A Non-Cooperative Game Theory Based Controller Tuning Method for Microgrid DC-DC Converters

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Jie Mei, James L.Kirtley
Department of Electrical Engineering and Computer Science
Massachusetts Institute of Technology
Cambridge, MA, USA

***Abstract*—PI controllers are widely used in converter control in Micro-grids. The PI controller tuning problem is usually modeled as a multi-objective optimization model because of its conflicting design specifications. This research introduces the idea of a new tuning method for PI controller of DC-DC converter in micro-grids based on non-cooperative game theory. Multi-objective optimization for the tuning problem will be transferred to a non-cooperative game. According to the solved Nash Equilibrium, best tuning parameters for the PI controller may be determined.**

***Index Terms*—Game theory, DC-DC power converters, Pareto optimization, Microgrids.**

I. Introduction

The connection of small generation units, with power ratings less than a few tens of kilowatts to low voltage networks potentially increases reliability to final consumers and brings additional benefits for global system operation and planning; namely, regarding investment reduction for future grid reinforcement and expansion [1]. In this context, Micro-grids, which can be defined as low voltage networks with micro-sources and Energy Storage connected and providing both power and heat to local loads, have been studied extensively in recent years. As more and more renewable energy sources, such as wind and solar power, are integrated to Micro-grids, fully controllable synchronous generators, which are normally responsible for voltage and frequency control in traditional power systems, become less common in Micro-grids. Those installed micro-sources, including Energy Storage, are not suitable for direct connection to Micro-grids due to their voltage level and other electricity characteristics. Thus power electronics interfaces (DC-DC or DC-AC) are essential in Micro-grid design. Control methods for these power electronic devices are important, as inevitable disturbances include variations and uncertainties from source, load, and circuit parameters can cause circuit operation to deviate from nominal. One fairly satisfactory and widely used control solution in Micro-grids to counteract departures from nominal is based on proportional-integral (PI) controller. The technical community shows continued interest in new tuning methods for finding optimal control parameters for the proportion and integration parts [2].

PI controller tuning can be viewed as the search for the best compromise between all the specifications and thereby many researchers applied the idea of multi-objective optimization as an alternative to resolve this problem [3]. Solving the multi-objective optimization can yield a set of solutions in which no objective can be improved without sacrificing at least one other objective. Graphically, this is often expressed as the 'Pareto Frontier'. The problem is how to choose the final solution among the Pareto set, since no solution is better according to multi-objective optimization definition. In another words, much decision choices in turn makes it difficult to make a decision. In reality, many of the objectives in the multi-objective optimization are conflicting. Thus how to meet all the objectives and show the importance of every objective, in another words, make final decision among elements of the Pareto set, is the core part of the multi-objective optimization, which has been studied for many years. Traditionally, there are generally two groups of methods to solve the problems. The first method is to set a single-objective evaluation function and using weighting factors to show the importance of every objective thus transferring multi-objective optimization to a single-objective optimization. The other group of methods uses the techniques of layering, grouping, and classifying methods to change multi-objective optimization to single-objective optimization. For those methods, the optimum of the weighted single objective depends on the preference of the designer a lot and thus lacks objectivity [4]. In fact, when a designer is making decision among conflicting objectives, the ultimate goal is to reconcile the conflict. Thus, a theory to reconcile the conflict between objectives is needed to avoid subjective arbitrariness. In this context, using Game Theory to solve multi-objective optimization seems rational and suitable as it is a theory for solving and reconciling conflict. In fact, game theory has been applied to solve power system decision-making problems in various research fields, mainly including power system planning and power markets [5]. Dating back to 1944, von Neumann and Morgenstern first put forward the problem of multi-objective optimization with conflicting objectives in the perspective of non-cooperative Game Theory [6]. For a multi-objective optimization problem, if the players, strategies set, utilities, and game rules can be defined, then the multi-objective optimization can be transferred to a game problem, and further the final solution for multi-objective optimization can be chosen from the Pareto set according to the Nash Equilibrium.

This research will study the tuning methods for the PI controller in Micro-grid converters based on multi-objective optimization and Game Theory. So far the most popular tuning method in use was proposed by Ziegler and

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Nichols as in [7]. However, further tuning is always required because the controller settings derived are rather aggressive and thus result in excessive overshoot and oscillatory response. And another problem is the parameters are difficult to estimate in noisy environment. Cohen and Coon design a tuning method in [8]. The main design objective is to reject load disturbances. One of the disadvantages of this method is that it can only be used for first-order models including large process delays. Followed the above two methods, more improved tuning methods were proposed [9]-[12]. However, all those tuning methods are based on the global optimal. In this research, how to set non-cooperative game based on multi-objective optimization will be explained in detail, and the controller parameters will be selected based on the solved Nash Equilibrium of the non-cooperative game. To authors' knowledge, this is the first time that non-cooperative game theory has been applied in DC-DC converter tuning problem of Micro-grid.

The remaining paper is organized as follows. A multi-objective optimization model based on conflicting control specifications for the controller tuning problem will be introduced in section II. In section III, a method to get the exact expressions for the design objective functions will be explained in detail. Then in section IV, method to transfer multi-objective optimization to a game will be introduced. Simulation results will be discussed in section V. And finally, section VI will give the conclusion and future work.

II. Multi-objective Optimization of Controller Tuning

The general model of multi-objective optimization can be described as follows:

$$\min_{x\in R^m}[f_1(\boldsymbol{x}), f_2(\boldsymbol{x}),..., f_n(\boldsymbol{x})] \tag{1}$$

Subject to

$$g_i(\boldsymbol{x}) \le 0, i = 1,2,\ldots,m \tag{2}$$

$$h_i(\boldsymbol{x}) = 0, i = 1,2,\ldots,p \tag{3}$$

where $\boldsymbol{x} = [x_1, \ldots, x_m]$ is the vector of decision variables, $f_i(\boldsymbol{x})$ ($i = 1, 2,\ldots, n$) are the objective functions, $g_i(\boldsymbol{x})$ ($i = 1, 2,\ldots, m$) and $h_i(\boldsymbol{x})$ ($i = 1, 2,\ldots, p$) are the corresponding inequality and equality constraints. The solution set of a multi-objective optimization is called the *Pareto Frontier*, in which no objective can be improved without sacrificing at least one other objective. All solutions in Pareto Frontier are acceptable according to multi-objective optimization definition and no solution can be seen to be better than any other. In recent years, controller tuning has been described as a multi-objective optimization problem. As stated in the background, designing and tuning a PI controller appears to be conceptually intuitive, but can be hard in practice, especially when multiple and conflicting objectives such as short transient and high stability are to be achieved at same time. There are usually four typical specifications in controller tuning [13]: rise time, peak overshoot, settling time, and steady-state error.

As it is difficult to write expressions for all the specifications above, especially for complicated plant to be controlled, two measure functions [14] are often applied to show the performance of the above four time domain specifications, which will here be treated as the objective functions in a multi-objective optimization model for the controller tuning problem. The first measure function is Integral of Time-weighted Square Error (*ITSE*).

$$ITSE = \int_0^\infty te^2(t)dt \tag{4}$$

The second one is Integral of Square Time-weighted Square Error (*ISTSE*) which can be express as

$$ISTSE = \int_0^\infty t^2e^2(t)dt \tag{5}$$

One characteristic of the *ITSE* criteria is that it may result in a response with quick rise time and relatively small overshoot but a long settling time because it integrates time linearly. With time weights squared, *ISTSE* will emphasis more on the settling time and steady-state error specifications. Those characteristics of *ITSE* and *ISTSE* are corresponding to our knowledge of controller tuning that transient stability and static stability are difficult to optimize at the same time as described in background, thus can be used to described the conflicting objectives.

Besides the four performance design specifications, nominal stability should also be considered in controller designing. A necessary and sufficient condition for BIBO stability is that the poles of the transfer function are strictly in the left half plane. For the nominal closed-loop system, this is equivalent to say that the sensitivity function is stable. The nominal stability requirement can be treated as a constraint on the decision variables.

With the objectives and constraint specified, the model of multi-objective optimization for controller tuning problem can be formulated as:

$$\min_{K_i,K_p}[ITSE, ISTSE] \tag{6}$$

subject to nominal stability requirement, where K_p and K_i are PI controller tuning parameters to be determined.

III. Calculation of Objective Functions

In this section, method to express *ITSE* and *ISTSE* with K_i and K_p will be explained, starting with the introduction of the Åström-Jury-Agniel algorithm.

A. Åström-Jury-Agniel Algorithm

According to [15]-[17], the Åström-Jury-Agniel algorithm can be used to evaluate the loss functions given in formula (7) below for continuous time systems.

$$I = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{B(s)}{A(s)}\frac{C(-s)}{A(-s)}ds \tag{7}$$

where the polynomials $A(s)$, $B(s)$, $C(s)$ have the forms of:

$$A(s) = a_0s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n \tag{8}$$

$$B(s) = b_1s^{n-1} + \ldots + b_{n-1}s + b_n \tag{9}$$

$$C(s) = c_1s^{n-1} + \ldots + c_{n-1}s + c_n \tag{10}$$

The algorithm operates in an iterative way as follows:

$$I_k = I_{k-1} + \frac{\beta_k\gamma_k}{2\alpha_k} \tag{11}$$

$$I_0 = 0 \tag{12}$$

$$I_k = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \frac{B_k(s)}{A_k(s)}\frac{C_k(-s)}{A_k(-s)}ds \tag{13}$$

where the kth order polynomials $A_k(s)$, $B_k(s)$ and $C_k(s)$ are:

$$A_k(s) = a_0^k s^k + a_1^k s^{k-1} + \ldots + a_{k-1}^k s + a_k^k \tag{14}$$

$$B_k(s) = b_1^k s^{k-1} + \ldots + b_{k-1}^k s + b_k^k \tag{15}$$

$$C_k(s) = c_1^k s^{k-1} + \ldots + c_{k-1}^k s + c_k^k \tag{16}$$

Starting from $k = n$, $A_n(s) = A(s)$, $B_n(s) = B(s)$, and $C_n(s) = C(s)$, and calculate to $k = 0$ according to the formulas (17)-(19) below, all values of a_i^k, b_i^k, c_i^k, β_k, α_k, and γ_k can be yielded. Then inserting the results in (11)-(13) one can get the final value of I.

$$a_i^{k-1} = \begin{cases} a_{i+1}^k & i\ even \\ a_{i+1}^k - \alpha_k a_{i+2}^k & i\ odd \end{cases}$$
$$i = 0,\ldots,k-1 \quad \alpha_k = \frac{a_0^k}{a_1^k} \tag{17}$$

$$b_i^{k-1} = \begin{cases} b_{i+1}^k & i\ even \\ b_{i+1}^k - \beta_k a_{i+2}^k & i\ odd \end{cases}$$
$$i = 1,\ldots,k-1 \quad \beta_k = \frac{b_1^k}{a_1^k} \tag{18}$$

$$c_i^{k-1} = \begin{cases} c_{i+1}^k & i\ even \\ c_{i+1}^k - \gamma_k a_{i+1}^k & i\ odd \end{cases}$$
$$i = 1,\ldots,k-1 \quad \gamma_k = \frac{c_1^k}{a_1^k} \tag{19}$$

The next step is to rewrite the objectives *ITSE* and *ISTSE* with the same structure of (7) and apply the algorithm, which is described in the next two subsections.

B. ITSE in Frequency Domian

Assume the discrepancy of output voltage with reference in frequency domain has the form of

$$e(s) = \frac{N(s)}{D(s)} \tag{20}$$

where $N(s)$ and $D(s)$ are polynomials. Then according to general Parseval formula [18], it shows

$$I = \int_0^{\infty} f(t)g(t)dt = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} F(s)G(-s)dt \tag{21}$$

where

$$f(t) = e(t) \quad g(t) = te(t) \tag{22}$$

and corresponding Laplace transforms are

$$F(s) = \frac{N(s)}{D(s)} \tag{23}$$

$$G(s) = \frac{N(s)\dfrac{dD(s)}{ds} - D(s)\dfrac{dN(s)}{ds}}{D^2(s)} \tag{24}$$

In order to match the objective *ITSE* with equation (7), defining the following relationships:

$$A(s) = D^2(s) \tag{25}$$

$$B(s) = D(s)N(s) \tag{26}$$

$$C(s) = N(s)\frac{dD(s)}{ds} - D(s)\frac{dN(s)}{ds} \tag{27}$$

Then apply the Åström-Jury-Agniel algorithm introduced earlier, an expression of the objective *ITSE* with only K_i and K_p as its variables is generated.

C. ISTSE in Frequency Domian

Similarly, *ISTSE* can be generated by the same steps. In this case, defining

$$f(t) = g(t) = te(t) \tag{28}$$

and the corresponding Laplace transforms are

$$F(s) = G(s) = \frac{N(s)\dfrac{dD(s)}{ds} - D(s)\dfrac{dN(s)}{ds}}{D^2(s)} \tag{29}$$

In order to match the objective *ISTSE* with equation (7), defining the following relationships:

$$A(s) = D^2(s) \tag{30}$$

$$B(s) = C(s) = N(s)\frac{dD(s)}{ds} - D(s)\frac{dN(s)}{ds} \tag{31}$$

Applying the Åström-Jury-Agniel algorithm, the expression of *ISTSE* with only K_i and K_p as its variables is generated.

IV. Game Theory of Controller Tuning

This section will explain the method to transfer multi-objective optimization to a non-cooperative game. For general multi-objective optimization model (1)-(3), Assume $m \geq n$, n objective functions $f_i(x)$ (i = 1, 2,…, n) can be treated as the objective function for each participant in a n-participant non-cooperative game. Further, variable vector $\boldsymbol{x}$ can be divided to n sub-vectors as following

$$x = [x_1, x_2, \ldots, x_n], x_i \in R^{m_i} \tag{32}$$

in which $\sum m_i = m$ and treat variable vector $\boldsymbol{x_i}$ as the decision variables for ith participant with fi as its objective function. The key technology of transforming multi-objective problems into game is to divide the variable set $\boldsymbol{x}$ into each player's strategies [x_1,…, x_n], which can be done by computing the factor index and fuzzy clustering [19]. Express the strategies in $\boldsymbol{x}$ except $\boldsymbol{x_i}$ as

$$x_{-i} = [x_1, x_2, \ldots, x_n] \tag{33}$$

Then the following non-cooperative game is yielded from the original multi-objective optimization.

Participants:

$$i = 1, \ldots, n. \quad (34)$$

Strategies:

$$x = [x_1, x_2, ..., x_n], x_i \in R^{m_i}, i = 1, \ldots, n. \quad (35)$$

Strategy Space:

$$X = X_1 \times X_2 \times \cdots \times X_n, X_i \in R^{m_i}, i = 1, \ldots, n. \quad (36)$$

Penalty Functions:

$$f_1(x), f_2(x), \ldots, f_n(x) \quad (37)$$

In this non-cooperative game, participant i always seeking to minimize its penalty function, thus the model can be expressed as

$$\left\{ \min_{x_i \in X_i} f_i(x_i, x_{-i}) \right\}, \forall i \quad (38)$$

from which the Nash Equilibrium can be calculated according to the iterative method described in [20]. During iteration, each participant will adopt a strategy according to its best response function and strategies taken by other participants. In the DC-DC buck converter case, the *ISTSE* and *ISTE* in would be the two objectives function and nominal stability will be the constraint. After it transferred to a two-player game, player one will treat *ISTE* as its objective function and can only change K_p to get lower *ISTE* value given whatever value of K_i, and player two will treat *ISTSE* as its objective function and can only change K_i to get lower *ISTSE* value given whatever value of K_p.

V. SIMULATION

In this section, the PI controller parameters of DC/DC Buck converter, as shown in Fig.1 [21], is determined by the proposed game theory based method. Other types of DC-DC converters can also apply the proposed method to get optimal controller parameters. The input voltage of the Buck converter is 36 V initially and drops to 24 V at 0.015s, and the output voltage is expected to be maintained at 15V.

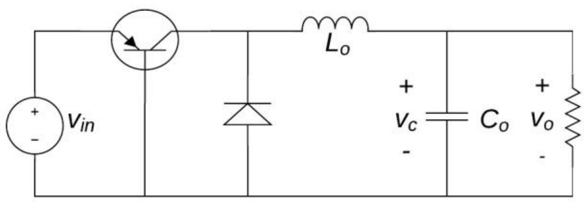

Figure 1. Circuit schematic of a buck converter.

The state space model of the Buck converter above is

$$\frac{di_{Lo}}{dt} = \frac{q(t)v_1}{Lo} - \frac{v_o(t)}{Lo} \quad (39)$$

$$\frac{dv_o}{dt} = \frac{i_{Lo}(t)}{Co} - \frac{v_o}{C_0 Ro} \quad (40)$$

where $q(t)$ is 1 when switch is on and 0 when off. Then following transfer function can be derived for the corresponding averaged model:

$$H_1(s) = \frac{\hat{v}_o(s)}{\hat{v}_1(s)} = \frac{D}{(s^2 LoCo + s\frac{Lo}{Ro} + 1)} \quad (41)$$

$$H_2(s) = \frac{\hat{v}_o(s)}{\hat{d}(s)} = \frac{V_1}{(s^2 LoCo + s\frac{Lo}{Ro} + 1)} \quad (42)$$

where D is the duration for switch is on. Assume the input to $H_1(s)$ is 1/s, a step response, which means a sudden change in the input voltage, the error of output voltage in frequency domain can be expressed as

$$e(s) = \frac{H_1(s)}{s[1 - (K_p + K_i)H_2(s)]} \quad (43)$$

To make the system stable, K_i and K_p should be picked up within the range of (44) below.

$$K_p < \frac{1}{V_1}, \quad K_I < 0 \quad (44)$$

Applying the method described in III, *ITSE* and *ISTSE* expressions made up by K_i and K_p can be generated. After 6 iterations of gaming, the tuning parameters can be determined to be K_p = -0.016 and K_I = -93.

Fig.2 plots the output voltage responses of the system in Fig.4 by using the proposed game theory based tuning method, compared with MATLAB self-optimal tuning and traditional ziegler–nichols tuning method, when input voltage drops to 24 V from 36 at 0.015s. The parameters of the circuit can be seen in Table I. As can be seen from Fig.2, by using the game theory based controller tuning method, the output voltage recovers much faster after the input voltage drops than using the MATLAB self-optimal tuning method and the traditional Ziegler–Nichols tuning method. Besides, the maximum output voltage drop is also the smallest among three tuning methods.

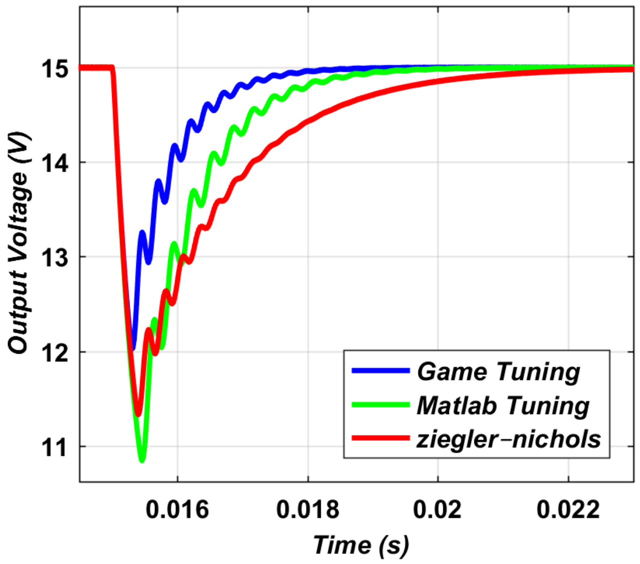

Figure 2. Comparison of game theory based tuning method with MATLAB optimal tuning and ziegler–nichols method.

TABLE I. BUCK CONVERTER PARAMETERS

L_o	10 μH
C_o	180 μF
R_o	4.5 Ω

In order to plot the Pareto Frontier of the multi-objective optimization model (6) of the DC-DC controller tuning problem, 6000 random combinations of K_i and K_p within the nominal stability range are generated by sweep over the two dimensional space to get 6000 values of corresponding *ISTE* and *ISTSE*. For better visualization, all the data are normalized such that minimum *ISTSE* happens at normalized *ISTE* equals 1 and minimum *ISTE* happens at normalized *ISTSE* equals 1. Only the points that can form Pareto Front are kept and are plotted in Fig.2, along with the Nash Equilibrium point generated from the proposed game theory based tuning method.

As can be seen in Fig.2, for the DC-DC buck converter controller tuning problem, the Nash Equilibrium locates at the Pareto front of the corresponding Multi-objective optimization. For other Multi-objective optimization problems with conflicting objectives and more variables, the Nash Equilibrium point might not be located on the corresponding Pareto front. In those cases, the final solution can be chosen from the Pareto front by finding the solution that is closest to the Nash Equilibrium. By doing this, the Nash Equilibrium is treated as a reference to help the selection of final solution, in which case, both global and individual optimal is stilled considered.

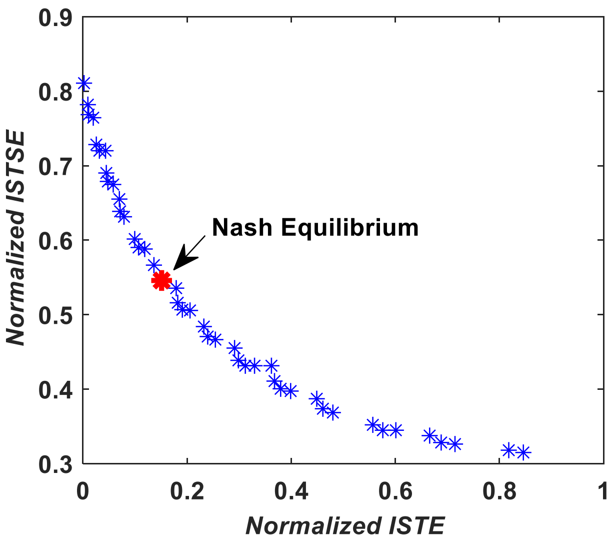

Figure 3. Plot of the Pareto Front and Nash Equilibrium point.

VI. CONCLUSION

We have proposed a new tuning method for a PI controller for a DC/DC converter based on non-cooperative game theory. The tuning problem is firstly modeled as multi-objective optimization problem according to conflicting design specifications. Then a method to transfer multi-objective optimization to non-cooperative game is introduced. According to the solved Nash Equilibrium, a final decision will be made among Pareto set. By using the game theory based tuning method, both global and individual optimal is considered, and the problem is solved based on the system conflicting characteristics, instead of making decision of the weights in transferring multi-objective optimization to single-objective optimization. As the simulation results shows, the tuning parameters yielded from game theory based method behaves better than traditional tuning methods. Besides DC-DC converters, it is expected that similar non-cooperative game theory based method will be developed to solve more conflicting multi-objective optimization problems in the future, such as the tuning problem for DC-AC converters. Moreover, the relationship between the solved Nash Equilibrium and Pareto frontier need to be explored.

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