Optimal Inventory Model for Managing Demand-Supply Mismatches for Perishables with Stochastic Supply

by

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ABSTRACT

While festivals bring a reason to cheer for everyone, businesses dealing with a spike in demand for perishables may have to live with the misery of lost sales and/or expired items. In the case of the dairy industry that deals with liquid milk, both raw material, and finished goods are perishable, which implies that merely stockpiling inventory of either item, without paying attention to potential inventory losses, cannot be an optimal strategy. In developing countries, the supplier base for perishables like milk, fruits, vegetables, flowers, etc. mostly comprise of small farmers instead of corporate/professional agencies, thus leading to supply variability. During special occasions like festivals, as individuals set aside more of the raw material for their own consumption, we encounter a reduction in supply. Around the same time, we notice a spike in customer demand, leading to a demand-supply mismatch. Companies dealing with perishables need an analytical approach to manage this.

In this thesis, we present a framework to address this problem of intermittent demand-supply mismatch using a 3-stage stochastic optimization model. We decide on the sourcing targets, the production plans based on supply realized, and finally, the dispatch plan based on orders received. As a case study, we analyze the operations and data from a private dairy company in eastern India, to understand the research problem and the applicability of the resulting model. We notice the impact of demand spikes and supply reduction in two areas: we increase supply targets in the periods preceding the demand spike; and we increase supply targets in periods when supply is expected to decrease, while demand is as usual. When there are multiple festival days within the time series, the compounding of impact depends on the sequencing of the events. Finally, when we introduce the realistic constraint that the supply target needs to be constant throughout the time series, we see a degradation in the profitability, as we need to tradeoff between lost sales and wasted products. While the focus of this case study is the dairy industry, the conclusions from this research are broadly applicable to other industries dealing with perishables.

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1. INTRODUCTION

Effective inventory control policies drive supply chain efficiency and reduce costs, and thus continue to be a leading area of operations research at both academic and practitioner levels (Talluri, Cetin, & Gardner, 2004; Nahmias, 2011). Businesses use safety stock of finished goods as one of the tools to manage demand variability. On similar lines, one of the options for dealing with supply uncertainty is also to carry safety stock (Talluri et al. 2004; van Kampen, van Donk, & van der Zee, 2010). This safety stock may be in the form of finished goods and/or raw materials. While the lifetime of safety stock is usually assumed to be infinite, this is not the case with perishable items, which results in loss of inventory (Nahmias, 2011). In supply chains with capacity limitations, the impact of demand surge has an important dynamic effect (Helo, 2000). These demand surges lead to intermittent demand-supply mismatches in the supply chain. Such mismatches have a detrimental effect on any company in the area of operations and inventory (Hendricks & Singhal, 2014).

In this research, we address the unique problem of inventory management for perishables with supply variability, specifically the scenario of dealing with intermittent demand surges.

1.1. Research Motivation

Seasonal events like festivals or special occasions often result in demand surges for some commodities. While this scenario should be a cause for cheer among most companies, the inability to service the increased demand might lead to lost sales, besides impacting customer goodwill. The food supply chain in developing countries like India involves a large number of small suppliers. At the individual level, these suppliers deal with capacity variability as well as yield variability (Prakash, Gupta, Gupta, Gandhi, & Kumar, 2015).

This research is focused on the dairy industry in Eastern India, where individual farmers form the bulk of the supplier base (Knips, 2005). These individuals offer surplus milk for sale to companies on a daily basis. Milk-based sweets form an integral part of festivities in India (Raman, Kumar, & Khetra, 2018). In this scenario, the surge in demand across the communities during festivals leads to an intermittent gap between customer intake and supplier offtake around the same time. Safety stock would usually serve the need to manage variability in supply and demand. However, in the case of perishables, stocking raw milk and/or packed milk may result in inventory losses.

MarketLine (2018) reports that the Indian dairy industry had a market size of about 19 billion USD in 2017, and is expected to reach about 28 billion USD by 2022. About 90% of the dairy market comprises of milk, with the rest composed of milk products like yogurt, butter, cottage cheese, etc. Also, cooperatives, comprising of smallholder farmers comprise the majority of suppliers.

It is ironic that increased demand during festivals results in pain instead of joy for the dairy industry. The inability to respond to the market during these periods of intermittent demand-supply mismatches results in lost sales (along with loss of customer goodwill) as well as inventory costs (stocking too much and material getting spoilt). At a social level, it seems unfair that while those who can afford the pricier alternatives (like milk in tetra packs or whole milk powder) can ignore the issue, the bulk of the population has to make do with reduced milk intake during those days. In effect, the problem impacts the most vulnerable part of the society (who cannot afford more expensive alternatives) and needs further research.

1.2. Thesis Objective

When supply chain for perishables is faced with the problem of intermittent demand-supply mismatches, it results in wastage (due to spoilage of excess stock) and/or lost sales (due to unfulfilled demand). Considering that demand and supply are both stochastic in the first place, we need an optimal inventory policy for raw material and/or finished goods to help address this problem.

An extensive literature review indicates that while supply chain for perishables is well researched, they mostly deal with demand variability, and thus the primary focus is on inventory policies for finished goods. Supply variability for perishables is primarily researched from the perspective of stochastic delivery timelines and supply disruption. Mismatches in demand-supply are mainly handled via information dissemination as well as risk transference. Unfortunately, the unique combination of all three scenarios (perishability, supply variability, and demand surge) faced by entities dealing with perishables is not well researched and forms the core focus of this thesis.

We approach this problem by using a mix of qualitative and quantitative methods. Interviews and surveys form the basis for understanding the processes and underlying challenges. Stochastic optimization is used as the primary tool to identify the optimal sourcing policy (to manage raw material inventory) and production policy (to manage finished goods inventory). Finally, we use simulation to assess the effectiveness of the policies, and sensitivity analysis to derive key insights about the applicability of this approach in different practical scenarios.

While this research centers around milk as a perishable commodity, the same can be generalized to other perishables with supply variability, such as fruits, vegetables, flowers, etc., where the primary supplier base consists of individuals rather than corporate/professional entities.

1.3. Thesis Outline

This thesis is organized as follows. Section 1 highlights the need for this research and outlines the objective of this research. Section 2 provides a detailed review of the existing literature to identify the gap that this thesis intends to cover. Section 3 covers the research methodology and outlines the qualitative and quantitative methods to be used to address the research question. Section 4 demonstrates the mathematical model and solution approach applicable to this situation. Section 5 elaborates on a case study covering a leading private dairy company in eastern India. Section 6 summarizes the results and associated insights. Section 7 concludes with the implications of this research and opportunities for further research.

2. LITERATURE REVIEW

This literature review summarizes key research papers focused on the following themes: inventory management of perishables; cause and impact of supply variability; and impact of demand surges leading to demand-supply mismatches. Additionally, it covers leading research in the area of the dairy supply chain. It concludes with a discussion of the gap in existing literature, which this research seeks to fill.

2.1. Supply Chain for Perishables

While most inventory policies assume an infinite lifetime of items in stock, in reality, some of the items may have their life restricted due to decay (continuous loss of inventory), obsolescence (item is superseded by a better version) and perishability (items are unusable after an expiration period) (Nahmias, 2011). In the context of this research, we deal with the perishability of both raw materials (which cannot be used for production beyond a time period) and finished goods (which cannot be sold beyond a time period).

Inventory models for perishables have been researched since the 1960s. Shukla and Jharkharia (2013) presented a literature review of the fresh produce supply chain management. While their study covered agri-produce (fruits, flowers, and vegetables), the same can be extended to other perishables like milk, etc. They highlighted the need for literature in this field, which is fragmented into silos, to move to a cross-functional coverage of different supply chain functions. Khanlarzade, Yegane, Kamalabadi, and Farughi (2013) presented another literature review covering inventory control of perishables. Their article separates obsolescence from perishability and covers single and multi-echelon supply chains. It also extracts the solving procedure used in different papers along with the key findings. The most recent literature review of this domain reviews and analyzes perishable inventory models along various dimensions such as evolution, scope, demand, shelf-life, replenishment policy, modeling techniques and research gaps (Chaudhary, Kulshrestha, & Routroy, 2018). They segregated the impact of a random lifetime from a fixed lifetime, which would be applicable for raw milk and packed milk respectively.

Blackburn and Scudder (2009) described the need to optimize the supply chain design for perishables by ensuring high responsiveness as a priority in early stages while ensuring high efficiency as a priority in later stages. Dillon, Oliveira, and Abbasi (2017) have addressed inventory management of blood as a perishable commodity. The collection, production, storage, and distribution functions are brought together with an intent to reduce the overall cost by balancing the storage and wastage costs. A two-stage stochastic programming model that can be set up in any commercial off-the-shelf (COTS) software is used to arrive at the suitable inventory model. The overall decision-making process is split into two stages (first defining the R,S policy, and then determining the number of units to order based on on-hand inventory) and optimized using stochastic programming. Hamdan and Diabat (2019) proposed stochastic programming to manage blood supply chain covering production, inventory and location-based decision-making parameters. They combined three objective functions, namely wastage, costs and service levels, and used two-stage stochastic modeling across the inventory echelons. The basic principles of stochastic optimization technique using discrete scenario modeling can very well be extended to the current research topic.

2.2. Supply Variability

Randomness in supply can be introduced by many factors: a process that relies on random production of raw material (e.g. harvesting, fishing, etc.), a production stage that depends on random yield produced by a previous stage, etc. (Khang and Fujiwara, 2000). In our research, we are mainly concerned with the variability in supply created by the suppliers' ability and willingness to adhere to supply targets.

Ignaciuk and Bartoszewicz (2011) addressed the supply variability that results from delivery latency and using multiple suppliers in procurement. Their solution is based on control theory and specifically addresses the challenges with perishables and storage capacity constraints. The overall objective is to minimize holding and shortage costs while ensuring minimal disruption in the flow of goods. Xiaoming and Yong (2013) investigated the impact of supply uncertainty on the ordering and pricing policy for perishables with finite sale horizon. They cover the scenarios where supply may be less than what was ordered, while the supplier is only paid for what was delivered. The same principles apply in the current research as well. Puranam, Novak, Lucas, and Fung (2017) outlined the approach to address the impact of having multiple suppliers leading to supply variability in blood, a perishable commodity. Their study uses multi-period cost optimization to arrive at an inventory policy that addresses irregular multi-source supply situations. The model is evaluated against empirical data along with standard inventory models.

2.3. Intermittent Demand-Supply Mismatch

Demand surge represents a significant increase in demand when compared to the regular situation, and could be an outcome of a natural or man-made situation (Huang, Song, & Tong 2016). In the absence of a planned equivalent increase in supply, such situations lead to intermittent demand-supply mismatches across the supply chain.

Datta and Christopher (2011) addressed the approach for managing uncertainty owing to unexpected large demand spikes. Their paper focuses on improving the information distribution and coordination to ensure all parties in the supply chain better respond to such spikes. However, their approach is mainly suitable for make-to-stock scenarios only. Kremer and Van Wassenhove (2014) explored the possibility of farming out inventory risk at a fixed/variable fee to other parties in the supply chain. This study does not aim to resolve the demand-supply mismatch but instead attempts to reduce the impact of such mismatch by negotiating lead times with supplier and customer. Roni, Eksioglu, Jin, and Mamun (2016) proposed a hybrid delivery policy to manage regular and surge demand separately. The recommended inventory model assumes that items are not perishable, and although they theorize that the model can be extended to perishables, there is no specific guidance for achieving the same.

2.4. Dairy Supply Chain

Prakash et al. (2015) presented a detailed study of various factors affecting milk yields over multiple lactation cycles. Their research provides an insight into the causes and extent of variance in milk production that drives the yield uncertainty at the upstream players of the dairy supply chain. Sel and Bilgen (2015) presented a meticulous review of quantitative models applied to various areas of the dairy supply chain - production planning, distribution planning, and vehicle route planning. This study covers analytical, approximation and simulation methods for production planning and inventory modeling. The overall uncertainty in supply is addressed via multi-stage stochastic programming solutions. Mor, Bhardwaj, and Singh (2018) covered the supply chain literature specific to dairy, specifically in the areas of distribution management, risk management, and decision-making strategies. Their paper indicates the limited presence of the organized sector and the potential to improve dairy practices via professional supply chain management policies.

2.5. Academic Contribution

Our extensive review of literature in this field indicates that most of the research has focused on these dimensions in isolation. Integrating two or more dimensions in the same research is mostly sparse. It is evident that there is a need to extend the existing research boundary to cover the unique problem of intermittent demand surges leading to demand-supply mismatches for perishables with supply variability, in the context of the dairy industry. Our exhaustive review of papers covering all three dimensions of this supply chain problem brings up no specific research done to address this challenge. In effect, this thesis makes the following contributions to literature in the area of perishables inventory management:

- Development of a mathematical model that captures the impact of demand-supply mismatch within a time series of interest to the operations team.
- Formulation of a three-stage stochastic programming approach to address the decisions at sourcing, production and dispatch stages.
- Usage of open source alternatives such as Python and Google OR-Tools to build a working code that solves a wide variety of scenarios of practical relevance within acceptable computational times.

3. METHODOLOGY

This section describes the approach taken to understand the research problem, formulate a solution to address the problem, and finally validate the effectiveness of the solution. Both qualitative and quantitative methods have been used to conduct the research. The process flow of the research methodology is depicted in figure 1.

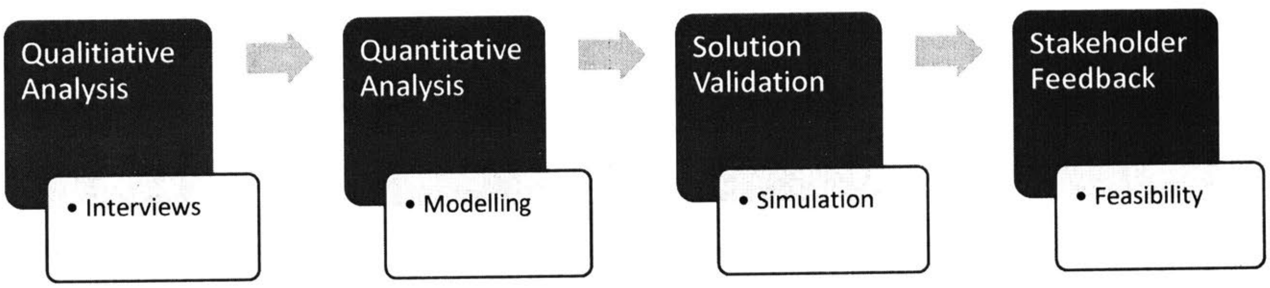

Figure 1: Research Methodology

3.1. Qualitative Analysis

To understand the supply chain processes better, close interaction with the key players was planned. While data and observations showed supply variability, the patterns, causes, and impact of this variability have not been formally captured in detail. To gather knowledge about variability, one-on-one interviews with a random sample of stakeholders is the most promising option. It gives us a chance to explore the motivation and feelings of the individuals involved, and explore related topics to better understand the problem and its cause/effect. This step involves interviewing the relevant stakeholders in the supply chain for perishables. The key stakeholders are people involved in supply (upstream suppliers, procurement team), production (operations team), and distribution (sales team). Of these, the supplier base is the most widely dispersed, and there is very little formal interaction with them, which limits the key insights being available upfront. Owing to the complexity of the supply variability dimension, more attention has been paid to interacting with suppliers. The interview sessions have been conducted across multiple time periods and geographies to provide insights from a broader supplier base. Pre-defined questionnaires covering both open-ended and closed-ended questions have been used. There has been no attempt to comment on the efficiency or effectiveness of the underlying processes during these interview sessions.

3.2. Quantitative Analysis

From an operational standpoint, our objective is to maximize profits while considering the potential losses on account of perishability and lost sales. In this step, we formulate the optimization problem, while realizing the stochastic nature of both demand and supply. We incorporate the impact of demand surge and supply reduction on specific time periods within the time series. This is indicative of the demand-supply mismatches expected around the specific time periods, i.e., festivals. We assume that the product is shipped to distributors based on firm orders received. To reflect the real-world situation, we model the need to specify the supply targets before we plan for production. Also, production needs to be based on the actual supply received, which is variable. Finally, we model the need to produce finished goods before knowing the actual order.

We make a key assumption with respect to perishability here as follows. The raw material received at the beginning of the time period needs to be consumed within the same time period. Any excess raw material not used in production needs to be discarded. Also, the finished goods produced during the time period needs to be shipped at the end of the time period. Any excess finished goods not needed to fulfill demand needs to be discarded. This situation closely represents the perishability challenges in the dairy supply chain as noted in section 5.

We explore different constraints on sourcing and production to evaluate the applicability of the model. Specifically, we evaluate the impact of specifying sourcing targets at an individual time period level, as well as a consistent sourcing target for the overall time series. Similarly, we evaluate the impact of having fixed production targets in advance, as well as flexible production targets based on actual supply received. As an outcome of this optimization, we obtain the recommended sourcing targets and production targets, that can be notified to the relevant stakeholders.

3.3. Solution Validation

An optimal solution derived in the previous section assumes the approximation of distribution for supply and demand. We further validate the model using simulation to build confidence in the solution. We test the solution using randomly generated data (as per the distribution) over different scenarios. This will ensure that while the optimization model is built using limited time series data, the principles are sound enough to be extended with enough confidence. Finally, we test the solution using actual historical data over multiple time periods (about three months). We present a summary of the results derived from simulations in section 6. The key measures are the improvement in overall profit, contributed by a reduction in wastage and lost sales.

3.4. Stakeholder Feedback

To ensure that the optimal inventory model is also feasible from an implementation perspective, feedback from procurement, production and sales team is taken to identify any key assumptions or constraints to be incorporated in the model.

4. MODEL DEVELOPMENT

In this section, we develop the mathematical model to represent the optimization problem discussed in section 3.2. We define the problem in mathematical terms and formulate the optimization model. We conclude by providing a solution approach for using the model to identify the decision variables and also provide a high-level algorithm (pseudocode) for the same.

4.1. Problem Statement

We have demand and supply variability in the supply chain. We have a delay between supply being realized and raw material being available for production. The supply is stochastic and centered around the supply target provided. We plan the production based on supply available while demand is yet unknown. The demand is also stochastic and centered around the forecast. Once the actual demand is realized in the form of orders, we dispatch the finished goods. The time series might comprise of some time periods when we have the simultaneous surge in demand and reduction in supply. Excess raw material (supply not used for production) and finished goods (production not used to fulfill demand) result in wastage, while a shortage of finished goods (demand not fulfilled by production) results in lost sales. Our intent is to maximize profits across the time series. In the absence of an analytical approach, we end up high wastage or lost sales, neither of which is desirable.

4.2. Model Notation

Following notation has been used for the model. We specify the symbols for sets/elements, parameters, random variables, and decision variables.

Table 1: Model Notation

Symbol	Explanation
SETS	
T	Set of time periods within the time series, indexed by t
I	Set of occurrences of a discrete probability of supply, indexed by i
J	Set of occurrences of a discrete probability of demand, indexed by j
PARAMETERS	
ΔD	Time periods between production and demand realization
ΔS	Time periods between supply realization and production
D	Demand expected per time period for the entire time series
C_1	The incremental cost of procurement of per unit of raw material
C_2	The incremental cost of production of per unit of finished goods
C_3	The incremental cost of selling per unit of finished goods to fulfill the demand
G_1	Salvage value per unit of excess raw material
G_2	Salvage value per unit of excess finished goods
P	Selling price realized for any unit sold
B	Shortage cost of unfulfilled demand per unit
p_i^S	Probability of the occurrence of a specific value of supply
δ_i^S	Scaling factor for the specific value of supply
p_j^D	Probability of the occurrence of a specific value of demand
δ_j^D	Scaling factor for the specific value of demand
Y_t	Binary value representing whether a specific time period has a simultaneous demand surge and supply shortage
α^D	The incremental surge in demand (+ve) during the relevant time period
α^S	The decremental wane in supply (-ve) during the relevant time period
RANDOM VARIABLES	
D_t	Demand realized for time period t
S_t	Supply realized for time period t
π_t	Profit for time period t
S_t^i	The specific value of supply for time period t
D_t^j	The specific value of demand for time period t
DECISION VARIABLES	
$S_t^{\sim}$	Supply target for time period t
$S^{\sim}$	Supply target per time period for the entire time series
ρ_t	Production realized for time period t
$\rho_t(S_{t-\Delta S}^i)$	Production realized for time period t as a function of supply realized
φ_t	The sale realized for time period t

4.3. Time Series Formulation

We formulate a time series comprising of multiple time periods (usually days). For each time period t, we expect a corresponding demand that is realized as an order received by the end of time period

$t-\Delta D$. On the supply side, for each time period t, we expect a corresponding supply that materializes and is available as raw material by the start of time period $t+\Delta S$. In effect, the production during time period t is based on supply received from time period $t-\Delta S$ and aims to fulfill demand from time period $t+\Delta D$. For simplicity, we assign ΔS and ΔD as 1 to arrive at a sample time series model in table 1. We denote the different values of realized supply, production and demand in keeping with the parameters mentioned earlier.

Table 2: Sample Time Series Model linking time periods across Supply-Production-Demand

Time Period	Supply	Raw Material	Production	Finished Goods	Demand
...	...				
99	500	...	...	...	
100	495	500	500	500	...
101	505	495	495	495	505
102	...	505	505	505	500
103		...	...	...	495
...					...

We note that during time period 101, production receives supply (495) that materialized during period 100, and produces finished goods (495) to cater to demand (500) during time period 102. As we can see, the time period of 101 results in lost sales, while time period 102 results in excess items.

4.4. Mathematical Model

As we see from the time series model in table 2, both supply and demand are variable. While producing more than demand may lead to excess inventory (which may lead to wastage in case of perishables), producing less than demand may lead to lost sales. We formulate a stochastic optimization model over the time series data of supply and demand to assess the optimal inventory policy to maximize profits while addressing the cost of wastage and lost sales.

The basic premise of a two-stage stochastic programming model is that we take an action at the first stage without knowledge of the outcome, and once the information is known, we take an action in the second stage to optimize the objective value (Shapiro and Philpott, 2007). However, many operational problems are more complex than this and may involve the cascading impact of decisions, which involves a multi-stage stochastic programming model (Birge and Louveaux, 2011). In the context of our research, we formulate this situation using a three-stage stochastic model. In the first stage, we decide on the target supply, that needs to be communicated to the upstream players. At this stage, we know neither the actual supply nor the actual demand. In the second stage, after we receive the supply, we decide on the target production. At this stage, we do not know the actual demand. However, we are constrained by the actual supply received. In the third stage, we know the actual demand based on orders and ship out the products. However, we are constrained by the actual demand, as well as actual production.

We model the overall optimization with an intent to increase total profit, which has the four elements: revenue, salvage values, cost of goods sold, and shortage costs. The revenue is driven by actual demand fulfilled. Salvage value may be applicable for raw material (excess supply not used in production) or finished goods (excess production not sold). In cases where we incur disposal costs, salvage value can be considered as negative. The cost can be broken down into procurement cost (driven by the actual supply of raw material received), production cost (driven by the actual quantity used in operations) and selling cost (driven by the actual demand fulfilled). Shortage costs are driven by unfulfilled demand and quantify the negative impact of not being able to meet the requirements of the distributor or retail outlet. The model parameters have been outlined in section 4.2.

We formulate the objective function as maximizing the overall profit across the time series.

Model 1: Optimization – Profit Maximization

$$\max \sum_{t \in T} \pi_t \tag{1}$$

$$\text{s.t.}\ \pi_t = \mathrm{P} * \varphi_t - (C_1 * S_{t-\Delta S} + C_2 * \rho_t + C_3 * \varphi_t) \qquad \forall\, t \in T \tag{2}$$
$$+ \big(G_1 * (S_{t-\Delta S} - \rho_t) + G_2 * (\rho_t - \varphi_t)\big)$$
$$- B * (D_{t+\Delta D} - \varphi_t)$$

$$\varphi_t \leq \rho_t \qquad \forall\, t \in T \tag{3}$$

$$\varphi_t \leq D_{t+\Delta D} \qquad \forall\, t \in T \tag{4}$$

$$\rho_t \leq S_{t-\Delta S} \qquad \forall\, t \in T \tag{5}$$

The objective function (1) intends to maximize the cumulative profit across the time series. The profit function for each time period (2) comprises of revenue, costs, salvage values and impact of lost sales. The constraints can be explained as follows: we cannot sell more than we can produce (3); we cannot sell more than the demand (4); we cannot produce more than what we receive as supply (5). In terms of decision variables, we primarily want to know the supply target to be set at the overall time series level or individual time period level for an expected demand and other cost parameters.

An astute reader would have noticed by now that we have not considered the impact of variability in demand and supply in this model. In order to do that, we realize that for an expected value of demand *D* over this time series, the occurrence of random demand during a production time period *t* is denoted by $D_{t+\Delta D}$. Similarly, given a supply target of $S^{\sim}$ throughout the time series, or $S_t^{\sim}$ for a time period, the occurrence of random supply during a production time period *t* is denoted by $S_{t-\Delta S}$.

In order to address the variability, we need to evaluate the objective function for all possible combinations of supply and demand. For a continuous normal distribution, this quickly makes the model non-linear, and unable to scale from a computational perspective. To address this concern, we propose to convert this into a stochastic linear programming model by converting the normal distribution into a discrete distribution having a set of values with assigned probabilities. To this effect, we introduce a few more model parameters into the optimization model to include the discrete probabilities for supply and demand as follows.

Model 2: Stochastic Optimization – Profit Maximization

$$max \sum_{t\in T}\sum_{i\in I} p_i^S * \sum_{j\in J} p_j^D * \pi_t \tag{6}$$

$$\begin{aligned} s.t.\, \pi_t = P * \varphi_t - \left(C_1 * S_{t-\Delta S}^i + C_2 * \rho_t\left(S_{t-\Delta S}^i\right) + C_3 * \varphi_t\right) \\ + \left(G_1 * \left(S_{t-\Delta S}^i - \rho_t\left(S_{t-\Delta S}^i\right)\right) + G_2 \right. \\ \left. * \left(\rho_t\left(S_{t-\Delta S}^i\right) - \varphi_t\right)\right) - B * \left(D_{t+\Delta D}^j - \varphi_t\right) \end{aligned} \qquad \forall\, t \in T, i \in I, j \in J \tag{7}$$

$$\varphi_t \le \rho_t\left(S_{t-\Delta S}^i\right) \qquad \forall\, t \in T, i \in I \tag{8}$$

$$\varphi_t \le D_{t+\Delta D}^j \qquad \forall\, t \in T, j \in J \tag{9}$$

$$\rho_t\left(S_{t-\Delta S}^i\right) \le S_{t-\Delta S}^i \qquad \forall\, t \in T, i \in I \tag{10}$$

$$\sum_{i\in I} p_i^S = 1 \tag{11}$$

$$\sum_{j\in J} p_j^D = 1 \tag{12}$$

The modified objective function (6) intends to maximize the cumulative profit across the time series weighted by the probability of supply and demand for all scenarios. The modified profit function for each time period (7) incorporates the supply and production realized for each scenario. The modified constraints can be explained as follows: we cannot sell more than we can produce, which depends on the probabilistic supply received (8); we cannot sell more than the probabilistic demand (9); we cannot produce more than we receive as the probabilistic supply (10). Finally, the sum of all probabilities must be 1 for both supply (11), and demand (12).

We now address the question about how to formulate the discrete probabilities and associated values that are needed for this model. We can represent any value of demand or supply realization as a scaled value of expected demand or target supply respectively. This can be represented as:

$$D_t^j = D * \delta_j^D \qquad \forall\, t \in T, j \in J \quad (13)$$

$$S_t^i = S_t^{\sim} * \delta_i^S \qquad \forall\, t \in T, i \in I \quad (14)$$

$$S_t^i = S^{\sim} * \delta_i^S \qquad \forall\, t \in T, i \in I \quad (15)$$

In other words, for all time periods, (13) is applicable for demand, and (14) or (15) is applicable for supply.

We draw inspiration from the work done by Barreiros (2005) as well as Miller and Rice (1983) to derive the approximation for probabilities and the scaling values, assuming a normal distribution of supply and demand. For a normal random variable with mean μ, standard deviation σ, and coefficient of variation CV= σ / μ, we use the matrix of probabilities and scaling factors outlined in table 3.

Table 3: Discrete Probability Matrix to approximate Normal Distribution

Scenario (i & j)	Probability (p_i^S & p_j^D)	Scaling Factor (δ_i & δ_j)
1	0.01	1-3*CV
2	0.06	1-2*CV
3	0.24	1-1*CV
4	0.38	1
5	0.24	1+1*CV
6	0.06	1+2*CV
7	0.01	1+3*CV

Next, we incorporate the impact of demand surge and supply shortage on specific days. We introduce a few more model parameters to enhance the optimization model to account for this feature as follows.

Model 3: Stochastic Optimization with Discrete Approximation and Surge Factor

$$max \sum_{t\in T}\sum_{i\in I} p_i^S * \sum_{j\in J} p_j^D * \pi_t \tag{16}$$

$$s.t. \pi_t = P * \varphi_t - \left(C_1 * S_{t-\Delta S}^i + C_2 * \rho_t\left(S_{t-\Delta S}^i\right) + C_3 * \varphi_t\right) \qquad \forall\, t \in T, i \in I, j \in J \tag{17}$$
$$+ \left(G_1 * \left(S_{t-\Delta S}^i - \rho_t\left(S_{t-\Delta S}^i\right)\right) + G_2\right.$$
$$\left. * \left(\rho_t\left(S_{t-\Delta S}^i\right) - \varphi_t\right)\right) - B * \left(D_{t+\Delta D}^j - \varphi_t\right)$$

$$\varphi_t \leq \rho_t\left(S_{t-\Delta S}^i\right) \qquad \forall\, t \in T, i \in I \tag{18}$$

$$\varphi_t \leq D_{t+\Delta D}^j \qquad \forall\, t \in T, i \in I \tag{19}$$

$$\rho_t\left(S_{t-\Delta S}^i\right) \leq S_{t-\Delta S}^i \qquad \forall\, t \in T, i \in I \tag{20}$$

$$D_t^j = \mathrm{D} * \delta_j^D * (1 + Y_t * \alpha^D) \qquad \forall\, t \in T, j \in J \tag{21}$$

$$S_t^i = S_t^{\sim} * \delta_i^S * (1 + Y_t * \alpha^S) \text{ OR} \qquad \forall\, t \in T, i \in I \tag{22}$$

$$S_t^i = S^{\sim} * \delta_i^S * (1 + Y_t * \alpha^S)$$

$$Y_t \in \{0,1\} \qquad \forall\, t \in T \tag{23}$$

$$\sum_{i\in I} p_i^S = 1 \tag{24}$$

$$\sum_{j\in J} p_j^D = 1 \tag{25}$$

The additional constraints can be explained as follows: The demand for any scenario during a specific time period (21) is first scaled for the specific occurrence and then the impact of demand surge is factored in. The same approach applies to supply for any scenario during a specific time period (22). We allow for the supply target to be defined at a specific time period level or entire time series level.

To summarize, we specify the expected demand, the distribution of demand variation with associated probability, the distribution of supply variation with associated probability, and the list of special days (with simultaneous demand surge and supply shortage) in the time series of interest. The optimization routine recommends the target supply for each time period or entire time series (to manage inventory of raw material), and the target production plan based on the realization of supply for each time period (to manage inventory of finished goods).

4.5. Solution Approach

The proposed model has overall profit as the objective function and uses a three-stage stochastic optimization approach. In order to solve this model for a specific context, we cover three different situations. In the first case, supply targets can be set up at time period level. We develop an array to represent the supply targets at time period level as decision variables. Next, we develop a matrix to represent production targets at time period level for each realization of supply. Finally, we develop an array to represent the sale realized at time period level. Using the probability distribution matrix in table 3, we develop a demand matrix representing possible values at time period level. We use a linear solver to evaluate the optimal profit across the time series and arrive at the recommended supply targets at time period level. In the second case, we set the same supply target for each time period across the time series. In the third case, we revert to supply targets at time period level, and we use an array of production targets at time period level. This implies that production targets need to be set before the supply is realized.

Gearhart, Adair, Durfee, Jones, Martin, & Detry (2013) provide an excellent summary of linear solvers that can be used for this research. To solve the model programmatically, we make use of open source options – Python language and Glop Linear Solver. This linear solver is available as part of Google-OR Tools, and merely needs installation of a package within the Python environment. Some of the efficient coding practices described by Kruk (2018) have been used in programming.

4.6. Pseudocode for Solving the Model

The pseudocode for solving the model using any programming logic and any linear solver is explained below. The implementation of this algorithm using Python and Glop Linear Solver is available in appendix E.

```
# initialization
initialize parameters
initialize CV for demand and supply
initialize arrays for probability of supply and demand scenarios
initialize arrays for scaling factors of supply and demand
initialize time series and specify time periods having demand surge & supply shortage

# prepare for optimization
expand time series for demand, production, and supply (add time gap for production to demand)
expand time series for demand, production, and supply (add time gap for supply to production)
generate matrix for demand scenarios across time series
initialize linear solver

# solver decision variables
define array for supply target
define matrix for production plan
define array for sales realized

# solver constraints
for each time period
  for all demand/supply scenarios
    sales realized should be least of demand and production
    production should be less than supply realized

# solver objective
define profit for each period as revenue - cost + salvage - lost sales
define total profit as sum of profit across all time periods
maximize total profit

# display results
display total profit across time series & average profit per time period
display supply targets across time series
```

This covers the scenario where supply targets can be specified at time period level. For the case where supply target needs to be consistent throughout the time series, we modify the decision variable for supply target from an array to a single variable. For the case where a production plan needs to be decided before the supply is realized, we modify the decision variable for the production plan from a matrix to an array, with one value per time period.

4.7. Validation Approach

In order to validate the solution, we simulate multiple scenarios of demand and supply for the same time series. All the scenarios follow the distribution of demand and supply, as well as the specific time periods of demand surge and supply shortfall. The profit for each scenario is computed using the cumulative profit across the time periods. Finally, the simulation is run multiple times to obtain the average overall profit for the time series. This is then compared with the expected value of overall profit for the time series (derived from stochastic optimization). Owing to the nature of simulation, there might be a slight deviation between the numbers. If the deviation is within a reasonable threshold, we consider this to be an acceptable approach. The implementation of the same using Python is available in appendix F.

5. CASE STUDY

This research makes use of data and insights about perishables supply chain available with Milk Mantra, a leading private dairy company in eastern India. In this case study, we conducted interviews to explore the different challenges related to sourcing, production, and distribution of liquid milk. We outline the dairy supply chain processes and the key stakeholders involved in the same. We also utilize the company data to draw key inferences about the application of the three-stage stochastic optimization model described in section 4.

5.1. Background

Milk Mantra is a private dairy company, operational since 2012, that sources raw milk from farmers Odisha, a state in eastern India; manufactures packed milk and other milk products in two plants within eastern and western Odisha; and distributes finished goods across the eastern states of India. As part of their ethical milk sourcing program, they primarily source milk directly from the farmers, thereby ensuring optimal price discovery and ensuring long term supplier loyalty.

On account of sourcing directly from a large number of small suppliers, the procurement team encounters the challenge of high supply variability. While supply targets are cascaded across successive tiers of the network, there are no strict contracts with suppliers that can be enforced. Another challenge noted during the festivals is the reduction in supply around the same time demand surge is noted. The supply shortage is usually an outcome of small suppliers increasing their own intake of milk to meet family needs during festivals. Since suppliers cannot be added at short notice, this usually results in lost sales. On the other hand, increasing the supplier base may result in wastage of raw material and/or finished goods.

5.2. Stakeholder Interviews

This analysis involved interviews and discussions with relevant stakeholders across the supply chain – suppliers, procurement team, operations team, and sales team. These in-person interview sessions were conducted using pre-defined questionnaires. The key insights gathered from this step have been used to formulate the process flow map and the data model for the quantitative analysis. We focused on these areas: a) Understanding Supply Variability, b) Understanding Demand Variability, and c) Understanding Production Constraints.

Interviews were conducted with relevant employees of the company as recommended by the department heads for procurement, sales, and production. Furthermore, interviews were conducted with company agents and independent suppliers randomly picked across the stakeholder population. The detailed questionnaires used for the interviews with each stakeholder group is available in appendix C.

5.2.1. Procurement

One of the key employees (Manager, Sourcing – Extension Services) of Sourcing department was interviewed to understand the key characteristics of suppliers and the supply of raw milk. Subsequently, 29 independent suppliers (farmers) across 4 geographically dispersed collection points were interviewed to understand the key influence on supply quantity and quality. In addition, 5 commercial dairy farms (CDFs), 5 rovers and 3 Group Associates (GAs) across multiple locations were interviewed to understand the different challenges applicable to high volume scenarios.

The upstream player in the supply chain is the small farmer with one or more cows. As this is the lowest level of granularity at which data is gathered at the company, we explored the characteristics of supply at this level. Besides the small farmers, the other key suppliers are the rovers (independent agents who collect milk from remote farmers) and commercial dairy farms. Understanding any differences in their supply characteristics was useful to compare and contrast the two different classes of suppliers. As milk is collected from small farmers at a collection point (CP) by GA, interviewing the GAs provided insight into the pattern of aggregated supplier data. Finally, the procurement team at the company level was able to provide insight into supply-related challenges across the company.

The primary suppliers are individual farmers, CDFs, and rovers. Collection Points (CPs) are established at the village level and are managed by a Group Associate (GA). Milk collected at this point is transported to Bulk Milk Coolers (BMCs) twice a day for cooling and storage. Finally, milk is transported on tankers to the factory once a day.

Following are the key insights about the supplier base derived from these discussions:

- Farmers with a limited number of cows form the bulk of suppliers. This results in yield variance at the individual farmer level.
- Farmers apportion an increased amount of milk for personal use during festivals. This results in a reduction of supply during those specific days. The same is noted for Rovers, whose primary source is other small farmers.
- Since CDFs produce a much higher volume of milk in comparison to personal needs, their supply does not get impacted by festivals.

Additional details about the outcome of interviews and surveys are available in appendix D.

To summarize, supply variability arises from the presence of multiple small suppliers, and the supply dips on special occasions on account of their need to apportion more of the yield for their own consumption.

5.2.2. Sales

The downstream player in the supply chain is the regional or local distributor who services multiple retail outlets. As this is the lowest level of granularity at which data is gathered at the company, we identify the characteristics of demand at this level. Interviewing the key sales staff provided a high-level understanding of the causes and extent of demand variability.

The head of the sales department was interviewed to understand the key characteristics of demand and its underlying variability. While festivals have an impact, it is expected that it can be predicted to some extent based on historical data. But the variability due to random factors inherent in the supply chain seems to have a higher impact on forecast accuracy.

Additional details about the outcome of interviews are available in appendix D.

5.2.3. Production

The production process usually involves the conversion of raw material into intermediate and then finished goods. The perishability of the item on hand changes based on the transformations that take place throughout the production process. Interviewing the production team provided a high-level understanding of production constraints related to perishability and storage capacity of items at each stage of production.

One of the key employees (Plant Manager) of the Operations department was interviewed to understand the key production processes from raw material receipt to finished goods dispatch. The key constraints on the production process are the perishability curve for each in-process item, the temperature of storage/transport, and the storage capacity at each process step. In general, the production capacity of the plant was considered more than adequate and thus not treated as a constraint.

The variable demand is witnessed as orders from distributors, which is received the evening before the day of delivery. In the case of supply, a target is set for the procurement team, which is communicated to upstream players (BMCs, GAs, and farmers). The supply that actually materializes is variable and is collected a day before it is received at the plant. On festival days, there is an additional impact on the demand (which goes up) and supply (which goes down).

5.3. Challenges in Dairy Supply Chain

The overall problem of intermittent demand-supply mismatches in case of the dairy industry has been analyzed across three distinct dimensions:

5.3.1. Perishability

Raw milk has a very low shelf life of around 4-6 hours in tropical climates, whereas cooling it down to 4°C extends the shelf life to around 16-24 hours (M. Mohapatra, personal communication, Oct 2018). Thus, overstocking raw material beyond a day might be counter-productive. Packed milk has a shelf life of around 48 hours, as long as it stays refrigerated at retail locations and consumer homes (S. Biswas, personal communication, Nov 2018). Since retailers need at least a day's worth shelf life at the point of sale (S. Misra, personal communication, Nov 2018), overstocking finished goods beyond a day might also be counter-productive.

5.3.2. Supply Variability

M. Mohapatra (personal communication, Oct 2018) described the key characteristics of milk supply variability. The bulk of milk supply in developing countries like India comes from individual farmers. This sourcing model implies that volumes are driven by informal agreements rather than formal contracts. These individual suppliers produce milk, set aside some of it for their own consumption, and sell the rest of it to the company. All of the above explains why the supply is variable.

5.3.3. Intermittent Demand-Supply Mismatch

Milk is one of the key ingredients for making sweets (which are a major part of festivals). This implies that demand for milk spikes around the festivals. At the same time, the individual suppliers tend to set aside more milk for their own consumption (individual farmers, personal communication, Oct 2018), which leads to a significant drop in supply.

The combination of these three factors makes inventory management a major challenge for dairy companies. This thesis aims to address this challenge using the methodology proposed in section 3.

5.4. Data Collection

The procurement data is captured at the farmer level as the milk gets collected at the collection point (CP). Besides this, procurement data is available at BMC (Bulk Milk Cooler) level where milk is aggregated from collection points, rovers, and commercial dairy farms via pickup trucks. Typically, collection at CP and BMC happens twice a day (milking sessions are during morning and evening). The intake at plant level happens typically once a day as milk from multiple BMCs is collected via tankers. It is pertinent to analyze the variation of supply at each level to understand the pattern of variation. The classification of such variation into common causes (which may cancel each other out) and special causes (significant increase or decrease) will provide information about the likelihood of shortage or excess at the plant level.

The sales data is available at distributor level as milk is supplied to them by the company early morning against final orders (also called indents) received the previous afternoon/evening. Direct sales occur for some large retailers (grocery stores with high volumes) too. The aggregated data at the plant level has been analyzed to identify any significant increase or decrease in demand and thereby assess the likelihood of demand-supply mismatches.

The data model for supply and demand is available in appendix B.

5.5. Procurement Process Overview

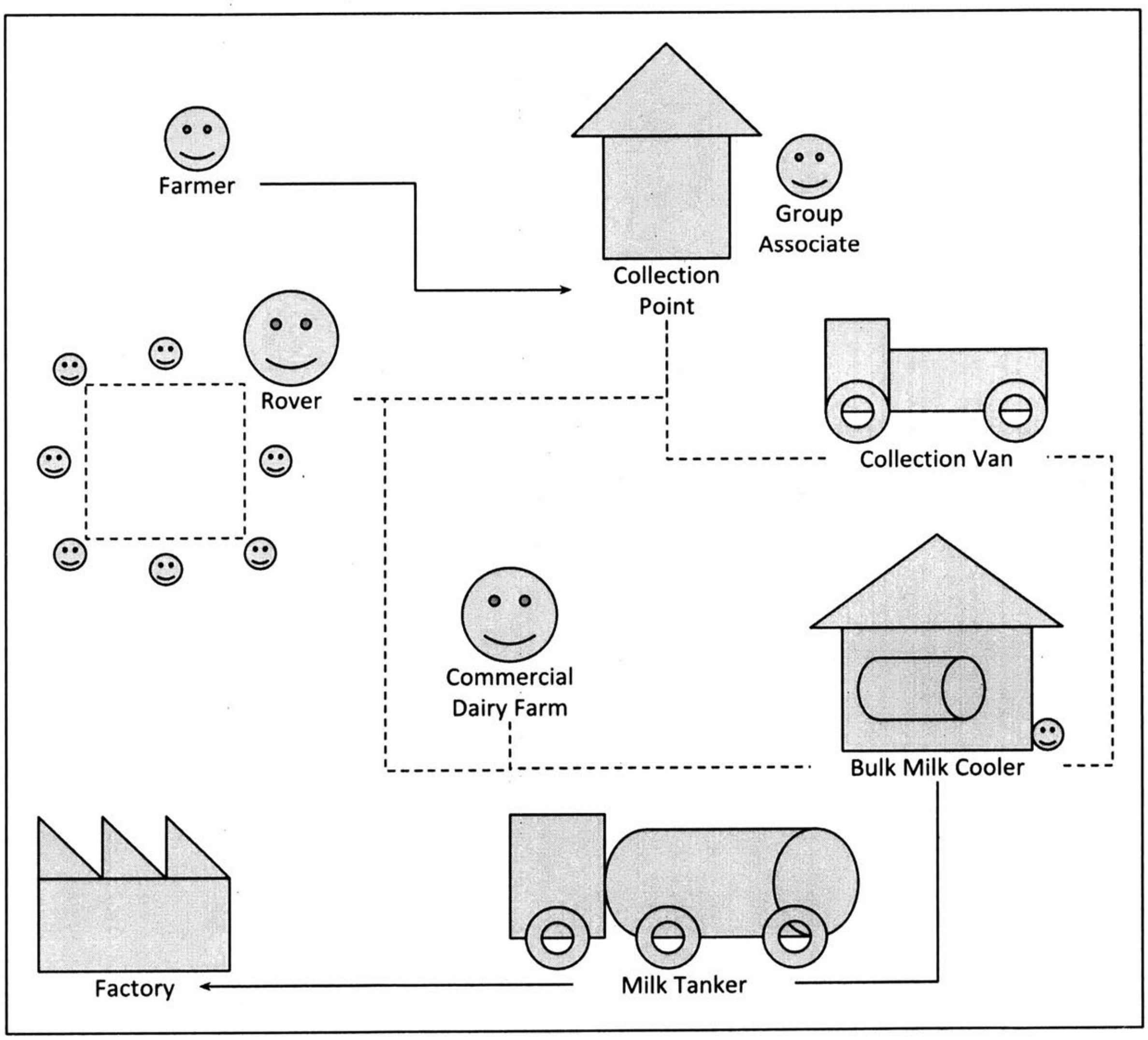

Figure 2: Process Flow for Milk Procurement

Figure 2 describes the process flow for milk procurement. Farmers within the company's network pour in milk at collection points, which are managed by Group Associates. Rovers collect milk from different farmers outside the company network. The milk collection van gathers milk from collection points, rovers, and commercial dairy farms to be brought over to BMC for cooling and storage. This process is done twice a day, as milk extraction happens in morning and evening. In some cases, rovers or farmers may directly bring milk to the BMC. Once a day, the milk from BMC is brought over to the factory for storage and processing.

The process flow for packaged milk production is available in appendix A.

6. RESULTS AND ANALYSIS

This section presents the outcomes of implementing the methodology described earlier. We consolidate our understanding of the supply variability, perishability, and mismatches in demand-supply that occur on festival days. Then we analyze the outcomes of applying stochastic optimization methods to arrive at optimal supply targets communicated to the upstream players. We evaluate the performance of the recommendations applied to simulated and historical supply and demand. We conclude by gathering stakeholder feedback on the recommendations to identify any limitations or additional opportunities for research.

6.1. Summary of Interviews

As part of this research, we conducted multiple interviews with stakeholders in sourcing, production and sales departments. Following are the key insights relevant for this research:

- Presence of a large number of small suppliers leads to high supply variability.
- Festivals lead to a reduction in supply as suppliers increase their own consumption.
- It is not easy to increase/decrease suppliers at very short notice to manage shortages.

6.2. Stochastic Optimization Outcomes

We use the optimization model developed in section 4 to identify the recommended decision variables for different scenarios. We generate a time series to represent all days of the week and identify some of the days as possible time periods of demand-supply mismatch. We interpret the optimization outcomes for different combinations and derive key insights.

Table 4: Sample values of parameters used in the Stochastic Optimization Model

Parameter	Symbol	Value
Time periods between production and demand realization	ΔD	1
Time periods between supply realization and production	ΔS	1
Demand expected per time period for the entire time series	D	100
The incremental cost of procurement of per unit of raw material	C_1	20
The incremental cost of production of per unit of finished goods	C_2	5
The incremental cost of selling per unit of finished goods to fulfill the demand	C_3	1
Salvage value per unit of excess raw material	G_1	-1
Salvage value per unit of excess finished goods	G_2	-2
Selling price realized for any unit sold	P	35
Shortage cost of unfulfilled demand per unit	B	50
The incremental surge in demand (+ve) during the relevant time period	α^D	20%
The decremental wane in supply (-ve) during the relevant time period	α^S	-10%
CV for demand (assumed normal distribution)	CV_D	0.1
CV for supply (assumed normal distribution)	CV_S	0.05

Key assumptions:

- We expect that the raw milk collected yesterday evening is brought into the plant today and the packed milk that is produced today to fulfill orders that will be dispatched tonight to meet the demand for tomorrow.
- We pay for any supply that is received, even if it is higher/lower than the target. We may choose not to produce, based on our assessment of demand. In this case, we may end up with raw material going waste. We may choose not to ship if there are not enough orders. In this case, we may end up with finished goods going waste.
- We expect negative salvage values, which represent disposal costs. While this applies in case of milk where there is no alternative usage of the expired items, in other instances, this may be zero or positive.
- Penalty for lost sale (B) is the best guess estimate of the impact of unfulfilled demand.
- The probability distribution matrix for supply and demand is as defined in table 3.

We solve the model for different scenarios of time series with or without festival days (denoted as 1 for festival day and 0 for the regular day) and arrive at the recommendations summarized in table 5.

Table 5: Recommended Supply Targets per Time Period

S.No.	Time Series	Recommended Supply Targets	Expected Objective Function (Total Profit & Average Profit)
1	[-, -, 0, 0, 0, 0, 0, 0, 0]	[105.263, 105.263, 105.263, 105.263, 105.263, 105.263, 105.263, -, -]	3758.185 536.884
2	[-, -, 1, 0, 0, 0, 0, 0, 0]	[126.316, 105.263, 116.959, 105.263, 105.263, 105.263, 105.263, -, -]	3865.562 552.223
3	[-, -, 0, 0, 0, 0, 1, 0, 0]	[105.263, 105.263, 105.263, 105.263, 126.316, 105.263, 116.959, -, -]	3865.562 552.223
4	[-, -, 0, 0, 1, 0, 0, 0, 0]	[105.263, 105.263, 126.316, 105.263, 116.959, 105.263, 105.263, -, -]	3865.562 552.223
5	[-, -, 0, 1, 0, 1, 0, 0, 0]	[105.263, 126.316, 105.263, 140.351, 105.263, 116.959, 105.263, -, -]	3972.938 567.563
6	[-, -, 0, 1, 1, 0, 0, 0, 0]	[105.263, 126.316, 126.316, 116.959, 116.959, 105.263, 105.263, -, -]	3972.938 567.563
7	[-, -, 1, 0, 0, 0, 1, 0, 0]	[126.316, 105.263, 116.959, 105.263, 126.316, 105.263, 116.959, -, -]	3972.938 567.563
8	[-, -, 1, 0, 1, 0, 1, 0, 0]	[126.316, 105.263, 140.351, 105.263, 140.351, 105.263, 116.959, -, -]	4080.315 582.902
9	[-, -, 0, 1, 1, 1, 0, 0, 0]	[105.263, 126.316, 126.316, 140.351, 116.959, 116.959, 105.263, -, -]	4080.315 582.902
10	[-, -, 1, 1, 1, 1, 1, 0, 0]	[126.316, 126.316, 140.351, 140.351, 140.351, 116.959, 116.959, -, -]	4295.069 613.581

It is important to note that there is a lag of two time periods between supply target and demand – this accounts for the fact that what is procured in time period t-2 is produced in time period t-1 to fulfill demand in period t.

We observe that any instance of demand-supply mismatch (occurring due to festival days) results in two spikes in supply targets: the first one occurs two days before the event, to account for increased supply in anticipation of expected increase in demand, and the second occurs on the day of event, to account for anticipation of a decrease in supply. As expected, there is also a compounding effect when multiple instances of the mismatch (accounting for multiple festival days) are noticed within the same time series.

One of the key assumptions we have made in this analysis is that supply targets can be set for every time period. However, in reality, this may not be as straight-forward. It takes time to acquire/eliminate suppliers, and thus supplier targets are expected to be stable for the entire time series. Applying this constraint degrades the objective function on account of the need to overstock or end up with lost sales. Table 6 summarizes the recommendations for the similar set of time series shown earlier:

Table 6: Recommended Supply Targets for entire Time series

S.No.	Time Series	Recommended Supply Target	Expected Objective Function (Total Profit & Average Profit)	Previous Objective Function (Total Profit & Average Profit)
1	[-, -, 0, 0, 0, 0, 0, 0, 0]	105.263	3758.185 536.884	3758.185 536.884
2	[-, -, 1, 0, 0, 0, 0, 0, 0]	110.000	3358.814 479.831	3865.562 552.223
3	[-, -, 0, 0, 0, 0, 1, 0, 0]	110.000	3358.814 479.831	3865.562 552.223
4	[-, -, 0, 0, 1, 0, 0, 0, 0]	110.000	3358.814 479.831	3865.562 552.223
5	[-, -, 0, 1, 0, 1, 0, 0, 0]	115.789	2823.145 403.306	3972.938 567.563

S.No.	Time Series	Recommended Supply Target	Expected Objective Function (Total Profit & Average Profit)	Previous Objective Function (Total Profit & Average Profit)
6	[-, -, 0, 1, 1, 0, 0, 0, 0]	116.402	3358.812 479.830	3972.938 567.563
7	[-, -, 1, 0, 0, 0, 1, 0, 0]	116.402	3358.812 479.830	3972.938 567.563
8	[-, -, 1, 0, 1, 0, 1, 0, 0]	122.222	2675.762 382.252	4080.315 582.902
9	[-, -, 0, 1, 1, 1, 0, 0, 0]	121.212	3183.358 454.765	4080.315 582.902
10	[-, -, 1, 1, 1, 1, 1, 0, 0]	132.000	3686.805 526.686	4295.069 613.581

The impact of the policy of consistent supply targets over the time series is pronounced. In all the cases the impact of festival days results in a reduction of profit over the duration. In the case of multiple festivals, the impact is more pronounced when it occurs with a gap of two days. This is due to the fact that besides compensating for the possible decrease in supply, we also need to compensate for the possible increase in demand that will occur when this supply is allocated to the same. These results clearly demonstrate that we should expect to place a premium on suppliers who can accept flexible targets.

The other key assumption we made is that the plant can decide on the production volumes after receiving the supply, which is then deterministic, while demand is still stochastic. This clearly depends on the flexibility of plant scheduling, and may not always be feasible. We conduct an analysis to investigate the impact of ensuring that the production targets are set for the day and cannot be changed based on supply received. Table 7 summarizes the recommendations for the similar set of time series shown earlier.

Table 7: Recommended Supply Targets for Fixed Production

S.No.	Time Series	Recommended Supply Targets	Expected Objective Function (Total Profit & Average Profit)	Previous Expected Objective Function (Total Profit & Average Profit)
1	[-, -, 0, 0, 0, 0, 0, 0, 0]	[117.647, 117.647, 117.647, 117.647, 117.647, 117.647, 117.647, -, -]	1358.082 194.012	3758.185 536.884
2	[-, -, 1, 0, 0, 0, 0, 0, 0]	[141.176, 117.647, 130.719, 117.647, 117.647, 117.647, 117.647, -, -]	1396.885 199.555	3865.562 552.223
3	[-, -, 0, 0, 0, 0, 1, 0, 0]	[117.647, 117.647, 117.647, 117.647, 141.176, 117.647, 130.719, -, -]	1396.885 199.555	3865.562 552.223
4	[-, -, 0, 0, 1, 0, 0, 0, 0]	[117.647, 117.647, 141.176, 117.647, 130.719, 117.647, 117.647, -, -]	1396.885 199.555	3865.562 552.223
5	[-, -, 0, 1, 0, 1, 0, 0, 0]	[117.647, 141.176, 117.647, 156.863, 117.647, 130.719, 117.647, -, -]	1435.687 205.098	3972.938 567.563
6	[-, -, 0, 1, 1, 0, 0, 0, 0]	[117.647, 141.176, 141.176, 130.719, 130.719, 117.647, 117.647, -, -]	1435.687 205.098	3972.938 567.563
7	[-, -, 1, 0, 0, 0, 1, 0, 0]	[141.176, 117.647, 130.719, 117.647, 141.176, 117.647, 130.719, -, -]	1435.687 205.098	3972.938 567.563
8	[-, -, 1, 0, 1, 0, 1, 0, 0]	[141.176, 117.647, 156.863, 117.647, 156.863, 117.647, 130.719, -, -]	1474.489 210.641	4080.315 582.902
9	[-, -, 0, 1, 1, 1, 0, 0, 0]	[117.647, 141.176, 141.176, 156.863, 130.719, 130.719, 117.647, -, -]	1474.489 210.641	4080.315 582.902
10	[-, -, 1, 1, 1, 1, 1, 0, 0]	[141.176, 141.176, 156.863, 156.863, 156.863, 130.719, 130.719, -, -]	1552.094 221.728	4295.069 613.581

The impact of the policy of setting a production target for the day does not have an impact on the sourcing targets. However, it does degrade the profitability as we decide to either produce more and suffer disposal cost or produce less and suffer the cost of lost sales. These results clearly demonstrate that we should expect to place a premium on flexibility in production scheduling.

6.3. Sensitivity Analysis

We assess the objective function's sensitivity to some of the key parameters. Primarily, we are interested in non-linear relationships that might provide some key insights. We show the graph of overall profit as a function of Cost of Lost Sales, Coefficient of Variation for Demand and for Supply.

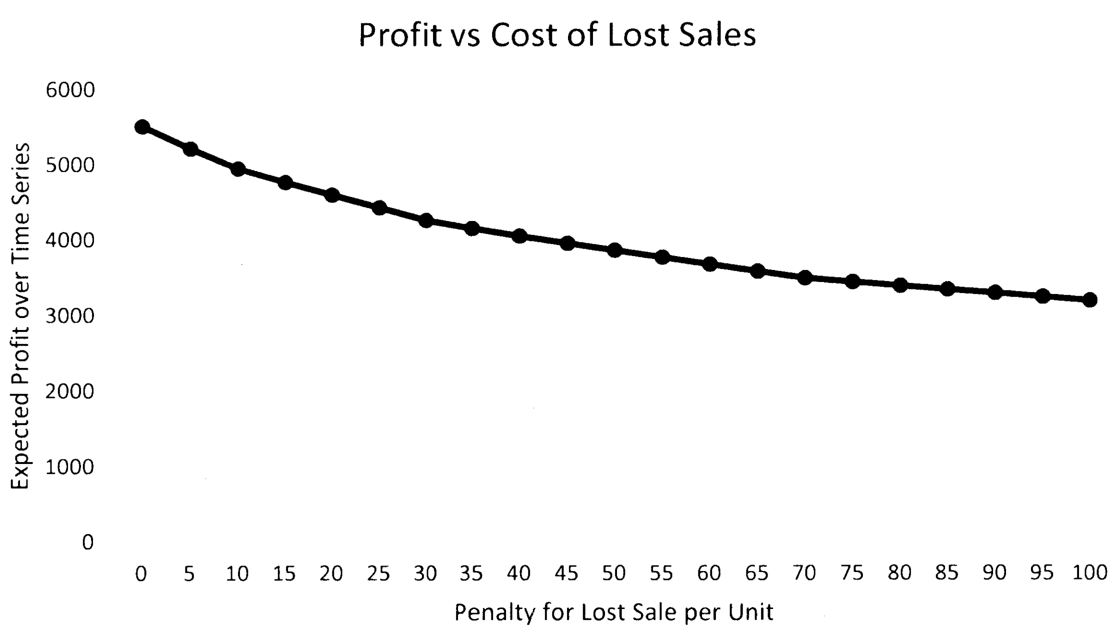

Figure 3: Sensitivity of Profit to Cost of Lost Sales

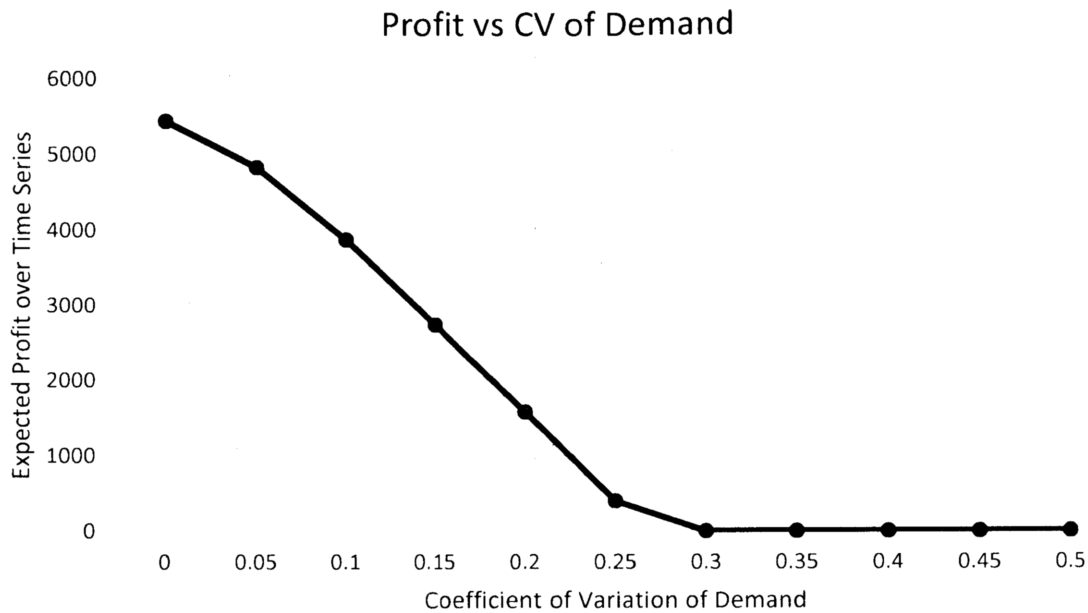

Figure 4: Sensitivity of Profit to the CV of Demand

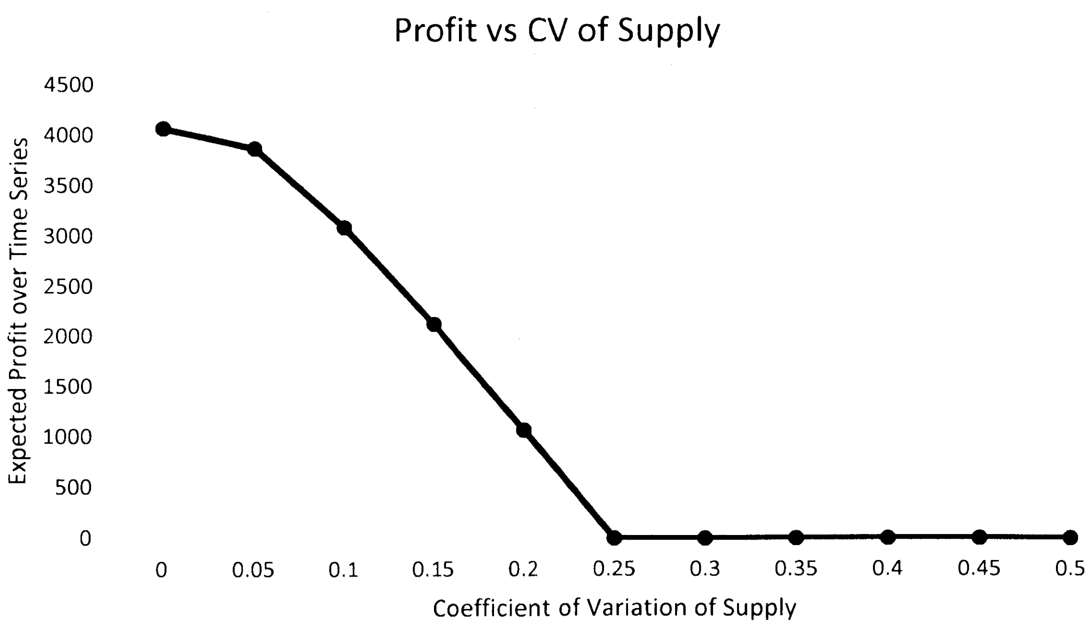

Figure 5: Sensitivity of Profit to the CV of Supply

6.4. Solution Validation

We evaluated the resulting decision variables regarding supplier targets and production plans using simulation. We chose time series #4 in table 4 for this assessment (with expected total profit value of 3865.562). We use a set of random values generated using a normal distribution for supply and demand during each time period, and make an adjustment for 3rd time period indicating demand surge and supply shortfall. We run this check over multiple iterations and increase the number of iterations until we reach convergence.

Table 8: Comparison of simulation results with expected values as per the optimization model

Iterations	Successive Outcomes	Deviation within outcomes	Deviation from Expected Value
1	3310.491 & 3844.372	-5.4%	-14.4%
10	4007.772 & 3634.942	-9.3%	+3.7%
100	3722.089 & 3907.222	-4.7%	+1.1%
1000	3849.763 & 3888.351	-1.0%	+0.6%
10000	3852.637 & 3857.602	-0.1%	-0.2%
100000	3844.372 & 3843.909	0%	-0.5%

6.5. Stakeholder Feedback

In terms of applicability of the model, it was noted that supply targets need to be consistent over the time series. This is to reflect the challenges in increasing/decreasing the supplier base within the time series. A time series of 10 days (instead of 7 days used in section 6.2) would be a closer representation of company policy regarding adding/removing suppliers. Production plan could very well be decided based on supply realized during the previous day. This is because the information about raw material availability is available, even though it would physically be available for production the next day. Although the plant capacity is not an immediate concern, to account for future growth, storage constraints for raw material and finished goods, as well as production capacity constraints could be added to the model. Considering the simplicity of the model, this does not present a major challenge.

6.6. Alternative Methodology

In this research, we have considered the variability as a discrete empirical distribution. The key advantage of this approach is the ability to use linear programming to model and solve the problem efficiently. Increasing the granularity of the probability distribution does not have a major impact on solver time. On the other hand, we could have considered a continuous normal distribution for both demand and supply. The key disadvantage is that it makes the model non-linear and thus unable to scale without the development of specialized algorithms.

In terms of solver, we have used Google-OR Tools, as it has relevant python libraries and is open-source. This implies that the solution can be further enhanced to suit specific scenarios without any licensing implications. The ease of coding as well as the speed of solving was also favorable. We could have used other commercial solvers using their academic license versions. However, it would limit further development and extension of this solution.

7. CONCLUSION

In this research, we address the unique problem faced by companies dealing with perishable products when they face an intermittent demand-supply mismatch on account of festivals or similar occasions. At the same time, we face demand and supply uncertainty, which increases the complexity of the problem. We look at the dairy industry in eastern India using the company Milk Mantra as a case study to understand the problem, its causes, and its impact. Interviews with Procurement, Production and Sales teams provide adequate insight into the scale and complexity of the problem. Interactions with the suppliers provided a better understanding of the supply variability that comes on account of small farmers constituting the bulk of the supplier base. The need to set aside additional production for self-consumption during the festival times leads to a reduction in supply. Around the same time, demand from the consumer side goes up.

Stochastic optimization over the time series is used to arrive at the recommended supply targets. In order to make use of the power of linear programming, the stochastic distribution for demand and supply is broken down into a discrete empirical distribution. When presented with a time series of interest where specific festival days are marked, the solution presents the supply targets to be communicated to upstream players. We also notice that ensuring constant supply target for the entire time series alters the supply targets dramatically and has a profound impact on the overall profitability. On the other hand, enforcing the policy of fixed production targets for a time period does not impact the supply targets, but it does reduce the overall profitability. These results indicate the premium that we can expect to place on flexible suppliers and flexible production options.

Further research in this area can focus on two key dimensions. The perishability of both raw material and finished goods can be further converted into a vector considering that for some cases, it is possible to have lifetimes that go across production time periods. This would enable modeling a FIFO policy and using up items nearing expiry before they get discarded. The other line of research can focus on enabling normal distribution to be included in the model and assess the impact of non-linearity induced. There might be merit in solving the same analytically so that the solution can have much wider use.

This thesis creates a framework for businesses and researchers dealing with similar situations and provides a starting point for implementing stochastic optimization via linear programming as a tool for dealing with uncertainty around perishables.

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APPENDIX A: Process Flows

Figure A6 describes the process flow for packaged milk production.

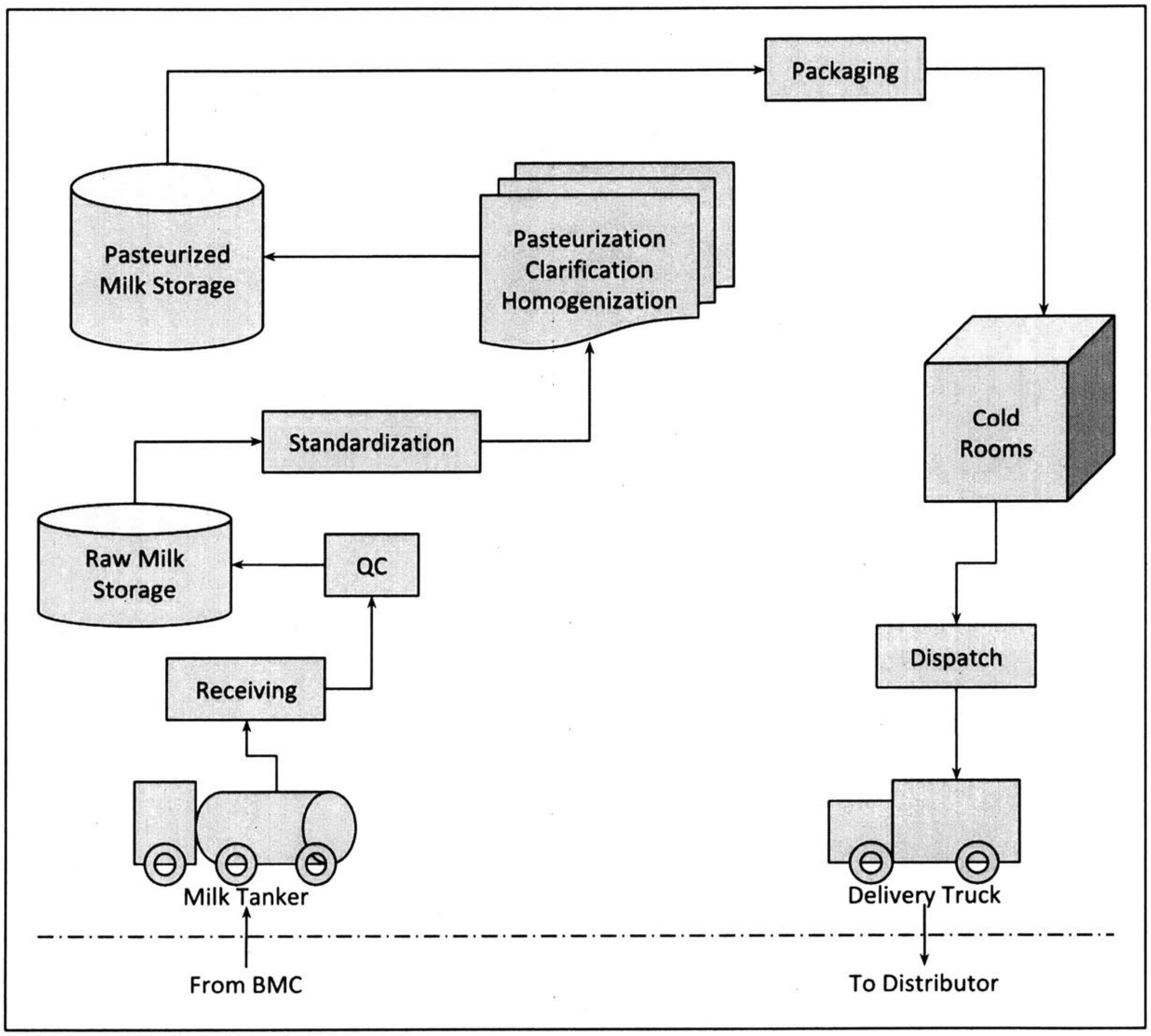

Figure A6: Process Flow for Milk Processing

Raw milk is received at the factory from BMCs on tankers. After Quality Check (QC) at Receiving dock, it is stored in Raw Milk Storage Tanks (RMST). After processing, it is stored in Pasteurized Milk Storage Tanks (PMST). Then it is packed in milk pouches (based on demand forecast for next day) and stored in cold room. Finally, packed milk is dispatched to distributors so as to reach them early morning for further dissemination to retailers.

APPENDIX B: Data Models

Figure B7 describes the data model for milk procurement.

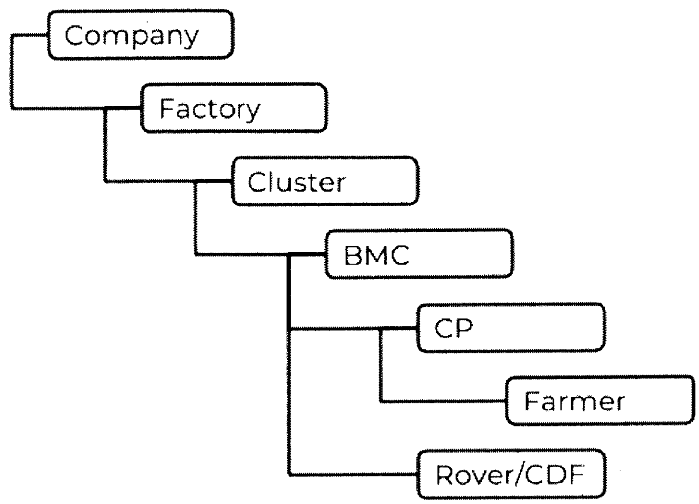

Figure B7: Data Model for Milk Procurement

Figure B8 describes the data model for milk distribution. HoReCa represents the customer segment for Hotels, Restaurants, and Caterers – these are bulk consumers where direct sales are feasible.

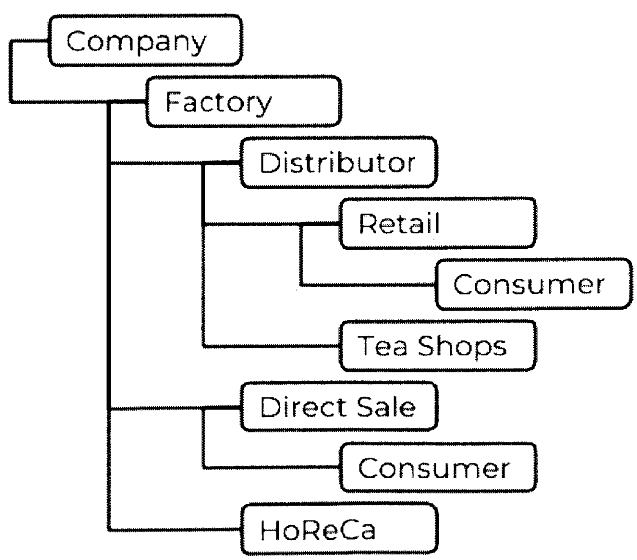

Figure B8: Data Model for Milk Distribution

APPENDIX C: Questionnaires for Interviews & Surveys

Questions for Sourcing Team

1. Who are the primary producers? (Farmers, Commercial Farms, others)
2. Who do we consider as suppliers? (producers, rovers, agents)
3. Do we have any contracts with primary producers?
4. What is the typical amount (range/median) procured from one supplier?
5. How many cows does a typical producer manage? (one or multiple)
6. Are there instances of producers grouping together to supply? (within family, rovers)
7. How many times does the production happen? (morning/evening)
8. How many times does the sourcing happen? (morning/evening)
9. How much is the variability in supply at the producer level?
10. What are the typical causes of variability? (cow's milk production cycle, weather, personal consumption, spoilage, better offers, others)
11. What are the reasons for someone to stop supplying temporarily or permanently?
12. What are the reasons for someone to start supplying for the first time?
13. What are the reasons for someone to begin supplying again after a lapse?
14. Are there any targets set for sourcing? (per session, per day)
15. How are the targets communicated to the producer level?
16. Are there any periods when supply is lower than usual?
17. What are the possible causes of intermittent lower supply?
18. Are there any periods when supply is higher than usual?
19. What are the possible causes of intermittent higher supply?
20. What happens to the excess supply received?
21. What are the ways to manage the shortfall in regular supply when it happens?
22. What are the capacity limits at the collection center level?
23. What are the capacity limits at BMC level?
24. What happens when the supply exceeds capacity at collection center or BMC?
25. What possible options can be explored to ensure consistent supply?

Questions for Suppliers (Farmer)

1. What is your primary occupation (agriculture, dairy, others)?
2. How many cows do you have?
3. What is the daily production (morning & evening) from the cows?
4. How much of the milk do you keep for yourself?
5. How many people are served by the milk you retain for yourself?
6. When did you first start supplying to Milk Mantra?
7. What attracted you to Milk Mantra?
8. What are some additional services from Milk Mantra you avail?
9. Do you supply to other companies or individuals?
10. What are the possible reasons for supplying less to Milk Mantra?
11. What do you do with surplus milk when such an event happens?

Questions for Suppliers (Commercial Dairy Farm)

1. How many cows do you have?
2. What is the daily production (morning & evening) from the cows?
3. How much of the milk do you keep for yourself?
4. How many people are served by the milk you retain for yourself?
5. Do you have any provision to store milk at your farm?
6. When did you first start supplying to Milk Mantra?
7. What attracted you to Milk Mantra?
8. What are some additional services from Milk Mantra you avail?
9. Do you supply to other companies or individuals?
10. What are the possible reasons for supplying less to Milk Mantra?
11. What do you do with surplus milk when such an event happens?

Questions for Suppliers (Rover)

1. How many people do you source milk from?
2. Do you have your own milk production too?
3. What is the daily collection (morning & evening) from the producers?
4. How much of the milk do you keep for yourself?
5. How many people are served by the milk you retain for yourself?
6. When did you first start supplying to Milk Mantra?
7. What attracted you to Milk Mantra?
8. Do you supply to other companies or individuals?
9. What are the possible reasons for supplying less to Milk Mantra?
10. What do you do with surplus milk when such an event happens?

Questions for Suppliers (GAs)

1. How many farmers do you source milk from?
2. Do you have your own milk production too?
3. What is the daily collection (morning & evening) from the producers?
4. What are the possible reasons for the reduction in supply from farmers?
5. What do you do with surplus milk when such an event happens?

Questions for Operations/Production

1. Describe timeline-based journey for milk from source to sink.
2. What is the process of receiving the demand from Sales?
3. How frequently is the demand adjusted (ice/slush/water)?
4. How is the product demand converted to supply requirement (BoM)?
5. How is the timing for supply requirement determined?
6. What is the process of sending requirements to Procurement?
7. How frequently is the requirement adjusted (ice/slush/water)?
8. What is the extent of adjustment done for perishability?
9. How is the perishability curve determined for finished goods?
10. What are the factors that determine the duration above?
11. How is the perishability curve determined for raw material?
12. What are the factors that determine the duration above?
13. Besides liquid milk, what are the other raw materials where perishability is a constraint?
14. What are the capacity constraints for raw materials (at factory/elsewhere)?
15. What are the capacity constraints for production (milk & products)?
16. What are the capacity constraints for finished goods (at factory/elsewhere)?
17. How are finished goods segmented based on time to expiry?
18. How are raw materials segmented based on time to expiry?
19. When supply is lower than the requirement, what approaches are taken?
20. When supply is higher than the requirement, what approaches are taken?
21. How much of production is fixed vs variable?
22. What is the impact of inconsistent production requirements?
23. What possible options can be explored to ensure consistent demand?
24. What possible options can be explored to ensure consistent supply?
25. What possible options can be explored to ensure consistent production?

APPENDIX D: Summary of Interviews & Surveys

Following are the key insights derived about small suppliers:

- Farmers with 1-2 cows form the bulk of small independent suppliers.
- Their key source of income is Dairy, followed by Agriculture.
- About a third of the farmers pour in 5-10 L per day.
- About half of the farmers have a long-term relationship (> 5yrs).
- Supplier loyalty is very high with almost no one having other clients.
- Soft factors (besides price) have a key role in attraction/retention.
- Extension services (cattle health & financial support) result in supplier loyalty.
- Festivals are a key reason for a dip in supply (increased need for self).
- About half of the farmers' supply dips 10-25% during festivals.
- Farmers do not have to worry about oversupply situations.

Following are the key insights derived about Rovers and CDFs:

- Rovers act as a buffer for uncertainty in supply as their loyalty is driven primarily by pricing.
- Premium pricing of high-fat milk & discounted pricing of low-fat milk influences the type of rovers that can be attracted.
- Suppliers with high volume (>100 L per day) tend to de-risk by having multiple clients.
- Excess supply results in wastage or distress sale and is a pain point.
- People having skills to convert excess milk to cheese curds are able to reduce the risk of wastage.
- The impact of festivals is minimal for CDFs owing to high base volume.
- For rovers, whose primary source is small farmers, the impact of festivals is significant.

Following are some of the key causes of demand variability indicated by sales team:

- Inherent random noise in demand (10% for milk, 30% for products)
- Oversupply or undersupply by competitors
- Possible impact of climate and/or disruptions, along with media coverage of the same
- Festival days or seasons (usually known in advance to some extent)
- Unorganized nature of distributor community

APPENDIX E: Python Script for Stochastic Optimization using Google OR-Tools

Scenario: Supply target can be set for every time period

```
# Optimization for Supply Targets at Time Period level

c_1=20 # Cost to procure RM
c_2=5 # Cost to produce FG
c_3=1 # Cost to ship (only if demand exists)
p=35 # Sale price
g_1=-1 # Salvage price of RM (Disposal cost)
g_2=-2 # Salvage price of FG (Disposal cost)
B=50 # Penalty cost of lost sale

opt_mu_supply=[] # to be optimized
cv_supply=0.05
prob_supply=[0.01, 0.06, 0.24, 0.38, 0.24, 0.06, 0.01]
rate_supply=[1-3*cv_supply, 1-2*cv_supply, 1-1*cv_supply, 1,\
             1+1*cv_supply, 1+2*cv_supply, 1+3*cv_supply]
opt_produce=[] # to be optimized
mu_demand=100
cv_demand=0.1
prob_demand=[0.01, 0.06, 0.24, 0.38, 0.24, 0.06, 0.01]
rate_demand=[1-3*cv_demand, 1-2*cv_demand, 1-1*cv_demand, 1,\
             1+1*cv_demand, 1+2*cv_demand, 1+3*cv_demand]
opt_sold_FG=[]  # to be optimized
days_special_start=[0,0,1,0,0] # time-series with surge days
produce_to_sell=1
procure_to_produce=1
special_supply_delta=-0.1
special_demand_delta=0.2

days_special_FG=days_special_start.copy()
days_special_PR=days_special_start.copy()
days_special_RM=days_special_start.copy()
for n in range(produce_to_sell):
    days_special_FG.append(0)
    days_special_PR.insert(0,0)
    days_special_RM.insert(0,0)
for n in range(procure_to_produce):
    days_special_FG.append(0)
    days_special_PR.append(0)
    days_special_RM.insert(0,0)

demand_FG=[[round(mu_demand*rate_demand[j]*\
                  (1+special_demand_delta*days_special_FG[t]),3) \
                for j in range(len(prob_demand))] \
                for t in range(len(days_special_FG))]

from ortools.linear_solver import pywraplp
s = pywraplp.Solver("THESIS",pywraplp.Solver.GLOP_LINEAR_PROGRAMMING)

total_profit = s.NumVar(0,s.infinity(),'Total Profit')
opt_mu_supply = [s.NumVar(0, s.infinity(),'mu_supply:%s' % ('day'+str(t))) \
                 for t in range(len(days_special_RM))]
opt_produce = [[s.NumVar(0, s.infinity(),'produce:%s' \
                        % ('supply'+str(i)+'&'+'day'+str(t))) \
                for i in range(len(prob_supply))] \
                for t in range(len(days_special_PR))]
opt_sold_FG = [[[s.NumVar(0, s.infinity(),'sold:%s' \
                        % ('demand'+str(j)+'&'+'supply'+str(i)+'&'+'day'+str(t))) \
                for j in range(len(prob_demand))] \
                for i in range(len(prob_supply))] \
                for t in range(len(days_special_FG))]

# Cannot sell more than demand or production
for t in range(len(days_special_FG)):
    for i in range(len(prob_supply)):
        for j in range(len(prob_demand)):
```

```
            s.Add(opt_sold_FG[t][i][j]<=demand_FG[t][j])
            s.Add(opt_sold_FG[t][i][j]<=opt_produce[t][i])

# Cannot produce more than supply
for t in range(len(days_special_RM)):
    for i in range(len(prob_supply)):
        s.Add(opt_produce[t][i]<=opt_mu_supply[t]*rate_supply[i]* \
              (1+special_supply_delta*days_special_RM[t]))

# Compute Total Profit across time horizon & maximize
s.Add(total_profit == sum(sum(prob_supply[i]*sum(prob_demand[j]* \
                              (p*opt_sold_FG[t][i][j] - \
                               (c_1*opt_mu_supply[t]*rate_supply[i]* \
                                (1+special_supply_delta*days_special_RM[t])+\
                                c_2*opt_produce[t][i]+ \
                                c_3*opt_sold_FG[t][i][j])+ \
                               (g_1*(opt_mu_supply[t]*rate_supply[i]* \
                                     (1+special_supply_delta*days_special_RM[t])-\
                                     opt_produce[t][i])+\
                                g_2*(opt_produce[t][i]-opt_sold_FG[t][i][j]))-\
                               B*(demand_FG[t][j]-opt_sold_FG[t][i][j])) \
                              for j in range(len(prob_demand))) \
                              for i in range(len(prob_supply))) \
                              for t in range(len(days_special_PR))))
s.Maximize(total_profit)

# Solve & Display
print('Number of variables =', s.NumVariables())
print('Number of constraints =', s.NumConstraints())
s.Solve()
print('Optimal objective value: %.3f' % total_profit.SolutionValue())
avg_daily_profit = total_profit.SolutionValue()/len(days_special_PR)
print('Average Daily Profit: %.3f' % avg_daily_profit)
opt_mu_supply=[round(opt_mu_supply[t].SolutionValue(),3) \
               for t in range(len(days_special_RM))]
print('Supply Target:', opt_mu_supply)
```

Scenario: Supply target has to be consistent for the entire time series

```
# Optimization for Supply Target at Time Series level

c_1=20 # Cost to procure RM
c_2=5 # Cost to produce FG
c_3=1 # Cost to ship (only if demand exists)
p=35 # Sale price
g_1=-1 # Salvage price of RM (Disposal cost)
g_2=-2 # Salvage price of FG (Disposal cost)
B=50 # Penalty cost of lost sale

opt_mu_supply=[] # to be optimized
cv_supply=0.05
prob_supply=[0.01, 0.06, 0.24, 0.38, 0.24, 0.06, 0.01]
rate_supply=[1-3*cv_supply, 1-2*cv_supply, 1-1*cv_supply, 1,\
             1+1*cv_supply, 1+2*cv_supply, 1+3*cv_supply]
opt_produce=[] # to be optimized
mu_demand=100
cv_demand=0.1
prob_demand=[0.01, 0.06, 0.24, 0.38, 0.24, 0.06, 0.01]
rate_demand=[1-3*cv_demand, 1-2*cv_demand, 1-1*cv_demand, 1,\
             1+1*cv_demand, 1+2*cv_demand, 1+3*cv_demand]
opt_sold_FG=[]  # to be optimized
days_special_start=[0,0,1,0,0] # time-series with surge days
produce_to_sell=1
procure_to_produce=1
special_supply_delta=-0.1
special_demand_delta=0.2

days_special_FG=days_special_start.copy()
days_special_PR=days_special_start.copy()
```

```
days_special_RM=days_special_start.copy()
for n in range(produce_to_sell):
    days_special_FG.append(0)
    days_special_PR.insert(0,0)
    days_special_RM.insert(0,0)
for n in range(procure_to_produce):
    days_special_FG.append(0)
    days_special_PR.append(0)
    days_special_RM.insert(0,0)

demand_FG=[[round(mu_demand*rate_demand[j]*\
                  (1+special_demand_delta*days_special_FG[t]),3) \
                for j in range(len(prob_demand))] \
                for t in range(len(days_special_FG))]

from ortools.linear_solver import pywraplp
s = pywraplp.Solver("THESIS",pywraplp.Solver.GLOP_LINEAR_PROGRAMMING)

total_profit = s.NumVar(0,s.infinity(),'Total Profit')
opt_mu_supply_overall = s.NumVar(0,s.infinity(),'mu_supply Overall')
opt_mu_supply = [s.NumVar(0, s.infinity(),'mu_supply:%s' % ('day'+str(t))) \
                 for t in range(len(days_special_RM))]
opt_produce = [[s.NumVar(0, s.infinity(),'produce:%s' \
                         % ('supply'+str(i)+'&'+'day'+str(t))) \
                for i in range(len(prob_supply))] \
                for t in range(len(days_special_PR))]
opt_sold_FG = [[[s.NumVar(0, s.infinity(),'sold:%s' \
                         % ('demand'+str(j)+'&'+'supply'+str(i)+'&'+'day'+str(t))) \
                for j in range(len(prob_demand))] \
                for i in range(len(prob_supply))] \
                for t in range(len(days_special_FG))]

# Ensure same supply target is applicable for entire duration
for t in range(len(days_special_RM)):
    s.Add(opt_mu_supply[t]==opt_mu_supply_overall)

# Cannot sell more than demand or production
for t in range(len(days_special_FG)):
    for i in range(len(prob_supply)):
        for j in range(len(prob_demand)):
            s.Add(opt_sold_FG[t][i][j]<=demand_FG[t][j])
            s.Add(opt_sold_FG[t][i][j]<=opt_produce[t][i])

# Cannot produce more than supply
for t in range(len(days_special_RM)):
    for i in range(len(prob_supply)):
        s.Add(opt_produce[t][i]<=opt_mu_supply[t]*rate_supply[i]* \
              (1+special_supply_delta*days_special_RM[t]))

# Compute Total Profit across time horizon & maximize
s.Add(total_profit == sum(sum(prob_supply[i]*sum(prob_demand[j]* \
                              (p*opt_sold_FG[t][i][j] - \
                               (c_1*opt_mu_supply[t]*rate_supply[i]* \
                                (1+special_supply_delta*days_special_RM[t])+\
                                c_2*opt_produce[t][i]+ \
                                c_3*opt_sold_FG[t][i][j])+ \
                               (g_1*(opt_mu_supply[t]*rate_supply[i]* \
                                     (1+special_supply_delta*days_special_RM[t])-\
                                     opt_produce[t][i])+\
                                g_2*(opt_produce[t][i]-opt_sold_FG[t][i][j]))-\
                               B*(demand_FG[t][j]-opt_sold_FG[t][i][j])) \
                              for j in range(len(prob_demand))) \
                              for i in range(len(prob_supply))) \
                              for t in range(len(days_special_PR))))
s.Maximize(total_profit)

# Solve & Display
print('Number of variables =', s.NumVariables())
print('Number of constraints =', s.NumConstraints())
s.Solve()
print('Optimal objective value: %.3f' % total_profit.SolutionValue())
```

```
avg_daily_profit = total_profit.SolutionValue()/len(days_special_PR)
print('Average Daily Profit: %.3f' % avg_daily_profit)
print('Supply Target Overall: %.3f' % opt_mu_supply_overall.SolutionValue())
opt_mu_supply=[round(opt_mu_supply[t].SolutionValue(),3) \
               for t in range(len(days_special_RM))]
print('Supply Target:', opt_mu_supply)
```

Scenario: Production plan has to be fixed in advance for a time period

```
# Optimization for Fixed Production Plan at Time Period level

c_1=20 # Cost to procure RM
c_2=5 # Cost to produce FG
c_3=1 # Cost to ship (only if demand exists)
p=35 # Sale price
g_1=-1 # Salvage price of RM (Disposal cost)
g_2=-2 # Salvage price of FG (Disposal cost)
B=50 # Penalty cost of lost sale

opt_mu_supply=[] # to be optimized
cv_supply=0.05
prob_supply=[0.01, 0.06, 0.24, 0.38, 0.24, 0.06, 0.01]
rate_supply=[1-3*cv_supply, 1-2*cv_supply, 1-1*cv_supply, 1,\
             1+1*cv_supply, 1+2*cv_supply, 1+3*cv_supply]
opt_produce=[] # to be optimized
mu_demand=100
cv_demand=0.1
prob_demand=[0.01, 0.06, 0.24, 0.38, 0.24, 0.06, 0.01]
rate_demand=[1-3*cv_demand, 1-2*cv_demand, 1-1*cv_demand, 1,\
             1+1*cv_demand, 1+2*cv_demand, 1+3*cv_demand]
opt_sold_FG=[]  # to be optimized
days_special_start=[0,0,1,0,0] # time-series with surge days
produce_to_sell=1
procure_to_produce=1
special_supply_delta=-0.1
special_demand_delta=0.2

days_special_FG=days_special_start.copy()
days_special_PR=days_special_start.copy()
days_special_RM=days_special_start.copy()
for n in range(produce_to_sell):
    days_special_FG.append(0)
    days_special_PR.insert(0,0)
    days_special_RM.insert(0,0)
for n in range(procure_to_produce):
    days_special_FG.append(0)
    days_special_PR.append(0)
    days_special_RM.insert(0,0)

demand_FG=[[round(mu_demand*rate_demand[j]*\
                  (1+special_demand_delta*days_special_FG[t]),3) \
                for j in range(len(prob_demand))] \
                for t in range(len(days_special_FG))]

from ortools.linear_solver import pywraplp
s = pywraplp.Solver("THESIS",pywraplp.Solver.GLOP_LINEAR_PROGRAMMING)

total_profit = s.NumVar(0,s.infinity(),'Total Profit')
opt_mu_supply = [s.NumVar(0, s.infinity(),'mu_supply:%s' % ('day'+str(t))) \
                 for t in range(len(days_special_RM))]
opt_produce = [s.NumVar(0, s.infinity(),'produce:%s' \
                        % ('day'+str(t))) \
                for t in range(len(days_special_PR))]
opt_sold_FG = [[[s.NumVar(0, s.infinity(),'sold:%s' \
                        % ('demand'+str(j)+'&'+'supply'+str(i)+'&'+'day'+str(t))) \
                for j in range(len(prob_demand))] \
                for i in range(len(prob_supply))] \
                for t in range(len(days_special_FG))]
```

```
# Cannot sell more than demand or production
for t in range(len(days_special_FG)):
    for i in range(len(prob_supply)):
        for j in range(len(prob_demand)):
            s.Add(opt_sold_FG[t][i][j]<=demand_FG[t][j])
            s.Add(opt_sold_FG[t][i][j]<=opt_produce[t])

# Cannot produce more than supply
for t in range(len(days_special_RM)):
    for i in range(len(prob_supply)):
        s.Add(opt_produce[t]<=opt_mu_supply[t]*rate_supply[i]* \
              (1+special_supply_delta*days_special_RM[t]))

# Compute Total Profit across time horizon & maximize
s.Add(total_profit == sum(sum(prob_supply[i]*sum(prob_demand[j]* \
                              (p*opt_sold_FG[t][i][j] - \
                               (c_1*opt_mu_supply[t]*rate_supply[i]* \
                                (1+special_supply_delta*days_special_RM[t])+\
                                c_2*opt_produce[t]+ \
                                c_3*opt_sold_FG[t][i][j])+ \
                               (g_1*(opt_mu_supply[t]*rate_supply[i]* \
                                     (1+special_supply_delta*days_special_RM[t])-\
                                     opt_produce[t])+\
                                g_2*(opt_produce[t]-opt_sold_FG[t][i][j]))-\
                               B*(demand_FG[t][j]-opt_sold_FG[t][i][j])) \
                              for j in range(len(prob_demand))) \
                              for i in range(len(prob_supply))) \
                              for t in range(len(days_special_PR))))
s.Maximize(total_profit)

# Solve & Display
print('Number of variables =', s.NumVariables())
print('Number of constraints =', s.NumConstraints())
s.Solve()
print('Optimal objective value: %.3f' % total_profit.SolutionValue())
avg_daily_profit = total_profit.SolutionValue()/len(days_special_PR)
print('Average Daily Profit: %.3f' % avg_daily_profit)
opt_mu_supply=[round(opt_mu_supply[t].SolutionValue(),3) \
               for t in range(len(days_special_RM))]
print('Supply Target:', opt_mu_supply)
```

APPENDIX F: Python Script for Solution Validation

Scenario: Simulation of time series using decision variables recommended by optimization

```
# Simulation of time series using optimal decision variables

c_1=20 # Cost to procure RM
c_2=5 # Cost to produce FG
c_3=1 # Cost to ship (only if demand exists)
p=35 # Sale price
g_1=-1 # Salvage price of RM (Disposal cost)
g_2=-2 # Salvage price of FG (Disposal cost)
B=50 # Penalty cost of lost sale

cv_supply=0.05
mu_demand=100
cv_demand=0.1
days_special_start=[0,0,1,0,0] # time-series with surge days
produce_to_sell=1
procure_to_produce=1
special_supply_delta=-0.1
special_demand_delta=0.2

opt_mu_supply=\
[105.263, 105.263, 126.316, 105.263, 116.959, 105.263, 105.263]
opt_produce=\
[[89.474, 94.737, 100.0, 105.263, 110.526, 115.789, 120.0],
 [89.474, 94.737, 100.0, 105.263, 110.526, 115.789, 120.0],
 [107.368, 113.684, 120.0, 126.316, 132.632, 138.947, 144.0],
 [89.474, 94.737, 100.0, 105.263, 110.526, 115.789, 120.0],
 [89.474, 94.737, 100.0, 105.263, 110.526, 115.789, 120.0],
 [89.474, 94.737, 100.0, 105.263, 110.526, 115.789, 120.0],
 [89.474, 94.737, 100.0, 105.263, 110.526, 115.789, 120.0]]

days_special_FG=days_special_start.copy()
days_special_PR=days_special_start.copy()
days_special_RM=days_special_start.copy()
for n in range(produce_to_sell):
    days_special_FG.append(0)
    days_special_PR.insert(0,0)
    days_special_RM.insert(0,0)
for n in range(procure_to_produce):
    days_special_FG.append(0)
    days_special_PR.append(0)
    days_special_RM.insert(0,0)

import numpy

iterations=1 # start with 1 and increase this in multiples of 10 till convergence
profit_series_iterations=0
for iter in range(iterations):
    profit_series=0
    for t in range(len(days_special_FG)):

        RM_actual=round(numpy.random.normal(opt_mu_supply[t],cv_supply*opt_mu_supply[t]),3)
        if days_special_RM[t]==1:
            RM_actual=round(RM_actual*(1+special_supply_delta),3)

        PR_actual=min(RM_actual,max(opt_produce[0]))
        RM_waste=round(RM_actual-PR_actual,3)

        demand_actual=round(numpy.random.normal(mu_demand,cv_demand*mu_demand),3)
        if days_special_FG[t]==1:
            demand_actual=round(demand_actual*(1+special_demand_delta),3)

        FG_sold=min(PR_actual,demand_actual)
        FG_waste=round(PR_actual-FG_sold,3)
        FG_lostsale=round(demand_actual-FG_sold,3)
```

```
        profit=p*FG_sold-(c_1*RM_actual+c_2*PR_actual+c_3*FG_sold)+\
        (g_1*RM_waste+g_2*FG_waste)-B*FG_lostsale
        profit=round(profit,3)
        profit_series=profit_series+profit

    profit_series=round(profit_series,3)
    profit_series_iterations=profit_series_iterations+profit_series

profit_series_avg=round(profit_series_iterations/iterations,3)
print(profit_series_avg)
```