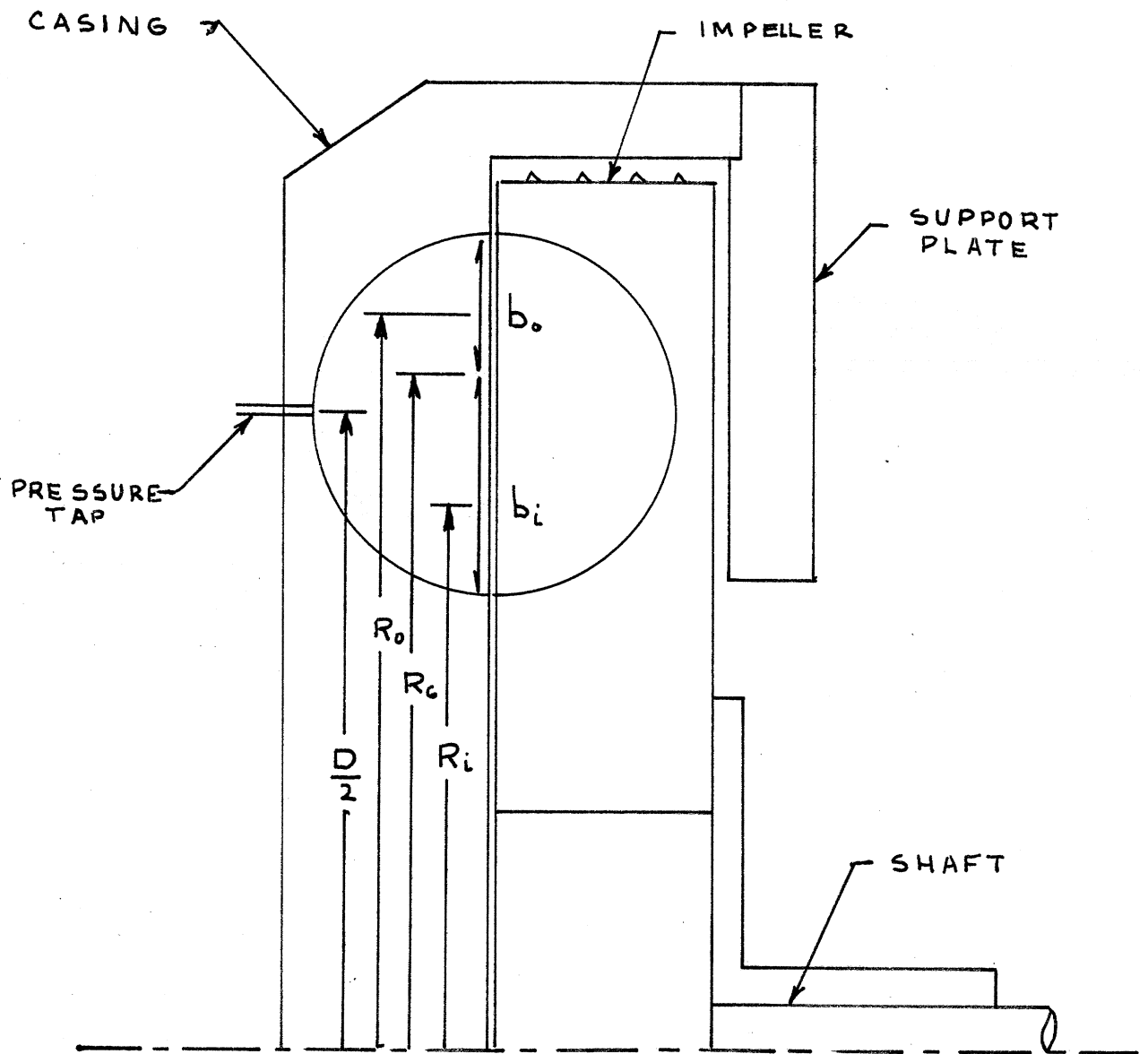
7. CONCLUSIONS AND RECOMMENDATIONS

From the preceding discussion it can be concluded that the inlet designed on the basis of the proposed model of the fluid mechanism in the inlet region of the experimental regenerative pump in question did not accomplish the expected results. However, some improvement was made in reducing the acceleration region. The author thinks that a further development of the proposed model can bring definite improvement to the inlet design since even though the present model was shown not to be very satisfactory, the trend is that the line of analysis followed was in the correct direction.



SCHEMATIC DIAGRAM OF EXPERIMENTAL UNIT

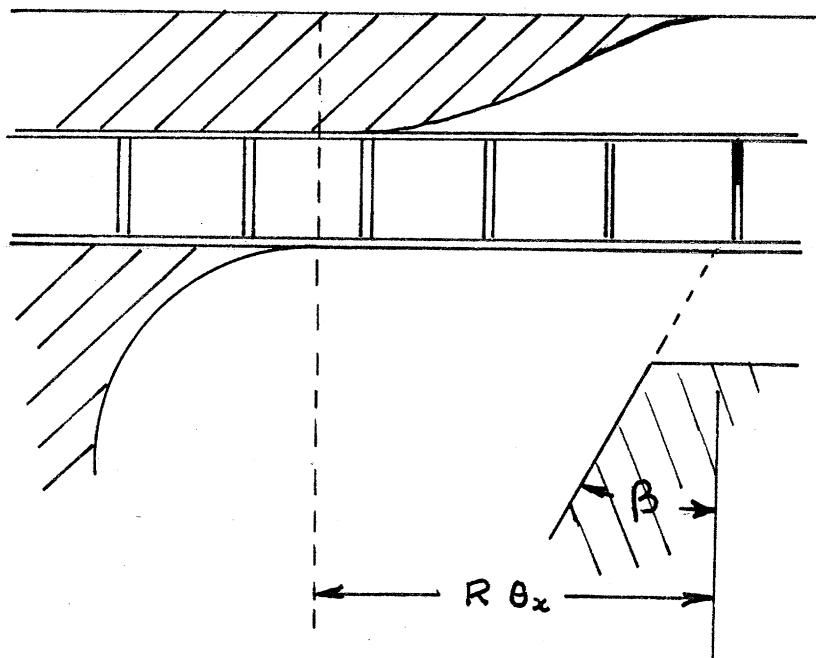
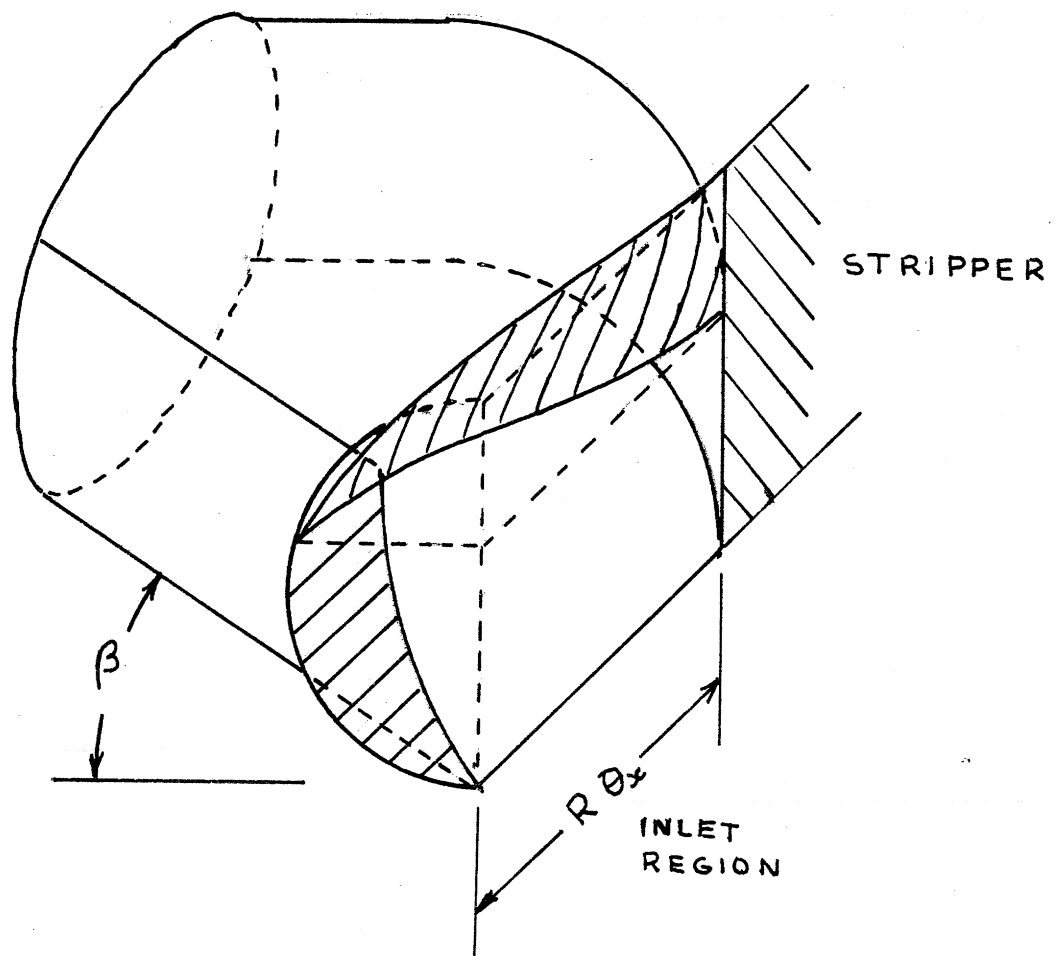
FIG 1.1



CROSS SECTION

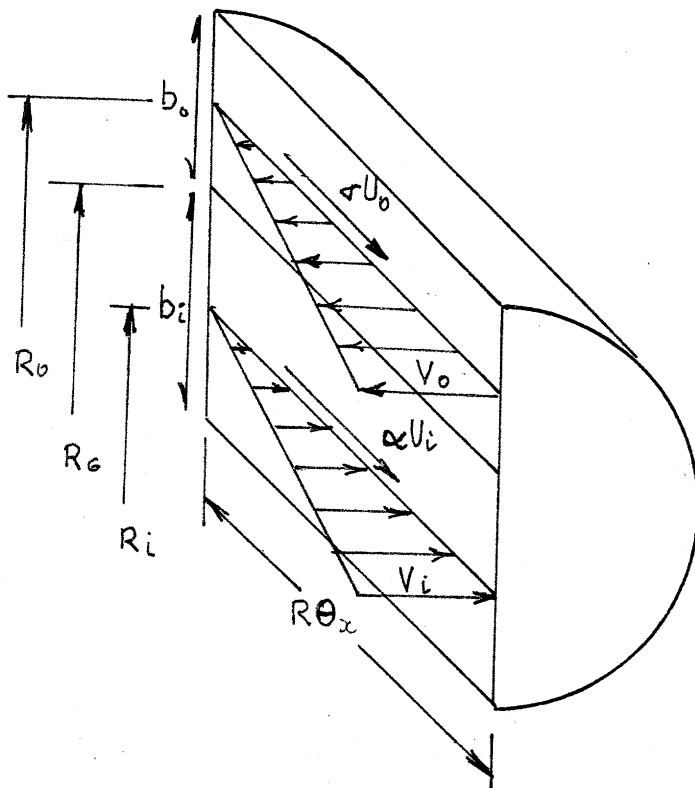
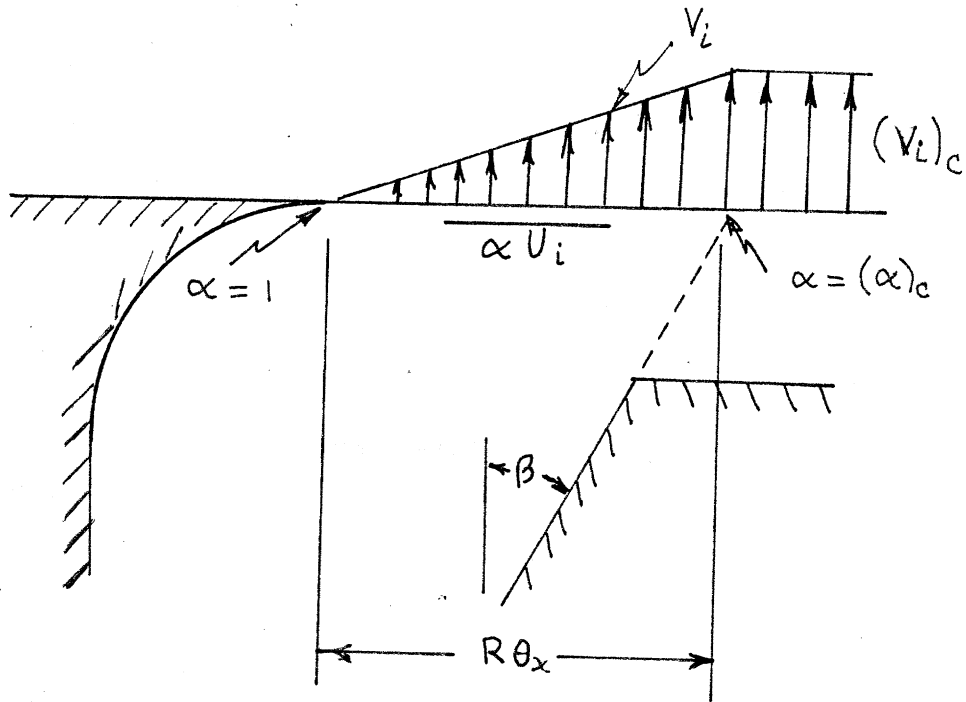
SCHEMATIC DIAGRAM OF EXPERIMENTAL UNIT

FIG 1.2



SCHEMATIC DIAGRAM OF NEW INLET

FIG 2

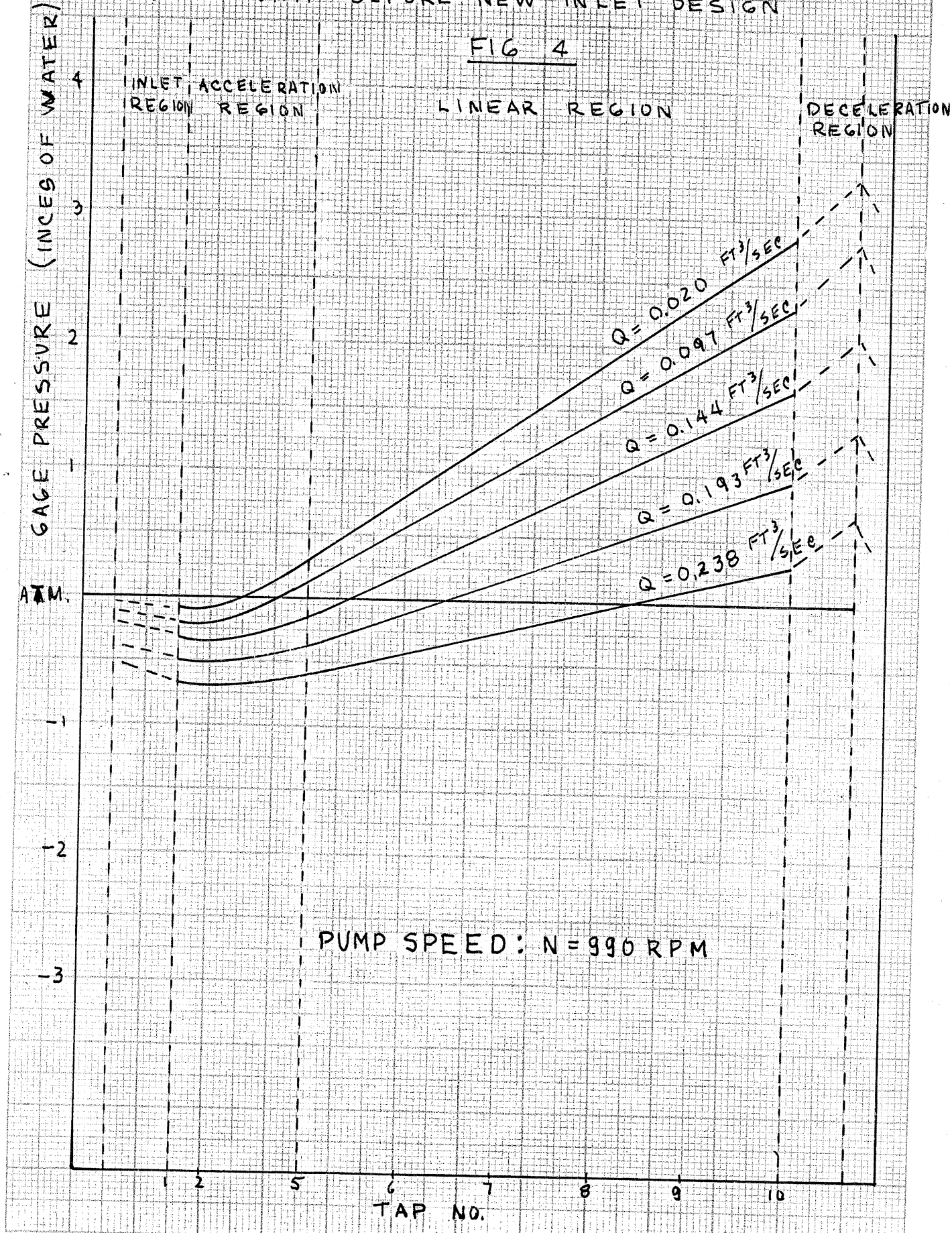


BASIC CONTROL VOLUME ASSUMED AT INLET

FIG 3

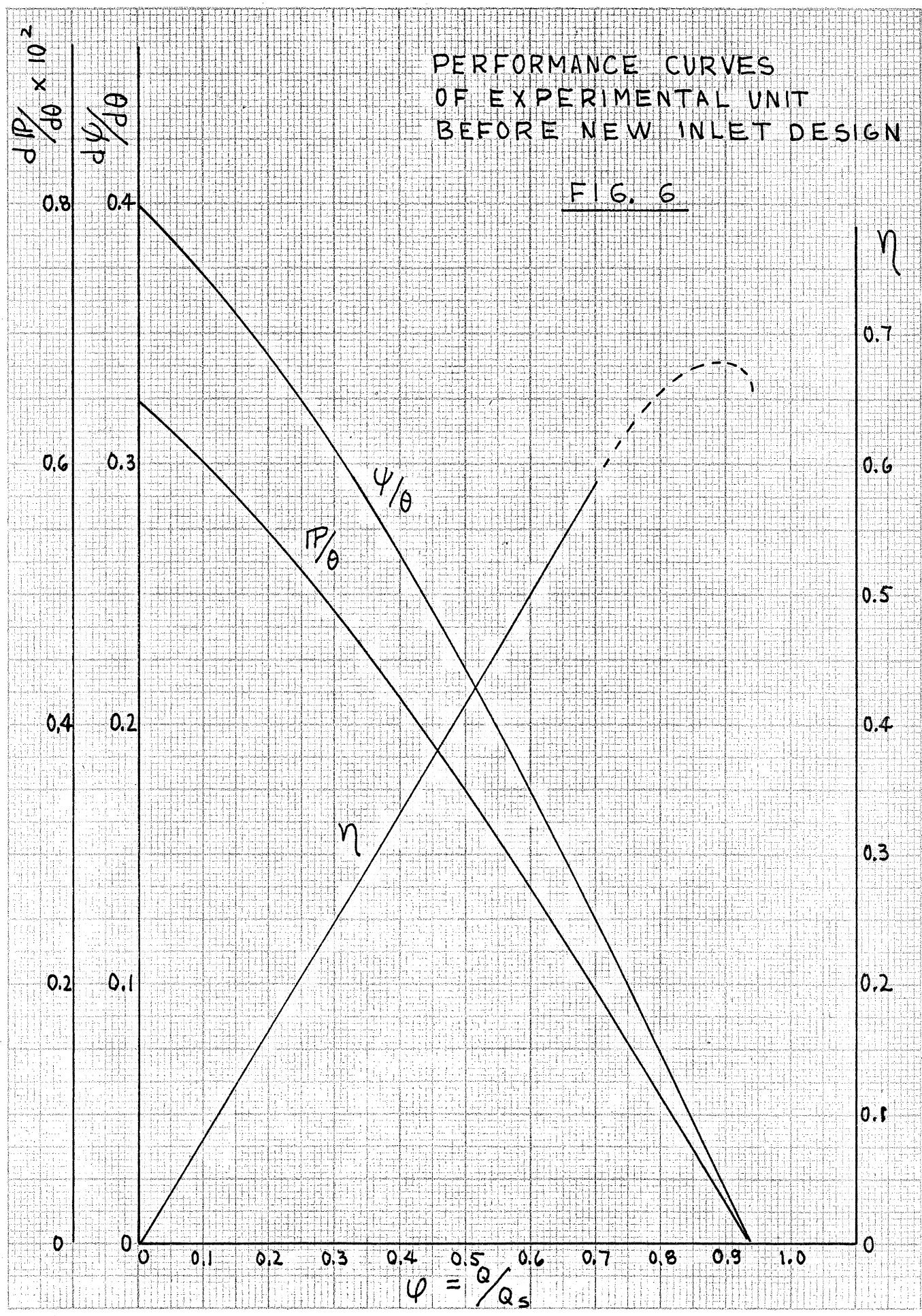
PRESSURE VARIATION WITHIN EXPERIMENTAL UNIT BEFORE NEW INLET DESIGN

FIG 4



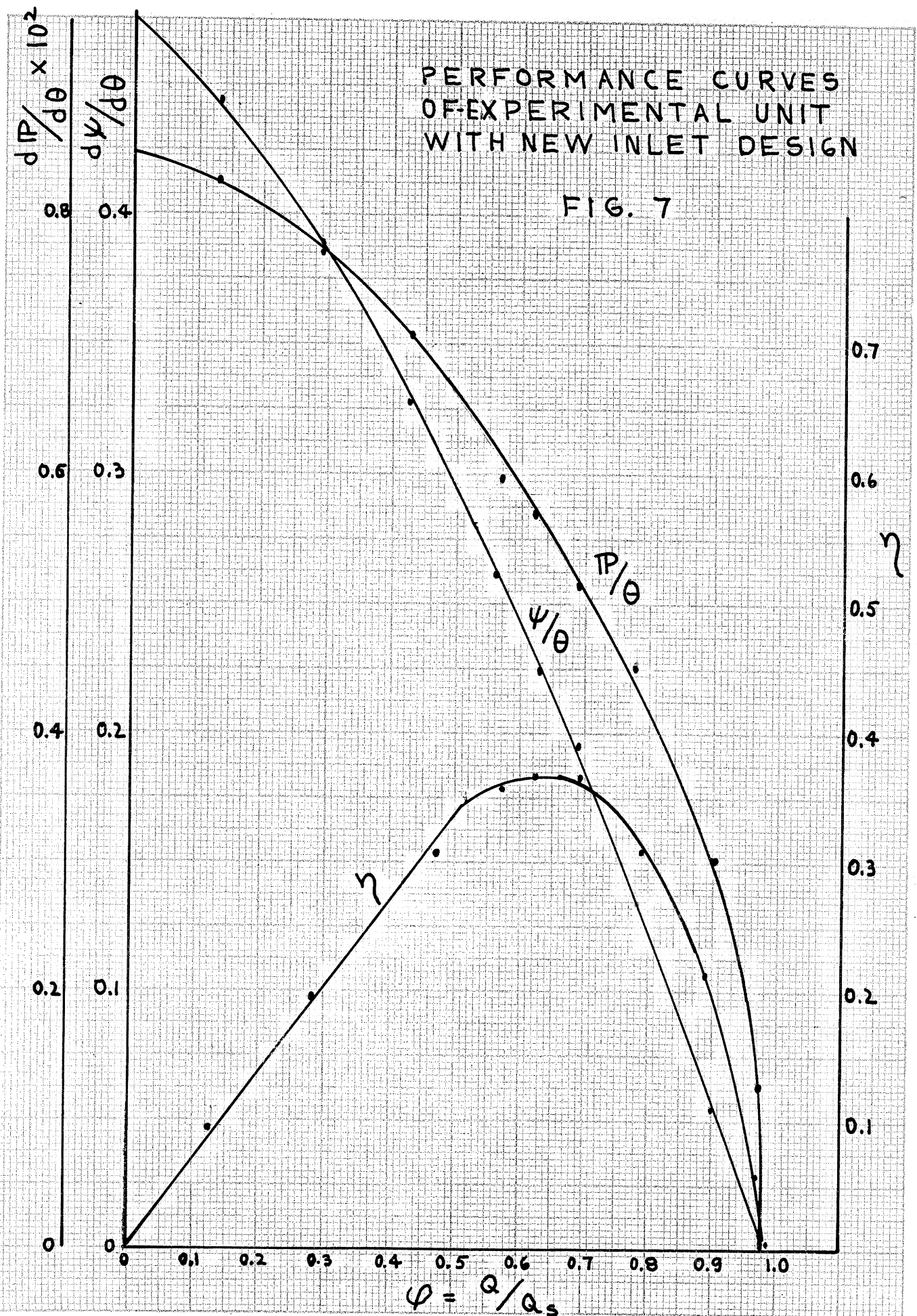
PERFORMANCE CURVES OF EXPERIMENTAL UNIT BEFORE NEW INLET DESIGN

FIG. 6



PERFORMANCE CURVES
OF EXPERIMENTAL UNIT
WITH NEW INLET DESIGN

FIG. 7



CONTENTS OF APPENDIX

TABLE I CONSTANTS USED IN CALCULATIONS

A.1 THE CIRCULATORY FLOW THEORY AND DERIVED
 EXPRESSIONS

A.2 CALCULATIONS FOR INLET DESIGN

TABLE I

CONSTANTS USED IN CALCULATIONS

A (in ²)	_____	1.54
R _G (in)	_____	3.87
R _O (in)	_____	4.45
R _i (in)	_____	3.20
R _i /R _G	_____	0.828
R _O /R _i	_____	1.39
B _O (in)	_____	0.840
B _i (in)	_____	1.16
C _i	_____	1.130
C ₂	_____	0.885
Δ	_____	0.820
K _c	_____	3.84

DESIGN CONSTANTS FOR INLET

$$(Q/Q_s)_{\text{design}} = 0.7$$

$$N = 1000 \text{ RPM}$$

$$\alpha = 0.46$$

APPENDIX A-1

THE CIRCULATORY FLOW THEORY

The theory and derived expressions exposed here are the work of D.P. Dewitt², and are based on the theory proposed by Wilson, Santalo and Oelrich¹. The following is only an abstract and is by no means complete.

A-1.1 Theory of Operation. The fluid mechanism

The flow within the pump is described as a helical motion by means of two perpendicular components of velocity at each point: the tangential component V_t and the component V_c in the plane perpendicular to V_t . The two terms introduced are:

- (1) Tangential or through flow $Q = \int_A V_t dA$ where A is the cross-sectional area of the channel.
- (2) Circulatory or meridional flow: Q_c with units of volume rate per radian arc length.

The angular momentum of the flow is increased radially through the impeller due to the guidance of the impeller blades. In the channel the flow loses angular momentum and thereby supports a tangential pressure gradient in the channel. The flow then reenters the impeller for another cycle. In order to increase the tangential pressure gradient, the rate of circulation must be increased and/or the tangential velocity

of the flow entering the impeller must decrease. Reduction of the tangential velocity will cause a smaller through-flow.

A-1.2 BASIC ASSUMPTIONS FOR ANALYSIS

- 1) Flow may be considered steady if time averages are used for pressures and velocity.
- 2) Fluid is incompressible.
- 3) There is no internal leakage.
- 4) All processes within the pump are adiabatic.
- 5) Characteristic flow is one dimensional in each major direction (radial, tangential and axial).
- 6) Tangential pressure gradient is independent of radius.

A-1.3 DIMENSIONLESS PERFORMANCE

The basic control volumes considered are shown in Figure A-1.

If we have:

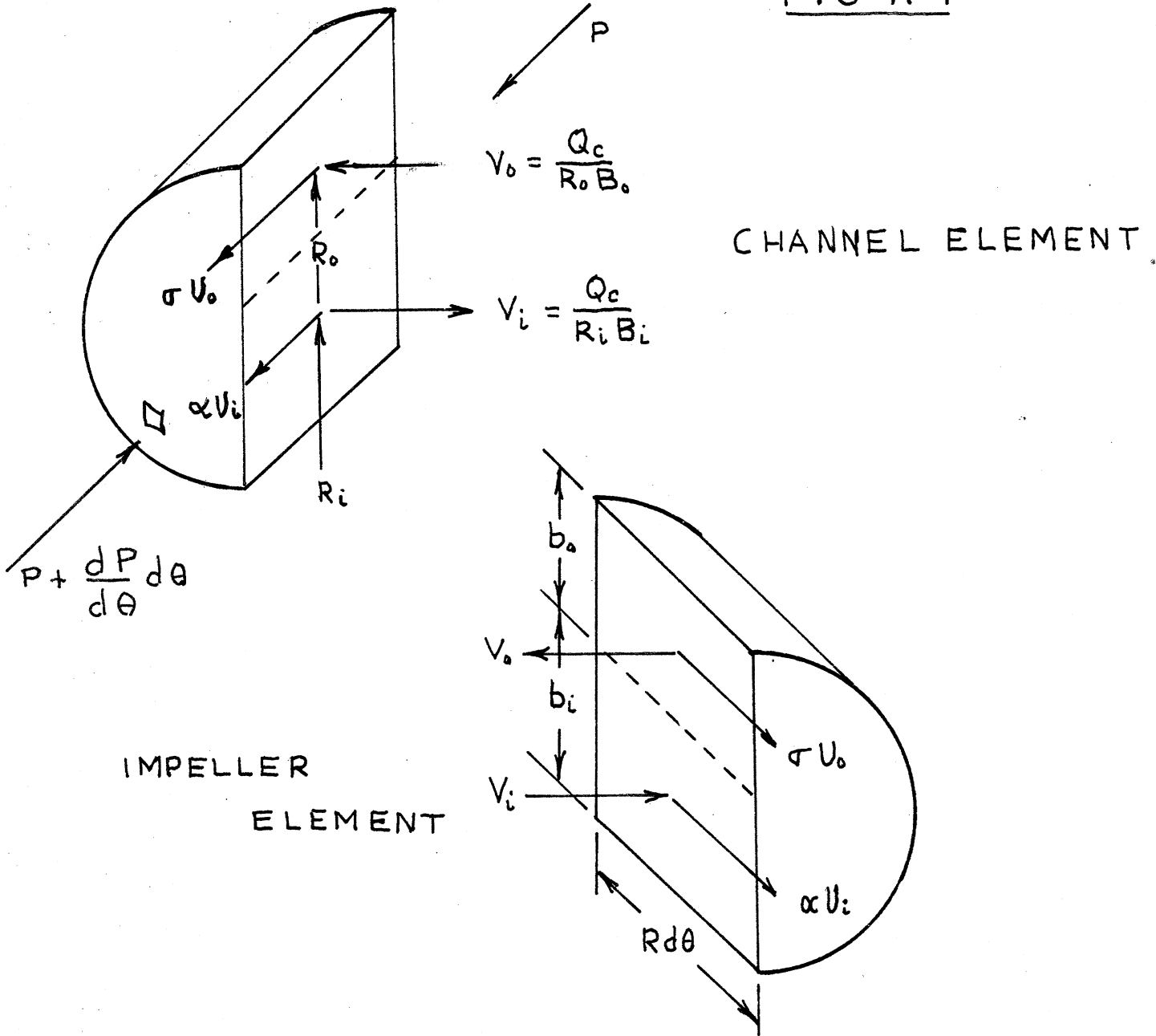
$$\sigma = \frac{V_{t0} \text{ (ACTUAL)}}{U_0} \quad (1)$$

$$\alpha = \frac{V_{t1} \text{ (ACTUAL)}}{U_1} \quad (2)$$

where σ is called the slip factor, and is a measure of degree of guidance which the blade offers to the flow leaving impeller, and is constant for a given pump geometry, and is measured experimentally. And α is a factor which depends on the flow as may be seen from equation (5).

BASIC CONTROL
VOLUMES

FIG A-1



The flow rate capacity is defined as

$$Q = \int_A V_t dA \cong \frac{\omega U_o + \alpha U_i}{2} A \quad (3)$$

and if the maximum theoretical flow capacity (solid body rotation) is defined as

$$Q_s = R_o A \omega \quad (4)$$

combining (3) and (4)

$$\varphi = \frac{1}{2} \left[\frac{R_o}{R_c} \omega + \frac{R_i}{R_c} \alpha \right] \quad (5)$$

The derived equation for the circulatory flow is:

$$Q_c = \frac{R_i B_i}{\sqrt{k_c}} \sqrt{(\omega U_o^2 - \alpha U_i^2) \left(1 - \frac{Q}{R_o A \omega}\right) - \frac{(1 - \alpha)^2 U_i^2}{2}} \quad (6)$$

then combining with equations (3) and (4) and (5):

$$\frac{Q_c}{Q_s} = \frac{R_i B_i}{A} \sqrt{\frac{2}{k_c}} \sqrt{\frac{R_i}{R_o}} \sqrt{(C_1 - \varphi)(1 - \varphi) - \frac{R_o}{R_i} (C_2 - \varphi)^2} \quad (7)$$

where

$$C_1 = \frac{R_o^2}{2 R_i R_o} \left(1 + \frac{R_i}{R_o}\right) \omega \quad C_2 = \frac{R_i}{2 R_o} \left(1 + \frac{R_o}{R_i}\right) \omega$$

The derived equation for the Head rise, or Pressure coefficient is:

$$\frac{d\psi}{d\theta} = \frac{1}{2} \frac{R_i}{R_c} \frac{Q_c}{Q_s} (C_1 - \varphi) \quad (8)$$

which comes from angular momentum relations.

A simplified angular moment relation, with the assumption of no friction losses is:

$$\frac{1}{\rho} \frac{dP}{d\theta} = \frac{Q_c}{R_o A} [\omega U_o R_o - \alpha U_i R_i] \quad (9)$$

APPENDIX A-2

CALCULATIONS FOR INLET DESIGN

A-2.1 CHOSEN DESIGN CONDITIONS

These conditions are:

$$\varphi = \frac{Q}{Q_s} = 0.7$$

The ratio of actual flow rate to solid body rotation of channel area

$$N = 1000 \text{ RPM}$$

Impeller speed

$$\omega = 104.7 \frac{\text{RAD}}{\text{SEC}}$$

or
Impeller angular velocity

A-2.2 CALCULATION OF $(\alpha)_c$

From equation:

$$\varphi = \frac{1}{2} \left[\frac{R_o}{R_c} \alpha + \frac{R_i}{R_c} \alpha \right]$$

and using values in Table I

$$(\alpha)_c = 0.46$$

A-2.3 CALCULATION OF INLET ANGLE

At end of inlet region ($\theta = \theta_\alpha$) we have:

$$(V_i)_c = \frac{(Q_c)_c}{R_i B_i} \quad \text{and} \quad (V_{ti})_c = (\alpha)_c U_i = (\alpha)_c R_i \omega$$

therefore the inlet angle is given by:

$$\beta = \tan^{-1} \frac{(\alpha)_c U_i}{(V_i)} = \frac{(\alpha)_c R_i \omega}{\frac{Q_c}{R_i B_i}}$$

and if:

$$\frac{Q_c}{Q_s} = \frac{R_i B_i}{A} \sqrt{\frac{2}{K_c}} \sqrt{\frac{R_i}{R_o} \sqrt{(C_1 - \varphi)(1 - \varphi) - \frac{R_o}{R_i} (C_2 - \varphi)^2}}$$

$$Q_s = R_i A W$$

for the chosen design conditions and the values given in Table I, the calculated inlet angle is:

$$\beta = 33.2^\circ$$

A-2.3 CALCULATION OF LENGTH OF INLET REGION

From continuity

$$Q = B_i R_i \theta_x (V)_x \text{ average velocity across inlet region} = \phi Q_s$$

and if this average velocity is approximated as:

$$(V)_{av.} = \sqrt{(\alpha U_i)_{x-AVE}^2 + (V_i)_{x-AVE}^2}$$

where

$$(\alpha U_i)_{x-AV} = \alpha_c U_i + \frac{2}{3} \sqrt{(1 - (\alpha)_c U_i)^2}$$

and

$$= \frac{2}{3} (V_i)_c = \frac{2(\alpha)_c Q_c}{3 (R_i B_i)}$$

combining we obtain

$$(R_i \theta_x) = \frac{\phi Q_s}{\frac{B_i}{3} \sqrt{U_i^2 (2 + (\alpha)_c^2) + 4 \left(\frac{Q_c}{R_i B_i} \right)^2}}$$

Then, substituting for the design conditions, and the constant values in Table I, the length of the inlet region is

$$R_i \theta_x = 1.35 \text{ inches}$$

or

$$\theta_x = 0.422 \text{ radians} = 24.2^\circ$$

A-2.4 PREDICTION OF PRESSURE ACROSS INLET REGION FOR DESIGN CONDITIONS

If friction losses are neglected, we have that for circulatory flow in the channel the pressure difference is given

by the relation

$$\frac{dP}{d\theta} = \frac{f Q_c}{g R_o A} \left[\Delta U_o R_o - \alpha U_i R_i \right]$$

If the flow would be fully developed circulatory flow, this expression would give the pressure difference without the need of further integration, because all quantities would be constant. However, in the inlet region, following the conditions of the model developed for the flow, we would have that Δ as well as α vary as a function of θ .

Therefore, as an approximation the average values of Q_c and α will be used, and therefore the following expression is obtained for the inlet region between $\theta = 0$ and $\theta = \theta_x$:

$$(\Delta P)_x = \frac{f (Q_c)_{x-AVE}}{g R_o A} \left[\Delta U_o R_o - (\alpha U_i)_{x-AV} R_i \right] \theta_x$$

or substituting

$$(\Delta P)_x = \frac{f \frac{2}{3} (Q_c)_c}{g R_o A} \left[\Delta U_o R_o - \left(\frac{2}{3} + \frac{(\alpha)_c}{3} \right) U_i R_i \right] \theta_x$$

and solving this equation for the design conditions and converting units, the following difference in head across the inlet region is obtained:

$$(\Delta P)_x = 2.56 \text{ ft. of air}$$

or

$$(\Delta P)_x = 0.036 \text{ in. of water}$$

BIBLIOGRAPHY

1. "A Theory of the Fluid-Dynamic Mechanism of Regenerative Pumps" by W.A. Wilson, M.A. Santalo, and J.A. Oelrich, A.S.M.E. Transactions, Volume 77, November 1955, pp. 1303-1311.
2. Dewitt, D.P., "Rational Design and Development of the Regenerative Pump," S.M. Thesis, M.I.T. January 1957.