A Cost Allocation Approach to Defining and Specifying Tolerances

by

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Submitted to the Department of Mechanical Engineering
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ABSTRACT

This thesis describes a methodology for specifying tolerances. It provides a way of concurrently evaluating product performance and production process options for determining and representing economical tolerances. The approach proposes a model for estimating the costs associated with performance loss, manufacturing process control, and quality control through inspection. These three cost elements are combined to form an objective function which can be optimized to yield the best combination of design nominals, statistical variation control specifications, and inspection control limits. These specifications fully prescribe expected product quality as well as process control requirements and product yield rates for evaluation during design. This approach differs from previous tolerancing methods in three ways:

1. Performance is used as a *criterion* for tolerancing as opposed to performance being predefined.
2. Tolerance is represented as a combination of statistical variation and inspection control limits.
3. Tolerances are determined by trading off product performance against the costs of controlling product variability.

The methodology has been implemented in two fastener joint examples at the Boeing Company, and one example derived from literature. The effectiveness of this methodology is presented in economic terms by comparing the costs of current tolerance specifications against the costs resulting from applying this methodology. New tolerances specified by the proposed methodology result in substantial cost savings.

Thesis Advisor: Karl T. Ulrich
Associate Professor, Sloan School of Management

DEDICATION

I dedicate this thesis to my parents. Without their encouragement and moral support, my endeavor would not have reached this end. The magnitude of their devotion to my quest would be incomprehensible to others. They have devoted the past 27 years of their lives for this moment. "Mom, Dad, I've done it!"

I dedicate this thesis also to my brother and sisters who have, near or far, provided continual moral support.

I dedicate this thesis also to the person who has always stood by me, in happiness and in sadness, in hope and in despair. Her love has allowed me to overcome many obstacles with ease. Thank you, Woo Sun. I also thank our newborn, Jun, for coming into my life. My love for my family is immeasurable.

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I thank my colleagues Bala Subramaniam, Rajan Ramaswamy, Brian Eberman, and numerous others at the MIT Artificial Laboratory for constantly feeding me new research ideas and keeping old ones in check.

I thank Dave Blundin for helping me put this thesis together. His friendship and humor has allowed me to keep my perspective. I also thank Jim Bevilacqua, Van Pham, Brian Mox, Johnny Byun, John Lunny, Roland Pan, and the rest of the brothers at ΘΔX for helping me keep my priorities straight. Their contribution to my personal growth has been immense. "A toast to wine. Cheers."

I thank the Leaders for Manufacturing program for providing me financial support and access to industrial corporations. I also thank people at Boeing, especially Mark Boberg, Stan Andrews, Ken Christie, Brandt Willson, and Harry Townsend for their generous time and technical support. This thesis would not have been possible without them.

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Contents

List of Figures

Chapter 1

Introduction

1.1 This thesis is about representing and determining tolerances

The ASME sponsored workshop on *Research Needs and Technological Opportunities in Mechanical Tolerancing* [Tip88] cites, "There is now little rational basis for deciding design tolerances from the functional specifications." Tolerances have direct and significant effect on the life cycle of a product, but their specification process is often inconsistent and ambiguous.

This thesis describes a methodology for specifying tolerances rationally. It provides a way of concurrently evaluating product performance and manufacturing costs to determine and represent economical tolerances.

1.1.1 Background

In product design, engineers constantly encounter the dilemma of whether to design products for better performance or lower cost. Better performing products are usually more difficult and more expensive to manufacture. Design is the process of constantly evaluating these conflicting goals and making tradeoffs between the different desirable qualities of a product.

In the era before the industrial revolution, the design and manufacture of products existed as a closely linked process. Often, this process consisted of a craftsman interacting closely with the commissioner of the product to define its desirable attributes. Because he knew how his customer valued each performance attribute, and because he knew the capacity of his crafting abilities, he could design and make the product so that his efforts would be efficiently spent. This tradeoff process often entailed weighing the *worth* of different designs against the effort and cost of conceiving them. This design process was inherently *concurrent* in the modern engineering sense.

In the late 19th century, design and manufacturing became specialized processes. This specialization evolved for three main reasons: 1) people could be trained to become experts in their respective fields, 2) it allowed people to concentrate on repetitive tasks, making job functions more efficient, and 3) design and manufacturing tasks became too large and complex for one person or organization to tackle. This specialization, however, is also accompanied by the inefficiencies of communicating design and manufacturing information. The absence of formal, quantitative methods to trade off one design attribute against another often results in designs that are either poor in quality or too expensive to produce.

1.1.2 Tolerancing as Allocation of Resources

One element of design where this problem is evident is in the specification of tolerances. Products cannot be made all alike. Variability inherent in manufacturing processes results in product properties that vary, which in turn result in variable product performance. Deviations from targeted performance levels are usually associated with high costs of repair, dissatisfaction, or other manifestations. Tighter tolerances improve performance, but they are usually associated with high manufacturing costs. Despite these consequential implications, we often witness less than ideal tolerance specification methods. The ASME sponsored workshop on *Research*

	Nominal [in]	± Tolerance [in]
Flushness F	0.006	0.003
Interference I	0.0030	0.0015

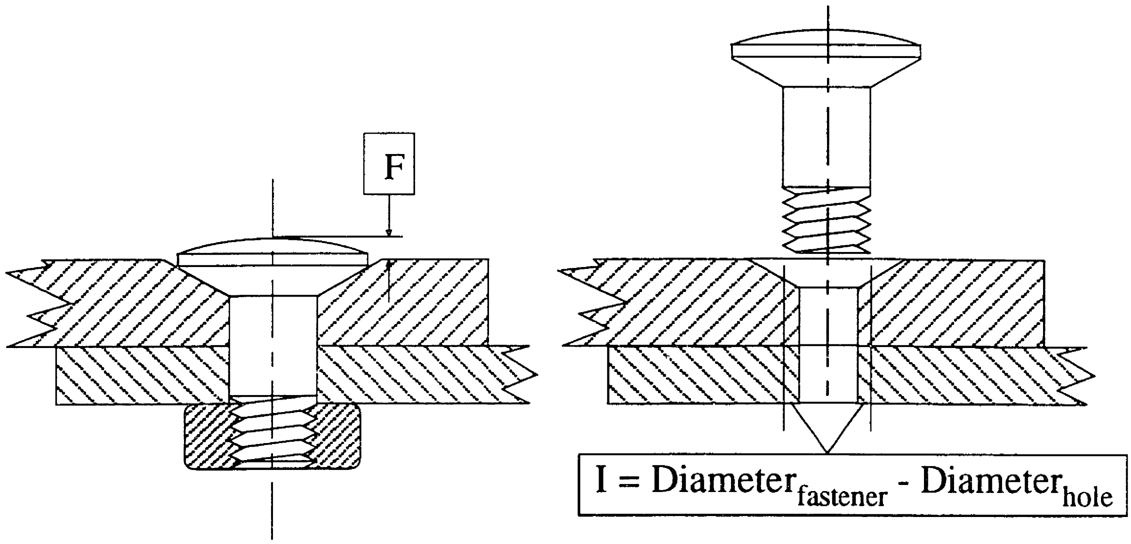


Figure 1.1: Tolerance Specifications for 747-400 Wing Skin Panel Splice Joint

Needs and Technological Opportunities in Mechanical Tolerancing [Tip88] cites, "It is also not uncommon for designers to specify tighter than necessary tolerances with the anticipation that manufacturing of parts and assembly would relax these tolerances anyhow."

This thesis is about specifying economical tolerances by strategically allocating design and manufacturing resources. It provides a methodology for modeling, evaluating, and making decisions about product performance and cost.

1.1.3 Examples

Consider the tolerance specifications for a high performance fastener joint at the Boeing Commercial Aircraft Group (BCAG)[1] (Figure 1.1).

[1]This research was conducted in close contact with BCAG. Examples used throughout this thesis are genuine, but numbers have been altered to protect proprietary information.

These fastener joints are used to splice aircraft skin panels. There are approximately 40,000 such fasteners on a Boeing 747-400 aircraft. Fastener tolerance specifications directly affect two key performance characteristics. First, because the upper surface shown in Figure 1.1 is exposed to high velocity airflow, controlling fastener head protrusion F is critical in limiting excrescence drag. If F is too high or too low, airflow is diverted, resulting in high drag and lift penalties. Second, because these joints are subjected to stress cycles during aircraft operation, fatigue resistance is a critical performance characteristic. Fastener interference I, the degree of pre-stress and material strengthening introduced on the joint through plastic deformation, is a primary design factor.

Each additional pound of required lift adds an estimated $600 in direct operating costs over the service life of an aircraft. A 747-400 depreciates as much as $60,000 on each flight cycle because of fatigue damage. The control of parameters F and I during design and manufacturing directly affects aircraft performance. The control of F and I also has direct implications on manufacturing cost. Therefore specifying tolerances requires balancing performance loss against manufacturing costs.

1.1.4 Tolerance Problems I Address

Current tolerance specification methods lack the ability to quantitatively trade off performance and manufacturability

Most literature devoted to tolerancing addresses the problem of allocating geometrical tolerances, the process of economically distributing geometric part tolerances subject to specified assembly tolerances. A precondition to this tolerancing method is the existence of a known, fixed assembly tolerance.[2]

But how do we determine these assembly tolerances in the first place? Sometimes, they are customer specifications. In many instances, however, they are decisions left to

[2] A review of existing tolerancing methods is presented in Chapter 2.

the designer and are flexible. Having pre-determined assembly tolerances implies that performance levels are fixed before evaluating the costs associated with attaining those performances. In effect, you buy something without asking its price. An informed shopper weighs the benefit of a product before paying its price. So should an engineer design tolerances.

Performance is often not properly evaluated in the presence of variability

We create models to represent reality. And because we have limited resources in reducing modeling uncertainties, we embed safety factors when designing. Sometimes we end up with a design whose specified tolerances demand very expensive procedures but whose performance expectations nevertheless are not well known.

In addition, models are often assumed to be deterministic representations of reality. Deterministic models are simpler to understand and analyze. However, there is a price for deterministic assumptions. Deterministic models are often derived from a body of imprecise, nondeterministic data. When this data is distilled down to deterministic models, probabilistic information that may be required in downstream analysis may be lost.

Current tolerance specification schemes do not adequately represent manufacturing variability control options for evaluation during design

In most industrial applications, allowable manufacturing variations are specified as tolerances. In the examples above, flushness F and interference I each are specified with nominals and tolerance limits. What are the intentions of these specifications and how may they be interpreted?

Tolerances generally represent one of two meanings. 1) Tolerances may represent design parameter ranges over which the product is considered acceptable. Often, these tolerances are called "goalpost" or binary tolerances. Nonconforming products are identified through inspection and corrected. 2) Tolerances otherwise represent statistical variation. A popular yet perhaps arbitrary convention used in industry is the association of tolerances with 6σ statistical variation where σ is the standard deviation of manufacturing processes. I will call this interpretation statistical variation.

When tolerances are specified with the current representation scheme, the meaning is often unclear. In the flushness example above, it is not apparent whether 0.006 ± 0.003 means goalpost tolerances or statistical variation. If design intent is not effectively communicated, the discrepancy between what is expected and what is delivered can have significant effects on product performance. Tolerance representation schemes not sufficiently specific in describing design and manufacturing constraints can result in design miscommunication and eventually in poor products.

We lack formal methods for determining cost effective quality assurance strategies

Product quality and variation may be controlled in one of two fundamental ways. The first method is to control the variability of the underlying distribution and is therefore called *variation* control. The second method is to inspect all products post-process for compliance to specified limits and implement measures to correct the defect. This is called *inspection* control.

Manufacturers generally use a combination of these control strategies to assure product quality. Variation control and inspection control both have direct effects on performance, manufacturing cost, and yield. For example, a fixed expected[3] quality level Q^* might be obtained by 1) implementing a tightly controlled manufacturing process followed by no inspection control or by 2) implementing a looser manufacturing process followed by tight inspection control. Although the same level of expected quality may be attained, the two strategies have significantly different implications on variability control costs. The first approach has no post-process inspection, rework, or scrap, but might have high process variation control costs. The second approach,

[3] *Expected* refers to the statistically expected quality level of a population of products.

by contrast, has lower process variation control costs, but has higher costs associated with inspection and correction. It is difficult to determine which strategy is economically preferable without actually investigating their cost implications. Such decisions have significant impact on quality assurance measures, but we lack formal methods for evaluating those strategies.

1.1.5 Highlight of the approach

To address the problems described above, I apply the following approaches to represent and determine tolerances.

1. Make economic tradeoffs between performance and cost.
2. Represent tolerances as a combination of statistical variation and inspection limit representations.
3. Evaluate statistical variation control costs and inspection control costs to determine the most economical combination of tolerances.
4. Apply nondeterministic modeling methods to estimate performance characteristics.
5. Design tolerances and nominals simultaneously when possible.

1.2 The Industrial Background of this Research

1.2.1 Identifying Key Tolerancing Issues at Boeing

The contents of this thesis are results of interaction with the Boeing Commercial Aircraft Group. I spent 5 months at Boeing working closely with their management, engineering, and manufacturing organizations to identify and address key issues in tolerancing.

Selecting Examples

I have selected the two fastener examples discussed above for several reasons. First, with approximately 4 million fasteners on a signle 747-400, fastening parts together is a principal concern at Boeing. One of Boeing's primary technologies is in fastening aircraft parts together. With a 747-400 being sold and delivered at the rate of one aircraft every four days, a penny saved on each fastener can result in over $2.6 million in annual savings.

Second, the chosen examples are representative of design attributes Boeing considers most important: performance, reliability, and cost.[4] Fastener joint tolerance specification may be looked upon as the process of allocating resources to control each one of these attributes. How efficiently resources are allocated determines how well the aircraft is designed. I have chosen these examples in particular because I believe resource allocation for these fastener attributes is currently unbalanced. Finding no formal methodology at Boeing dedicated to this allocation task, I took on the challenge of making one.

Potential Benefits

Case studies in Chapters 6, 7, and 8 show the potential benefits.

Fastener Flushness Example: Cost analysis indicates that specifying tolerances with my proposed methodology results in visible savings of as much as $17,000 per aircraft for the 40,000 fasteners considered. Considering there are a total of about 120,000 wing skin fasteners on a 747-400 and considerably more on the body, the potential benefits of applying the same methodology to all the fasteners are conservatively estimated at $100,000 per aircraft. Expecting more than one thousand 747 aircraft to be sold, this results in estimated savings of about $100M for the 747 fleet

[4]Safety is considered to be *the* most important design attribute. But because safety is an attribute that cannot be compromised, it is less subject to tolerancing and will not be discussed.

disregarding discount rates.

Fastener Interference Example: The interference specifications from previous 747 models have been tightened for the 747-400 and 777 models because engineering models predicted high performance gains from tighted specifications. Yet, my analysis indicates that the newer specifications produce only a small improvement in fatigue performance. Service records obtained from previous aircraft models indicate that the joints already perform extremely well. I estimated that the newly specified process will increase manufacturing costs by about $2,300 for the 40,000 fasteners considered, yet I believe there is no significant performance gain. Considering the potential impact on the several millions fasteners installed on each aircraft, I recommend that better performance estimates be obtained before specifying expensive variability control procedures.

There are also invisible cost savings resulting from my methodology. When properly implemented, the method can eliminate expensive design iterations often required in specifying producible tolerances. For example, design changes are so expensive to make that looser and more producible flushness tolerances specified on the more modern 777 aircraft have not been, and are never expected to be implemented on the 747-400 models. It is beyond the scope of this thesis to estimate the quantitative impact of my tolerancing scheme on design and manufacturing operations. Nevertheless, in light of current research in concurrent engineering[NW89], the impact is thought to be significant.

1.3 Thesis Contents

1.3.1 What to Expect from this Thesis

This thesis presents a collection of concepts that collectively make up a new methodology for product tolerance design. The ideas provide a practical and logical way of thinking about design and manufacturing strategies. The thesis provides a methodology for collecting information relevant to specifying tolerances and presents guidelines to determine and represent those tolerances. The ideas are backed by examples demonstrating applicability and impact. The following are the contents of this thesis.

Chapter 2: Review of Existing Tolerancing Methods

In this chapter I review a selection of research literature and industrial design practice relevant to tolerancing. First, I present a background of tolerance representations widely used in industry and the accumulation models prevalently used in literature. I also survey some of the models that have been used to relate tolerances to cost. Then I describe how researchers and designers have used these models to allocate tolerances.

Chapter 3: Key Ideas of Approach and Representation

In this chapter, I describe the key ideas of my approach. I first define terms relevant to this approach, outline the criteria for tolerance specification, and describe the proposed scheme for representing tolerances.

Chapter 4: Tolerance Determination Methodology

My approach to determining tolerances is based on the premise that decisions made in the presence of relevant information and analytical techniques should result in economical designs. In this chapter I describe my proposed methodology for specifying tolerances.

Chapter 5: Implementation Guidelines

This chapter describes some difficulties we might encounter when implementing the methodology, and demonstrates how tolerances may be specified in practice.

Chapters 6, 7, and 8: Examples

I demonstrate the effectiveness of my methodology by applying it to three design examples. Impact analyses are also presented.

Chapter 9: Summary and Recommendations

In this chapter I summarize my results, describe the concerns relating to my methodology, and provide a list of recommendations for future research in this area.

Chapter 2

Review of Existing Tolerancing Methods

In this chapter I review a selection of research literature and industrial design practice in tolerancing. First, I present a background of tolerance representations widely used in industry and the accumulation models prevalently used in literature. I also survey some of the models that have been used to relate tolerances to cost. Then I describe how researchers and designers have used these models to allocate tolerances.

2.1 Background

2.1.1 Prevalent Variability Models

Variability is the dispersion of a parameter around a deterministic value and exists in many shapes and forms. Because in practice we can not obtain the true underlying variability of a design characteristic, we use statistical models to represent it. The accuracy of variability estimations depends on many factors. Gauge accuracy and precision affect variability estimations as well as estimation reliability. The integrity of variability models is also affected by the frequency and number of samples measured. Better gauges and more frequent samples yield models that are more reliable and accurate, but also more expensive.

Variability models may be categorized into two major groups: parametric and non-parametric. Parametric variability models are essentially mathematical models whose parameters are estimated from collected data. Non-parametric models are less distilled models often still in frequency data form. Parametric models are by far the most widely used models in design.

Why Parametric?

Parametric variability models are advantageous for three main reasons: 1) Variability parameters are often quantifiers having physical relevance to variability. For example, the variance parameter of Gaussian distributions is an abstraction of the distances of sample points to the mean of the distribution. 2) Parametric variability descriptions greatly enhance our ability to perform analytical manipulations. 3) Parametric models are concise compared to non-parametric models which are often associated with large bodies of data.

On the other hand, parametric variability models are disadvantageous because there are uncertainties involved in associating data to parameterized models. For example, it is widely accepted that fastener flushness characteristics follow Gaussian distributions. But we can never be certain. Also, because there are only a handful of convenient parametric models, designers often manipulate data so that they fit parameteric models. I observed several statistical modeling cases in industry where selective data discrimination was used to fit data to speculated models.

In practice, the choice of the representation scheme boils down to practicality and convenience. Because of the overwhelming preference for simpler models and the resource intensive nature of non-parametric models, parametric models are dominantly used in tolerancing. I will now describe a parametric model widely used in industry.

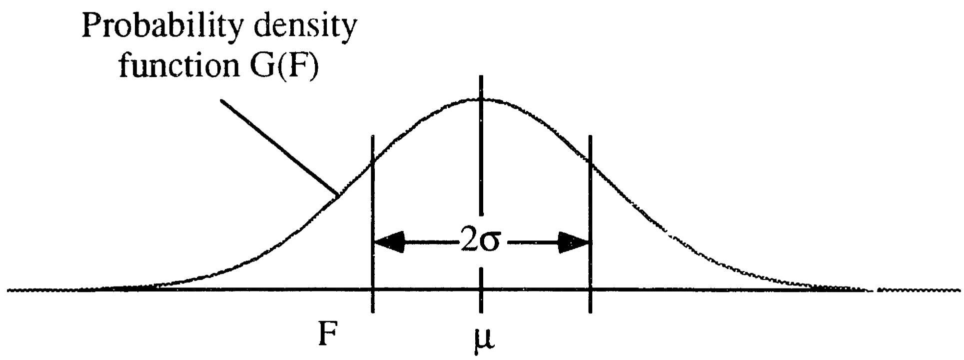


Figure 2.1: The Gaussian distribution

Gaussian Distribution

The Gaussian distribution is by far the most widely used model in tolerancing today. In mathematical form the Gaussian function is described by

$$G(F) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(F-\mu)^2/\sigma^2} \qquad \{-\infty \leq F \leq \infty\}. \tag{2.1}$$

Its general shape is shown in Figure 2.1. The probability density function $G(F)$ is described by the standard deviation σ and mean μ. μ represents the distribution mean while σ describes the dispersion of the population around μ. The Gaussian distribution is applied in many tolerancing models because 1) It is convenient to use, and 2) it suitably represents a wide variety of variability phenomena. See Section 3.4.1 for elaboration. Also, Gaussian distributions are sometimes used as variability approximations when accurate variability data is not available [CP91].

By popular yet arbitrary convention, tolerances have been linked to σ by the relation:

$$\Delta F = \pm 3\sigma \tag{2.2}$$

Tolerances specified by this convention generate yield rates of 99.7% considered to be acceptable in many engineering applications. See Figure 2.2.

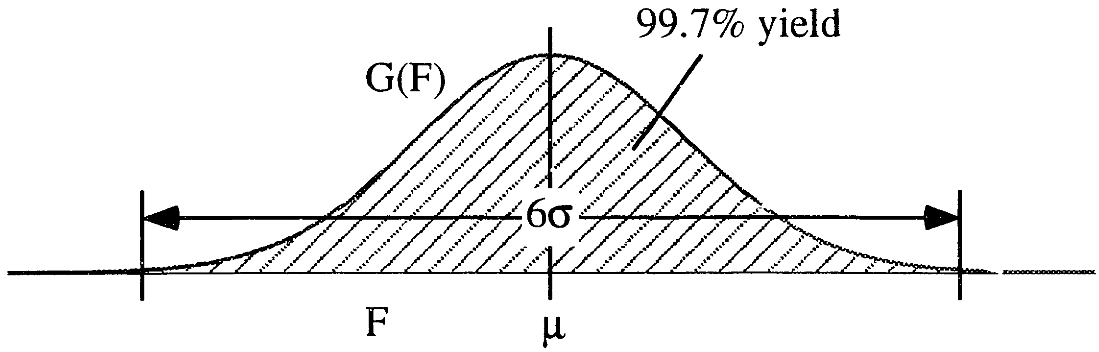


Figure 2.2: $\pm 3\sigma$ tolerances

2.1.2 Stack-up Modeling

Tolerance stack-up is the accumulation of component tolerances. A significant portion of tolerancing research is dedicated to the *analysis* of component tolerance stack-up to evaluate whether the resulting assembly tolerance is functionally or economically acceptable. Tolerance *allocation* uses stack-up models to allocate economical component tolerances from predetermined assembly tolerances. Among the innumerable models describing tolerance stack-up, there are three that are widely used in practice.

- Worst case (WC)
- Statistical (ST) and
- Mean shift (MS)

Worst Case Accumulation

The WC tolerance stack-up model accommodates the fact that the worst component dimensions can occur simultaneously. In 1-dimension,

$$T_{asm} = \Sigma T_i \quad i = \{1, \ldots, n\} \tag{2.3}$$

T_{asm} is the assembly tolerance; T_i, the component tolerances; and n, the number of stacked parts in the assembly. The WC model is usually applied when designers require all assemblies to meet specified tolerances. The WC model is often considered conservative because it does not take into account the ***likelihood*** of worst case combinations occurring. For example, when two components exhibiting Gaussian characteristics are toleranced under the WC model, the theoretical expected rate of tolerance nonconformance is only approximately 22 parts per million.

Statistical Accumulation

The ST model takes into account the statistical likelihood of component combination. In the statistical accumulation model, tolerances add as *root sum squared.*

$$T_{asm} = [\Sigma T_i^2]^{1/2} \qquad i = \{1, \ldots, n\} \tag{2.4}$$

When used with $\pm 3\sigma$ tolerance representations, the ST model provides 99.7% conformance rates for the assembly and its components.

Mean shift

In reality, manufacturing processes drift because of tool wear or inconsistent setup. These factors result in biased variability distributions or shifted means, creating lower yield rates than expected by the ST model [Spo78, Eva75]. To account for these uncertainties, [CG88, CGLH90, Man63] have used hybrid accumulation models consisting of both the WC and ST models.

$$T_{asm} \geq \Sigma m_i T_i + [\Sigma(1 - m_i)T_i^2]^{1/2} \qquad \begin{array}{l} i = \{1, \ldots, n\} \\ 0 \leq m_i \leq 1.0 \end{array} \tag{2.5}$$

m_i is a weighting factor that can be chosen anywhere between 0 and 1.0. When $m_i = 0$, the result is an ST model. When $m_i = 1.0$, the result is a WC model. The

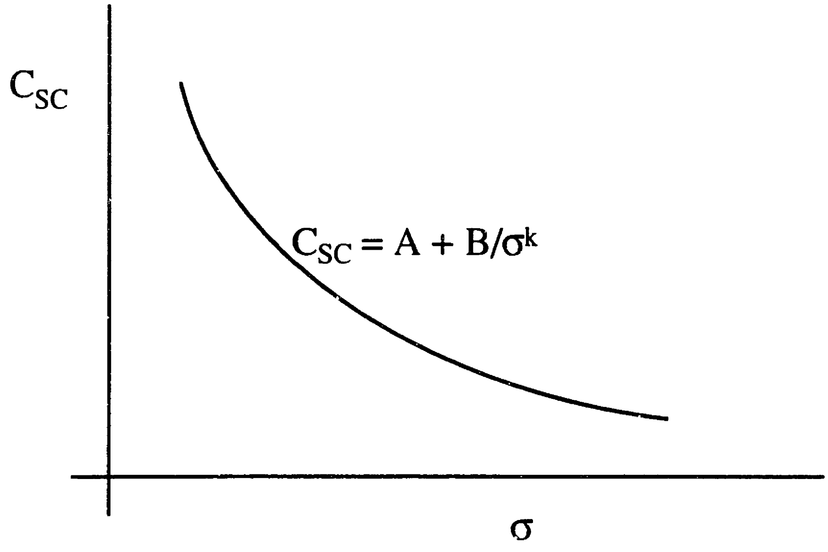


Figure 2.3: Typical cost vs statistical variation control relation

appropriate combination of m_i is chosen to account for the uncertainty involved in characterizing individual processes.

2.1.3 Cost-Tolerance Relationship

In an effort to quantify costs of controlling manufacturing variability, [Spo78, CG88, SR75, Bjo89], among several others, have created parametric cost models for process variability. Their cost models generally take the form:

$$C_{SC} = A + \frac{B}{\sigma^k} \tag{2.6}$$

where C_{SC} is the cost of manufacturing with variability σ; A is the fixed cost associated with the process; and B and k are parameters chosen to fit empirical cost data. C_{SC} is a decreasing function of σ. Figure 2.3 reflects the fact that the resources required to obtain tighter variation increase nonlinearly. This phenomena may be attributed to the fact that per unit labor and equipment costs increase nonlinearly with tighter variation.

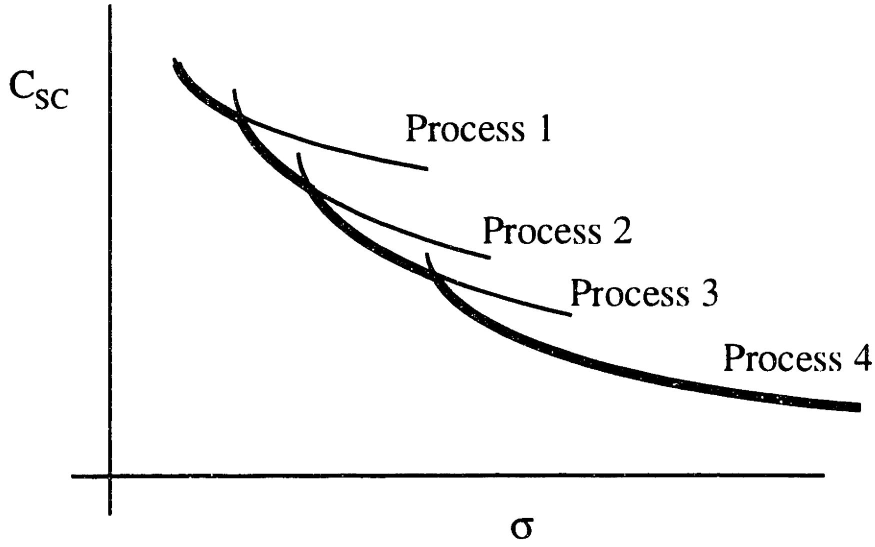


Figure 2.4: Processes and the optimal cost curve

Usually there are many processes available for any given manufacturing task. The rational manufacturer will always choose to operate at the lowest cost possible as shown in Figure 2.4

2.2 Existing Tolerancing Methods

Figure 2.5 shows the traditional sequential process Boeing has followed in the past for designing aircraft. Under this scheme, aircraft requirements are first determined by *marketing.* These requirements are then converted into general aircraft definitions by *configurations* and *aeronautics.* After the major outlines of the aircraft are designed, *structural* and *systems* details are designed. Upon completion, designs are "thrown over the wall" to manufacturing, where specifications are converted into equivalent manufacturing process specifications.

This design scheme has drawbacks significantly affecting product quality and cost. Design decisions made upstream without proper knowledge of their implications on downstream activities may result in specifications that are expensive to produce or

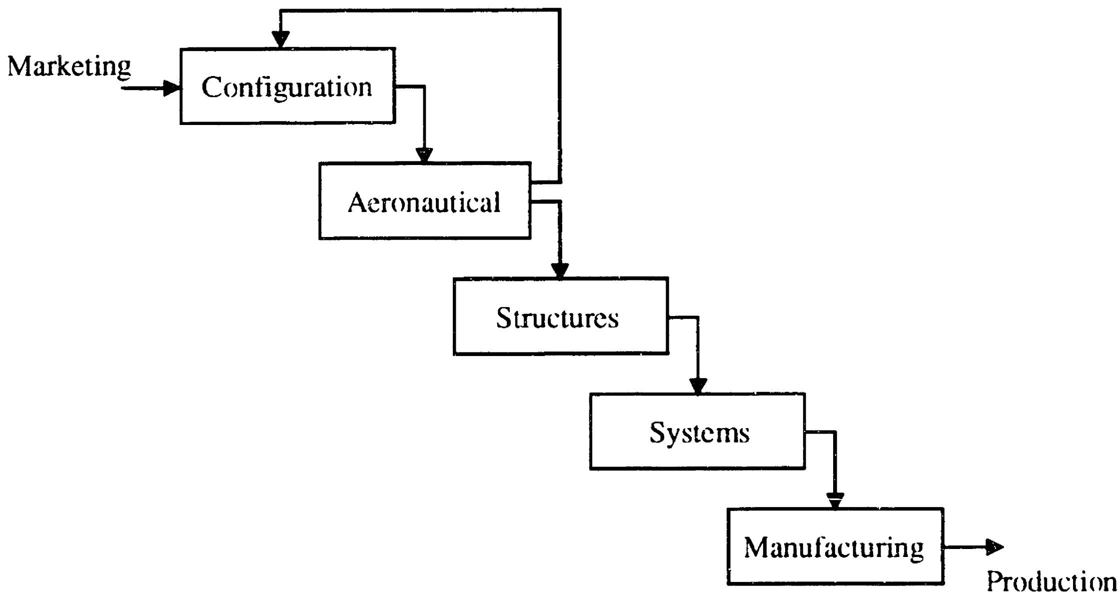


Figure 2.5: The sequential design process

impractical to follow. Products designed and manufactured under this scheme exhibit high cost or low quality.

This section describes some tolerancing methods applied in industry to address this problem.

2.2.1 Traditional Methods

Iterative Design

Products designed under the sequential scheme often require design iterations to address concerns that emerge in downstream processes. For example, excessively demanding tolerances need to reconsidered. This usually involves making compromises on performance, or redesigning products so that performance becomes less sensitive to tolerances. These redesign activities are extremely expensive. Often, even design changes known to reduce manufacturing costs significantly are not approved simply because design changes are expensive to carry out in practice. For example, highly expensive flushness tolerances originally specified on the 747-400 model have been relaxed for the newer 777 model, but the original 747-400 tolerances have not been relaxed because of the costs associated with changing already established designs.

Expert Tolerancing

Tolerancing requires a vast amount of knowledge relating to product performance and manufacturability. Tolerances affect many factors in product design, manufacturing, and product life cycle in general. Because of the complexity of the problem, tolerancing has long been considered a job for the experts. Experts use their years of acquired knowledge to evaluate the innumerable factors related to tolerances. Experts may well be indispensable in designing quality products. However, design decisions are often made amid so much uncertainty and nondeterminism that tolerance specifications become highly conservative. One objective of this thesis is to provide designers with effective analytical tools so that they have to rely less on intuition and more on quantitative analysis methods for tolerancing.

Tolerance Tables

Tolerance tables inform designers of typical tolerances associated with desired design features. Such tables allow designers to assess the relative manufacturability of different specifications. Figure 2.6 shows examples of tolerance tables for hole location and machining finish from [Loc]. As demonstrated, however, these tables provide only normalized scales for manufacturability. Although they are capable of assessing the relative difficulty of each process, they still lack the information required to make quantitative tradeoffs between the manufacturing cost and performance.

2.2.2 Geometrical Tolerance Allocation

Geometrical tolerance allocation is a method for allocating economical component tolerances from predetermined assembly tolerances. It requires: 1) determining an adequate assembly tolerance from performance requirements, 2) defining a stack-up

	Method			
	Cast	Lathe	Grind	Hone
Finish [μin]	to 2000	2000-32	125-16	32-16
Relative cost factor	1	2-4	3-5	4-10

	Method			
	Hand Drill	Drill Press	Jig Bore	Special Equipment
Tolerance [in]	to 0.015	0.015-0.010	0.010-0.001	0.00-0.0002
Relative cost factor	1	2	3-4	3-15

Figure 2.6: Tolerance table for selected processes

model to relate component tolerances to the assembly tolerance, 3) defining the tolerance allocation criteria, and 4) allocating tolerances. Most literature in geometrical tolerance allocation is devoted to the third and fourth items: defining allocation criteria and implementing the allocation.

Design Heuristics

Proportional Scaling: In proportional scaling, designers determine initial component tolerances from intuition. Then stack-up models are used to investigate whether resulting assembly tolerances meet design requirements. If found unacceptable, each component tolerance is scaled down by a constant proportion so that the resulting assembly satisfies the requirements. When certain components are known to be cost or performance critical, weighting factors may be attached so that they are scaled accordingly [HS88].

Cube Root of Nominal: This rule-of-thumb approach is based on the assumption that the difficulty of controlling tolerances increases as the cube root of the nominal value [CP91]. The rule originates from the tolerancing standards used in designing cylindrical fits where initial tolerances are selected as the cube root of the component nominal [For67]. If stack-up results in unacceptable assembly tolerances, component tolerances are scaled proportionally to meet assembly requirements.

Difficulty Factors: In this approach critical design characteristics (such as dimensions, material properties, shape) are identified for each component in an assembly. Manufacturing difficulty factors are evaluated for each design characteristic of every component, then summed for aggregate difficulty assessment for each component. These values are then used as weighting factors to distribute assembly tolerances among the components [For85].

Minimum Cost Allocation

When empirical cost-tolerance relations can be found for each dimension in an assembly, optimization techniques may be used to search for the minimum cost combination of component tolerances leading to an acceptable assembly tolerance. Chase [CP91] surveys various optimization algorithms and cost models for this tolerancing approach. Extensions to this approach include 1) applications where manufacturing processes are selected in conjunction with tolerances [LW89, OH77], 2) tolerance allocation for multidimensional assemblies [SR75, Par85, MD82], and 3) application on nonlinear assemblies.

Comments on Current Tolerance Allocation Research The tolerance allocation methods described above require the existence of predetermined and known assembly tolerances as constraints for optimization. Fixed assembly tolerances imply performance levels have already been specified and are not negotiable. They also imply performance specifications are not determined with explicit consideration of manufacturability issues. Geometric tolerance allocation therefore acts only as a local optimization in comparison to the global performance-manufacturability optimization.

2.2.3 Robust Design

The objective of robust design is to minimize product performance variations by controlling design parameter targets and their variations [Hau80, Mis89]. Often, performance sensitivity to tolerances is analyzed only after the design is complete and all nominal values have been determined. By considering the effects of variation during the design of nominals, a *robust* design, whose performance is relatively insensitive to design parameter variation, can be found.

Taguchi Experimental Design

The Taguchi approach is perhaps the best known robust design approach. It implements designed experiments to directly investigate performance sensitivity to design variations. Taguchi separates design into three separate stages. First, *system* design determines general configurations and shapes. Second, *parameter* design determines design nominals that result in relatively invariant performances. Third, *tolerance* design adjusts tolerance parameters such that product variation is minimized.

Taguchi advocates the use of initial "inexpensive" tolerances for parameter design. These tolerances are applied to investigate performance sensitivities so that the nominal with the least performance variability can be selected. Once the nominal is determined, tolerances are tightened during tolerance design.

Comments on Taguchi Design

As the approach emphasizes, expected product performance depends on both the selection of the nominal and the specified tolerances. Taguchi proposes separate parameter and tolerance design stages because of the high costs of conducting experiments. Repetitive exploration of the design space is impractical in experimental design. It is noted, however, that the simultaneous design of nominals and tolerances would indeed yield better results.

Robust design methods provide little guidance in balancing performance against

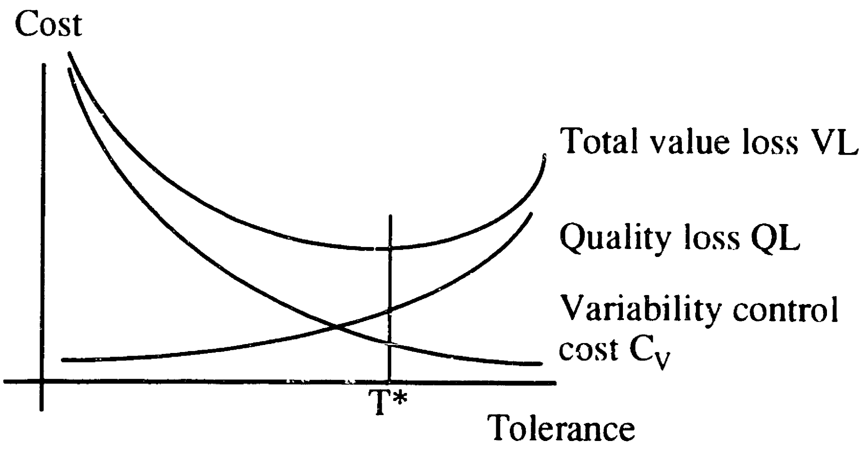


Figure 2.7: The economic model for performance-cost tradeoff

manufacturability. They tell us how to minimize performance variability by resourcefully controlling nominal selection and tolerance specification. But they do not provide quantitative methods for determining optimal combinations of nominals and tolerances to achieve well balanced performance and cost.

2.2.4 Performance-Cost Tradeoff Model

There are theoretical models for trading off performance against manufacturing costs. Figure 2.7 shows one such model from [Jur88, Bjo89]. Performance degradation resulting from poor tolerances is represented by the expected loss QL. Quality loss increases as product variability increases. The cost of controlling variability C_V decreases as tolerance increases. The total value loss VL is the sum of the two cost elements.

The objective is to find tolerance T^* where the total value loss is minimized. In practice, however, very few researchers have attempted to implement this theory on industrial tolerancing problems. One reason is that cost functions have not been available in the past. There has been little use for manufacturing cost models. My hope is that the cost impact analyses shown in this thesis will motivate manufacturers to document their processes. A second reason for the model's lack of use is that the definition of tolerances has been ambiguous. It is unclear, for example, whether tolerance T in Figure 2.7 represents statistical variation, inspection limits, or both. Sometimes T even represents nonconformance rates [CA81].

2.3 Where This Thesis Fits

This thesis can be viewed in one of several ways:

1. Extending tolerance allocation: I extend the boundaries of current tolerance allocation methods by including product performance as an additional criterion for allocating tolerances. Performance has traditionally been used as a fixed input in tolerance allocation thus far. Instead I include performance models in the optimization and treat performance as an *outputs*.

2. Bridging the gap between theoretical performance-cost tradeoff models and industrial tolerancing practice: I use industrial examples to demonstrate how performance and manufacturability may be traded off for better product design.

3. Extending robust design: Experimental design methods result in suboptimal design specifications because design spaces cannot be fully investigated. I demonstrate that in cases where mathematical performance models are available, better specification methods exist.

4. Extending the tolerancing problem by defining two distinct tolerance representations I define statistical variation and inspection control limits. This representation scheme, described in Section 3.4, builds on each of three previous items by adding another control factor to product variability.

Chapter 3

Key Ideas of Approach and Representation

In this chapter, I describe the key ideas of my approach. I first define terms relevant to this approach, then outline the criteria for tolerance specification, and finally describe the proposed scheme for representing tolerances.

I emphasize four points in my tolerancing approach.

1. Tolerancing is making economic decisions about product performance and cost.
2. Tolerancing is determining and representing constraints on variability sources.
3. Tolerancing is selecting the most economical combination of statistical variation and inspection control options.
4. Caution must accompany deterministic performance modeling.

3.1 Definitions

Variability

Variability describes the tendency of parameters to statistically distribute themselves. Variability may result from inherent imprecision in manufacturing processes and/or uncertainties in modeling.

Design parameters

Design parameters are the variables designers control to obtain desired product performances. In the fastener joint design case, examples of design parameters are fastener flushness F and fastener interference I.

Performance loss

Performance *attributes* are the characteristics of a product considered to bring satisfaction or dissatisfaction to product users. In this thesis, I measure performance in quantifiable performance *losses*. An example of performance loss in fastener joint design is drag D_F resulting from improper control of flushness F.

$$D_F = D_F(F) \tag{3.1}$$

Quality loss

Quality loss L is the *monetary* cost incurred by using products with performance losses. An example of quality loss in fastener joint design is the cost of operating an aircraft with aerodynamic drag.

$$\begin{aligned} L_F &= L_F(D_F(F)) && (3.2) \\ &= L_F(F) && (3.3) \end{aligned}$$

Expected quality loss

Expected quality loss QL is the total expected monetary loss resulting from probabilistic quality losses. $L_F(F)$ above varies for two main reasons. 1) Manufacturing imprecision causes variations in F. 2) Modeling uncertainties make the relationship between L_F and F nondeterministic. We will neglect modeling uncertainties for now but will return to them in Chapter 7.

QL for fastener drag may be obtained by the relation:

$$QL_F = \int_F P(F)\, L_F(F)\, dF \tag{3.4}$$
$$= E_F\left[L_F(F)\right] \tag{3.5}$$

where $P(F)$ is the probability density function describing the dispersion of F. E_F is the notation for probabilistic expectancy operation over the parameter F [Dra88]. QL increases as variability increases.

Variability control cost

Variability control cost C_V is the expected cost of controlling variability. Variability can be controlled in two ways. 1) Control the shape and location of the probability distribution. I call this statistical variation control. The associated control cost is defined as C_{SC}. 2) Truncate the distribution by identifying limits and screen for nonconforming elements. I call this *inspection control.* The associated control cost is defined as C_{IC}. My use of these two control methods will become more clear in Section 3.4

Value loss

Value represents the level of satisfaction derived from a particular combination of product characteristics [Man85]. In this thesis I assume values are measurable and significant. I define *value loss* as the monetary cost of *acquiring* and *using* a product. An example of total value loss VL in the airline industry is the total cost of owning and operating an aircraft. The lower the cost, the higher the value of the aircraft.

Design characteristics

Design characteristics represent the universal set of all parameters associated with a

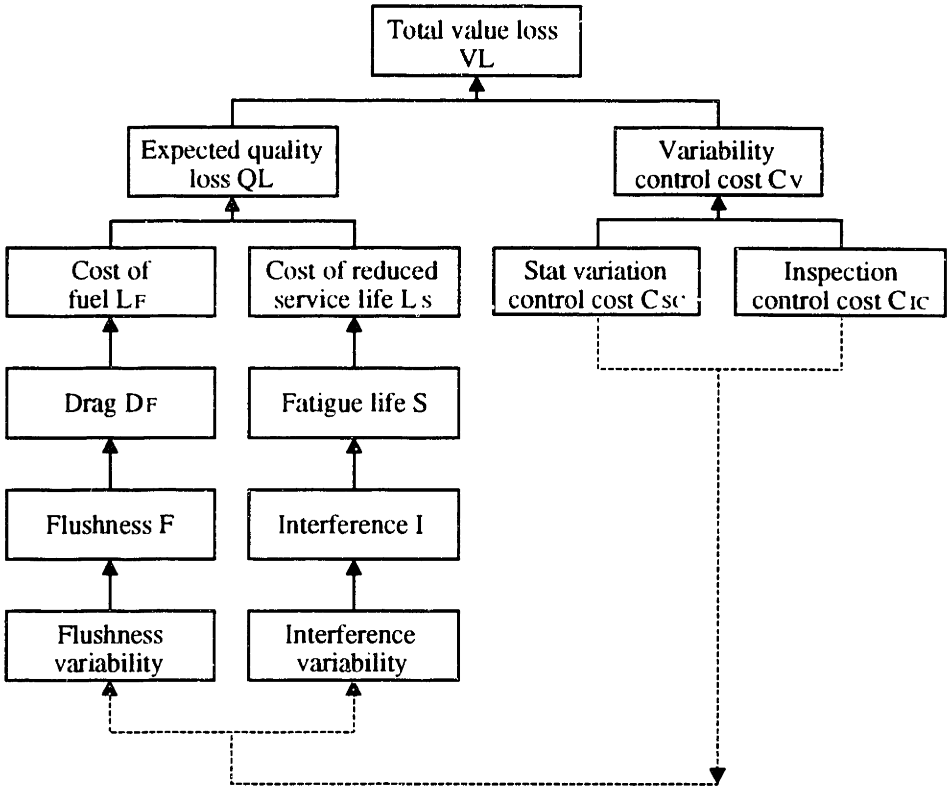


Figure 3.1: Design causality tree for wing skin splice joint examples

product. Design characteristics include all design parameters, performance quantifiers, and costs described above.

3.2 The relationship among design characteristics

In any design task, there are design objectives. The most important design objective is arguably to design a well performing product for low cost. In tolerancing this objective might be represented in a design causality tree as exampled in Figure 3.1 for the fastener joint design case. This tree begins with VL as the objective value loss to optimize. Two cost elements affect VL: 1) expected quality loss QL and 2) expected variability control cost C_V. QL is in turn determined by quality loss

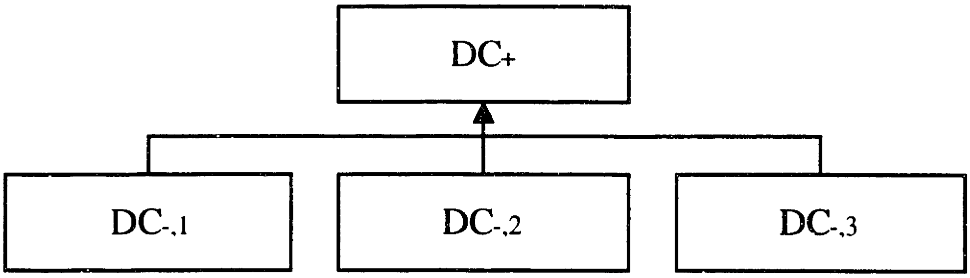


Figure 3.2: Definition of DC_+ and $\mathbf{DC}_-$

elements associated with aircraft fuel consumption and service life. C_V is affected by the combination of statistical variation control cost C_{SC} and inspection control cost C_{IC}. The causality tree generates further nodes and branches until it adequately represents the fastener joint tolerancing problem. For conciseness, only the nodes associated with fastener flushness and interference are shown. The construction of this tree parallels the human tendency to think ***top-down***, or from the general functions and forms to the details.

The causal direction, however, is ***bottom-up***. In reality, the only directly controllable design characteristics are the lower level design characteristics. The objective of fastener joint tolerancing is to find an economical combination of flushness and interference variabilities such that the upper level constraints and objectives are satisfied.

3.2.1 Relevant causality tree properties

In this section, I will discuss two important properties associated with design causality trees. I will first define some conventions useful to this discussion.

If DC_+ is a design characteristic at an arbitrary level of detail, $\mathbf{DC}_-$ is a vector of design characteristics $\{DC_{-,1}, \ldots, DC_{-,n}\}$ directly affecting DC_+. See Figure 3.2. For example, if VL in Figure 3.1 were designated DC_+, cost elements QL and C_V would constitute vector $\mathbf{DC}_-$ with elements $DC_{-,1}$ and $DC_{-,2}$ respectively:

$$\begin{aligned} DC_+ &= VL \\ \mathbf{DC}_- &= \begin{bmatrix} DC_{-,1} \\ DC_{-,2} \end{bmatrix} = \begin{bmatrix} QL \\ C_V \end{bmatrix} \end{aligned} \tag{3.6}$$

The relationship between design characteristics DC_+ and $\mathbf{DC}_-$ exists in many different forms. Sometimes, as is the case when design characteristics are defined in terms of cost, DC_+ is a simple sum of the individual elements of $\mathbf{DC}_-$. For example, VL may be written as:

$$VL = QL + C_V \tag{3.7}$$

In other cases, the relationship is not as simple. For example, the effect of fastener flushness and paint on aircraft drag is a highly complex relation involving, among other things, statistical modeling. This type of performance modeling must be done on a case by case basis.

Design tradeoffs

An important property of causality trees is that there are many, if not infinite combinations of $DC_{-,i}$ that will describe a unique value of DC_+. In Equation 3.7, there are infinite combinations of QL and C_V that would result in the same VL. Similarly, there are many combinations of flushness and paint characteristics that will result in the same drag. This property allows designers to trade off one design characteristic against another. Most tolerance allocation methods discussed in Section 2.2.2 use this property to allocate part tolerances from fixed assembly tolerances.

Variability generation

There are variabilities associated with each design characteristic. Design characteristics accumulate variability from two basic sources. First, variabilities introduced during manufacturing processes propagate up the causality tree. Second, when we model relations between design characteristics, we introduce modeling uncertainties. The variability of each design characteristic DC_+ is the accumulated result of variabilities in $\mathbf{DC}_-$ and the uncertainties in modeling the relation between DC_+ and $\mathbf{DC}_-$. For example, fatigue life variability can be traced to two sources. 1) Inherent manufacturing imprecision in producing wing skin holes and fasteners translates to interference variability. Interference variability in turn affects fatigue life variability. This is the variability propagation property. 2) Furthermore, there are uncertainties in modeling the relationship between interference and fatigue life. Even when interference levels can be precisely controlled, the presence and influence of other unmodeled factors results in fatigue life variabilities which approximately follow Weibull distributions[1].

What are tolerances?

Tolerances are constraints placed on these variabilities. The primary objective of this thesis is to provide a methodology for specifying economical design and manufacturing tolerances.

3.3 Criteria for tolerance specification

3.3.1 Representation criteria

We have seen in Section 1.1.4 that tolerances are often represented with the specification of a nominal value and a set of tolerance limits placed in relation to that nominal. We have also seen that this representation scheme is ambiguous. Because of the the lack of manufacturing information during design and miscommunication of design intent during manufacturing, we often find manufacturing applying variability

[1]The Weibull probability function is described in Section 3.4.1

control techniques different from what designers had originally anticipated.

To eliminate these problems we need to provide a representation scheme

- that fully and unambiguously specifies variability constraints and
- whose specification represents what manufacturing can actually do.

3.3.2 Determination criteria

Let us assume Q represents some quality indicator for a manufactured product. Customers purchase the product only when their perceived value $V(Q)$ exceeds or at least equals its price P. Consumer surplus S may be written as

$$S(Q,P) = V(Q) - P \tag{3.8}$$

The objective of the consumer is to maximize this surplus.

Manufacturers' surplus is commonly known as profits. The manufacturer continues production only when profits are generated. His objective is to maximize profits

$$\pi(Q,P) = P - C(Q) \tag{3.9}$$

where $C(Q)$ is the cost of manufacturing products at quality level Q.

As shown, customer and manufacturer objectives are contradictory: high prices mean high manufacturer profits but low consumer surplus. From a societal point of view, however, the objective is to maximize the global surplus $S(Q,P) + \pi(Q,P)$ which is the value created V_c for the society by producing and consuming the product.

$$V_C(Q) = S(Q,P) + \pi(Q,P) \tag{3.10}$$

$$= V(Q) - C(Q) \tag{3.11}$$

The objective is to specify Q such that V_C is maximized. This can be achieved in one of three ways.

1. Maximize $V(Q)$ with C fixed. In aircraft design and manufacturing, this is equivalent to optimizing aircraft performance with a fixed amount of manufacturing resources. This might involve maximizing performance gains by reallocating manufacturing effort and cost.

2. Minimize $C(Q)$ with perceived product value V fixed. This minimizes manufacturing costs while delivering equivalent aircraft quality. This might be done by trading off one aircraft performance attribute against another to minimize aggregate manufacturing costs while maintaining an equivalent amount of total expected quality loss QL.

3. Simultaneously adjust $V(Q)$ and $C(Q)$ so that V_c is maximized. Here we are free to balance aircraft quality against the costs of delivering that level of quality. While the two previous approaches can yield some design improvement, the simultaneous adjustment of $V(Q)$ and $C(Q)$ is expected to yield even better designs.

My approach for determining tolerances is to use these approaches to design high performing, low cost products.

3.4 My approach to tolerance representation

When faced with a task, we tend to take one of the following two approaches: 1) do it right the first time and 2) do a decent job the first time and correct the mistakes later. As long as we satisfactorily complete the task, our strategy rests entirely on the efficiency of each approach. The first approach requires higher initial effort than the second, but the second approach has a higher correction effort associated with it. The desirability of each approach is strictly a resource evaluation and allocation problem.

My thesis is that tolerancing is a similar problem. We know product quality is usually highly dependent on variability. When determining variability control methods, we face the same dilemma: do we control variation tightly the first time, or do we apply only adequate initial variation control and fix the nonconforming products later?

I define tolerances in terms of two variability control methods commonly used in industry: statistical variation control and inspection control. Statistical variation control is the technique of controlling the the underlying distribution by controlling the *shape* and *location* of the variability function. Inspection control is the technique of identifying and correcting products not conforming to specified requirements after the product has already been made.

These two variability control approaches are commonly used as manufacturing quality assurance techniques in industry, but their coexistent application in design has not been recognized as a valid tolerance specification approach. One reason is that the notion of using inspection control as a quality assurance strategy conflicts with the literature devoted to *continual quality improvement.* Continual quality improvement [Dem82] advocates zero defects as the optimal conformance level. It prescribes continual movement towards zero variability under the belief that higher quality products cost less to produce per unit. This belief originates from circumstantial yet compelling evidence that many Japanese companies implementing continual quality improvement to produce higher quality products and have lower costs at the same time [Gar83]. This belief, however, is also under the pretense that the cost of variability control is virtually a constant or negative function of statistical variation and time [Fin86]: in other words "quality is free". However, in industries where manufacturing technologies are relatively stable yet still state-of-the-art, as is the case in fastening technologies, this pretense is no longer valid.

In addition, inspection control is an important option for manufacturers whose product time-to-market is critical. For example, it is common to find semiconductor manufacturers initially implementing low yield processes to beat competitors to the market. Because the cost of inspecting and discarding defective chips is relatively inexpensive compared to the opportunity cost of not capturing market share, inspection control becomes a strong, economical strategy. The manufacturer may eventually choose to tighten the statistical variation of his processes as he acquires more knowledge about his processes. He will increase yield and reduce the costs associated with rejection and discarding. Still, he will tighten his processes only to the point where statistical variation control costs are economically balanced against the costs of implementing inspection control. For example, although lower particle count in semiconductor fabrication clean rooms results in tighter statistical variation, it is simply not practical to control particle count to better than class 1 per federal standard FED-STD-209E. Although better control methods are conceivable, they are not economically justifiable against the option of implementing looser statistical variation control and relying on inspection control. Inspection control is a strong option in these circumstances.

I represent tolerances as a combination of statistical variation specifications and inspection control limits. These representations, described in the next two sections, are used in Chapters 6, 7, and 8 to discuss the effectiveness of the combined representation.

3.4.1 Statistical variation

I use two parametric models to represent statistical variation:

- Gaussian distributions to represent dimensional variability and
- Weibull functions for reliability distributions.

I use these parametric models because of the three advantages outlined in Section 2.1.1, and because there is significant evidence these distributions correctly represent the fastener joint design variabilities I investigated at Boeing. However, I

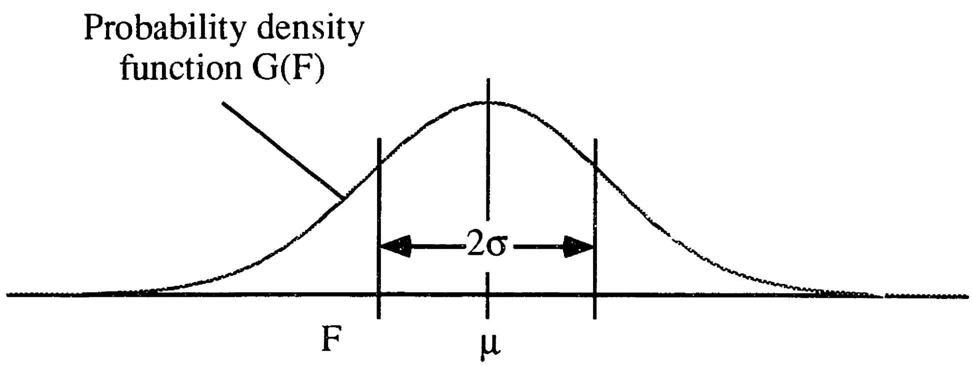


Figure 3.3: The Gaussian distribution

recognize these models may not be appropriate in other applications. Although I use Gaussian and Weibull models to demonstrate the effectiveness of my tolerancing approach, I have made my methodology generalizable to accommodate other variability models.

The Gaussian distribution

The Gaussian distribution $G(F)$ and its cumulative function $\Phi_G(F)$ are described by:

$$\begin{aligned} G(F) &= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(F-\mu)^2/\sigma^2} && \{-\infty \leq F \leq \infty\} \\ \Phi_G(F) &= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{F} e^{-\frac{1}{2}(t-\mu)^2/\sigma^2} dt && \{-\infty \leq F \leq \infty\} \end{aligned} \tag{3.12}$$

where F represents the random variable; μ, the mean; and σ the standard deviation of the distribution. $G(F)$ takes on the general shape shown in Figure 3.3, and is fully constrained by specifying μ and σ.

The Gaussian distribution is convenient because it regenerates upon combination. The statistical combination of multiple Gaussian functions results in another Gaussian function whose variance is the sum of individual variances. This property makes mathematical manipulations convenient.

The Gaussian distribution is also convenient because it represents the variability characteristics of a wide variety of problems. Many design characteristics including fastener flushness and interference exhibit variability shapes closely resembling Gaussian distributions. This is especially interesting because the variability sources contributing to fastener flushness and interference are not entirely Gaussian. Fastener hole diameters contributing to interference, for example, exhibit a high degree of skewness and kurtosis. But upon combination with other variability sources such as reamer variability, fastener shank variability, and coldworking variability[2], final interference variability approximately follows the Gaussian distribution. This is explained by the central limit theorem, which shows that when there are many sources of variability contributing to a parameter, the resulting combined variability approaches a Gaussian distribution regardless of the shapes of the source variabilities [Gan83, CP91].

The Weibull distribution

The Weibull function $W(x)$ and its cumulative function $\Phi_W(x)$ are described by:

$$\begin{aligned} W(x) &= \frac{\alpha}{\beta}\left(\frac{x-\gamma}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x-\gamma}{\beta}\right)^{\alpha}} & \{x \geq \gamma\} \\ \Phi_W(x) &= 1 - e^{\left(\frac{x-\gamma}{\beta}\right)^{\alpha}} & \{x \geq \gamma\} \end{aligned} \tag{3.13}$$

where α is called the shape parameter; β, the scale parameter; and γ, the lower bound. $W(x)$ takes on the general shape shown in Figure 3.4.

The Weibull distribution is effective in representing a large variety of variability characteristics associated with reliability problems. Parameters α, β, and γ are derived from empirical tests and service data. In fatigue life design, the probability density function $W(S)$ describes the probability of failure at life S. The α parameter is assumed to be fixed and invariant given a constant material type. In the case of aluminum skin splice design, α is assumed to be constant at $\alpha = 4.0$. The β parameter, otherwise known as characteristic life, is determined from design parameters. In fastener joint fatigue, β is a function of fastener interference I. The γ parameter is

[2]Coldworking is the process of plastically conditioning fastener holes for better fatigue performance.

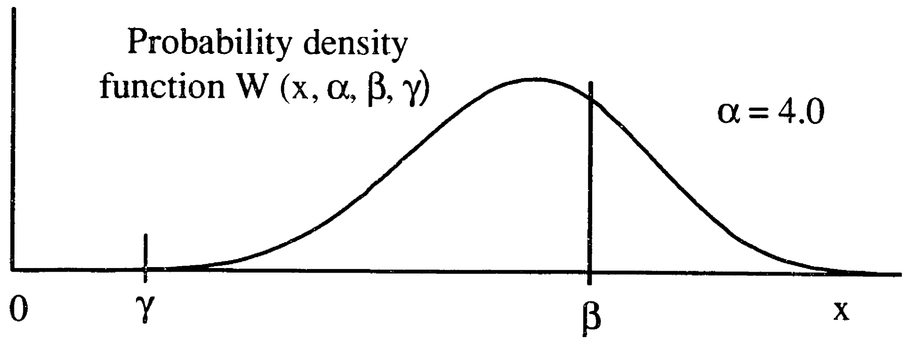


Figure 3.4: The Weibull distribution

the bounding life constant at which initial failure starts occurring. In Boeing fatigue models $\gamma = 0$, meaning there is always a chance fatigue failure will occur.

3.4.2 Inspection limits

Tolerances have been traditionally expressed as lower and upper bounds placed on design characteristics. In the "goalpost" interpretation of tolerances, products whose characteristics fall within these limits are accepted while those that fall outside are identified and corrected or discarded. Application of inspection control requires the following tasks:

1. Determination of inspection limits
2. Identification of nonconforming products
3. Correction of rejected products

Inspection control implies 100% inspection of parts. It can be implemented in many different phases of a product life cycle. Inspection control can take place as early as immediately following production, or can take place within the service life of the product. How and when to implement inspection control depends on inspectability, expected yield, and correction costs among other factors. In Chapters 6, 7, and 8 I will show inspection control applied both immediately following production and during service life.

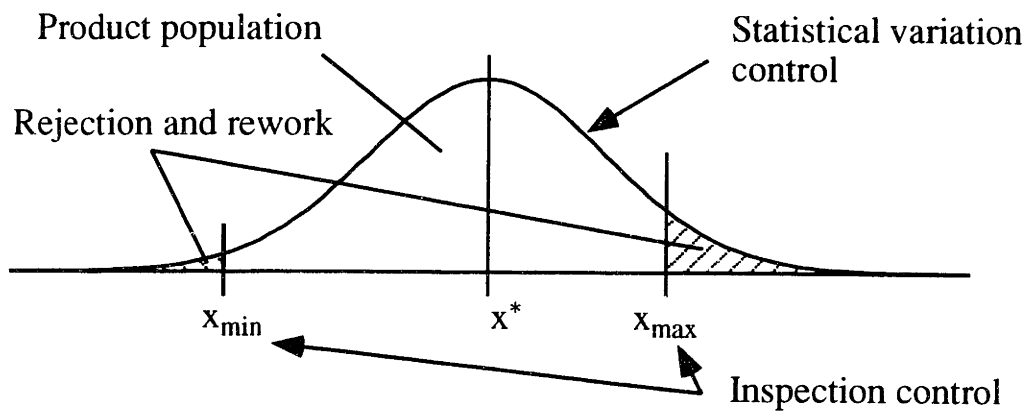


Figure 3.5: Product population variability resulting from combined representation

Inspection control immediately following production requires nondestructive inspection of 100% of the parts. On mechanical parts this usually involves measuring part or assembly dimensions, while on electronic products simple circuit continuity tests might be implemented. Usually, required inspection control resources at this stage are limited to inspection, rejection, rework, and scrap.

Inspection control may also be implemented during the product service life. Inspection is performed comprehensively on all products, but in this case, it is either done by the customer or at a service location. Inspection control at this stage again includes the costs of inspection, rejection, rework, and scrap, but also includes the costs associated with customer dissatisfaction.

The decision of when and how to implement inspection control depends entirely on the economic implications.

3.4.3 The combined tolerance representation

The combination of statistical variation control and inspection control representations completely defines the variability of a product population. It represents product variabilities as shown in Figure 3.5. The specification of tolerances under this scheme fully prescribes manufacturing process control requirements, inspection control requirements, as well as the expected product quality as a population. This combination represents all variability control options available during production so that their respective desirability can be evaluated during design.

Chapter 4

Tolerance Determination Methodology

My approach to determining tolerances is based on the premise that decisions made in the presence of relevant information and analytical techniques should result in economical designs. In this chapter I describe my proposed methodology for specifying tolerances.

The following is an outline of my methodology.

1. Define objective value loss VL.
2. Model quality loss.
3. Identify the significant sources of variability.
4. Model expected quality loss QL.
5. Model variability control cost C_V.
6. Minimize VL with respect to variability control parameters.

Step 1: Define objective value loss VL.

As discussed in Section 3.3.2, the sole criterion for tolerance determination is the maximization of a single parameter - created value V_c. When defined in terms of cost, created value V_c may be measured by its complement: total value loss VL. Minimizing value loss is equivalent to maximizing created value.

The two major components of VL

VL consists of two cost elements. The first element is the expected quality loss, QL, which represents the aggregate cost of using imperfectly performing products. The second element is the expected variability control cost C_V required to deliver that level of expected quality loss.

$$VL = QL + C_V \tag{4.1}$$

QL and C_V are probabilistic functions describing *expected* costs of variability. Steps 4 and 5 describe how these may be modeled. By defining QL and C_V as functions of statistical variation control and inspection control parameters, we can optimize VL for the most economic combination of those parameters.

Multiple performance attributes

When there are several performance attributes in consideration, their corresponding expected quality losses are additive.

$$QL = \sum_i QL_i \tag{4.2}$$

where QL_i represents the expected quality loss for performance attribute i. In the fastener joint design example, the total expected quality loss is the sum of expected quality losses resulting from aircraft drag and fatigue reliability, QL_D and QL_R respectively.

$$QL = QL_D + QL_R \tag{4.3}$$

Similarly, their corresponding expected variability control costs are also additive:

$$C_V = \sum_i C_{V,i} \tag{4.4}$$

Hence,

$$VL = QL + C_V \tag{4.5}$$

$$= \sum_i [QL_i + C_{V,i}] \tag{4.6}$$

Step 2: Model quality loss.

The quality loss function may be obtained by following these two steps.

2.1 Model the relationship between performance loss and design parameters.

2.2 Model the relationship between quality loss and performance loss.

As defined in Section 3.1, performance loss describes the loss of product performance as design parameters vary while quality loss translates these losses to *monetary* cost. Modeling these relationships is one of the most important and difficult steps in determining economical tolerances. First, there are sometimes significant uncertainties in modeling the relationship between performance loss and design parameters. Second, the relationship between quality loss and performance loss is often absent.

Step 2.1: Model the relationship between performance loss and design parameters.

Modeling the relationship between performance loss and design parameters requires three acivities.

1. Identify the performance losses we care about: Identifying significant performance losses involves investigating what customers care about. It is listening to the "voice of the customer" [HG92]. In the fastener joint example, fastener excresence drag D_F and fatigue service life S are two performance attributes considered highly significant by Boeing customers.

2. Identify the design parameters we care about: There are innumerable parameters affecting a single performance loss. We need to identify those design parameters we care to tolerance. Designers usually know what they have to tolerance through experience or intuition. It is usually the design parameter whose effect on product quality loss is visible over the range of possible design parameter variations. In the aircraft fastener joint design example, design parameters are fastener head flushness F and fastener interference I.

3. Model the relationship between performance attributes and design parameters: In most cases the relationship between performance attributes and design parameters is available to the designer. They are the result of analytical modeling or experimental correlations, or both. For the fastener flushness tolerancing problem,

$$D_F = D_F(F) \tag{4.7}$$

These relations will be described in more detail in Chapters 6 and 7.

Step 2.2: Model the relationship between quality loss and performance loss.

In order to perform economic analyses of variability control options, we need to describe product performance loss in terms of one commensurate variable: cost. We need to transform performance losses into quality losses. Defining this relationship is equivalent to modeling the indifference curve between monetary value and performance. This relationship is not readily available in many cases, but can be obtained by posing the question: how much is an extra unit of performance attribute valued by the customer? For example, how much is an airline willing to pay for an added flight cycle of service life? How much is a reduced pound of aircraft weight worth to an airline? The answers may be used to model *quality losses* from performance losses. [LT92, VCK92, Hau83, HG92] have devoted much research towards this task.

Quality loss for fastener head flushness may be written as

$$
\begin{aligned}
L_F &= L_F(D_F(F)) && (4.8)\\
&= L_F(F) && (4.9)
\end{aligned}
$$

Step 3: Identify the significant sources of variability.

As discussed in Section 3.2.1, we know there are variabilities associated with each design characteristic and modeling step. We also know, however, that probabilistic evaluations are computationally expensive and difficult. It is therefore necessary to trim the problem by identifying only the variability sources that have significant impact on quality and cost. Sensitivity analysis is one way of identifying these sources. Otherwise, there is no universal methodology for identifying these variabilities. Here we appeal to the experience and knowledge of the designer.

When we have quantitative variabilities known to be significant, we can mathematically incorporate them into quality and cost analyses. Unfortunately, however, most variabilities we encounter in design are qualitative. For example, it is difficult to quantify fastener drag model uncertainties. I have interviewed several aeronautical specialists at Boeing. I have also reviewed papers dedicated to fastener drag modeling. Depending on the assumptions made, drag models can vary by as much as a factor of 10 or even more in drastic conditions. The truth is difficult to ascertain because fastener drag models cannot be verified in practice. Under these circumstances, we can create upper and lower bound estimations on the uncertainty involved, and perform sensitivity analysis to investigate their effect on overall costs.

Step 4: Model expected quality loss QL.

Taguchi[Tag86] emphasizes that product quality is determined by how far design characteristics deviate from targets, not how they conform to tolerance specifications. Under this model, the expected quality loss resulting from fastener flushness variability is defined by

$$QL_F = \int_{-\infty}^{\infty} L_F(F)\, P(F)\, dF \tag{4.10}$$

where $P(F)$ is the probability density function representing the statistical variation of F. See Figure 4.1.

Some manufacturers are highly dependent on inspection control to limit expected quality loss. Based on my observations at Boeing and elsewhere, I would argue that the combination of statistical variation and inspection limit representations more closely follows what actually takes place in industry. For the flushness example,

$$QL_F = R \int_{F_{min}}^{F_{max}} L_F(F)\, P(F)\, dF \tag{4.11}$$

F_{min} and F_{max} are the lower and upper inspection limits for F, and R is a normal-

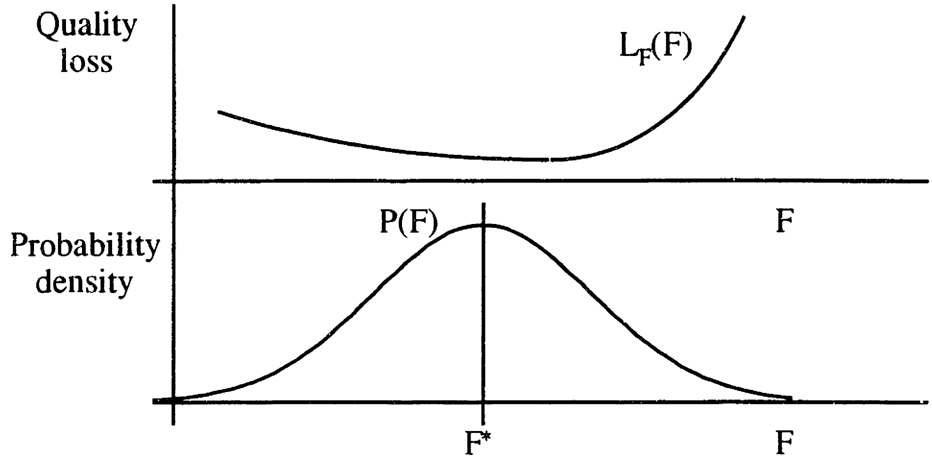


Figure 4.1: The Taguchi quality loss model

izing factor to account for rejection. See Figure 4.2. Inspection control determines the location of F_{min} and F_{max} while statistical variation control determines the shape and location of $P(F)$. The location of $P(F)$ essentially determines what is traditionally known as the design *nominal.* As discussed in Section 2.2.3, most design methods treat nominal design and tolerance design separately. Yet we know from Equation 4.11 that expected quality loss depends highly on the *combination* of nominal values, statistical variation, and inspection limits. In the application of my tolerancing approach in Chapter 6, I demonstrate the significant improvements to be had by applying simultaneous nominal and tolerance design.

Caution when modeling QL

As discussed in Section 3.2.1, there are uncertainties associated with each modeled *relationship* and variabilities associated with each *design characteristic.* Because humans are better at dealing with deterministic models, we often lump variabilities and relations into deterministic models. For example, we often find ourselves fitting deterministic curves to a set of experimental data points because deterministic models are more convenient to use and simpler to understand and communicate. However, extreme care is need when converting nondeterministic models into deterministic ones.

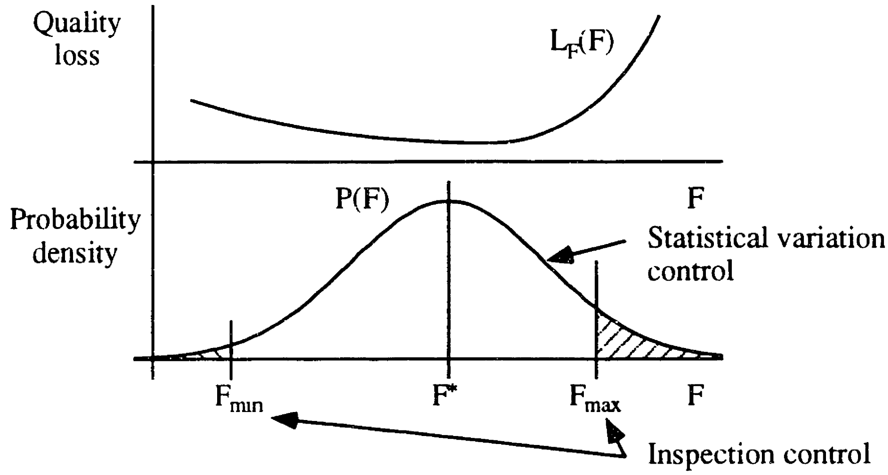


Figure 4.2: Quality loss under the combined representation

Probability law tells us that the expected value of a function does not simply equal the function evaluated at its expected operating point.

$$E[L(x)] \neq L(E[x]) \tag{4.12}$$

E is the probabilistic expectancy operator. The expected value loss of $L(x)$ is often not equivalent to the value loss of the expected x. When we convert from a nondeterministic model to a deterministic one, we often mistakenly make that assumption.

We often define intermediate parameters to simplify design tasks. In complex design environments, it is often difficult to identify which are intermediate parameters and which are the objective performance value losses. It is sometimes confusing to decide on which parameters to perform expectancy operations for deterministic modeling. By identifying expected quality losses as objective parameters for design, we can readily identify which are intermediate parameters. We know where we can perform deterministic modeling. I will demonstrate how much of a difference this can make in Chapter 7.

Step 5: Model expected variability control cost C_V.

C_V is the cost of controlling variability to deliver a desired level of quality loss. In many cases, it is simply the cost of production. In other cases, as we will see in Chapter 7, it includes other costs associated with product service life such as maintenance costs.

C_V is defined by:

$$C_V = C_{SC} + C_{IC} \tag{4.13}$$

where C_{SC} is the cost of controlling the statistical variation of manufacturing processes and C_{IC}, the cost of implementing inspection control.

Step 5.1: Model statistical variation control cost C_{SC}.

C_{SC} may be obtained from empirical manufacturing data. Otherwise we may use cost models to represent C_{SC}. Section 2.1.3 describes a widely applied statistical variation control cost model:

$$C_{SC} = A + \frac{B}{\sigma^k} \tag{4.14}$$

Statistical variation control affects both the variation parameter σ and the nominal μ. However, C_{SC} is a function of σ and no other parameter because there is often little or no cost associated with operating at a different nominal.

Step 5.2: Model expected inspection control cost C_{IC}.

Inspection control cost C_{IC} is the cost required to identify and correct products exhibiting design characteristics outside of the specified inspection control limits:

$$C_{IC} = C_{insp} + C_{corr} \tag{4.15}$$

C_{insp} defines inspection costs including the costs of inspection equipment and labor:

$$\begin{aligned} C_{insp} &= C_{insp\,equip} + C_{insp\,labor} \\ &= N \cdot c_{insp} \end{aligned} \tag{4.16}$$

where c_{insp} is the unit inspection cost and N, the number of total units produced.

C_{corr} includes all costs associated with correcting rejected products. C_{corr} defines costs associated with rework and scrap.

$$\begin{aligned} C_{corr} &= C_{rew} + C_{scrap} \\ &= N_{rew} \cdot c_{rew} + N_{scrap} \cdot c_{scrap} \end{aligned} \tag{4.17}$$

where N_{rew} and N_{scrap} are the number of reworked and scrapped products respectively; and c_{rew} and c_{scrap}, the unit costs of rework and scrap respectively.

Step 6: Minimize *VL* with respect to variability control parameters.

We have thus far identified and modeled cost components defining the total expected value loss VL. The task now is to use this model to determine the most economical combination of variability control specifications.

Tolerancing becomes an optimization problem where the objective is to select a combination of statistical variation and inspection control parameters such that the total expected value loss is minimized:

$$Minimize[VL] = Minimize\,[QL + C_V] \tag{4.18}$$

$$= Minimize \sum_i [QL_i + C_{SC,i} + C_{IC,i}] \tag{4.19}$$

Optimization Illustrated

For illustration, consider the following generic design case. Assume:

- Design variable x follows the Gaussian variation distribution $G(x, \sigma, \mu)$.
- Statistical variation control cost per manufactured unit is $C_{SC} = A + \frac{B}{\sigma^k}$
- Quality loss function for x is quadratic: $L(x) = (x - \mu)^2$.
- The nominal is predetermined at μ_0.
- Inspection limits $\mu_0 - \frac{\Delta x}{2}$ and $\mu_0 + \frac{\Delta x}{2}$ are symmetrical about the nominal.
- All rejected parts are reworked at a fixed unit cost of c_{rew} with no scrap. All reworked parts are assumed to follow $G(x, \sigma, \mu)$.

Then we may rewrite each cost element in Equation 4.19:

$$
\begin{aligned}
QL(\sigma, \Delta x) &= N \frac{1}{1 - R_{rej}} \int_{\mu_0 - \frac{\Delta x}{2}}^{\mu_0 + \frac{\Delta x}{2}} G(x, \sigma \mu_0)\, L(x)\, dx && (4.20) \\
C_{SC}(\sigma, \Delta x) &= N \cdot (A + \frac{B}{\sigma^k}) && (4.21) \\
C_{IC}(\sigma, \Delta x) &= N \cdot (c_{insp} + R_{rej} c_{rew}) && (4.22)
\end{aligned}
$$

where R_{rej} is the ratio of rejected parts resulting from applying inspection control:

$$
R_{rej} = 1 - \int_{\mu_0 - \frac{\Delta x}{2}}^{\mu_0 + \frac{\Delta x}{2}} G(x, \sigma, \mu_0)\, dx \tag{4.23}
$$

The objective function is the sum of these elements:

$$
VL(\sigma, \Delta x) = N \cdot [A + \frac{B}{\sigma^k} + \frac{1}{1 - R_{rej}} \int_{\mu_0 - \frac{\Delta x}{2}}^{\mu_0 + \frac{\Delta x}{2}} G(x, \sigma, \mu_0)\, L(x)\, dx + c_{insp} + c_{rew}(1 - R_{rej})] \tag{4.24}
$$

Examples of each of these costs are illustrated in Figures 4.3, 4.4, 4.5, and 4.6 for the following parameter values: $N = 100$, $\mu_0 = 100$, $K = 2$, $A = 0$, $B = 3$, $c_{insp} = 1$, $c_{rew} = 10$.

Tolerancing is finding the optimal combination of σ and Δx such that VL is minimized.

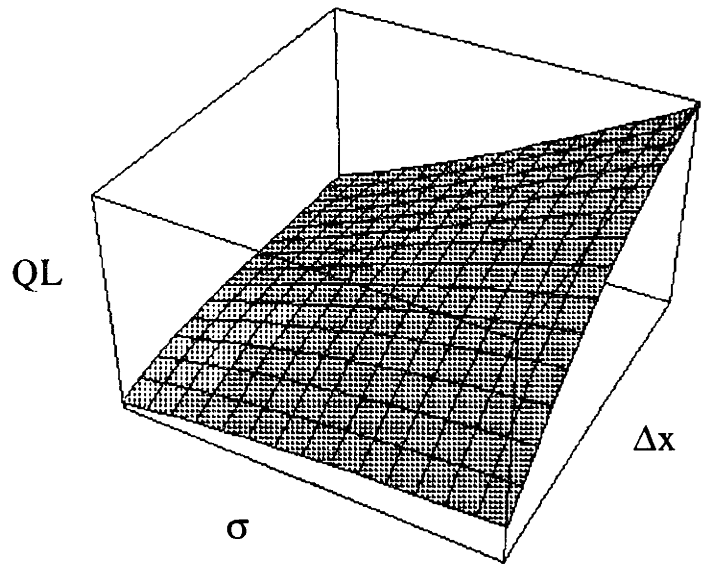


Figure 4.3: Expected quality loss: QL

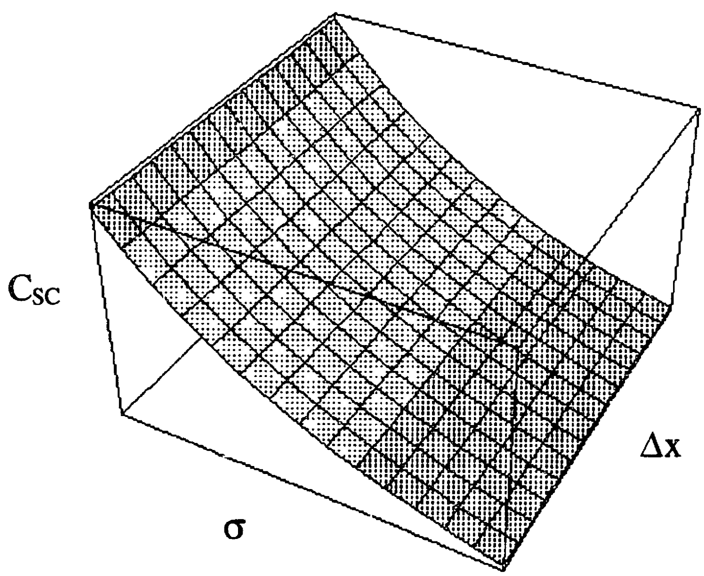


Figure 4.4: Statistical variation control cost: C_{SC}

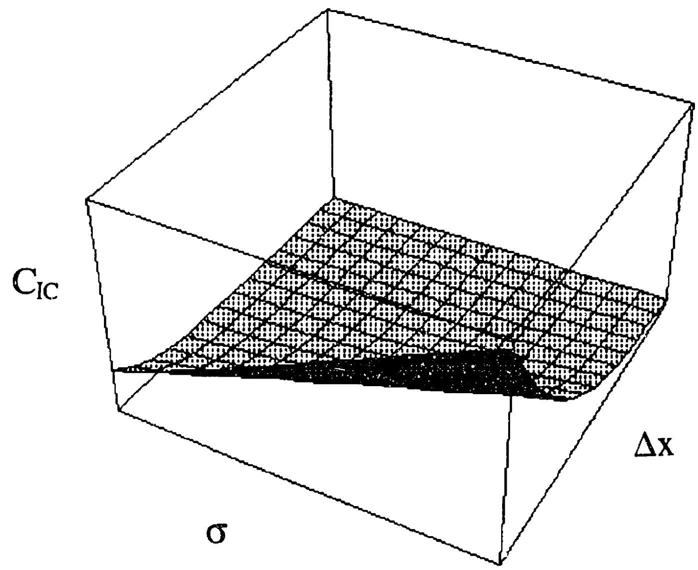


Figure 4.5: Inspection control cost: C_{IC}

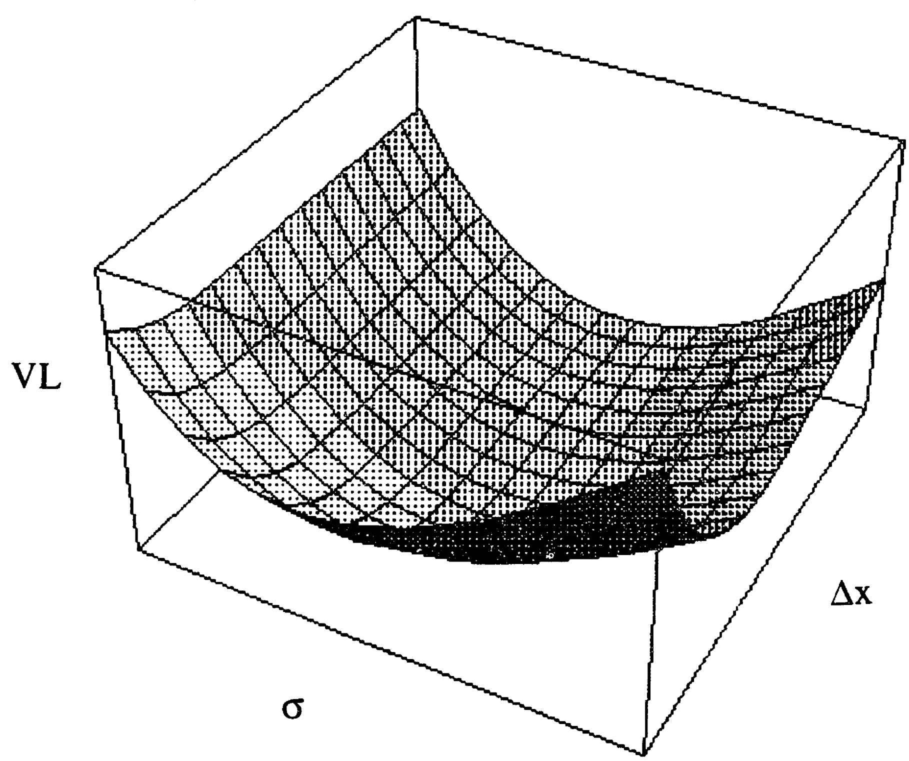


Figure 4.6: Total value loss: $VL = QL + C_{SC} + C_{IC}$

The optimization problem modeled above is different from existing tolerancing procedures for four reasons.

1. The objective function includes both quality loss and variability control costs. Tolerancing had traditionally been a segmented process where the determination of quality requirements and the assessment of variability control costs were conducted separately. This approach allowed designers to make design tradeoffs among performances or among manufacturing options, but not *between* performances and manufacturing options. My proposed methodology allows quantitative tradeoffs between and among performance and manufacturing costs.

2. The variability representation treats statistical variation control and inspection control options separately. This distinction allows the explicit evaluation of quality assurance costs *during* design. The representation also fully prescribes product quality loss expectations.

3. Design of nominals and tolerances can be performed in parallel. Although in the generic example I chose to constrain the nominal at μ_0 for the purpose of graphical illustration, we can choose to leave the nominal as an optimization variable. This allows us to design nominals and tolerances simultaneously.

4. Identifying expected quality loss QL as the objective parameter during modeling eliminates the ambiguity as to which are intermediate design parameters and which are objective parameters. We can readily identify where deterministic modeling operations can be performed.

Difficulties

The application of this methodology in reality is quite difficult. The optimization problem is computationally demanding because objective functions are often nonlinear and involve many variables. However, there are practical guidelines that make the problem more manageable. These guidelines are discussed in Chapter 5.

Chapter 5

Implementation Guidelines

Chapters 3 and 4 present the methodology for representing and specifying economical tolerances. This chapter describes some difficulties we might encounter when implementing the methodology, and demonstrates how tolerances may be specified in practice.

5.1 The independence assumption

The optimization problem had been defined as:

$$Minimize[VL] = Minimize \sum_{i} [QL_i + C_{SC,i} + C_{IC,i}] \tag{5.1}$$

The objective value loss VL consists of several expected quality losses and associated variability control costs. There are many parameters to tolerance. In the current form, this optimization is impractical because the problem is too large to manage computationally.

We can, however, take advantage of the fact that many expected quality losses and variability control costs are independent of others. In the aircraft fastener joint design example, the objective value loss VL consists of two parts: the part describing the fastener flushness tolerancing problem and the part describing the fastener interference tolerancing problem.

$$\begin{aligned} Minimize\,[VL] &= Minimize\,[\; QL_F + C_{SC,F} + C_{IC,F} + \\ &\qquad QL_I + C_{SC,I} + C_{IC,I}] \end{aligned} \tag{5.2}$$

Because costs are strictly additive, we may treat the optimization problem separately if there is evidence that the cost elements are decoupled.

$$\begin{aligned} Minimize\,[VL_F] &= Minimize\,[\; QL_F + C_{SC,F} + C_{IC,F}] \\ Minimize\,[VL_I] &= Minimize\,[\; QL_I + C_{SC,I} + C_{IC,I}] \end{aligned} \tag{5.3}$$

This way we can sometimes break up large problems into smaller, more manageable optimization problems.

5.2 Requirement for only relative costs

Absolute value loss and cost functions are often difficult or even impossible to formulate. For example, it is difficult to model exactly how much a manufacturing process costs. We can model the costs of equipment, labor, and materials, but many companies in industry have no methods for estimating and allocating costs such as overhead of different manufacturing functions. It is relatively simple, on the other hand, to obtain *relative* costs that describe the cost differences *between* different manufacturing process options.

The objective of my tolerancing methodology is to maximize the value created V_C, or to minimize the total value loss VL. The implication is that I assume V_C (VL) is already positive (negative), and the design task is to merely maximize V_C (minimize VL) without having to quantify V_C (VL) in absolute terms. Whether we use absolute or relative costs, we arrive at the same optimal tolerance specification. This methodology requires only the modeling of relative quality losses and costs.

5.3 Estimating relative manufacturing costs

When costs are not immediately available, good estimates may be obtained by observing the manufacturing processes first hand. Manufacturing activities may be documented and used for cost modeling. The following questions are very helpful in quickly assessing the relative costs of different manufacturing processes. What is the objective of the process? What affects the process? How many workers are involved? What type of equipment in needed? How long does the process take? What else would do the job? What does that alternative cost?

5.4 Discrete variability

5.4.1 Discrete process variation

I have proposed that the expected quality loss resulting from performance degradation needs to be weighed against the costs of attaining that level of quality loss. Determining tolerances for multi-component assemblies with this approach can quickly become a large optimization problem. Consider the task of tolerancing an assembly. In theory there are many if not infinite combinations of component variabilities resulting in the same assembly variability. This implies that local optimizations are required to first find the minimum variability control cost combination of component variabilities for each level of assembly variability. Then global optimizations are required to find the minimum combination of expected assembly quality loss and variability control cost. Considering the number of components often found in an assembly and considering the combinatoric growth of their combinations, this nested optimization problem suddenly gets very large.

For example, given the task of specifying fastener flushness tolerance ΔF, we need to compare fastener drag costs against the costs of manufacturing. We know that component tolerances of fastener head height HH and panel countersink depth

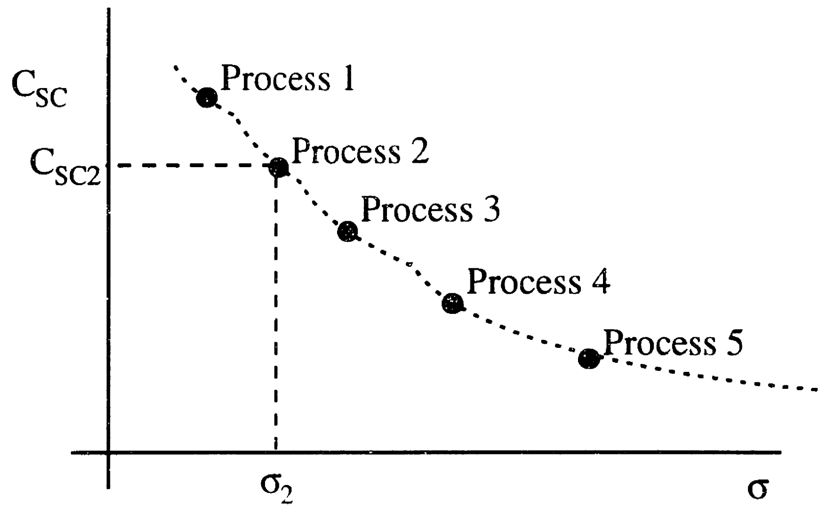


Figure 5.1: Discrete process control cost

CD directly affect the assembly dimension of flushness F. In theory there are infinite combinations of ΔHH and ΔCD resulting in the same flushness tolerances ΔF. For each level of ΔF, we need to perform optimization to determine the most cost-effective combination of ΔHH and ΔCD. Once we find the minimum cost combinations for each level of ΔF, we need to conduct another optimization to minimize *VL* by trading off drag quality loss against the corresponding levels of ΔF. A seemingly simple tolerancing problem has turned out to be quite complex.

Note, however, that in reality there are only a limited number of process variability options. Manufacturers have only a limited number of production processes available to them. There are discrete manufacturing processes capable of providing discrete levels of process control. Instead of the traditional continuous cost curves shown as a dotted line in Figure 5.1, we have a limited number of process control options as shown by the dark dots.

This discretized model is especially helpful when considering assembly tolerance specifications because discrete component variabilities limit the number of assembly variability options. For example, if there are n_{HH} and n_{CD} numbers of variability options for head height and countersink depth respectively, there are only $n_{HH} \cdot n_{CD}$

number of flushness variability options to consider. Typically, n_i is quite small. For example, there are three basic ways to control fastener hole diameters: 1) drill, 2) drill and ream, 3) drill, ream, and bore. The discrete nature of process variability considerably condenses the optimization problem. In this sense, tolerancing becomes a *selection* problem instead of an involved optimization problem.

5.4.2 Discrete inspection control limits

Inspection control limits may also be considered discrete. It is not practical in many cases to inspect parts to extended decimal places. Limitations such as gauge precision allow us to effectivly model inspection limits in discrete increments.

5.5 Symmetric performance and variability models

Symmetry in quality loss and variability functions can considerably alleviate the tolerance optimization task. Quality loss functions can be assumed to be symmetric for practical purposes when the range of assumed symmetry can be bounded by the interval where the significant portion of the expected manufacturing probability function lies. See Figure 5.2. Then we may fix the nominal x^* at the minimum loss point on the line of symmetry, and represent tolerance limits (x_{min}, x_{max}) by a single variable Δx:

$$\Delta x \quad = \quad 2 \cdot (x^* - x_{min}) \quad = \quad 2 \cdot (x_{max} - x^*) \tag{5.4}$$

We have narrowed three parameters down to just one by taking advantage of the problem's natural near-symmetry.

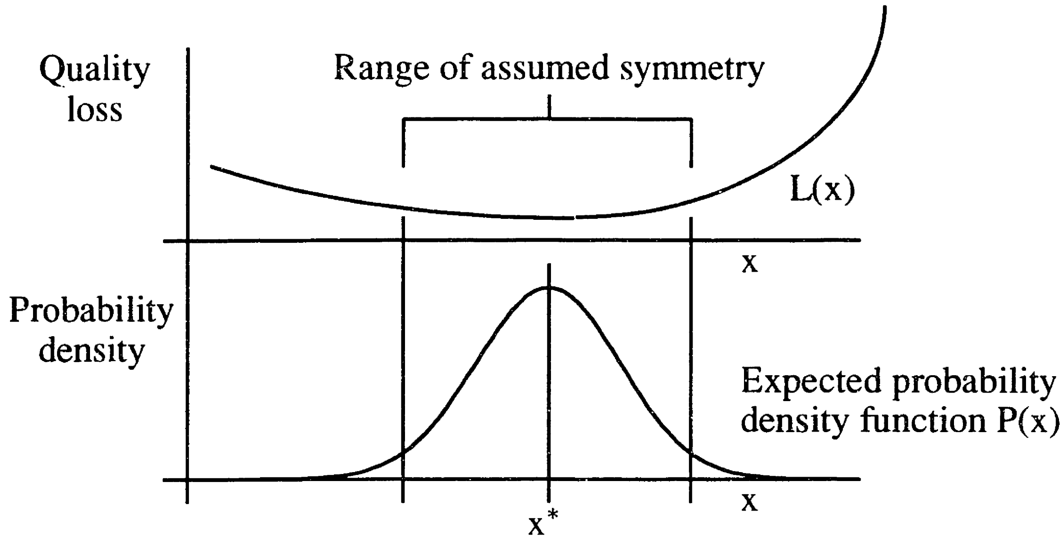


Figure 5.2: Criteria for symmetry

5.6 Uncertain variability models

When variability models are unavailable or vague, it is difficult to represent and use them in mathematical analyses. In these situations, Chase [CP91] advocates using uniform distributions to represent design uncertainties and Gaussian distributions to represent manufacturing variation. He claims these distributions are adequate for performing first-cut analyses. As better models are found, they may be substituted.

Another way to deal with uncertain variability models is to form lower and upper bound limits for design parameters and conduct bounded analyses to investigate the range of cost variation. If the uncertainty leads to large variations in performance and cost, it is a good indication that better models are needed. In that case, resource allocation for better modeling is advised.

5.7 Predetermined specifications

One of this thesis' main themes is balancing performance against manufacturing cost. Sometimes, however, performance specifications are binary and not negotiable. For instance, when we lack proper performance models, we sometimes form discrete limits on design characteristics and conduct bounded analysis to evaluate performance. In other cases, such binary specifications are customer defined. It is common for people to specify, "it has to be accurate to x," or "it can't weigh more than y," or "it can't cost more than $z."

Sometimes we fix performance specifications for practical reasons. When optimization problems get too large, or when several optimized performance variables end up nullifying each other, predetermined performance specifications are necessary to keep the design tasks in focus. Chapter 7 demonstrates one such case.

Fixed performance specifications limit our freedom to explore the design space that would otherwise be available, but such specifications also alleviate much of the design problem by narrowing down our search space. The first example in Chapter 6 demonstrates a case where performance is treated as a flexible variable. The second example in Chapter 7, describes a case where design requirements are fixed for practical reasons. The third example in Chapter 8 describes a case where even the "goalpost" tolerances are pre-specified.

Chapter 6

Flushness Tolerancing Example

This chapter implements the tolerancing methodology discussed in Chapters 3 and 4 on the fastener flushness tolerancing example. I will compare the costs of current tolerances against those resulting from the application of my methodology.

What this example demonstrates

1. Concurrent design of nominals and tolerances: This example demonstrates the impact of concurrent nominal and tolerance design.

2. Effectiveness of dual tolerance representation: The combination of statistical variation control and inspection control methods result in more economical tolerances.

3. Tradeoff between performance and cost: Performance is a variable in this example. The flexibility to trade off performance against manufacturing cost results in more economical tolerances.

Current specifications and manufacturing processes

Figure 6.1 shows the current flushness tolerance specifications for 747-400 wing skin splice joints. When assembly mechanics install wing fasteners, they apply the manufacturing procedures shown in Figure 6.2 for controlling fastener flushness. All processes are conducted manually.

	Nominal [in]	± Tolerance [in]
Flushness F	0.006	0.003

Figure 6.1: Current flushness specifications

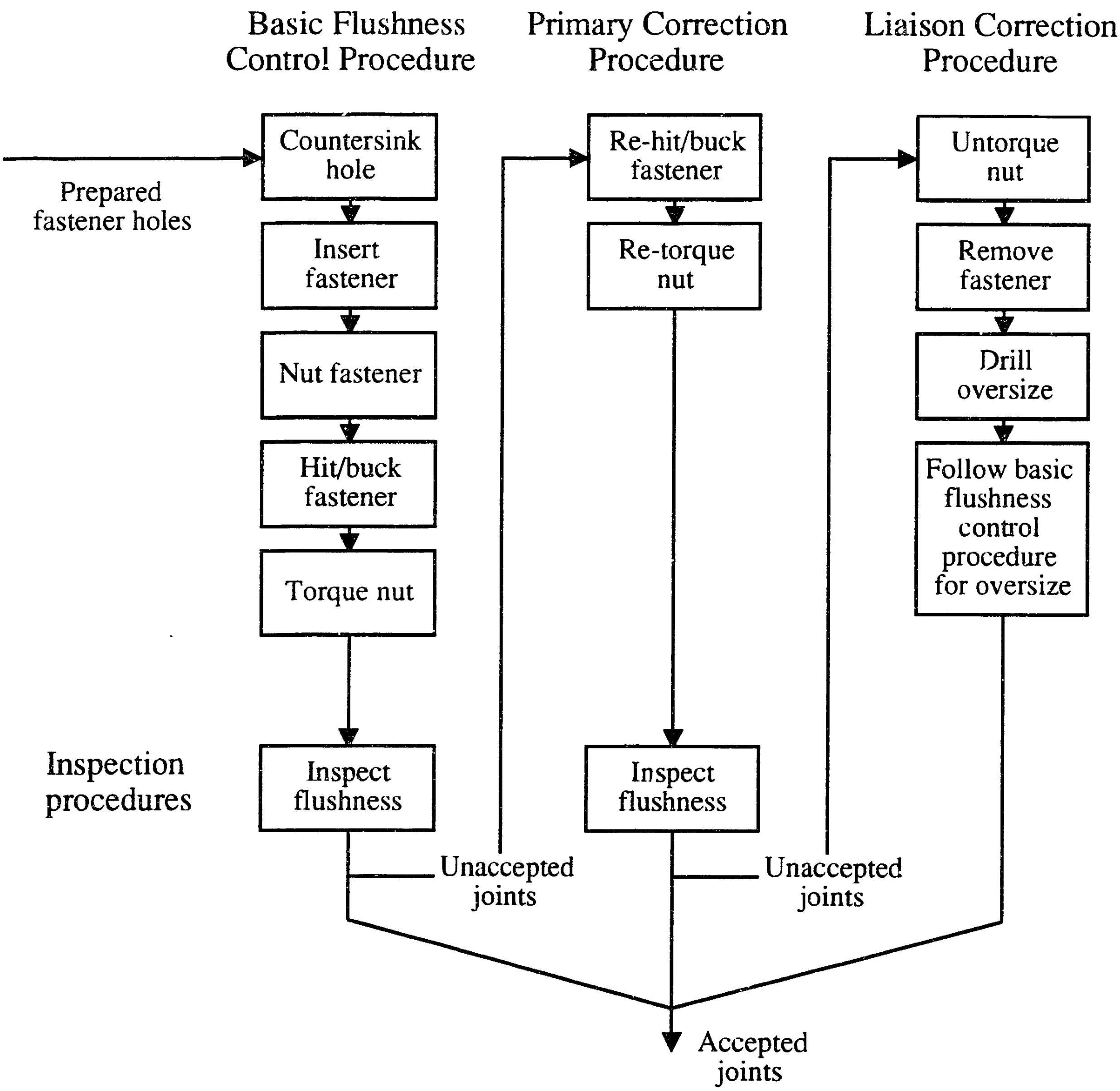


Figure 6.2: Production processes for flushness control

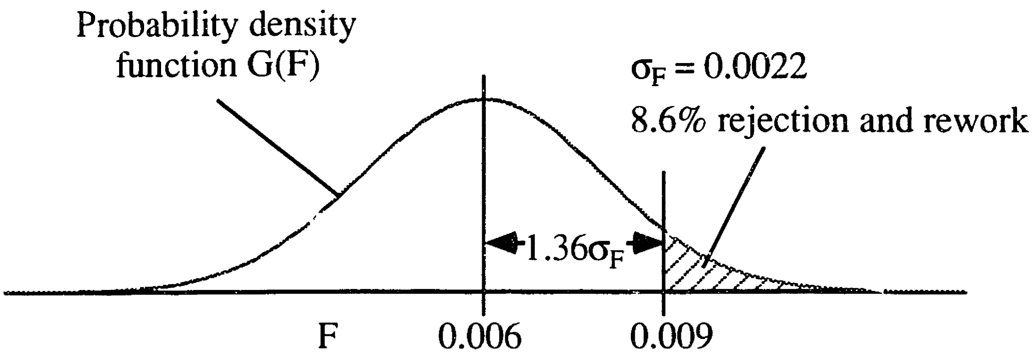


Figure 6.3: Currently applied inspection control

Basic flushness control procedure

The illustrated *basic flushness control procedure* essentially defines the statistical variation of fastener flushness. The *basic* procedure consists of: the countersinking of predrilled holes, inserting fasteners, putting nuts onto fasteners, bucking[1] to set fasteners, then torqueing the nut twice to control clamping forces.

The *basic* procedure currently applied on the 747-400 exhibits a Gaussian distribution targeting the nominal of $F^* = 0.006$ with a standard deviation $\sigma_F = 0.0022$. From the resulting flushness distribution, we expect theoretical nonconformance rates of about 17%, divided equally between the lower and upper tails of the distribution. In practice, lower tail nonconforming fasteners are tolerated while upper tail fasteners are identified and reworked. See Figure 6.3. Lower end nonconforming fasteners are tolerated for two reasons. First, it is extremely difficult and expensive to rework fasteners that have been driven too far into the panel. It involves removing the original fastener, drilling a larger diameter hole, countersinking, and finally installing an oversize fastener. Considering there are approximately 3,400 lower end nonconforming fasteners that would have to be reworked, Boeing chooses to tolerate them. Second, fasteners that have been driven in too far result in less drag than fasteners

[1]Bucking is the manufacturing process of hammering fasteners into fastener holes using pneumatic rivet guns.

protruding out too far. Given the smaller drag penalties associated with lower tail nonconforming fasteners, Boeing chooses to divert its efforts to controlling upper tail nonconforming fasteners.

Primary correction procedure

The upper tail nonconforming fasteners are inspected, identified, and reworked first by the *primary correction procedure.* Although mathematically we expect about 8.6% of the fasteners to be reworked, Boeing actually reworks about 10% of the fasteners. This difference might be attributed to the fact that fastener flushness inspection is not a well controlled process itself. Flushness is usually inspected visually, or by running a hand over the protruding fastener. Compliance is left to the judgment of quality assurance inspectors. For modeling purposes, we note that 10% rejection rate is equivalent to placing the inspection limit at $F_{max} = 0.0088$. Also note here that flushness inspection is a binary activity: nonconforming fasteners are identified and documented, but the *degree* of noncompliance is not recorded.

Approximately 4,000 upper tail nonconforming fasteners are inspected out and reworked per aircraft. The *primary correction procedure* is relatively simple: 1) non-conforming fasteners are identified and marked, 2) fasteners are bucked with rivet guns to drive their heads farther into the panel, and 3) fastener nuts are retorqued.

Engineering liaison correction procedure

Approximately 10% of the 4,000 reworked fasteners or about 1% of the entire population of 40,000 fasteners fail to meet the upper inspection limit even after the *primary correction procedure.* These 400 fasteners fail inspection requirements the second time because the *primary correction procedure* cannot correct fastener installations that exhibit excessively high flushness profiles. This is because bucking excessively protruding fasteners to acceptable flushness results in panel material deformations that create "volcanos" around the fastener. Note that these fasteners could have been identified immediately after the *basic* procedure if the inspection technique had not been binary.

The 400 nonconforming fasteners are processed through the *liaison correction procedure*, so called because this process typically requires interaction between manufacturing and engineering to assure structural integrity of the reworked joint. As shown in Figure 6.2, this process requires significant manufacturing effort. Nonconforming fasteners are removed. Holes are re-drilled to larger diameters and countersunk to larger dimensions. Then larger and heavier fasteners are put into the hole.

Step 1: Define objective value loss VL.

Boeing's objective is to design and produce an aerodynamic aircraft at low cost. As described in Section 3.3.2, the objective is to maximize the value created V_c, or to minimize the value loss VL. VL consists of two parts. First, the expected quality loss QL describes the cost of operating an aircraft with high drag. Second, the variability control cost C_V describes the cost of delivering that level of QL. The sum of these two components defines the objective value loss:

$$VL = QL + C_V \tag{6.1}$$

Step 2: Model quality loss.

The model for drag penalty as a function of flushness has been supplied by the Boeing Aeronautics group and is shown in Figure 6.4. Numbers have been omitted to protect proprietary information. This figure indicates there is an optimum flushness level at F'. As fasteners deviate from this flushness level, drag force $D_F(F)$ increases nonlinearly as airflow streamlines are diverted. D_F is represented in units of weight [lb], the lift loss resulting from additional excrescence drag. The vertical axis may be thought of as equivalent amounts of additional aircraft weight resulting from fastener

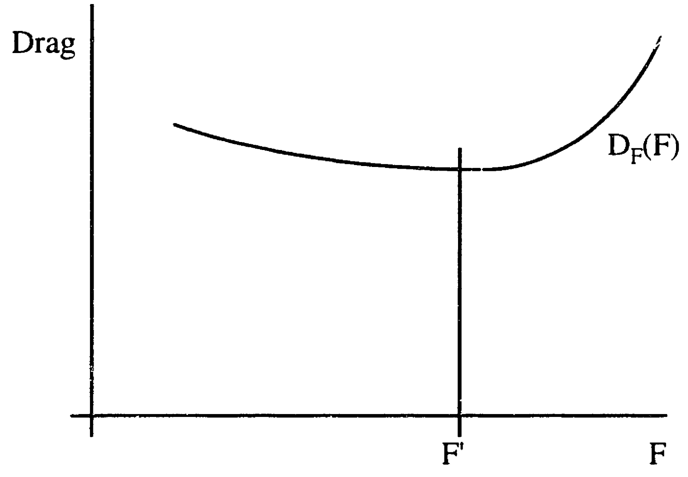


Figure 6.4: Drag function for fastener flushness

drag.

Boeing customers claim each additional pound of weight added onto an aircraft results in additional fuel costs of approximately \$600 over the service life of the aircraft. Quality loss can then be modeled as

$$L_F(F) = 600\, D_F(F) \tag{6.2}$$

Step 3: Identify the significant sources of variability.

There are innumerable sources of variability contributing to the final flushness variability. First, there are fastener variabilities. Almost a dozen different vendors supply these fasteners. Because of the diverse manufacturing control techniques used, the general population of fasteners exhibits high variability. Fastener heads exhibit a statistical variation of: $6\sigma_{HH} = 0.0045$ inch. There are also fastener thread runout and nut thread runout variabilities leading to clamp force variability, which in turn affects flushness variability.

There are also variabilities associated with the installation process. The first is tool variability. Although mechanics use the same *types* of tools, individual tool variability and setup imprecision result in variabilities that affect countersink travel distance, rivet gun power, and nutting torque. The second source of installation variability is human variability. The individual styles of each mecahnic lead to variabilities in countersink depth, bucking forces, and torqueing effort.

However, it is impractical to address each of these variabilities individually. Rather, we look at the generalized form of flushness variability. The probability density function describing fastener flushness is

$$P(F) = G(\sigma_F, F^*) \tag{6.3}$$

Step 4: Model expected quality loss QL.

When we consider flushness variability without inspection control, the expected quality loss QL resulting from drag is

$$\begin{aligned} QL &= E[L_F(F)] && (6.4) \\ &= E[600\, D_F(F)] && (6.5) \\ &= 600\, E[D_F(F)] && (6.6) \\ &= 600\, N \int_{-\infty}^{\infty} D_F(F)\, G(\sigma_F, F^*)\, dF && (6.7) \end{aligned}$$

where N is the total number of installed fasteners. The cost coefficient of 600 may be brought out of the integral because it is a constant. This is a very simple instance where $E[f(x)] = f(E[x])$ because f is a linear function of x.

When we consider inspection control, QL gets considerably more complex. We must first make some assumptions.

Assumptions

1. I assume fasteners corrected by the *primary correction procedure* end up at

the upper specification limit of F_{max}. This is a reasonable assumption since mechanics re-buck rejected fasteners just enough to pass inspection the second time.

2. I also assume the *primary correction procedure*'s maximum effective flushness correction range is 0.0015 inch. Fasteners initially protruding more than $F_{max}+0.0015$ will ultimately require *liaison correction*. This assumption is consistent with current rejection and rework rates.

3. I assume that fasteners corrected by the *liaison correction procedure* follow a flushness distribution similar to the *basic* installation distribution. This is also reasonable because the same installation procedures are followed for the oversize fasteners, only with different tools.

4. For simplicity, I assume that oversize fasteners exhibit drag characteristics similar to the original fastener drag $D_F(F)$.

5. I also assume that inspection continues to be binary: there is no way of telling which fasteners will require *liaison correction* until the *primary correction procedure* is completed.

6. I also assume no fasteners are inspected or reworked after the *liaison correction procedure.*

The number of fastener joints rejected immediately following *basic* installation is described by

$$N_p = N \int_{F_{max}}^{\infty} G(\sigma_F, F^*)\, dF \tag{6.8}$$

The subscript p in N_p is used to indicate that this is the number of fastener joints processed by the *primary correction procedure.*

The number of fastener joints rejected after the *primary correction procedure* is

$$N_l = N \int_{F_{max}+0.0015}^{\infty} G(\sigma_F, F^*)\, dF \tag{6.9}$$

The subscript l in N_l is used to indicate that this is the number of fastener joints processed by the *liaison correction procedure.*

Activity	Time [hours]
Basic procedure	$t_b = 0.0125$
Primary correction	$t_p = 0.0083$
Liaison correction	$t_l = 0.1250$

Figure 6.5: Required processing time

Then we model expected quality loss as

$$\begin{aligned} QL &= 600\,(N - N_p)\frac{N}{N - N_p}\int_{-\infty}^{F_{max}} D_F(F)\,G(\sigma_F, F^*)\,dF + \\ &\quad 600\,(N_p - N_l)\,D_F(F_{max}) + \\ &\quad 600\,N_l \int_{-\infty}^{\infty} D_F(F)\,G\,(\sigma_F, F^*)\,dF \end{aligned} \tag{6.10}$$

The first term represents the drag penalty resulting from $N - N_p$ fasteners that pass the first inspection. The second term represents fasteners that are corrected by the *primary correction procedure* and are considered acceptable. The third term represents the population of oversize fasteners.

Step 5: Model expected variability control cost C_V.

Step 5.1: Model statistical variation control cost C_{SC}.

I created cost models for the three procedures used in controlling flushness. Figure 6.5 shows the estimated time (per fastener) required for each of the three procedures. The cost models require a few assumptions.

Assumptions: I assume the only practical way to control statistical variation is to control the skill level of the mechanics. The tools, materials, and parts are all assumed to remain constant. I have gathered data showing how better trained mechanics yield better statistical variation control. These mechanics are often rewarded

Skill level	statistical variation control	Labor rate [$/hour] (Including overhead)
Skilled	$\sigma_F = 0.0018$	40
Current	$\sigma_F = 0.0022$	30
Unskilled	$\sigma_F = 0.0024$	25

Figure 6.6: Cost of labor by skill levels

by being promoted to better programs such as the 777 program, or are monetarily compensated. Figure 6.6 shows the estimated cost of labor according to different skill and statistical variation control levels.

Under these assumptions, the relative[2] cost of statistical variation control C_{SC} depends on the time required to complete the *basic* installation procedure t_b, the labor rate $w(\sigma_F)$ and the total number of fasteners N:

$$C_{SC}(\sigma_F) = N \cdot t_b \cdot w(\sigma_F) \tag{6.11}$$

$$= 500 \cdot w(\sigma_F) \tag{6.12}$$

Step 5.2: Model expected inspection control cost C_{IC}.

The cost of inspection control C_{IC} has two components: the cost of inspection and the cost of correction. There are no special tools used in inspecting flushness. Therefore, the cost of inspection depends only on the labor required. Assuming a fixed inspection labor rate of $40/hour and an inspection rate of 720 fasteners an hour,

$$C_{insp} = N \cdot c_{insp} \tag{6.13}$$

$$= N\frac{40}{720} \tag{6.14}$$

$$= 0.056N \tag{6.15}$$

[2]As discussed in Section 5.2, we care only about *relative* costs.

Thus it costs approximately \$2,200 to complete inspection immediately following the *basic* procedure. It also costs approximately $0.056 \cdot N_p$ to inspect fasteners following the *primary correction procedure.* Total inspection cost is then

$$C_{insp} = 2,200 + 0.056 N_p \tag{6.16}$$

As long as we choose to implement inspection control, we can eliminate the fixed cost of \$2,200 because we again assume that only relative costs are significant. Furthermore, the variable part is hardly significant. In the current application where $N_p \approx 4000$, C_{insp} turns out to be a little over \$220. We can therefore consider C_{insp} negligible.

We proceed to the second element of C_{IC}: the cost of correction. C_{corr} has two parts: 1) the *primary correction* part and 2) the *liaison correction* part.

$$C_{corr} = C_{corr,p} + C_{corr,l} \tag{6.17}$$

$\mathbf{C_{corr,p}}$: Since there is no scrap in the *primary correction procedure*, $C_{corr,p}$ consists of only the costs associated with labor. Assuming a fixed rework labor rate w_f of \$30/hour,

$$C_{corr,p} = N_p \cdot t_p \cdot w_f \tag{6.18}$$

$$= 0.25\, N_p \tag{6.19}$$

$\mathbf{C_{corr,l}}$: $C_{corr,l}$ includes the cost of rework labor and additional parts. Replacement oversize fasteners cost about \$4/unit. Assuming a fixed rework labor rate w_f of \$30/hour,

$$C_{corr,l} = N_l \cdot (w_f \cdot t_l + 4) \tag{6.20}$$

$$= 7.75 \cdot N_l \tag{6.21}$$

Combining Manufacturing Cost Components

We combine variability control cost elements:

$$\begin{aligned} C_V &= C_{SC} + C_{IC} && (6.22) \\ &= 500\, w\,(\sigma_F) + 0.25\, N_p + 7.75\, N_l && (6.23) \end{aligned}$$

In expanded form,

$$\begin{aligned} C_V = \; & 500w(\sigma_F) + \\ & 10,000 \int_{F_{max}}^{\infty} G\,(\sigma_F, F^*)\, dF + \\ & 310,000 \int_{F_{max}+0.0015}^{\infty} G\,(\sigma_F, F^*)\, dF \end{aligned} \tag{6.24}$$

Step 6: Minimize VL with respect to variability control parameters.

Combining the performance and manufacturing cost elements we obtain:

$$\begin{aligned} VL(\sigma_F, F_{max}, F^*) = \; & 500\, w\,(\sigma_F) + 0.25\, N_p + 7.75\, N_l + \\ & 600\,(N - N_p) \frac{N}{N - N_p} \int_{-\infty}^{F_{max}} D_F(F)\, G\,(\sigma_F, F^*)\, dF + \\ & 600\,(N_p - N_l)\, D_F(F_{max}) + \\ & 600\, N_l \int_{-\infty}^{\infty} D_F(F)\, G\,(\sigma_F, F^*)\, dF \end{aligned} \tag{6.25}$$

The objective is to select a combination of $\{\sigma_F, F_{max}, F^*\}$ such that VL is minimized.

I have used Mathematica[3] to symbolically and numerically evaluate tolerance options. The following sections describe the results.

[3]Software from Wolfram Research, Inc.

σ_F [in]	F_{max} [in]	F^* [in]
0.0022	0.0088	0.006

Figure 6.7: Current flushness variability control practice

C_{SC} [$]	C_{IC} [$]	QL [$]	VL [$]
15,000	8,689	20,348	44,036

Figure 6.8: Estimated costs of current variability control practice

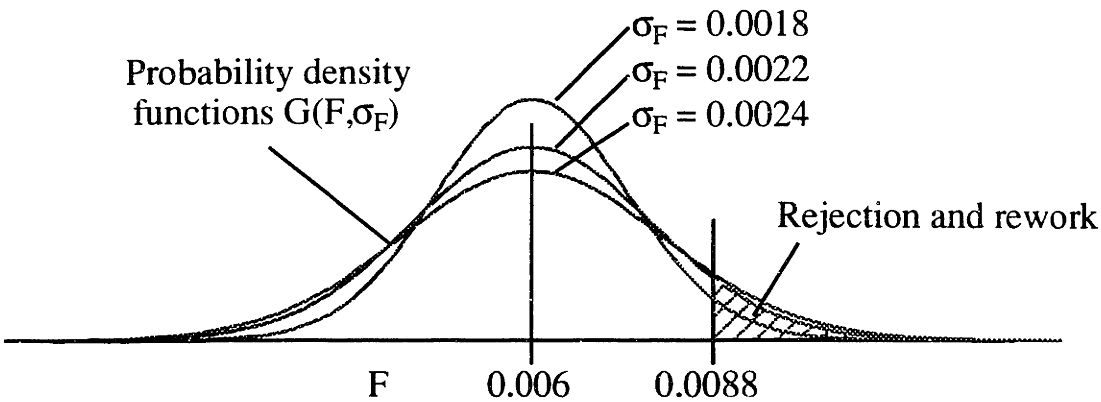
Current practice

Figure 6.7 tables the tolerance parameters currently applied for the production of the 747-400. Using these values, we obtain the cost elements shown in Figure 6.8. These costs are intended as references for benchmarking. Their absolute values are not significant.

Variable statistical variation control

To minimize value loss VL, we will examine a variety of parameter values and check the results. First we look at the simplest alternative to current practice. We allow statistical variation control to vary according to the three discrete statistical variation control processes shown in Figure 6.6. We leave F_{max} and F^* fixed at current values of 0.0088 and 0.006 respectively. Figure 6.9 shows the statistical variation control options and their resulting costs.

The middle row with $\sigma_F = 0.0022$ represents current practice at Boeing. We observe that by implementing a tighter statistical variation control of $\sigma_F = 0.0018$, we can reduce total value loss VL by approximately $1,900 for the 40,000 fasteners considered. Implementing tighter statistical variation control increases C_{SC}, but we obtain larger savings in inspection control cost C_{IC}. Rework is significantly reduced by implementing tighter statistical variation control. We also obtain savings from



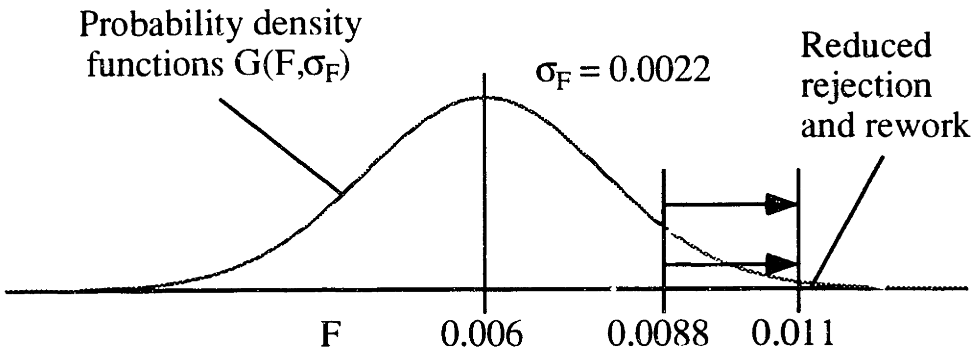
statistical variation control σ_F	C_{SC} [$]	C_{IC} [$]	QL [$]	VL [$]
0.0018	20,000	3,130	19,028	42,158
0.0022	15,000	8,689	20,348	44,036
0.0024	12,500	12,345	20,910	45,755

Figure 6.9: Cost alternatives for variable statistical variation control

reduction of the expected quality loss QL. If we move to a looser statistical variation control of $\sigma_F = 0.0024$, the opposite is true.

Variable inspection control

We now look at another simple alternative to current practice. We allow the upper inspection control limit F_{max} to vary while fixing σ_F and F^* at current values of 0.0022 and 0.006 respectively. This leads to the estimated costs shown in Figure 6.10. The upper row with $F_{max} = 0.0088$ represents current practice. Observe that by moving the upper inspection limit to $F_{max} = 0.0112$, we can reduce total value loss VL by approximately $4,300. Increasing the inspection limit penalizes performance loss, but we obtain almost twice the savings in reduced rework.



Optimum inspection limit F_{max}	C_{SC} [$]	C_{IC} [$]	QL [$]	VL [$]
0.0088	15,000	8,689	20,348	44,036
0.0112	15,000	459	24,256	39,715

Figure 6.10: Cost alternative for variable inspection control

statistical variation control σ_F	Inspection limit Optimum F_{max}	C_{SC} [$]	C_{IC} [$]	QL [$]	VL [$]
0.0018	0.0106	20,000	154	20,619	40,773
0.0022	0.0112	15,000	459	24,256	39,715
0.0024	0.0114	12,500	749	26,251	39,500

Figure 6.11: Cost alternatives with variable statistical variation control and inspection control

Variable statistical variation control and inspection control

I have also investigated the alternative of varying both statistical variation control and inspection control while fixing F^* at 0.006. Figure 6.11 shows the estimated costs. The least expensive option is the statistical variation control of $\sigma_F = 0.0024$ and inspection control limit of $F_{max} = 0.0114$. There is a small gain from the added flexibility of varying both process and inspection control.

Variable statistical variation control and F*

So far, F^* had been held constant at its specified value of 0.006. Now we look at the effect of varying both σ_F and F^* simultaneously.

First we observe one characteristic common to all scenarios discussed above. In every optimization where F_{max} is allowed to vary, the inspection limit is pushed out towards the upper tail of the distribution where its effectiveness in assuring product quality is reduced. This occurs because inspection control costs are relatively high. My initial analysis for this section included varying F_{max} as well as σ_F and F^*. Upon investigation, however, I observed that inspection control has virtually zero effect on total value loss VL when F^* is allowed to vary around its optimum operating range. For example, I conducted two separate optimizations for the current statistical variation control of $\sigma_F = 0.0022$:

1. varying both the nominal F^* and inspection limit F_{max} and

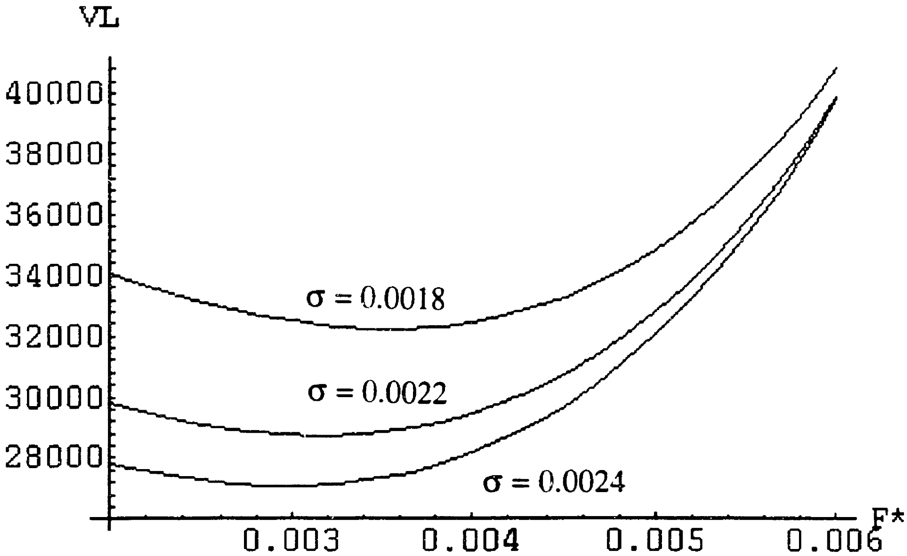
2. varying only F^* with F_{max} placed at effectively ∞.

In the first case, I found the optimum combination of F^* and F_{max} to be 0.0032 and 0.0106 respectively. In the second case, I found optimum F^* also to be 0.0032. The difference between the two is very small about the optimum area of $F^* = 0.0032$.

In consideration of the negligible difference, F_{max} was placed at ∞ for this analysis. When there is no inspection control, Equation 6.25 may be rewritten as:

$$VL(\sigma_F, F^*) \quad = \quad 500\, w(\sigma_F) + 600\, N \int_{-\infty}^{\infty} D_F(F)\, G(\sigma_F, F^*)\, dF \qquad (6.26)$$

Figure 6.12 shows VL as a function of F^* for different values of σ_F. The optimum process specification is $\sigma_F = 0.0024$ and $F^* = 0.0030$. Boeing may save approximately \$15,300 by changing the target nominal F^* to 0.0032, or approximately \$17,000 by changing the target nominal F^* to 0.0030 and statistical variation control σ_F to 0.0024.



statistical variation control σ_F	Inspection limit Optimum F^*	C_{SC} [$]	C_{IC} [$]	QL [$]	VL [$]
0.0018	0.0035	20,000	0	12,204	32,204
0.0022	0.0032	15,000	0	13,726	28,726
0.0024	0.0030	12,500	0	14,571	27,071

Figure 6.12: Variable nominal and statistical variation control

The important point to note here is that as statistical variation changes, so does the optimal nominal point. As discussed in Section 2.2.3 design methods often treat nominal design and tolerance design separately. Most robust design techniques advocate using liberal initial tolerances to first determine and fix the nominal, then to determine the tolerances. This example demonstrates that there are benefits to designing nominals and tolerances simultaneously.

Recommendations

Eliminate inspection control

As discussed above, the inspection control limit is pushed out towards the flushness range where its effect on quality becomes negligible. When we eliminate inspection control altogether, we obtain savings in inspection, labor, and most importantly, the *fixed costs* associated with implementing inspection control. Although unmodeled in this example, the additional cost savings resulting from eliminating inspection control entirely is expected to be significant. It is therefore recommend that Boeing eliminate inspection control of flushness altogether.

Shift the nominal

A valid response to the analyses presented above is: what accounts for the large discrepancy between how Boeing currently controls fastener flushness and what my optimizations recommend, particularly in terms of the specification of F^*?

One factor is that the drag penalty model $D_F(F)$ has evolved over time. The original design was specified when initial models for $D_F(F)$ exhibited minimum drag at $F = 0.0055$. Over time, engineers realized fastener drag characteristics change significantly when wing surfaces are painted. Then, the minimum drag flushness value shifts approximately 0.0012 inch lower to the current minimum drag flushness of 0.0043. The drag function also changes such that it becomes significantly more

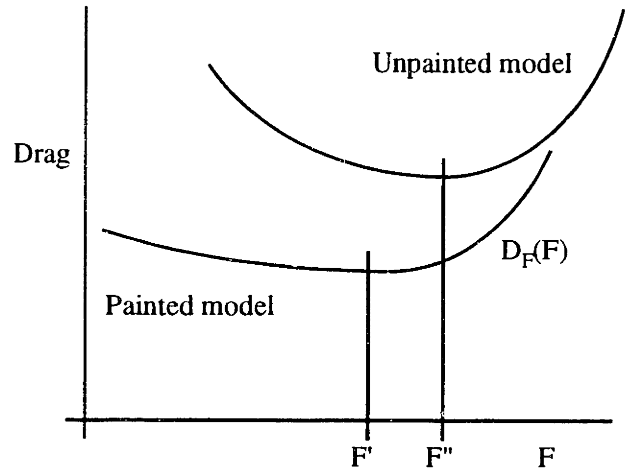


Figure 6.13: Different drag models

asymmetrical. See Figure 6.13. As indicated in Section 5.5, non-symmetric quality loss functions require concurrent design of nominals and tolerances.

Another reason why the current nominal differs significantly from my recommendation is that fatigue performance is also affected by flushness. Best fatigue performance may be obtained when $F = 0.006$. To limit the size of the problem, I have chosen to decouple flushness from fatigue. Nevertheless, we can use the cost analyses presented above to get an indication of how suboptimal current flushness specifications are in terms of aerodynamic performance. If designers are willing to spend \$15,300 to obtain the fatigue performance they are getting by specifying $F^* = 0.006$, the current nominal is justifiable. If not, they will reach a more economically beneficial point by trading one aspect of performance for another. I highly recommend that Boeing investigate the fatigue issues in designing the fastener flushness nominal.

Total value loss is highly sensitive to small changes in the nominal. Shifting the nominal creates hugh savings in terms of fuel costs, and allows for looser statistical variation control. I recommend that Boeing seriously consider re-specifying the nominal.

Chapter 7

Fastener Interference Tolerancing Example

This chapter implements the tolerancing methodology discussed in Chapters 3 and 4 on the fastener interference tolerancing example.

What this example demonstrates

1. Nondeterministic modeling: This example shows how important it can be to model performance characteristics nondeterministically.

2. Inspection control during service life: This example shows an instance where inspection control takes place during product service life.

3. Impact of modeling uncertainty: Uncertainty in performance modeling results in widely varying performance expectations. This example discusses how we might balance manufacturing uncertainty and modeling uncertainty.

4. Fixed Constraints: Although the ability to trade off performance against manufacturability results in economical tolerances, design performance requirements are sometimes predetermined for practical reasons.

This example is intended to give a twist to the methodology described in Section 4. Inspection control generally takes place during the manufacturing stage of the product life cycle. But what happens after the product is sold? Inspection and correction procedures must be applied during product service life to maintain product performance.

We again face the problem: do we tightly control manufacturing processes to provide long lasting fastener joints, or do we apply looser processes and choose to fix failures later? This type of decision takes place all the time in design. If Boeing spent enough, it could conceivably manufacture aircraft fastener joints that never fail. But is this desirable? This example models maintenance operations as inspection control during service life and compares the resulting costs against manufacturing process costs.

The Physical Problem

When the shank of a fastener is inserted into a smaller diameter hole, the *interference* creates a zone of plastic deformation immediately surrounding the hole. This interference, I, introduces cold-work as well as preload on the surrounding alloy. Controlled levels of cold-work and preload are known to improve fastener joint fatigue reliability because they suppress crack propagation. Too little or too much interference, however, results in poor joint reliability.

Step 1: Define objective value loss VL.

The objective is to specify tolerances that provide long aircraft lives at low cost. In this example, V_c is the value created by making long lasting aircraft. Converting to the convention of value loss, let VL be the total value loss associated with aircraft reliability. As before, VL consists of two major costs of variability: QL representing the expected quality loss resulting from a service fatigue life of S, and C_V representing the cost of controlling variability to obtain that level of service fatigue life.

$$VL = QL + C_V \tag{7.1}$$

The objective is to minimize VL by trading off QL against C_V.

Step 2: Model quality loss.

Step 2.1: Model the relationship between performance loss and design parameters.

Modeling the relationship between S and I is a complex process involving nondeterministic modeling. Because I first need to discuss the sources of variability, I will delay this modeling process until Step 4.

Step 2.2: Model the relationship between quality loss and performance loss.

An added ground-air-ground (GAG)[1] cycle of a 747-400 is worth approximately \$60,000, or about \$1,875 per fatigue cycle. Under grossly simple assumptions, this means airlines are indifferent whether they own a \$150M 747-400 for 80K fatigue cycles of service or a 747-X00 costing \$100K that would be disposable after just 53 fatigue cycles. I assume that this indifference is valid in the vicinity of the current design fatigue life of $S = 80K$. Then quality loss for shortened service fatigue life is:

$$L_S = L_S(S) \tag{7.2}$$

$$= V_\infty - 1875\,S \tag{7.3}$$

where V_∞ is the unknown value of an everlasting aircraft. Since we care about relative costs only, we need not model V_∞.

[1]Note that GAG cycles are different from fatigue life S. An aircraft structure experiences several fatigue cycles within a GAG cycle. In this fastener joint, each GAG cycle is equivalent to about 32 fatigue cycles. I will use service fatigue cycle S as the life variable in this example.

Step 3: Identify the significant sources of variability.

There are two major sources of variability in this problem.

Statistical variation of interference I

First, there is the manufacturing statistical variation of interference I. It has been shown that interference dimensions follow Gaussian distributions despite the fact that drilled holes exhibit highly skewed distributions. The reason is that hole drilling variability is only one of many variabilities leading to the final interference variability: varying drill bits, drilling processes, reamer sizes, reaming processes, and fastener shank diameters all contribute to the overall interference variability. We represent the statistical variation of I with

$$G(\sigma_I, I^*) \quad = \quad \frac{1}{\sigma_I\sqrt{2\pi}}\, e^{-\frac{1}{2}(I-I^*)^2/\sigma_I^2} \qquad \{-\infty \leq I \leq \infty\}\,. \tag{7.4}$$

where σ_I and I^* are the standard deviation and the mean of I.

Statistical variation of fatigue life s

There are also uncertainties associated with modeling the relationship between interference and fatigue life. Even if we had perfect manufacturing processes yielding zero statistical variation of I, fastener joint fatigue lives still exhibit variabilities because there are innumerable unmodeled and uncontrolled factors contributing to the life of a joint. If we included every conceivable variable associated with fastener fatigue life and modeled their relations correctly, we would obtain a complex deterministic model relating fastener joint life to interference. However, this is neither possible nor practical. Instead, Boeing uses the Weibull function to represent fatigue life variability.

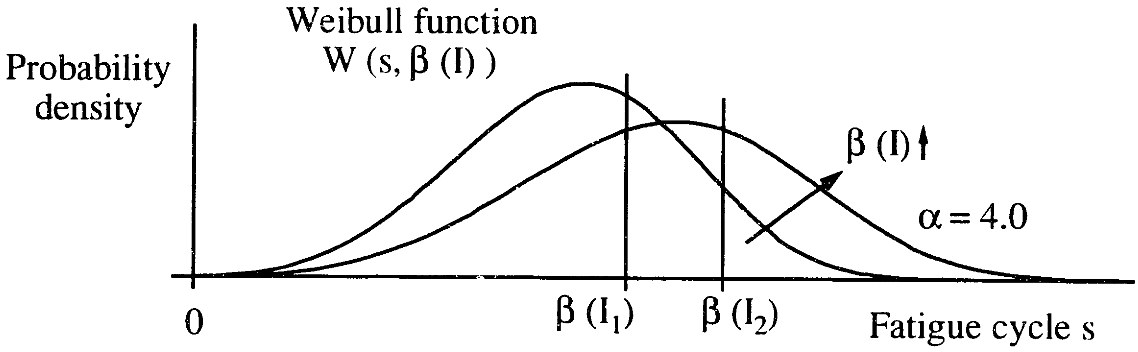


Figure 7.1: Fatigue reliability

Under this model, the probabilistic failure of a fastened joint is described by

$$W(s) = \frac{\alpha}{\beta(I)}\left(\frac{s-\gamma}{\beta(I)}\right)^{\alpha-1} e^{-\left(\frac{s-\gamma}{\beta(I)}\right)^{\alpha}} \qquad \{S \geq \gamma\} \tag{7.5}$$

For fastener joint fatigue reliability in aluminum structures, Boeing fixes α and γ at 4.0 and 0 respectively. β, otherwise known as the characteristic life, is a function of I. Fatigue reliability increases as β increases. See Figure 7.1.

Assumptions

I assume that only the statistical variation of I is controllable. Since there are no practical methods for nondestructively inspecting fastener interference, inspection control of I is not a valid option.

Step 4: Model expected quality loss QL.

Part of the design objective to minimize VL is maximizing service fatigue life S. Because γ is fixed at 0 in the Weibull probability function above, there is always a chance that failure will occur - even when we can control interference I to optimum values. Therefore we always have to accept some failures to obtain non-zero design fatigue lives. Given a deterministic value for I, the probability of failure R_I is equivalent to

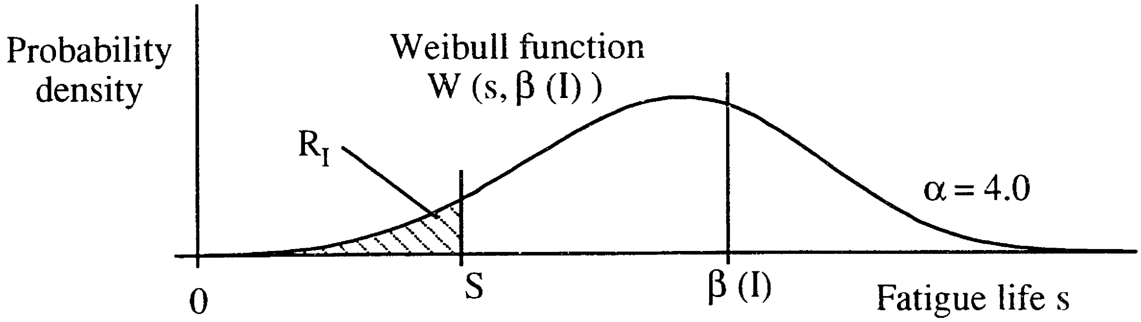


Figure 7.2: Probability of failure

the shaded area under the Weibull distribution shown in Figure 7.2. Mathematically,

$$R_I = \Phi_W(S, \beta(I)) \tag{7.6}$$

$$= \int_0^S W(s, \beta(I))\, ds \tag{7.7}$$

For fixed I, higher values of S result in higher values of R_I: the longer we expect to use the aircraft, the more joints we need to repair.

Now we incorporate the variability of I. The expected failure rate R is

$$R = E_I[R_I] \tag{7.8}$$

$$= E_I[\Phi_W(S, \beta(I))] \tag{7.9}$$

$$= E_I[\int_0^S W(s, \beta(I))\, ds] \tag{7.10}$$

$$= \int_{-\infty}^{\infty} G(\sigma_I, I^*) \int_0^S W(s, \beta(I))\, ds\, dI \tag{7.11}$$

We see that R is ultimately dependent on S, σ_I, and I^*. Solving figuratively for S we obtain

$$S = S_{ND}(R, \sigma_I, I^*) \tag{7.12}$$

The subscript ND stands for nondeterministic modeling.

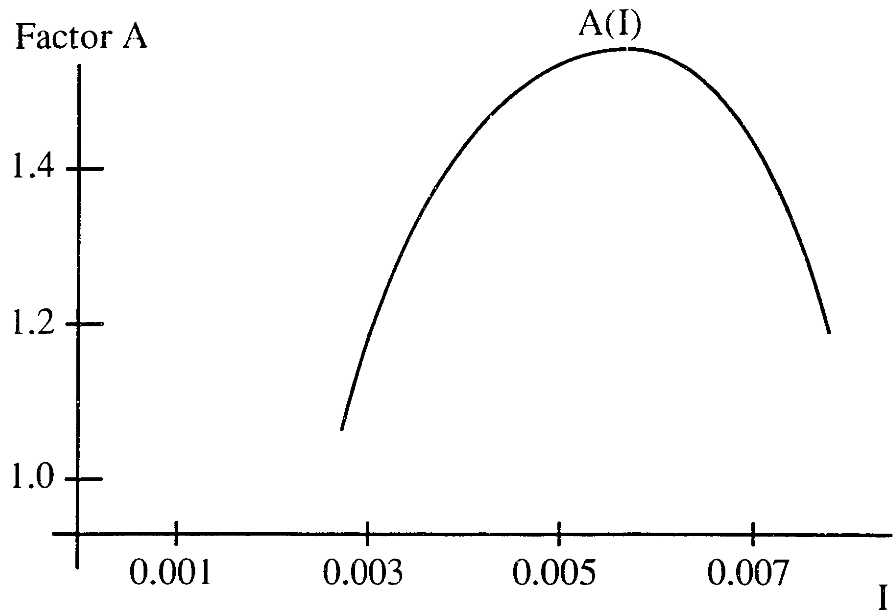


Figure 7.3: $A(I)$ - Boeing hole filling interference factor

Boeing models

We now take a closer look at how Boeing already models fatigue life so that we can compare it to our model. Boeing defines an intermediate design parameter: factor A is the hole filling interference factor describing the intermediate quality of interference fit.

$$A = A(I) \tag{7.13}$$

Factor A behaves as shown in Figure 7.3[2]. Characteristic life β is a function of A[3].

$$\beta = \beta(A(I)) \tag{7.14}$$

$$= \beta(I) \tag{7.15}$$

An increase in A results in an increase of the characteristic life β. Therefore, larger values of A result in better fatigue reliability.

[2]The derivation of this model is shown in Appendix A.
[3]The function cannot be disclosed for proprietary reasons.

Boeing separates the evaluation of fatigue reliability into three segments. First, the expected value of factor A is found by evaluating

$$A_{exp} = E_I\left[A(I)\right] \tag{7.16}$$

$$= \int_{-\infty}^{\infty} A(I)\, G(\sigma_I, I^*)\, dI \tag{7.17}$$

Then the expected characteristic life is found by evaluating

$$\beta_{exp} = \beta(A_{exp}) \tag{7.18}$$

Finally, the expected failure rate is evaluated:

$$R = \int_0^S W(s, \beta_{exp})\, ds \tag{7.19}$$

$$= \Phi_W(S, \beta_{exp}) \tag{7.20}$$

$$= \Phi_W(S, \beta(E_I[A(I)])) \tag{7.21}$$

Solving figuratively for S we obtain

$$S = S_D(R, \sigma_I, I^*) \tag{7.22}$$

The subscript D stands for deterministic modeling.

Comparison of fatigue reliability models

The significant difference between this model and the model shown previously is that here, the deterministic expectancy operation takes place at an early stage of modeling. Instead of performing the deterministic expectancy operation on I over the entire failure probability function as was done in Equation 7.10, Boeing performs the deterministic expectancy operation on I over factor A so that a deterministic value of A_{exp} may be used in subsequent modeling steps.

We know from probability theory that

$$E\left[f\left(x\right)\right] \neq f\left(E\left[x\right]\right) \tag{7.23}$$

when f is a nonlinear function of x. Hence,

$$S_{ND}\left(R, \sigma_I, I^*\right) \neq S_D\left(R, \sigma_I, I^*\right) \tag{7.24}$$

When upper level design characteristics are nonlinear functions of lower level design characteristics, deterministic modeling cannot be performed at the lower levels. Deterministic operations may be performed only on design characteristics identified as critial characteristics.

The fatigue reliability model described by Equation 7.10 is significantly different from the one in Equation 7.21. To demonstrate the difference, consider the following: fix the required life S at 80K cycles and evaluate the expected failure rates for the two models for different values of σ_I. We then obtain Figure 7.4 showing the discrepancy between the deterministic model and the nondeterministic model. The nondeterministic model actually predicts higher failure rates. In theory, this means fastener wing joints are being underdesigned. We will see if this is true later in this chapter.

Having derived the fatigue reliability model for fastener interference, we now observe a practical constraint that renders this fatigue failure model unnecessary in modeling quality loss. We will, however, use the model later when we estimate inspection control costs.

Practical considerations

In theory, the quality loss estimate shown in Equation 7.3 may be used to model QL. In practice, however, QL is specified as a requirement rather than a variable. The reason is that reliability problems affect every part and assembly of the aircraft. In

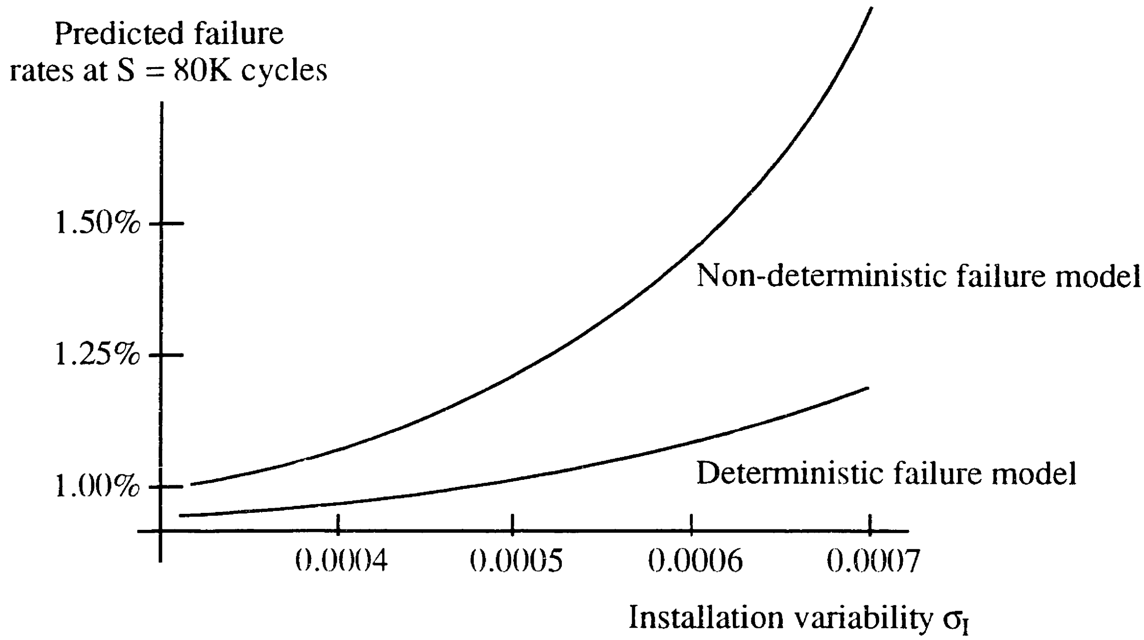


Figure 7.4: Difference in failure predictions

essence, every part and assembly has a relation like Equation 7.1 associated with it. Optimizing for each part and assembly would not only be a near impossible task, but would also result in component design fatigue lives that vary from part to part and assembly to assembly. Component design lives that vary from component to component are suboptimal. For example, the optimal life for wing joints might be 2500 GAG cycles, but for landing gears it might be 3000 GAG cycles. The 500 extra GAG cycles designed into landing gears is of little value if the airplane is to be treated as a single unit. Rather, for practical purposes, we define design fatigue life S as a constraint for all components on the aircraft. In the wing skin fastener joint example, $S = 2500$ GAG cycles or $S = 80K$ fatigue cycles.

Because S is fixed, the expected quality loss QL is also fixed. Since we are concerned only about the variable part of Equation 7.1 we may rewrite it as:

$$VL = C_V \tag{7.25}$$

Available specs	Process control	Relative costs c_{SC} [$]
Previous	$\sigma_I = 0.00050$	0.000
Current (747-400)	$\sigma_I = 0.00045$	0.058
Theoretically possible	$\sigma_I = 0.00038$	0.258

Figure 7.5: Cost of statistical variation control

Step 5: Model expected variability control cost C_V.

We now model the resources involved in controlling variability. C_V is composed of two parts as before: the cost of controlling the statistical variation of applied manufacturing processes C_{SC} and the cost of implementing inspection control C_{IC}.

$$C_V = C_{SC} + C_{IC} \tag{7.26}$$

Step 5.1: Model statistical variation control cost C_{SC}.

I assume there are only discrete statistical variation control options for controlling fastener interference. Figure 7.5 shows the the available statistical variation control options and their costs (per fastener). Appendix B shows how these costs have been derived. Note that the relative cost of zero in the first row representing a previously applied specification does not mean the process costs Boeing nothing. Since we care only about the *relative* costs when choosing process options, we can arbitrarily fix that cost to zero and define other statistical variation control costs relative to that process cost.

We may form the statistical variation control cost for N number of fasteners:

$$C_{SC} = N \cdot c_{SC}(\sigma_I) \tag{7.27}$$

Step 5.2: Model expected inspection control cost C_{IC}.

Fastener joint failures occurring on wing skin joints are difficult to repair. I estimate that on the average it costs airlines approximately $420 to repair a failed joint. Appendix B shows how this figure is derived. From equation 7.10,

$$R = E_I\left[\Phi_W(S, \beta(I))\right] \tag{7.28}$$

The cost of inspection control is

$$\begin{aligned}
C_{IC} &= 420 \cdot R \cdot N & (7.29)\\
&= 420 \cdot N E_I\left[\Phi_W(S, \beta(I))\right] & (7.30)\\
&= 420 \cdot N \int_{-\infty}^{\infty} G(\sigma_I, I^*) \int_0^S W(s, \beta(I))\, ds\, dI & (7.31)
\end{aligned}$$

Step 6: Minimize VL with respect to variability control parameters.

We write the objective value loss by combining the relevant cost factors:

$$\begin{aligned}
VL &= C_V & (7.32)\\
&= C_{SC} + C_{IC} & (7.33)\\
&= N \cdot c_{SC}(\sigma_I) + 420 \cdot N \int_{-\infty}^{\infty} G(\sigma_I, I^*) \int_0^S W(s, \beta(I))\, ds\, dI & (7.34)
\end{aligned}$$

Because the $A(I)$ function in Figure 7.3 is approximately symmetrical, we can fix I^* at the line of symmetry as proposed in Section 5.5. Then the objective is to select σ_I such that VL is minimized. When we implement loose statistical variation control, we decrease manufacturing costs but increase the number of expected fastener repairs and associated costs. If we implement tight statistical variation control, we increase manufacturing costs but expect lower costs of inspection control.

Process control σ_F	Inspection limit C_{SC} [\$]	C_{IC} [\$]	VL [\$]
0.00050	0	169,360	169,360
0.00045	2,320	162,170	164,490
0.00038	10,320	154,490	164,810

Figure 7.6: Costs of variability control

Boeing applies 1% failure criteria on fastener joint designs. This means Boeing expects 1% of all fasteners to have failed at the end of the design life. For example, if the design life for the 40,000 fasteners in consideration is 80K fatigue cycles, Boeing expects about 400 fastener joints to have failed by the time structures reach 80K cycles.

To investigate whether this 1% failure criteria results in economical design specifications in terms of total costs, we apply the three statistical variation control options listed in Figure 7.5 and obtain the cost elements shown in Figure 7.6. Observe that the current specification results in the optimum combination of statistical variation control and inspection control costs. Also note that the overall cost differences are not significant. Given the uncertainties in cost and performance modeling, we cannot confidently propose a better solution.

Discussion

In this case, the theoretical application of my methodology is somewhat inconclusive. But before we leave the topic, let us take a look at some of the more practical aspects of the design problem. Theoretically, we expect about 400 out of 40,000 fasteners to fail during the design service life. How correct is this assumption?

Fatigue service data is not available for the relatively new 747-400 fleet. However, my investigation of the 737 and 757 models reveal that only about 0.1% of all wing skin fasteners show structural deterioration during service life. Most of these failures are caused by corrosion, not fatigue. Those that show corrosion deterioration are sometimes replaced, but in most cases they are just buffed and re-painted.

If we apply the same statistics to the 747-400, which incidentally is manufactured by higher design standards, this means that only about 40 fastened joints out of 40,000 require maintenance during service life, most of them requiring only simple corrosion treatment. This means that wing fastener joint reliability models are extremely conservative. Wing joint reliability is not as critical a problem as the models lead engineers to believe. Boeing continually strives to improve wing skin fastener joint designs despite the questionable gains resulting from tighter design specifications. Failure data indicates that current designs are already at near-optimum in terms of performance. The records are an indication that resources need to be diverted to estimating better engineering models instead of to specifying and applying tighter variability control.

Chapter 8

Model Airplane Engine Example

This chapter describes the design of a model airplane engine. I will analyze current tolerance specifications for this engine, and estimate cost models to demonstrate how statistical variation control and inspection control costs may be traded off to minimize overall cost.

What this example demonstrates

1. Generalizability: I have formulated my thesis on design examples gathered at the Boeing Company. I show the generalizability of the approach by applying it to a non-Boeing example.

2. Using the dual tolerance representation: This example specifies a set of predefined goalpost tolerances. Even under this constraint, statistical variation control and inspection control may be optimized for cost.

The Design Problem

Consider the design of a model airplane engine.[1] The engine costs about $20 each. It is a single cylinder, air cooled, two cycle engine developing approximately 0.06 hp at 12,500 rpm. The piston is approximately 0.420 inch in diameter and has no piston rings. The diametrical clearance between the piston and the cylinder is of

[1] This example originates from [Bry71]

critical importance because it determines engine wear and compression loss. The engine performs best at a clearance D of 150μin to 250μin. It has been shown that within these limits, the engine performs well, while outside of these limits performance deteriorates rapidly. If the clearance is too tight, friction wears down the piston and the cylinder wall. If the clearance is too loose, compression loss results in lower power and less reliable initial ignition.

We are given goalpost tolerance limits at $D_{min} = 150\mu$in and $D_{max} = 250\mu$in as design *specifications.*

Current Processes

Pistons are roughed out on screw machines, centerless ground, heat treated, and centerless ground three more times in different stages to control workpiece temperature. Cylinders are roughed out of bar stock, bored by a special machine, and hand honed to final dimensions. Current piston and cylinder diameters exhibit Gaussian variability characteristics showing remarkably tight statistical variations of:

$$\sigma_P = 25\mu in \tag{8.1}$$

$$\sigma_C = 25\mu in \tag{8.2}$$

σ_P and σ_C are the standard deviations representing piston and cylinder distributions respectively. The piston-cylinder assemblies are comprehensively inspected and sorted to conform to the specified tolerances.

The piston-cylinder assembly exhibits high rates of rejection because the assembly statistical variation is high compared to the specified goalpost tolerances. 100% inspection is required to identify nonconforming assemblies. Inspection is performed by mating the piston-cylinder assembly to a differential air gauge has been calibrated to show assembly clearance directly.

Rejected parts are sorted into bins to keep probabilistically smaller components separate from probabilistically larger components. For example, if an assembly clearance is too small, it is probable that 1) the piston is large, and 2) the cylinder is small. This sorting greatly enhances the probability that reassembly of cross-matching components will result in satisfactory clearance dimensions. See Figure 8.1. Something remarkable about this type of sorting is that populations of sorted parts very closely resemble Gaussian distributions. Sorted components exhibit shifted means, but as long as initial piston and cylinder distributions are identical, cross-matching of sorted parts results in matching piston and cylinder means for reassembly. See Figure 8.1. Currently the manufacturer conducts two iterations of inspection and reassemblies for every batch of initial assembly. These cascading processes result in near-perfect tolerance conformance.

Step 1: Define objective value loss VL.

Performance loss is assumed to be flat within the tolerance region and infinite on the outside of that region. There is no performance degradation resulting from variation within those tolerances. Therefore, as long as we comply to the specified goalpost tolerances, we can disregard QL. Then we have

$$VL = C_V \tag{8.3}$$

The objective is to design process and inspection control specifications for minimum cost while complying to the goalpost specifications.

Step 2: Model quality loss.

Because QL vanishes from our objective function, we may skip Steps 2 and 4.

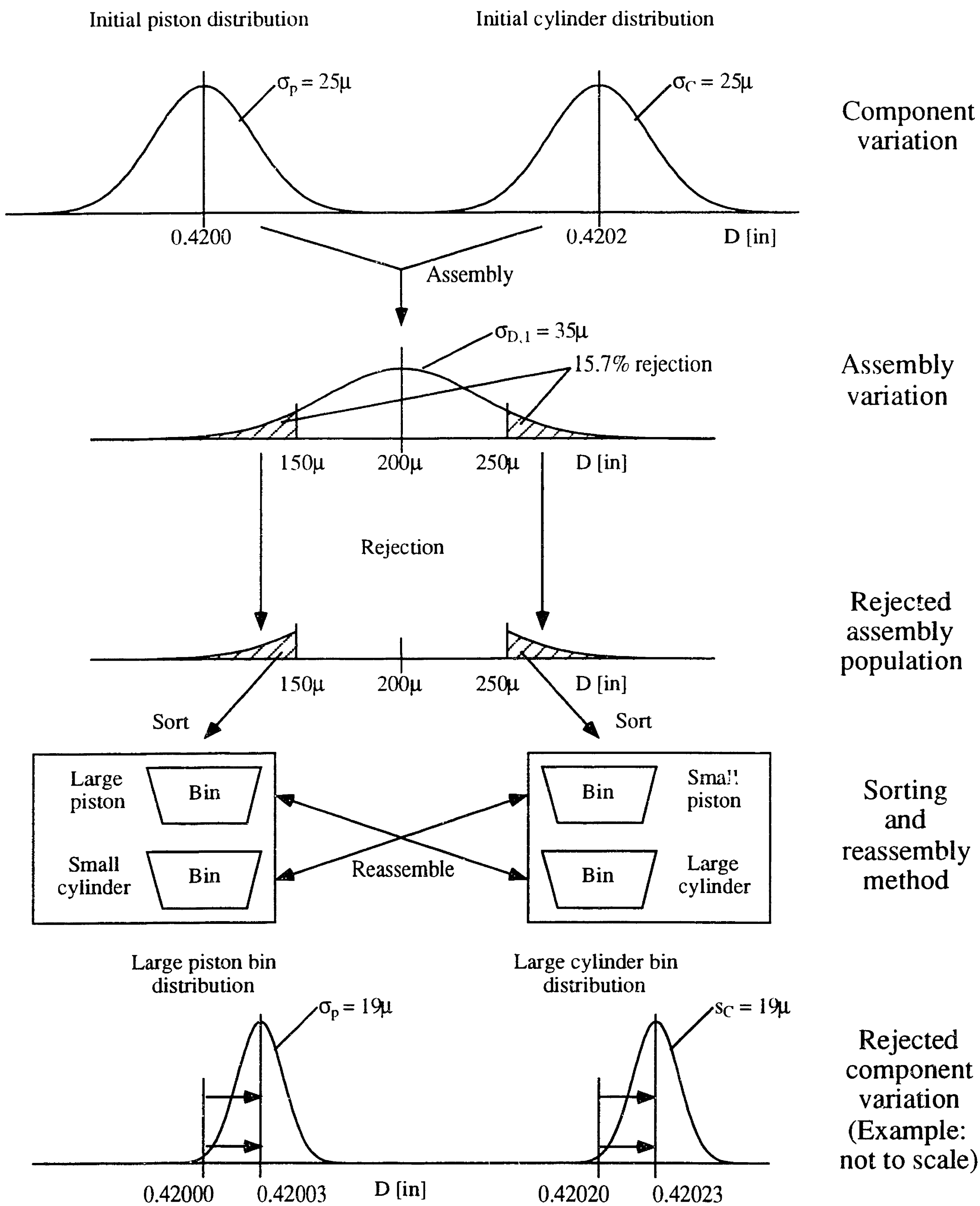


Figure 8.1: Statistical variation control and inspection control via sorting

Step 3: Identify the significant sources of variability.

The major sources of variability are the statistical variations of the piston and the cylinder. Pistons and cylinders are both currently manufactured to the same statistical variation of $\sigma_P = \sigma_C = 25\mu\text{in}$. I speculate this is done for one of two reasons. 1) Because the rational manufacturer always operates where marginal utilities are equal [Man85], we might conclude that the resources required in controlling piston and cylinder statistical variations are about equal. Otherwise, the manufacturer would gain by relaxing the more expensive process and tightening up the cheaper process.[2] 2) The manufacturer recognizes that identical statistical variations greatly enhance sorting and reassembly conformance rates as described above. Even when marginal utilities are not equal, it might be worth the extra cost to make statistical variations identical.

Step 4: Model expected quality loss QL.

Because QL vanishes from our objective function, we may skip Steps 2 and 4.

Step 5: Model expected variability control cost C_V.

Step 5.1: Model statistical variation control cost C_{SC}.

I assume statistical variation control costs vary as

$$C_{SC} = A + \frac{B}{\sigma_P} \tag{8.4}$$

[2] I am assuming here that the manufacturer is not operating at the boundary of technology.

Chase [CG88] has shown this model closely follows material removal processes such as turning and grinding operations. The current statistical variation of $\sigma_P = 25\mu$in is obtained by grinding the piston in several steps to control thermal expansion during grinding operations. I estimate that without this additional handling, pistons would exhibit a statistical variation of $\sigma_P = 50\mu$in. The machining cost for pistons at 25μin is estimated at \$0.75 apiece, whereas nonhandled pistons at 50μin are estimated at \$0.60 apiece. Using these figures to determine the unknown parameters in Equation 8.4, we obtain a cost model for piston statistical variation control:

$$C_{SC,P} = 0.45 + \frac{7.5 \cdot 10^{-6}}{\sigma_P} \tag{8.5}$$

For the purpose of illustration, I assume statistical variation control costs are the same for the piston and cylinder machining processes. Then the statistical variation control cost for the assembly clearance is

$$C_{SC,D} = 2 \cdot C_{SC,P} \tag{8.6}$$

and engine clearance variation may be described by

$$\sigma_D = \sqrt{2}\sigma_P \tag{8.7}$$

Step 5.2: Model expected inspection control cost C_{IC}.

We define c_i as the unit cost of inspection and c_r as the unit cost of reassembly. We formulate the costs associated with the first inspection and reassembly cycle:

$$C_{IC,1} = c_i \cdot N + c_r \cdot N_{r,1} \tag{8.8}$$

N is the total number of piston-cylinder assemblies. $N_{r,1}$ is the number of rejected parts identified by the first inspection cycle and is estimated by:

$$N_{r,1} = N \cdot [1 - \int_{D^*-50\mu}^{D^*+50\mu} G(\sigma_{D,1}, D^*)\, dD] \tag{8.9}$$

where $\sigma_{D,1}$ is the initial assembly standard deviation equaling $\sqrt{2}\sigma_P$ and D^* is the clearance target of 200μin.

The mathematical probability density function describing the population of rejected pistons suspected of being small is

$$P(p) = \frac{2}{N_{r,1}} G(\sigma_p, p^*) \cdot [1 - \Phi_G(\sigma_P, p^* + 50\mu)] \tag{8.10}$$

The fraction $\frac{2}{N_{r,1}}$ is a normalizing coefficient. As mentioned above, $P(p)$ closely resembles a Gaussian distribution. For practical purposes, I will assume it is a Gaussian.

We proceed to formulate the costs associated with the second inspection and reassembly cycle:

$$C_{IC,2} = c_i \cdot N_{r,1} + c_r \cdot N_{r,2} \tag{8.11}$$

$N_{r,2}$ is the number of rejected assemblies identified by the second inspection cycle and is defined by:

$$N_{r,2} = N_{r,1} \cdot [1 - \int_{D^*-50\mu}^{D^*+50\mu} G(\sigma_{D,2}, D^*)\, dD] \tag{8.12}$$

$\sigma_{D,2}$ is the standard deviation of the reassembled engine clearance. $\sigma_{D,2}$ depends on initial $\sigma_{D,1}$, which in turn depends on initial component variability σ_P.

Further inspection cycles may be modeled similarly if needed.

It is estimated that each inspection with the air gauge costs the manufacturer about 8 cents. It is also estimated that each sorting, handling, and reassembling cycle costs about 50 cents per rejected assembly. Then the cost of applying two inspection cycles is

$$C_{IC} = 0.58 \cdot N_{r,1} + 0.50 \cdot N_{r,2} \tag{8.13}$$

The cost of inspecting the initial batch of N assemblies has been eliminated because it is not variable ($c_i \cdot N = constant$). Two inspection cycles are adequate for this example.

Step 6: Minimize VL with respect to variability control parameters.

Combining the cost elements and dropping fixed costs we obtain:

$$\begin{aligned} VL &= C_V \quad (8.14) \\ &= C_{SC} + C_{IC} \quad (8.15) \\ &= \frac{7.5 \cdot 10^{-6}}{\sigma_P} + 0.58\,(N_{r,1} + N_{r,2}) + 0.50 \cdot N_{r,3} \quad (8.16) \end{aligned}$$

There is only one variability parameter in the equation above. Initial component variability σ_P is the denominator in the first term and it determines $N_{r,1}$, $N_{r,2}$, and $N_{r,3}$. The objective is to determine σ_P such that the total variability control cost is minimized. Figure 8.2 shows variability control cost elements as a function of σ_p. The cost models lead to an optimum tolerance specification of $\sigma_P \approx 27\mu$in. This is the location where the cost combination of inspection, reassembly, and statistical variation control is at a minimum.

Discussion

This example demonstrates that there is always an optimum combination of statistical variation control and inspection control. Even when the simplest form of design requirements - goalpost tolerances - is specified, the costs of statistical variation control and inspection control need to be fully evaluated to find that optimum combination. This is contrary to the tolerancing approach proposed by *continual*

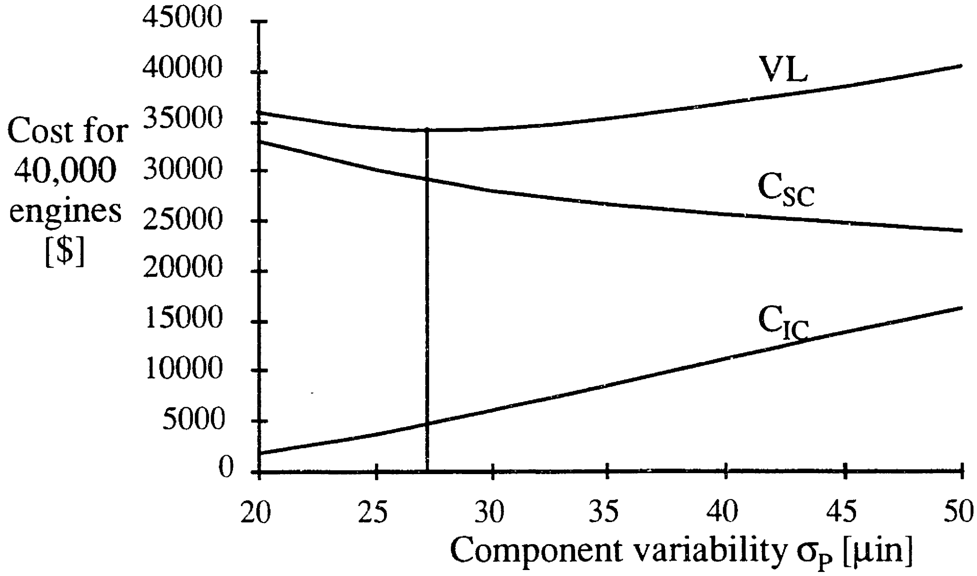


Figure 8.2: Variability control costs

quality improvement which advocates continual improvement of statistical variation control. This example demonstrates that there is a point of diminishing return for statistical variation control, and a point of increasing return for inspection control.

Chapter 9

Summary and Recommendations

9.1 Summary

Designers often encounter the following questions. Do I design for better performance or for lower cost? What is the best production strategy for delivering that performance? How do I determine and represent that strategy? Design is the process of constantly evaluating these questions in presence of conflicting goals and uncertainties.

Current methods for tolerancing fall short of providing satisfactory answers to the questions above. Some methods provide probabilistically rigorous methods for evaluating performance in the presence of variability. Others focus on geometrical tolerance allocation for manufacturability. Still others work on complex tolerance accumulation models. But they lack a common design objective. There has been no successful attempt at combining the different tolerancing approaches together to produce an effective overall tolerancing method.

This thesis describes a methodology for concurrently evaluating product performance and production process options during tolerance design. The methodology emphasizes the following ideas:

1. **Performance-Cost Tradeoff:** Tolerancing is purely an economic decision.

The best tolerance specification is one that balances product performance against the cost of attaining it.

2. **Tolerance Representation:** Tolerance specifications must unambiguously represent the variability control options available to the manufacturer. I represent tolerances as a combination of statistical variation control and inspection control specifications.

3. **Determining Variability Control Options:** The most economical variability control option can be obtained by evaluating and trading off statistical variation control and inspection control costs.

4. **Cost Modeling:** I provide guidelines for modeling variability control costs and quality losses. In presence of variability, modeling must be done carefully. Several simplification techniques make this task practical.

5. **Parallel Design of Nominals:** When analytical engineering models are available, nominal design and tolerance design may be conducted simultaneously to yield even better robust design specifications.

The ideas presented in this thesis are not complex. The methodology basically proposes a structure for formulating an objective function, and provides guidelines for optimizing it. Yet application of even a simple methodology on real design problems can severely complicate tolerancing. The methodology incorporates several design issues traditionally not included in tolerance design. The addition of flexible performance criteria, the distinction between goalpost and statistical tolerances, the notion of continually variable nominals, the presence of modeling uncertainties, and the emphasis on nondeterministic modeling approaches all sum up to a significantly more complex design problem. Although I provide implementation guidelines to simplify some of the modeling and optimizing tasks, this methodology in the end makes tolerancing more difficult.

The goal of this thesis never was to simplify the design process. Rather, it was to make designers more aware of the different issues in tolerancing, and to provide a methodology for making intelligent tolerancing decisions in complex design environments. The objective was to give designers the tools for making difficult but economical tolerancing decisions the first time around, at perhaps the cost of expending more initial design effort.

9.2 Conclusion

Conclusions for each design example are included in their respective chapters. Here I will describe my general conclusion.

I describe three cost elements in the formulation of my overall cost function: expected quality loss, statistical variation control cost, and inspection control cost. Modest gains may be obtained by optimizing each cost element individually. The power of the proposed methodology, however, is in varying all cost elements simultaneously whenever possible. I allow tradeoffs between product performance and manufacturability. I also select the most economical combination of variability control options. The effectiveness of the proposed methodology is demonstrated by applied examples in Chapters 6, 7, and 8. These examples demonstrate that:

1. Trading off quality against variability control cost results in economical tolerances.
2. Statistical variation control and inspection control representations allow the explicit evaluation of production options during design.
3. Parallel design of nominals and tolerances result in higher product quality at lower cost.

9.3 Thesis contributions

The following is a list of my contributions:

1. I provide a methodology for making quantitative tradeoffs between performance and cost.
2. I provide a tolerance representation scheme that fully prescribes all tolerance related costs for evaluation during design.
3. I provide a framework for developing nondeterministic performance models for tolerancing.
4. I provide practical guidelines for solving complex tolerancing problems.
5. I provide applied examples to demonstrate how the methodology may be implemented.

9.4 Future Challenges

Form manufacturing cost models: In most cases, manufacturing cost models are not available. There has been little effort spent on documenting how much manufacturing processes cost. Even when cost models do exist, they are closely guarded as company secrets and are rarely used during design. It has been a personal challenge to create cost models for the two Boeing examples used in this thesis. It is my sincere hope that this thesis motivates further research into cost modeling and encourages manufacturers to document process related resource requirements.

Develop ways to model monetary loss functions from performance characteristics: One of the most difficult steps in quantitatively trading off performance against cost is modeling the monetary loss of performance degradation. Although some of these models do exist, they are absent in the most part. Without these models, we can only resort to qualitative evaluations which often result in suboptimal designs. Researchers such as [Hau83, HG92] address some of these modeling issues, but this area is largely unexplored.

Investigate applicability in dynamic manufacturing technologies: My methodology is most effective in relatively mature manufacturing industries. It would be a challenge to modify the methodology to accommodate some of the dynamic aspects of manufacturing systems such as the existence of learning curves and supply and demand.

Consider how to distribute benefits: My optimization criterion is to minimize the total value loss associated with a product, or to maximize the value created by manufacturing a product. The notion of created value is in reality quite vague. Created value is a sum to be distributed between the consumer and the manufacturer. How it is distributed essentially defines the supply and demand of products. The current methodology does not address these social and economic implications.

Expand to multi-dimensional and multi-attribute design This thesis presents a way of specifying single dimensional, single attribute tolerances. I have identified some of the critical issues in tolerancing for optimum performance and cost. I have demonstrated the importance of these issues by applying single-dimensional, single-attribute tolerancing methodology on design examples. In reality, however, designs are multi-dimensional and multi-attributed. The next step would be to expand this methodology to multi-dimensional and multi-attribute design problems.

Address computer implementation: No design methodology can replace the judgment of a good designer. Tolerancing especially requires human interaction for decision making because it is inherently a negotiation process. In the past, tolerancing was done mostly by experts with the knowledge to make critical decisions about product performance and cost. It is not the intent of my proposed methodology to displace that expertise. Rather, it is to enhance the designers' ability to make decisions during product design by providing the analytical tools capable of evaluating the economic implications of his decisions. I recognize that methodologies and analytic tools are effective only when they are easy to use. It would be a worthwhile challenge to transform this methodology into a computer tool that is easy to use.

Appendix A

Modeling Factor A

Factor A is a function of interference I. At Boeing, the relationship between factor A and interference I is provided in tables in fastener design manuals. Figure A.1 shows one such table[1] This table shows the expected values of factor A resulting from a set of interference ranges. For example, fastener installations exhibiting interference tolerances ranging from 0.0015 to 0.0055 yield an expected factor A of 1.19.

How do we interpret the table for factor A? The table lists a set of tolerances and their corresponding values for A. This is represented by the discrete horizontal lines in Figure A.2.

[1]Numbers have been changed to protect proprietary information.

	Interference I Range [in] for Tension Structure				
	0.0010 to 0.0040	0.0015 to 0.0055	0.0020 to 0.0050	0.0030 to 0.006	0.0040 to 0.0075
Factor A for titanium steel fasteners	1.09	1.19	1.25	1.37	1.31

Figure A.1: An example of factor A table

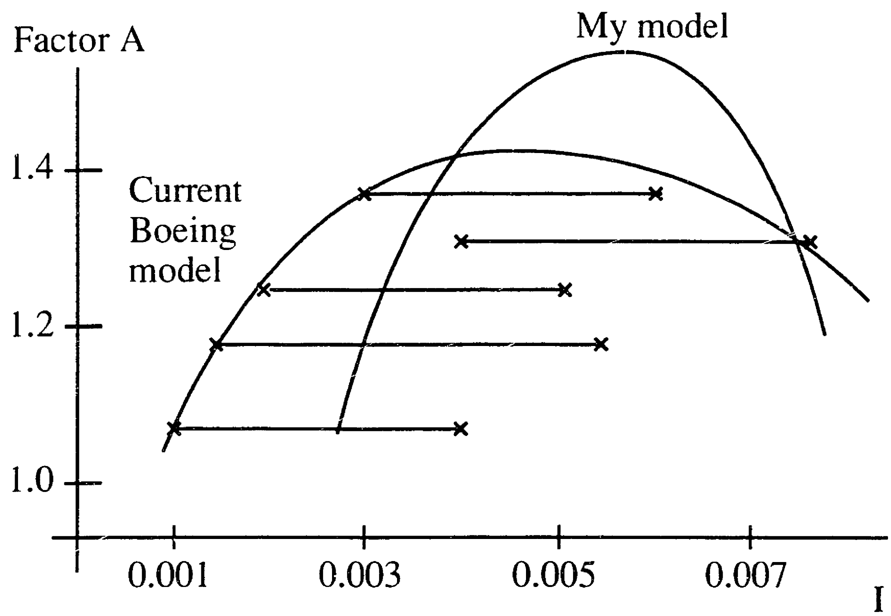


Figure A.2: Graphical representation of factor A

What happens when we want to apply tolerances other than the ones listed in the table? What do we use for factor A, for example, when we want to use tighter tolerances? Engineers at Boeing have addressed these questions with several solutions. One method is to represent factor A with a line placed through the centers of listed tolerances. Another method implemented in the design of 747-400 was to represent factor A as the line that connects the outer limits of the listed tolerances. This curve, shown as the flatter curve in Figure A.2, is more lenient than the first in terms of manufacturability. But does it make more sense?

Observe that the problem is to develop a continuous model for A from a discrete and nondeterministic set of representations for A. A continuous model would allow us to interpolate and use combinations of nominals and tolerances not supplied by the design manual.

In the absence of the original body of data used in forming the table for factor A, I attempt to formulate another model for factor A by assuming that there exists an exact function describing factor A as a function of interference. I assume that this function may be represented by a fourth order polynomial.

$$A(I) = A_0 + A_1 I + A_2 I^2 + A_3 I^3 + A_4 I^4 \tag{A.1}$$

I find the coefficients by repeatedly applying the expectancy operation on each of the tolerance regions found in the table for factor A:

$$A_i = \int_{I_{min,i}}^{I_{max,i}} G_i(I)\, A(I)\, dI \tag{A.2}$$

where A_i is the listed value for factor A for the tolerance region i; $I_{min,i}$ and $I_{max,i}$, the lower and upper limits of tolerance region i; $G_i(I)$, the Gaussian probability function governing the distribution of I in tolerance region i; and $A(I)$, the fourth order polynomial estimate of the A function. I assume the distribution $G_i(I)$ follows the tolerancing convention ($\pm 3\sigma_i = I_{max,i} - I_{min,i}$). Then I solve for the coefficients. Figure A.2 shows the resulting function as the more acute curve compared to the one used by Boeing.

Appendix B

Cost Modeling for Interference Variability

In this appendix, I describe how I derived the cost models in Figure 7.5.

B.1 Statistical variation control

I assume there are only three discrete statistical variation control options for controlling fastener interference.

Allowable specifications	Statistical variation control [in]	Relative costs c_{PC} [$]
Previous	$\sigma_I = 0.00050$	0.000
Current (747-400)	$\sigma_I = 0.00045$	0.058
Possible	$\sigma_I = 0.00038$	0.258

Figure B.1: Estimated costs of statistical variation control

Added costs per fastener [cents]				
Reamer	Gage	Training	Labor	Total
0.5	0.3	0.2	4.8	5.8

Figure B.2: Increased cost for current process

Previous Specification

The first option is representative of a manufacturing process that had been applied on previous 747 models. It exhibits a statistical variation of $\sigma_I = 0.00050$. This assembly statistical variation is a result of component statistical variations: the fastener shank diameter statistical variation is $\sigma_S = 0.00036$ and the fastener hole diameter statistical variation is $\sigma_H = 0.00035$. Statistically combined, they result in $\sigma_I = 0.00050$. I use this process as the benchmark to compare other process costs, and therefore assign the relative cost of zero.

B.1.1 Current specification

Current 747-400 models are being manufactured to $\sigma_I = 0.00045$. This is a result of applying tighter manufacturing processes in fastener hole preparation: the resulting hole diameter statistical variation is $\sigma_H = 0.00027$. Figure B.2 shows the increased costs of applying this process. 1) Sorting of reamers and improved reamer maintenance results in an additional cost of approximately \$15 per reamer. With each reamer lasting about 3000 reamed holes, this results in an added cost of 0.5 cents per hole. 2) Total gauge costs including the computers required to keep track of measurements sum to approximately \$40,000. Estimating these tools to last about 5 years, this amounts to about 0.3 cents a fastener. 3) Additional training of mechanics results in about an additional 0.2 cents a fastener. 4) The largest cost increase is from the addition of two more mechanics to account for the extra time needed in obtaining tighter statistical variation control. At \$30 per hour of labor and a processing rate of about 56 fasteners an hour, this amounts to about 4.8 cents of additional cost.

Altogether better hole preparation processes result in a reduced statistical variation of $\sigma_I = 0.00045$ at a total additional cost of 5.8 cents per fastener.

Theoretically possible specification

Hole making processes are as tight as Boeing can practically manage with current technologies. Additional reduction in statistical variation can only result from reducing fastener statistical variation. Fasteners are supplied by over a dozen vendors. Boeing can reduce fastener shank variability by: 1) reducing the number of suppliers, and 2) keeping the supplier lots separate.[1] The combination of these two tactics are expected to cut shank statistical variation by as much as 25% to $\sigma_S = 0.00026$ while increasing fastener and related handling costs by as much as 20 cents per fastener.

The combination of better hole preparation processes and tighter fastener control results in a combined variability of $\sigma_I = 0.00038$ with an increased cost of about 25.8 cents a fastener.

B.2 Inspection Control

Fastener failures on wing skin joints are expensive to repair. There are basically two types of failures. The first type is local failure contained around the immediate area (within 1/8 inch) of the fastener. This type of failures accounts for approximately 90% of all of all fastener failures found on previous aircraft models. These failures require removal of fasteners, drilling of larger holes and countersinks, and installation of oversize fasteners. Although this seems simple, the repair requires work on both surfaces of the wing skin. Maintenance personnel need to climb into the wing structure, find the opposite end of the failed joint, and work with personnel on the outer surface of the wing. It is estimated that this type of repair work costs airlines

[1] A third option is to tighten vendor variability, but this option is not considered here.

approximately $400 per failed joint.

The second type of failure is more serious. Sometimes fatigue cracks extend over 1/8 inch from the fastened joint. This type of failure requires the consulting of Boeing engineers for repair. Usually, the work involves 1) removing several fasteners around the failure joint, 2) placing steel shims to guard stringers during repair[2], 3) machining holes as large as two inches in diameter in some cases to remove the entire area affected by the fatigue crack, 4) placing a plug larger than the machined hole to create an interference fit, 5) machining and buffing the area flat, and 6) re-fastening the stringer to the wing skin. It is estimated that this repair costs as much as $1,600 per repair.

Then, on a per fastener basis, the expected cost of repair is:

$$E\left[C_{corr}\right] \approx 400 \cdot 0.9 + 1600 \cdot 0.1 \tag{B.1}$$

$$\approx 420 \tag{B.2}$$

[2] Or to guard the skin when stringers develop cracks.

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