FIGURE 5-7

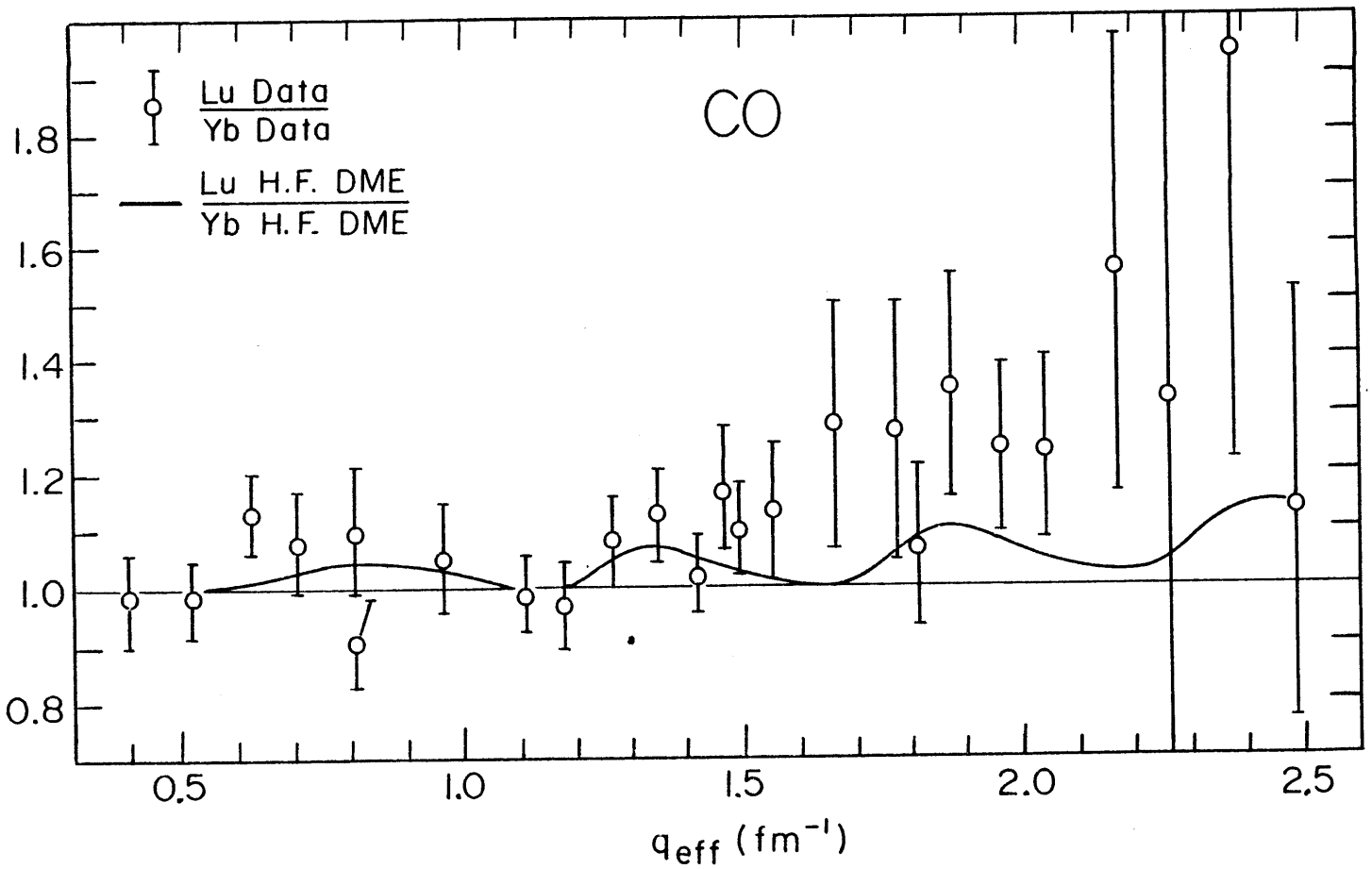

Lu Data / Yb Data
Lu H.F. DME / Yb H.F. DME
CO
1.8
1.6
1.4
1.2
1.0
0.8
0.5
1.0
1.5
2.0
2.5
q_{eff} (fm^{-1})

FIGURE 5-8

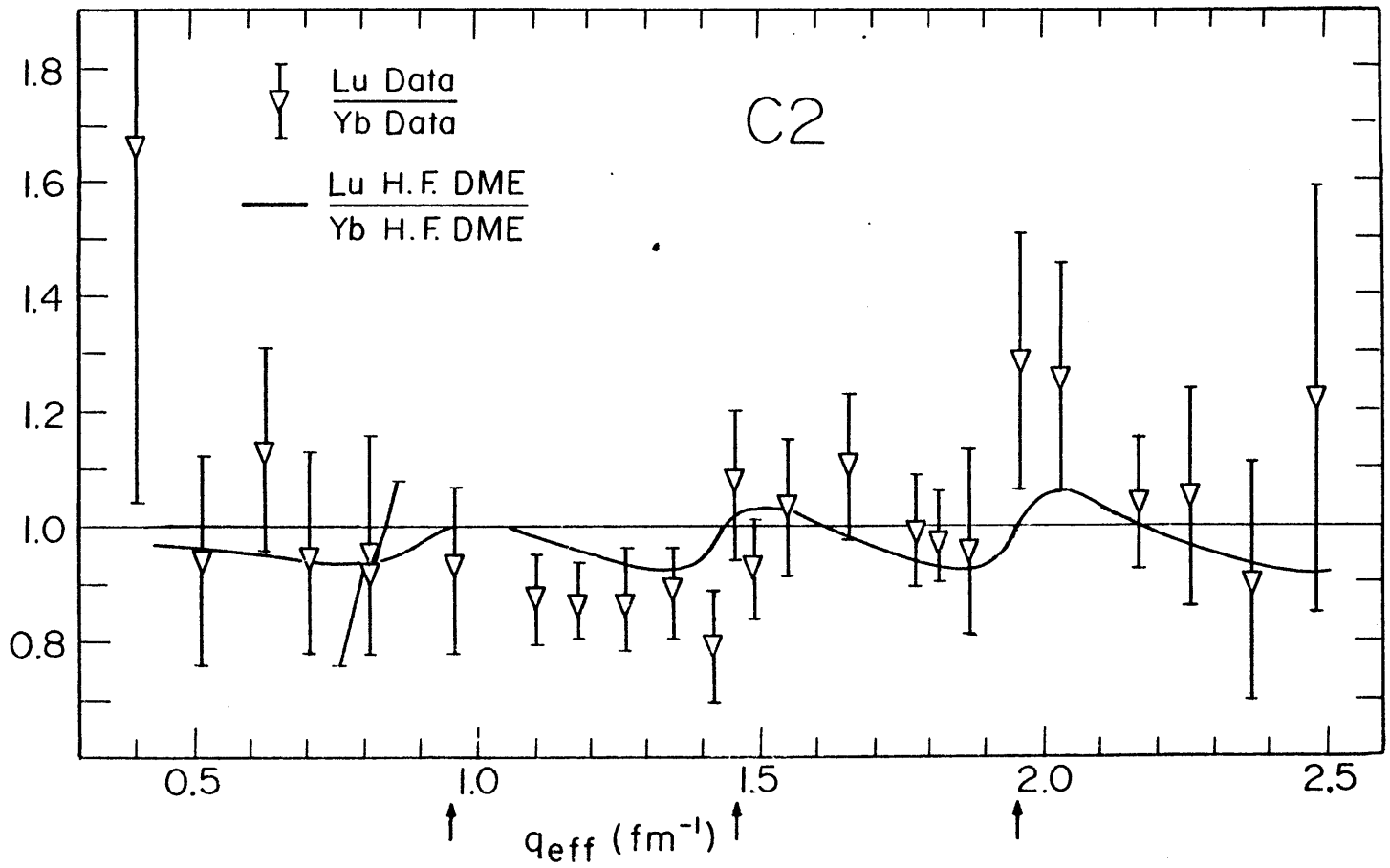

Lu Data
Yb Data
Lu H.F. DME
Yb H.F. DME
C2
1.8
1.6
1.4
1.2
1.0
0.8
0.5
1.0
1.5
2.0
2.5
q_{eff} (fm^{-1})

FIGURE 5-9

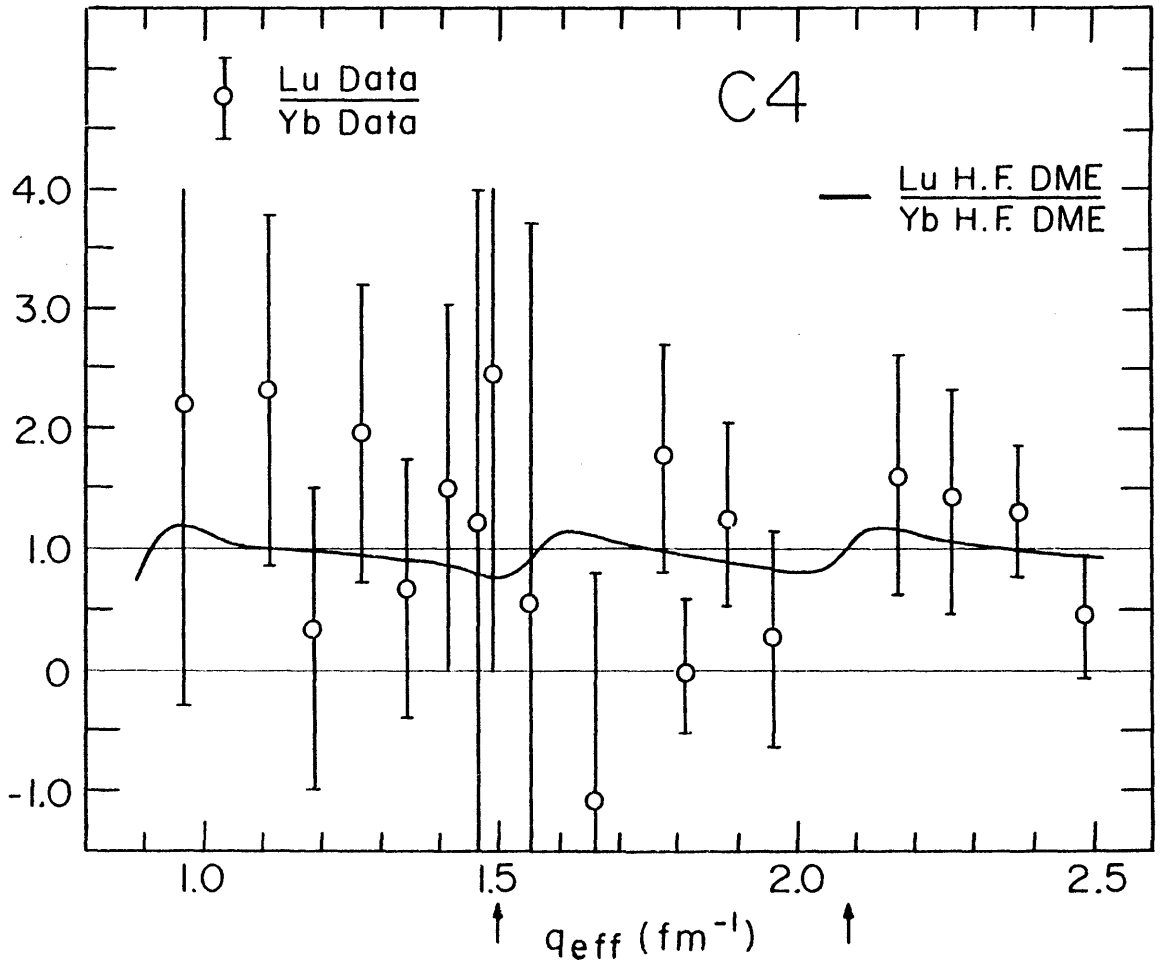
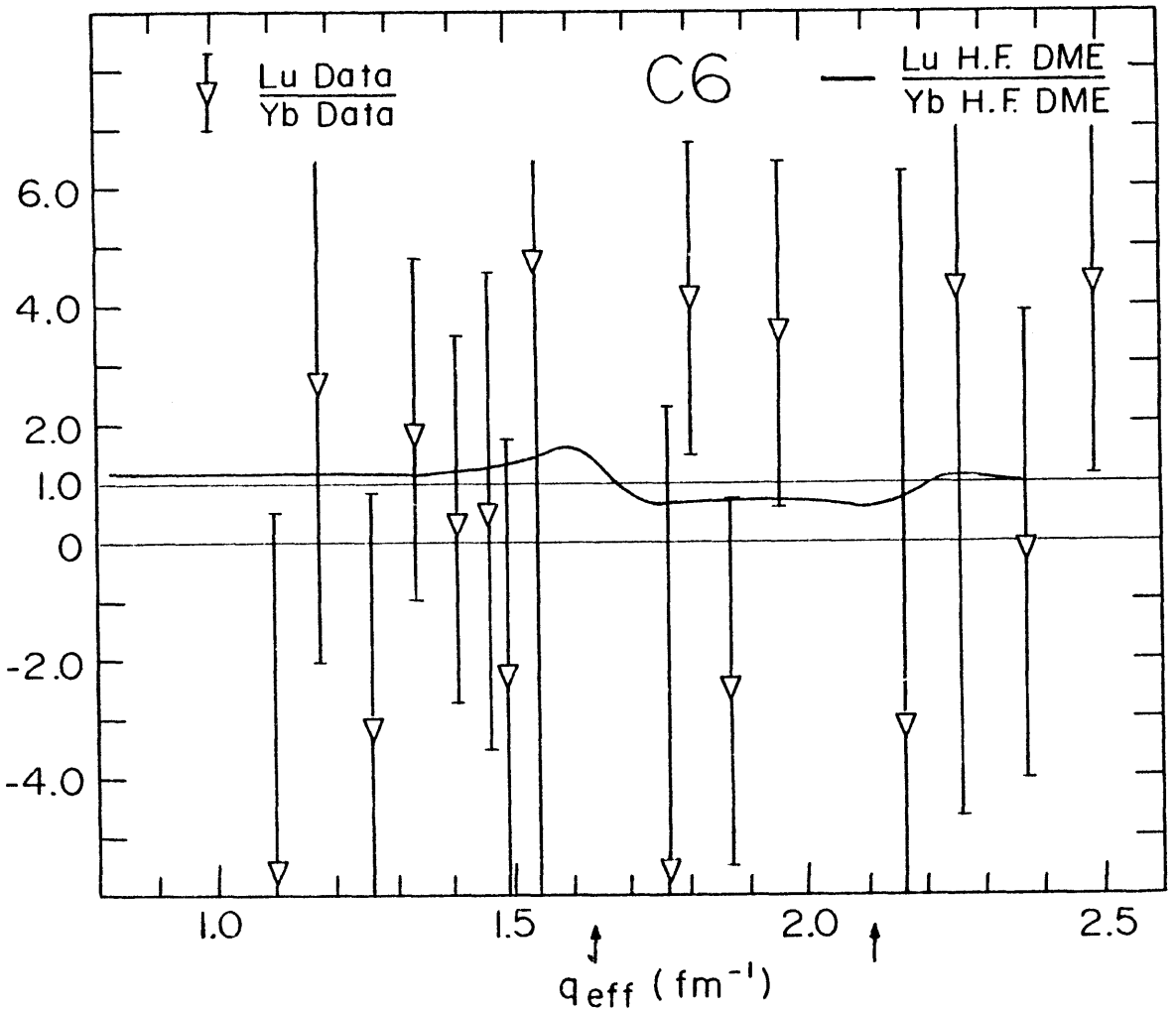

Lu Data
Yb Data
C4
Lu H.F. DME
Yb H.F. DME
4.0
3.0
2.0
1.0
0
-1.0
1.0
1.5
2.0
2.5
q_{eff} (fm^{-1})

FIGURE 5-10


Lu Data / Yb Data
C6
Lu H.F. DME / Yb H.F. DME
6.0
4.0
2.0
1.0
0
-2.0
-4.0
1.0
1.5
2.0
2.5
q eff (fm⁻¹)

over 1 and also larger than expected on the basis of the theory. The general features for F_2 are similar. The deviations of the ratios from 1 in the region of 1.0 $fm^{-1} < q_{eff} < 1.5$ fm^{-1} might be due to the polarization of the core. Extraction of F_4^2 and F_6^2 are more difficult and we can only conclude that there are no major discrepancies between F_4^2's and F_6^2's of the two nuclei.

3. MODEL INDEPENDENT ANALYSIS

Since the deformed Fermi function was found to be inadequate to extract nuclear charge densities from electron scattering data[5-10], a model independent analysis (MIA) on the data was performed for this thesis. In this analysis, as described in Section IV-9, the transition charge densities have the form of

$$\rho_0(r) = \sum_{\nu=1}^{N} A_\nu j_0\left(\frac{\nu\pi r}{R}\right)\theta(r-R) \qquad \text{for } L=0$$

$$\rho_L(r) = \sum_{\nu=1}^{N} A_{L\nu} q_{L-1\,\nu} j_L(q_{L-1\,\nu} r)\,\theta(r-R) \qquad \text{for } L\geq 1$$

In the equation, $(q_{L-1\nu}R)$ gives the νth zero of the L-1 th spherical Bessel function $j_{L-1}(q_{L-1}R) = 0$ and $A_{L\nu}$'s are adjusted to get the best fit for the data with the constraint of $\rho_L(r) = 0$ at r=R. The intrinsic form factors F_L^2 extracted from the data of ^{175}Lu using the method discussed in Section V-2 are used in order to perform this MIA on the ^{175}Lu data. Tables 5-6 and 5-7 contain the list of the coefficients of the Bessel function, B(EL) values, transition radii and χ^2 values for the best fit to ^{174}Yb and ^{175}Lu

TABLE 5-6

^{174}Yb Fourier-Bessel fits (R=11 fm)

Parameter	0^+	2^+	4^+	6^+
A1	0.684	-1.134	-1.508	-1.814
A2	0.503	-1.309	-3.790	-3.511
A3	-0.573	0.515	-3.426	-1.186
A4	-0.263	0.606	0.657	1.327
A5	0.257	-0.383	1.710	0.253
A6	0.034	-0.163	-0.645	-0.646
A7	-0.076	0.159	-0.909 ...a	0.090 ...a
A8	-0.002	0.041	0.497 ...a	0.232 ...a
A9	0.015	0.001 ...a	0.515 ...a	
A10		-0.003 ...a	-0.296 ...a	
R_{tr} (fm)	5.41±0.03	6.91±0.08	7.58±0.11	7.57±0.27
B(EL)		5.20×10^4	1.05×10^6	3.58×10^9
χ^2/ν	30/25	35/25	57/21	35/19

a -- Not varied.

TABLE 5-7

^{175}Lu Fourier-Bessel fits (R=11 fm)

Parameter	0	2
A1	0.699	-1.141
A2	0.531	-1.349
A3	-0.566	0.468
A4	-0.245	0.583
A5	0.283	-0.422
A6	0.049	-0.168
A7	-0.091	0.173
A8	0.004	0.033
A9	-0.003	0.001 ...a
A10		-0.003 ...a
R_{tr} (fm)	5.37±0.03	6.88±0.18
B(EL)		4.82×10^4
χ^2/ν	10/25	15/25

a -- Not varied.

respectively. This analysis could not be performed on F_4^2 and F_6^2 of ^{175}Lu because the errors associated with the extracted intrinsic form factors are too large (Figures 5-9 and 5-10). Figures 5-11 and 5-12 show the calculated form factors from charge densities using the results of this MIA along with the data of ^{174}Yb and ^{175}Lu. A good fit has been obtained for all states for each nucleus. For the 4^+ state of ^{174}Yb, large portion of χ^2 comes from the two data points in the low q region and data points in the region of the minima of the form factor. Similarly for the 6^+ state, the fit is not so good in the region of the minima. The coupled channel calculations by Cardman, et al.[5-8] indicates that the dispersive effect is expected to fill in the minima of the form factors for L=4 and 6, and that this inability to fit the data in the region of the minima of the form factors of 4^+ and 6^+ using a MIA is due to the neglect of the dispersive effect in this analysis.

Figures 5-13 and 5-14 show the extracted transition charge densities $\rho_L(r)$ from the data using the results of the MIA along with the charge densities generated by the Hartree-Fock calculations of ^{174}Yb and ^{175}Lu. As discussed in Section III-1, the intrinsic charge density $\rho(\vec{r})$ can be reconstructed in the following way.

$$\rho(\vec{r}) = \sum_L \rho_L(r) Y_{L0}(\Omega)$$

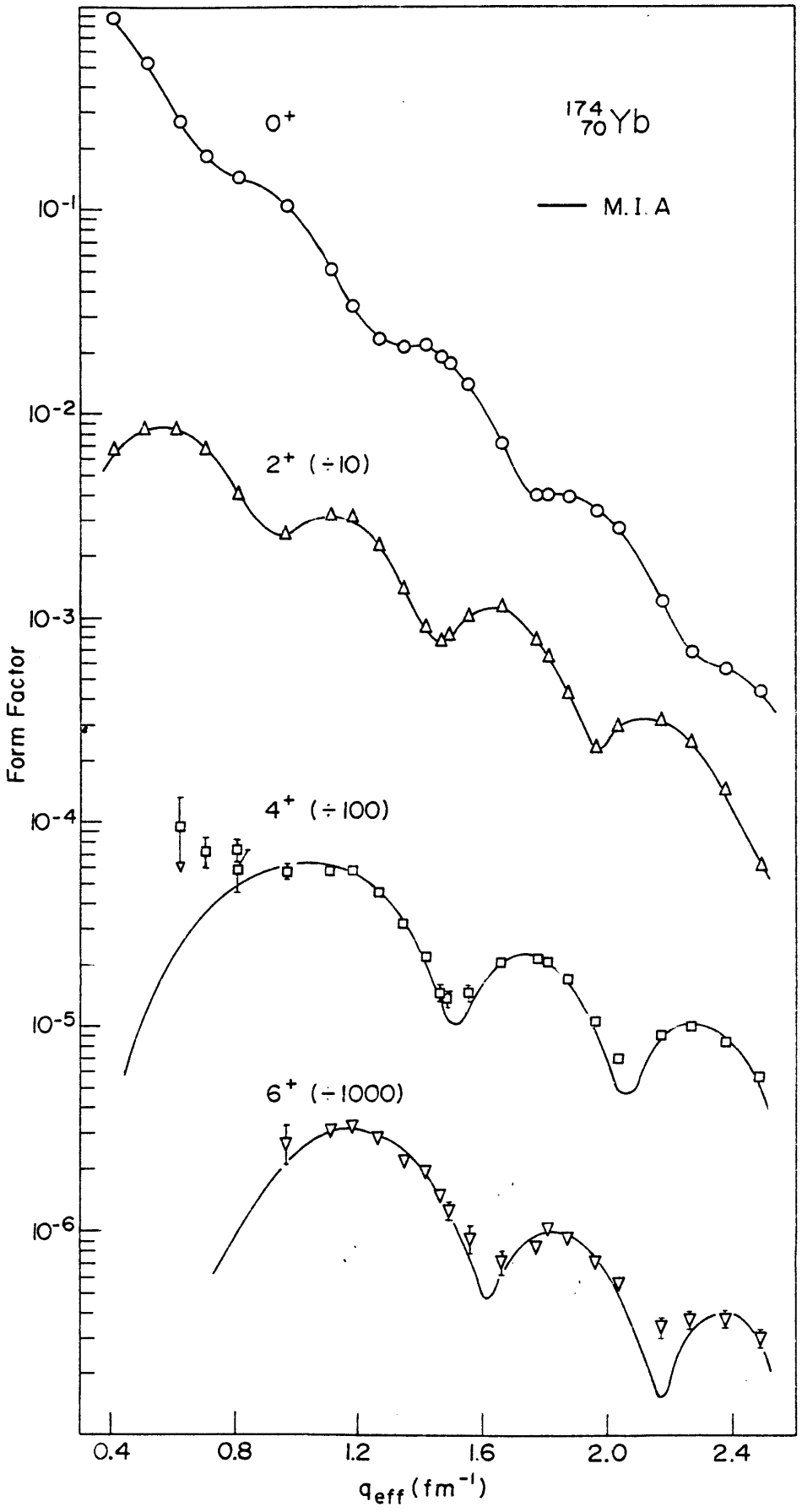

$^{174}_{70}$Yb
0+
M.I.A
2+ (÷10)
4+ (÷100)
6+ (÷1000)
Form Factor
10-1
10-2
10-3
10-4
10-5
10-6
0.4
0.8
1.2
1.6
2.0
2.4
q_{eff} (fm^{-1})

FIGURE 5-11

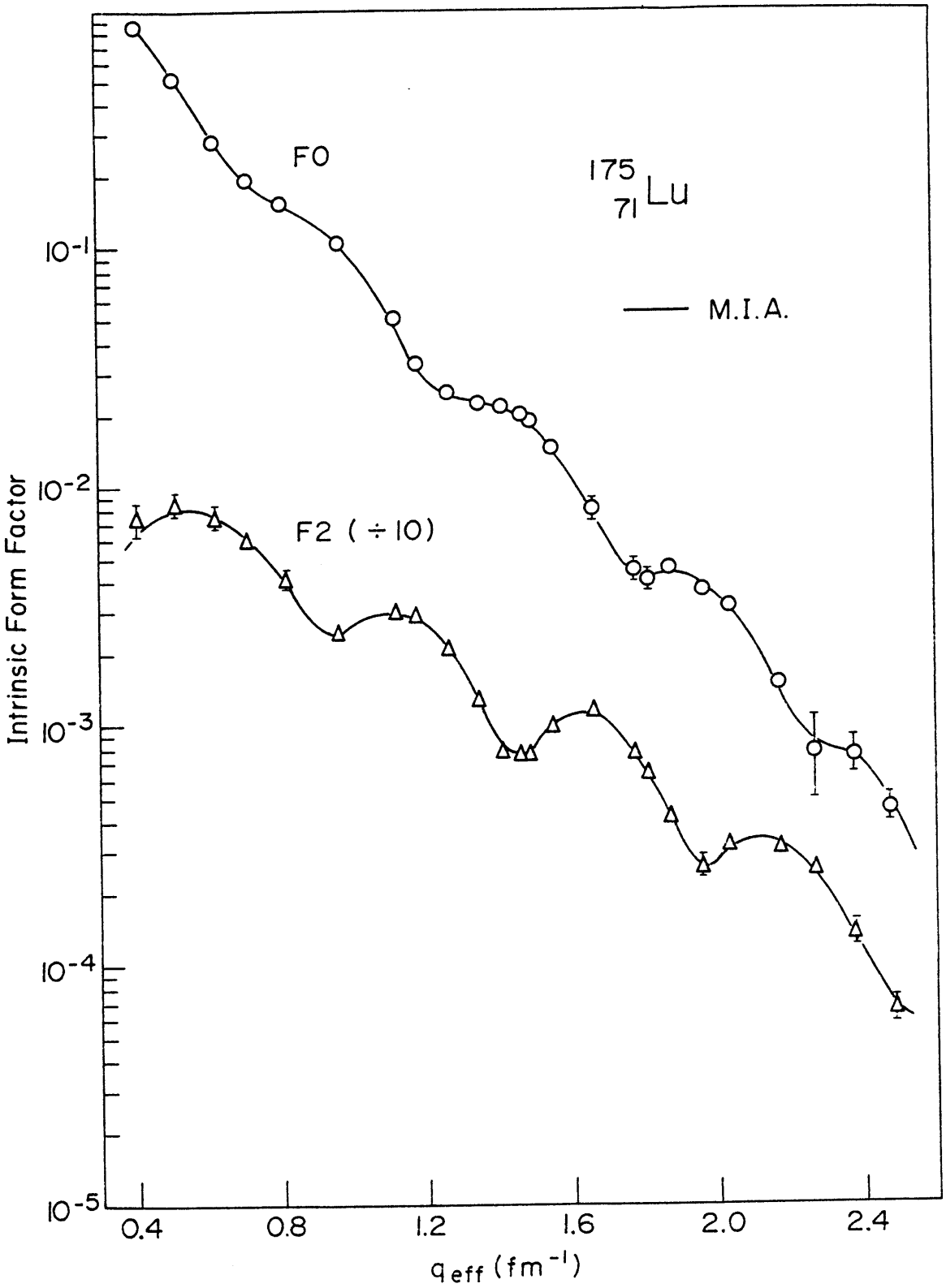

F0
175 71 Lu
M.I.A.
F2 (÷ 10)
Intrinsic Form Factor
10-1
10-2
10-3
10-4
10-5
0.4
0.8
1.2
1.6
2.0
2.4
q eff (fm-1)

FIGURE 5-12

FIGURE 5-13 a

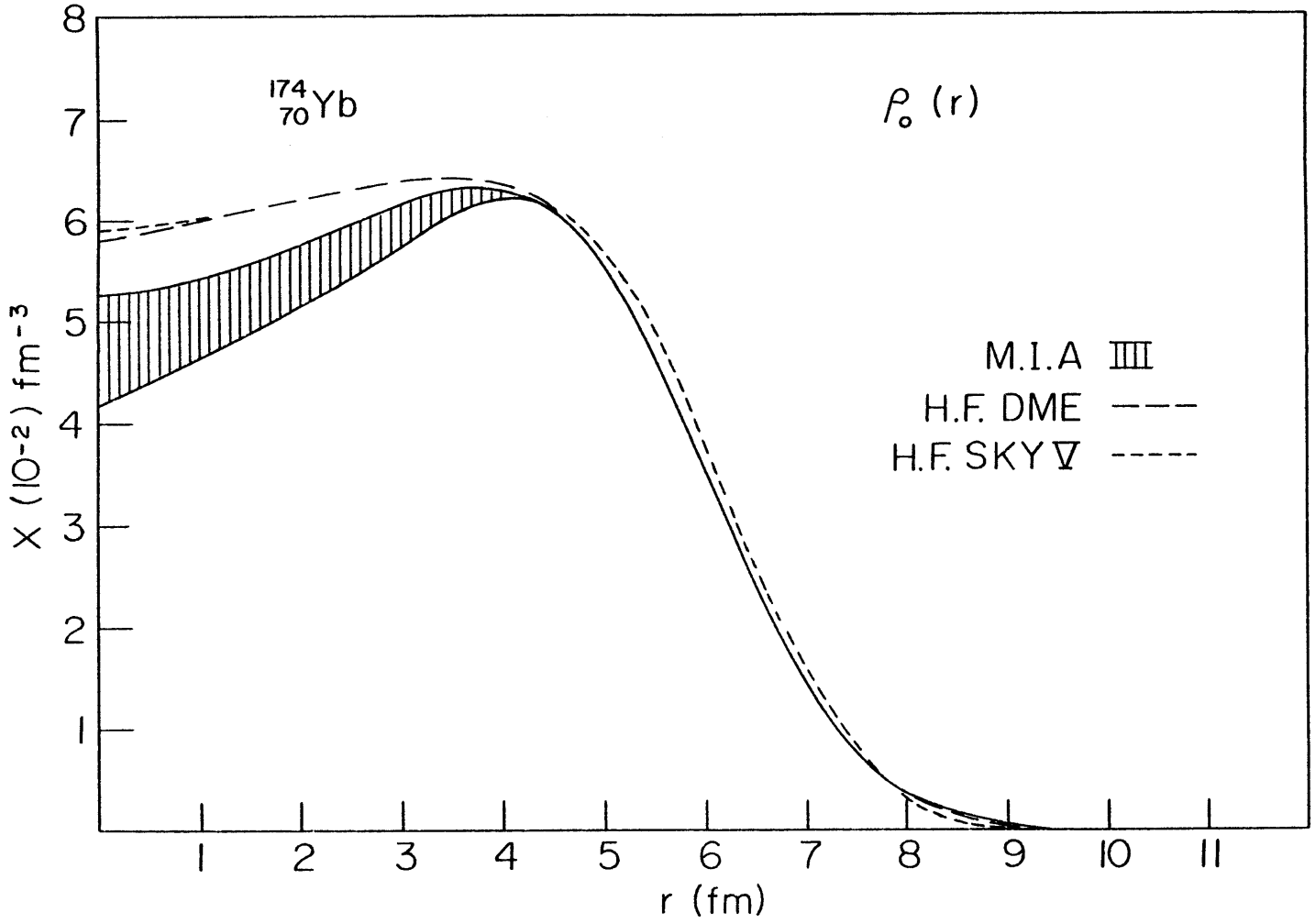

$^{174}_{70}$Yb
ρ_o (r)
M.I.A III
H.F. DME ---
H.F. SKY V -----
X (10-2) fm-3
r (fm)
1
2
3
4
5
6
7
8
9
10
11

FIGURE 5-13b

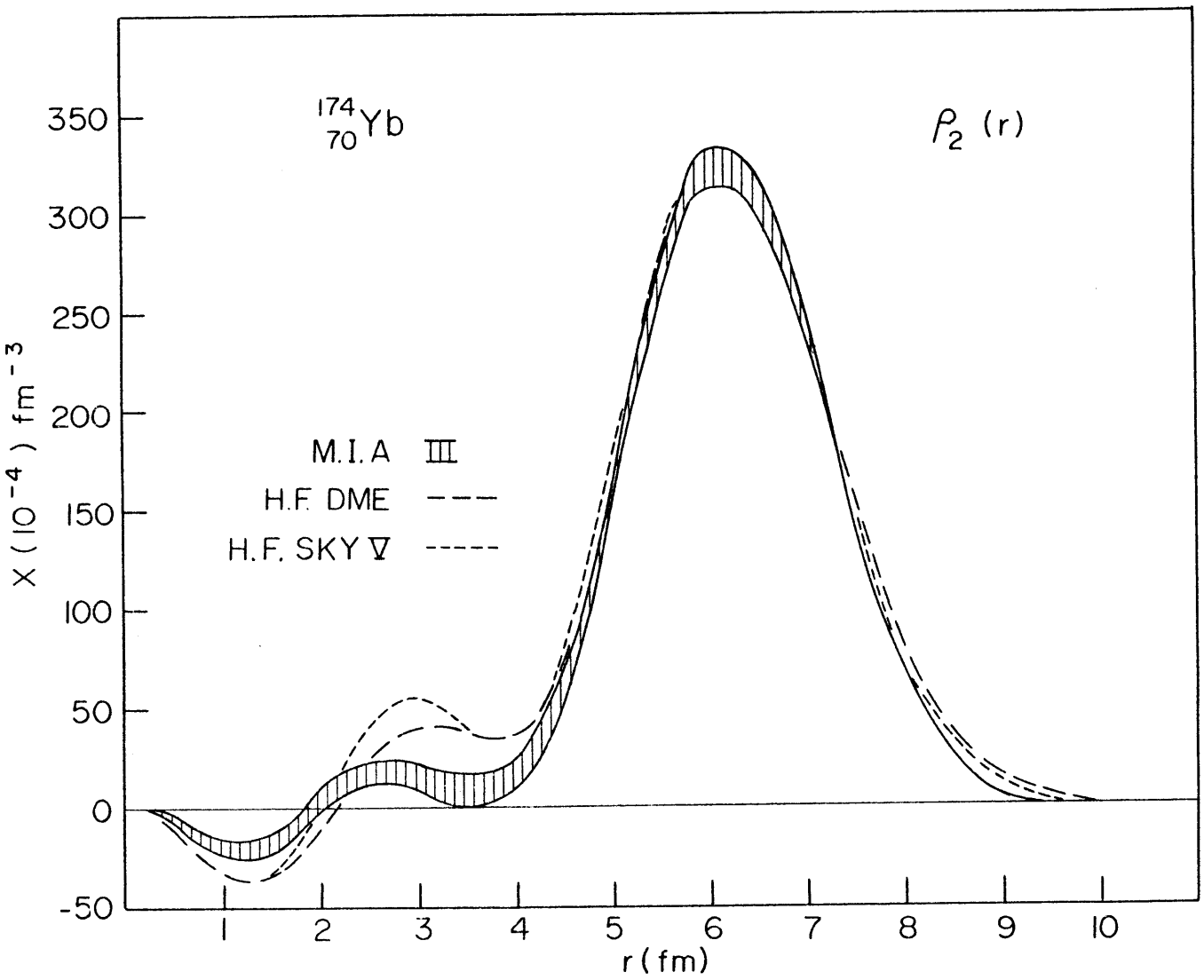

$^{174}_{70}Yb$
ρ_2 (r)
M.I.A III
H.F. DME
H.F. SKY V
X (10^{-4}) fm^{-3}
350
300
250
200
150
100
50
0
-50
1
2
3
4
5
6
7
8
9
10
r (fm)

FIGURE 5-13c

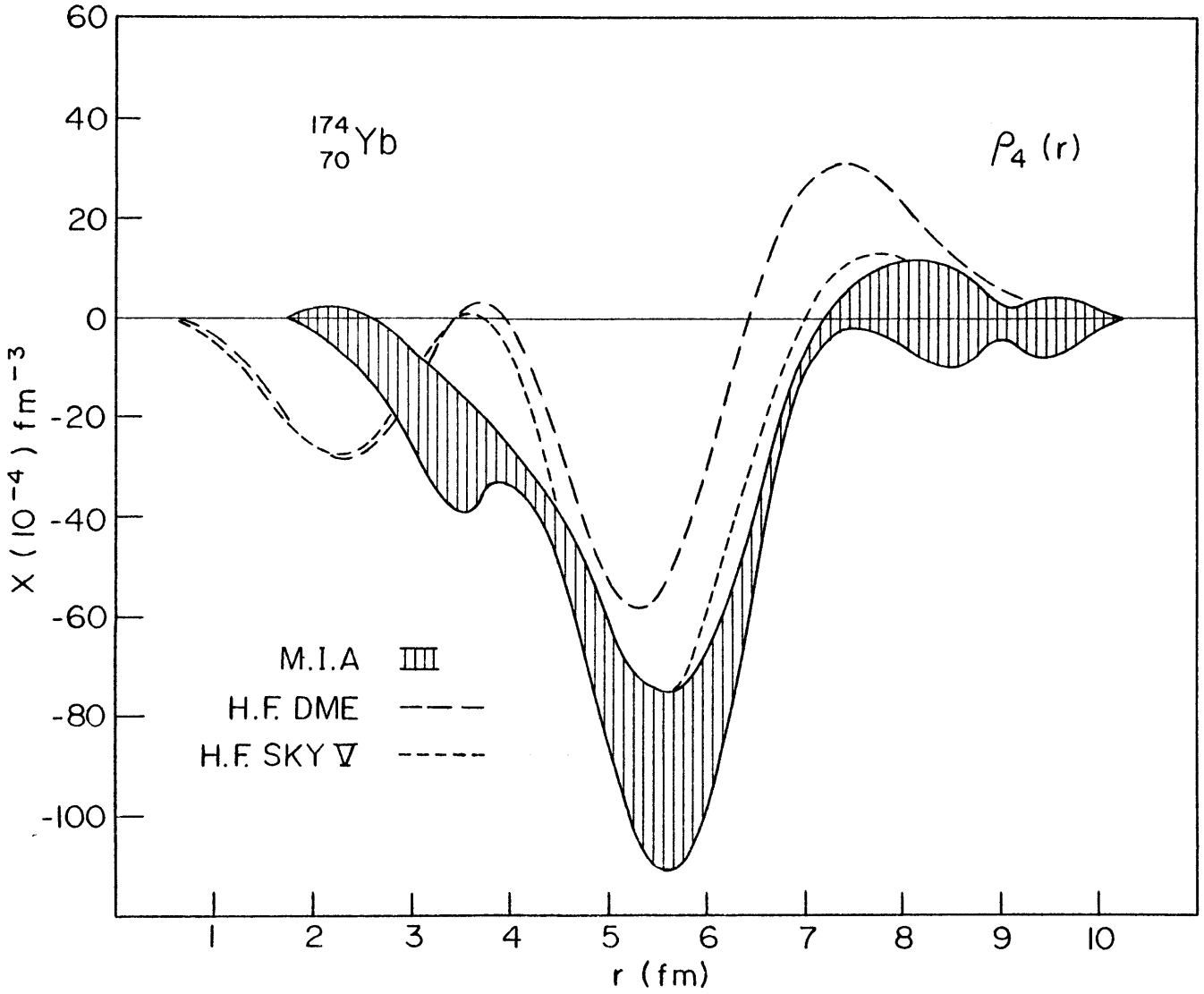

60
40
20
0
-20
-40
-60
-80
-100
$^{174}_{70}Yb$
ρ_4 (r)
M.I.A
H.F. DME
H.F. SKY V
X (10^{-4}) fm^{-3}
1
2
3
4
5
6
7
8
9
10
r (fm)

FIGURE 5-13d

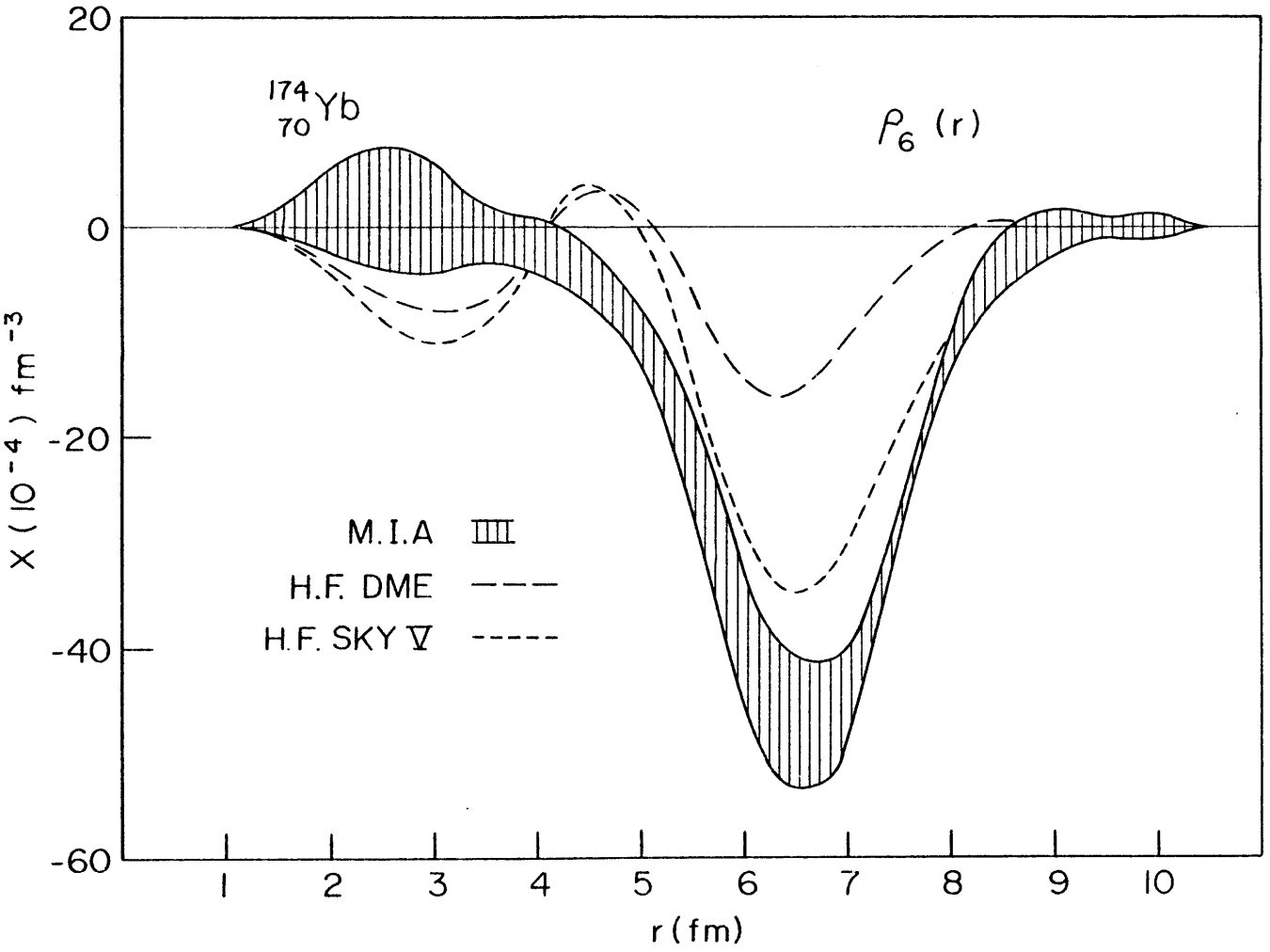

20
0
-20
-40
-60
$^{174}_{70}Yb$
ρ_6 (r)
M.I.A
H.F. DME
H.F. SKY V
X (10^{-4}) fm^{-3}
1
2
3
4
5
6
7
8
9
10
r (fm)

FIGURE 5-14a

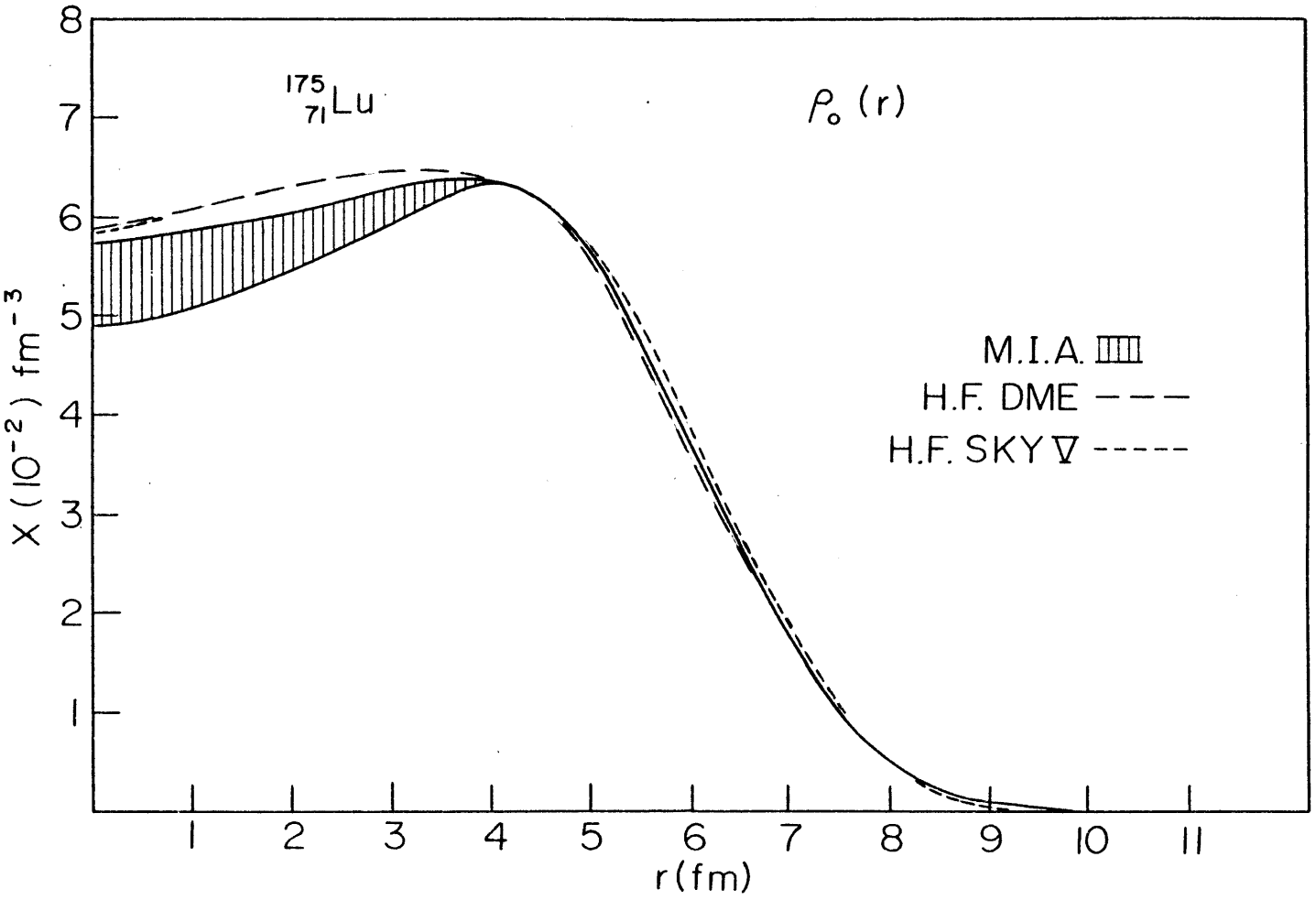
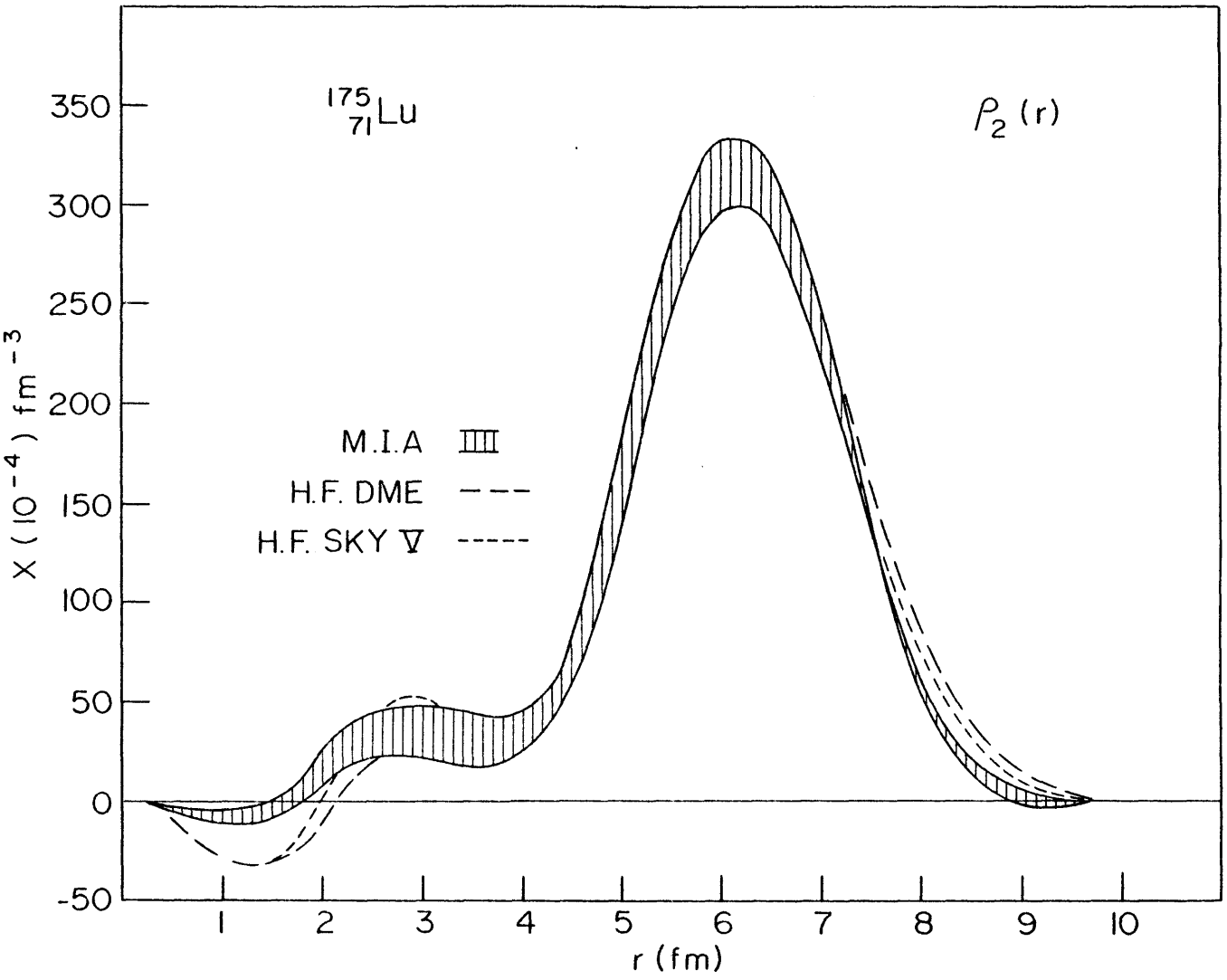


FIGURE 5-14b


$^{175}_{71}$Lu
ρ_2 (r)
M.I.A
H.F. DME
H.F. SKY
X (10^{-4}) fm^{-3}
350
300
250
200
150
100
50
0
-50
1
2
3
4
5
6
7
8
9
10
r (fm)

The error bands of charge densities by the MIA contain uncertainties only in the experimental data and do not include the completeness error. The error band, which is determined by the Fourier-Bessel analysis must not be interpreted such that any curve within this band is compatible with the electron scattering data. To the contrary, there exist strong correlations between the value of $\rho_L(r)$ for different radii.

In order to take account of the charge of the odd proton, $\rho_0(r)$ for each nucleus is normalized in the following way.

$$\int \rho_0(r) r^2 dr = 1$$

The charge density $\rho_0(r)$ generated by the Hartree-Fock calculation using the DME approximation agrees well with the charge density extracted from the data between 4 fm<r<8 fm. Though the error band is wide, it seems to be a depression of charge density $\rho_0(r)$ in the center of ^{174}Yb and ^{175}Lu nuclei. Data of higher q region are needed to find the behavior of the interior of the nuclei more accurately. The agreement between the charge densities of $\rho_2(r)$ generated by the Hartree-Fock calculations and extracted from the data is very good. For the $\rho_4(r)$ and $\rho_6(r)$, the Hartree-Fock calculation predicts too many wiggles in the interior of the nucleus, though the theory does a better job of predicting the position of the dominant peak in $\rho_L(r)$.

Figure 5-15 shows the difference of the transition charge densities of $\rho_0(r)$ and $\rho_2(r)$ between ^{175}Lu and ^{174}Yb. As expected from

FIGURE 5-15a

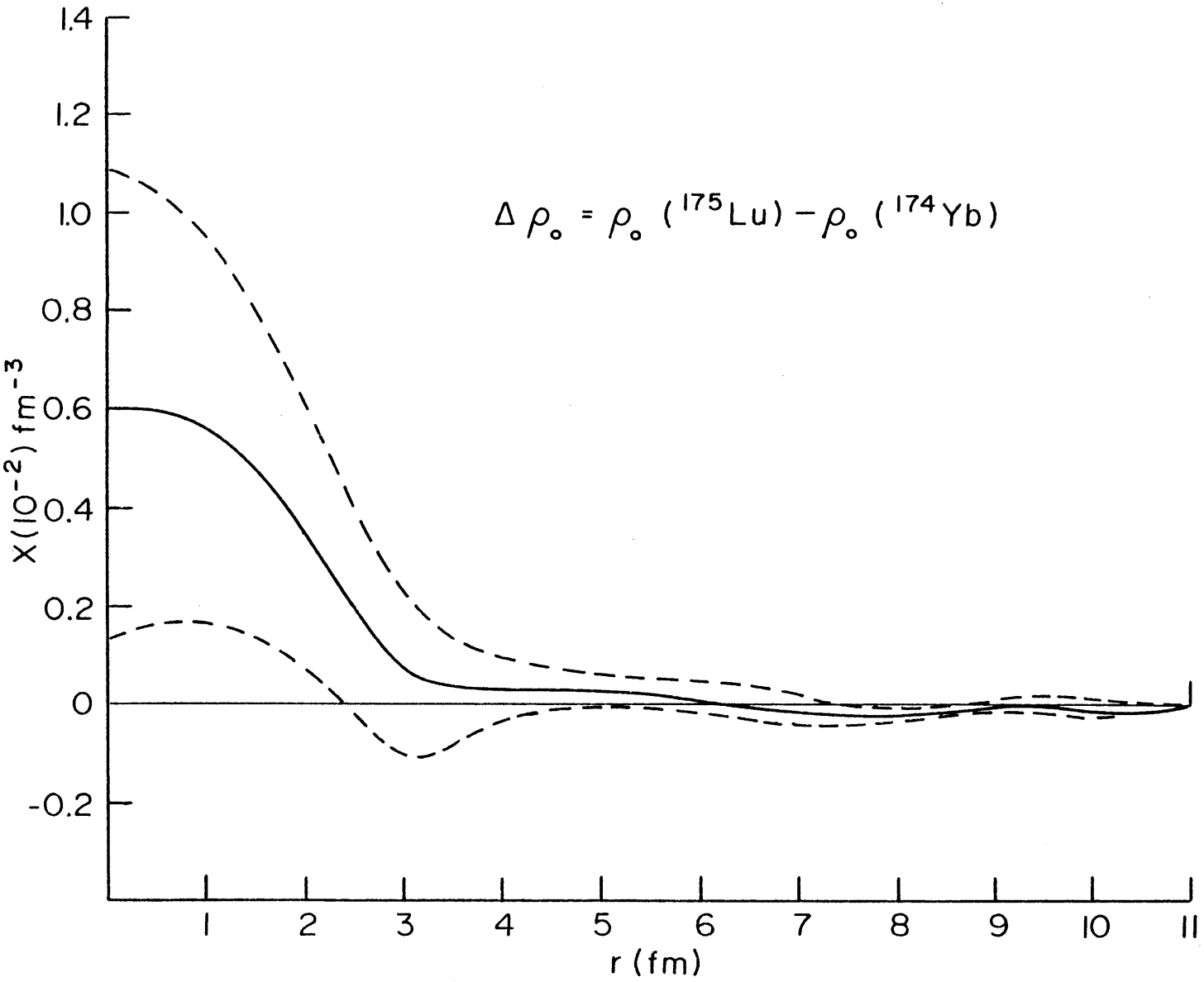
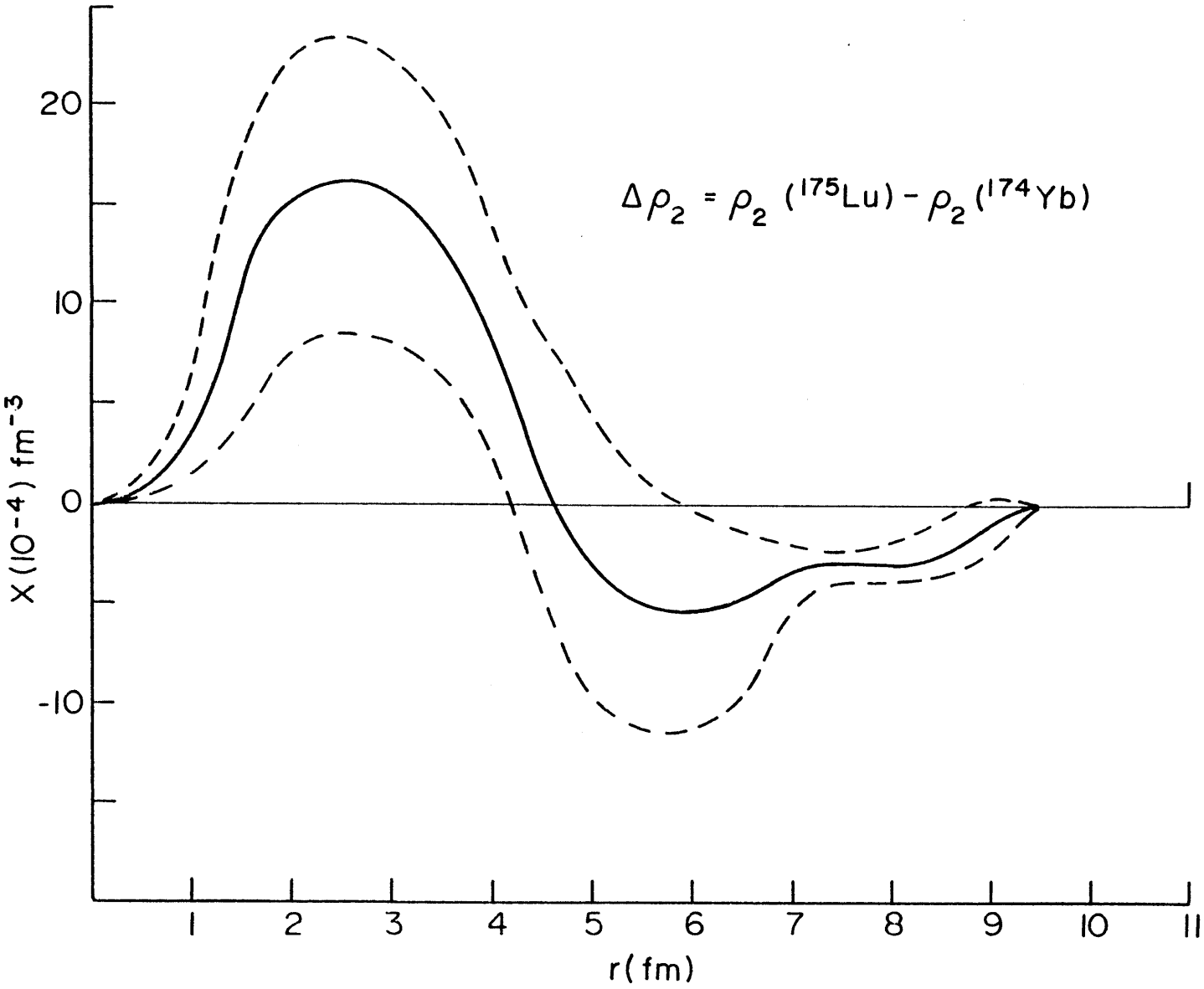

1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
-0.2
X(10^-2) fm^-3
Δρ₀ = ρ₀ (175Lu) − ρ₀ (174Yb)
1
2
3
4
5
6
7
8
9
10
11
r (fm)

FIGURE 5-15b


$\Delta\rho_2 = \rho_2(^{175}Lu) - \rho_2(^{174}Yb)$
20
10
0
-10
X (10-4) fm-3
1
2
3
4
5
6
7
8
9
10
11
r(fm)

the comparison between the intrinsic form factors of ^{175}Lu and ^{174}Yb in Section V-2 (Figures 5-7 and 5-8), there is almost no difference between the two nuclei for $\rho_0(r)$ except the interior where very small change is seen, even though the error is large. There is also only very small difference for $\rho_2(r)$ except in the region near r=3 fm.

4. SUMMARY AND CONCLUSION

Electron scattering from the ground state rotational bands of ^{174}Yb and ^{175}Lu has been performed at M.I.T.-Bates Linear Accelerator. Transition charge densities were extracted from the experimental data using a model independent analysis. The values of chi-square obtained in our fits indicate that this is an adequate description of our data, however the fits in the region of the minima of form factors for L=4 and 6 are poor. As indicated by the coupled channel calculations[5-8], the inability to fit the data in the region of the minima of the form factors is probably due to the neglect of the dispersive effects in the analysis.

The Hartree-Fock calculations using both realistic (DME) and phenomenological (SKYRME) nuclear interactions were performed. The cross sections obtained by the Hartree-Fock calculations within the zeroth order of the rotational model: the total wave function as a product of a wave function describing the intrinsic and static core and a wave function describing the rotation, were compared with the electron scattering data. The agreement between the theory

and the experiment for the Coulomb multipoles C0 and C2 is good. In the case of the Hartree-Fock calculation using the DME approximation, this agreement is impressive considering the properties of the force are derived directly from realistic two body interaction. The agreement worsens as one proceeds to higher multipolarity in the ground state rotational band.

By using the data of ^{175}Lu, the rotational model without any assumption about the intrinsic charge density was tested. The results show the data are consistent with the fundamental rotational assumption for all range of q even around the minima of form factors where the dispersive effects are supposed to be significant. This independent check is not sensitive enough to see the dispersive effects. We can only conclude the discrepancies observed between the electron scattering data and predictions of the best available microscopic theories (mainly for L=4 and 6) are probably due to a poor representation of the intrinsic charge density predicted by the theory in the region of the maxima of form factors and the combination of that and the dispersive effects in the region of minima.

The comparison between the intrinsic form factors derived from the data of ^{175}Lu and ^{174}Yb shows the polarization of the ^{174}Yb core due to the odd proton is very small for $\rho_0(r)$ and $\rho_2(r)$ except the interior of the nuclei. The comparison between the two nuclei for $\rho_4(r)$ and $\rho_6(r)$ cannot be performed because the errors associated with the extracted intrinscic form factors of ^{175}Lu are too large.

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Last, but definitely not lease, special thanks go to my parents who encouraged me in everything I undertook.

BIOGRAPHICAL NOTE

The author was born in Saitama, Japan in 1949 and attended Azabu High School from 1964 to 1967. He came to the United States in 1969 and attended Whittier College until 1970. He attended the University of California at Berkeley from 1970 to 1973 when he received the B.A. degree with Distinction in General Scholarship in physics. He has been a graduate student in the Department of Physics at the Massachusetts Institute of Technology since September, 1973.

The author is a member of the honorary society Sigma-Xi.