C.E. Thesis 1928

A THEORETICAL AND EXPERIMENTAL INVESTIGATION of the PROPERTIES AND BEHAVIOR OF HYDRAULIC-FILL DAMS

by

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To

Dr. CHARLES TERZAGHI

in appreciation of his most valuable advice and inspiration.

ACKNOWLEDGMENT

The writer wishes to express his indebtedness to the Miami Conservancy District for active cooperation in providing material for experimental research. Special credit is due to Mr. C. H. Eiffert, Chief Engineer of the District, under whose competent direction the difficult task of obtaining core samples was successfully accomplished.

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INTRODUCTION

The hydraulic-fill method of dam construction is an outgrowth and extension of the hydraulic sluicing process used extensively in mining operations in the West. This method is especially applicable in moving and placing earth for dam construction, since it affords a relatively economical means of transporting large quantities of heterogeneous material which is automatically separated by the sorting action of the flowing water in such a manner that the finer constituents are segregated into a central impervious core, while the coarser particles are left in the side slopes to afford weight and stability to the structure as a whole. The basic principle involved is perfectly simple; but the actual design and construction is a very complicated problem from an engineering standpoint. In the first place, the dam is built of natural earth, the commonest and at the same time the most complex construction material with which the engineer has to deal. Furthermore, this earth is removed from its natural bed, transported, segregated, and deposited with an excess of water. It immediately begins to readjust itself to its new conditions, and this readjustment process, instead of occupying weeks or months, as with concrete, occupies years, during which time the character of the structure and the forces acting within it are continually changing.

Engineering literature contains detailed accounts of observations made on the behavior of hydraulic-fill dams, during construction and after completion, together with the opinions of the various writers as to the suitability of certain materials and methods and the advantages or disadvantages of various items of design. The value of these empirical data and of the opinions derived therefrom, widely different though they are, should not be underestimated. There seems, however, to have been no attempt to evolve a basic theory for the design of such structures, probably on account of the extreme complexity of the mechanics involved. In addition, there has been no concerted research into the fundamental properties and behavior of the construction material in the light of recent advances in Soil Mechanics.

Appreciating this lack of basic information, Dr. Charles Terzaghi opened negotiations in May, 1927, with Mr. Charles H. Paul, then Chief Engineer of the Miami Conservancy District, suggesting that the District obtain samples from one of their five hydraulic-fill dams and ship them to the Soil Mechanics Laboratory of M. I. T. for investigation by the writer. The District agreed whole-heartedly to this plan, and the preliminary details were at once worked out, in the early summer of 1927.

The work at the dam was carried forward as rapidly as possible, under the personal direction of Mr. C. H. Eiffert, who succeeded Mr. Paul as Chief Engineer. The task of obtaining undisturbed samples proved much more difficult than had been anticipated. The samples were finally procured, substantially in accordance with the original plan, and were shipped to the Institute.

Research work at the laboratory was carried on continuously during the past academic year. Many improvements in apparatus and methods were effected to overcome difficulties which constantly arose during the progress of the investigation. The development of the basic theory, especially that of the consolidation of the core after completion, proved to be a rather complex problem. At first glance the final result may appear too hopelessly theoretical to be of any practical value. But the mathematics has been simplified as far as possible, and certain specific cases worked out to illustrate the application.

It is hoped that the theoretical and empirical results herein set forth will be of service to the engineer designing hydraulic-fill dams in affording at least an approximate idea of the behavior of these structures and of the importance of the various factors influencing that behavior; and that these researches will be considered as only the first step, a stimulus to further and more widespread investigations upon this subject.

SCOPE OF INVESTIGATION

The studies described herein fall naturally into three main sections: first, a theoretical investigation of the behavior of the dam under various conditions; second, a series of experimental studies of the physical properties of the core material of the Germantown Dam; third, illustrative applications of the theory on the basis of the tests performed, with additional notes on behavior in the light of present knowledge.

Section I. The fundamental theory has been investigated for the following conditions:

A. Stability of the dam as a whole with the core in a state of hydrostatic stress.

B. Rate of consolidation of the core.

Section II. Experimental Data. This section may be subdivided into:

A. Description of apparatus and methods.

B. Enumeration of tests performed and results obtained.

C. Analysis of results.

Section III. Applications and Discussion.

A. Examples of the application of the theories of Section I to the actual case in point.

B. Discussion of the principles and their applications, with notes upon the controlling factors.

SECTION I. FUNDAMENTAL THEORY

GENERAL CONSIDERATIONS

A dam, in order to be useful, must hold water; and in order to be safe, it must be stable under all conditions. These two propositions are self-evident. In an earth dam, they are closely interrelated, but not necessarily interdependent. In the hydraulic-fill dam the two functions are partially separated; the central core is intended to furnish the watertight element, whereas the coarse and heavy shell furnishes the weight required to support the core and to insure the stability of the structure.

To fix the ideas, a brief review of the various stages through which a hydraulic-fill dam passes during and after construction will be useful.

At any given time during construction the dam looks in cross-section something like Fig. 1. The borrow-pit material, thoroughly mixed with water, is delivered to the edges of the dam in sluicing pipes or flumes P. As the mixture leaves the pipe, its velocity is suddenly checked by coming in contact with the outer dike, and the heaviest stones and gravel are dropped. The water then flows inward toward the center, spreading out as it goes, and dropping still more of the coarser constituents. When the stream reaches the edge of the core pool C, its velocity is considerably reduced by contact with the larger body of still water, so that only

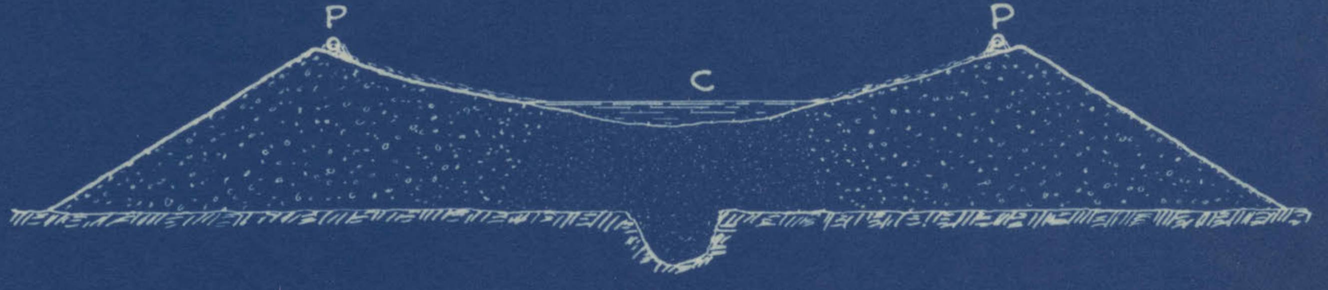

Fig. 1

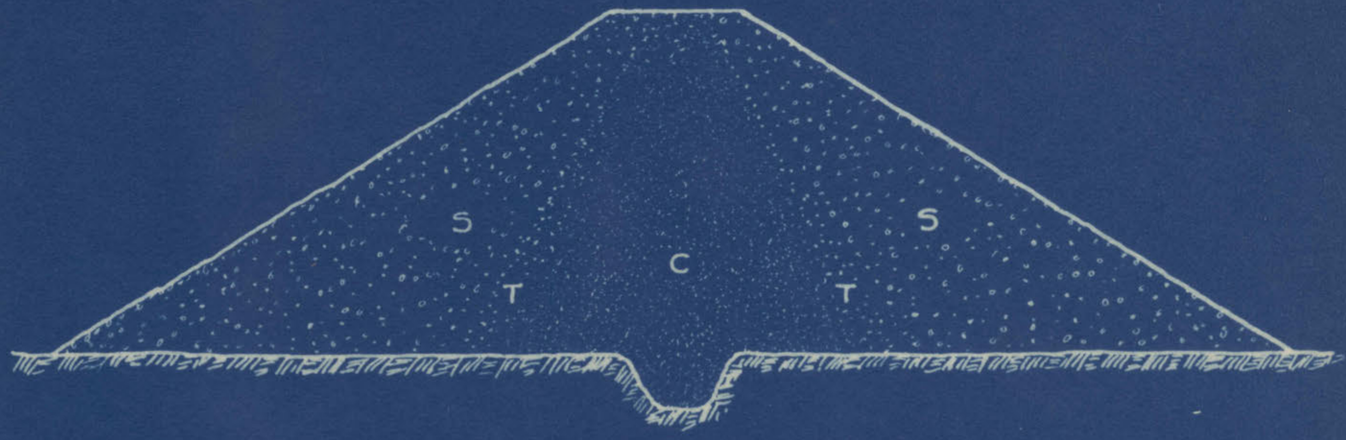

Fig. 2

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the finest particles are carried into the pool. Here the water is quiet, and the fine particles gradually settle out. The width of core is controlled by the width of the pool, and the character of the core material, especially as to size, depends principally upon the detention period in the pool. The pool level is maintained by an overflow, so that the detention period is inversely proportional to the rate at which the sluicing is carried on. It is obvious, therefore, that rapid construction means a coarser and more pervious core, with higher waste of fine material, than would obtain if the rate of sluicing were decreased. So that here, at the very beginning of the discussion, one is forced to realize the importance of proper construction methods and intelligent supervision.

The work proceeds in this manner to completion, the finished structure having a cross-section like that shown in Fig. 2. The dam thus consists of three elements: the central impervious core, C, of fine material; the transition zone, T, relatively narrow, where the material grades from fine to coarse; and the heavy, pervious shell, S, consisting of coarse material. The term "shell" used in this connection seems to the writer to be simpler and more descriptive than the terms "side slopes", "toes","heavy pervious outer sections," etc., encountered in engineering literature, and will be employed when referring to this section of the dam.

As previously noted, the core is built up by allowing the fine particles to settle out in still water. The sediment thus obtained is initially in a soft liquid state, very similar to the mud at the bottom of a lake whose water is continuously supplied with fine silt and clay. It is therefore to be expected that the material in this state will exert upon surfaces in contact with it a hydrostatic pressure equivalent to that of a liquid somewhat heavier than water. The shell of the dam may therefore be considered to play the part during this time of two dams retaining between them a heavy viscous liquid exerting hydrostatic pressure upon their inner faces. If the shell is sufficiently strong to sustain this action, the dam is stable. If not, the core will push out one side or the other and flow away.

As construction proceeds, and after completion, any element of the core is subjected to the pressure of the material above. This is equivalent to saying that the pressure varies with the depth, which, of course, is the hydrostatic condition previously mentioned. In a true liquid, this pressure distribution represents equilibrium. In the mixture of soil and water forming the core of the dam, the pressure exerted upon any element causes a decrease in volume of that element, with a corresponding loss of water. The only way this water can escape is by traveling through the surrounding material. The total amount of water forced out of the given element is dependent upon the compressibility of the soil and the pressure applied. The rate at which the water escapes is dependent upon the permeability of the soil. This phenomenon of the forcing out of excess water by pressure is known as the consolidation of the soil. These considerations therefore indicate that the rate of consolidation of the material depends directly upon its permeability and its compressibility.

The consolidation of hydraulic-fill cores is of extreme importance with respect to the internal resistance of the structure. A slightly different view of the process may serve to illustrate this point. Consider a sample of saturated soil to be subjected to a certain pressure. This pressure corresponds to a definite decrease in volume, which means that a definite amount of water must be forced out of the sample. If there is no opportunity for the water to escape, all the pressure will be taken up by excess hydrostatic pressure in the water, and none of it transmitted from grain to grain of the soil. The soil would then appear to have no frictional resistance; not because the coefficient of friction between individual grains is zero, but because the pressure exerted by one grain on another is zero. If drainage is now provided, the excess water will escape, at a rate dependent upon the permeability of the soil. As the pressure in the water is relieved. more and more of the applied pressure is transferred to the grains, until after the lapse of a certain time (theoretically infinite) all the pressure is transmitted from grain to grain, and there is no tendency for water to flow out of the sample. It is said that the material has reached its final state of consolidation under this particular pressure.*

This variation in the pressure exerted by one soil grain on another can thus be seen to have an important bearing on the mechanics of the material and its behavior under stress. At the outset, the material has practically no resistance to shear; during consolidation, this resistance increases gradually, finally attaining a maximum dependent upon the angle of friction between the grains. The foregoing is merely an approximate description of the actual process involved, but will afford a general idea of the phenomenon of consolidation and of its influence on the internal resistance.

It is evident, then, that fundamental theories must be developed to determine the behavior of the material under two specific cases: first, the stability of the structure with the core in a liquid state; and second, the process of consolidation of the core under the pressures exerted within it. The first applies particularly to the shape and dimensions of the cross-section, while the second affords a measure of the total internal resistance after the lapse of any length of time, and fixes the degree of influence of the various factors involved, and determines the type of material which would be applicable to the conditions arising in any particular case.

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*A simple and interesting mechanical analogy to this action is included in Dr. Terzaghi's paper "Principles of Final Soil Classification," Public Roads, Vol. 8, No. 3, May, 1927.

There are many factors other than those mentioned which affect the stability of the structure after completion. Joel D. Justin* enumerates six criteria for the design of earth dams, all of which are applicable to the case of hydraulic-fill dams. It would be superfluous to reproduce any of Justin's results here. Suffice it to say that his criteria are well established and should be fulfilled. The two specific cases mentioned above, to which the following discussion will be confined, are peculiar to hydraulic-fill dams as opposed to other types of earth dams. In other words, Justin's safety requirements must be fulfilled in the hydraulic-fill dam; and in addition, the design must be investigated on the basis of the two theories herein developed.

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*The Design of Earth Dams. Transactions, Am. Soc. C. E., 1924.

I.A. ANALYSIS OF STRESS IN SHELL OF HYDRAULIC-FILL DAM WITH CORE IN LIQUID STATE

Hypothesis. In order to permit of a solution, the following assumptions will be made:

1. The dam consists of two well-defined and distinctly separated elements, the core and the shell, each homogeneous and isotropic.

   (This amounts to neglecting variations in the material of the two sections, and to assuming the thickness of the transition zone as zero.)

2. The core is in a liquid state, exerting hydrostatic pressure upon the inner surface of the shell.

   (The transition zone is thus considered to be an impervious layer of negligible thickness serving to transmit the pressure of the core to the shell, and to prevent the core as a whole from flowing freely through the voids of the shell.)

3. The outer slopes of the shell are of constant inclination.

4. The surface of demarcation between core and shell is a definite but unknown function of depth.

5. The stability of the shell is due entirely to its own weight, with no cohesion existing and no excess hydrostatic pressure within the shell itself.

Derivation of Equation of Ideal Core Section.

Assume coördinate axes OX and OZ, OZ being the center line of the cross-section and OX lying along the top of the dam.

Let z = depth to any horizontal plane.

x = horizontal distance on this plane from OZ to the intersection of the plane with the surface $x = f(z)$ between the core and the shell. The length x is thus equal to half the width of the core on the given plane.

x_o = half width of top of dam.

β = angle between outer slope and horizontal.

s_1 = specific mass gravity of core.

s_2 = specific mass gravity of shell.

ϕ = angle of internal friction of shell material.

The above quantities are indicated schematically in Fig. 3.

Assuming a unit length of dam, the forces acting on a horizontal element of the shell of thickness dz, lying at a depth z below the top, are as indicated in Fig. 4. V is the vertical component of the force exerted on the element by the superimposed mass of the shell. $V + dV$ is the reaction exerted by the material below. T and $T + dT$ are the corresponding shearing forces. dW is the weight of the element. $s_1z\ dz$ and $s_1z\ dx$ are the horizontal and vertical components, respectively, of the pressure exerted on the element by the fluid core.

It is now possible to set up an equation expressing the relation existing between these various forces.

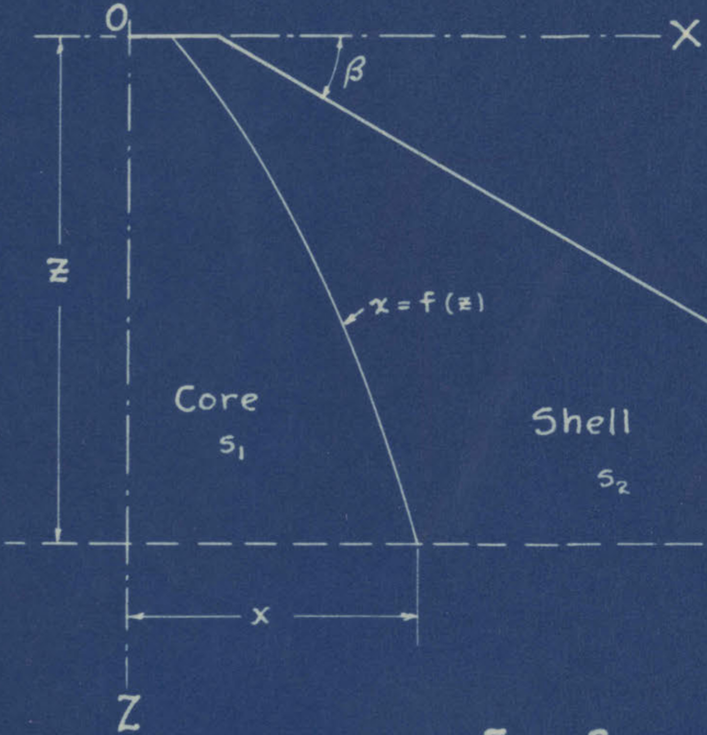

Fig. 3

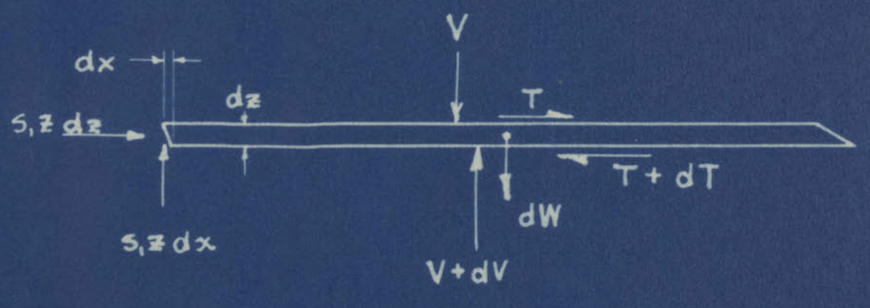

Fig. 4

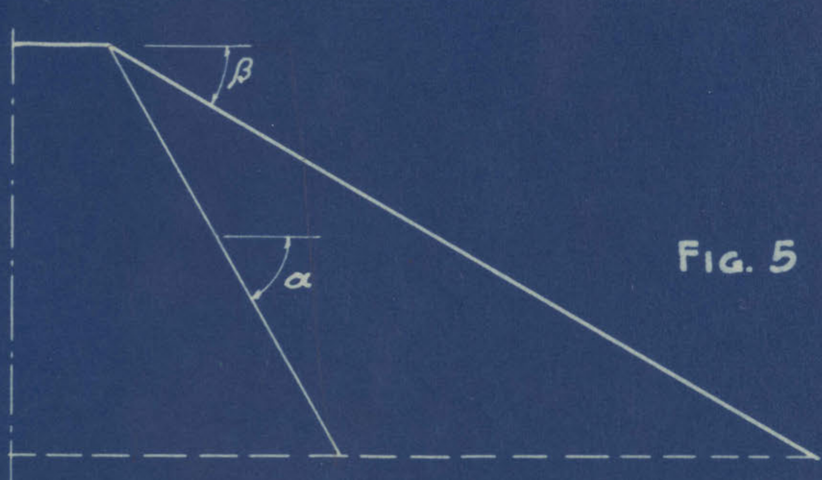

Fig. 5

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For equilibrium, $V + dW = V + dV + s_1 z\ dx$ (1)

$$T + dT = T + s_1 z\ dz \tag{2}$$

From (1), $dW = dV + s_1 z\ dx$ (3)

From (2), $dT = s_1 z\ dz$ (4)

Referring to Fig. 3,

$$dW = s_2\ dz\ [\ x_\circ + z \cot \beta - x\ ] \tag{5}$$

If the shell is just in equilibrium,

$$T = V \tan \phi, \text{ and}$$

$$T + dT = (V + dV) \tan \phi = T + dV \tan \phi$$

Whence $dV = dT \cot \phi$ (6)

From (4) and (6), $dV = s_1 z\ dz \cot \phi$ (7)

Substituting the values of dW and dV from (5) and (7) in (3),

$$s_2\ dz\ [x_\circ + z \cot \beta - x] = s_1 z\ dz \cot \phi + s_1 z\ dx \tag{8}$$

which may be written

$$\frac{dx}{dz} + \frac{s_2}{s_1 z} x = \frac{s_2}{s_1 z} x_\circ + \frac{s_2}{s_1} \cot \beta - \cot \phi \tag{9}$$

Equation (9) is a linear equation of first order, the solution of which is

$$x = e^{-\int \frac{s_2 dz}{s_1 z}} \left[\int e^{\int \frac{s_2 dz}{s_1 z}} \left(\frac{s_2}{s_1 z} x_\circ + \frac{s_2 \cot \beta - s_1 \cot \phi}{s_1} \right) dz + C \right]$$

The above expression is readily integrable, and reduces to

$$x = x_\circ + \frac{s_2 \cot \beta - s_1 \cot \phi}{s_2 + s_1} z + C z^{-\frac{s_2}{s_1}} \tag{10}$$

If $C \neq 0$, $f(0) = \infty$, which is impossible. Equation (10) therefore becomes

$$x = x_o + \frac{s_2 \cot \beta - s_1 \cot \phi}{s_2 + s_1} z \tag{11}$$

the equation of a straight line.

If

$$\cot \alpha = \frac{s_2 \cot \beta - s_1 \cot \phi}{s_2 + s_1} \tag{12}$$

Then

$$x = x_o + z \cot \alpha \tag{13}$$

According to equation (13), the core should have a top width equal to the width of the top of the dam, and be bounded by planes making an angle α with the horizontal, α being defined by (12). This result is shown in Fig. 5.

State of stress on any plane through Shell. The equation of the preceding paragraph was derived to satisfy the conditions of equilibrium on a horizontal plane through the shell, and shows that the core slope in section should be a straight line intersecting the top of the dam at the outer edge. It will now be advisable to assume an idealized cross-section on this basis, with top width zero for simplicity, and investigate the state of stress in the shell on a plane parallel to OY and making any angle θ with the horizontal.

In Fig. 6, let QR be the trace of the given plane, and θ the angle between the plane and the horizontal.

Let z = vertical distance from top of dam to intersection of QR with core slope.

W = weight of shell above QR (shaded portion).

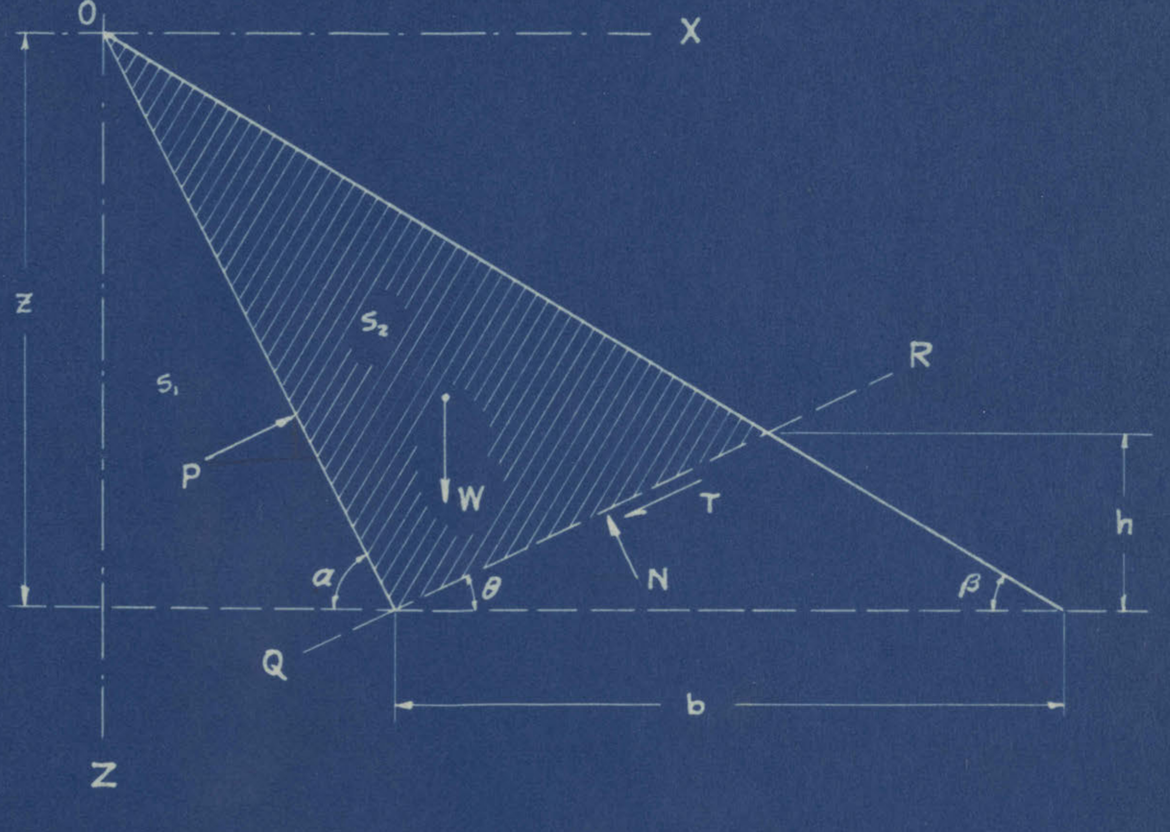

FIG. 6

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P = hydrostatic pressure of core against shell.

T = total shearing stress on QR.

N = total normal stress on QR.

Other quantities as indicated and previously described.

The hydrostatic pressure may at once be written

$$P = \frac{s_1 z^2}{2} \csc \alpha$$

For equilibrium,

$$N \cos \theta - T \sin \theta + \frac{s_1 z^2}{2} \cot \alpha - W = 0 \qquad (1)$$

$$N \sin \theta + T \cos \theta - \frac{s_1 z^2}{2} = 0 \qquad (2)$$

Solving the above equations simultaneously gives the following values for T and N:

$$T = \frac{s_1 z^2}{2} (\cos \theta + \cot \alpha \sin \theta) - W \sin \theta \qquad (3)$$

$$N = \frac{s_1 z^2}{2} (\sin \theta - \cot \alpha \cos \theta) + W \cos \theta \qquad (4)$$

W may be evaluated as follows:

In Fig. 6, $W = \frac{s_2}{2} (bz - bh) = \frac{s_2 b}{2} (z - h)$

Also, $b = z (\cot \beta - \cot \alpha)$, and

$h \cot \theta + h \cot \beta = b$, or $h = \dfrac{z(\cot \beta - \cot \alpha)}{\cot \beta + \cot \theta}$

So that $$W = \frac{s_2 z^2}{2} (\cot \beta - \cot \alpha) \frac{\cot \theta + \cot \alpha}{\cot \theta + \cot \beta} \qquad (5)$$

The expressions for tangential and normal stresses on QR therefore become

$$T = \frac{s_1 z^2}{2}(\cos\theta + \cot\alpha \sin\theta) - \frac{s_2 z^2}{2}(\cot\beta - \cot\alpha)\frac{\cot\theta + \cot\alpha}{\cot\theta + \cot\beta}\sin\theta$$

$$N = \frac{s_1 z^2}{2}(\sin\theta - \cot\alpha \cos\theta) + \frac{s_2 z^2}{2}(\cot\beta - \cot\alpha)\frac{\cot\theta + \cot\alpha}{\cot\theta + \cot\beta}\cos\theta$$

In general, stability depends not upon the absolute value of either of the above stresses, but upon the obliquity of their resultant to the normal to the plane, which must not exceed the angle of internal friction at any point. The essential problem, then, is to determine the maximum angle of obliquity of the resultant by first finding the plane upon which this maximum angle occurs. If the angle of obliquity is denoted by ψ, then $\tan\psi = T/N$, and the plane upon which maximum obliquity occurs is determined by

$$\frac{d}{d\theta}\left(\frac{T}{N}\right) = 0$$

An inspection of the values of T and N previously determined will show that the evaluation of this derivative is a matter of extreme complexity. This fact has indicated the advisability of a graphical solution, the principles of which will now be developed.

<u>Graphical Solution.</u> Any triangular section of the shell, such as the shaded portion in Fig. 6, stands in equilibrium under the action of three forces:

P - the hydrostatic pressure of the core,

W - its own weight, and

$\sqrt{N^2 + T^2}$ - the resultant force on plane QR.

For a given core section composed of a given material, P is determined both in magnitude and direction.

If h represents the vertical distance from the Z-plane to the intersection of plane QR with the outer slope, then

$W = \frac{s_2 b}{2}(z - h)$. (<u>b</u> being the common base of both triangles).

Referring to Fig. 7, if the distance z is taken to some scale to represent the value $\overline{W}_o = \frac{s_2 b}{2} z$, i.e., the weight of the entire prism of shell material above the Z-plane, then the weight of the material above plane QR is represented by z - h, or $\overline{W}$, as indicated in the diagram at the right.

If $\overline{P}$ is now laid off to the scale determined above, perpendicular to the slope of the core, from the upper end of $\overline{W}$, three vertices of the force polygon are determined.

The directions of $\overline{T}$ and $\overline{N}$ being known, the force polygon may be completed, as indicated in Fig. 7.

The position of the force polygon plotted in this way is rather awkward, inasmuch as it affords no direct means of comparison of obliquity of resultant stress as θ varies. A simple modification of the plot is therefore advisable.

Referring to Fig. 8, the stress diagram at the right, identical with that shown in Fig. 7, is transferred parallel

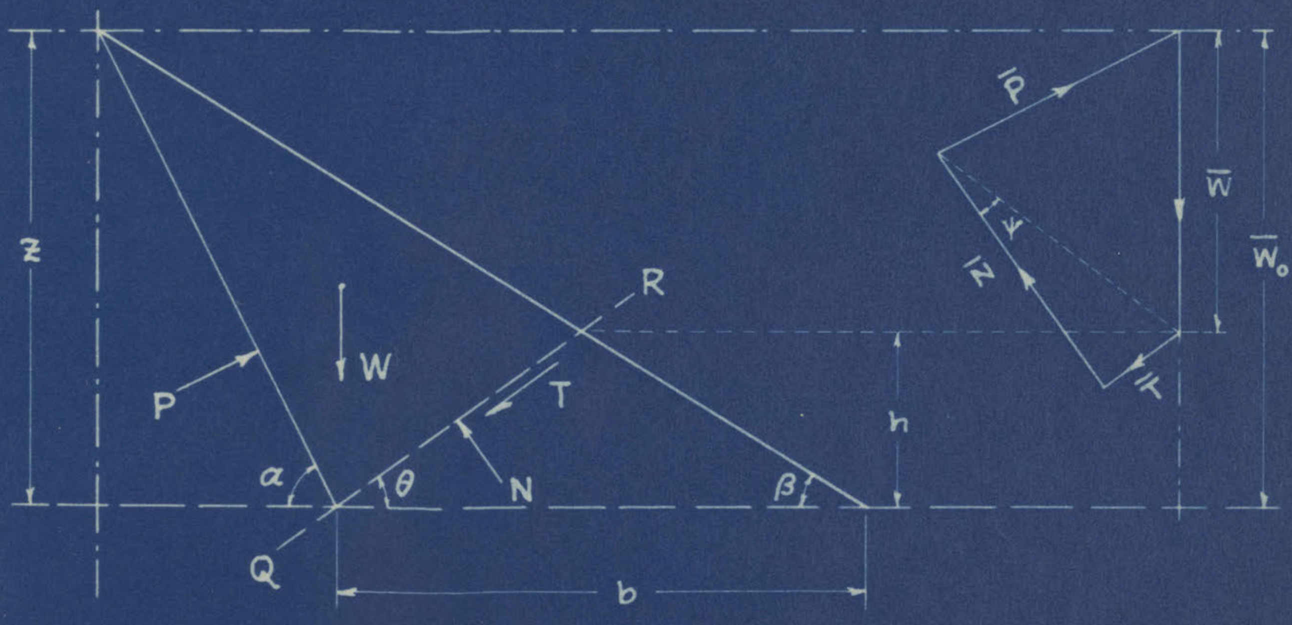

FIG. 7

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to itself so that point a falls at a', the intersection of the core slope with the Z-plane. A vertical through c' determines the new line of action of $\overline{W}$. The diagram is then rotated clockwise through an angle θ, bringing the line of action of $\overline{N}$ into a vertical position. This is most easily effected by drawing a horizontal ef, then a line fd' parallel to Oc', d' being determined by the intersection of fd' with the vertical through c'. A circle with center at a' and radius a'd' is then described, and the chord m laid off clockwise from d', determining d". The angle of obliquity ψ is then the angle between the vertical and a'd".

An inspection of the analytical equations for T and N shows that in the quotient T/N the coordinate z vanishes. That is, the obliquity of resultant stress on any plane is independent of depth.

The scale of the force polygon is determined by the depth z and the weight of the shell material above the Z-plane. This scale fixes the length of line $\overline{P}$, which may be determined as follows:

$$z \text{ units} = \frac{s_2 bz}{2}, \text{ or } 1 \text{ unit} = \frac{s_2 b}{2}$$

$$P = \frac{s_1 z^2}{2} \csc \alpha$$

$$\text{Therefore } \overline{P} = \frac{s_1 z^2}{2} \csc \alpha \div \frac{s_2 b}{2} = z \frac{s_1}{s_2} \frac{z \csc \alpha}{b}$$

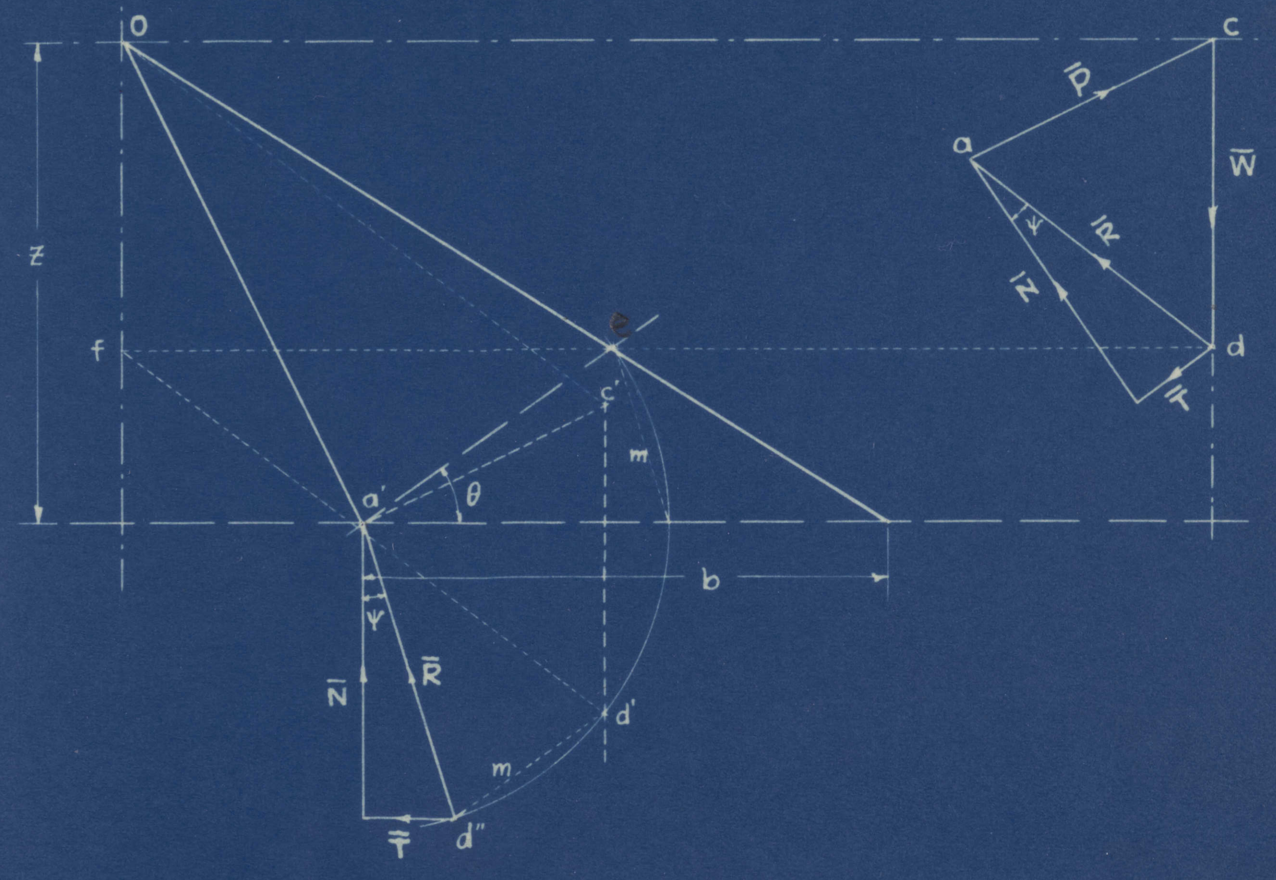

FIG. 8

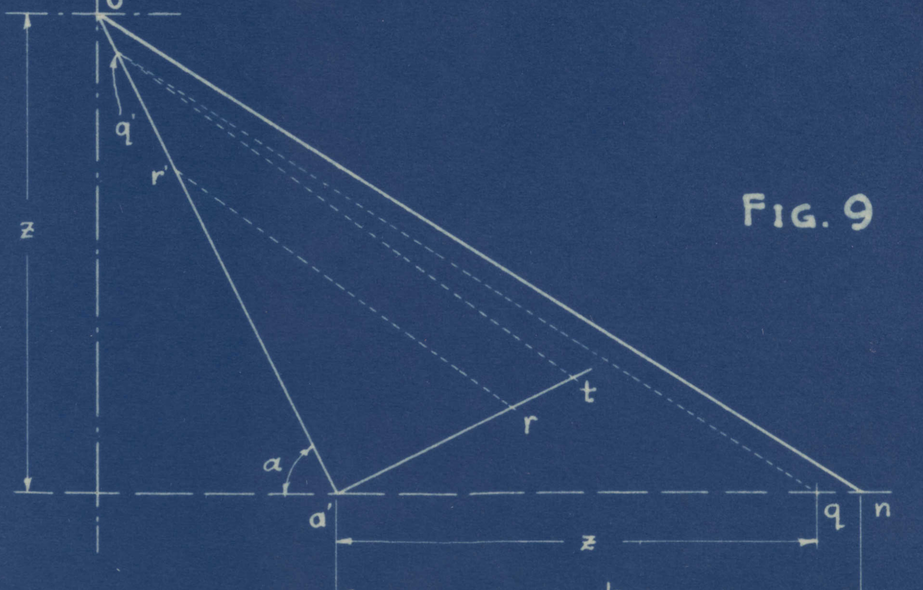

FIG. 9

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The length of $\overline{P}$ may be determined graphically as shown in Fig. 9, in the following steps:

On the base, lay off $a'q = z$.

Draw through q parallel to outer slope a line intersecting Oa' in q'.

On a line from a' perpendicular to Oa' lay off to any convenient scale $a'r = s_1$

On the core slope to the same scale lay off $a'r' = s_2$.

Draw rr', then q't parallel to rr'.

Then a't is $\overline{P}$.

It is evident that the direction of the vector $\overline{P}$ determined in this manner is correct, since the line a't is by construction perpendicular to the core slope. That its magnitude is also correct may be proved as follows:

In triangles a'rr' and a'tq', $a't:a'q' = a'r:a'r' = s_1:s_2$.

Or
$$a't = \frac{s_1}{s_2}(a'q').$$

In triangles Oa'n and q'a'q, $\dfrac{a'q'}{a'q} = \dfrac{a'O}{a'n} = \dfrac{z \csc \alpha}{b}$

Or $$a'q' = a'q \frac{z \csc \alpha}{b} = z \frac{z \csc \alpha}{b}$$

Whence $$a't = z \frac{s_1}{s_2} \frac{z \csc \alpha}{b} = \overline{P}$$

The procedure to be carried out in the graphical analysis of the stress conditions existing within a shell of given or assumed section, with the weights of the shell material and the core material given or assumed, would consist essentially of the following steps:

1. Lay out the section accurately to scale, selecting some convenient value of z.
2. Using the given values of s_1 and s_2 determine the vector $\overline{P}$ as described.
3. Assume successive values of θ and determine graphically the corresponding angles obliquity, all referred to a vertical line, by drawing the force polygon in the rotated position.
4. If for any particular value of θ the angle of obliquity is greater than a given or assumed value of ϕ for the shell material, a condition of instability exists.

Supplementary Remarks. The values of the constants s_1, s_2 and ϕ should be determined experimentally for any particular design. It is not as important to obtain exact values (in fact, it is impossible, strictly speaking) as it is to determine limiting values. The most unfavorable case can thus be investigated and allowance made therefor.

The assumption of an idealized cross-section was found necessary in order to obtain a general solution. It will be observed, however, that this idealized section does not differ a great deal, in the last analysis, from the actual section of the average dam. As the height of the dam increases, the actual section approaches more and more closely to the idealized section, except in regard to the outer slopes, which are usually flatter at the bottom than at the top in the case of a high dam. Under these conditions, an average value of the slope angle can be assumed which closely approximates the shape of the upper part, and the error made in the lower part will be on the side of safety. As far as the assumption of top width zero is concerned, any dam may be considered as a section of zero top width to which is added a layer of material on both slopes. Here again, the error in the idealized section is on the side of safety.

The writer does not recommend the use of a factor of safety in this part of the design, for two reasons:

1. The additional material required in the shell to give a reasonable factor of safety will probably cost much more than the damage done by minor slides. This contention may be open to criticism as unsound engineering; but slides under these conditions occur only during construction, so that there is no damage to life or property involved.*

2. The assumption of full hydrostatic stress acting within the completed section of the dam is too severe. This

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*A similar argument is presented by Mr. John E. Field in his discussion of Allen Hazen's paper "Hydraulic-Fill Dams", Transactions, Am. Soc. C. E., 1920.

condition could occur only if the dam were thrown up instantaneously. In the actual case, by the time the structure is completed, the lowermost portions of the core are well on their way to consolidation, since the first part of the consolidation process takes place quite rapidly. The decrease in pressure on the shell due to the increased stability of the core may therefore be considered as an adequate factor of safety for the design.

I.B. THEORY OF CONSOLIDATION OF HYDRAULIC-FILL CORES

Nature of the Problem. The phenomenon of the gradual change of the core of a hydraulic-fill dam from its initial soft liquid state to the state of a solid mass has been observed, commented upon, and tested in a more or less approximate manner by all the engineers who have been engaged in this type of construction. There is perhaps no other single item involved upon which there has been such a widespread difference of opinion. As an illustration, Allen Hazen* recommends the exclusion from the core of all particles smaller than 0.01 mm., while Charles H. Paul** states that 40 or 50 percent of the core material may be finer than this limiting size without endangering stability, a contention which has been borne out by the success of the Miami Conservancy Dams. Mr. Paul makes no attempt to explain his opinion on a theoretical basis, merely stating the observed facts, which, of course, are incontrovertible. Mr. Hazen, on the other hand, tries to illustrate his view theoretically. He assumes that material with 50% of voids is unstable, and that material with 40% of voids is stable, and computes the length of time it would take for the water corresponding to that decrease in volume

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*Hydraulic-Fill Dams. Transactions, Am. Soc. C. E., 1920.

**Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District. Transactions, Am. Soc. C. E., 1922.

to move through a layer of material of an effective size of 0.002 mm. (about that of the Miami cores), under an assumed hydraulic gradient of 10%. He arrives at the result that "for a large dam, complete drainage and consolidation may be a matter of many years." Without attempting to analyze Mr. Hazen's assumptions, which are certainly open to criticism, it may be remarked that the problem is in fact of such a nature that a crude solution of this type will not afford even an approximate idea of the actual behavior of the material. The truth of this statement will be brought home immediately and forcibly to any investigator who attempts to evolve a mathematical theory of the consolidation of earth under various conditions.

The questions which are of immediate interest are somewhat as follows: In what manner does consolidation proceed? What is the underlying cause of the increase of resistance of the soil due to consolidation? Is it possible to obtain complete consolidation of a mass of earth such as a hydraulic-fill core? If not, what degree of consolidation will produce a reasonable amount of internal resistance? What are the factors affecting consolidation, and how much importance has any one of these factors? Finally, if a theoretical solution of the problem is obtained, is it possible to determine the constants of the material, experimentally or otherwise, so that a practical solution may be evolved for a given case?

In order to obtain intelligent answers to these questions, the theory underlying the phenomenon of consolidation must be developed and investigated. Without such a definite statement of basic principles, one man's guess is as good as another's. It therefore becomes necessary to first set up a fundamental equation which will cover the phenomenon in general, and then to solve this equation for the special case in hand, stating any particular simplifying assumptions which are found necessary in order that a solution may be obtained.

In the analysis which follows, it becomes at once apparent that an absolutely rigid solution for the hydraulic-fill core involves hopeless complexities from a mathematical point of view. Solutions have been obtained, however, for the three simplest cases, each of which may be considered an element in the actual case. Examination and comparison of these solutions cannot fail to afford a clearer picture of the phenomenon and a better idea of the importance of the factors involved than has hitherto been possible.

Definitions and Nomenclature. In the development of the equations the following quantities will be employed:

k = the coefficient of permeability of the soil.

k_o = the reduced coefficient of permeability.

e_n = the voids ratio of any elementary volume of soil.

p = the portion of the pressure existing at any point in the mass which is transmitted directly from grain to grain of the soil.

w = the portion of the pressure which is carried as excess hydrostatic pressure in the voids water.

a = the coefficient of compressibility of the soil, or the change in voids ratio per unit change in pressure.

c = k_o/a, the coefficient of consolidation.

c_2 = the index of compression of the soil.

The usual mathematical symbols, x, y, z, r, t, etc., expressing distance, time, etc., will be referred to as occasion arises. The meaning of the preceding terms will be made clearer in the succeeding paragraph.

Preliminary Considerations. In all phenomena relating to consolidation the flow of water plays the major role. Darcy's law states that the quantity of water flowing through a layer of soil is directly proportional to the time, the area of the layer, and the hydraulic gradient; the constant of proportionality is known as the coefficient of permeability of the material. This coefficient may therefore be defined as the quantity of water percolating in a unit of time through a layer of unit area and unit thickness, under a unit hydraulic head. This quantity is actually constant for a given sample of soil in a given condition, when the flow is laminary, as is usually the case. It is different not only for different soils but also for differences in condition of the same soil, these differences being changes in structure and arrangement of grains, variations in temperature, and so on.

The process of consolidation as a phenomenon in space may best be studied experimentally, by determining the relation between the volume of a given sample of soil and the pressure applied to the sample. Diagrams similar to the familiar stress-strain diagrams of steel and concrete have been obtained for a great number of soils, both natural samples and artificial mixtures. These diagrams all have a characteristic shape when plotted as indicated in Fig. 10. The strain in a material under stress is ordinarily considered as the change of length per unit of length. In the case of soil samples, however, the unit is taken as the length of the sample would have if its volume of voids were zero. For a given sample, this unit, called the reduced thickness or reduced height, is obviously constant, since the total volume of solid matter cannot change; and all changes in height or thickness of the sample due to the applied pressure are referred to this unit. The stress-strain diagram for a soil is therefore a plot of pressure against voids ratio, where the

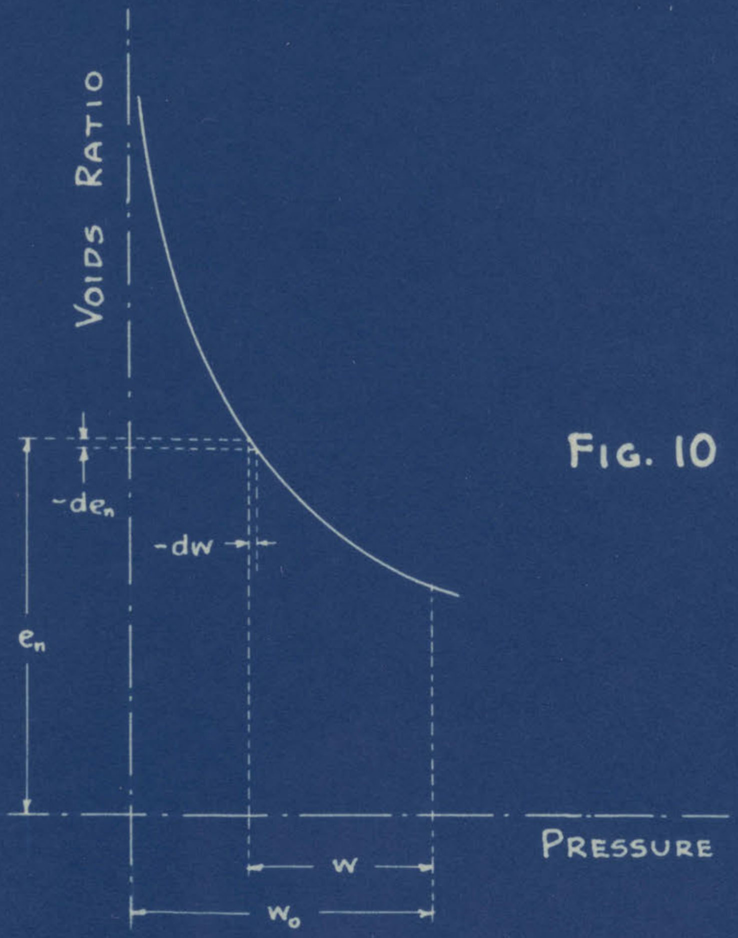

Fig. 10

Fig. 11

voids ratio is the volume of voids (variable) per unit of volume of solid matter (constant).

In the usual type of diagram, pressure is denoted by p and voids ratio by e. Confusion may arise in the use of the symbol e to represent voids ratio, inasmuch as it ordinarily denotes the base of the natural system of logarithms, and as such will enter into the theory which follows. The writer will therefore use e_n to represent voids ratio, and the usual e for the natural base.

The simplest mathematical expression so far discovered to express the relation between e_n and p is that given by Dr. Terzaghi.* The modulus of elasticity is approximately proportional to the pressure, or

$$\frac{dp}{de_n} = - c_2 p \qquad (1)$$

the minus sign being used to denote a decrease of voids ratio with an increase in pressure. The constant c_2 is called the index of compression. It must be distinctly borne in mind that this constant refers to increasing pressure. If the pressure on the sample be decreased, after a previous increase, the expansion does not occur along the same curve as the compression, because the material is not perfectly elastic. The present problem, however, has to deal only with increases of pressure, so that a consideration of the phenomenon of expansion will not be necessary.

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*Erdbaumechanik, Section 12.

Consider a given pressure to be applied to a sample of saturated soil. This increased pressure corresponds to a decrease in voids ratio. The only way in which the voids ratio can decrease is by the reduction of the total quantity of water in the sample. When the pressure is first applied, there is no reduction in voids ratio. Consequently the entire increase of pressure is carried as excess hydrostatic stress in the voids water. If the sample is provided with surface drainage, this excess hydrostatic stress corresponds to a definite instantaneous hydraulic gradient through the sample, and the water flows out in accordance with Darcy's law. As time goes on, more and more water is forced out, and the hydrostatic excess pressure gradually decreases to zero. If p_1 represents the total increase of pressure at any point, and p and w are as previously defined, $p_1 = p + w$ represents the distribution of p_1 between the grains and the water, respectively, at any time. As time increases from 0 (the instant of application of the increase of pressure p_1) to infinity, p increases from 0 to p_1 and w decreases from p_1 to 0. so that in any given interval of pressure represented by p_1

$$\frac{dp}{de_n} = - c_2 p = - \frac{dw}{de_n}, \quad \text{or} \quad \frac{dw}{de_n} = c_2 (p_1 - w). \qquad (2)$$

If w_o is the hydrostatic pressure at time $t = 0$, then w_o is obviously equal to p_1. So that

$$\frac{dw}{de_n} = c_2 (w_o - w) \qquad (3)$$

In this manner the process of consolidation may be translated into terms of the excess hydrostatic pressure.

During consolidation, the voids ratio of a given sample of soil changes with pressure and with time, hence the linear dimensions are variable. Equations involving the true linear dimensions are therefore hopelessly complicated, and it becomes necessary to operate with reduced dimensions, that is, with the dimensions the sample would have if its volume of voids were zero. This fact necessitates the introduction of the reduced coefficient of permeability, which is defined as the quantity of water percolating in a unit of time through a unit of area of a sample of unit <u>reduced</u> thickness under a unit hydrostatic head. An example will serve to illustrate the meaning of this reduced coefficient.

Consider a sample of soil of area A, thickness d, whose volume of voids is e_n, and through which water is flowing under a head h. Then, by Darcy's law, the rate of percolation Q is

$$Q = k\,A\,\frac{h}{d}.$$

The reduced thickness of the sample, or the thickness it would have if its voids ratio were zero, is evidently

$$d_o = \frac{d}{1 + e_n}.$$

From the foregoing definition of the reduced coefficient of permeability, the rate of percolation may be expressed as

$$Q = k_o A\,\frac{h}{d_o} = k_o\,A\,\frac{h}{d}\,(1 + e_n)$$

The two expressions for Q must be equal, so that

$$k_e = \frac{k}{1 + e_n}$$

Since the value of Q computed from either expression will be the same, it is apparent that the introduction of the reduced coefficient does not affect the validity of results derived from operations in which it is involved.

Fundamental Equation for the Rate of Consolidation of Saturated Earth.

Let x, y, z, be the coordinates of any infinitesimal element of volume dx dy dz, remembering that all dimensions are reduced dimensions, as previously explained.

Consider a region T of the material, enclosing the given element, and bounded by a surface S.

By Darcy's law, the quantity of water flowing in time dt across an element of surface dS is

$$- k_\circ \frac{dw}{dn} dS\, dt \tag{1}$$

wherein $\frac{dw}{dn}$ is the directional derivative of w with respect to the outward drawn normal to the surface, hence is the hydraulic gradient in the direction of the normal. The negative sign indicates that a decrease of pressure in an outward direction corresponds to outgoing flow.

The total quantity of water flowing across the surface S in time dt, which is the total amount lost from T in time dt, is

$$- k_\circ\, dt \iint_{(S)} \frac{dw}{dn}\, dS \tag{2}$$

which may be written

$$-k_\circ\, dt \iint_{(S)} \left[\frac{\partial w}{\partial x} \cos \alpha + \frac{\partial w}{\partial y} \cos \beta + \frac{\partial w}{\partial z} \cos \gamma \right] dS \tag{3}$$

In (3), cos α, cos β, and cos γ are the direction cosines of the normal.

By Green's theorem, (3) is equivalent to

$$- k_\circ \, dt \iiint\limits_{(T)} \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right] dx\, dy\, dz$$

or, in abbreviated form,

$$- k_\circ \, dt \iiint\limits_{(T)} \nabla^2 w \, dx\, dy\, dz \qquad (4)$$

If e_n is the voids ratio of the element, the total quantity of water in the element at time t is

$$e_n \, dx\, dy\, dz \qquad (5)$$

After the lapse of an interval of time dt, the total quantity is

$$\left(e_n + \frac{\partial e_n}{\partial t} dt \right) dx\, dy\, dz \qquad (6)$$

Therefore the loss of water from the element in time dt is (6) - (5), or

$$- dx\, dy\, dz \frac{\partial e_n}{\partial t} dt \qquad (7)$$

and the total loss from T in time dt is

$$- dt \iiint\limits_{(T)} \frac{\partial e_n}{\partial t} dx\, dy\, dz \qquad (8)$$

The two expressions (4) and (8) for loss of water must be equal. Hence

$$k_{\circ}\, dt \iiint_{(T)} \nabla^2 w \, dx\, dy\, dz = dt \iiint_{(T)} \frac{\partial e_n}{\partial t} \, dx\, dy\, dz \tag{9}$$

Or

$$\iiint_{(T)} \left[k_{\circ} \nabla^2 w - \frac{\partial e_n}{\partial t} \right] dx\, dy\, dz = 0 \tag{10}$$

This equation is true for any region T whatsoever. Hence the integrand must vanish at all points, otherwise a region T could be assumed so small that the integral expression would not satisfy (10). Therefore

$$k_{\circ} \nabla^2 w = \frac{\partial e_n}{\partial t} \tag{11}$$

It has been shown from the equation of the pressure-compression diagram (equation (3) of preceding section) that

$$\frac{dw}{de_n} = c_2\, (w_{\circ} - w), \quad \text{or}$$

$$\frac{dw}{c_2\, (w_{\circ} - w)} = de_n$$

The solution of this equation is

$$e_n = -\frac{1}{c_2} \ln\, (w_{\circ} - w) + c_3 \tag{12}$$

wherein c_3 is an arbitrary constant.

Differentiating (12) with respect to t,

$$\frac{\partial e_n}{\partial t} = \frac{1}{c_2\, (w_{\circ} - w)} \frac{\partial w}{\partial t} \tag{13}$$

Whence (11) becomes

$$k_\circ \nabla^2 w = \frac{1}{c_2 \, (w_\circ - w)} \frac{\partial w}{\partial t} \tag{14}$$

If the quantity $\frac{1}{c_2 \, (w_\circ - w)}$ is called a, a constant, known as the coefficient of compressibility, and if, in addition, the coefficient of consolidation c is defined by

$$c = \frac{k_\circ}{a},$$

equation (14) becomes

$$c \nabla^2 w = \frac{\partial w}{\partial t} \tag{15}$$

Equation (15) is the fundamental equation for the rate of consolidation. The objection might be raised that it is of very limited value on account of the fact that the quantities $\underline{k_\circ}$ and $\underline{a}$, which are known to vary with the voids ratio, have been assumed constant. Actually, however, the assumption of a constant value of $\underline{c}$ is not at all as unwarranted as might be at first supposed. The coefficient of permeability decreases with decreasing voids ratio. The truth of this statement is self-evident, since a decrease in the volume of voids must correspond to an increased resistance to percolation. The quantity $\underline{a}$, which is the decrease in voids ratio per unit increase of pressure, also decreases with decreasing voids ratio, as seen in Fig. 10. The quantity $\underline{c}$, being the quotient $k_\circ/a$, is therefore more nearly constant, if the phrase may be used, than either $\underline{k_\circ}$ or $\underline{a}$. As a matter of fact, tests have shown that c actually is very nearly constant, especially for compressible materials with a low degree of permeability, such as clays. One may therefore feel confident that a solution of the fundamental equation for a special case will not be far removed from the truth.

Application of the Fundamental Equation to the Hydraulic-Fill Core.

The following assumptions will be made:

1. The core is homogeneous, isotropic, and symmetrical with respect to the center line; hence, there is no flow across the center line.
2. There is no flow through the bottom of the dam.
3. There is no flow parallel to the axis of the dam.
4. The material is at some time subject to hydrostatic stress throughout; that is, the consolidation process is considered to begin at the same time in all parts of the core.
5. The shell material offers no appreciable resistance to flow.

To attempt to evaluate the equation exactly, that is, to assume a triangular core with varying initial hydrostatic pressure throughout, is, for the present at least, an insoluble problem. An examination of the lines of flow which will be set up in accordance with the foregoing assumptions leads to a consideration of three simple cases wherein a solution is possible. If Fig. 11 represents a cross-section of half the core, the above assumptions require that the flow shall take place along lines somewhat as shown, the lines emerging perpendicular to the core slope. It may be seen that along the base the flow is horizontal; along the center line the flow is vertical; and in general, the flow is approximately radial. The following simple cases may therefore be investigated.

Case 1. Consolidation of a layer of soil within which the flow of water takes place in one direction, and the initial hydrostatic pressure is constant throughout.

Case 2. Consolidation of a layer of soil within which the flow of water takes place in one direction, and the initial hydrostatic pressure is a linear function of the co-ordinate in the direction of flow.

Case 3. Consolidation of a cylinder of soil within which the flow of water takes place radially, and the initial hydrostatic pressure is a function of the distance from the center, or constant.

Case 1. Linear flow in a layer, constant initial hydrostatic pressure. This case is particularly applicable to the lowermost stratum of the core, wherein the initial pressure is equal to the weight per unit volume of core material multiplied by the height of the dam, and is uniform throughout the layer.

Let the X-direction be the direction of flow.

Let $2x_1$ = total length of layer,

w_o = initial hydrostatic pressure.

Choose the coordinate axes such that one end of the layer is at the origin, remembering that free drainage is provided at both ends. As applied to the bottom layer of the dam, the origin would be at the foot of the core slope, as shown in Fig. 12.

The fundamental equation becomes

$$c \frac{\partial^2 w}{\partial x^2} = \frac{\partial w}{\partial t} \qquad (1)$$

Subject to the boundary conditions

1. $w = 0$ when $x = 0$, for all values of t
2. $w = 0$ when $x = 2x_1$, for all values of t
3. $w = w_o$ when $t = 0$, for all values of x

The equation and the boundary conditions are exactly analogous to those applying to the cooling of a plate of infinite area and thickness $2x_1$, with an initial temperature w_o and the temperatures of both surfaces held at zero. A solution has been obtained for this case which can at once be applied to the equation of consolidation.

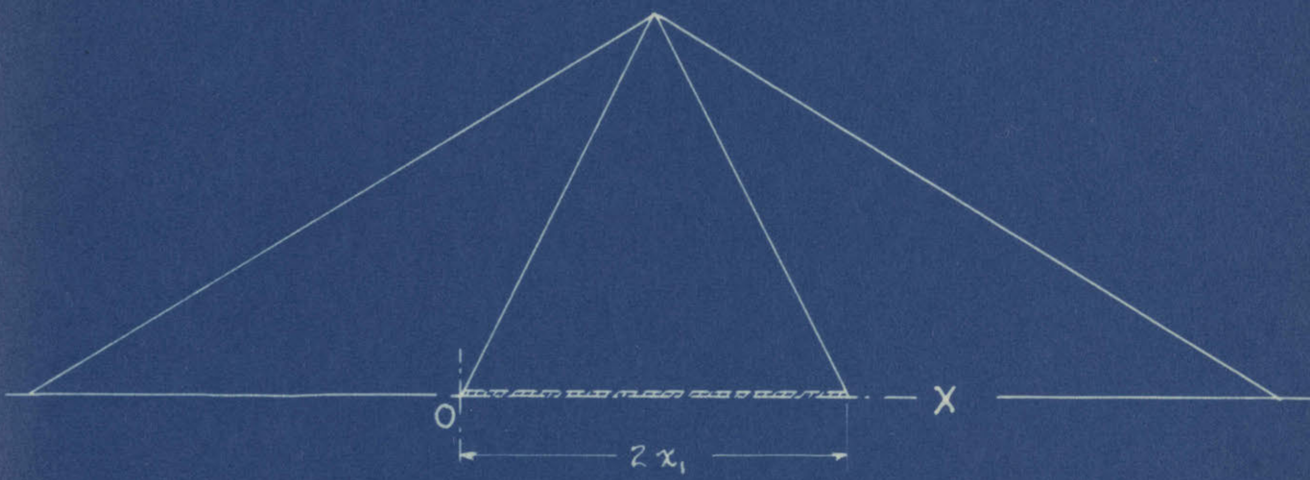

FIG. 12

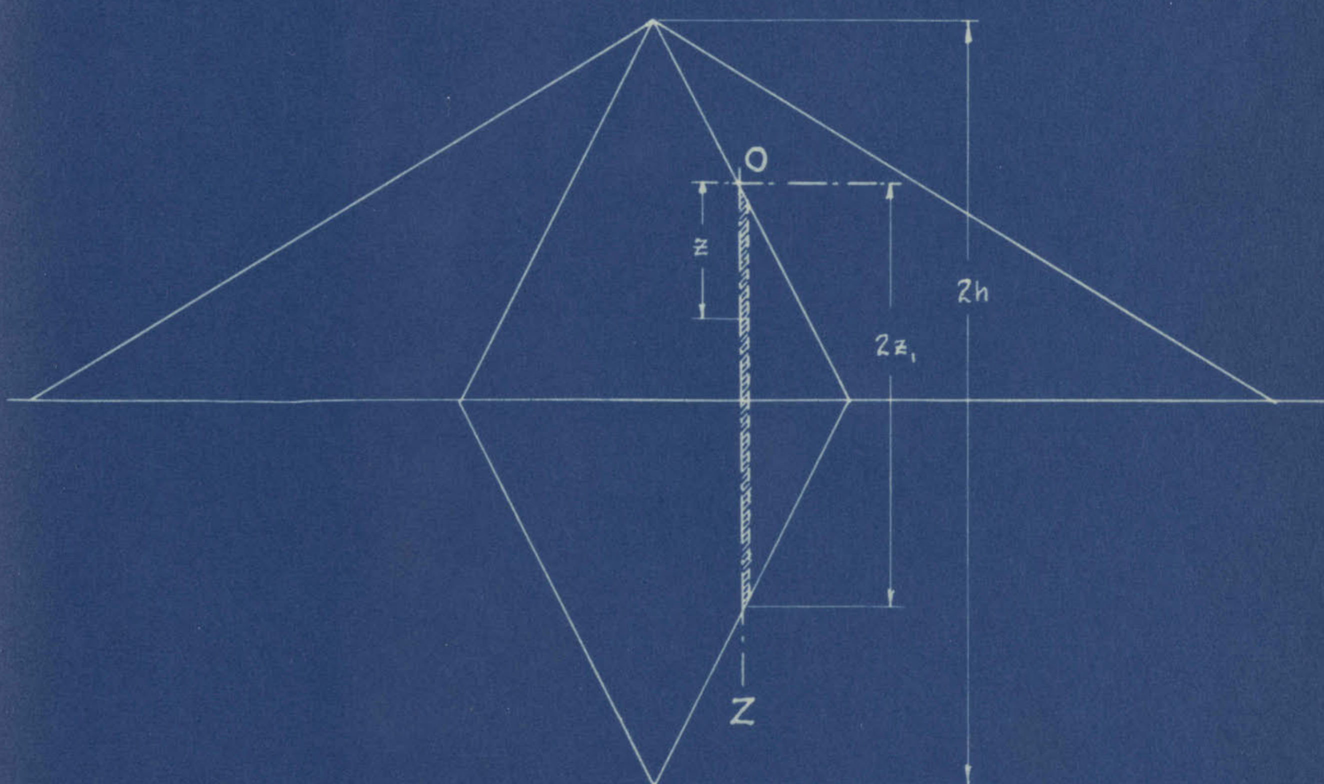

FIG. 13

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The solution is as follows:*

$$w = \frac{1}{x_1} \sum_{m=1}^{m=\infty} \left[e^{-\frac{cm^2\pi^2 t}{4x_1^{\,2}}} \sin \frac{m\pi x}{2x_1} \int_0^{2x_1} w_o \sin \frac{m\pi x}{2x_1} dx \right] \qquad (2)$$

which readily integrates into

$$w = \frac{2w_o}{\pi} \sum_{m=1}^{m=\infty} e^{-\frac{cm^2\pi^2 t}{4x_1^{\,2}}} \left(-\frac{1}{m}\right) \sin \frac{m\pi x}{2x_1} (\cos m\pi - 1) \qquad (3)$$

An inspection of the last factor shows that for even values of _m_ the entire quantity vanishes. The series will therefore contain only the odd values of _m_.

Let $m = 2n - 1$.

Since $\cos (2n - 1)\pi = -1$ for any integral value of _n_, the last factor becomes -2. Therefore

$$w = \frac{4w_o}{\pi} \sum_{n=1}^{n=\infty} \frac{1}{2n - 1} e^{-\frac{(2n-1)^2 c\pi^2 t}{4x_1^{\,2}}} \sin (2n-1)\pi \frac{x}{2x_1} \qquad (4)$$

The quantity w/w_o is the proportion of the initial hydrostatic pressure which is still carried by the water after a lapse of time _t_. This ratio is therefore a

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* See Ingersoll and Zobel - Mathematical Theory of Heat Conduction. Ginn & Co. 1913.

Riemann, Bernhard - Partielle Differentialgleichungen und deren Anwendung auf physikalischen Fragen. F. Vieweg und Sohn, Braunschweig. 1882.

measure of the state of consolidation at any time. If $w/w_o = 0.4$, for example, it means that at that particular time and in that particular spot 60% of the excess hydrostatic pressure has been relieved. It is obvious that theoretically the ratio becomes zero throughout the layer only after the lapse of an infinite length of time.

<u>Case 2.</u> Linear flow in a layer, initial hydrostatic pressure varying as a linear function of the distance in the direction of flow.

Let the Z-direction be the direction of flow.

Consider another mass of material exactly similar to the core to be placed below the core in reversed position, as indicated in Fig. 13. It is apparent from the symmetry of the figure that this does not change the conditions of flow in the actual core.

Choose any vertical element, as indicated, and place the origin of coordinates at the upper end of the element.

Let $2z_1$ be the total length of the element (free drainage at both ends), and 2h be twice the height of the dam. Let the specific mass gravity of the core material be s_1, as before.

The fundamental equation becomes

$$c \frac{\partial^2 w}{\partial z^2} = \frac{\partial w}{\partial t} \tag{1}$$

Subject to the boundary conditions

1. $w = 0$ when $z = 0$
2. $w = 0$ when $z = 2z_1$
3. $w = s_1(h-z_1+z)$, for $0<z<z_1$, when $t = 0$
4. $w = s_1(h-z+z_1)$, for $z_1<z<2z_1$, when $t = 0$

The previous solution now becomes

$$w = \frac{s_1}{z_1}\sum_{m=1}^{m=\infty} e^{-\frac{cm^2\pi^2 t}{4z_1^2}} \sin\frac{m\pi z}{2z_1}\left[\int_0^{z_1}(h-z_1+z)\sin\frac{m\pi z}{2z_1}\,dz + \int_{z_1}^{2z_1}(h+z_1-z)\sin\frac{m\pi z}{2z_1}\,dz\right] \quad (2)$$

The term in brackets may be expressed as

$$(h - z_1)\int_0^{z_1}\sin\frac{m\pi z}{2z_1}\,dz + \int_0^{z_1} z\sin\frac{m\pi z}{2z_1}\,dz$$

$$+ (h + z_1)\int_{z_1}^{2z_1}\sin\frac{m\pi z}{2z_1}\,dz - \int_{z_1}^{2z_1} z\sin\frac{m\pi z}{2z_1}\,dz \quad (3)$$

Evaluating each integral of (3) separately, the first becomes

$$(h-z_1)\left(\frac{2z_1}{m\pi}\right)\left(1 - \cos\frac{m\pi}{2}\right) \quad (4)$$

The third is

$$-(h+z_1)\left(\frac{2z_1}{m\pi}\right)\left(\cos m\pi - \cos\frac{m\pi}{2}\right) \quad (5)$$

The second is

$$\frac{4z_1^2}{m^2\pi^2}\sin\frac{m\pi}{2} - \frac{2z_1^2}{m\pi}\cos\frac{m\pi}{2} \quad (6)$$

The fourth is

$$-\frac{4z_1^2}{m^2\pi^2}\left(\sin m\pi - \sin\frac{m\pi}{2}\right) + \frac{2z_1}{m\pi}\left(2z_1\cos m\pi - z_1\cos\frac{m\pi}{2}\right) \quad (7)$$

Expanding and adding (4), (5), (6), and (7), the original term in brackets in (2) reduces to

$$\frac{2z_1}{m\pi}(h-z_1)(1-\cos m\pi) + \frac{8z_1^2}{m^2\pi^2}\sin\frac{m\pi}{2} - \frac{4z_1^2}{m^2\pi^2}\sin m\pi \qquad (8)$$

For all integral values of m, the third term vanishes.

If m is an even integer, the first and second terms vanish. The series will therefore contain only odd values of m.

For all odd values of m, the first term becomes $\frac{4z_1}{m\pi}(h-z_1)$

Let $m = 2n - 1$.

If n is odd, the second term becomes $+\frac{8z_1^2}{m^2\pi^2}$

If n is even, the second term becomes $-\frac{8z_1^2}{m^2\pi^2}$

(8) may therefore be written

$$\frac{4z_1}{(2n-1)\pi}\left[h - z_1 - \frac{(-1)^n\, 2z_1}{(2n-1)\pi}\right] \qquad (9)$$

So that the original solution (2) becomes

$$w = \frac{4s_1}{\pi}\sum_{n=1}^{n=\infty}\frac{1}{2n-1}\left[h-z_1-(-1)^n\frac{2z_1}{(2n-1)\pi}\right] e^{-\frac{(2n-1)^2 c\pi^2 t}{4z_1^2}} \sin\frac{(2n-1)\pi z}{2z_1} \qquad (10)$$

The solution is seen to be quite analogous to that obtained for Case 1, the only essential difference being the presence of the term in brackets.

In particular, if the layer is assumed to lie at the center line of the core, $z_1 = h$, and the solution reduces to

$$w = \frac{8s_1h}{\pi^2} \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1}}{(2n-1)^2} e^{-\frac{(2n-1)^2 c\pi^2 t}{4h^2}} \sin\frac{(2n-1)\pi z}{2h} \qquad (11)$$

Case 3. Radial flow in a cylinder, initial hydrostatic pressure varying as a function of the distance from the center.

If the X and Z directions are taken as the directions of flow, the fundamental equation becomes

$$c \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} \right] = \frac{\partial w}{\partial t} \tag{1}$$

Consider the origin of coordinates to be at the center of the cylinder, the Y-axis being the axis of the cylinder.

Let r_1 be the radius of the cylinder.

Replacing Cartesian by cylindrical coordinates,

$$r^2 = x^2 + z^2$$

and the fundamental equation becomes

$$c \left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right] = \frac{\partial w}{\partial t} \tag{2}$$

subject to the boundary conditions

1. $w = f(r)$ when $t = 0$
2. $w = 0$ when $r = r_1$

A method will be employed which is often successful in obtaining particular solutions of equations of this type. Assume

$$w = RT$$

wherein R is a function of r alone, and T is a function of t alone.

Equation (2) then becomes

$$\frac{1}{R}\left[\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr}\right] = \frac{1}{cT}\frac{dT}{dt} \tag{3}$$

Now, a change in r will not affect the right hand member of the equation, and a change in t will not affect the left hand member. Therefore each member must be equal to a constant, which may be called for convenience

$$- A^2 .$$

Taking the right hand member first,

$$\frac{dT}{T} = - A^2c\ dt \tag{4}$$

which integrates to

$$T = B\ e^{-A^2ct} \tag{5}$$

B being an arbitrary constant of integration.

Setting the left hand member of (3) equal to $-A^2$, transposing, and clearing of fractions,

$$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} + A^2r^2R = 0 \tag{6}$$

This is evidently a Bessel equation with parameter zero, so that the solution may at once be written

$$R = c_3\ J_o(Ar) \tag{7}$$

wherein c_3 is an arbitrary constant, and $J_o(Ar)$ indicates the Bessel function of first kind in (AR) with parameter zero.

A particular solution of (2) is therefore

$$w = K\, e^{-A^2 ct}\, J_o(Ar) \tag{8}$$

wherein K is an arbitrary constant.

The sum of any number of such particular solutions is also a solution. Therefore a series of the form

$$w = \sum_{n=1}^{n=\infty} K_n\, e^{-A_n{}^2 ct}\, J_o(A_n r) \tag{9}$$

is also a solution.

It remains to determine a series of this form to satisfy the boundary conditions. These conditions give

1. $f(r) = \sum_{n=1}^{n=\infty} K_n\, J_o(A_n r)$ for any value of r,

2. $\sum_{n=1}^{n=\infty} K_n\, J_o(A_n r_1) = 0$ for any value of t .

If $u_1, u_2, u_3, \ldots\ldots u_n \ldots$ are the roots, real, positive, and infinite in number, of the equation $J_o(u) = 0$, then a series of the form

$$\sum_{n=1}^{n=\infty} K_n\, J_o\left(u_n \frac{r}{r_1}\right)$$

satisfies condition 2, since when $r = r_1$ every term in the series vanishes.

Let $v = \dfrac{r}{r_1}$. Then $f(r) = f(r_1 v) = F(v)$

It remains to find an expansion of the form

$$F(v) = \sum_{n=1}^{n=\infty} K_n J_\circ(u_n v) \tag{10}$$

by determining the constants K_n .

If u_i and u_j are two different roots of the equation

$$J_\circ(u) = 0$$

then

$$\int_0^1 v\, J_\circ(u_i v)\, J_\circ(u_j v)\, dv = 0 \quad * \tag{11}$$

Now, if both sides of equation (10) are multiplied by

$$v\, J_\circ(u_n v)\, dv$$

and integrated between 0 and 1, every term in the series composing the right hand member, except the $\underline{n}^{th}$ term, assumes the form of (11), and therefore vanishes. Under these conditions (10) reduces to

$$\int_0^1 F(v)\, J_\circ(u_n v)\, v\, dv = K_n \int_0^1 J_\circ{}^2(u_n v)\, v\, dv$$

So that the general expression for K_n becomes

$$K_n = \frac{\int_0^1 F(v)\, J_\circ(u_n v)\, v\, dv}{\int_0^1 J_\circ^2(u_n v)\, v\, dv} \tag{12}$$

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* For proof of this relation, as well as other valuable information on Bessel Functions, see Gray, Mathews, and MacRobert, Bessel Functions, MacMillan & Co., London, 1922.

Noting that the constants A_n in (9) have been replaced by u_n/r_1, a complete solution of the equation may now be written:

$$w = \sum_{n=1}^{n=\infty} \frac{\int_0^1 F(v)\, J_0(u_n v)\, v\, dv}{\int_0^1 J_0^2(u_n v)\, v\, dv}\, e^{-\frac{u_n^2 ct}{r_1^2}}\, J_0(u_n v) \tag{13}$$

In particular, if $F(v)$ is a constant w_0,

$$w = w_0 \sum_{n=1}^{n=\infty} \frac{\int_0^1 J_0(u_n v)\, v\, dv}{\int_0^1 J_0^2(u_n v)\, v\, dv}\, e^{-\frac{u_n^2 ct}{r_1^2}}\, J_0(u_n v) \tag{14}$$

It may be well to note the values of $J_0(u_n v)$ and u_n

$$J_0(u_n v) = \sum_{m=0}^{m=\infty} (-1)^m \frac{(u_n v)^{2m}}{2^{2m}\,(m!)^2} = 1 - \frac{(u_n v)^2}{2^2} + \frac{(u_n v)^4}{2^4 (2!)^2} - \ldots .$$

The first six roots of $J_0(u) = 0$ are, to three places, as follows:

n	u_n
1	2.405
2	5.520
3	8.654
4	11.792
5	14.931
6	18.071

A table of the first 40 roots of $J_0(u) = 0$ is given on page 300 of the text referred to at the bottom of the preceding page. The above values have been copied from this table.

Summary and comparison of special cases. The three solutions are

Case 1. $$w = \frac{4w_o}{\pi} \sum_{n=1}^{n=\infty} \frac{1}{2n-1} e^{-\frac{(2n-1)^2 c\pi^2 t}{4{x_1}^2}} \sin \frac{(2n-1)\pi x}{2x_1}$$

Case 2. $$w = \frac{8s_1 h}{\pi^2} \sum_{n=1}^{n=\infty} \frac{(-1)^{n-1}}{(2n-1)^2} e^{-\frac{(2n-1)^2 c\pi^2 t}{4h^2}} \sin \frac{(2n-1)\pi z}{2h}$$

Case 3. $$w = w_o \sum_{n=1}^{n=\infty} \frac{\int_0^1 J_o(u_n v)\ v\ dv}{\int_0^1 J_o^2(u_n v)\ v\ dv} e^{-\frac{{u_n}^2 ct}{{r_1}^2}} J_o(u_n v)$$

These three equations make it possible to compute the state of consolidation at any point after the lapse of any time, under the three assumed conditions of flow.

It will now be advisable to develop equations to express the total volume change, or total consolidation, occurring during any given lapse of time. Such equations and the curves derived therefrom afford an easy method of computation, and at the same time furnish a basis for comparing the rates of consolidation furnished by each of the three cases.

The most convenient type of equation or plot is one expressing the percentage of total consolidation as a function of time. An equation of this type will be developed for each of the three cases, and the results compared.

Let q = percentage of consolidation (expressed as a ratio) of the mass after a lapse of time t.

$e_\circ$ = voids ratio of the mass at time 0.

e_t = voids ratio of the mass at time t.

e_i = voids ratio of the mass at time ∞.

Then $$q = \frac{e_\circ - e_t}{e_\circ - e_i}$$

It is desired to express q as a function of t.

Case 1. Consider any element of volume of unit cross-sectional area, whose reduced thickness in the direction of flow is dx. Let the excess hydrostatic pressure in the element at time t be w, as before. If the voids ratio of this element at time t is e_n (variable) its water content at time t is evidently $e_n\,dx$.

Hence, in a layer of unit cross-sectional area and reduced length x_1, the total water content at time t is

$$e_t\, x_1 = \int_0^{x_1} e_n\, dx \tag{1}$$

So that $$e_t = \frac{1}{x_1} \int_0^{x_1} e_n\, dx \tag{2}$$

In this case, the initial hydrostatic pressure is a constant, $w_\circ$. If the coefficient of compressibility a is constant, as has been assumed, then

$$e_i = e_\circ - a\, w_\circ \tag{3}$$

and

$$e_n = e_\circ - aw_\circ + aw \tag{4}$$

Therefore (2) becomes

$$e_t = \frac{1}{x_1}\int_0^{x_1}(e_o - aw_o + aw)\,dx = e_o - aw_o + \frac{a}{x_1}\int_0^{x_1} w\,dx \quad (5)$$

The percentage consolidation in the mass at time t is therefore

$$q = \frac{e_o - [e_o - aw_o + \frac{a}{x_1}\int_0^{x_1} w\,dx]}{e_o - (e_o - aw_o)} = 1 - \frac{1}{w_o x_1}\int_0^{x_1} w\,dx \quad (6)$$

Substituting the value of w previously obtained and integrating,

$$q = 1 - \frac{8}{\pi^2}\sum_{n=1}^{n=\infty}\frac{1}{(2n-1)^2}\, e^{-\frac{(2n-1)^2 c\pi^2 t}{4x_1^2}} \quad (7)$$

<u>Case 2.</u> The analysis is essentially the same as for Case 1, except that the initial hydrostatic pressure varies with z.

The total water content at time t of a layer of reduced length h is

$$e_t\,h = \int_0^h e_n\,dz \quad (1)$$

Whence

$$e_t = \frac{1}{h}\int_0^h e_n\,dz \quad (2)$$

Expressing the initial hydrostatic pressure as $s_1 z$,

$$e_i = e_o - \frac{a}{h}\int_0^h s_1 z\,dz = e_o - \frac{as_1h}{2} \quad (3)$$

and

$$e_n = e_o - \frac{as_1h}{2} + aw \quad (4)$$

So that (2) becomes

$$e_t = \frac{1}{h}\int_0^h \left(e_\circ - \frac{as_1h}{2} + aw\right) dz = e_\circ - \frac{as_1h}{2} + \frac{a}{h}\int_0^h w\ dz \qquad (5)$$

Therefore

$$q = \frac{e_\circ - \left(e_\circ - \frac{as_1h}{2} + \frac{a}{h}\int_0^h w\ dz\right)}{e_\circ - \left(e_\circ - \frac{as_1h}{2}\right)} = 1 - \frac{2}{s_1h^2}\int_0^h w\ dz \qquad (6)$$

Substituting the previously determined value of w and integrating,

$$q = 1 - \frac{32}{\pi^3}\sum_{n=1}^{n=\infty} \frac{(-1)^{n-1}}{(2n-1)^3}\, e^{-\frac{(2n-1)^2 c\pi^2 t}{4h^2}} \qquad (7)$$

<u>Case 3.</u> Consider a sector of the cylinder of unit length, bounded by planes enclosing an angle $d\theta$.

The water content at time t of an infinitesimal element of this sector a distance r from the center, whose reduced thickness is dr and whose voids ratio is e_n is given by

$$e_n\ r\ d\theta\ dr$$

The total water content of the sector at time t is

$$e_t\int_0^{r_1} r\ d\theta\ dr = e_t\frac{r_1^{\,2}}{2}\,d\theta = \int_0^{r_1} e_n\ r\ d\theta\ dr \qquad (1)$$

Whence

$$e_t = \frac{2}{r_1^{\,2}}\int_0^{r_1} e_n\ r\ dr \qquad (2)$$

Since the initial hydrostatic pressure is $w_\circ$,

$$e_1 = e_\circ - aw_\circ \qquad (3)$$

and

$$e_n = e_\circ - aw_\circ + aw \qquad (4)$$

Therefore (2) becomes

$$e_t = \frac{2}{r_1^2}\int_0^{r_1}(e_\circ - aw_\circ + aw)\ r\ dr = e_\circ - aw_\circ + \frac{2a}{r_1^2}\int_0^{r_1} wr\ dr \qquad (5)$$

And the percentage consolidation at time t is

$$q = \frac{e_\circ - (e_\circ - aw_\circ + \frac{2a}{r_1^2}\int_0^{r_1} w\ r\ dr\ )}{e_\circ - (e_\circ - aw_\circ)} = 1 - \frac{2}{w_\circ r_1^2}\int_0^{r_1} w\ r\ dr \qquad (6)$$

Substituting $v = \frac{r}{r_1}$ as before,

$$q = 1 - \frac{2}{w_\circ}\int_0^1 w\ v\ dv \qquad (7)$$

Substituting the value previously determined for w for this case, and performing the integration,

$$q = 1 - 2\sum_{n=1}^{n=\infty} \frac{[\int_0^1 J_\circ(u_n v)\ v\ dv\ ]^2}{\int_0^1 J_\circ^2(u_n v)\ v\ dv}\ e^{-\frac{u_n^2 ct}{r_1^2}} \qquad (8)$$

It now becomes necessary to evaluate the definite integrals. Taking first the one in the numerator,

$$[\int_0^1 J_\circ(u_n v)\ v\ dv\ ]^2 = \left[\int_0^1 \left(\sum_{m=0}^{m=\infty} (-1)^m \frac{u_n^{2m}\ v^{2m}}{2^{2m}\ (m!)^2}\right) v\ dv\right]^2$$

$$= \left[\sum_{m=0}^{m=\infty} (-1)^m \frac{u_n^{2m}}{(2m+2)\ 2^{2m}\ (m!)^2}\right]^2 \qquad (9)$$

Let K_m be the coefficient of u_n^{2m} in the series within the brackets in (9). Then the coefficient of the m-th term of the series resulting from the square will be

$$K_0K_m + K_1K_{m-1} + \dots + K_{m-1}K_1 + K_mK_0 \quad .$$

$$= \sum_{p=0}^{p=m} K_pK_{m-p}$$

So that (9) may be written

$$\sum_{m=0}^{m=\infty} (-1)^m \left(\sum_{p=0}^{p=m} K_p\, K_{m-p}\right) u_n^{2m} \tag{10}$$

From (9),

$$K_p = \frac{1}{(p+1)\ 2^{2p+1}\ (p!)^2} \tag{11}$$

and

$$K_{m-p} = \frac{1}{(m-p+1)\ 2^{2m-2p+1}\ [(m-p)!]^2} \tag{12}$$

So that

$$K_p\, K_{m-p} = \frac{1}{(p+1)(m-p+1)\ 2^{2(m+1)}\ [\ p!\ (m-p)!]^2} \tag{13}$$

Whence (10) becomes

$$\sum_{m=0}^{m=\infty} (-1)^m \frac{u_n^{2m}}{2^{2(m+1)}} \sum_{p=0}^{p=m} \frac{1}{(p+1)(m-p+1)[\ p!\ (m-p)!\ ]^2} \tag{14}$$

.

(14) being equivalent to the term

$$[\ \int_0^1 J_0(u_n v)\ v\ dv\ ]^2$$

Next taking the integral in the denominator of (8),

$$\int_0^1 J_0^2(u_n v)\ v\ dv = \int_0^1 \left[\sum_{m=0}^{m=\infty} (-1)^m \frac{u_n^{2m}\ v^{2m}}{2^{2m}\ (m!)^2} \right]^2 v\ dv \qquad (15)$$

Let K_m be the coefficient of the m-th term of $J_0(u_n v)$
Then the coefficient of the m-th term of $J_0^2(u_n v)$ is

$$K_0K_m + K_1K_{m-1} + \ldots\ldots + K_{m-1}K_1 + K_mK_0$$

$$= \sum_{p=0}^{p=m} K_p\ K_{m-p} \qquad (16)$$

From (15) $\quad K_p = \dfrac{1}{2^{2p}\ (p!)^2}$, and $\quad K_{m-p} = \dfrac{1}{2^{2(m-p)}[(m-p)!]^2}$

Therefore (16) becomes $\quad \displaystyle\sum_{p=0}^{p=m} \frac{1}{2^{2m}\ [\ p!\ (m-p)!]^2} \qquad (17)$

And the integral (15) becomes

$$\int_0^1 \left[\sum_{m=0}^{m=\infty} (-1)^m \frac{u_n^{2m}}{2^{2m}} \left(\sum_{p=0}^{p=m} \frac{1}{[p!\ (m-p)!]^2} \right) v^{2m+1} \right] dv \qquad (18)$$

Which integrates to

$$\sum_{m=0}^{m=\infty} \frac{(-1)^m}{2(m+1)} \frac{u_n^{2m}}{2^{2m}} \left(\sum_{p=0}^{p=m} \frac{1}{[p!\ (m-p)!]^2} \right) \qquad (19)$$

(19) being equivalent to $\int_0^1 J_0^2(u_n v)\ dv$

The original equation (8) may now be written

$$q = 1 - \sum_{n=1}^{n=\infty} \frac{\sum_{m=0}^{m=\infty} (-1)^m \left(\frac{u_n}{2}\right)^{2m} \left(\sum_{p=0}^{p=m} \frac{1}{(p+1)(m-p+1)[p!(m-p)!]^2}\right)}{\sum_{m=0}^{m=\infty} \frac{(-1)^m}{m+1} \left(\frac{u_n}{2}\right)^{2m} \left(\sum_{p=0}^{p=m} \frac{1}{[p!(m-p)!]^2}\right)} e^{-\frac{u_n^2 ct}{r_1^2}} \tag{20}$$

The three equations for q express the relation between q and t for the three assumed cases, and it is purely a matter of arithmetic to determine q for any value of t, assuming that the constants in the exponential term are known. Any desired degree of accuracy may be obtained by taking a sufficient number of terms in the expansions.

In order to make the solutions applicable to a general case, the writer has assumed

$$\frac{c}{x_1^2} = \frac{c}{h^2} = \frac{c}{r_1^2} = 1$$

and computed the values of q for various values of t, accurate to within less than one-half of one percent. These values are given in Table I and plotted in Fig. 14.

The computations are rather long and tedious, especially in Case 3. As previously stated, they are purely arithmetic in nature, so that it would not add any information to include them here in detail.

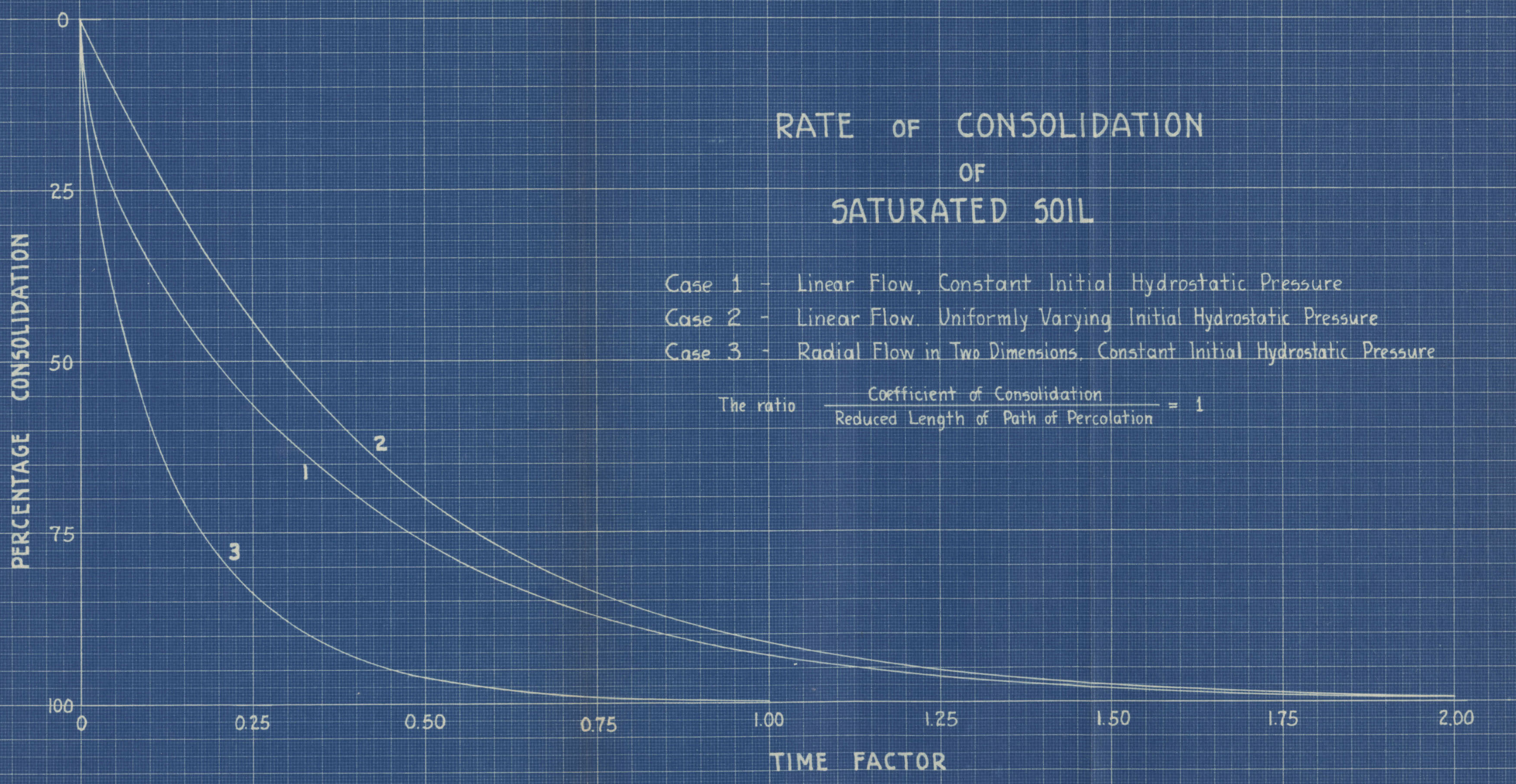
TABLE I

VALUES OF PERCENTAGE CONSOLIDATION (100q) AS A FUNCTION OF TIME

$$\text{The ratio } \frac{\text{Coefficient of consolidation}}{\text{Square of reduced length of path of percolation}} = 1$$

TIME	PERCENTAGE CONSOLIDATION		
	Case 1 Linear Flow Constant Initial Hydrostatic Pressure	Case 2 Linear Flow Uniformly Varying Initial Hydrostatic Pressure	Case 3 Radial Flow in two dimensions Constant Initial Hydrostatic Pressure
0	0.0	0.0	0.0
0.02	16.2	4.1	
0.04	22.5	8.0	35.5
0.06	27.6	12.0	43.9
0.08	32.0	15.7	52.1
0.10	35.7	19.8	58.7
0.15	43.7	28.8	70.4
0.20	50.4	37.0	78.2
0.25	56.2	44.3	83.7
0.30	61.3	50.7	87.8
0.40	69.6	61.5	93.2
0.50	76.4	70.0	96.2
0.60	81.6	76.5	97.8
0.70	85.6	81.7	98.8
0.80	88.7	85.7	99.3
0.90	91.2	88.8	99.6
1.00	93.1	91.2	99.8
1.20	95.8	94.7	
1.40	97.4	96.7	
1.60	98.4	98.0	
1.80	99.0	98.8	
2.00	99.4	99.3	
∞	100.0	100.0	100.0

FIG. 14



G. Gilboy 5-15-28

TABLE II

VALUES OF TIME FACTOR AS A FUNCTION OF PERCENTAGE CONSOLIDATION

Constant Initial Hydrostatic Pressure

PERCENTAGE CONSOLIDATION	TIME FACTOR	
	Case 1 Linear Flow	Case 3 Radial Flow
0	0	0
10	0.008	0.005
15	0.017	0.008
20	0.031	0.014
25	0.049	0.021
30	0.072	0.029
35	0.097	0.038
40	0.126	0.048
45	0.159	0.060
50	0.195	0.073
55	0.238	0.088
60	0.287	0.105
65	0.342	0.124
70	0.405	0.147
75	0.475	0.176
80	0.565	0.214
85	0.684	0.264
90	0.845	0.333
95	1.127	0.453
100	∞	∞

Fig. 14 clearly shows the difference between the rates of consolidation in the three different cases. It now remains to translate the information obtainable from these curves into a simple system which will furnish the basis for the computation of the rate of consolidation of the core of a dam as a whole.

From now on, attention will be concentrated upon Cases 1 and 3, as these two cases furnish limiting conditions. It is evident that Case 1 expresses the rate of consolidation of a horizontal slab with an impermeable lower surface, infinite in extent, and of reduced thickness x_1, since such a slab is composed of an infinite number of layers of the type assumed in the original computation, the layers standing side by side and the water flowing vertically upward in each layer. If the reduced radius of the cylinder in Case 3 is assumed equal to the thickness of the slab, then

$$r_1 = x_1 = B$$

the quantity B being introduced for convenience. Therefore the curves of Cases 1 and 3, Fig. 14, show the relation between the rate of consolidation of a slab and of a cylinder of radius equal to the height of the slab, provided $c/B^2 = 1$, wherein c is the coefficient of consolidation of the material of which the slab and cylinder are composed.

It will be noted that the time scale in Fig. 14 is designated as Time Factor, instead of using minutes or seconds. The significance of this usage will be apparent later.

The curves of Fig. 14 furnish values of the time factor as a function of percentage consolidation. Values of the time factor for each 5 percent consolidation have been read from Fig. 14 (from fine pencil lines before the curves were inked) and are given in Table II, for Cases 1 and 3. This process is equivalent to a graphical solution for t in the original equations, a direct solution being, of course, impossible.

Consider any given percentage consolidation, q_1. It is desired to find the time, t_1, it will take for a slab of any height B_1, of a material whose coefficient of consolidation is any value c_1, to reach this percentage consolidation. From the curve for Case 1, Fig. 14, the given value q_1 determines a certain time factor T_1. Now, the given value q_1 will result from the original equation for one and only one value of the exponent e. The time factor T_1 furnishes this particular value of the exponent in case $c/B^2 = 1$. If c_1/B_1^2 is different from 1, as will usually be the case, then the time t_1 must be such that the value of the exponent will remain unchanged. Therefore

$$1 \cdot T_1 = \frac{c_1}{B_1^2} \cdot t_1$$

Whence
$$t_1 = T_1 \frac{B_1^2}{c_1}$$

So that the procedure is: first, determine from the curve, or from Table II, the value of the time factor corresponding to the given percentage consolidation; multiply this factor by the ratio of the square of the reduced height of the slab to the coefficient of consolidation; the result is the time it will take for the slab to reach the given percentage consolidation, time being measured from the beginning of the consolidation process. The same procedure applies to the cylinder, substituting "radius" for "height".

As far as units are concerned, any units may be used as long as they are consistent within themselves. If voids ratio is expressed as grams of water per cubic centimeter of solid; pressures as grams per square centimeter; coefficient of permeability as centimeters per second; thickness of slab or radius of cylinder in centimeters; the resulting value of time will be in seconds. If the coefficient of permeability is given in centimeters per minute, all other quantities remaining as above, the resulting time will be in minutes. As a check upon the use of other units, it may be well to note that the quantity

$$\frac{ct}{B^2}$$

must be an abstract number, since the exponent of e must be an abstract number.

It is thus possible to compute the rate of consolidation of any cylinder or slab whatsoever by a simple arithmetical operation. But the core of a hydraulic-fill dam is neither a cylinder nor a slab. So that it will be necessary to develop a means of applying the basic information furnished by the two cases to computations involving the actual section.

Consider a core section of the type shown in Fig. 15, with the core slopes at an angle of 45° with the horizontal. If the actual triangle be replaced by a semi-circle of equal area, and the computations based on a cylinder whose cross-section is this circle, there is little doubt that the resulting computed rate of consolidation would very closely approach that of the actual case.

If the core slopes are at any other angle, as shown in Fig. 16, a semi-cylinder of circular section could hardly be said to approximate the actual triangle. However, a semi-cylinder of elliptical section, of area equal to that of the triangle, with the ratio of major to minor axis the same as the ratio of height to half-width of core, would closely approximate this case.

Referring to Fig. 17, a section of a slab of height B, a semi-circle of radius B, and a semi-ellipse of semi-minor axis B and semi-major axis A are shown. Now the rectangular section of the slab is nothing more than a semi-ellipse whose major axis is infinite in length. The time factor for any given percentage consolidation of the elliptical cylinder must lie somewhere between that of the circular cylinder and that of the slab. It is therefore possible to interpolate between the two curves (Case 1 and Case 3, Fig. 14), or between the two proper values in Table II, to determine the time factor for the ellipse.

There is no way of determining exactly how this interpolation should be performed. The simplest method seems to

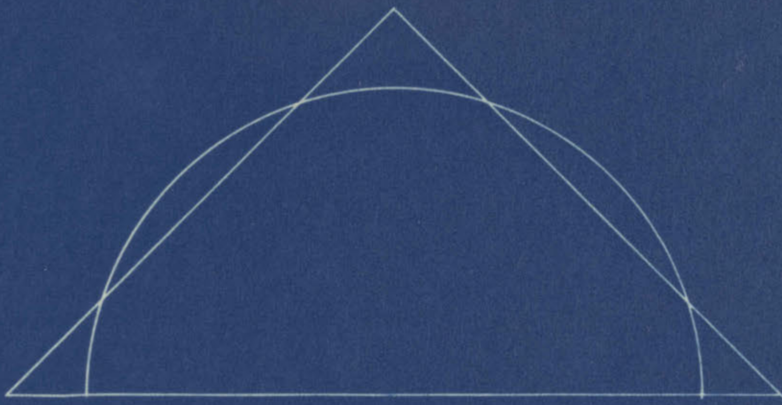


FIG. 15

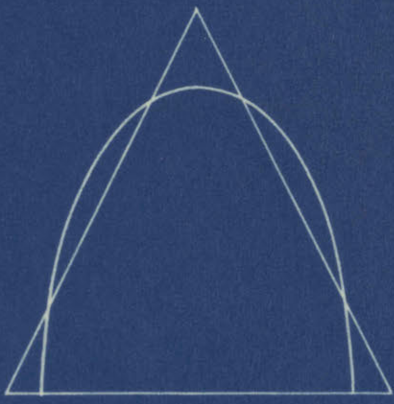


FIG. 16

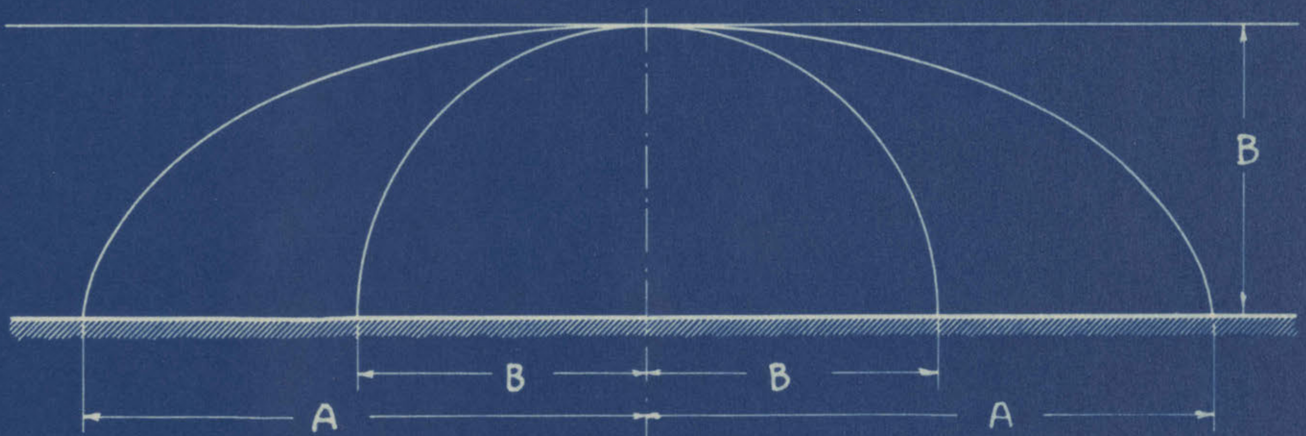


FIG. 17

G

be a linear interpolation as a function of the ellipticity. The ellipticity E is defined as

$$E = \frac{A - B}{A}$$

When $A = B$, the ellipse is a circle, and $E = 0$. When $A = \infty$, the ellipse is a rectangle, and $E = 1$. Therefore, for any intermediate shape of the ellipse, E will lie somewhere between 0 and 1. The basis of interpolation suggested is to take as the time factor of the elliptical cylinder a value intermediate between that for the circular cylinder and that for the slab, as a linear function of the ellipticity. In particular, if the time factors for the circular cylinder and the slab for a given q_1 are T_c and T_s, respectively, then the time factor for the elliptical cylinder T_e is given by

$$T_e = T_c + E(T_s - T_c)$$

wherein E is the ellipticity of the ellipse in question.

It is quite evident that this method will not furnish an exact solution. But it is believed that a better idea of the rate of consolidation can be obtained upon the basis herein laid down than is possible by any other means at present existing.

Summarized Procedure for Computing Rate of Consolidation.

1. The coefficient of consolidation of the material forming the core, and the voids ratio of the material in an unconsolidated state, must be determined experimentally. The tests required to obtain these values will be described in the following section.

2. The cross-section of the actual core is replaced by a semi-ellipse of area equal to that of the core whose semi-major and semi-minor axes are in the same ratio as that of the height of the core to half its bottom width. The lengths of the semi-axes and the ellipticity are computed.

3. Using the value of voids ratio previously noted, the reduced length of the semi-minor axis is obtained. The reduced length is equal to true length divided by (1 + voids ratio).

4. With the previously determined value of the ellipticity, the time factors for various percentages of consolidation are obtained by interpolating between proper values of the factors for Case 1 and Case 3, as found in Table II.

5. The time factor for any given percentage consolidation multiplied by the square of the reduced length of the semi-minor axis and divided by the coefficient of consolidation gives the time required for the core to reached the assumed percentage consolidation.

Expressed in symbols, the procedure is:

1. The values c and e_n are determined experimentally.

2. If h is the height of the core and b is half its width at the bottom, an ellipse of semi-major axis A and semi-minor axis B is determined to satisfy the equations

$$\frac{A}{B} = \frac{h}{b} \quad \text{and} \quad \pi AB = 2bh$$ π

Simultaneous solution of these equations gives values of A and B. The ellipticity E is then

$$E = \frac{A - B}{A}$$

3. The reduced length of the semi-minor axis, B_e, is given by

$$B_e = \frac{B}{1 + e_n}$$

4. If the time factors for any given percentage consolidation for the circular cylinder and the slab, obtained from Table II, are T_c and T_s, respectively, then the time factor for the same percentage consolidation for the elliptical cylinder is

$$T_e = T_c + E(T_s - T_c)$$

5. The time t_1 required for the core to reach the given percentage consolidation is then

$$t_1 = T_e \frac{B_e^2}{c}$$

SECTION II

EXPERIMENTAL DATA

The Germantown Dam. This structure is the southernmost of the five large hydraulic-fill dams built by the Miami Conservancy District for the protection of the City of Dayton and the other towns in the Miami Valley from damage due to floods. The dam lies across the valley of Twin Creek, a tributary of the Miami River, at a point about 2 miles northwest of the town of Germantown. The valley at this point is fairly narrow, with steep side hills, so that the dam is rather high, with a comparatively low yardage. More specifically, the top of the dam is at elevation 830 (above mean sea level), while the old channel of Twin Creek is at elevation 720. The dam is therefore 110 feet high at its maximum section. The total yardage is 835,000 cu. yd., of which about 800,000 were placed by hydraulic fill, and the remainder by dragline from the outlet conduit excavation. The overall length of the structure is 1156 feet.

About ninety percent of the material forming the dam was obtained from a borrow pit 40 acres in extent located 4000 feet upstream from the dam site. The formation consisted of a glacial deposit of sand and gravel, on top of which was a later alluvial deposit of clay and rock flour. The layer of fine material was thin in the bottom of the valley, so that when this portion of the borrow pit was being worked the deficiency in fines had to be supplied from an auxiliary borrow pit on top of the hill, where the shale and limestone were overlaid by glacial till.

The borrow pit material was excavated by dragline, loaded into cars, and hauled by train to a hog box of 250 cu. yd. capacity at the pumping plant, just upstream from the dam site. The material was thoroughly mixed in the hog box by hydraulic giants, and passed through a revolving screen 4' in diameter, 9' long, with 7" diameter openings. The oversize rocks retained on the screen dropped into a skip, and were loaded by a derrick into cars and hauled to the upstream face of the dam to be used as rip-rap. The sluiced material and the water passed into a sump, where it was picked up by a 15" dredge pump and delivered to the embankment through 15" pipe lines.

The outside levees were built up by draglines, defining the outer slopes. The pipe lines were parallel to and inside the levees, so that the mixture of water and earth flowed down the inner slope of the levees, the heavier material being dropped on the beaches and the finer constituents carried into the core pool. An excess of fine material was supplied in order to facilitate control of the core width and to permit of the waste of the extremely fine particles whose presence would serve to seriously retard the process of consolidation of the core. The overflow from the core pool, carrying this wasted material, passed through the embankment to the upstream toe, where the fine material formed an impervious blanket and the returned water was of assistance in augmenting the supply.

In common with the other Miami dams, the core of the Germantown Dam was designed such that its width at any point was equal to the height of the dam above that point. This makes the core fairly narrow, decreasing the danger of failure of the shell by reason of the core pressures, while at the same time it is wide enough so that its dimensions and composition can be very well controlled.

As the core becomes narrower and narrower with increasing height of dam, the control of its width becomes more and more difficult. At Germantown all attempts at holding to a definite core line were abandoned at elevation 815 (15 feet below the top). A dragline was run over the entire length of the dam at this point, excavating the bottom of the core pool to a considerable depth to mix up the material and break up any previous strata which might have formed by raveling of sand and gravel from the beaches into the pool. It is evident, then, that a core of triangular section, as designed by the District and as assumed in Section I, is purely of a theoretical nature, impossible to attain in practice.

The Germantown Dam was selected by the District as the proper one from which to take samples for investigation for two reasons: first, it is the second largest of the five dams in point of height; and second, the work of obtaining the samples could be accomplished with a minimum of inconvenience, as the highway across the dam is of relatively small importance.

II. A. APPARATUS AND METHODS

The procedure followed in obtaining the samples from the dam and testing their physical properties in the laboratory will be described in the following order:

1. Sampling.
    - a. Undisturbed Samples from Shaft.
    - b. Drillhole Samples.
2. Laboratory Tests.
    - a. Direct Permeability Test.
    - b. Standard Compression Test.
        - (1) Compression alone.
        - (2) Compression and Permeability combined,
    - c. Test of Compressive Strength Unconfined.
    - d. Limits of Consistency.
    - e. Mechanical Analysis.
    - f. Specific Gravity Test.
    - g. Natural Water Content Determination.

<u>1.a. Undisturbed Samples from Shaft</u>. In order to obtain samples of the core material in their natural state, or as near to that state as possible, a shaft four feet in diameter, lined with concrete, was sunk into the core of the dam at the side of the road, in a section of maximum height. The excavation was done with hand tools by a man at the bottom of the shaft, the excavated material being lifted out in buckets by means of a small hand derrick. The concrete lining sunk of its own weight at first as excavation progressed, additional sections 3 feet in length being poured at the top as occasion required. During the latter part of the work the lining had to be forced down by jacks.

Samples were in all cases taken in pairs, in the manner described below. It was the original intention to take samples every ten feet, sinking the shaft to the bottom of the dam, so that in all there would be ten pairs of samples. Due to unforeseen difficulties, however, the total depth of the shaft was limited to 80 feet, and only four pairs of samples were obtained.

Each sample was taken by hand, by means of a 4" diameter sampling tube, 12" long, made of seamless steel tubing. One end of the tube was turned down to a knife edge, and this edge burnished in one one-thousandth of an inch. The tube was forced into the core material by hand, the excess material around the outside of the tube being carefully removed as fast as the tube penetrated. In this manner, alterations in the structure due to the resistance of the surrounding earth were reduced to a minimum. The burnished cutting edge allowed a thousandth of an inch clearance between the sample and the inside of the tube, and in addition, the tube was wiped inside and out with a piece of oily waste just before being used; so that there was little or no tendency for the sample to deform on account of friction against the inside of the tube.

As soon as the tube had penetrated its full length into the material, it was carefully removed by cutting into the soil ahead of the cutting edge. The projecting material was sliced off flush with the ends of the tube by means of a fine wire in a small hack-saw frame, the ends capped, and the tube lifted to the surface. This operation was performed twice at any sampling level, two tubes being used. One sample was taken vertically, the other horizontally, in a direction perpendicular to the axis of the dam.

At the head of the shaft a tool house was erected for storage of material and for the carrying out of the operations subsequent to the actual sampling. Inside the tool house was a small work table on whose top were fixed two hollow cast iron bases, 3/4" high and 4-1/2" in diameter, with smooth top surfaces, a hole being bored in the table top just below each base. When a pair of samples was to be taken, the surfaces of these bases were cleaned and covered with a thin layer of melted paraffin. In readiness were two cylindrical containers, 4-1/2" in diameter and 13-1/2" long, made of 24-gauge galvanized iron; a kettle of melted paraffin; and a small quantity of modeling clay.

When the sampling tubes filled with soil came up from the shaft, they were taken into the tool house. The caps were removed from one of the tubes, and the tube stood on end on one of the cast iron bases. The sample being clear of the inside of the tube by a thousandth of an inch, it was possible to lift the tube completely off, and allow the sample to stand on the base without any support other than the capillary tension of the water in the voids. One of the galvanized iron containers was then placed over the sample and carried down until it rested on the table top, its lower end then being 3/4" (the height of the cast iron) below the bottom of the sample, and its upper end therefore extending 3/4" above the top of the sample. The annular space of 1/4" between sample and container was then poured full of melted paraffin. The level of the paraffin was carried up to the top of the container, thus sealing the upper end with a 3/4" thickness of wax. Leakage of paraffin between table top and container was prevented by applying the modeling clay at the proper points, or all around the container if necessary. The same procedure was then repeated with the other sample of the pair.

When the paraffin had thoroughly hardened, a flame was applied to the under side of the cast-iron base, melting the paraffin which had been previously applied to its surface. The container and enclosed sample could then be lifted off, reversed, and the lower end sealed with paraffin.

The sample in its final form therefore consisted of a cylinder of soil, 4" in diameter and 12" high, enclosed in a solid shell of paraffin 1/4" thick on the sides and 3/4" thick on the ends, the whole being contained in a stiff cylinder of galvanized iron. The cylinders were packed carefully into wooden boxes and shipped to the Institute.

1.b. Drillhole Samples. A drillhole was sunk into the core of the dam near the shaft, with an ordinary commercial well-drilling outfit, carrying a four-inch casing vertically downward. Samples were taken at intervals of about 10 feet, by driving down a 2" diameter seamless steel pipe 2 feet long, provided with a cutting edge at its lower end. The sampling pipe was driven down without twisting, as previous experience in the laboratory has indicated that twisting the sampler disturbs the soil to such an extent that any stratification which might exist becomes almost unrecognizable. When the pipe was full it was drawn up to the surface, unscrewed from the drill rod, about 3/4" of soil scraped out of each end, and the space thus created filled with paraffin. With the exception of the use of the large size sampling pipes, sealed immediately upon removal, no special provisions were made which require detailed description. The intention was to duplicate as far as possible the standard commercial practice in drilling for underground explorations.

Designation and Location of Samples. It will be most convenient in the following discussion to refer to the samples by their laboratory number. Every sample brought into the laboratory is given a number consisting of three parts, separated by hyphens. The first is the key number, referring to the job, project, firm, etc., to which the sample pertains. The second is the number of the drillhole, test pit, excavation, etc. The third is the number of the particular sample from the given location, the samples being numbered in order of depth. The key number of the Miami Conservancy District, Germantown Dam operation, is 30; the shaft is called test boring No. 1, the drillhole No. 2. The suffixes V and H are added to the sample number to indicate vertical and horizontal, respectively. The second horizontal sample taken from the shaft would then bear the number 30-1-2H; the fifth sample from the drillhole would be designated 30-2-5; and so on.

The tests made on any one sample are numbered in chronological order, irrespective of the type of test. The third test made on the fourth vertical sample from the shaft would have the test number 30-1-4V-3. The entire filing system is carefully cross-indexed for quick reference.

The location and numbers of the samples are indicated in Fig. 18. Inasmuch as all subsequent discussion will refer to the Germantown Dam samples, the key number 30 may be omitted for the sake of brevity.

MIAMI CONSERVANCY DISTRICT
GERMANTOWN DAM

LOCATION OF
CORE SAMPLES

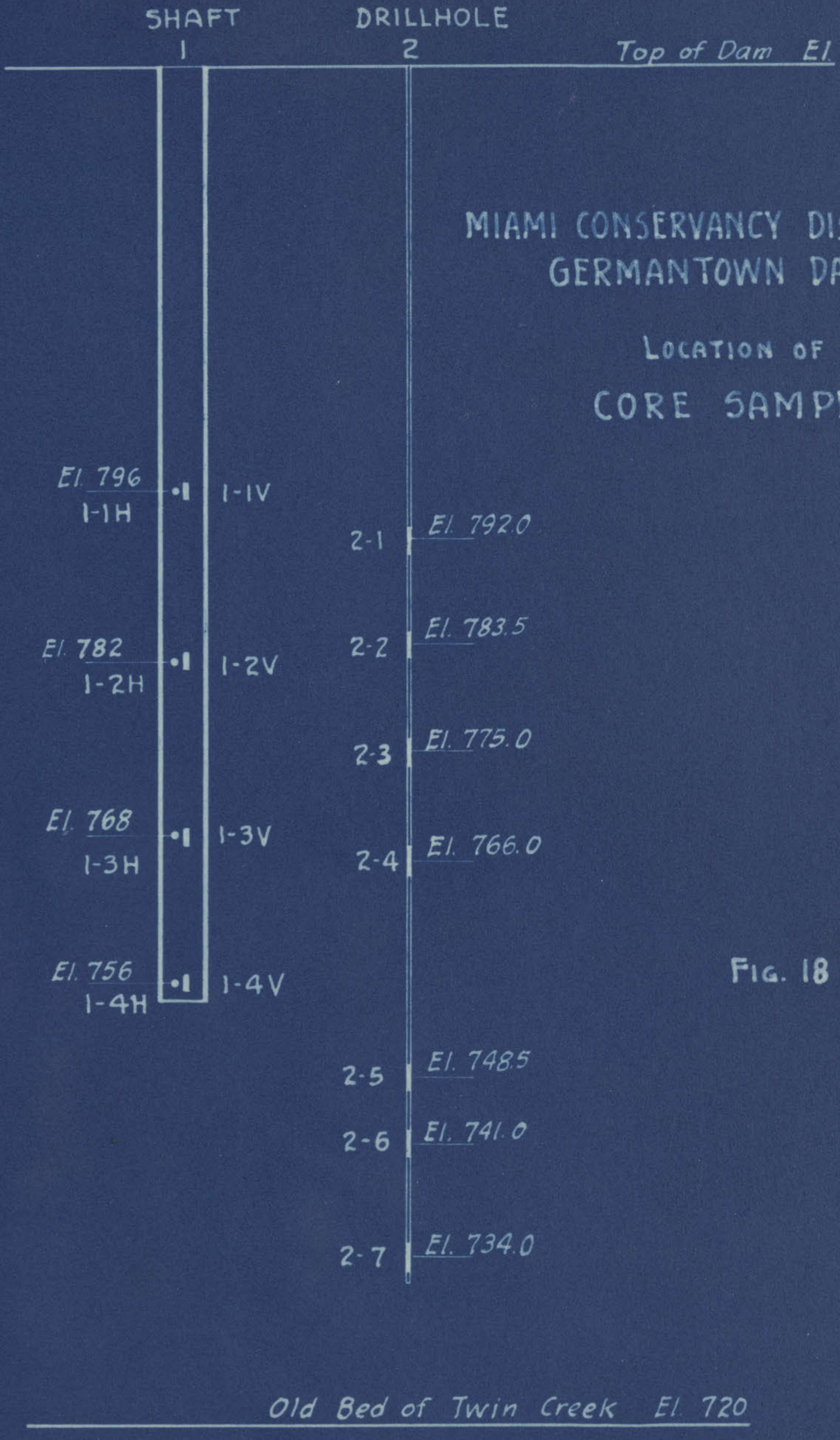


FIG. 18

G

2. Laboratory Tests. In order to minimize the inclusion of non-essential details, tests which are standard in the laboratory and which have been described in publications will be outlined here as briefly as possible. Tests especially devised or developed for this particular work will be described at greater length.

2.a. The Direct Permeability Test. There are in general two methods of determining the permeability of a soil by direct observation. The first is to measure the quantity of water which percolates through a sample of known dimensions in a given length of time under a given constant head. The second is to measure the rate at which a column of water in a graduated standpipe falls by reason of the water percolating through the sample. The first method is applicable to the case of coarse-grained materials, which allow a comparatively large quantity of water to pass in a comparatively short length of time. In dealing with fine-grained soils, however, this method cannot be used, as the rate of percolation under any reasonable head is so slow that the water will evaporate about as fast as it percolates, so that a measurement of quantity is not possible. A conception of the slow rate of percolation in fine-grained sediments may be gained by noting that in many cases it took 24 hours for 2 cubic centimeters of water to flow through one of the Miami samples. The falling-head permeamter, then, is the type adapted to these tests.

A special type of falling-head permeameter was designed for the Miami tests. Details and assembly are shown in Fig. 19. The sample is contained in the cylinder 2, free drainage being provided at the bottom surface through the Norton fine grade porous disc set into the recess in the base 1. A tight joint between the top of the cylinder 2 and the cover 3 is obtained by means of a rubber packing (not shown). The standpipe upon which observations are made is fitted tightly by means of a rubber stopper or hose connection, depending on the diameter of the pipe, to the top of inlet 6. The standpipe is held in a vertical position by means of a brass rod (Fig. 19a) clamped to the inlet.

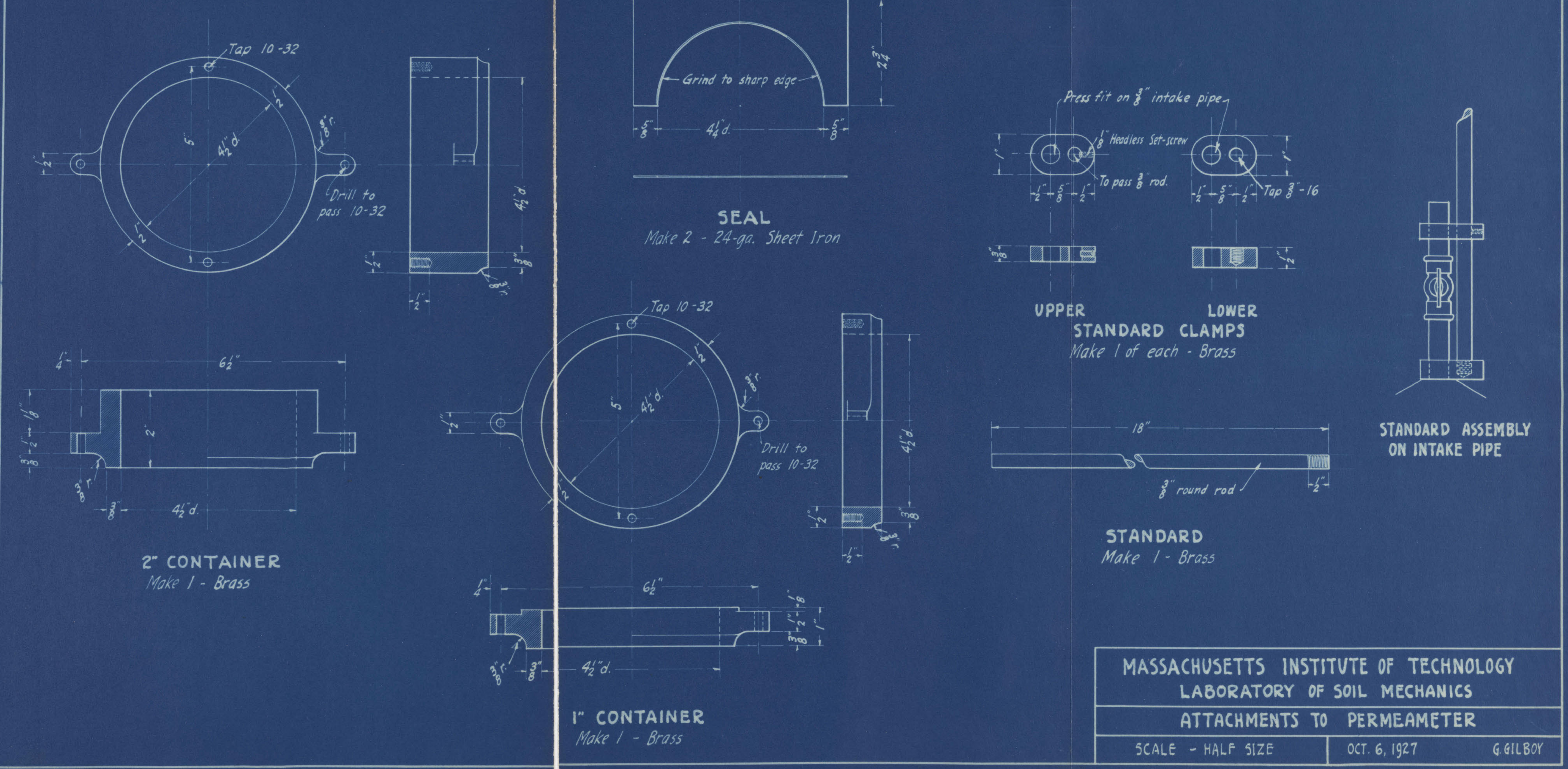
As its dimensions will indicate, this instrument was particularly designed for tests on the undisturbed samples. The original intention was to turn off on a potter's wheel about 1/8" of the paraffin for a length of a little more than 6", in the center of the sample; to place the cylinder of the permeameter around the sample and support it in such a manner that the turned length fell entirely within the cylinder; to fill the 1/8" annular space between cylinder and sample with melted paraffin; and finally to cut off the sample flush with top and bottom of the cylinder, assemble the apparatus, and make the observations.

It became apparent at the very outset that this procedure could not be followed. The paraffin shell was tight against the outer surface of the sample, but there was little or no bond between the soil and the paraffin. As a result,


Tap 10-32, 1/2 deep
NOTE:- Dimension "A" and 3/8" depth controlled by actual finished dimensions of porous disc.
"A" (4 1/2" d. approx.)
5 1/4" d.
7" d.
1/4" drill
① BASE
Make 1 - Brass
Tap 10-32
5 1/2" d.
4 1/2" d.
5 1/4" d.
② CYLINDER
Make 1 - Brass
Drill to pass 10-32
3/8" Pipe Tap
30°
2 1/4"
2 1/2"
5 1/2" d.
③ COVER
Make 1 - Brass
⑤ THUMB SCREW
Make 2 - B=1" and 2 - B=3/4" - Brass
10-32
Tap 6-32
Drill & Countersink to pass 6-32
Drill to pass 10-32
Drill to pass 10-32
5" d.
6 1/2"
④ CLAMP RING
Make 2 - Brass - to fit together
Petcock
3/8" Pipe
5"
⑥ INLET
Make 1 - Brass
Porous Disc
ASSEMBLY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LABORATORY OF SOIL MECHANICS
PERMEAMETER
SCALE - HALF SIZE
SEPT. 23, 1927
G. GILBOY

FIG. 19

FIG. 19A



Tap 10-32
Drill to pass 10-32
2" CONTAINER
Make 1 - Brass
SEAL
Make 2 - 24-ga. Sheet Iron
Grind to sharp edge
Press fit on 3/8" intake pipe
1/8" Headless Set-screw
To pass 3/8" rod.
Tap 3/8"-16
UPPER
LOWER
STANDARD CLAMPS
Make 1 of each - Brass
3/8" round rod
STANDARD
Make 1 - Brass
STANDARD ASSEMBLY ON INTAKE PIPE
1" CONTAINER
Make 1 - Brass
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LABORATORY OF SOIL MECHANICS
ATTACHMENTS TO PERMEAMETER
SCALE - HALF SIZE
OCT. 6, 1927
G. GILBOY

the percolating water soon created a path for itself all around the outside of the sample, and the test was useless.

It was then found that the original paraffin could be completely removed from the sample without injuring or disturbing the soil in the slightest. Lines of weakness in the shell were created by cutting part way through the paraffin with a knife, both horizontally and vertically, and the small squares thus formed were removed without any trouble.

With the paraffin covering removed, it was possible to apply a new thin coat of paraffin to the surface of the sample by means of a paint brush. Experience indicates that this is the only method of using the paraffin which will effectually prevent the water from short circuiting. That it is effective will be learned from the fact that when it is removed, after the test, the inner surface of the paraffin is covered with a thin coating of soil which clings tenaciously to the wax and is very hard to remove without cutting into the paraffin.

The procedure finally adopted is therefore as follows:

(1) The sample is removed from the container, heating the outside of the container if necessary.

(2) The sample is placed on end in a small wooden box, and the permeameter cylinder placed around it. The box is of such a height that with the cylinder resting on the upper edge of the box it encloses the central portion of the sample.

(3) A cut is turned out of the paraffin flush with the top of the cylinder by means of a straight-bladed kitchen knife.

(4) The projecting upper part of the sample is freed by means of a violin wire, and the cut surfaces painted with paraffin.

(5) The cylinder is reversed and the lower end of the sample removed in the same manner. In this case only the cut surface of the unused portion is painted. The cut surface of the permeability sample is allowed to rest on a glass plate.

(6) The cylinder is lifted away from the sample.

(7) Horizontal and vertical lines of weakness are created in the original paraffin shell by cutting part way through with a knife blade. The small sections of the shell are then readily removed.

(8) The entire outside surface of the sample is carefully but rapidly painted with paraffin, care being taken to seal the sample tightly to the glass plate.

(9) The cylinder is replaced around the sample and centered.

(10) The annular space (1/4") between sample and cylinder is filled with melted paraffin, poured in layers not more than 1/4" thick. Each layer is allowed to solidify before the next is poured.

(11) The base of the permeameter is made ready by immersing it in a tank of distilled water, allowing it to rest on the bottom. The porous disc is lowered into the water to drive out the air in the voids, and then fitted into its recess in the base.

(12) When the sealing process is completed, the thin coating of paraffin on the top surface of the sample is removed by cutting it around with a knife and lifting it off. A thin layer of soil will adhere to the paraffin, but with proper care the coating may be removed without injury to the sample.

(13) The glass plate is slid free of the sample. The shearing strength of the bond between paraffin and glass is so low that no trouble is encountered in this operation.

(14) The cylinder is lowered in a horizontal position into the water tank, care being taken that no air is trapped on the surfaces. The tank used in the laboratory is 20" long, 10" wide, and 10" deep. It must in any event be of such dimensions as to permit the entire assembling process to be carried on under water.

(15) The cylinder is placed in position on the base and screwed down.

(16) The cover, with stop-cock open, is immersed slowly, in an inclined position, so as to drive the air out through the inlet pipe. When completely immersed, the stop-cock is closed, unless the depth of water in the tank happens to be great enough to accommodate the entire apparatus.

(17) The cover is lifted over the cylinder, placed in position, and screwed down. If the top of the inlet projects above the water surface, the stop-cock must be closed during the first part of the process. When a sufficient seal has been obtained, the stop-cock is opened, and the screws seated

tightly. The open stop-cock is necessary to avoid the creation of excessive pressure in the water between the top of the sample and the inside of the cover. With reasonably impermeable material, such as that composing the Miami samples, the water level will remain practically stationary at the top of the inlet pipe for a considerable length of time.

(18) The glass standpipe, projecting a short distance through the rubber stopper, is filled by immersion in the tank. A finger is placed over the upper end of the standpipe to hold the water in when the pipe is lifted.

(19) The standpipe is inserted into the top of the inlet, and the stopper pressed in to a tight fit. By proper manipulation, this operation can easily be accomplished without trapping any air in the apparatus.

(20) The apparatus is lifted out of the water and placed on a stand in the tank, the stand being of such a height that the water level in the tank is at about the level of the top of the base. The essential item is to keep the outlet holes always well immersed.

(21) The standpipe is filled to the zero mark by means of a capillary tube with a rubber bulb on the end. These tubes may be easily made in the laboratory by drawing out a short length of 1/4" glass tubing and fitting the bulb of a medicine dropper on the end.

(22) The necessary measurements are made and recorded, as described later, to furnish the constants for computation.

(23) Observations of time and position of water level in the standpipe are made and recorded at intervals as required. It will be found easier to read the time at which the level reaches a certain mark, rather than to try to estimate distances between graduations.

Some supplementary information will be necessary to clarify certain points in the above procedure. It will be better, however, to first develop the theory upon which the computations are based.

Let dQ = quantity of water flowing through sample in time dt.

i = hydraulic gradient through sample at time t.

A = area of sample (perpendicular to direction of flow).

a = area of standpipe.

L = length of sample (in direction of flow).

H = head on sample at time t. (i.e., difference in height between water level in standpipe and water level in tank at time t).

k = coefficient of permeability.

By Darcy's law, $$dQ = k\ i\ A\ dt \qquad (1)$$

The hydraulic gradient may be written

$$i = \frac{H}{L} \qquad (2)$$

So that

$$dQ = \frac{k\ A}{L} H\ dt \qquad (3)$$

This quantity of water flows out of the standpipe in time dt, causing a drop in the water level. If this drop is $- dH$ (the minus sign indicating that a lowering in level corresponds to flow through the sample toward the outlet, which is the case) then

$$dQ = - a\, dH \tag{4}$$

Equating the values for dQ from (3) and (4), and transposing,

$$\frac{a}{H}\, dh + \frac{k\, A}{L}\, dt = 0 \tag{5}$$

Which integrates to

$$a \ln H + \frac{k\, A\, t}{L} = C \tag{6}$$

wherein C is an arbitrary constant.

If the head is H_1 at time t_1, and H_2 at time t_2, substituting these values in (6) and subtracting the two equations gives

$$k = \frac{a\, L}{A(t_2 - t_1)} \ln \frac{H_1}{H_2} \tag{7}$$

The values of head must be expressed in the same units, otherwise the logarithm has no meaning. If the areas are expressed in the same units, then k will be in the units corresponding to L and t. In particular, if L is in centimeters, and t in minutes, k will be in centimeters per minute, which is the usual unit.

Equation (7) makes it possible to compute the coefficient of permeability by the observation on the time interval required for the head to drop a given amount. Actual application of the equation to a given set of computations can be simplified by grouping all the constant terms together in a single constant of the apparatus. The writer also introduced into this constant the value 2.3026, to change the natural logarithm into the common logarithm. Mr. J. V. Davila, in connection with his experiments on the permeability of filter sands performed in the laboratory, developed a nomographic chart for the solution of equation (7). Other simplified methods of computation will suggest themselves to experimenters.

Cylinders one inch and two inches in height were also constructed for the permeameter, in order to make possible the determination of variations in permeability in a given sample. The standpipes used in these tests were a 2 c.c. pipette, area 0.11 sq. cm., for the less permeable samples, and a 10 c.c. burette tube, area 0.482 sq. cm., for those samples where the permeability was so great that the smaller tube emptied too rapidly.

A few words must be said about the use of the filter stone under the sample. In the preceding discussion no mention was made of lost heads in the apparatus, all lost head being considered to be used up in forcing the water through the sample proper. It was originally intended to use a layer of filter paper between the soil and the stone, in order to prevent the fine particles of soil from clogging the voids of the stone. Preliminary experiments on a sample of silt from the foundation of the new Aeronautical Laboratory at M. I. T. were run to determine the effect of the presence of filter paper, as well as any other lost heads in the apparatus. Observations were made under constant head by reading standpipes let into the side of the permeameter cylinder, one an inch from the top and another an inch from the bottom, each standpipe communicating with a thin layer of sand running completely around the inside of the cylinder, the remainder of the space between cylinder and sample being sealed with paraffin. It was found that when the filter paper was used it silted up rather rapidly, so that the lost head in the last inch of flow was about 70 percent of the total drop through the sample, whereas it should have been exactly one-sixth. The filter paper was removed, and the standpipe readings became just as they should have been, namely one-sixth of the total head lost in the first and last inch, and the remaining four-sixths in the intervening four inches. Losses in the stop-cock, the filter stone, etc., were so small that they gave no indication of their presence.

It was feared that this procedure would eventually result in the filling of the pores of the stone with fine particles of soil. The experiments were continued long enough, however, to show that this effect was not appreciable. A certain amount of fine material probably does penetrate in the first few moments; but the soil seems to be able to arch over the voids in the stone, which, though small, are large in comparison with the size of the finest constituents. Thus a small amount of fine material is probably washed out of the very bottom of the sample, whereupon the bottom layer forms a kind of filter, preventing any further leaching of the fine particles. The use of filter paper in connection with porous stones has been entirely discarded in the laboratory as a result of these tests. And the stones, after long continued use, even under pressure in the compression tests, seem to show no ill effects, being just about as permeable at present as they were originally. It is important, of course, to wash each stone out thoroughly after a test by passing through it a current of water at a fairly high velocity in the direction opposite to that in which water flowed during the test.

A characteristic of behavior, rather disconcerting at first, was observed in practically every one of the permeability tests. When the test was first started, the permeability was determined from the first readings and found to be considerably higher than was expected. As time went on, the permeability became lower and lower. It was observed that this lowering was a function of flow, because if the test were stopped in the evening and started again the next morning the last and first values, respectively, agreed closely with one another. It was feared that the porous stone was silting up after all; but subsequent checks showed that this was not the case. The permeability dropped to a value which was only a small fraction of that found at the start, and there remained constant indefinitely.

The obvious explanation of this behavior is that the sample in its original condition is full of minute seams and cracks, which allow a more or less free passage of the water. The velocity of flow is very low, even in this case; but it is evident that this value is not by any means the true permeability of the sample. As time goes on, small amounts of material are washed into the seams, so that they gradually close up. These seams must be very small, otherwise, the flowing water would tend to open them up, as has been noticed in experimenting with natural silts. There is little doubt, however, that the final constant value obtained really represents the true coefficient of permeability of the material. The experimenter can satisfy himself of this by running the test for a long enough time, and observing how the final value remains quite constant.

It is apparent that this unforeseen behavior complicated matters considerably in regard to time. Every permeability test had to be run for about a week, and some of the first for two weeks in order to obtain a conception of the rate of variation. It would be possible to make much better progress on an investigation of this character by having a half-dozen permeameters running simultaneously.

A smaller permeameter which has been used in the laboratory for some time was also employed in certain of these tests, notably for the drillhole samples. The principle of this apparatus is exactly the same as that of the larger instrument so that nothing new would be added by describing it in detail.

<u>2.b. Standard Compression Test</u>. This test is probably the most important of all the tests at present in use, inasmuch as it furnishes a means of determining the constants of the material and its behavior under load. The test was originated by Dr. Terzaghi,* and has been improved continuously until at the present time the apparatus and methods have reached a point where very accurate results can be obtained.

The principle of the test is that of observing the compression of a confined sample of soil under load. The sample is introduced at a consistency at or somewhat above the liquid limit (see <u>d</u> below) into a brass cylinder 7 cm. in diameter. A sketch of the apparatus is shown in Fig. 20. Within the cylinder is a closely fitting piston, which is capable of sliding freely up and down. The cylinder is fastened to a base very similar to that of the permeameter shown in Fig. 19, but smaller, the diameter of the porous stone being equal to the inside diameter of the cylinder. Another porous stone is carried in the bottom surface of the piston. Load is applied by means of a dead-load testing machine, consisting of a lever upon the free end of which weights are hung. The load is transferred to the piston through a steel ball resting in the center of the top of the piston. A yoke on the cylinder carries two 1/10,000-inch Ames dials, diametrically opposed, in reversed position, the ends of the spindles bearing on a crossbar fastened rigidly

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* Erdbaumechanik, p. 82.

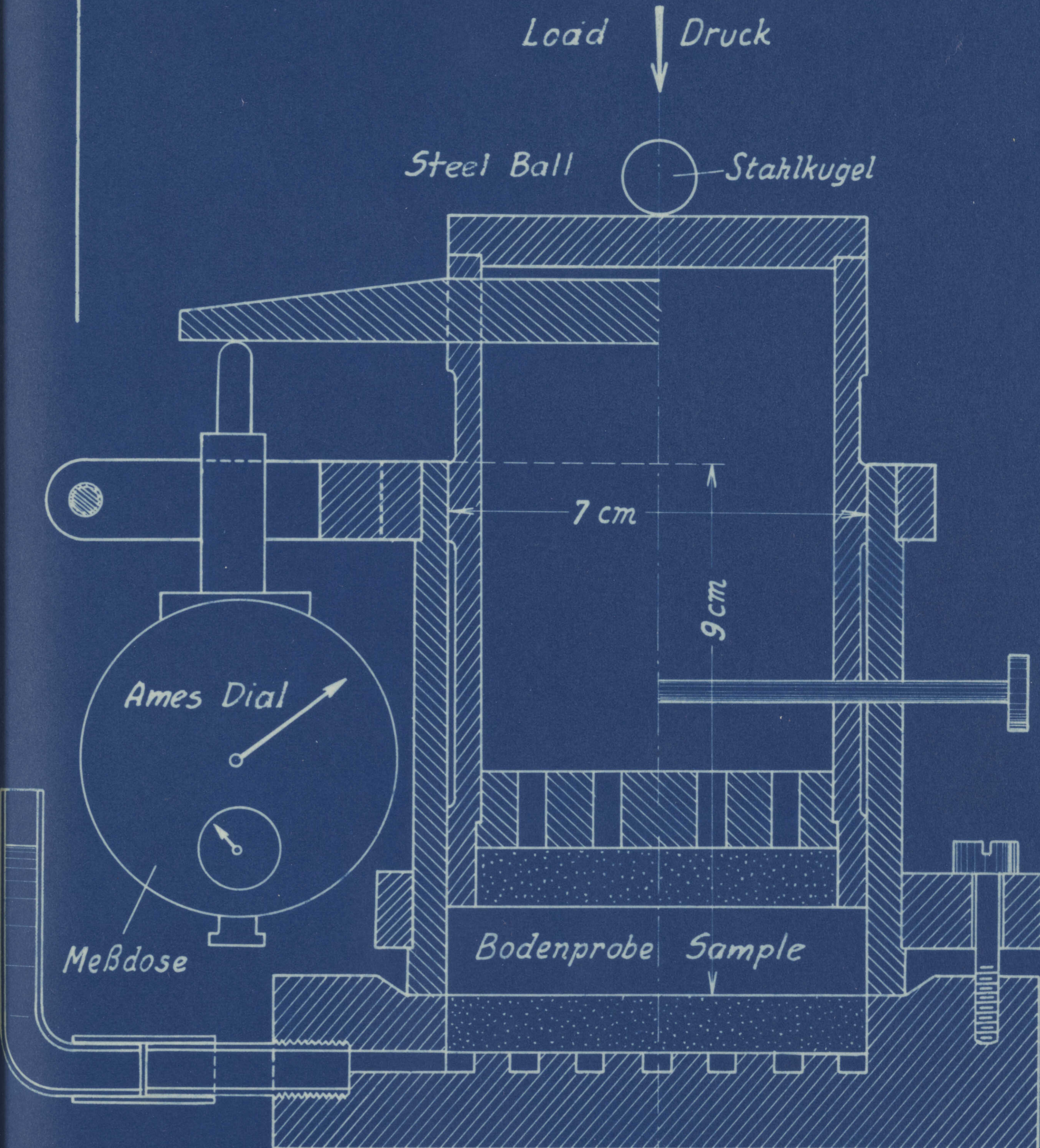
Load
Druck
Steel Ball
Stahlkugel
7 cm
9 cm
Ames Dial
Meßdose
Bodenprobe
Sample

FIG. 20

to the piston. The dials record the up or down movement of the piston with reference to the cylinder, which is equivalent to the compression and expansion of the sample. Two dials are used to permit of a correction for any slight inequality in settlement which may occur. The initial height of the sample is fixed at 1.1 cm. by means of a stop pin which passes in a close fit through holes in the piston and cylinder, and which is removed as soon as the test is ready to begin.

Load is applied in definite increments, as follows: increasing from 0 to 0.4, 0.8, 1.6, and 3.2 kg. per sq. cm. then decreasing by the same increments to zero again. It will be observed that the first zero load is actually zero, since the stop pin is still in position when the dials are set at zero. The last zero load is not really zero, but consists of the weight of the piston plus friction between the piston and cylinder, both of which tend to interfere with the expansion of the sample to the full amount of its elastic energy.

The load steps given above are designated as 0, I, II, III, and IV. Each container and each compression machine has been carefully calibrated, so that the exact value of any of the load increments may be taken from a table on file in the laboratory.

It has been noted that drainage has been provided on the top and bottom of the sample by means of porous stones, which are always kept under a small quantity of free water;

the base has two glass standpipes communicating with the space under the stone, which are kept about half full at all times, while the hollow interior of the piston is kept filled with water up to the level of the stop-pin holes. Therefore, as load is increased the excess water in the sample can freely escape; and as load is decreased the sample can freely soak up water.

When load is first applied, or another increment added, the corresponding compression of the sample does not occur instantaneously, since it takes time for the excess water to escape. The rate at which compression proceeds as a function of time is dependent upon the permeability of the sample. The more permeable the material, the faster will compression proceed. Exactly the same process goes on in this sample as was considered under Case 1 of the previous Section. Therefore, by observing the compression, measured by the dials, as a function of time at constant load, a curve can be constructed which is similar in form to that of Case 1, Fig. 14. Knowing the time scale and the reduced height of the sample (computed from its water content after the test) the coefficient of consolidation may be immediately computed. The method of computing this coefficient is not quite as simple as for the theoretical case of Section I, inasmuch as there is a certain lag due to the internal friction of the material. The procedure has been worked out in detail by Dr. Terzaghi, and is available in published form.*

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* Principles of Final Soil Classification, Public Roads, Vol. 8, No. 3, May, 1927.

The values of voids ratio of the material in a consolidated state are plotted as a function of the corresponding pressure, as will be learned from the curves of the tests included in the next sub-section. These curves fix the value of the coefficient of compressibility for any given pressure range. Knowing the coefficient of compressibility and the coefficient of consolidation, the coefficient of permeability is simply the product of the two.

The direct results of the tests are (1) a curve of variation of voids ratio with pressure, for compression and expansion, and (2) a curve or curves of consolidation under constant pressure as a function of time. The value of the information obtained from this test is manifold. In the first place, it furnishes the coefficient of consolidation and the coefficient of permeability of the material. Furthermore, the voids-ratio pressure curve fixes the equilibrium conditions of the soil. An immediate practical application is that of determining the state of consolidation of a natural sample. If the depth of a sample is known, the pressure acting upon it can be computed, at least approximately. This pressure corresponds to a definite water content, as shown by the pressure-compression diagram. If the water content of the natural sample is greater than this, it indicates that the natural deposit is as yet in an unconsolidated state, and further indicates the degree of consolidation already reached. One test of this type furnishes more real information as to the properties of a given soil than a hundred mechanical analyses.

A simple assembly has been devised whereby direct permeability tests can be run on the sample in the container. A graduated standpipe is attached to the container in such a manner that water from the standpipe enters the base, passes up through the sample, and comes out into the central part of the piston, constant level being automatically maintained at the last point by reason of the stop-pin holes forming an overflow. It is, of course, essential that there be no leakage anywhere in the system, since the permeability is measured by the fall of level in the standpipe as a function of time, similar to the method in a above. This requirement nesessitates the introduction of a packing between cylinder and base, and the test in general is somewhat more complicated and takes a longer time to perform. The direct permeability observations are useful as a check upon the computed permeability, and are absolutely necessary in the case of an incompressible and fairly permeable soil, i.e., a soil whose coefficient of consolidation is unduly large, because in this case the time-compression diagram is not sufficiently accurate to permit of close computation.

2.c. Test of Compressive Strength Unconfined. These tests have been made in the laboratory ever since its inception, and have now become well standardized. The basic principle is the observation of the compression of a cylinder of soil 1" in diameter and 1" high under load. During the test the sample is completely enclosed in a humidifier, in order to prevent evaporation. Load is applied by means of a counterbalanced lever upon which rides a traveling weight. Compression is measured by means of a 1/10,000-inch Ames dial, and is expressed in percent of the original height.

Load is applied in increments at intervals of 30 sec., the increments being such that each addition corresponds to a deflection of about 50 divisions on the dial. Considerable practice is required for the proper manipulation of the load in order to obtain a smooth series of points. Load is increased from zero in four increments, making a total deflection of 200 divisions. Time is allowed at this point for the dial to come to rest. Load is then reduced in four increments to zero, increased again in the same manner, and continued until failure occurs. With plastic materials, there is no actual break, but the test is stopped when the total compression is 20 percent of the original height.

As the sample deforms, its area increases in proportion to its decrease in height. Pressures as plotted are therefore not the pressures read from the machine, but reduced to the average area of the sample at that time. If the pressure per unit of original area is P, then the pressure per unit of average area, p, is

$$p = P\left(1 - \frac{d}{h}\right)$$

where d/h is the compression in percent of original height corresponding to the given pressure. For a 1" sample, d/h is one ten-thousandth of the dial reading. The reduction can be made very simply on a slide rule by setting the right end of the slide at P on the scale and reading the value of d/h backward on the slide, the corresponding value on the scale being p.

The curve of pressure against compression resulting from the observations is continuous except for one hysteresis loop taken at a low pressure. The curve furnishes a value of the ultimate compressive strength, while the modulus of elasticity may be computed from the slope of the axis of the hysteresis loop.

The test was originally intended as a measure of the "consistency" of the material, the stiffer materials having in general the higher compressive strengths and moduli of elasticity. Quite recently, however, a further application of this test has developed. It will be noticed that after a test of this kind the specimen contains more or less well developed shear elements, arising from failure in shear occurring along certain inclined planes. These planes usually occur in pairs, symmetrical about a vertical axis, intersecting at a certain angle. Dr. Joseph Janicsek* has called attention to the fact that a measurement of this angle affords a very close determination of the angle of internal friction of the material, the angle of intersection of the planes being equal to 90° minus the angle of internal friction. The measurement of this intersection angle is now part of the routine of the test. Immediately after the test is completed,

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* Alkalmas-e a Kocka a Törőszilárdság Megállapitására? "Technika", Vol. VIII, Sec. 3-4, Budapest, 1927.

the specimen is placed on a small square glass plate and weighed. Then the intersection angle is measured with a protractor laid out especially for this purpose, recorded, and the angle of friction computed. The angle as thus obtained is not exact, but closely approximate. Investigations are now under way to determine whether modifications of the test can be made which will furnish a still better value of the friction angle.

2.d. Limits of Consistency. As the water content of a clay varies from a high value to a low value (due to evaporation, for instance) the mass changes from a liquid form into a plastic state, from plastic to semi-solid, and from semi-solid to solid. The water content at which these changes take place are known as the Liquid Limit, the Plastic Limit, and the Shrinkage Limit, respectively. These limits have been defined by Atterberg* and are useful in studying the variations in the properties of the soil as the water content varies.

The liquid limit test as originally prescribed by Atterberg consisted of mixing up the material in an evaporating dish, cutting a V-shaped groove in a pat about 1 cm. high, and jarring the dish against the hand. The water content was adjusted by adding water or dry powder, as necessary, until the material flowed together in the bottom of the groove to a height of 1 mm. The water content at this point was defined as the liquid limit.

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* Die Plastizität der Tone. Internationale Mitteilungen für Bodenkunde, 1911, Heft. 1.

This test was obviously rather crude, and results of determinations on the same material by different experimenters were widely different. At the suggestion of Dr. Terzaghi, Mr. Arthur Casagrande, of the Bureau of Public Roads, made an exhaustive study of the determination of this constant. As a result of his researches he has developed a standard method which gives reliable results, and which is now followed. The basis of the test is the same as the original. A standard brass cup is substituted for the dish, and the groove in the sample is made of standard section by means of a special tool. The cup is mounted on a small stand which carries a cam and crank so arranged that for each turn of the crank the cam lifts the cup and lets it drop suddenly through a height of exactly 1 cm. The material is mixed up to about the liquid limit, placed in the machine, the groove formed, and the crank turned until the groove closes over a length of 1 cm. The number of blows and the water content of the material is recorded. Several points are obtained in this way by changing the water content, so that a curve of water content against number of blows may be plotted. The intersection of this curve with the line denoting 25 blows is defined as the liquid limit.

The plastic limit is better defined by the German name "Ausrollgrenze." It is the water content at which the material can no longer be rolled out by hand on a sheet of paper into threads 3 mm. in diameter. It is evident that this test must be started at a water content slightly higher than the limit, allowing the rolling to bring the water content down to the desired value.

The shrinkage limit is the water content at which the material ceases to shrink on evaporation. It corresponds to the color change caused by the recession of the surface water into the voids. Below the shrinkage limit, the voids of the soil are no longer entirely filled with water. Atterberg's original method of determination was to measure the approach of two sharp lines on a small prismatic sample, weighing the sample after each measurement. This procedure is very tedious, and not particularly accurate. The method used in the laboratory is that recommended by Dr. Terzaghi, namely, the determination of volume and weight of a dried sample of the soil, its specific gravity being known. If V_o is the dry volume, and W_o the dry weight, then the shrinkage limit is

$$\frac{V_o}{W_o} - \frac{1}{s}$$

wherein s is the specific gravity of the dry matter. The volume determination is made by mercury immersion, usually on a sample of soil which has been subjected to the confined compression test, as these samples are in the form of a round thin disc and are especially suitable for volume determinations.

<u>2.e. Mechanical Analysis</u>. The determination of the distribution of the particles with respect to size is made according to the method devised by Prof. G. Wiegner.* The basic principle is the utilization of Stokes' Law of the velocity of sinking particles, and the measurement of the difference in density between clear water and a suspension of soil in water. The apparatus consists of a glass tube 2" in diameter, 44" high, closed at the bottom, with a standpipe 1/2" ~~high~~ in diameter close beside the tube, the standpipe being connected through a stopcock to the tube at a point about 10" from the bottom. The tube is filled with a suspension of soil and water, about 60 grams of dry soil being used. The mixture is thoroughly shaken to obtain uniform dispersion. The standpipe is filled with distilled water, and the stopcock closed during the shaking operation. When the suspension has been thoroughly mixed, the apparatus is hung is a vertical position, stoppers removed from the tube and the standpipe, and the stopcock opened. Due to the fact that the suspension above the point of junction of tube and standpipe is heavier than pure water, the water in the standpipe stands at a somewhat higher level (about 8 mm.) than that in the tube. As time goes on, the particles of soil in the tube gradually settle out, so that the difference in level of the water

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* Ueber eine neue Methode der Schlämmanalyse. Zentralblatt für die gesamte Landwirtschaft, Band I, No. 1, 1920. A brief but comprehensive discussion of the theory and principle is included as an appendix to Terzaghi's Erdbaumechanik.

surfaces gradually decreases, becoming zero when the suspension has entirely cleared. The difference in level is observed as a function of time and a curve plotted with time as abscissae and level differences in percent of total as ordinates.

If H denoted the height of the water (suspension) in the tube above the junction of the standpipe, then the time t_1 required for a particle of diameter d_1 to sink through a height H can be computed by Stokes' law. It can be shown theoretically* that if a point is taken on the Wiegner curve with an abscissa t_1, and a tangent drawn to the curve at this point, the tangent will intersect the vertical coördinate axis in a point which corresponds to the percentage of the total sample smaller than d_1. The distribution curve can then be constructed on the same sheet as the Wiegner curve, by taking different values of the grain diameter and determining the percentages by tangents to the Wiegner curve at the proper points.

The apparatus as originally constructed was sound in principle, but possessed one marked defect. On account of the small difference in level between the water surfaces in tube and standpipe, it is impossible to measure the difference with a sufficient degree of accuracy. And inasmuch as the entire test depends directly upon the curve derived from this measurement, the results were anything but accurate. The same sample could be shaken up three times, and three different distribution curves would be obtained. Unless some way were discovered to remedy the difficulty, the apparatus was practically useless.

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* Terzaghi, Erdbaumechanik. Appendix.

The writer finally hit upon a remedy which seems to have solved the problem. Briefly, it is nothing more than magnifying the level difference by means of an inclined graduated glass tube. The tube, a 25 c.c. burette 18" long, is fixed rigidly to the wall near the hook from which the apparatus hangs. The tube is on a slope of about 1:20 (more exactly, 1:20.8) so that the level differences are magnified over twenty times. The tube is placed with the graduations uppermost, as the water at this point forms a sharp, distinct line, which can be easily read. The lower end of the tube is connected by a rubber hose with the upper end of the standpipe, tube, hose, and standpipe being filled with distilled water. A stopcock is introduced at the lower end of the burette to kill the water hammer induced by shaking the apparatus before the test.

Results of tests performed in the modified apparatus check with gratifying precision. As it now stands, the system is practical and useful. Certain improvements, however, are necessary before a maximum of utility is attained. At present it is necessary to guard against any disturbance of the sedimentation tube or the hose, as the measuring tube is sensitive to a marked degree. Furthermore, the greatest of care must be used to guard against bubbles of air collecting in the bottom of the standpipe during the shaking, and finding their way into the distilled water line when the stopcocks are opened. The writer intends to design an apparatus which will eliminate these complications, and to publish an account of his findings.

<u>2.f. Specific Gravity Tests</u>. A very important constant of a soil is the specific gravity of the solid material. With this value known, the weight per unit of volume of the soil with any water content can be easily determined. Many methods have been tried, with more or less success, the best know of which is probably the use of the pycnometer bottle, whereby the specific gravity is measured by displacement of water or other liquid. The pycnometer method, especially as modified by Dr. Terzaghi, gives good results for fairly coarse materials. But it does not seem to work satisfactorily with the finer soils.

The writer, in collaboration with Mr. Arthur Casagrande, made a series of experiments on the determination of specific gravity by determining the water content of a known volume of saturated material. It was found that the results obtained in this manner were quite consistent within themselves; the method was therefore adopted, and is now used in all cases for fine-grained, saturated soils.

The test is performed in a Coors porcelain capsule, 25 cc. capacity. The weight of the capsule is determined, and its volume measured by filling with mercury. The soil is placed in a soft plastic condition into the capsule, and the top surface smoothed over with a spatula so that it is flush with, or slightly below, the ground edge of the capsule. The capsule is then immersed in clean water, and covered with a small square of glass of known weight. If properly manipulated, the glass can be lifted out of the water with the capsule adhering to its lower surface. The exposed faces of capsule and plate are carefully dried, and the weight determined. The capsule is then dried in the oven at 105° to constant weight, and the dry weight determined. The weight of capsule and plate being known, it is possible to determine the wet and dry weight of the sample.

Let W = weight of sample wet, in grams

W_o = weight of sample dry, in grams

V = volume of capsule = volume of sample wet, in c.c.

$W - W_o$ is evidently the weight of the water in the wet sample, which is also equal to the volume of the water. The volume of the solid must therefore be

$$V - (W - W_o)$$

The weight of solid is W_o. Therefore the specific gravity, or weight per unit volume, is

$$s = \frac{W_o}{V + W_o + W}$$

It will be observed that if the wet sample is not exactly flush with the top of the capsule, the result is not affected provided the remaining space is filled with water. The essential point is that the measured volume of the capsule must be filled with soil and water. The proportion of one to the other is in general immaterial. The greatest accuracy is obtained by making the quantity of solid as great as possible, consistent with ease of manipulation.

<u>2.g. Natural Water Content Determination.</u> In common with water content determinations incident to other tests, such as liquid and plastic limit tests, the determination of the natural water content of a soil sample is made by weighing the sample accurately and drying in an oven at 105° C. to constant weight, then weighing again. The water content is the loss of weight on drying expressed as a percentage of the dry weight. The laboratory is equipped with a number of 50 mm. and 65 mm. watch glasses, in pairs, with ground edges to fit closely together, and a clip for each pair to hold the glasses in place while weighing. These glasses are necessary to prevent evaporation of the water while the first weighing is made. A sensitive balance, accurate to one ten-thousandth of a gram, is used for all weights, although the nearest thousandth is usually close enough for practical purposes. The drying oven is regulated to keep a minimum temperature of 105° C. and a maximum of 110° . Closer control than this is unnecessary.

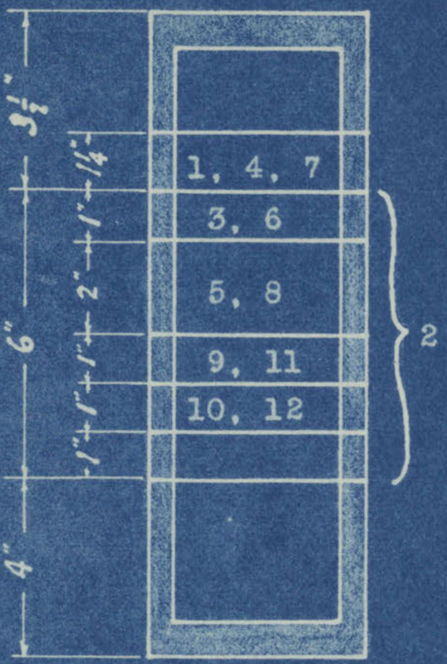
II. B. TEST DATA AND RESULTS.

In the laboratory files all data sheets pertaining to a specific test are bound together, the first sheet at the bottom. Above these are bound the plot or plots, if the test is such that a plot is obtained. On the top is bound a summary sheet, upon which is written in brief all the information afforded by the test. In this manner it is very easy to run through a whole series of tests and obtain at a glance all the essential results. Copies of the summary sheets and of the plots pertaining thereto are included in the following pages. Preceding the group of summaries and plots for each sample will be found a schedule sheet, upon which the tests are enumerated and their position in reference to the original sample indicated in a sketch.

The sheets are arranged in order of sample numbers and test numbers, irrespective of the type of test. In later pages the results of the same type of test on different samples have been grouped together for easy reference and comparison.

30-1-2H

SCHEDULE SHEET



1. Compressive Strength, Natural State. Modulus of Elasticity. (3 tests). Water Content (5 tests).

2. Permeability, undisturbed.

3. Permeability, undisturbed.

4. Combined Compression and Permeability, remolded. Specific Gravity (6 tests). Shrinkage Limit.

5. Permeability, undisturbed. Specific Gravity (5 tests).

6. Mechanical Analysis, Wiegner Method. Specific Gravity (4 tests).

7. Liquid and Plastic Limits (9 and 2 tests, respectively).

8. Combined Compression and Permeability, undisturbed after Permeability Test #5.

9. Permeability, Undisturbed.

10. Permeability, Undisturbed.

11. Combined Compression and Permeability, undisturbed after Permeability Test #9.

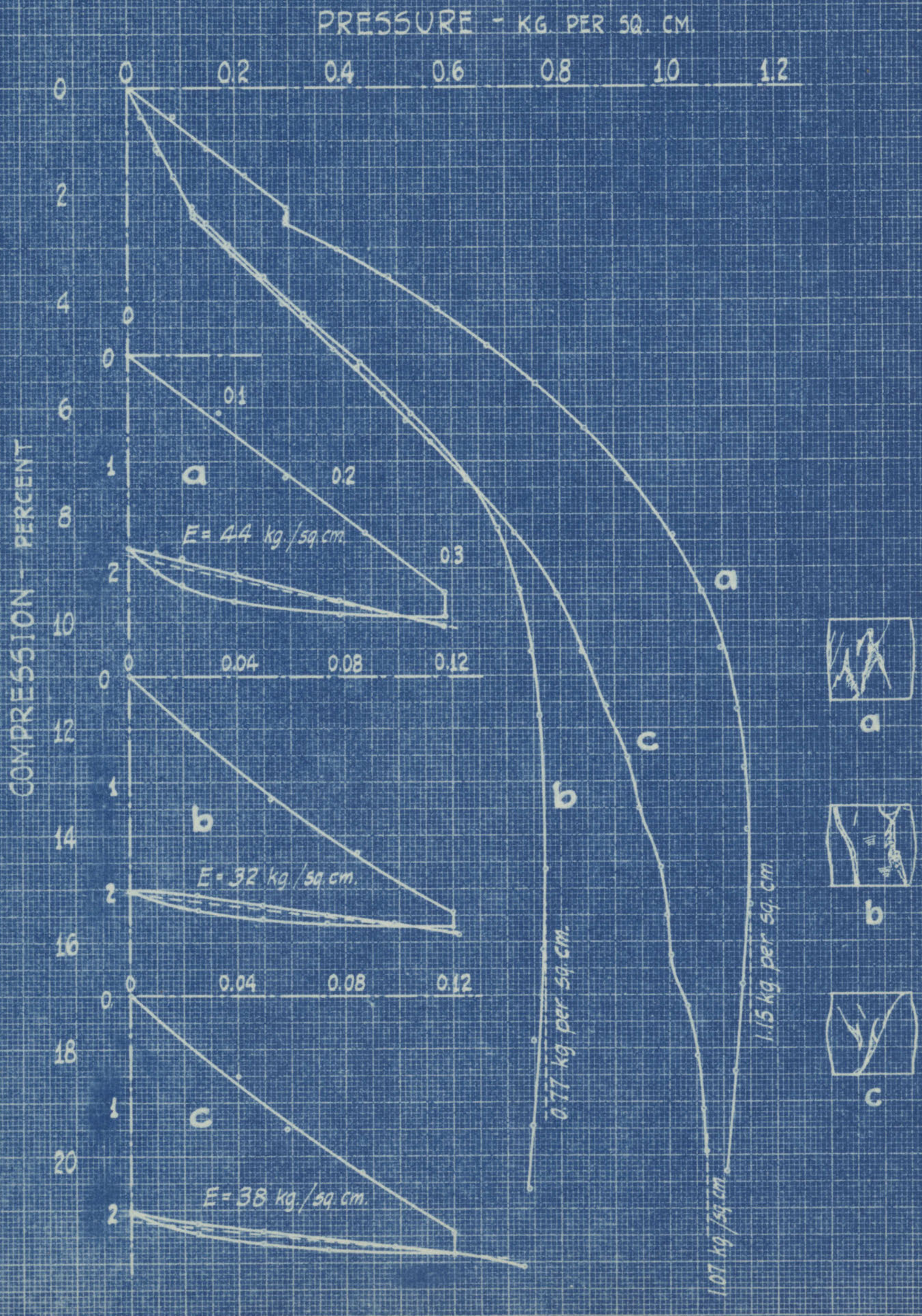
12. Permeability, remolded.

30-1-2H-1

SUMMARY SHEET

2/6/28. Compressive Strength in Natural State.
Modulus of Elasticity (3 tests, a, b, and c)
Water Content Determinations (5 tests)

	a	b.	c.	Average
Comp. Str. Kg./sq.cm.	1.15	0.77	1.07	1.00
Mod. of E. " " "	44	32	38	39
Water Content	24.9	27.4	25.4	
Water Content, surrounding material	27.1, 26.0			26.2


PRESSURE - KG. PER SQ. CM.
COMPRESSION - PERCENT
E = 44 kg./sq.cm.
E = 32 kg./sq.cm.
E = 38 kg./sq.cm.
0.77 kg per sq.cm.
1.07 kg/sq.cm
1.15 kg per sq.cm.
a
b
c

30-1-2H-2

SUMMARY SHEET

2/8/28. Permeability Test, undisturbed.

6" Sample from center of container.

Coefficient of Permeability (minimim) 350×10^{-7} cm./min.

Note: This sample cut into slices for subsequent tests.
From results of later tests, above value of k is too high.
Run not continued long enough. Water temperature 20° C.

30-1-2H-3

SUMMARY SHEET

2/9/28. Permeability, undisturbed.

1" slice from top of 2.

$k = 87 \times 10^{-7}$ cm./min. at 20° C.

Water Content at end of Test.

				Average
Upper Third	27.6	26.1	25.8	26.5
Middle Third	28.7	26.6	26.0	27.1
Lower Third	29.1	26.7	26.3	27.4
Average of whole sample				27.0

Run - 6 days.

30-1-2H-4

SUMMARY SHEET

2/13/28. Combined Compression and Permeability.

Specific Gravity. Shrinkage Limit.

Material from 1, molded from natural state.

From Compression Test.

$c_1 = 124$

$c_2 = 12.3$

k at 1.5 kg./sq.cm. (e = 0.654), 21° C.

14.3×10^{-7} cm./min.

From Permeability Test.

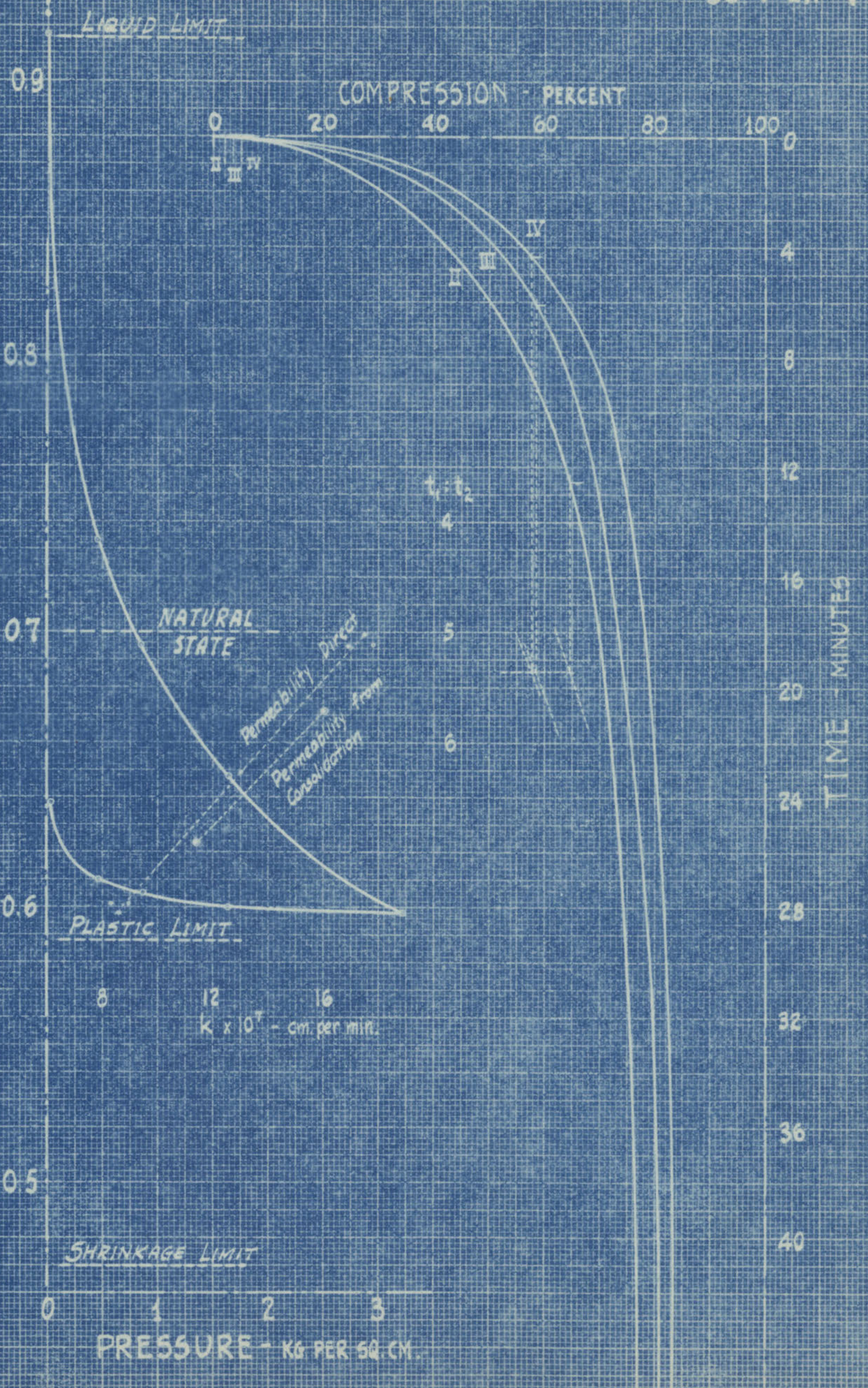
k as above, 13.5×10^{-7} cm./min.

Liquid Limit	33.7)	from #7.
Plastic Limit	21.6)	
Shrinkage Limit	17.3	

Specific Gravity (6 tests) 2.72

Coefficient of Consolidation

Range of Voids Ratio	c.
0.733 - 0.695	0.0077
0.695 - 0.648	0.0161
0.648 - 0.598	0.0221



LIQUID LIMIT
0.9
COMPRESSION - PERCENT
0
20
40
60
80
100
II III IV
IV
III
II
0
4
0.8
8
12
$t_1 : t_2$
4
VOIDS RATIO
16
NATURAL STATE
0.7
5
Permeability Direct
Permeability from Consolidation
20
TIME - MINUTES
6
24
0.6
28
PLASTIC LIMIT
8
12
16
$k \times 10^7$ - cm per min.
32
36
0.5
40
SHRINKAGE LIMIT
0
1
2
3
PRESSURE - KG PER SQ. CM.

30-1-2H-5

SUMMARY SHEET

2/15/28. Permeability, undisturbed. Specific Gravity.

2" slice from 2, below 3.

$k = 109 \times 10^{-7}$ cm./min. at 21° C.

Water Content after Test - 26.3

Specific Gravity (5 tests) - 2.675

Run - 4 days.

30-1-2H-6

SUMMARY SHEET

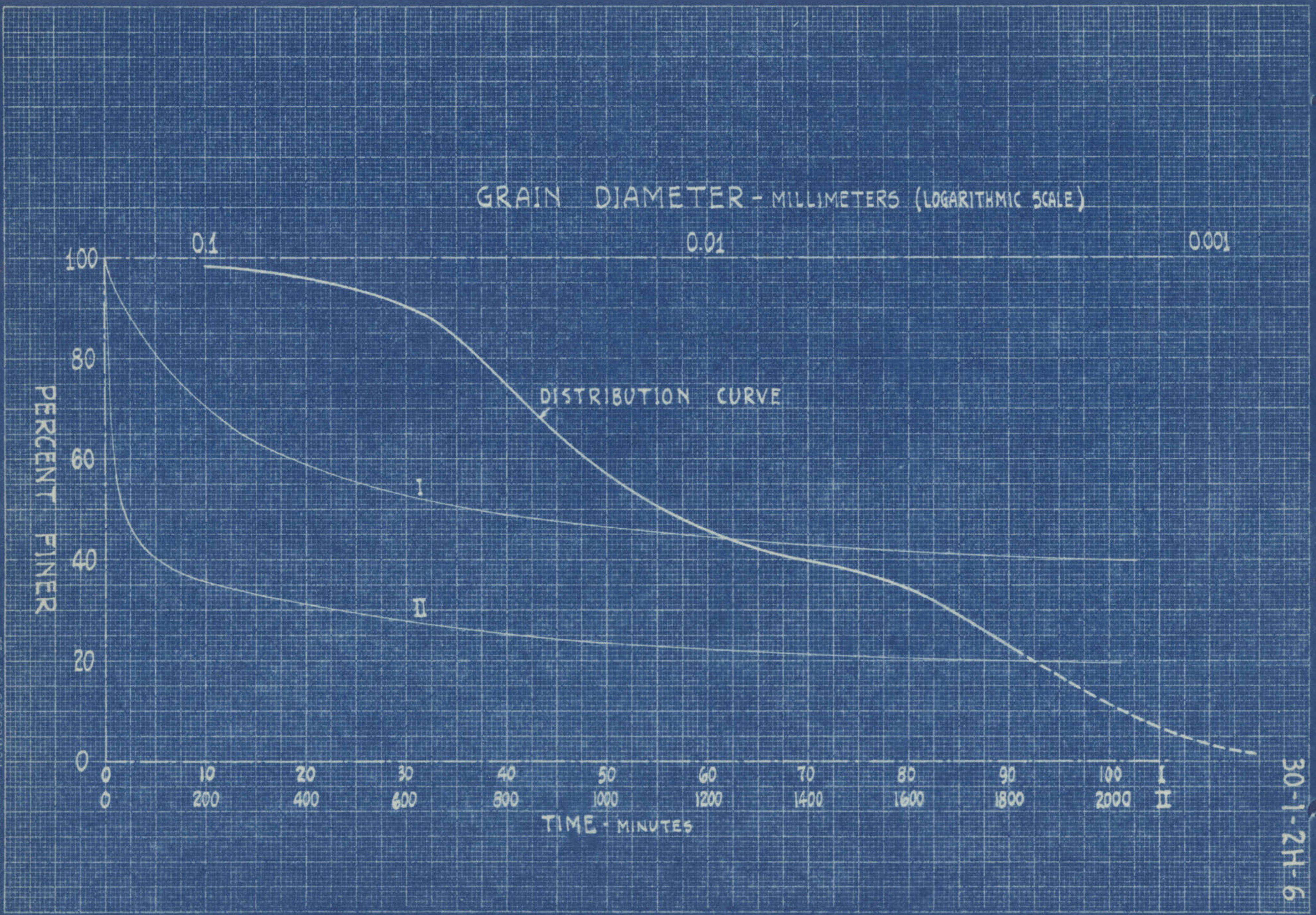
2/18/28. Mechanical Analysis (Wiegner) of material from 3. Specific Gravity.

Effective Size - 0.0015 mm. (approximately)

Uniformity Coefficient - 12 (approximately)

Note: Value of diameter at 10 percent obtained by extrapolation. Test not continued long enough to reach this point.

Specific Gravity (4 tests) - 2.67


GRAIN DIAMETER - MILLIMETERS (LOGARITHMIC SCALE)
0.1
0.01
0.001
100
80
60
40
20
0
PERCENT FINER
DISTRIBUTION CURVE
I
II
0 10 20 30 40 50 60 70 80 90 100 I
0 200 400 600 800 1000 1200 1400 1600 1800 2000 II
TIME - MINUTES
30-1-2H-6

30-1-2H-7

SUMMARY SHEET

2/23/28. Liquid and Plastic Limits (Casagrande)

Material from 4.

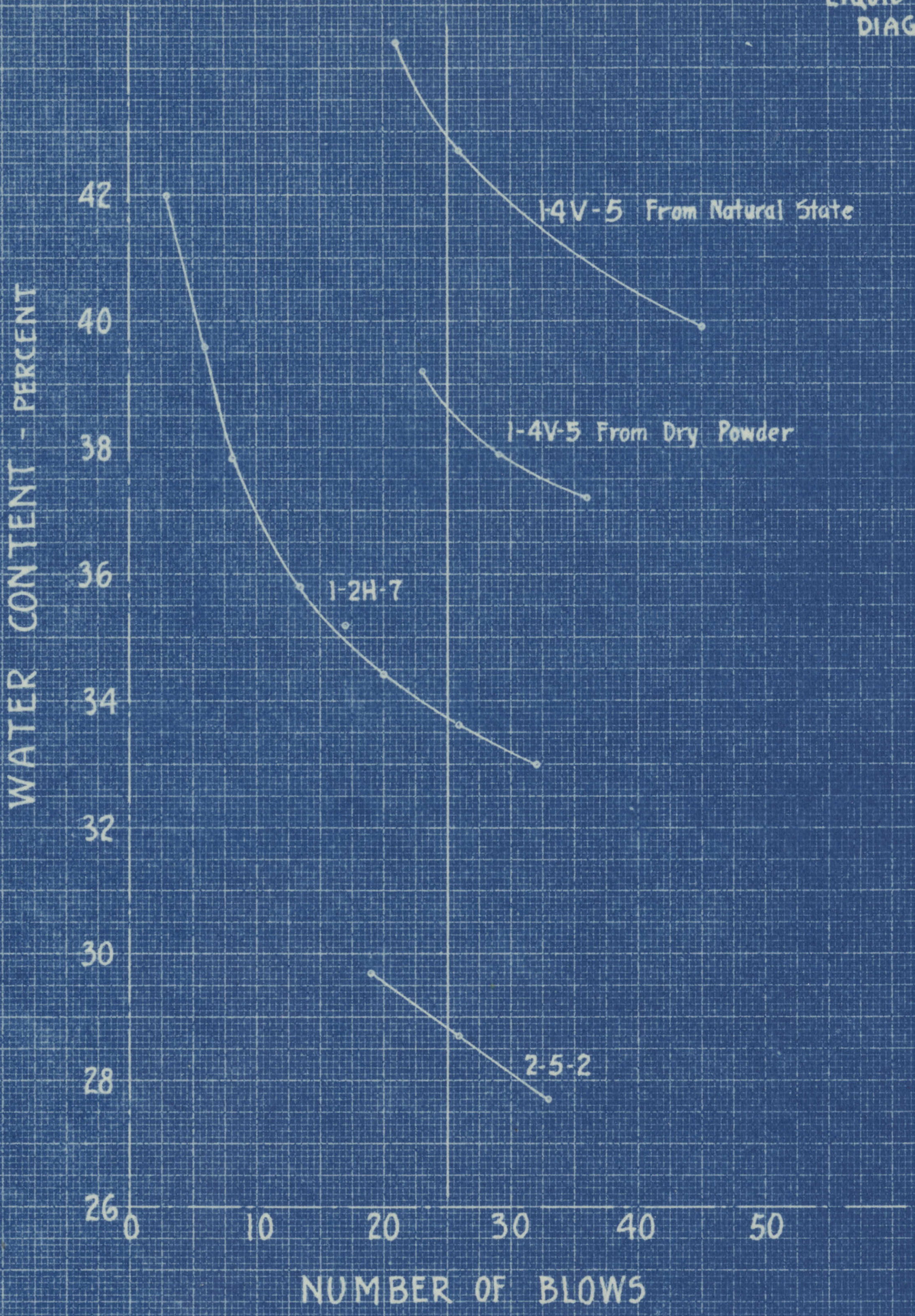
Liquid Limit = 33.7

Plastic Limit - 21.6

Plasticity Index - 12.1

Color - Wet: Olive Brown
Dry: Clay Color.

LIQUID LIMIT
DIAGRAMS



30-1-2H-8

SUMMARY SHEET

2/24/28. Combined Compression and Permeability.
Material from 5, undisturbed.

Compression Test worthless.. Seems to have been soil particles squeezed between piston and cylinder.

Shrinkage Limit - 16.4

Specific Gravity - 2.67

From Permeability Test, k varies with voids ratio as follows:

e	$k \times 10^7$ cm./min.
0.695	54.4 (average of 3)
0.663	40.1 (average of 4)

30-1-2H-9

SUMMARY SHEET

2/24/28. Permeability, undisturbed.

1" slice from 2, below 5.

$k = 124 \times 10^{-7}$ cm./min. at 21° C.

Water Content after Test - 26.2

Run - 4 days.

30-1-2H-10

SUMMARY SHEET

2/29/28. Permeability, undisturbed.

1" slice from 2, below 9.

$k = 96 \times 10^{-7}$ cm./min. at 19° C.

Water Content after Test - 26.3

Run - 4 days.

30-1-2H-11

SUMMARY SHEET

2/28/29. Combined Compression and Permeability.

Material from 9, placed undisturbed in container.

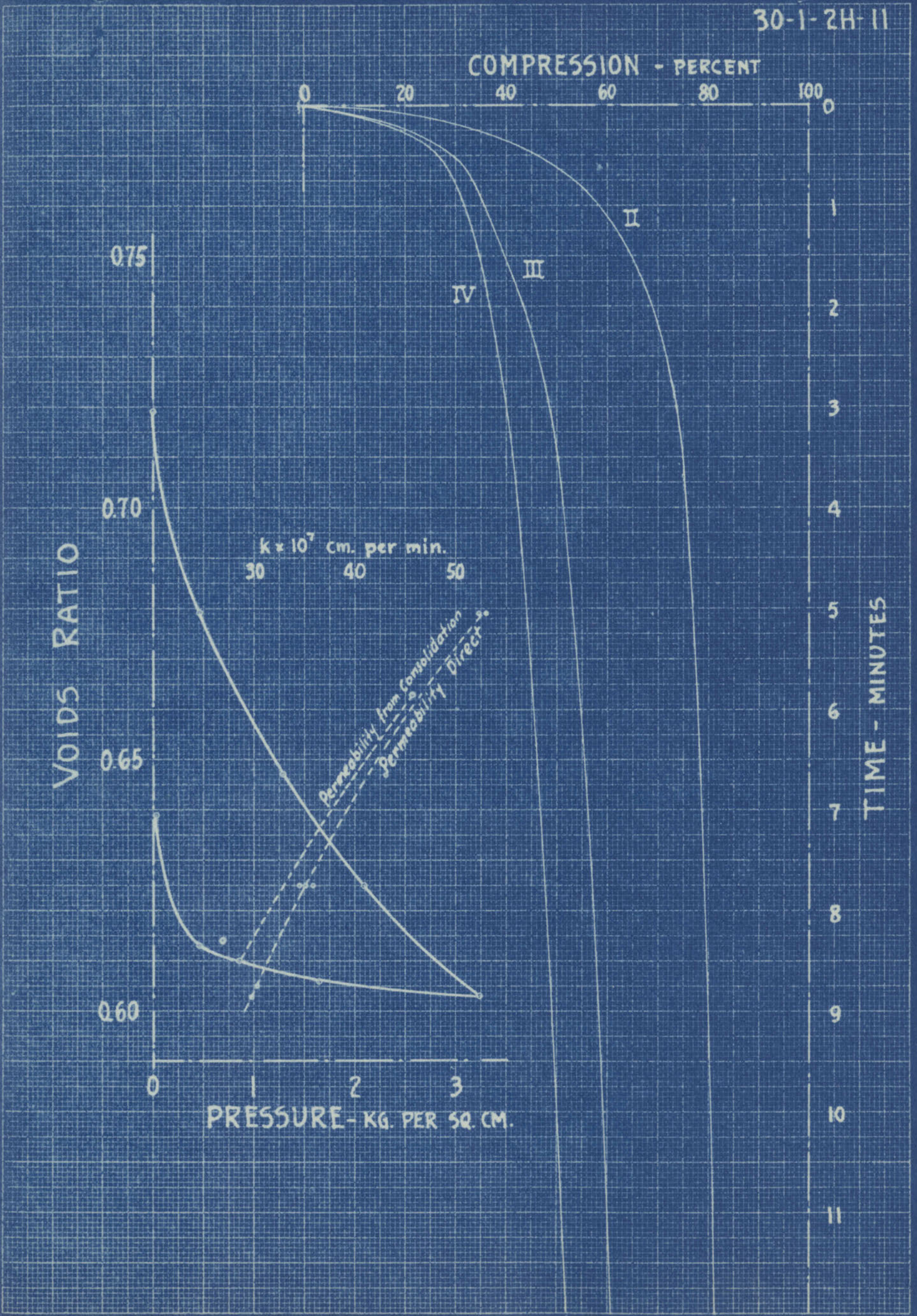
From Permeability Test, k varies as follows:

e	10^7 k, cm./min.
0.604	30.0
0.625	35.0
0.647	41.5
0.679	52.5

Coefficient k determined from consolidation curves agrees well with above values (see plot).

This test intended to show how permeability is affected by closing of drainage channels which probably exist in sample. Permeability of test 9 was 124×10^{-7} cm./min. Pressure of 0.4 kg./sq.cm. caused k to drop to 52.5×10^{-7}.

Coefficients of Pressure-Voids Ratio diagram meaningless, as original pressure not really zero.


COMPRESSION - PERCENT
0
20
40
60
80
100
0
1
2
3
4
5
6
7
8
9
10
11
TIME - MINUTES
II
III
IV
VOIDS RATIO
0.75
0.70
0.65
0.60
k x 10⁷ cm. per min.
30
40
50
Permeability from Consolidation
Permeability Direct
0
1
2
3
PRESSURE - KG. PER SQ. CM.

30-1-2H-12

SUMMARY SHEET

3/7/28. Permeability, remolded.

Material from 10.

$k = 44 \times 10^{-7}$ cm./min. at 21° C.

Water Content after Test - 29.2

Run - 8 days.

30-1-2V

SCHEDULE SHEET

NOTE: Slice for Test #1 distinctly stratified on planes sloping about 1:3.4. Top and central portion firm plastic clay, upper layer containing distinct band of black organic matter. Between firm layers a thin layer of soft, lighter colored clay. At bottom of slice a thin layer of sandy material.

1. Compressive Strength, Natural State. Modulus of Elasticity. (2tests). Water Content (5 tests).

2. Permeability, undisturbed.

30-1-2V-1

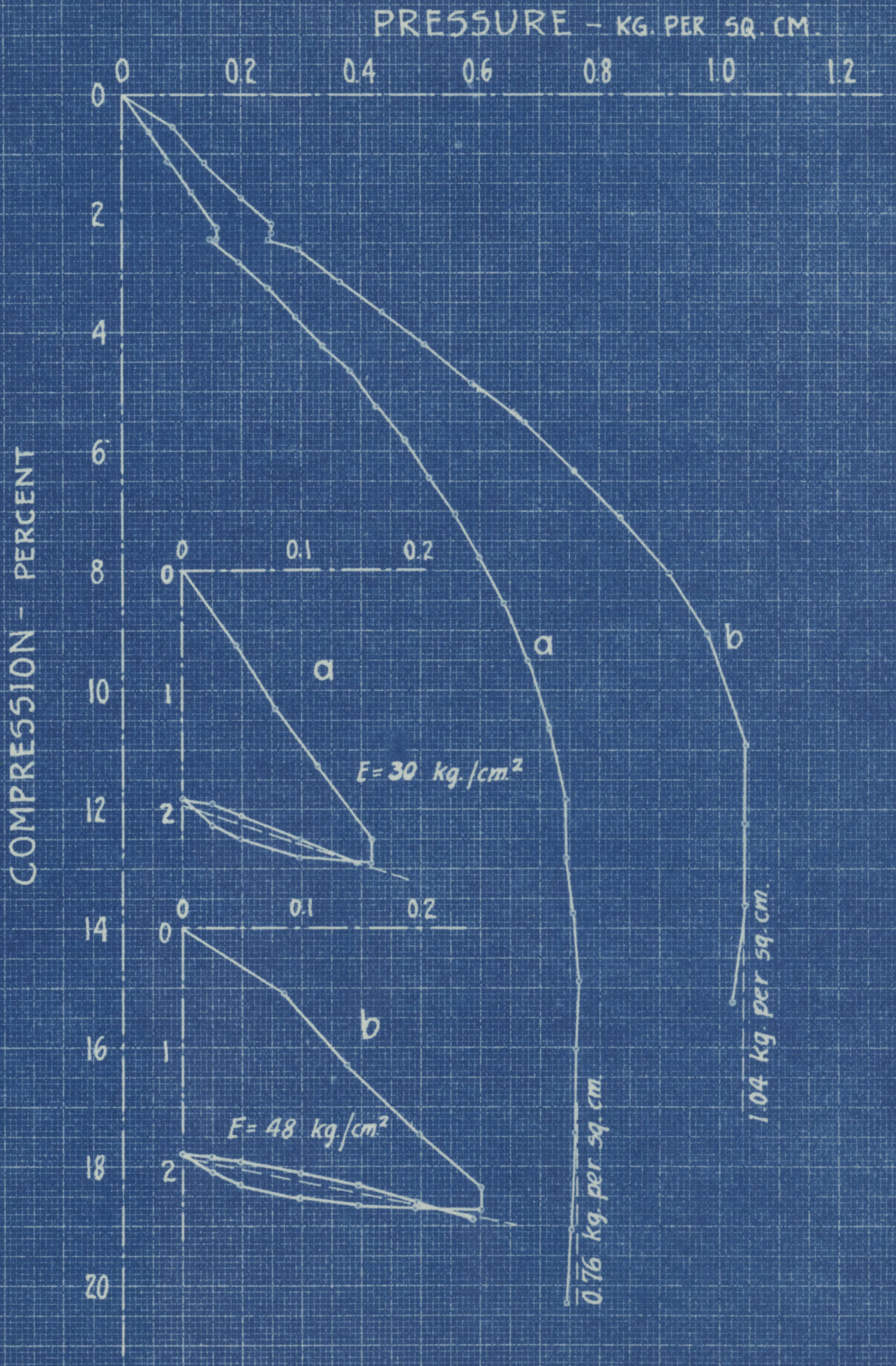
SUMMARY SHEET

3/6/28. Compressive Strength. Natural State. (2 tests)

	a	b
Compressive Strength, kg./sq.cm.	0.76	1.04
Modulus of Elasticity, " " "	30	48

Water Content, Natural State, 26.7

Sandy Layer 30.4


PRESSURE - KG. PER SQ. CM.
0
0.2
0.4
0.6
0.8
1.0
1.2
COMPRESSION - PERCENT
0
2
4
6
8
10
12
14
16
18
20
a
b
0.76 kg. per sq. cm.
1.04 kg. per sq. cm.
0
0.1
0.2
0
1
2
a
E = 30 kg./cm²
0
0.1
0.2
0
1
2
b
E = 48 kg./cm²

30-1-2V-2

SUMMARY SHEET

3/8/28. Permeability, undisturbed.

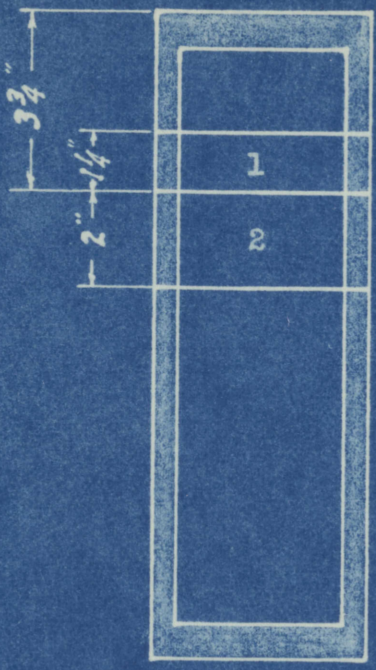
6" specimen from center of sample.

$k = 400 \times 10^{-7}$ cm./min. at 21° C.

Run - 14 days.

30-1-4H

SCHEDULE SHEET



1. Compressive Strength, undisturbed (2 tests) and remolded (2 tests). Modulus of Elasticity. Angle of Internal Friction. Water Content (6 tests).

2. Permeability, undisturbed.

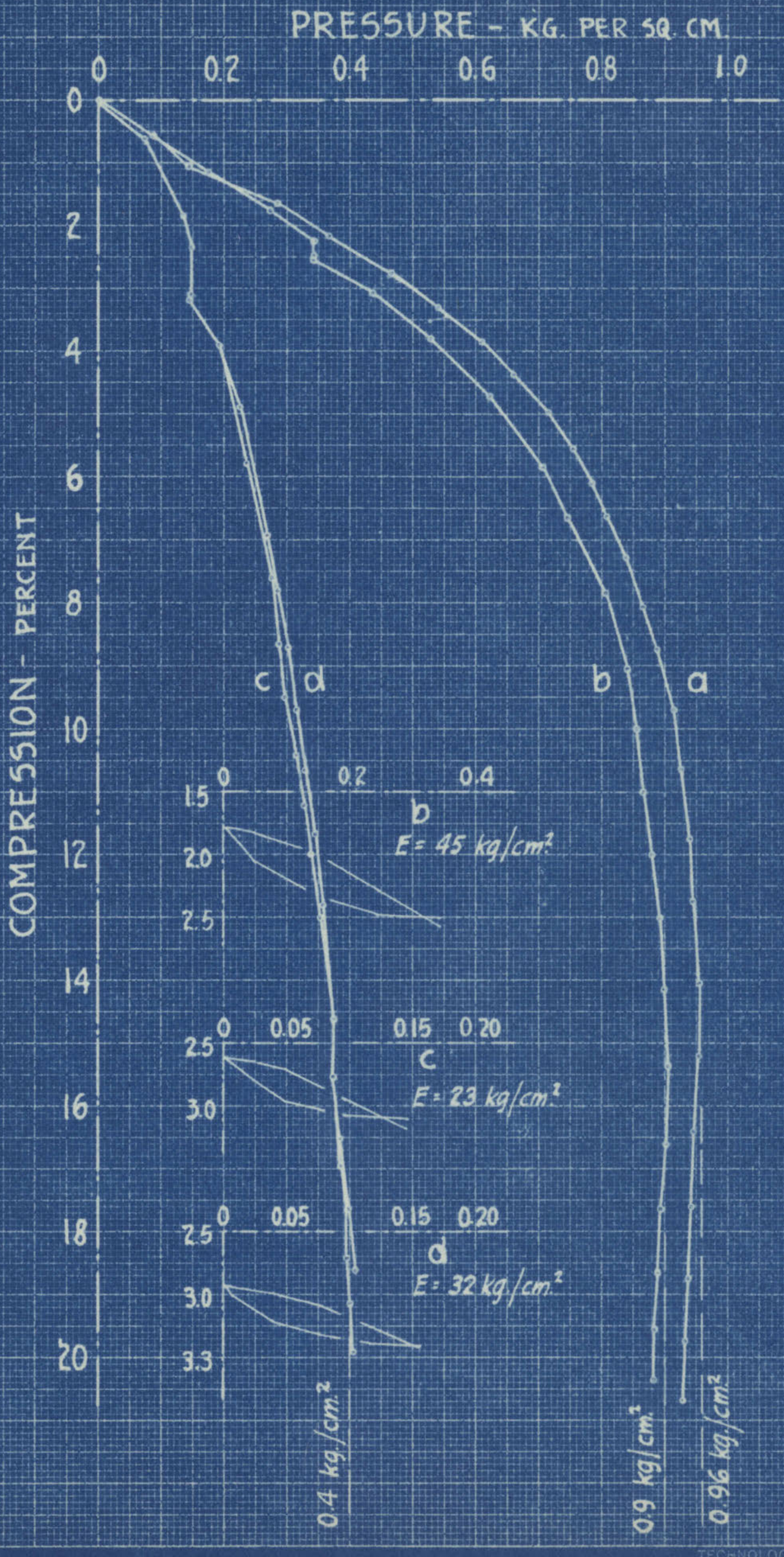
30-1-4H-1

4/17/28. Compressive Strength, Natural and Remolded.

	a	b	c	d
	Natural	Natural	Remolded	Remolded
Comp. Str., kg./sq.cm.	0.96	0.90	0.40	0.40
Mod. Elas., kg./sq.cm.		45	23	32
Water Content	34.2	34.2	27.4	29.3
Friction Angle, degrees	25	25	27	24

Water Content, surrounding material, 31.1, 34.2

Average of 6, 32.1


PRESSURE - KG. PER SQ. CM.
0
0.2
0.4
0.6
0.8
1.0
COMPRESSION - PERCENT
0
2
4
6
8
10
12
14
16
18
20
c
d
b
a
0
0.2
0.4
1.5
2.0
2.5
b
E = 45 kg/cm²
0
0.05
0.15
0.20
2.5
3.0
c
E = 23 kg/cm²
0
0.05
0.15
0.20
2.5
3.0
3.3
d
E = 32 kg./cm²
0.4 kg./cm.²
0.9 kg/cm.²
0.96 kg./cm²

30-1-4H-2

SUMMARY SHEET

4/20/28. Permeability, undisturbed.

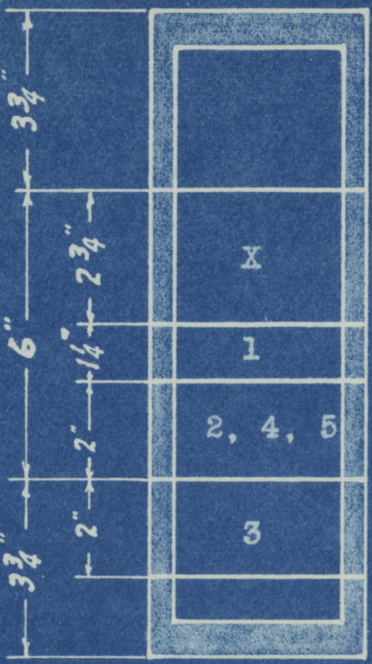
2" slice below 1.

$k = 270 \times 10^{-7}$ cm./min.

Run - 5 days.

30-1-4V

SCHEDULE SHEET



NOTE: Sample a very stiff, hard clay, apparently uniform and homogeneous. Section marked X contained several horizontal cracks and other irregularities which caused it to be discarded.

1. Compressive Strength, Modulus of Elasticity, Angle of Internal Friction, natural state (3 tests) and remolded (2 tests). Water Content (6 tests).

2. Permeability, undisturbed.

3. Permeability, undisturbed.

4. Mechanical Analysis, Wiegner Method.

5. Combined Compression and Permeability. Specific Gravity (4 tests). Liquid and Plastic Limits from natural state and from dried powder.

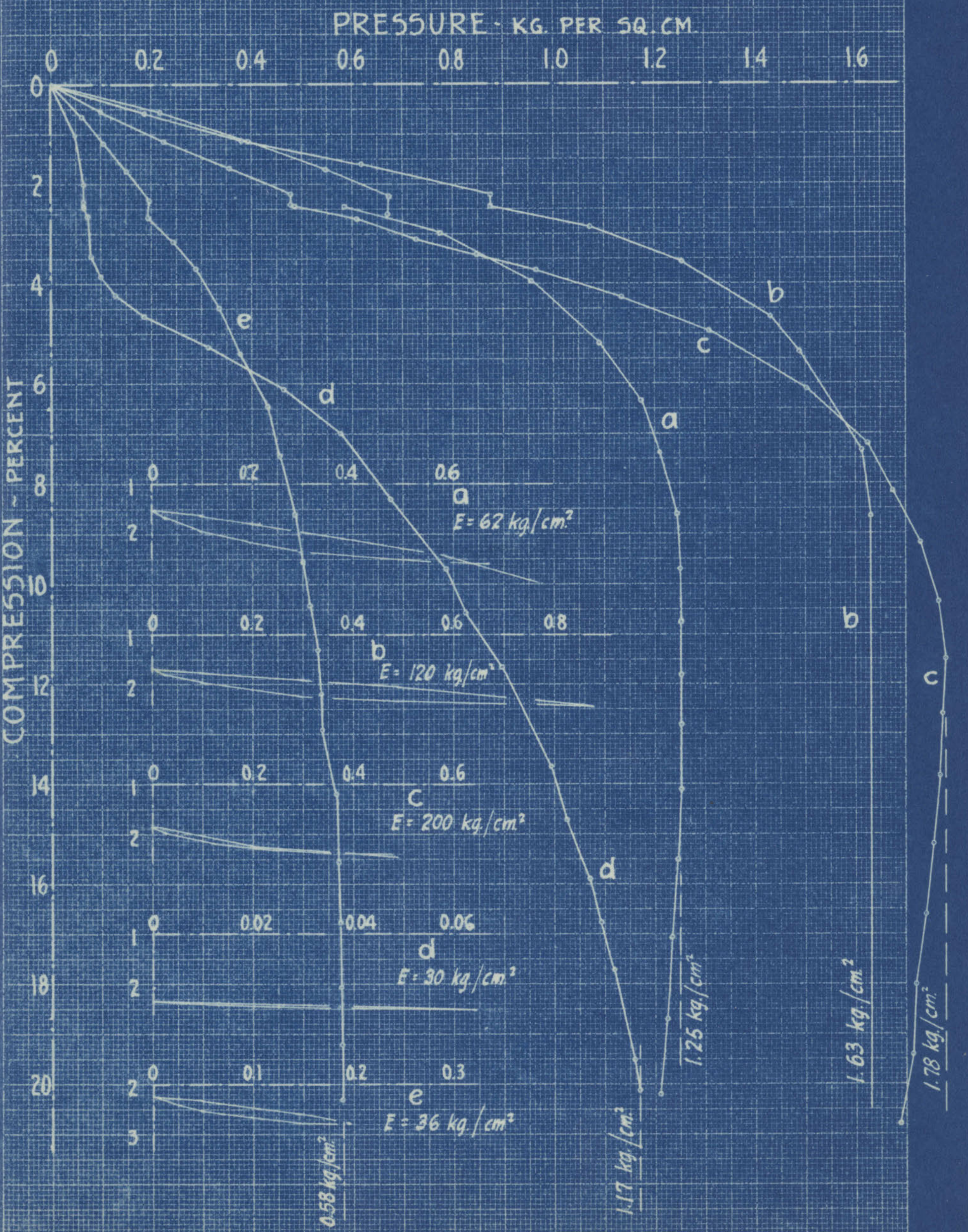
30-1-4V-1

3/24/28. Compressive Strength, Natural and Remolded.

	a	b	c	d	e
	Natural	Natural	Natural	Remolded	Remolded
C.S., kg./sq.cm.	1.25	1.63	1.78	1.17	0.58
E., kg./sq.cm.	62	120	200	30	36
Water Content	16.8 ?	31.8	32.4	29.9	33.8
Friction Angle	26	26	26	27	26

Water Content, Natural, surrounding material, 32.8, 29.7

Water Content, Average of 6, omitting a - 31.7


PRESSURE - KG. PER SQ. CM.
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
COMPRESSION - PERCENT
0
2
4
6
8
10
12
14
16
18
20
a
b
c
d
e
a
E = 62 kg./cm²
b
E = 120 kg./cm²
c
E = 200 kg./cm²
d
E = 30 kg./cm²
0.02
0.04
0.06
e
E = 36 kg./cm²
0.1
0.2
0.3
0.58 kg./cm²
1.17 kg./cm²
1.25 kg./cm²
1.63 kg./cm²
1.78 kg./cm²

30-1-4V-2

SUMMARY SHEET

3/23/28. Permeability, undisturbed.

2" sample.

$k = 20 \times 10^{-7}$ cm./min.

Water Content after Test, 34.8, 32.7, Av. 33.8

Run - 8 days.

30-1-4V-3

SUMMARY SHEET

4/3/28. Permeability, undisturbed.

2" sample.

$k = 70 \times 10^{-7}$ cm./min. at 21°

Note: Sample contains a large percentage of sand, very fine, sufficient to seriously affect plasticity. Sandy material lies in horizontal layers between usual clay-like deposits.

Water Content after Test - 26.6%

Run - 13 days.

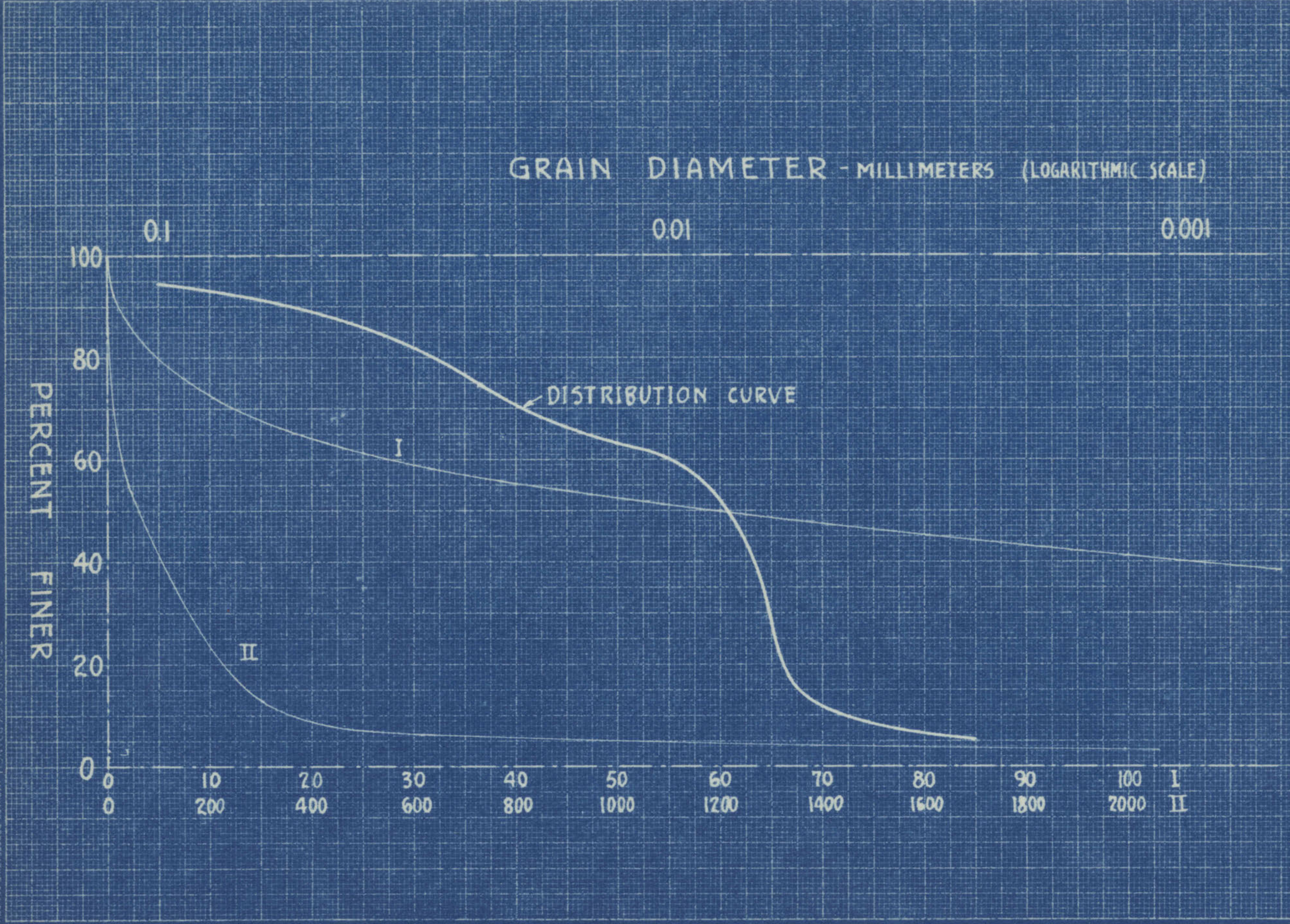
30-1-4V-4

SUMMARY SHEET

4/12/28. Mechanical Analysis (Wiegner).
Material from 2.

Effective Size - 0.0047 mm.
Uniformity Coefficient - 2.1

Note: Above value of uniformity does not give proper indication of distribution. Sample not at all uniform. See plot.



GRAIN DIAMETER - MILLIMETERS (LOGARITHMIC SCALE)
0.1
0.01
0.001
PERCENT FINER
100
80
60
40
20
0
DISTRIBUTION CURVE
I
II
0 10 20 30 40 50 60 70 80 90 100 I
0 200 400 600 800 1000 1200 1400 1600 1800 2000 II

30-1-4V-4

30-1-4V-5.

SUMMARY SHEET

4/13/28. Combined Compression and Permeability.

Material from 2, remolded.

From Compression Test

C_1 = 66

C_2 = 7.5

From Permeability Test

k at 1.5 kg./sq.cm.(e = 0.787) , 21° C

7.6 x 10^{-7} cm./min.

Liquid Limit -	From natural state -	42.9
	From dry powder	38.6
Plastic Limit -	From natural state	23.8
	From dry powder	21.8

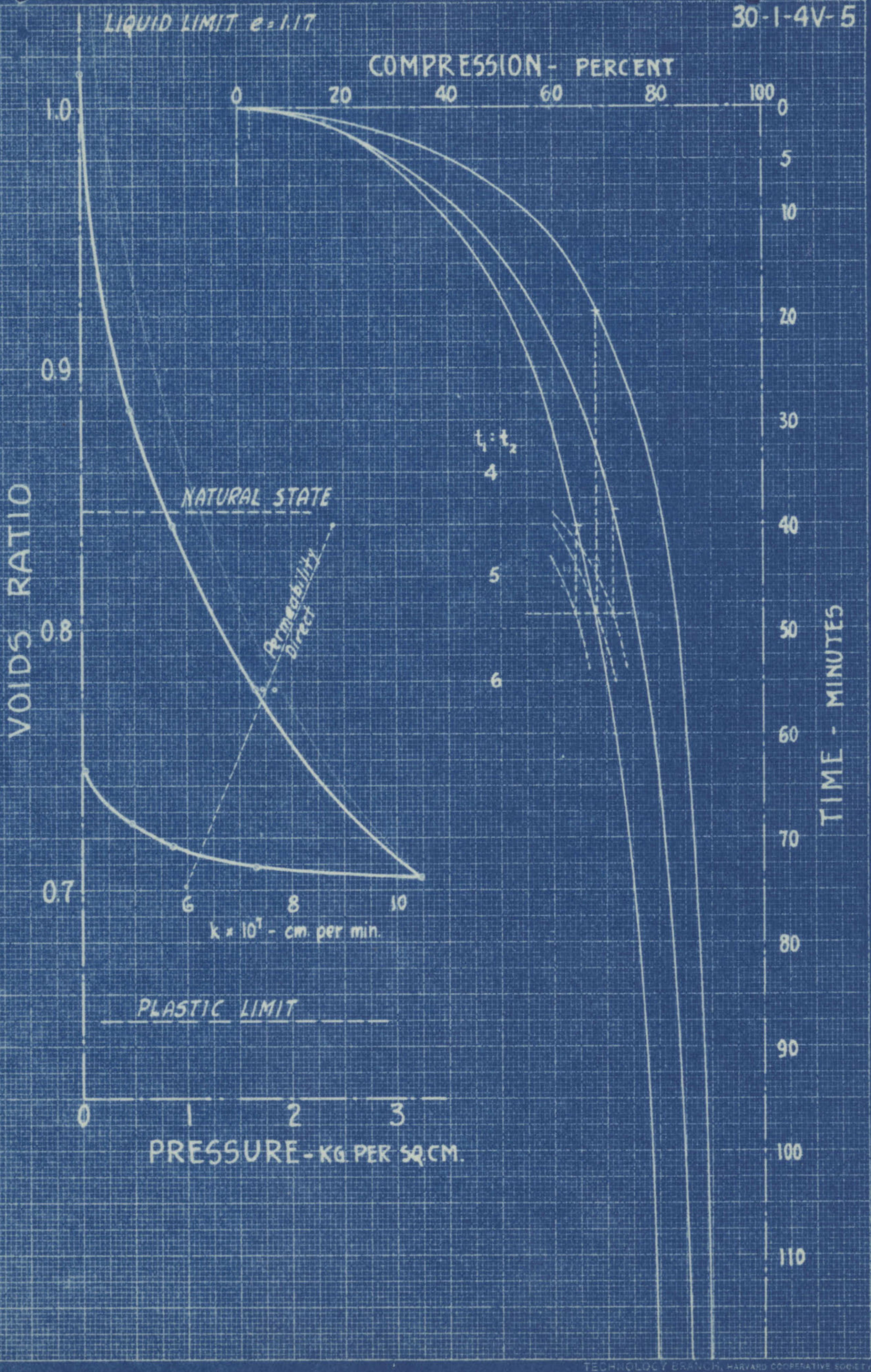
Shrinkage Limit undetermined, due to cracking of sample in oven.

Specific Gravity - 2.73

Note: Consolidation curves erratic, so that determination of k from consolidation uncertain.

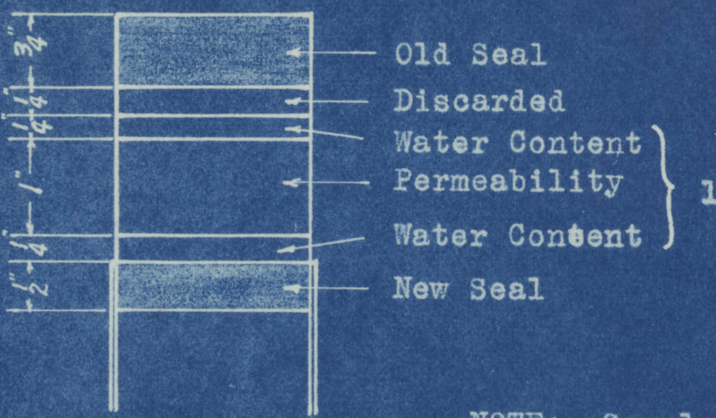
Coefficient of Consolidation

Range in Voids Ratio	c
0.884 - 0.840	0.002025
0.840 - 0.777	0.002135
0.777 - 0.705	0.00416


LIQUID LIMIT e = 1.17
COMPRESSION - PERCENT
0
20
40
60
80
100
VOIDS RATIO
1.0
0.9
0.8
0.7
NATURAL STATE
Permeability Direct
$t_1 : t_2$
4
5
6
6
8
10
k × 10^7 - cm. per min.
PLASTIC LIMIT
0
1
2
3
PRESSURE - KG PER SQ.CM.
TIME - MINUTES
0
5
10
20
30
40
50
60
70
80
90
100
110

30-2-2

SCHEDULE SHEET



NOTE: Sample apparently in excellent condition. Slid out of pipe easily, leaving perfectly smooth side surfaces. Portion tested contains a noticeable quantity of fine sand. Enough clay to cause strong cohesion, allowing samples to be cut off in layers. Slight tendency to disintegrate when handled, especially on one side where sand content was excessive.

1. Permeability, undisturbed. Water Content (3 tests).

30-2-2-1

SUMMARY SHEET

3/17/28. Permeability, undisturbed.

2.4 cm. sample from threaded end of pipe.

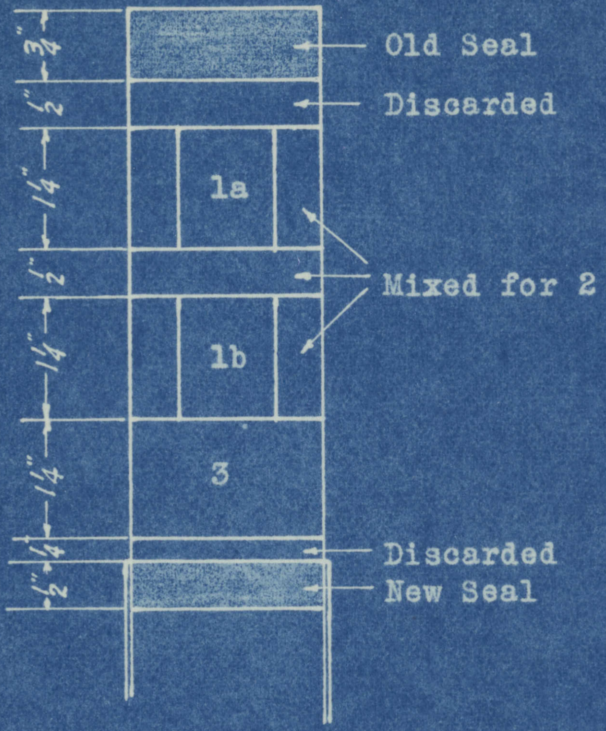
$k = 400 \times 10^{-7}$ cm./min.

Water Content - Natural, above sample - 17.9
Natural, below sample - 24.8
Sample after test - 23.6

Run - 13 days.

30-2-5

SCHEDULE SHEET



NOTE: Material fairly homogeneous and uniform.

1. Compressive Strength, Angle of Internal Friction, natural state (2 tests). Water Content (6 tests).

2. Compression Test, carried to 6.3 kg./sq.cm. Limits. Specific Gravity.

3. Permeability, undisturbed.

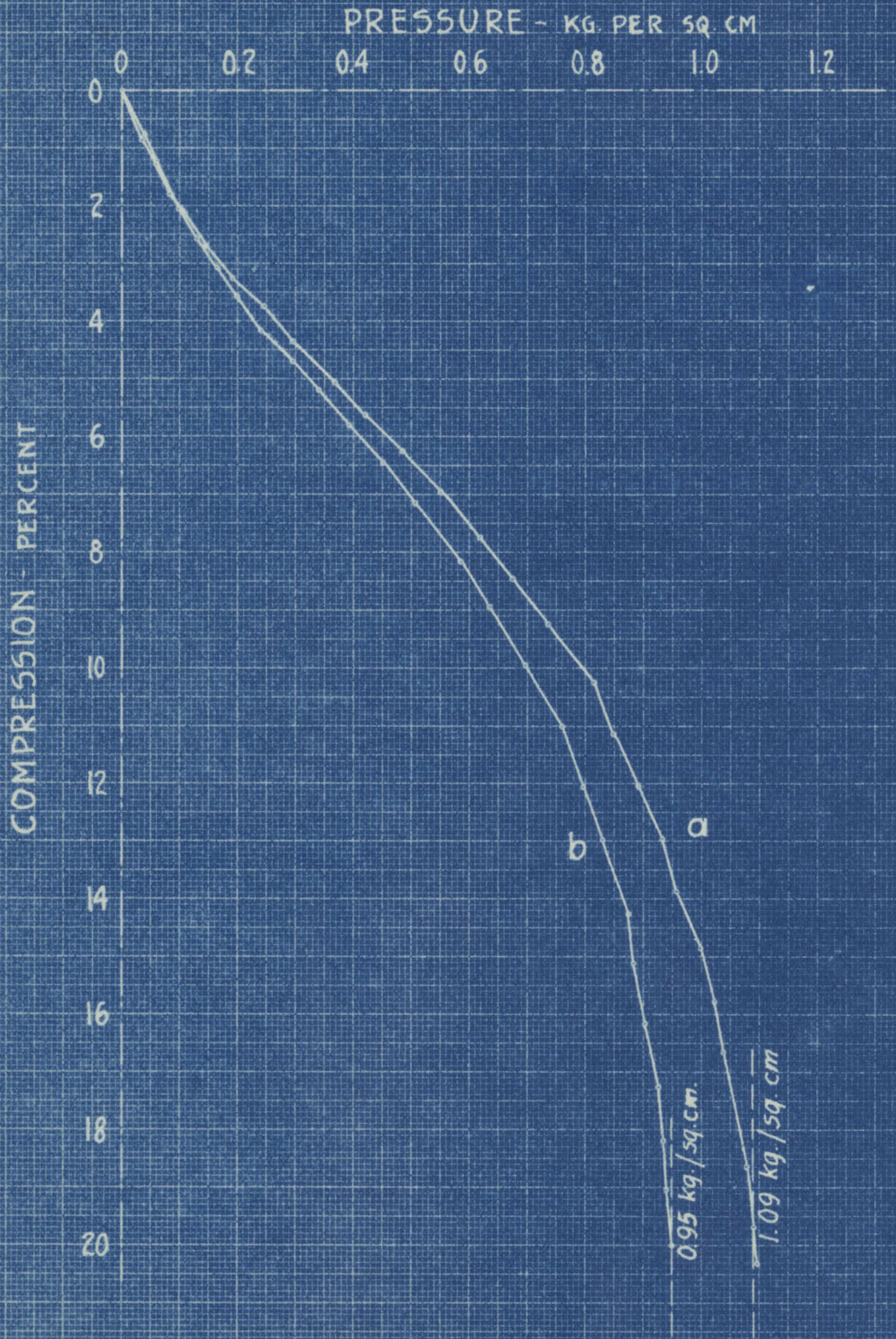
30-2-5-1

SUMMARY SHEET

4/30/28. Compressive Strength, Natural State.

	a	b
Compressive Strength, kg./sq.cm.	1.09	0.95
Angle of Internal Friction, degrees	30	32

Water Content, natural state, average of 6 tests - 22.5


PRESSURE - KG. PER SQ. CM
0
0.2
0.4
0.6
0.8
1.0
1.2
COMPRESSION - PERCENT
0
2
4
6
8
10
12
14
16
18
20
a
b
0.95 kg./sq.cm.
1.09 kg./sq.cm

30-2-5-2

SUMMARY SHEET

5/1/28. Compression Test, carried to 6.3 kg./sq.cm.

$C_1 = 152$

$C_2 = 11.9$

k at 1.5 kg./sq.cm. (e = 0.584) and 21° C.

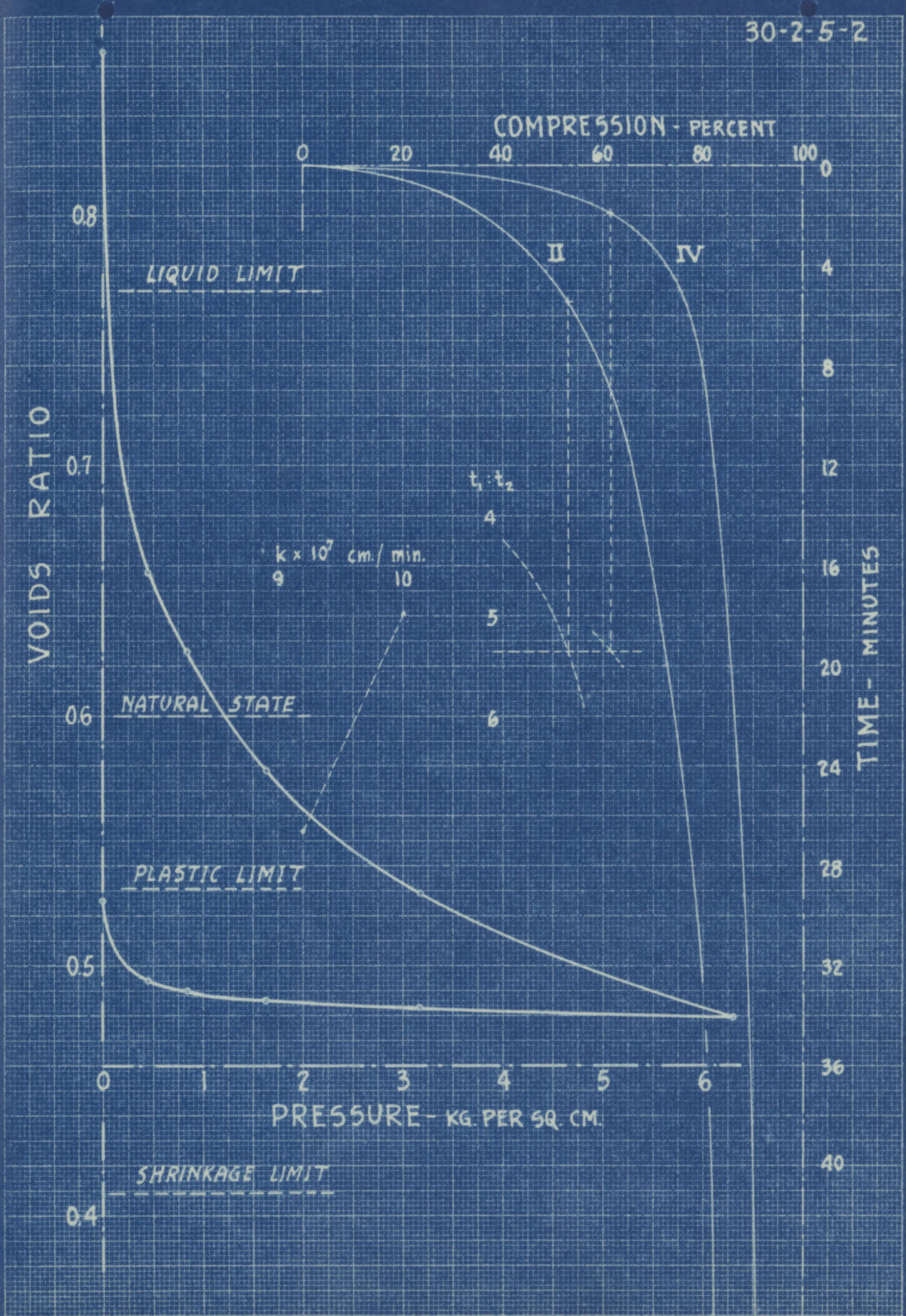
9.5×10^{-7} cm./min.

Liquid Limit	28.8
Plastic Limit	19.9
Shrinkage Limit	15.3
Natural State	22.5

Specific Gravity - 2.67

Coefficient of Consolidation

Range in Voids Ratio	c
0.657 - 0.626	0.0077
0.578 - 0.529	0.0188



TECHNOLOGY BRANCH, HARVARD COOPERATIVE SOCIETY

30-2-5-3

SUMMARY SHEET

5/2/28. Permeability, undisturbed.

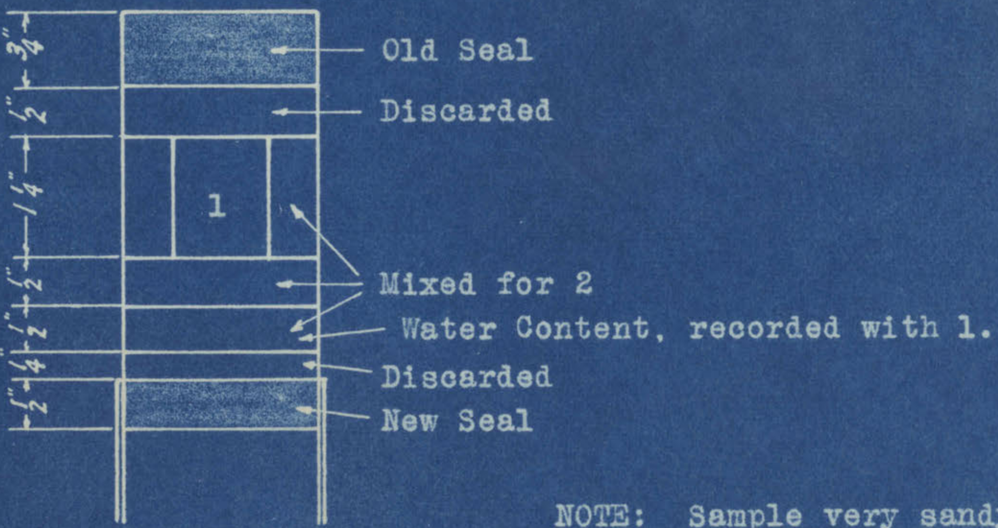
2.4 cm. slice, located as shown on schedule sheet.

k about 2000×10^{-7} cm./min.

Note: Test not accurate, as channels formed in sample or between sample and paraffin in the early part of the test.

30-2-7

SCHEDULE SHEET



NOTE: Sample very sandy. High friction in pipe. Difficult to remove by cutting with wire. Impossible to force out further specimens undisturbed.

1. Compressive Strength, Angle of Internal Friction, natural state. Water Content (4 tests).

2. Standard Compression Test.

30-2-7-1

SUMMARY SHEET

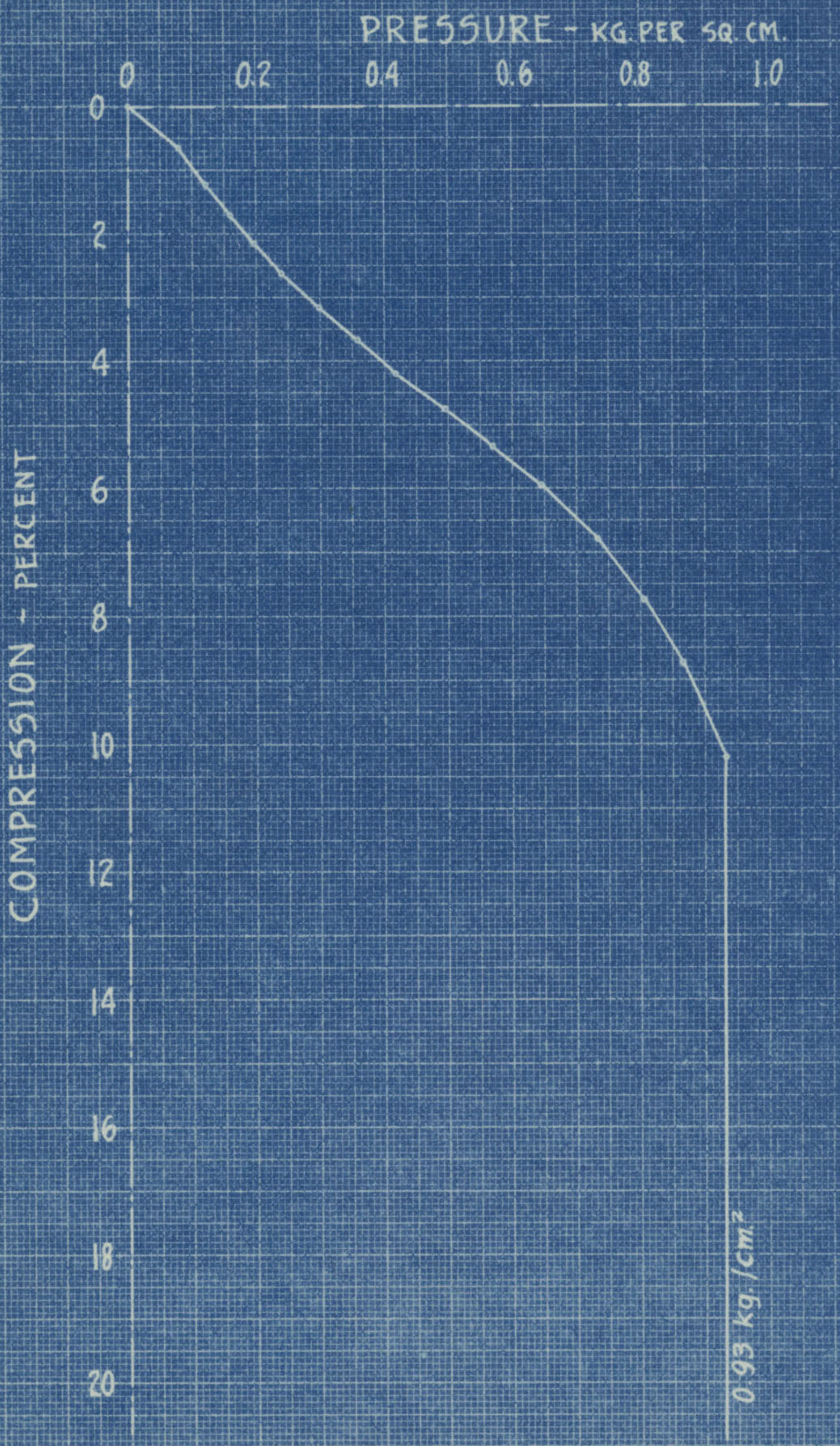
5/1/28. Compressive Strength, Natural State.

Compressive Strength, 0.93 kg./sq.cm.

Angle of Internal Friction - 30°

Natural Water Content - 17.6%

30-2-7-1



PRESSURE - KG. PER SQ. CM.
0
0.2
0.4
0.6
0.8
1.0
COMPRESSION - PERCENT
2
4
6
8
10
12
14
16
18
20
0.93 kg./cm.²

30-2-7-2

SUMMARY SHEET

5/2/28. Standard Compression Test.

$C_1 = 455$

$C_2 = 25.8$

Most of the time lag due to internal friction. Impossible to compute permeability accurately. Values determined by approximate method.

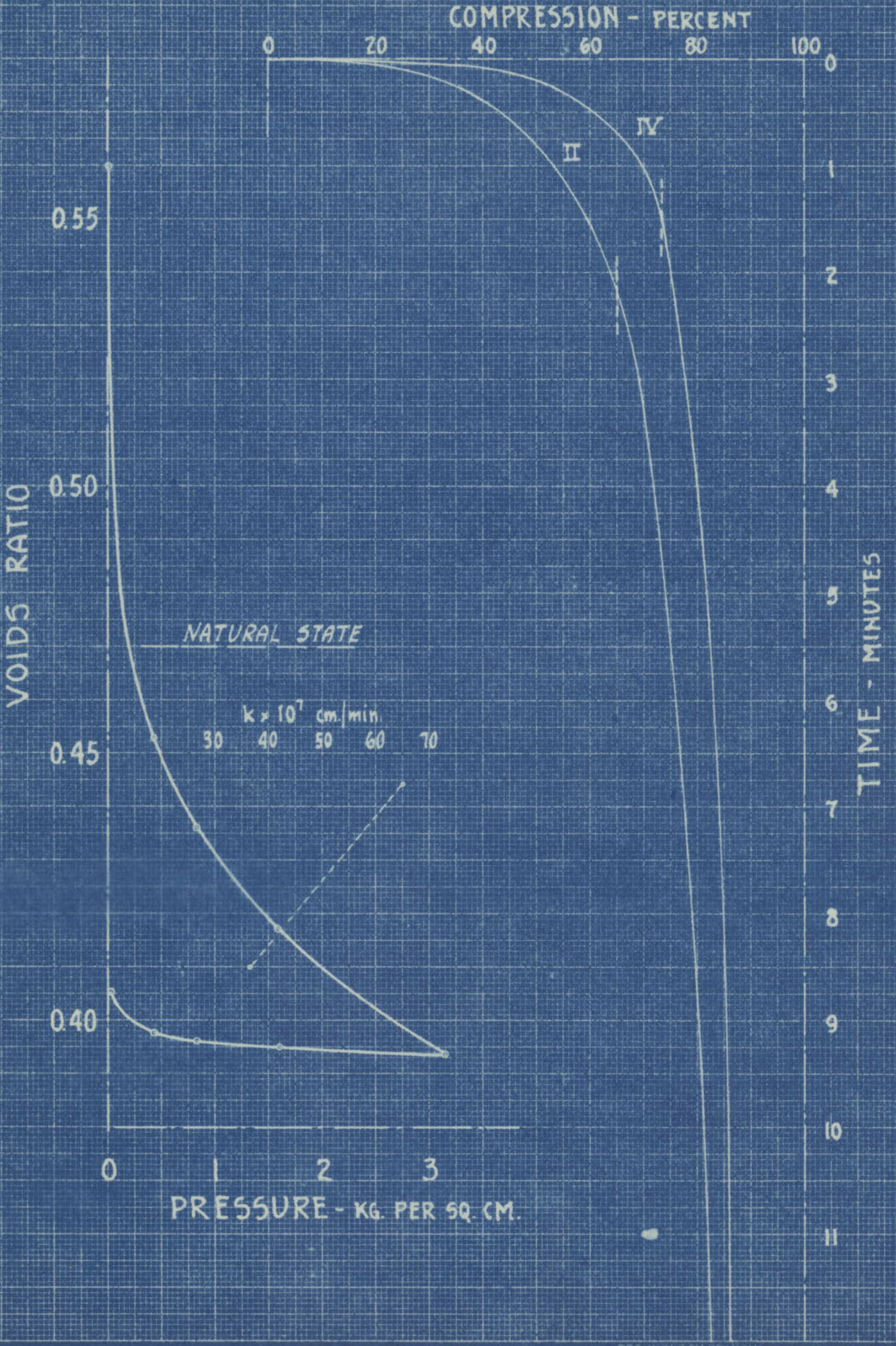
k at 1.5 kg./sq.cm. and 21° C. = 44×10^{-7} cm./min.

Limits of consistency indeterminate.

Plasticity index zero.

Coefficient of Consolidation

Range of Voids Ratio	c
0.453 - 0.436	0.1025
0.417 - 0.394	0.299


COMPRESSION - PERCENT
0
20
40
60
80
100
IV
II
VOIDS RATIO
0.55
0.50
0.45
0.40
NATURAL STATE
k × 10⁷ cm./min.
30
40
50
60
70
0
1
2
3
PRESSURE - KG. PER SQ. CM.
TIME - MINUTES
0
1
2
3
4
5
6
7
8
9
10
11

In the following pages the results are grouped with reference to tests in the following order:

1. Specific Gravity of Solid.
2. Water Content in Natural State.
3. Limits of Consistency.
4. Compressive Strength, Modulus of Elasticity, Internal Friction.
5. Permeability, undisturbed.
6. Constants for Classification.
7. Coefficient of Consolidation.
8. Special Tests.

1. SPECIFIC GRAVITY OF SOLID.

Test Number	Specific Gravity
1-2H-4	2.72
1-2H-5	2.675
1-2H-6	2.67
1-2H-8	2.67
1-4V-5	2.73
2-5-2	2.67
Best Value	2.67

The last statement is justified by the fact that in the two tests which differ from this value the capsules were allowed to stand in air until the main test was over. Errors of unknown magnitude may thus have been introduced, causing the tests to be somewhat inconsistent within themselves and to differ from others performed under normal conditions.

2. WATER CONTENT IN NATURAL STATE

Test Number	Water Content
1-2H-1	26.2
1-2V-1	26.7
1-4H-1	32.1
1-4V-1	31.7
2-2-1	23.6
2-5-1	22.5
2-5-2	22.5
2-7-1	17.6

Water Contents are in all cases expressed as percentage of water to dry matter, or solid, by weight.

3. LIMITS OF CONSISTENCY

Test Number	Liquid Limit	Plastic Limit	Shrinkage Limit
1-2H-4	33.7	21.6	17.3
1-2H-8			16.4
1-4V-5	42.9	23.8	
2-5-2	28.8	19.9	15.3

4. COMPRESSIVE STRENGTH - MODULUS OF ELASTICITY

ANGLE OF INTERNAL FRICTION

Test Number	C. S. kg./sq.cm.	E kg./sq.cm.	Friction Angle Degrees
1-2H-1 a	1.15	44	
b	0.77	32	
c	1.07	38	
1-2V-1 a	0.76	30	
b	1.04	48	
1-4H-1			
a, Natural	0.96		25
b, Natural	0.90	45	25
c, Remolded	0.40	23	27
d, Remolded	0.40	32	24
1-4V-1			
a, Natural	1.25	62	26
b, Natural	1.63	120	26
c, Natural	1.78	200	26
d, Remolded	1.17	30	27
e, Remolded	0.58	36	26
2-5-1 a	1.09		30
b	0.95		32
2-7-1	0.93		30

5. PERMEABILITY, UNDISTURBED

Test Number	$k \times 10^7$ cm./min.
1-2H-3	87
1-2H-5	109
1-2H-9	124
1-2H-10	96
1-2V-2	400
1-4H-2	270
1-4V-2	20
1-4V-3	70
2-2-1	400
2-5-3	2000

6. CONSTANTS FOR CLASSIFICATION

Test Number	C_1	C_2	$k \times 10^7$ Standard Conditions
1-2H-4	124	12.3	10.2
1-4V-5	66	7.5	5.7
2-5-2	152	11.9	7.1
2-7-2	455	25.8	44.

7. COEFFICIENT OF CONSOLIDATION

Test Number	Mean Voids Ratio	Coefficient
1-2H-4	0.713	0.0077
	0.672	0.0161
	0.623	0.0221
1-4V-5	0.862	0.0020
	0.808	0.0021
	0.741	0.0042
2-5-2	0.641	0.0077
	0.554	0.0188
2-7-2	0.444	0.1025
	0.410	0.299

8. SPECIAL TESTS

1-2H-8. Combined Compression and Permeability.

Material from 5, undisturbed.

Voids Ratio	$k \times 10^7$ cm./min.
0.695	54.4
0.663	40.1

1-2H-11. Combined Compression and Permeability.

Material from 9, undisturbed.

Voids Ratio	$k \times 10^7$ cm./min.
0.604	30.0
0.625	35.0
0.647	41.5
0.679	52.5

1-2H-12. Permeability, Remolded

Material from 10.

$$k = 44 \times 10^{-7} \text{ cm./min.}$$

II. C. ANALYSIS OF RESULTS.

Unit Weight. From the specific gravity and water content determinations it is a simple matter to compute the weight per cubic unit of the material.

Let s = specific gravity of solid (2.67 in this case)

s_1 = specific gravity of mass.

W = water content expressed as a ratio.

Then
$$s_1 = \frac{Ws + s}{Ws + 1} .$$

Performing this computation for the case in hand, it may be seen that s_1 varies from 1.90 to 2.14, averaging 2.00. The last value corresponds to a weight of 125 pounds per cubic foot.

Permeability. The undisturbed permeability tests indicate the wide range of variation which is to be expected in a mass of this kind. With the exception of 2-5-3, however, the variation is not as important as it appears at first glance, and it is an open question as to whether 2-5-3 could be considered representative enough to be included in the average.

In connection with the undisturbed permeability tests, the results included under the heading Special Tests throw an interesting light upon the behavior of the material when subjected to a flow of water. It has been previously noted that in practically all the undisturbed permeability tests the value of the coefficient was unusually high at the beginning, gradually decreasing as time went on and finally attaining a substantially constant value; and the explanation was offered that very small fissures originally existing within the sample were sealed up by the action of the flowing water in bringing solid particles into the cracks. The special tests indicate that this is the case, and also that there are some of the fissures which never are sealed up by the flowing water. When the sample is thoroughly mixed, as in 1-2H-12, the fissures disappear, of course, and the permeability is seen to drop to a value less than half of the original.

It might be argued that this mixing breaks up layers of more permeable material which formerly acted as channels to carry the major part of the water; but the two other tests, 1-2H-8 and 1-2H-11, do not bear out this theory. In these tests the material taken out of the permeameter was carefully cut into a cylinder just large enough to fit into the clay container, without disturbing the bulk of the sample in the slightest. The container was flooded, and allowed to stand for a day in order to permit the sample to absorb water if required. No movement of the dials was noticed. It was impossible to run a direct permeability test under no load, on account of the fact that the pressure of the water would lift the sample and piston within the cylinder. There is no reason to believe, however, that such a test would yield values very much different from the original permeability test, assuming, of course, that the sample fitted the cylinder accurately enough so that there would be no leakage between soil and brass under zero load; a condition which is practically impossible to attain. As soon as the first increment of load was applied (0.4 kg./sq.cm.) the permeability dropped to somewhere in the neighborhood of one half of the original value. It is hardly conceivable that this pressure would produce any mixing or breaking up of pervious strata; but it is certain that the pressure would first of all tend to close up any fissures which might exist. The evidence, then, points to the existence of fissures within the sample; and their presence must be taken into account when drawing conclusions from permeability tests of this character.

There seems to be no definite relation between the horizontal and vertical permeability. In one pair of samples the horizontal is greater than the vertical, while in the other pair the opposite is the case.

The best agreement in the whole series was obtained with 1-2V-2 and 2-2-1, both vertical samples from the same depth, the former from the shaft and the latter from the drillhole. That they should check exactly is a coincidence; but the writer feels convinced that for homogeneous material a good drillhole sample will furnish results as reliable and consistent as an undisturbed sample. The principle advantage of the undisturbed sample lies in the fact that it can be chosen with care, while in the drillhole one is obliged to take whatever comes along. Furthermore, the drillhole sample will not afford a means of determining the permeability in any but a vertical direction. The thing to remember, however, is that permeability is dependent principally upon structure; and if drillhole samples are properly taken, so as to disturb the structure of the material as little as possible, and if, in addition, the material is of such a character that it cannot change its essential structure rapidly, the drillhole sample will probably turn out to be very satisfactory.

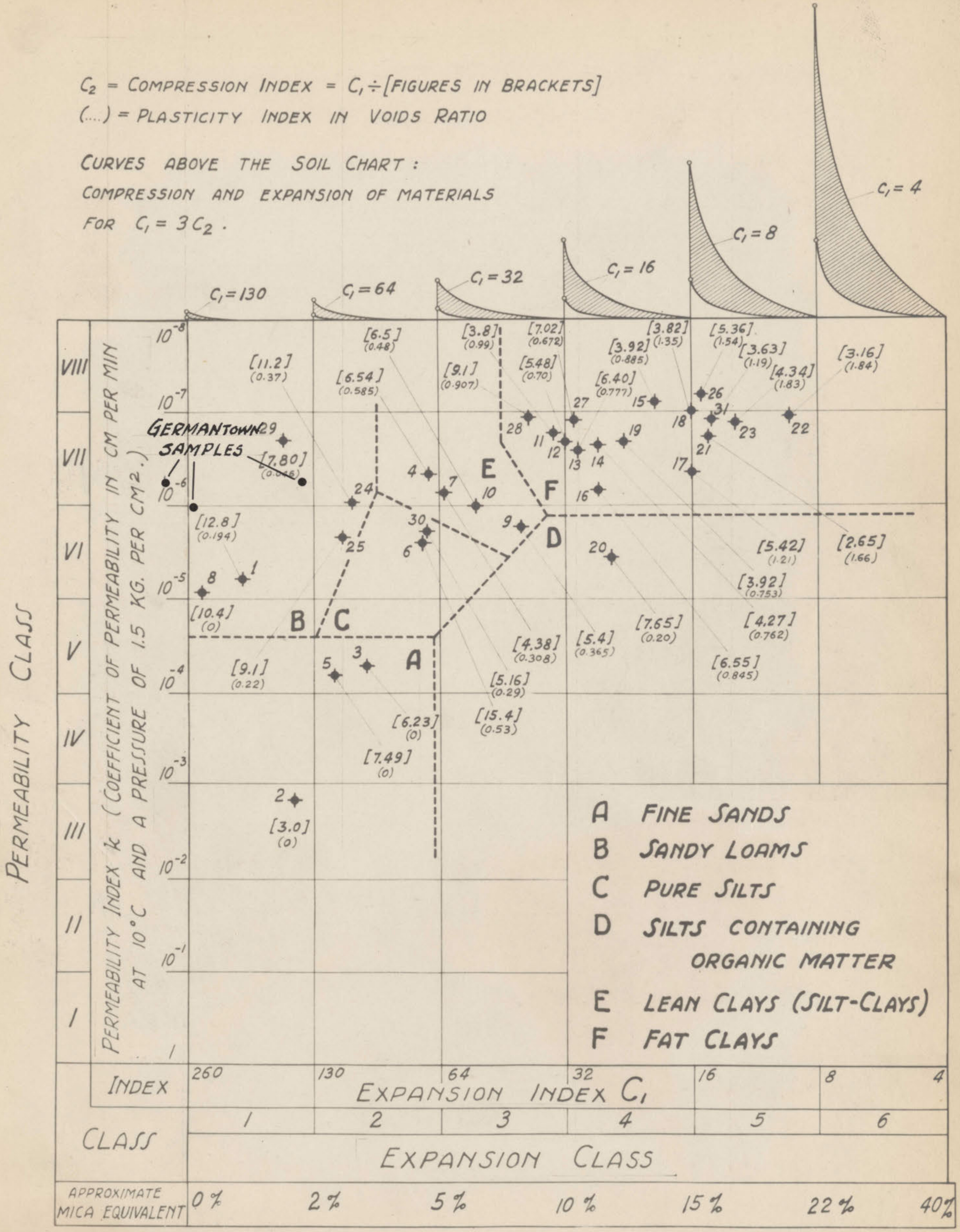
If the permeability were to be used in computations, the writer would be inclined to take the value 100×10^{-7} cm./min. as a round average figure.

<u>Mechanical Analysis</u>. The two distribution curves obtained from the Wiegner apparatus are interesting in that they illustrate the factor which especially influences the peculiar behavior of this material, namely, the lack of uniformity. The soil contains a little of everything from fine sand down to clay. Inasmuch as mixtures of different grain sizes seem to exhibit the peculiar power of retaining the good points of each size and eliminating the bad, it is obvious that this kind of grading is very much to be desired. The tests further show that the water-filtration terms "effective size" and "uniformity coefficient" tend to lose their significance when applied to materials of this character.

<u>Limits of Consistency</u>. These values afford a simple and convenient means for classifying plastic soils and for obtaining a general idea of their properties. Their significance has already been explained, and inasmuch as they are of lesser importance in the present problem it will not be necessary to discuss them further.

<u>Coefficient of Consolidation</u>. The values determined for this coefficient will be utilized in Section III, so that discussion on this point may be postponed for the present.

<u>Constants for Classification.</u> The information obtained from these values is of the utmost importance, inasmuch as it enables an immediate comparison of the material at hand with a great many other soils whose properties have been tested. The constants C_1 and C_2 are the parameters of logarithmic curves which approximate the expansion and compression curves, respectively. The use of these approximate curves has already been noted in Section I B. In order to obtain a conception of the factors influencing the behavior of soils in general, Dr. Terzaghi designed a "Soil Chart", a copy of which is included as Fig. 21. The value of k reduced to arbitrary standard conditions (pressure of 1.5 kg./sq.cm. and temperature of 10° C.) and the value of C_1 determine the position of the point on the chart representing any given soil. It was found that certain very pointed deductions could be drawn from this chart, one of the most important of which is that



C2 = COMPRESSION INDEX = C1 ÷ [FIGURES IN BRACKETS]
(....) = PLASTICITY INDEX IN VOIDS RATIO
CURVES ABOVE THE SOIL CHART:
COMPRESSION AND EXPANSION OF MATERIALS
FOR C1 = 3C2.
C1 = 130
C1 = 64
C1 = 32
C1 = 16
C1 = 8
C1 = 4
GERMANTOWN SAMPLES
PERMEABILITY CLASS
PERMEABILITY INDEX k (COEFFICIENT OF PERMEABILITY IN CM PER MIN AT 10°C AND A PRESSURE OF 1.5 KG. PER CM2.)
VIII
VII
VI
V
IV
III
II
I
INDEX
EXPANSION INDEX C1
260
130
64
32
16
8
4
CLASS
1
2
3
4
5
6
EXPANSION CLASS
APPROXIMATE MICA EQUIVALENT
0%
2%
5%
10%
15%
22%
40%
A FINE SANDS
B SANDY LOAMS
C PURE SILTS
D SILTS CONTAINING ORGANIC MATTER
E LEAN CLAYS (SILT-CLAYS)
F FAT CLAYS

FIG. 21

the stable materials find their position toward the upper left hand corner, whereas unstable soils tend toward the lower right hand corner. More specifically, a stiff, solid mixture of sand and clay would be in the upper left corner, while a sample from a peat bog would fall in the lower right. The chart has been in use for so long, and checked repeatedly with so many tests, that the information it furnishes is not at all a matter of conjecture. If the position of a given material is plotted, one familiar with the chart can tell very closely whether the material is going to cause trouble.

An inspection of the constants obtained from the Germantown samples shows that this material is well up in the left hand corner. It is not too much to say that a more suitable material for the core of a dam could scarcely be found.

State of Consolidation. The tests of compressive strength and the pressure-voids ratio diagrams afford a means for estimating the state of consolidation prevailing in the core at the time the samples were removed. A few computations will be made to illustrate the application of these tests.

Knowing the depths from which the various samples were taken, and on the basis of the previously determined average value of the weight per unit volume of the core, the pressures exerted at the various points may be computed. Consider levels 2 and 4 in the shaft and 5 in the drillhole. The depths of these points below the top of the dam are 48', 74' and 81.5', respectively. The corresponding pressures are 2.92, 4.51, and 4.97 kg./sq.cm. Referring to the pressure-voids ratio diagrams for 1-2H-4, 1-4V-5, and 2-5-2, the pressures corresponding to the natural water content are 0.8, 0.8 and 1.22 kg./sq.cm. That for 1-4V-5 is incorrect, as the test was inadvertently started at a point considerably below the liquid limit, so that the zero axis of pressures does not represent a true zero pressure within the sample on the compression curve. The curve is well defined, however, and by passing a similar curve through the actual liquid limit point a correction can be made with a fair degree of accuracy. The true pressure is about 1.15 kg./sq.cm. The three pressures are therefore 0.8, 1.15, and 1.22 kg./sq.cm. The ratios of these pressures to the three computed actual pressures will obviously be the percentage consolidation at the three given points. The ratios are 27.2, 25.5, and 24.6 percent, respectively. The values are consistent, in that they show a decrease with increasing depth. They indicate that the core as a whole is about 25% consolidated.

Next consider the compressive strength tests for the undisturbed samples, 1-2H-1 and 1-4V-1. The angle of internal friction is about 25°. The computed pressure at level 2 was 2.92 kg./sq. If all this pressure were carried by the soil, the compressive strength of 1-2H-1 should be,

$$2.92 \frac{1 + \sin 25^\circ}{1 - \sin 25^\circ} - 2.92 = 4.28 \text{ kg./sq.cm.}$$

The compressive strength of 1-4V-1 should be,

$$4.51 \frac{1 + \sin 25^\circ}{1 - \sin 25^\circ} - 4.51 = 6.60 \text{ kg./sq.cm.}$$

The measured compressive strengths of the two samples are not nearly as high as these computed values. Consolidation is evidently far from complete. Performing the same operation with the true pressures, the compressive strength of 1-2H-1 should be,

$$0.8\,\frac{1+\sin 25^\circ}{1-\sin 25^\circ} - 0.8 = 1.11 \text{ kg./sq.cm.}$$

and of 1-4V-1,

$$1.15\,\frac{1+\sin 25^\circ}{1-\sin 25^\circ} - 1.15 = 1.69 \text{ kg./sq.cm.}$$

The actual tests of 1-2H-1 yield one value of 1.07 and one of 1.15. Of 1-4V-1, one value 1.63 and one 1.78. This substantial agreement further checks the value of percentage consolidation determined in the preceding paragraph.

Brief Summary of Test Results.

The specific gravity of the solid is 2.67.

The specific mass gravity of the core is approximately 2, corresponding to a weight of 125 pounds per cubic foot.

The permeability of the undisturbed material varies considerably, both on account of variations between samples and on account of the presence of minute seams in the samples themselves. A rough average is 100×10^{-7} cm./min.

Mechanical analyses show that the material is a mixture of all sizes of grain from fine sand to clay.

The position of the material on the Soil Chart indicates that it possesses an unusual degree of inherent stability, and is therefore peculiarly suited to this type of construction.

Tests of compressive strength, natural water content, and variation of voids ratio with pressure show that the core as a whole is about 25% consolidated.

SECTION III. APPLICATIONS AND DISCUSSION

III. A. APPLICATIONS OF THEORY

Analysis of Stress in Shell, Core in Liquid State. The application of the graphical method of determination of the maximum obliquity of stress in the shell, described in Section I A, requires that the weight per unit volume of core and shell material be determined, as well as the dimensions of the idealized section.

The specific mass gravity of the core, s_1, may be taken as 1.75. This value corresponds to a voids ratio slightly higher than the liquid limit of 1-4V-5, or to a volume of voids of 53.5%. If in error, it is probably somewhat too large.

The weight per unit volume of the shell material may be assumed as 2. This figure corresponds to the specific gravity of the core as it stands at present, and is certainly not too high.

The core slope is fixed by the design at 2:1. The idealized outer slope is taken as the dotted line shown in the cross-section of the actual dam in the lower right corner of Fig. 22. It will be noted that this line touches the outer slope in only one point and lies at all points entirely within the shell. The error in idealization is therefore on the side of safety. The slope of the line is 1:2.4.

With these values given, it is a simple matter to perform the graphical analysis. Fig. 22 shows the result, all the construction lines being inked in for the case of a plane at 30° to the horizontal. To ink in the other construction lines would make the drawing too complicated in appearance. It may be seen that the maximum obliquity occurs on a plane making an angle of 4° with the horizontal, and the value of the maximum angle is 31.5°,not at all excessive for the coarse materials of which the shell is composed.

It should be remarked that it is poor policy to draw in each resultant as soon as its line of action is determined. The better way is to determine the ends of the resultants in rotated position without drawing the actual lines, then to draw a curve passing through these end points. This curve is shown running through the broken border line at the bottom of the figure. A line from the foot of the core slope tangent to the curve will obviously enclose with the vertical the angle of maximum obliquity. The plane corresponding to this angle is then found by reversing the rotating process.

The procedure is very simple, and the entire process occupies only a few minutes. It may thus be seen that a large number of different cases may be investigated without undue loss of time.

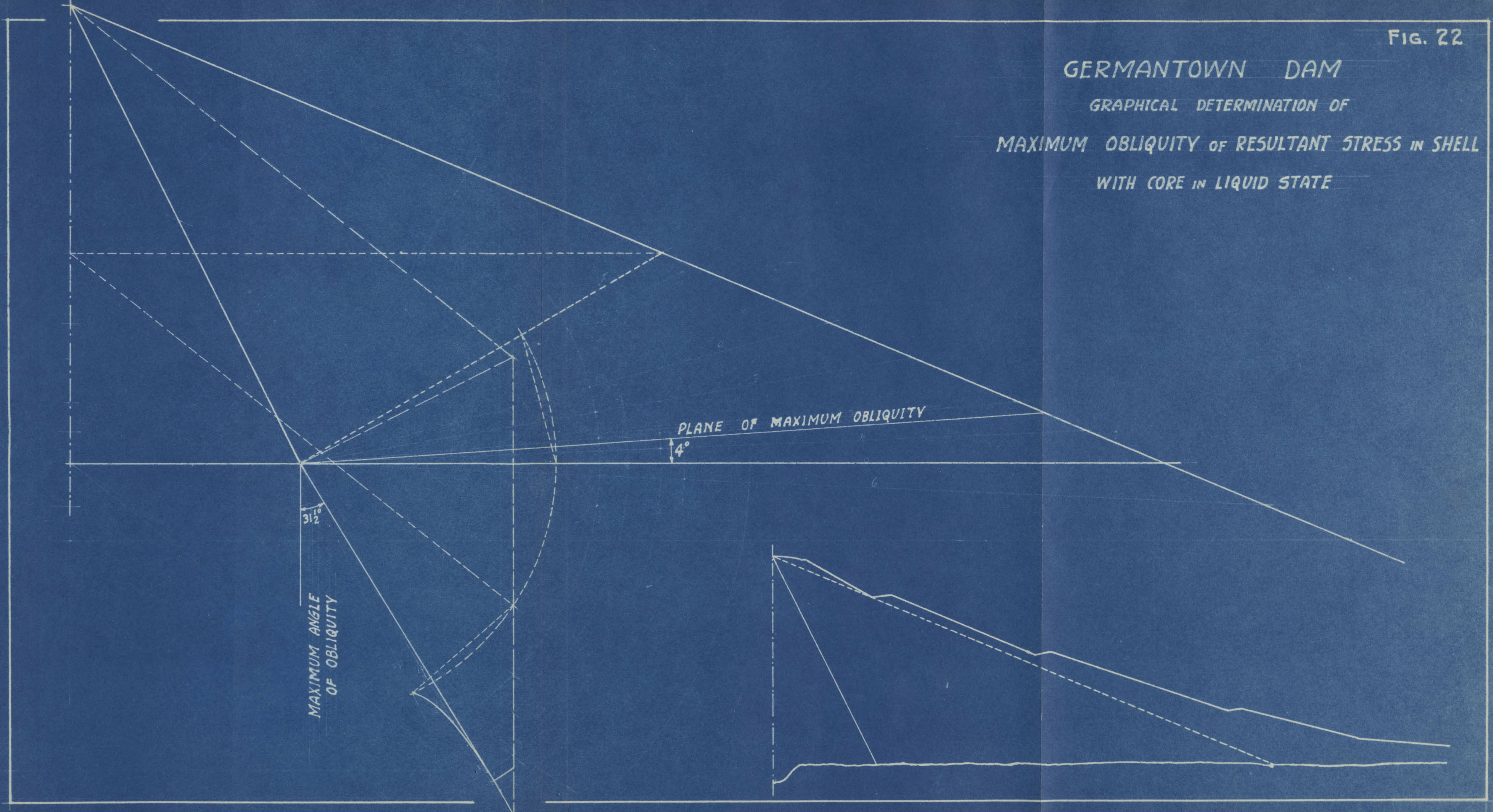


FIG. 22

GERMANTOWN DAM

GRAPHICAL DETERMINATION OF

MAXIMUM OBLIQUITY OF RESULTANT STRESS IN SHELL

WITH CORE IN LIQUID STATE

<u>Theoretical Rate of Consolidation of Core</u>. Referring to Section I B, the first step is to determine an elliptical section which will approximate the given core section.

The height of the core is 110 ft., or 3,350 cm.

The ratio of height to half width is 2:1.

If A is the semi-major axis and B the semi-minor axis of the ellipse, the equations become

$$\pi A\,B = (3{,}350)^2 = 11.2 \times 10^6 \text{ sq.cm.}$$

and $A = 2B.$

whence $B = 1{,}340$ cm.

An average value of the voids ratio is 0.68, so that the reduced length of the path is

$$B_o = 1{,}340 \div 1.68 = 798, \quad \text{say } 800 \text{ cm.}$$

By plotting the various values of the coefficient of consolidation against voids ratio and noting the intersection of the curves with the 0.68 line, a fair average value of the coefficient may be obtained. The value thus obtained is

$$c = 0.0065 \text{ sq.cm./min.}$$

The ellipticity E is evidently 0.5.

With the foregoing data, it is possible to plot a curve of consolidation against time for the actual core.

Using the value of E = 0.5, and interpolating in Table II, the following relation is found.

TIME FACTORS FOR ELLIPTICAL CYLINDER, E = 0.5

Percentage Consolidation	Time Factor
0	0
10	0.0065
15	0.0125
20	0.0225
25	0.0350
30	0.0505
35	0.0675
40	0.0870
45	0.1095
50	0.1340
55	0.1630
60	0.1960
65	0.2330
70	0.2760
75	0.3255
80	0.3895
85	0,4740
90	0.5890
95	0.7900
100	∞

These values are plotted in Fig. 23.

GERMANTOWN DAM

MIAMI CONSERVANCY DISTRICT

THEORETICAL RATE OF CONSOLIDATION OF CORE

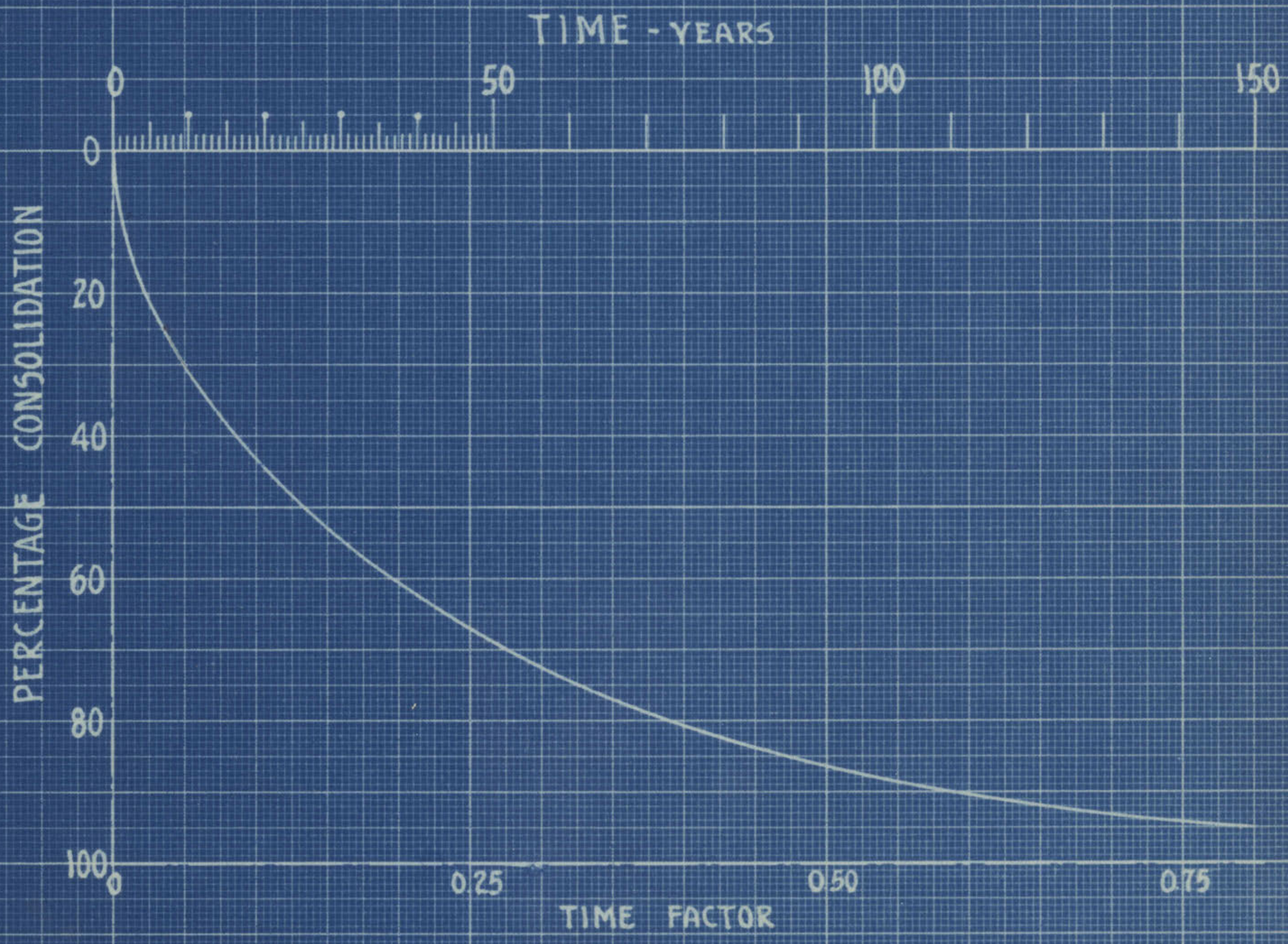


FIG. 23

G. GILBOY

To find the time corresponding to any given percentage consolidation, it is only necessary to multiply the proper time factor by the ratio B_o^2/c, as previously explained. Since this ratio is a constant for the particular case in question, an auxiliary time scale can be constructed and placed in Fig. 23, as follows:

Taking the time factor for 95% consolidation, the corresponding time is

$$0.7900 \times \frac{(800)^2}{0.0065} = 0.78 \times 10^8 \text{ minutes,}$$

or 148 years.

Laying off a uniform scale such that the zero point coincides with the zero axis and the point representing 148 years lies at time factor 0.7900, the curve of Fig. 23 permits the rate of consolidation of the actual core to be read off directly for any desired interval of time.

The Germantown Dam had been in existence 7 years at the time the samples were taken. It is interesting to note that the percentage consolidation from the graph corresponding to the time 7 years is 25%, again checking the previously determined value. It will be still more instructive to follow the process of consolidation as time goes on, to ascertain just how far the theoretical curve can be relied upon. One item in favor of the accuracy of the curve is the fact that in the lowest range of time the points are least accurate. Inasmuch as the curve checks experiments for a rather low value of time, it is not too much to expect that it represents the consolidation process reasonably well throughout the entire range.

III. B. DISCUSSION

Stability during Construction. The graphical method of analyzing the state of stress in the shell while the core exerts full hydrostatic pressure, developed in Section I A and illustrated in III A, affords a simple means for studying the stability of various sections composed of various kinds of material. It is not claimed that this method is absolutely accurate, inasmuch as sliding, if it occurs, will probably occur along a curved surface rather than along a plane. The assumption of curved surfaces would complicate the problem to such an extent that the solution would hardly be of practical value. If for any given section the angle of the resultant as determined graphically is always below the angle of friction of the shell material, there is every reason to believe that the shell will be stable, especially as a liberal factor of safety is provided by the initial stage of consolidation of the core which occurs very rapidly while the dam is being built, so that the assumption of full hydrostatic stress within the completed section is much too severe. If it is found that the obliquity of the resultant is much in excess of the angle of internal friction of the shell, there is grave danger that the section will not be stable. It may be that in the given case the core will consolidate to a sufficient degree so that failure will not occur; but it would be much better engineering to redesign the section, rather than to take a chance on the pressures becoming so low that the shell resistance will not be exceeded.

The theory underlying the graphical solution illustrates the importance of accurate determinations of the specific mass gravity and the angle of internal friction of the shell material. These factors are liable to be neglected in actual design, the assumption being made that the material is about average, with an average weight and an average angle of friction. This assumption is usually justified; but in an isolated case the material might depart just enough from the average to produce disastrous consequences. At any rate, there is no excuse for omitting a determination of these constants, now that their influence has been definitely established.

These conditions further emphasize the importance of including nothing but clean, coarse material within the shell. The presence of any appreciable quantity of fine plastic soil may cause local lubrication due to the hydrodynamic stresses and the low angle of internal friction characteristic of this material, thus creating areas of low resistance within the shell which are a constant source of danger. The Calaveras dam, for example, was built by true hydraulic-fill methods to a height of 60 feet. Above this level, the core was enclosed between two dumped embankments, no attempt being made to wash the finer constituents out of the dumps. The result of this procedure is too well-known to engineers to warrant any further comment. The Calaveras failure has been used as argument against the hydraulic-fill method; but the portion of the dam which failed was not by any means a true hydraulic-fill structure. The lower section, which was true hydraulic-fill, remained intact after the slip.

The theory shows that the narrow core section is to be preferred. As an illustration, if the unit weight of core and shell were the same, the outer slopes made an angle with the horizontal equal to the angle of friction of the shell material, the core slopes should be vertical. This is, of course, a limiting case, and would not ordinarily occur in practice. At the same time, it is well to remember that the narrow core is the inherently stable section. The analysis of the Germantown core given in Section III A shows that this section is stable for the material used. It was found during construction that the core was wide enough to permit of good control of the pool. The core slope of 1:2 used in all the Miami dams is a good figure to keep in mind.

<u>Rate of Consolidation</u>. The theory developed in Section I B furnishes a very useful means of computing the rate of consolidation of any given core section composed of any given material. The mathematical analysis is rather complicated, but the results have been concentrated to a point where the actual computation of a particular case is a very simple matter. Referring to the second part of Section III A, it will be seen that the computations for the Germantown core, including the tabulation of time factors, would scarcely fill a single sheet of paper. It has been pointed out that

an exact determination of the rate of consolidation is not only a mathematical impossibility, but also quite unnecessary from a practical point of view, as the constants of the material vary so much within themselves that exact results could not be obtained even from an exact equation. The method herein presented is therefore sufficiently accurate, and is at the same time easy to apply. As a matter of fact, it has proved to be more accurate in the case of the Germantown Dam than the writer originally expected. To determine from tests that the core was approximately 25% consolidated, and then to find from the theoretical curve that the time at which the samples were removed corresponded to a consolidation of 25%, is a gratifying proof of the validity of both the tests and the theory.

The writer hopes that the theoretical curve will be further checked experimentally in the future, both by tests on the Miami dams at intervals of ten or twenty years and by tests on other dams which were built some time ago.

Consolidation during Construction. It was observed during the construction of the Miami dams that the core reached a very fair degree of stability after the lapse of a comparatively brief time.* If the Germantown Dam were thrown up instantaneously, the theoretical curve indicates that at the end of two years the core would have been about 12% consoli-

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* Paul, C.H. Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District. Trans., A.S.C.E., Vol. 85, 1922.

dated. Inasmuch as the dam was built up gradually, the consolidation of the core at completion would be considerably greater than this, since the initial stage of consolidation proceeds much more rapidly than the general curve would indicate.* The percentage consolidation at completion would certainly be over 15%, perhaps as much as 20%. A computation of the settlement of the dam, assuming 20% consolidation at completion and 25% at the end of 7 years, indicates that the settlement should have been somewhat less than one foot in the maximum section. Actual levels show the settlement to have been smaller than this, about 6 inches on the average. The apparent discrepancy is accounted for by the fact that the rate of consolidation of the vertical elements in the center line of the cross-section is much less rapid than the rate for the section as a whole, as may be learned from the curve for Case 2, Fig. 14. The difference is even greater than the 50% difference noted between computed and observed settlements, so that the assumed percentage consolidation of 20% at completion is apparently somewhat high. At any rate, the core evidently reached a consolidation value of something between 15% and 20% during construction. It is too bad that samples of the core could not have been removed and tested immediately upon completion, to obtain a more accurate idea of the percentage to be expected.

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* Örtenblad, A.L. Mathematical Theory of the Consolidation of Mud Deposits. M.I.T. C.E.Dept. Thesis, 1926.

<u>Factors influencing Consolidation</u>. The fact that the coefficient of permeability of the material is not the only item which determines its rate of consolidation cannot be too strongly emphasized. The coefficient of compressibility of the material enters into the process with exactly the same weight as the coefficient of permeability. If two materials, A and B, have the same permeability, but B is twice as compressible as A, it will take B twice as long to reach a given state of consolidation under given conditions as it will A. The essential point is that the determination of one property of a given material is of little value if the behavior of the material is influenced by other undetermined properties. An investigator may make a hundred mechanical analyses of a certain soil, and think that he had studied the situation thoroughly; as a matter of fact, at the end of his exhaustive investigation he is just as ignorant of the essential factors governing the behavior of the material as he was when he started. From that point on, he must use his engineering judgment, and predict in a general way the behavior of his material, basing his conclusions on the behavior of other materials with which he is familiar and which are similar in grain distribution to the one just investigated. The validity of his conclusions will vary with his common sense and the amount and character of his experience; but he has no way of assuring himself or others that his conclusions are correct, simply because he has not taken advantage of all available means for obtaining definite information.

Tests upon the physical properties of a soil, similar to those described in Section II, are of vital importance in studying the behavior of the material, and in analysing the importance of the influencing factors. It has been shown that the rate of consolidation of a mass of soil is directly influenced by its coefficient of consolidation. It is only logical, therefore, that the first step in the attack of a problem involving consolidation should be the experimental determination of this coefficient. With the value known, definite computations can be made and definite results obtained, with the assurance that the results are reasonably correct; without it, all conclusions are pure guesswork.

If it is desired to obtain a material which will attain a certain degree of consolidation in a given length of time, the foregoing considerations indicate that it is fundamentally incorrect to specify that the material should contain particles of a certain grain size. Such a specification is about as accurate as to specify that a bridge should be built of metal. There is no denying the fact that sands are more permeable and less compressible than clays; but between these two extremes there is an infinite number of materials the properties of which differ widely one from the other; and the specification of any one factor alone is by no means sufficient to define a given soil. With far more accurate methods now available, there is no longer an excuse for the vagueness and lack of precision which has characterized all phases of earthwork engineering in the past.

<u>Factors affecting Stability</u>. In the past, the assumption has universally been made that in order for a hydraulic-fill core to be stable it must attain practically complete consolidation within a comparatively short time. Nothing could be more erroneous. In the first place, consolidation as used in soil mechanics does not mean solidity. A material may be fairly solid, and yet very incompletely consolidated. Consolidation of saturated earth under pressure is the phenomenon of the gradual transfer of the pressure from the water to the soil particles. Now, the resistance of a material depends essentially upon its shearing strength. The shearing strength is a function of the intrinsic pressure and the angle of internal friction of the material. If the friction angle is low, but the intrinsic pressure high, the material may have a very high shearing strength; this is the case with steel, for example. The two governing factors, pressure and internal friction, are inseparable. As far as the resistance of the material is concerned, it makes no difference what the origin of the intrinsic pressure may be. It might be due to intermolecular attraction, as in metals; to the capillary pressure of water, as in clays; or to an actual applied external pressure.

With these fundamental facts in mind, the effect of consolidation upon the stability of a soil can be very clearly analyzed. Assume a material with a certain angle of internal friction, so saturated with water that the grains are not touching one another. To all intents and purposes, the material is a liquid; not because the angle of friction between the grains is zero, but because there is no contact, and therefore no opportunity for friction to act. Now let a load be applied to the material, in such a manner that the soil particles are retained and the water allowed to escape. In the first moment, the shearing strength of the mixture is still zero, all the pressure being carried by the water. As the water gradually escapes, more and more of the pressure is transferred to the soil. The angle of internal friction is appreciably constant; but the shearing strength of the mass gradually increases, since the effective pressure on the grains is increasing.

It is perfectly evident, then, that in the design of a hydraulic-fill dam (or any other structure involving the phenomenon of consolidation) it is not necessary nor sufficient to use a material which will completely consolidate within a certain time; it is necessary to use a material whose angle of internal friction and coefficient of consolidation are such that it will, within the given length of time, consolidate sufficiently to attain the requisite stability, or shearing strength. Consolidation beyond this point merely adds strength where it is not needed.

Mention has been made of the effect of intermolecular attraction in metals. This attraction exists, of course, between two soil grains in contact with one another, and corresponds in its effect exactly to an applied pressure, causing a certain definite shearing strength on the plane of contact. In coarse materials, the effect is negligible; but in the finer soils, there are a large number of inter-molecular bonds per unit of volume, and their effect is very marked. This property is called the true cohesion of the soil. It is evident that true cohesion causes a shearing strength over and above that due to the net applied pressure between grains. So that if it is found that to attain a certain shearing strength a material must reach a certain percentage consolidation, the actual shearing strength at this point is greater by the amount due to true cohesion. Computations made on the basis of applied pressure alone are therefore in error on the side of safety.

These conceptions provide a definite basis for the selection of suitable core material for a hydraulic-fill dam. In the original design, a computation should be made of the total shearing strength necessary in the core in order to balance the applied forces for the worst condition. It may also be assumed that a certain definite time may elapse before the dam will come into use. The core material must be so selected that at the end of this time the mass will have reached such a state of consolidation that the requisite shearing strength is obtained. It will, of course, be necessary to make tests on various mixtures of the available borrow pit material, in order to determine just what composition will consolidate at this rate. And it is utter folly to waste fine material, thereby sacrificing water-tightness, if the inclusion of this material will not decrease the rate of consolidation beyond that determined.

In general, a dam core should have a low permeability. The coefficient of consolidation is the quotient of the coefficient of permeability divided by the coefficient of compressibility. So that with the same permeability, more rapid consolidation will be obtained with less compressible material.

The Ideal Core. The foregoing considerations make it possible to obtain an idea of the best type of material for a hydraulic-fill core. First, it should be impermeable; second, it should be as incompressible as possible; and third, it should have a high angle of internal friction. These conditions are met, in general, by graded mixtures of particles, ranging from coarse silt to clay. A high degree of true cohesion is desirable. Therefore, the amount of fine sand present, if any, should be small, as any appreciable quantity seriously interferes with the true cohesion of the mass. A uniform mixture is to be avoided, as uniform materials are apt to be unstable under pressure, exhibiting the quicksand effect due to hydrodynamic lubrication upon a slight change in volume.

The presence of any appreciable percentage of flat-grained particles makes the material entirely unsuitable, as even a small amount of mica or other flat-grained constituent increases the compressibility enormously.*

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* See paper by the writer, "The Compressibility of Sand-Mica Mixtures." Proceedings, Am.Soc. C.E., February, 1928.

The Germantown core complies very well with the foregoing specifications. The actual optimum composition for a given borrow pit material can be determined only by tests.

In connection with the above remarks, it will be well to note that Mr. Allen Hazen's* recommendations that all particles smaller than 0.01 mm. be excluded from the core will not provide as good a dam as would be obtained with the inclusion of finer material. First, such a core would be more uniform than if finer material were used. Second, it would be much more permeable. Third, the mass cohesion would be so far reduced as to be almost negligible. It is true that the coarser core would consolidate more rapidly than the finer: but this high rate of consolidation is quite unnecessary to afford stability, and is obtained at the expense of the other important items mentioned.

A distinctly opposite view seems to have been taken by Mr. Thomas H. Wiggin. In his discussion of Mr. Paul's paper,** Mr. Wiggin states that "In the future, some one may add a little of a chemical -- an electrolyte -- to the sluicing water and precipitate core material which otherwise would be unsuitable." It is hard to imagine a more dangerour procedure. No one would build a dam out of jelly; and colloidal clay coagulated by the addition of an electrolyte

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* Hydraulic-Fill Dams. Transactions, Am.Soc. C.E., 1920.

** Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District. Transactions, Am. Soc. C.E., 1922.

is nothing more or less than a colloidal gel. It is porous, containing up to 95% voids; compressible as a sponge; low in internal resistance; in short, the least suitable material to form a part of any structure which is expected to be stable.

No definite program of tests can at present be formulated for the determination of the ideal core material, beyond that of trying out various mixtures and finally obtaining one which fulfills the conditions. The writer has just received a substantial quantity of the borrow pit material from the site of the Germantown Dam, and intends to make a thorough study of the material in the immediate future. If possible, a definite program will be worked out as a result of this study, in order to minimize the amount of work which will be required in subsequent tests.

Notes on Construction. In order to obtain a core which complies in composition with the requirements previously mentioned, it is necessary that special care be taken during construction. The sloughing of sand and gravel into the core pool is a dangerous item, and must be guarded against at all times. The use of floating baffles anchored to the beaches was effective in the Miami work and is to be recommended.

All true colloidal material must be excluded. This will usually be accomplished automatically, as the colloids do not precipitate and are therefore carried off through the overflow. The pool should never be allowed to evaporate if construction is stopped, but should be drained off quickly. Otherwise, a thin layer of colloidal gel will be deposited entirely across the core, introducing a definite plane of weakness.

The rate of sluicing should be determined beforehand as accurately as possible, and maintained. Continuous sluicing will produce the most homogeneous core and is to be recommended whenever feasible.

Supplementary Remarks. It is hoped that these researches will be only the first step in a thorough and long-continued study of the properties and behavior of hydraulic-fill dams. The hydraulic-fill method of construction is sound and economical, and if used properly will give excellent results. The necessary tests upon the physical properties of the material, and the manner of making these tests, are well established. A fundamental theory has been developed, and found to check in one case investigated. Upon this basis, future investigations may be carried on in an intelligent manner, throwing more and more light on the phenomena of cohesion, friction, and consolidation, which are very complicated effects, and at the same time very important.

One type of study which should always be made is that of testing samples from the interior of the core during construction. The progress of consolidation during this time is very difficult to treat on a theoretical basis. The steep slope of the theoretical curve at the zero axis of time shows that the rate is very fast; and if the constants of the material are known, a fair idea of its state of consolidation and its corresponding shearing resistance may be determined from the curve. Greater accuracy, however, is desirable; and with this end in view, it is advisable to provide for the removal and testing of samples during construction. This procedure would be of far more practical and scientific value than the present tests with rods and cannon-balls, and not very expensive.

SUMMARY OF RESULTS AND CONCLUSIONS

SECTION I. The fundamental theory underlying the behavior of hydraulic-fill dams has been investigated and the following developments obtained:

A. A graphical method for analyzing the state of stress on any plane in the shell, with the core exerting full hydrostatic pressure upon the inner surface of the shell.

B. Theoretical rates of consolidation of the core for various conditions, including:

The general equation for the rate of consolidation of saturated earth.

Case 1. Theoretical rate of consolidation of a layer of soil in which the flow of water is linear and the initial hydrostatic pressure constant throughout the layer.

Case 2. Theoretical rate of consolidation of a layer of soil in which the flow of water is linear and the initial hydrostatic pressure varies as a linear function of distance in the direction of flow.

Case 3. Theoretical rate of consolidation of a circular cylinder of soil in which the flow of water is radial and the initial hydrostatic pressure is a function of the distance from the center, or constant.

The consolidation curves resulting from the theory have been accurately plotted, and the values of time factors tabulated.

A method of interpolation by the use of an elliptical cylinder as an approximation to the actual core section has been obtained, whereby the time factors and the proper corresponding time scale may be determined for any given core section. In this manner the more complicated theoretical work has been concentrated to a point where the computation of an actual case is a very simple process.

<u>SECTION II.</u> Undisturbed samples of the core material of the Germantown Dam have been obtained from a shaft four feet in diameter sunk into the core; additional samples have been taken from an ordinary drillhole nearby. The samples have been subjected to various tests upon their physical properties, in an endeavor to obtain an idea of the permeability and the state of consolidation of the core.

<u>A.</u> In connection with these tests, the following new developments in apparatus and methods have been made:

1. A special type of permeameter for the testing of undisturbed samples in their natural state.

2. An improved method for measuring the permeability of a sample during a compression test by direct observations.

3. A notable improvement in the Wiegner apparatus for the mechanical analysis of fine-grained fractions by simultaneous sedimentation.

B. The tests performed may be enumerated briefly as follows:

1. Direct permeability determinations, in natural state and remolded.

2. Standard compression tests for the determination of the physical constants of the material, combined in certain cases with direct permeability observations during the test.

3. Tests of compressive strength unconfined, in natural state and remolded, with determinations of the modulus of elasticity and the angle of internal friction.

4. Determinations of natural water content and limits of consistency.

5. Mechanical analyses.

6. Specific gravity tests.

C. The tests have furnished the following information:

1. The permeability in natural state varies considerably, the general order of magnitude being from 10^{-5} to 10^{-6} cm. per minute. It is shown that the values obtained are in general higher than the true coefficient on account of the presence of minute seams within the material which close only

partially after the lapse of a considerable length of time.

2. The material is quite resistant, having a compressive strength of from one to two kg./sq.cm., and an angle of internal friction of about 25°.

3. The material is a graded mixture, containing particles of all sizes from the finest sands fo clays, with a very small percentage of true colloids.

4. The position of the material on the Soil Chart, as determined by the physical constants, indicates that it is inherently stable.

5. The tests show that the core is in general about 25% consolidated.

SECTION III. The information obtained in the preceding sections has been utilized as follows:

A. 1. A graphical determination of the state of stress in the shell of the Germantown Dam has been made. The result indicates that the dam section as built was inherently stable during construction.

A. 2. A theoretical curve for the consolidation of the Germantown core has been constructed. The theoretical percentage consolidation corresponding to the time at which samples were removed is 25%, which agfees with the results of the tests.

<u>B.</u> The following specific conclusions may be drawn:

1. The angle of internal friction and the specific gravity of the mass of the shell are of the utmost importance with regard to stability during construction.

2. The shell must include only clean, coarse material, all fine clay particles being eliminated by proper sluicing.

3. The core slope should in general be steep to insure stability. The slope of 1:2 used in the Miami cores is sound in principle.

4. The theory of consolidation herein developed furnishes a convenient basis for computation, and checks exactly with the case investigated.

5. The permeability of the material is not the only factor influencing its rate of consolidation. Its compressibility is of the same importance as its permeability. An accurate idea of the rate of consolidation of a material can be obtained only by tests.

6. The specification that core material should contain only particles between certain limiting grain sizes is fundamentally inexact and erroneous.

7. A high degree of consolidation is neither attainable nor necessary.

8. The ideal core material is impermeable; it is as incompressible as possible; its angle of internal friction is high; and it possesses strong true cohesion. Therefore it should be non-uniform as to size of grains; should contain no colloids nor flat-grained particles; should have a very small percentage, if any, of fine sand; and should be so constructed as to attain a maximum of homogeneity. The actual optimum composition for a given borrow pit material can be determined only by tests.

9. The true hydraulic-fill dam, if designed and constructed according to the principles herein laid down, is far superior to any other type of sluiced dam.

BIBLIOGRAPHY

TERZAGHI, Charles. Erdbaumechanik auf bodenphysikalischer Grundlage. Franz Deuticke, Leipzig und Wien. 1925.

TERZAGHI, Charles. Principles of Final Soil Classification. Public Roads, Vol. 8, No. 3, May, 1927.

PAUL, Charles H. Core Studies in the Hydraulic-Fill Dams of the Miami Conservancy District. Transactions, American Society of Civil Engineers, 1922.

HAZEN, Allen. Hydraulic-Fill Dams. Transactions, American Society of Civil Engineers, 1920.

JUSTIN, Joel D. The Design of Earth Dams. Transactions, American Society of Civil Engineers, 1924.

INGERSOLL, L. R., and ZOBEL, O. J. An Introduction to the Mathematical Theory of Heat Conduction. Ginn and Company, 1913.

RIEMANN, Bernhard. Partielle Differentialgleichungen und deren Anwendung auf physikalischen Fragen. F. Vieweg und Sohn, Braunschweig. 1882.

FOURIER, Joseph. La Théorie Analytique de la Chaleur. English Translation, annotated, by Alexander Freeman. Cambridge University Press. 1878.

GRAY, Andrew, MATHEWS, G. B., and MACROBERT, T.M. Bessel Functions. MacMillan and Co., Limited. London. 1922.

WOODS, Fredrick H. Advanced Calculus. Ginn & Co. 1926.

JANICSEK, Jozsef. Alkalmas-e a Kocka a Törőszilárdság Megállapítására? Technika, Vol. VIII, Sec. 3-4, Budapest. 1927.

ATTERBERG, A. Die Plastizität der Tone. Internationale Mitteilungen für Bodenkunde. 1911. Vol. 1.

WIEGNER, G. Ueber eine neue Methode der Schlämmanalyse. Zentralblatt für die gesamte Landwirtschaft, Band I, No. 1. 1920.

ÖRTENBLAD, A. L. Mathematical Theory of the Consolidation of Mud Deposits. M.I.T. C.E. Dept. Thesis. 1926.

GILBOY, G. The Compressibility of Sand-Mica Mixtures. Proceedings, American Society of Civil Engineers, February, 1928.