Structurally Constrained Control Systems Using a Factorization Approach

by

Jose Elias Lopez

B.S. in Electrical Engineering, University of Minnesota (1980)
M.S. in Electrical Engineering, University of Colorado (1985)

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

October 1993

Author ....
Department of Electrical Engineering and Computer Science
October 29, 1993

Certified by ...
....
Michael Athans
Professor of Electrical Engineering
Thesis Supervisor

Accepted by .
....
Frederic R. Morgenthaler
Chairman, Departmental Committee on Graduate Students

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Abstract

Quite often, in the control of physical systems, structural constraints are placed on the feedback controller. Issues of complexity, computation, ease of implementation and physical dimensions play a role in the decision to select structurally constrained controllers. This thesis is devoted to an analysis and investigation of fundamental properties of structurally constrained controllers from an input/output framework. The type of structurally constrained controllers considered consist of fully decentralized controllers and partially decentralized controllers. The point of view taken is to analyze the controllers as a specific subset of the set of all stabilizing controllers in the modern control paradigm. This involves examining the parameterized set of structurally constrained controllers via their stable factors and associated stable factor constraints. The thesis first examines the issue of a suitable stable factor parameterization for fully decentralized controllers. An analysis identifies a subclass of controllers useful in autonomous designs and design methods based on iteration. An autonomous design method for decentralized control is developed by bounding parameters of the controller in terms of stable factors of the plant. The method is developed for both stable and unstable plants. Issues in developing decentralized controllers for robust performance are examined. Fundamental obstacles to concurrent design of robust subcontrollers are detailed and a D-K methodology for sequential design of subcontrollers is developed. Computation issues for this sequential D-K methodology are outlined along with some anticipated difficulties. Finally, a methodology for incorporating partially decentralized controllers into the input/output framework is developed. A transformation applicable to partially decentralized systems is developed which allows partially decentralized controllers to be designed using input/output methods developed specifically for fully decentralized systems.

Thesis Supervisor: Michael Athans
Title: Professor of Electrical Engineering

Acknowledgments

I deeply express my gratitude to my thesis advisor Professor Michael Athans. Under his mentorship, formidable technical skills, and outstanding intuition I have learned and benefited a great deal. I would like to thank Professor Munther Dahleh for his assistance over the course of my graduate program. In addition, his clarity and precision in presenting technical material have proved invaluable in both my studies and research. I owe a great debt of gratitude to Dr. John Dowdle for his support and help in motivating and funding this research. In addition, I would like to thank him for the his time and effort in meeting with me, reviewing the research and providing useful critiques of the thesis. Finally I would like to thank Professor Gunter Stein. He met with me a number of times during the course of the thesis and his help and insight are greatly appreciated.

In addition there are some other individuals associated with Draper Laboratories that I would like to thank. Dr. Karl Flueckiger who also served as a technical monitor on this research and Timothy Henderson who early on provided assistance with the SBL finite element model.

I would like to thank the following graduate students whom I have had the privilege of interacting with: Kirk Gilpin, John Wissinger, Leonard Lublin, Joel Douglas, Alan Chao, Ignacio Diaz Bobillo, Jon How, Frank Aguirre, Ed Bielecki, Steve Patek, Nicola Elia, Mitch Livstone, Mike Branicky, Marcos Escobar, Wesley McDermott.

I would like to thank my wife, Jennifer Leigh Lopez whose beauty captivated me, whose talent inspired me and whose humor actually makes me laugh. Finally, I would like to thank a very precious member of my family, my daughter, Katarina Lynn Lopez. Katarina you have brought me great joy and tons of love, your contribution to this effort is without equal.

This research was conducted at the MIT Laboratory for Information and Decision Systems with support initially provided by the Patricia Roberts Harris Fellowship. Subsequent funding was provided by IRAD funds from the C.S. Draper Laboratory Inc., Dr. J.R. Dowdle and Dr. K.W. Flueckiger of CSDL are the technical monitors.

Dedication

I dedicate this thesis to my brother Rafael Luiz Lopez. His courage and endurance go beyond anything I have ever known. Through his struggles I have rediscovered the need to always persist in the valiant fight. Que Dios te bendiga mi querido hermano.

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Chapter 1

Introduction

1.1 Motivation

The last decade has witnessed a phenomenal growth in computing power. This accessible computing capability has been exploited by new control methodologies which rely on it to develop sophisticated controllers capable of robustness in the face of model and disturbance uncertainty. These technical advances have led to increased application in ever widening domains of complexity and scale. Unfortunately, the control of many large scale systems still present prohibitive costs in terms of instrumenting centralized control solutions. Issues of complexity, limitations on computation, ease of implementation and physical dimension continue to play a significant role in forcing the control engineer to place structural constraints on the feedback controller eventually used to control large scale systems.

The most familiar structurally constrained controller is the fully decentralized controller. A decentralized control structure imposes a partitioning and pairing of system inputs and outputs. The resulting controller is constrained to be block diagonal thereby providing an individual controller for each channel of the partitioned system. The characteristic advantages of decentralized control over centralized control must, to a degree, be the characteristic advantages of any structurally constrained controller in order to be beneficial. Namely, the implementation of the controller should be simplified relative to a centralized controller, the effective block diagonal nature of structurally constrained controllers should provide for a set of lower order subcontrollers, a corresponding reduction in communication bottle necks and an inherent parallel processing advantage over any centralized solution. In addition, structurally constrained controllers can often be designed to provide reliable/strongly stabilizable control strategies which provide for graceful degradation of system control in the event of subcontroller failures [1], [2], [3], [4]. One disadvantage to structurally constrained control is overall performance degradation due the structural constrains placed on the controller (i.e. loss of full information feedback due to the partitioning of the feedback controller into sub-blocks). Another disadvantage is that the design and synthesis of structurally constrained controllers which satisfy overall robustness measures and performance objectives is considerably more difficult relative to centralized controller designs.

In the last decade major strides have been made in the development of an elegant input/output control framework based on the concept of stable factors [5] [6]. The plant is decomposed into stable factors (i.e. the origin of the terminology "fractional approach"). These factors can be used in a parameterization of all closed-loop stabilizing controllers. The controller synthesis then reduces to selecting the controller from the set of all stabilizing controllers that satisfies a performance metric. In the case of optimal design, the problem is formulated in such a way that the search for the optimal controller is conducted over the parameter set which defines the set of all stabilizing controllers. The parameter itself is often referred to as the Youla parameter as a result of one the original papers discussing this parameterization method [7]. This framework allows for the inclusion of both model and disturbance uncertainty in the input/output sense and has lead to the solution of a number of important centralized control problems [8]. The salient features of this centralized control framework are the following:

- The parameterization of all stabilizing centralized controllers.
- The controller parameter is unconstrained over the stable ring it is defined on.
- The nominal plant optimization problem results in solving a model matching

problem where the optimization equation is affine in the controller parameter.

- This framework is applicable to both continuous-time and discrete-time, lumped parameter system models.
- Model, disturbance and parameter uncertainty can be represented via structured perturbations.
- Robust controller design methodologies can be developed using this framework.

The main emphasis in this thesis is to study structurally constrained controllers in the input/output framework and to provide a unification of structurally constrained controller methodologies under the modern control paradigm currently employed in centralized control systems. This study uses as a starting point a parameterization of all stabilizing decentralized controllers [9] which helps to expose the rich underlying algebraic structure of these problems. A number of results follow from this investigation and are more fully detailed in section 1.3.

1.2 Background

The evolution of structurally constrained controllers lies within a vast body of literature devoted to decentralized systems and control. A plethora of design strategies and incremental adaptations have been developed over the last couple of decades [10], [11]. The voluminous variety and often ad-hoc methods that have been developed is understandable given the fact that a general methodology for the design of linear time invariant decentralized controllers, which takes into account overall performance metrics and robustness to uncertainty, still does not exist. The purpose of this section is to provide highlights of some of the more notable trends developed for decentralized control and to put into perspective the methods developed in this thesis with respect to work done in the past.

A important point noted early on in connection with stability under a decentralized control was the observation that under decentralized control the condition of plant observability and controllability (in a state space setting) was not sufficient to guarantee arbitrary pole placement. Wang and Davison [12], [13] pioneered the notion of fixed modes under decentralized information structure. Effectively, fixed modes generalize the notion of observability and controllability. Under centralized control fixed modes would be the unobservable and uncontrollable system poles. Since fixed modes are invariant under static and dynamic decentralized control, the existence of a stabilizing decentralized controller is dependent on the absence of unstable fixed modes. Other researches have provided alternative characterizations of fixed modes. Anderson and Clements [14] and Tarokh [15] have provided algebraic time domain characterizations. Seraji [16] has provided a frequency domain characterization. Glover and Silverman [17], Reinschke [18] and Sezer and Siljak [19] [20] have provided graph theoretic characterizations. These various characterizations provide greater insight into the nature and cause of fixed modes and provide specific methods for testing the system for fixed modes, given the decentralized partitioning. A definition and more detail of fixed modes in terms of stable factors [21], which fits the input/output development in this thesis, will be give in section 2.5.

Some of the first decentralized design approaches relied on pole placement for decentralized stabilization and control. For systems with no fixed modes or only stable fixed modes decentralized feedback schemes where developed to place the poles of the closed loop systems [12], [22], [23], [24]. Richter and DeCarlo [25] presented a solution to the decentralized pole placement problem where nonlinear pole placement equations where formulated and a numerical solution developed using homotopy methods. Although pole placement methods would seem to have diminished impact on practical applications due to the difficulty of translating eigenstructure assignment to performance measures of the closed loop system, designers still rely on these methods for decentralized control of large flexible structures [26], [27].

Due to the difficulties in designing decentralized controllers for generalized systems, many methods are based on exploiting a special property of the plant. A broad break down of these plant characteristics which spawn individual decentralized design methods would include plants with special asymptotic properties and plants composed of a interconnection of similar subsystems. Plants possessing special asymptotic properties lead to a set of design methods know as nonsingular and singular perturbation techniques [10]. Nonsingular perturbation techniques can be applied to plants in which the off block diagonal terms of the system A matrix are small in some measurable sense allowing the system to be approximated by decoupled subsystems. Designs are then based on these decoupled subsystems [28], [29], [30]. Issues associated with this technique involve developing an appropriate measure of the coupling, generating decoupled subsystems (i.e. ignore coupling, use some form of series expansion and ignore all but zero order terms etc.) and developing a bounds which guarantees stability of the closed loop system.

Singular perturbation techniques rely on detecting separation between multiple time scales within the plant [31]. The simplest case is a plant which is composed of slow and fast dynamics [32]. By using the singular perturbation decomposition and utilizing interconnection properties the plant is partitioned into several smaller subsystems for which controllers are designed. Problems with this method include restrictions on the closed loop systems to be multi-timescale and requirements of intermediate dynamics to exhibit good asymptotic separation for successful implementation of controllers [33].

Plants composed of a interconnection of similar subsystems (also referred to as a composite system) result from identifying physical or mathematical geometries of the system which will then allow a decomposition into similarly connected subsystems. A variety of decentralized control techniques have been developed for plants satisfying this condition [34], [35], [36]. The majority of the results tend to be concerned with global stability once the individual subsystems have been stabilized [37], [38]. The usual global stability criterion for these systems is developed from Lyapunov methods [39] [40]. Some work has been done to develop global stability criterions and a measure of subcontroller performance degradation due to the interconnections [41], [42].

The problem with many of the methods mentioned so far deal with the assumptions made. The plant is assumed perfect and often overall performance measures are not directly dealt with. Even on the subsystem level, performance is defined only in terms of the nominal subsystem operator. Very few examples are given which incorporate model and performance uncertainties. This is primarily due to the ad-hoc and specialized techniques developed which make a systematic treatment of uncertainty difficult.

Recently there have been some attempts to align decentralized control methods with the modern centralized control framework which allows for a systematic treatment of model and performance uncertainty [43] [44]. An interesting example which seeks to extend this modern framework in a relatively straight forward fashion to the case of interconnected or composite plant systems is the work by Tan and Ikeda [45]. In their formulation of decentralized control the centralized input/output fractional framework mentioned in section 1.1 is applied directly to the individual subsystems. The individual Youla controller parameter is selected in such a manner as not to destabilize the entire system when the individual subsystem is connected. Subsystems are connected sequentially and the set of selected Youla parameters for the decentralized control system will provide for closed-loop stability of the overall system. Note however that this is not a full extension of the centralized fractional control methodology. Closed-loop stability is only one part.

As will be discussed in section 2.2 an essential component to extending decentralized methods to a modern control paradigm is a set of stabilizing decentralized compensators over which a search can be performed in order to determine the decentralized controller which comes closest to satisfying a prespecified performance metric. The selection of decentralized controller parameterization makes a difference in how much analysis can be done and how far along one can develop synthesis tools. Manousiouthakis has developed a parameterization for decentralized controllers [46] by relying on the traditional parameterization available for centralized controllers and constraining the Youla parameter via a quadratic operator constraint. Unfortunately, this particular approach to decentralized controller parameterization is very limiting due to the lack of direct decentralizing information in the parameterization. For the moment, the only analytical result available from this parameterization is an approximation to the value of the overall performance norm achievable for the l_1 case using suboptimal decentralized control [47].

The decentralized controller parameterization used as a starting point in this thesis is a parameterization developed by Gundes and Desoer [9]. This parameterization was further embellished by Date and Chow [48]. More details concerning these parameterizations will be provide in sections 3.2-3.3. The essential point is that the Gundes/Desoer parameterization is useful for analysis precisely because information concerning the decentralized information structure imposed on the plant is directly available in the formulation of the decentralized parameterization.

1.3 Contribution of Thesis

The focus of this thesis has been to investigate and provide an analytical framework for structural controllers from an input/output point of view and to align the design of structurally constrained controllers to the methodology of centralized control systems under the modern robust stability/robust performance paradigm [49]. This is achieved by applying a parameterized factorization approach to the problem of structurally constrained controllers. A number of theoretical results follow from this method as will be discussed in this section. A robust design method is developed for decentralized controllers in chapter 5 using this parameterized factorization approach. An analogous approach based on previous design methods developed for decentralized controllers could also have been developed. These issues are further elaborated on both in this section and in section 5.5.

The type of structurally constrained controllers considered consist of fully decentralized controllers and partially decentralized controllers. The point of view taken is to analyze the controllers as a specific subset of the set of all stabilizing controllers in a modern control paradigm as mentioned in section 1.1. As such, this involves examining the parameterized set of structural controllers via their stable factors and associated stable factor constraints.

The starting point for the thesis is the examination of a recently developed stable factor parameterization for fully decentralized controllers [48]. From this investigation a new proof (section 3.4) of some critical auxiliary stable factor identities is developed.

This proof helps clarify the relationship of these auxiliary identities to a fundamental stable factor stability identity (see section 3.2) upon which the parameterization of all stabilizing decentralized controllers is built. Based on these auxiliary stable factor identities, developed in section 3.4, a class of stabilizing decentralized controllers is developed (see section 3.5). This class of controllers is shown to be useful for the development of a new autonomous design method for subcontrollers (chapter 4) and in the development of an adaptation of the D-K methodology for the sequential design of robust decentralized controllers (chapter 5). The usefulness of this set of decentralized controllers results from a simplifying assumption developed in section 3.5 which imposes a unimodular restriction on a subset of the design parameters of the decentralized controllers. Such a restriction has the benefit of generating a set of decentralized controllers whose individual subcontrollers have the form of the basic Youla parameterization (see section 2.2) which results in only one design parameter per subcontroller.

A new method for the autonomous design of subcontrollers is developed in this thesis. Chapter 4 presents this work along with a brief discussion outlining the usefulness of autonomous design methods. This autonomous method relies on the class of decentralized controllers developed in section 3.4. From this set of controllers the associated unimodular stability constraint is exploited to develop a simplified norm bound stability guarantee for the set of subcontroller parameters. For the case of stable plants, the subcontroller parameter bounds are in terms of the stable off-diagonal elements of the plant (see sections 4.2-4.4). One of the distinct advantages of this autonomous formulation based on stable factors is that it allows for the development of a similar bound for the case of unstable plants. The development for the unstable plant case is carried out in this thesis and presented in section 4.5. An additional result produced by this approach is the quantification of weak-coupling within a input/output plant operator point of view (see section 4.2).

The important issues concerning the design of robust stability/robust performance are analyzed in this thesis for the design of robust decentralized controllers. Using the stable factor formulation for decentralized controllers a stable factor decentralized

$M(\cdot)$ operator is developed. Specialized decentralized interaction properties, given in section 3.4, are used to produce a simplified decentralized $M(\cdot)$ operator. The $M(\cdot)$ operator is used in conjunction with the structured singular value metric to assess whether robust stability/performance requirements have been achieved and it plays a integral role in the synthesis of robust controllers (see section 5.2). An analysis indicating the difficulties in producing concurrent design methods for the parameters of robust decentralized controllers is presented (see section 5.3).

An adaptation of the D-K methodology for the sequential design of decentralized controllers is developed in this thesis (see section 5.4). The advantage of this adaptation for the design of decentralized controllers resides with the specific robustness guarantees available from the μ-framework. Decentralized controllers designed to insure a specific μ criterion for a specific closed loop operator extracted from the plant and involving the decentralized controllers result in decentralized controllers which are robust from both a stability and performance point of view. This is an improvement over the guarantees available from other decentralized controller methodologies (see section 1.2).

The utility of developing computational methods directly from this parameterized D-K methodology for the sequential design of robust decentralized controllers is outlined in section 5.5. Embedded within this sequential D-K method is a step involving the iteration of subcontroller parameters. An argument can be made that the iteration of subcontroller parameters is analogous to the iteration of individual subcontrollers. Given this point of view the K step in the D-K method can be viewed in the time domain framework and in an input/output framework as a form of sequential loop closing where the iteration taking place is among the individual subcontrollers. This point of view is relied upon (see section 5.5) in developing an alternative method of computation for this algorithm in terms of commercially available software. Additional computation issues for this sequential D-K methodology are outlined along with some anticipated difficulties.

Finally, a methodology for incorporating partially decentralized controllers into the input/output framework is developed. A novel unimodular transformation applicable to partially decentralized systems is developed (see section 6.3) which allows partially decentralized controllers to be designed using input/output methods developed specifically for fully decentralized systems. The method is demonstrated for a number of canonical forms of partial decentralized controllers developed in this thesis. This unimodular transformation is sufficiently broad enough that other structurally constrained controllers not specifically characterized can also be handled using these same methods. The application of decentralized design methods, developed in this thesis, to the design of partially decentralized robust controllers is presented (see section 6.7).

1.4 Thesis Organization

Chapter 2 gives definitions and some essentials of the notation used in the thesis. A brief synopsis of a modern control paradigm basics for centralized systems is given. The definitions of fully decentralized systems are given. The decentralized partitioning problem is mentioned and the important role of decentralized fixed modes is detailed along with a definition compatible with the input/output framework being developed.

Chapter 3 presents a parameterization of stabilizing decentralized controllers in terms of stable factors and a unimodular constraint. Auxiliary properties associated with this parameterization along with a direct proof of these properties is developed. An analysis identifies a subclass of controllers which have special auxiliary Bezout identity properties. These properties will be used in Chapter 4 for autonomous design methods and Chapter 5 for methods based on iteration.

Chapter 4 presents an autonomous design method for decentralized control based on a simplified bound of the controller parameters in terms of the stable factors of the plant. Connections to small gain results, extension to the multichannel case and bounds for the unstable plant case are presented.

Chapter 5 places the decentralized control problem in a μ-framework. Difficulties which prevent simultaneous design of robust controllers are presented. A D-K methodology for the sequential design of robust decentralized controllers is presented.

Chapter 6 presents a methodology for handling partially decentralized controllers. Canonical forms of partially decentralized controllers are given. A unimodular transformation applicable to partially decentralized systems is developed which allows partially decentralized controllers to be designed using input/output methods developed specifically for fully decentralized systems. Application of this method to the defined canonical forms of partially decentralized controllers is detailed.

Chapter 7 provides a research summary and recommendations for future work.

Chapter 2

Notation and Preliminary Concepts

2.1 Ring Notation Essentials

This section will present some basic notation used in the thesis. Parameterizing the set of all stabilizing decentralized controllers using rings reduces the need for repeated duplication since the parameterization based on rings remains the same whether the rings represent continuous time or discrete-time, lumped parameter systems. The same can be said for parameterizations based on normed linear vector spaces as done in [6], [50]. However, most of the earlier work in developing input/output approaches to decentralized control have relied upon ring theory in their developments [9], [46], [48], [44]. Therefore, this thesis will present parameterizations built upon rings. What follows is the essential ring notation consistent with that found in the Gundes/Desoer text [9]. For an explanation of basic ring properties which are applicable to control systems the reader is directed to appendices A and B of the Vidyasagar text [5] and for a more mathematical treatment see texts [51], [52].

H	principle ideal domain
$U \subset H$	is the group of units of H
G	is the ring of fractions associated with H

$m(H)$	set of matrices with elements in H
$m(G)$	set of matrices with elements in G
$m(0)$	set of matrices whose elements are 0
$\|F\|$	determinant of F
unimodular	$F \in m(H)$ is unimodular iff $\|F\| \in U$
$\|\| \cdot \|\|$	refers to the H_∞ norm of enclosed operator

As an example to illustrate the use of the above ring notation, continuous time definitions are associated with their appropriate ring notation.

H	set of real-rational, stable, proper transfer functions
$U \subset H$	transfer functions whose inverse is stable and proper
G	set of all real-rational transfer functions (stable and unstable)
$m(H)$	set of stable, proper transfer matrices
$m(G)$	set of real-rational transfer matrices
unimodular	$F \in m(H)$ is unimodular iff F has a stable, proper inverse

2.2 Modern Control Paradigm Basics for Centralized Systems

In this section a brief review of a modern control paradigm for centralized systems is presented. The reason for such a review lies with the central theme of this thesis which is to align structured controller design methods with this framework. This modern centralized control paradigm has been actively developed over the last decade. The paradigm provides for model, disturbance and parameter uncertainty via structured perturbations. Robust controller design methodologies have been developed using this framework. A number of sources provide thorough treatments of this modern control paradigm [6], [53], [8], [50]. Some of the highlights of this methodology will be presented here.

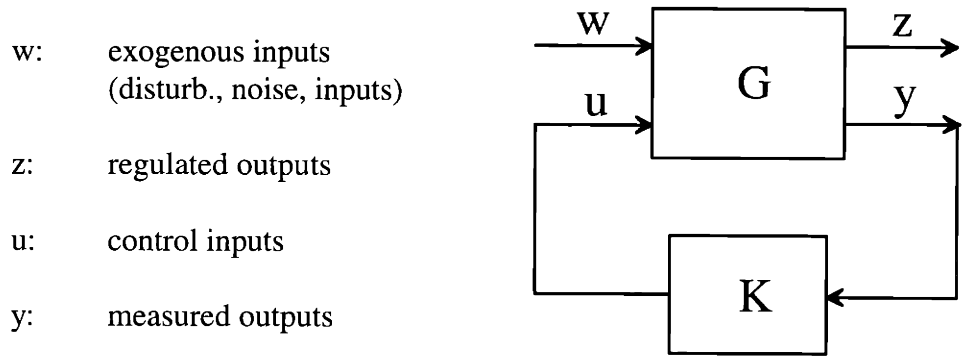

Figure 2-1: Block Diagram of Nominal Formulation for the General Control Problem

Figure 2-1 illustrates the nominal formulation of the modern control paradigm in block diagram form. Virtually any linear time invariant control problem relying on feedback control can be placed in this formulation. The open loop transfer function matrix (TFM) for G consists of four individual operators

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \tag{2.1}$$

The loop equations for figure 2-1 becomes

$$\begin{aligned} z &= G_{11}w + G_{12}u \\ y &= G_{21}w + G_{22}u \\ u &= Ky \end{aligned} \tag{2.2}$$

Effectively, the original plant is contained in G_{22}. The other TFM's G_{11}, G_{12}, G_{21} represent fictitious operators developed to represent the relationship between exogenous inputs to regulated and measured outputs. These fictitious operators will couple into the performance TFM when closed loop control is implemented as indicated in figure 2-1. The set of equations (2.2) will be well-posed if for any w there exist unique u, y, and $z's$ satisfying these equations. The equations will be well-posed if and only if the inverse, $(I - G_{22}K)^{-1}$, exists. This inverse also plays an important role in the performance operator.

H_∞ Formulation	l_1 Formulation
$w \in L_2 \qquad \|w\|_2 \leq 1$	$w \in l_\infty \qquad \|w\|_\infty \leq 1$
$\sup_w \|z\|_2 = \|T_{zw}\|_{H_\infty}$	$\sup_w \|z\|_\infty = \|T_{zw}\|_1$
$\min_{K-stab} \|T_{zw}\|_{H_\infty}$	$\min_{K-stab} \|T_{zw}\|_1$

Table 2.1: Nominal problem formulations which optimizes a given induced norm

The performance operator is denoted T_{zw}. It is the operator which maps the exogenous inputs to the the regulated outputs.

$$z = T_{zw} w \tag{2.3}$$

Without closed loop control T_{zw} consists solely of G_{11}. However, under closed loop control the performance operator is a linear fractional transformation in terms of G and K and is given by

$$F_l(G, K) := T_{zw} = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21} \tag{2.4}$$

The general nominal design objectives under this modern control paradigm are two fold:

1. Maintain closed loop system stability
2. Minimize the effect of "w", in some quantifiable way, on "z"

The first objective requires establishing a set of all stabilizing compensators for a given plant. The second objective requires defining performance in terms of a norm bound on the performance operator T_{zw}. Table 2.1 gives two examples of nominal problem formulation under this control paradigm for different induced operator norms. The H_∞ formulation is concerned with providing the best performance in terms of rejecting or minimizing the impact of bounded energy disturbances on the closed loop system. This reduces to finding a stabilizing compensator K which minimizes the

H_∞ norm of the performance operator T_{zw}. Likewise, the l_1 formulation is concerned with providing the best performance in terms of rejecting or minimizing the impact of bounded and persistent disturbances on the closed loop system. This reduces to finding a stabilizing compensator K which minimizes the l_1 norm of the performance operator T_{zw}. Of course there is a difficulty in searching for an optimal K given the form of the performance operator in eq. (2.4). The performance eq. (2.4) is nonconvex with respect to the compensator operator K. In addition, a set of stabilizing compensators based on a given plant has not been established. Both of these problems are solved under this modern control paradigm via the parameterization of all stabilizing compensators using stable factors.

An important fact, relative to developing a set of stabilizing controllers, is that K stabilizes G (of eq. (2.1)) *iff* K stabilizes G_{22}, [6]. This then allows the development of a set of stabilizing controllers to be based on stable factors of the actual plant G_{22}. Before describing how a parameterization for compensators is developed a few definitions involving coprime factors and stable factor decompositions need to be given.

Definition 1 (Right Coprime) *For $S, T \in m(H)$ which have the same number of columns, S and T are right-coprime* iff *there exist $C, D \in m(H)$ such that*

$$CS + DT = I \tag{2.5}$$

Definition 2 (Left Coprime) *For $\tilde{S}, \tilde{T} \in m(H)$ which have the same number of rows, $\tilde{S}$ and $\tilde{T}$ are right-coprime* iff *there exist $C, D \in m(H)$ such that*

$$\tilde{S}C + \tilde{T}D = I \tag{2.6}$$

Equations 2.5-2.6 are respectively referred to as individual left and right Bezout identities. Two operators, S and T, which are right coprime, effectively can be thought of as a stacked operator such as

$$F = \begin{bmatrix} S \\ T \end{bmatrix} \tag{2.7}$$

where the operator is invertible from the left. Additionally, right coprime implies that the operators S and T do not share any common zeros over the stable matrix ring $m(H)$. Equivalent concepts apply to operators which are left coprime.

Using these notions of right and left coprime, definitions for the decomposition of LTI operators into stable factors can be given. As given in reference [5] any LTI plant G_{22} can be decomposed into the following stable factors

$$G_{22} = ND^{-1} = \tilde{D}^{-1}\tilde{N} \tag{2.8}$$

Where N, D are right coprime and $\tilde{D}, \tilde{N}$ are left coprime with both D, $\tilde{D}$ square and $|D|, |\tilde{D}| \neq 0$. (N, D) is referred to as a right coprime factorization (r.c.f.) and $(\tilde{D}, \tilde{N})$ is referred to as a left coprime factorization (l.c.f.). The coprime stable factor decompositions of G_{22} are unique to within a unimodular factor. For example, given (N, D) are right coprime factors of G_{22}, the right coprime factors (NR, DR) are also a right coprime factorization of G_{22} where R is unimodular.

Given these definitions for stable factor plant decompositions and individual Bezout identities an important theorem which will be instrumental in parameterizing the set of all stabilizing compensators can now be given.

Theorem 1 *For $G_{22} \in m(G)$ with (N, D), $(\tilde{D}, \tilde{N})$ any r.c.f. and l.c.f. of G_{22}. Given $\tilde{U}$, $\tilde{V} \in m(H)$ satisfy*

$$\tilde{V}D + \tilde{U}N = I \tag{2.9}$$

Then there exists U, $V \in m(H)$ such that

$$\begin{bmatrix} \tilde{V} & \tilde{U} \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & -U \\ N & V \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{2.10}$$

Equation 2.10 is known as a doubly coprime Bezout identity (or doubly coprime factorization) for the plant G_{22}. The proof of Theorem 1 is available in [5], p. 79. The fundamental importance of the doubly coprime Bezout identity is that any compensator which stabilizes the plant G_{22} has a doubly coprime factorization of the form found in eq. (2.10). The proof of this is available in [6]. An interesting interpretation of this is that due to internal stability requirements in the classic two block problem (i.e. plant and compensator form the two blocks of a closed loop MIMO feedback system), where no assumptions are made on plant stability, the closed loop map of the system will require that four MIMO transfer functions be stable. However, this requirement really reduces to a requirement that two stable factor return difference matrices must have stable inverses or in other words must be unimodular. For example, a general stable factorization decomposition for a compensator could be denoted $K = U_1V_1^{-1} = \tilde{V}_1^{-1}\tilde{U}_1$. The associated return difference matrices for the two block system would be

$$\begin{aligned} \tilde{V}_1 D + \tilde{U}_1 N &= R \\ \tilde{N} U_1 + \tilde{D} V_1 &= \tilde{R} \end{aligned} \tag{2.11}$$

Due to internal stability requirements R and $\tilde{R}$ must have stable inverses (i.e. be unimodular) in order for K to be a stabilizing compensator. Rewriting eq. (2.11), assuming K is stabilizing or equivalently R and $\tilde{R}$ are unimodular, the following equations are obtained.

$$\begin{aligned} R^{-1}\tilde{V}_1 D + R^{-1}\tilde{U}_1 N &= I \\ \tilde{N} U_1 \tilde{R}^{-1} + \tilde{D} V_1 \tilde{R}^{-1} &= I \end{aligned} \tag{2.12}$$

Hence, the source of the individual Bezout identities in eq. (2.10) for a stabilizing compensator become apparent.

Some authors [9] refer to eq. (2.11) as denominator matrices. This terminology follows since it can be shown (see [9], p.45) that

$$|R| = |\tilde{V}_1 D + \tilde{U}_1 N|$$

$$|\tilde{R}| \;=\; |\tilde{N}U_1 + \tilde{D}V_1| \tag{2.13}$$

form characteristic determinants of the closed loop system. However, the use of characteristic determinant also applies to $|D|$ and $|\tilde{D}|$ where D and $\tilde{D}$ might be used to denote stable factors of an open loop system. To avoid confusion the term *return difference matrices* is used to refer to equations of the form found in eq. (2.11).

An important side note is that the two stable factor return difference matrices of eq. (2.11) are an equivalence relation with respect to the relationship that a given compensator stabilizes a given plant. Formalizing this statement in the form of a theorem generates the following.

Theorem 2 *For $G_{22}, K \in m(G)$, let $(N, D), (\tilde{D}, \tilde{N})$ be any r.c.f. and any l.c.f. of G_{22}, and let $(U_1, V_1), (\tilde{V}_1, \tilde{U}_1)$ be any r.c.f. and l.c.f. of K. Under these conditions, the following are equivalent:*

1. *The pair (G_{22}, K) is stable.*

2. *The matrix $\tilde{V}_1 D + \tilde{U}_1 N = R$ is unimodular.*

3. *The matrix $\tilde{N}U_1 + \tilde{D}V_1 = \tilde{R}$ is unimodular.*

The proof for this theorem is available in [5], pp. 105-106. The point to be made here is that only one return difference matrix, eq. (2.11), or equivalently one Bezout identity, eq. (2.12) is needed to generate the results for the set of all stabilizing compensators. Results then generated from the other Bezout identity are equilvalent and form a complementary set. Doubly Coprime Bezout identities are useful in steamlining the duplication efforts in generating these equilvalent results.

Theorem 1 is used directly in parameterizing the set of all stabilizing compensators. The details are found in [5] and [6] but a rough outline of the method is as follows. Multiplying the doubly coprime Bezout identity (eq. (2.10)) on the left by

$$\begin{bmatrix} I & Q \\ 0 & I \end{bmatrix} \tag{2.14}$$

and on the right by

$$\begin{bmatrix} I & -Q \\ 0 & I \end{bmatrix} \tag{2.15}$$

the following parameterized doubly coprime Bezout identity is obtained.

$$\begin{bmatrix} \tilde{V} - Q\tilde{N} & \tilde{U} + Q\tilde{D} \\ -\tilde{N} & \tilde{D} \end{bmatrix} \begin{bmatrix} D & -U - DQ \\ N & V - NQ \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{2.16}$$

The factors of this doubly coprime Bezout identity parameterized by $Q \in m(H)$ form left and right parameterized factorizations of compensators which stabilized G_{22}. To show these factors characterize all stabilizing compensators, a proof is done to illustrate that any arbitrary stabilizing compensator of G_{22} can be written using these parameterized stable factors. To summarize, the following theorem presents the parameterized set of all stabilizing compensators.

Theorem 3 *Given $G_{22} = ND^{-1} = \tilde{D}^{-1}\tilde{N}$ where the r.c.f. (N, D), (U, V) and the l.c.f. $(\tilde{D}, \tilde{N})$, $(\tilde{V}, \tilde{U})$ satisfy a doubly coprime Bezout identity, the parameterized set of stabilizing compensators is given by*

$$\begin{aligned} K &= (U + DQ)(V - NQ)^{-1}, \qquad |V - NQ| \neq 0 \\ &= (\tilde{V} - Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{D}), \qquad |\tilde{V} - Q\tilde{N}| \neq 0 \end{aligned} \tag{2.17}$$

for some $Q \in m(H)$.

Proof, as mentioned earlier, is available in [5] and [6]. This parameterization is often referred to as the Youla or Q parameterization. A principle advantage obtained by using this parameterization is that it will allow for a simplified reformulation of centralized control problems such as the aforementioned H_∞ and l_1 formulations.

As discussed earlier in this section, the performance operator in terms of K and G_{22} is a linear fractional transformation of the form

$$T_{zw} = G_{11} - G_{12}K(I + G_{22}K)^{-1}G_{21} \tag{2.18}$$

Substituting the controller parameterization, eq. (2.17), into $K(I+G_{22}K)^{-1}$ gives

$$K(I+G_{22}K)^{-1} = (U+DQ)\tilde{D} \tag{2.19}$$

The performance operator, eq. (2.18), then becomes

$$T_{zw} = G_{11} - G_{12}U\tilde{D}G_{21} - G_{12}DQ\tilde{D}G_{21} \tag{2.20}$$

Defining the following operators

$$\begin{aligned} H &:= G_{11} - G_{12}U\tilde{D}G_{21} \\ A &:= G_{12}D \\ B &:= \tilde{D}G_{21} \end{aligned} \tag{2.21}$$

the performance operator can be simplified to

$$T_{zw} = H - AQB \tag{2.22}$$

where $H, A, B \in m(H)$ and $Q \in m(H)$ is arbitrary. The form of eq. (2.22) is referred to as a model matching equation and more importantly it is affine in the parameter Q which is an element of the stable ring $m(H)$. This effectively transforms the nominal performance formulations from a search over a nonconvex set involving stabilizing controllers K and a rather complex linear fractional formulation for the performance operator to a search over a convex set involving the parameter Q and a simplified model matching formulation for the performance operator.

2.3 Definitions for Fully Decentralized Systems

A fully decentralized control strategy has the following properties.

- The plant is partitioned into input/output pairs. Sets of actuators are associated with sets of measured outputs.

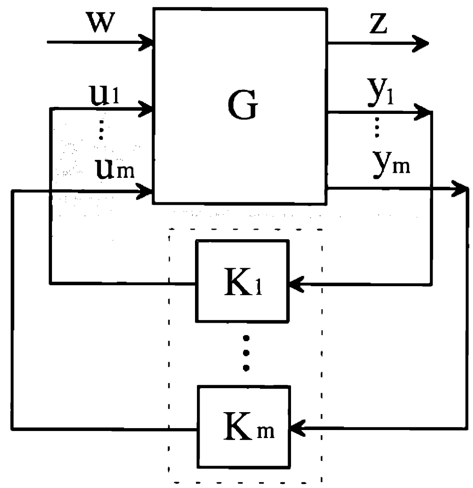

Figure 2-2: Decentralized Control within Framework of Modern Control Paradigm

- Individual controllers are associated with each I/O pair.
- No sharing of information occurs between the individual controllers.

Figure 2-2 illustrates, in a block diagram form, the fully decentralized control configuration as realized in the general control framework. The actual plant, G_{22}, is an element of $m(G)$ and has dimensions $p \times q$. To implement a fully decentralized control strategy the plant is partitioned into m channels. The structure of the input and output take on the following form.

$$\begin{aligned} U &= [U_1^T, U_2^T, \cdots, U_m^T]^T \\ Y &= [Y_1^T, Y_2^T, \cdots, Y_m^T]^T \end{aligned} \tag{2.23}$$

This then results in a decentralized controller which is block diagonal.

$$K = blkdiag[K_1, K_2, \cdots, K_m] \tag{2.24}$$

Individual subcontrollers map to their respective I/O pairs.

$$K_i : Y_i \longmapsto U_i \tag{2.25}$$

And the overall dimension is accounted for in the following fashion

$$\sum_i p_i = p \quad \text{and} \quad \sum_i q_i = q \tag{2.26}$$

where p_i is the dimension of Y_i and q_i is the dimension of U_i.

2.4 The Partitioning Problem

One of the first issues facing a designer developing a decentralized control strategy is that of partitioning the plant into appropriate I/O pairs. Quite often the spatial properties will dictate a particular plant partition. For example the geographic extent of a packet switched network, where dynamic flow control is used to solve problems of routing and congestion, can often require partitioning to be based on subsystem location. Particularly when the subsystems tie into the network at locations separated from one another by large physical distances [54]. Another natural partitioning strategy relies on geometric properties of the plant. For example, in [55] a segmented reflector telescope was partitioned according to a geometry suggested by the symmetric construction of the reflector from identically constructed subsystems.

In the absence of a naturally dictated partition, selecting a partition without some sort of analytic tool can become somewhat overwhelming due to the large number of possible pairing combinations. For example, in a centralized control strategy only one pairing combination is possible (i.e. all outputs are interconnected to all inputs). In a scalar decentralized strategy, where the individual subcontrollers are SISO, there are $n!$ pairing combinations for the case of a $n \times n$ plant.

A number of analytic tools have been developed over the years to help select a partition which may be viable. Some of the first tools developed and used primarily by the process control industry to select a pairing strategy for decentralized controllers composed of SISO subcontrollers are the Relative Gain Array (RGA) by Bristol [56] and the Niederlinski Index [57]. Interestingly, Niederlinski developed with heuristics a selection method which reduces to the same pairing selections given by a quantitative method with additional interaction conditions known as Decentralized Integral Controllability (DIC) which was developed a number of years later by Morari [58], [59]. Arkun and Manousiouthakis [60],[61], generalized the Bristol method to decentralized controllers whose individual subcontrollers are MIMO or in other words block diagonal decentralized controllers. These methods are referred to as Block Relative Gain (BRG) and Dynamic Block Relative Gain (DBRG). Nett and Manousiouthakis [62] attempted to develop partitioning strategies based less on qualitative and empirical validation and more on theoretical and quantitative validation. Their attempt results in making a connection between the Block Relative Gain methods and the Euclidean condition number. They go on to make conjectures which they hope in the future will become fact but at the moment are the qualitative reasons for basing partitions on these methods. Recently, Chen et al [63] have shown that in the face of structured uncertainties the RGA and BRG methods are less reliable indicators for partition selection than the Euclidean condition number. This development helps to strengthen the conjectures of Nett and Manousiouthakis [62]. However, this results primarily because the RGA and BRG depend on steady state matrices and don't account for possible large off diagonal elements at frequencies of interest. It would be interesting to see the performance of the DBRG method in this context since it incorporates frequency related information and not just steady state information in its formulation.

The above discussion illustrates some of the tools available to the designer for partition selection. As can be seen a good deal of engineering judgment, qualitative and quantitative information can be involved in the selection process. In this thesis, an underlying assumption is that a partition structure has been selected and that it is viable. The discussion of what constitutes a viable partition for decentralized control is the topic of the next section.

2.5 Definitions and the Role of Decentralized Fixed Modes

As mentioned in section 1.2 fixed modes generalize the notion of observability and controllability under a decentralized information structure. Fixed modes arise as a result of the constraints on the controller structure which limits how feedback information is interconnected to control or actuation signals. Wang and Davison [12] pioneered the notion of fixed modes and reported that fixed modes are invariant under static and dynamic linear time invariant decentralized control. Hence, the condition for the existence of a stabilizing LTI decentralized controller is that the plant under a given decentralized partition must not have any unstable fixed modes.

Although there are a number of ways to characterize fixed modes [14]-[20] a characterization in terms of stable factors developed by Vidyasagar and Viswanadham [21] will be given here.

Theorem 4 (Fixed Modes) *Given $G_{22} \in m(G^{p \times q})$, let (D, N) be any r.c.f. of G_{22}, and observe that $D \in m(H^{q \times q})$, $N \in m(H^{p \times q})$. Partition D, N as*

$$D = \begin{bmatrix} D_1 \\ \vdots \\ D_s \end{bmatrix} \qquad N = \begin{bmatrix} N_1 \\ \vdots \\ N_s \end{bmatrix} \tag{2.27}$$

where $D_i \in m(H^{q_i \times q})$, $N \in m(H^{p_i \times q})$. Finally, define

$$F_i = \begin{bmatrix} D_i \\ N_i \end{bmatrix} \in m(H^{(p_i+q_i)\times q}) \qquad F = \begin{bmatrix} F_1 \\ \vdots \\ F_s \end{bmatrix} \in m(H^{(p+q)\times q}) \tag{2.28}$$

and let β denote the greatest common divisor (g.c.d.) of all $q \times q$ minors of F obtained by choosing exactly q_i rows from F_i. Then G_{22} can be stabilized by a decentralized controller if and only if $\beta = 1$.

Proof of this theorem is available in [21]. The characterization of fixed modes resides with the factor β. This term, β, is referred to as the decentralized fixed determinant. The unstable zeros of β correspond to the unstable decentralized fixed modes for the plant G_{22} under the given partition.

In reference [21] the proof of theorem 4 relies on developing conditions on whether there exists a stable inversion of F as follows

$$EF = I \tag{2.29}$$

where E has a certain pattern of its minors constrained to be zero. It turns out that an E which satisfies the constraint on its minors can always be constructed from stable factors of a block diagonal controller with dimensions compatible with the plant partition of G_{22}. The formulation of E is as follows: Let

$$A = [A_1 \cdots A_s], \qquad B = [B_1 \cdots B_s] \tag{2.30}$$

where $A_i \in m(H^{q\times q})$ and $B_i \in m(H^{q\times(p+q)})$. Construct E as

$$\begin{aligned} E_i &= \begin{bmatrix} A_i & B_i \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \tilde{V}_i \\ 0 \end{bmatrix} & \begin{bmatrix} \tilde{U}_i \\ 0 \end{bmatrix} \end{bmatrix} \in m(H^{q\times(q_i+p_i)}) \\ E &= [E_1 \cdots E_s] \in m(H^{q\times(q+p)}) \end{aligned} \tag{2.31}$$

Equation (2.29) has a very familiar form. To illustrate this eq. (2.29) is given for a plant partitioned into two channels.

$$EF = \begin{bmatrix} \tilde{V}_1 & \tilde{U}_1 & 0 & 0 \\ 0 & 0 & \tilde{V}_2 & \tilde{U}_2 \end{bmatrix} \begin{bmatrix} D_1 \\ N_1 \\ D_2 \\ N_2 \end{bmatrix} = I \tag{2.32}$$

It is straight forward to verify that eq. (2.32) is equivalent to

$$EF = \begin{bmatrix} \tilde{V}_1 & 0 & \tilde{U}_1 & 0 \\ 0 & \tilde{V}_2 & 0 & \tilde{U}_2 \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ N_1 \\ N_2 \end{bmatrix} = I \tag{2.33}$$

which of course is the two channel, partitioned form of the Bezout identity.

$$\tilde{V}D + \tilde{U}N = I \tag{2.34}$$

Note, if such a E is found to provide a stable inverse of F this is equivalent to satisfying the Bezout identity of eq. (2.34). However, satisfying the Bezout identity of eq. (2.34) implies (see theorem 2) that there exists a stabilizing decentralized controller for the plant or equivalently that no unstable fixed modes exist for the given partition.

This illustrates the connection between theorem 4 defining fixed modes and decentralized stability conditions in terms of stable factors. Interestingly, Vidyasagar noted in the conclusion section of [21] that an open problem would be to characterize all stable inverses of eq. (2.29), but that such a task would effectively be *"the problem of characterizing all decentralized stabilizing controllers, which is known to be highly intractable"*. This is no longer an open problem, the next chapter presents a recent parameterization of all stabilizing decentralized controllers [9] along with some new results associated with this parameterization.

Chapter 3

Parameterization of All Stabilizing Decentralized Controllers

3.1 Introduction

As mentioned in section 2, the problem of parameterizing the set of all stabilizing controllers has been solved [9]. This parameterization represents the starting point for the developments in this thesis. This chapter will present this parameterization for the two channel case to make the notation manageable. The notation and formulation of the parameterization given in this thesis draws upon two sources, [9] and [48] which are equivalent versions of the same parameterization. Section 3.2 will introduce the decentralizing stability property known as the decentralized doubly coprime Bezout identity (DDCBI). The parameterization will be built upon identities extracted from the DDCBI. Section 3.3 will present all the pertinent details concerning the decentralized controller parameterization and the associated unimodular constraint on the design parameters. In section 3.4 a class of identities known as the auxiliary doubly coprime Bezout identities (ADCBI) will be presented along with their importance and fundamental role played in establishing the decentralized controller parameterization. A new proof of the ADCBI will be given in this section along with a clarification of their direct relation to the decentralized doubly coprime Bezout identity. Based on the ADCBI developed in section 3.4 a new class of stabilizing decentralized controllers will be characterized in section 3.5. This class of decentralized controllers are shown to be useful for the development of a new autonomous design method for subcontrollers (chapter 4) and in the development of an adaptation of the D-K methodology for the sequential design of robust decentralized controllers (chapter 5).

3.2 Decentralized Bezout Identity

Before laying out the pieces necessary for the decentralized parameterization, the following notion will be used to define an appropriate two channel partition of the plant P.

Definition 3 (Two Channel Partition) *For a plant, $P \in m(G^{p\times q})$ with l.c.f. $(\tilde{D}_d, \tilde{N}_d)$ and r.c.f. (N_d, D_d) the following represents a two channel partition of a plant where the input channel dimensions are q_1 and q_2 with $q = q_1 + q_2$ and the output channel dimensions are p_1 and p_2 with $p = p_1 + p_2$*

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{3.1}$$

$$D_d = \begin{bmatrix} D_{d1} \\ D_{d2} \end{bmatrix} = \begin{bmatrix} D_{d_{11}} & D_{d_{12}} \\ D_{d_{21}} & D_{d_{22}} \end{bmatrix}$$

$$N_d = \begin{bmatrix} N_{d1} \\ N_{d2} \end{bmatrix} = \begin{bmatrix} N_{d_{11}} & N_{d_{12}} \\ N_{d_{21}} & N_{d_{22}} \end{bmatrix}$$

$$\tilde{D}_d = \left[\tilde{D}_{d1}, \tilde{D}_{d2}\right] = \begin{bmatrix} \tilde{D}_{d_{11}} & \tilde{D}_{d_{12}} \\ \tilde{D}_{d_{21}} & \tilde{D}_{d_{22}} \end{bmatrix}$$

$$\tilde{N}_d = \left[\tilde{N}_{d1}, \tilde{N}_{d2}\right] = \begin{bmatrix} \tilde{N}_{d_{11}} & \tilde{N}_{d_{12}} \\ \tilde{N}_{d_{21}} & \tilde{N}_{d_{22}} \end{bmatrix} \tag{3.2}$$

where $P_{11} \in m(G^{p_1\times q_1})$, $P_{22} \in m(G^{p_2\times q_2})$, $D_{d_{11}} \in m(H^{q_1\times q_1})$, $D_{d_{22}} \in m(H^{q_2\times q_2})$, $\tilde{D}_{d_{11}} \in m(H^{p_1\times p_1})$, $\tilde{D}_{d_{22}} \in m(H^{p_2\times p_2})$, $N_{d_{11}}, \tilde{N}_{d_{11}} \in m(H^{p_1\times q_1})$, $N_{d_{22}}, \tilde{N}_{d_{22}} \in m(H^{p_2\times q_2})$ and the other blocks have conforming dimensions.

Definition 3 applies to any arbitrary two channel partition of a plant P, and for the moment the subscript notation $(\cdot)_d$ carries no special significance. However, this will change after the discussion of decentralized Bezout identities to follow. The derivation of block diagonal stable factors for a block diagonal compensator is defined as follows.

Definition 4 (Block Diagonal Compensator Factors) *Given a compensator $C_d = blkdiag[C_1, C_2] \in m(G^{q\times p})$, left and right coprime block diagonal factorizations can be constructed as follows*

1. *For subcompensators, $C_i \in m(G^{q_i\times p_i})$ find l.c.f. $(\tilde{V}_i, \tilde{U}_i)$ and r.c.f. (U_i, V_i) where $\tilde{V}_i \in m(H^{q_i\times q_i})$, $V_i \in m(H^{p_i\times p_i})$, $\tilde{U}_i, U_i \in m(H^{q_i\times p_i})$*

2. *Define $\tilde{V}_{bd} = blkdiag[\tilde{V}_1, \tilde{V}_2]$, $\tilde{U}_{bd} = blkdiag[\tilde{U}_1, \tilde{U}_2]$, $V_{bd} = blkdiag[V_1, V_2]$ and $U_{bd} = blkdiag[U_1, U_2]$.*

3. *$(\tilde{V}_{bd}, \tilde{U}_{bd})$ and (U_{bd}, V_{bd}) are respectively l.c.f. and r.c.f. of C_d. These block diagonal factorizations are unique to within multiplication by a block diagonal unimodular operator. (i.e. $(\tilde{R}_d\tilde{V}_{bd}, \tilde{R}_d\tilde{U}_{bd})$ and $(U_{bd}R_d, V_{bd}R_d)$ are l.c.f. and r.c.f. of C_d where $\tilde{R}_d$ and R_d are block diagonal unimodular operators appropriately dimensioned to preserve the block diagonal nature of the factorization).*

Definition 4 illustrates by construction that every block diagonal compensator has a corresponding set of stable block diagonal factorizations. However, not all factorizations of a block diagonal compensator are themselves block diagonal. For a given block diagonal l.c.f. $(\tilde{V}_{bd}, \tilde{U}_{bd})$ and r.c.f. (U_{bd}, V_{bd}) of C_d, the following factorizations

$$\begin{aligned} (\tilde{V}, \tilde{U}) &= (\tilde{R}\tilde{V}_{bd}, \tilde{R}\tilde{U}_{bd}) \\ (U, V) &= (U_{bd}R, V_{bd}R) \end{aligned} \tag{3.3}$$

are also factorizations of C_d (see section 2.2) where R and $\tilde{R}$ are arbitrary unimodular operators. But, the factorizations $(\tilde{V}, \tilde{U})$ and (U, V) will not necessarily be block diagonal. An interesting note is although the stable factors, $(\tilde{V}, \tilde{U})$ and (U, V), will not necessarily be block diagonal a certain pattern of minors (corresponding to the decentralized partition) will be constrained to equal zero (see section 2.5) and these constraints on the minors are the same whether the stable factors are in block diagonal form or not.

As in section 2.2 the use of a doubly coprime Bezout identity is instrumental in establishing the set of all stabilizing controllers. For the decentralized case, construction of a special form of Bezout identity denoted decentralized doubly coprime Bezout identity (DDCBI) significantly simplifies the synthesis of the set of all stabilizing decentralized controllers. Given the form of stable factors in definitions 3 and 4 the DDCBI has the following form.

$$\begin{bmatrix} \tilde{V}_{bd} & \tilde{U}_{bd} \\ -\tilde{N}_d & \tilde{D}_d \end{bmatrix} \begin{bmatrix} D_d & -U_{bd} \\ N_d & V_{bd} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{3.4}$$

As mentioned in section 2.5 the absence of unstable fixed modes guarantees the existence of stable factors satisfying the DDCBI (eq. (3.4)). Understanding how to construct a DDCBI leads to an explanation of the subscript $(\cdot)_d$ found on the plant stable factors in eq. (3.4).

To construct a DDCBI, find a least one stabilizing block diagonal compensator, C_d, for the plant, P. Note, there is an assumption here that the decentralized partition has been verified not to induce any unstable fixed modes on the plant P. Form block diagonal l.c.f. and r.c.f. of C_d as given in definition 4. For any arbitrary r.c.f. of the plant P given by (N, D) the following operator

$$R^{-1} := \tilde{V}_{bd}D + \tilde{U}_{bd}N \tag{3.5}$$

is unimodular. This is a direct consequence of C_d stabilizing P (see theorem 2, section 2.2). Equation (3.5) can then be rewritten as

$$I = \tilde{V}_{bd}DR + \tilde{U}_{bd}NR \tag{3.6}$$

The following plant factors of the DDCBI are then defined as

$$(N_d, D_d) := (NR, DR) \tag{3.7}$$

Hence, the significance of the subscript $(\cdot)_d$ is to denote a special form of the plant stable factors which satisfy eq. (3.6) (and likewise the appropriate single Bezout identities in eq. (3.4)) which are referred to as decentralized stable plant factors (DSPF). Similar construction exists for the left coprime DSPF $(\tilde{D}_d, \tilde{N}_d)$. DSPF are unique to within appropriately dimensioned block diagonal unimodular operators. To illustrate this consider the unimodular operator $R_d = blkdiag[R_1, R_2]$ where $R_1 \in m(H^{q_1 \times q_1})$ and $R_2 \in m(H^{q_2 \times q_2})$. Select the following Bezout identity from eq. (3.4)

$$\tilde{V}_{bd} D_d + \tilde{U}_{bd} N_d = I \tag{3.8}$$

Apply operator R_d to the right of eq. (3.8).

$$\tilde{V}_{bd} D_d R_d + \tilde{U}_{bd} N_d R_d = R_d \tag{3.9}$$

Apply the inverse of R_d to the left of eq. (3.9).

$$R_d^{-1} \tilde{V}_{bd} D_d R_d + R_d^{-1} \tilde{U}_{bd} N_d R_d = I \tag{3.10}$$

Since $(R_d^{-1} \tilde{V}_{bd}, R_d^{-1} \tilde{U}_{bd})$ are block diagonal stable factors of C_d then $(N_d R_d, D_d R_d)$ represents another right coprime DSPF of the plant P which varies from (N_d, D_d) by a properly dimensioned, block diagonal, unimodular operator, R_d.

3.3 Parameterized Controllers and Unimodular Constraint

Now that the decentralized doubly coprime Bezout identity (DDCBI) has been defined (eq. (3.4) in the previous section) the parameterization of all stabilizing decentralized controllers can be stated. A compact version will be given first to illustrate the connection to the centralized parameterization and then an expanded version in terms of individual parameters which tends to be useful for analysis will be given.

Theorem 5 (Compact Parameterization of Decentralized Controllers) *For plant P which satisfies the partition of definition 3 and for which a DDCBI as in eq. (3.4) can be established, the parameterized set of stabilizing decentralized controllers is given by*

$$\begin{aligned} K &= (U_{bd} + D_d Q_{dp})(V_{bd} - N_d Q_{dp})^{-1}, & |V_{bd} - N_d Q_{dp}| \neq 0 \\ &= (\tilde{V}_{bd} - Q_{dp}\tilde{N}_d)^{-1}(\tilde{U}_{bd} + Q_{dp}\tilde{D}_d), & |\tilde{V}_{bd} - Q_{dp}\tilde{N}_d| \neq 0 \end{aligned} \tag{3.11}$$

where W_{12} and W_{21} are composed of stable factors from the partitioned plant as follows

$$\begin{aligned} W_{12} &= -\tilde{N}_{d_{11}} D_{d_{12}} + \tilde{D}_{d_{11}} N_{d_{12}} = \tilde{N}_{d_{12}} D_{d_{22}} - \tilde{D}_{d_{12}} N_{d_{22}} \\ W_{21} &= -\tilde{N}_{d_{22}} D_{d_{21}} + \tilde{D}_{d_{22}} N_{d_{21}} = \tilde{N}_{d_{21}} D_{d_{11}} - \tilde{D}_{d_{21}} N_{d_{11}} \end{aligned} \tag{3.12}$$

and the decentralizing parameter, Q_{dp}, can be a member of either of the equivalent sets

$$Q_{dp} \in Q_u^{-1} \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} \quad or \quad Q_{dp} \in \begin{bmatrix} \hat{Q}_1 & 0 \\ 0 & \hat{Q}_2 \end{bmatrix} \hat{Q}_u^{-1} \tag{3.13}$$

where Q_u and $\hat{Q}_u$ are both constrained to be unimodular and consist of the following terms

$$Q_u = \begin{bmatrix} Q_{11} & Q_1 W_{12} \\ Q_2 W_{21} & Q_{22} \end{bmatrix} \qquad \hat{Q}_u = \begin{bmatrix} \hat{Q}_{11} & W_{12}\hat{Q}_2 \\ W_{21}\hat{Q}_1 & \hat{Q}_{22} \end{bmatrix} \tag{3.14}$$

with the individual parameters being members of the following stable matrix rings

$$\begin{aligned} &Q_{11} \in m(H^{q_1 \times q_1}), \quad Q_{22} \in m(H^{q_2 \times q_2}), \quad \hat{Q}_{11} \in m(H^{p_1 \times p_1}) \\ &\hat{Q}_{22} \in m(H^{p_2 \times p_2}), \quad Q_1, \hat{Q}_1 \in m(H^{q_1 \times p_1}), \quad Q_2, \hat{Q}_2 \in m(H^{q_2 \times p_2}) \end{aligned} \tag{3.15}$$

Equation (3.11) shows directly the relationship between the parameterization of stabilizing decentralized compensators and the parameterization associated with centralized controllers eq. (2.17). The decentralizing parameter, Q_{dp}, is constrained to a subset of the stable matrix ring $m(H^{q\times p})$. The sets over which Q_{dp} can vary are specifically given by eq. (3.13). Note that the decentralizing parameter Q_{dp} is unimodularly related to block diagonal parameters $blkdiag[Q_1, Q_2]$ and $blkdiag[\hat{Q}_1, \hat{Q}_2]$ (see eq. (3.13)). The constraint on the parameter selection effectively resides with eq. (3.14) and these constraints are referred to as the unimodular constraints. The reason two constraints are given is just a reflection of being able to write the parameterization in an expanded left and right coprime form for the decentralized compensator. This expanded form is written in the following manner.

Definition 5 (Expanded Form of Decentralized Parameterization) *Expansion of the parameterization into left coprime parameterized factors is as follows*

$$C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \begin{bmatrix} \tilde{D}_{C_1}^{-1}\tilde{N}_{C_1} & 0 \\ 0 & \tilde{D}_{C_2}^{-1}\tilde{N}_{C_2} \end{bmatrix} \tag{3.16}$$

$$\tilde{D}_{C_1}^{-1}\tilde{N}_{C_1} = (Q_{11}\tilde{V}_1 - Q_1\tilde{N}_{d_{11}})^{-1}(Q_{11}\tilde{U}_1 + Q_1\tilde{D}_{d_{11}}), \quad |Q_{11}\tilde{V}_1 - Q_1\tilde{N}_{d_{11}}| \neq 0 \tag{3.17}$$

$$\tilde{D}_{C_2}^{-1}\tilde{N}_{C_2} = (Q_{22}\tilde{V}_2 - Q_2\tilde{N}_{d_{22}})^{-1}(Q_{22}\tilde{U}_2 + Q_2\tilde{D}_{d_{22}}), \quad |Q_{22}\tilde{V}_2 - Q_2\tilde{N}_{d_{22}}| \neq 0 \tag{3.18}$$

where the individual parameters must be selected such that the following operator is unimodular

$$Q_u = \begin{bmatrix} Q_{11} & Q_1W_{12} \\ Q_2W_{21} & Q_{22} \end{bmatrix} \tag{3.19}$$

Expansion of the parameterization into right coprime parameterized factors is as follows

$$C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \begin{bmatrix} N_{C_1}D_{C_1}^{-1} & 0 \\ 0 & N_{C_2}D_{C_2}^{-1} \end{bmatrix} \tag{3.20}$$

$$N_{C_1}D_{C_1}^{-1} = (U_1\hat{Q}_{11} + D_{d_{11}}\hat{Q}_1)(V_1\hat{Q}_{11} - N_{d_{11}}\hat{Q}_1)^{-1}, \quad |V_1\hat{Q}_{11} - N_{d_{11}}\hat{Q}_1| \neq 0 \tag{3.21}$$

$$N_{C_2}D_{C_2}^{-1} = (U_2\hat{Q}_{22} + D_{d_{22}}\hat{Q}_2)(V_2\hat{Q}_{22} - N_{d_{22}}\hat{Q}_2)^{-1}, \quad |V_2\hat{Q}_{22} - N_{d_{22}}\hat{Q}_2| \neq 0 \tag{3.22}$$

where the individual parameters must be selected such that the following operator is unimodular

$$\hat{Q}_u = \begin{bmatrix} \hat{Q}_{11} & W_{12}\hat{Q}_2 \\ W_{21}\hat{Q}_1 & \hat{Q}_{22} \end{bmatrix} \tag{3.23}$$

The expanded derivation can be found from the compact formulation by substitution of partitioned values as given in definition 3 and 4 into eq. (3.11) along with substituting in the values from eq. (3.12), eq. (3.13), eq. (3.14) and strategic use of a set of decentralized stable factor properties which arise from the DDCBI. These decentralized stable factor properties will be examined in section 3.4. In the expanded form the connection of the parameterization to the centralized parameterization is obscured due to the distribution of the individual parameters. However, as will be seen in chapter 4 the expanded form is useful in analysis.

In order to provide a self contained listing of the various parameterizations the special case of stable plant (i.e. $P \in m(H)$) will be examined. This special form of the parameterization will also be used in chapter 4. A valid DDCBI for the stable plant case takes on the following form

$$\begin{bmatrix} I & 0 \\ -P & I \end{bmatrix} \begin{bmatrix} I & 0 \\ P & I \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{3.24}$$

The initial compensator used is the zero compensator since the plant is stable. Note no special adjustment is needed on plant factorization for the stable case. In other words any stable plant will satisfy a DDCBI which implies that for the stable case no unstable fixed modes are possible. Of course this is exactly what must occur since there exists no partition of a stable plant which can induce any unstable modes (see section 2.5). Also of interest is that for the stable case the DDCBI is indistinguishable from a valid DCBI used in the centralized controller case. Decentralized expanded parameterization for the stable case is given by definition 6.

Definition 6 (Expanded Decentralized Parameterization for $P \in m(H)$) *Expansion of the parameterization into left coprime parameterized factors is as follows*

$$C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \begin{bmatrix} \tilde{D}_{C_1}^{-1}\tilde{N}_{C_1} & 0 \\ 0 & \tilde{D}_{C_2}^{-1}\tilde{N}_{C_2} \end{bmatrix} \tag{3.25}$$

$$\tilde{D}_{C_1}^{-1}\tilde{N}_{C_1} = (Q_{11} - Q_1 P_{11})^{-1} Q_1, \quad |Q_{11} - Q_1 P_{11}| \neq 0 \tag{3.26}$$

$$\tilde{D}_{C_2}^{-1}\tilde{N}_{C_2} = (Q_{22} - Q_2 P_{22})^{-1} Q_2, \quad |Q_{22} - Q_2 P_{22}| \neq 0 \tag{3.27}$$

where the individual parameters must be selected such that the following operator is unimodular

$$Q_u = \begin{bmatrix} Q_{11} & Q_1 P_{12} \\ Q_2 P_{21} & Q_{22} \end{bmatrix} \tag{3.28}$$

Expansion of the parameterization into right coprime parameterized factors is as follows

$$C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \begin{bmatrix} N_{C_1} D_{C_1}^{-1} & 0 \\ 0 & N_{C_2} D_{C_2}^{-1} \end{bmatrix} \tag{3.29}$$

$$N_{C_1} D_{C_1}^{-1} = \hat{Q}_1 (\hat{Q}_{11} - P_{11}\hat{Q}_1)^{-1}, \quad |\hat{Q}_{11} - P_{11}\hat{Q}_1| \neq 0 \tag{3.30}$$

$$N_{C_2} D_{C_2}^{-1} = \hat{Q}_2 (\hat{Q}_{22} - P_{22}\hat{Q}_2)^{-1}, \quad |\hat{Q}_{22} - P_{22}\hat{Q}_2| \neq 0 \tag{3.31}$$

where the individual parameters must be selected such that the following operator is unimodular

$$\hat{Q}_u = \begin{bmatrix} \hat{Q}_{11} & P_{12}\hat{Q}_2 \\ P_{21}\hat{Q}_1 & \hat{Q}_{22} \end{bmatrix} \tag{3.32}$$

3.4 Reliance on Auxiliary Bezout Identities

The proof of theorem 5, section 3.3, is dependent on the use of auxiliary doubly coprime Bezout identities (ADCBI) which follow directly from the decentralized doubly coprime Bezout identity (DDCBI), eq. (3.4). These auxiliary doubly coprime Bezout identities are given in the following corollary.

Corollary 1 (Auxiliary Doubly Coprime Bezout Identities) *The stable factors which satisfy the DDCBI (eq. (3.4)) also satisfy the following auxiliary doubly coprime Bezout identities (ADCBI)*

$$\begin{bmatrix} \tilde{V}_1 & \tilde{U}_1 \\ -\tilde{N}_{d_{11}} & \tilde{D}_{d_{11}} \end{bmatrix} \begin{bmatrix} D_{d_{11}} & -U_1 \\ N_{d_{11}} & V_1 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$
$$\begin{bmatrix} \tilde{V}_2 & \tilde{U}_2 \\ -\tilde{N}_{d_{22}} & \tilde{D}_{d_{22}} \end{bmatrix} \begin{bmatrix} D_{d_{22}} & -U_2 \\ N_{d_{22}} & V_2 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \qquad (3.33)$$

These auxiliary identities indicate that not only does the overall compensator, expressed by say the stable factors $U_{bd} = blkdiag[U_1, U_2]$ and $V_{bd} = blkdiag[V_1, V_2]$, stabilize the plant P as indicated by DDCBI, eq. (3.4), but the individual subcompensators by satisfying the ADCBI of corollary 1 stabilize fictitious plant operators formed from the main diagonal (see eq. (3.2)) of the decentralized stable plant factors, (i.e. $(N_{d_{11}}, D_{d_{11}})$ and $(N_{d_{22}}, D_{d_{22}})$). Note, that if the plant was decoupled it would be immediately obvious that the above auxiliary doubly coprime Bezout identities would be satisfied. This follows since the aforementioned fictitious plant operators would no longer be fictitious. They would correspond to the stable factors associated with the individual plant operators P_{11} and P_{22} of the decoupled plant, and the individual subcontrollers would be the respective stabilizing controllers for P_{11} and P_{22}. It is less obvious that the auxiliary doubly coprime identities should hold for a plant with coupling, but when the plant stable factors are placed in the DSPF form the above auxiliary properties can be shown to be true.

In reference [48] the proof of corollary 1 obscures its direct connection to the DDCBI. Corollary 1, (ADCBI) follows directly from the DDCBI in a straight forward manner and proof of this is given below.

Proof Substituting eq. (3.2) into the DDCBI eq. (3.4) yields

$$\begin{bmatrix} \tilde{V}_1 & 0 & \tilde{U}_1 & 0 \\ 0 & \tilde{V}_2 & 0 & \tilde{U}_2 \\ -\tilde{N}_{d_{11}} & -\tilde{N}_{d_{12}} & \tilde{D}_{d_{11}} & \tilde{D}_{d_{12}} \\ -\tilde{N}_{d_{21}} & -\tilde{N}_{d_{22}} & \tilde{D}_{d_{21}} & \tilde{D}_{d_{22}} \end{bmatrix} \begin{bmatrix} D_{d_{11}} & D_{d_{12}} & -U_1 & 0 \\ D_{d_{21}} & D_{d_{22}} & 0 & -U_2 \\ N_{d_{11}} & N_{d_{12}} & V_1 & 0 \\ N_{d_{21}} & N_{d_{22}} & 0 & V_2 \end{bmatrix} = I \tag{3.34}$$

The following three equations are then immediately available from eq. (3.34).

$$\begin{aligned} \begin{bmatrix} \tilde{V}_1 & \tilde{U}_1 \end{bmatrix} \begin{bmatrix} D_{d_{11}} \\ N_{d_{11}} \end{bmatrix} &= I \\ \begin{bmatrix} -\tilde{N}_{d_{11}} & \tilde{D}_{d_{11}} \end{bmatrix} \begin{bmatrix} -U_1 \\ V_1 \end{bmatrix} &= I \\ \begin{bmatrix} \tilde{V}_1 & \tilde{U}_1 \end{bmatrix} \begin{bmatrix} -U_1 \\ V_1 \end{bmatrix} &= 0 \end{aligned} \tag{3.35}$$

Also directly available from eq. (3.34) is the following relation

$$\tilde{N}_{d_{12}} U_2 + \tilde{D}_{d_{12}} V_2 = 0 \tag{3.36}$$

Operating on the left by $V_2^{-1} N_{d_{21}}$ gives

$$\tilde{N}_{d_{12}} U_2 V_2^{-1} N_{d_{21}} + \tilde{D}_{d_{12}} N_{d_{21}} = 0 \tag{3.37}$$

Using the relation $C_2 = \tilde{V}_2^{-1} \tilde{U}_2 = U_2 V_2^{-1}$ we obtain

$$\tilde{N}_{d_{12}} \tilde{V}_2^{-1} \tilde{U}_2 N_{d_{21}} + \tilde{D}_{d_{12}} N_{d_{21}} = 0 \tag{3.38}$$

Applying the following relation (which is also from the DDCBI, eq. (3.34))

$$\tilde{V}_2 D_{d_{21}} + \tilde{U}_2 N_{d_{21}} = 0 \Longrightarrow \tilde{U}_2 N_{d_{21}} = -\tilde{V}_2 D_{d_{21}} \tag{3.39}$$

to eq. (3.38) yields

$$-\tilde{N}_{d_{12}}D_{d_{21}}+\tilde{D}_{d_{12}}N_{d_{21}}=0 \tag{3.40}$$

From eq. (3.34) we have that

$$-\tilde{N}_{d_{11}}D_{d_{11}}+\tilde{D}_{d_{11}}N_{d_{11}}-\tilde{N}_{d_{12}}D_{d_{21}}+\tilde{D}_{d_{12}}N_{d_{21}}=0 \tag{3.41}$$

By application of eq. (3.40) to eq. (3.41) we obtain

$$\begin{bmatrix} -\tilde{N}_{d_{11}} & \tilde{D}_{d_{11}} \end{bmatrix}\begin{bmatrix} D_{d_{11}} \\ N_{d_{11}} \end{bmatrix}=0 \tag{3.42}$$

Combining eq. (3.42) with eq. (3.35) gives the following

$$\begin{bmatrix} \tilde{V}_1 & \tilde{U}_1 \\ -\tilde{N}_{d_{11}} & \tilde{D}_{d_{11}} \end{bmatrix}\begin{bmatrix} D_{d_{11}} & -U_1 \\ N_{d_{11}} & V_1 \end{bmatrix}=\begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

□

The proof for the other auxiliary Bezout identity in corollary 1 is completely analogous.

As mentioned earlier the ADCBI are used in the proofs of theorem 5. In section 3.5 they will be used in the parameterization of a special class of decentralized compensators. In addition the ADCBI are used in establishing a set of relations between the interaction terms, W_{12} and W_{21}, and the decentralized stable plant factors (DSPF) (see eq.(3.2)). These relations are used in the necessary and sufficient parts of the proofs for theorem 5 and will be used in chapter 5 to simplify stable factor terms. Although, the relationships were never given explicitly in reference [48] an analysis of the proofs given in that reference indicate that a number of algebraic relationships in the proofs relied on these properties being true. For completeness these properties will be collected here in the following table 3.1 and will be referred to as the decentralized interaction properties (DIP). These properties are derived by applying the definitions of W_{12}, W_{21}, (see eq. (3.12)) and the ADCBI (see corollary 1).

Left DIP	Right DIP
$W_{12}\tilde{V}_2 = \tilde{N}_{d_{12}}$	$U_1 W_{12} = -D_{d_{12}}$
$W_{21}\tilde{V}_1 = \tilde{N}_{d_{21}}$	$U_2 W_{21} = -D_{d_{21}}$
$W_{12}\tilde{U}_2 = -\tilde{D}_{d_{12}}$	$V_1 W_{12} = N_{d_{12}}$
$W_{21}\tilde{U}_1 = -\tilde{D}_{d_{21}}$	$V_2 W_{21} = N_{d_{21}}$

Table 3.1: Decentralized Interaction Properties (DIP)

3.5 Class of Decentralized Controllers Which Always Satisfy ADCBI

This section is devoted to characterizing a subclass of stabilizing decentralized controllers which are useful in autonomous design methods and in design methods based on iteration. These controllers will be used in chapter 4 and chapter 5. The subclass of controllers is defined by imposing a unimodular restriction on the parameters used in the expanded form of the decentralized parameterization of definition 5. The parameters affected by this unimodular restriction are given in the following definition.

Definition 7 (Unimodular Parameter Restriction (UPR)) *For the set of parameters satisfying the expanded form of decentralized parameterization, (definition 5), unimodular parameter restriction (UPR) refers to constraining the parameters,* Q_{11}*,* Q_{22}*,* $\hat{Q}_{11}$*, and* $\hat{Q}_{22}$ *to being unimodular.*

An important relationship between the parameters established in [48] is the following.

$$Q_1\hat{Q}_{11} = Q_{11}\hat{Q}_1 \tag{3.43}$$

$$Q_2\hat{Q}_{22} = Q_{22}\hat{Q}_2 \tag{3.44}$$

For the case involving UPR these relationships become

$$Q_{11}^{-1}Q_1 = \hat{Q}_1\hat{Q}_{11}^{-1} \qquad \text{where} \qquad Q_{11}^{-1}Q_1,\ \hat{Q}_1\hat{Q}_{11}^{-1} \in m(H) \tag{3.45}$$

$$Q_{22}^{-1}Q_2 \quad = \quad \hat{Q}_2\hat{Q}_{22}^{-1} \qquad \text{where} \qquad Q_{22}^{-1}Q_2,\ \hat{Q}_2\hat{Q}_{22}^{-1} \in m(H) \tag{3.46}$$

The following theorem 6 shows that the UPR leads to a set of subcontrollers which always satisfies an ADCBI.

Theorem 6 (Subcontrollers Which Always Satisfy ADCBI) *Given the expanded form of decentralized parameterization, (definition 5), selecting a subset of the parameters to satisfy UPR, (definition 7), results in subcontrollers which satisfy a parameterized ADCBI, (corollary 1).*

Proof Starting with the following ADCBI

$$\begin{bmatrix} \tilde{V}_i & \tilde{U}_i \\ -\tilde{N}_{d_{ii}} & \tilde{D}_{d_{ii}} \end{bmatrix} \begin{bmatrix} D_{d_{ii}} & -U_i \\ N_{d_{ii}} & V_i \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \tag{3.47}$$

for $i = 1, 2$. Operating on the left by

$$\begin{bmatrix} Q_{ii} & Q_i \\ 0 & I \end{bmatrix} \tag{3.48}$$

and on the right by

$$\begin{bmatrix} I & -\hat{Q}_i \\ 0 & \hat{Q}_{ii} \end{bmatrix} \tag{3.49}$$

yields the following

$$\begin{bmatrix} Q_{ii}\tilde{V}_i - Q_i\tilde{N}_{d_{ii}} & Q_{ii}\tilde{U}_i + Q_i\tilde{D}_{d_{ii}} \\ -\tilde{N}_{d_{ii}} & \tilde{D}_{d_{ii}} \end{bmatrix} \begin{bmatrix} D_{d_{ii}} & -(D_{d_{ii}}\hat{Q}_i + U_i\hat{Q}_{ii}) \\ N_{d_{ii}} & V_i\hat{Q}_{ii} - N_{d_{ii}}\hat{Q}_i \end{bmatrix} =$$

$$\begin{bmatrix} Q_{ii} & Q_i\hat{Q}_{ii} - Q_{ii}\hat{Q}_i \\ 0 & \hat{Q}_{ii} \end{bmatrix} \tag{3.50}$$

Equations (3.43)-(3.44) imply

$$Q_i\hat{Q}_{ii} - Q_{ii}\hat{Q}_i = 0 \tag{3.51}$$

Since Q_{ii} and $\hat{Q}_{ii}$ are unimodular by UPR, the operator $blkdiag[Q_{ii}, \hat{Q}_{ii}]$ is unimodular. Operating on eq. (3.50) from the right by the stable inverse of $blkdiag[Q_{ii}, \hat{Q}_{ii}]$ yields

$$\begin{bmatrix} Q_{ii}\tilde{V}_i - Q_i\tilde{N}_{d_{ii}} & Q_{ii}\tilde{U}_i + Q_i\tilde{D}_{d_{ii}} \\ -\tilde{N}_{d_{ii}} & \tilde{D}_{d_{ii}} \end{bmatrix} \begin{bmatrix} D_{d_{ii}}Q_{ii}^{-1} & -(D_{d_{ii}}\hat{Q}_i + U_i\hat{Q}_{ii})\hat{Q}_{ii}^{-1} \\ N_{d_{ii}}Q_{ii}^{-1} & (V_i\hat{Q}_{ii} - N_{d_{ii}}\hat{Q}_i)\hat{Q}_{ii}^{-1} \end{bmatrix} = I$$

which is a parameterized version of ADCBI for both $i = 1, 2$.

□

The following theorem gives the subclass of decentralized controllers which always satisfy ADCBI and result from applying the unimodular parameter restriction.

Theorem 7 (Unimodular Parameter Restricted Controllers (UPRC)) *Given the expanded form of stabilizing decentralized controllers, (definition 5), applying the UPR, (definition 7), leads to the following subset of stabilizing decentralized controllers.*

Expansion of the parameterization into left coprime parameterized factors is as follows

$$C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \begin{bmatrix} \tilde{D}_{C_1}^{-1}\tilde{N}_{C_1} & 0 \\ 0 & \tilde{D}_{C_2}^{-1}\tilde{N}_{C_2} \end{bmatrix} \tag{3.52}$$

$$\tilde{D}_{C_1}^{-1}\tilde{N}_{C_1} = (\tilde{V}_1 - \bar{Q}_1\tilde{N}_{d_{11}})^{-1}(\tilde{U}_1 + \bar{Q}_1\tilde{D}_{d_{11}}), \quad |\tilde{V}_1 - \bar{Q}_1\tilde{N}_{d_{11}}| \neq 0 \tag{3.53}$$

$$\tilde{D}_{C_2}^{-1}\tilde{N}_{C_2} = (\tilde{V}_2 - \bar{Q}_2\tilde{N}_{d_{22}})^{-1}(\tilde{U}_2 + \bar{Q}_2\tilde{D}_{d_{22}}), \quad |\tilde{V}_2 - \bar{Q}_2\tilde{N}_{d_{22}}| \neq 0 \tag{3.54}$$

where the individual parameters must be selected such that the following operator is unimodular

$$\begin{bmatrix} I & \bar{Q}_1 W_{12} \\ \bar{Q}_2 W_{21} & I \end{bmatrix} \tag{3.55}$$

Expansion of the parameterization into right coprime parameterized factors is as follows

$$C_d = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} = \begin{bmatrix} N_{C_1}D_{C_1}^{-1} & 0 \\ 0 & N_{C_2}D_{C_2}^{-1} \end{bmatrix} \tag{3.56}$$

$$N_{C_1}D_{C_1}^{-1} = (U_1 + D_{d_{11}}\bar{Q}_1)(V_1 - N_{d_{11}}\bar{Q}_1)^{-1}, \quad |V_1 - N_{d_{11}}\bar{Q}_1| \neq 0 \tag{3.57}$$

$$N_{C_2}D_{C_2}^{-1} = (U_2 + D_{d_{22}}\bar{Q}_2)(V_2 - N_{d_{22}}\bar{Q}_2)^{-1}, \quad |V_2 - N_{d_{22}}\bar{Q}_2| \neq 0 \tag{3.58}$$

where the individual parameters must be selected such that the following operator is unimodular

$$\begin{bmatrix} I & W_{12}\bar{Q}_2 \\ W_{21}\bar{Q}_1 & I \end{bmatrix} \tag{3.59}$$

Where $\bar{Q}_1 \in m(H^{q_1 \times p_1})$ *and* $\bar{Q}_2 \in m(H^{q_2 \times p_2})$

Proof The form of the left coprime parameterized subcompensators of eq. (3.17) and eq. (3.18) can be rewritten

$$\begin{aligned} C_i &= (Q_{ii}\tilde{V}_i - Q_i\tilde{N}_{d_{ii}})^{-1}(Q_{ii}\tilde{U}_i + Q_i\tilde{D}_{d_{ii}}) \qquad i = 1,2 \\ &= (Q_{ii}(\tilde{V}_i - Q_{ii}^{-1}Q_i\tilde{N}_{d_{ii}}))^{-1}(Q_{ii}\tilde{U}_i + Q_i\tilde{D}_{d_{ii}}) \\ &= (\tilde{V}_i - Q_{ii}^{-1}Q_i\tilde{N}_{d_{ii}})^{-1}(\tilde{U}_i + Q_{ii}^{-1}Q_i\tilde{D}_{d_{ii}}) \\ &= (\tilde{V}_i - \bar{Q}_i\tilde{N}_{d_{ii}})^{-1}(\tilde{U}_i + \bar{Q}_i\tilde{D}_{d_{ii}}) \end{aligned} \tag{3.60}$$

where $\bar{Q}_i = Q_{ii}^{-1}Q_i \in m(H)$, for $i = 1, 2$, since Q_{ii} is constrained to be unimodular. The form of eq. (3.60) is UPRC, (see eq. (3.53-3.54)), the unimodular operator constraint, (eq. (3.55)), is obtained as follows. By rewriting the unimodular operator of eq. (3.19) the following is obtained

$$\begin{aligned} Q_u &= \begin{bmatrix} Q_{11} & Q_1W_{12} \\ Q_2W_{21} & Q_{22} \end{bmatrix} \\ &= \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} \begin{bmatrix} I & Q_{11}^{-1}Q_1W_{12} \\ Q_{22}^{-1}Q_2W_{21} & I \end{bmatrix} \end{aligned} \tag{3.61}$$

Since Q_{11} and Q_{22} are unimodular, $blkdiag[Q_{11}, Q_{22}]$ is also unimodular. Hence the above unimodular constraint becomes

$$Q_u \quad \text{unimodular} \quad \Longleftrightarrow \quad \begin{bmatrix} I & \bar{Q}_1 W_{12} \\ \bar{Q}_2 W_{21} & I \end{bmatrix} \quad \text{unimodular} \tag{3.62}$$

□

The proof for the right coprime UPRC, (see eq. (3.57)-(3.59)), is completely analogous.

3.6 Summary

In chapter 3 the parameterization of all stabilizing decentralized controllers along with a number of associated properties and useful subclasses of decentralized controllers has been presented. In section 3.2 the decentralizing stability property was presented in the form of a specially constructed decentralized doubly coprime Bezout identity (DDCBI). The special form of decentralized stable plant factors (DSPF) and their role in the DDCBI were explained. Section 3.3 was devoted to a through overview of the parameterized decentralized controllers and the associated unimodular constraint. Parameterizations were presented in a compact form, expanded form and expanded form for the special case of a stable plant. Section 3.4 presented auxiliary doubly coprime Bezout identities (ADCBI) and indicated their uses and importance. A new proof for the ADCBI was given and it established a more direct link of the ADCBI to their respective DDCBI. A summary, in table form, of a set of decentralized interaction properties (DIP), which follow from definitions of the interaction operators and ADCBI was presented. These decentralized interaction properties will find use in chapter 5 to simplify complex stable factor terms. Finally, section 3.5 ends the chapter by characterizing a new subset of stabilizing decentralized controllers which will be used in chapter 4 for autonomous design methods and in chapter 5 for methods based on iteration. The characterization involves defining the type of parameter restriction, which was denoted as a unimodular parameter restriction (UPR). A link was established to ADCBI for the individual subcontrollers. The resulting expanded form of specialized parameterization for the controllers, (designated, unimodular parameter restricted controllers (UPRC)), was presented.

Chapter 4

Autonomous Design of Subcontrollers

4.1 Introduction

The method of autonomous design for subcontrollers deals with the design of individual subcontrollers for individually identified subsystems of the plant. Effectively the individual subsystems are associated with the main diagonal of the plant. This is usually the case when the plant is stable. The difficulty arises in developing a methodology which allows designing the individual subcontrollers and then guaranteeing that the effective overall decentralized controller does not destabilize the nominal plant. Essentially, one desires a means of accounting for the coupling elements in the plant with out needlessly complicating the design of the individual subcontrollers. These types of design methods are often used in process control [11] and are associated with nonsingular perturbation design techniques [10] and design techniques aimed at plants composed of similar interconnected subsystems [36].

A recent method [45] developed for the autonomous design of subcontrollers involves the use of Youla parameterization for the individual subcontrollers. The stable factors of the subsystem plant operator are used to parameterize the class of all stabilizing controllers for this subsystem in a manner identical to that used with centralized design (see section 2.2, eq. 2.17). The Youla parameter selected for each subsystem is done in a sequential fashion where the interconnection operators constrain the choice of parameter as each loop is closed. In this way the set of selected Youla parameters for the decentralized control system will provide closed loop stability for the overall system.

In this chapter a method will be develop which allows for the autonomous design of subcontrollers in a nonsequential fashion. In other words, design of subcontroller i does not rely on subcontrollers 1 through $i-1$ having been designed first. In this chapter the decentralized parameterization, eq. (3.17-3.18), will be recast into the more familiar Youla parameterization form for the individual subcontrollers. This formulation of the controllers will effectively turn out to be the stable version of the UPRC developed in section 3.5, theorem 7. The sequential loop closing and sequential selection of Youla parameters of [45] will be avoided by using the unimodular constraint, eq. (3.19). From the unimodular constraint, eq. (3.19), imposed on the parameters for the class of all stabilizing decentralized compensators a simple norm bound will be derived which constrains the Youla parameters of the individual subsystems in terms of the plant off diagonal operators (i.e. the interaction operators of the plant not accounted for in stabilization of the individual subsystems). The bound serves as an interaction measure and provides a upper threshold which when met by the set of subsystem Youla parameters provides a stability guarantee for the overall closed loop system. Section 4.2 derives these results for the two channel case. The interaction measure in the form of a norm bound effectively quantifies the notion of weak coupling which is a condition for nonsingular perturbation design of decentralized control [64]. These issues will be elaborated on in section 4.2. Section 4.3 compares the bound derived for the two channel case with a small gain bound derived by placing the problem in a robust stability type framework. It will be shown that any pair of Youla parameters which satisfy this small gain bound will also satisfy the bound derived from the unimodular constraint of section 4.2. The converse of this is not true, thereby making the aforementioned bound less conservative than the small gain bound in the two channel case. In section 4.4 extensions to the multiple channel case are developed. Finally, in section 4.5 a compatible bound is developed for the unstable plant case, (i.e. $P \in m(G)$).

4.2 Parameter Bound of Two Channel, Stable Plant Case

Figure 4-1 illustrates the two channel decentralized control problem. The parameterization of all stabilizing decentralized compensators for the two channel case with stable plant, $P \in m(H)$, was given by definition 6, section 3.3. In this section we will use the left coprime parameterization of the compensator from definition 6 which has the form

$$C_d = \begin{bmatrix} (Q_{11} - Q_1 P_{11})^{-1} Q_1 & 0 \\ 0 & (Q_{22} - Q_2 P_{22})^{-1} Q_2 \end{bmatrix} \tag{4.1}$$

for some Q_{11}, Q_{22}, Q_1, $Q_2 \in m(H)$ such that

$$Q = \begin{bmatrix} Q_{11} & Q_1 P_{12} \\ Q_2 P_{21} & Q_{22} \end{bmatrix} \quad \text{is unimodular} \tag{4.2}$$

If the plant P is initially decoupled the interaction constraint (eq. 4.2) reduces to Q_{11} and Q_{22} being unimodular. To prove this the following lemma will be useful.

Lemma 1 *(see [5, p. 393, Fact B.1.26] for proof) $F \in m(H)$ (where $m(H)$ corresponds to the matrix ring of proper stable systems) is unimodular iff $|F|$ is a unit in H (where H corresponds to the ring of proper stable transfer functions).*

For a decoupled plant, $P_{12} = 0$ and $P_{21} = 0$, the interaction constraint reduces as follows

$$Q = diag(Q_{11}, Q_{22}) \quad \text{is unimodular} \quad \Leftrightarrow \quad |Q| \quad \text{is a unit}$$

Since $|Q| = |Q_{11}|\,|Q_{22}|$ and Q_{11}, Q_{22} are elements of $m(H)$ then $|Q_{11}|, |Q_{22}|$ must be units in H which by Lemma 2 implies Q_{11} and Q_{22} are unimodular. The lack of coupling in the plant will allow reformulating the individual compensator parameterizations in eq. (4.1) to the one parameter Youla form [6]. The observation that Q_{11} and

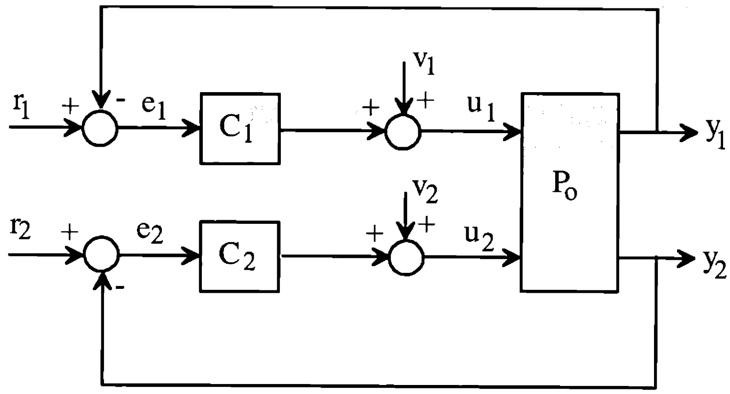

Figure 4-1: The Two Channel Decentralized Control Problem

Q_{22} must become unimodular as the coupling vanishes facilitates this reformulation. Specifically, for $C_d = diag(C_1, C_2)$ both compensators can be rewritten as

$$\begin{aligned} C_i &= (Q_{ii} - Q_i P_{ii})^{-1} Q_i \qquad \text{for } i = 1 \text{ or } 2 \\ &= \left(Q_{ii}(I - Q_{ii}^{-1} Q_i P_{ii})\right)^{-1} Q_i \\ &= \left(I - Q_{ii}^{-1} Q_i P_{ii}\right)^{-1} Q_{ii}^{-1} Q_i \\ &= \left(I - \tilde{Q}_i P_{ii}\right)^{-1} \tilde{Q}_i \end{aligned}$$

where
$$\tilde{Q}_i = Q_{ii}^{-1} Q_i \tag{4.3}$$

Since Q_{ii} is unimodular, $Q_{ii}^{-1} Q_i$ is an element in $m(H)$ and therefore $\tilde{Q}_i$ is also an element in $m(H)$. This then places eq. (4.3) in the Youla parameterization form for the case of stable plant operators P_{11} and P_{22}.

When the plant is coupled (i.e. $P_{12}, P_{21} \neq 0$) the above parameterization can be extended by accounting for the effect of the cross coupling on the $\tilde{Q}_1, \tilde{Q}_2$ terms. This effect will be accounted for in terms of a norm bound on the $\tilde{Q}_1, \tilde{Q}_2$ parameters. The following induced operator norm will be used

$$\|P\| = \sup_{\omega \in \Re} \bar{\sigma}(P(i\omega)) \tag{4.4}$$

Before deriving the bound on the Youla parameters the following lemma will prove useful.

Lemma 2 *(see [5, p. 22, Lemma 2.2.19] for proof) For $R \in m(H)$ if $\|R\| < 1$ then $|I - R|$ is a unit in H.*

To derive the bound we begin with the interaction constraint from eq. (4.2).

$$Q = \begin{bmatrix} Q_{11} & Q_1 P_{12} \\ Q_2 P_{21} & Q_{22} \end{bmatrix} \quad \text{is unimodular} \tag{4.5}$$

Invoking lemma 1 and using the well known Schur determinantal formula [65], constraint 4.5 becomes

$$\begin{aligned} |Q| &= |Q_{11}| \left| Q_{22} - (Q_2 P_{21})(Q_{11}^{-1})(Q_1 P_{12}) \right| \qquad \text{is a unit} & (4.6) \\ &= |Q_{11}| \, |Q_{22}| \left| I - Q_{22}^{-1} Q_2 P_{21} Q_{11}^{-1} Q_1 P_{12} \right| & (4.7) \end{aligned}$$

Requiring Q_{11} and Q_{22} to be unimodular ensures parameterization given by eq. (4.3). In addition, since Q_{11}, Q_{22} are elements of $m(H)$, $|Q|$ will be a unit if and only if $\left| I - Q_{22}^{-1} Q_2 P_{21} Q_{11}^{-1} Q_1 P_{12} \right|$ is a unit. Substituting $\tilde{Q}_1$ for $Q_{11}^{-1} Q_1$ and $\tilde{Q}_2$ for $Q_{22}^{-1} Q_2$ this constraint becomes

$$|Q| \quad \text{is a unit} \quad \Leftrightarrow \quad \left| I - \tilde{Q}_2 P_{21} \tilde{Q}_1 P_{12} \right| \quad \text{is a unit} \tag{4.8}$$

$\tilde{Q}_2 P_{21} \tilde{Q}_1 P_{12}$ is a element of $m(H)$ and invoking lemma 2 means $\left| I - \tilde{Q}_2 P_{21} \tilde{Q}_1 P_{12} \right|$ will be a unit if $\|\tilde{Q}_2 P_{21} \tilde{Q}_1 P_{12}\| < 1$. Use of the submultiplicative property of induced operator norms gives

$$\|\tilde{Q}_2 P_{21} \tilde{Q}_1 P_{12}\| \leq \|\tilde{Q}_2\| \|P_{21}\| \|\tilde{Q}_1\| \|P_{12}\| \tag{4.9}$$

Therefore, forcing $\|\tilde{Q}_2\|\|P_{21}\|\|\tilde{Q}_1\|\|P_{12}\| < 1$ ensures $|Q|$ is a unit and provides the following bound on the design parameters in terms of the off diagonal plant operators

$$\|\tilde{Q}_2\|\|P_{21}\|\|\tilde{Q}_1\|\|P_{12}\| \; < \; 1 \tag{4.10}$$

$$\|\tilde{Q}_2\|\|\tilde{Q}_1\| \; < \; \frac{1}{\|P_{21}\|\|P_{12}\|} \tag{4.11}$$

Thus the controller parameterization of eq. (4.3) will provide for closed loop stability if the above bound, eq. (4.11), is satisfied.

The following set of remarks provide interpretation and checks on the bound of eq. (4.11).

Remark 1 As $\|P_{12}\| \rightarrow 0$ and $\|P_{21}\| \rightarrow 0$, effectively the restrictions on the parameters $\tilde{Q}_1$ and $\tilde{Q}_2$ disappear. That is the upper bound of eq. (4.11) becomes virtually infinite and the set of Youla parameters expands to encompass the entire matrix ring of proper stable systems $m(H)$. This is the expected result and quantifies the notion of weak coupling. Specifically, the bound of eq. (4.11) specifies an upper bound on the Youla parameters in terms of the off diagonal operators P_{12} and P_{21}. The expectation is that as the cross coupling in the plant becomes small (i.e. $\|P_{12}\| \rightarrow \varepsilon$ and $\|P_{21}\| \rightarrow \varepsilon$ where $\varepsilon \ll 1$) stabilization of the overall system is not compromised by simply ensuring that the individual compensators for P_{11} and P_{22} provide stabilization for these individual loops. As this coupling goes to zero the expectation is that the set of individual stabilizing compensators for P_{11} and P_{22} grows to encompass the entire set of all stabilizing compensators for P_{11} and P_{22} (i.e. the parameterization in eq. (4.3)). In effect this is exactly what the bound of eq. (4.11) provides. Precisely how the set of stabilizing compensators grow to encompass the entire set, is quantified by the upper bound placed on the Youla parameters in terms of P_{12} and P_{21}.

Remark 2 It is expected for a block triangular plant (i.e. $\|P_{12}\| = 0$ or $\|P_{21}\| = 0$) that no restriction should exist on the individual stabilizing controllers that can be applied to P_{11} and P_{22}. Stabilization of the overall system is once again not compromised by simply stabilizing the individual subsystems, P_{11} and P_{22}. The bound of eq. (4.11) satisfies this condition. For example as $\|P_{12}\|$ goes to zero the bound on the Youla parameters $\tilde{Q}_1$ and $\tilde{Q}_2$ disappears. An interesting aspect of this is that the bound in the face of weak triangular coupling behaves similarly to case of weak coupling discussed in remark 1. That is as $\|P_{12}\| \rightarrow \varepsilon$ (or $\|P_{21}\| \rightarrow \varepsilon$) for $\varepsilon \ll 1$ there exist a rather large set of stabilizing compensators for P_{11} and P_{22} which also do not destabilize the overall system. As $\varepsilon \rightarrow 0$ this set grows to encompass the entire set of all stabilizing compensators for P_{11} and P_{22}.

4.3 Connection to Small Gain Methods

The problem can be approached from a robust stability point of view where the decoupled plant is treated as the nominal plant and the off diagonal plant operators, P_{12} and P_{21}, become an additive perturbation. Using the parameterization of eq. (4.3) we seek the constraints placed on the Youla parameters by the Small Gain Theorem. It will be shown that any Youla parameters which satisfy this small gain constraint will also satisfy the bound of eq. (4.11). This is reassuring in the sense that the bound of eq. (4.11) is derived via small gain arguments (see lemma 2). It is extended beyond the small gain bound only as a consequence of the existence of a simple determinantal formula (eq. 4.7) for the two channel case which allows separation of the Youla parameters from the off diagonal plant operators. As will be seen in the multichannel case (section 4.4), when using the same line of reasoning as in section 4.2, the absence of a similar simple determinantal formula results in a bound from the multi-channel unimodular constraint which is identical to a small gain bound derived using only the robust stability framework of this section.

The Plant P can be decomposed in the following manner

$$P = P_0 + \Delta = \begin{bmatrix} P_{11} & 0 \\ 0 & P_{22} \end{bmatrix} + \begin{bmatrix} 0 & P_{12} \\ P_{21} & 0 \end{bmatrix} \tag{4.12}$$

Gundes and Desoer [9] formulation of the two channel decentralized control problem (see figure 4-1) is in the form of the two block problem where the controller is constrained to be block diagonal (see figure 4-2). The parameterization of eq. (4.3),

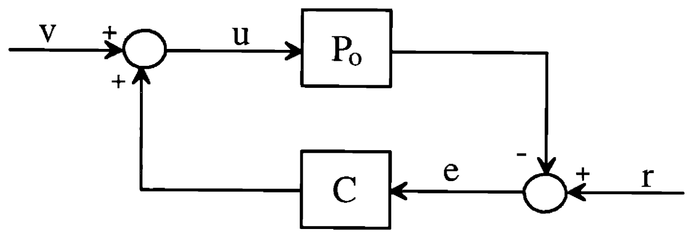

Figure 4-2: Centralized Two Block Problem

where $C = C_d = diag(C_1, C_2)$ guarantees internal stability for the closed loop map of the two block problem illustrated in figure 4-2. That is for the closed loop map

$$H(C, P_0) : \begin{bmatrix} r \\ v \end{bmatrix} \longrightarrow \begin{bmatrix} e \\ u \end{bmatrix}$$

where

$$H(C, P_0) = \begin{bmatrix} (I + P_0 C)^{-1} & -(I + P_0 C)^{-1} P_0 \\ (I + C P_0)^{-1} C & (I + C P_0)^{-1} \end{bmatrix} \tag{4.13}$$

all transfer functions which are elements of $H(C, P)$ are in $m(H)$ (i.e. they are stable). By applying the additive perturbation to the two block problem and performing the linear fractional transformation indicated in figure 4-3 the closed loop system is now in a form where the Small Gain Theorem can be applied directly. Note that the operator M is defined as

$$M : \quad d \longrightarrow u \qquad \text{where} \qquad M = -(I + C P_0)^{-1} C \tag{4.14}$$

and $M \in m(H)$ by internal stability, also $\Delta \in m(H)$ since $P \in m(H)$. Because M and Δ are both stable the Small Gain Theorem [66] provides that the closed loop remains stable as long as

$$\|M\Delta\| < 1 \tag{4.15}$$

Substituting $C = diag(C_1, C_2)$ and $P_0 = diag(P_{11}, P_{22})$, M becomes

$$M \;=\; -(I + C P_0)^{-1} C$$

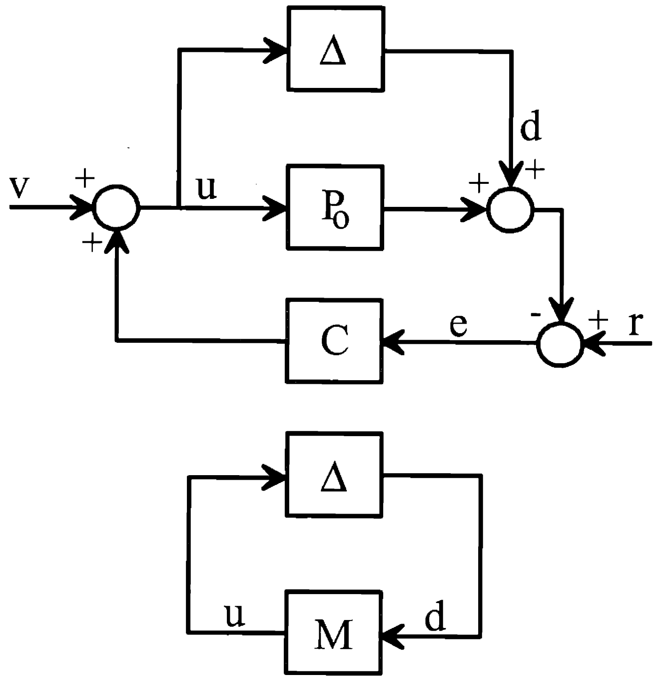

Figure 4-3: Transformation to Small Gain Loop

$$= \begin{bmatrix} -(I+C_1P_{11})^{-1}C_1 & 0 \\ 0 & -(I+C_2P_{22})^{-1}C_2 \end{bmatrix} \tag{4.16}$$

Substituting in the Youla parameterization from eq. (4.3) each term in M reduces as follows

$$\begin{aligned} -(I+C_iP_{ii})^{-1}C_i &= -(I+(I-\tilde{Q}_iP_{ii})^{-1}\tilde{Q}_iP_{ii})^{-1}(I-\tilde{Q}_iP_{ii})^{-1}\tilde{Q}_i \\ &= -\left((I-\tilde{Q}_iP_{ii})(I+(I-\tilde{Q}_iP_{ii})^{-1}\tilde{Q}_iP_{ii})\right)^{-1}\tilde{Q}_i \\ &= -\left((I-\tilde{Q}_iP_{ii})+\tilde{Q}_iP_{ii}\right)^{-1}\tilde{Q}_i \\ &= -\tilde{Q}_i \qquad \text{for } i = 1 \text{ or } 2 \end{aligned} \tag{4.17}$$

And $M\Delta$ becomes

$$M\Delta = \begin{bmatrix} -\tilde{Q}_1 & 0 \\ 0 & -\tilde{Q}_2 \end{bmatrix} \begin{bmatrix} 0 & P_{12} \\ P_{21} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\tilde{Q}_1 P_{12} \\ -\tilde{Q}_2 P_{21} & 0 \end{bmatrix} \tag{4.18}$$

To find the constraint on the $\tilde{Q}_1$ and $\tilde{Q}_2$ the following lemma will prove useful

Lemma 3

$$For \quad H \quad = \begin{bmatrix} 0 & H_{12} \\ H_{21} & 0 \end{bmatrix}$$

$$\|H\| \quad < \quad 1 \quad \textit{iff} \quad \|H_{12}\| < 1 \quad \textit{and} \quad \|H_{21}\| < 1 \tag{4.19}$$

Proof

$$\begin{aligned}
\|H\| &= \sup_{\omega\in\Re} \bar{\sigma}(H(i\omega)) \\
&= \sup_{\omega\in\Re} [\lambda_{\max}(H^*(i\omega)H(i\omega))]^{1/2} \\
&= \sup_{\omega\in\Re} \left[\lambda_{\max}\left(\begin{bmatrix} 0 & H_{12} \\ H_{21} & 0 \end{bmatrix}^* \begin{bmatrix} 0 & H_{12} \\ H_{21} & 0 \end{bmatrix}\right)\right]^{1/2} \\
&= \sup_{\omega\in\Re} \left[\lambda_{\max}\begin{pmatrix} H_{21}^* H_{21} & 0 \\ 0 & H_{12}^* H_{12} \end{pmatrix}\right]^{1/2} \\
&= \sup_{\omega\in\Re} \left[\max\left[\lambda_{\max}(H_{21}^* H_{21})^{1/2}, \lambda_{\max}(H_{12}^* H_{12})^{1/2}\right]\right] \\
&= \max\left[\sup_{\omega\in\Re}\left[\lambda_{\max}(H_{21}^* H_{21})^{1/2}\right], \sup_{\omega\in\Re}\left[\lambda_{\max}(H_{12}^* H_{12})^{1/2}\right]\right] \\
&= \max\left[\sup_{\omega\in\Re} \bar{\sigma}(H_{21}(i\omega)), \sup_{\omega\in\Re} \bar{\sigma}(H_{12}(i\omega))\right] \\
&= \max[\|H_{21}\|, \|H_{12}\|]
\end{aligned}$$

Therefore $\quad \|H\| < 1 \iff \|H_{12}\| < 1 \quad \text{and} \quad \|H_{21}\| < 1$

□

Thus to find the constraints on $\tilde{Q}_1$ and $\tilde{Q}_2$ we invoke lemma 3. That is $\|M\Delta\| < 1$

iff $\|\tilde{Q}_1 P_{12}\| < 1$ and $\|\tilde{Q}_2 P_{21}\| < 1$. This then leads to the following constraint on the Youla parameters due to the Small Gain Theorem

$$\|\tilde{Q}_1\| < \frac{1}{\|P_{12}\|} \qquad \text{and} \qquad \|\tilde{Q}_2\| < \frac{1}{\|P_{21}\|} \tag{4.20}$$

From the above bound we can derive the bound in equation 4.11 as follows

$$\begin{aligned}
\|\tilde{Q}_2\| &< \frac{1}{\|P_{21}\|} \\
\|\tilde{Q}_2\|\|\tilde{Q}_1\| &< \frac{\|\tilde{Q}_1\|}{\|P_{21}\|} \\
&< \frac{1}{\|P_{21}\|\|P_{12}\|}
\end{aligned} \tag{4.21}$$

This then says that any Youla parameters which satisfy the Small Gain bound of eq. (4.20) also satisfy the bound found earlier in section 4.2 given by eq. (4.11). However, the converse is not true. This is seen by considering the following example. If $\|P_{12}\| \rightarrow 0$ the bound on $\tilde{Q}_1$ and $\tilde{Q}_2$ from eq. (4.11) disappears but as can be seen from eq. (4.20) the Small Gain bound still constrains $\tilde{Q}_2$ when $\|P_{21}\| \neq 0$. Hence arbitrary parameters $\tilde{Q}_1$ and $\tilde{Q}_2$ which satisfy the bound given by eq. (4.11) may not satisfy the bound imposed by the Small Gain condition given by eq. (4.20). This illustrates that the bound of eq. (4.11) encompasses a larger set of Youla parameters which will stabilize the closed loop system then is given by the small gain bound eq. (4.20). Note however that in a fundamental sense these two seemingly different frameworks (unimodular constraint v.s. stability robustness) give precisely the same conditions for stability and hence result in the same bound. This is seen as follows, stability of the closed loop involving the stable operators Δ and M in figure 4-3 is guaranteed as long as $|I - M\Delta|$ is a unit. Substituting in the matrix values for $M\Delta$ from eq. (4.18) results in precisely the constraint of eq. (4.8) which is derived from the unimodular constraint of eq. (4.5).

4.4 Extension to Multichannel Case

From [9] the unimodular constraint for the m channel case where $P \in m(H)$ is

$$\mathcal{Q} = \begin{bmatrix} Q_{11} & Q_1P_{12} & Q_1P_{13} & \cdots & Q_1P_{1m} \\ Q_2P_{21} & Q_{22} & Q_2P_{23} & \cdots & Q_2P_{2m} \\ Q_3P_{31} & Q_3P_{32} & Q_{33} & \cdots & Q_3P_{1m} \\ \vdots & \vdots & \vdots & & \vdots \\ Q_mP_{m1} & Q_mP_{m2} & Q_mP_{m3} & \cdots & Q_{mm} \end{bmatrix} \quad (4.22)$$

Directly generalizing the method in section 4.2 for finding an interaction measure in the form of a norm bound on the Youla parameters for the individual subsystem compensators would require finding a determinantal formula for the following matrix

$$\tilde{\mathcal{Q}} = \begin{bmatrix} I & \tilde{Q}_1P_{12} & \tilde{Q}_1P_{13} & \cdots & \tilde{Q}_1P_{1m} \\ \tilde{Q}_2P_{21} & I & \tilde{Q}_2P_{23} & \cdots & \tilde{Q}_2P_{2m} \\ \tilde{Q}_3P_{31} & \tilde{Q}_3P_{32} & I & \cdots & \tilde{Q}_3P_{1m} \\ \vdots & \vdots & \vdots & & \vdots \\ \tilde{Q}_mP_{m1} & \tilde{Q}_mP_{m2} & \tilde{Q}_mP_{m3} & \cdots & I \end{bmatrix} \quad (4.23)$$

And deriving a norm which would allow separation of the $\tilde{Q}_i$'s and P_{ij}'s in the form of an inequality which provides that $|\tilde{\mathcal{Q}}|$ is a unit (see section 4.2 eq. (4.5) through eq. (4.11)). The complexity of determinantal formula for the m channel case precludes this approach. Another approach which generalizes the intent of the bound in eq.(4.11) for the multi-channel case and takes advantage of the equivalence of the stability constraint in both the robust stability framework and unimodular interaction setting (as noted at the end of section 4.3) is as follows. Rewriting eq. (4.23) we obtain

$$\tilde{Q} = \begin{bmatrix} I & & & \\ & I & & \\ & & \ddots & \\ & & & I \end{bmatrix} + \begin{bmatrix} \tilde{Q}_1 & & & \\ & \tilde{Q}_2 & & \\ & & \ddots & \\ & & & \tilde{Q}_m \end{bmatrix} \begin{bmatrix} 0 & P_{12} & \cdots & P_{1m} \\ P_{21} & 0 & \cdots & P_{2m} \\ \vdots & \vdots & & \vdots \\ P_{m1} & P_{m2} & \cdots & 0 \end{bmatrix} \quad (4.24)$$

By application of lemma 2, $|\tilde{Q}|$ will be a unit if

$$\left\| \begin{bmatrix} \tilde{Q}_1 & & & \\ & \tilde{Q}_2 & & \\ & & \ddots & \\ & & & \tilde{Q}_m \end{bmatrix} \begin{bmatrix} 0 & P_{12} & \cdots & P_{1m} \\ P_{21} & 0 & \cdots & P_{2m} \\ \vdots & \vdots & & \vdots \\ P_{m1} & P_{m2} & \cdots & 0 \end{bmatrix} \right\| < 1 \quad (4.25)$$

or equivalently

$$\left\| diag(\tilde{Q}_1, \cdots, \tilde{Q}_m) \right\| < \left\| \begin{matrix} 0 & P_{12} & \cdots & P_{1m} \\ P_{21} & 0 & \cdots & P_{2m} \\ \vdots & \vdots & & \vdots \\ P_{m1} & P_{m2} & \cdots & 0 \end{matrix} \right\|^{-1} \quad (4.26)$$

or by lemma 3

$$\left\| \tilde{Q}_i \right\| < \left\| \begin{matrix} 0 & P_{12} & \cdots & P_{1m} \\ P_{21} & 0 & \cdots & P_{2m} \\ \vdots & \vdots & & \vdots \\ P_{m1} & P_{m2} & \cdots & 0 \end{matrix} \right\|^{-1} \quad \forall \; i \quad (4.27)$$

Note however that this bound is identical to that given by the Small Gain Theorem for the multi-channel case. For the multi-channel case

$$M = \begin{bmatrix} \tilde{Q}_1 & & & \\ & \tilde{Q}_2 & & \\ & & \ddots & \\ & & & \tilde{Q}_m \end{bmatrix} \qquad \Delta = \begin{bmatrix} 0 & P_{12} & \cdots & P_{1m} \\ P_{21} & 0 & \cdots & P_{2m} \\ \vdots & \vdots & & \vdots \\ P_{m1} & P_{m2} & \cdots & 0 \end{bmatrix} \tag{4.28}$$

The Small Gain bound requirement $\|M\Delta\| < 1$ is equivalent to eq.(4.25). Remarks from section 4.2 extend in an analogous fashion to the above multi-channel bound.

4.5 Parameter Bound for Unstable Plants

A similar bound to the one developed for stable plants (i.e. $P \in m(H)$) in section 4.2 can be developed for the more general case $P \in m(G)$. Or in other words, the case were the plant, P, could be unstable. A close examination of the constraints placed on Q_{11} and Q_{22} in section 4.2 reveals that this is the same constraint as UPR (unimodular parameter constraint) of definition 7, section 3.5. Assuming $P \in m(G)$ is partitioned such that no unstable fixed modes exist, the associated left coprime UPRC (see eq. (3.53)-(3.55)) is

$$\begin{aligned} C_d &= \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \\ &= \begin{bmatrix} (\tilde{V}_1 - \tilde{Q}_1\tilde{N}_{d_{11}})^{-1}(\tilde{U}_1 + \tilde{Q}_1\tilde{D}_{d_{11}}) & 0 \\ 0 & (\tilde{V}_2 - \tilde{Q}_2\tilde{N}_{d_{22}})^{-1}(\tilde{U}_2 + \tilde{Q}_2\tilde{D}_{d_{22}}) \end{bmatrix} \end{aligned} \tag{4.29}$$

with the following associated unimodular constraint

$$\mathcal{Q} = \begin{bmatrix} I & \tilde{Q}_1 W_{12} \\ \tilde{Q}_2 W_{21} & I \end{bmatrix} \text{ is unimodular} \tag{4.30}$$

The operators W_{12} and W_{21} are formed from stable factors of the plant P in the following fashion.

$$\begin{aligned} W_{12} &= -\tilde{N}_{d_{11}} D_{d_{12}} + \tilde{D}_{d_{11}} N_{d_{12}} = \tilde{N}_{d_{12}} D_{d_{22}} - \tilde{D}_{d_{12}} N_{d_{22}} \\ W_{21} &= -\tilde{N}_{d_{22}} D_{d_{21}} + \tilde{D}_{d_{22}} N_{d_{21}} = \tilde{N}_{d_{21}} D_{d_{11}} - \tilde{D}_{d_{21}} N_{d_{11}} \end{aligned} \tag{4.31}$$

Since W_{12} and W_{21} are stable, (i.e. $W_{12}, W_{21} \in m(H)$), eqs. (4.8)-(4.10) apply with P_{12} and P_{21} replaced by W_{12} and W_{21}. This then gives the following bound for the case $P \in m(G)$.

$$\begin{aligned} \|\tilde{Q}_2\|\|W_{21}\|\|\tilde{Q}_1\|\|W_{12}\| &< 1 \\ \|\tilde{Q}_2\|\|\tilde{Q}_1\| &< \frac{1}{\|W_{21}\|\|W_{12}\|} \end{aligned} \tag{4.32}$$

Thus the controller parameterization of eq. (4.29) will provide for closed loop stability if the above bound, eq. (4.32), is satisfied. The use of this bound provides for a simplified method of designing autonomous subcontrollers when the plant is no longer strictly stable. This is a natural extension (from the symmetry of the input/output stable factor point of view) for the methods developed in section 4.2. However, because the off-diagonal operators of P may no longer be strictly stable, the interpretation used in section 4.3 is not applicable to this case.

4.6 Summary

The set of stabilizing compensators for a decoupled, two channel, plant consists of a compensator of the form $C_d = diag(C_1, C_2)$ where the individual compensators C_1 and C_2 have a Youla parameterization. For coupled stable plants this parameterization can be extended by constraining the norm of the Youla parameters by the norm of the off diagonal plant operators P_{12} and P_{21} as was done in section 4.2, eq. (4.11). This bound was derived from the unimodular interaction constraint associated with the parameterization of stabilizing compensators found in section 4.2, eq. (4.2). One result from such a bound is the quantification of weak coupling with respect to stabilizing decentralized compensators. This bound provides for the recovery of the entire set of stabilizing compensators for the individual plant operators P_{11} and P_{22} as the coupling goes to zero. A relationship is made to a bound derived using the Small Gain Theorem. It is shown that Youla parameters for the decentralized controllers which satisfy this small gain bound (eq. 4.20) will also satisfy the bound derived via the interaction constraint (eq. 4.11). It is noted that fundamentally the robust stability framework setup in section 4.3 effectively produces the same stability constraint as the unimodular interaction constraint of section 4.2. This observation is then used in section 4.4 when extending the bound to the multichannel case. Finally in section 4.5 a similar bound is developed for the unstable plant case (i.e. $P \in m(G)$). However an equivalent interpretation in terms of off diagonal stable perturbations as developed in section 4.3, for the case $P \in m(H)$, is no longer possible for the more general case, $P \in m(G)$.

Chapter 5

Developing Decentralized Controllers for Robust Performance

5.1 Introduction

One of the purposes of the modern control paradigm, as given in section 2.2 and illustrated by figure 2-1, is to provide not only for nominal stability and nominal performance of the plant but to also have a methodology which can provide for robust stability and robust performance. This is accomplished by accounting for model uncertainties and model errors in terms of norm bounded perturbations. The nominal plant when coupled with a modeling device for uncertainties in the form of say additive uncertainty, input multiplicative uncertainty, output multiplicative uncertainty, etc. (these terms will be defined in the next section), generates a set of plants, $\mathcal{F}$. If the selected compensator stabilizes the nominal plant G and in addition all plants which are elements of $\mathcal{F}$ the system is said to be robustly stable. A more profitable way to view this, as will be done in section 5.2, is to develop a set of closed loop transfer matrices associated with each perturbation in an input/output sense. The controller selected is said to be robustly stabilizing if the closed loop maps remain stable in the face of the feedback perturbation used to model the uncertainties. The notion of robust performance is added to this framework by scaling the norm bound of the performance operator, T_{zw}, appropriately such that it can be connected to a fictitious unity norm bounded perturbation which is augmented to the previous perturbation structure. The machinery developed for robust stability extends to provide a notion of robust performance via this use of a fictitious performance perturbation.

An important point to mention is that the design methodology used in developing robust MIMO controllers is important. For example selecting a controller which provides for robust stability and good nominal performance does not in general guarantee that the overall system will achieve robust performance [49]. The reason for this is that exclusive consideration of robust stability and nominal performance neglect closed loop transfer matrices associated with the aggregated uncertainty/performance perturbation structure which affect the robust performance of the closed loop system. The use of an appropriate analysis metric which accounts for all relevant closed loop transfer matrices associated with the perturbation structure is an important element in a framework for developing robust controllers. Such a metric exists and it is known as the structured singular value (its definition will be presented in section 5.2). The structured singular value as an analysis tool accounts for all relevant closed loop transfer matrices and a synthesis method for robust controllers exists based on this analysis tool.

It is possible to use a more general metric referred to as the structured norm. The structured norm identifies the type of perturbation allowed, (linear time varying, nonlinear time varying) along with the sense in which the perturbation is bounded (i.e. bounded in the sense of an l_p induced norm). The use of the structured norm in this chapter will be avoided primarily for two reasons. First the focus in this chapter will be on stable, LTI, bounded perturbations. Secondly the necessary development and definitions for the appropriate norm bounded vector spaces has not been made and such a development will not necessarily further illuminate the ideas to be presented in this chapter. The interested reader is referred to [50] for information concerning the use of structured norms.

Clearly, making a connection to the modern centralized methods for robust controller design will be beneficial to developing robust decentralized controllers. Many of the issues remain the same with the fundamental difference being that the selected robust controller must also satisfy a decentralized structural constraint. Section 5.2 defines some of the essentials of the μ-framework (i.e. the structured singular value framework) and then places the decentralized problem in this framework. Section 5.3 details the problems encountered in trying to develop robust decentralized controllers. Section 5.4 develops a D-K methodology for the sequential design of decentralized controllers. And finally, in section 5.5, computation methods are discussed for the D-K sequential design algorithm for robust decentralized controllers along with some anticipated difficulties.

5.2 Essentials of Robust Stability/Performance Methodology

In this section the essential tools needed for using the μ-framework will be defined. The source for this material and many more of the details is available from [49], [67]. Placing the decentralized problem in this framework will then be demonstrated with the use of a specific example in section 5.3.

One of the first elements needed for a robust framework is a means to incorporate model uncertainties and modeling errors with the nominal plant model. Figure 5-1 illustrates various methods of formulating model uncertainty. Typically the uncertainty is modeled via a norm bounded perturbation and a scalar weight. Figure 5-2 illustrates this perturbation approach to uncertainty modeling. The weight is usually restricted to be a unit, $W_i \in U$, which for continuous time definitions implies that the weight is usually restricted to be a real-rational transfer function which is stable, minimum phase, proper and has an inverse which is also stable, minimum phase and proper [68]. The uncertainty modeling perturbations in this chapter will be restricted to $\Delta_i \in m(H)$ which for continuous time systems implies LTI, stable. In addition the perturbations will be constrained to satisfy a unity norm bound.

$$||\Delta_i|| \leq 1 \tag{5.1}$$

Given this form of uncertainty modeling the plants modeled using multiplicative input uncertainty can be represented by

$$P \in \{P_n(I + W_I\Delta_I)\} \tag{5.2}$$

Plants modeled using multiplicative output uncertainty are given by

$$P \in \{(I + W_O\Delta_I)P_n\} \tag{5.3}$$

And plants modeled using additive uncertainty are given by

$$P \in \{P_n + W_A\Delta_A\} \tag{5.4}$$

Now that a means of mathematically reflecting uncertainty in the nominal plant has been established, figure 2-1 which represents the nominal formulation for the generalized control problem can be modified to include the set of plants formed by the perturbation approach to modeling uncertainty. Figure 5-3 represents this pictorially and this formulation is simply referred to as the general control problem formulation.

Using the notation of Linear Fractional Transformations (LFT) from section 2.2, eq. (2.4), important operators can be described. For example, viewing the plant G as being partitioned in the following manner

$$\begin{bmatrix} \begin{bmatrix} b \\ z \end{bmatrix} \\ y \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} a \\ w \end{bmatrix} \\ u \end{bmatrix} \tag{5.5}$$

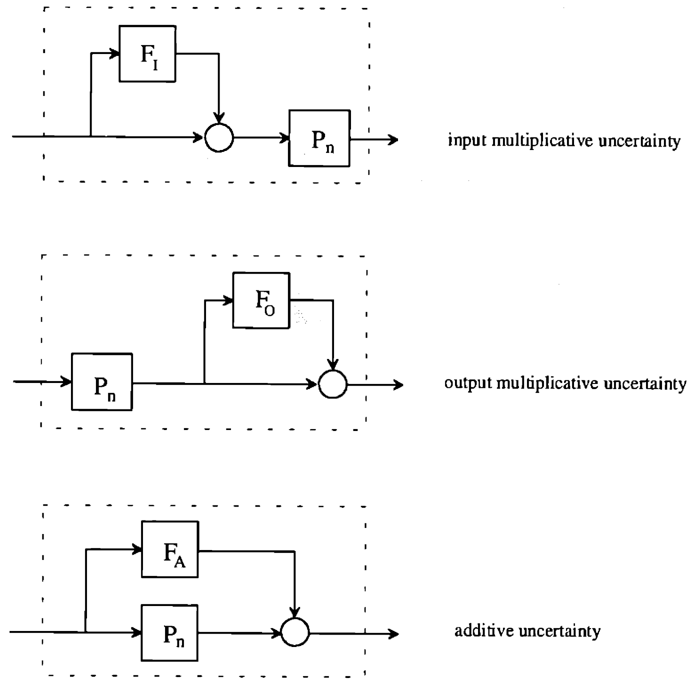

Figure 5-1: Typical Forms of Input/Output Model Uncertainty

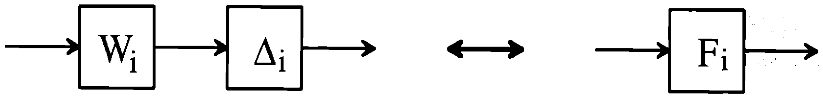

Figure 5-2: Standard Method of Representing Uncertainty Blocks

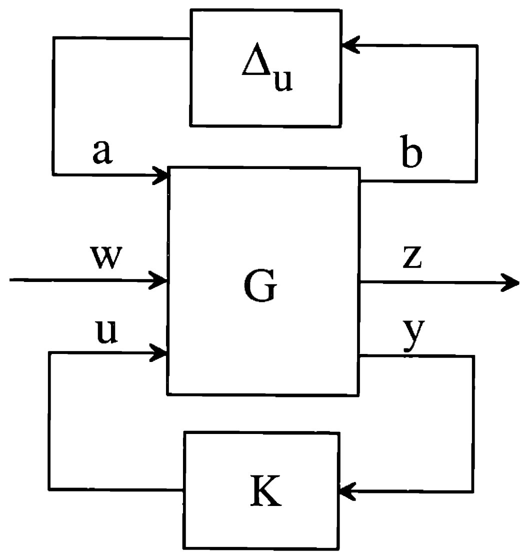

Figure 5-3: Block Diagram of the Formulation for the General Control Problem

allows formulating the set of systems to be controlled in the following way

$$\{F_u(G,\Delta_u) : \Delta_u \in m(H), ||\Delta_u|| \leq 1\} \tag{5.6}$$

where Δ_u is the perturbation used to model uncertainty in the plant. An important LFT which can be extracted from the general formulation using the 2×2 partition of the general plant G, eq. (5.5), is

$$M(G,K) := F_l(G,K) \tag{5.7}$$

The $M(\cdot)$ designation is commonly used for this LFT in the literature and this particular operator will be the one use in analysis tests to determine whether the system is meeting the desired robust stability and performance under closed loop control. Figure 5-4 illustrates the general control formulation in terms of the $M(\cdot)$ operator. The following LFT

$$F_u(M(G,K),\Delta_u) = M_{22} + M_{21}\Delta_u(I - M_{11}\Delta_u)^{-1}M_{12} \tag{5.8}$$

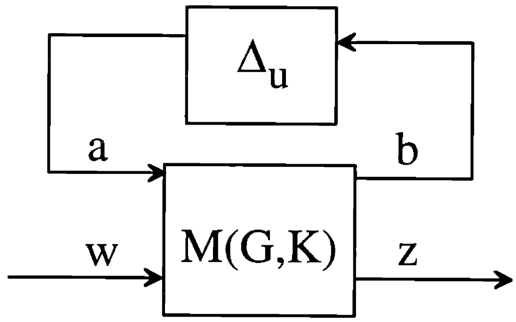

Figure 5-4: M-system With Uncertainty Perturbation Loop Closed

represents the nominal performance operator perturbed by the model uncertainty. If there is no model uncertainty, (i.e. $||\Delta_u|| = 0$), eq. (5.8) reduces to the original nominal performance operator, T_{zw}. Or in other words

$$M_{22} = T_{zw} \tag{5.9}$$

Other elements of the $M(\cdot)$ operator yield the following facts [67]

- Nominal performance is satisfied if and only if

$$||M_{22}|| < 1 \tag{5.10}$$

- System is robustly stable (meaning all plants in the set of plants formed by the uncertainty modeling are stabilized by the selected compensator K) if and only if

$$||M_{11}|| < 1 \tag{5.11}$$

Finally, since the nominal performance objective is $||T_{zw}|| < 1$, the robust performance objective is to try and maintain this performance in the face of the uncertainty perturbation, or specifically the LFT of eq. (5.8) should be less than one for all unity

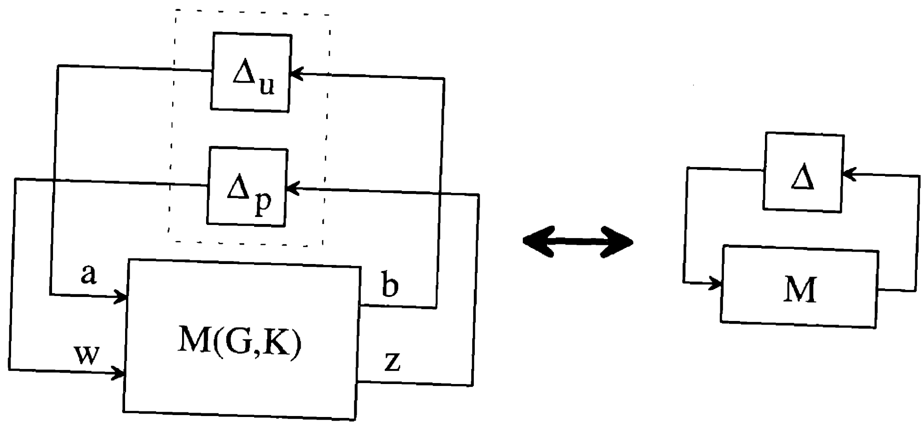

Figure 5-5: M-system With Uncertainty and Performance Perturbation Loops Closed

norm bound perturbations. This is expressed by the following equation.

$$||F_u(M(G,K),\Delta_u)|| < 1 \quad \text{for all} \quad ||\Delta_u|| \leq 1 \tag{5.12}$$

Equation (5.12) represents our optimization objective. In terms of analysis however the norm bounded LFT of eq. (5.12) represents a difficulty due to the dependence on the uncertainty perturbation, Δ_u. What is needed is an analysis tool which operates on an expression independent of Δ_u but indicates when the objective of eq. (5.12) is satisfied.

Such a tool exists, it is referred to as the structured singular value. To provide a definition useful for the robust control problems we desire to solve, the perturbation structure used in the general control formulation must be augmented. Figure 5-5 shows how connecting a fictitious perturbation, (denoted Δ_p for performance perturbation) between the performance output, z, and the performance input, w, produces overall a closed loop consisting of a structured perturbation operator, Δ, and the $M(\cdot)$ operator which represents the "known" closed loop system. By "known" closed loop system we mean that $M(\cdot)$ has the performance and uncertainty weighting functions reflected into it, contains the nominal performance operator, the nominal plant operator and other operators resulting from the uncertainty structure. The structured perturbation, Δ, is an element of the following set

$$\mathit{\Delta} = \left\{ \begin{bmatrix} \Delta_u & 0 \\ 0 & \Delta_p \end{bmatrix} \mid \Delta_u \in \mathit{\Delta}_u, \Delta_p \in m(H^{n_w \times n_z}) \right\} \tag{5.13}$$

Placing the generalized control problem in the form of a $M(\cdot)$-Δ closed loop will allow applying a metric on the $M(\cdot)$ operator to assess whether the desired robustness properties under closed loop control have been achieved. The following definition is a operator equivalent definition for the structured singular value.

Definition 8 (Structured Singular Value) *For $\Delta \in \mathit{\Delta}$ and $M \in m(H)$ the Structured Singular Value is a map from the matrix ring, $m(H)$, of stable operators to the positive reals and is defined as*

$$\mu_{\mathit{\Delta}}(M) = \left[\inf_{\Delta} \{ ||\Delta|| \mid (I - M\Delta) \quad \textit{is no longer unimodular} \} \right]^{-1} \tag{5.14}$$

If for every $\Delta \in \mathit{\Delta}$, $(I - M\Delta)$ is unimodular, then $\mu_{\mathit{\Delta}}(M) := 0$.

Using definition 8 the following robustness theorem is obtained.

Theorem 8 (Robust Stability/Performance Test) *The generalized control system, figure 5-3, is stable and satisfies the perturbed performance objective of eq. (5.12) for all $\Delta_u \in \mathit{\Delta}_u$ iff $M(G, K)$ is an element of $m(H)$ and the following condition holds*

$$\mu_{\mathit{\Delta}}(M(G, K)) < 1 \tag{5.15}$$

Due to a "maximum-modulus-like" theorem associated with linear fractional transformations, [69], μ-robustness tests for continuous time systems reduces to one dimensional searches along the $j\omega$ axis. The robustness theorem for this continuous time case then becomes

Theorem 9 *Robust Stability/Performance is guaranteed iff*

$$\max_{\omega} \mu_{\mathit{\Delta}}(M(G, K)(j\omega)) < 1 \tag{5.16}$$

Note the structured singular value used in thm. 9 is defined in terms of complex matrices at each frequency. This definition is commonly given as

Definition 9 (μ_{Δ} in terms of complex matrices) *For $M \in \mathbf{C}^{n \times n}$*

$$\mu_{\Delta}(M) = [\min\{\bar{\sigma}(\Delta) \mid \Delta \in \mathit{\Delta}, det(I - M\Delta) = 0\}]^{-1} \tag{5.17}$$

If for every $\Delta \in \mathit{\Delta}$, $(I - M\Delta)$ nonsingular, then $\mu_{\Delta}(M) := 0$.

The set, $\mathit{\Delta}$, for definition 9 is defined as, $\mathit{\Delta} \in \mathbf{C}^{n \times n}$ with

$$\begin{aligned} \mathit{\Delta} = \{ & blkdiag[\delta_1 I_{r_1}, \cdots, \delta_s I_{r_s}, \Delta_1, \cdots, \Delta_f] \mid \delta_i \in \mathbf{C} \\ & \Delta_j \in \mathbf{C}^{m_j \times m_j}, \quad 1 \leq i \leq s, 1 \leq j \leq f\} \end{aligned} \tag{5.18}$$

The formulation of the structured singular value in terms of complex matrices plays an important role in numerical computations. In section 5.4 a convex upper bound calculation for $\mu_{\Delta}(\cdot)$ will be given. This upper bound also plays an additional role in the synthesis of robust controllers and this also will be discussed in section 5.4.

5.3 Placing the Decentralized Problem in the μ-Framework

Given the background provided in section 5.2, the decentralized control problem can now be placed in the μ-framework through the use of a specific example. Figure 5-6 is a representative robust control problem. Model uncertainty is given in the form of output multiplicative uncertainty perturbation indicated by the scalar weight W_u and the uncertainty perturbation Δ_u. The performance operator will effectively be a input sensitivity transfer function matrix scaled by W_p. In order to develop the M operator to be used for robust analysis, as indicated in section 5.2, a fictitious unity norm bound performance perturbation, Δ_p, is included in the control setup of figure 5-6. M is a map of the perturbation outputs to the perturbation inputs and is

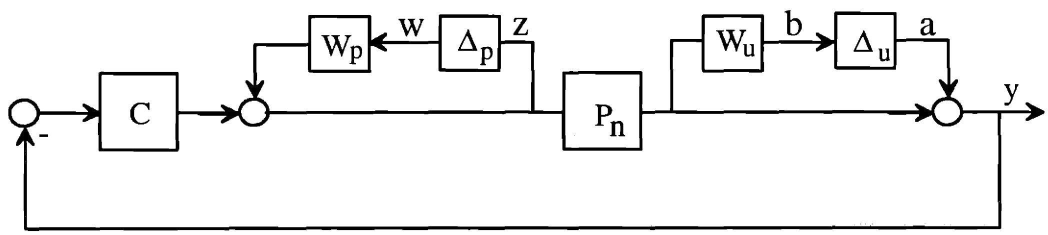

Figure 5-6: Robust Control Problem

defined as

$$M : \begin{bmatrix} a \\ w \end{bmatrix} \longrightarrow \begin{bmatrix} b \\ z \end{bmatrix} \tag{5.19}$$

For LTI systems, the elements of the 2×2 M operator can be found by breaking the loops associated with the perturbations and finding the transfer matrices, T_{ba}, T_{bw}, T_{za}, and T_{zw} which result from the four input/output combinations of the uncertainty and performance perturbations, Δ_p and Δ_u. These individual transfer matrices, which are elements of M, take on the following values.

$$\begin{aligned} M_{11} &= T_{ba} = -W_u P_n (I - C P_n)^{-1} C \\ M_{12} &= T_{bw} = W_u P_n (I + C P_n)^{-1} W_p \\ M_{21} &= T_{za} = -(I + C P_n)^{-1} C \\ M_{22} &= T_{zw} = (I + C P_n)^{-1} W_p \end{aligned} \tag{5.20}$$

Which implies the M operator has the following form.

$$M = \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -P_n(I - CP_n)^{-1}C & P_n(I + CP_n)^{-1} \\ -(I + CP_n)^{-1}C & (I + CP_n)^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \tag{5.21}$$

By satisfying the conditions of theorem 3 in section 2.2 the M operator can be written in terms of stable coprime factors of the nominal plant, P_n. To prove this the following equivalence between a sensitivity transfer matrix and its stable factor form must be observed.

$$(I + CP_n)^{-1} = D(\tilde{V} - Q\tilde{N}) \tag{5.22}$$

This equivalence is readily obtained by application of the Youla parameterization for all stabilizing controllers, eq. (2.17) and properties available from the doubly coprime Bezout identity, eq. (2.10). Application of eq. (5.22) to the elements of the M operator, eq. (5.21), along with parameterized stable factor form of C, eq. (2.17) and stable factors of P_n, eq. (2.8), yield the following stable factor form for M.

$$M = \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N(\tilde{U} + Q\tilde{D}) & N(\tilde{V} - Q\tilde{N}) \\ -D(\tilde{U} + Q\tilde{D}) & D(\tilde{V} - Q\tilde{N}) \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \tag{5.23}$$

The M operator associated with the decentralized problem is obtained by assuming the nominal plant, $P_n \in m(G)$, satisfies the two channel partition of eq. (3.1) without inducing any unstable fixed modes. A decentralized doubly coprime Bezout identity (DDCBI) of the form found in eq. (3.4) then exists for the nominal plant, P_n. The expression for the operator M, eq. (5.23), can then be rewritten in terms of the decentralized stable factors satisfying the DDCBI.

$$M = \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_d & N_d \\ -D_d & D_d \end{bmatrix} \begin{bmatrix} (\tilde{U}_{bd} + Q\tilde{D}_d) & 0 \\ 0 & (\tilde{V}_{bd} - Q\tilde{N}_d) \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \tag{5.24}$$

Substituting in the decentralizing parameters from eq. (3.13) and eq. (3.14) yields the following expressions for $(\tilde{U}_{bd} + Q\tilde{D}_d)$ and $(\tilde{V}_{bd} - Q\tilde{N}_d)$.

$$(\tilde{U}_{bd} + Q\tilde{D}_d) = Q_u^{-1} \begin{bmatrix} Q_{11}\tilde{U}_1 + Q_1\tilde{D}_{d_{11}} & Q_1[W_{12}\tilde{U}_2 + \tilde{D}_{d_{12}}] \\ Q_2[W_{21}\tilde{U}_1 + \tilde{D}_{d_{21}}] & Q_{22}\tilde{U}_2 + Q_2\tilde{D}_{d_{12}} \end{bmatrix} \tag{5.25}$$

$$(\tilde{V}_{bd} - Q\tilde{N}_d) = Q_u^{-1} \begin{bmatrix} Q_{11}\tilde{V}_1 + Q_1\tilde{N}_{d_{11}} & Q_1[W_{12}\tilde{V}_2 - \tilde{N}_{d_{12}}] \\ Q_2[W_{21}\tilde{V}_1 - \tilde{N}_{d_{21}}] & Q_{22}\tilde{V}_2 + Q_2\tilde{N}_{d_{12}} \end{bmatrix} \tag{5.26}$$

Application of left decentralized interaction properties from table 3.1 will produce a simplification for eqs. (5.25)-(5.26). Applying the following pair of left decentralizing interaction properties

$$\begin{aligned} W_{12}\tilde{U}_2 &= -\tilde{D}_{d_{12}} \\ W_{21}\tilde{U}_1 &= -\tilde{D}_{d_{21}} \end{aligned} \tag{5.27}$$

to eq. (5.25) and applying the following pair of left decentralizing interaction properties

$$\begin{aligned} W_{12}\tilde{V}_2 &= \tilde{N}_{d_{12}} \\ W_{21}\tilde{V}_1 &= \tilde{N}_{d_{21}} \end{aligned} \tag{5.28}$$

to eq. (5.26) results in the following simplifications.

$$(\tilde{U}_{bd} + Q\tilde{D}_d) = Q_u^{-1} \begin{bmatrix} Q_{11}\tilde{U}_1 + Q_1\tilde{D}_{d_{11}} & 0 \\ 0 & Q_{22}\tilde{U}_2 + Q_2\tilde{D}_{d_{12}} \end{bmatrix} \tag{5.29}$$

$$(\tilde{V}_{bd} - Q\tilde{N}_d) = Q_u^{-1} \begin{bmatrix} Q_{11}\tilde{V}_1 + Q_1\tilde{N}_{d_{11}} & 0 \\ 0 & Q_{22}\tilde{V}_2 + Q_2\tilde{N}_{d_{12}} \end{bmatrix} \tag{5.30}$$

Hence the M operator for the decentralized version of the robust problem in figure 5-6 takes the following form.

$$M = \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_dQ_u^{-1} & N_dQ_u^{-1} \\ -D_dQ_u^{-1} & D_dQ_u^{-1} \end{bmatrix} \begin{bmatrix} T_{d_1} & 0 \\ 0 & T_{d_2} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \tag{5.31}$$

Where

$$\begin{aligned} T_{d_1} &= \begin{bmatrix} Q_{11}\tilde{U}_1 + Q_1\tilde{D}_{d_{11}} & 0 \\ 0 & Q_{22}\tilde{U}_2 + Q_2\tilde{D}_{d_{12}} \end{bmatrix} \\ T_{d_2} &= \begin{bmatrix} Q_{11}\tilde{V}_1 + Q_1\tilde{N}_{d_{11}} & 0 \\ 0 & Q_{22}\tilde{V}_2 + Q_2\tilde{N}_{d_{12}} \end{bmatrix} \end{aligned} \tag{5.32}$$

As will be shown in the following section 5.4 the M operator is used in a standard H_∞ formulation for the synthesis of robust controllers. For centralized controller problems there exists a solution methodology [70], but for the decentralized form of the M operator given in eq. (5.31) a difficulty exists. The inverse of the unimodular constraint, Q_u^{-1}, effects every element of the M operator (see eq. (5.31)). The formulation of a convex, concurrent solution to generate simultaneously the design parameters Q_{11}, Q_1, Q_{22}, Q_2 is hindered by the presence of the Q_u^{-1} term associated with each element of M. To see this more clearly consider the nominal performance operator, M_{22}. For the centralized case this operator takes the form

$$M_{22} = D_d(\tilde{V} - Q\tilde{N}_d)W_p \tag{5.33}$$

This equation clearly takes the affine form, $T_1 - T_2QT_3$, where

$$\begin{aligned} T_1 &= D_d\tilde{V}W_p \\ T_2 &= D_d \\ T_3 &= \tilde{N}_dW_p \end{aligned} \tag{5.34}$$

The centralized nominal performance problem

$$\inf_Q ||T_1 - T_2QT_3|| \tag{5.35}$$

is solvable for $Q \in m(H)$. However, the M_{22} operator for the decentralized problem, using the unimodular parameter restriction of section 3.5, is of the form

$$M_{22} = D_dQ_u^{-1}\begin{bmatrix} \tilde{V}_1 + Q_1\tilde{N}_{d_{11}} & 0 \\ 0 & \tilde{V}_2 + Q_2\tilde{N}_{d_{12}} \end{bmatrix}W_p \tag{5.36}$$

where the unimodular constraint takes the form

$$Q_u = \begin{bmatrix} I & Q_1W_{12} \\ Q_2W_{21} & I \end{bmatrix} \tag{5.37}$$

Now the elements in the bracketed center term of eq. (5.36) take on an affine structure, but because the Q_u shares similar terms (namely, Q_1 and Q_2) the overall M_{22} term is not convex with respect to the design parameters Q_1 and Q_2 and hence a convex solution algorithms for the following type of concurrent design problem

$$\inf_{Q_1,Q_2} ||M_{22}|| \tag{5.38}$$

is not available. This is the same difficulty associated with concurrent design problems for decentralized M operator, eq. (5.31). In the next section a iterative strategy will be introduced which will restore a convexity property for the parameter searches and allow the problem formulation to remain in the μ-framework, thereby providing for the synthesis of robust stability/robust performance decentralized controllers.

5.4 D-K Methodology for Sequential Design of Decentralized Controllers

In this section a methodology for the sequential design of decentralized controllers is developed. It is adapted from a centralized synthesis technique for the design of robust controllers known as the D-K synthesis technique and represents a natural extension of these centralized methods to the case of decentralized control. The D-K method is developed using the structured singular analysis tools outlined in section 5.2. The advantage of developing a method of synthesizing decentralized controllers via an adapted version of D-K resides with maintaining a design method within the confines of the μ-framework. Hence, decentralized controllers developed to satisfy the robustness constraints of the μ-framework are then guaranteed to be robustly stabilizing and provide robust performance for a defined global objective. Before developing the method for decentralized controllers some essential elements of the D-K method for centralized systems must be presented.

The μ-synthesis methods result from an upper bound developed to compute $\mu_\Delta(\cdot)$. The following notation will be used for norm-bounded subsets of the perturbation set

$\mathit{\Delta}$ given in eq. (5.18).

$$\mathbf{B}_\Delta = \{\Delta \in \mathit{\Delta} \mid \bar{\sigma}(\Delta) \leq 1\} \tag{5.39}$$

The following subset of $\mathbf{C}^{n\times n}$ will shortly be shown to be rather useful.

$$\mathbf{D} = \Big\{ blkdiag[D_1, \cdots, D_s, d_{s+1} I_{m_1}, \cdots, d_{s+f} I_{m_f}] \mid \\ D_i \in \mathbf{C}^{r_j \times r_j}, D_i = D_i^* > 0, d_{s+j} > 0 \Big\} \tag{5.40}$$

Where for any $\Delta \in \mathit{\Delta}$, and $D \in \mathbf{D}$, $D\Delta = \Delta D$. From these definitions it can be shown, [49], that the following are tight upper and lower bounds for the computation of $\mu_{\mathit{\Delta}}(\cdot)$.

$$\max_{\Delta \in \mathbf{B}_\Delta} \rho(\Delta M) = \mu_{\mathit{\Delta}}(M) \leq \inf_{D \in \mathbf{D}} \bar{\sigma}(DMD^{-1}) \tag{5.41}$$

The upper and lower bounds of $\mu_{\mathit{\Delta}}(\cdot)$ allow it to be numerically tractable and the upper bound has convex properties which make it computationally attractive (see [49] for details).

The synthesis method relies on developing from the upper bound, frequency domain scales denoted $D(s)$. This is accomplished as follows. From the μ robust stability/performance test of eq. (5.16) the following synthesis equation can be formulated.

$$\min_K \max_\omega \mu_{\mathit{\Delta}}[M(G,K)(j\omega)] \tag{5.42}$$

This equation formulates the following objective, find the controller, from the set of all stabilizing controllers, which minimizes the peak value $\mu_{\mathit{\Delta}}(M(G,K))$. Where $M(G,K)$ represents the closed loop system transfer matrices of the general control problem. Equation (5.42) can be approximated using the $\mu_{\mathit{\Delta}}(\cdot)$ upper bound as follows

$$\min_K \max_\omega \min_{D_\omega \in \mathbf{D}} \bar{\sigma}[D_\omega M(G,K)(j\omega) D_\omega^{-1}] \tag{5.43}$$

where D_w is chosen from the set of scalings, $\mathbf{D}$, independently at every ω. From these D_w scalings frequency domain scalings $\hat{D}(s)$ can be constructed. These scalings are usually restricted to real-rational, stable, minimum-phase transfer functions and the optimization becomes

$$\min_{K} \min_{\hat{D}(s)\in \mathbf{D}} ||\hat{D}M(G,K)(j\omega)\hat{D}|| \tag{5.44}$$

We are now in a position to describe the D-K synthesis. The robust controllers are synthesized under D-K method by performing a number of iterations where alternately the $\hat{D}(s)$ scales or the compensator $K(s)$ are held fixed. Holding the $\hat{D}(s)$ scales fixed it is readily established [49] that the following equation

$$\min_{K} ||\hat{D}M(G,K)(j\omega)\hat{D}|| \tag{5.45}$$

is equivalent to

$$\min_{K} ||M(G_D,K)|| \tag{5.46}$$

Where the frequency scales $\hat{D}(s)$ and $\hat{D}(s)^{-1}$ are absorbed directly into the generalized plant, G. The form of eq. (5.46) is in a standard H_∞ formulation for which a solution algorithm exists, [70].

Holding the compensator, $K(s)$, fixed, the following upper bound calculation of $\mu_\Delta(\cdot)$ is performed.

$$\min_{D_\omega \in D} \bar{\sigma}[D_\omega M(G,K)(j\omega)D_\omega^{-1}] \tag{5.47}$$

From the set of D_w found at each discrete frequency point evaluated, a set of $\hat{D}(s)$ scale transfer functions are constructed. Reflecting these $\hat{D}(s)$ back into the generalized plant is the mechanism by which the H_∞ minimization is forced to focus its efforts over specific frequency ranges to try an lower the peak value of the $\mu_\Delta(\cdot)$ for the closed loop generalized system. Iterating back and forth between the steps of fixing the $\hat{D}(s)$ scales and the compensator, $K(s)$, comprises the D-K methodology. Although, as indicated in [67], the D-K method does not necessarily converge to a global minimum, it has proven quite successful in practice for synthesizing robust controllers [49].

Now adapting the D-K method to synthesizing robust decentralized controllers is accomplished as follows. Having placed the decentralized control problem in the

μ-framework (see section 5.2), the decentralized problem is positioned to develop a set of $\hat{D}(s)$ scalings in an identical fashion to the centralized case by holding the decentralized compensator fixed during the D step of the D-K iteration.

The difficulty resides in the step where the $\hat{D}(s)$ scales are held fixed and a decentralized compensator is sought out to satisfy eq. (5.46). The way this can be resolved is to impose the unimodular parameter restriction, definition 7, section 3.5. This then reduces the number of design parameters to be found for each subcompensator to one. The design parameter of the subcompensator is individually found by holding the other subcompensator's parameters fixed. After a new design parameter is found, the decentralized doubly coprime Bezout identity, DDCBI, is recomputed so that the new parameterized subcompensator becomes the factorized subcompensator for the newly adjusted DDCBI. After this step, the design parameter for the second subcompensator is sought out, while holding the first subcompensator fixed. In order for this algorithm to be effective two issues must be resolved.

1. If at each step the resulting $M(\cdot)$ operator can be shown to be convex in the individual design parameter sought, the problem can then be reduced to a solvable algorthm using convex methods.

2. Iterating between the controllers must reduce the over all optimization problem in a monotonic decreasing fashion.

Both properties will be demonstrated for this sequential design method for decentralized controllers.

5.4.1 Convexity of the $M(\cdot)$ Operator

To demonstrate the resulting convexity of the $M(\cdot)$ in terms of the single design parameter when sequentially designing subcontrollers we will work with the $M(\cdot)$ operator developed for the two channel decentralized control problem in section 5.2.

The M operator for the decentralized control problem, eq. (5.31), can be rewritten as

$$M = \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_d & N_d \\ -D_d & D_d \end{bmatrix} \begin{bmatrix} Q_u^{-1}T_{d_1} & 0 \\ 0 & Q_u^{-1}T_{d_2} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \tag{5.48}$$

where

$$\begin{aligned} Q_u^{-1}T_{d_1} &= \begin{bmatrix} Q_{11} & Q_1 W_{12} \\ Q_2 W_{21} & Q_{22} \end{bmatrix}^{-1} \begin{bmatrix} Q_{11}\tilde{U}_1 + Q_1 \tilde{D}_{d_{11}} & 0 \\ 0 & Q_{22}\tilde{U}_2 + Q_2 \tilde{D}_{d_{12}} \end{bmatrix} \\ Q_u^{-1}T_{d_2} &= \begin{bmatrix} Q_{11} & Q_1 W_{12} \\ Q_2 W_{21} & Q_{22} \end{bmatrix}^{-1} \begin{bmatrix} Q_{11}\tilde{V}_1 + Q_1 \tilde{N}_{d_{11}} & 0 \\ 0 & Q_{22}\tilde{V}_2 + Q_2 \tilde{N}_{d_{12}} \end{bmatrix} \end{aligned} \tag{5.49}$$

For the case of finding subcontroller one, C_1, impose the unimodular restriction on the parameters Q_{11} , Q_{22} and set $||Q_2|| = 0$. Equation (5.49) becomes

$$\begin{aligned} Q_u^{-1}T_{d_1} &= \begin{bmatrix} I & -Q_1 W_{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{U}_1 + Q_1 \tilde{D}_{d_{11}} & 0 \\ 0 & \tilde{U}_2 \end{bmatrix} \\ &= \begin{bmatrix} \tilde{U}_1 & 0 \\ 0 & \tilde{U}_2 \end{bmatrix} + \begin{bmatrix} Q_1 & 0 \\ 0 & Q_1 \end{bmatrix} \begin{bmatrix} \tilde{D}_{d_{11}} & -W_{12}\tilde{U}_2 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{5.50}$$

$$\begin{aligned} Q_u^{-1}T_{d_2} &= \begin{bmatrix} I & -Q_1 W_{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{V}_1 + Q_1 \tilde{N}_{d_{11}} & 0 \\ 0 & \tilde{V}_2 \end{bmatrix} \\ &= \begin{bmatrix} \tilde{V}_1 & 0 \\ 0 & \tilde{V}_2 \end{bmatrix} + \begin{bmatrix} Q_1 & 0 \\ 0 & Q_1 \end{bmatrix} \begin{bmatrix} \tilde{N}_{d_{11}} & -W_{12}\tilde{V}_2 \\ 0 & 0 \end{bmatrix} \end{aligned} \tag{5.51}$$

The middle term of eq. (5.48) can now be written

$$\begin{bmatrix} Q_u^{-1}T_{d_1} & 0 \\ 0 & Q_u^{-1}T_{d_2} \end{bmatrix} = \begin{bmatrix} U_{bd} & 0 \\ 0 & V_{bd} \end{bmatrix} + \begin{bmatrix} Q_{1_d} & 0 \\ 0 & Q_{1_d} \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \tag{5.52}$$

with

$$
\begin{aligned}
U_{bd} &= \begin{bmatrix} U_1 & 0 \\ 0 & U_2 \end{bmatrix} \\
V_{bd} &= \begin{bmatrix} V_1 & 0 \\ 0 & V_2 \end{bmatrix} \\
Q_{1_d} &= \begin{bmatrix} Q_1 & 0 \\ 0 & Q_1 \end{bmatrix} \\
S_1 &= \begin{bmatrix} \tilde{D}_{d_{11}} & -W_{12}\tilde{U}_2 \\ 0 & 0 \end{bmatrix} \\
S_2 &= \begin{bmatrix} \tilde{N}_{d_{11}} & -W_{12}\tilde{V}_2 \\ 0 & 0 \end{bmatrix}
\end{aligned} \tag{5.53}
$$

Given the form of eq. (5.52) the $M(\cdot)$ operator becomes

$$M = T_1 + T_2\hat{Q}_1T_3 \tag{5.54}$$

where the expressions T_1, T_2 and T_3 take on the values

$$
\begin{aligned}
T_1 &= \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_d & N_d \\ -D_d & D_d \end{bmatrix} \begin{bmatrix} U_{bd} & 0 \\ 0 & V_{bd} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \\
T_2 &= \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_d & N_d \\ -D_d & D_d \end{bmatrix} \\
T_3 &= \begin{bmatrix} S_1 & 0 \\ 0 & S_2 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \\
\hat{Q}_1 &= \begin{bmatrix} Q_{1_d} & 0 \\ 0 & Q_{1_d} \end{bmatrix} = blkdiag[Q_1, Q_1, Q_1, Q_1]
\end{aligned} \tag{5.55}
$$

The $M(\cdot)$ operator is convex in the design parameter Q_1. The proof of this is as follows.

Proof M is affine in $\hat{Q}_1$ (see eq. (5.54)) and $\hat{Q}_1$ is convex in the parameter Q_1 which implies M is convex in the design parameter Q_1.

□

This means that the optimization problem eq. (5.46) is solvable using convex algorthmic methods, [71].

Assume the optimization problem eq. (5.46) for the individual subcontroller parameterized by Q_1 is solved and the selected parameter is Q_1^*. Using the stable factors of the new subcontroller, C_1, the decentralized doubly coprime Bezout identity, eq. (3.4) can be adjusted, as indicated in section 3.2. This will preserve the stable factor structure of $M(\cdot)$ operator, eq. (5.48), with the old stable factors replaced by the appropriate new factors from the adjusted DDCBI. This will then allow a design iteration for subcontroller two, C_2, by once again enforcing a unimodular parameter restriction for parameters Q_{11}, Q_{22} and setting $||Q_1|| = 0$ to obtain a $M(\cdot)$ operator which is convex in Q_2.

Following a similar method as used with controller one, C_1, the $M(\cdot)$ in terms of Q_2 has the following form. The middle term of eq. (5.48) will become

$$\begin{bmatrix} Q_u^{-1}T_{d_1} & 0 \\ 0 & Q_u^{-1}T_{d_2} \end{bmatrix} = \begin{bmatrix} U_{bd}^{(1)} & 0 \\ 0 & V_{bd}^{(1)} \end{bmatrix} + \begin{bmatrix} Q_{2_d} & 0 \\ 0 & Q_{2_d} \end{bmatrix} \begin{bmatrix} \hat{S}_1 & 0 \\ 0 & \hat{S}_2 \end{bmatrix} \tag{5.56}$$

with

$$U_{bd}^{(1)} = \begin{bmatrix} U_1^{(1)} & 0 \\ 0 & U_2 \end{bmatrix}$$

$$V_{bd}^{(1)} = \begin{bmatrix} V_1^{(1)} & 0 \\ 0 & V_2 \end{bmatrix}$$

$$Q_{2_d} = \begin{bmatrix} Q_2 & 0 \\ 0 & Q_2 \end{bmatrix}$$

$$\hat{S}_1 = \begin{bmatrix} 0 & 0 \\ -W_{21}\tilde{V}_1^{(1)} & \tilde{N}_{d_{12}}^{(1)} \end{bmatrix}$$

$$\hat{S}_2 = \begin{bmatrix} 0 & 0 \\ -W_{21}\tilde{V}_1^{(1)} & \tilde{N}_{d_{12}}^{(1)} \end{bmatrix} \tag{5.57}$$

Where the superscript, $(\cdot)^{(1)}$, refers to the new stable factors resulting from the first iteration which designed a new controller for the first channel. The $M(\cdot)$ operator will once again have the form

$$M^{(1)} = T_1 + T_2\hat{Q}_2T_3 \tag{5.58}$$

where the expressions T_1, T_2 and T_3 take on the values

$$\begin{aligned} T_1 &= \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_d^{(1)} & N_d^{(1)} \\ -D_d^{(1)} & D_d^{(1)} \end{bmatrix} \begin{bmatrix} U_{bd}^{(1)} & 0 \\ 0 & V_{bd}^{(1)} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \\ T_2 &= \begin{bmatrix} W_u & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} -N_d^{(1)} & N_d^{(1)} \\ -D_d^{(1)} & D_d^{(1)} \end{bmatrix} \\ T_3 &= \begin{bmatrix} \hat{S}_1 & 0 \\ 0 & \hat{S}_2 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & W_p \end{bmatrix} \\ \hat{Q}_2 &= \begin{bmatrix} Q_{2_d} & 0 \\ 0 & Q_{2_d} \end{bmatrix} = blkdiag[Q_2, Q_2, Q_2, Q_2] \end{aligned} \tag{5.59}$$

Due to the similar form of $M(\cdot)^{(1)}$ in eq. (5.58) to eq. (5.54), $M(\cdot)^{(1)}$ is convex in the subcontroller parameter Q_2.

5.4.2 Monotonic Decreasing Property of Iterative Subcontroller Design

In order for the iteration between subcontrollers to be useful, the overall norm bound of the optimization equation should decrease in a monotonic fashion. The optimization problem in terms of $M(\cdot)$ is given by eq. (5.46). For the sequential design of subcontrollers, eq. (5.46) is rewritten in the following manner

$$\min_{Q_i} ||M^{(j)}(Q_1, Q_2)|| \quad \text{for} \quad i = 1 \text{ or } 2 \quad j = 0, 1, 2, \cdots \tag{5.60}$$

where only one design parameter is being sought during a given minimization. In other words if the minimization is over the entire set of $Q_1 \in m(H)$, then $||Q_2|| = 0$ and vice-versa. The superscript, j, is an iteration index to keep track of what iteration is currently proceeding. The alternating between controller parameters Q_1 and Q_2 has the desirable effect of monotonically decreasing the H_∞ norm bound of the $M(\cdot)$ operator. To see this consider the following, before any iteration takes place, the DDCBI has assigned stable factors for a stable compensator. Using these factors the value of the $M(\cdot)$ before any iteration is

$$||M^{(0)}(0,0)|| = \delta \tag{5.61}$$

The first iteration optimization problem is

$$\min_{Q_1} ||M^{(0)}(Q_1, 0)|| \tag{5.62}$$

Since the above is convex in Q_1, we'll assume Q_1^* is the minimum of eq. (5.62). Define

$$||M^{(0)}(Q_1^*, 0)|| =: \delta_0 \tag{5.63}$$

By definition we have that

$$\delta_0 \leq \delta \tag{5.64}$$

The subcontroller one obtained from Q_1^* is absorbed back in to a newly adjusted DDCBI and we obtain the following

$$||M^{(1)}(0,0)|| = ||M^{(0)}(Q_1^*, 0)|| \tag{5.65}$$

Now on this next iteration we are looking to solve

$$\min_{Q_2} ||M^{(1)}(0, Q_2)|| \tag{5.66}$$

Once again since the above is convex in Q_2, we'll assume Q_2^* is the minimum of eq. (5.66). Define

$$||M^{(1)}(0, Q_2^*)|| =: \delta_1 \tag{5.67}$$

By definition we have that

$$\delta_1 \leq \delta_0 \tag{5.68}$$

Continued iteration proceeds in a similar fashion and hence we have established the monotonic decreasing property for iterating between the subcontrollers.

5.5 Computation Methods Using Existing D-K Tools

Ideally, the D-K sequential design algorithm for robust decentralized controllers presented in section 5.4 could be reduced to computation via direct state-space interpretations of the DDCBI and associated unimodular restricted parameterization of decentralized controllers, theorem 7, section 3.5. The reparameterizations, which must occur between parameter iterations, could also be reduced to a systematic computation in this state-space setting. The remaining problem to be solved resides with the reduction of the optimization problem, eq. (5.60), to a computable algorithm. One method might be to formulate the sequential $M(\cdot)$ operator (see eq. (5.54)), which is convex with respect to an individual design parameter (see section 5.4.1), in terms of a linear matrix inequality, LMI [72]. A LMI formulation lends itself to a direct implementation in terms of numerical convex optimization algorithms which should be solvable in polynomial-time. In any event, however one chooses to numerically solve the convex optimization problem, eq. (5.60), the appeal of the sequential D-K method as specified in section 5.4 is as follows:

- The method as developed is defined in terms of a general class of stable rings, meaning the method as specified corresponds to continuous time and discrete time lumped parameter systems and can be used in conjunction with a number of norm bounds, (H_2, H_∞, l_1).

- The sequential method being posed in terms of decentralized stable factors retains a link to the more difficult concurrent decentralized stable factor problem (see section 5.3) and direct implementations of this method may provide a window to a possible concurrent algorithm.

- Finally, development of subcontrollers based on iterating design parameters provide aggregated decentralized controllers from an identified subset of the set of all possible stabilizing decentralized controllers (see section 3.5).

Another approach is to reformulate or effectively approximate the method of section 5.4 in such a way as to take advantage of existing commercial software. Currently, the elementary methods of D-K synthesis for centralized design is widely available in toolbox form from a commercial vendor of control software [73]. These tool boxes provide the necessary software for continuous time problems, specifically D-K robust controller synthesis where the K step optimization is based on induced operator norms for finite energy signals, i.e. the H_∞ norm. An argument can be made that the sequential design method based on parameter iteration suggested in section 5.4 can be effectively implemented through the direct iteration of the subcontrollers which form the overall decentralized controller. To illustrate this, figure 5-7 shows the generalized control formulation for the case of a two channel decentralized controller. If a sequential design method based on iterating the subcontrollers is pursued, this generates a series (dependent on the number of iterations) of two distinct generalized control formulations. Each one associated with its respective subcontroller. Figure 5-8 illustrates the two generalized control formulations which result from iterating the subcontrollers associated with the original two channel control problem in figure 5-7. During any given iteration sequence the optimization problems being solved take the

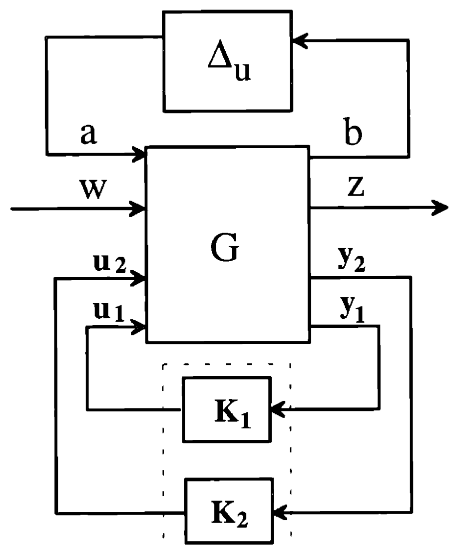

Figure 5-7: Formulation for the Two Channel Generalized Control Problem

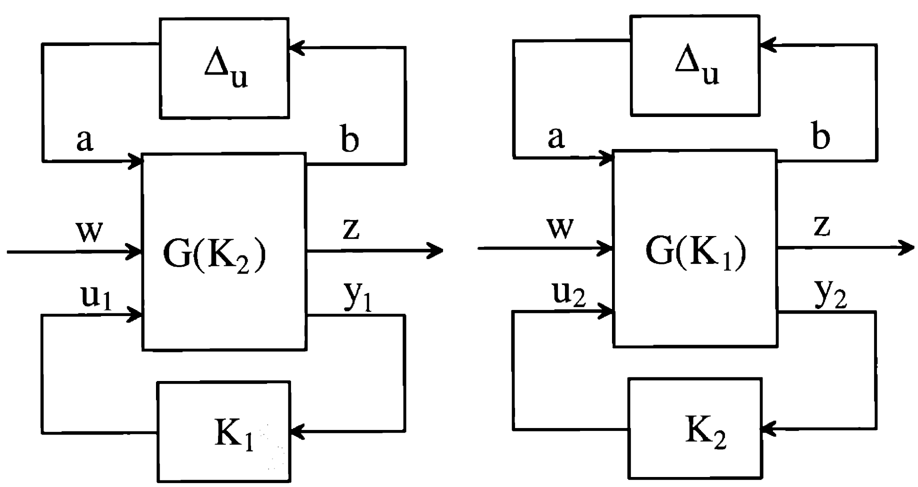

Figure 5-8: Resulting Individual Control Problems from Iterating Subcontrollers

form

$$\min_{K_1} ||M(G_D(K_2^*), K_1)|| \qquad \text{or} \qquad \min_{K_2} ||M(G_D(K_1^*), K_2)|| \tag{5.69}$$

depending of course on which subcontroller is currently being sought. The optimization problems in eq. (5.69) are individually in a form similar to eq. (5.46) which implies they are in a standard H_∞ form for which a solution algorithm exists (namely the Doyle et al [70] algorithm). Note, K_i^* represents the optimal subcontroller found in a previous iteration step. Hence, the proof for a monotonic decreasing property when iterating between subcontrollers follows in a similar fashion to the proof given in section 5.4.2 for the case of iterating between design parameters. Also notice that selection of a given subcontroller, say K_1^* for example, updates the respective $G_D(\cdot)$ operator which in turn causes the respective $M(\cdot)$ operator to be updated between iterations. The updating of these respective operators between iterations is directly analogous to the reparameterizations required when iterating between design parameters in the stable factor formulation.

Finally, a difficulty that becomes apparent in this sequential method is the potential for quite a large growth in dimensionality of the subcontrollers. This occurs due to the iteration process. As each subcontroller is designed it is reabsorbed into the closed loop system upon successive iterations. The dimension of the subcontroller increases the overall dimension of the closed loop system which in turn has the effect of increasing the dimension of the successively designed subcontrollers within this iteration process. There is of course no guarantee, or even a mechanism, in the algorithm as currently outlined to provide for a minimal realization of robust decentralized controllers. One possible solution to this problem would be to take the current subcontroller found in a given iteration step and project it to a subcontroller of fixed lower dimension [74]. This reduced order subcontroller would then only be accepted if it represented an improvement in its respective norm bound, eq. (5.69), over the prior subcontroller.

5.6 Summary

Chapter 5 covered a number of issues concerning the development of decentralized controllers for robust performance. Section 5.2 provided the necessary structures and definitions for developing controllers under the μ-framework. Section 5.3 demonstrated how the decentralized control problem could be placed in the μ-framework. Use of decentralizing interaction properties from section 3.4 helped to simplify the decentralized stable factor formulation of the $M(\cdot)$ operator. An examination of the difficulties in developing a concurrent method for generating design parameters for the decentralized controllers in the robust framework was also provided. Finally, section 5.4 provided a methodology for developing sequentially robust decentralized controllers in the μ-framework. As illustrated in this chapter development of decentralized controllers under this framework provides the benefit of specifically trying to synthesize decentralized controllers which satisfies the structured Singular Value robustness tests for the closed loop system. The net result is decentralized control with robust stability and robust performance properties. The sequential design method consisted of an adaptation of the D-K synthesis for centralized systems. Effectively, the D scales developed from an upper bound estimate of the structured singular values were developed in a fashion consistent with the way they are used in centralized problem except that the controllers providing the closed loop feedback are decentralized. An iteration scheme was developed using the unimodular parameter restriction technique developed in chapter 3. This simplified the number of design parameters per subcontroller to one. Based on this restriction and searching for the design parameters sequentially lead to convex properties for the $M(\cdot)$ operator which makes the problem solvable via convex algorithm methods. It is shown that as the controllers are iterated in the K step the norm of the generalized closed loop system exhibits a monotonic decreasing property. Finally, the computation issues associated with this sequential D-K method for robust decentralized controllers is discussed along with some anticipated difficulties.

Chapter 6

Partially Decentralized Controllers

6.1 Introduction

Partially decentralized control structures can be characterized by the way in which local information between channels is shared. In general, any constraint on feedback information will lead to some form of a structurally constrained controller. Partially decentralized controllers are singled out because of their resemblance in form to fully decentralized controllers and this in turn leads to some practical applications. The use of partially decentralized controllers usually arises out of physical systems where strong local interactions of subsystems exist. For example, reference [75] demonstrates the benefits of using a partially decentralized controller over a fully decentralized controller in terms of the performance obtainable in the simulated closed loop systems. These controllers were used in the design of the Laser Demonstration Facility (LDF) laser alignment control system at the Lawrence Livermore National Laboratory. The subsystems of the laser transport scheme comprised a chain-like structure. The partially decentralized controller used in this system employs a local information sharing structure which consists of individual subsystems sharing the feedback channel information with the adjacent subsequent subsystem in the chain-like system structure.

In this chapter theoretical issues associated with developing partially decentralized controllers via stable factor methods are examined. Section 6.2 characterizes three types of partially decentralized controllers. The characterization is limited to a natural order in which local information between channels would most likely be shared, however the method used to generate controllers with such structure can be applied to any particular combinations of local information sharing between feedback channels. Section 6.3 develops a novel unimodular transformation applicable to the partially decentralized systems which permits partially decentralized controllers to be designed using input/output methods developed specifically for fully decentralized systems. Sections 6.4 and 6.6 provide the details of applying this method to the canonical partially decentralized forms given in section 6.2. Section 6.5 discusses the issue of coupling for partially decentralized controllers. Finally, section 6.7 discusses the application of decentralized methods developed in chapters 4 and 5 to the design of partially decentralized controllers.

6.2 Developing a Set of Partially Decentralized Controllers

Figure 6-1 gives the representation of the standard two block problem. The plant is represented by G where $G : u \longmapsto y$, is an element of $m(G)$ and has dimension $p \times q$. The compensator is represented by C where $C : e \longmapsto k$, is an element of $m(G)$ and has dimension $q \times p$. For a plant G partitioned into m channels the associated fully decentralized controller has the following structure:

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_m \end{bmatrix} = \begin{bmatrix} C_1 & & & \\ & C_2 & & \\ & & \ddots & \\ & & & C_m \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} \tag{6.1}$$

With $\sum_i p_i = p$ and $\sum_i q_i = q$ where p_i is the dimension of e_i and q_i is the dimension of k_i. The feedback channels $e_1, e_2 \cdots e_m$ are independent of one another or

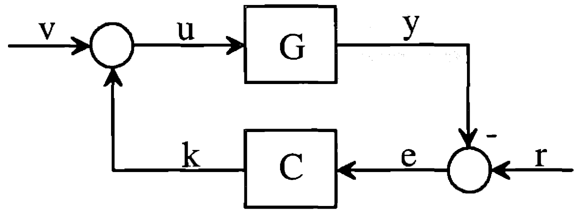

Figure 6-1: Two Block Control Problem

in other words the channels share no information between one another. Three partially decentralized controller structures based on the local sharing of information among the feedback channels are characterized in the following manner. A Type 1 partially decentralized controller, $\mathcal{C}_1 : e \longmapsto k$, is defined to have the following structure:

$$\mathcal{C}_1 = \begin{bmatrix} C_{11} & 0 & \cdots & & & 0 \\ C_{21} & C_{22} & 0 & \cdots & & 0 \\ 0 & C_{32} & C_{33} & 0 & \cdots & 0 \\ \vdots & & \ddots & & & \vdots \\ 0 & \cdots & & 0 & C_{m,m-1} & C_{mm} \end{bmatrix} \tag{6.2}$$

Where the local sharing of information in the feedback channel with respect to the k_i output channel of the controller consist of information in channels e_{i-1} and e_i. A Type 2 partially decentralized controller, $\mathcal{C}_2 : e \longmapsto k$, is defined to have the following structure:

$$\mathcal{C}_2 = \begin{bmatrix} C_{11} & C_{12} & 0 & \cdots & & 0 \\ 0 & C_{22} & C_{23} & 0 & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & \cdots & & & 0 & C_{mm} \end{bmatrix} \tag{6.3}$$

Where the local sharing of information in the feedback channel with respect to the k_i output channel of the controller consist of information in channels e_i and e_{i+1}. And finally a Type 3 partially decentralized controller, $\mathcal{C}_3 : e \longmapsto k$, is defined to have the

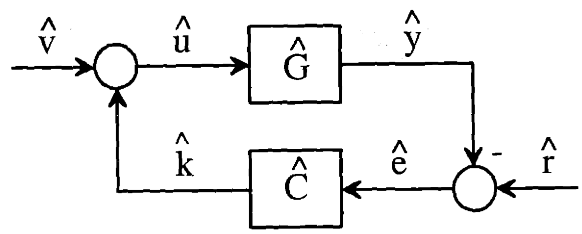

Figure 6-2: Two Block Control Problem for $\hat{G}$

following structure:

$$C_3 = \begin{bmatrix} C_{11} & C_{12} & 0 & \cdots & & 0 \\ C_{21} & C_{22} & C_{23} & 0 & \cdots & 0 \\ 0 & C_{32} & C_{33} & C_{34} & \cdots & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & \cdots & & C_{m-1,m-2} & C_{m-1,m-1} & C_{m-1,m} \\ 0 & \cdots & & 0 & C_{m,m-1} & C_{m,m} \end{bmatrix} \tag{6.4}$$

Where the local sharing of information in the feedback channel with respect to the k_i output channel of the controller consist of information in channels e_{i-1}, e_i and e_{i+1}.

In the case of the fully decentralized compensator the structure of the compensator factorization into stable factors is readily apparent. For example, one factorization of the block diagonal compensator could be $C_d = V^{-1}U$ where V and U are coprime and also block diagonal (see section 3.2, definition 4). However, the complex structure of the partially decentralized controllers (as exhibited by eqs. (6.2)-(6.4)) do not simplify into a readily recognizably stable factors structure and hence the parameterization of partially decentralized controllers using stable factors directly becomes difficult. The method developed in this chapter takes advantage of the stable factor parameterization indirectly by transforming the original plant operator via left and right unimodular transformations and then lifting or effectively repartitioning the resulting operator into a multichannel operator which can be stabilized by the set of parameterized fully decentralized controllers as given in chapter 3. The desired partial decentralized controller will then be recovered from the fully decentralized controllers by the reciprocal left and right unimodular transformations. Before detailing the precise steps involved in this method section 6.3 presents some needed definitions and theorems.

6.3 Unimodular Transformations

A Left unimodular operator is defined as follows:

Definition 10 (Left Unimodular) *An operator N an element of $m(H)$ is left unimodular if there exists an operator $Z \in m(H)$ such that $ZN = I$.*

Likewise a right unimodular operator is defined as follows:

Definition 11 (Right Unimodular) *An operator M an element of $m(H)$ is right unimodular if there exists an operator $W \in m(H)$ such that $MW = I$.*

Synthesis of structurally constrained controllers from say fully decentralized controllers is dependent on establishing a relation between the original plant operator G and an operator $\hat{G}$. For example figure 6-2 shows the two block problem for operator $\hat{G}$ and stabilizing controller $\hat{C}$. By requiring that the relation $G = M\hat{G}N$ holds, where M is right unimodular with $MW = I$ and N is left unimodular with $ZN = I$, a two block diagram can be written as shown in figure 6-3. Where $M : \hat{r} \mapsto r$ and $Z : \hat{v} \mapsto v$. This leads to the following theorem which will be instrumental in recovering partially decentralized controllers from fully decentralized controllers.

Theorem 10 *Given $\hat{C}$ stabilizes $\hat{G}$ and $G = M\hat{G}N$ where M is right unimodular with $MW = I$ and N is left unimodular with $ZN = I$, $C \stackrel{def}{=} Z\hat{C}W$ stabilizes G.*

Proof

$$H(\hat{C},\hat{G}) : \quad \begin{bmatrix} \hat{r} \\ \hat{v} \end{bmatrix} \longrightarrow \begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} \quad \text{where}$$

$$H(\hat{C},\hat{G}) = \begin{bmatrix} (I+\hat{G}\hat{C})^{-1} & -(I+\hat{G}\hat{C})^{-1}\hat{G} \\ (I+\hat{C}\hat{G})^{-1}\hat{C} & (I+\hat{C}\hat{G})^{-1} \end{bmatrix} = \begin{bmatrix} \hat{H}_{11} & \hat{H}_{12} \\ \hat{H}_{21} & \hat{H}_{22} \end{bmatrix}$$

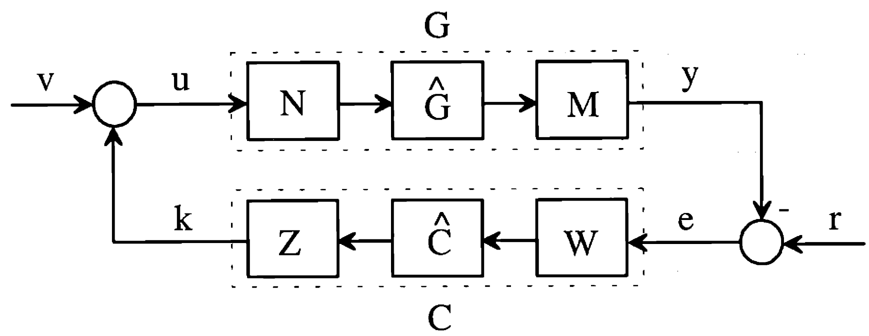

Figure 6-3: Two Block Problem Transformed Using Left and Right Unimodular Operators

$$\hat{C} \quad \text{stabilizes} \quad \hat{G} \Longrightarrow \hat{H}_{ij} \in m(H) \quad \forall\, i,j$$

$G = M\hat{G}N$ where M is right unimodular with $MW = I$ and N is left unimodular with $ZN = I$, and with C defined as $C \overset{def}{=} Z\hat{C}W$ the following maps are defined:

$$\begin{aligned} W: &\ r \mapsto \hat{r} \\ M: &\ \hat{e} \mapsto e \\ N: &\ v \mapsto \hat{v} \\ Z: &\ \hat{u} \mapsto u \end{aligned}$$

Using these above maps the mapping corresponding to $H(\hat{C},\hat{G})$ can be rewritten as

$$\begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} = H(\hat{C},\hat{G}) \begin{bmatrix} \hat{r} \\ \hat{v} \end{bmatrix}$$

$$\begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \hat{e} \\ \hat{u} \end{bmatrix} = \begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} H(\hat{C},\hat{G}) \begin{bmatrix} W & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix}$$

$$\begin{bmatrix} e \\ u \end{bmatrix} = \begin{bmatrix} M\hat{H}_{11}W & M\hat{H}_{12}N \\ Z\hat{H}_{21}W & Z\hat{H}_{22}N \end{bmatrix} \begin{bmatrix} r \\ v \end{bmatrix}$$

Note however that the map from $\begin{bmatrix} r \\ v \end{bmatrix}$ to $\begin{bmatrix} e \\ u \end{bmatrix}$ is the closed loop map H(C,G).

Therefore

$$H(C,G) = \begin{bmatrix} M\hat{H}_{11}W & M\hat{H}_{12}N \\ Z\hat{H}_{21}W & Z\hat{H}_{22}N \end{bmatrix}$$

Since $M, W, N, Z \in m(H)$ this implies $M\hat{H}_{11}W$, $M\hat{H}_{12}N$, $Z\hat{H}_{21}W$, $Z\hat{H}_{22}N$ are elements of $m(H)$ and that C stabilizes G.

□

Using Theorem 10, in general a number of structurally constrained controller can be synthesized. The focus here will be on synthesizing partially decentralized controls as given in eqs. (6.2)-(6.4). To illustrate this method the synthesis of a three channel, type 3 controller will be developed since the type 3 structure is more complex then the other two controller types. Extensions to the multichannel case proceeds directly along the lines outlined in the next section for the three channel case.

6.4 Synthesizing Type 3 Controllers

For the following three channel plant, using the notation described above and in Figure 6-3:

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{6.5}$$

The structure corresponding to a three channel, type 3 controller takes the form:

$$\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{21} & C_{22} & C_{23} \\ 0 & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{6.6}$$

As will be demonstrated a m-channel, type 3 controller can be recovered from a $(m-1)$-channel fully decentralized controller where the channel dimension has been appropriately increased. For the 3-channel, type 3 controller, it will be recovered from a 2-channel fully decentralized controller in the following manner:

$$C = Z\hat{C}W \tag{6.7}$$

Where $\hat{C}$ has the following 2-channel structure

$$\begin{bmatrix} \hat{C}^{(1)} & 0 \\ 0 & \hat{C}^{(2)} \end{bmatrix} = \begin{bmatrix} \hat{C}^{(1)}_{11} & \hat{C}^{(1)}_{12} & 0 & 0 \\ \hat{C}^{(1)}_{21} & \hat{C}^{(1)}_{22} & 0 & 0 \\ 0 & 0 & \hat{C}^{(2)}_{11} & \hat{C}^{(2)}_{12} \\ 0 & 0 & \hat{C}^{(2)}_{21} & \hat{C}^{(2)}_{22} \end{bmatrix} \tag{6.8}$$

The type 3 controller C can be recovered from $\hat{C}$ by using the following right and left unimodular operators:

$$Z = \begin{bmatrix} I_{i1} & 0 & 0 & 0 \\ 0 & I_{i2} & I_{i2} & 0 \\ 0 & 0 & 0 & I_{i3} \end{bmatrix} \tag{6.9}$$

$$W = \begin{bmatrix} I_{o1} & 0 & 0 \\ 0 & I_{o2} & 0 \\ 0 & I_{o2} & 0 \\ 0 & 0 & I_{o3} \end{bmatrix} \tag{6.10}$$

The identity operator I_{ij} is compatible with the input dimension of the plant operator G_{ij} and likewise I_{oi} is compatible with the output dimension of G_{ij}. Applying Z and W to $\hat{C}$ as given in eq. (6.7) gives the following:

$$C = Z\hat{C}W = \begin{bmatrix} \hat{C}^{(1)}_{11} & \hat{C}^{(1)}_{12} & 0 \\ \hat{C}^{(1)}_{21} & \hat{C}^{(1)}_{22} + \hat{C}^{(2)}_{11} & \hat{C}^{(2)}_{12} \\ 0 & \hat{C}^{(2)}_{21} & \hat{C}^{(2)}_{22} \end{bmatrix} \tag{6.11}$$

Which has the desired structure of a type 3, 3-channel controller. C will be stabilizing, according to theorem 10 as long as $G = M\hat{G}N$ where M is right unimodular with

$MW = I$ and N is left unimodular with $ZN = I$. Based on Z and W as given in eqs. (6.9)-(6.10), M and N can have the following form:

$$M = \begin{bmatrix} I_{o1} & 0 & 0 & 0 \\ 0 & .5I_{o2} & .5I_{o2} & 0 \\ 0 & 0 & 0 & I_{i3} \end{bmatrix} \tag{6.12}$$

$$N = \begin{bmatrix} I_{i1} & 0 & 0 \\ 0 & .5I_{i2} & 0 \\ 0 & .5I_{i2} & 0 \\ 0 & 0 & I_{i3} \end{bmatrix} \tag{6.13}$$

What needs to be determined is the structure of $\hat{G}$.

Since $\hat{G}$ must satisfy

$$G = M\hat{G}N \tag{6.14}$$

$\hat{G}$ can be obtained in the following fashion

$$\hat{G} = WGZ + S \tag{6.15}$$

Applying eq. (6.14) we obtain:

$$\begin{aligned} G &= M(WGZ + S)N \\ &= G + MSN \end{aligned} \tag{6.16}$$

Which is satisfied if $MSN \in m(0)$ where $m(0)$ is the matrix ring whose elements are all zero. For the three channel case WGZ has the following form:

$$WGZ = \begin{bmatrix} G_{11} & G_{12} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{22} & G_{23} \\ G_{21} & G_{22} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{32} & G_{33} \end{bmatrix} \tag{6.17}$$

Before lifting WGZ to a two channel plant for which a set of parameterized fully decentralized controllers can be developed using stable factors, an S operator can be added which will minimize the size of the coupling operator, [76], for $\hat{G}$. An operator S having the following form will achieve this:

$$S = \begin{bmatrix} 0 & G_{12} & -G_{12} & 0 \\ G_{21} & G_{22} & -G_{22} & -G_{23} \\ -G_{21} & -G_{22} & G_{22} & G_{23} \\ 0 & -G_{32} & G_{32} & G_{33} \end{bmatrix} \tag{6.18}$$

And $\hat{G}$ takes the following form:

$$\begin{aligned} \hat{G} &= WGZ + S \\ &= \begin{bmatrix} G_{11} & 2G_{12} & 0 & G_{13} \\ 2G_{21} & 2G_{22} & 0 & 0 \\ 0 & 0 & 2G_{22} & 2G_{23} \\ G_{31} & 0 & 2G_{32} & G_{33} \end{bmatrix} \end{aligned} \tag{6.19}$$

Now $\hat{G}$ can be lifted or equivalently repartitioned into a two channel plant having the structure:

$$\hat{G} = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} \\ \hat{G}_{21} & \hat{G}_{22} \end{bmatrix} \tag{6.20}$$

where

$$\begin{aligned} \hat{G}_{11} &= \begin{bmatrix} G_{11} & 2G_{12} \\ 2G_{21} & 2G_{22} \end{bmatrix} \\ \hat{G}_{12} &= \begin{bmatrix} 0 & G_{13} \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$\hat{G}_{21} = \begin{bmatrix} 0 & 0 \\ G_{13} & 0 \end{bmatrix}$$

$$\hat{G}_{22} = \begin{bmatrix} 2G_{22} & 2G_{23} \\ 2G_{32} & 2G_{33} \end{bmatrix} \tag{6.21}$$

Now if $\hat{G}$ has no unstable fixed modes [13] a parameterized set of two channel fully decentralized controllers exists based on the stable factor method presented in chapter 3. And from each two channel fully decentralized controller a type 3 partially decentralized controller can be recovered as given in eq. (6.11).

6.5 Coupling in Partially Decentralized Controllers

As demonstrated in reference [76] weak coupling can be quantified in terms of the norm of the off-diagonal elements of a stable plant operator for which fully decentralized controllers are to be designed. The effect as the coupling goes to zero is that the unimodular constraint which restricts the design parameters used in the selection of fully decentralized controllers disappears. These notions serve to provide a measure of the improvement obtainable via the use of partially decentralized controllers versus applying a fully decentralized control strategy directly to the plant operator G. For example, a stable three channel plant as given in eq. (6.5) would have the following coupling operator norm if a three channel fully decentralized controller where to be designed:

$$\|G_c\| = \left\| \begin{matrix} 0 & G_{12} & G_{13} \\ G_{21} & 0 & G_{23} \\ G_{31} & G_{32} & 0 \end{matrix} \right\| \tag{6.22}$$

However, when a three channel, type 3 controller is designed for G, a fully decentralized controller is designed using the associated $\hat{G}$ operator as given by eq. (6.20).

The coupling operator norm for a stable $\hat{G}$ is

$$\begin{aligned} \left\|\hat{G}_c\right\| &= \left\| \begin{matrix} 0 & \hat{G}_{12} \\ \hat{G}_{21} & 0 \end{matrix} \right\| \\ &= \max\left[\|\hat{G}_{21}\|, \|\hat{G}_{12}\|\right] \end{aligned} \tag{6.23}$$

Due to eq. (6.21), it follows that $\|\hat{G}_{21}\| = \|G_{31}\|$ and $\|\hat{G}_{12}\| = \|G_{13}\|$. Hence the coupling norm when using a type 3 controller is given by $\|\hat{G}_c\| = \max[\|G_{31}\|, \|G_{13}\|]$. This implies (as outlined in section 4.2 for the general two channel fully decentralized case) a quantification of weak coupling can be developed using only the norms of the G_{13} and G_{31} operators of the three channel plant when controlled by a type 3 controller. This simplification with respect to the more complicated coupling operator for the fully decentralized 3 channel controller (see eq. (6.22)) is not unexpected considering information sharing occurs between adjacent channels in the type 3 controller (see eq. (6.6)) unlike the fully decentralized 3 channel controller where there is no sharing of information among the feedback channels.

6.6 Synthesizing Type 1 and Type 2 Controllers

Synthesizing Type 1 and Type 2 controllers follows the same basic pattern as performed for the type 3 controller. To demonstrate this a 3 channel type 1 controller will be developed. The type 2 controllers are developed in a complementary fashion. In general a type 1 or type 2, m-channel controller can be synthesized from a m-channel fully decentralized controller where $(m-1)$ of the fully decentralized channels have an increased dimension. The structure of the 3 channel, type 1 controller is

$$C = \begin{bmatrix} C_{11} & 0 & 0 \\ C_{21} & C_{22} & 0 \\ 0 & C_{32} & C_{33} \end{bmatrix} \tag{6.24}$$

Such a controller can be recovered from a three block fully decentralized controller with the following form:

$$\hat{C} = \begin{bmatrix} \hat{C}^{(1)} & 0 & 0 \\ 0 & \hat{C}^{(2)} & 0 \\ 0 & 0 & \hat{C}^{(3)} \end{bmatrix} \tag{6.25}$$

$$= \begin{bmatrix} \hat{C}^{(1)} & 0 & 0 & 0 & 0 \\ 0 & \hat{C}_1^{(2)} & \hat{C}_2^{(2)} & 0 & 0 \\ 0 & 0 & 0 & \hat{C}_1^{(3)} & \hat{C}_2^{(3)} \end{bmatrix} \tag{6.26}$$

The right and left unimodular operators Z and W have the following form

$$Z = I \tag{6.27}$$

$$W = \begin{bmatrix} I_{o1} & 0 & 0 \\ I_{o1} & 0 & 0 \\ 0 & I_{o2} & 0 \\ 0 & I_{o2} & 0 \\ 0 & 0 & I_{o3} \end{bmatrix} \tag{6.28}$$

And the recovered type 1 controller has the following form:

$$C = \begin{bmatrix} \hat{C}^{(1)} & 0 & 0 \\ \hat{C}_1^{(2)} & \hat{C}_2^{(2)} & 0 \\ 0 & \hat{C}_1^{(3)} & \hat{C}_2^{(3)} \end{bmatrix} \tag{6.29}$$

Once again $\hat{G}$ is found from $\hat{G} = WGZ + S$ where

$$WGZ = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \tag{6.30}$$

$$S = \begin{bmatrix} G_{11} & -G_{12} & G_{13} \\ -G_{11} & G_{12} & -G_{13} \\ -G_{21} & G_{22} & -G_{23} \\ G_{21} & -G_{22} & G_{23} \\ 0 & 0 & 0 \end{bmatrix} \tag{6.31}$$

Resulting in a $\hat{G}$ of the following form:

$$\hat{G} = \begin{bmatrix} 2G_{11} & 0 & 2G_{13} \\ 0 & 2G_{12} & 0 \\ 0 & 2G_{22} & 0 \\ 2G_{21} & 0 & 2G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \tag{6.32}$$

Now $\hat{G}$ can be repartitioned into a three channel plant having the structure:

$$\hat{G} = \begin{bmatrix} \hat{G}_{11} & \hat{G}_{12} & \hat{G}_{13} \\ \hat{G}_{21} & \hat{G}_{22} & \hat{G}_{23} \\ \hat{G}_{31} & \hat{G}_{32} & \hat{G}_{33} \end{bmatrix} \tag{6.33}$$

where

$$\hat{G}_{11} = 2G_{11} \tag{6.34}$$

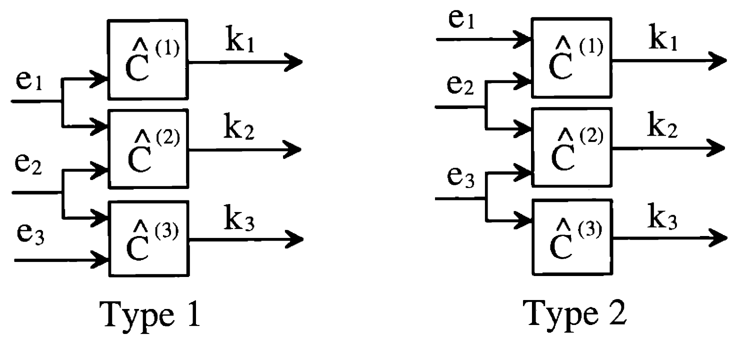

Figure 6-4: Type 1 and Type 2 Controller Structure

$$\hat{G}_{13} = 2G_{13} \tag{6.35}$$

$$\hat{G}_{22} = \begin{bmatrix} 2G_{12} \\ 2G_{22} \end{bmatrix} \tag{6.36}$$

$$\hat{G}_{31} = \begin{bmatrix} 2G_{21} \\ G_{31} \end{bmatrix} \tag{6.37}$$

$$\hat{G}_{32} = \begin{bmatrix} 0 \\ G_{32} \end{bmatrix} \tag{6.38}$$

$$\hat{G}_{33} = \begin{bmatrix} 2G_{23} \\ G_{33} \end{bmatrix} \tag{6.39}$$

And $\hat{G}_{12}, \hat{G}_{21}, \hat{G}_{23} \in m(0)$. Now once again if $\hat{G}$ has no unstable fixed modes [13] a parameterized set of three channel fully decentralized controllers exists based on the stable factors method presented in chapter 3. And from each three channel fully decentralized controller a type 1 partially decentralized controller can be recovered as given in eq. (6.29). Finally figures 6-4 and 6-5 illustrate how the recovered partially decentralized controllers maintain the desirable property of parallel processing with (in the case of a type 3 controller) the small additional overhead of output channel summations.

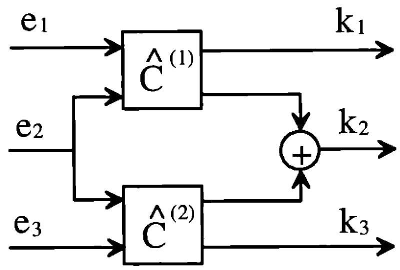

Figure 6-5: Type 3 Controller Structure

6.7 Application of Decentralized Design Methods to Partially Decentralized Controllers

The methods developed in this chapter rely on recovering the desired partially decentralized controller via left and right unimodular transformations applied to an appropriately designed fully decentralized controller. This section discusses how the autonomous design methods of chapter 4 and the robust design methods of chapter 5, originally developed for fully decentralized systems, would impact the design of partially decentralized controllers.

6.7.1 Autonomous Design for Partially Decentralized Controllers

Applying the autonomous design method of chapter 4 to the design of partially decentralized controllers is relatively straight forward. For example, in section 4.2, figure 4-1 illustrates the two channel, fully decentralized problem. Equation (4.11) gives the simplified design bound for the subcontroller parameters for the case of a stable plant, $P \in m(H)$ and eq. (4.32) gives this bound for the more generalized case, $P \in m(G)$. In section 6.4, a type 3, partially decentralized controllers is developed for a three channel plant. The partially decentralized controller, eq. (6.6), will be recovered from a two channel, fully decentralized controller (see eq. (6.8)). The simplified bounds for the subcontroller parameters will be in terms of the transformed plant, $\hat{G}$. For the case, $G \in m(H)$, (i.e. the original plant is stable) the subcontroller parameters must satisfy

$$||\tilde{Q}_2||||\tilde{Q}_1|| < \frac{1}{||\hat{G}_{21}||||\hat{G}_{12}||} \tag{6.40}$$

In the more generalized case, $G \in m(G)$, the subcontroller parameters must satisfy

$$||\tilde{Q}_2||||\tilde{Q}_1|| < \frac{1}{||\hat{W}_{21}||||\hat{W}_{12}||} \tag{6.41}$$

where $\hat{W}_{21}$ and $\hat{W}_{12}$ are formed from the two channel partitioned stable factors of $\hat{G}$ as given by eq. (4.31). In both cases, the subcontrollers can be designed in an autonomous fashion as long as the appropriate design bounds, eq. (6.40) or eq. (6.41) are observed. The resulting subcontrollers are then aggregated into a stabilizing, fully decentralized controller $\hat{C}$. A stabilizing partially decentralized controller of the form given by eq. (6.6) will be recovered from $\hat{C}$ via left and right unimodular transformation as given by eq. (6.11).

6.7.2 Robust Design for Partially Decentralized Controllers

The application of autonomous design methods to partially decentralized controllers presented essentially no technical difficulties because the overall objective was the development of a stabilizing partially decentralized controller for a nominal plant. This situation changes when the partially decentralized controller is required to provide for an overall performance criterion while simultaneously stabilizing a family of plants which reflect model uncertainty and errors of the original nominal plant. This is of course the robust partially decentralized control problem. The methods of chapter 5 can be adapted to provide a framework for developing robust partially decentralized controllers. The key is providing a connection between the robust formulation of the problem when synthesizing a robust fully decentralized controller, $\hat{C}$, for the transformed plant $\hat{P}_n$ and the original system which had the model uncertainty perturbation topology along with performance criterion defined in terms of a partially decentralized controller and the original nominal plant. This connection is provided by relating the LFT, $M(G, C)$, to the LFT, $\hat{M}(\hat{G}, \hat{C})$ which is used to synthesize a

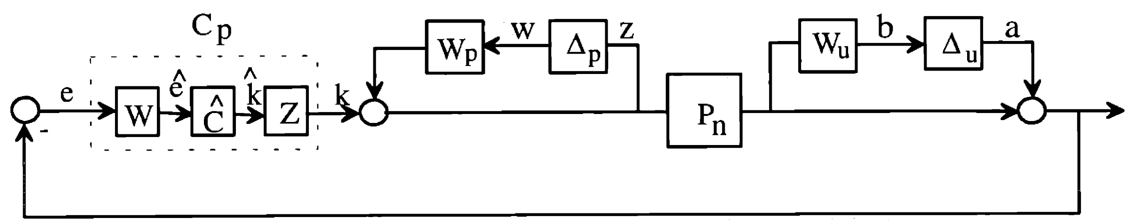

Figure 6-6: Robust Control Problem For Partially Decentralized Controller

decentralized controller $\hat{C}$. To illustrate this the robust control problem of section 5.3 will be used.

Figure 6-6 is the robust control problem from section 5.3 with scalar performance and uncertainty weights Δ_p and Δ_u. The controller C_p for this problem is constrained to be a partially decentralized controller. The $M(\cdot)$ operator is the closed loop map

$$M(G, C_p) : \begin{bmatrix} a \\ w \end{bmatrix} \longrightarrow \begin{bmatrix} b \\ z \end{bmatrix} \tag{6.42}$$

and is constructed with respect the structured perturbation $\Delta = blkdiag[\Delta_u, \Delta_p]$. By relying on theorem 10, namely $\hat{C}$ stabilizes $\hat{P}_n$ implies $C_p \stackrel{def}{=} Z\hat{C}W$ stabilizes P_n where

$$\begin{aligned} MW &= I \\ ZN &= I \end{aligned} \tag{6.43}$$

figure 6-6 is rewritten using these unimodular operators to formulate the robust control problem in terms of $\hat{P}_n$ and $\hat{C}$ (see figure 6-7). For the moment ignore the elements inside the dotted boxes of figure 6-7. The block diagram of figure 6-7 defines the structure for following LFT

$$\hat{M}(\hat{G}, \hat{C}) : \begin{bmatrix} \hat{a} \\ \hat{w} \end{bmatrix} \longrightarrow \begin{bmatrix} \hat{b} \\ \hat{z} \end{bmatrix} \tag{6.44}$$

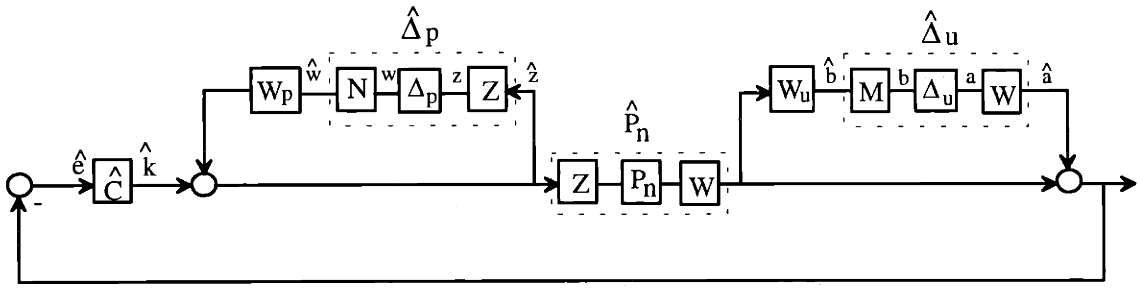

Figure 6-7: Transformed Partially Decentralized Robust Control Problem

where the structured perturbation is $\hat{\Delta} = blkdiag[\hat{\Delta}_u, \hat{\Delta}_p]$ and $\hat{G}$ is the transformed generalized plant where $\hat{G}_{22} = \hat{P}_n$. The operator $\hat{M}(\hat{G}, \hat{C})$ is important since the compensator synthesis component of any D-K method (and this includes the sequential D-K method of section 5.4) will be done in terms of this operator. This is how compensator synthesis in terms of a fully decentralized compensator, $\hat{C}$, is applied to the synthesis of robust partially decentralized systems. The robust partially decentralized compensator, C_p, will be recovered from the fully decentralized compensator obtained from solving the optimization problem involving the $\hat{M}(\hat{G}, \hat{C})$ operator (see eq. (5.46)).

By defining the perturbations in figure 6-7 in the following manner

$$\begin{aligned} \hat{\Delta}_u &= W\Delta_u M \\ \hat{\Delta}_p &= N\Delta_p Z \end{aligned} \tag{6.45}$$

a connection between the μ-framework for $(\Delta, M(\cdot))$ and $(\hat{\Delta}, \hat{M}(\cdot))$ can be made in the form of a unimodular transformation. This connection is made in the following way. From eq. (6.45) the following signal transformations are obtained.

$$\begin{aligned} \begin{bmatrix} b \\ z \end{bmatrix} &= \begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{z} \end{bmatrix} \\ \begin{bmatrix} \hat{a} \\ \hat{w} \end{bmatrix} &= \begin{bmatrix} W & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} a \\ w \end{bmatrix} \end{aligned} \tag{6.46}$$

This leads to the following correspondence between $\hat{M}(\hat{G}, \hat{C})$ and $M(G, C_p)$.

$$\begin{bmatrix} \hat{b} \\ \hat{z} \end{bmatrix} = \hat{M}(\hat{G}, \hat{C}) \begin{bmatrix} \hat{a} \\ \hat{w} \end{bmatrix}$$

$$\begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} \begin{bmatrix} \hat{b} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} M & 0 \\ 0 & Z \end{bmatrix} \hat{M}(\hat{G}, \hat{C}) \begin{bmatrix} W & 0 \\ 0 & N \end{bmatrix} \begin{bmatrix} a \\ w \end{bmatrix}$$

$$\begin{bmatrix} b \\ z \end{bmatrix} = \begin{bmatrix} M\hat{M}_{11}W & M\hat{M}_{12}N \\ Z\hat{M}_{21}W & Z\hat{M}_{22}N \end{bmatrix} \begin{bmatrix} a \\ w \end{bmatrix} \tag{6.47}$$

Note however that the map from $\begin{bmatrix} a \\ w \end{bmatrix}$ to $\begin{bmatrix} b \\ z \end{bmatrix}$ is the operator $M(G, C_p)$. Hence the correspondence between $\hat{M}(\hat{G}, \hat{C})$ and $M(G, C_p)$ is established.

An interesting open issue exists with the implementation of a D-K algorithm as applied to the synthesis of robust partially decentralized controllers. Only the controller synthesis (or K step) of the D-K algorithm requires the use of the $\hat{M}(\hat{G}, \hat{C})$ operator. The actual μ computation and reflecting of D scales back into the plant could be carried out in the framework which uses the $(\hat{\Delta}, \hat{M}(\hat{G}, \hat{C}))$ combination or the framework which uses the $(\Delta, M(G, C_p))$ combination since the compensator is fixed during the D step. Performing the D step using the operators $(\Delta, M(G, C_p))$ would intuitively seem the wiser choice since this is the original problem setting and contains the original uncertainty and performance topology. Determination of the advantages, if any, of either of these approaches remains an area available for future research.

6.8 Summary

Partially decentralized controllers can be beneficial for physical systems where strong local interactions of subsystems exist. LDF laser alignment control system [75] exhibited improved performance through the use of a partially decentralized control. This chapter classifies three types of partially decentralized controllers (eqs. (6.2)-(6.4)) and presents a method of controller synthesis linked to stable factor methods. The method developed in this chapter is sufficiently broad enough that other structurally constrained controllers not specifically contained in the three classifications can also be synthesized. The general notion of left and right unimodular transformations are developed for the recovery of a stabilizing compensator C for the plant G as given by theorem 10 in section 6.3. Using these ideas a method of synthesizing partially decentralized controllers from the parameterized set of fully decentralized controllers is developed. Section 6.4 illustrates the method for type 3 partially decentralized controllers and section 6.6 illustrates the method for type 1 and type 2 partially decentralized controllers. A discussion quantifying the concept of weak plant coupling under partial decentralized control is examined in section 6.5 and insight is gained by contrasting this against weak plant coupling in the more familiar sense [76] under fully decentralized control. Finally, section 6.7 discussed the application of decentralized methods developed in chapters 4 and 5 to the design of partially decentralized controllers.

Chapter 7

Conclusions

7.1 Research Summary

The focus of thesis has been to develop a framework for the design of structurally constrained controllers which is aligned with the methodology of centralized control systems under the modern robust stability/robust performance paradigm. To that end a number of significant results have been contributed by the work in this thesis.

In section 2.4 an important discussion concerning the decentralized partitioning problem was presented. This was augmented by section 2.5 where the fundamental condition for a partition to be viable for decentralized control was presented and the development presented for fixed modes was selected to fit the input/output stable factors approached used in this thesis.

A starting point for the thesis resided with the use of a recently developed stable factor parameterization of stabilizing decentralized controllers for all admissible partitioned plants. The pertinent details of this parameterization are given in sections 3.2-3.3. In section 3.4 a class of identities known as the auxiliary doubly coprime Bezout identities (ADCBI) were presented along with their importance and fundamental role played in establishing the decentralized controller parameterization. A new proof of the ADCBI was given in this section along with a clarification of their direct relation to the decentralized doubly coprime Bezout identity. Based on the ADCBI developed in section 3.4 a new class of stabilizing decentralized controllers were developed in section 3.5. This class of controllers are shown to be useful for the development of a new autonomous design method for subcontrollers (chapter 4) and in the development of a new decentralized D-K methodology for the sequential design of decentralized controllers (chapter 5). The usefulness of this set of decentralized controllers results from a simplifying assumption developed in section 3.5 which imposes a unimodular restriction on a subset of the design parameters of the decentralized controllers. Such a restriction has the benefit of generating a set of decentralized controllers whose individual subcontrollers have the form of the basic Youla parameterization (see section 2.2) which results in only one design parameter per subcontroller.

In chapter 4 a new method for autonomous design for subcontrollers was developed. Section 4.1 details the applications for which these type of controllers are most often used. The germane issues associated with any autonomous design method are ease of design and a stability guarantee so that aggregating the subcontrollers into a fully decentralized control scheme provides for closed loop stability. The new method developed in this thesis relies on the set of decentralized controllers developed in section 3.5. From the set's associated unimodular constraint a simplified norm bound stability guarantee is developed for the set of subcontroller parameters. Section 4.2 gives this bound for the two channel, stable plant case. Section 4.3 compares this bound to bounds developed using the stable off diagonal plant operators and small gain methods. Section 4.5 develops a similar bound on the design parameters for the case of unstable plants. This highlights the unique nature of this approach since by employing a stable factor approach symmetric results are obtainable for the unstable plant case. For comparison purposes an autonomous design method developed in [77], relying on the multivariable Nyquist criterion for a stability guarantee, made an attempt to extend the result to unstable plants. However, the restrictions on the plant were so specialized that the researchers conceded that the method effectively only held for stable plants. The method developed in this thesis avoided such problems by relying on a unimodular stability criterion and by using stable factor methods.

In chapter 5.1 the decentralized problem is placed in the μ-framework. Or in other words issues concerning the design of robust stability/robust performance are examined for decentralized controllers. Tying the development of decentralized controllers to this generalized framework, initially developed for centralized controllers, as opposed to other decentralized controller methodologies (see section 1.2), has the advantage of a specific guarantee. That is, if a specific closed loop operator extracted from the plant and involving the decentralized controllers satisfies a given μ criterion the synthesized decentralized controllers are guaranteed to be robust from both a stability and performance point of view. In section 5.3 a decentralized $M(\cdot)$ operator in a stable factor form is developed. The difficulties of synthesizing concurrently the design parameters of decentralized controllers satisfying a μ constraint are detailed. Section 5.4 presents a novel adaptation of the D-K methodology for the sequential design of decentralized controllers. The critical step, shown in section 5.4.1, involved the development of a iteration scheme using the parameter constraints of section 3.5 to develop a $M(\cdot)$ operator which would be convex in the design parameters for the decentralized subcontrollers. Section 5.4.2 then presented the result that iteration between the subcontrollers produced a monotonic decrease in the norm bound of the $M(\cdot)$ operator which then produces a viable decentralized controller synthesis for the D-K framework. Finally, in section 5.5 the computation issues associated with this sequential D-K method for robust decentralized controllers is discussed along with some anticipated difficulties.

In chapter 6 an important connection between decentralized controllers and a collection of structurally constrained controllers denoted, *partially decentralized controllers*, was made. It effectively allows the extension of decentralized stable factor methods to be extended to the domain of partially decentralized controller design. Section 6.2 develops canonical forms for partial decentralized controllers. In section 6.3 a novel transformation in terms of unimodular operators is developed. This unimodular transformation provides the key to extending decentralized design methods to the development of partially decentralized controllers. A fully detailed application of this novel unimodular transformation to partially decentralized controllers was provided. The unimodular transformation method developed in this thesis is sufficiently broad enough that other structurally constrained controllers not characterized in section 6.2 can also be handled using these same methods.

7.2 Recommendations for Future Research

The results and framework developed in this thesis provide additional motivation for future research.

An important area for future work would involve the coding of a practical implementation of the D-K Methodology, developed in chapter 5, for the sequential design of robust decentralized controllers. A number of practical issues associated with the coding of this algorithm need to be worked out. These would include, for example, the number of iterations among subcontrollers between each pass in the D-K iteration and a mechanism for handling or constraining the growth of the individual dimensions of the subcontrollers due to the iteration process. Many of these issues will naturally be problem/application dependent.

A difficult open problem still remains for problems in the robust framework of chapter 5 which involves the development (if possible) of a concurrent algorithm (i.e. an algorithm which would lead to the synthesis of decentralized design parameters simultaneously) for the synthesis of decentralized controllers. The envisioned advantage of such an algorithm would be the development of minimal forms for the dimensions of the individual subcontrollers. However, the tradeoff may be that a stringent simplifying condition may need to be imposed on the diagonalizing parameter, eq. (3.13), which in turn would lead to a synthesis method for decentralized controllers which would be highly restricted in terms of the overall performance obtainable.

Another interesting area to be investigated would be to develop some criteria for the selection of uncertainty and fictitious performance perturbation structure along with their associated weights which would take advantage of the decentralized information pattern to be imposed on the plant. The idea here is that when the plant is decoupled, one effectively has a set of individual subplant operators along the main diagonal of the original plant. Essentially, a μ formulation could be developed for these individual subplant operators by treating them as individual plants. By aggregating the uncertainty and performance perturbation structures of these individual subplants an effective decoupled uncertainty and performance perturbation structure has been defined. The questions become what are the interpretation of these decoupled perturbation structures once coupling in the plant is reintroduced? Does this decoupled perturbation structure lead to any useful, simplified design methods within the robust framework? Or in other words does a simplified methodology for the design of robust decentralized controllers in the face of weak coupling result?

Finally, an interesting open issue exists with the implementation of a D-K algorithm as applied to the synthesis of robust partially decentralized controllers. The specific issues involved in this future research area are outline at the end of section 6.7.2.

Bibliography

[1] Ünyelioğlu, K.A. Decentralized blocking zeros-part 1: Decentralized strong stabilization problem. *31st IEEE Conference on Decision and Control, Tucson, Arizona*, 1992.

[2] Özgüler, A.B. and Hiraoglu, M. Implications of a characterization result on strong and reliable decentralized control. *NATO ASI Series, Vol. F34, pp. 425-450*, 1992.

[3] Date, R.A. and Chow, J.H. A reliable coordinated decentralized control system design. *28th IEEE Conference on Decision and Control, Tampa, Florida*, 1989.

[4] Locatelli, A. Scattolini, R. and Schiavoni, N. On the design of reliable robust decentralized regulators for linear systems. *Large Scale Systems, Vol. 10, pp. 95-113*, 1986.

[5] Vidyasagar, M. *Control System Synthesis: A Factorization Approach.* MIT Press, 1985.

[6] Francis, B. *A Course in H_∞ Control Theory: Lecture Notes in Control and Information Sciences, Vol. 88.* Springer-Verlag, 1987.

[7] Youla, D.C. Jabr, H.A. and Bongiorno, J.J. Modern wiener-hopf design of optimal controllers, part ii: The multivariable case. *IEEE Trans. AC, Vol. 21, pp. 319-338*, 1976.

[8] Doyle, J. Francis, B. and Tannenbaum, A. *Feedback Control Theory.* Macmillan, 1992.

[9] Gundes, A.N. and Desoer, C.A. *Algebraic Theory of Linear Feedback Systems with Full and Decentralized Compensators: Lecture Notes in Control and Information Sciences, Vol. 142*. Springer-Verlag, 1990.

[10] Sandell Jr., N.R. Varaiya, P. Athans, M. and Safonov, M.G. Survey of decentralized control methods for large scale systems. *IEEE Trans. AC, Vol. 23, pp. 108-128*, 1978.

[11] Bakule, L. and Lunze, J. Decentralized design of feedback control for large-scale systems. *Kybernetika, Vol. 24, pp. 1-97*, 1988.

[12] Davison, E.J. and Wang, S.H. On the stabilization of decentralized control systems. *IEEE Trans. AC, Vol. 18, No. 5, pp. 473-478*, 1973.

[13] Davison, E.J. and Wang, S.H. A characterization of decentralized fixed modes in terms of transmission zeros. *IEEE Trans. AC, Vol. 30, pp. 81-82*, 1985.

[14] Anderson, B.D. and Clements, D.J. Algebraic characterization of fixed modes in decentralized control. *Automatica, Vol. 17, No. 5, pp. 703-712*, 1981.

[15] Tarokh, M. Fixed modes in multivariable systems using constrained controllers. *Automatica, Vol. 21, No. 4, pp.495-497*, 1985.

[16] Seraji, H. On fixed modes in decentralized control systems. *Int. J. Control, Vol. 35, No. 5, pp 775-784*, 1982.

[17] Glover, K. and Silverman, L.M. Characterization of structural controllability. *IEEE Trans. AC Vol. 21, pp. 534-537*, 1976.

[18] Reinschke, K. Graph theoretic characterization of fixed modes in centralized and decentralized control. *Int. J. Control, Vol. 39, No. 4, pp. 715-729*, 1984.

[19] Sezer, M.E. and Siljak, D.D. Structurally fixed modes. *Systems and Control Letters, 1, pp. 60-64*, 1981.

[20] Sezer, M.E. and Siljak, D.D. On structurally fixed modes. *Proc. IEEE Inter. Symposium on Ckts. and Systems, Chicago, IL, pp. 558-565*, 1981.

[21] Vidyasagar, M. and Viswanadham, N. Construction of inverses with prescribed zero minors and applications to decentralized stabilization. *Linear Algebra and Its Applications, Vol. 83, pp. 103-115*, 1986.

[22] Vidyasagar, M. and Viswanadham, N. Algebraic characterization of decentralized fixed modes and pole assignment. *Proc. 21st Conf. on Decision and Control, Orlando, Florida, pp. 501-505*, 1982.

[23] Davison, E.J. The robust control of servomechanism problem for linear time invariant multivariable systems. *IEEE Trans. AC, Vol. 21, pp. 25-34*, 1976.

[24] Corfmat, J.P. and Morse, A.S. Decentralized control of linear multivariable systems. *Automatica, Vol. 12, pp. 479-495*, 1976.

[25] Richter, S. and DeCarlo, R. A homotopy method for eigenvalue assignment using decentralzied state feedback. *IEEE Trans. AC, Vol. 29, pp. 148-158*, 1984.

[26] West-Vukovich, G.S. Davison, E.J. and Hughes, P.C. The decentralized control of large flexible space structures. *IEEE Trans. AC, Vol. 29, pp. 866-879*, 1984.

[27] Ryaciotaki-Boussalis, H.A. Griggs, H.C. and Charles, C. Dynamic performance modeling and stability analysis of a segmented reflector telescope. *Proc. American Control Conf., Boston, MA, pp. 1705-1706*, 1991.

[28] Ozguner, U. and Perkins, W.R. A series solution to the nash strategy for large scale interconnected systems. *Automatica, Vol. 13, pp. 313-315*, 1977.

[29] Kokotovic, P.V. Subsystems time scales and multimodeling. *Automatica, Vol. 17, No. 6, pp. 789-795*, 1981.

[30] Chow, J.H. and Kokotovic, P.V. Time scale modeling of dynamic networks with sparse and weak connections. *in J.H. Chow ed., Time Scale Modeling of Dynamic Networks with Applications to Power Systems,*, New York: Springer-Verlag, pp. 310-353, 1982.

[31] Peres, P. Yamakami, A. and Geromel, J.C. Decentralized control of large scale systems with multiple time scales. *IFAC 10th Triennial World Congress, Munich, FRG, pp. 57-60*, 1987.

[32] Chow, J.H. and Kokotovic, P.V. A decomposition of near-optimum regulators for systems with slow and fast modes. *IEEE Trans. AC, Vol. 12, pp 321-335*, 1976.

[33] Mercadal, M. *H_2, Fixed Architecture, Control Design for Large Scale Systems.* Ph.D. Thesis, MIT, 1990.

[34] Geromel, J.C. Gernusson, J. An algorithm for optimal decentralized regulation of linear quadratic interconnected systems. *Automatica, Vol. 15, pp. 489-491*, 1979.

[35] Ioannou, P. Decentralized adaptive control of interconnected systems. *IEEE Trans. AC, Vol 31, No. 4, pp. 291-298*, 1986.

[36] Mao, C. and Lin, W. Decentralized control of interconnected systems with unmodelled nonlinearity and interaction. *Automatica, Vol. 26, No. 2, p. 263-268*, 1990.

[37] Callier, F.M. Chan, W.S. and Desoer, C.A. Input-output stability of interconnected systems using decompositions: An improved formulation. *IEEE Trans. AC, Vol 23, No. 2, pp. 150-163*, 1978.

[38] Lunze, J. Stability analysis of large scale systems composed of strongly coupled similar subsystems. *10th IFAC Congress, Vol. 6, Munich, FRG*, 1987.

[39] Ikeda, M. and Siljak, D.D. Generalized decompositions of dynamic systems and vector lyapunov functions. *IEEE Trans. AC, Vol. 26, pp. 1118-1125*, 1980.

[40] Sezer, M.E. and Siljak, D.D. On decentralized stabilization and structure of linear large scale systems. *Automatica, Vol. 17, pp. 641-644*, 1981.

[41] Ikeda, M. and Siljak, D.D. On decentrally stabilizable large-scale systems. *Automatica, Vol. 16, pp. 331-334*, 1980.

[42] Ikeda, M. and Siljak, D.D. Decentralized stabilization of linear time-varying systems. *IEEE Trans. AC, Vol. 25, pp. 106-107*, 1980.

[43] Tan, X.-L. and Ikeda, M. Decentralized stabilization of large-scale interconnected systems: A stable factorization approach. *Proc. 26th IEEE Conf. Decision Contr., pp. 2295-2300*, 1987.

[44] Ozguler, A.B. Decentralized control: A stable proper fractional approach. *IEEE Trans. AC, Vol. 35, pp. 1109-1117*, 1990.

[45] Tan, X.-L. and Ikeda, M. Decentralized stabilization for expanding construction of large-scale systems. *IEEE Trans. AC, Vol. 35, pp. 644-651*, 1990.

[46] Manousiouthakis, V. On parametrization of all decentralized stabilizing controllers. *Proc. of American Control Conf., pp. 2108-2111*, 1989.

[47] Sourlas, D. and Manousiouthakis, V. On simultaneously optimal decentralized performance. *Submitted for Publication in the Int. J. of Control*, 1992.

[48] Date, R. and Chow, J. Decentralized stable factors and parametrization of decentralized controllers. *Proc. of American Control Conf., Boston, MA, pp. 904-909*, 1990.

[49] Packard, A. Doyle, J. and Balas, G. Linear, multivariable robust control with a μ perspective. *Trans. of ASME, Vol. 115, pp. 426-438*, 1993.

[50] Dahleh, M.A. and Diaz-Bobillo, I.J. *Course Notes for Advanced Linear Control Systems, 6.242.* Dept. EECS, Massachusetts Institute of Technology, 1993.

[51] Nagpaul, S.R. Bhattacharya, P.B., Jain, S.K. *Basic Abstract Algebra.* Cambridge University Press, 1986.

[52] Rowen, L.H. *Ring Theory.* Academic Press, 1991.

[53] Maciejowski, J.M. *Multivariable Feedback Design.* Addison-Wesley, 1989.

[54] Filipiak, J. *Modelling and Control of Dynamic Flows in Communication Networks.* Springer-Verlag, 1988.

[55] Ryaciotaki-Boussalis, H.A. and Wang, S.J. A decentralized approach to vibration suppression in segmented reflector telescopes. *Proc. American Control Conf., Pittsburgh, PA pp. 2548-2550*, 1989.

[56] Bristol, E.H. On a new measure of interaction for multivariable process control. *IEEE Trans. AC, Vol. 11, pp. 133-134*, 1966.

[57] Niederlinski, A. A heuristic approach to the design of linear multivariable interaction control systems. *Automatica, Vol. 7, pp. 691-701*, 1971.

[58] Morari, M. Robust stability of systems with integral control. *IEEE Trans. AC, Vol. 30, pp. 574-577*, 1985.

[59] Nwokah, O.D. Frazho, A.E. and Le, D.K. A note on decentralized integral controllability. *Int. J. Control, Vol. 57, No. 2, pp. 485-494*, 1993.

[60] Manousiouthakis, V. Savage, R. and Arkun, Y. Synthesis of decentralized process control structures using the concept of block relative gain. *AIChE J., Vol. 32, No. 6, pp. 991-1003*, 1986.

[61] Arkun, Y. Dynamic block relative gain and its connection with the performance and stability of decentralized control structures. *Int. J. Control, Vol. 57, No. 2, pp. 485-494*, 1993.

[62] Nett, C. and Manousiouthakis, V. Euclidean condition and block relative gain: Connections, conjectures, and clarificaitons. *IEEE Trans. AC, Vol. 32, pp. 405-407*, 1987.

[63] Chen, J. Freudenberg, J.S. and Nett, C. On relative gain array and condition number. *Proc. 31st Conf. on Decision and Control, Tucson, Arizona, pp. 219-224*, 1992.

[64] Gajic, Z. Petkovski, D. and Shen, X. *Singularly Perturbed and Weakly Coupled Linear Control Systems: Lecture Notes in Control and Information Sciences, Vol. 140.* Springer-Verlag, 1990.

[65] Horn, R.A. and Johnson, C.R. *Matrix Analysis.* Cambridge University Press, 1985.

[66] Desoer, C.A. and Vidyasagar, M. *Feedback Systems: Input-Output Properties.* Academic Press, 1975.

[67] Stein, G. and Doyle, J.C. Beyond singular values and loop shapes. *J. Guidance, Vol. 14, No. 1, pp 5-16*, 1991.

[68] Moser, A.N. Designing controllers for flexible structures with h-infinity/μ-synthesis. *IEEE Control Systems, pp. 79-89, April*, 1993.

[69] Packard, A. and Balsamo, W. A maximum modulus theorem for linear fractional transformations. *Systems and Control Letters, Vol. 11, pp. 365-367*, 1988.

[70] Doyle, J.C. Glover, K. Khargonekar, P. and Francis, B.A. State-space solutions to standard h_2 and h_∞ control problems. *IEEE Trans. Auto. Control, Vol. 34, No. 8, pp. 831-847*, 1989.

[71] Pecaric, J.E. Proschan, F. and Tong, Y.L. *Convex Functions, Partial Orderings, and Statistical Applications.* Academic Press, Inc., 1992.

[72] Boyd, S. Ghaoui, L.E. Eric Feron and Balakrishman, V. *Linear Matrix Inequalities in System and Control Theory.* Version 0.0, Draft, February, 1993.

[73] Balas, G. Doyle, J. Glover, K. Packard, A. and Smith, R. *The μ Analysis and Synthesis Toolbox.* Math Works and MUSYN, 1991.

[74] Hyland, D.C. and Bernstein, D.S. The optimal projection equations for fixed-order dynamic compensation. *IEEE Trans. Auto. Control, Vol. 29, No. 11, pp. 1034-1037*, 1984.

[75] Long, T.W. and Bliss, E.S. Application of decentralized control to a laser alignment system. *Proc. of 1992 ACC, pp. 1623-1627*, 1992.

[76] Lopez, J.E. and Athans, M. Connection between unimodular interaction constraint for decentralized controllers and small gain bounds. *Proc. of 1992 ACC, pp. 3289-3293*, 1992.

[77] Skogestad, S. and Morari, M. Robust performance of decentralized control systems by independent designs. *Automatica, Vol. 25, No. 1, pp. 119-125*, 1989.