ESSAYS ON FINANCE

by

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Submitted to the
Sloan School of Management
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Management

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 1993

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Abstract

This thesis, entitled "Essays on Finance", consists of three independent essays.

Essay one considers the structuring and pricing of options that protect holders of corporate bonds (either zero coupon or coupon bearing) from a reduction in their value. For zero coupon bonds, the options are structured in the context of a model developed by Merton (1974); for coupon bonds, they are structured in the context of a model developed by Black and Cox (1976). Two types of options are considered: A put on the bond's price and a call on the bond's spread above the risk-free rate. It is shown that in the case of coupon bonds, both options must be of the barrier type; also the call on the coupon bonds spread must be capped. The partial differential equation and boundary conditions that such options' price must obey is derived and solved numerically using finite difference methods thereby enabling us to establish the pricing properties of these options. Among other properties, it is shown that

1. the put on the bond price will never be exercised early unless it is a forced exercise;
2. the price of the call on the spread may exceed current value of the spread and
3. there are circumstances, in which higher volatility may reduce the price of the call on the spread of the coupon bond.

Essay two considers the pricing of lookback call and put options on treasury bond prices. Two valuation models for such options are developed; one is based on the Cox, Ingersoll and Ross (1985) square root interest rate process, and the other on the extended Vasicek interest rate process. The option prices given by the two models are compared and the differences are linked to the different assumptions about the evolution of the interest rate in each model. Also, the pricing properties of these options are established and compared to those of Goldman, Sosin and Gatto (1979) for lookback options on stocks. It is shown that a meaningful comparison necessitates the performance of a particular type of test that investigates the effect of changes in the underlying bond price on the option price, while keeping the discount factor constant.

Essay three considers asset securitization by commercial banks. A model that rationalizes asset securitization by commercial banks as a response to an asymmetric information problem facing bank managers and investors is developed. The model is similar to the Myers and Majluf (1984) model. It is shown that both asset securitization and an equity

issue improve a bank's equity to assets ratio. Note that bank regulators required such an improvement in the mid-1980's, the period in which banks began securitizing assets. The model characterizes the circumstances under which banks will

prefer to boost the equity to assets ratio through securitization rather than through an equity issue.

Thesis Committee:
John C. Cox, Nomura Professor of Finance (Chairman)
Stewart C. Myers, Gordon Y. Billard Professor of Finance
Julio J. Rotemberg, Professor of Applied Economics

ACKNOWLEDGMENT

I am deeply indebted to my two principal advisors, Professor John Cox for Essays I and II and Professor Stewart Myers for Essay III for their guidance through all phases of this thesis.

It is rarely that one sees an individual with a knowledge of option pricing theory as deep as that of John Cox. The typical pattern of my conversations with him was as follows: I began explaining what my question was, and halfway through my explanation, he would stop me and give me the answer! His help was truly invaluable to me. Furthermore, John was very patient with me even when I requested something that was probably unreasonable. Our conversations always ended up being both productive and entertaining because John possesses a unique sense of humor. I will definitely miss our discussions.

Stewart Myers guidance was equally indispensable to me. Stewart possesses a unique sense of intuition as to which results one ought to try to prove formally. Fortunately for me, he was never wrong. He was always very friendly and relaxed and spent many hours trying to untangle my arguments. Stewart has also been a source of support through the ups and downs of the early years of my Ph.D. studies.

Julio Rotemberg deserves special thanks for agreeing to serve as the "outside reader" of this thesis. He made a number of suggestions that improved all three essays. I regret not having had the opportunity to pursue more fully some of his suggestions.

Professor Jim Orlin, Ph.D. program Chairman, and Sharon Cayley, Ph.D. program coordinator helped me a lot in dealing with administrative matters. I am most grateful to both of them.

Patrick Edsparr, Arnout Eikenboom, Kazu Ohashi, Angel Serrat Tubert and Sung-Hwan Shin have been great colleagues and friends. Our discussions have always been a source of ideas for me, and hopefully for them too!

Sally Santiago did an excellent job of typing the final document. Without her help, it would have taken much longer to complete this thesis.

Finally, my parents Antonis and Nedy Pierides have been a constant source of support throughout my student years. I dedicate this thesis to them.

Contents

I The Structuring and Pricing of Credit Risk Derivatives 11

1 Introduction 11

2 Merton's Model of Zero-Coupon Corporate Bond Pricing 13

3 Structuring and Pricing Credit Risk Derivatives for Zero-Coupon Bonds . . 15

4 Coupon Bonds: The Black-Cox Model 46

5 Structuring and Pricing Credit Risk Derivatives for Coupon Bonds 57

6 Conclusion 83

Appendix.A. 85

Appendix.B. 90

Appendix.C. 96

II The Pricing of Lookback Call and Put Options on Treasury Bond Prices 143

1 Introduction 143

2 Contractual Features of Lookback Call and Put Options on Treasury Bond Prices 145

3 A Valuation Model for Lookback Call and Put Options on Zero-Coupon Treasury Bond Prices Based on the Cox, Ingersoll and Ross (1985) Square Root Process for the Short Term Interest Rate 147

4 A Valuation Model for Lookback Call and Put Options On Treasury Bond Prices Based on the Extended Vasicek Process for the Short Term Interest Rate 155

5 A Comparison of Cox, Ingersoll and Ross and Extended Vasicek Lookback Option Prices 163
6 Pricing Properties of Lookback Options at Time 0 170
7 Pricing Properties of Lookback Options at Time > 0 178
8 Conclusion 183
Appendix.A. 184

III Asset Securitization by Commercial Banks: An Asymmetric Information Explanation **187**

1 Introduction 187
2 The Main Features of Asset Securitization Transactions 189
3 Prior Research 193
4 A Binomial Model of the Issue-Securitize Decision 199
5 CASE 1: Management has no Inside Information About Asset 1 (i.e., $\delta = 0$), Debt is Risk Free ($D_M = D_B$), and $r = 0$ 207
6 CASE 2: Both Assets Risky But Asset 1 Less Risky Than Asset 2 ($0 < \delta < 1$), Debt Risk-Free, and $r = +1$ 213
7 CASE 3: Debt is Risk-Free and Both Assets Are Equally Risky, i.e. $\delta = 1$ and $r = +1$ 215
8 CASE 4: Both Assets Risky but Asset 1 Less Risky than Asset 2, Debt Risk-Free and $r = -1$ 218
9 CASE 5: Both Assets Equally Risky ($\delta = 1$), Debt Risk-Free, and $r = -1$. 225
10 CASE 6: Debt is Risky, $\delta = 0$, i.e. Management Has No Inside Information About Asset 1 and $r = 0$ 229
11 CASE 7: Both Assets Risky But Asset 1 Less Risky Than Asset 2 ($0 < \delta < 1$), Debt Risky and $r = +1$ 230
12 CASE 8: Both Assets Equally Risky, i.e. $\delta = 1$ and Debt Risky and $r = +1$ 239
13 CASE 9: Asset 1 Less Risky than Asset 2, i.e., $\delta < 1$, Debt Risky and $r = -1$ 242
14 CASE 10: Both Assets Equally Risky, i.e., $\delta = 1$, Debt Risky and $r = -1$. 249
15 Conclusion 250
Appendix.A. 253

Appendix.B. 258

Essay I

The Structuring and Pricing of Credit Risk Derivatives

1 Introduction

Recently, a number of Wall Street firms began considering the possibility of offering a new breed of derivative technology, known as credit risk derivatives (see Investment Dealers Digest, July 6, 1992). These are instruments whose payoff is tied to the credit characteristics of a particular asset. For example, a portfolio manager holding some AA rated bonds could buy a credit risk derivative that would compensate him if the securities are downgraded to, say, A. This paper examines the pricing of credit risk derivatives. It focuses on the pricing of derivatives that protect the purchasers of corporate bonds from downgradings. To price credit risk derivatives, we need a model of the determinants of a corporate bond's price. Accordingly Section 2 presents a model of the determinants of a corporate bond's price that was developed by Merton (1974). Section 3 shows how to define and price credit risk derivatives in the context of Merton's model.

Merton's model covers the pricing of zero-coupon corporate bonds. This paper also considers the pricing of credit risk derivatives on coupon bonds using a model of corporate coupon bond pricing developed by Black and Cox (1976). Section 4 of the paper describes

the Black and Cox model and Section 5 shows how to define and price credit risk derivatives in the context of their model. Section 6 offers concluding remarks.

2 Merton's Model of Zero-Coupon Corporate Bond Pricing

The following assumptions were made:

1. r = riskless interest rate is constant;

2. Firm value (V) dynamics: $dV = aVdt + sVdz$; dz is a standard Wiener process;

3. Firm has outstanding a single issue of debt promising B at T_B. If payment is not made, the bondholders take over the company; $f(V,t)$ = debt value at time t, is the solution to the following p.d.e

$$0.5s^2V^2f_{vv} + rVf_v - rf + f_t = 0 \tag{I.1}$$

The boundary conditions are:

$$\begin{aligned} f(0,t) &= 0 \\ f(V,t) &\leq V \quad \text{for any } t \\ f(V,T_B) &= \min(V,B) \end{aligned}$$

The closed form solution can be written as

$$f(V,t) = B\exp(-r(T_B-t))(\Phi(h_2(d,s^2(T_B-t))) + d^{-1}\Phi(h_1(d,s^2(T_B-t)))) \tag{I.2}$$

where $\Phi(x)$ = standard normal distribution function

$$
\begin{aligned}
d &= V^{-1} B \exp(-r(T_B - t)) \\
h_1(d, s^2(T_B - t)) &= -s^{-1}(T_B - t)^{-.5}(0.5s^2(T_B - t) - \log d) \\
h_2(d, s^2(T_B - t)) &= -s^{-1}(T_B - t)^{-.5}(0.5s^2(T_B - t) + \log d)
\end{aligned}
$$

The above is the solution for $t < T_B$, with $f(V, T_B) = \min(V(T_B), B)$.

Defining $e(V, t)$ as the spread above the risk-free rate at which the debt trades at t, we can rewrite the solution for $t < T_B$ as

$$f(V, t) = B \exp(-(r + e(V, t))(T_B - t)) \tag{I.3}$$

where

$$e(V, t) = -(T_B - t)^{-1} \log(\Phi(h_2(d, s^2(T_B - t))) + d^{-1}\Phi(h_1(d, s^2(T_B - t)))).$$

Note that $e(V, t)$ is not defined for $t = T_B$.

3 Structuring and Pricing Credit Risk Derivatives for Zero-Coupon Bonds

We now consider the structuring and pricing of derivatives which protect holders of zero-coupon bonds from credit rating downgradings. One such derivative is a put option on the bond's price; another is a call option on the spread above the risk-free rate at which the bond trades. We begin with the put option.

3.1 An American Put Option on the Zero-Coupon Bond Price

Consider the following American put option: At any $t < T$, where T = option maturity $< T_B$ its owner can sell the bond at a time varying exercise price $K(t) = B\exp(-(r+g)(T_B - t))$, i.e., the exercise price is the price of the bond for a fixed spread $g(> 0)$ above the risk free rate. To see why this option qualifies as a credit risk derivative, suppose that we know that a AA credit rating implies that the bond is priced at a spread g; furthermore, assume that a portfolio manager bought the bond when it was rated AA. If this portfolio manager also purchases this American put, he is protected from a downgrading: For example, if the bond is downgraded to A, its price will decrease and hence the put will be in the money; by exercising it, the portfolio manager is guaranteed a value equal to the price at which the bond should trade if it had not been downgraded.

I now present an early exercise result.

Proposition 1 *Early exercise of this American Option can never be optimal. Hence, its price will be the same as the price of an otherwise identical European Option.*

PROOF: It is shown that the price of this put can never be lower than, or equal to, the

exercise value. Hence the put should never be exercised early.

To show that the put price can never be lower than, or equal to, the exercise value, we show that if it is, an arbitrage opportunity exists. Assume, therefore, that the put price is lower than, or equal to, the exercise value at some time t prior to option maturity, i.e., assume that

$$F(t) \leq K(t) - f(V,t) \tag{I.4}$$

where

$$\begin{aligned} F(t) &= \text{put price} \\ K(t) &= \text{exercise price} = B\exp(-(r+g)(T_B - t)) \\ f(V,t) &= \text{bond price} \end{aligned}$$

Adopt the following arbitrage strategy: Buy the put, buy the bond and borrow an amount $K(t)$ at the risk-free rate. The time t payoff is clearly non-negative. We wait until the maturity date of the put and then close our position.

Assume that the option is in the money at maturity. We exercise it by selling the bond which gives us a payoff of $K(T)$, the exercise price. To repay our time t borrowing of $K(t)$ we need $K(t)\exp(r(T-t))$. Therefore,

$$\begin{aligned} \text{Net payoff at } T &= K(T) - K(t)\exp(r(T-t)) \\ &= B\exp(-(r+g)(T_B - T)) - B\exp(-(r+g)(T_B - t) + r(T-t)) \\ &= B\exp(-(r+g)(T_B - T)) - B\exp(-(r+g)(T_B) + rT + gt) \end{aligned} \tag{I.5}$$

$$= B\exp(-(r+g)(T_B - T))[1 - \exp(-(r+g)(T) + gt + rT)]$$

$$= B\exp(-(r+g)(T_B - T))[1 - \exp(g(t-T))].$$

Note that since $g > 0$ and $t < T$, $\exp(g(t-T)) < 1$. Hence net payoff at T is positive. Therefore, an arbitrage opportunity exists.

Now suppose that the option is not in the money at T. It follows that the bond is worth more than $K(T)$. We sell the bond for an amount greater than $K(T)$ and repay $K(t)\exp(r(T-t))$ for the amount we borrowed at time t. Hence,

$$\text{Net Payoff at} T > K(T) - K(t)\exp(r(T-t)). \tag{I.6}$$

Since we showed earlier that the right hand side of the above inequality is positive, it follows that our net payoff at T is positive. Again, an arbitrage opportunity exists.

The implication of the above result is that to avoid arbitrage opportunities we need

$$F(t) > K(t) - f(V,t). \tag{I.7}$$

Hence, the price of the put can never be lower than or equal to the exercise value. The put should never be exercised early.

We now derive the p.d.e. and boundary conditions that this option must satisfy. Let $F[f(V,t),t]$ be the option price at time t. It is assumed that the option price depends on debt value and time. Since debt value depends on the value of the firm, V, we can also

think of option price as a function of V and t, i.e.,

$$F[f(V,t),t] = M[V,t]. \tag{I.8}$$

Consider forming a portfolio consisting of a short position in one option contract and a long position in M_v contracts of a portfolio consisting of the firm's debt and equity.

Let I = portfolio value

$$I = -M + M_v V \tag{I.9}$$

$$dI = -dM + M_v dV. \tag{I.10}$$

Applying Ito's lemma to $M[V,t]$ we get:

$$dM = (M_v aV + M_t + 0.5M_{vv}s^2V^2)dt + M_v sV\ dz. \tag{I.11}$$

Hence,

$$dI = (-0.5M_{vv}s^2V^2 - M_t)dt. \tag{I.12}$$

Since the portfolio is riskless,

$$dI = rI\ dt = (-rM + rM_v V)dt. \tag{I.13}$$

Hence, $M(V,t)$ must obey the following p.d.e:

$$0.5M_{vv}s^2V^2 + M_t - rM + rM_v V = 0. \tag{I.14}$$

The boundary conditions are:

$$M[V,T] = \max[0, K(T) - f(V,T)] \tag{I.15}$$

$$M[V,t] \geq \max[0, K(t) - f(V,t)] \tag{I.16}$$

$$\lim_{v \to 0} \frac{\partial M}{\partial f} = -1 \tag{I.17}$$

$$\lim_{v \to \infty} \frac{\partial M}{\partial f} \text{ is finite.} \tag{I.18}$$

Eq. (I.15) says that at option maturity the option value will be equal to its exercise value if the latter is positive and zero otherwise. Eq. (I.16) says that the price of an American put cannot fall below its exercise value. Eq. (I.17) says that when firm value is very low and hence bond value is very low, a \$1.00 increase in bond value will reduce option value by \$1.00. The reason for this is that under such circumstances, exercise of the option is virtually guaranteed. Eq. (I.18) says that when firm and bond value are very high and hence the option is way out of the money, a marginal increase in bond value will only have a finite impact on option value.

No closed form solution exists to this valuation problem. For this reason, we employ a numerical method of solution.

3.2 Numerical Method of Solution for American Put on Zero-Coupon Bond

The above p.d.e. has variable coefficients. This means that an explicit method of solution (which is computationally easier than an implicit method) may not converge (Ames 1992). For this reason we use a transformation of the state variable which will give us a p.d.e. with constant coefficients. For such a p.d.e., we can give conditions such that an explicit method of solution converges.

Define

$$Y = \log V \tag{I.19}$$

$$M(V,t) = H(Y,t) \tag{I.20}$$

so that

$$\begin{aligned} M_v &= H_y \exp(-Y) \\ M_{vv} &= (H_{yy} - H_y)\exp(-2Y) \\ M_t &= H_t. \end{aligned} \tag{I.21}$$

Making the appropriate substitutions, we obtain the transformed equation

$$0.5s^2 H_{yy} + (r - 0.5s^2)H_y + H_t - rH = 0. \tag{I.22}$$

This is a p.d.e. with constant coefficients.

The boundary conditions in terms of Y are:

$$H[Y,T] = \max[0, K(T) - f(Y,T)] \tag{I.23}$$

where $f(Y,t) = B\exp(-r(T_B - t))(\Phi(h_2) + d^{-1}\Phi(h_1))$

$$\begin{aligned} d &= B\exp(-r(T_B - t) - Y) \\ h_1 &= -s^{-1}(T_B - t)^{-0.5}(0.5s^2(T_B - t) - \log d) \\ h_2 &= -s^{-1}(T_B - t)^{-0.5}(0.5s^2(T_B - t) + \log d) \end{aligned}$$

$$H[Y,t] \geq \max[0, K(t) - f(Y,t)] \tag{I.24}$$

$$\lim_{y \to -\infty} \frac{\partial H}{\partial f} = -1 \tag{I.25}$$

$$\lim_{y \to +\infty} \frac{\partial H}{\partial f} \text{ is finite.} \tag{I.26}$$

The interpretation of these boundary conditions is identical to the earlier interpretation provided for the boundary conditions in terms of V.

To solve the p.d.e. numerically, we consider a finite number of discrete values of Y and t. Let $Y_{\max}(Y_{\min})$ be the maximum (minimum) value of Y considered and, also, consider $n-1$ other discrete values between $Y_{\max}$ and $Y_{\min}$ such that dy represents the difference between two consecutive values, i.e., the step size. Then, any discrete value of Y can be represented as $Y = Y_{\min} + j(dy)$, where j consists of the integers between 0 and n. Likewise, let T be the maximum value of t and 0 the minimum value. Consider a number of discrete

values between 0 and T, such that dt is the step size. Then, any discrete value of t can be represented as $t = k(dt)$ where k consists of the integers between 0 and $T/(dt)$. The choice of dy and dt will be discussed later.

The value of H for the discrete values of Y and t can be written as

$$H(t, Y) = H(k(dt), Y_{\min} + j(dy)) = H(k, j). \tag{I.27}$$

Explicit Numerical Scheme:

For the explicit scheme, the partial derivatives are approximated by the following finite differences:

$$\begin{aligned} H_y &= (H(k+1, j+1) - H(k+1, j-1))/2(dy) \\ H_{yy} &= (H(k+1, j+1) - 2H(k+1, j) + H(k+1, j-1))/(dy)^2 \\ H_t &= (H(k+1, j) - H(k, j))/(dt). \end{aligned}$$

Substituting these finite differences into the p.d.e. we obtain the following difference equation

$$H(k, j) = \frac{(P_u H(k+1, j+1) + P_o H(k+1, j) + P_d H(k+1, j-1))}{(1 + r(dt))} \tag{I.28}$$

where

$$\begin{aligned} P_u &= \left(s^2 + dy\left(r - \left(\frac{s^2}{2}\right)\right)\right)\left(\frac{dt}{2(dy)^2}\right) \\ P_d &= \left(s^2 - dy\left(r - \left(\frac{s^2}{2}\right)\right)\right)\left(\frac{dt}{2(dy)^2}\right) \end{aligned}$$

$$P_o = 1 - s^2 \frac{dt}{(dy)^2}.$$

The boundary condition at maturity eq. (I.23) can be approximated by

$$H(T, j) = \max[0, K(T) - f(T, j)] \tag{I.29}$$

Boundary condition eq. (I.24) can be approximated by

$$H(k, j) \geq \max[0, K(k) - f(k, j)] \tag{I.30}$$

where

$$K(k) = B \exp(-(r + g)(T_B - k(dt)))$$

$$f(k, j) = B \exp(-r(T_B - k(dt)))(\Phi(h_2) + d^{-1}\Phi(h_1))$$

$$\begin{aligned} d &= B \exp(-r(T_B - k(dt))) - (Y_{\min} + j(dy)) \\ h_1 &= -s^{-1}(T_B - k(dt))^{-0.5}(0.5s^2(T_B - k(dt)) - \log d) \\ h_2 &= -s^{-1}(T_B - k(dt))^{-0.5}(0.5s^2(T_B - k(dt)) + \log d). \end{aligned}$$

Since this is an explicit method of solution, boundary conditions eq. (I.25) and eq. (I.26) describing the behavior of H for very large and very small Y are not used in the numerical

procedure.

The above difference equation and maturity boundary condition eq. (I.29) give the time t option price only if their solution exceeds the exercise value given by boundary condition eq. (I.30). Otherwise, the option price is set equal to the exercise value. In particular, the solution is obtained as follows: Given $H(T, j)$ for all j from the maturity boundary condition, we obtain $H(T - dt, j)$ for all but two of the j's we considered at time T from the finite difference equation. $H(T - dt, j)$ can be thought of as the price of the option at $T - dt$ for given j, assuming that we decide to keep the option alive, i.e., it is a temporary option price. We then compare the latter with the exercise value of the option at $T - dt$ for given j to take account of boundary condition eq. (I.30). We set the option price at $T - dt$ for given j, denoted $C(T - dt, j)$ equal to the maximum of the two. We then use $C(T - dt, j)$ as the value for $H(T - dt, j)$ in the difference equation to obtain the temporary option price at $T - 2(dt)$, $H(T - 2(dt), j)$. We compare the latter with the exercise value at $T - 2(dt)$ and set the option price at $T - 2(dt), C(T - 2(dt), j)$ equal to their maximum. We proceed in this way until we reach time 0. Note that, in general, at time t we obtain $H(t, j)$ for all but two of the j's for which we know $H(t + dt, j)$ at time $t + dt$. Hence, by time 0, we get $W(0, j)$ for all but $2T/dt$ of the j's considered at time T.

From Ames (1992), a sufficient condition for the convergence and stability of this numerical scheme is the positivity of P_u, P_d, and P_o. To ensure this, dy and dt must be chosen so that

$$\frac{dt}{(dy)^2} < \frac{1}{s^2}; \quad dy < \frac{s^2}{\left(r - \frac{s^2}{2}\right)} \tag{I.31}$$

Implicit Numerical Scheme(Cranl.-Nicholson)

For the implicit scheme the partial derivatives are approximated by the following finite differences;

$$\begin{aligned} H_y &= \frac{(H(k+1,j+1)-H(k+1,j-1))+(H(k,j+1)-H(k,j-1))}{4(dy)} \\ H_{yy} &= \frac{(H(k+1,j+1)-2H(k+1,j)+H(k+1,j-1))+(H(k,j+1)-2H(k,j)+H(k,j-1))}{2(dy)^2} \\ H_t &= \frac{H(k+1,j)-H(k,j)}{dt}. \end{aligned}$$

Substituting these finite differences into the p.d.e. we obtain the following set of $n-1$ equations in $n+1$ unknowns $H(k,j), j=0,\ldots,n$:

$$\begin{aligned} &\alpha_1 H(k+1,j+1)+b_1 H(k+1,j)+\alpha_2 H(k+1,j-1) \\ &+\alpha_1 H(k,j+1)+b_2 H(k,j)+\alpha_2 H(k,j-1)=0 \quad \text{for } j=1,\ldots,n-1 \end{aligned} \tag{I.32}$$

where

$$\begin{aligned} \alpha_1 &= \frac{\sigma^2}{4(dy)^2}+\frac{1}{4(dy)}\left(r-\frac{\sigma^2}{2}\right) \\ \alpha_2 &= \frac{\sigma^2}{4(dy)^2}-\frac{1}{4(dy)}\left(r-\frac{\sigma^2}{2}\right) \\ b_1 &= -\frac{\sigma^2}{2(dy)^2}+\frac{1}{dt} \\ b_2 &= -\frac{\sigma^2}{2(dy)^2}-\frac{1}{dt}-r. \end{aligned}$$

Boundary conditions eq. (I.23) and eq. (I.24) can be approximated as in the explicit

scheme. Boundary condition eq. (I.25) tells us that

$$\lim_{y \to -\infty} \frac{\partial H}{\partial f} = -1.$$

Appendix A shows that this condition is equivalent to

$$\lim_{y \to -\infty} \frac{\partial H}{\partial y} = 0. \tag{I.33}$$

This condition is approximated by

$$H(k,1) = H(k,0) \quad \forall k. \tag{I.34}$$

Boundary condition eq. (I.26) tells us that

$$\lim_{y \to +\infty} \frac{\partial H}{\partial f} \text{ is finite.}$$

Appendix A shows that this condition is equivalent to

$$\lim_{y \to +\infty} \frac{\partial H}{\partial y} = 0. \tag{I.35}$$

This condition is approximated by

$$H(k.n) = H(k,n-1) \quad \forall k. \tag{I.36}$$

Define

$$H^k = \begin{bmatrix} H(k,0) \\ H(k,1) \\ \vdots \\ H(k,n) \end{bmatrix}. \tag{I.37}$$

We can write the system of equations derived above and boundary conditions eq. (I.34) and eq. (I.36) as:

$$BH^k + AH^{k+1} = 0 \tag{I.38}$$

where A and B are the following tridiagonal matrices:

$$A = \begin{bmatrix} 0 & 0 & & & \\ \alpha_2 & \beta_1 & \alpha_1 & & \\ & \alpha_2 & \ddots & \alpha_1 & \\ & & \alpha_2 & \beta_1 & \alpha_1 \\ & & & 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 & & & \\ \alpha_2 & \beta_2 & \alpha_1 & & \\ & \alpha_2 & \ddots & \alpha_1 & \\ & & \alpha_2 & \beta_2 & \alpha_1 \\ & & & 1 & -1 \end{bmatrix}. \tag{I.39}$$

We can rewrite this system of equations as:

$$BH^k = -AH^{k+1}. \tag{I.40}$$

This system of equations together with the maturity boundary condition eq. (I.23) gives the time t option price only if their solution exceeds the exercise value given by boundary condition eq. (I.24). Otherwise, the option price is set equal to the exercise value. In particular, the solution is obtained as follows: Given H^T from the maturity boundary condition, we solve the above system of equations to obtain H^{T-dt}. As before, H^{T-dt} can be thought of as the price of the option at $T - dt$ (for various j's) assuming that we decide

to keep the option alive, i.e., it is a temporary option price. We then compare the latter with the exercise value of the option at time $T-dt$ (for given j) to take account of boundary condition eq. (I.24). We set the option price at $T-dt$ for given j, denoted C^{T-dt} equal to the maximum of the two. We then use C^{T-dt} as the value for H^{T-dt} in the above system of equations to obtain the temporary option price at $T-2(dt)$, $H^{T-2(dt)}$. We compare the latter with the exercise value at $T-2(dt)$ and set $C^{T-2(dt)}$ equal to their maximum. We proceed in this way until we reach time 0.

From Ames(1992) this scheme is unconditionally stable.

3.3 Numerical Results for American Put on Zero-Coupon Bond

A computer program that implements the above described numerical valuation techniques was developed in Baakkonen (1993). Numerical results were obtained using this computer program for various parameter values. For each set of parameter values both the implicit and explicit valuation schemes were implemented to check the consistency of the output.

The output is presented in Tables A1-A5 of Appendix C. Table A1 presents results from the explicit scheme for the following parameter values:

s=0.24 per year

T = 1 year

T_B = 1.2 years

r = 0.06 per year

B = 20

g = 0.01

The first column gives the value grid point j, the second column gives firm value V at that

point and the third column gives the bond price f (all at time 0). As expected, f increases with V. Furthermore, note that at low values of V, f is only slightly less than V. The reason for this is that when V is low, the probability that it will increase above 20 (the bond's face value) by time 1.2 (the bond maturity) is very small. The time 0 put option price for different firm values and bond prices is given in the fourth column. As expected, the put option price is a decreasing function of V and f.

Column five gives the time 0 temporary option price (temp) for different firm values; as defined above, this is the option price assuming that the option is not exercised. Column six gives the exercise value of the option at time 0. Note that the option price is set equal to the maximum of temp and exercise value.

At the bottom of program output, the early exercise boundary is printed. This gives, for different timeslices, the value of the firm (the boundary column) and the value of the bond below which the option is exercised early. Note that this American option is never exercised early. This confirms our earlier no early exercise result (proposition 1). To check this further, a European option version of the program was run and the option prices obtained were identical to the ones obtained for the American option.

Effect of Option Maturity - Table A2

The parameters used in Table A2 are the same as those in Table A1 except for a lower maturity ($T = 0.5$). Note that the option price is consistently lower for a lower maturity.

This result is similar to a result in Cox and Rubinstein (1985) (Proposition 3, page 146) which states that for American put options higher maturity results in higher price. However, Cox and Rubinstein (1985) point out that this result does not hold for European

puts with constant exercise prices. In the case of a European put of the type defined in this paper, this result will hold as shown below.

Proposition 2 *The higher the maturity of the put option (European or American), the higher its price.*

PROOF: First consider American puts. Let $H(t,Y(t),T)$ and $H(t,Y(t),T^1)$ be the prices at time t of two American puts with maturities T and T^1 respectively, where $T^1 > T$. Suppose $H(t,Y(t),T) > H(t,Y(t),T^1)$. Adopt the following arbitrage strategy: Sell the option with maturity T and buy the option with maturity T^1. The time t payoff is positive. If the option you sold is exercised at any time $\tau, t < \tau \leq T$, your position is worth

$$-\max[0, K(\tau) - f(Y(\tau),\tau)] + H(\tau,Y(\tau),T^1 - \tau).$$

If this payoff is positive, sell the option you purchased and close the position at a profit. If this payoff is not positive, exercise the American put you bought and receive

$$\max[0, K(\tau) - f(Y(\tau),\tau)]$$

so that no further gain or loss results. Hence, an arbitrage opportunity exists. In equilibrium, we must have $H(t,Y(t),T) < H(t,Y(t),T^1)$

Now consider European puts. Let $H(t,Y(t),T)$ and $H(t,Y(t),T^1)$ be the prices at time t of two European puts with maturities T and $T^1(T^1 > T)$. Suppose $H(t,Y(t),T) > H(t,Y(t),T^1)$. Adopt the following arbitrage strategy: Sell the option with maturity T and buy the option with maturity T^1: the time t payoff is positive. At the maturity of the

option you sold T, your position is worth

$$-\max[0, K(T) - f(T, Y(T))] + H[T, Y(T), T^1 - T].$$

From Proposition 1, know that

$$H[T, Y(T), T^1 - T] > \max[0, K(T) - f(T, Y(T))].$$

Hence, you can close your position at a profit. An arbitrage opportunity exists. Therefore, in equilibrium, need

$$H(t, Y(t), T) < H(t, Y(t), T^1).$$

Effect of Volatility - Table A3

Merton (1974) shows analytically that higher volatility reduces bond prices. This is confirmed by comparing the results in Table A2 with those in Table A3: For given V, the bond price for s =0.324 (Table A3) is lower than the bond price for s =0.24 (Table A2). Also comparing Tables A2 and A3, we see that for given firm value, the option price is higher, the higher the volatility. This is the result of two separate effects. The first effect relates to the previously mentioned relation between bond price and volatility. For given firm value, the higher the volatility, the lower the bond price. Since this is a put option, lower bond price increases the option price. The second effect is a direct effect of higher volatility on put prices analogous to the effect of higher volatility on the price of a put option on a common stock.

Effect of Interest Rate - Table A4

The influence of interest rates on put option prices for given firm value is ambiguous. The reason for this is that a higher interest rate has two opposing effects on option price: First, a higher interest rate reduces bond prices (Merton (1974)) and hence increases put prices for given firm value; second, a higher interest rate reduces both the exercise price and the present value of the exercise price, thereby reducing put prices. It is not clear which effect will dominate. A situation where a higher interest rate results in lower option prices is illustrated in Table A4.

Table A4 incorporates a higher interest rate than Table A3; otherwise the parameters are the same. Comparing the two tables, we see that a higher interest rate results in lower option prices for given firm value (as expected, it also results in lower bond prices).

Effect of Striking Price - Table A5

Table A5 incorporates a higher g, i.e., a lower exercise price than Table A4; otherwise, the parameters are the same. As expected, the lower the striking price (i.e., the higher g), the lower the option price.

3.4 Another Credit Risk Derivative: A Call Option on the Zero-Coupon Bond's Spread $e(V,t)$

In the context of Merton's model described above, consider an American call option on $e(V,t)$ with maturity $T < T_B$ and exercise price $K = g$. If the option is exercised, the payoff is $\max(0, e(V,t) - g)$. To avoid confusing percentages with monetary units we assume that $e(V,t)$ and g are expressed in numerical form and, hence, the payoff is in monetary units.

This option qualifies as a credit risk derivative because its value increases with increases in the bond's spread, i.e., with decreases in the bond's price.

We cannot obtain a no early exercise result for this call option because the option is written on $e(V,t)$ which is not traded. In the earlier case of the put option, we managed to get a no early exercise result because the option was written on the corporate bond—a traded asset; of course, $e(V,t)$ is a function of the bond price f and we would be able to get a no early exercise result if $e(V,t)$ were a linear function of f—since it is non-linear, we cannot. This is stated formally below.

Proposition 3 *The Price of the American call can be equal to its exercise value at any time, in particular, even before the expiration date. In such a case, the call should be immediately exercised.*

PROOF: It is shown that when price equals exercise value, no arbitrage opportunity exists. Hence, this can be an equilibrium situation.

Suppose that at $t_1 < T$,

$$F[e(V,t_1),t_1] = e(V,t_1) - g > 0$$

where F = call price at t_1. To investigate whether arbitrage opportunities exist, at time t_1, buy the call short $e(V,t_1)/f(V,t_1)$ corporate bonds worth $e(V,t_1)$ and lend g. The time t_1 payoff is 0. To see what type of payoff we can expect from this strategy in the future, suppose we exercise the call at $t_2 > t_1$. We pay the exercise price by using the time t_2 proceeds from lending g at time t_1. Clearly, we gain the interest rate on g. By exercising the call we receive $e(V,t_2)$ which enables us to buy $e(V,t_2)/f(V,t_2)$ corporate bonds at t_2.

This may be lower or higher than the number of bonds we need to buy to cover our short position. Hence, the time t_2 payoff can be either positive or negative. It follows that no arbitrage opportunity exists at t_1.

Note that if $e(V,t)$ were a linear function of f, i.e., if $e(V,t) = cf(V,t)$ where c = constant, we would always be assured a profit from this strategy and hence an arbitrage opportunity would exist. The reason is that at t_1 we short $e(V,t_1)/f(V,t_1) = c$ bonds and at t_2 we buy $e(V,t_2)/f(V,t_2) = c$ bonds, i.e., the number of bonds bought is equal to the number of bonds shorted. Hence the time t_2 payoff upon exercising the call will be positive and equal to the interest earned on g. ∎

Let $F(e(V,t),t)$ be the option price at time t. It is assumed that the option price depends on the value of the spread $e(V,t)$ and time. Since the spread depends on firm value, we can also think of option price as a function of V and t, i.e.,

$$F(e(V,t),t) = Z(V,t). \tag{I.41}$$

Using an argument similar to the one employed in the previous section, we can show that $Z[V,t]$ must obey the following p.d.e.

$$0.5Z_{vv}s^2V^2 + Z_t - rZ + rVZ_v = 0. \tag{I.42}$$

In addition, Z must satisfy the following boundary conditions:

$$Z(V,T) = \max[e(V,T) - g, 0] \tag{I.43}$$

$$Z(V,t) \geq \max[e(V,t) - g, 0] \tag{I.44}$$

$$\lim_{v \to 0} \frac{\partial Z}{\partial e} = 1 \tag{I.45}$$

$$\lim_{v \to \infty} \frac{\partial Z}{\partial e} \text{ is finite} \tag{I.46}$$

Equation I.43 says that at option maturity, the option value will be equal to its exercise value if the latter is positive and zero otherwise. Equation I.44 says that the price of an American call can never fall below its exercise value. Equation I.45 says that when firm value is very low and hence the spread is very high, a \$1.00 increase in the spread will increase option value by \$1.00. The reason for this is that under such circumstances, exercise of the option is virtually guaranteed. Equation I.46 says that when firm value is very high and the spread is very low, a marginal increase in the spread will only have a finite impact on option value because the option is way out of the money.

3.5 Numerical Method of Solution for an American Call on Spread of Zero-Coupon Bond

To solve this valuation problem, a numerical procedure similar to the one employed earlier for the put is employed. We first transform the p.d.e. to one with constant coefficients by using a logarithmic transformation of the state variable. The p.d.e. becomes

$$0.5s^2 L_{yy} + (r - 0.5s^2)L_y + L_t - rL = 0 \tag{I.47}$$

where

$$Y = \log V$$

$$Z(V,t) = L(Y,t).$$

The boundary conditions in terms of Y are:

$$L(Y,T) = \max[e(Y,T) - g, 0] \tag{I.48}$$

$$L(Y,t) \geq \max[e(Y,t) - g, 0] \tag{I.49}$$

where

$$e(Y,t) = -(T_B - t)^{-1} \log(\Phi(h_2) + d^{-1}\Phi(h_1))$$

$$\text{where } d = B\exp(-r(T_B - t) - Y)$$

h_1 and h_2 are as defined earlier.

$$\lim_{Y \to -\infty} \frac{\partial L}{\partial e} = 1 \tag{I.50}$$

$$\lim_{Y \to \infty} \frac{\partial L}{\partial e} \text{ is finite.} \tag{I.51}$$

The interpretation of these boundary conditions is identical to the earlier interpretation of the boundary conditions in terms of V.

To solve the p.d.e. numerically we consider the same finite number of discrete values of Y and t as Section 3.2 in the derivation of eq. (I.27).

The value of L for the discrete values of Y and t can be written as:

$$L(t,Y) = L(k(dt), Y_{\min} + j(dy)) = L(k,j).$$

Explicit Numerical Scheme

The partial derivatives are approximated by the finite differences used earlier in the case of the put. Using the same procedure, we obtain the following difference equation:

$$L(k,j) = \frac{(P_u L(k+1,j+1) + P_o L(k+1,j) + P_d L(k+1,j-1))}{1+r(dt)} \tag{I.52}$$

where P_u, P_d, and P_o are as defined in eq. (I.28) .

The boundary condition eq. (I.48) can be approximated by

$$L(T,j) = \max[0, -(T_B - T)^{-1} \log(\Phi(h_2) + d^{-1}\Phi(h_1)) - g] \tag{I.53}$$

where

$$d = B \exp(-r(T_B - T) - (Y_{\min} + j(dy))).$$

h_1 and h_2 are as defined earlier.

The boundary condition in eq. (I.49) can be approximated by

$$L(k,j) \geq \max[0, -(T_B - k(dt))^{-1} \log(\Phi(h_2) + d^{-1}\Phi(h_1)) - g] \tag{I.54}$$

where

$$d = B \exp(-r(T_B - k(dt)) - (Y_{\min} + j(dy))).$$

h_1 and h_2 are as defined earlier.

Since this is an explicit method of solution, boundary conditions eq. (I.50) and eq. (I.51) describing the behavior of L for very large and very small Y are not used in the numerical

procedure.

The above difference equation and maturity boundary condition eq. (I.53) give the time t option price only if their solution exceeds the exercise value given by boundary condition eq. (I.54). Otherwise, the option price is set equal to the exercise value. The solution is obtained by the same recursive technique described for the put in Section 3.2.

Implicit Numerical Scheme (Crank-Nicholson)

For the implicit scheme, the partial derivatives are approximated by the same finite differences described for the put in Section 3.2.

Substituting these finite differences into the p.d.e. we obtain the following set of $n-1$ equations in $n+1$ unknowns $L(k,j), j=0,\ldots,n$:

$$\begin{aligned}&\alpha_1 L(k+1,j+1)+b_1 L(k+1,j)+\alpha_2 L(k+1,j-1)\\&+\alpha_1 L(k,j+1)+b_2 L(k,j)+\alpha_2 L(k,j-1)=0 \quad \text{for } j=1,\ldots,n-1\end{aligned} \tag{I.55}$$

where α_1, α_2, b_1, and b_2 are as defined in eq. (I.32). Boundary conditions eq. (I.48) and eq. (I.49) can be approximated as in the explicit scheme.

Boundary condition eq. (I.50) tells us that

$$\lim_{Y\to-\infty}\frac{\partial L}{\partial e}=1.$$

Appendix B shows that this condition is equivalent to

$$\lim_{Y\to-\infty}\frac{\partial L}{\partial Y}=-\frac{1}{T_B-t}. \tag{I.56}$$

This condition is approximated by

$$L(k,1) = L(k,0) - (dy)\frac{1}{T_B - k(dt)} \quad \forall k. \tag{I.57}$$

Boundary condition eq. (I.51) tells us that

$$\lim_{Y\to\infty} \frac{\partial L}{\partial e} \text{ is finite.}$$

Appendix B shows that this condition is equivalent to

$$\lim_{Y\to\infty} \frac{\partial L}{\partial Y} = 0. \tag{I.58}$$

This condition is approximated by

$$L(k,n) = L(k,n-1) \quad \forall k. \tag{I.59}$$

Define the vector

$$L^k = \begin{bmatrix} L(k,0) \\ L(k,1) \\ \vdots \\ L(k,n) \end{bmatrix}.$$

We can rewrite the system of equations derived above and boundary conditions eq. (I.57) and eq. (I.59) as:

$$BL^k + AL^{k+1} = b^k \tag{I.60}$$

where A and B are as defined in eq. (I.39), and b^k is defined as

$$b^k = \begin{bmatrix} \frac{dy}{T_B - k(dt)} \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{I.61}$$

We can rewrite this system of equations as

$$BL^k = -(AL^{k+1} - b^k). \tag{I.62}$$

This system of equations together with the maturity boundary condition eq. (I.53) give the time t option price only if their solution exceeds the exercise value given by boundary condition eq. (I.54). Otherwise the option price is set equal to the exercise value. The solution is obtained by the same recursive technique described for the put in Section 3.2.

3.6 Numerical Results for American Call on Spread of Zero-Coupon Bond

Numerical results were obtained for various parameter values using the computer program in Baakkonen (1993). Both the implicit and explicit valuation schemes were implemented to check the consistency of the output.

The output is presented in Tables B1-B6 of Appendix C. Table B1 presents results from the implicit scheme for the following parameter values:

s=0.3 per year

$T = 1$ year

$T_B = 2$ years

$r = 0.1$ per year

$B = 10$

$g = 0.02$

The first column gives the value grid point j, the second column gives firm value V at that point and the third column gives the value of the spread e (all at time 0). As expected, the spread e decreases with V since the bond price increases with V. The time 0 call option price for different values of e and V is given in column four. As expected the call option price is an increasing function of e. Column five gives the time 0 temporary option price (temp), which, as defined above is the option price assuming that the option is not exercised. Column six, gives the exercise value of the option at time 0. Note that option price is set equal to the maximum of temp and exercise value.

At the bottom of program output, the early exercise boundary is printed. This gives, for different timeslices, the value of the firm in terms of j (the boundary column) *below* which the option is exercised and the value of the spread $e(V,t)$ *above* which the option is exercised. Note that this American option is never exercised early for the parameter values considered in Table B1.

A distinguishing feature of the results in Table B1 is the fact that the option price sometimes exceeds the current value of the spread. This is due to the non-linear payoff function of this option. For an American call on a stock with constant exercise price we know from Cox and Rubinstein (1985) that the option price can never exceed the contemporaneous

price of the stock. Nevertheless, the arbitrage proof of this result cannot be extended to situations in which the payoff is a nonlinear function of the underlying state variable as shown below.

Proposition 4 *The price of the call can exceed the current value of the spread.*

PROOF: It is shown that when call price exceeds the spread, no arbitrage opportunity exceeds. Hence, this can be an equilibrium situation.

Suppose that at any $t_1 < T$,

$$L(Y, t_1) > e(Y, t_1)$$

i.e., option price exceeds the spread.

To investigate whether an arbitrage opportunity exists, let us try to follow the arbitrage strategy described in Cox and Rubinstein (1985) to exploit situations where the price of a call on a stock exceeds the current stock price. In our context, that strategy involves selling the call and buying $e(Y, t_1)/f(Y, t_1)$ corporate bonds (note $f(Y, t_1)$ = bond price) worth $e(Y, t_1)$. The time t_1 payoff is clearly positive. However, if the call we sold is exercised on or before the expiration date we may be in trouble: We have to pay an amount $e(Y, t_2) - g$ where t_2 equals time of exercise of the option and it is not clear that the bonds we purchase at t_1 will be worth this much at t_2. Hence our payoff may be negative and no arbitrage opportunity exists.∎

Table B2 presents results from the implicit scheme for different parameter values which entail early exercise of the option under particular circumstances. The new parameter values are:

s=0.3 per year

$T = 1$ year

$T_B = 10$ years

$r = 0.15$ per year

$B = 5$

$g = 0.0012$

Note that unlike Table B1, the option price in Table B2 is always lower than the contemporaneous value of the spread e. Hence, no general relationship between the option price and the contemporaneous value of the spread can be established.

The early exercise boundary printed at the bottom of the program output should be read as follows: take timeslice 0; the option should be exercised early if firm value is below value grid point 66; an alternative way of expressing this is to say that the option should be exercised early if the value of the spread is above 5.3362×10^{-3}. This result is confirmed by looking at program output above the early exercise boundary. Note that when firm value is below $j = 66$ or the spread is above 5.3362×10^{-3} the exercise value of the option exceeds the temp price and vice-versa; the interpretation of the early exercise boundary for different timeslices is the same.

A distinguishing feature of the early exercise boundary in terms of the spread ($e(Y,t)$) is that as time increases the value of the spread above which we exercise early sometimes increases and sometimes falls. This is different from the results obtained in Schwartz (1977): in the case of an American call on a stock with a constant exercise price, the boundary in terms of the stock price (i.e., the stock price above which we exercise early) decreases with time. The interpretation of this result in Schwartz (1977) is that early in the option's life

we need a high exercise value to exercise early. However, the non-linear payoff in the case of the option considered in this paper leads to a different result.

Effect of Volatility

Table B3 presents program output for the same parameter values as Table B2 except that volatility is lower. As expected this reduces option prices for given firm value. Furthermore note that the early exercise boundary in terms of the spread $e(Y,t)$ is lower than the one in Table B2, i.e., for a given timeslice the value of $e(Y,t)$ above which we exercise early falls. This is similar to a comparable result in Schwartz (1977). Its interpretation is that since volatility reduces option price we now need a lower exercise value to exercise early.

Effect of Option Maturity

Table B4 presents program output for the same parameter values as Table B2 except that option maturity is lower. As expected, this reduces option prices. Note that in this case, the two early exercise boundaries cannot be compared because a given timeslice in Table B2 corresponds to a different point in calendar time than the same timeslice in Table B4.

Effect of Exercise Price

Table B5 presents program output for the same parameter values as Table B2 except that the exercise price of the call is higher. As expected, this results in lower option prices and also, for a given timeslice, the value of $e(Y,t)$ above which we exercise early increases.

Effect of Interest Rate

Table B6 presents program output for the same parameter values as Table B2 except that the interest rate is higher. To analyze the effect of the interest rate on option prices consider first its effect on the value of e for given firm value. The equation linking the two is derived from Merton's model eq. (I.3) as

$$e(Y,t) = \frac{1}{T_B - t}\{\log B - \log f(Y,t) - r(T_B - t)\}.$$

An increase in r has two effects on e: A direct negative effect through the last term in the above equation and an indirect positive effect through the negative effect of higher r on the bond price f. These two effects oppose each other and as a result, an increase in r could either increase or reduce $e(Y,t)$ for given firm value Y.

In the case illustrated in Table B6, an increase in r results in lower e for given firm value. This should tend to lower option prices for given firm value. However there is another opposing effect to consider: An increase in r reduces the present value of the striking price which tends to increase option prices for given firm value. It is not clear which of these two opposing forces will prevail. In the case illustrated in Table B6, the effect of higher r on e prevails and as a result the option price is lower (compared to Table B2). Also, the boundary in terms of $e(Y,t)$ is lower for all timeslices in Table B6. The interpretation is that since a higher r reduces option prices, we now need a lower exercise value to exercise early.

4 Coupon Bonds: The Black-Cox Model

The above analysis considered the structuring and pricing of options on zero-coupon bonds. We now consider coupon bonds. We first present a pricing model for coupon bonds.

Merton's model assumed that the firm had outstanding a single issue of debt promising B at T_B. Black and Cox (1976) extended Merton's model to allow for the existence of coupon paying debt. They make the same assumptions as Merton except that they postulate the existence of coupon paying debt promising B at T_B and discrete coupons in the amount c (fixed) at discrete points in time. For simplicity, assume that coupons in the amount c have been promised at times $0.2T_B, 0.4T_B, 0.6T_B$ and $0.8T_B$. Furthermore, assume that these coupon payments must be financed by issuing new subordinated securities, i.e., no asset sales are allowed. If a coupon payment is not made, the firm is in default and the promised payment of B becomes due immediately. Black and Cox show that the value of the coupon paying debt at time t, denoted $f(V,t)$ will satisfy the following p.d.e.

$$0.5s^2V^2f_{vv} + rVf_v - rf + f_t + \sum_{i=1}^{4} c\delta(t - (0.2)iT_B) = 0 \tag{I.63}$$

where $\delta()$ is the Dirac delta function. The boundary conditions are:

$$f(V,T_B) = \min(B, V(T_B)). \tag{I.64}$$

$$\lim_{v\to 0} f = v \tag{I.65}$$

$$\lim_{v\to\infty} \frac{\partial f}{\partial v} = 0. \tag{I.66}$$

Boundary condition eq. (I.64) says that at bond maturity, the bond value is equal to the promised amount B if the latter exceeds firm value and to firm value otherwise. Boundary condition eq. (I.65) says that when firm value is very low, bond value equals firm value. Boundary condition eq. (I.66) says that when firm value is very high and the bond is effectively risk-free, a marginal increase in firm value will have no impact on bond value.

The above described stopping condition completes the specification of the program. In light of this stopping condition, no closed form solution exists. For this reason we use a numerical solution.

4.1 Numerical Solution of the Black and Cox Model

We first transform the p.d.e. into one with constant coefficients. Define

$$\begin{aligned} Y &= \log V \\ f(V,t) &= W(Y,t) \\ f_v &= W_y \exp(-Y) \\ f_{vv} &= (W_{yy} - W_y)\exp(-2Y) \\ f_t &= W_t. \end{aligned}$$

Making the appropriate substitutions, we obtain the transformed equation

$$0.5s^2 W_{yy} + (r - 0.5s^2)W_y + W_t - rW + \sum_{i=1}^{4} c\delta(t - (0.2)iT_B) = 0. \qquad \text{(I.67)}$$

The boundary conditions in terms of Y are

$$W(Y, T_B) = \min(\exp(Y), B) \tag{I.68}$$

$$\lim_{Y \to -\infty} W = \exp(Y) \tag{I.69}$$

$$\lim_{Y \to \infty} \frac{\partial W}{\partial Y} = 0. \tag{I.70}$$

The interpretation of these boundary conditions is identical to the interpretation provided earlier for the boundary conditions in terms of V. The stopping condition is the same as the one described above.

To solve the p.d.e. numerically, we consider a finite number of discrete values of Y and t. Let $Y_{\max}(Y_{\min})$ be the maximum (minimum) value of Y considered and also consider $n-1$ other discrete values between $Y_{\max}$ and $Y_{\min}$ such that dy represents the difference between two consecutive values, i.e., the step size. Then, any discrete value of Y can be represented as $Y = Y_{\min} + j(dy)$, where j consists of the integers between 0 and n. Likewise, let T_B be the maximum value of t and 0 the minimum value. Consider a number of discrete values between 0 and T_B such that dt is the step size. Then, any discrete value of t can be represented at $t = k(dt)$ where k consists of the integers between 0 and T_B/dt. The choice of dy, dt will be discussed later.

The value of W for the discrete values of Y and t can be written as

$$W(t, Y) = W(k(dt), Y_{\min} + j(dy)) = W(k, j).$$

Explicit Numerical Scheme for the Black and Cox Model

For the explicit scheme, the partial derivatives are approximated by the finite differences described in Section 3.2. We substitute these finite differences into the p.d.e., to obtain a difference equation. Between coupon dates, the Dirac delta function term does not affect the evolution of W. Hence, between coupon dates, the evolution of W can be described by this p.d.e. without the Dirac delta term, i.e., by

$$0.5s^2W_{yy} + (r - 0.5s^2)W_y + W_t - rW = 0. \tag{I.71}$$

This p.d.e. describes the evolution of W between coupon dates, but not on coupon dates. The behavior of W on coupon dates will be described below. After substituting the finite differences we obtain

$$W(k,j) = \frac{(P_u W(k+1,j+1) + P_o W(k+1,j) + P_d W(k+1,j-1))}{(1 + r(dt))} \tag{I.72}$$

where

$$\begin{aligned} P_u &= \left(s^2 + dy\left(r - \left(\frac{s^2}{2}\right)\right)\right)\left(\frac{dt}{2(dy)^2}\right) \\ P_d &= \left(s^2 - dy\left(r - \left(\frac{s^2}{2}\right)\right)\right)\left(\frac{dt}{2(dy)^2}\right) \\ P_o &= 1 - s^2\frac{dt}{(dy)^2}. \end{aligned}$$

For stability, need $P_u > 0, P_d > 0, P_o > 0$. Hence,

$$\frac{dt}{(dy)^2} < \frac{1}{s^2}; \quad dy < \frac{s^2}{r - \frac{s^2}{2}}.$$

The boundary conditions at low and high firm values are not used in the explicit scheme. The boundary condition at bond maturity can be approximated by

$$W(T_B, j) = \min[B, \exp(Y_{\min} + j(dy)]. \tag{I.73}$$

To analyze the behavior of W on coupon dates, assume that the coupon payment dates coincide with discrete points on the time grid of the numerical scheme. In particular, assume that between time T_B and time $T_B - 5(dt)$ only time $T_B - 3(dt)$ is a coupon payment date as shown below

Time	Coupon
T_B	No
$T_B - dt$	No
$T_B - 2(dt)$	No
$T_B - 3(dt)$	Yes
$T_B - 4(dt)$	No
$T_B - 5(dt)$	No

The solution is obtained as follows: Given $W(T_B, j)$ from the maturity boundary condition, we obtain $W(T_B - dt, j)$ from the difference equation for all but two of the j's considered at T_B. At time $T_B - 2(dt)$, given $W(T_B - dt, j)$, we obtain $W(T_B - 2(dt), j)$ from the difference equation for all but two of the j's considered at time $T_B - dt$. At time $T_B - 3(dt)$, given

$W(T_B - 2(dt))$ we obtain $W(T_B - 3(dt))$ from the difference equation for all but two of the j's we considered at time $T_B - 2(dt)$. Furthermore, given $T_B - 3(dt)$ is a coupon date, we calculate the total payoff to the bond at $T_B - 3(dt)$ defined as $G(T_B - 3(dt), j)$ as follows[1]:

$$\begin{aligned} G(T_B - 3(dt), j) &= W(T_B - 3(dt), j) + c \\ \text{if } \exp[Y_{\min} + j(dy)] &\geq W(T_B - 3(dt), j) + c \text{ i.e., if firm can pay } c. \qquad \text{(I.74)} \\ G(T_B - 3(dt), j) &= \exp[Y_{\min} + j(dy)] \\ \text{if } \exp[Y_{\min} + j(dy)] &< W(T_B - 3(dt), j) + c \text{ i.e., if firm bankrupt.} \qquad \text{(I.75)} \end{aligned}$$

Note that $W(T_B - 3(dt), j)$ will be the ex-coupon bond price at time $T_B - 3(dt)$ only if the firm does not go bankrupt, i.e., if firm value $> W + c$. If the firm does go bankrupt (i.e., if firm value $< W + c$) at $T_B - 3(dt)$ because the firm cannot pay the coupon, the ex-coupon bond price cannot be defined; in such circumstances only the total payoff $G(T_B - 3(dt), j)$ is defined.

At time $T_B - 4(dt)$, we calculate $W(T_B - 4(dt), j)$ from the difference equation using $G(T_B - 3(dt), j)$ instead of $W(T_B - 3(dt), j)$ in the difference equation. As before, we do this for all but two of the j's considered at $T_B - 3(dt)$. Note that since $T_B - 4(dt)$ is not a coupon date the total payoff at $T_B - 4(dt)$, i.e., $G(T_B - 4(dt), j) = W(T_B - 4(dt), j)$. At time $T_B - 5(dt)$, given $W(T_B - 4(dt), j)$ we calculate $W(T_B - 5(dt), j)$ from the difference equation for all but two of the j's considered at $T_B - 4(dt)$. Again, $G(T_B - 5(dt), j) = W(T_B - 5(dt), j)$.

The calculations proceed in this way and depend on whether the time grid point at which they are performed is a coupon date of not. We proceed in this way until we reach

[1] An explanation follows in the next section; on non-coupon dates, $G = W$.

time 0.

Explanation of Total Payoff Calculation on a Coupon Date

The bond contract requires that shareholders finance the coupon payment through a new equity issue. The shareholders will be able to pay the coupon only if, after a cash infusion from new equity in the amount c (equals coupon) and payment of the coupon, the combined value of the old and new stock (which will be equal to $V - W$) exceeds or is equal to c, i.e., only if

$$V - W \geq c.$$

To see this, note that if $V - W < c$, new shareholders will not be willing to buy stock at any price and hence the firm goes bankrupt. The reason is that even if new shareholders are offered 100% of the company, their shares' value after the coupon is paid ($= V - W$) will be worth less than c and, clearly, they are not willing to pay c for such a claim. In such a case, the firm goes bankrupt, and the total payoff to the bond is V.

Alternatively, consider a situation in which $V - W$ exceeds c by a tiny amount. In this case, the old shareholders will be able to finance the coupon provided they allow new shareholders to buy shares at a price such that the stake of old shareholders is diluted to a tiny amount. Of course, this is better than bankruptcy.

Finally, if V -- W exceeds c by a big amount, old shareholders will be able to finance the coupon without diluting their stake substantially.

The above discussion suggests that the coupon will be determined as follows:

$$\text{coupon} = c \quad \text{if} V - W \geq c$$

firm bankrupt if $V - W < c$.

Hence,

$$\begin{aligned} \text{total payoff to bond} &= c + W, \text{ if } V - W \geq c \\ &= V \text{ if } V - W < c. \end{aligned} \tag{I.76}$$

Implicit Numerical Scheme for Black-Cox Model

For the implicit scheme the partial derivatives are approximated by the finite differences that were used earlier in the derivation of the implicit scheme for the option on the bond and the option on the spread.

We substitute these finite differences into the p.d.e. ignoring the p.d.e. term involving the Dirac delta function to get the following *system of equations that describes the behavior of* W between coupon dates; this system of equations consists of $n-1$ equations in $n+1$ unknowns $W(k,j), j = 0, \ldots, n$.

$$\begin{aligned} &\alpha_1 W(k+1, j+1) + b_1 W(k+1, j) + \alpha_2 W(k+1, j-1) \\ &+\alpha_1 W(k, j+1) + b_2 W(k, j) + \alpha_2 W(k, j-1) = 0 \quad \text{for } j = 1, \ldots, n-1 \end{aligned} \tag{I.77}$$

where

$$\begin{aligned} \alpha_1 &= \frac{\sigma^2}{4(dy)^2} + \frac{1}{4(dy)}\left(r - \frac{\sigma^2}{2}\right) \\ \alpha_2 &= \frac{\sigma^2}{4(dy)^2} - \frac{1}{4(dy)}\left(r - \frac{\sigma^2}{2}\right) \\ b_1 &= -\frac{\sigma^2}{2(dy)^2} + \frac{1}{dt} \end{aligned}$$

$$b_2 = -\frac{\sigma^2}{2(dy)^2} - \frac{1}{dt} - r.$$

Boundary condition eq. (I.68) can be approximated as in the explicit scheme. Boundary condition eq. (I.69) can be approximated by

$$W(k,0) = \exp(Y_{\min}) \quad \forall k. \tag{I.78}$$

Boundary condition eq. (I.70) can be approximated by

$$W(k,n) = W(k,n-1) \quad \forall k. \tag{I.79}$$

The above system of equations together with boundary conditions eq. (I.78) and eq. (I.79) can be written as the following system of equations:

$$DW^k + AW^{k+1} - u = 0 \tag{I.80}$$

where

$$W^k = \begin{bmatrix} W(k,0) \\ W(k,1) \\ \vdots \\ W(k,n) \end{bmatrix} \quad u = \begin{bmatrix} \exp(Y_{\min}) \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 0 & 0 & & & \\ \alpha_2 & \beta_1 & \alpha_1 & & \\ & \alpha_2 & \ddots & \alpha_1 & \\ & & \alpha_2 & \beta_1 & \alpha_1 \\ & & & 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 0 & & & \\ \alpha_2 & \beta_2 & \alpha_1 & & \\ & \alpha_2 & \ddots & \alpha_1 & \\ & & \alpha_2 & \beta_2 & \alpha_1 \\ & & & -1 & 1 \end{bmatrix}.$$

We can rewrite this system of equations as

$$DW^k = -(AW^{k+1} - u). \tag{I.81}$$

This system of equations describes the behavior of W on non-coupon dates. To analyze the behavior of W on coupon dates, assume, as in the explicit scheme that *the coupon payment dates coincide with discrete points on the time grid of the numerical scheme.* In particular, assume that between time T_B and time $T_B - 5(dt)$ only time $T_B - 3(dt)$ is a coupon payment date as shown below.

Time	Coupon
T_B	No
$T_B - dt$	No
$T_B - 2(dt)$	No
$T_B - 3(dt)$	Yes
$T_B - 4(dt)$	No
$T_B - 5(dt)$	No

The solution is obtained as follows: Given W^{T_B} from the maturity boundary condition, we solve the above system of equations to get $W^{T_B - dt}$ as

$$W^{T_B - dt} = -(D)^{-1}(AW^{T_B} - u). \tag{I.82}$$

Note that this involves inverting the matrix D. At $T_B - 2(dt)$, given $W^{T_B - dt}$ we again solve the same system of equations to get $W^{T_B - 2(dt)}$. At $T_B - 3(dt)$, given $W^{T_B - 2(dt)}$ we solve the same system of equations to get $W^{T_B - 3(dt)}$. Furthermore, given $T_B - 3(dt)$ is a coupon date, we calculate the total payoff to the bond at $T_B - 3(dt)$ defined as $G^{T_B - 3(dt)}$ where G^k is given by

$$G^k = \begin{bmatrix} G(k,0) \\ G(k,1) \\ \vdots \\ G(k,n) \end{bmatrix}.$$

The calculation of $G^{T_B - 3(dt)}$ from $W^{T_B - 3(dt)}$ is identical to the same calculation performed in the explicit scheme and the procedure is not repeated here. At time $T_B - 4(dt)$, we calculate $W^{T_B - 4(dt)}$ from the same system of equations by using $G^{T_B - 3(dt)}$ instead of $W^{T_B - 3(dt)}$ in this system of equations, i.e.,

$$W^{T_B - 4(dt)} = -(D)^{-1}(AG^{T_B - 3(dt)} - u). \tag{I.83}$$

Note that since $T_B - 4(dt)$ is not a coupon date, $G^{T_B - 4(dt)} = W^{T_B - 4(dt)}$.

At time $T_B - 5(dt)$, given $W^{T_B - 4(dt)}$ we calculate $W^{T_B - 5(dt)}$ from the same system of equations. Again $G^{T_B - 5(dt)} = W^{T_B - 5(dt)}$.

The calculations proceed in this way and depend on whether the time grid point at which they are performed is a coupon date or not. We proceed in this way until we reach time 0.

5 Structuring and Pricing Credit Risk Derivatives for Coupon Bonds

We now consider the structuring and pricing of credit risk derivatives which protect holders of coupon bonds from credit rating downgradings. Unlike zero-coupon bonds, coupon bonds can default prior to their maturity and as a result *any credit risk derivative on a coupon bond will necessarily involve a barrier option which ceases to exist in the event of bankruptcy.* We consider two types of barrier options: A put on the coupon bond's price and a call on the coupon bond's spread. We begin with the put.

5.1 An American put Option on the Cum-Coupon Bond Price (G)

As explained above, the ex-coupon bond price is not defined on a coupon date if the firm goes bankrupt. For this reason, any option contract must be based on the cum-coupon bond price.

Consider the following American Put: At any $t < T$, where T = option maturity $< T_B$, assuming the firm is not bankrupt, the option's owner can sell the bond at a time varying exercise price $K(t)$ equal to the cum-coupon price of the bond for a fixed spread g above the risk free rate. For example, consider the coupon bond defined earlier: It promised B at T_B and coupons in the amount c at $0.2T_B, 0.4T_B, 0.6T_B$, and $0.8T_B$. Then

$$\begin{aligned} K(0) &= c\exp(-(r+g)(0.2T_B)) + c\exp(-(r+g)(0.4T_B)) + c\exp(-(r+g)(0.6T_B)) \\ &\quad + c\exp(-(r+g)(0.8T_B)) + B\exp(-(r+g)(T_B)) \qquad \text{(I.84)} \\ K(0.2T_B) &= c + c\exp(-(r+g)(0.4T_B - 0.2T_B)) + c\exp(-(r+g)(0.6T_B - 0.2T_B)) \\ &\quad + c\exp(-(r+g)(0.8T_B - 0.2T_B)) + B\exp(-(r+g)(T_B - 0.2T_B)). \qquad \text{(I.85)} \end{aligned}$$

Note that $K(t)$ is defined in a way consistent with the bond price on which the option is defined—they are both cum-coupon.

Furthermore, the option contract provides that if the firm goes bankrupt at any time prior to option maturity, the option will be immediately exercised. This is a necessary condition to impose since the bond will cease to exist if the firm goes bankrupt and hence there will be no way of calculating the payoffs to the option. This provision makes this option contract a barrier option. Since the firm can only go bankrupt on a coupon date prior to bond maturity, it is only on coupon dates that the option can be terminated. Furthermore, from the discussion of the coupon bond pricing model above, we know that the firm is bankrupt on a coupon date whenever $G = V$, i.e., whenever the cum-coupon bond price equals firm value. Hence, whenever $G = V$ on a coupon date, the option is terminated.

We now present an early exercise result.

Proposition 5 *This American put will only be exercised early if it is a forced exercise, i.e., if the option is terminated. Hence, American and European put values will be the same.*

PROOF: For simplicity, assume current time is t, bond pays a coupon at t_c, option matures at T and bond matures at T_B with

$$t < t_c < T < T_B.$$

We shall prove that at either t or t_c the option will never be exercised early (unless it is a forced exercise at t_c).

Proof that option will not be exercised early at t

Suppose that at t, put price $< K(t) - G(t)$ where

$$\begin{aligned} K(t) &= \text{exercise price} \\ &= B\exp(-(r+g)(T_B - t)) + c\exp(-(r+g)(t_c - t)) \\ G(t) &= \text{cum-coupon bond price.} \end{aligned}$$

Arbitrage strategy: Buy put, buy bond and borrow $K(t)$. The time t payoff is positive. Wait until time t_c.

Two possibilities at time t_c:

1. If forced to exercise at time t_c, we deliver the bond and receive the option's exercise price ($K(t_c)$) if the latter exceeds the bond price; if the option is not in the money, the bond must be worth at least $K(t_c)$. Therefore, our payoff from our bond and option position is at least $K(t_c)$. Also, we repay our borrowings which amount to $K(t)\exp(r(t_c - t))$ at time t_c.

   Hence, payoff at t_c

$$\begin{aligned} &\geq K(t_c) - K(t)\exp(r(t_c - t)) \\ &= B\exp(-(r+g)(T_B - t_c)) + c - B\exp(-(r+g)(T_B) + gt + rt_c) \\ &\quad -c\exp(-g(t_c - t)) \\ &= c(1 - \exp(-g(t_c - t))) + B\exp(-(r+g)(T_B - t_c))(1 - \exp(-g(t_c - t))). \end{aligned}$$

   Clearly, $1 - \exp(-g(t_c - t)) > 1$, hence payoff at $t_c \geq 0$. An arbitrage opportunity exists. We must have put price $> K(t) - G(t)$.

2. If we are not forced to exercise at time t_c, we wait until the expiration date T. Since we were not forced to exercise at t_c, the firm did not go bankrupt and therefore we received a coupon of c at t_c from our long bond position. By T, this coupon is worth $c\exp(r(T-t_c))$.

   At time T, our option and bond position must be worth at least $K(T)$. The reason for this is that if the option is in the money, we exercise it by delivering the bond and receive $K(T)$. If the option is not in the money, the bond must be worth at least $K(T)$. Also, we repay our borrowings at time T which amount to $K(t)\exp(r(T-t))$. Finally, as explained in the previous paragraph, we have an additional amount $c\exp(r(T-t_c))$ at T. Therefore, payoff at T

$$\begin{aligned}
&\geq K(T) + c\exp(r(T-t_c)) - K(t)\exp(r(T-t)) \\
&= B\exp(-(r+g)(T_B-T)) + c\exp(r(T-t_c)) \\
&\quad -B\exp(-(r+g)(T_B-T) - g(T-t)) \\
&\quad -c\exp(r(T-t_c) - g(t_c-t)) \\
&= B\exp(-(r+g)(T_B-T))(1-\exp(-g(T-t))) \\
&\quad +c\exp(r(T-t_c))(1-\exp(-g(t_c-t))).
\end{aligned}$$

   Clearly

$$\exp(-g(T-t)) < 1$$

$$\exp(-g(t_c-t)) < 1.$$

Hence, payoff at $T \geq 0$. An arbitrage opportunity exists. We must have put price $> K(t) - G(t)$.

Proof that option will not be exercised early at t_c unless it is a forced exercise.

Suppose at t_c option is not terminated and

$$\text{put price} < K(t_c) - G(t_c).$$

Adopt the following arbitrage strategy: Buy put, buy bond, borrow $K(t_c)$. The time t_c payoff is clearly positive. We wait until the expiration date T. Note that we bought the bond cum-coupon at t_c and hence receive a payoff of c at t_c.

At T, our long bond and long option positions will be worth at least $K(T)$. As before, the reason is that if the option is in the money, we exercise it by delivering the bond and receive the exercise price $K(T)$; if the option is not in the money, the bond must be worth at least $K(T)$. We repay our borrowings at T which amount to $K(t_c)\exp(r(T-t_c))$. Also the coupon of c we received at t_c is now worth $c\exp(r(T-t_c))$. Hence payoff at T

$$\begin{aligned}
&\geq K(T) + c\exp(r(T-t_c)) - K(t_c)\exp(r(T-t_c)) \\
&= B\exp(-(r+g)(T_B-T)) + c\exp(r(T-t_c)) \\
&\quad -B\exp(-(r+g)(T_B-t_c) + r(T-t_c)) - c\exp(r(T-t_c)) \\
&= B\exp(-(r+g)(T_B-T))[1-\exp(-g(T-t_c))].
\end{aligned}$$

Clearly, $\exp(-g(T-t_c)) < 1$. Hence payoff at $T \geq 0$. An arbitrage opportunity exists and

we must have

$$\text{Put price} > K(t_c) - G(t_c).$$

Since we have shown that the put price must exceed the exercise value at both t and t_c (unless the latter is a forced exercise situation), we have established that the put will never be exercised early unless it is a forced exercise.

We now consider the pricing of this put. Let $F[G,t]$ be the option price at time t. It is assumed that the option price depends on the cum-coupon bond price G and time t. Since G depends on firm value V, we can also think of option price as a function of V and t. For example,

$$F[G,t] = N[V,t].$$

Using the usual arbitrage argument, we can show that the option price will satisfy the following p.d.e.

$$0.5N_{vv}s^2V^2 + N_t - rN + rN_vV = 0. \tag{I.86}$$

The boundary conditions are:

$$N[V,T] = \max[0, K(T) - G(V,T)] \tag{I.87}$$

$$N[V,t] \geq \max[0, K(t) - G(V,t)] \tag{I.88}$$

$$\lim_{V\to 0} \frac{\partial N}{\partial G} = -1 \tag{I.89}$$

$$\lim_{V\to\infty} \frac{\partial N}{\partial G} \text{ is finite.} \tag{I.90}$$

Equation (I.87) says that option value at maturity will be the exercise value if the latter

is positive and zero otherwise. Equation (I.88) says that the price of an American put can never fall below its exercise value. Equation (I.89) says that when firm and bond value are very low, a \$1.00 increase in bond value will reduce option value by \$1.00 since the option is way in the money. Equation (I.90) says that when firm and bond value are very high, a marginal increase in the bond price will only have a finite impact on option price because the option is way out of the money.

No closed form solution exists to this valuation problem. For this reason, we employ a numerical method of solution. Note that a fundamental difference between the put on the coupon bond and the put on the zero-coupon bond is that in the case of the latter there exists a closed form solution for the zero-coupon bond price whereas no such closed form solution exists for the coupon bond. For this reason, to price the put on the coupon bond we need two numerical schemes: The first scheme, as described above, gives us the coupon bond price G; the second scheme uses the G calculated from the first scheme to calculate the option price. We now explain how the second numerical scheme works.

5.2 Numerical Method of Solution for the Put on Coupon Bond

As earlier mentioned, we transform the p.d.e. into a p.d.e. with constant coefficients using the transformation $Y = \log V, N(V,t) = P(Y,t)$. Making the appropriate substitutions, we obtain the transformed p.d.e

$$0.5s^2 P_{yy} + (r - 0.5s^2)P_y + P_t - rP = 0. \tag{I.91}$$

The boundary conditions in terms of Y are:

$$P[Y,T] = \max[0, K(T) - G(Y,T)] \tag{I.92}$$

$$P[Y,t] \geq \max[0, K(t) - G(Y,t)] \tag{I.93}$$

$$\lim_{Y \to -\infty} \frac{\partial P}{\partial G} = -1 \tag{I.94}$$

$$\lim_{Y \to \infty} \frac{\partial P}{\partial G} \text{ is finite.} \tag{I.95}$$

To solve numerically, we consider the same finite number of discrete values of Y and t as in Section 3.2 in the derivation of eq. (I.27).

The value of P for the discrete values of Y and t can be written as

$$P(t,Y) = P(k(dt), Y_{\min} + j(dy)) = P(k,j).$$

where

$$j = 0, 1, \ldots, n; k = 0, 1, \ldots, \frac{T}{dt}$$

As explained above, the second numerical scheme that calculates the option price relies on the first numerical scheme to provide the cum-coupon bond price G. For this reason, we *assume that the value grid and the time grid are identical in the two numerical schemes*, i.e., that Y and t are discretized the same way in both schemes.

Explicit Numerical Scheme

Using the same method as in the case of the put on the zero-coupon bond, we obtain the following difference equation:

$$P(k,j) = \frac{(P_u P(k+1,j+1) + P_o P(k+1,j) + P_d P(k+1,j-1))}{(1+r(dt))} \tag{I.96}$$

where P_u, P_d and P_o are as defined in eq. (I.28). The boundary condition eq. (I.92) at option maturity can be approximated by

$$P(T,j) = \max[0, K(T) - G(T,j)]. \tag{I.97}$$

$G(T,j)$, the cum-coupon bond price at option maturity for different values of j is obtained from the coupon bond numerical pricing scheme. Boundary condition eq. (I.93) can be approximated by

$$P(k,j) \geq \max[0, K(k(dt)) - G(k(dt),j)]. \tag{I.98}$$

Again, $G(k(dt),j)$ is obtained from the coupon bond numerical pricing scheme. Since this is an explicit method of solution, the boundary conditions in eq. (I.94) and eq. (I.95) describing the behavior of P for very large and very small Y are not used in the numerical procedure.

The above difference equation and maturity boundary condition eq. (I.97) give the time t option price only if their solution exceeds the exercise value given by boundary condition eq. (I.97). Otherwise, the option price is set equal to the exercise value. The solution is obtained by the same recursive technique described earlier for the put on the zero-coupon bond.

Implicit Numerical Scheme (Crank-Nicholson)

As in the case of the put on the zero-coupon bond, making the appropriate substitutions of finite differences (approximating the partial derivatives) into the p.d.e. we obtain the following set of $n-1$ equations in $n+1$ unknowns $P(k,j) j = 0, 1, \ldots, n$.

$$\begin{aligned} &\alpha_1 P(k+1, j+1) + b_1 P(k+1, j) + \alpha_2 P(k+1, j-1) \\ &+\alpha_1 P(k, j+1) + b_2 P(k, j) + \alpha_2 P(k, j-1) = 0 \quad \text{for } j = 1, \ldots, n-1 \end{aligned} \tag{I.99}$$

where α_1, α_2, b_1, and b_2 are as defined in eq. (I.32). Boundary conditions in eq. (I.92) and eq. (I.93) can be approximated as in the explicit scheme. Boundary condition eq. (I.94) tells us that

$$\lim_{Y \to -\infty} \frac{\partial P}{\partial G} = -1.$$

Using the analytical results from the put on the zero-coupon bond, we *assume* that this condition is equivalent to

$$\lim_{Y \to -\infty} \frac{\partial P}{\partial Y} = 0. \tag{I.100}$$

Note that in the case of the coupon bond, we cannot establish this result analytically because there is no closed form expression linking G and Y.

This condition is approximated by

$$P(k, 1) = P(k, 0) \quad \forall k. \tag{I.101}$$

Boundary condition eq. (I.95) tells us that

$$\lim_{Y\to\infty} \frac{\partial P}{\partial G} \text{ is finite.}$$

This is *assumed* to be equivalent to

$$\lim_{Y\to\infty} \frac{\partial P}{\partial Y} = 0 \tag{I.102}$$

and is approximated by

$$P(k,n) = P(k,n-1) \quad \forall k. \tag{I.103}$$

Define

$$\boldsymbol{P}^k = \begin{bmatrix} P(k,0) \\ P(k,1) \\ \vdots \\ P(k,n) \end{bmatrix}.$$

We can rewrite the system of equations derived above and boundary conditions in eq. (I.101) and eq. (I.103) as:

$$B\boldsymbol{P}^k + A\boldsymbol{P}^{k+1} = 0 \tag{I.104}$$

where A and B are as defined in eq. (I.39)

As in the case of the put on the zero-coupon bond, this system or equations together with maturity boundary condition eq. (I.97) give the time t option price only if their solution exceeds the exercise value given by boundary condition eq. (I.98). Otherwise, option price is set equal to exercise value. The solution is obtained using the recursive technique described

earlier.

5.3 Numerical Results

Numerical results were obtained for various parameter values using the computer program in Baakkonen (1993).

The output is presented in Tables C1-C5 of Appendix C. Table C1 presents results from the implicit scheme for the following parameter values:

s=0.24 per year

T =0.8 years

T_B = 1.2 years

r = 0.06 per year

B = 20

c = 3 at t=0.3,0.6 and 0.9

g = 0.01

The first column gives the value grid point j, the second column gives firm value at that point and the third column gives the cum-coupon bond price G (all at time 0). As expected, G increases with V. Furthermore, note that at low values of V, G is only slightly less than V (note that for some low $j, G = V$ because of the number of decimal points shown in program output, i.e., if we showed a larger number of decimal points G would always be less than V on a non-coupon date such as time 0 because the firm can never go bankrupt at any time other than a coupon date or bond maturity). The reason that G is very close to V when firm value is very low is that in such circumstances, the probability that the firm will avoid bankruptcy, either on the next coupon date or at maturity, is very low. The time

0 put option price for different firm values and option prices is shown in column four. As expected, the put option price is a decreasing function of G and V. Columns five and six give the temporary option price and exercise value—these are defined the same way as in Section 3.3.

At the bottom of program output the early exercise boundary is printed. This provides the same information as the corresponding boundary in Section 3.3.

Below the early exercise boundary, program output indicates the circumstances under which this barrier option is terminated. It indicates that the option is terminated at time step 15 when $j < 73$ and at time step 30 when $j < 72$. Note that both time step 15 and time step 30 correspond to coupon dates.

Note also that the early exercise boundary output suggests that the option is exercised early only when it is terminated, i.e., when it is a forced exercise. The option is never exercised early at any time step other than 15 and 30 and when it is exercised early at these time steps, it is for precisely the same values of the value grid point j for which the option is terminated. This confirms Proposition 5 which proved analytically that this American put will only be exercised early if it is a forced exercise, i.e., if the option is terminated. To check this further a European option version of the program was run and the option prices obtained were identical to the ones obtained for the American option; also the European put was terminated under the same circumstances as the American put.

Effect of Option Maturity—Table C2

The parameters used in Table C2 are the same as those in Table C1 except for a lower maturity ($T = 0.4$). Note that the option price is consistently lower for a lower maturity.

Moreover, this result holds for both American and European puts for the reasons given in the Proof of Proposition 2. This leads to:

Proposition 6 *The higher the maturity of the put option (European or American) the higher its price.*

PROOF: Omitted since it is similar to Proof 2.

Effect of volatility - Table C3

The parameters used in Table C3 are the same as those in Table C1 except for a lower volatility. Note that the cum-coupon bond price is lower the higher the volatility. This is analogous to Merton's analytical result that higher volatility reduces zero coupon bond prices; however, in the case of the coupon bond, no such result can be established analytically because there is no closed form formula for the coupon bond price. Also, comparing Tables C1 and C3, we see that for given firm value, the option price is higher, the higher the volatility. However, note that this effect is significant only at high firm values.

Effect of Interest Rate - Table C4

As in the case of the zero-coupon bond, a higher interest rate has two opposing effects on option price for given firm value: First, a higher interest rate reduces bond price (this can be shown numerically but not analytically) and hence increases put prices for given firm value; second, a higher interest rate reduces both the exercise price and the present value of the exercise price, thereby reducing put prices. It is not clear which effect will dominate.

A situation where a higher interest rate results in lower option prices is illustrated in Table C4. Table C4 depicts program output for the same parameter values as Table C1

except that the interest rate is higher. Note that the option prices are consistently lower in Table C4. As expected the coupon bond price is also lower in Table C4.

Effect of Striking Price - Table C5

Table C5 incorporates a higher g, i.e., a lower exercise price than Table C1, otherwise the parameters are the same. As expected, the lower the striking price (i.e., the higher g), the lower the option price.

5.4 Another Credit Risk Derivative for Coupon Bonds: A Call Option on the Bond's Spread

Consider the following American call option: at any $t \leq T$, where T = option maturity $< T_B$, assuming the firm is not bankrupt, the option's owner can exercise the option and receive an amount equal to $\max(0, \epsilon(V,t) - g)$ where $\epsilon(V,t)$ is the spread above the risk-free rate at which the coupon bond trades at time t (on a cum-coupon basis) and g is a fixed exercise price. For example, consider the coupon bond defined earlier. It promised B at T_B and coupons in the amount c at $0.2T_B$, $0.4T_B, 0.6T_B$, and $0.8T_B$. The spread on a non-coupon date, for example when $t = 0$, defined as $\epsilon(V,0)$ solves the equation:

$$\begin{aligned} G(V,0) &= c\exp(-(r+\epsilon(V,0))(0.2T_B)) \\ &\quad + c\exp(-(r+\epsilon(V,0))(0.4T_B)) \\ &\quad + c\exp(-(r+\epsilon(V,0))(0.6T_B)) \\ &\quad + c\exp(-(r+\epsilon(V,0))(0.8T_B)) \\ &\quad + B\exp(-(r+\epsilon(V,0))(T_B)) \end{aligned} \tag{I.105}$$

where $G(V,0)$ equals cum-coupon bond price at time 0. Hence, $\epsilon(V,0)$ equates the right-hand side of equation (I.105) to the numerically calculated cum-coupon bond price $G(V,0)$. The spread on a coupon date, for example when $t = 0.2(T_B), \epsilon(V, 0.2(T_B))$ solves the following equation:

$$\begin{aligned} G(V,0.2(T_B)) &= c + c\exp(-(r+\epsilon(V,0.2(T_B)))(0.2T_B)) \\ &\quad + c\exp(-(r+\epsilon(V,0.2(T_B)))(0.4T_B)) \\ &\quad + c\exp(-(r+\epsilon(V,0.2(T_B)))(0.6T_B)) \\ &\quad + B\exp(-(r+\epsilon(V,0.2(T_B)))(0.8T_B)) \end{aligned} \tag{I.106}$$

where $G(V,0.2(T_B))$ equals cum-coupon bond price at time $0.2T_B$.

Note that a fundamental difference between the call option on the zero-coupon bond spread and the call option on the coupon bond spread is that in the case of the former, the spread can be calculated from the closed form solution, whereas in the case of the latter, the spread must be calculated from equations (I.105) and (I.106) using the coupon bond price provided by the numerical scheme described earlier.

Equations (I.105) and (I.106) can be solved for $\epsilon(V,t)$ using the Newton-Raphson root finding scheme.

Furthermore, the option contract provides that if the firm goes bankrupt at any time prior to option maturity, the option will be immediately terminated and the payoff will be the exercise value. This is a necessary condition to impose since the bond will cease to exist if the firm goes bankrupt and hence there will be no way of calculating the payoffs to the option. This provision makes this option contract a barrier option. Since the firm can only go bankrupt on a coupon date prior to bond maturity, it is only on coupon dates

that the option can be terminated. Furthermore, from the discussion of the coupon bond pricing model above, we know that the firm is bankrupt on a coupon date whenever $G = V$, i.e., whenever the cum-coupon bond price equals firm value. Hence, whenever $G = V$ on a coupon date, the option is immediately exercised.

Another complication to consider is that on a coupon date, eq. (I.106) may have no root; this will happen whenever $G < c$ (as is obvious from inspection of eq. (I.106)). In such circumstances, the value of $\epsilon(V,t)$ is set equal to an arbitrarily defined upper bound $\epsilon^{\max}$ and option payoffs are calculated using $\epsilon^{\max}$.

Equation I.105 which gives the spread on a non-coupon date will always have a root. Nevertheless, for consistency reasons, whenever the calculated value of ϵ exceeds $\epsilon^{\max}$, ϵ is automatically set to $\epsilon^{\max}$ and we calculate option payoffs using $\epsilon^{\max}$.

We now consider the pricing of this put. Let $F[\epsilon(V,t),t]$ be its price at t. It is assumed that option price depends on the spread $\epsilon(V,t)$ and time t. Since ϵ depends on V, we can think of option price as a function of V and t i.e.,

$$F[\epsilon(V,t),t] = Q[V,t]. \tag{I.107}$$

Using the usual arbitrage argument we can show that the option price will satisfy the following p.d.e.

$$0.5Q_{vv}s^2V^2 + Q_t - rQ + rQ_vV = 0. \tag{I.108}$$

The boundary conditions are

$$Q[V,T] = \max[0, \epsilon(V,T) - g] \tag{I.109}$$

$$Q[V,t] \geq \max[0, \epsilon(V,t) - g] \tag{I.110}$$

$$\lim_{V \to 0} \frac{\partial Q}{\partial \epsilon} = 1 \tag{I.111}$$

$$\lim_{V \to \infty} \frac{\partial Q}{\partial \epsilon} \text{ is finite.} \tag{I.112}$$

The interpretation is similar to the one provided earlier for the boundary conditions of the call on the zero-coupon bond spread.

No closed form solution exists to this valuation problem. For this reason, we employ a numerical method of solution.

5.5 Numerical Method of Solution for Call on Coupon Bond's Spread

As earlier,

$$Y = \log V$$

$$Q(V,t) = \Omega(Y,t)$$

Transformed p.d.e.

$$0.5s^2\Omega_{yy} + (r - 0.5s^2)\Omega_y + \Omega_t - r\Omega = 0. \tag{I.113}$$

Boundary conditions in terms of Y:

$$\Omega[Y,T] = \max[0, \epsilon(Y,T) - g] \tag{I.114}$$

$$\Omega[Y,t] \geq \max[0, \epsilon(Y,t) - g] \tag{I.115}$$

$$\lim_{Y \to -\infty} \frac{\partial \Omega}{\partial \epsilon} = 1 \tag{I.116}$$

$$\lim_{Y \to \infty} \frac{\partial \Omega}{\partial \epsilon} \text{ is finite.} \tag{I.117}$$

We discretize Y and t in the usual way, and set

$$\Omega(t, Y) = \Omega(k, j)$$

$$\text{where} \qquad j = 0, 1, \ldots, n$$

$$k = 0, 1, \ldots, \frac{T}{dt}.$$

Explicit Numerical Scheme

Using the same method as earlier we obtain the difference equation:

$$\Omega(k, j) = \frac{(P_u \Omega(k+1, j+1) + P_o \Omega(k+1, j) + P_d \Omega(k+1, j-1))}{(1 + r(dt))} \tag{I.118}$$

where P_u, P_d, and P_o are as defined in eq. (I.28).

The boundary condition eq. (I.114) can be approximated by

$$\Omega(T, j) = \max[0, \epsilon(T, j) - g]. \tag{I.119}$$

Likewise eq. (I.115) can be approximated by

$$\Omega(k, j) \geq \max[0, \epsilon(k(dt), j) - g] \tag{I.120}$$

The difference eq. (I.118)and eq. (I.119) give the time t option price only if their solution exceeds the exercise value given by eq. (I.120). Otherwise the option price is set equal to the exercise value. The same recursive technique employed earlier is used here too.

Implicit Numerical Scheme (Crank-Nicholson)

Using the usual approach, we obtain the following set of $n-1$ equations in $n+1$ unknowns $\Omega(k,j), j = 0, 1 \ldots, n$.

$$\begin{aligned} &\alpha_1\Omega(k+1,j+1) + b_1\Omega(k+1,j) + \alpha_2\Omega(k+1,j-1) \\ &+\alpha_1\Omega(k,j+1) + b_2\Omega(k,j) + \alpha_2\Omega(k,j-1) = 0 \quad \text{for } j = 1,\ldots,n-1 \end{aligned} \tag{I.121}$$

where α_1, α_2, b_1, and b_2 are as defined in eq. (I.32).

Boundary conditions in equations (I.114) and (I.115) can be approximated as in the explicit scheme. Boundary condition eq. (I.116) tells us that

$$\lim_{Y \to -\infty} \frac{\partial \Omega}{\partial \epsilon} = 1.$$

Using analytical results from the call on the spread of the zero-coupon bond, we *assume* (since we cannot get analytical results here) that this condition is equivalent to

$$\lim_{Y \to -\infty} \frac{\partial \Omega}{\partial Y} = -\frac{1}{T_B - t} \tag{I.122}$$

This condition is the same as the comparable condition for the call option on the spread of the zero-coupon bond. However, in the latter case, the spread was never set equal to a maximum value i.e., there was no equivalent to $\epsilon^{\max}$ in the zero-coupon bond spread

calculation. For this reason, the boundary condition for the coupon bond spread is adjusted to

$$\lim_{Y \to -\infty} \frac{\partial \Omega}{\partial Y} = 0 \tag{I.123}$$

whenever

$$\epsilon(0,k) = \epsilon(1,k) = \epsilon^{\max}.$$

This is a reasonable condition to impose since the option price should not change if the spread is not changing.

Equation I.122 can be approximated by

$$\Omega(k,1) - \Omega(k,0) = -\frac{1}{T_B - k(dt)}(dy) \tag{I.124}$$

Equation I.123 can be approximated by

$$\Omega(k,0) = \Omega(k,1) \tag{I.125}$$

Boundary condition eq. (I.117) tells us that

$$\lim_{Y \to \infty} \frac{\partial \Omega}{\partial \epsilon} \text{ is finite.}$$

Using analytical results from the call on the spread of the zero-coupon bond, we assume (since we cannot get analytical results here) that this condition is equivalent to

$$\lim_{Y \to \infty} \frac{\partial \Omega}{\partial Y} = 0 \tag{I.126}$$

This condition is approximated by

$$\Omega(k,n) = \Omega(k,n-1) \quad \forall k. \tag{I.127}$$

Note that we need not adjust eq. (I.126) for situations where the spread is set to $\epsilon^{\max}$ because this never happens as $Y \to \infty$.

Define

$$\boldsymbol{\Omega}^k = \begin{bmatrix} \Omega(k,0) \\ \Omega(k,1) \\ \vdots \\ \Omega(k,n) \end{bmatrix}.$$

We can rewrite the system of equations derived above and the boundary conditions in eq. (I.124) or (I.125) and (I.127) as

$$B\Omega^k + A\Omega^{k+1} = q^k \tag{I.128}$$

where A and B are defined in eq. (I.39).

$$q^k = \begin{bmatrix} \frac{dy}{T_B - k(dt)} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \text{ if } \epsilon(0,k) \neq \epsilon(1,k); \quad q^k = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \text{ if } \epsilon(0,k) = \epsilon(1,k) = \epsilon^{\max}.$$

If $\epsilon(0,k) = \epsilon(1,k) = \epsilon^{\max}$ we use the same system with q^k being a null vector since eq. (I.125) and not eq. (I.124) becomes applicable.

This system of equations together with the maturity boundary condition eq. (I.119) give the time t option price only if their solution exceeds the exercise value given by eq. (I.120). Otherwise the option price is set equal to the exercise value. The solution is obtained by the usual recursive technique

5.6 Numerical Results for Call on Coupon Bond's Spread

Numerical results were obtained for various parameter values using the computer program in Baakkonen (1993). The output is presented in Tables D1-D6 of Appendix C. Table D1 presents results from the implicit scheme for the following parameter values:

s=100% per year

T =0.8 years

T_B = 1.2 years

r = 0.06 per year

B = 20

g = 4

$\epsilon^{\max}$ = 5

c = 3 at t=0.3,0.6 and 0.9

The first column gives the value grid point j, the second column gives firm value V at that point and the third column gives the value of the spread ϵ (all at time 0). As expected, at low firm values the spread is set equal to its maximum value $\epsilon^{\max}$. This occurs for $j \leq 43$; for these values of j, the spread does not vary with V for purposes of calculating the value of this capped call option. For $j > 43$ the spread decreases with V since the bond price increases with V. The time 0 call price for different values of ϵ and V is given in column

four. As expected, the call option price is an increasing function of ϵ. Column five gives the temporary option price and column six the exercise value. At the bottom of the program output, the early exercise boundary is printed.

Note that the option price never exceeds the current value of the spread. This is purely coincidental. For a different set of parameter values, we can get a situation where the option price always exceeds the current value of the spread for the reasons discussed in Section 3.6.

The early exercise boundary shows that the option is terminated at time step 15 (a coupon date) when $j < 72$ and at time step 30 (another coupon date) when $j < 72$. Note that at time step 15 and 30 the option is exercised early only when it is terminated, i.e., at these time steps there is no voluntary early exercise. Nevertheless, at every other time step between 0 and 39 there is voluntary early exercise. Note that the termination of the option creates an early exercise boundary that is non-monotonic as a function of time.

Effect of Volatility - Table D2

Table D2 incorporates a lower volatility than Table D1; otherwise, the parameters are the same. Comparing the two tables, we see that when the option is in the money, *lower volatility increases the option price* for given firm value. Note that this result holds even though lower volatility increases bond price and reduces the spread for given firm value—the latter should tend to reduce option price for given firm value. This result is obtained because of the capped nature of the option we are considering. Note that for purposes of calculating the option price, the spread is capped at 5. Since the exercise price is at 4, when the option is in the money, the upside potential is limited whereas the downside risk is unlimited. In such circumstances, high volatility reduces option prices because it implies that it is more

likely that the option will end up out of the money without providing significant upward potential to counterbalance the former effect.

Higher volatility also reduces option prices for given firm value when the option is out of the money provided the option is not way out of the money. Comparing Tables D1 and D2, we see that for $48 \leq j \leq 57$, this comparative static result holds. However, when $j > 58$, i.e., when the option is way out of the money, higher volatility always increases option price for given firm value. The explanation is similar to the one provided above for in the money options.

Finally, comparing Tables D1 and D2, we see that lower volatility increases the range of firm values for which the option is terminated on a coupon date because it increases bond prices (recall that the condition for option termination is bond price equals firm value).

Effect of Option Maturity - Table D3

Table D3 incorporates a lower maturity than Table D1; otherwise, the parameters are the same. Comparing the two, we see that lower maturity reduces option prices for given firm value. However, note that the effect is significant only for large firm values. This result is a general property of American calls (see Cox and Rubinstein (1985), Proposition 3, Page 138).

Effect of Max Spread (ϵ^{max}) - Table D4

Table D4 incorporates a higher value for the max spread than Table D1; otherwise, the parameters are the same. Comparing the two tables, we see that, as expected, a higher max spread increases option prices for given firm value. Furthermore, note that the effect

of max spread is quite significant, especially when the option is in the money or not way out of the money.

Also note that the higher the max spread, the higher the value of the spread above which we exercise the option early at any time period. The reason is that a higher max spread increases option prices and hence increases the exercise value necessary to induce us to exercise early.

Effect of Exercise Price - Table D5

Table D5 incorporates a lower exercise price than Table D1; otherwise the parameters are the same. As expected, a lower exercise price increases option prices.

Effect of Interest Rate - Table D6

Table D6 incorporates a higher interest rate than Table D1; otherwise, the parameters are the same. For the reasons given in Section 3.6, an increase in r can either increase or reduce the spread for given firm value. In the case illustrated in Table D6, a higher r results in lower spread for given firm value. This should tend to lower option prices for given firm value. However, there is another opposing effect to consider: An increase in r reduces the present value of the exercise price, which tends to increase option prices for given firm value. It is not clear which of these two opposing forces will prevail. In the case illustrated in Table D6, the effect of higher r on the spread prevails and as a result the option price is lower compared to Table D1.

6 Conclusion

This paper examined the structuring and pricing of credit risk derivatives for both zero-coupon and coupon bonds. Both puts on bond prices and calls on bond spreads can serve as credit risk derivatives. For coupon bonds, these put and call options must be of the barrier type. Furthermore, the call on the coupon bond's spread must be capped; otherwise there will be no way of calculating its price.

The pricing of these options was investigated using a numerical method of solution for the partial differential equation and associated boundary conditions. For zero-coupon bonds, both the bond price and the bond spread can be calculated analytically; the calculated price and spread are then used in the numerical scheme that gives the option prices. For coupon bonds, there is no way of calculating the bond price and spread analytically; instead, they must be calculated numerically. The numerically calculated price and spread are then used in a second numerical scheme that gives the option prices. We therefore have two numerical schemes that are interdependent. As shown in the paper, the relevant variables in the two schemes (firm value and time) must be discretized in a consistent way; in addition, it is convenient to assume that coupon dates coincide with discrete time points of both schemes.

The pricing properties of these options were established. A number of unexpected results were obtained. American puts on the zero-coupon bond price will never be exercised early. American puts on the coupon bond price will be exercised early only if it is a forced exercise. The price of the call on the spread can exceed the current value of the spread. There are circumstances in which higher volatility can reduce the price of the call option on the spread of the coupon bond. The influence of the interest rate on the price of these

options is ambiguous.

Appendix A. Boundary Conditions for Put option on Zero-Coupon Bond

We first establish the limiting behavior of $\frac{\partial f}{\partial Y}$ as Y tends to $+\infty$ and $-\infty$. We rewrite the equation for the price of the bond eq. (I.2) in term of Y as

$$f(Y,t) = B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)] \tag{A.1}$$

where

$$\begin{aligned} d &= B\exp(-r(T_B - t) - Y) \\ h_1 &= -\frac{0.5s^2(T_B - t) - \log d}{s\sqrt{T_B - t}} \\ h_2 &= -\frac{0.5s^2(T_B - t) + \log d}{s\sqrt{T_B - t}}. \end{aligned}$$

We calculate the following derivatives for use in the calculation of $\frac{\partial f}{\partial Y}$

$$\begin{aligned} \frac{\partial h_1}{\partial Y} &= -\frac{1}{s\sqrt{T_B - t}} \\ \frac{\partial h_2}{\partial Y} &= \frac{1}{s\sqrt{T_B - t}} \\ \frac{\partial}{\partial Y}[d^{-1}] &= \frac{\exp(r(T_B - t) + Y)}{B} \\ \frac{\partial}{\partial Y}[\Phi(h_2)] &= \frac{\exp(-(0.5)h_2^2)}{s\sqrt{2\pi(T_B - t)}} \\ \frac{\partial}{\partial Y}[\Phi(h_1)] &= -\frac{\exp(-(0.5)h_1^2)}{s\sqrt{2\pi(T_B - t)}}. \end{aligned}$$

Using the above,

$$\frac{\partial f}{\partial Y} = \frac{B\exp(-r(T_B-t))\exp(-(0.5)h_2^2)}{s\sqrt{2\pi(T_B-t)}} - \frac{\exp(Y)\exp(-(0.5)h_1^2)}{s\sqrt{2\pi(T_B-t)}} + \Phi(h_1)\exp(Y) \tag{A.2}$$

Lemma 1

$$\lim_{Y\to-\infty}\frac{\partial f}{\partial Y} = 0 \tag{A.3}$$

PROOF:

$$\lim_{Y\to-\infty} d = \infty$$

$$\lim_{Y\to-\infty} \log d = \infty$$

$$\lim_{Y\to-\infty} h_1 = \infty$$

$$\lim_{Y\to-\infty} h_2 = -\infty.$$

Hence,

$$\lim_{Y\to-\infty}\frac{\partial f}{\partial Y} = (\text{constant})\lim_{Y\to-\infty}\exp(-(0.5)h_2^2) - (\text{constant})\left[\lim_{Y\to-\infty}\exp(Y)\right]\left[\lim_{Y\to-\infty}\exp(-(0.5)h_1^2)\right] + \left[\lim_{Y\to-\infty}\Phi(h_1)\right]\left[\lim_{Y\to-\infty}\exp(Y)\right] = 0$$

Lemma 2

$$\lim_{Y\to\infty}\frac{\partial f}{\partial Y} = 0 \tag{A.4}$$

PROOF:

$$\begin{aligned}
\lim_{Y\to\infty} d &= 0 \\
\lim_{Y\to\infty} \log d &= -\infty \\
\lim_{Y\to\infty} h_1 &= -\infty \\
\lim_{Y\to\infty} h_2 &= \infty.
\end{aligned}$$

$$\begin{aligned}
\lim_{Y\to\infty} \frac{B\exp(-r(T_B-t))\exp(-(0.5)h_2^2)}{s\sqrt{2\pi(T_B-t)}} &= (\text{constant}) \lim_{Y\to\infty} \exp(-(0.5)h_2^2) = 0 \qquad \text{(A.5)} \\
\lim_{Y\to\infty} \frac{\exp(Y)\exp(-(0.5)h_1^2)}{s\sqrt{2\pi(T_B-t)}} &= \frac{1}{s\sqrt{2\pi(T_B-t)}} \lim_{Y\to\infty} \frac{\exp(Y)}{\exp(0.5h_1^2)}.
\end{aligned}$$

Note that we can rewrite h_1 as

$$\begin{aligned}
h_1 &= -\frac{0.5s^2(T_B-t) - \log B + r(T_B-t) + Y}{s\sqrt{T_B-t}} \\
&= \frac{c_1 + Y}{c_2}
\end{aligned}$$

where c_1 and c_2 are constants. Hence,

$$h_1^2 = \frac{c_1^2 + 2c_1Y + Y^2}{c_2^2} = c_3 + c_4Y + c_5Y^2$$

where c_3, c_4 and c_5 are constants. Therefore

$$\lim_{Y\to\infty} \frac{exp(Y)\exp(-(0.5)h_1^2)}{s\sqrt{2\pi(T_B-t)}} = (C_8) \lim_{Y\to\infty} \frac{\exp(Y)}{\exp(c_6Y + c_7Y^2)}$$

$$= C_8 \left[\lim_{Y\to\infty} \frac{1}{\exp(c_6 Y)}\right]\left[\lim_{Y\to\infty} \exp(Y - C_7 Y^2)\right]$$

$$= C_8(0)(\mathcal{J}) = 0 \qquad \text{(A.6)}$$

since

$$\lim_{Y\to\infty}(Y - C_7 Y^2) = \lim_{Y\to\infty} Y(1 - C_7 Y) = -\infty$$

$$\lim_{Y\to\infty} \Phi(h_1)\exp(Y) = \lim_{Y\to\infty} \frac{\Phi(h_1)}{\exp(-Y)} = \lim_{Y\to\infty} \frac{\frac{1}{s\sqrt{2\pi(T_B-t)}}\exp(-0.5h_1^2)}{\exp(-Y)}.$$

(using L'Hopital's rule)

$$= (\text{constant}) \lim_{Y\to\infty} \frac{\exp(Y)}{\exp(0.5h_1^2)} = 0 \qquad \text{(A.7)}$$

using previous result in eq. (A.6).

Combining equations (A.2),(A.5),(A.6), and (A.7) we get eq. (A.4). Having established the limiting behavior of $\frac{\partial f}{\partial Y}$, we consider the appropriate boundary conditions for the put option.

Lemma 3

$$\lim_{Y\to-\infty} \frac{\partial H}{\partial Y} = 0 \qquad \text{(A.8)}$$

PROOF:

$$\lim_{Y\to-\infty} \frac{\partial H}{\partial Y} = \left(\lim_{Y\to-\infty} \frac{\partial H}{\partial f}\right)\left(\lim_{Y\to-\infty} \frac{\partial f}{\partial Y}\right).$$

From eq. (I.25)

$$\lim_{Y\to-\infty} \frac{\partial H}{\partial f} = -1.$$

Combining this with Lemma 1 completes the proof.

Lemma 4

$$\lim_{Y\to\infty} \frac{\partial H}{\partial Y} = 0 \tag{A.9}$$

PROOF:

$$\lim_{Y-\infty} \frac{\partial H}{\partial Y} = \left(\lim_{Y\to\infty} \frac{\partial H}{\partial f}\right)\left(\lim_{Y\to\infty} \frac{\partial f}{\partial Y}\right).$$

From eq. (I.26)

$$\lim_{Y\to\infty} \frac{\partial H}{\partial f} \text{is finite.}$$

Combining this with Lemma 2 completes the proof.

Appendix B. Boundary Conditions for Call Option on Zero-Coupon Bond Spread

We first establish the limiting behavior of $\frac{\partial e}{\partial Y}$ as Y tends to $+\infty$ and $-\infty$. We rewrite the equation for the spread (e), i.e., eq. (I.3), in terms of Y as

$$\begin{aligned} f(Y,t) &= B\exp(-(r+e(Y,t))(T_B-t)) \\ \Rightarrow e(Y,t) &= \frac{\log B - r(T_B-t) - \log f(Y,t)}{T_B-t}. \end{aligned} \tag{B.1}$$

Hence,

$$\frac{\partial e}{\partial f} = -\frac{1}{T_B-t}\left(\frac{1}{f(Y,t)}\right).$$

By definition

$$\begin{aligned} \frac{\partial e}{\partial Y} &= \frac{\partial e}{\partial f}\frac{\partial f}{\partial Y} \\ \Rightarrow \frac{\partial e}{\partial Y} &= -\frac{1}{T_B-t}\left(\frac{1}{f}\right)\left(\frac{\partial f}{\partial Y}\right). \end{aligned} \tag{B.2}$$

Lemma 5

$$\lim_{Y\to\infty}\frac{\partial e}{\partial Y} = 0 \tag{B.3}$$

PROOF: It is obvious that $\lim_{Y\to\infty} f$ is finite. Using this fact, Lemma 2 and eq. (B.2) establishes eq. (B.3).

Lemma 6

$$\lim_{Y\to-\infty}\frac{\partial e}{\partial Y} = -\frac{1}{T_B-t} \tag{B.4}$$

PROOF: Note that

$$\lim_{Y \to -\infty} f = 0.$$

Using this fact, eq. (B.2) and Lemma 1 is clearly inadequate in establishing Lemma 6. To prove Lemma 6, we must establish the limiting behavior of $\left(\frac{1}{f}\right)\frac{\partial f}{\partial Y}$ in some other way.

From eqs. (A.1) and (A.2)

$$\begin{aligned}\lim_{Y \to -\infty} \left(\frac{1}{f}\right)\left(\frac{\partial f}{\partial Y}\right) &= \lim_{Y \to -\infty} \frac{\exp(-(0.5)h_2^2)}{s\sqrt{2\pi(T_B - t)}(\Phi(h_2) + d^{-1}\Phi(h_1))} \\ &\quad - \lim_{Y \to -\infty} \frac{\exp(Y - (0.5)h_1^2)}{s\sqrt{2\pi(T_B - t)}B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]} \\ &\quad + \lim_{Y \to -\infty} \frac{\Phi(h_1)\exp(Y)}{B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]}. \end{aligned} \tag{B.5}$$

We shall evaluate each of the three limits on the right hand side of eq. (B.5) separately. We begin with the last term.

Lemma 7

$$\lim_{Y \to -\infty} \frac{\Phi(h_1)\exp(Y)}{B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]} = 1 \tag{B.6}$$

PROOF: Using the definition of d (eq. (A.1)),

$$\begin{aligned}\lim_{Y \to -\infty} \frac{\Phi(h_1)\exp(Y)}{B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]} &= \\ \lim_{Y \to -\infty} \frac{\Phi(h_1)\exp(Y)}{B\exp(-r(T_B - t))\Phi(h_2) + \Phi(h_1)\exp(Y)} &= \\ \lim_{Y \to -\infty} \frac{1}{\delta + 1} & \end{aligned} \tag{B.7}$$

where

$$\delta = \frac{B\exp(-r(T_B - t))\Phi(h_2)}{\Phi(h_1)\exp(Y)}$$

$$\lim_{Y\to-\infty}\delta = (\text{constant})\lim_{Y\to-\infty}\frac{\Phi(h_2)}{\exp(Y)} \tag{B.8}$$

since from Appendix A., we know (see Proof of Lemma 1) that

$$\lim_{Y\to-\infty} h_1 = \infty \Rightarrow \lim_{Y\to-\infty}\Phi(h_1) = 1.$$

Applying L'Hopital's rule to eq. (B.8) we get

$$\lim_{Y\to-\infty}\delta = \lim_{Y\to-\infty}\frac{\exp(-(0.5)h_2^2)}{s\sqrt{2\pi(T_B-t)}\exp(Y)}. \tag{B.9}$$

From Appendix A.,

$$h_2 = -\frac{0.5s^2(T_B-t)+\log B - r(T_B-t)-Y}{s\sqrt{T_B-t}} \tag{B.10}$$

$$= -\frac{C_9-Y}{C_{10}} \tag{B.11}$$

where C_9 and C_{10} are constants. Hence

$$h_2^2 = \left(\frac{C_9-Y}{C_{10}}\right)^2 = C_{11} - C_{12}Y + C_{13}Y^2$$

where C_{11}, C_{12}, and C_{13} are constants. From eq. (B.9),

$$\lim_{Y\to-\infty}\delta = (\text{constant})\left[\lim_{Y\to-\infty}\frac{\exp(-(0.5)h_2^2)}{\exp Y}\right]$$

$$= (\text{constant})\lim_{Y\to-\infty}\exp(-Y + C_{14}Y - C_{15}Y^2) \tag{B.12}$$

where C_{14} and C_{15} are constants.

$$\lim_{Y \to -\infty} -Y + C_{14}Y - C_{15}Y^2 = \lim_{Y \to -\infty} Y(-1 + C_{14} - C_{15}Y) = -\infty. \tag{B.13}$$

since C_{15} is positive. From eq. (B.12) and (B.13)

$$\lim_{Y \to -\infty} \delta = 0. \tag{B.14}$$

Eq. (B.14) and (B.7) give eq. (B.6) to complete the proof.

Lemma 8

$$\lim_{Y \to -\infty} \frac{\exp(Y - (0.5)h_1^2)}{s\sqrt{2\pi(T_B - t)}B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]} = 0 \tag{B.15}$$

PROOF: LHS of eq. (B.15) is

$$\begin{aligned}&[\lim_{Y \to -\infty} \exp(-(0.5)h_1^2)] \left[\lim_{Y \to -\infty} \frac{\exp(Y)}{s\sqrt{2\pi(T_B - t)}B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]}\right] = \\ &(0)\left(\lim_{Y \to -\infty} \frac{\exp(Y)}{s\sqrt{2\pi(T_B - t)}B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]}\right)\end{aligned} \tag{B.16}$$

since $\lim_{Y \to -\infty} h_1 = \infty$ (see Appendix A.).

From Lemma 7, using the fact that

$$\lim_{Y \to -\infty} \Phi(h_1) = 1,$$

$$\lim_{Y \to -\infty} \frac{\exp(Y)}{B\exp(-r(T_B - t))[\Phi(h_2) + d^{-1}\Phi(h_1)]} = 1$$

$$\Rightarrow \lim_{Y\to-\infty} \frac{\exp(Y)}{s\sqrt{2\pi(T_B-t)}B\exp(-r(T_B-t))[\Phi(h_2)+d^{-1}\Phi(h_1)]} = \frac{1}{s\sqrt{2\pi(T_B-t)}}. \quad \text{(B.17)}$$

Eq. (B.17) and (B.16) give eq. (B.15).

Lemma 9

$$\lim_{Y\to-\infty} \frac{\exp(-0.5)h_2^2)}{s\sqrt{2\pi(T_B-t)}[\Phi(h_2)+d^{-1}\Phi(h_1)]} = 0 \quad \text{(B.18)}$$

PROOF: LHS of eq. (B.18) is

$$(constant)\left[\lim_{Y\to-\infty}\exp(-0.5h_2^2-Y)\right]\left[\lim_{Y\to-\infty}\frac{\exp(Y)}{\Phi(h_2)+d^{-1}\Phi(h_1)}\right] \quad \text{(B.19)}$$

From eq. (B.12) and (B.14)

$$\lim_{Y\to-\infty}\exp(-0.5h_2^2-Y)=0. \quad \text{(B.20)}$$

From Lemma 7 using the fact that

$$\lim_{Y\to-\infty}\Phi(h_1)=1$$

$$\lim_{Y\to-\infty}\frac{\exp(Y)}{B\exp(-r(T_B-t))[\Phi(h_2)+d^{-1}\Phi(h_1)]}=1$$

$$\Rightarrow \lim_{Y\to-\infty}\frac{\exp(Y)}{\Phi(h_2)+d^{-1}\Phi(h_1)} = B\exp(-r(T_B-t)). \quad \text{(B.21)}$$

Eqs. (B.19),(B.20), and (B.21) give eq. (B.18). Lemmas 7, 8 and 9 with eqs. (B.5) and (B.2) give Lemma 6. Having established the limiting behavior of $\frac{\partial e}{\partial Y}$, we consider the appropriate boundary conditions for the call option.

Lemma 10

$$\lim_{Y \to -\infty} \frac{\partial L}{\partial Y} = -\frac{1}{T_B - t} \tag{B.22}$$

PROOF:

$$\lim_{Y \to -\infty} \frac{\partial L}{\partial Y} = \left(\lim_{Y \to -\infty} \frac{\partial L}{\partial e} \right) \left(\lim_{Y \to -\infty} \frac{\partial e}{\partial Y} \right).$$

From eq. (I.50),

$$\lim_{Y \to -\infty} \frac{\partial L}{\partial e} = 1.$$

Combining this with Lemma 6 completes the proof.

Lemma 11

$$\lim_{Y \to \infty} \frac{\partial L}{\partial Y} = 0 \tag{B.23}$$

PROOF:

$$\lim_{Y \to \infty} \frac{\partial L}{\partial Y} = \left(\lim_{Y \to \infty} \frac{\partial L}{\partial e} \right) \left(\lim_{Y \to \infty} \frac{\partial e}{\partial Y} \right).$$

From eq. (I.51), $\lim_{Y \to \infty} \frac{\partial L}{\partial e}$ is finite. Combining this with Lemma 5 completes the proof.

Appendix C. Numerical Results

Program output for various parameter values is shown in the tables that follow.

	TABLE A1	Time Steps:	80	Min Firm Value:	1
	PUT	Value Steps:	215	Bond Face Value:	20
	BOND	Bond Maturity:	1.2	Volatility:	0.24
	ZERO	Option Maturity:	1	Riskless Rate:	0.06
		Max Firm Value:	325	g:	0.01
J	Firm	Bond	Option	Temp	Exercise
80	8.6031E+00	8.6015E+00	9.9722E+00	9.9722E+00	9.7871E+00
81	8.8377E+00	8.8354E+00	9.7382E+00	9.7382E+00	9.5532E+00
82	9.0786E+00	9.0754E+00	9.4982E+00	9.4982E+00	9.3132E+00
83	9.3262E+00	9.3216E+00	9.2519E+00	9.2519E+00	9.0670E+00
84	9.5805E+00	9.5740E+00	8.9994E+00	8.9994E+00	8.8146E+00
85	9.8417E+00	9.8327E+00	8.7406E+00	8.7406E+00	8.5559E+00
86	1.0110E+01	1.0098E+01	8.4756E+00	8.4756E+00	8.2909E+00
87	1.0386E+01	1.0369E+01	8.2043E+00	8.2043E+00	8.0197E+00
88	1.0669E+01	1.0646E+01	7.9269E+00	7.9269E+00	7.7425E+00
89	1.0960E+01	1.0929E+01	7.6437E+00	7.6437E+00	7.4593E+00
90	1.1259E+01	1.1218E+01	7.3548E+00	7.3548E+00	7.1705E+00
91	1.1566E+01	1.1512E+01	7.0606E+00	7.0606E+00	6.8764E+00
92	1.1881E+01	1.1811E+01	6.7616E+00	6.7616E+00	6.5775E+00
93	1.2205E+01	1.2114E+01	6.4582E+00	6.4582E+00	6.2742E+00
94	1.2538E+01	1.2421E+01	6.1512E+00	6.1512E+00	5.9672E+00
95	1.2880E+01	1.2731E+01	5.8414E+00	5.8414E+00	5.6574E+00
96	1.3231E+01	1.3043E+01	5.5295E+00	5.5295E+00	5.3455E+00
97	1.3592E+01	1.3356E+01	5.2167E+00	5.2167E+00	5.0325E+00
98	1.3962E+01	1.3669E+01	4.9040E+00	4.9040E+00	4.7196E+00
99	1.4343E+01	1.3981E+01	4.5926E+00	4.5926E+00	4.4080E+00
100	1.4734E+01	1.4290E+01	4.2838E+00	4.2838E+00	4.0989E+00
101	1.5136E+01	1.4595E+01	3.9790E+00	3.9790E+00	3.7937E+00
102	1.5548E+01	1.4895E+01	3.6795E+00	3.6795E+00	3.4938E+00
103	1.5972E+01	1.5188E+01	3.3869E+00	3.3869E+00	3.2005E+00
104	1.6408E+01	1.5473E+01	3.1023E+00	3.1023E+00	2.9153E+00
105	1.6855E+01	1.5749E+01	2.8273E+00	2.8273E+00	2.6396E+00
106	1.7315E+01	1.6014E+01	2.5630E+00	2.5630E+00	2.3744E+00
107	1.7787E+01	1.6267E+01	2.3106E+00	2.3106E+00	2.1211E+00
108	1.8272E+01	1.6508E+01	2.0712E+00	2.0712E+00	1.8807E+00
109	1.8770E+01	1.6735E+01	1.8456E+00	1.8456E+00	1.6540E+00
110	1.9282E+01	1.6947E+01	1.6344E+00	1.6344E+00	1.4417E+00
111	1.9808E+01	1.7144E+01	1.4383E+00	1.4383E+00	1.2443E+00
112	2.0348E+01	1.7327E+01	1.2574E+00	1.2574E+00	1.0621E+00
113	2.0903E+01	1.7493E+01	1.0919E+00	1.0919E+00	8.9522E-01
114	2.1472E+01	1.7645E+01	9.4149E-01	9.4149E-01	7.4350E-01
115	2.2058E+01	1.7782E+01	8.0619E-01	8.0619E-01	6.0665E-01
116	2.2659E+01	1.7904E+01	6.8510E-01	6.8510E-01	4.8421E-01
117	2.3277E+01	1.8013E+01	5.7803E-01	5.7803E-01	3.7553E-01
118	2.3912E+01	1.8109E+01	4.8375E-01	4.8375E-01	2.7989E-01
119	2.4564E+01	1.8192E+01	4.0188E-01	4.0188E-01	1.9641E-01
120	2.5234E+01	1.8264E+01	3.3098E-01	3.3098E-01	1.2416E-01
121	2.5922E+01	1.8326E+01	2.7055E-01	2.7055E-01	6.2187E-02
122	2.6629E+01	1.8379E+01	2.1913E-01	2.1913E-01	9.4783E-03
123	2.7355E+01	1.8424E+01	1.7613E-01	1.7613E-01	0.0000E+00
124	2.8101E+01	1.8461E+01	1.4019E-01	1.4019E-01	0.0000E+00
125	2.8867E+01	1.8491E+01	1.1073E-01	1.1073E-01	0.0000E+00
126	2.9654E+01	1.8517E+01	8.6549E-02	8.6549E-02	0.0000E+00
127	3.0462E+01	1.8537E+01	6.7144E-02	6.7144E-02	0.0000E+00
128	3.1293E+01	1.8554E+01	5.1501E-02	5.1501E-02	0.0000E+00
129	3.2146E+01	1.8567E+01	3.9217E-02	3.9217E-02	0.0000E+00
130	3.3023E+01	1.8577E+01	2.9498E-02	2.9498E-02	0.0000E+00
131	3.3923E+01	1.8585E+01	2.2036E-02	2.2036E-02	0.0000E+00
132	3.4848E+01	1.8592E+01	1.6243E-02	1.6243E-02	0.0000E+00
133	3.5798E+01	1.8597E+01	1.1897E-02	1.1897E-02	0.0000E+00
134	3.6774E+01	1.8600E+01	8.5875E-03	8.5875E-03	0.0000E+00

135	3.7777E+01	1.8603E+01	6.1640E-03	6.1640E-03	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		40	Never Exercise	
1	Never Exercise		41	Never Exercise	
2	Never Exercise		42	Never Exercise	
3	Never Exercise		43	Never Exercise	
4	Never Exercise		44	Never Exercise	
5	Never Exercise		45	Never Exercise	
6	Never Exercise		46	Never Exercise	
7	Never Exercise		47	Never Exercise	
8	Never Exercise		48	Never Exercise	
9	Never Exercise		49	Never Exercise	
10	Never Exercise		50	Never Exercise	
11	Never Exercise		51	Never Exercise	
12	Never Exercise		52	Never Exercise	
13	Never Exercise		53	Never Exercise	
14	Never Exercise		54	Never Exercise	
15	Never Exercise		55	Never Exercise	
16	Never Exercise		56	Never Exercise	
17	Never Exercise		57	Never Exercise	
18	Never Exercise		58	Never Exercise	
19	Never Exercise		59	Never Exercise	
20	Never Exercise		60	Never Exercise	
21	Never Exercise		61	Never Exercise	
22	Never Exercise		62	Never Exercise	
23	Never Exercise		63	Never Exercise	
24	Never Exercise		64	Never Exercise	
25	Never Exercise		65	Never Exercise	
26	Never Exercise		66	Never Exercise	
27	Never Exercise		67	Never Exercise	
28	Never Exercise		68	Never Exercise	
29	Never Exercise		69	Never Exercise	
30	Never Exercise		70	Never Exercise	
31	Never Exercise		71	Never Exercise	
32	Never Exercise		72	Never Exercise	
33	Never Exercise		73	Never Exercise	
34	Never Exercise		74	Never Exercise	
35	Never Exercise		75	Never Exercise	
36	Never Exercise		76	Never Exercise	
37	Never Exercise		77	Never Exercise	
38	Never Exercise		78	Never Exercise	
39	Never Exercise		79	Never Exercise	

	TABLE A2	Time Steps:	80	Min Firm Value:	1
	PUT	Value Steps:	215	Bond Face Value:	20
	BOND	Bond Maturity:	1.2	Volatility:	0.24
	ZERO	Option Maturity:	0.5	Riskless Rate:	0.06
		Max Firm Value:	325	g:	0.01
J	Firm	Bond	Option	Temp	Exercise
80	8.6031E+00	8.6015E+00	9.8794E+00	9.8794E+00	9.7871E+00
81	8.8377E+00	8.8354E+00	9.6455E+00	9.6455E+00	9.5532E+00
82	9.0786E+00	9.0754E+00	9.4055E+00	9.4055E+00	9.3132E+00
83	9.3262E+00	9.3216E+00	9.1593E+00	9.1593E+00	9.0670E+00
84	9.5805E+00	9.5740E+00	8.9068E+00	8.9068E+00	8.8146E+00
85	9.8417E+00	9.8327E+00	8.6481E+00	8.6481E+00	8.5559E+00
86	1.0110E+01	1.0098E+01	8.3831E+00	8.3831E+00	8.2909E+00
87	1.0386E+01	1.0369E+01	8.1119E+00	8.1119E+00	8.0197E+00
88	1.0669E+01	1.0646E+01	7.8346E+00	7.8346E+00	7.7425E+00
89	1.0960E+01	1.0929E+01	7.5515E+00	7.5515E+00	7.4593E+00
90	1.1259E+01	1.1218E+01	7.2626E+00	7.2626E+00	7.1705E+00
91	1.1566E+01	1.1512E+01	6.9685E+00	6.9685E+00	6.8764E+00
92	1.1881E+01	1.1811E+01	6.6696E+00	6.6696E+00	6.5775E+00
93	1.2205E+01	1.2114E+01	6.3663E+00	6.3663E+00	6.2742E+00
94	1.2538E+01	1.2421E+01	6.0593E+00	6.0593E+00	5.9672E+00
95	1.2880E+01	1.2731E+01	5.7494E+00	5.7494E+00	5.6574E+00
96	1.3231E+01	1.3043E+01	5.4375E+00	5.4375E+00	5.3455E+00
97	1.3592E+01	1.3356E+01	5.1246E+00	5.1246E+00	5.0325E+00
98	1.3962E+01	1.3669E+01	4.8117E+00	4.8117E+00	4.7196E+00
99	1.4343E+01	1.3981E+01	4.5001E+00	4.5001E+00	4.4080E+00
100	1.4734E+01	1.4290E+01	4.1910E+00	4.1910E+00	4.0989E+00
101	1.5136E+01	1.4595E+01	3.8859E+00	3.8859E+00	3.7937E+00
102	1.5548E+01	1.4895E+01	3.5860E+00	3.5860E+00	3.4938E+00
103	1.5972E+01	1.5188E+01	3.2928E+00	3.2928E+00	3.2005E+00
104	1.6408E+01	1.5473E+01	3.0077E+00	3.0077E+00	2.9153E+00
105	1.6855E+01	1.5749E+01	2.7321E+00	2.7321E+00	2.6396E+00
106	1.7315E+01	1.6014E+01	2.4672E+00	2.4672E+00	2.3744E+00
107	1.7787E+01	1.6267E+01	2.2143E+00	2.2143E+00	2.1211E+00
108	1.8272E+01	1.6508E+01	1.9744E+00	1.9744E+00	1.8807E+00
109	1.8770E+01	1.6735E+01	1.7484E+00	1.7484E+00	1.6540E+00
110	1.9282E+01	1.6947E+01	1.5370E+00	1.5370E+00	1.4417E+00
111	1.9808E+01	1.7144E+01	1.3409E+00	1.3409E+00	1.2443E+00
112	2.0348E+01	1.7327E+01	1.1604E+00	1.1604E+00	1.0621E+00
113	2.0903E+01	1.7493E+01	9.9567E-01	9.9567E-01	8.9522E-01
114	2.1472E+01	1.7645E+01	8.4666E-01	8.4666E-01	7.4350E-01
115	2.2058E+01	1.7782E+01	7.1314E-01	7.1314E-01	6.0665E-01
116	2.2659E+01	1.7904E+01	5.9468E-01	5.9468E-01	4.8421E-01
117	2.3277E+01	1.8013E+01	4.9068E-01	4.9068E-01	3.7553E-01
118	2.3912E+01	1.8109E+01	4.0039E-01	4.0039E-01	2.7989E-01
119	2.4564E+01	1.8192E+01	3.2291E-01	3.2291E-01	1.9641E-01
120	2.5234E+01	1.8264E+01	2.5724E-01	2.5724E-01	1.2416E-01
121	2.5922E+01	1.8326E+01	2.0230E-01	2.0230E-01	6.2187E-02
122	2.6629E+01	1.8379E+01	1.5695E-01	1.5695E-01	9.4783E-03
123	2.7355E+01	1.8424E+01	1.2007E-01	1.2007E-01	0.0000E+00
124	2.8101E+01	1.8461E+01	9.0504E-02	9.0504E-02	0.0000E+00
125	2.8867E+01	1.8491E+01	6.7184E-02	6.7184E-02	0.0000E+00
126	2.9654E+01	1.8517E+01	4.9085E-02	4.9085E-02	0.0000E+00
127	3.0462E+01	1.8537E+01	3.5274E-02	3.5274E-02	0.0000E+00
128	3.1293E+01	1.8554E+01	2.4921E-02	2.4921E-02	0.0000E+00
129	3.2146E+01	1.8567E+01	1.7299E-02	1.7299E-02	0.0000E+00
130	3.3023E+01	1.8577E+01	1.1792E-02	1.1792E-02	0.0000E+00
131	3.3923E+01	1.8585E+01	7.8892E-03	7.8892E-03	0.0000E+00
132	3.4848E+01	1.8592E+01	5.1779E-03	5.1779E-03	0.0000E+00
133	3.5798E+01	1.8597E+01	3.3322E-03	3.3322E-03	0.0000E+00
134	3.6774E+01	1.8600E+01	2.1017E-03	2.1017E-03	0.0000E+00

135	3.7777E+01	1.8603E+01	1.2985E-03	1.2985E-03	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		40	Never Exercise	
1	Never Exercise		41	Never Exercise	
2	Never Exercise		42	Never Exercise	
3	Never Exercise		43	Never Exercise	
4	Never Exercise		44	Never Exercise	
5	Never Exercise		45	Never Exercise	
6	Never Exercise		46	Never Exercise	
7	Never Exercise		47	Never Exercise	
8	Never Exercise		48	Never Exercise	
9	Never Exercise		49	Never Exercise	
10	Never Exercise		50	Never Exercise	
11	Never Exercise		51	Never Exercise	
12	Never Exercise		52	Never Exercise	
13	Never Exercise		53	Never Exercise	
14	Never Exercise		54	Never Exercise	
15	Never Exercise		55	Never Exercise	
16	Never Exercise		56	Never Exercise	
17	Never Exercise		57	Never Exercise	
18	Never Exercise		58	Never Exercise	
19	Never Exercise		59	Never Exercise	
20	Never Exercise		60	Never Exercise	
21	Never Exercise		61	Never Exercise	
22	Never Exercise		62	Never Exercise	
23	Never Exercise		63	Never Exercise	
24	Never Exercise		64	Never Exercise	
25	Never Exercise		65	Never Exercise	
26	Never Exercise		66	Never Exercise	
27	Never Exercise		67	Never Exercise	
28	Never Exercise		68	Never Exercise	
29	Never Exercise		69	Never Exercise	
30	Never Exercise		70	Never Exercise	
31	Never Exercise		71	Never Exercise	
32	Never Exercise		72	Never Exercise	
33	Never Exercise		73	Never Exercise	
34	Never Exercise		74	Never Exercise	
35	Never Exercise		75	Never Exercise	
36	Never Exercise		76	Never Exercise	
37	Never Exercise		77	Never Exercise	
38	Never Exercise		78	Never Exercise	
39	Never Exercise		79	Never Exercise	

	TABLE A3	Time Steps:	80	Min Firm Value:	1
	PUT	Value Steps:	215	Bond Face Value:	20
	BOND	Bond Maturity:	1.2	Volatility:	0.324
	ZERO	Option Maturity:	0.5	Riskless Rate:	0.06
		Max Firm Value:	325	g:	0.01
J	Firm	Bond	Option	Temp	Exercise
80	8.6031E+00	8.5798E+00	9.9010E+00	9.9010E+00	9.8089E+00
81	8.8377E+00	8.8084E+00	9.6723E+00	9.6723E+00	9.5802E+00
82	9.0786E+00	9.0422E+00	9.4385E+00	9.4385E+00	9.3464E+00
83	9.3262E+00	9.2810E+00	9.1997E+00	9.1997E+00	9.1076E+00
84	9.5805E+00	9.5247E+00	8.9560E+00	8.9560E+00	8.8640E+00
85	9.8417E+00	9.7731E+00	8.7075E+00	8.7075E+00	8.6155E+00
86	1.0110E+01	1.0026E+01	8.4545E+00	8.4545E+00	8.3625E+00
87	1.0386E+01	1.0283E+01	8.1971E+00	8.1971E+00	8.1051E+00
88	1.0669E+01	1.0545E+01	7.9356E+00	7.9356E+00	7.8437E+00
89	1.0960E+01	1.0810E+01	7.6704E+00	7.6704E+00	7.5784E+00
90	1.1259E+01	1.1079E+01	7.4017E+00	7.4017E+00	7.3098E+00
91	1.1566E+01	1.1351E+01	7.1300E+00	7.1300E+00	7.0381E+00
92	1.1881E+01	1.1625E+01	6.8558E+00	6.8558E+00	6.7639E+00
93	1.2205E+01	1.1901E+01	6.5795E+00	6.5795E+00	6.4876E+00
94	1.2538E+01	1.2179E+01	6.3018E+00	6.3018E+00	6.2098E+00
95	1.2880E+01	1.2457E+01	6.0231E+00	6.0231E+00	5.9311E+00
96	1.3231E+01	1.2736E+01	5.7442E+00	5.7442E+00	5.6522E+00
97	1.3592E+01	1.3015E+01	5.4657E+00	5.4657E+00	5.3737E+00
98	1.3962E+01	1.3292E+01	5.1884E+00	5.1884E+00	5.0963E+00
99	1.4343E+01	1.3568E+01	4.9129E+00	4.9129E+00	4.8207E+00
100	1.4734E+01	1.3841E+01	4.6400E+00	4.6400E+00	4.5478E+00
101	1.5136E+01	1.4110E+01	4.3706E+00	4.3706E+00	4.2783E+00
102	1.5548E+01	1.4376E+01	4.1052E+00	4.1052E+00	4.0129E+00
103	1.5972E+01	1.4636E+01	3.8448E+00	3.8448E+00	3.7524E+00
104	1.6408E+01	1.4891E+01	3.5901E+00	3.5901E+00	3.4975E+00
105	1.6855E+01	1.5140E+01	3.3418E+00	3.3418E+00	3.2491E+00
106	1.7315E+01	1.5381E+01	3.1005E+00	3.1005E+00	3.0076E+00
107	1.7787E+01	1.5615E+01	2.8670E+00	2.8670E+00	2.7738E+00
108	1.8272E+01	1.5840E+01	2.6417E+00	2.6417E+00	2.5484E+00
109	1.8770E+01	1.6057E+01	2.4254E+00	2.4254E+00	2.3316E+00
110	1.9282E+01	1.6264E+01	2.2184E+00	2.2184E+00	2.1242E+00
111	1.9808E+01	1.6462E+01	2.0211E+00	2.0211E+00	1.9264E+00
112	2.0348E+01	1.6650E+01	1.8340E+00	1.8340E+00	1.7385E+00
113	2.0903E+01	1.6828E+01	1.6571E+00	1.6571E+00	1.5608E+00
114	2.1472E+01	1.6995E+01	1.4908E+00	1.4908E+00	1.3934E+00
115	2.2058E+01	1.7152E+01	1.3351E+00	1.3351E+00	1.2364E+00
116	2.2659E+01	1.7299E+01	1.1900E+00	1.1900E+00	1.0898E+00
117	2.3277E+01	1.7435E+01	1.0556E+00	1.0556E+00	9.5342E-01
118	2.3912E+01	1.7561E+01	9.3154E-01	9.3154E-01	8.2721E-01
119	2.4564E+01	1.7678E+01	8.1775E-01	8.1775E-01	7.1089E-01
120	2.5234E+01	1.7784E+01	7.1394E-01	7.1394E-01	6.0419E-01
121	2.5922E+01	1.7882E+01	6.1977E-01	6.1977E-01	5.0675E-01
122	2.6629E+01	1.7970E+01	5.3484E-01	5.3484E-01	4.1818E-01
123	2.7355E+01	1.8051E+01	4.5873E-01	4.5873E-01	3.3805E-01
124	2.8101E+01	1.8123E+01	3.9095E-01	3.9095E-01	2.6590E-01
125	2.8867E+01	1.8187E+01	3.3100E-01	3.3100E-01	2.0124E-01
126	2.9654E+01	1.8245E+01	2.7832E-01	2.7832E-01	1.4358E-01
127	3.0462E+01	1.8296E+01	2.3238E-01	2.3238E-01	9.2405E-02
128	3.1293E+01	1.8341E+01	1.9260E-01	1.9260E-01	4.7210E-02
129	3.2146E+01	1.8381E+01	1.5843E-01	1.5843E-01	7.4916E-03
130	3.3023E+01	1.8416E+01	1.2930E-01	1.2930E-01	0.0000E+00
131	3.3923E+01	1.8446E+01	1.0468E-01	1.0468E-01	0.0000E+00
132	3.4848E+01	1.8472E+01	8.4042E-02	8.4042E-02	0.0000E+00
133	3.5798E+01	1.8495E+01	6.6896E-02	6.6896E-02	0.0000E+00
134	3.6774E+01	1.8514E+01	5.2779E-02	5.2779E-02	0.0000E+00

135	3.7777E+01	1.8531E+01	4.1264E-02	4.1264E-02	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		40	Never Exercise	
1	Never Exercise		41	Never Exercise	
2	Never Exercise		42	Never Exercise	
3	Never Exercise		43	Never Exercise	
4	Never Exercise		44	Never Exercise	
5	Never Exercise		45	Never Exercise	
6	Never Exercise		46	Never Exercise	
7	Never Exercise		47	Never Exercise	
8	Never Exercise		48	Never Exercise	
9	Never Exercise		49	Never Exercise	
10	Never Exercise		50	Never Exercise	
11	Never Exercise		51	Never Exercise	
12	Never Exercise		52	Never Exercise	
13	Never Exercise		53	Never Exercise	
14	Never Exercise		54	Never Exercise	
15	Never Exercise		55	Never Exercise	
16	Never Exercise		56	Never Exercise	
17	Never Exercise		57	Never Exercise	
18	Never Exercise		58	Never Exercise	
19	Never Exercise		59	Never Exercise	
20	Never Exercise		60	Never Exercise	
21	Never Exercise		61	Never Exercise	
22	Never Exercise		62	Never Exercise	
23	Never Exercise		63	Never Exercise	
24	Never Exercise		64	Never Exercise	
25	Never Exercise		65	Never Exercise	
26	Never Exercise		66	Never Exercise	
27	Never Exercise		67	Never Exercise	
28	Never Exercise		68	Never Exercise	
29	Never Exercise		69	Never Exercise	
30	Never Exercise		70	Never Exercise	
31	Never Exercise		71	Never Exercise	
32	Never Exercise		72	Never Exercise	
33	Never Exercise		73	Never Exercise	
34	Never Exercise		74	Never Exercise	
35	Never Exercise		75	Never Exercise	
36	Never Exercise		76	Never Exercise	
37	Never Exercise		77	Never Exercise	
38	Never Exercise		78	Never Exercise	
39	Never Exercise		79	Never Exercise	

	TABLE A4	Time Steps:	80	Min Firm Value:	1
	PUT	Value Steps:	215	Bond Face Value:	20
	BOND	Bond Maturity:	1.2	Volatility:	0.324
	ZERO	Option Maturity:	0.5	Riskless Rate:	0.1
		Max Firm Value:	325	g:	0.01
J	Firm	Bond	Option	Temp	Exercise
80	8.6031E+00	8.5700E+00	9.0447E+00	9.0447E+00	8.9569E+00
81	8.8377E+00	8.7965E+00	8.8181E+00	8.8181E+00	8.7303E+00
82	9.0786E+00	9.0278E+00	8.5868E+00	8.5868E+00	8.4990E+00
83	9.3262E+00	9.2636E+00	8.3510E+00	8.3510E+00	8.2632E+00
84	9.5805E+00	9.5038E+00	8.1107E+00	8.1107E+00	8.0230E+00
85	9.8417E+00	9.7483E+00	7.8662E+00	7.8662E+00	7.7785E+00
86	1.0110E+01	9.9967E+00	7.6178E+00	7.6178E+00	7.5302E+00
87	1.0386E+01	1.0249E+01	7.3657E+00	7.3657E+00	7.2781E+00
88	1.0669E+01	1.0504E+01	7.1103E+00	7.1103E+00	7.0227E+00
89	1.0960E+01	1.0762E+01	6.8520E+00	6.8520E+00	6.7643E+00
90	1.1259E+01	1.1023E+01	6.5910E+00	6.5910E+00	6.5034E+00
91	1.1566E+01	1.1286E+01	6.3281E+00	6.3281E+00	6.2405E+00
92	1.1881E+01	1.1551E+01	6.0636E+00	6.0636E+00	5.9760E+00
93	1.2205E+01	1.1816E+01	5.7981E+00	5.7981E+00	5.7105E+00
94	1.2538E+01	1.2082E+01	5.5323E+00	5.5323E+00	5.4446E+00
95	1.2880E+01	1.2348E+01	5.2667E+00	5.2667E+00	5.1790E+00
96	1.3231E+01	1.2612E+01	5.0020E+00	5.0020E+00	4.9143E+00
97	1.3592E+01	1.2876E+01	4.7390E+00	4.7390E+00	4.6513E+00
98	1.3962E+01	1.3136E+01	4.4783E+00	4.4783E+00	4.3906E+00
99	1.4343E+01	1.3394E+01	4.2207E+00	4.2207E+00	4.1329E+00
100	1.4734E+01	1.3648E+01	3.9669E+00	3.9669E+00	3.8790E+00
101	1.5136E+01	1.3897E+01	3.7176E+00	3.7176E+00	3.6297E+00
102	1.5548E+01	1.4141E+01	3.4736E+00	3.4736E+00	3.3855E+00
103	1.5972E+01	1.4379E+01	3.2355E+00	3.2355E+00	3.1473E+00
104	1.6408E+01	1.4611E+01	3.0041E+00	3.0041E+00	2.9157E+00
105	1.6855E+01	1.4836E+01	2.7798E+00	2.7798E+00	2.6913E+00
106	1.7315E+01	1.5052E+01	2.5634E+00	2.5634E+00	2.4746E+00
107	1.7787E+01	1.5261E+01	2.3554E+00	2.3554E+00	2.2662E+00
108	1.8272E+01	1.5460E+01	2.1561E+00	2.1561E+00	2.0665E+00
109	1.8770E+01	1.5651E+01	1.9661E+00	1.9661E+00	1.8760E+00
110	1.9282E+01	1.5832E+01	1.7856E+00	1.7856E+00	1.6948E+00
111	1.9808E+01	1.6003E+01	1.6149E+00	1.6149E+00	1.5233E+00
112	2.0348E+01	1.6165E+01	1.4542E+00	1.4542E+00	1.3617E+00
113	2.0903E+01	1.6317E+01	1.3036E+00	1.3036E+00	1.2099E+00
114	2.1472E+01	1.6459E+01	1.1632E+00	1.1632E+00	1.0680E+00
115	2.2058E+01	1.6591E+01	1.0329E+00	1.0329E+00	9.3594E-01
116	2.2659E+01	1.6713E+01	9.1255E-01	9.1255E-01	8.1358E-01
117	2.3277E+01	1.6826E+01	8.0203E-01	8.0203E-01	7.0071E-01
118	2.3912E+01	1.6930E+01	7.0107E-01	7.0107E-01	5.9705E-01
119	2.4564E+01	1.7025E+01	6.0937E-01	6.0937E-01	5.0230E-01
120	2.5234E+01	1.7111E+01	5.2656E-01	5.2656E-01	4.1609E-01
121	2.5922E+01	1.7189E+01	4.5225E-01	4.5225E-01	3.3802E-01
122	2.6629E+01	1.7259E+01	3.8599E-01	3.8599E-01	2.6765E-01
123	2.7355E+01	1.7322E+01	3.2728E-01	3.2728E-01	2.0452E-01
124	2.8101E+01	1.7379E+01	2.7563E-01	2.7563E-01	1.4817E-01
125	2.8867E+01	1.7429E+01	2.3050E-01	2.3050E-01	9.8097E-02
126	2.9654E+01	1.7473E+01	1.9137E-01	1.9137E-01	5.3832E-02
127	3.0462E+01	1.7512E+01	1.5769E-01	1.5769E-01	1.4889E-02
128	3.1293E+01	1.7546E+01	1.2893E-01	1.2893E-01	0.0000E+00
129	3.2146E+01	1.7576E+01	1.0458E-01	1.0458E-01	0.0000E+00
130	3.3023E+01	1.7601E+01	8.4127E-02	8.4127E-02	0.0000E+00
131	3.3923E+01	1.7624E+01	6.7101E-02	6.7101E-02	0.0000E+00
132	3.4848E+01	1.7643E+01	5.3054E-02	5.3054E-02	0.0000E+00
133	3.5798E+01	1.7659E+01	4.1571E-02	4.1571E-02	0.0000E+00
134	3.6774E+01	1.7673E+01	3.2273E-02	3.2273E-02	0.0000E+00

135	3.7777E+01	1.7684E+01	2.4818E-02	2.4818E-02	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		40	Never Exercise	
1	Never Exercise		41	Never Exercise	
2	Never Exercise		42	Never Exercise	
3	Never Exercise		43	Never Exercise	
4	Never Exercise		44	Never Exercise	
5	Never Exercise		45	Never Exercise	
6	Never Exercise		46	Never Exercise	
7	Never Exercise		47	Never Exercise	
8	Never Exercise		48	Never Exercise	
9	Never Exercise		49	Never Exercise	
10	Never Exercise		50	Never Exercise	
11	Never Exercise		51	Never Exercise	
12	Never Exercise		52	Never Exercise	
13	Never Exercise		53	Never Exercise	
14	Never Exercise		54	Never Exercise	
15	Never Exercise		55	Never Exercise	
16	Never Exercise		56	Never Exercise	
17	Never Exercise		57	Never Exercise	
18	Never Exercise		58	Never Exercise	
19	Never Exercise		59	Never Exercise	
20	Never Exercise		60	Never Exercise	
21	Never Exercise		61	Never Exercise	
22	Never Exercise		62	Never Exercise	
23	Never Exercise		63	Never Exercise	
24	Never Exercise		64	Never Exercise	
25	Never Exercise		65	Never Exercise	
26	Never Exercise		66	Never Exercise	
27	Never Exercise		67	Never Exercise	
28	Never Exercise		68	Never Exercise	
29	Never Exercise		69	Never Exercise	
30	Never Exercise		70	Never Exercise	
31	Never Exercise		71	Never Exercise	
32	Never Exercise		72	Never Exercise	
33	Never Exercise		73	Never Exercise	
34	Never Exercise		74	Never Exercise	
35	Never Exercise		75	Never Exercise	
36	Never Exercise		76	Never Exercise	
37	Never Exercise		77	Never Exercise	
38	Never Exercise		78	Never Exercise	
39	Never Exercise		79	Never Exercise	

	TABLE A5	Time Steps:	80	Min Firm Value:	1
	PUT	Value Steps:	215	Bond Face Value:	20
	BOND	Bond Maturity:	1.2	Volatility:	0.324
	ZERO	Option Maturity:	0.5	Riskless Rate:	0.1
		Max Firm Value:	325	g:	0.02
J	Firm	Bond	Option	Temp	Exercise
80	8.6031E+00	8.5700E+00	8.9218E+00	8.9218E+00	8.7478E+00
81	8.8377E+00	8.7965E+00	8.6952E+00	8.6952E+00	8.5212E+00
82	9.0786E+00	9.0278E+00	8.4639E+00	8.4639E+00	8.2900E+00
83	9.3262E+00	9.2636E+00	8.2281E+00	8.2281E+00	8.0541E+00
84	9.5805E+00	9.5038E+00	7.9878E+00	7.9878E+00	7.8139E+00
85	9.8417E+00	9.7483E+00	7.7434E+00	7.7434E+00	7.5695E+00
86	1.0110E+01	9.9967E+00	7.4950E+00	7.4950E+00	7.3211E+00
87	1.0386E+01	1.0249E+01	7.2429E+00	7.2429E+00	7.0690E+00
88	1.0669E+01	1.0504E+01	6.9875E+00	6.9875E+00	6.8136E+00
89	1.0960E+01	1.0762E+01	6.7291E+00	6.7291E+00	6.5553E+00
90	1.1259E+01	1.1023E+01	6.4682E+00	6.4682E+00	6.2944E+00
91	1.1566E+01	1.1286E+01	6.2052E+00	6.2052E+00	6.0314E+00
92	1.1881E+01	1.1551E+01	5.9407E+00	5.9407E+00	5.7669E+00
93	1.2205E+01	1.1816E+01	5.6753E+00	5.6753E+00	5.5014E+00
94	1.2538E+01	1.2082E+01	5.4094E+00	5.4094E+00	5.2356E+00
95	1.2880E+01	1.2348E+01	5.1439E+00	5.1439E+00	4.9700E+00
96	1.3231E+01	1.2612E+01	4.8792E+00	4.8792E+00	4.7053E+00
97	1.3592E+01	1.2876E+01	4.6163E+00	4.6163E+00	4.4422E+00
98	1.3962E+01	1.3136E+01	4.3556E+00	4.3556E+00	4.1815E+00
99	1.4343E+01	1.3394E+01	4.0981E+00	4.0981E+00	3.9238E+00
100	1.4734E+01	1.3648E+01	3.8445E+00	3.8445E+00	3.6700E+00
101	1.5136E+01	1.3897E+01	3.5954E+00	3.5954E+00	3.4206E+00
102	1.5548E+01	1.4141E+01	3.3517E+00	3.3517E+00	3.1765E+00
103	1.5972E+01	1.4379E+01	3.1139E+00	3.1139E+00	2.9383E+00
104	1.6408E+01	1.4611E+01	2.8829E+00	2.8829E+00	2.7066E+00
105	1.6855E+01	1.4836E+01	2.6593E+00	2.6593E+00	2.4822E+00
106	1.7315E+01	1.5052E+01	2.4437E+00	2.4437E+00	2.2655E+00
107	1.7787E+01	1.5261E+01	2.2365E+00	2.2365E+00	2.0571E+00
108	1.8272E+01	1.5460E+01	2.0385E+00	2.0385E+00	1.8575E+00
109	1.8770E+01	1.5651E+01	1.8499E+00	1.8499E+00	1.6669E+00
110	1.9282E+01	1.5832E+01	1.6712E+00	1.6712E+00	1.4858E+00
111	1.9808E+01	1.6003E+01	1.5026E+00	1.5026E+00	1.3143E+00
112	2.0348E+01	1.6165E+01	1.3443E+00	1.3443E+00	1.1526E+00
113	2.0903E+01	1.6317E+01	1.1966E+00	1.1966E+00	1.0008E+00
114	2.1472E+01	1.6459E+01	1.0593E+00	1.0593E+00	8.5893E-01
115	2.2058E+01	1.6591E+01	9.3264E-01	9.3264E-01	7.2688E-01
116	2.2659E+01	1.6713E+01	8.1632E-01	8.1632E-01	6.0452E-01
117	2.3277E+01	1.6826E+01	7.1019E-01	7.1019E-01	4.9164E-01
118	2.3912E+01	1.6930E+01	6.1398E-01	6.1398E-01	3.8799E-01
119	2.4564E+01	1.7025E+01	5.2734E-01	5.2734E-01	2.9324E-01
120	2.5234E+01	1.7111E+01	4.4986E-01	4.4986E-01	2.0703E-01
121	2.5922E+01	1.7189E+01	3.8107E-01	3.8107E-01	1.2896E-01
122	2.6629E+01	1.7259E+01	3.2045E-01	3.2045E-01	5.8585E-02
123	2.7355E+01	1.7322E+01	2.6745E-01	2.6745E-01	0.0000E+00
124	2.8101E+01	1.7379E+01	2.2147E-01	2.2147E-01	0.0000E+00
125	2.8867E+01	1.7429E+01	1.8193E-01	1.8193E-01	0.0000E+00
126	2.9654E+01	1.7473E+01	1.4821E-01	1.4821E-01	0.0000E+00
127	3.0462E+01	1.7512E+01	1.1971E-01	1.1971E-01	0.0000E+00
128	3.1293E+01	1.7546E+01	9.5838E-02	9.5838E-02	0.0000E+00
129	3.2146E+01	1.7576E+01	7.6035E-02	7.6035E-02	0.0000E+00
130	3.3023E+01	1.7601E+01	5.9765E-02	5.9765E-02	0.0000E+00
131	3.3923E+01	1.7624E+01	4.6529E-02	4.6529E-02	0.0000E+00
132	3.4848E+01	1.7643E+01	3.5871E-02	3.5871E-02	0.0000E+00
133	3.5798E+01	1.7659E+01	2.7378E-02	2.7378E-02	0.0000E+00
134	3.6774E+01	1.7673E+01	2.0683E-02	2.0683E-02	0.0000E+00

135	3.7777E+01	1.7684E+01	1.5461E-02	1.5461E-02	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		40	Never Exercise	
1	Never Exercise		41	Never Exercise	
2	Never Exercise		42	Never Exercise	
3	Never Exercise		43	Never Exercise	
4	Never Exercise		44	Never Exercise	
5	Never Exercise		45	Never Exercise	
6	Never Exercise		46	Never Exercise	
7	Never Exercise		47	Never Exercise	
8	Never Exercise		48	Never Exercise	
9	Never Exercise		49	Never Exercise	
10	Never Exercise		50	Never Exercise	
11	Never Exercise		51	Never Exercise	
12	Never Exercise		52	Never Exercise	
13	Never Exercise		53	Never Exercise	
14	Never Exercise		54	Never Exercise	
15	Never Exercise		55	Never Exercise	
16	Never Exercise		56	Never Exercise	
17	Never Exercise		57	Never Exercise	
18	Never Exercise		58	Never Exercise	
19	Never Exercise		59	Never Exercise	
20	Never Exercise		60	Never Exercise	
21	Never Exercise		61	Never Exercise	
22	Never Exercise		62	Never Exercise	
23	Never Exercise		63	Never Exercise	
24	Never Exercise		64	Never Exercise	
25	Never Exercise		65	Never Exercise	
26	Never Exercise		66	Never Exercise	
27	Never Exercise		67	Never Exercise	
28	Never Exercise		68	Never Exercise	
29	Never Exercise		69	Never Exercise	
30	Never Exercise		70	Never Exercise	
31	Never Exercise		71	Never Exercise	
32	Never Exercise		72	Never Exercise	
33	Never Exercise		73	Never Exercise	
34	Never Exercise		74	Never Exercise	
35	Never Exercise		75	Never Exercise	
36	Never Exercise		76	Never Exercise	
37	Never Exercise		77	Never Exercise	
38	Never Exercise		78	Never Exercise	
39	Never Exercise		79	Never Exercise	

	TABLE B1	Time Steps:	35	Min Firm Value:	0.001
	CALL	Value Steps:	90	Bond Face Value:	10
	SPREAD	Bond Maturity:	2	Volatility:	0.3
	ZERO	Option Maturity:	1	Riskless Rate:	0.1
		Max Firm Value:	100	Exercise price:	0.02
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-03	4.5052E+00	8.0808E+00	8.0808E+00	4.4852E+00
1	1.1365E-03	4.4412E+00	8.0168E+00	8.0168E+00	4.4212E+00
2	1.2915E-03	4.3772E+00	7.9269E+00	7.9269E+00	4.3572E+00
3	1.4678E-03	4.3133E+00	7.8226E+00	7.8226E+00	4.2933E+00
4	1.6681E-03	4.2493E+00	7.7114E+00	7.7114E+00	4.2293E+00
5	1.8957E-03	4.1854E+00	7.5972E+00	7.5972E+00	4.1654E+00
6	2.1544E-03	4.1214E+00	7.4820E+00	7.4820E+00	4.1014E+00
7	2.4484E-03	4.0574E+00	7.3663E+00	7.3663E+00	4.0374E+00
8	2.7826E-03	3.9935E+00	7.2506E+00	7.2506E+00	3.9735E+00
9	3.1623E-03	3.9295E+00	7.1349E+00	7.1349E+00	3.9095E+00
10	3.5938E-03	3.8656E+00	7.0191E+00	7.0191E+00	3.8456E+00
11	4.0842E-03	3.8016E+00	6.9033E+00	6.9033E+00	3.7816E+00
12	4.6416E-03	3.7376E+00	6.7876E+00	6.7876E+00	3.7176E+00
13	5.2750E-03	3.6737E+00	6.6718E+00	6.6718E+00	3.6537E+00
14	5.9948E-03	3.6097E+00	6.5560E+00	6.5560E+00	3.5897E+00
15	6.8129E-03	3.5458E+00	6.4403E+00	6.4403E+00	3.5258E+00
16	7.7426E-03	3.4818E+00	6.3245E+00	6.3245E+00	3.4618E+00
17	8.7992E-03	3.4178E+00	6.2088E+00	6.2088E+00	3.3978E+00
18	1.0000E-02	3.3539E+00	6.0930E+00	6.0930E+00	3.3339E+00
19	1.1365E-02	3.2899E+00	5.9772E+00	5.9772E+00	3.2699E+00
20	1.2915E-02	3.2260E+00	5.8615E+00	5.8615E+00	3.2060E+00
21	1.4678E-02	3.1620E+00	5.7457E+00	5.7457E+00	3.1420E+00
22	1.6681E-02	3.0980E+00	5.6299E+00	5.6299E+00	3.0780E+00
23	1.8957E-02	3.0341E+00	5.5142E+00	5.5142E+00	3.0141E+00
24	2.1544E-02	2.9701E+00	5.3984E+00	5.3984E+00	2.9501E+00
25	2.4484E-02	2.9062E+00	5.2826E+00	5.2826E+00	2.8862E+00
26	2.7826E-02	2.8422E+00	5.1669E+00	5.1669E+00	2.8222E+00
27	3.1623E-02	2.7782E+00	5.0511E+00	5.0511E+00	2.7582E+00
28	3.5938E-02	2.7143E+00	4.9353E+00	4.9353E+00	2.6943E+00
29	4.0842E-02	2.6503E+00	4.8196E+00	4.8196E+00	2.6303E+00
30	4.6416E-02	2.5863E+00	4.7038E+00	4.7038E+00	2.5663E+00
31	5.2750E-02	2.5224E+00	4.5880E+00	4.5880E+00	2.5024E+00
32	5.9948E-02	2.4584E+00	4.4723E+00	4.4723E+00	2.4384E+00
33	6.8129E-02	2.3945E+00	4.3565E+00	4.3565E+00	2.3745E+00
34	7.7426E-02	2.3305E+00	4.2408E+00	4.2408E+00	2.3105E+00
35	8.7992E-02	2.2665E+00	4.1250E+00	4.1250E+00	2.2465E+00
36	1.0000E-01	2.2026E+00	4.0092E+00	4.0092E+00	2.1826E+00
37	1.1365E-01	2.1386E+00	3.8935E+00	3.8935E+00	2.1186E+00
38	1.2915E-01	2.0747E+00	3.7777E+00	3.7777E+00	2.0547E+00
39	1.4678E-01	2.0107E+00	3.6619E+00	3.6619E+00	1.9907E+00
40	1.6681E-01	1.9467E+00	3.5462E+00	3.5462E+00	1.9267E+00
41	1.8957E-01	1.8828E+00	3.4304E+00	3.4304E+00	1.8628E+00
42	2.1544E-01	1.8188E+00	3.3146E+00	3.3146E+00	1.7988E+00
43	2.4484E-01	1.7549E+00	3.1989E+00	3.1989E+00	1.7349E+00
44	2.7826E-01	1.6909E+00	3.0831E+00	3.0831E+00	1.6709E+00
45	3.1623E-01	1.6269E+00	2.9673E+00	2.9673E+00	1.6069E+00
46	3.5938E-01	1.5630E+00	2.8516E+00	2.8516E+00	1.5430E+00
47	4.0842E-01	1.4990E+00	2.7358E+00	2.7358E+00	1.4790E+00
48	4.6416E-01	1.4351E+00	2.6201E+00	2.6201E+00	1.4151E+00
49	5.2750E-01	1.3711E+00	2.5043E+00	2.5043E+00	1.3511E+00
50	5.9948E-01	1.3071E+00	2.3885E+00	2.3885E+00	1.2871E+00
51	6.8129E-01	1.2432E+00	2.2728E+00	2.2728E+00	1.2232E+00
52	7.7426E-01	1.1792E+00	2.1570E+00	2.1570E+00	1.1592E+00
53	8.7992E-01	1.1153E+00	2.0412E+00	2.0412E+00	1.0953E+00
54	1.0000E+00	1.0513E+00	1.9255E+00	1.9255E+00	1.0313E+00

55	1.1365E+00	9.8733E-01	1.8097E+00	1.8097E+00	9.6733E-01
56	1.2915E+00	9.2337E-01	1.6939E+00	1.6939E+00	9.0337E-01
57	1.4678E+00	8.5941E-01	1.5782E+00	1.5782E+00	8.3941E-01
58	1.6681E+00	7.9546E-01	1.4624E+00	1.4624E+00	7.7546E-01
59	1.8957E+00	7.3152E-01	1.3467E+00	1.3467E+00	7.1152E-01
60	2.1544E+00	6.6762E-01	1.2310E+00	1.2310E+00	6.4762E-01
61	2.4484E+00	6.0382E-01	1.1154E+00	1.1154E+00	5.8382E-01
62	2.7826E+00	5.4023E-01	1.0000E+00	1.0000E+00	5.2023E-01
63	3.1623E+00	4.7710E-01	8.8520E-01	8.8520E-01	4.5710E-01
64	3.5938E+00	4.1482E-01	7.7140E-01	7.7140E-01	3.9482E-01
65	4.0842E+00	3.5402E-01	6.5951E-01	6.5951E-01	3.3402E-01
66	4.6416E+00	2.9555E-01	5.5085E-01	5.5085E-01	2.7555E-01
67	5.2750E+00	2.4047E-01	4.4727E-01	4.4727E-01	2.2047E-01
68	5.9948E+00	1.8994E-01	3.5099E-01	3.5099E-01	1.6994E-01
69	6.8129E+00	1.4503E-01	2.6444E-01	2.6444E-01	1.2503E-01
70	7.7426E+00	1.0659E-01	1.8979E-01	1.8979E-01	8.6586E-02
71	8.7992E+00	7.5048E-02	1.2860E-01	1.2860E-01	5.5048E-02
72	1.0000E+01	5.0387E-02	8.1421E-02	8.1421E-02	3.0387E-02
73	1.1365E+01	3.2102E-02	4.7636E-02	4.7636E-02	1.2102E-02
74	1.2915E+01	1.9315E-02	2.5478E-02	2.5478E-02	0.0000E+00
75	1.4678E+01	1.0925E-02	1.2352E-02	1.2352E-02	0.0000E+00
76	1.6681E+01	5.7847E-03	5.4000E-03	5.4000E-03	0.0000E+00
77	1.8957E+01	2.8565E-03	2.1260E-03	2.1260E-03	0.0000E+00
78	2.1544E+01	1.3114E-03	7.5483E-04	7.5483E-04	0.0000E+00
79	2.4484E+01	5.5830E-04	2.4255E-04	2.4255E-04	0.0000E+00
80	2.7826E+01	2.1994E-04	7.0859E-05	7.0859E-05	0.0000E+00
81	3.1623E+01	8.0046E-05	1.8919E-05	1.8919E-05	0.0000E+00
82	3.5938E+01	2.6876E-05	4.6411E-06	4.6411E-06	0.0000E+00
83	4.0842E+01	8.3151E-06	1.0514E-06	1.0514E-06	0.0000E+00
84	4.6416E+01	2.3684E-06	2.2108E-07	2.2108E-07	0.0000E+00
85	5.2750E+01	6.2051E-07	4.3344E-08	4.3344E-08	0.0000E+00
86	5.9948E+01	1.4944E-07	7.9580E-09	7.9580E-09	0.0000E+00
87	6.8129E+01	3.3060E-08	1.3739E-09	1.3739E-09	0.0000E+00
88	7.7426E+01	6.7152E-09	2.2477E-10	2.2477E-10	0.0000E+00
89	8.7992E+01	1.2517E-09	4.0598E-11	4.0598E-11	0.0000E+00
90	1.0000E+02	2.1402E-10	4.0598E-11	4.0598E-11	0.0000E+00
Time	Boundary	Spread	Time	Boundary	Spread
0	Never Exercise		18	Never Exercise	
1	Never Exercise		19	Never Exercise	
2	Never Exercise		20	Never Exercise	
3	Never Exercise		21	Never Exercise	
4	Never Exercise		22	Never Exercise	
5	Never Exercise		23	Never Exercise	
6	Never Exercise		24	Never Exercise	
7	Never Exercise		25	Never Exercise	
8	Never Exercise		26	Never Exercise	
9	Never Exercise		27	Never Exercise	
10	Never Exercise		28	Never Exercise	
11	Never Exercise		29	Never Exercise	
12	Never Exercise		30	Never Exercise	
13	Never Exercise		31	Never Exercise	
14	Never Exercise		32	Never Exercise	
15	Never Exercise		33	Never Exercise	
16	Never Exercise		34	Never Exercise	
17	Never Exercise				

	TABLE B2	Time Steps:	35	Min Firm Value:	0.001
	CALL	Value Steps:	90	Bond Face Value:	5
	SPREAD	Bond Maturity:	10	Volatility:	0.3
	ZERO	Option Maturity:	1	Riskless Rate:	0.15
		Max Firm Value:	100	Exercise price:	0.0012
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-03	7.0172E-01	7.0052E-01	6.9968E-01	7.0052E-01
1	1.1365E-03	6.8893E-01	6.8773E-01	6.8689E-01	6.8773E-01
2	1.2915E-03	6.7614E-01	6.7494E-01	6.7411E-01	6.7494E-01
3	1.4678E-03	6.6334E-01	6.6214E-01	6.6134E-01	6.6214E-01
4	1.6681E-03	6.5055E-01	6.4935E-01	6.4856E-01	6.4935E-01
5	1.8957E-03	6.3776E-01	6.3656E-01	6.3579E-01	6.3656E-01
6	2.1544E-03	6.2497E-01	6.2377E-01	6.2302E-01	6.2377E-01
7	2.4484E-03	6.1217E-01	6.1097E-01	6.1024E-01	6.1097E-01
8	2.7826E-03	5.9938E-01	5.9818E-01	5.9747E-01	5.9818E-01
9	3.1623E-03	5.8659E-01	5.8539E-01	5.8469E-01	5.8539E-01
10	3.5938E-03	5.7380E-01	5.7260E-01	5.7192E-01	5.7260E-01
11	4.0842E-03	5.6101E-01	5.5981E-01	5.5915E-01	5.5981E-01
12	4.6416E-03	5.4821E-01	5.4701E-01	5.4637E-01	5.4701E-01
13	5.2750E-03	5.3542E-01	5.3422E-01	5.3360E-01	5.3422E-01
14	5.9948E-03	5.2263E-01	5.2143E-01	5.2082E-01	5.2143E-01
15	6.8129E-03	5.0984E-01	5.0864E-01	5.0805E-01	5.0864E-01
16	7.7426E-03	4.9705E-01	4.9585E-01	4.9528E-01	4.9585E-01
17	8.7992E-03	4.8425E-01	4.8305E-01	4.8250E-01	4.8305E-01
18	1.0000E-02	4.7146E-01	4.7026E-01	4.6973E-01	4.7026E-01
19	1.1365E-02	4.5867E-01	4.5747E-01	4.5695E-01	4.5747E-01
20	1.2915E-02	4.4588E-01	4.4468E-01	4.4418E-01	4.4468E-01
21	1.4678E-02	4.3308E-01	4.3188E-01	4.3141E-01	4.3188E-01
22	1.6681E-02	4.2029E-01	4.1909E-01	4.1863E-01	4.1909E-01
23	1.8957E-02	4.0750E-01	4.0630E-01	4.0586E-01	4.0630E-01
24	2.1544E-02	3.9471E-01	3.9351E-01	3.9309E-01	3.9351E-01
25	2.4484E-02	3.8192E-01	3.8072E-01	3.8031E-01	3.8072E-01
26	2.7826E-02	3.6913E-01	3.6793E-01	3.6754E-01	3.6793E-01
27	3.1623E-02	3.5634E-01	3.5514E-01	3.5477E-01	3.5514E-01
28	3.5938E-02	3.4356E-01	3.4236E-01	3.4200E-01	3.4236E-01
29	4.0842E-02	3.3078E-01	3.2958E-01	3.2924E-01	3.2958E-01
30	4.6416E-02	3.1800E-01	3.1680E-01	3.1648E-01	3.1680E-01
31	5.2750E-02	3.0523E-01	3.0403E-01	3.0373E-01	3.0403E-01
32	5.9948E-02	2.9248E-01	2.9128E-01	2.9100E-01	2.9128E-01
33	6.8129E-02	2.7974E-01	2.7854E-01	2.7828E-01	2.7854E-01
34	7.7426E-02	2.6703E-01	2.6583E-01	2.6558E-01	2.6583E-01
35	8.7992E-02	2.5435E-01	2.5315E-01	2.5292E-01	2.5315E-01
36	1.0000E-01	2.4171E-01	2.4051E-01	2.4030E-01	2.4051E-01
37	1.1365E-01	2.2913E-01	2.2793E-01	2.2774E-01	2.2793E-01
38	1.2915E-01	2.1663E-01	2.1543E-01	2.1525E-01	2.1543E-01
39	1.4678E-01	2.0423E-01	2.0303E-01	2.0286E-01	2.0303E-01
40	1.6681E-01	1.9194E-01	1.9074E-01	1.9059E-01	1.9074E-01
41	1.8957E-01	1.7981E-01	1.7861E-01	1.7847E-01	1.7861E-01
42	2.1544E-01	1.6785E-01	1.6665E-01	1.6653E-01	1.6665E-01
43	2.4484E-01	1.5610E-01	1.5490E-01	1.5479E-01	1.5490E-01
44	2.7826E-01	1.4460E-01	1.4340E-01	1.4331E-01	1.4340E-01
45	3.1623E-01	1.3339E-01	1.3219E-01	1.3210E-01	1.3219E-01
46	3.5938E-01	1.2250E-01	1.2130E-01	1.2122E-01	1.2130E-01
47	4.0842E-01	1.1196E-01	1.1076E-01	1.1069E-01	1.1076E-01
48	4.6416E-01	1.0183E-01	1.0063E-01	1.0057E-01	1.0063E-01
49	5.2750E-01	9.2123E-02	9.0923E-02	9.0869E-02	9.0923E-02
50	5.9948E-01	8.2882E-02	8.1682E-02	8.1634E-02	8.1682E-02
51	6.8129E-01	7.4133E-02	7.2933E-02	7.2890E-02	7.2933E-02
52	7.7426E-01	6.5899E-02	6.4699E-02	6.4661E-02	6.4699E-02
53	8.7992E-01	5.8200E-02	5.7000E-02	5.6966E-02	5.7000E-02
54	1.0000E+00	5.1049E-02	4.9849E-02	4.9819E-02	4.9849E-02

55	1.1365E+00	4.4454E-02	4.3254E-02	4.3228E-02	4.3254E-02
56	1.2915E+00	3.8418E-02	3.7218E-02	3.7195E-02	3.7218E-02
57	1.4678E+00	3.2936E-02	3.1736E-02	3.1716E-02	3.1736E-02
58	1.6681E+00	2.7999E-02	2.6799E-02	2.6782E-02	2.6799E-02
59	1.8957E+00	2.3592E-02	2.2392E-02	2.2377E-02	2.2392E-02
60	2.1544E+00	1.9694E-02	1.8494E-02	1.8482E-02	1.8494E-02
61	2.4484E+00	1.6280E-02	1.5080E-02	1.5070E-02	1.5080E-02
62	2.7826E+00	1.3320E-02	1.2120E-02	1.2112E-02	1.2120E-02
63	3.1623E+00	1.0782E-02	9.5818E-03	9.5759E-03	9.5818E-03
64	3.5938E+00	8.6300E-03	7.4300E-03	7.4259E-03	7.4300E-03
65	4.0842E+00	6.8275E-03	5.6275E-03	5.6260E-03	5.6275E-03
66	4.6416E+00	5.3362E-03	4.1488E-03	4.1488E-03	4.1362E-03
67	5.2750E+00	4.1185E-03	2.9638E-03	2.9638E-03	2.9185E-03
68	5.9948E+00	3.1376E-03	2.0308E-03	2.0308E-03	1.9376E-03
69	6.8129E+00	2.3584E-03	1.3180E-03	1.3180E-03	1.1584E-03
70	7.7426E+00	1.7485E-03	7.9841E-04	7.9841E-04	5.4850E-04
71	8.7992E+00	1.2781E-03	4.4475E-04	4.4475E-04	7.8092E-05
72	1.0000E+01	9.2083E-04	2.2484E-04	2.2484E-04	0.0000E+00
73	1.1365E+01	6.5371E-04	1.0223E-04	1.0223E-04	0.0000E+00
74	1.2915E+01	4.5716E-04	4.1620E-05	4.1620E-05	0.0000E+00
75	1.4678E+01	3.1487E-04	1.5172E-05	1.5172E-05	0.0000E+00
76	1.6681E+01	2.1354E-04	4.9666E-06	4.9666E-06	0.0000E+00
77	1.8957E+01	1.4257E-04	1.4667E-06	1.4667E-06	0.0000E+00
78	2.1544E+01	9.3695E-05	3.9291E-07	3.9291E-07	0.0000E+00
79	2.4484E+01	6.0600E-05	9.6044E-08	9.6044E-08	0.0000E+00
80	2.7826E+01	3.8568E-05	2.1547E-08	2.1547E-08	0.0000E+00
81	3.1623E+01	2.4152E-05	4.4609E-09	4.4609E-09	0.0000E+00
82	3.5938E+01	1.4879E-05	8.5674E-10	8.5674E-10	0.0000E+00
83	4.0842E+01	9.0174E-06	1.5338E-10	1.5338E-10	0.0000E+00
84	4.6416E+01	5.3754E-06	2.5709E-11	2.5709E-11	0.0000E+00
85	5.2750E+01	3.1516E-06	4.0514E-12	4.0514E-12	0.0000E+00
86	5.9948E+01	1.8173E-06	6.0250E-13	6.0250E-13	0.0000E+00
87	6.8129E+01	1.0305E-06	8.4852E-14	8.4852E-14	0.0000E+00
88	7.7426E+01	5.7460E-07	1.1390E-14	1.1390E-14	0.0000E+00
89	8.7992E+01	3.1504E-07	1.6961E-15	1.6961E-15	0.0000E+00
90	1.0000E+02	1.6983E-07	1.6961E-15	1.6961E-15	0.0000E+00
Time	Boundary	Spread	Time	Boundary	Spread
0	66	5.3362E-03	18	66	5.8493E-03
1	66	5.3631E-03	19	66	5.8797E-03
2	66	5.3902E-03	20	66	5.9103E-03
3	66	5.4175E-03	21	66	5.9411E-03
4	66	5.4449E-03	22	66	5.9721E-03
5	66	5.4725E-03	23	66	6.0034E-03
6	66	5.5003E-03	24	66	6.0348E-03
7	66	5.5283E-03	25	66	6.0665E-03
8	66	5.5565E-03	26	66	6.0985E-03
9	66	5.5849E-03	27	66	6.1306E-03
10	66	5.6135E-03	28	66	6.1630E-03
11	66	5.6423E-03	29	66	6.1956E-03
12	66	5.6712E-03	30	67	4.7721E-03
13	66	5.7004E-03	31	67	4.7962E-03
14	66	5.7298E-03	32	67	4.8204E-03
15	66	5.7593E-03	33	67	4.8448E-03
16	66	5.7891E-03	34	67	4.8694E-03
17	66	5.8191E-03			

	TABLE B3	Time Steps:	35	Min Firm Value:	0.001
	CALL	Value Steps:	90	Bond Face Value:	5
	SPREAD	Bond Maturity:	10	Volatility:	0.2
	ZERO	Option Maturity:	1	Riskless Rate:	0.15
		Max Firm Value:	100	Exercise price:	0.0012
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-03	7.0172E-01	7.0052E-01	6.9961E-01	7.0052E-01
1	1.1365E-03	6.8893E-01	6.8773E-01	6.8682E-01	6.8773E-01
2	1.2916E-03	6.7614E-01	6.7494E-01	6.7404E-01	6.7494E-01
3	1.4678E-03	6.6334E-01	6.6214E-01	6.6127E-01	6.6214E-01
4	1.6681E-03	6.5055E-01	6.4935E-01	6.4849E-01	6.4935E-01
5	1.8957E-03	6.3776E-01	6.3656E-01	6.3572E-01	6.3656E-01
6	2.1544E-03	6.2497E-01	6.2377E-01	6.2295E-01	6.2377E-01
7	2.4484E-03	6.1217E-01	6.1097E-01	6.1017E-01	6.1097E-01
8	2.7826E-03	5.9938E-01	5.9818E-01	5.9740E-01	5.9818E-01
9	3.1623E-03	5.8659E-01	5.8539E-01	5.8462E-01	5.8539E-01
10	3.5938E-03	5.7380E-01	5.7260E-01	5.7185E-01	5.7260E-01
11	4.0842E-03	5.6101E-01	5.5981E-01	5.5908E-01	5.5981E-01
12	4.6416E-03	5.4821E-01	5.4701E-01	5.4630E-01	5.4701E-01
13	5.2750E-03	5.3542E-01	5.3422E-01	5.3353E-01	5.3422E-01
14	5.9948E-03	5.2263E-01	5.2143E-01	5.2075E-01	5.2143E-01
15	6.8129E-03	5.0984E-01	5.0864E-01	5.0798E-01	5.0864E-01
16	7.7426E-03	4.9705E-01	4.9585E-01	4.9521E-01	4.9585E-01
17	8.7992E-03	4.8425E-01	4.8305E-01	4.8243E-01	4.8305E-01
18	1.0000E-02	4.7146E-01	4.7026E-01	4.6966E-01	4.7026E-01
19	1.1365E-02	4.5867E-01	4.5747E-01	4.5688E-01	4.5747E-01
20	1.2916E-02	4.4588E-01	4.4468E-01	4.4411E-01	4.4468E-01
21	1.4678E-02	4.3308E-01	4.3188E-01	4.3133E-01	4.3188E-01
22	1.6681E-02	4.2029E-01	4.1909E-01	4.1856E-01	4.1909E-01
23	1.8957E-02	4.0750E-01	4.0630E-01	4.0579E-01	4.0630E-01
24	2.1544E-02	3.9471E-01	3.9351E-01	3.9301E-01	3.9351E-01
25	2.4484E-02	3.8192E-01	3.8072E-01	3.8024E-01	3.8072E-01
26	2.7826E-02	3.6912E-01	3.6792E-01	3.6746E-01	3.6792E-01
27	3.1623E-02	3.5633E-01	3.5513E-01	3.5469E-01	3.5513E-01
28	3.5938E-02	3.4354E-01	3.4234E-01	3.4192E-01	3.4234E-01
29	4.0842E-02	3.3075E-01	3.2955E-01	3.2914E-01	3.2955E-01
30	4.6416E-02	3.1796E-01	3.1676E-01	3.1637E-01	3.1676E-01
31	5.2750E-02	3.0516E-01	3.0396E-01	3.0359E-01	3.0396E-01
32	5.9948E-02	2.9237E-01	2.9117E-01	2.9082E-01	2.9117E-01
33	6.8129E-02	2.7958E-01	2.7838E-01	2.7805E-01	2.7838E-01
34	7.7426E-02	2.6679E-01	2.6559E-01	2.6527E-01	2.6559E-01
35	8.7992E-02	2.5400E-01	2.5280E-01	2.5250E-01	2.5280E-01
36	1.0000E-01	2.4121E-01	2.4001E-01	2.3973E-01	2.4001E-01
37	1.1365E-01	2.2842E-01	2.2722E-01	2.2696E-01	2.2722E-01
38	1.2916E-01	2.1563E-01	2.1443E-01	2.1419E-01	2.1443E-01
39	1.4678E-01	2.0286E-01	2.0166E-01	2.0143E-01	2.0166E-01
40	1.6681E-01	1.9009E-01	1.8889E-01	1.8869E-01	1.8889E-01
41	1.8957E-01	1.7735E-01	1.7615E-01	1.7596E-01	1.7615E-01
42	2.1544E-01	1.6465E-01	1.6345E-01	1.6328E-01	1.6345E-01
43	2.4484E-01	1.5201E-01	1.5081E-01	1.5066E-01	1.5081E-01
44	2.7826E-01	1.3947E-01	1.3827E-01	1.3813E-01	1.3827E-01
45	3.1623E-01	1.2707E-01	1.2587E-01	1.2574E-01	1.2587E-01
46	3.5938E-01	1.1486E-01	1.1366E-01	1.1355E-01	1.1366E-01
47	4.0842E-01	1.0292E-01	1.0172E-01	1.0163E-01	1.0172E-01
48	4.6416E-01	9.1337E-02	9.0137E-02	9.0059E-02	9.0137E-02
49	5.2750E-01	8.0195E-02	7.8995E-02	7.8929E-02	7.8995E-02
50	5.9948E-01	6.9592E-02	6.8392E-02	6.8336E-02	6.8392E-02
51	6.8129E-01	5.9623E-02	5.8423E-02	5.8377E-02	5.8423E-02
52	7.7426E-01	5.0375E-02	4.9175E-02	4.9137E-02	4.9175E-02
53	8.7992E-01	4.1924E-02	4.0724E-02	4.0693E-02	4.0724E-02
54	1.0000E+00	3.4324E-02	3.3124E-02	3.3099E-02	3.3124E-02

55	1.1365E+00	2.7610E-02	2.6410E-02	2.6390E-02	2.6410E-02
56	1.2916E+00	2.1790E-02	2.0590E-02	2.0574E-02	2.0590E-02
57	1.4678E+00	1.6848E-02	1.5648E-02	1.5637E-02	1.5648E-02
58	1.6681E+00	1.2744E-02	1.1544E-02	1.1536E-02	1.1544E-02
59	1.8957E+00	9.4163E-03	8.2163E-03	8.2109E-03	8.2163E-03
60	2.1544E+00	6.7857E-03	5.5857E-03	5.5833E-03	5.5857E-03
61	2.4484E+00	4.7621E-03	3.5742E-03	3.5742E-03	3.5621E-03
62	2.7826E+00	3.2500E-03	2.1181E-03	2.1181E-03	2.0500E-03
63	3.1623E+00	2.1540E-03	1.1114E-03	1.1114E-03	9.5395E-04
64	3.5938E+00	1.3846E-03	4.8306E-04	4.8306E-04	1.8460E-04
65	4.0842E+00	8.6229E-04	1.6076E-04	1.6076E-04	0.0000E+00
66	4.6416E+00	5.1974E-04	4.0770E-05	4.0770E-05	0.0000E+00
67	5.2750E+00	3.0293E-04	8.1122E-06	8.1122E-06	0.0000E+00
68	5.9948E+00	1.7061E-04	1.3094E-06	1.3094E-06	0.0000E+00
69	6.8129E+00	9.2787E-05	1.7651E-07	1.7651E-07	0.0000E+00
70	7.7426E+00	4.8702E-05	2.0347E-08	2.0347E-08	0.0000E+00
71	8.7992E+00	2.4660E-05	2.0445E-09	2.0445E-09	0.0000E+00
72	1.0000E+01	1.2041E-05	1.8186E-10	1.8186E-10	0.0000E+00
73	1.1365E+01	5.6669E-06	1.4505E-11	1.4505E-11	0.0000E+00
74	1.2916E+01	2.5702E-06	1.0483E-12	1.0483E-12	0.0000E+00
75	1.4678E+01	1.1230E-06	6.9280E-14	6.9280E-14	0.0000E+00
76	1.6681E+01	4.7259E-07	4.2186E-15	4.2186E-15	0.0000E+00
77	1.8957E+01	1.9150E-07	2.3826E-16	2.3826E-16	0.0000E+00
78	2.1544E+01	7.4709E-08	1.2554E-17	1.2554E-17	0.0000E+00
79	2.4484E+01	2.8054E-08	6.2020E-19	6.2020E-19	0.0000E+00
80	2.7826E+01	1.0138E-08	2.8859E-20	2.8859E-20	0.0000E+00
81	3.1623E+01	3.5253E-09	1.2699E-21	1.2699E-21	0.0000E+00
82	3.5938E+01	1.1794E-09	5.3036E-23	5.3036E-23	0.0000E+00
83	4.0842E+01	3.7957E-10	2.1091E-24	2.1091E-24	0.0000E+00
84	4.6416E+01	1.1750E-10	8.0098E-26	8.0098E-26	0.0000E+00
85	5.2750E+01	3.4980E-11	2.9129E-27	2.9129E-27	0.0000E+00
86	5.9948E+01	1.0015E-11	1.0169E-28	1.0169E-28	0.0000E+00
87	6.8129E+01	2.7567E-12	3.4154E-30	3.4154E-30	0.0000E+00
88	7.7426E+01	7.2986E-13	1.1062E-31	1.1062E-31	0.0000E+00
89	8.7992E+01	1.8593E-13	3.7246E-33	3.7246E-33	0.0000E+00
90	1.0000E+02	4.5852E-14	3.7246E-33	3.7246E-33	0.0000E+00
Time	Boundary	Spread	Time	Boundary	Spread
0	61	4.7621E-03	18	62	3.9041E-03
1	61	4.8101E-03	19	62	3.9445E-03
2	61	4.8586E-03	20	62	3.9854E-03
3	61	4.9076E-03	21	62	4.0268E-03
4	61	4.9572E-03	22	62	4.0686E-03
5	61	5.0073E-03	23	62	4.1110E-03
6	61	5.0580E-03	24	62	4.1538E-03
7	61	5.1093E-03	25	62	4.1972E-03
8	61	5.1612E-03	26	62	4.2410E-03
9	61	5.2136E-03	27	62	4.2854E-03
10	61	5.2666E-03	28	62	4.3303E-03
11	61	5.3203E-03	29	62	4.3757E-03
12	61	5.3745E-03	30	62	4.4216E-03
13	61	5.4294E-03	31	62	4.4681E-03
14	61	5.4849E-03	32	62	4.5151E-03
15	61	5.5410E-03	33	62	4.5627E-03
16	62	3.8246E-03	34	62	4.6109E-03
17	62	3.8641E-03			

	TABLE B4	Time Steps:	35	Min Firm Value:	0.001
	CALL	Value Steps:	90	Bond Face Value:	5
	SPREAD	Bond Maturity:	10	Volatility:	0.3
	ZERO	Option Maturity:	0.5	Riskless Rate:	0.15
		Max Firm Value:	100	Exercise price:	0.0012
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-03	7.0172E-01	7.0052E-01	7.0010E-01	7.0052E-01
1	1.1365E-03	6.8893E-01	6.8773E-01	6.8730E-01	6.8773E-01
2	1.2915E-03	6.7614E-01	6.7494E-01	6.7452E-01	6.7494E-01
3	1.4678E-03	6.6334E-01	6.6214E-01	6.6174E-01	6.6214E-01
4	1.6681E-03	6.5055E-01	6.4935E-01	6.4896E-01	6.4935E-01
5	1.8957E-03	6.3776E-01	6.3656E-01	6.3617E-01	6.3656E-01
6	2.1544E-03	6.2497E-01	6.2377E-01	6.2339E-01	6.2377E-01
7	2.4484E-03	6.1217E-01	6.1097E-01	6.1061E-01	6.1097E-01
8	2.7826E-03	5.9938E-01	5.9818E-01	5.9782E-01	5.9818E-01
9	3.1623E-03	5.8659E-01	5.8539E-01	5.8504E-01	5.8539E-01
10	3.5938E-03	5.7380E-01	5.7260E-01	5.7226E-01	5.7260E-01
11	4.0842E-03	5.6101E-01	5.5981E-01	5.5947E-01	5.5981E-01
12	4.6416E-03	5.4821E-01	5.4701E-01	5.4669E-01	5.4701E-01
13	5.2750E-03	5.3542E-01	5.3422E-01	5.3391E-01	5.3422E-01
14	5.9948E-03	5.2263E-01	5.2143E-01	5.2112E-01	5.2143E-01
15	6.8129E-03	5.0984E-01	5.0864E-01	5.0834E-01	5.0864E-01
16	7.7426E-03	4.9705E-01	4.9585E-01	4.9556E-01	4.9585E-01
17	8.7992E-03	4.8425E-01	4.8305E-01	4.8278E-01	4.8305E-01
18	1.0000E-02	4.7146E-01	4.7026E-01	4.6999E-01	4.7026E-01
19	1.1365E-02	4.5867E-01	4.5747E-01	4.5721E-01	4.5747E-01
20	1.2915E-02	4.4588E-01	4.4468E-01	4.4443E-01	4.4468E-01
21	1.4678E-02	4.3308E-01	4.3188E-01	4.3164E-01	4.3188E-01
22	1.6681E-02	4.2029E-01	4.1909E-01	4.1886E-01	4.1909E-01
23	1.8957E-02	4.0750E-01	4.0630E-01	4.0608E-01	4.0630E-01
24	2.1544E-02	3.9471E-01	3.9351E-01	3.9330E-01	3.9351E-01
25	2.4484E-02	3.8192E-01	3.8072E-01	3.8052E-01	3.8072E-01
26	2.7826E-02	3.6913E-01	3.6793E-01	3.6773E-01	3.6793E-01
27	3.1623E-02	3.5634E-01	3.5514E-01	3.5496E-01	3.5514E-01
28	3.5938E-02	3.4356E-01	3.4236E-01	3.4218E-01	3.4236E-01
29	4.0842E-02	3.3078E-01	3.2958E-01	3.2941E-01	3.2958E-01
30	4.6416E-02	3.1800E-01	3.1680E-01	3.1664E-01	3.1680E-01
31	5.2750E-02	3.0523E-01	3.0403E-01	3.0388E-01	3.0403E-01
32	5.9948E-02	2.9248E-01	2.9128E-01	2.9114E-01	2.9128E-01
33	6.8129E-02	2.7974E-01	2.7854E-01	2.7841E-01	2.7854E-01
34	7.7426E-02	2.6703E-01	2.6583E-01	2.6570E-01	2.6583E-01
35	8.7992E-02	2.5435E-01	2.5315E-01	2.5303E-01	2.5315E-01
36	1.0000E-01	2.4171E-01	2.4051E-01	2.4040E-01	2.4051E-01
37	1.1365E-01	2.2913E-01	2.2793E-01	2.2783E-01	2.2793E-01
38	1.2915E-01	2.1663E-01	2.1543E-01	2.1534E-01	2.1543E-01
39	1.4678E-01	2.0423E-01	2.0303E-01	2.0294E-01	2.0303E-01
40	1.6681E-01	1.9194E-01	1.9074E-01	1.9067E-01	1.9074E-01
41	1.8957E-01	1.7981E-01	1.7861E-01	1.7854E-01	1.7861E-01
42	2.1544E-01	1.6785E-01	1.6665E-01	1.6659E-01	1.6665E-01
43	2.4484E-01	1.5610E-01	1.5490E-01	1.5485E-01	1.5490E-01
44	2.7826E-01	1.4460E-01	1.4340E-01	1.4335E-01	1.4340E-01
45	3.1623E-01	1.3339E-01	1.3219E-01	1.3214E-01	1.3219E-01
46	3.5938E-01	1.2250E-01	1.2130E-01	1.2126E-01	1.2130E-01
47	4.0842E-01	1.1196E-01	1.1076E-01	1.1073E-01	1.1076E-01
48	4.6416E-01	1.0183E-01	1.0063E-01	1.0060E-01	1.0063E-01
49	5.2750E-01	9.2123E-02	9.0923E-02	9.0895E-02	9.0923E-02
50	5.9948E-01	8.2882E-02	8.1682E-02	8.1657E-02	8.1682E-02
51	6.8129E-01	7.4133E-02	7.2933E-02	7.2911E-02	7.2933E-02
52	7.7426E-01	6.5899E-02	6.4699E-02	6.4680E-02	6.4699E-02
53	8.7992E-01	5.8200E-02	5.7000E-02	5.6983E-02	5.7000E-02
54	1.0000E+00	5.1049E-02	4.9849E-02	4.9834E-02	4.9849E-02

55	1.1365E+00	4.4454E-02	4.3254E-02	4.3241E-02	4.3254E-02
56	1.2915E+00	3.8418E-02	3.7218E-02	3.7206E-02	3.7218E-02
57	1.4678E+00	3.2936E-02	3.1736E-02	3.1726E-02	3.1736E-02
58	1.6681E+00	2.7999E-02	2.6799E-02	2.6791E-02	2.6799E-02
59	1.8957E+00	2.3592E-02	2.2392E-02	2.2385E-02	2.2392E-02
60	2.1544E+00	1.9694E-02	1.8494E-02	1.8488E-02	1.8494E-02
61	2.4484E+00	1.6280E-02	1.5080E-02	1.5075E-02	1.5080E-02
62	2.7826E+00	1.3320E-02	1.2120E-02	1.2116E-02	1.2120E-02
63	3.1623E+00	1.0782E-02	9.5818E-03	9.5788E-03	9.5818E-03
64	3.5938E+00	8.6300E-03	7.4300E-03	7.4279E-03	7.4300E-03
65	4.0842E+00	6.8275E-03	5.6275E-03	5.6262E-03	5.6275E-03
66	4.6416E+00	5.3362E-03	4.1362E-03	4.1362E-03	4.1362E-03
67	5.2750E+00	4.1185E-03	2.9308E-03	2.9308E-03	2.9185E-03
68	5.9948E+00	3.1376E-03	1.9711E-03	1.9711E-03	1.9376E-03
69	6.8129E+00	2.3584E-03	1.2273E-03	1.2273E-03	1.1584E-03
70	7.7426E+00	1.7485E-03	6.8210E-04	6.8210E-04	5.4850E-04
71	8.7992E+00	1.2781E-03	3.2407E-04	3.2407E-04	7.8092E-05
72	1.0000E+01	9.2083E-04	1.2758E-04	1.2758E-04	0.0000E+00
73	1.1365E+01	6.5371E-04	4.1335E-05	4.1335E-05	0.0000E+00
74	1.2915E+01	4.5716E-04	1.1126E-05	1.1126E-05	0.0000E+00
75	1.4678E+01	3.1487E-04	2.5280E-06	2.5280E-06	0.0000E+00
76	1.6681E+01	2.1354E-04	4.9325E-07	4.9325E-07	0.0000E+00
77	1.8957E+01	1.4257E-04	8.3952E-08	8.3952E-08	0.0000E+00
78	2.1544E+01	9.3695E-05	1.2638E-08	1.2638E-08	0.0000E+00
79	2.4484E+01	6.0600E-05	1.7031E-09	1.7031E-09	0.0000E+00
80	2.7826E+01	3.8568E-05	2.0756E-10	2.0756E-10	0.0000E+00
81	3.1623E+01	2.4152E-05	2.3082E-11	2.3082E-11	0.0000E+00
82	3.5938E+01	1.4879E-05	2.3605E-12	2.3605E-12	0.0000E+00
83	4.0842E+01	9.0174E-06	2.2347E-13	2.2347E-13	0.0000E+00
84	4.6416E+01	5.3754E-06	1.9701E-14	1.9701E-14	0.0000E+00
85	5.2750E+01	3.1516E-06	1.6258E-15	1.6258E-15	0.0000E+00
86	5.9948E+01	1.8173E-06	1.2618E-16	1.2618E-16	0.0000E+00
87	6.8129E+01	1.0305E-06	9.2485E-18	9.2485E-18	0.0000E+00
88	7.7426E+01	5.7460E-07	6.4282E-19	6.4282E-19	0.0000E+00
89	8.7992E+01	3.1504E-07	4.6152E-20	4.6152E-20	0.0000E+00
90	1.0000E+02	1.6983E-07	4.6152E-20	4.6152E-20	0.0000E+00
Time	Boundary	Spread	Time	Boundary	Spread
0	67	4.1185E-03	18	67	4.3015E-03
1	67	4.1284E-03	19	67	4.3120E-03
2	67	4.1383E-03	20	67	4.3225E-03
3	67	4.1483E-03	21	67	4.3330E-03
4	67	4.1583E-03	22	67	4.3436E-03
5	67	4.1683E-03	23	67	4.3542E-03
6	67	4.1784E-03	24	67	4.3648E-03
7	67	4.1884E-03	25	67	4.3755E-03
8	67	4.1985E-03	26	67	4.3862E-03
9	67	4.2087E-03	27	67	4.3970E-03
10	67	4.2189E-03	28	67	4.4078E-03
11	67	4.2291E-03	29	67	4.4186E-03
12	67	4.2393E-03	30	67	4.4294E-03
13	67	4.2496E-03	31	67	4.4403E-03
14	67	4.2599E-03	32	67	4.4512E-03
15	67	4.2703E-03	33	67	4.4622E-03
16	67	4.2806E-03	34	67	4.4732E-03
17	67	4.2910E-03			

	TABLE B5	Time Steps:	35	Min Firm Value:	0.001
	CALL	Value Steps:	90	Bond Face Value:	5
	SPREAD	Bond Maturity:	10	Volatility:	0.3
	ZERO	Option Maturity:	1	Riskless Rate:	0.15
		Max Firm Value:	100	Exercise price:	0.02
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-03	7.0172E-01	6.8172E-01	6.8096E-01	6.8172E-01
1	1.1365E-03	6.8893E-01	6.6893E-01	6.6817E-01	6.6893E-01
2	1.2915E-03	6.7614E-01	6.5614E-01	6.5539E-01	6.5614E-01
3	1.4678E-03	6.6334E-01	6.4334E-01	6.4262E-01	6.4334E-01
4	1.6681E-03	6.5055E-01	6.3055E-01	6.2985E-01	6.3055E-01
5	1.8957E-03	6.3776E-01	6.1776E-01	6.1707E-01	6.1776E-01
6	2.1544E-03	6.2497E-01	6.0497E-01	6.0430E-01	6.0497E-01
7	2.4484E-03	6.1217E-01	5.9217E-01	5.9152E-01	5.9217E-01
8	2.7826E-03	5.9938E-01	5.7938E-01	5.7875E-01	5.7938E-01
9	3.1623E-03	5.8659E-01	5.6659E-01	5.6597E-01	5.6659E-01
10	3.5938E-03	5.7380E-01	5.5380E-01	5.5320E-01	5.5380E-01
11	4.0842E-03	5.6101E-01	5.4101E-01	5.4043E-01	5.4101E-01
12	4.6416E-03	5.4821E-01	5.2821E-01	5.2765E-01	5.2821E-01
13	5.2750E-03	5.3542E-01	5.1542E-01	5.1488E-01	5.1542E-01
14	5.9948E-03	5.2263E-01	5.0263E-01	5.0210E-01	5.0263E-01
15	6.8129E-03	5.0984E-01	4.8984E-01	4.8933E-01	4.8984E-01
16	7.7426E-03	4.9705E-01	4.7705E-01	4.7656E-01	4.7705E-01
17	8.7992E-03	4.8425E-01	4.6425E-01	4.6373E-01	4.6425E-01
18	1.0000E-02	4.7146E-01	4.5146E-01	4.5101E-01	4.5146E-01
19	1.1365E-02	4.5867E-01	4.3867E-01	4.3823E-01	4.3867E-01
20	1.2915E-02	4.4588E-01	4.2588E-01	4.2546E-01	4.2588E-01
21	1.4678E-02	4.3308E-01	4.1308E-01	4.1269E-01	4.1308E-01
22	1.6681E-02	4.2029E-01	4.0029E-01	3.9991E-01	4.0029E-01
23	1.8957E-02	4.0750E-01	3.8750E-01	3.8714E-01	3.8750E-01
24	2.1544E-02	3.9471E-01	3.7471E-01	3.7437E-01	3.7471E-01
25	2.4484E-02	3.8192E-01	3.6192E-01	3.6159E-01	3.6192E-01
26	2.7826E-02	3.6913E-01	3.4913E-01	3.4882E-01	3.4913E-01
27	3.1623E-02	3.5634E-01	3.3634E-01	3.3605E-01	3.3634E-01
28	3.5938E-02	3.4356E-01	3.2356E-01	3.2328E-01	3.2356E-01
29	4.0842E-02	3.3078E-01	3.1078E-01	3.1052E-01	3.1078E-01
30	4.6416E-02	3.1800E-01	2.9800E-01	2.9776E-01	2.9800E-01
31	5.2750E-02	3.0523E-01	2.8523E-01	2.8501E-01	2.8523E-01
32	5.9948E-02	2.9248E-01	2.7248E-01	2.7228E-01	2.7248E-01
33	6.8129E-02	2.7974E-01	2.5974E-01	2.5956E-01	2.5974E-01
34	7.7426E-02	2.6703E-01	2.4703E-01	2.4686E-01	2.4703E-01
35	8.7992E-02	2.5435E-01	2.3435E-01	2.3420E-01	2.3435E-01
36	1.0000E-01	2.4171E-01	2.2171E-01	2.2158E-01	2.2171E-01
37	1.1365E-01	2.2913E-01	2.0913E-01	2.0902E-01	2.0913E-01
38	1.2915E-01	2.1663E-01	1.9663E-01	1.9653E-01	1.9663E-01
39	1.4678E-01	2.0423E-01	1.8423E-01	1.8414E-01	1.8423E-01
40	1.6681E-01	1.9194E-01	1.7194E-01	1.7187E-01	1.7194E-01
41	1.8957E-01	1.7981E-01	1.5981E-01	1.5975E-01	1.5981E-01
42	2.1544E-01	1.6785E-01	1.4785E-01	1.4781E-01	1.4785E-01
43	2.4484E-01	1.5610E-01	1.3610E-01	1.3608E-01	1.3610E-01
44	2.7826E-01	1.4460E-01	1.2470E-01	1.2470E-01	1.2460E-01
45	3.1623E-01	1.3339E-01	1.1372E-01	1.1372E-01	1.1339E-01
46	3.5938E-01	1.2250E-01	1.0309E-01	1.0309E-01	1.0250E-01
47	4.0842E-01	1.1196E-01	9.2797E-02	9.2797E-02	9.1963E-02
48	4.6416E-01	1.0183E-01	8.2879E-02	8.2879E-02	8.1827E-02
49	5.2750E-01	9.2123E-02	7.3366E-02	7.3366E-02	7.2123E-02
50	5.9948E-01	8.2882E-02	6.4292E-02	6.4292E-02	6.2882E-02
51	6.8129E-01	7.4133E-02	5.5686E-02	5.5686E-02	5.4133E-02
52	7.7426E-01	6.5899E-02	4.7578E-02	4.7578E-02	4.5899E-02
53	8.7992E-01	5.8200E-02	3.9990E-02	3.9990E-02	3.8200E-02
54	1.0000E+00	5.1049E-02	3.2944E-02	3.2944E-02	3.1049E-02

55	1.1365E+00	4.4454E-02	2.6461E-02	2.6461E-02	2.4454E-02
56	1.2915E+00	3.8418E-02	2.0573E-02	2.0573E-02	1.8418E-02
57	1.4678E+00	3.2936E-02	1.5330E-02	1.5330E-02	1.2936E-02
58	1.6681E+00	2.7999E-02	1.0805E-02	1.0805E-02	7.9994E-03
59	1.8957E+00	2.3592E-02	7.0929E-03	7.0929E-03	3.5921E-03
60	2.1544E+00	1.9694E-02	4.2639E-03	4.2639E-03	0.0000E+00
61	2.4484E+00	1.6280E-02	2.3122E-03	2.3122E-03	0.0000E+00
62	2.7826E+00	1.3320E-02	1.1192E-03	1.1192E-03	0.0000E+00
63	3.1623E+00	1.0782E-02	4.8124E-04	4.8124E-04	0.0000E+00
64	3.5938E+00	8.6300E-03	1.8389E-04	1.8389E-04	0.0000E+00
65	4.0842E+00	6.8275E-03	6.2679E-05	6.2679E-05	0.0000E+00
66	4.6416E+00	5.3362E-03	1.9165E-05	1.9165E-05	0.0000E+00
67	5.2750E+00	4.1185E-03	5.2907E-06	5.2907E-06	0.0000E+00
68	5.9948E+00	3.1376E-03	1.3276E-06	1.3276E-06	0.0000E+00
69	6.8129E+00	2.3584E-03	3.0475E-07	3.0475E-07	0.0000E+00
70	7.7426E+00	1.7485E-03	6.4391E-08	6.4391E-08	0.0000E+00
71	8.7992E+00	1.2781E-03	1.2593E-08	1.2593E-08	0.0000E+00
72	1.0000E+01	9.2083E-04	2.2915E-09	2.2915E-09	0.0000E+00
73	1.1365E+01	6.5371E-04	3.8982E-10	3.8982E-10	0.0000E+00
74	1.2915E+01	4.5716E-04	6.2262E-11	6.2262E-11	0.0000E+00
75	1.4678E+01	3.1487E-04	9.3739E-12	9.3739E-12	0.0000E+00
76	1.6681E+01	2.1354E-04	1.3351E-12	1.3351E-12	0.0000E+00
77	1.8957E+01	1.4257E-04	1.8050E-13	1.8050E-13	0.0000E+00
78	2.1544E+01	9.3695E-05	2.3231E-14	2.3231E-14	0.0000E+00
79	2.4484E+01	6.0600E-05	2.8546E-15	2.8546E-15	0.0000E+00
80	2.7826E+01	3.8568E-05	3.3574E-16	3.3574E-16	0.0000E+00
81	3.1623E+01	2.4152E-05	3.7885E-17	3.7885E-17	0.0000E+00
82	3.5938E+01	1.4879E-05	4.1107E-18	4.1107E-18	0.0000E+00
83	4.0842E+01	9.0174E-06	4.2975E-19	4.2975E-19	0.0000E+00
84	4.6416E+01	5.3754E-06	4.3370E-20	4.3370E-20	0.0000E+00
85	5.2750E+01	3.1516E-06	4.2326E-21	4.2326E-21	0.0000E+00
86	5.9948E+01	1.8173E-06	4.0010E-22	4.0010E-22	0.0000E+00
87	6.8129E+01	1.0305E-06	3.6691E-23	3.6691E-23	0.0000E+00
88	7.7426E+01	5.7460E-07	3.2724E-24	3.2724E-24	0.0000E+00
89	8.7992E+01	3.1504E-07	3.1662E-25	3.1662E-25	0.0000E+00
90	1.0000E+02	1.6983E-07	3.1662E-25	3.1662E-25	0.0000E+00
Time	Boundary	Spread	Time	Boundary	Spread
0	44	1.4460E-01	18	43	1.7150E-01
1	44	1.4536E-01	19	43	1.7241E-01
2	44	1.4613E-01	20	43	1.7333E-01
3	44	1.4690E-01	21	43	1.7425E-01
4	44	1.4768E-01	22	43	1.7518E-01
5	44	1.4846E-01	23	43	1.7612E-01
6	44	1.4925E-01	24	43	1.7706E-01
7	44	1.5004E-01	25	43	1.7801E-01
8	44	1.5084E-01	26	43	1.7897E-01
9	44	1.5164E-01	27	43	1.7993E-01
10	44	1.5245E-01	28	43	1.8090E-01
11	44	1.5326E-01	29	43	1.8187E-01
12	44	1.5408E-01	30	43	1.8286E-01
13	44	1.5491E-01	31	43	1.8385E-01
14	44	1.5574E-01	32	43	1.8484E-01
15	44	1.5658E-01	33	43	1.8584E-01
16	44	1.5742E-01	34	43	1.8685E-01
17	43	1.7060E-01			

	TABLE B6	Time Steps:	35	Min Firm Value:	0.001
	CALL	Value Steps:	90	Bond Face Value:	5
	SPREAD	Bond Maturity:	10	Volatility:	0.3
	ZERO	Option Maturity:	1	Riskless Rate:	0.2
		Max Firm Value:	100	Exercise price:	0.0012
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-03	6.5172E-01	6.5052E-01	6.4885E-01	6.5052E-01
1	1.1365E-03	6.3893E-01	6.3773E-01	6.3605E-01	6.3773E-01
2	1.2915E-03	6.2614E-01	6.2494E-01	6.2330E-01	6.2494E-01
3	1.4678E-03	6.1334E-01	6.1214E-01	6.1054E-01	6.1214E-01
4	1.6681E-03	6.0055E-01	5.9935E-01	5.9779E-01	5.9935E-01
5	1.8957E-03	5.8776E-01	5.8656E-01	5.8503E-01	5.8656E-01
6	2.1544E-03	5.7497E-01	5.7377E-01	5.7227E-01	5.7377E-01
7	2.4484E-03	5.6217E-01	5.6097E-01	5.5952E-01	5.6097E-01
8	2.7826E-03	5.4938E-01	5.4818E-01	5.4676E-01	5.4818E-01
9	3.1623E-03	5.3659E-01	5.3539E-01	5.3401E-01	5.3539E-01
10	3.5938E-03	5.2380E-01	5.2260E-01	5.2125E-01	5.2260E-01
11	4.0842E-03	5.1101E-01	5.0981E-01	5.0849E-01	5.0981E-01
12	4.6416E-03	4.9821E-01	4.9701E-01	4.9574E-01	4.9701E-01
13	5.2750E-03	4.8542E-01	4.8422E-01	4.8298E-01	4.8422E-01
14	5.9948E-03	4.7263E-01	4.7143E-01	4.7023E-01	4.7143E-01
15	6.8129E-03	4.5984E-01	4.5864E-01	4.5747E-01	4.5864E-01
16	7.7426E-03	4.4705E-01	4.4585E-01	4.4472E-01	4.4585E-01
17	8.7992E-03	4.3425E-01	4.3305E-01	4.3196E-01	4.3305E-01
18	1.0000E-02	4.2146E-01	4.2026E-01	4.1920E-01	4.2026E-01
19	1.1365E-02	4.0867E-01	4.0747E-01	4.0645E-01	4.0747E-01
20	1.2915E-02	3.9588E-01	3.9468E-01	3.9369E-01	3.9468E-01
21	1.4678E-02	3.8309E-01	3.8189E-01	3.8094E-01	3.8189E-01
22	1.6681E-02	3.7030E-01	3.6910E-01	3.6819E-01	3.6910E-01
23	1.8957E-02	3.5751E-01	3.5631E-01	3.5543E-01	3.5631E-01
24	2.1544E-02	3.4472E-01	3.4352E-01	3.4268E-01	3.4352E-01
25	2.4484E-02	3.3194E-01	3.3074E-01	3.2994E-01	3.3074E-01
26	2.7826E-02	3.1917E-01	3.1797E-01	3.1720E-01	3.1797E-01
27	3.1623E-02	3.0640E-01	3.0520E-01	3.0447E-01	3.0520E-01
28	3.5938E-02	2.9364E-01	2.9244E-01	2.9175E-01	2.9244E-01
29	4.0842E-02	2.8090E-01	2.7970E-01	2.7904E-01	2.7970E-01
30	4.6416E-02	2.6819E-01	2.6699E-01	2.6636E-01	2.6699E-01
31	5.2750E-02	2.5550E-01	2.5430E-01	2.5371E-01	2.5430E-01
32	5.9948E-02	2.4286E-01	2.4166E-01	2.4111E-01	2.4166E-01
33	6.8129E-02	2.3028E-01	2.2908E-01	2.2856E-01	2.2908E-01
34	7.7426E-02	2.1777E-01	2.1657E-01	2.1608E-01	2.1657E-01
35	8.7992E-02	2.0536E-01	2.0416E-01	2.0370E-01	2.0416E-01
36	1.0000E-01	1.9306E-01	1.9186E-01	1.9144E-01	1.9186E-01
37	1.1365E-01	1.8091E-01	1.7971E-01	1.7932E-01	1.7971E-01
38	1.2915E-01	1.6893E-01	1.6773E-01	1.6737E-01	1.6773E-01
39	1.4678E-01	1.5717E-01	1.5597E-01	1.5563E-01	1.5597E-01
40	1.6681E-01	1.4564E-01	1.4444E-01	1.4414E-01	1.4444E-01
41	1.8957E-01	1.3440E-01	1.3320E-01	1.3292E-01	1.3320E-01
42	2.1544E-01	1.2348E-01	1.2228E-01	1.2202E-01	1.2228E-01
43	2.4484E-01	1.1291E-01	1.1171E-01	1.1148E-01	1.1171E-01
44	2.7826E-01	1.0274E-01	1.0154E-01	1.0133E-01	1.0154E-01
45	3.1623E-01	9.2990E-02	9.1790E-02	9.1606E-02	9.1790E-02
46	3.5938E-01	8.3706E-02	8.2506E-02	8.2341E-02	8.2506E-02
47	4.0842E-01	7.4911E-02	7.3711E-02	7.3563E-02	7.3711E-02
48	4.6416E-01	6.6629E-02	6.5429E-02	6.5298E-02	6.5429E-02
49	5.2750E-01	5.8881E-02	5.7681E-02	5.7565E-02	5.7681E-02
50	5.9948E-01	5.1679E-02	5.0479E-02	5.0378E-02	5.0479E-02
51	6.8129E-01	4.5034E-02	4.3834E-02	4.3745E-02	4.3834E-02
52	7.7426E-01	3.8946E-02	3.7746E-02	3.7670E-02	3.7746E-02
53	8.7992E-01	3.3414E-02	3.2214E-02	3.2148E-02	3.2214E-02
54	1.0000E+00	2.8428E-02	2.7228E-02	2.7172E-02	2.7228E-02

55	1.1365E+00	2.3973E-02	2.2773E-02	2.2726E-02	2.2773E-02
56	1.2915E+00	2.0030E-02	1.8830E-02	1.8790E-02	1.8830E-02
57	1.4678E+00	1.6572E-02	1.5372E-02	1.5340E-02	1.5372E-02
58	1.6681E+00	1.3572E-02	1.2372E-02	1.2347E-02	1.2372E-02
59	1.8957E+00	1.0997E-02	9.7971E-03	9.7771E-03	9.7971E-03
60	2.1544E+00	8.8115E-03	7.6115E-03	7.5965E-03	7.6115E-03
61	2.4484E+00	6.9786E-03	5.7786E-03	5.7678E-03	5.7786E-03
62	2.7826E+00	5.4605E-03	4.2605E-03	4.2533E-03	4.2605E-03
63	3.1623E+00	4.2194E-03	3.0194E-03	3.0177E-03	3.0194E-03
64	3.5938E+00	3.2183E-03	2.0481E-03	2.0481E-03	2.0183E-03
65	4.0842E+00	2.4221E-03	1.3216E-03	1.3216E-03	1.2221E-03
66	4.6416E+00	1.7980E-03	7.9758E-04	7.9758E-04	5.9800E-04
67	5.2750E+00	1.3160E-03	4.4210E-04	4.4210E-04	1.1599E-04
68	5.9948E+00	9.4940E-04	2.2147E-04	2.2147E-04	0.0000E+00
69	6.8129E+00	6.7491E-04	9.9131E-05	9.9131E-05	0.0000E+00
70	7.7426E+00	4.7264E-04	3.9436E-05	3.9436E-05	0.0000E+00
71	8.7992E+00	3.2599E-04	1.3943E-05	1.3943E-05	0.0000E+00
72	1.0000E+01	2.2140E-04	4.3964E-06	4.3964E-06	0.0000E+00
73	1.1365E+01	1.4803E-04	1.2433E-06	1.2433E-06	0.0000E+00
74	1.2915E+01	9.7423E-05	3.1738E-07	3.1738E-07	0.0000E+00
75	1.4678E+01	6.3102E-05	7.3619E-08	7.3619E-08	0.0000E+00
76	1.6681E+01	4.0221E-05	1.5618E-08	1.5618E-08	0.0000E+00
77	1.8957E+01	2.5224E-05	3.0487E-09	3.0487E-09	0.0000E+00
78	2.1544E+01	1.5563E-05	5.5071E-10	5.5071E-10	0.0000E+00
79	2.4484E+01	9.4460E-06	9.2534E-11	9.2534E-11	0.0000E+00
80	2.7826E+01	5.6394E-06	1.4532E-11	1.4532E-11	0.0000E+00
81	3.1623E+01	3.3115E-06	2.1421E-12	2.1421E-12	0.0000E+00
82	3.5938E+01	1.9124E-06	2.9757E-13	2.9757E-13	0.0000E+00
83	4.0842E+01	1.0861E-06	3.9096E-14	3.9096E-14	0.0000E+00
84	4.6416E+01	6.0651E-07	4.8744E-15	4.8744E-15	0.0000E+00
85	5.2750E+01	3.3305E-07	5.7845E-16	5.7845E-16	0.0000E+00
86	5.9948E+01	1.7982E-07	6.5521E-17	6.5521E-17	0.0000E+00
87	6.8129E+01	9.5451E-08	7.1023E-18	7.1023E-18	0.0000E+00
88	7.7426E+01	4.9812E-08	7.4015E-19	7.4015E-19	0.0000E+00
89	8.7992E+01	2.5555E-08	8.5614E-20	8.5614E-20	0.0000E+00
90	1.0000E+02	1.2888E-08	8.5614E-20	8.5614E-20	0.0000E+00
Time	Boundary	Spread	Time	Boundary	Spread
0	64	3.2183E-03	18	65	2.7778E-03
1	64	3.2431E-03	19	65	2.7993E-03
2	64	3.2681E-03	20	65	2.8210E-03
3	64	3.2933E-03	21	65	2.8429E-03
4	64	3.3187E-03	22	65	2.8651E-03
5	64	3.3444E-03	23	65	2.8874E-03
6	64	3.3703E-03	24	65	2.9099E-03
7	64	3.3965E-03	25	65	2.9326E-03
8	64	3.4229E-03	26	65	2.9555E-03
9	64	3.4495E-03	27	65	2.9787E-03
10	64	3.4764E-03	28	65	3.0021E-03
11	64	3.5035E-03	29	65	3.0257E-03
12	64	3.5309E-03	30	65	3.0495E-03
13	64	3.5586E-03	31	66	2.2684E-03
14	64	3.5864E-03	32	66	2.2859E-03
15	64	3.6146E-03	33	66	2.3035E-03
16	64	3.6430E-03	34	66	2.3212E-03
17	65	2.7565E-03			

	TABLE C1	Time Steps:	40	Bond Face Value:	20
	PUT	Bond Time Steps	20	Coupon:	3
	BOND	Value Steps:	75	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	0.24
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value:	50	g:	0.01
		Min Firm Value:	0.00001		
J	Firm	Bond	Option	Temp	Exercise
0	1.0000E-05	1.0000E-05	2.7101E+01	2.7101E+01	2.7020E+01
1	1.2283E-05	1.2283E-05	2.7101E+01	2.7101E+01	2.7020E+01
2	1.5088E-05	1.5088E-05	2.7101E+01	2.7101E+01	2.7020E+01
3	1.8534E-05	1.8534E-05	2.7101E+01	2.7101E+01	2.7020E+01
4	2.2766E-05	2.2766E-05	2.7101E+01	2.7101E+01	2.7020E+01
5	2.7964E-05	2.7964E-05	2.7101E+01	2.7101E+01	2.7020E+01
6	3.4349E-05	3.4349E-05	2.7101E+01	2.7101E+01	2.7020E+01
7	4.2193E-05	4.2193E-05	2.7101E+01	2.7101E+01	2.7020E+01
8	5.1827E-05	5.1827E-05	2.7101E+01	2.7101E+01	2.7020E+01
9	6.3661E-05	6.3661E-05	2.7101E+01	2.7101E+01	2.7020E+01
10	7.8198E-05	7.8198E-05	2.7101E+01	2.7101E+01	2.7020E+01
11	9.6054E-05	9.6054E-05	2.7101E+01	2.7101E+01	2.7020E+01
12	1.1799E-04	1.1799E-04	2.7101E+01	2.7101E+01	2.7020E+01
13	1.4493E-04	1.4493E-04	2.7101E+01	2.7101E+01	2.7020E+01
14	1.7802E-04	1.7802E-04	2.7101E+01	2.7101E+01	2.7020E+01
15	2.1867E-04	2.1867E-04	2.7101E+01	2.7101E+01	2.7020E+01
16	2.6860E-04	2.6860E-04	2.7101E+01	2.7101E+01	2.7019E+01
17	3.2994E-04	3.2994E-04	2.7101E+01	2.7101E+01	2.7019E+01
18	4.0528E-04	4.0528E-04	2.7101E+01	2.7101E+01	2.7019E+01
19	4.9782E-04	4.9782E-04	2.7101E+01	2.7101E+01	2.7019E+01
20	6.1149E-04	6.1149E-04	2.7101E+01	2.7101E+01	2.7019E+01
21	7.5112E-04	7.5112E-04	2.7100E+01	2.7100E+01	2.7019E+01
22	9.2264E-04	9.2264E-04	2.7100E+01	2.7100E+01	2.7019E+01
23	1.1333E-03	1.1333E-03	2.7100E+01	2.7100E+01	2.7019E+01
24	1.3921E-03	1.3921E-03	2.7100E+01	2.7100E+01	2.7018E+01
25	1.7100E-03	1.7100E-03	2.7099E+01	2.7099E+01	2.7018E+01
26	2.1004E-03	2.1004E-03	2.7099E+01	2.7099E+01	2.7018E+01
27	2.5801E-03	2.5801E-03	2.7099E+01	2.7099E+01	2.7017E+01
28	3.1692E-03	3.1692E-03	2.7098E+01	2.7098E+01	2.7017E+01
29	3.8929E-03	3.8929E-03	2.7097E+01	2.7097E+01	2.7016E+01
30	4.7818E-03	4.7818E-03	2.7096E+01	2.7096E+01	2.7015E+01
31	5.8736E-03	5.8736E-03	2.7095E+01	2.7095E+01	2.7014E+01
32	7.2148E-03	7.2148E-03	2.7094E+01	2.7094E+01	2.7013E+01
33	8.8623E-03	8.8623E-03	2.7092E+01	2.7092E+01	2.7011E+01
34	1.0886E-02	1.0886E-02	2.7090E+01	2.7090E+01	2.7009E+01
35	1.3372E-02	1.3372E-02	2.7088E+01	2.7088E+01	2.7006E+01
36	1.6425E-02	1.6425E-02	2.7085E+01	2.7085E+01	2.7003E+01
37	2.0176E-02	2.0176E-02	2.7081E+01	2.7081E+01	2.7000E+01
38	2.4782E-02	2.4782E-02	2.7076E+01	2.7076E+01	2.6995E+01
39	3.0441E-02	3.0441E-02	2.7071E+01	2.7071E+01	2.6989E+01
40	3.7392E-02	3.7392E-02	2.7064E+01	2.7064E+01	2.6982E+01
41	4.5931E-02	4.5931E-02	2.7055E+01	2.7055E+01	2.6974E+01
42	5.6419E-02	5.6419E-02	2.7045E+01	2.7045E+01	2.6963E+01
43	6.9302E-02	6.9302E-02	2.7032E+01	2.7032E+01	2.6950E+01
44	8.5126E-02	8.5126E-02	2.7016E+01	2.7016E+01	2.6935E+01
45	1.0456E-01	1.0456E-01	2.6997E+01	2.6997E+01	2.6915E+01
46	1.2844E-01	1.2844E-01	2.6973E+01	2.6973E+01	2.6891E+01
47	1.5777E-01	1.5777E-01	2.6943E+01	2.6943E+01	2.6862E+01
48	1.9379E-01	1.9379E-01	2.6907E+01	2.6907E+01	2.6826E+01
49	2.3805E-01	2.3805E-01	2.6863E+01	2.6863E+01	2.6782E+01
50	2.9240E-01	2.9240E-01	2.6809E+01	2.6809E+01	2.6727E+01
51	3.5917E-01	3.5917E-01	2.6742E+01	2.6742E+01	2.6661E+01
52	4.4118E-01	4.4118E-01	2.6660E+01	2.6660E+01	2.6579E+01

53	5.4192E-01	5.4192E-01	2.6559E+01	2.6559E+01	2.6478E+01
54	6.6567E-01	6.6567E-01	2.6435E+01	2.6435E+01	2.6354E+01
55	8.1767E-01	8.1767E-01	2.6283E+01	2.6283E+01	2.6202E+01
56	1.0044E+00	1.0044E+00	2.6097E+01	2.6097E+01	2.6015E+01
57	1.2337E+00	1.2337E+00	2.5867E+01	2.5867E+01	2.5786E+01
58	1.5154E+00	1.5154E+00	2.5586E+01	2.5586E+01	2.5504E+01
59	1.8615E+00	1.8615E+00	2.5240E+01	2.5240E+01	2.5158E+01
60	2.2865E+00	2.2865E+00	2.4814E+01	2.4814E+01	2.4733E+01
61	2.8086E+00	2.8086E+00	2.4292E+01	2.4292E+01	2.4211E+01
62	3.4500E+00	3.4500E+00	2.3651E+01	2.3651E+01	2.3570E+01
63	4.2377E+00	4.2377E+00	2.2863E+01	2.2863E+01	2.2782E+01
64	5.2054E+00	5.2054E+00	2.1895E+01	2.1895E+01	2.1814E+01
65	6.3940E+00	6.3940E+00	2.0707E+01	2.0707E+01	2.0626E+01
66	7.8540E+00	7.8540E+00	1.9246E+01	1.9246E+01	1.9166E+01
67	9.6475E+00	9.6475E+00	1.7453E+01	1.7453E+01	1.7372E+01
68	1.1850E+01	1.1850E+01	1.5250E+01	1.5250E+01	1.5169E+01
69	1.4556E+01	1.4556E+01	1.2544E+01	1.2544E+01	1.2463E+01
70	1.7880E+01	1.7874E+01	9.2275E+00	9.2275E+00	9.1461E+00
71	2.1963E+01	2.1854E+01	5.2500E+00	5.2500E+00	5.1653E+00
72	2.6978E+01	2.5910E+01	1.2222E+00	1.2222E+00	1.1098E+00
73	3.3138E+01	2.7077E+01	1.8069E-01	1.8069E-01	0.0000E+00
74	4.0705E+01	2.7265E+01	2.2584E-02	2.2584E-02	0.0000E+00
75	5.0000E+01	2.7265E+01	2.2584E-02	2.2584E-02	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		20	Never Exercise	
1	Never Exercise		21	Never Exercise	
2	Never Exercise		22	Never Exercise	
3	Never Exercise		23	Never Exercise	
4	Never Exercise		24	Never Exercise	
5	Never Exercise		25	Never Exercise	
6	Never Exercise		26	Never Exercise	
7	Never Exercise		27	Never Exercise	
8	Never Exercise		28	Never Exercise	
9	Never Exercise		29	Never Exercise	
10	Never Exercise		30	72	2.5081E+01
11	Never Exercise		31	Never Exercise	
12	Never Exercise		32	Never Exercise	
13	Never Exercise		33	Never Exercise	
14	Never Exercise		34	Never Exercise	
15	73	2.7719E+01	35	Never Exercise	
16	Never Exercise		36	Never Exercise	
17	Never Exercise		37	Never Exercise	
18	Never Exercise		38	Never Exercise	
19	Never Exercise		39	Never Exercise	
	Option terminated at time 15 below J= 73				
	Option terminated at time 30 below J= 72				

	TABLE C2	Time Steps:	20	Bond Face Value:	20
	PUT	Bond Time Steps	40	Coupon:	3
	BOND	Value Steps:	75	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	0.24
		Option Maturity:	0.4	Riskless Rate:	0.06
		Max Firm Value:	50	g:	0.01
		Min Firm Value:	0.00001		
J	Firm	Bond	Option	Temp	Exercise
0	1.0000E-05	1.0000E-05	2.7101E+01	2.7101E+01	2.7020E+01
1	1.2283E-05	1.2283E-05	2.7101E+01	2.7101E+01	2.7020E+01
2	1.5088E-05	1.5088E-05	2.7101E+01	2.7101E+01	2.7020E+01
3	1.8534E-05	1.8534E-05	2.7101E+01	2.7101E+01	2.7020E+01
4	2.2766E-05	2.2766E-05	2.7101E+01	2.7101E+01	2.7020E+01
5	2.7964E-05	2.7964E-05	2.7101E+01	2.7101E+01	2.7020E+01
6	3.4349E-05	3.4349E-05	2.7101E+01	2.7101E+01	2.7020E+01
7	4.2193E-05	4.2193E-05	2.7101E+01	2.7101E+01	2.7020E+01
8	5.1827E-05	5.1827E-05	2.7101E+01	2.7101E+01	2.7020E+01
9	6.3661E-05	6.3661E-05	2.7101E+01	2.7101E+01	2.7020E+01
10	7.8198E-05	7.8198E-05	2.7101E+01	2.7101E+01	2.7020E+01
11	9.6054E-05	9.6054E-05	2.7101E+01	2.7101E+01	2.7020E+01
12	1.1799E-04	1.1799E-04	2.7101E+01	2.7101E+01	2.7020E+01
13	1.4493E-04	1.4493E-04	2.7101E+01	2.7101E+01	2.7020E+01
14	1.7802E-04	1.7802E-04	2.7101E+01	2.7101E+01	2.7020E+01
15	2.1867E-04	2.1867E-04	2.7101E+01	2.7101E+01	2.7020E+01
16	2.6860E-04	2.6860E-04	2.7101E+01	2.7101E+01	2.7019E+01
17	3.2994E-04	3.2994E-04	2.7101E+01	2.7101E+01	2.7019E+01
18	4.0528E-04	4.0528E-04	2.7101E+01	2.7101E+01	2.7019E+01
19	4.9782E-04	4.9782E-04	2.7101E+01	2.7101E+01	2.7019E+01
20	6.1149E-04	6.1149E-04	2.7101E+01	2.7101E+01	2.7019E+01
21	7.5112E-04	7.5112E-04	2.7100E+01	2.7100E+01	2.7019E+01
22	9.2264E-04	9.2264E-04	2.7100E+01	2.7100E+01	2.7019E+01
23	1.1333E-03	1.1333E-03	2.7100E+01	2.7100E+01	2.7019E+01
24	1.3921E-03	1.3921E-03	2.7100E+01	2.7100E+01	2.7018E+01
25	1.7100E-03	1.7100E-03	2.7099E+01	2.7099E+01	2.7018E+01
26	2.1004E-03	2.1004E-03	2.7099E+01	2.7099E+01	2.7018E+01
27	2.5801E-03	2.5801E-03	2.7099E+01	2.7099E+01	2.7017E+01
28	3.1692E-03	3.1692E-03	2.7098E+01	2.7098E+01	2.7017E+01
29	3.8929E-03	3.8929E-03	2.7097E+01	2.7097E+01	2.7016E+01
30	4.7818E-03	4.7818E-03	2.7096E+01	2.7096E+01	2.7015E+01
31	5.8736E-03	5.8736E-03	2.7095E+01	2.7095E+01	2.7014E+01
32	7.2148E-03	7.2148E-03	2.7094E+01	2.7094E+01	2.7013E+01
33	8.8623E-03	8.8623E-03	2.7092E+01	2.7092E+01	2.7011E+01
34	1.0886E-02	1.0886E-02	2.7090E+01	2.7090E+01	2.7009E+01
35	1.3372E-02	1.3372E-02	2.7088E+01	2.7088E+01	2.7006E+01
36	1.6425E-02	1.6425E-02	2.7085E+01	2.7085E+01	2.7003E+01
37	2.0176E-02	2.0176E-02	2.7081E+01	2.7081E+01	2.7000E+01
38	2.4782E-02	2.4782E-02	2.7076E+01	2.7076E+01	2.6995E+01
39	3.0441E-02	3.0441E-02	2.7071E+01	2.7071E+01	2.6989E+01
40	3.7392E-02	3.7392E-02	2.7064E+01	2.7064E+01	2.6982E+01
41	4.5931E-02	4.5931E-02	2.7055E+01	2.7055E+01	2.6974E+01
42	5.6419E-02	5.6419E-02	2.7045E+01	2.7045E+01	2.6963E+01
43	6.9302E-02	6.9302E-02	2.7032E+01	2.7032E+01	2.6950E+01
44	8.5126E-02	8.5126E-02	2.7016E+01	2.7016E+01	2.6935E+01
45	1.0456E-01	1.0456E-01	2.6997E+01	2.6997E+01	2.6915E+01
46	1.2844E-01	1.2844E-01	2.6973E+01	2.6973E+01	2.6891E+01
47	1.5777E-01	1.5777E-01	2.6943E+01	2.6943E+01	2.6862E+01
48	1.9379E-01	1.9379E-01	2.6907E+01	2.6907E+01	2.6826E+01
49	2.3805E-01	2.3805E-01	2.6863E+01	2.6863E+01	2.6782E+01
50	2.9240E-01	2.9240E-01	2.6809E+01	2.6809E+01	2.6727E+01
51	3.5917E-01	3.5917E-01	2.6742E+01	2.6742E+01	2.6661E+01
52	4.4118E-01	4.4118E-01	2.6660E+01	2.6660E+01	2.6579E+01

53	5.4192E-01	5.4192E-01	2.6559E+01	2.6559E+01	2.6478E+01
54	6.6567E-01	6.6567E-01	2.6435E+01	2.6435E+01	2.6354E+01
55	8.1767E-01	8.1767E-01	2.6283E+01	2.6283E+01	2.6202E+01
56	1.0044E+00	1.0044E+00	2.6097E+01	2.6097E+01	2.6015E+01
57	1.2337E+00	1.2337E+00	2.5867E+01	2.5867E+01	2.5786E+01
58	1.5154E+00	1.5154E+00	2.5586E+01	2.5586E+01	2.5504E+01
59	1.8615E+00	1.8615E+00	2.5240E+01	2.5240E+01	2.5158E+01
60	2.2865E+00	2.2865E+00	2.4814E+01	2.4814E+01	2.4733E+01
61	2.8086E+00	2.8086E+00	2.4292E+01	2.4292E+01	2.4211E+01
62	3.4500E+00	3.4500E+00	2.3651E+01	2.3651E+01	2.3570E+01
63	4.2377E+00	4.2377E+00	2.2863E+01	2.2863E+01	2.2782E+01
64	5.2054E+00	5.2054E+00	2.1895E+01	2.1895E+01	2.1814E+01
65	6.3940E+00	6.3940E+00	2.0707E+01	2.0707E+01	2.0626E+01
66	7.8540E+00	7.8540E+00	1.9246E+01	1.9246E+01	1.9166E+01
67	9.6475E+00	9.6475E+00	1.7453E+01	1.7453E+01	1.7372E+01
68	1.1850E+01	1.1850E+01	1.5250E+01	1.5250E+01	1.5169E+01
69	1.4556E+01	1.4556E+01	1.2544E+01	1.2544E+01	1.2463E+01
70	1.7880E+01	1.7874E+01	9.2275E+00	9.2275E+00	9.1461E+00
71	2.1963E+01	2.1854E+01	5.2494E+00	5.2494E+00	5.1653E+00
72	2.6978E+01	2.5910E+01	1.2170E+00	1.2170E+00	1.1098E+00
73	3.3138E+01	2.7077E+01	1.5661E-01	1.5661E-01	0.0000E+00
74	4.0705E+01	2.7265E+01	1.4697E-02	1.4697E-02	0.0000E+00
75	5.0000E+01	2.7265E+01	1.4697E-02	1.4697E-02	0.0000E+00
Time	Boundary	Bond Price			
0	Never Exercise				
1	Never Exercise				
2	Never Exercise				
3	Never Exercise				
4	Never Exercise				
5	Never Exercise				
6	Never Exercise				
7	Never Exercise				
8	Never Exercise				
9	Never Exercise				
10	Never Exercise				
11	Never Exercise				
12	Never Exercise				
13	Never Exercise				
14	Never Exercise				
15	73	2.7719E+01			
16	Never Exercise				
17	Never Exercise				
18	Never Exercise				
19	Never Exercise				
	Option terminated at time 15 below j= 73				

	TABLE C3	Time Steps:	40	Bond Face Value:	20
	PUT	Bond Time Steps	20	Coupon:	3
	BOND	Value Steps:	75	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	0.84
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value:	50	g:	0.01
		Min Firm Value:	0.00001		
J	Firm	Bond	Option	Temp	Exercise
0	1.0000E-05	1.0000E-05	2.7101E+01	2.7101E+01	2.7020E+01
1	1.2283E-05	1.2282E-05	2.7101E+01	2.7101E+01	2.7020E+01
2	1.5088E-05	1.5085E-05	2.7101E+01	2.7101E+01	2.7020E+01
3	1.8534E-05	1.8529E-05	2.7101E+01	2.7101E+01	2.7020E+01
4	2.2766E-05	2.2760E-05	2.7101E+01	2.7101E+01	2.7020E+01
5	2.7964E-05	2.7957E-05	2.7101E+01	2.7101E+01	2.7020E+01
6	3.4349E-05	3.4341E-05	2.7101E+01	2.7101E+01	2.7020E+01
7	4.2193E-05	4.2182E-05	2.7101E+01	2.7101E+01	2.7020E+01
8	5.1827E-05	5.1814E-05	2.7101E+01	2.7101E+01	2.7020E+01
9	6.3661E-05	6.3646E-05	2.7101E+01	2.7101E+01	2.7020E+01
10	7.8198E-05	7.8179E-05	2.7101E+01	2.7101E+01	2.7020E+01
11	9.6054E-05	9.6030E-05	2.7101E+01	2.7101E+01	2.7020E+01
12	1.1799E-04	1.1796E-04	2.7101E+01	2.7101E+01	2.7020E+01
13	1.4493E-04	1.4489E-04	2.7101E+01	2.7101E+01	2.7020E+01
14	1.7802E-04	1.7798E-04	2.7101E+01	2.7101E+01	2.7020E+01
15	2.1867E-04	2.1862E-04	2.7101E+01	2.7101E+01	2.7020E+01
16	2.6860E-04	2.6854E-04	2.7101E+01	2.7101E+01	2.7019E+01
17	3.2994E-04	3.2986E-04	2.7101E+01	2.7101E+01	2.7019E+01
18	4.0528E-04	4.0518E-04	2.7101E+01	2.7101E+01	2.7019E+01
19	4.9782E-04	4.9770E-04	2.7101E+01	2.7101E+01	2.7019E+01
20	6.1149E-04	6.1134E-04	2.7101E+01	2.7101E+01	2.7019E+01
21	7.5112E-04	7.5094E-04	2.7100E+01	2.7100E+01	2.7019E+01
22	9.2264E-04	9.2241E-04	2.7100E+01	2.7100E+01	2.7019E+01
23	1.1333E-03	1.1330E-03	2.7100E+01	2.7100E+01	2.7019E+01
24	1.3921E-03	1.3918E-03	2.7100E+01	2.7100E+01	2.7018E+01
25	1.7100E-03	1.7096E-03	2.7099E+01	2.7099E+01	2.7018E+01
26	2.1004E-03	2.0999E-03	2.7099E+01	2.7099E+01	2.7018E+01
27	2.5801E-03	2.5794E-03	2.7099E+01	2.7099E+01	2.7017E+01
28	3.1692E-03	3.1684E-03	2.7098E+01	2.7098E+01	2.7017E+01
29	3.8929E-03	3.8919E-03	2.7097E+01	2.7097E+01	2.7016E+01
30	4.7818E-03	4.7806E-03	2.7096E+01	2.7096E+01	2.7015E+01
31	5.8736E-03	5.8722E-03	2.7095E+01	2.7095E+01	2.7014E+01
32	7.2148E-03	7.2131E-03	2.7094E+01	2.7094E+01	2.7013E+01
33	8.8623E-03	8.8601E-03	2.7092E+01	2.7092E+01	2.7011E+01
34	1.0886E-02	1.0883E-02	2.7090E+01	2.7090E+01	2.7009E+01
35	1.3372E-02	1.3368E-02	2.7088E+01	2.7088E+01	2.7006E+01
36	1.6425E-02	1.6421E-02	2.7085E+01	2.7085E+01	2.7003E+01
37	2.0176E-02	2.0171E-02	2.7081E+01	2.7081E+01	2.7000E+01
38	2.4782E-02	2.4776E-02	2.7076E+01	2.7076E+01	2.6995E+01
39	3.0441E-02	3.0434E-02	2.7071E+01	2.7071E+01	2.6989E+01
40	3.7392E-02	3.7383E-02	2.7064E+01	2.7064E+01	2.6982E+01
41	4.5931E-02	4.5919E-02	2.7055E+01	2.7055E+01	2.6974E+01
42	5.6419E-02	5.6405E-02	2.7045E+01	2.7045E+01	2.6963E+01
43	6.9302E-02	6.9284E-02	2.7032E+01	2.7032E+01	2.6950E+01
44	8.5126E-02	8.5105E-02	2.7016E+01	2.7016E+01	2.6935E+01
45	1.0456E-01	1.0454E-01	2.6997E+01	2.6997E+01	2.6915E+01
46	1.2844E-01	1.2841E-01	2.6973E+01	2.6973E+01	2.6891E+01
47	1.5777E-01	1.5773E-01	2.6943E+01	2.6943E+01	2.6862E+01
48	1.9379E-01	1.9375E-01	2.6907E+01	2.6907E+01	2.6826E+01
49	2.3805E-01	2.3799E-01	2.6863E+01	2.6863E+01	2.6782E+01
50	2.9240E-01	2.9233E-01	2.6809E+01	2.6809E+01	2.6727E+01
51	3.5917E-01	3.5908E-01	2.6742E+01	2.6742E+01	2.6661E+01
52	4.4118E-01	4.4107E-01	2.6660E+01	2.6660E+01	2.6579E+01

53	5.4192E-01	5.4179E-01	2.6559E+01	2.6559E+01	2.6478E+01
54	6.6567E-01	6.6550E-01	2.6436E+01	2.6436E+01	2.6354E+01
55	8.1767E-01	8.1747E-01	2.6284E+01	2.6284E+01	2.6202E+01
56	1.0044E+00	1.0041E+00	2.6097E+01	2.6097E+01	2.6016E+01
57	1.2337E+00	1.2334E+00	2.5868E+01	2.5868E+01	2.5786E+01
58	1.5154E+00	1.5151E+00	2.5586E+01	2.5586E+01	2.5505E+01
59	1.8615E+00	1.8610E+00	2.5240E+01	2.5240E+01	2.5159E+01
60	2.2865E+00	2.2860E+00	2.4815E+01	2.4815E+01	2.4734E+01
61	2.8086E+00	2.8079E+00	2.4293E+01	2.4293E+01	2.4212E+01
62	3.4500E+00	3.4490E+00	2.3652E+01	2.3652E+01	2.3571E+01
63	4.2377E+00	4.2360E+00	2.2865E+01	2.2865E+01	2.2784E+01
64	5.2054E+00	5.2012E+00	2.1900E+01	2.1900E+01	2.1819E+01
65	6.3940E+00	6.3814E+00	2.0720E+01	2.0720E+01	2.0638E+01
66	7.8540E+00	7.8141E+00	1.9288E+01	1.9288E+01	1.9206E+01
67	9.6475E+00	9.5266E+00	1.7578E+01	1.7578E+01	1.7493E+01
68	1.1850E+01	1.1515E+01	1.5595E+01	1.5595E+01	1.5504E+01
69	1.4556E+01	1.3720E+01	1.3400E+01	1.3400E+01	1.3300E+01
70	1.7880E+01	1.6004E+01	1.1131E+01	1.1131E+01	1.1016E+01
71	2.1963E+01	1.8174E+01	8.9800E+00	8.9800E+00	8.8458E+00
72	2.6978E+01	2.0024E+01	7.1483E+00	7.1483E+00	6.9953E+00
73	3.3138E+01	2.1384E+01	5.8031E+00	5.8031E+00	5.6354E+00
74	4.0705E+01	2.2123E+01	5.0721E+00	5.0721E+00	4.8963E+00
75	5.0000E+01	2.2123E+01	5.0721E+00	5.0721E+00	4.8963E+00

Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		20	Never Exercise	
1	Never Exercise		21	Never Exercise	
2	Never Exercise		22	Never Exercise	
3	Never Exercise		23	Never Exercise	
4	Never Exercise		24	Never Exercise	
5	Never Exercise		25	Never Exercise	
6	Never Exercise		26	Never Exercise	
7	Never Exercise		27	Never Exercise	
8	Never Exercise		28	Never Exercise	
9	Never Exercise		29	Never Exercise	
10	Never Exercise		30	71	2.0419E+01
11	Never Exercise		31	Never Exercise	
12	Never Exercise		32	Never Exercise	
13	Never Exercise		33	Never Exercise	
14	Never Exercise		34	Never Exercise	
15	71	2.0851E+01	35	Never Exercise	
16	Never Exercise		36	Never Exercise	
17	Never Exercise		37	Never Exercise	
18	Never Exercise		38	Never Exercise	
19	Never Exercise		39	Never Exercise	
	Option terminated at time 15 below J= 71				
	Option terminated at time 30 below J= 71				

	TABLE C4	Time Steps:	40	Bond Face Value:	20
	PUT	Bond Time Steps	20	Coupon:	3
	BOND	Value Steps:	75	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	0.24
		Option Maturity:	0.8	Riskless Rate:	0.2
		Max Firm Value:	50	g:	0.01
		Min Firm Value:	0.00001		
J	Firm	Bond	Option	Temp	Exercise
0	1.0000E-05	1.0000E-05	2.3563E+01	2.3563E+01	2.3490E+01
1	1.2283E-05	1.2283E-05	2.3563E+01	2.3563E+01	2.3490E+01
2	1.5088E-05	1.5088E-05	2.3563E+01	2.3563E+01	2.3490E+01
3	1.8534E-05	1.8534E-05	2.3563E+01	2.3563E+01	2.3490E+01
4	2.2766E-05	2.2766E-05	2.3563E+01	2.3563E+01	2.3490E+01
5	2.7964E-05	2.7964E-05	2.3563E+01	2.3563E+01	2.3490E+01
6	3.4349E-05	3.4349E-05	2.3563E+01	2.3563E+01	2.3490E+01
7	4.2193E-05	4.2193E-05	2.3563E+01	2.3563E+01	2.3490E+01
8	5.1827E-05	5.1827E-05	2.3563E+01	2.3563E+01	2.3490E+01
9	6.3661E-05	6.3661E-05	2.3563E+01	2.3563E+01	2.3490E+01
10	7.8198E-05	7.8198E-05	2.3563E+01	2.3563E+01	2.3490E+01
11	9.6054E-05	9.6054E-05	2.3563E+01	2.3563E+01	2.3490E+01
12	1.1799E-04	1.1799E-04	2.3563E+01	2.3563E+01	2.3490E+01
13	1.4493E-04	1.4493E-04	2.3563E+01	2.3563E+01	2.3490E+01
14	1.7802E-04	1.7802E-04	2.3563E+01	2.3563E+01	2.3490E+01
15	2.1867E-04	2.1867E-04	2.3563E+01	2.3563E+01	2.3490E+01
16	2.6860E-04	2.6860E-04	2.3563E+01	2.3563E+01	2.3490E+01
17	3.2994E-04	3.2994E-04	2.3563E+01	2.3563E+01	2.3490E+01
18	4.0528E-04	4.0528E-04	2.3563E+01	2.3563E+01	2.3490E+01
19	4.9782E-04	4.9782E-04	2.3563E+01	2.3563E+01	2.3489E+01
20	6.1149E-04	6.1149E-04	2.3563E+01	2.3563E+01	2.3489E+01
21	7.5112E-04	7.5112E-04	2.3563E+01	2.3563E+01	2.3489E+01
22	9.2264E-04	9.2264E-04	2.3562E+01	2.3562E+01	2.3489E+01
23	1.1333E-03	1.1333E-03	2.3562E+01	2.3562E+01	2.3489E+01
24	1.3921E-03	1.3921E-03	2.3562E+01	2.3562E+01	2.3489E+01
25	1.7100E-03	1.7100E-03	2.3562E+01	2.3562E+01	2.3488E+01
26	2.1004E-03	2.1004E-03	2.3561E+01	2.3561E+01	2.3488E+01
27	2.5801E-03	2.5801E-03	2.3561E+01	2.3561E+01	2.3487E+01
28	3.1692E-03	3.1692E-03	2.3560E+01	2.3560E+01	2.3487E+01
29	3.8929E-03	3.8929E-03	2.3559E+01	2.3559E+01	2.3486E+01
30	4.7818E-03	4.7818E-03	2.3559E+01	2.3559E+01	2.3485E+01
31	5.8736E-03	5.8736E-03	2.3557E+01	2.3557E+01	2.3484E+01
32	7.2148E-03	7.2148E-03	2.3556E+01	2.3556E+01	2.3483E+01
33	8.8623E-03	8.8623E-03	2.3554E+01	2.3554E+01	2.3481E+01
34	1.0886E-02	1.0886E-02	2.3552E+01	2.3552E+01	2.3479E+01
35	1.3372E-02	1.3372E-02	2.3550E+01	2.3550E+01	2.3477E+01
36	1.6425E-02	1.6425E-02	2.3547E+01	2.3547E+01	2.3474E+01
37	2.0176E-02	2.0176E-02	2.3543E+01	2.3543E+01	2.3470E+01
38	2.4782E-02	2.4782E-02	2.3539E+01	2.3539E+01	2.3465E+01
39	3.0441E-02	3.0441E-02	2.3533E+01	2.3533E+01	2.3459E+01
40	3.7392E-02	3.7392E-02	2.3526E+01	2.3526E+01	2.3453E+01
41	4.5931E-02	4.5931E-02	2.3517E+01	2.3517E+01	2.3444E+01
42	5.6419E-02	5.6419E-02	2.3507E+01	2.3507E+01	2.3434E+01
43	6.9302E-02	6.9302E-02	2.3494E+01	2.3494E+01	2.3421E+01
44	8.5126E-02	8.5126E-02	2.3478E+01	2.3478E+01	2.3405E+01
45	1.0456E-01	1.0456E-01	2.3459E+01	2.3459E+01	2.3385E+01
46	1.2844E-01	1.2844E-01	2.3435E+01	2.3435E+01	2.3361E+01
47	1.5777E-01	1.5777E-01	2.3405E+01	2.3405E+01	2.3332E+01
48	1.9379E-01	1.9379E-01	2.3369E+01	2.3369E+01	2.3296E+01
49	2.3805E-01	2.3805E-01	2.3325E+01	2.3325E+01	2.3252E+01
50	2.9240E-01	2.9240E-01	2.3271E+01	2.3271E+01	2.3198E+01
51	3.5917E-01	3.5917E-01	2.3204E+01	2.3204E+01	2.3131E+01
52	4.4118E-01	4.4118E-01	2.3122E+01	2.3122E+01	2.3049E+01

53	5.4192E-01	5.4192E-01	2.3021E+01	2.3021E+01	2.2948E+01
54	6.6567E-01	6.6567E-01	2.2897E+01	2.2897E+01	2.2824E+01
55	8.1767E-01	8.1767E-01	2.2745E+01	2.2745E+01	2.2672E+01
56	1.0044E+00	1.0044E+00	2.2559E+01	2.2559E+01	2.2486E+01
57	1.2337E+00	1.2337E+00	2.2329E+01	2.2329E+01	2.2256E+01
58	1.5154E+00	1.5154E+00	2.2047E+01	2.2047E+01	2.1975E+01
59	1.8615E+00	1.8615E+00	2.1701E+01	2.1701E+01	2.1628E+01
60	2.2865E+00	2.2865E+00	2.1276E+01	2.1276E+01	2.1203E+01
61	2.8086E+00	2.8086E+00	2.0754E+01	2.0754E+01	2.0681E+01
62	3.4500E+00	3.4500E+00	2.0112E+01	2.0112E+01	2.0040E+01
63	4.2377E+00	4.2377E+00	1.9324E+01	1.9324E+01	1.9252E+01
64	5.2054E+00	5.2054E+00	1.8356E+01	1.8356E+01	1.8285E+01
65	6.3940E+00	6.3940E+00	1.7167E+01	1.7167E+01	1.7096E+01
66	7.8540E+00	7.8540E+00	1.5706E+01	1.5706E+01	1.5636E+01
67	9.6475E+00	9.6475E+00	1.3912E+01	1.3912E+01	1.3842E+01
68	1.1850E+01	1.1850E+01	1.1709E+01	1.1709E+01	1.1640E+01
69	1.4556E+01	1.4551E+01	9.0117E+00	9.0117E+00	8.9390E+00
70	1.7880E+01	1.7788E+01	5.7817E+00	5.7817E+00	5.7019E+00
71	2.1963E+01	2.1242E+01	2.3634E+00	2.3634E+00	2.2479E+00
72	2.6978E+01	2.3488E+01	2.2801E-01	2.2801E-01	1.5965E-03
73	3.3138E+01	2.3719E+01	1.3509E-02	1.3509E-02	0.0000E+00
74	4.0705E+01	2.3733E+01	7.1568E-04	7.1568E-04	0.0000E+00
75	5.0000E+01	2.3733E+01	7.1568E-04	7.1568E-04	0.0000E+00

Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		20	Never Exercise	
1	Never Exercise		21	Never Exercise	
2	Never Exercise		22	Never Exercise	
3	Never Exercise		23	Never Exercise	
4	Never Exercise		24	Never Exercise	
5	Never Exercise		25	Never Exercise	
6	Never Exercise		26	Never Exercise	
7	Never Exercise		27	Never Exercise	
8	Never Exercise		28	Never Exercise	
9	Never Exercise		29	Never Exercise	
10	Never Exercise		30	72	2.3551E+01
11	Never Exercise		31	Never Exercise	
12	Never Exercise		32	Never Exercise	
13	Never Exercise		33	Never Exercise	
14	Never Exercise		34	Never Exercise	
15	72	2.5095E+01	35	Never Exercise	
16	Never Exercise		36	Never Exercise	
17	Never Exercise		37	Never Exercise	
18	Never Exercise		38	Never Exercise	
19	Never Exercise		39	Never Exercise	
	Option terminated at time 15 below J= 72				
	Option terminated at time 30 below J= 72				

	TABLE C5	Time Steps:	40	Bond Face Value:	20
	PUT	Bond Time Steps	20	Coupon:	3
	BOND	Value Steps:	75	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	0.24
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value:	50	g:	0.012
		Min Firm Value:	0.00001		

J	Firm	Bond	Option	Temp	Exercise
0	1.0000E-05	1.0000E-05	2.7063E+01	2.7063E+01	2.6965E+01
1	1.2283E-05	1.2283E-05	2.7063E+01	2.7063E+01	2.6965E+01
2	1.5088E-05	1.5088E-05	2.7063E+01	2.7063E+01	2.6965E+01
3	1.8534E-05	1.8534E-05	2.7063E+01	2.7063E+01	2.6965E+01
4	2.2766E-05	2.2766E-05	2.7063E+01	2.7063E+01	2.6965E+01
5	2.7964E-05	2.7964E-05	2.7063E+01	2.7063E+01	2.6965E+01
6	3.4349E-05	3.4349E-05	2.7063E+01	2.7063E+01	2.6965E+01
7	4.2193E-05	4.2193E-05	2.7063E+01	2.7063E+01	2.6965E+01
8	5.1827E-05	5.1827E-05	2.7063E+01	2.7063E+01	2.6965E+01
9	6.3661E-05	6.3661E-05	2.7063E+01	2.7063E+01	2.6965E+01
10	7.8198E-05	7.8198E-05	2.7063E+01	2.7063E+01	2.6965E+01
11	9.6054E-05	9.6054E-05	2.7063E+01	2.7063E+01	2.6965E+01
12	1.1799E-04	1.1799E-04	2.7063E+01	2.7063E+01	2.6965E+01
13	1.4493E-04	1.4493E-04	2.7063E+01	2.7063E+01	2.6965E+01
14	1.7802E-04	1.7802E-04	2.7063E+01	2.7063E+01	2.6965E+01
15	2.1867E-04	2.1867E-04	2.7063E+01	2.7063E+01	2.6965E+01
16	2.6860E-04	2.6860E-04	2.7063E+01	2.7063E+01	2.6965E+01
17	3.2994E-04	3.2994E-04	2.7063E+01	2.7063E+01	2.6965E+01
18	4.0528E-04	4.0528E-04	2.7063E+01	2.7063E+01	2.6965E+01
19	4.9782E-04	4.9782E-04	2.7062E+01	2.7062E+01	2.6965E+01
20	6.1149E-04	6.1149E-04	2.7062E+01	2.7062E+01	2.6965E+01
21	7.5112E-04	7.5112E-04	2.7062E+01	2.7062E+01	2.6965E+01
22	9.2264E-04	9.2264E-04	2.7062E+01	2.7062E+01	2.6964E+01
23	1.1333E-03	1.1333E-03	2.7062E+01	2.7062E+01	2.6964E+01
24	1.3921E-03	1.3921E-03	2.7062E+01	2.7062E+01	2.6964E+01
25	1.7100E-03	1.7100E-03	2.7061E+01	2.7061E+01	2.6964E+01
26	2.1004E-03	2.1004E-03	2.7061E+01	2.7061E+01	2.6963E+01
27	2.5801E-03	2.5801E-03	2.7060E+01	2.7060E+01	2.6963E+01
28	3.1692E-03	3.1692E-03	2.7060E+01	2.7060E+01	2.6962E+01
29	3.8929E-03	3.8929E-03	2.7059E+01	2.7059E+01	2.6961E+01
30	4.7818E-03	4.7818E-03	2.7058E+01	2.7058E+01	2.6961E+01
31	5.8736E-03	5.8736E-03	2.7057E+01	2.7057E+01	2.6959E+01
32	7.2148E-03	7.2148E-03	2.7056E+01	2.7056E+01	2.6958E+01
33	8.8623E-03	8.8623E-03	2.7054E+01	2.7054E+01	2.6957E+01
34	1.0886E-02	1.0886E-02	2.7052E+01	2.7052E+01	2.6954E+01
35	1.3372E-02	1.3372E-02	2.7050E+01	2.7050E+01	2.6952E+01
36	1.6425E-02	1.6425E-02	2.7046E+01	2.7046E+01	2.6949E+01
37	2.0176E-02	2.0176E-02	2.7043E+01	2.7043E+01	2.6945E+01
38	2.4782E-02	2.4782E-02	2.7038E+01	2.7038E+01	2.6941E+01
39	3.0441E-02	3.0441E-02	2.7032E+01	2.7032E+01	2.6935E+01
40	3.7392E-02	3.7392E-02	2.7026E+01	2.7026E+01	2.6928E+01
41	4.5931E-02	4.5931E-02	2.7017E+01	2.7017E+01	2.6919E+01
42	5.6419E-02	5.6419E-02	2.7006E+01	2.7006E+01	2.6909E+01
43	6.9302E-02	6.9302E-02	2.6994E+01	2.6994E+01	2.6896E+01
44	8.5126E-02	8.5126E-02	2.6978E+01	2.6978E+01	2.6880E+01
45	1.0456E-01	1.0456E-01	2.6958E+01	2.6958E+01	2.6861E+01
46	1.2844E-01	1.2844E-01	2.6934E+01	2.6934E+01	2.6837E+01
47	1.5777E-01	1.5777E-01	2.6905E+01	2.6905E+01	2.6808E+01
48	1.9379E-01	1.9379E-01	2.6869E+01	2.6869E+01	2.6772E+01
49	2.3805E-01	2.3805E-01	2.6825E+01	2.6825E+01	2.6727E+01
50	2.9240E-01	2.9240E-01	2.6770E+01	2.6770E+01	2.6673E+01
51	3.5917E-01	3.5917E-01	2.6704E+01	2.6704E+01	2.6606E+01
52	4.4118E-01	4.4118E-01	2.6622E+01	2.6622E+01	2.6524E+01

53	5.4192E-01	5.4192E-01	2.6521E+01	2.6521E+01	2.6423E+01
54	6.6567E-01	6.6567E-01	2.6397E+01	2.6397E+01	2.6300E+01
55	8.1767E-01	8.1767E-01	2.6245E+01	2.6245E+01	2.6148E+01
56	1.0044E+00	1.0044E+00	2.6058E+01	2.6058E+01	2.5961E+01
57	1.2337E+00	1.2337E+00	2.5829E+01	2.5829E+01	2.5732E+01
58	1.5154E+00	1.5154E+00	2.5547E+01	2.5547E+01	2.5450E+01
59	1.8615E+00	1.8615E+00	2.5201E+01	2.5201E+01	2.5104E+01
60	2.2865E+00	2.2865E+00	2.4776E+01	2.4776E+01	2.4679E+01
61	2.8086E+00	2.8086E+00	2.4254E+01	2.4254E+01	2.4157E+01
62	3.4500E+00	3.4500E+00	2.3613E+01	2.3613E+01	2.3515E+01
63	4.2377E+00	4.2377E+00	2.2825E+01	2.2825E+01	2.2728E+01
64	5.2054E+00	5.2054E+00	2.1857E+01	2.1857E+01	2.1760E+01
65	6.3940E+00	6.3940E+00	2.0668E+01	2.0668E+01	2.0571E+01
66	7.8540E+00	7.8540E+00	1.9208E+01	1.9208E+01	1.9111E+01
67	9.6475E+00	9.6475E+00	1.7415E+01	1.7415E+01	1.7318E+01
68	1.1850E+01	1.1850E+01	1.5211E+01	1.5211E+01	1.5115E+01
69	1.4556E+01	1.4556E+01	1.2506E+01	1.2506E+01	1.2409E+01
70	1.7880E+01	1.7874E+01	9.1893E+00	9.1893E+00	9.0917E+00
71	2.1963E+01	2.1854E+01	5.2125E+00	5.2125E+00	5.1109E+00
72	2.6978E+01	2.5910E+01	1.1905E+00	1.1905E+00	1.0554E+00
73	3.3138E+01	2.7077E+01	1.7515E-01	1.7515E-01	0.0000E+00
74	4.0705E+01	2.7265E+01	2.1983E-02	2.1983E-02	0.0000E+00
75	5.0000E+01	2.7265E+01	2.1983E-02	2.1983E-02	0.0000E+00
Time	Boundary	Bond Price	Time	Boundary	Bond Price
0	Never Exercise		20	Never Exercise	
1	Never Exercise		21	Never Exercise	
2	Never Exercise		22	Never Exercise	
3	Never Exercise		23	Never Exercise	
4	Never Exercise		24	Never Exercise	
5	Never Exercise		25	Never Exercise	
6	Never Exercise		26	Never Exercise	
7	Never Exercise		27	Never Exercise	
8	Never Exercise		28	Never Exercise	
9	Never Exercise		29	Never Exercise	
10	Never Exercise		30	72	2.5081E+01
11	Never Exercise		31	Never Exercise	
12	Never Exercise		32	Never Exercise	
13	Never Exercise		33	Never Exercise	
14	Never Exercise		34	Never Exercise	
15	73	2.7719E+01	35	Never Exercise	
16	Never Exercise		36	Never Exercise	
17	Never Exercise		37	Never Exercise	
18	Never Exercise		38	Never Exercise	
19	Never Exercise		39	Never Exercise	
	Option terminated at time15 below J=73				
	Option terminated at time 30 below J=72				

	TABLE D1	Time Steps:	40	Bond Face Value:	20
	CALL	Bond Time Steps	20	Coupon:	3
	SPREAD	Value Steps:	80	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	1
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value:	40	Exercise price:	4
		Min Firm Value:	0.01	Max Spread:	5
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
1	1.1092E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
2	1.2304E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
3	1.3648E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
4	1.5139E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
5	1.6793E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
6	1.8627E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
7	2.0662E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
8	2.2920E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
9	2.5423E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
10	2.8201E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
11	3.1281E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
12	3.4698E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
13	3.8489E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
14	4.2693E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
15	4.7357E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
16	5.2531E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
17	5.8269E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
18	6.4634E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
19	7.1695E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
20	7.9527E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
21	8.8215E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
22	9.7851E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
23	1.0854E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
24	1.2040E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
25	1.3355E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
26	1.4814E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
27	1.6432E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
28	1.8227E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
29	2.0218E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
30	2.2427E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
31	2.4877E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
32	2.7595E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
33	3.0609E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
34	3.3953E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
35	3.7662E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
36	4.1776E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
37	4.6340E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
38	5.1402E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
39	5.7017E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
40	6.3246E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
41	7.0155E-01	5.0000E+00	1.0000E+00	9.9878E-01	1.0000E+00
42	7.7818E-01	5.0000E+00	1.0000E+00	9.9872E-01	1.0000E+00
43	8.6319E-01	5.0000E+00	1.0000E+00	9.9848E-01	1.0000E+00
44	9.5749E-01	4.7991E+00	9.9748E-01	9.9748E-01	7.9905E-01
45	1.0621E+00	4.5621E+00	9.9615E-01	9.9615E-01	5.6209E-01
46	1.1781E+00	4.3336E+00	9.9349E-01	9.9349E-01	3.3364E-01
47	1.3068E+00	4.1137E+00	9.8999E-01	9.8999E-01	1.1368E-01
48	1.4496E+00	3.9021E+00	9.8465E-01	9.8465E-01	0.0000E+00
49	1.6079E+00	3.6986E+00	9.7680E-01	9.7680E-01	0.0000E+00
50	1.7836E+00	3.5031E+00	9.6537E-01	9.6537E-01	0.0000E+00
51	1.9784E+00	3.3150E+00	9.4925E-01	9.4925E-01	0.0000E+00
52	2.1945E+00	3.1342E+00	9.2719E-01	9.2719E-01	0.0000E+00

53	2.4342E+00	2.9600E+00	8.9805E-01	8.9805E-01	0.0000E+00
54	2.7002E+00	2.7923E+00	8.6087E-01	8.6087E-01	0.0000E+00
55	2.9951E+00	2.6304E+00	8.1512E-01	8.1512E-01	0.0000E+00
56	3.3223E+00	2.4741E+00	7.6079E-01	7.6079E-01	0.0000E+00
57	3.6853E+00	2.3231E+00	6.9856E-01	6.9856E-01	0.0000E+00
58	4.0878E+00	2.1770E+00	6.2983E-01	6.2983E-01	0.0000E+00
59	4.5344E+00	2.0356E+00	5.5661E-01	5.5661E-01	0.0000E+00
60	5.0297E+00	1.8987E+00	4.8138E-01	4.8138E-01	0.0000E+00
61	5.5792E+00	1.7662E+00	4.0684E-01	4.0684E-01	0.0000E+00
62	6.1887E+00	1.6381E+00	3.3562E-01	3.3562E-01	0.0000E+00
63	6.8647E+00	1.5146E+00	2.6999E-01	2.6999E-01	0.0000E+00
64	7.6146E+00	1.3957E+00	2.1167E-01	2.1167E-01	0.0000E+00
65	8.4464E+00	1.2819E+00	1.6170E-01	1.6170E-01	0.0000E+00
66	9.3691E+00	1.1733E+00	1.2040E-01	1.2040E-01	0.0000E+00
67	1.0393E+01	1.0703E+00	8.7511E-02	8.7511E-02	0.0000E+00
68	1.1528E+01	9.7345E-01	6.2259E-02	6.2259E-02	0.0000E+00
69	1.2787E+01	8.8305E-01	4.3582E-02	4.3582E-02	0.0000E+00
70	1.4184E+01	7.9950E-01	3.0279E-02	3.0279E-02	0.0000E+00
71	1.5734E+01	7.2314E-01	2.1158E-02	2.1158E-02	0.0000E+00
72	1.7452E+01	6.5427E-01	1.5137E-02	1.5137E-02	0.0000E+00
73	1.9359E+01	5.9312E-01	1.1309E-02	1.1309E-02	0.0000E+00
74	2.1474E+01	5.3990E-01	8.9581E-03	8.9581E-03	0.0000E+00
75	2.3819E+01	4.9480E-01	7.5558E-03	7.5558E-03	0.0000E+00
76	2.6421E+01	4.5801E-01	6.7373E-03	6.7373E-03	0.0000E+00
77	2.9308E+01	4.2976E-01	6.2684E-03	6.2684E-03	0.0000E+00
78	3.2509E+01	4.1038E-01	6.0113E-03	6.0113E-03	0.0000E+00
79	3.6061E+01	4.0035E-01	5.8954E-03	5.8954E-03	0.0000E+00
80	4.0000E+01	4.0035E-01	5.8954E-03	5.8954E-03	0.0000E+00

Time	Boundary	Spread	Time	Boundary	Spread
0	44	4.7991E+00	20	50	4.8373E+00
1	45	4.7832E+00	21	51	4.8213E+00
2	46	4.7658E+00	22	52	4.8034E+00
3	47	4.7465E+00	23	53	4.7834E+00
4	48	4.7252E+00	24	54	4.7608E+00
5	49	4.7014E+00	25	55	4.7351E+00
6	50	4.6747E+00	26	56	4.7057E+00
7	51	4.6447E+00	27	57	4.6718E+00
8	52	4.6105E+00	28	58	4.6323E+00
9	52	4.9840E+00	29	59	4.5859E+00
10	53	4.9712E+00	30	72	7.8069E-01
11	54	4.9559E+00	31	51	4.7832E+00
12	55	4.9373E+00	32	52	4.7679E+00
13	56	4.9142E+00	33	53	4.7510E+00
14	58	4.2582E+00	34	54	4.7325E+00
15	72	6.9597E-01	35	54	4.9840E+00
16	46	4.8877E+00	36	55	4.9762E+00
17	47	4.8768E+00	37	56	4.9674E+00
18	48	4.8649E+00	38	58	4.6361E+00
19	49	4.8518E+00	39	59	4.6043E+00
	Option terminated at time 15 below j=72				
	Option terminated at time 30 below j=72				

	TABLE D2	Time Steps:	40	Bond Face Value:	20
	CALL	Bond Time Steps	20	Coupon :	3
	SPREAD	Value Steps:	80	Coupons dates:	0.3,0.6,0.9
	COUPON	Bond Maturity:	1.2	Volatility:	0.3
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value	40	Exercise price:	4
		Min Firm Value	0.01	Max Spread:	5
j	Firm	Spread	Option	Temp	Exercise
0	1.0000E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
1	1.1092E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
2	1.2304E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
3	1.3648E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
4	1.5139E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
5	1.6793E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
6	1.8627E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
7	2.0662E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
8	2.2920E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
9	2.5423E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
10	2.8201E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
11	3.1281E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
12	3.4698E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
13	3.8489E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
14	4.2693E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
15	4.7357E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
16	5.2531E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
17	5.8269E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
18	6.4634E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
19	7.1695E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
20	7.9527E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
21	8.8215E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
22	9.7851E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
23	1.0854E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
24	1.2040E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
25	1.3355E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
26	1.4814E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
27	1.6432E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
28	1.8227E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
29	2.0218E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
30	2.2427E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
31	2.4877E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
32	2.7595E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
33	3.0609E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
34	3.3953E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
35	3.7662E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
36	4.1776E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
37	4.6340E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
38	5.1402E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
39	5.7017E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
40	6.3246E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
41	7.0155E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
42	7.7818E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
43	8.6319E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
44	9.5749E-01	4.7988E+00	9.9870E-01	9.9870E-01	7.9881E-01
45	1.0621E+00	4.5619E+00	9.9750E-01	9.9750E-01	5.6186E-01
46	1.1781E+00	4.3334E+00	9.9630E-01	9.9630E-01	3.3342E-01
47	1.3068E+00	4.1135E+00	9.9510E-01	9.9510E-01	1.1346E-01
48	1.4496E+00	3.9019E+00	9.9390E-01	9.9390E-01	0.0000E+00
49	1.6079E+00	3.6984E+00	9.9263E-01	9.9263E-01	0.0000E+00
50	1.7836E+00	3.5029E+00	9.9117E-01	9.9117E-01	0.0000E+00
51	1.9784E+00	3.3148E+00	9.8945E-01	9.8945E-01	0.0000E+00
52	2.1945E+00	3.1339E+00	9.8784E-01	9.8784E-01	0.0000E+00

53	2.4342E+00	2.9598E+00	9.8574E-01	9.8574E-01	0.0000E+00
54	2.7002E+00	2.7919E+00	9.8066E-01	9.8066E-01	0.0000E+00
55	2.9951E+00	2.6300E+00	9.6470E-01	9.6470E-01	0.0000E+00
56	3.3223E+00	2.4735E+00	9.1764E-01	9.1764E-01	0.0000E+00
57	3.6853E+00	2.3221E+00	8.0536E-01	8.0536E-01	0.0000E+00
58	4.0878E+00	2.1754E+00	6.0548E-01	6.0548E-01	0.0000E+00
59	4.5344E+00	2.0330E+00	3.6015E-01	3.6015E-01	0.0000E+00
60	5.0297E+00	1.8947E+00	1.6581E-01	1.6581E-01	0.0000E+00
61	5.5792E+00	1.7600E+00	6.0104E-02	6.0104E-02	0.0000E+00
62	6.1887E+00	1.6288E+00	1.7647E-02	1.7647E-02	0.0000E+00
63	6.8647E+00	1.5007E+00	4.3155E-03	4.3155E-03	0.0000E+00
64	7.6146E+00	1.3755E+00	9.0072E-04	9.0072E-04	0.0000E+00
65	8.4464E+00	1.2529E+00	1.6379E-04	1.6379E-04	0.0000E+00
66	9.3691E+00	1.1328E+00	2.6401E-05	2.6401E-05	0.0000E+00
67	1.0393E+01	1.0150E+00	3.8272E-06	3.8272E-06	0.0000E+00
68	1.1528E+01	8.9926E-01	5.0514E-07	5.0514E-07	0.0000E+00
69	1.2787E+01	7.8547E-01	6.1342E-08	6.1342E-08	0.0000E+00
70	1.4184E+01	6.7348E-01	6.9157E-09	6.9157E-09	0.0000E+00
71	1.5734E+01	5.6323E-01	7.2948E-10	7.2948E-10	0.0000E+00
72	1.7452E+01	4.5488E-01	7.2478E-11	7.2478E-11	0.0000E+00
73	1.9359E+01	3.4929E-01	6.8230E-12	6.8230E-12	0.0000E+00
74	2.1474E+01	2.4910E-01	6.1260E-13	6.1260E-13	0.0000E+00
75	2.3819E+01	1.6001E-01	5.5010E-14	5.5010E-14	0.0000E+00
76	2.6421E+01	9.0180E-02	9.3556E-15	9.3556E-15	0.0000E+00
77	2.9308E+01	4.4928E-02	7.0765E-15	7.0765E-15	0.0000E+00
78	3.2509E+01	2.0973E-02	5.7720E-15	5.7720E-15	0.0000E+00
79	3.6061E+01	1.1353E-02	4.1695E-15	4.1695E-15	0.0000E+00
80	4.0000E+01	1.1353E-02	4.1695E-15	4.1695E-15	0.0000E+00

Time	Boundary(j)	Spread	Time	Boundary(j)	Spread
0	44	4.7988E+00	20	50	4.8371E+00
1	45	4.7830E+00	21	51	4.8211E+00
2	46	4.7656E+00	22	52	4.8033E+00
3	47	4.7463E+00	23	53	4.7832E+00
4	48	4.7250E+00	24	54	4.7606E+00
5	49	4.7012E+00	25	55	4.7350E+00
6	50	4.6745E+00	26	56	4.7056E+00
7	51	4.6445E+00	27	57	4.6717E+00
8	51	4.9946E+00	28	58	4.6323E+00
9	52	4.9838E+00	29	59	4.5859E+00
10	53	4.9710E+00	30	76	3.0491E-02
11	54	4.9557E+00	31	51	4.7830E+00
12	55	4.9372E+00	32	52	4.7676E+00
13	56	4.9142E+00	33	52	4.9972E+00
14	57	4.8850E+00	34	53	4.9908E+00
15	77	2.5031E-02	35	54	4.9838E+00
16	46	4.8874E+00	36	55	4.9760E+00
17	47	4.8766E+00	37	56	4.9672E+00
18	48	4.8647E+00	38	57	4.9574E+00
19	49	4.8516E+00	39	58	4.9463E+00
	Option terminated at time 15 below j=77				
	Option terminated at time 30 below j=76				

	TABLE D3	Time Steps:	20	Bond Face Value:	20
	CALL	Bond Time Steps	40	Coupon:	3
	SPREAD	Value Steps:	80	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	1
		Option Maturity:	0.4	Riskless Rate:	0.06
		Max Firm Value:	40	Exercise price:	4
		Min Firm Value:	0.01	Max Spread:	5
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
1	1.1092E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
2	1.2304E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
3	1.3648E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
4	1.5139E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
5	1.6793E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
6	1.8627E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
7	2.0662E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
8	2.2920E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
9	2.5423E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
10	2.8201E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
11	3.1281E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
12	3.4698E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
13	3.8489E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
14	4.2693E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
15	4.7357E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
16	5.2531E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
17	5.8269E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
18	6.4634E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
19	7.1695E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
20	7.9527E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
21	8.8215E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
22	9.7851E-02	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
23	1.0854E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
24	1.2040E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
25	1.3355E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
26	1.4814E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
27	1.6432E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
28	1.8227E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
29	2.0218E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
30	2.2427E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
31	2.4877E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
32	2.7595E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
33	3.0609E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
34	3.3953E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
35	3.7662E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
36	4.1776E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
37	4.6340E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
38	5.1402E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
39	5.7017E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
40	6.3246E-01	5.0000E+00	1.0000E+00	9.9880E-01	1.0000E+00
41	7.0155E-01	5.0000E+00	1.0000E+00	9.9878E-01	1.0000E+00
42	7.7818E-01	5.0000E+00	1.0000E+00	9.9872E-01	1.0000E+00
43	8.6319E-01	5.0000E+00	1.0000E+00	9.9848E-01	1.0000E+00
44	9.5749E-01	4.7991E+00	9.9748E-01	9.9748E-01	7.9905E-01
45	1.0621E+00	4.5621E+00	9.9615E-01	9.9615E-01	5.6209E-01
46	1.1781E+00	4.3336E+00	9.9349E-01	9.9349E-01	3.3364E-01
47	1.3068E+00	4.1137E+00	9.8999E-01	9.8999E-01	1.1368E-01
48	1.4496E+00	3.9021E+00	9.8465E-01	9.8465E-01	0.0000E+00
49	1.6079E+00	3.6986E+00	9.7680E-01	9.7680E-01	0.0000E+00
50	1.7836E+00	3.5031E+00	9.6537E-01	9.6537E-01	0.0000E+00
51	1.9784E+00	3.3150E+00	9.4925E-01	9.4925E-01	0.0000E+00
52	2.1945E+00	3.1342E+00	9.2719E-01	9.2719E-01	0.0000E+00

53	2.4342E+00	2.9600E+00	8.9805E-01	8.9805E-01	0.0000E+00
54	2.7002E+00	2.7923E+00	8.6087E-01	8.6087E-01	0.0000E+00
55	2.9951E+00	2.6304E+00	8.1511E-01	8.1511E-01	0.0000E+00
56	3.3223E+00	2.4741E+00	7.6077E-01	7.6077E-01	0.0000E+00
57	3.6853E+00	2.3231E+00	6.9854E-01	6.9854E-01	0.0000E+00
58	4.0878E+00	2.1770E+00	6.2979E-01	6.2979E-01	0.0000E+00
59	4.5344E+00	2.0356E+00	5.5654E-01	5.5654E-01	0.0000E+00
60	5.0297E+00	1.8987E+00	4.8126E-01	4.8126E-01	0.0000E+00
61	5.5792E+00	1.7662E+00	4.0665E-01	4.0665E-01	0.0000E+00
62	6.1887E+00	1.6381E+00	3.3532E-01	3.3532E-01	0.0000E+00
63	6.8647E+00	1.5143E+00	2.6953E-01	2.6953E-01	0.0000E+00
64	7.6146E+00	1.3957E+00	2.1100E-01	2.1100E-01	0.0000E+00
65	8.4464E+00	1.2819E+00	1.6075E-01	1.6075E-01	0.0000E+00
66	9.3691E+00	1.1733E+00	1.1910E-01	1.1910E-01	0.0000E+00
67	1.0393E+01	1.0703E+00	8.5794E-02	8.5794E-02	0.0000E+00
68	1.1528E+01	9.7345E-01	6.0063E-02	6.0063E-02	0.0000E+00
69	1.2787E+01	8.8305E-01	4.0862E-02	4.0862E-02	0.0000E+00
70	1.4184E+01	7.9950E-01	2.7014E-02	2.7014E-02	0.0000E+00
71	1.5734E+01	7.2314E-01	1.7357E-02	1.7357E-02	0.0000E+00
72	1.7452E+01	6.5427E-01	1.0843E-02	1.0843E-02	0.0000E+00
73	1.9359E+01	5.9312E-01	6.5875E-03	6.5875E-03	0.0000E+00
74	2.1474E+01	5.3990E-01	3.8953E-03	3.8953E-03	0.0000E+00
75	2.3819E+01	4.9480E-01	2.2445E-03	2.2445E-03	0.0000E+00
76	2.6421E+01	4.5801E-01	1.2643E-03	1.2643E-03	0.0000E+00
77	2.9308E+01	4.2976E-01	7.0417E-04	7.0417E-04	0.0000E+00
78	3.2509E+01	4.1038E-01	4.0436E-04	4.0436E-04	0.0000E+00
79	3.6061E+01	4.0035E-01	2.7335E-04	2.7335E-04	0.0000E+00
80	4.0000E+01	4.0035E-01	2.7335E-04	2.7335E-04	0.0000E+00
Time	Boundary	Spread			
0	44	4.7991E+00			
1	45	4.7832E+00			
2	46	4.7658E+00			
3	47	4.7465E+00			
4	48	4.7252E+00			
5	49	4.7014E+00			
6	50	4.6747E+00			
7	51	4.6447E+00			
8	52	4.6105E+00			
9	52	4.9840E+00			
10	53	4.9712E+00			
11	54	4.9559E+00			
12	55	4.9373E+00			
13	56	4.9142E+00			
14	58	4.2582E+00			
15	72	6.9597E-01			
16	46	4.8877E+00			
17	47	4.8768E+00			
18	48	4.8649E+00			
19	49	4.8518E+00			
	Option terminated at time 15 below j= 72				

	TABLE D4	Time Steps:	40	Bond Face Value:	20
	CALL	Bond Time Steps	20	Coupon:	3
	SPREAD	Value Steps:	80	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	1
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value:	40	Exercise price:	4
		Min Firm Value:	0.01	Max Spread:	10
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
1	1.1092E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
2	1.2304E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
3	1.3648E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
4	1.5139E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
5	1.6793E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
6	1.8627E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
7	2.0662E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
8	2.2920E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
9	2.5423E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
10	2.8201E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
11	3.1281E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
12	3.4698E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
13	3.8489E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
14	4.2693E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
15	4.7357E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
16	5.2531E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
17	5.8269E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
18	6.4634E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
19	7.1695E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
20	7.9527E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
21	8.8215E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
22	9.7851E-02	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
23	1.0854E-01	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
24	1.2040E-01	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
25	1.3355E-01	1.0000E+01	6.0000E+00	5.9928E+00	6.0000E+00
26	1.4814E-01	1.0000E+01	6.0000E+00	5.9926E+00	6.0000E+00
27	1.6432E-01	9.8022E+00	5.9920E+00	5.9920E+00	5.8022E+00
28	1.8227E-01	9.4763E+00	5.9894E+00	5.9894E+00	5.4763E+00
29	2.0218E-01	9.1525E+00	5.9869E+00	5.9869E+00	5.1525E+00
30	2.2427E-01	8.8311E+00	5.9828E+00	5.9828E+00	4.8311E+00
31	2.4877E-01	8.5124E+00	5.9794E+00	5.9794E+00	4.5124E+00
32	2.7595E-01	8.1967E+00	5.9761E+00	5.9761E+00	4.1967E+00
33	3.0609E-01	7.8844E+00	5.9725E+00	5.9725E+00	3.8844E+00
34	3.3953E-01	7.5759E+00	5.9690E+00	5.9690E+00	3.5759E+00
35	3.7662E-01	7.2715E+00	5.9654E+00	5.9654E+00	3.2715E+00
36	4.1776E-01	6.9717E+00	5.9617E+00	5.9617E+00	2.9717E+00
37	4.6340E-01	6.6770E+00	5.9578E+00	5.9578E+00	2.6770E+00
38	5.1402E-01	6.3880E+00	5.9537E+00	5.9537E+00	2.3880E+00
39	5.7017E-01	6.1050E+00	5.9493E+00	5.9493E+00	2.1050E+00
40	6.3246E-01	5.8288E+00	5.9443E+00	5.9443E+00	1.8288E+00
41	7.0155E-01	5.5596E+00	5.9382E+00	5.9382E+00	1.5596E+00
42	7.7818E-01	5.2981E+00	5.9305E+00	5.9305E+00	1.2981E+00
43	8.6319E-01	5.0444E+00	5.9198E+00	5.9198E+00	1.0444E+00
44	9.5749E-01	4.7991E+00	5.9045E+00	5.9045E+00	7.9905E-01
45	1.0621E+00	4.5621E+00	5.8819E+00	5.8819E+00	5.6209E-01
46	1.1781E+00	4.3336E+00	5.8486E+00	5.8486E+00	3.3364E-01
47	1.3068E+00	4.1137E+00	5.7998E+00	5.7998E+00	1.1368E-01
48	1.4496E+00	3.9021E+00	5.7301E+00	5.7301E+00	0.0000E+00
49	1.6079E+00	3.6986E+00	5.6329E+00	5.6329E+00	0.0000E+00
50	1.7836E+00	3.5031E+00	5.5015E+00	5.5015E+00	0.0000E+00
51	1.9784E+00	3.3150E+00	5.3294E+00	5.3294E+00	0.0000E+00
52	2.1945E+00	3.1342E+00	5.1112E+00	5.1112E+00	0.0000E+00

53	2.4342E+00	2.9600E+00	4.8441E+00	4.8441E+00	0.0000E+00
54	2.7002E+00	2.7923E+00	4.5279E+00	4.5279E+00	0.0000E+00
55	2.9951E+00	2.6304E+00	4.1664E+00	4.1664E+00	0.0000E+00
56	3.3223E+00	2.4741E+00	3.7673E+00	3.7673E+00	0.0000E+00
57	3.6853E+00	2.3231E+00	3.3417E+00	3.3417E+00	0.0000E+00
58	4.0878E+00	2.1770E+00	2.9034E+00	2.9034E+00	0.0000E+00
59	4.5344E+00	2.0356E+00	2.4675E+00	2.4675E+00	0.0000E+00
60	5.0297E+00	1.8987E+00	2.0487E+00	2.0487E+00	0.0000E+00
61	5.5792E+00	1.7662E+00	1.6601E+00	1.6601E+00	0.0000E+00
62	6.1887E+00	1.6381E+00	1.3117E+00	1.3117E+00	0.0000E+00
63	6.8647E+00	1.5146E+00	1.0101E+00	1.0101E+00	0.0000E+00
64	7.6146E+00	1.3957E+00	7.5770E-01	7.5770E-01	0.0000E+00
65	8.4464E+00	1.2819E+00	5.5364E-01	5.5364E-01	0.0000E+00
66	9.3691E+00	1.1733E+00	3.9420E-01	3.9420E-01	0.0000E+00
67	1.0393E+01	1.0703E+00	2.7382E-01	2.7382E-01	0.0000E+00
68	1.1528E+01	9.7345E-01	1.8598E-01	1.8598E-01	0.0000E+00
69	1.2787E+01	8.8305E-01	1.2405E-01	1.2405E-01	0.0000E+00
70	1.4184E+01	7.9950E-01	8.1863E-02	8.1863E-02	0.0000E+00
71	1.5734E+01	7.2314E-01	5.4108E-02	5.4108E-02	0.0000E+00
72	1.7452E+01	6.5427E-01	3.6464E-02	3.6464E-02	0.0000E+00
73	1.9359E+01	5.9312E-01	2.5614E-02	2.5614E-02	0.0000E+00
74	2.1474E+01	5.3990E-01	1.9142E-02	1.9142E-02	0.0000E+00
75	2.3819E+01	4.9480E-01	1.5377E-02	1.5377E-02	0.0000E+00
76	2.6421E+01	4.5801E-01	1.3223E-02	1.3223E-02	0.0000E+00
77	2.9308E+01	4.2976E-01	1.2009E-02	1.2009E-02	0.0000E+00
78	3.2509E+01	4.1038E-01	1.1350E-02	1.1350E-02	0.0000E+00
79	3.6061E+01	4.0035E-01	1.1055E-02	1.1055E-02	0.0000E+00
80	4.0000E+01	4.0035E-01	1.1055E-02	1.1055E-02	0.0000E+00

Time	Boundary	Spread	Time	Boundary	Spread
0	27	9.8022E+00	20	37	9.6377E+00
1	29	9.7682E+00	21	39	9.5729E+00
2	31	9.7293E+00	22	41	9.4944E+00
3	33	9.6844E+00	23	43	9.3978E+00
4	35	9.6319E+00	24	44	9.9840E+00
5	37	9.5700E+00	25	46	9.9314E+00
6	39	9.4957E+00	26	48	9.8589E+00
7	40	9.9885E+00	27	50	9.7527E+00
8	42	9.9476E+00	28	52	9.5843E+00
9	44	9.8947E+00	29	54	9.2861E+00
10	46	9.8239E+00	30	72	7.8069E-01
11	48	9.7244E+00	31	31	9.8740E+00
12	50	9.5747E+00	32	33	9.8482E+00
13	52	9.3273E+00	33	35	9.8191E+00
14	54	8.8583E+00	34	37	9.7862E+00
15	72	6.9597E-01	35	39	9.7487E+00
16	29	9.8133E+00	36	41	9.7054E+00
17	31	9.7785E+00	37	43	9.6550E+00
18	33	9.7386E+00	38	45	9.5957E+00
19	35	9.6922E+00	39	47	9.5250E+00
	Option terminated at time15 below j= 72				
	Option terminated at time30 below j= 72				

	TABLE D5	Time Steps:	40	Bond Face Value:	20
	CALL	Bond Time Steps	20	Coupon:	3
	SPREAD	Value Steps:	80	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	1
		Option Maturity:	0.8	Riskless Rate:	0.06
		Max Firm Value:	40	Exercise price:	3
		Min Firm Value:	0.01	Max Spread:	5
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
1	1.1092E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
2	1.2304E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
3	1.3648E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
4	1.5139E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
5	1.6793E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
6	1.8627E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
7	2.0662E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
8	2.2920E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
9	2.5423E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
10	2.8201E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
11	3.1281E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
12	3.4698E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
13	3.8489E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
14	4.2693E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
15	4.7357E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
16	5.2531E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
17	5.8269E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
18	6.4634E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
19	7.1695E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
20	7.9527E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
21	8.8215E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
22	9.7851E-02	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
23	1.0854E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
24	1.2040E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
25	1.3355E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
26	1.4814E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
27	1.6432E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
28	1.8227E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
29	2.0218E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
30	2.2427E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
31	2.4877E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
32	2.7595E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
33	3.0609E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
34	3.3953E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
35	3.7662E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
36	4.1776E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
37	4.6340E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
38	5.1402E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
39	5.7017E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
40	6.3246E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
41	7.0155E-01	5.0000E+00	2.0000E+00	1.9976E+00	2.0000E+00
42	7.7818E-01	5.0000E+00	2.0000E+00	1.9975E+00	2.0000E+00
43	8.6319E-01	5.0000E+00	2.0000E+00	1.9970E+00	2.0000E+00
44	9.5749E-01	4.7991E+00	1.9952E+00	1.9952E+00	1.7991E+00
45	1.0621E+00	4.5621E+00	1.9929E+00	1.9929E+00	1.5621E+00
46	1.1781E+00	4.3336E+00	1.9884E+00	1.9884E+00	1.3336E+00
47	1.3068E+00	4.1137E+00	1.9827E+00	1.9827E+00	1.1137E+00
48	1.4496E+00	3.9021E+00	1.9742E+00	1.9742E+00	9.0208E-01
49	1.6079E+00	3.6986E+00	1.9620E+00	1.9620E+00	6.9865E-01
50	1.7836E+00	3.5031E+00	1.9442E+00	1.9442E+00	5.0308E-01
51	1.9784E+00	3.3150E+00	1.9189E+00	1.9189E+00	3.1505E-01
52	2.1945E+00	3.1342E+00	1.8840E+00	1.8840E+00	1.3417E-01

53	2.4342E+00	2.9600E+00	1.8372E+00	1.8372E+00	0.0000E+00
54	2.7002E+00	2.7923E+00	1.7763E+00	1.7763E+00	0.0000E+00
55	2.9951E+00	2.6304E+00	1.6997E+00	1.6997E+00	0.0000E+00
56	3.3223E+00	2.4741E+00	1.6067E+00	1.6067E+00	0.0000E+00
57	3.685?E+00	2.3231E+00	1.4975E+00	1.4975E+00	0.0000E+00
58	4.0[illegible]78E+00	2.1770E+00	1.3736E+00	1.3736E+00	0.0000E+00
59	4.5344E+00	2.0356E+00	1.2378E+00	1.2378E+00	0.0000E+00
60	5.0297E+00	1.8987E+00	1.0940E+00	1.0940E+00	0.0000E+00
61	5.5792E+00	1.7652E+00	9.4708E-01	9.4708E-01	0.0000E+00
62	6.1887E+00	1.6381E+00	8.0200E-01	8.0200E-01	0.0000E+00
63	6.8647E+00	1.5146E+00	6.6372E-01	6.6372E-01	0.0000E+00
64	7.6146E+00	1.3957E+00	5.3647E-01	5.3647E-01	0.0000E+00
65	8.4464E+00	1.2819E+00	4.2344E-01	4.2344E-01	0.0000E+00
66	9.369?E+00	1.1733E+00	3.2657E-01	3.2657E-01	0.0000E+00
67	1.0393E+01	1.0703E+00	2.4649E-01	2.4649E-01	0.0000E+00
68	1.1528E+01	9.7345E-01	1.8267E-01	1.8267E-01	0.0000E+00
69	1.2787E+01	8.8305E-01	1.3367E-01	1.3367E-01	0.0000E+00
70	1.4184E+01	7.9950E-01	9.7454E-02	9.7454E-02	0.0000E+00
71	1.5734E+01	7.2314E-01	7.1713E-02	7.1713E-02	0.0000E+00
72	1.7452E+01	6.5427E-01	5.4127E-02	5.4127E-02	0.0000E+00
73	1.9359E+01	5.9312E-01	4.2581E-02	4.2581E-02	0.0000E+00
74	2.1474E+01	5.3990E-01	3.5288E-02	3.5288E-02	0.0000E+00
75	2.3819E+01	4.9480E-01	3.0841E-02	3.0841E-02	0.0000E+00
76	2.6421E+01	4.5801E-01	2.8210E-02	2.8210E-02	0.0000E+00
77	2.9308E+01	4.2976E-01	2.6698E-02	2.6698E-02	0.0000E+00
78	3.2509E+01	4.1038E-01	2.5876E-02	2.5876E-02	0.0000E+00
79	3.6061E+01	4.0035E-01	2.5508E-02	2.5508E-02	0.0000E+00
80	4.0000E+01	4.0035E-01	2.5508E-02	2.5508E-02	0.0000E+00

Time	Boundary	Spread	Time	Boundary	Spread
0	44	4.7991E+00	20	50	4.8373E+00
1	45	4.7832E+00	21	51	4.8213E+00
2	46	4.7658E+00	22	52	4.8034E+00
3	47	4.7465E+00	23	53	4.7834E+00
4	48	4.7252E+00	24	54	4.7608E+00
5	49	4.7014E+00	25	55	4.7351E+00
6	50	4.6747E+00	26	56	4.7057E+00
7	51	4.6447E+00	27	57	4.6718E+00
8	52	4.6105E+00	28	58	4.6323E+00
9	53	4.5716E+00	29	59	4.5859E+00
10	53	4.9712E+00	30	72	7.8069E-01
11	54	4.9559E+00	31	51	4.7832E+00
12	55	4.9373E+00	32	52	4.7679E+00
13	57	4.3427E+00	33	53	4.7510E+00
14	58	4.2582E+00	34	54	4.7325E+00
15	72	6.9597E-01	35	55	4.7122E+00
16	46	4.8877E+00	36	55	4.9762E+00
17	47	4.8768E+00	37	56	4.9674E+00
18	48	4.8649E+00	38	58	4.6361E+00
19	49	4.8518E+00	39	59	4.6043E+00
	Option terminated at time15 below j=72				
	Option terminated at time30 below j=72				

	TABLE D6	Time Steps:	40	Bond Face Value:	20
	CALL	Bond Time Steps	20	Coupon:	3
	SPREAD	Value Steps:	80	Coupon dates:	0.3,0.6,0.9.
	COUPON	Bond Maturity:	1.2	Volatility:	1
		Option Maturity:	0.8	Riskless Rate:	0.3
		Max Firm Value:	40	Exercise price:	4
		Min Firm Value:	0.01	Max Spread:	5
J	Firm	Spread	Option	Temp	Exercise
0	1.0000E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
1	1.1092E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
2	1.2304E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
3	1.3648E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
4	1.5139E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
5	1.6793E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
6	1.8627E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
7	2.0662E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
8	2.2920E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
9	2.5423E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
10	2.8201E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
11	3.1281E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
12	3.4698E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
13	3.8489E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
14	4.2693E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
15	4.7357E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
16	5.2531E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
17	5.8269E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
18	6.4634E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
19	7.1695E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
20	7.9527E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
21	8.8215E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
22	9.7851E-02	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
23	1.0854E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
24	1.2040E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
25	1.3355E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
26	1.4814E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
27	1.6432E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
28	1.8227E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
29	2.0218E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
30	2.2427E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
31	2.4877E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
32	2.7595E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
33	3.0609E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
34	3.3953E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
35	3.7662E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
36	4.1776E-01	5.0000E+00	1.0000E+00	9.9404E-01	1.0000E+00
37	4.6340E-01	5.0000E+00	1.0000E+00	9.9403E-01	1.0000E+00
38	5.1402E-01	5.0000E+00	1.0000E+00	9.9403E-01	1.0000E+00
39	5.7017E-01	5.0000E+00	1.0000E+00	9.9402E-01	1.0000E+00
40	6.3246E-01	5.0000E+00	1.0000E+00	9.9395E-01	1.0000E+00
41	7.0155E-01	5.0000E+00	1.0000E+00	9.9371E-01	1.0000E+00
42	7.7818E-01	5.0000E+00	1.0000E+00	9.9273E-01	1.0000E+00
43	8.6319E-01	4.8042E+00	9.8883E-01	9.8883E-01	8.0419E-01
44	9.5749E-01	4.5588E+00	9.8461E-01	9.8461E-01	5.5881E-01
45	1.0621E+00	4.3219E+00	9.7694E-01	9.7694E-01	3.2186E-01
46	1.1781E+00	4.0934E+00	9.6947E-01	9.6947E-01	9.3418E-02
47	1.3068E+00	3.8735E+00	9.6004E-01	9.6004E-01	0.0000E+00
48	1.4496E+00	3.6619E+00	9.4853E-01	9.4853E-01	0.0000E+00
49	1.6079E+00	3.4585E+00	9.3378E-01	9.3378E-01	0.0000E+00
50	1.7836E+00	3.2629E+00	9.1480E-01	9.1480E-01	0.0000E+00
51	1.9784E+00	3.0749E+00	8.9040E-01	8.9040E-01	0.0000E+00
52	2.1945E+00	2.8941E+00	8.5950E-01	8.5950E-01	0.0000E+00

53	2.4342E+00	2.7200E+00	8.2120E-01	8.2120E-01	0.0000E+00
54	2.7002E+00	2.5524E+00	7.7502E-01	7.7502E-01	0.0000E+00
55	2.9951E+00	2.3907E+00	7.2098E-01	7.2098E-01	0.0000E+00
56	3.3223E+00	2.2348E+00	6.5978E-01	6.5978E-01	0.0000E+00
57	3.6853E+00	2.0843E+00	5.9279E-01	5.9279E-01	0.0000E+00
58	4.0878E+00	1.9389E+00	5.2197E-01	5.2197E-01	0.0000E+00
59	4.5344E+00	1.7986E+00	4.4969E-01	4.4969E-01	0.0000E+00
60	5.0297E+00	1.6633E+00	3.7851E-01	3.7851E-01	0.0000E+00
61	5.5792E+00	1.5329E+00	3.1087E-01	3.1087E-01	0.0000E+00
62	6.1887E+00	1.4077E+00	2.4888E-01	2.4888E-01	0.0000E+00
63	6.8647E+00	1.2878E+00	1.9406E-01	1.9406E-01	0.0000E+00
64	7.6146E+00	1.1736E+00	1.4731E-01	1.4731E-01	0.0000E+00
65	8.4464E+00	1.0653E+00	1.0885E-01	1.0885E-01	0.0000E+00
66	9.3691E+00	9.6322E-01	7.8324E-02	7.8324E-02	0.0000E+00
67	1.0393E+01	8.6784E-01	5.4964E-02	5.4964E-02	0.0000E+00
68	1.1528E+01	7.7942E-01	3.7723E-02	3.7723E-02	0.0000E+00
69	1.2787E+01	6.9824E-01	2.5452E-02	2.5452E-02	0.0000E+00
70	1.4184E+01	6.2449E-01	1.7031E-02	1.7031E-02	0.0000E+00
71	1.5734E+01	5.5831E-01	1.1457E-02	1.1457E-02	0.0000E+00
72	1.7452E+01	4.9972E-01	7.8946E-03	7.8946E-03	0.0000E+00
73	1.9359E+01	4.4870E-01	5.6912E-03	5.6912E-03	0.0000E+00
74	2.1474E+01	4.0518E-01	4.3653E-03	4.3653E-03	0.0000E+00
75	2.3819E+01	3.6903E-01	3.5827E-03	3.5827E-03	0.0000E+00
76	2.6421E+01	3.4014E-01	3.1250E-03	3.1250E-03	0.0000E+00
77	2.9308E+01	3.1843E-01	2.8595E-03	2.8595E-03	0.0000E+00
78	3.2509E+01	3.0386E-01	2.7117E-03	2.7117E-03	0.0000E+00
79	3.6061E+01	2.9649E-01	2.6446E-03	2.6446E-03	0.0000E+00
80	4.0000E+01	2.9649E-01	2.6446E-03	2.6446E-03	0.0000E+00

Time	Boundary	Spread	Time	Boundary	Spread
0	43	4.8042E+00	20	49	4.8782E+00
1	44	4.8002E+00	21	50	4.8779E+00
2	45	4.7957E+00	22	51	4.8775E+00
3	46	4.7908E+00	23	52	4.8771E+00
4	47	4.7852E+00	24	53	4.8766E+00
5	48	4.7790E+00	25	54	4.8760E+00
6	49	4.7720E+00	26	55	4.8754E+00
7	50	4.7639E+00	27	56	4.8746E+00
8	51	4.7546E+00	28	57	4.8737E+00
9	52	4.7438E+00	29	58	4.8726E+00
10	53	4.7310E+00	30	72	6.3983E-01
11	54	4.7157E+00	31	50	4.7682E+00
12	55	4.6972E+00	32	51	4.7630E+00
13	56	4.6742E+00	33	52	4.7572E+00
14	57	4.6450E+00	34	53	4.7508E+00
15	72	5.4769E-01	35	54	4.7438E+00
16	45	4.8792E+00	36	55	4.7360E+00
17	46	4.8790E+00	37	56	4.7273E+00
18	47	4.8787E+00	38	57	4.7175E+00
19	48	4.8785E+00	39	58	4.7063E+00
	Option terminated at time 15 below J= 72				
	Option terminated at time 30 below J= 72				

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Essay II

The Pricing of Lookback Call and Put Options on Treasury Bond Prices

1 Introduction

In a path breaking article, Goldman Sosin and Gatto (1979) established the pricing properties of lookback call and put options on stocks. Somewhat surprisingly, little research has been done on the pricing of lookback options on instruments other than stocks. The only exception is Garman (1989) who studied the pricing of lookback options on currencies. This paper considers the pricing of lookback call and put options on treasury bond prices. Section 2 describes the contractual features of such options. Section 3 presents a valuation model for such options that is based on the assumption that the short term interest rate follows the square root process introduced by Cox, Ingersoll and Ross (1985). Section 4 presents an alternative valuation model for such options based on the assumption that the short term interest rate follows the extended Vasicek model introduced by Hull and White (1990). Section 5 compares the prices generated by the models of Sections 3 and 4. Section 6 establishes the pricing properties of these options at the time at which they are written, i.e., at time 0; it also compares these properties with those established by Goldman et. al.

(1979) for stocks. Section 7 establishes the pricing properties of these options after they have been outstanding for some time and as in Section 6, compares them with those of Goldman et. al (1979) for stocks. Section 8 offers concluding remarks.

2 Contractual Features of Lookback Call and Put Options on Treasury Bond Prices

A lookback put option on a zero coupon treasury bond, written at time 0 and expiring at time $T > 0$, entitles its owner to receive at T the difference between the maximum bond price in the time interval 0 to T and the bond price at T. It is obvious that a lookback put has all the features of a regular put except that the exercise price is equal to the maximum bond price during the option's life. For future reference, we define

0 = time at which option written

T = option maturity at time 0

TB = maturity of the bond on which the option is written at time 0

t = current time

$P(t,TB)$ = price at t of a zero coupon Treasury bond paying \$1 at TB

$M(t) = \max_{0 \le \tau \le t} P(\tau, TB)$

PMAX $(P(t,TB), M(t), t, T)$ = Price at time t of a lookback put option written at time 0 and maturing at time $T (0 \le t \le T)$, when the current bond price at t is $P(t,TB)$ and the maximum bond price to date i.e., between time 0 and time t is $M(t)$. The bond had maturity TB at time 0, with $TB > T > t$.

From contract definition, it is obvious that at option maturity, i.e., when $t = T$,

$$\text{PMAX}\,(P(T,TB), M(T), T, T) = \left[\max_{0 \le \tau \le T} P(\tau, TB) \right] - P(T,TB).$$

The value of PMAX for $t < T$ will be determined using the valuation models that will be

introduced in Section 3 and 4.

A lookback call option on a zero-coupon Treasury bond, written at time 0 and expiring at time $T > 0$, entitles its owner to receive at T the difference between the current bond price and the minimum bond price in the interval 0 to T. It is obvious that a lookback call has all the features of a regular call except that the exercise price is not fixed but instead is equal to the minimum bond price during the option's life.

For future reference, we define $t, T, TB, P(t,TB)$ as above and assume that the lookback call is written at time 0. Also, we define

$$\begin{aligned} Q(t) &= \min_{0 \le \tau \le t} P(\tau, TB) \\ \text{PMIN}\,(P(t,TB), Q(t), t, T) &= \text{price at time } t \text{ of a lookback call option, written at time 0 and expiring at } T (0 \le t \le T), \text{ when the current bond price at time } t \text{ is } P(t,TB) \text{ and the minimum bond price to date i.e., between time 0 and time } t \text{ is } Q(t) \text{ (the bond had maturity } TB \text{ at time 0, with } T_B > T > t). \end{aligned}$$

From contract definition it is obvious that at $t = T$ i.e., at option maturity

$$\text{PMIN}\,(P(T,TB), Q(T), T, T) = P(T,TB) - \left[\begin{array}{ll} \min & P(\tau, TB) \\ 0 \le \tau \le T & \end{array} \right].$$

The value of PMIN for $t < T$ will be determined using the valuation models of subsequent sections.

3 A Valuation Model for Lookback Call and Put Options on Zero-Coupon Treasury Bond Prices Based on the Cox, Ingersoll and Ross (1985) Square Root Process for the Short Term Interest Rate

From the work of Harrison and Kreps (1979) and Huang (1991), we know that the lookback options defined in Section 2 can be priced using a no-arbitrage methodology. As is common in the bond and bond option pricing literature, we specify a process for the short term interest rate and derive the bond and option prices as a function of the interest rate. In this section, we assume that the short term interest rate $r(t)$ follows the square root process introduced by Cox, Ingersoll and Ross (1985) i.e.,

$$dr = k(\theta - r)dt + \sigma\sqrt{r}\, dw$$

where

$k =$ intensity of mean reversion

$\theta =$ long run mean of interest rate

$\sigma =$ volatility of r

$w =$ standard Brownian motion under a probability measure P.

Huang (1991) shows that the (normalized) price of any interest rate sensitive security paying a random payoff $\tilde{x}(\tau)$ at time $\tau > 0$ can be calculated as the conditional expectation of the (normalized) payoff under an equivalent martingale measure. In particular, let

$x(t)$ = price at t of security paying random payoff $\tilde{x}(\tau)$ at $\tau > t$.

$B(t)$ = price process for the instantaneously riskless asset that is used as the numeraire.

By definition,

$$B(t) = \exp\left(\int_0^t r(s)ds\right).$$

$w^*(t)$ = standard Brownian motion under a measure Q equivalent to P, where

$$w^*(t) = w(t) + \int_0^t \frac{\lambda}{\sigma}\sqrt{r(s)}\,ds$$

where λ is a scalar. Using the last equation to substitute for $w(t)$ in the interest rate process we get

$$dr = (k\theta - (k+\lambda)r)dt + \sigma\sqrt{r}\,dw^*. \tag{II.1}$$

This is the interest rate process under the equivalent martingale measure and is also known as the risk adjusted interest rate process. Then,

$$\frac{x(t)}{B(t)} = E^*\left[\frac{\tilde{x}(\tau)}{B(\tau)}\right] \tag{II.2}$$

where the expectation operator E^* denotes expectation under the risk adjusted interest rate process.

Since a zero-coupon Treasury bond of maturity TB pays \$1.00 with certainty at TB, its price $P(t,TB)$ at time t will be given by

$$\frac{P(t,TB)}{B(t)} = E^*\left[\frac{1}{B(TB)}\right] \Rightarrow P(t,TB) = E^*\left[\exp\left(-\int_t^{TB} r(s)ds\right)\right]. \tag{II.3}$$

Cox, Ingersoll and Ross (1985) calculate this conditional expectation analytically and pro-

vide the following closed form formula for $P(t,TB)$

$$P(t,TB) = A(t,TB)\exp(-r(t)G(t,TB)) \tag{II.4}$$

where

$$A(t,TB) = \left[\frac{2\gamma \exp\left[(b+\gamma)\left(\frac{TB-t}{2}\right)\right]}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}} \tag{II.5}$$

$$G(t,TB) = \frac{2(\exp(\gamma(TB-t))-1)}{(\gamma+b)(\exp(\gamma(TB-t))-1)+2\gamma} \tag{II.6}$$

$$b = k+\lambda$$

$$c = k\theta$$

$$\gamma = \sqrt{b^2+2\sigma^2}$$

3.1 Pricing Lookback Puts

From Section 3 above we know that

$$\text{PMAX}\,(P(0,TB),M(0),0,T) = E^*\left[\frac{M(T)-P(T,TB)}{B(T)}\right]. \tag{II.7}$$

By definition,

$$E^*\left[\frac{P(T,TB)}{B(T)}\right] = \frac{P(0,TB)}{B(0)} = P(0,TB). \tag{II.8}$$

Hence,

$$\text{PMAX}\,(P(0,TB),M(0),0,T) = E^*\left[\frac{M(T)}{B(T)}\right] - P(0,TB)$$

$$= E^* \left[M(T) \left(\exp\left(- \int_0^T r(s) ds \right) \right) \right] - P(0, TB) \quad \text{(II.9)}$$

Alternatively, if we want to price the option at time $t > 0$,

$$\begin{aligned} \text{PMAX}\left(P(t,TB), M(t), t, T\right) &= E^* \left[\frac{\max\{M(t), \max_{t \le \tau \le T} P(\tau, TB)\} - P(T, TB)}{B(T)} \right] \\ &= E^* \left[\frac{\max\{M(t), \max_{t \le \tau \le T} P(\tau, TB)\}}{B(T)} \right] - P(t, TB) \quad \text{(II.10)} \end{aligned}$$

There is no way to calculate analytically the conditional expectation that gives us the lookback put price. For this reason we calculate this price using Monte Carlo simulation.

To calculate PMAX $(P(0,TB), M(0), 0, T)$ we divide the time interval between 0 and T into m subintervals such that

$$0 = t_0 < t_1 < \ldots < t_m = T. \quad \text{(II.11)}$$

We simulate the risk adjusted interest rate process using the stochastic analogue of Euler's integration method; the latter converges uniformly in quadratic mean as shown by Rumelin (1982). In particular, we simulate using the discrete approximation

$$r_{t_i} = r_{t_{i-1}} + (k\theta - (k+\lambda) r_{t_{i-1}})(t_i - t_{i-1}) + \sigma \sqrt{r_{t_{i-1}}} \sqrt{t_i - t_{i-1}}\ \tilde{\epsilon} \qquad \text{for } i = 1, \ldots m \quad \text{(II.12)}$$

where $\tilde{\epsilon}$ is normally distributed with mean 0 and variance 1.

We repeat the simulation N times and let $r_n(t_i)$ denote the simulated value of $r(t_i)$ in the nth simulation, $n = 1, 2, \ldots, N$.

Let $\widehat{\text{PMAX}}(P(0,TB), M(\upsilon), 0, T)$ = estimate of PMAX using this Monte Carlo technique

$$\widehat{\text{PMAX}}(\text{P}(0,\text{TB}),\text{M}(0),0,\text{T}) = \left\{\frac{1}{\text{N}}\sum_{\text{n=1}}^{\text{N}}\hat{\text{M}}_{\text{n}}(\text{T})\exp\left[-\sum_{\text{i=1}}^{\text{m}}\text{r}_{\text{n}}(\text{t}_{\text{i-1}})(\text{t}_{\text{i}}-\text{t}_{\text{i-1}})\right]\right\} - \text{P}(0,\text{TB}) \tag{II.13}$$

where $\hat{M}_n(T)$ = simulated value of $M(T)$ in nth simulation.

Rubinstein (1981) shows that as m and N increase to infinity, $\widehat{\text{PMAX}}$ converges to PMAX . A similar procedure is used for the estimation of PMAX $(P(t,TB), M(t), t, T)$.

In practice, we don't know how large m and N must be to get convergence to the true price. For this reason, we build a confidence interval around our estimate. To do this, we first calculate

$$\hat{s}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(\text{PMAX}\,(i) - \widehat{\text{PMAX}}\,)^2 \tag{II.14}$$

where PMAX (i) = estimate of value of PMAX calculated in the ith simulation. Rubinstein (1981) shows that

$$\frac{\widehat{\text{PMAX}} - \text{PMAX}}{\sqrt{\frac{\hat{s}^2}{N}}} \tag{II.15}$$

tends asymptotically to a standard normal distribution for large N. We can therefore calculate a confidence interval for PMAX . Note that since the standard deviation of $\widehat{\text{PMAX}}$ is $\hat{s}/\sqrt{N}$, the confidence limits can be reduced by increasing N. However, Rubinstein (1981) points out that instead of focusing on increasing N, it is more efficient in terms of computer time usage, to focus on reducing $\hat{s}$. One variance reduction technique that can be used is known as the antithetic variable technique. In this technique, each simulation run involves calculating two values for the option. The first value is calculated in the usual

way by drawing random numbers from a standard normal distribution. The second value is calculated by changing the sign of all the standard normal distribution samples drawn in calculating the first value. The estimate of the value of the option for each simulation run is then the average of these two values. This works well because when one value is above the true value, the other tends to be below it. As a result, the value of $\hat{s}$ is reduced.

All option prices presented in this paper were calculated by using this antithetic variable technique to increase the precision of the estimates.

3.2 Pricing Lookback Calls

From Section 3 we know that

$$\text{PMIN}\,(P(0,TB),Q(0),0,T) = E^*\left[\frac{P(T,TB)-Q(T)}{B(T)}\right]. \tag{II.16}$$

By definition,

$$E^*\left[\frac{P(T,TB)}{B(T)}\right] = P(0,TB).$$

Hence,

$$\begin{aligned}\text{PMIN}\,(P(0,TB),Q(0),0,T) &= P(0,TB) - E^*\left[\frac{Q(T)}{B(T)}\right] \\ &= P(0,TB) - E^*\left[Q(T)\left(\exp\left(-\int_0^T r(s)ds\right)\right)\right] \end{aligned} \tag{II.17}$$

Alternatively, if we want to price the option at time $t > 0$,

$$\text{PMIN}\,(P(t,TB),Q(t),t,T) = E^*\left[\frac{P(T,TB)-\min\{Q(t),\min_{t\le\tau\le T}P(\tau,TB)\}}{B(T)}\right]$$

$$= P(t,TB) - E^*\left[\frac{\min\{Q(t), \min_{t\le\tau\le T} P(\tau,TB)\}}{B(T)}\right] . \text{(II.18)}$$

As in the case of the lookback put option, there is no way to calculate this conditional expectation analytically. For this reason, we use Monte Carlo simulation.

Let $\widehat{\text{PMIN}}(P(0,TB),Q(0),0,T)$ be the estimate of PMIN . We simulate the risk adjusted interest rate process as described in Section 3.1 and calculate $\widehat{\text{PMIN}}$ by

$$\widehat{\text{PMIN}}(\text{P}(0,\text{TB}),\text{Q}(0),0,\text{T}) = \text{P}(0,\text{TB}) - \left\{\frac{1}{\text{N}}\sum_{\text{n}=1}^{\text{N}} \hat{\text{Q}}_\text{n}(\text{T}) \exp\left[-\sum_{\text{i}=1}^{\text{m}} \text{r}_\text{n}(\text{t}_{\text{i}-1})(\text{t}_\text{i} - \text{t}_{\text{i}-1})\right]\right\} \quad \text{(II.19)}$$

where $\hat{Q}_n(T)$ = simulated value of $Q(T)$ in nth simulation.

A similar procedure is used for the estimation of PMIN $(P(t,TB),Q(t),t,T)$.

3.3 Computer Programs that Implement Above Valuation Model

A computer program that carries out the simulations described in Sections 3.1 and 3.2 was written in the C programming language and implemented on a HP 9000 workstation.

In writing this program, particular attention was given to the need for a random number generator that is "truly random." As Press, et. al. (1992) point out, many system supplied random number generators are very suspicious. For this reason, I did not use the internal random number generator that is available in C, but instead used a routine that Press, et. al. (1992) provide that is known to be unbiased. This routine is known as the Gasdev routine for generating random samples from a standard normal distribution and is described in Press, et. al. (1992) Chapter 7.

To check the accuracy of the results obtained, I calculated numerically various bond prices and compared them to the bond prices given by the closed form formula. The differences were extremely small, usually of an order of magnitude of 10^{-6}.

4 A Valuation Model for Lookback Call and Put Options On Treasury Bond Prices Based on the Extended Vasicek Process for the Short Term Interest Rate

In this section, we present an alternative valuation model for lookback options based on the Extended Vasicek interest rate process. The latter was introduced by Hull and White (1990). It is called the Extended Vasicek interest rate process because it is a generalization of a process introduced by Vasicek (1977). Under the Extended Vasicek Process, the evolution of the short term interest rate is described by the stochastic differential equation

$$dr = [\theta(t) + a(t)(b - r)]dt + \sigma(t)dw(t) \tag{II.20}$$

where $\theta(t)$ = time dependent drift

$a(t)$ = time dependent reversion rate

b = long-term level of interest rate

$\sigma(t)$ = time dependent volatility

$w(t)$ = Standard Brownian motion under measure P.

As in the case of the square root interest rate process, the (normalized) price of any interest sensitive security paying a random interest dependent payoff $\tilde{x}(\tau)$ at $\tau > 0$ can be calculated as the conditional expectation of the (normalized) payoff under an equivalent martingale measure. In particular, let

$x(t)$ = price at t of security paying $\tilde{x}(\tau)$ at $\tau > t$.

$B(t)$ = price process for instantaneously riskless asset that is used as the numeraire, where

$B(t)$ = $\exp(\int_0^t r(s)ds)$

$w^{**}(t)$ = standard Brownian motion under a measure Q equivalent to P, where

$w^{**}(t)$ = $w(t) + \int_0^t \lambda(s)ds$

$\lambda(s)$ = bounded function of time.

Using the equation for $w^{**}(t)$ to substitute for $w(t)$ in the interest rate process, we get

$$dr = (\phi(t) - a(t)r)dt + \sigma(t)dw^{**} \tag{II.21}$$

where, $\phi(t)$ is given by

$$\phi(t) = \theta(t) + a(t)b - \lambda(t)\sigma(t). \tag{II.22}$$

This stochastic differential equation describes the risk adjusted Extended Vasicek interest rate process. Then,

$$\frac{x(t)}{B(t)} = E^{**}\left[\frac{\tilde{x}(\tau)}{B(\tau)}\right] \tag{II.23}$$

where, the expectation operator E^{**} denotes expectation under the risk adjusted Extended Vasicek interest rate process.

Since a zero-coupon Treasury bond of maturity TB pays \$1.00 with certainty at TB, its price $P(t,TB)$ at time t will be given by

$$\frac{P(t,TB)}{B(t)} = E^{**}\left[\frac{1}{B(TB)}\right] \Rightarrow P(t,TB) = E^{**}\left[\exp\left(-\int_t^{TB} r(s)ds\right)\right]. \tag{II.24}$$

Hull and White (1990) provide a closed form solution for $P(t,TB)$ of the form

$$P(t,TB) = A(t,TB)\exp(-r(t)G(t,TB)) \tag{II.25}$$

where, $A(t,TB)$ and $G(t,TB)$ are given by

$$G(t,TB) = \frac{G(0,TB) - G(0,t)}{\partial G(0,t)/\partial t} \tag{II.26}$$

$$\hat{A}(t,TB) = \log A(t,TB) \tag{II.27}$$

$$\begin{aligned}\hat{A}(t,TB) = {} & \hat{A}(0,TB) - \hat{A}(0,t) - G(t,TB)\frac{\partial \hat{A}(0,t)}{\partial t} \\ & -\frac{1}{2}\left[G(t,TB)\frac{\partial G(0,t)}{\partial t}\right]^2 \int_0^t \left[\frac{\sigma(\tau)}{\partial G(0,\tau)/\partial \tau}\right]^2 d\tau.\end{aligned} \tag{II.28}$$

Note that to use this formula we must know $A(0,t)$ and $G(0,t)$ for all t. Hull and White (1990) show that these two functions are uniquely defined by $\sigma(0)$, the current term structure of interest rates and the current term structure of spot or forward interest rate volatilities. Hence, their formula is usable once $A(0,t)$ and $G(0,t)$ are estimated (for all t) from market data. Furthermore, the functions $\phi(t)$ and $a(t)$ that appear in the risk adjusted interest rate process, are related to $A(0,t)$ and $G(0,t)$ through the following equations

$$a(t) = -\frac{\partial^2 G(0,t)/\partial t^2}{\partial G(0,t)/\partial t} \tag{II.29}$$

$$\phi(t) = -a(t)\frac{\partial \hat{A}(0,t)}{\partial t} - \frac{\partial^2 \hat{A}(0,t)}{\partial t^2} + \left[\frac{\partial G(0,t)}{\partial t}\right]^2 \int_0^t \left[\frac{\sigma(\tau)}{\partial G(0,\tau)/\partial \tau}\right]^2 d\tau. \tag{II.30}$$

4.1 Fitting the Extended Vasicek Model to the Cox, Ingersoll and Ross Model

One of our objectives is to price lookback options using the Extended Vasicek model and to compare the prices obtained with those of Section 3 for the Cox, Ingersoll and Ross square root process. A meaningful way to carry out this comparison is to ensure that both models are specified in such a way that they result in the same term structure at time 0, the time at which the options are written. One way of achieving this is to assume that the Cox, Ingersoll and Ross model describes the true evolution of the term structure and to fit the Extended Vasicek model to the term structure generated by the Cox, Ingersoll and Ross model. In particular, note that the assumption that the Cox, Ingersoll and Ross model is correct implies that the $A(0,t)$ and $G(0,t)$ functions that would be estimated for the Extended Vasicek model from historical data are given by eqs. (II.5) and (II.6) of Section 3 i.e.,

$$A(0,t) = \left[\frac{2\gamma \exp\left[(b+\gamma)\left(\frac{t}{2}\right)\right]}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}} \tag{II.31}$$

$$G(0,t) = \frac{2(\exp(\gamma t)-1)}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma} \tag{II.32}$$

$$\begin{aligned} b &= k+\lambda \\ c &= k\theta \\ \gamma &= \sqrt{b^2+2\sigma^2}. \end{aligned} \tag{II.33}$$

Note that by assumption, k, θ, λ and σ are the parameters of the Cox, Ingersoll and

Ross model. Also, we set $\sigma(t)$ equal to $\sigma\sqrt{r(0)}$ to ensure that the initial short term interest rate volatility in the Extended Vasicek model is equal to that in the Cox, Ingersoll and Ross model.

$A(t,TB), G(t,TB), \phi(t)$ and $a(t)$ can be calculated from equations (II.26)–(II.30) using equations (II.31) and (II.32) and $\sigma(t) = \sigma\sqrt{r(0)}$.

4.2 Pricing Lookback Puts

From Section 4 above we know that to calculate the price of the lookback put, we must calculate the conditional expectation of its payoff under the risk adjusted Extended Vasicek interest rate process. Hence

$$\text{PMAX}\left(P(0,TB), M(0), 0, T\right) = E^{**}\left[\frac{M(T) - P(T,TB)}{B(T)}\right]. \tag{II.34}$$

By definition,

$$E^{**}\left[\frac{P(T,TB)}{B(T)}\right] = \frac{P(0,TB)}{B(0)} = P(0,TB). \tag{II.35}$$

Hence,

$$\begin{aligned}\text{PMAX}\left(P(0,TB), M(0), 0, T\right) &= E^{**}\left[\frac{M(T)}{B(T)}\right] - P(0,TB) \\ &= E^{**}\left[M(T)\left(\exp\left(-\int_0^T r(s)ds\right)\right)\right] - P(0,TB)\end{aligned} \tag{II.36}$$

Alternatively, if we want to price the option at time $t > 0$,

$$\text{PMAX}\left(P(t,TB), M(t), t, T\right) = E^{**}\left[\frac{\max\{M(t), \max_{t\le\tau\le T} P(\tau,TB)\} - P(T,TB)}{B(T)}\right]$$

$$= E^{**}\left[\frac{\max\{M(t), \max_{t\le\tau\le T} P(\tau, TB)\}}{B(T)}\right] - P(t, TB) \quad \text{(II.37)}$$

As in the case of the square root process, there is no way to calculate analytically the conditional expectation that gives the lookback put price. For this reason, we again use Monte Carlo simulation. As in Section 3, we divide the time interval between 0 and T into m subintervals such that eq. (II.11) of Section 3 is satisfied.

We simulate the risk adjusted Extended Vasicek interest rate process using the stochastic analogue of Euler's integration method. In particular, we simulate using the discrete approximation

$$r_{t_i} = r_{t_{i-1}} + (\phi(t_{i-1}) - a(t_{i-1})r_{t_{i-1}})(t_i - t_{i-1}) + \sigma\sqrt{r(0)}\sqrt{t_i - t_{i-1}}\,\tilde{\epsilon} \quad \text{(II.38)}$$

where $\tilde{\epsilon}$ is normally distributed with mean 0 and variance 1.

The Monte Carlo simulation is carried out in exactly the same way as described in Section 3.1. The only thing that changes here is the use of a different risk adjusted interest rate process.

4.3 Pricing Lookback Calls

From Sections 2 and 4 we know that

$$\text{PMIN}\,(P(0,TB), Q(0), 0, T) = E^{**}\left[\frac{P(T,TB) - Q(T)}{B(T)}\right]. \quad \text{(II.39)}$$

By definition,

$$E^{**}\left[\frac{P(T,TB)}{B(T)}\right] = P(0,TB).$$

Hence,

$$\begin{aligned}\text{PMIN}\,(P(0,TB),Q(0),0,T) &= P(0,TB) - E^{**}\left[\frac{Q(T)}{B(T)}\right] \\ &= P(0,TB) - E^{**}\left[Q(T)\left(\exp\left(-\int_0^T r(s)ds\right)\right)\right] \quad \text{(II.40)}\end{aligned}$$

Alternatively, if we want to price the option at time $t > 0$,

$$\begin{aligned}\text{PMIN}\,(P(t,TB),Q(t),t,T) &= E^{**}\left[\frac{P(T,TB) - \min\{Q(t), \min_{t\le\tau\le T} P(\tau,TB)\}}{B(T)}\right] \\ &= P(t,TB) - E^{**}\left[\frac{\min\{Q(t), \min_{t\le\tau\le T} P(\tau,TB)\}}{B(T)}\right] \quad \text{(II.41)}\end{aligned}$$

As in the case of the lookback put option, there is no way to calculate this conditional expectation analytically. For this reason, we use Monte Carlo simulation, using the discrete approximation to the risk adjusted Extended Vasicek interest rate process described in Section 4.2. The procedure is the same as the one described in Section 3.1 except for the use of a different risk adjusted process.

4.4 Computer Programs that Implement Above Valuation Model

A computer program that takes as input the parameters of the Cox, Ingersoll and Ross model and gives as output the Extended Vasicek risk adjusted interest rate process was written in the C programming language. A second computer program takes as input the

output of the first program, i.e., the Extended Vasicek risk adjusted interest rate process and gives as output the results of the Monte Carlo simulations described in Sections 4.2 and 4.3.

As in the case of Section 3, the program was tested by calculating numerically various bond prices and comparing them to the bond prices given by the closed form formula. The differences were very small, usually of an order of magnitude of 10^{-6}.

5 A Comparison of Cox, Ingersoll and Ross and Extended Vasicek Lookback Option Prices

Table II.1 presents prices for lookback put options on Treasury bonds under both the Cox, Ingersoll and Ross and Extended Vasicek models. As explained in Section 4, the prices are generated by assuming that the Cox, Ingersoll and Ross model describes the evolution of the term structure and fitting the Extended Vasicek model to it. The parameters of the Cox, Ingersoll and Ross model are assumed to be those estimated by Chan, et. al (1992). These parameter estimates are:

$$\begin{aligned} k\theta &= 0.0189 \\ k+\lambda &= 0.2339 \\ \sigma &= 0.0085 \end{aligned}$$

Note that since we are fitting the Extended Vasicek model to the Cox, Ingersoll and Ross term structure, the bond prices given by the two models at time 0 are the same (for a given interest rate). This explains why we have only one column for the Bond Price in Table II.1. However, since the two models make different assumptions about the evolution of the interest rate, they will give different bond prices at future times and, hence, different lookback put prices.

Table 1 also gives prices for ordinary puts under the Cox, Ingersoll and Ross model. It is assumed that the exercise price of the put is 105% of the current bond price. Note that, as expected, the lookback put price is higher than the ordinary put price for any given initial

r	Bond Price	Lookback Put Price		Ordinary Put Price - CIR
		CIR	EV	
0.20	42.62	0.4420	0.7493	0.0000
0.19	43.98	0.4595	0.7651	0.0000
0.18	45.39	0.4778	0.7803	0.0000
0.17	46.84	0.4968	0.7946	0.0000
0.16	48.35	0.5166	0.8077	0.0000
0.15	49.91	0.5371	0.8194	0.0000
0.14	51.50	0.5582	0.8291	0.0001
0.13	53.15	0.5800	0.8364	0.0001
0.12	54.86	0.6027	0.8409	0.0002
0.11	56.61	0.6264	0.8419	0.0003
0.10	58.43	0.6511	0.8385	0.0005
0.09	60.30	0.6766	0.8293	0.0008
0.08	62.24	0.7028	0.8131	0.0014
0.07	64.23	0.7300	0.7877	0.0022
0.06	66.29	0.7583	0.7504	0.0037
0.05	68.42	0.7875	0.6974	0.0061
0.04	70.61	0.8173	0.6236	0.0098
0.03	72.88	0.8475	0.5217	0.0154
0.02	75.22	0.8774	0.3821	0.0222
0.01	77.62	0.9056	0.1971	0.0235

Table II.1: Lookback put prices under CIR and Ext. Vasicek models. Ordinary put price under CIR model. Current time = 0, $T = 3, TB = 6$, bond face value = 100, exercise price of ordinary put = 105% of current bond price.

interest rate.

Note that the Extended Vasicek model (EV model) gives higher lookback put prices than the Cox, Ingersoll and Ross model (CIR model) for interest rates above or very close to the long-run mean of the interest rate process, which was estimated by Chan et. al (1992) to be 0.08. However, for interest rates below 0.07, the EV model gives lower lookback put prices.

To explain these results, consider the benefit and cost of exercising this option at maturity. The benefit is the maximum bond price over the option's life, and the cost is the

bond price at maturity. The option will always be in the money at maturity.

For comparison purposes, let us take the CIR model as the base case and investigate how the benefit and cost of exercising the option at maturity change under the EV model. Since we are comparing the current (i.e., time 0) prices under the two models, we should examine the present value of the benefit and the present value of the cost of exercising the option at maturity. The present value of the cost of exercising is the same in both models: it is the current price of the bond with maturity $TB = 6$ which is the same in both models. As a result, the differences in the current option prices arise because of differences in the present value of the benefit.

To investigate how the present value of the benefit might differ in the two models, let us examine the nature of the stochastic process for the interest rate in the two models. We begin by considering the drift term. Both the CIR and EV interest rate processes are mean reverting. The long-run mean estimated by Chan et. al (1992) is 0.08. This means that whenever the interest rate is (below) above 0.08, the drift term will tend to (increase) decrease it. Furthermore, the volatility terms in the two models are different. The volatility of the interest rate in CIR at time t is $\sigma\sqrt{r(t-1)}$ whereas the volatility term in EV at all t is $\sigma\sqrt{r(0)}$. The reason for this is that we fitted the EV model in such a way, that its initial (i.e., time 0) interest rate volatility is the same as in the CIR model. A direct consequence of this, is that, for a given initial interest rate, the volatility term implies a higher probability of a lower interest rate in EV and a higher probability of a higher interest rate in CIR.

The key to understanding why the EV model gives higher (lower) lookback put prices when the interest rate is above (below) its long-run mean is to realize that a low interest rate in either the CIR or the EV model is much more useful in establishing a high maximum

bond price over the option's life if it occurs late during the option's life. The reason for this, is that for a given interest rate, bond prices in both models increase with the passage of time.

Consider the case where $r = 0.01$, i.e., the interest rate is substantially below its long-run mean of 0.08. Early in the option's life, the EV model will be more likely to give a lower interest rate than the CIR model. However, for the reason given in the previous paragraph, this will not be very useful in establishing a high maximum bond price over the option's life. Furthermore, note that the drift term will tend to increase the interest rate in both models over time; as a result, by the time the option has been outstanding for, say 80% of its life, the interest rate will be, on average, much higher in both models. The crucial thing to realize is that at that time, the volatility of the interest rate will be much higher in the CIR model—recall that time t volatility in CIR is given by $\sigma\sqrt{r(t-1)}$, whereas, volatility in EV at all t is $\sigma\sqrt{0.01}$. A direct consequence of this is that towards the end of the option's life the higher volatility in CIR will imply a higher probability of a lower interest rate in CIR (compared to EV). As explained in the previous section, a low interest rate will be much more useful in establishing a high maximum bond price over the option's life if it occurs late in the option's life. Since we established that the EV (CIR) model will imply a higher probability of a lower interest rate early (late) in the option's life, the probability of a higher maximum bond price is higher in CIR and, therefore, the option's benefit is higher in CIR. This explains why the CIR model gives higher lookback put prices for an interest rate of 0.01. Similar considerations apply for interest rates in the 0.02 to 0.06 range, i.e., for interest rates below the long-run mean. The CIR model gives higher lookback put prices in these cases too.

Let us now consider the case where $r = 0.20$. Early in the option's life, the EV model will imply a higher probability of a lower interest rate. As explained above, this will not be very useful in establishing a high maximum bond price over the option's life. The important thing to realize is that, unlike the case where $r = 0.01$, the EV model will also imply a higher probability of a lower interest rate late in the option's life. The reason for this is that the drift term will tend to reduce the interest rate. On average, the interest rate will reach a much lower level by the time the option has been outstanding for, say 80% of its life. At that time, the volatility of the interest rate will be much higher in the EV model. (Recall that volatility at time t is $\sigma\sqrt{r(t-1)}$ in CIR and $\sigma\sqrt{0.2}$ in EV). Hence, towards the end of the option's life, the higher volatility in EV will imply a higher probability of a lower interest rate (compared to CIR). The EV model will therefore imply a higher probability of a lower interest rate both early and late in the option's life; as a result, the probability of a higher maximum bond price over the option's life and therefore the option's benefit is higher under EV. This explains why the EV model gives a higher price for a lookback put option when $r = 0.20$. Similar considerations apply for interest rates in the 0.19 to 0.08 range, i.e., for interest rates above the long-run mean. The EV model gives higher lookback put prices in these cases too.

Table II.2 presents the same comparison for lookback calls. Also, it gives prices for ordinary calls under the CIR model. As expected, the lookback call prices are higher than the ordinary call prices for any given initial interest rate.

Table II.2 shows that the CIR model gives consistently higher prices than the EV model for lookback calls. To explain this result, consider the benefit and cost of exercising this

r	Bond Price	Lookback Call Price		Ordinary Call Price - CIR
		CIR	EV	
0.20	42.62	17.26	17.22	16.65
0.19	43.98	17.25	17.21	16.61
0.18	45.39	17.21	17.16	16.53
0.17	46.84	17.14	17.09	16.42
0.16	48.35	17.03	16.97	16.27
0.15	49.91	16.88	16.81	16.07
0.14	51.50	16.68	16.62	15.84
0.13	53.15	16.45	16.37	15.55
0.12	54.86	16.16	16.07	15.21
0.11	56.61	15.81	15.72	14.81
0.10	58.43	15.41	15.30	14.35
0.09	60.30	14.95	14.82	13.83
0.08	62.24	14.42	14.27	13.24
0.07	64.23	13.81	13.64	12.57
0.06	66.29	13.13	12.92	11.82
0.05	68.42	12.36	12.10	10.98
0.04	70.61	11.49	11.17	10.05
0.03	72.88	10.53	10.11	9.02
0.02	75.22	9.44	8.88	7.88
0.01	77.62	8.21	7.42	6.64

Table II.2: Lookback call prices under CIR and Ext. Vasicek models. Ordinary call price under CIR model. Current time $= 0, T = 3, TB = 6$, bond face value $= 100$, exercise price of ordinary call $=$ current bond price

option at maturity. The benefit is the bond price at maturity and the cost is the minimum bond price over the option's life. The option will always be in the money at maturity. Since we are comparing the current (i.e., time 0) prices under the two models, we should examine the present value of the benefit and the present value of the cost. The present value of the benefit is the same in both models: It is the current bond price for the bond with maturity $TB = 6$. As a result, the differences in the current option prices arise because of differences in the present value of the cost.

The key to understanding why the CIR model gives higher lookback call prices for all initial interest rates, is to realize that a high interest rate in either model is much more useful in establishing a low minimum bond price over the option's life—and hence a lower cost of exercising the option—if it occurs early in the option's life. The reason for this is that for a given interest rate, bond prices in both models increase with the passage of time. Furthermore, note that irrespective of whether the interest rate is above or below its long-run mean, the CIR model implies a higher probability of a higher interest rate early in the option's life. Hence, the CIR model implies a higher probability of a lower minimum bond price over the option's life and, therefore, a higher probability of a lower cost of exercising the option. As a result, the CIR model gives higher prices for lookback call options.

6 Pricing Properties of Lookback Options at Time 0

In this section, we examine how changes in the underlying bond price effect option prices at time 0, i.e., at the time at which the option is written.

6.1 Pricing Properties of Lookback Puts at Time 0

Table II.1 of Section 5 shows that as the interest rate falls and the price of the underlying (maturity TB) bond increases, lookback put prices increase under the CIR model; under the EV model, they sometimes increase and sometimes decrease. The reason for the difference is related to the different assumptions about the volatility of the interest rate made by the two models and will be discussed later in this section.

We begin by discussing the results for CIR. The rise in the lookback put price as the bond price increases is analogous to the following comparative static result established by Goldman, Sosin and Gatto (1979): as the stock price increases, the price of the lookback put on the stock increases. Note that Goldman et. al. (1979) established their results by examining how the price of the lookback put on the stock changed as they increased the stock price, while keeping all other factors affecting option prices constant. In particular the discount factor was kept constant in their experiment. Unfortunately, the same is not true in the case of the experiment presented in Table II.1. In this case, as we increase the (TB maturity) bond price by reducing the interest rate, we also increase the (T maturity) bond price and, hence, reduce the underlying discount factor for the option's payoff.

To investigate the effect of this reduction in the discount factor on option prices, consider the benefit and cost of exercising this option at maturity. The benefit is the maximum bond

price over the option's life, and the cost is the bond price at maturity. Since we are pricing the options at time 0, we should investigate the effect of the reduction in the discount factor on the present value of the benefit and cost. By definition, the present value of the underlying (TB maturity) bond price is the current bond price, irrespective of what the discount factor is. Hence, the present value of the cost is not influenced by the discount factor. However, the present value of the benefit—the maximum bond price over the option's life—is clearly higher under a lower discount factor. As a result, the rise in the lookback put price as the interest rate falls might be due to this discount factor effect on the benefit, and not to the effect of the change in the underlying (TB maturity) bond price on the option price.

We now consider the results in Table II.1 of Section 5 for the EV model. In the case of the EV model, the factors mentioned above for the CIR model are also in operation. However, the lookback put price in EV does not always increase as the interest rate falls and the underlying bond price increases because as the interest rate falls, the volatility of the interest rate is also reduced throughout the option's life—recall that interest rate volatility at all t is equal to $\sigma\sqrt{r(0)}$ in EV.

In light of the above, it would be useful to devise a test that will establish whether the lookback put price increases with the underlying (maturity TB) bond price when both the discount factor and the volatility of the interest rate are kept constant. Such a test is presented in Section 6.1. The results of this test will show whether the properties established by Goldman et. al (1979) for lookback puts on stocks carry over to lookback puts on bonds.

A Test to Establish the Pricing Properties of Lookback Puts at time 0 While Keeping the Discount Factor Constant

The first step in carrying out the new test for the same option maturity as in Section 6.1 ($T = 3$) is to specify two term structures at time 0: both term structures give the same bond prices for bond maturities between 0 and 3 years; however, the two term structures give different prices for bond maturities beyond 3 years.

The first term structure is chosen to be the CIR term structure at time 0, i.e., the bond price for maturity t, $P(0,t)$ is given by

$$P(0,t) = A(0,t)\exp(-r(0)G(0,t))$$

where

$$\begin{aligned} A(0,t) &= \left[\frac{2\gamma \exp\left[(b+\gamma)\left(\frac{t}{2}\right)\right]}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma}\right]^{\frac{2c}{\sigma^2}} \\ G(0,t) &= \frac{2(\exp(\gamma t)-1)}{(\gamma+b)(\exp(\gamma t)-1)+2\gamma} \end{aligned} \tag{II.42}$$

$$\begin{aligned} b &= k+\lambda \\ c &= k\theta \\ \gamma &= \sqrt{b^2+2\sigma^2} \end{aligned}$$

The second term structure is chosen as follows: for maturities between 0 and 3 years it is the CIR term structure i.e., the $P(0,t)$ function for $0 \leq t \leq 3$ will be the one specified

above for the CIR term structure at time 0. For maturities beyond 3 years, i.e., for $t > 3$ the $P(0,t)$ function in the second term structure is given by

$$P(0,t) = \text{ ANEW } (0,t)\exp(-r(0)G(0,t)) \quad \text{for } t > 3$$

where

$$\text{ANEW } (0,t) = \left[\frac{2\gamma \exp\left[(b+\gamma)\left(\frac{t^2}{12}+0.75\right)\right]}{(\gamma+b)\left(\exp\left(\gamma\left(\frac{t^2}{6}+1.5\right)\right)-1\right)+2\gamma}\right]^{\frac{2c}{\sigma^2}} \tag{II.43}$$

$G(0,t), b, c, \gamma$ are as specified above.

The Appendix shows that the second term structure is a legitimate term structure in that it implies continuous bond prices and forward rates.

From now on, the first term structure will be referred to as term structure #1; the second will be referred to as term structure #2.

Term structure #2 will, by construction, give lower bond prices for a given initial interest rate for maturities beyond 3 years (compared to term structure #1).

The new test we propose to carry out consists of fitting the Extended Vasicek (EV) model to both term structures and pricing lookback puts under the EV model in both term structures. We assume, as in the previous section, that the lookback puts are written on a bond with maturity $TB = 6$ and have a maturity $T = 3$. We compare for a given interest rate, the lookback put prices given by fitting the EV model to term structure #1 with those given by fitting the EV model to term structure #2. By construction, for a given interest rate, the maturity TB bond price will be lower in term structure #2; However, the maturity

r	Term Structure #1		Term Structure #2	
	Bond Price	Lookback Put Price	Bond Price	Lookback put Price
0.20	42.62	0.7493	38.80	0.6823
0.19	43.98	0.7651	40.05	0.6966
0.18	45.39	0.7803	41.33	0.7105
0.17	46.84	0.7946	42.65	0.7235
0.16	48.35	0.8077	44.03	0.7355
0.15	49.91	0.8194	45.43	0.7461
0.14	51.50	0.8291	46.89	0.7549
0.13	53.15	0.8364	48.39	0.7616
0.12	54.86	0.8409	49.95	0.7657
0.11	56.61	0.8419	51.55	0.7666
0.10	58.43	0.8385	53.20	0.7635
0.09	60.30	0.8293	54.91	0.7551
0.08	62.24	0.8131	56.67	0.7403
0.07	64.23	0.7877	58.48	0.7172
0.06	66.29	0.7504	60.36	0.6832
0.05	68.42	0.6974	62.29	0.6350
0.04	70.61	0.6236	64.29	0.5679
0.03	72.88	0.5217	66.35	0.4750
0.02	75.22	0.3821	68.48	0.3479
0.01	77.62	0.1971	70.68	0.1794

Table II.3: Lookback put prices for different initial interest rates given by fitting EV model to Term Structure #1 and Term Structure #2. Current time = 0, $T = 3, TB = 6$, bond face value = 100.

T bond price—and hence, the discount factor— will be the same in the two term structures. Consequently, we will establish how the price of the lookback put changes as we change the underlying (maturity TB) bond price, while keeping the discount factor constant.

Table II.3 presents the results of this test. It shows that for a *given interest rate*, the term structure that gives a higher bond price (term structure #1) also gives a higher lookback put price. This result holds even though the discount factors are the same (for a given interest rate) in the two term structures.

The conclusion to be drawn from the above analysis is that a higher bond price leads

to a higher lookback put price when the discount factor is kept constant. Note that in Section 6.1, we established the same result even when the discount factor was not kept constant; however, we were not sure whether the underlying cause was a discount factor effect. The results in this section show that the underlying cause was not a discount factor effect. Hence, the result in Goldman et. al (1979)—the price of a lookback put on a stock increases as the stock price increases when everything else, including the discount factor, is kept constant—carries over to lookback puts on bonds.

6.2 Pricing Properties of Lookback Calls at Time 0

Table II.2 of Section 5 shows that as the interest rate falls and the bond price increases, lookback call prices fall. Goldman et. al. (1979) established the opposite result for lookback calls on a stock: as the stock price increased, the lookback call price increased too. Note that Goldman et. al (1979) established their result by examining how the price of the option changed as they changed the stock price while keeping all other factors affecting option prices constant. In particular, they kept the discount factor constant. Unfortunately, the same is not true in the case of the experiment presented in Table II.2: as the interest rate falls, the discount factor falls too.

To investigate the effect of this reduction in the discount factor on option prices, consider the benefit and cost of exercising this option at maturity. The benefit is the bond price at maturity and the cost is the minimum bond price over the option's life. Since we are pricing the options at time 0, we should investigate the effect of the reduction in the discount factor as the underlying bond price increases, on the present value of the benefit and cost. By definition, the present value of the underlying (maturity TB) bond price is the current bond

price irrespective of what the discount factor is. Hence, the present value of the benefit is not influenced by the discount factor. However, the present value of the cost (the minimum bond price) is clearly higher the lower the discount factor. Hence, the reduction in the lookback call price as the interest rate falls might be due to this discount factor effect on the present value of the cost of exercising the option and not to the effect of the rise in the underlying (TB maturity) bond price on the option price.

In light of the above, we carry out as in Section 6.1 for lookback puts, a test to establish the pricing properties of lookback calls at time 0 when the discount factor is kept constant.

A Test to Establish the Pricing Properties of Lookback Calls at time 0 while Keeping the Discount Factor Constant

The test is identical to the one described in Section 6.1 except that we use it for lookback calls, i.e., instead of pricing lookback puts under the EV model in both term structures, we price lookback calls under the EV model in both term structures.

Table II.4 presents the results of this test. It shows that for a *given interest rate*, the term structure that gives a higher bond price (term structure #1) also gives a higher lookback call price. The discount factors are the same (for a given interest rate) in the two term structures.

It follows that a higher underlying (maturity TB) bond price leads to a higher lookback call price when the discount factor is kept constant. Note that in Section 6.2 we got the *opposite* result when the discount factor was not kept constant—as the underlying bond price increased, the lookback call price fell; however, we were not sure whether this was

r	Term Structure #1		Term Structure #2	
	Bond Price	Lookback Call Price	Bond Price	Lookback Call Price
0.20	42.62	17.22	38.80	15.68
0.19	43.98	17.21	40.05	15.67
0.18	45.39	17.16	41.33	15.63
0.17	46.84	17.09	42.65	15.56
0.16	48.35	16.97	44.03	15.45
0.15	49.91	16.81	45.43	15.31
0.14	51.50	16.62	46.89	15.13
0.13	53.15	16.37	48.39	14.91
0.12	54.86	16.07	49.95	14.64
0.11	56.61	15.72	51.55	14.31
0.10	58.43	15.30	53.20	13.93
0.09	60.30	14.82	54.91	13.49
0.08	62.24	14.27	56.67	12.99
0.07	64.23	13.64	58.48	12.42
0.06	66.29	12.92	60.36	11.76
0.05	68.42	12.10	62.29	11.02
0.04	70.61	11.17	64.29	10.17
0.03	72.88	10.11	66.35	9.21
0.02	75.22	8.88	68.48	8.09
0.01	77.62	7.42	70.68	6.76

Table II.4: Lookback call prices for different initial interest rates given by fitting EV model to Term Structure #1 and Term Structure #2. Current time = 0, $T = 3, TB = 6$, bond face value = 100.

caused by a discount factor effect. The results in this section show that it was indeed caused by a discount factor effect. Hence, the result in Goldman et. al (1979)—the price of a lookback call on a stock increases as the stock price increases when everything else including the discount factor is kept constant—carries over to lookback calls on bonds.

7 Pricing Properties of Lookback Options at Time > 0

In this section we investigate the relation between lookback option prices and underlying (maturity TB) bond prices at a time greater than time 0, i.e., after the options have been outstanding for some time (recall they were written at time 0). In particular, we establish the pricing properties of these options at time 1. Note that by time 1 a maximum or minimum bond price different from the current (time 1) bond price will have been established. The properties in this section are established using the CIR model. However, identical results hold for the EV model. We begin with lookback puts.

7.1 Pricing Properties of Lookback Puts at Time > 0

Table II.5 presents lookback put prices at time 1 for different option maturities under the CIR model. The parameters of this model are assumed to be those estimated by Chan et. al. (1992). The underlying bond is assumed to have a maturity $TB = 10$ years. The lookback put was written at time 0. The maximum bond price until time 1 is assumed to be 53.10.

r	Bond Price	Lookback Put Price		
		$T = 2$	$T = 4$	$T = 6$
0.11	44.28	3.991	1.314	0.745
0.10	45.90	3.166	1.299	0.771
0.09	47.60	2.475	1.291	0.799
0.08	49.36	1.964	1.312	0.828
0.07	51.19	1.662	1.335	0.858
0.06	53.07	1.559	1.369	0.890

Table II.5: Lookback put prices for different bond prices, and different option maturities. Current time = $1, TB = 10$, Bond Face Value = 100, Max Bond Price until time 1 = 53.10.

We begin by discussing the results for $T = 4$ in Table II.5. When the current bond price is substantially below the maximum bond price to date, for example, when $r = 0.11$ and the bond price $= 44.28 < 53.10$, increases in the bond price result in decreases in the option price. The reason is that since the bond price is much lower than the maximum bond price to date, not enough time remains to establish a new maximum through increases in the bond price, and to then establish a larger option value at option maturity through decreases in the bond price. Hence, increases in the bond price result in a smaller expected option value at maturity and, therefore, in a lower option price now. However, note that when the bond price is close to the maximum bond price to date—for example, when $r = 0.08$ and bond price $= 49.36 < 53.10$— increases in the bond price result in increases in the option price. The reason is that when the bond price is sufficiently close to the maximum bond price to date, sufficient time remains to establish a new maximum through increases in the bond price and to then establish a larger option value at option maturity through decreases in the bond price. Hence, increases in the bond price increase the option price.

Let us now consider the results for $T = 2$ in Table II.5, i.e., for a lower option maturity. Note that in this case, irrespective of how close to the maximum bond price to date the current bond price is, increases in the bond price result in decreases in the option price. The reason is that since the option is expiring very soon, not enough time remains to establish a new maximum through increases in the bond price and to then establish a larger option value at maturity through decreases in the bond price. As a result, an increase in the bond price decreases the option price.

Finally let us consider the results for $T = 6$. In this case, irrespective of how close to the maximum bond price to date the current bond price is, increases in the bond price

always result in increases in the option price. The reason for this is that even when the current bond price is substantially lower than the maximum bond price to date, given that the option will be outstanding for a long period of time, enough time remains to establish a new maximum bond price through increases in the bond price and to then establish a larger option value at maturity through decreases in the bond price. As a result, higher bond prices lead to higher option prices.

Identical results were established by Goldman et. al. (1979) for lookback puts on stocks. In this case too, the relation between the option price and the bond price depends not only on the maximum bond price to date, but also on the option's maturity.

7.2 Effect of Lookback Put Maturity on Price for a Given Bond Price

Another comparative static result that can be established from Table II.5 is that for given interest rate and bond price, the lookback put price is lower, the higher the option maturity. The reason behind this result can be understood by considering an option with maturity equal to the bond's maturity, i.e., $T = TB$. The option is worthless at maturity because the bond price at maturity is 1 and the maximum bond price during the option's life cannot exceed 1. As a result, the option is worthless at any time prior to maturity. It follows that for a lookback put to have any value, it must have a maturity lower than that of the underlying bond. In light of this, it is intuitively plausible that the lower the maturity of the lookback put, the higher the price; the underlying reason is that on average, bond prices increase with the passage of time.

7.3 Pricing Properties of Lookback Calls at Time > 0

Table II.6 presents lookback call prices at time 1 for different option maturities under the CIR model. The lookback call was written at time 0. The minimum bond price until time 1 is assumed to be 44.27.

r	Bond Price	Lookback Call Price		
		$T = 2$	$T = 4$	$T = 6$
0.11	44.28	5.94	12.76	17.87
0.10	45.90	6.36	13.04	18.21
0.09	47.60	7.32	13.72	18.84
0.08	49.36	8.57	14.61	19.65
0.07	51.19	9.98	15.62	20.56
0.06	53.07	11.48	16.72	21.53

Table II.6: Lookback call prices for different bond prices, and different option maturities. Current time = $1, TB = 10$, Bond Face Value = 100, Minimum Bond Price until time 1= 44.27.

The results in Table II.6 show that irrespective of how close to the minimum bond price to date the current bond price is, the option price increases with increases in the bond price. The identical result was established by Goldman, Sosin and Gatto (1979) for stocks. Option maturity does not affect the relation either for bonds or for stocks.

7.4 Effect of Lookback Call Maturity on Price for a Given Bond Price

Another comparative static result that can be established from Table II.6 is that the higher the maturity of the option, the higher its price. The reason for this is that, on average, bond prices increase with the passage of time. One direct consequence of this is that, on average, the bond's minimum price over the option's life is more likely to be established early in

the option's life; another direct consequence of this is that an option of longer maturity is more likely to have a higher bond price at maturity compared to a shorter maturity option. However, the longer maturity option is more likely to have a minimum bond price that is almost equal to that of a shorter maturity option (both minimums, established early during the option's life). It follows that, on average, the longer maturity option will have a higher payoff at maturity, and hence, a higher price prior to maturity.

8 Conclusion

This paper compared the prices of lookback options on Treasury bonds under two different valuation models, one based on the Cox, Ingersoll and Ross square root interest rate process and the other based on the Extended Vasicek interest rate process. The comparison was carried out in a meaningful way by ensuring that the two models are specified in such a way that they give the same term structure at time 0, the time at which the options are written. It was established that the Extended Vasicek model consistently underprices lookback calls compared to the Cox, Ingersoll and Ross model. Also the Extended Vasicek model overprices lookback puts for high interest rates (compared to the long-run mean of the interest rate process) and underprices lookback puts for low interest rates. The differences between the lookback option prices given by the two models were linked to the different assumptions that the two models make about the evolution of interest rates.

This paper also established the pricing properties of lookback options on bonds, both at the time at which they are written and after they have been outstanding for some time. Its contribution has been to show that the pricing properties established by Goldman, Sosin and Gatto (1979) for lookback options on stocks carry over to lookback options on Treasury bonds. The demonstration of this necessitated the performance of a particular type of test that enabled us to investigate the effect of changes in the underlying bond price on the option price while keeping the discount factor constant.

Appendix A. Proof of Continuity of Bond Prices and Forward Rates in Term Structure #2 of Section 6.1

The time 0 bond price for maturity $t = 3$ (the maturity at which the functional form of the $A(0,t)$ function changes in term structure # 2) is continuous since $A(0,3) =$ ANEW $(0,3)$.

To prove continuity of forward rates define

$$F(0,t,t+\Delta t) = \text{Forward rate as seen at time 0 for period between } t \text{ and } (t+\Delta t).$$

By definition

$$F(0,t,t+\Delta t) = \frac{\log P(0,t) - \log P(0,t+\Delta t)}{\Delta t}.$$

By using the functional form of $P(0,t)$, we calculate the following limits:

$$\begin{aligned}
\lim_{\Delta t \to 0} F(0,3-\Delta t,3) &= \frac{2\gamma e^{3\gamma} - \frac{2(e^{3\gamma}-1)(\gamma+b)\gamma e^{3\gamma}}{(\gamma+b)(e^{3\gamma}-1)+2\gamma}}{(\gamma+b)(e^{3\gamma}-1)+2\gamma} \\
&\quad - \frac{c(\gamma+b)}{\sigma^2} \\
&\quad + \frac{2c}{\sigma^2}\left\{\frac{\gamma(\gamma+b)e^{3\gamma}}{(\gamma+b)(e^{3\gamma}-1)+2\gamma}\right\} \\
&= \lim_{\Delta t \to 0} F(0,3,3+\Delta t).
\end{aligned}$$

Hence, forward rates are continuous.

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Essay III

Asset Securitization by Commercial Banks: An Asymmetric Information Explanation

1 Introduction

Commercial banks began securitizing assets in 1985. The structure of a $100 million securitization transaction is illustrated below.

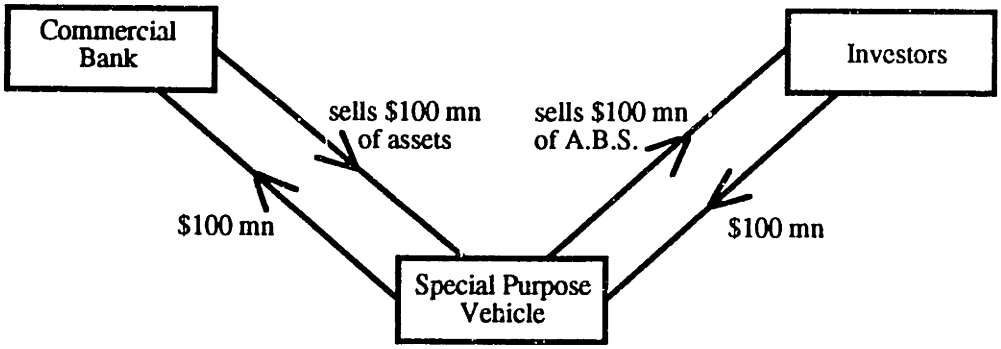


Figure III-1: Structure of a typical asset securitization transaction

The commercial bank with the help of an investment bank sets up a separate, special purpose corporate entity to which it sells $100 million of assets it wants to securitize. The special purpose vehicle finances the purchase through the sale of $100 million of Asset Backed Securities (A.B.S.) to investors. The latter rely on the assets in the special purpose

vehicle for payment of interest and principal on the securities which they have purchased. It is for this reason that these securities are called asset backed.

The volume of asset backed securities issued by commercial banks since 1985 has been substantial. According to *Asset Sales Report* (an American Banker Newsletter), total A.B.S. issuance by commercial banks between 1985 and 1991 exceeded $40 billion.

This essay is organized as follows: Section 2 describes the main features of asset securitization transactions from the point of view of a commercial bank; Section 3 summarizes prior research and relates it to the research in this paper; Section 4 presents the assumptions underlying a model that is developed to analyze securitization transactions; Sections 5 to 14 analyze different versions of this model; Section 15 offers concluding remarks.

2 The Main Features of Asset Securitization Transactions

Asset securitization transactions are arranged in such a way that a bankruptcy of the commercial bank will have no effect whatsoever on the performance of the asset backed securities. This is achieved through the establishment of the special purpose vehicle mentioned above as a separate corporate entity. Investors purchase securities from this separate corporate entity and not from the commercial bank. Hence, the credit risk assumed by investors depends entirely on the assets of the special purpose vehicle.

The banks which have been most active in this field are large money center banks like Citicorp, Bank of America, Chase, Chemical, Security Pacific, and Marine Midland. The assets which have been securitized are those whose cash flows are fairly predictable in an actuarial sense. Such assets include mortgage, automobile and credit card loans. As will become evident later, for the purposes of this paper, the important feature of such assets is that there is a low degree of information asymmetry about their value, i.e. the management of the commercial bank which originated such assets does not have that much more information about them than outside investors. This statement will be made precise through a formal model later.

An asset securitization transaction has important regulatory implications for commercial banks. To analyze them we must draw a distinction between Generally Accepted Accounting Principles (GAAP) and Regulatory Accounting Principles (RAP). Commercial banks prepare their annual reports to shareholders on the basis of GAAP. However, they must also satisfy federal regulatory requirements, outlined in RAP. The latter require commercial banks to hold a certain percentage of equity capital against certain bank assets. This

required ratio is specified in book value terms. In calculating it, different assets receive different weights depending on their riskiness. Risk-free assets such as cash receive no weight. On average, this ratio has been approximately 6% in the last decade. Almost all asset securitization transactions have been structured in such a way that the securitized assets are removed from a commercial bank balance sheet under both GAAP and RAP. This results in an improvement in the equity to asset ratio monitored by federal regulators as illustrated in Table III.1.

(a) Initial Balance Sheet

A	L
Asset 1: 40	Debt: 97
Asset 2: 60	Equity: 3
100	100

$\frac{E}{A^*} = \frac{3}{100} = 3\%$

(b) Balance Sheet after Securitization of Asset 1

A	L
Cash:40	Debt: 97
Asset 2: 60	Equity: 3
100	100

$\frac{E}{A^*} = \frac{3}{60} = 5\%$

(c) Balance sheet after securitization of asset 1 and use of proceeds to repay debt.

A	L
Asset 2: 60	Debt: 57
	Equity: 3
	60

$\frac{E}{A^*} = \frac{3}{60} = 5\%$

Table III.1: Commercial bank balance sheet in book value terms prior to and after securitization of Asset 1. Note that the assets (A^*) used in calculating the regulatory equity to asset (E/A^*) ratio exclude liquid assets.

The result of the asset securitization transaction illustrated in Table III.1 is an improvement in the equity to asset ratio from 3% to 5%.

This sort of improvement was very valuable to commercial banks in the mid 1980s

(the period in which they began securitizing assets). The reason is that commercial banks experienced major problems with profitability and asset quality in the mid-1980s. Such problems which persist even today include:

- Less developed country (LDC) debt. Many commercial banks were forced to build huge reserves amounting to 40% to 100% of their exposure to LDC debt.
- Other problematic loans, e.g. real estate. Again many banks were forced to make huge loan loss provisions.

The effect of the above mentioned problems has been a rise in loan charge-offs and an erosion of bank profitability. As a result, equity to asset ratios in the mid 1980's had deteriorated in both book and market value terms. For example, loan charge-offs which cannot be covered by earnings erode the book value of equity and hence the book value equity to asset ratio because banks are highly levered institutions. Likewise, loan charge-offs and declining profitability reduce the market value of the capital adequacy ratio.

In some cases equity to asset ratios, also referred to as capital adequacy ratios, fell below the minimum levels required by RAP. In such cases the federal banking regulators usually required commercial banks to boost their ratios to acceptable levels over a relatively short adjustment period of approximately one year. In the case of the stronger banks, i.e. those which had traditionally maintained capital adequacy ratios substantially higher than the regulatory minimum, the erosion in these ratios did not result in a violation of RAP. Nevertheless, even in these cases, it is reasonable to assume that stronger banks wished to boost their capital adequacy ratios to their traditional higher levels in order to preserve their credit ratings and images as well capitalized institutions.

In addition, it should be pointed out that beginning in 1985, federal bank regulators in the major industrialized nations began considering the whole issue of how much capital a commercial bank should be required to hold against assets. The idea was to make the requirements consistent across countries. It was widely expected that this would result in an increase in the required ratio for American banks, and indeed this is what ended up happening with the publication of the new rules by the Bank for International Settlements in 1988. Obviously banks were aware of this tendency, and took it into account in managing their balance sheets. Accordingly, it is safe to assume that commercial banks wanted to find ways of boosting their profitability and increasing their equity to asset ratios in the mid-1980s.

3 Prior Research

Goldberg and Rogers [6] outline the various ways of structuring the cash flows of the Asset Backed Securities to satisfy different investor clienteles. For example, a mortgage backed security may be structured to eliminate prepayment risk by guaranteeing a fixed prepayment rate. When prepayments exceed this fixed rate, they are invested in a guaranteed investment contract; when prepayments fall short of the guaranteed rate, the special purpose corporation draws on a borrowing facility provided by a commercial bank.

Rosenberg and Kravitt [8] discuss the legal issues involved in securitization. They argue that the principal legal obstacle has been the difficulty in removing assets from originating banks' balance sheets for regulatory accounting purposes.

Bryan [1]—Bryan is a partner of McKinsey & Co.—argues that securitization enables banks to improve their profitability by offering the possibility of greater specialization in risk-taking. He points out that under the traditional lending system, the same bank would originate the loan, structure the terms, absorb the credit risk, fund the asset, and service the collection of principal and interest payments. Under securitized credit, several different institutions might be involved, each playing a different role. For example, one institution could originate the loan, a second could structure it into a security, a third could insure the credit risk, a fourth could place the security with an investor, and a fifth could service the underlying loan. Bryan identifies four major benefits from unbundling these different functions:

1. Improvements in the credit underwriting process, because not only originators, but also third-party credit enhancers play a role in credit evaluation;

2. Diversification of credit risk (banks and thrifts with geographic concentrations can lay off "catastrophic risk" to others);

3. Increased liquidity;

4. Reduction in interest rate risk; banks can eliminate the mismatch from funding longer-term fixed rate assets with short-term liabilities by no longer warehousing such assets.

Bryan [2] argues that securitization should be used to unbundle or break apart the bank into its component functions. Specifically, this means separat'..g deposit taking from credit risk taking and from loan servicing. He argues that the current system that bundles functions together prevents banks from assessing their cost effectiveness in each function and inevitably leads to some customers cross-subsidizing others or being cross-subsidized by others. The unbundled bank can assess its cost effectiveness in each role and eliminate through sale or shutdown any function that other players are performing more cost effectively. In this way overall industry overcapacity would be reduced.

Ocampo and Rosenthal [11, 12]—both work at McKinsey & Co.—elaborate on Bryan's ideas and argue that securitization is fundamentally more efficient than conventional lending. To quote

> "Risk in lending comes from two sources. First, the borrower may not repay on time, or may not repay at all (credit rist). Second, interest on the debt assumed to fund the loans may not match the terms and pricing of the loans, and this mismatching exposes the lender to interest rate or prepayment risk. Credit securitization transactions manage these risks more explicitly and therefore more efficiently, than does conventional lending. It makes these risks more transparent

and it also allocates them far more precisely to the players who are best able to absorb them"[11], page 32.

Ocampo and Rosenthal [11, 12] also suggest a methodology for quantifying the economic benefits of securitization. However, they point out that their methodology is more relevant to transactions undertaken by unregulated financial institutions. To quote,

> "Nor will we focus in this article on issues by commercial banks, which may have special regulatory motives for securitizing credit. Instead, we will examine in some detail, an innovative transaction executed by an unregulated financial institution—General Motors Acceptance Corporation (GMAC). By looking at a transaction that is uncomplicated by implicit governmental subsidies or regulations, we are able to see more clearly the economic power and flexibility of the credit securitization technology." [11], page 36

Ocampo and Rosenthal [13] argue that securitized credit will become the dominant technology for lending and will solve the problem of excess capacity that now plagues the financial services industry.

Goldberg [5], which presents the views of Moody's Investor Service Inc., offers a more balanced view of securitization. He argues that in some cases, the securitization of high quality assets could lead to the deterioration of the selling bank's credit; on the other hand, the effect of securitization on the management of interest rate and other risks may actually improve the credit position of some banks.

Thomas (1992) performs an empirical study of pricing efficiency in the resale market for credit card portfolios. Note that this market differs from the market for securitized assets

in one important sense: Whereas securitized assets are repackaged and sold to secondary market investors, the credit card portfolios examined in this study are sold intact to one firm. He presents evidence concerning the pricing efficiency of credit card backed securities. Using an event study methodology, he finds a significant negative stock price reaction for purchases around the announcement date, which he interprets as evidence of overpricing. Likewise, he finds a significant negative stock price reaction for sellers around the announcement date. He points out that this is not necessarily evidence that the sale was a negative NPV transaction for sellers—even if the securities are overpriced or correctly priced, investors may view the sale as a signal that the selling firm is in financial distress. In such a case, the positive stock price effect resulting from overpricing may be offset by a negative information effect. In fact, it turns out that the sample of sellers contains a disproportionate number of firms in financial distress. When separate tests are performed on sellers based on financial condition, the significant negative stock price effect is observed only for distressed firms; for non-distressed firms the stock price effect is positive suggesting overpricing, or some good news about the company, i.e., a positive information effect.

3.1 Prior Research as it Relates to This Paper

The above described research is undoubtedly useful in explaining some of the factors driving asset securitization. Nevertheless, this research is based on the presumption that securitization is essentially a financing cost phenomenon. For example, Ocampo and Rosenthal [11, 12] argue that securitization is more efficient than traditional lending because it makes more transparent the interest rate and prepayment risks inherent in lending and allocates them far more precisely to players best able to absorb them. This argument, depends cru-

cially on the way the special purpose vehicle (See Figure 1) finances the purchase of the assets. For example, if the special purpose vehicle financed the purchase through the sale of plain vanilla debt, and not through the sale of different types of debt securities, each entailing different degrees of interest rate and prepayment risk, this argument would no longer have much merit to it.

This essay adopts a different approach: It looks at securitization as an asset sale phenomenon on the presumption that the financing cost implications of securitization are of second degree importance especially for commercial banks. In particular, recall from the earlier numerical example that a securitization transaction results in a rise in the equity to asset ratio because the sale of assets reduces the denominator of this ratio. Alternatively, an equity issue in the context of the same numerical example would result in the balance sheet illustrated in Table III.2 on the next page. The effect on the equity to asset ratio is the same as in the case of asset securitization—a rise from 3% to 5%.

It is clear from the above that both asset securitization (viewed as an asset sale phenomenon) and an equity issue achieve the same thing. One might therefore argue that if an equity issue is costly because of an asymmetric information problem similar to the one in Myers and Majluf (1984), asset securitization may be beneficial because it eliminates the need for a costly equity issue. To analyse this, I develop a formal model in the next section. The model attempts to answer the following question: Under what circumstances should a commercial bank prefer to improve its capital adequacy ratio by using asset securitization rather than an equity issue?

Note that the argument presented in this paper does not depend on the type of securities issued by the special purpose vehicle to finance the purchase of assets. Even if the special

purpose vehicle simply issued plain vanilla debt or equity, the approach in this essay would still be valid.

(a) Initial Balance Sheet

A	L
Asset 1: 40	Debt: 97
Asset 2: 60	Equity: 3
100	100

$\frac{E}{A^*} = \frac{3}{100} = 3\%$

(b) Balance Sheet after \$2.00 equity issue

A	L
Cash:2	Debt: 97
Asset 1: 40	Equity: 5
Asset 2: 60	102
102	

$\frac{E}{A^*} = \frac{5}{100} = 5\%$

Table III.2: Commercial bank balance sheet in book value terms after \$2.00 equity issue (same context as previous example). Note that the assets (A^*) used in calculating the regulatory equity to asset (E/A^*) ratio exclude liquid assets.

4 A Binomial Model of the Issue-Securitize Decision

The model has two periods. It relates to a bank whose balance sheet in period 1 is illustrated in the following diagram.

Assets	Liabilities
A_{B1}	D_B
A_{B2}	E_B

(Book Value)

Assets	Liabilities
A_{M1}	D_M
A_{M2}	E_M

(Market Value)

Definitions:

A_{B1} = book value of asset 1.

A_{B2} = book value of asset 2.

A_{M1} = market value of asset 1.

A_{M2} = market value of asset 2.

D_B = book value (face value) of debt.

E_B = book value of equity.

D_M = market value of debt.

E_M = market value of equity.

For ease of exposition, we assume $A_{B1} = A_{B2}, A_{M1} = A_{M2}$. It is assumed that this commercial bank must boost its equity-to-assets ratio from its current level

$$\frac{E_B}{(A_{B1} + A_{B2})} = d$$

to a new level $c > d$. This can be achieved by pursuing one of the following four strategies:

(i) Issue E of new equity;

(ii) Securitize proportion b of asset 1;

(iii) Securitize proportion b of asset 2;

(iv) Securitize some combination of assets 1 and 2, e.g. for $0 < w < 1$ securitize proportion wb of asset 1 and proportion $(1-w)b$ of asset 2.

E and b are determined such that

$$\begin{aligned} c &= \frac{E + E_B}{A_{B1} + A_{B2}} \\ &= \frac{E_B + b(A_{M1} - A_{B1})}{A_{B2} + (1-b)A_{B1}} \\ &= \frac{E_B + b(A_{M2} - A_{B2})}{A_{B1} + (1-b)A_{B2}} \rightarrow E = \frac{b(2A_{M1} - D_B)}{(2-b)}. \end{aligned}$$

Note that $b(A_{M1} - A_{B1})$ is the gain (if $A_{M1} > A_{B1}$) or loss (if $A_{M1} < A_{B1}$) on the sale of asset 1 which is reflected in the book value of equity; similarly for asset 2.

At time 1, the period to which the previous balance sheets relate, the management of the bank learns the values of assets 1 and 2. The market does not observe these values until time 2.

We assume that the two asset values follow a two state (binomial) process with a correlation coefficient (r) of either $+1$ or -1.

4.1 Binomial Model with $r = +1$

We assume that A_{M1} can equal $A_{M1}(1 + \delta a)$ with probability $\frac{1}{2}$ (the good state) and $A_{M1}(1 - \delta a)$ with probability $\frac{1}{2}$ (the bad state), with $0 \leq \delta < 1$ and $a > 0$. A_{M2} can equal $A_{M2}(1 + a)$ with probability $\frac{1}{2}$ (the good state) and $A_{M2}(1 - a)$ with probability $\frac{1}{2}$ (the bad state). It is obvious that these assumptions imply that the correlation coefficient is $+1$. Figure III-2 illustrates the possible values of the two assets in period 2.

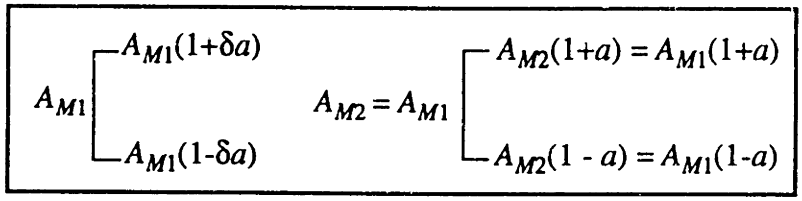


Figure III-2: Asset values in good and bad states. $r = +1$

The upper branch in the bracket represents the good state and the lower branch the bad state.

4.2 Binomial Model with $r = -1$

We assume that A_{M2} can equal $A_{M2}(1+a)$ with probability $\frac{1}{2}$ (the good state) and $A_{M2}(1-a)$ with probability $\frac{1}{2}$ (the bad state). A_{M1} can equal $A_{M1}(1-\delta a)$ with probability $\frac{1}{2}$ in the good state and $A_{M1}(1+\delta a)$ with probability $\frac{1}{2}$ in the bad state. These assumptions imply that the correlation coefficient is -1. Note that with $r = -1$, the value of asset 1 falls in the good state and rises in the bad state. Nevertheless, the good state is "good" in the sense that the combined value of the two assets is higher in that state (recall, $0 \leq \delta < 1$). Figure III-3 illustrates the possible values of the two assets in period 2.

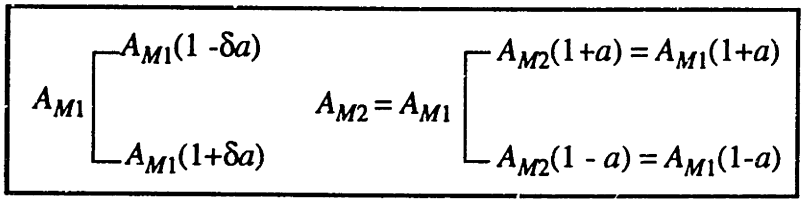


Figure III-3: Asset values in good and bad states. $r = -1$.

As in the case of Figure III-2, the upper branch in the bracket of Figure III-3 represents the good state, and the lower branch the bad state.

4.3 Measure of Risk

Note that with $0 \leq \delta < 1, a > 0$ asset 1 is less risky than asset 2 in the sense that the random variable representing the value of asset 1 once management's inside information is revealed has a smaller variance than the corresponding variable for asset 2. If $\delta = 1$, the two assets are equally risky. Throughout this paper we adopt this definition of risk.

4.4 Management's Objectives

At time 1, the management of the bank must decide which one of the four above mentioned strategies to pursue in order to boost the capital adequacy ratio. To analyze this decision define:

$P_I =$ market value of old shareholders' shares at $t = 1$ if the bank issues new equity at $t = 1$.

$P_{S1} =$ market value of old shareholders' shares at $t = 1$ if the bank securitizes proportion b of asset 1 at $t = 1$.

$P_{S2} =$ Market value of old shareholders' shares at $t = 1$ if the bank securitizes proportion b of asset 2 at $t = 1$.

$P_{SCW} =$ market value of old shareholders' shares at $t = 1$ if the bank securitizes proportion wb of asset 1 and proportion $(1 - w)b$ of asset 2 at $t = 1$.

$V_I =$ Market value of old shareholders' shares at $t = 2$ if the bank issues equity at $t = 1$.

V_{S1} = Market value of old shareholders' shares at $t = 2$if the bank securitizes proportion b of asset 1 at $t = 1$.

V_{S2} = Market value of old shareholders' shares at $t = 2$ if the bank securitizes proportion b of asset 2 at $t = 1$.

V_{SCW} = Market value of old shareholders' shares at $t = 2$ if the bank securitizes proportion wb of asset 1 and proportion $(1 - w)b$ of asset 2 at $t = 1$.

V_{IG} = Value of V_I in good state.

V_{IB} = Value of V_I in bad state.

V_{S1G} = Value of V_{S1} in good state.

V_{S1B} = Value of V_{S1} in bad state.

$V_{S2G}, V_{S2B}, V_{SCWG}$ and V_{SCWB} are similarly defined.

Since the management of a bank has inside information about the realization of V_I, V_{S1}, V_{S2}, and V_{SCW}, i.e., since the management of each bank knows whether it is managing a good or bad bank, it will take (at $t = 1$) that course of action that maximizes old shareholders wealth at $t = 2$.

For example, if $V_{S1G} > V_{IG}$, the management of a good bank will prefer to securitize asset 1. An interesting question is what happens if two courses of action lead to the same old shareholder wealth in period 2. In this case, we assume that management chooses that course of action that maximizes old shareholder wealth in period 1. For example, suppose that issuing equity and securitizing asset 1 entail the same wealth for the bad bank's old shareholders in period 2. Furthermore, assume that the good bank prefers to securitize asset 1. In this situation the bad bank will prefer to securitize asset 1 too, in order to delay revelation of its type for one more period, thereby maximizing old shareholder wealth in period 1. The reason that the bad bank delays revelation of its type by securitizing asset 1

is that since the good bank is also securitizing asset 1, the market is not able to distinguish among the two banks. As a result, the market assigns a higher market value to the bad bank in period 1 (compared to the value it would assign to it if it knew that it was, indeed, the bad bank).

We will now construct the equilibria of this binomial model. To get a better understanding of the factors driving the results we will focus on the ten separate cases shown in Table III.3 on Page 205.

4.5 Restrictive Assumptions of Binomial Model: Their Relaxation Necessitates the Development of a Model with a Continuum of Types

It should be pointed out that the binomial model is restrictive in the sense that it only allows for a correlation coefficient between the two assets of either $+1$ or -1. Ideally, we would like to have a more general model that allowed for any correlation coefficient between -1 and $+1$. A limited analysis of such a model is presented in the Appendix.

The model in the Appendix assumes that there are two assets each of which follows a normal distribution: Both have a mean of 0, but asset 1 has a lower standard deviation (σ_1) than asset 2 (σ_2). The correlation coefficient between the two assets is r, where r can take any value between -1 and $+1$, i.e., $-1 \leq r \leq +1$. We let V_1 and V_2 denote these two asset values.

The model has two periods; in period 1 the market does not know the realization of V_1 and V_2 whereas the owner of the two assets—for example, a commercial bank—does. In period 2, the market observes the realization of V_1 and V_2. For regulatory reasons, the

owner of the assets is forced to sell one of them in order to raise cash. This sale must take place in period 1, i.e., at a time when the market does not know the values of V_1 and V_2.

The analysis of this model in the Appendix shows that whenever $-1 \leq r < \frac{\sigma_1}{\sigma_2}$, both pooling and separating equilibria exist. In a pooling equilibrium, the less risky asset (asset 1) is sold, whereas, by definition, in the separating case both asset 1 and asset 2 are sold in equilibrium. However, for $\frac{\sigma_1}{\sigma_2} \leq r \leq +1$, no separating equilibria exist; for this range of values of r we only get a pooling equilibrium in which the less risky asset (asset 1) is sold.

These results agree with the results of the binomial model, i.e., in the binomial model too, whenever $r = +1$ we only get pooling equilibria whereas, whenever $r = -1$, we get both pooling and separating equilibria.

The analysis in the paper focuses on the binomial model since it is much more tractable. Nevertheless, it is encouraging to know that a richer model would give similar conclusions.

Case	Assets	r	Debt Before Equity Issue	Debt After Equity Issue	Equilibrium
Case 1	Asset 1 safe ($\delta = 0$) Asset 2 risky	0	Risk-free	Risk-free	Pooling: Asset 1
Case 2	Both assets risky but asset 1 less risky ($0 < \delta < 1$)	+1	Risk-free	Risk-free	Pooling: Asset 1
Case 3	Both assets equally risky ($\delta = 1$)	+1	Risk-free	Risk-free	Pooling: Asset 1 or Pooling: Asset 2 or Pooling: Equity
Case 4	Both assets risky but asset 1 less risky ($0 < \delta < 1$)	−1	Risk-free	Risk-free	Only separating equilibria
Case 5	Both assets Equally risky ($\delta = 1$)	−1	Risk-free	Risk-free	Both pooling and separating equilibria
Case 6	Asset 1 safe ($\delta = 0$) Asset 2 risky	0	Risky	Risk-free	Pooling: Asset 1
				Risky	
Case 7	Both assets risky but asset 1 less risky ($0 < \delta < 1$)	+1	Risky	Risk-free	Pooling: Asset 1
				Risky	Pooling: Asset 1 or Pooling: equity
Case 8	Both assets equally risky ($\delta = 1$)	+1	Risky	Risk-free	Pooling: Asset 1 or Pooling: Asset 2 or pooling:Equity
				Risky	Pooling:Equity
Case 9	Both assets risky but asset 1 less risky ($0 < \delta < 1$)	-1	Risky	Risk-free	Pooling:Asset 1
				Risky	
Case 10	Both assets equally risky ($\delta = 1$)	−1	Risky	Not a meaningful case to analyse—see text.	

Table III.3: The ten different versions of the model.

5 CASE 1: Management has no Inside Information About Asset 1 (i.e., $\delta = 0$), Debt is Risk Free ($D_M = D_B$), and $r = 0$

In this case it is easy to show that both good and bad banks will prefer to securitize the safe asset.

Proposition 1: Under the conditions of Case 1, the unique equilibrium of this model is a pooling equilibrium in which both good and bad banks, securitize asset 1. This equilibrium is supported by the following out-of-equilibrium belief: If a bank deviates by pursuing any one of the other three strategies, the market thinks it is bad.

A formal proof of this proposition is presented in the Appendix. We present below a sketch of the proof and provide some intuition as to why it holds.

The first step in the proof is to show that the equilibrium postulated above exists. After showing this, we show that no other equilibrium (pooling or separating) exists, thereby establishing the uniqueness of the above specified pooling equilibrium.

If a good bank securitizes asset 1 as required by the postulated pooling equilibrium,

$$V_{S1G} = \begin{matrix}\text{Cash proceeds} \\ \text{from securitization}\end{matrix} + \begin{matrix}\text{Value of that part} \\ \text{of asset 1 not securi-} \\ \text{tized}\end{matrix} + \text{Value of asset 2} - \text{Value of Debt}$$

$$\begin{aligned} &= bA_{M1} + (1-b)A_{M1}(1+\delta a) + A_{M2}(1+a) - D_B \\ &= bA_{M1} + (1-b)A_{M1} + A_{M2}(1+a) - D_B \quad [\text{since } \delta = 0] \\ &= A_{M1} + A_{M2}(1+a) - D_B. \end{aligned}$$

Similarly, given the out of equilibrium beliefs, if a good bank deviates and pursues any of

the other three strategies, the payoffs to its old shareholders at $t = 2$ will be:

$$
\begin{aligned}
V_{S2G} &= A_{M1} + A_{M2}(1 + a - 2ab) - D_B \\
V_{SCWG} &= A_{M1} + A_{M2}(1 + a - 2ab(1 - w)) - D_B, 0 < w < 1 \\
V_{IG} &= \begin{array}{l}\text{\% of firm owned by} \\ \text{old shareholders}\end{array} \times \begin{array}{l}\text{Value of good firm at} \\ t = 2 \text{ after equity is-} \\ \text{sue of } E.\end{array} \\
&= \frac{P_I}{P_I + E}[A_{M1}(1 + \delta a) + A_{M2}(1 + a) + E - D_B] \\
&= \left(1 - \frac{E}{P_I + E}\right)[A_{M1} + A_{M2}(1 + a) + E - D_B]
\end{aligned}
$$

where

$$
\begin{aligned}
P_I &= A_{M1}(1 - \delta a) + A_{M2}(1 - a) - D_B \\
&= A_{M1} + A_{M2}(1 - a) - D_B
\end{aligned}
$$

Using elementary algebra, it can be easily shown that V_{S1G} exceeds V_{S2G}, V_{SCWG} and V_{IG}. For example,

$$
\begin{aligned}
V_{S1G} - V_{IG} &= -E + \frac{E}{P_I + E}[A_{M1} + A_{M2}(1 + a) + E - D_B] \\
&= -E + E\left[\frac{A_{M1} + A_{M2}(1 + a) + E - D_B}{A_{M1} + A_{M2}(1 - a) + E - D_B}\right] > 0
\end{aligned}
$$

Hence, the good bank follows the strategy postulated in the pooling equilibrium.

Similarly, it is easy to show that the bad firm does not deviate. To complete the proof, we use similar arguments to show that no other equilibrium can exist (see Appendix).

We now provide an intuitive explanation of these results, using Figure III-4.

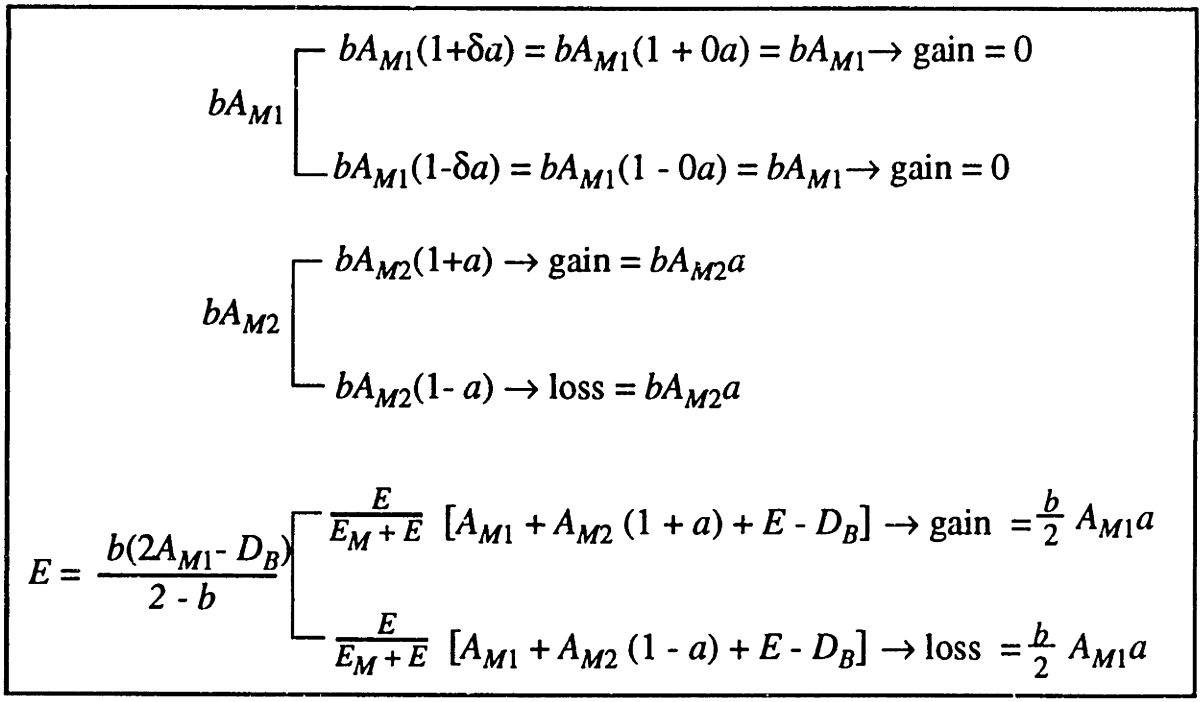


Figure III-4: Gain/loss to purchasers of different securities in the good and bad states. Securities are purchased in period 1 and purchasers don't know if they are buying from a good or a bad bank.

Note that the gain/loss to the purchasers of different securities shown in Figure III-4 is calculated on the assumption that each of these securities is sold in a pooling equilibrium in which both good and bad banks sell the same security.

The calculation of the gain/loss to the purchasers of part of asset 1 or asset 2 is straightforward and needs no further discussion.

The calculation of the gain/loss to the purchasers of new equity is easy once we substitute the expression for E and E_M. In particular, the gain to the purchasers of new equity is given by

$$\begin{array}{l}\text{\% of firm owned by} \\ \text{new shareholders}\end{array} \times \begin{array}{l}\text{value of good firm at} \\ t = 2 \text{ after issue of } E\end{array} - \begin{array}{l}\text{what new share-} \\ \text{holders paid for} \\ \text{their shares}\end{array} =$$

$$= \frac{E}{E_M + E}[A_{M1} + A_{M2}(1+a) + E - D_B] - E$$
$$= \frac{E}{A_{M1} + A_{M2} - D_B + E}[A_{M1} + A_{M2}(1+a) + E - D_B - E_M - E]$$
$$= \frac{E}{2A_{M1} - D_B + E}[A_{M1}a] \qquad (\text{using } A_{M1} = A_{M2})$$
$$= \frac{b(2A_{M1} - D_B)}{(2-b)(2A_{M1} - D_B) + b(2A_{M1} - D_B)}[A_{M1}a] = \frac{b}{2}(A_{M1}a).$$

Note that Figure III-4 shows the expected gains/losses to the purchasers of securities from management's perspective, i.e., it reflects management's inside information. In the case of asset 1, since management has no inside information about its value, it expects its value in period 2 to be the same as in period 1 and hence, it expects the purchasers of bA_{M1} to suffer no gains or losses. In the case of equity, management expects the value of new equity issued in period 1 to increase if the good state (upper branch) occurs and decrease if the bad state (lower branch) occurs. Figure III-4 assumes that new equity can be sold at $t = 1$ at its then market value of E_M, i.e. it assumes that the issue of new equity provides no signal to the market as to whether it is the good or the bad bank that is issuing.

Figure III-4 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, the preferences of each bank in *descending* order of preference, would be as follows:

	Good		**Bad**
1)	Asset 1	1)	Asset 2
2)	Equity	2)	Equity
3)	Asset 2	3)	Asset 1

For example, the good bank prefers to securitize asset 1 rather than issue equity because the purchasers of part of asset 1 reap no gain, whereas the purchasers of equity do reap a

gain. Likewise, the good bank prefers to issue equity rather than securitize asset 2 because the gain to the purchasers of equity (gain $= \frac{b}{2}A_{M1}a = \frac{b}{2}A_{M2}a$) is lower than the gain to the purchasers of part of asset 2 (gain $= bA_{M2}a$). Similar considerations enable us to deduce the bad bank's order of preference.

We can now provide an intuitive explanation as to why only one pooling equilibrium—securitize asset 1—exists.

5.1 Pooling Equilibria

Let us try to construct a pooling equilibrium in which asset 1 is securitized. Since asset 1 is most (least) prefereed by the good (bad) bank, if a bank deviates from this equilibrium by selling any other security, the market thinks it is bad. The good bank will not deviate: It gains or loses nothing in this pooling equilibrium, whereas, if it deviates by selling equity or asset 2 it would suffer a loss since it would be selling at a price reflecting the bad bank's value. The bad bank will not deviate either: It gains or loses nothing in this pooling equilibrium; of course if it deviates, it would identify itself as the bad bank and sell equity or asset 2 at a price reflecting the bad bank's value; as a result, it would not gain anything by deviating; furthermore, it prefers to pool with the good bank to avoid revelation of its type for one more period, i.e., if the bad bank sold equity or asset 2 at $t = 1$, the payoff to its old shareholders at $t = 2$ would be the same as in the case in which it securitizes asset 1 at $t = 1$; however, if the bad bank issues equity or securitizes asset 2 at $t = 1$, the value of its old shareholders shares at $t = 1$ is lower than if it securitizes asset 1 (i.e., pools with the good bank).

Let us try to support a pooling equilibrium in which equity is issued. If a bank deviates

by securitizing asset 1, the market makes no inference as to which bank it is, since asset 1 is safe. If a bank deviates by securitizing asset 2, the market thinks it is the bad bank since it is the bad bank that has the strongest incentive to securitize asset 2. This equilibrium cannot be supported. The good bank will deviate by selling asset 1. The reason for this is that in this pooling equilibrium, the good bank suffers a loss by selling equity, whereas, if it securitized asset 1, it would suffer no gain or loss.

Finally, let us try to support a pooling equilibrium in which asset 2 is securitized. If a bank deviates, the market thinks it is the good bank since asset 2 is most (least) preferred by the bad (good) bank. This equilibrium cannot be supported. The good bank will deviate by selling asset 1. The reason is that the good bank suffers a loss in this pooling equilibrium, whereas, if it securitized asset 1, it would suffer no gain or loss.

We now consider separating equilibria.

5.2 Separating Equilibria

Let us try to support a separating equilibrium in which the good bank securitizes asset 1 and the bad bank securitizes asset 2. This equilibrium cannot be supported because the bad bank will deviate. The bad bank does not gain or lose anything by selling asset 2 in a separating equilibrium since it identifies itself. Likewise, it does not gain or lose anything by deviating and selling asset 1 since the latter is safe. However, it prefers to deviate and securitize asset 1 in order to avoid revelation of its type for one more period.

Using similar arguments, we can show that no separating equilbria exist.

6 CASE 2: Both Assets Risky But Asset 1 Less Risky Than Asset 2 $(0 < \delta < 1)$, Debt Risk-Free, and $r = +1$

In this case it is easy to show that both good and bad banks will prefer to securitize the less risky asset.

Proposition 2: Under the conditions of case 2, the unique equilibrium of this model is a pooling equilibrium in which both good and bad banks securitize asset 1. This equilibrium is supported by the following out of equilibrium belief: if a bank deviates by pursuing any one of the other three strategies, the market thinks it is bad.

To explain the rationale for Proposition 2, Figure III-5 shows the gain/loss to the purchasers of different securities.

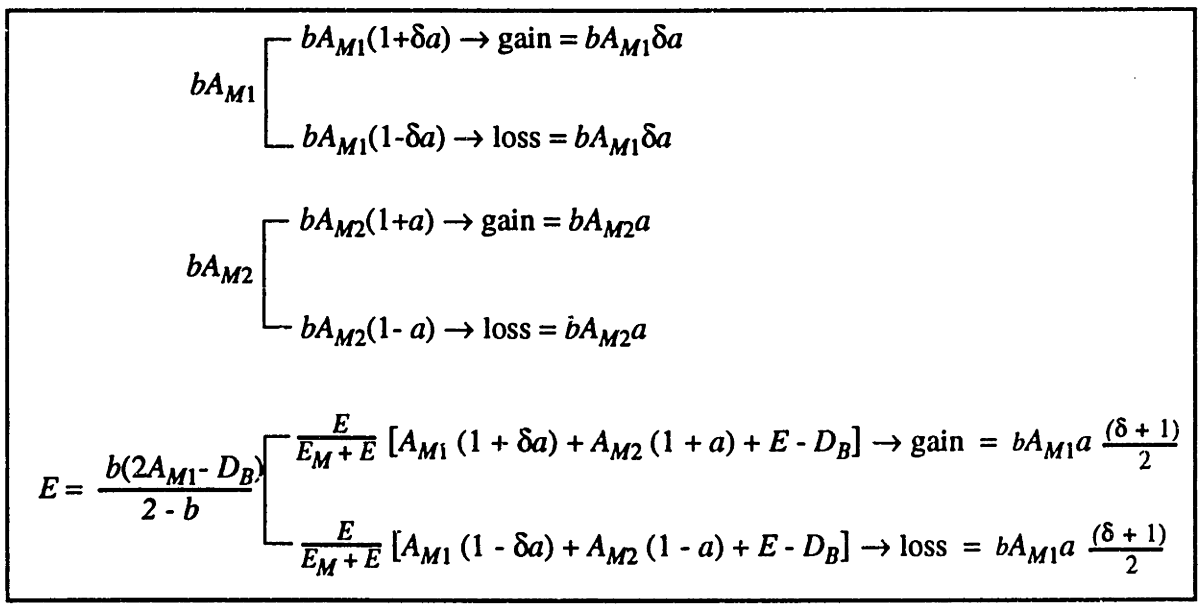


Figure III-5: Gain/loss to purchasers of different securities in the good and bad states. Securities are purchased in period 1 and purchasers don't know if they are buying from a good or a bad bank.

As in the case of Figure III-4, the calculation of the gains and losses in Figure III-5 is

based on the assumption that each of these securities is sold in a pooling equilibrium, in which both good and bad banks sell the same security. Figure III-5 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, the preferences of each bank in descending order of preference would be as follows:

	Good		**Bad**
1)	Asset 1	1)	Asset 2
2)	Equity	2)	Equity
3)	Asset 2	3)	Asset 1

For example, the good bank prefers to securitize asset 1 rather than issue equity because the purchasers of part of asset 1 gain $bA_{M1}\delta a$, whereas the purchasers of equity gain $bA_{M1}a^{\frac{(\delta+1)}{2}} > bA_{M1}\delta a$ with $0 < \delta < 1$. Similar considerations enable us to deduce the bad bank's preferences.

Note that each bank's preferences in this case (i.e., case 2) are identical to its preferences in case 1 of Section 5. As a result, it is not surprising that we get the same type of pooling equilibrium in which both banks securitize asset 1. The reasoning is identical to the one given in the previous section for case 1.

7 CASE 3: Debt is Risk-Free and Both Assets Are Equally Risky, i.e. $\delta = 1$ and $r = +1$

With $\delta = 1$, the two assets become identical. Hence, the strategy of securitizing asset 1 is identical to the strategy of securitizing asset 2 which, in turn, is identical to the strategy of issuing equity since selling equity is equivalent to selling a weighted average of the two assets. In light of this, it is not surprising that we get multiple pooling equilibria in this case, as stated formally in Proposition 3.

Proposition 3: Under the conditions of case 3, there exist three pooling equilibria:

1. securitize asset 1;
2. securitize asset 2; and
3. issue equity.

Each of these three equilibria is supported by the following out-of-equilibrium belief: if a bank deviates by pursuing any one of the other three strategies, the market makes no inference as to whether it is a good or a bad bank.

To explain the rationale for Proposition 3, Figure III-6 shows the gain/loss to the purchasers of different securities. Note that Figure III-6 is obtained by substituting $\delta = 1$ in all equations of Figure III-5 of Section 6.

Figure III-6 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, both the good and the bad bank would be indifferent as to which security they sell. The reason for this is that the loss to the good bank and the gain to the bad bank is the same irrespective of the security they sell.

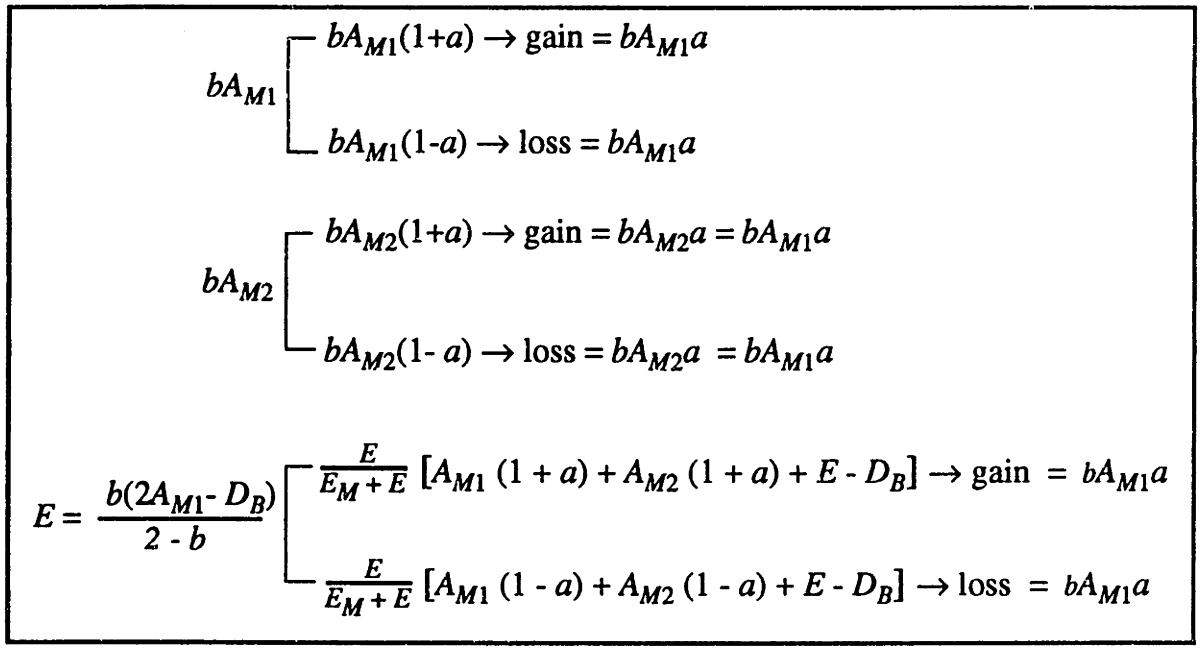


Figure III-6: Gain/loss to purchasers of different securities in the good and bad states. Securities are purchased in period 1 and purchasers don't know if they are buying from a good or a bad bank.

We can now provide an intuitive explanation as to why we get these multiple pooling equilibria.

7.1 Pooling Equilibria

Let us try to construct a pooling equilibrium in which asset 1 is securitized. Since both the good and the bad bank are indifferent as to which security they sell, if a bank deviates from this pooling equilibrium, the market makes no inference as to whether it is the good or the bad bank. The good bank will not deviate from this equilibrium: even though it loses $bA_{M1}a$, if it deviates by securitizing asset 2 or issuing equity it would lose the same amount, given the out of equilibrium beliefs. Likewise, the bad bank will not deviate: it gains $bA_{M1}a$ in this equilibrium; if it deviates by securitizing asset 2 or issuing equity it would gain the same amount, given the out of equilibrium beliefs. It follows that this pooling equilibrium

is a viable one.

Similar arguments can be used to show that the other two pooling equilibria mentioned in Proposition 3 are also viable.

We now show that no separating equilibria exist.

7.2 Separating Equilibria—They Don't Exist

Let us try to construct a separating equilibrium in which the good bank securitizes asset 1 and the bad bank securitizes asset 2. The bad bank will deviate: if it does not deviate, it sells asset 2 at a price reflecting the bad bank's value and, as a result, gains or loses nothing; if it deviates by securitizing asset 1 it reaps a gain since it sells asset 1 at a price reflecting the good bank's value. The bad bank will therefore deviate.

Similar arguments can be used to show that no other separating equilibrium exists. The underlying reason is that the bad bank will always want to deviate in order to sell a security at a price higher than its true value.

8 CASE 4: Both Assets Risky but Asset 1 Less Risky than Asset 2, Debt Risk-Free and $r = -1$

In this case, we only get separating equilibria.

Proposition 4: Under the conditions of Case 4, there exist three possible separating equilibria

1. good bank securitizes asset 1 and bad bank issues equity;

2. good bank securitizes asset 1 and bad bank securitizes asset 2;

3. good bank securitizes asset 1 and bad bank securitizes proportion w^*b of asset 1 and proportion $(1 - w^*)b$ of asset 2, where $w^* = \frac{1}{1+\delta}$.

To explain the rationale for Proposition 4, Figure III-7 shows the gain/loss to the purchasers of different securities.

The gains/losses in Figure III-7 are calculated on the assumption that each security is sold in a pooling equilibrium in which both good and bad banks sell the same security. The calculation of the gain/loss to the purchasers of part of asset 1 or asset 2 is straightforward and needs no further discussion.

The calculation of the gain/loss to the purchasers of new equity is easy once we substitute the expression for E and E_M. For example, the gain to the purchasers of new equity in the good state is given by

$$\begin{array}{l}\text{\% of firm owned by} \\ \text{new shareholders}\end{array} \times \begin{array}{l}\text{value of good firm at} \\ t = 2 \text{ after issue of } E\end{array} - \begin{array}{l}\text{what new share-} \\ \text{holders paid for} \\ \text{their shares}\end{array} =$$

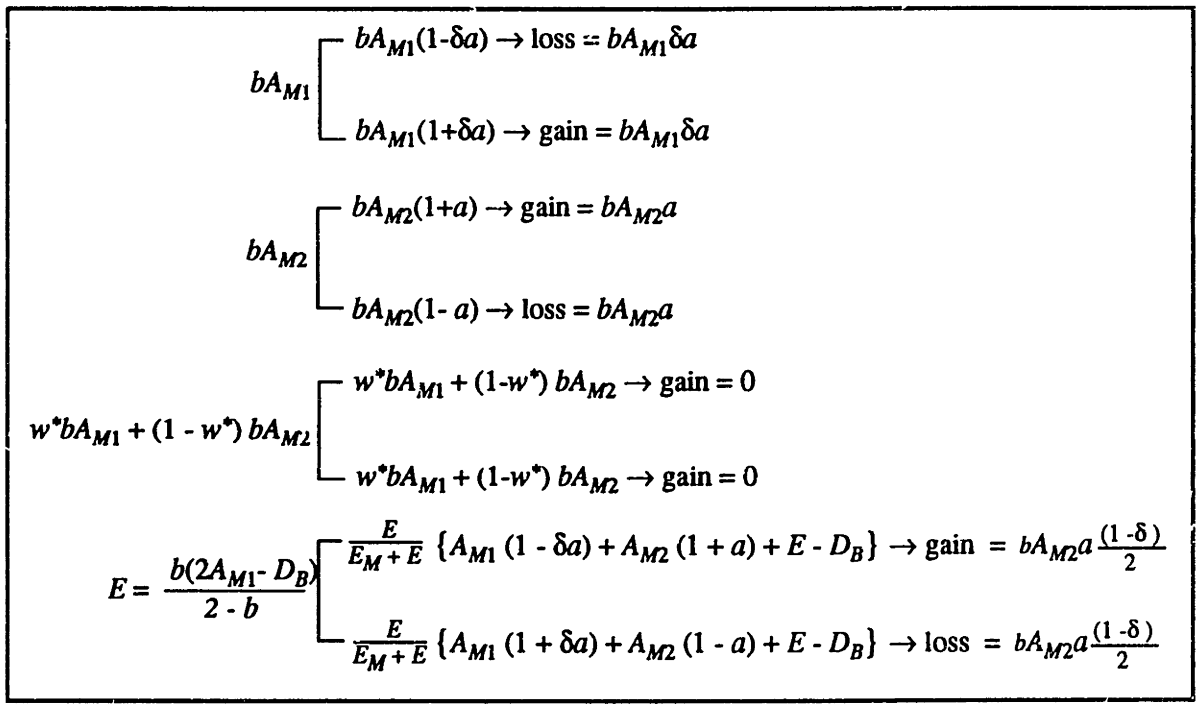


Figure III-7: Gain/loss to purchasers of different securities in the good and bad states. Securities are purchased in period 1 and purchasers don't know if they are buying from a good or a bad bank.

$$
\begin{aligned}
&= \frac{E}{E_M + E}[A_{M1}(1-\delta a) + A_{M2}(1+a) + E - D_B] - E \\
&= \frac{E}{A_{M1} + A_{M2} - D_B + E}[A_{M1}(1-\delta a) + A_{M2}(1+a) + E - D_B - E_M - E] \\
&= \frac{E}{2A_{M1} - D_B + E}[A_{M1}a(1-\delta)] \\
&= \frac{b(2A_{M1} - D_B)}{(2-b)(2A_{M1} - D_B) + b(2A_{M1} - D_B)}[A_{M1}a(1-\delta)] \\
&= \frac{b}{2}A_{M1}a(1-\delta) \\
&= \frac{b}{2}A_{M2}a(1-\delta).
\end{aligned}
$$

Note that with a correlation coefficient of -1 there exists a combination of the two assets with weight w^*b on asset 1 ($w^* = \frac{1}{1+\delta}$) and $(1-w^*)b$ on asset 2 that is riskless. To prove this, note that if a bad bank securitizes a combination of the two assets with these weights, the value of this combination in period 2 (in the bad state) is given by

$$
\begin{aligned}
\text{value of } w^*b, (1-w^*)b \text{ in bad state} &= w^*bA_{M1}(1+\delta a) + (1-w^*)bA_{M2}(1-a) \\
&= w^*bA_{M1} + (1-w^*)bA_{M2} + w^*bA_{M1}\delta a - (1-w^*)bA_{M2}a \\
&= w^*bA_{M1} + (1-w^*)bA_{M2} + bA_{M1}a\frac{\delta}{1+\delta} - bA_{M2}a\frac{\delta}{1+\delta} \\
&= w^*bA_{M1} + (1-w^*)bA_{M2}.
\end{aligned}
$$

Likewise, if a good bank securitizes the same combination, its value in period 2 (in the good state) is given by

$$
\begin{aligned}
\text{value of } w^*b, (1-w^*)b \text{ in good state} &= w^*bA_{M1}(1-\delta a) + (1-w^*)bA_{M2}(1+a) \\
&= w^*bA_{M1} + (1-w^*)bA_{M2} - w^*bA_{M1}\delta a + (1-w^*)bA_{M2}a \\
&= w^*bA_{M1} + (1-w^*)bA_{M2} - bA_{M1}a\frac{\delta}{1+\delta} + \frac{\delta}{1+\delta}bA_{M2}a
\end{aligned}
$$

$$= \quad w^* b A_{M1} + (1 - w^*) b A_{M2}.$$

Since the value of this combination of assets in period 1 (when it is sold in a pooling equilibrium) is equal to its value in period 2 in either the good or the bad state, this combination of assets is riskless. Note that no such riskless combination of assets exists when the correlation coefficient is $+1$.

Figure III-7 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, the preferences of each bank in descending order of preference, would be as follows:

Good	**Bad**
1) Asset 1	1) Asset 2
2) $w^*, 1 - w^*$	2) Equity
3) Equity	3)$w^*, 1 - w^*$
4) Asset 2	4) Asset 1

For example, the good bank prefers to securitize asset 1 rather than securitize combination $(w^*, 1 - w^*)$ because the purchasers of asset 1 suffer a loss whereas the purchasers of combination $(w^*, 1 - w^*)$ suffer no gain or loss. Likewise, the good bank prefers to securitize combination $(w^*, 1 - w^*)$ rather than issue equity because the purchasers of this combination do not gain or lose anything whereas the purchasers of equity do reap a gain. Finally, the good bank prefers to issue equity rather than securitize asset 2 because (with $0 < \delta < 1$) the gain that accrues to the purchasers of equity ($bA_{M2}a\frac{(1-\delta)}{2}$) is lower than the gain that accrues to the purchasers of asset 2 ($bA_{M2}a$). Similar considerations enable us to deduce the bad bank's order of preference.

We can now provide an explanation as to why only separating equilbria exist. We begin by showing that no pooling equilibrium exists.

8.1 Pooling Equilibria—They do not Exist

Let us try to construct a pooling equilibrium where asset 1 is securitized. If a bank deviates, the market thinks it is bad since it is the bad bank that has the least incentive to securitize asset 1. This equilibrium is not sustainable. The bad bank will deviate by, say, securitizing asset 2. In so doing, it will identify itself and get a fair deal, thereby avoiding the loss it suffers in the pooling equilibrium—recall the bad bank loses $bA_{M1}\delta a$ by securitizing asset 1 in a pooling equilibrium.

Let us try to construct a pooling equilibrium where asset 2 is securitized. If a bank deviates, the market thinks it is the good bank since it is the good bank that has the least incentive to securitize asset 2. This equilibrium is not sustainable. The good bank will deviate by, say, securitizing asset 1. In so doing, it will identify itself and get a fair deal, thereby avoiding the loss it suffers in the pooling equilibrium.

Let us try to construct a pooling equilibrium in which equity is issued. If a bank deviates by securitizing asset 1, the market thinks it is good since it is the good bank that has the strongest incentive to sell asset 1. If a bank deviates by securitizing asset 2, the market thinks it is bad, since it is the bad bank that has the strongest incentive to sell asset 2. For the reason given in the previous paragraph, the good bank will deviate by securitizing asset 1. As a result, this equilibrium is not sustainable.

Finally, let us try to construct a pooling equilibrium in which the combination $(w^*, 1 - w^*)$ is securitized. If a bank deviates by issuing asset 2 or equity, the market thinks it is

bad since it is the bad bank that has the strongest incentive to sell these securities. If a bank deviates by securitizing asset 1, the market thinks it is good since it is the good bank that has the strongest incentive to sell asset 1. This equilibrium is not sustainable. The good bank will deviate by securitizing asset 1; in so doing, the good bank does not improve its payoff in period 2 since it gets a fair deal by either securitizing asset 1 (given it identifies itself once it announces securitization of asset 1) or securitizing combination $(w^*, 1 - w^*)$ (given the latter is safe). However, the good bank improves its period 1 stock price by deviating since it reveals its type earlier. For this reason, the good bank will deviate.

We now consider separating equilibria.

8.2 Separating Equilibria that Exist

Let us try to construct a separating equilibrium in which the good bank securitizes asset 1 and the bad bank issues equity. The good bank does not deviate because if it does, it issues equity at a price reflecting the bad bank, and as a result, suffers a loss. The bad bank does not deviate either. If it deviates, it sells asset 1 at a price reflecting the good bank's asset 1 value—which is lower than the value of the bad bank's asset 1—and suffers a loss.

Let us try to construct a separating equilibrium in which the good bank securitizes asset 1 and the bad bank securitizes asset 2. The good bank does not deviate because if it deviates, it sells asset 2 at a price reflecting the bad bank's asset 2 value—which is lower than the value of the good bank's asset 2—and suffers a loss. The bad bank does not deviate either, because if it deviates, it sells asset 1 at a price reflecting the good bank's asset 1 value—which is lower than the bad bank's asset 1 value—and suffers a loss.

Finally, let us try to construct a separating equilibrium in which the good bank secu-

ritizes asset 1 and the bad bank securitizes the combination $(w^*, 1 - w^*)$. The good bank does not deviate, because the strategy $(w^*, 1 - w^*)$ entails no gain or loss to it. The bad bank does not deviate, because if it deviates, it securitizes asset 1 at a price reflecting the good bank's asset 1 value—which is lower than the bad bank's asset 1 value—and suffers a loss.

8.3 Separating Equilibria that do not Exist

No separating equilibria other than the ones presented in the previous section exist.

For example, let us try to construct a separating equilibrium in which we switch the strategies that the two banks play in the sustainable separating equilibrium of section 8.2, paragraph 2. In other words, we will try to construct a separating equilibrium in which the good bank securitizes asset 2 and the bad bank securitizes asset 1. The bad bank will deviate, because if it deviates, it sells asset 2 at a price reflecting the value of the good bank's asset 2—which is higher than the value of the bad bank's asset 2—and reaps a gain.

Using similar arguments, it can be shown that no other separating equilibria exist.

9 CASE 5: Both Assets Equally Risky ($\delta = 1$), Debt Risk-Free, and $r = -1$

From Figure III-3 of Section 4.2 note that when $\delta = 1$, it is no longer valid to refer to a "good" bank and a "bad" bank. The reason is that both banks total asset value in period 2 is the same, i.e., $A_{M1} + A_{M2}$. As a result, in the analysis of case 5, we refer to the bank whose asset values are shown in the upper (lower) branch of Figure III-3 as bank X (bank Y).

This has an important implication. No bank will deviate from a pooling equilibrium by playing a strategy that does not improve its period 2 payoffs if the purpose of such a deviation is simply to reveal its type in period 1. The reason is that such a revelation will have no effect on its period 1 market value. This, in turn, leads to the existence of some types of equilibrium that cannot exist in case 4, where early revelation of type is a revelant consideration. The equilibria obtained in case 5 are stated in Proposition 5.

Proposition 5: Under the conditions of case 5, there exist two pooling equilibria (issue equity and securitize combination $(w^*, 1 - w^*)$) and seven separating equilibria:

1. X securitizes asset 1 and Y issues equity.

2. X securitizes asset 1 and Y securitizes asset 2.

3. X issues equity and Y securitizes asset 2.

4. X securitizes asset 1 and Y securitizes combination $(w^*, 1 - w^*)$.

5. X securitizes combination $(w^*, 1 - w^*)$ and Y securitizes asset 2.

6. X issues equity and Y securitizes combination $(w^*, 1 - w^*)$.

7. X securitizes combination $(w^*, 1 - w^*)$ and Y issues equity.

We now provide an intuitive explanation of these results using Figure III-8

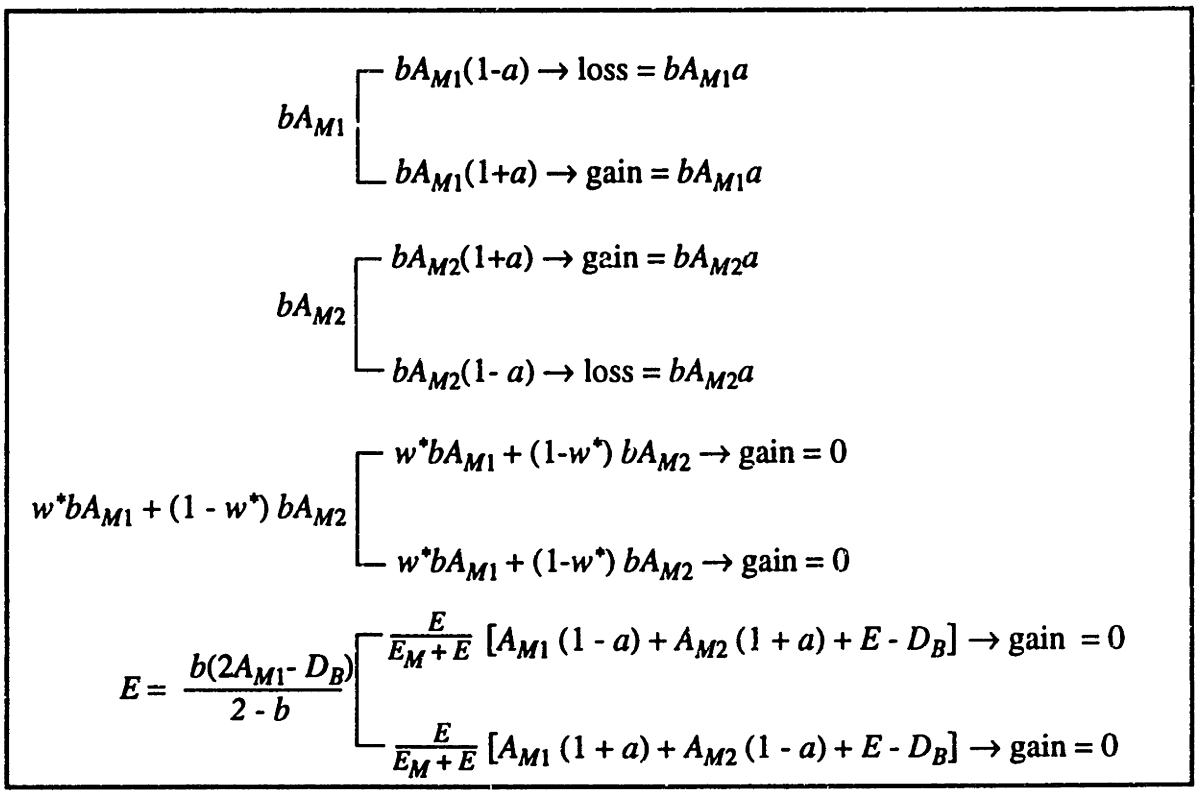


Figure III-8: Gain/loss to purchasers of different securities in state X and state Y. Securities are purchased in period 1 and purchasers don't know who they are buying from.

The gains/losses in Figure III-8 are calculated by substituting $\delta = 1$ in Figure III-7 of Section 8. Note that with $\delta = 1, w^* = \frac{1}{1+\delta} = \frac{1}{2}$.

Figure III-8 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, the preferences of each bank in descending order of preference would be as follows:

Bank X	Bank Y
1) Asset 1	1) Asset 2
2) Equity	2) Equity
2) w^*	2) w^*
3) Asset 2	3) Asset 1

We now discuss the types of equilibria obtained.

9.1 Pooling Equilibria that Exist

Let us try to construct a pooling equilibrium in which both banks issue equity. If a bank deviates by securitizing asset 1 (asset 2) the market thinks it is bank X (bank Y) since it is bank X (bank Y) that has the strongest incentive to securitize asset 1 (asset 2).If a bank deviates by securitizing combination $(w^*, 1-w^*)$ the market makes no inference as to which bank it is. Bank X does not deviate. It gains or loses nothing in this pooling equilibrium; if it deviates by securitizing asset 1, it identifies itself as bank X and gains or loses nothing either; furthermore, early identification of type is of no value for the reason mentioned above. As a result, bank X does not deviate. Likewise, bank Y does not deviate either. It gains or loses nothing in this pooling equilibrium; if it deviates by securitizing asset 2, it identifies itself as bank Y and gains or loses nothing; furthermore, early identification of type is of no value. Consequently, this pooling equilibrium is sustainable.

Similar arguments can be used to show that a pooling equilibrium in which combination $(w^*, 1-w^*)$ is securitized is also sustainable.

9.2 Pooling Equilibria that don't Exist

Let us try to support a pooling equilibrium in which asset 1 is sold. If a bank deviates by securitizing asset 2, the market thinks it is bank Y, since it is bank Y that has the strongest incentive to securitize asset 2. Bank Y will deviate. It loses in this pooling equilibrium, whereas if it deviates by securitizing asset 2, it would identify itself and suffer no loss.

Similarly, there can be no pooling equilibrium in which asset 2 is securitized.

9.3 Separating Equilibria

Let us try to support a separating equilibrium in which bank X securitizes asset 1 and bank Y issues equity. Bank X will not deviate: it gains or loses nothing by securitizing asset 1 (given it identifies itself) and gains or loses nothing by deviating and issuing equity since equity is safe. Bank Y will not deviate either: it gains or loses nothing by issuing equity; if it deviates by securitizing asset 1 it would suffer a loss because it would be selling asset 1 at a price reflecting the value of bank X's asset 1—which is lower than the value of bank Y's asset 1.

Using similar arguments, it can be shown that the other separating equilibria mentioned in Proposition 5 exist.

10 CASE 6: Debt is Risky, $\delta = 0$, i.e. Management Has No Inside Information About Asset 1 and $r = 0$

We introduce risky debt in the model by assuming that the debt (face value D_B) has to be repaid in full in period 2 and that only the good bank can repay it in full, i.e.

$$\begin{aligned} A_{M1}(1+\delta a) + A_{M2}(1+a) &= A_{M1} + A_{M2}(1+a) > D_B \\ A_{M1}(1-\delta a) + A_{M2}(1-a) &= A_{M1} + A_{M2}(1-a) < D_B \end{aligned}$$

We impose the restriction $2A_{M1} > D_B$ to ensure that a positive amount of new equity has to be issued (since $E = b(2A_{M1} - D_B)/(2-b)$). Otherwise, there is no need to increase the capital adequacy ratio.

Proposition 6: Under the conditions of case 6, the unique equilibrium is pooling in which both good and bad banks securitize asset 1. If a bank deviates by pursuing any other strategy, the market thinks it is bad.

The rationale for Proposition 6 is very similar to the rationale for Proposition 7 of case 7 which is presented in Section 11 that follows. Note that if we set $\delta = 0$ in Proposition 7, $bA_{M1}\delta a = 0$ and hence $bA_{M1}\delta a < E$. Thus only Proposition 7(a) and the part of Proposition 7(b) that assumes $bA_{M1}\delta a < E$ become applicable. The results of the latter are the same as the result in Proposition 6. Consequently, we focus on Case 7.

11 CASE 7: Both Assets Risky But Asset 1 Less Risky Than Asset 2 $(0 < \delta < 1)$, Debt Risky and $r = +1$

The model used is the same as the one in Case 6 except that $0 < \delta < 1$ here. Debt has to be repaid in full in period 2 and only the good bank can repay it in full, i.e.

$$A_{M1}(1 + \delta a) + A_{M2}(1 + a) > D_B$$

$$A_{M1}(1 - \delta a) + A_{M2}(1 - a) < D_B$$

We impose the restriction $2A_{M1} > D_B$ to ensure that a positive amount of new equity has to be issued (since $E = b(2A_{M1} - D_B)/(2 - b)$). Otherwise, there is no need to boost the capital adequacy ratio.

The equilibrium of this case will depend on the assumptions we make about the bad bank's ability to repay its debt in full at $t = 2$ if it issues equity at $t = 1$, assuming that the equity is issued in a pooling equilibrium so that the purchasers don't know if they are buying from a good or a bad bank.

Proposition 7(a): Under the conditions of case 7, assume that the bad bank can repay its debt in full at $t = 2$ if it issues equity at $t = 1$. The unique equilibrium is pooling in which both good and bad banks securitize asset 1. If a bank deviates by pursuing any one of the other three strategies, the market thinks it is bad.

Proposition 7(b): Under the conditions of case 7, assume that the bad bank cannot repay its debt in full at $t = 2$ if it issues equity at $t = 1$. Then, if $bA_{M1}\delta a < E$, the unique equilibrium is pooling in which both types securitize asset 1—if a bank deviates, the market thinks it is bad; alternatively, if $bA_{M1}\delta a > E$, the unique equilibrium is pooling in which

both types issue equity—if a bank deviates the market thinks it is bad.

11.1 Discussion of Proposition 7(a)

We begin by discussing Proposition 7(a). To explain the rationale for it, Figure III-9 shows the gains/losses to the purchasers of different securities.

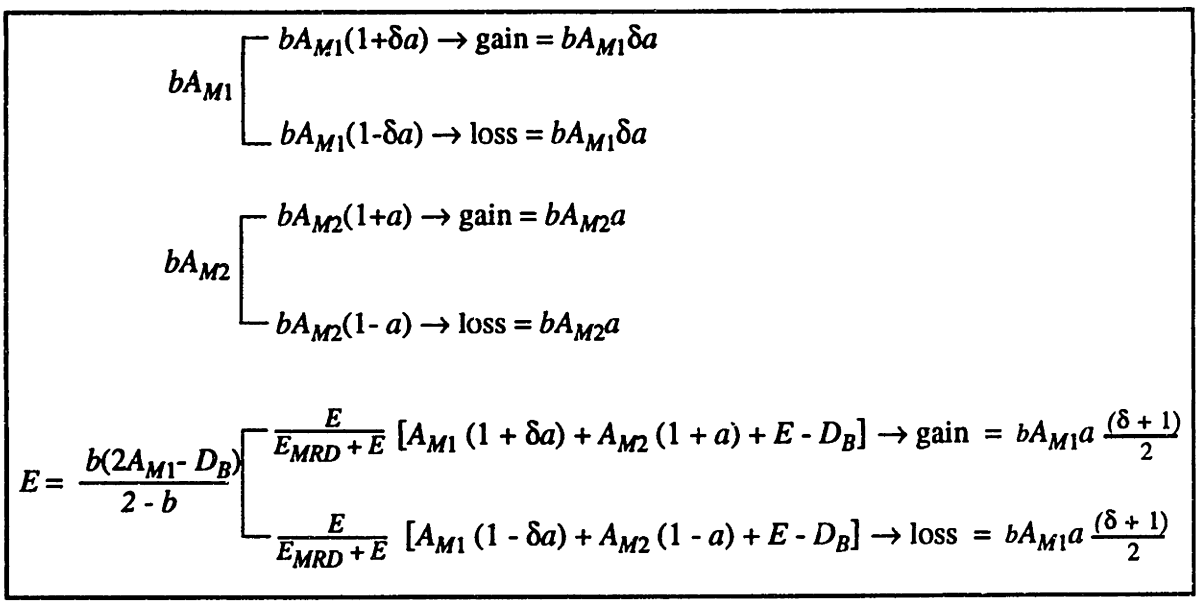


Figure III-9: Gains/losses to purchasers of different securities in the good and bad states. Securities are purchased in period 1 and purchasers don't know if they are buying from a good or a bad bank.

The calculation of the gains/losses to the purchasers of equity deserves some discussion. The purchasers of new equity realize that the value of debt increases when new equity is issued. Hence, the price E_{MRD} at which they are willing to buy new equity reflects this. In particular

$$E_{MRD} = A_{M1} + A_{M2} - D_M - (\Delta D_M)$$

where

$$\begin{aligned}
\Delta D_M &= \text{increase in debt value due to equity issue} \\
&= \frac{1}{2}[\text{increase in value of debt in good state}] + \\
&\quad \frac{1}{2}[\text{increase in value of debt in bad state}] \\
&= \frac{1}{2}[0] + \frac{1}{2}[D_B - A_{M1}(1-\delta a) - A_{M2}(1-a)] \\
D_M &= \frac{1}{2}[D_B] + \frac{1}{2}[A_{M1}(1-\delta a) + A_{M2}(1-a)] \\
&= \text{market value of debt}
\end{aligned}$$

Note that the increase in debt value in the bad state is calculated on the basis of the assumption that the bad bank can repay its debt in full if it issues new equity.

Hence,

$$E_{MRD} = A_{M1} + A_{M2} - D_B = 2A_{M1} - D_B.$$

This shows that investors realize that the equity issue makes debt risk free and buy new equity at a price reflecting this.

If a good bank issues equity it receives E for new equity worth

$$\frac{E}{E_{MRD} + E}[A_{M1}(1+\delta a) + A_{M2}(1+a) + E - D_B]$$

By using the expression for E_{MRD} and E, it can be shown that the good bank's loss is

$$\frac{E}{E_{MRD} + E}[A_{M1}(1+\delta a) + A_{M2}(1+a) + E - D_B] - E$$

$$
\begin{aligned}
&= \frac{E}{E_{MRD}+E}[A_{M1}(1+\delta a)+A_{M2}(1+a)+E-D_B-(E_{MRD}+E)] \\
&= \frac{b(2A_{M1}-D_B)}{(2-b)(2A_{M1}-D_B)+b(2A_{M1}-D_B)}[A_{M1}(\delta a+a)] \\
&= bA_{M1}a\left[\frac{(\delta+1)}{2}\right]
\end{aligned}
$$

Likewise, the bad bank's gain if it issues equity is given by

$$E-\frac{E}{E_{MRD}+E}[A_{M1}(1-\delta a)+A_{M2}(1-a)+E-D_B]=bA_{M1}a\frac{(\delta+1)}{2}.$$

Figure III-9 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, the preferences of each bank in decending order of preference would be as follows:

Good	**Bad**
1) Asset 1	1) Asset 2
2) Equity	2) Equity
3) Asset 2	3) Asset 1

We can now consider the existence of pooling and separating equilibria.

Pooling Equilibria

Let us try to construct a pooling equilibrium in which asset 1 is securitized. If a bank deviates by issuing equity or securitizing asset 2 the market thinks it is bad, since it is the bad bank that has the strongest incentive to issue these securities. The good bank does not deviate because the loss it suffers in this pooling equilibrium ($bA_{M1}\delta a$) is lower than the loss it would suffer by selling asset 2 or equity at a price reflecting the bad bank's value.

For example if it sells asset 2 at $bA_{M2}(1-a)$ it loses

$$bA_{M2}(1+a) - bA_{M2}(1-a) = 2bA_{M2}a > bA_{M1}\delta a.$$

The bad bank does not deviate because if it deviates it identifies itself and sells securities at their fair value, whereas in this pooling equilibrium it gains $bA_{M1}\delta a$. Of course, another consideration that is relevant for the bad bank is whether it can repay its debt in period 2. Assume, that the bad bank cannot repay it if it securitizes asset 1 in a pooling equilibrium. Then, it would clearly prefer to issue equity in a pooling equilibrium, since, by assumption, it can then repay its debt.

However, note that since the good bank prefers to securitize asset 1, an equity issue reveals that it is the bad bank that is issuing. If the bad bank issues new equity, the market value of both old and new shareholders' stake at $t = 2$ is

$$A_{M1}(1-\delta a) + A_{M2}(1-a) + E - D_B.$$

This value is less than E since,

$$A_{M1}(1-\delta a) + A_{M2}(1-a) < D_B.$$

Therefore, new shareholders will not buy new equity even if they are offered 100% ownership. Hence, the bad bank cannot sell new equity and is forced to securitize asset 1 at $t = 1$. We have a pooling equilibrium in which both types securitize asset 1.

Let us try to support a pooling equilibrium in which asset 2 is securitized. If a bank

deviates by selling asset 1, the market thinks it is good since it is the good bank that has the strongest incentive to sell asset 1. The good bank will deviate. In the pooling equilibrium it suffers a loss of $bA_{M2}a$ whereas, if it deviates b selling asset 1, it identifies itself and suffers no gain or loss.

Using similar arguments, it can be shown that no other pooling equilibria exist.

Separating Equilibria

No such equilibria exist. The bad bank will always deviate from such an equilibrium to avoid identifying itself, since, by identifying itself, it sells securities at their fair value which, in turn, implies that the bad bank is bankrupt in period 2.

11.2 Discussion of Proposition 7(b)

Figure III-10 shows the gains/losses to the purchasers of different securities under the assumption of Proposition 7(b), i.e., under the assumption that the bad bank cannot repay its debt if it issues equity in a pooling equilibrium.

The price E_{MRD} at which investors are willing to buy new equity is given by

$$E_{MRD} = A_{M1} + A_{M2} - D_M - (\Delta D_M)$$

where

$$\begin{aligned} \Delta D_M &= \frac{1}{2}[\text{increase in value of debt in good state}] + \\ &\quad \frac{1}{2}[\text{increase in value of debt in bad state}] \end{aligned}$$

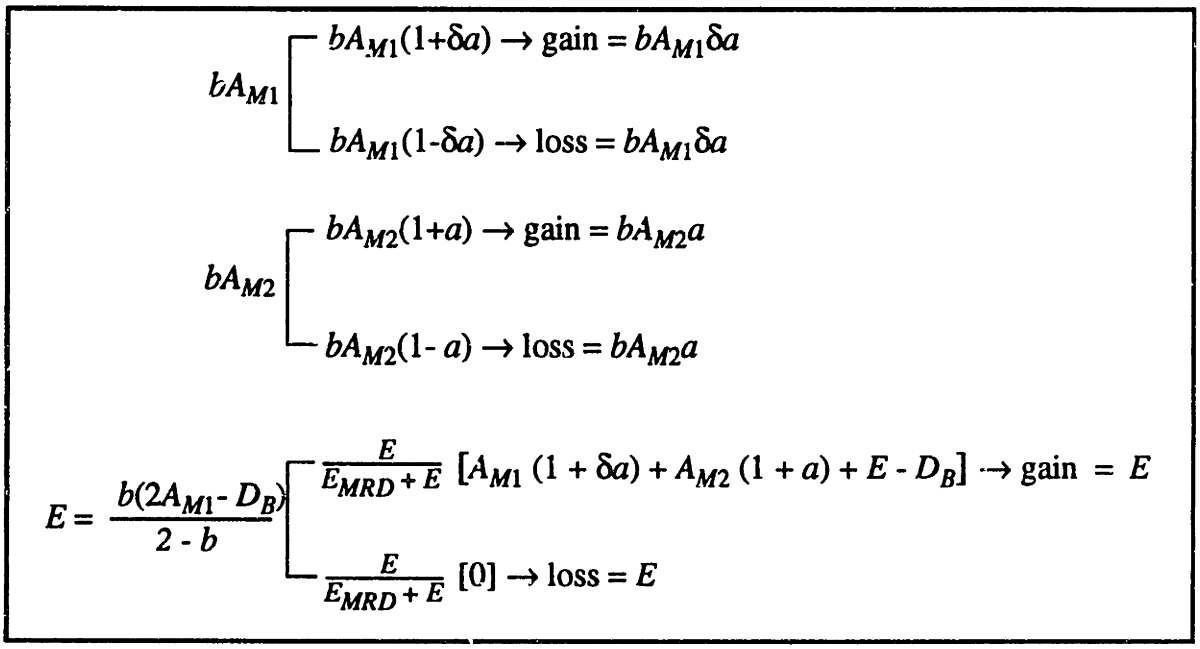


Figure III-10: Gains/losses to purchasers of different securities in the good and bad states. Securities are purchased in period 1 and purchasers don't know if they are buying from a good or a bad bank.

$$= \frac{1}{2}[0] + \frac{1}{2}[E]$$

$$D_M = \frac{1}{2}[D_B] + \frac{1}{2}[A_{M1}(1-\delta a) + A_{M2}(1-a)]$$

Note that in this case, the increase in the value of the debt in the bad state is E since the bad bank cannot repay its debt in full by issuing equity.

Hence,

$$E_{MRD} = \frac{1}{2}[A_{M1}(1+\delta a) + A_{M2}(1+a) - D_B - E]$$

$$= \frac{1}{2}[A_{M1}(2+\delta a + a) - D_B - E]$$

If a good bank issues equity it receives E for new equity worth

$$\frac{E}{E_{MRD}+E}[A_{M1}(1+\delta a)+A_{M2}(1+a)+E-D_B]$$

By using the expression for E_{MRD} derived above, it can be shown that the new equity is worth

$$\frac{2E}{A_{M1}(2+\delta a+a)-D_B+E}[A_{M1}(2+\delta a+a)+E-D_B]=2E$$

Hence the good bank looses $2E-E=E$ by issuing equity.

An alternative way to explain this is to note that since investors know that the equity of the bad bank is worthless, they require $2E$ in the good state to ensure that they get a fair deal, i.e.

$$\frac{1}{2}\begin{bmatrix}\text{payoff to investors in}\\ \text{good state}\end{bmatrix}+\frac{1}{2}\begin{bmatrix}\text{payoff to investors in}\\ \text{bad state}\end{bmatrix}=E$$

Since payoff to investors in bad state = 0, payoff to investors in good state = $2E$.

The above analysis shows that the good bank loses $bA_{M1}\delta a$ if it securitizes asset 1 and E if it issues equity. Hence, if $bA_{M1}\delta a < E(bA_{M1}\delta a > E)$ the good bank prefers to securitize asset 1 (issue equity).

Pooling Equilibria

Given the conclusion in the previous paragraph, it is not surprising that whenever $bA_{M1}\delta a < E(bA_{M1}\delta a > E)$ we get a pooling equilibrium in which asset 1 (equity) is issued. The bad bank does not deviate from this equilibrium for reasons similar to those given in Section 11.1.

Separating Equilibria

No such equilibria exist for reasons similar to those given in Section 11.1.

12 CASE 8: Both Assets Equally Risky, i.e. $\delta = 1$ and Debt Risky and $r = +1$

As in case 7, we assume that only the good bank can repay its debt in period 2. I first state the results and then explain them.

Proposition 8(a):

Under the conditions of case 8, assume that the bad bank can repay its debt in full at $t = 2$ if it issues equity at $t = 1$. Then, there exist three pooling equilibria:

1. securitize asset 1;

2. securitize asset 2;

3. Issue equity.

Proposition 8(b):

Under the conditions of case 8, assume that the bad bank cannot repay its debt in full at $t = 2$ if it issues equity at $t = 1$. Then, the unique equilibrium is pooling in which both types issue equity—if a bank deviates, the market thinks it is bad.

12.1 Discussion of Proposition 8(a)

Figure III-11 shows the gains/losses to the purchasers of different securities.

Figure III-11 is obtained by setting $\delta = 1$ in Figure III-9.

Figure III-11 shows that if the sale of any security provided no signal as to whether it is the good or the bad bank that is issuing, each bank would be indifferent as to which security it sells. The reason is that the loss to the good bank and the gain to the bad bank

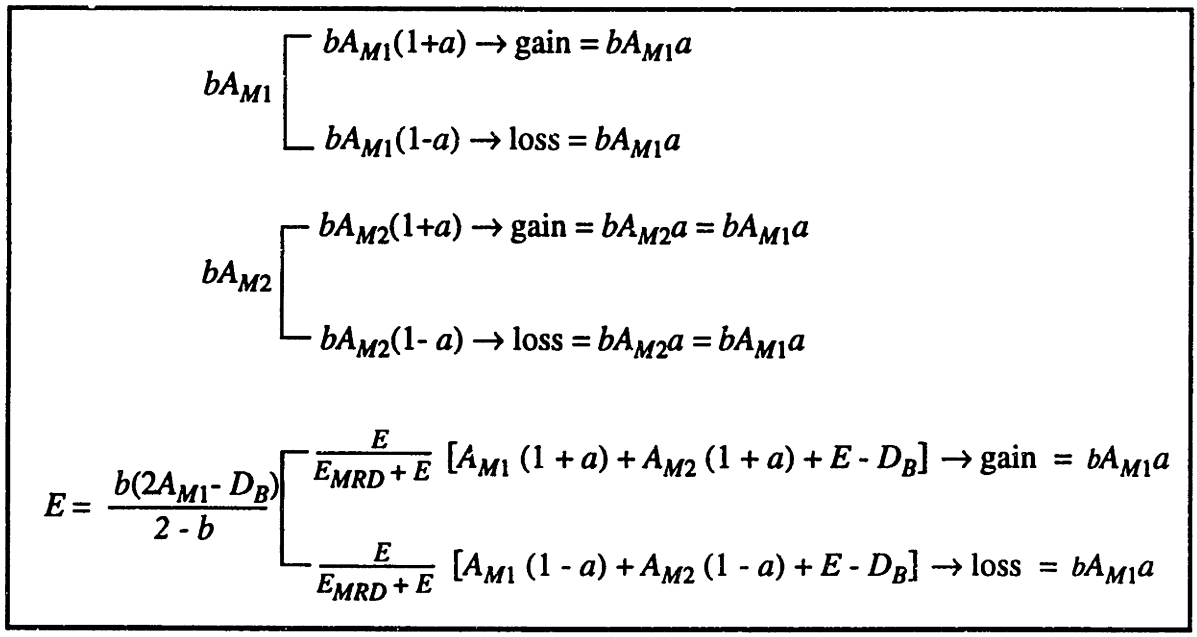


Figure III-11: Gains/losses to purchasers of different securities

is the same irrespective of security issued.

In light of the above, we get the three pooling equilibria mentioned in Proposition 8(a). The bad bank does not deviate from these pooling equilibria because if it does deviate, it identifies itself and sells securities at their fair value which, means, that it is bankrupt in period 2.

12.2 Discussion of Proposition 8(b)

Figure III-12 shows the gains/losses to the purchasers of different securities under the assumption that the bad bank cannot repay its debt if it issues equity.

Figure III-12 is obtained by setting $\delta = 1$ in Figure III-10.

Given our assumptions, the following parameter restrictions hold:

$$A_{M1}(1-a) + A_{M1}(1-a) + E < D_B \Rightarrow 2A_{M1} - D_B < 2A_{M1}a - E.$$

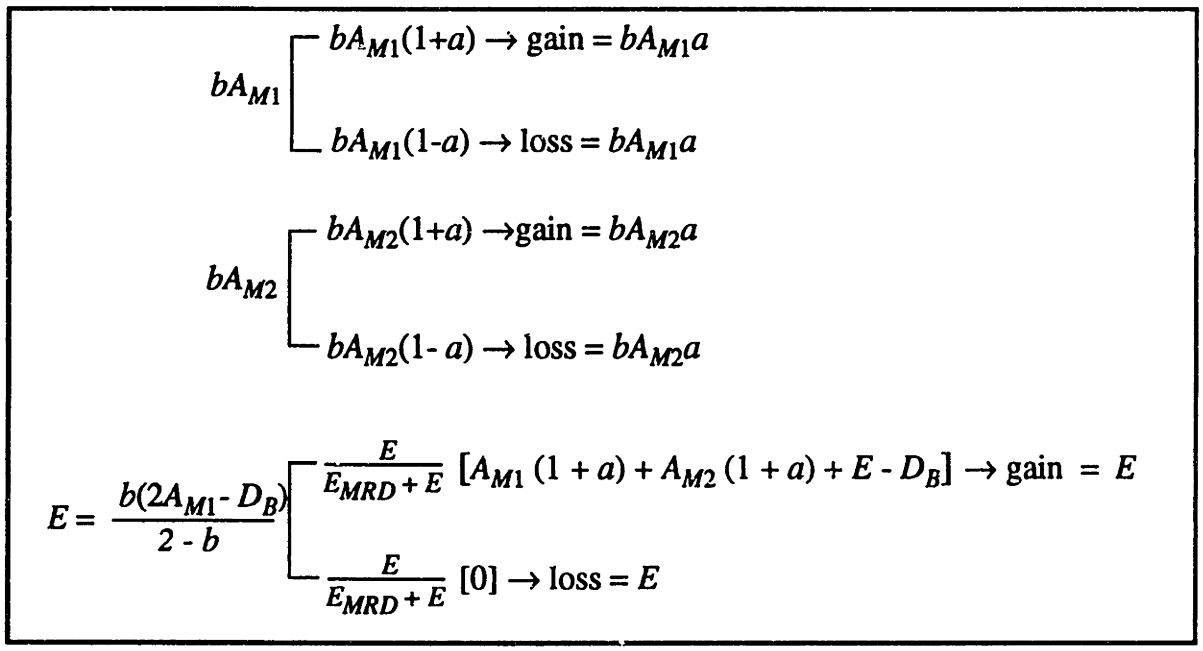


Figure III-12: Gains/losses to purchasers of different securities

Hence, using

$$
\begin{aligned}
E &= \frac{b(2A_{M1} - D_B)}{2-b} \\
\frac{E}{bA_{M1}a} &= \frac{2A_{M1} - D_B}{(2-b)A_{M1}a} < \frac{2A_{M1}a - E}{(2-b)A_{M1}a} \\
&\Rightarrow \frac{E}{bA_{M1}a}\left[1 + \frac{b}{(2-b)}\right] < \frac{2}{(2-b)} \\
&\Rightarrow \frac{E}{bA_{M1}a}\left[\frac{2}{(2-b)}\right] < \frac{2}{(2-b)} \\
&\Rightarrow E < bA_{M1}a.
\end{aligned}
$$

Hence, the good bank prefers to issue equity since the loss it suffers (E) is lower than the loss from securitizing asset 1 or 2. In light of this we only get a pocling equilibrium in which equity is issued. The bad bank does not deviate for the reason given in Section 12.1.

13 CASE 9: Asset 1 Less Risky than Asset 2, i.e., $\delta < 1$, Debt Risky and $r = -1$

The assumptions in case 9 imply that

$$\begin{aligned} A_{M1}(1-\delta a) + A_{M2}(1+a) &> D_B \\ A_{M1}(1+\delta a) + A_{M2}(1-a) &< D_B \end{aligned}$$

The first equation shows that the good bank can repay its debt; the second shows that the bad bank cannot repay its debt.

Proposition 9: Under the conditions of case 9, the unique equilibrium is pooling in which both banks securitize asset 1. If a bank deviates, the market thinks it is bad.

Note that in Proposition 9, the assumption we make about the bad bank's ability to repay its debt if it issues equity makes no difference as to the equilibrium obtained. Nevertheless, the analysis is facilitated if we present it first for a situation where the bad bank can repay its debt by issuing equity, and then for a situation where the bad bank cannot repay if it issues equity.

13.1 Analysis of Case 9 Under Assumption that Bad Bank can Repay by Issuing Equity

Figure III-13 shows the gains/losses to the purchasers of different securities.

The calculation of the gains/losses arising from the sale of either asset 1 or asset 2 or combination $(w^*, 1-w^*)$ where $w^* = \frac{1}{1+\delta}$ is straightforward. The calculation of the

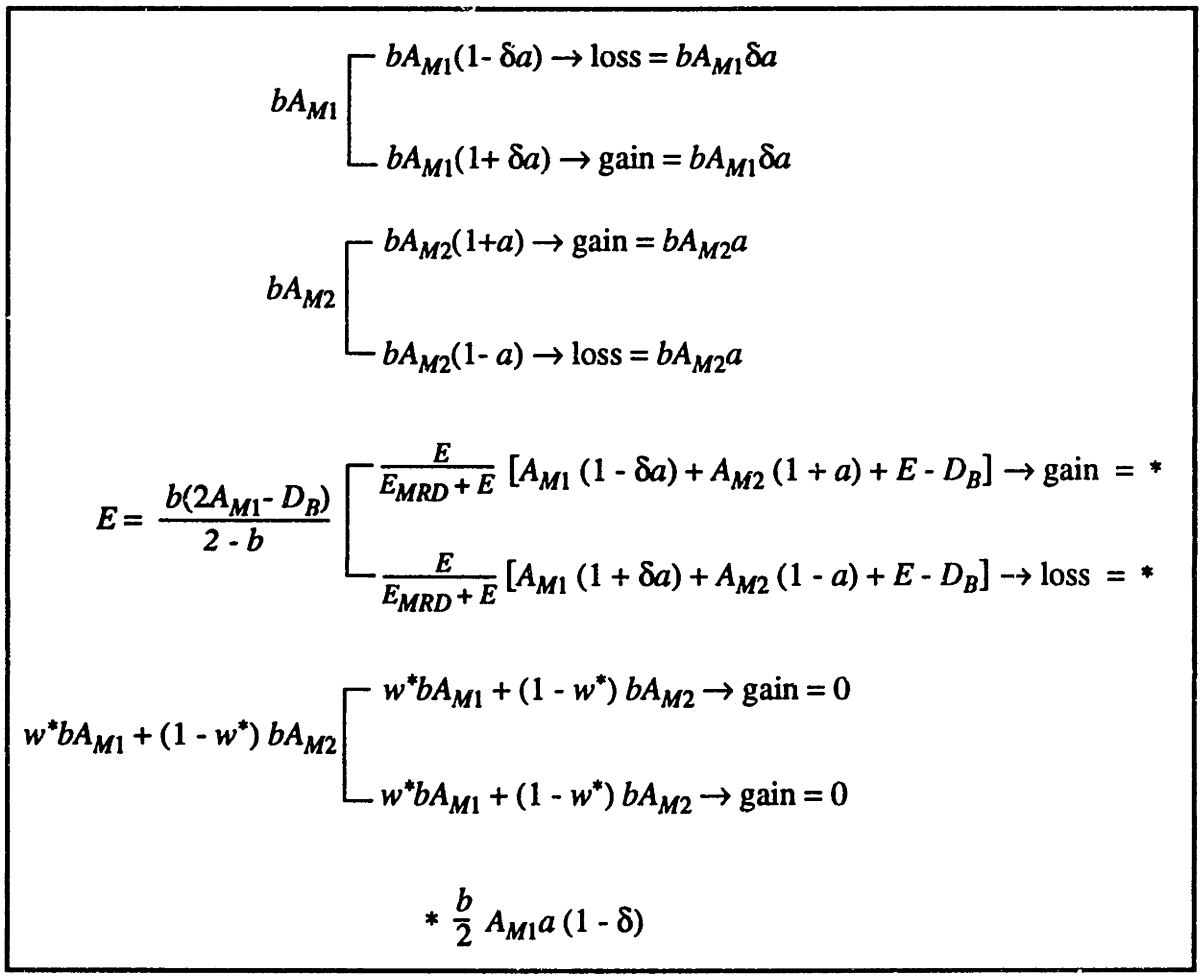


Figure III-13: Gains/losses to purchasers of different securities

gains/losses from issuing equity relies on the recognition that the issue of equity increases the value of debt; as a result, new equity holders buy at a price E_{MRD} reflecting this. In particular

$$E_{MRD} = A_{M1} + A_{M2} - D_M - (\Delta D_M)$$

where

$$\begin{aligned} D_M &= \text{Market value of debt} \\ &= \frac{1}{2}[D_B] + \frac{1}{2}[A_{M1}(1+\delta a) + A_{M2}(1-a)] \\ \Delta D_M &= \text{Increase in debt value due to equity issue} \\ &= \frac{1}{2}[0] + \frac{1}{2}[D_B - A_{M1}(1+\delta a) - A_{M2}(1-a)] \end{aligned}$$

By substituting for D_M and ΔD_M, we get

$$E_{MRD} = A_{M1} + A_{M2} - D_B$$

This shows that new equity holders recognize that the equity issue makes debt risk free and buy at a price reflecting this.

The good bank's loss if it issues equity in a pooling equilibrium is given by

$$\frac{E}{E_{MRD} + E}[A_{M1}(1-\delta a) + A_{M2}(1+a) + E - D_B] - E$$

Using the expression for E_{MRD} and E, it can be shown that this loss is equal to

$$\frac{b}{2}A_{M1}a(1-\delta).$$

Likewise, it can be shown that the bad bank's gain is

$$\frac{b}{2}A_{M1}a(1-\delta).$$

Figure III-13 shows that if the issue of any security provided no signal as to which bank was issuing, the preferences of each bank in descending order of preference would be as follows:

Good	**Bad**
1) Asset 1	1) Asset 2
2) $w^*, 1-w^*$	2) Equity
3) Equity	3)$w^*, 1-w^*$
4) Asset 2	4) Asset 1

Pooling Equilibria

Let us try to construct a pooling equilibrium in which asset 1 is issued. If a bank deviates the market thinks it is bad since it is the bad bank that has the least incentive to securitize asset 1. The good bank does not deviate because it gains in this pooling equilibrium, whereas if it deviates, it would be selling securities at a price reflecting the bad bank's value, i.e., at a loss. The bad bank does not deviate either because if it deviates, it identifies itself which means it gets a fair deal on the securities it sells; as a result it is bankrupt in period 2.

Furthermore, note that if the bad bank attempts to issue equity, nobody will buy for the same reason as mentioned in the discussion of case 7—new shareholders cannot recover their investment even if offered 100% of the bank.

Let us try to support a pooling equilibrium in which asset 2 is sold. If a bank deviates by selling asset 1, the market thinks it is good since it is the good bank that has the strongest incentive to sell asset 1. The good bank will deviate since it suffers a loss in this pooling equilibrium, whereas, if it deviates by securitizing asset 1, it identifies itself and, hence, gets a fair deal.

Using similar arguments, it can be shown that no other pooling equilibrium exists.

Separating Equilibria

No separating equilibria exist. The bad bank deviates from a separating equilibrium, because, it identifies itself in a separating equilibrium which means it is bankrupt in period 2.

13.2 Analysis of Case 9 Under Assumption that Bad Bank Cannot Repay by Issuing Equity

Figure III-14 shows the gains/losses to the purchasers of different securities.

The purchasers of new equity buy it at a price E_{MRD} given by

$$E_{MRD} = A_{M1} + A_{M2} - D_M - \Delta D_M$$

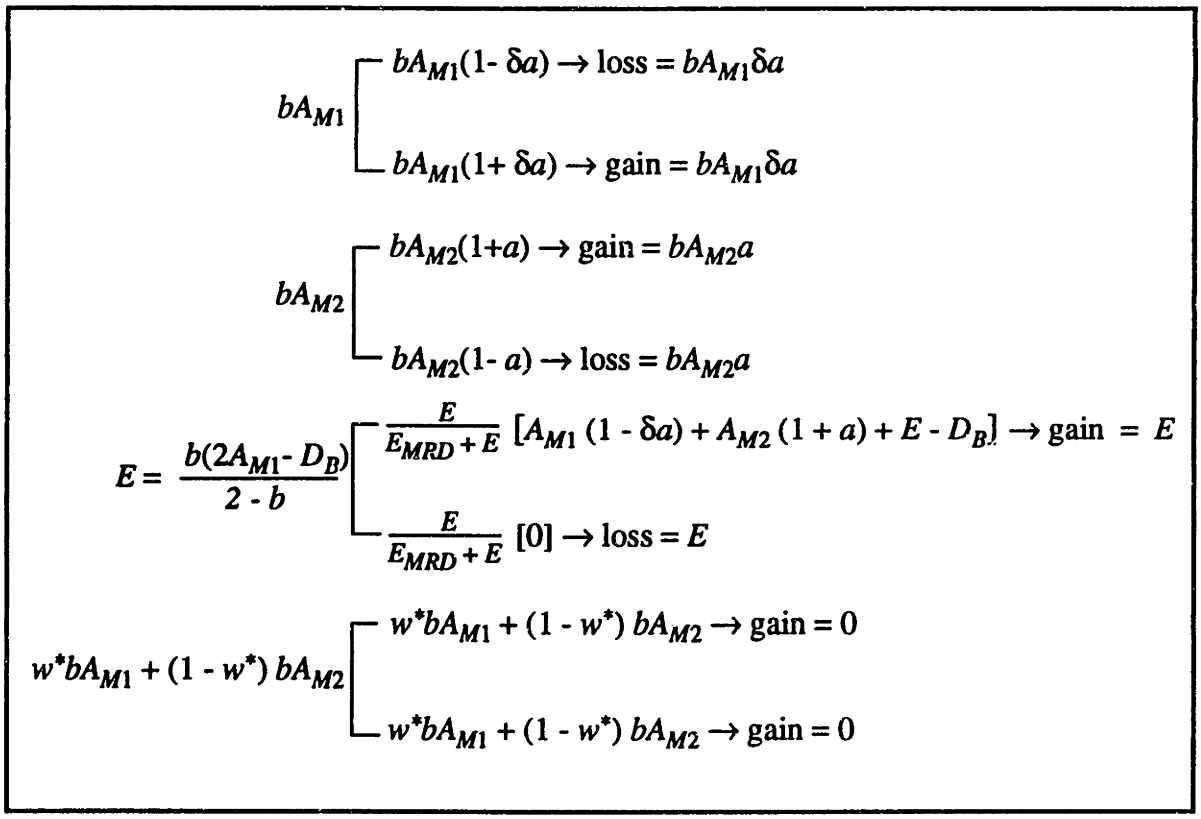


Figure III-14: Gains/losses to purchasers of different securities

where

$$\begin{aligned} D_M &= \text{market value of debt} \\ &= \frac{1}{2}[D_B] + \frac{1}{2}[A_{M1}(1+\delta a) + A_{M2}(1-a)] \\ \Delta D_M &= \text{increase in debt value} \\ &= \frac{1}{2}[0] + \frac{1}{2}[E] \end{aligned}$$

The loss to the good bank from the issue of equity is given by

$$\frac{E}{E_{MRD}+E}[A_{M1}(1-\delta a) + A_{M2}(1+a) + E - D_B] - E.$$

Using the expression for E_{MRD} and E, we can show that this loss is equal to E.

Figure III-14 shows that if the issue of any security provided no signal as to which bank is issuing, the good bank would always prefer to securitize asset 1 since by doing so it gains $bA_{M1}\delta a$, whereas by pursuing any one of the other strategies, it either reaps no gain or suffers a loss. In light of this, we get a pooling equilibrium in which asset 1 is securitized because for reasons very similar to those given in Section 13.1, the bad bank does not deviate from such a pooling equilibrium.

14 CASE 10: Both Assets Equally Risky, i.e., $\delta = 1$, Debt Risky and $r = -1$

The assumptions in case 10 imply that

$$A_{M1}(1-a) + A_{M2}(1+a) > D_B$$

$$A_{M1}(1+a) + A_{M2}(1-a) < D_B$$

The first equation states that bank X (recall the discussion in Case 5—when $r = -1$ and $\delta = 1$, we cannot refer to a good and a bad bank) can repay its debt whereas the second equation states that bank Y cannot repay its debt. Note that these two equations are incompatible since $A_{M1} = A_{M2}$. Hence, this is not a meaningful case to analyze.

15 Conclusion

This essay developed a model that characterizes the circumstances under which bank managers will prefer to boost their banks' equity to asset ratios by securitizing (selling) assets, rather than by issuing equity. This model is similar to the Myers-Majluf model of equity issues under asymmetric information. That model shows that when the firm's management has information outside investors do not have, the firm will be forced to follow a pecking order of financing: it will use its internally generated cash-flow to finance investment outlays, and if such cash-flow is insufficient, it will reduce its marketable securities portfolio. If both these sources prove insufficient, i.e. if external financing is needed, firms issue the safest security first, i.e. debt, then convertible bonds, and perhaps equity only as a last resort.

Case	Assets	r	Debt Before Equity Issue	Debt After Equity Issue	Equilibrium
Case 2	Both assets risky but asset 1 less risky ($0 < \delta < 1$)	+1	Risk-free	Risk-free	Pooling: Asset 1
Case 7	Both assets risky but asset 1 less risky ($0 < \delta < 1$)	+1	Risky	Risk-free	Pooling: Asset 1
				Risky	Pooling: Asset 1 or Pooling: equity
Case 8	Both assets equally risky ($\delta = 1$)	+1	Risky	Risk-free	Pooling: Asset 1 or Pooling: Asset 2 or pooling:Equity
				Risky	Pooling:Equity

Table III.4: Cases 2, 7, and 8 of the model.

The equivalent of the Myers-Majluf marketable securities in the model developed here are the securitizable assets of a bank. The main difference between the two models is that the Myers-Majluf model assumes that there is no information asymmetry about the value of the marketable securities, whereas here we explicitly model such asymmetry. For this reason, we get qualitatively different results. For example, in Myers-Majluf, firms always prefer to finance investment projects by selling marketable securities (when they have them) rather than issuing equity, whereas in this paper's model this is not necessarily true.

Table III.4 summarizes the results obtained in cases 2, 7, and 8 of the model which, I believe, are the most interesting ones to comment on. Note that with $r = 1$, risk-free debt and one asset safer than the other as in case 2, we get a pooling equilibrium in which the less risky asset is securitized. Risky debt does not lead to an equilibrium in which equity is issued as long as debt becomes risk-free after the equity issue (case 7). However, if debt remains risky after the equity issue, we might get an equilibrium in which equity is issued depending on model parameters. Furthermore, case 8 shows that with both assets equally risky and debt risky both before and after an equity issue, we only get a pooling equilibrium in which equity is issued.

The main prediction of this model– that banks will prefer to securitize a safe asset—is supported by the evidence. In almost all asset securitization transactions, an insurance company provides credit enhancement for a fee. Credit enhancement refers to a guarantee provided to the investors by the insurance company for, say, 5% of the aggregate amount of the asset that is securitized — i.e. the insurance company agrees to cover any losses not exceeding 5% of the asset's aggregate amount, thereby creating a safer asset for investors.

The ideas presented in this essay are also relevant to the analysis of other issues in

corporate finance. For example, consider a company in financial distress that must raise funds to continue operating. One way of achieving this would be through asset sales; another, would obviously be through the sale of debt and equity securities. If this company possesses some assets which are safe in the sense that their cash flows are fairly predictable, it might be better off selling those rather than issuing securities. However, note that this problem is more difficult to analyse than the one considered in this paper, because in the case of the latter the amount of assets that must be sold is fixed by regulatory considerations.

Two other financing phenomena that can be explained using ideas similar to those presented in this essay are project financing (see, for example, [3]) and asset backed loans. In both cases, investors or lenders make funds available for specific uses because the alternative of unsecured loans or general purpose financing entails a high degree of information asymmetry.

Appendix A. Proof of Proposition 1

The proof is given in two steps.

Step 1 We show that the pooling equilibrium postulated in the Proposition exists.

Step 2 We show that no other equilibrium can exist.

Proposition 1: Under the conditions of case 1 ($\delta = 0$ and debt risk free), the unique equilibrium is a pooling equilibrium in which both good and bad banks securitize asset 1. This equilibrium is supported by the following out of equilibrium belief: if a bank deviates by pursuing any one of the other three strategies, the market thinks it is bad.

Step 1: Proof that the postulated equilibrium exists.

$$\begin{aligned}
V_{S1G} &= bA_{M1} + (1-b)A_{M1}(1+\delta a) + A_{M2}(1+a) - D_B \\
&= A_{M1}(2+a) - D_B \\
V_{S2G} &= bA_{M2}(1-a) + (1-b)A_{M2}(1+a) + A_{M1} - D_B \\
&= A_{M1} + A_{M2}(1+a-2ab) - D_B \\
V_{SCWG} &= wbA_{M1} + (1-w)bA_{M2}(1-a) + (1-wb)A_{M1} + (1-(1-w)b)A_{M2}(1+a) - D_B \\
&= A_{M1} + A_{M2}[1+a-2ab(1-w)] - D_B; \quad 0 < w < 1.
\end{aligned}$$

Clearly, $V_{S1G} > V_{S2G}$. Also, for any $0 < w < 1, V_{S1G} > V_{SCWG}$. Finally,

$$\begin{aligned}
V_{IG} &= \frac{P_I}{P_I + E}[A_{M1}(1+\delta a) + A_{M2}(1+a) + E - D_B] \\
&= \left(1 - \frac{E}{P_I + E}\right)[A_{M1}(2+a) + E - D_B]
\end{aligned}$$

where

$$\begin{aligned} P_I &= A_{M1} + A_{M2}(1-a) - D_B \\ &= A_{M1}(2-a) - D_B \\ V_{S1G} - V_{IG} &= -E + E\left[\frac{A_{M1}(2+a) + E - D_B}{A_{M1}(2-a) + E - D_B}\right] > 0. \end{aligned}$$

Hence, $V_{S1G} > V_{IG}$. The good bank does not deviate.

Now, consider the bad bank:

$$\begin{aligned} V_{S1B} &= bA_{M1} + (1-b)A_{M1}(1-\delta a) + A_{M2}(1-a) - D_B \\ &= A_{M1}(2-a) - D_B \\ V_{S2B} &= bA_{M2}(1-a) + (1-b)A_{M2}(1-a) + A_{M1} - D_B \\ &= A_{M1} + A_{M2}(1-a) - D_B \\ &= A_{M1}(2-a) - D_B \\ V_{SCWB} &= wbA_{M1} + (1-w)bA_{M2}(1-a) + (1-wb)A_{M1} + (1-(1-w)b)A_{M2}(1-a) - D_B \\ &= A_{M1} + A_{M2}(1-a) - D_B \\ &= A_{M1}(2-a) - D_B \\ V_{IB} &= \frac{P_I}{P_I + E}[A_{M1}(1-\delta a) + A_{M2}(1-a) + E - D_B = \\ &= P_I. \end{aligned}$$

Hence,

$$V_{S1B} = V_{S2B} = V_{SCWB} = V_{IB} \quad 0 < w < 1.$$

Since the time 2 payoffs to the bad bank's old shareholders are the same irrespective of the strategy it pursues, the bad bank will try to maximize the time 1 payoffs to its old shareholders. These are given by

$$\begin{aligned} P_{S1} &= A_{M1} + A_{M2} - D_B \\ P_{S2} &= A_{M1} + A_{M2}(1-a) - D_B \\ P_{SCW} &= A_{M1} + A_{M2}(1-a) - D_B, 0 < w < 1 \\ P_I &= A_{M1} + A_{M2}(1-a) - D_B \end{aligned}$$

Clearly,

$$P_{S1} > P_{S2} = P_{SCW} = P_I.$$

Hence, the bad bank follows the strategy postulated in the pooling equilibrium.

Step 2: The uniqueness of this pooling equilibrium will be established by showing that no other equilibrium exists. For example, let us try to construct a pooling equilibrium in which good, bad issue; if a good bank deviates and securitizes asset 1,

$$\begin{aligned} V_{S1G} &= bA_{M1} + (1-b)A_{M1} + A_{M2}(1+a) - D_B \\ &= A_{M1}(2+a) - D_B \\ V_{IG} &= \frac{P_I}{P_I + E}[A_{M1} + A_{M2}(1+a) + E - D_B] \\ &= \left(1 - \frac{E}{P_I + E}\right)[A_{M1}(2+a) + E - D_B] \end{aligned}$$

where

$$\begin{aligned} P_I &= A_{M1} + A_{M2} - D_B \\ &= A_{M1}(2) - D_B \\ V_{S1G} - V_{IG} &= -E + E\left[\frac{A_{M1}(2+a) + E - D_B}{A_{M1}(2) + E - D_B}\right] > 0 \end{aligned}$$

Hence, $V_{S1G} > V_{IG}$ and good deviates.

Alternatively, let us try to support a separating equilibrium in which good securitizes asset 1 and bad issues

$$V_{IB} = \frac{P_I}{P_I + E}[A_{M1} + A_{M2}(1-a) + E - D_B] = P_I$$

where

$$P_I = A_{M1} + A_{M2}(1-a) - D_B.$$

If bad deviates,

$$\begin{aligned} V_{S1B} &= bA_{M1} + (1-b)A_{M1} + A_{M2}(1-a) - D_B \\ &= A_{M1} + A_{M2}(1-a) - D_B \end{aligned}$$

Hence

$$V_{IB} = V_{S1B}.$$

However,

$$P_{S1} = A_{M1} + A_{M2}(1+a) - D_B > P_I$$

and the bad bank deviates.

Alternatively, let us try to support a separating equilibrium in which good issues and bad securitizes asset 1.

$$\begin{aligned} V_{S1B} &= bA_{M1} + (1-b)A_{M1} + A_{M2}(1-a) - D_B \\ &= A_{M1}(2-a) - D_B \\ V_{IB} &= \frac{P_I}{P_I + E}[A_{M1} + A_{M2}(1-a) - D_B + E] \\ &= \left(1 - \frac{E}{P_I + E}\right)[A_{M1}(2-a) + E - D_B] \end{aligned}$$

where

$$P_I = A_{M1} + A_{M2}(1+a) - D_B = A_{M1}(2+a) - D_B$$

$$\begin{aligned} V_{S1B} - V_{IB} &= -E + \frac{E}{P_I + E}[A_{M1}(2-a) + E - D_B] \\ &= -E + E\left[\frac{A_{M1}(2-a) + E - D_B}{A_{M1}(2+a) + E - D_B}\right] < 0 \\ &\Rightarrow V_{IB} > V_{S1B} \Rightarrow \text{ bad deviates.} \end{aligned}$$

Using similar arguments it can be shown that no other equilibrium exists.

Appendix B. Building a Model with a continuum of types; Some Results (Technical)

A deficiency of the above model is the assumption that the correlation coefficient between the two assets is 1. This is a consequence of our use of a two state model.

To build a richer model, assume that there are two assets each of which follows a normal distribution: both have mean = 0 but asset 1 has a lower standard deviation (σ_1) than asset 2 (σ_2). The correlation coefficient between the two assets is r. We assume that r can take any value between -1 and 1. Let V_1 and V_2 denote these two asset values.

The model has two periods; in period 1 the market does not know the realization of V_1 and V_2 whereas the owner of the two assets—for example, a commercial bank—does. In period 2, the market observes the realization of V_1 and V_2. For regulatory reasons, the owner of the assets is forced to sell one of them in order to raise cash. This sale must take place in period 1, i.e., at a time when the market does not know the values of V_1 and V_2.

We investigate whether there exist prices P_1 and P_2 (for asset 1 and asset 2 respectively), such that the buyer agrees to buy unconditionally (i.e., without the right of refusal) any one of the two assets that the seller wants to sell at the above mentioned prices, P_1 and P_2. Implicit in this formulation is the assumption that the seller has the right to decide which asset to sell and the buyer can only influence this decision through the choice of P_1 and P_2.

Both buyers and sellers are assumed to be risk neutral. The buyer will agree to play this game and buy unconditionally if his expected profit on the sale, defined as asset value less purchase price, is zero. Since the buyer's profit is the seller's loss in equilibrium, the seller should also earn an expected profit of zero.

By showing the existence of such prices P_1 and P_2, we establish the existence of a separating equilibrium. This equilibrium is separating in the sense that the seller decides to sell asset 1 on some occasions and asset 2 on other occasions (depending on model parameters).

To analyze this model, define

$$x = V_1 - V_2, q = P_1 - P_2.$$

For the buyer to earn zero expected profits, two conditions must hold simultaneously:

$$\begin{aligned} P_1 &= E[V_1 | P_1 - V_1 > P_2 - V_2] \\ P_2 &= E[V_2 | P_2 - V_2 > P_1 - V_1] \end{aligned}$$

The first condition captures the fact that the seller will decide to sell asset 1 only if $P_1 - V_1 > P_2 - V_2$, i.e., only if the seller's gain from selling asset 1 exceeds the gain from selling asset 2. To ensure zero expected profits for the buyer, P_1 must equal the expected value of V_1 assuming that the seller behaves in this way. The second condition has a similar interpretation. As explained above if the buyer earns zero expected profits, so does the seller. It follows that there is no need to impose additional restrictions.

Using the definition of x and q, we can rewrite these two conditions as

$$\begin{aligned} P_1 &= E[V_1 | x < q] \\ P_2 &= E[V_2 | x > q]. \end{aligned}$$

To evaluate these conditional expectations, we regress V_1 and V_2 on x. Let

$$V_1 = bx + e$$

$$V_2 = (b-1)x + e$$

where $E(e) = 0$.

Note that these two equations are consistent with the definitions of V_1, V_2 and x. In particular, by subtracting the second equation from the first, we get $V_1 - V_2 = x$.

By taking conditional expectations on both sides of these two equations, we can derive the following expressions for P_1 and P_2.

$$P_1 = E(V_1|x < q) = bE(x|x < q)$$

$$P_2 = E(V_2|x > q) = (b-1)E(x|x > q).$$

By definition,

$$b = \text{cov}\,(V_1, x)/\,\text{var}\,(x).$$

Since V_1 and V_2 are normally distributed, $x = V_1 - V_2$ is also normally distributed with mean 0 and variance equal to $(\sigma_x)^2 = (\sigma_1)^2 + (\sigma_2)^2 - 2r\sigma_1\sigma_2$. Therefore, it is relatively straightforward to calculate $E(x|x < q)$ and $E(x|x > q)$ as functions of q. In particular,

$$E(x|x < q) = \int_{-\infty}^{q} \frac{xf(x)}{\text{prob}\,(x < q)} dx$$

where

$$f(x) = \frac{1}{\sigma_x\sqrt{2\pi}} \exp\left(-0.5\left(\frac{x}{\sigma_x}\right)^2\right)$$

Hence, after integrating we get:

$$E(x|x < q) = \frac{-\sigma_x \exp\left(-0.5\left(\frac{q}{\sigma_x}\right)^2\right)}{\text{prob}\,(x < q)\sqrt{2\pi}}$$

Similarly,

$$E(x|x > q) = \frac{\sigma_x \exp\left(-0.5\left(\frac{q}{\sigma_x}\right)^2\right)}{\text{prob}\,(x > q)\sqrt{2\pi}}$$

Using these calculations we get,

$$\begin{aligned}
q &= P_1 - P_2 \Rightarrow \\
q &= bE(x|x < q) - (b-1)E(x|x > q) \Rightarrow \\
b &= \frac{q - E(x|x > q)}{(E(x|x < q) - E(x|x > q))} \Rightarrow \\
b &= N\left(\frac{q}{\sigma_x}\right) - \frac{qN\left(\frac{q}{\sigma_x}\right)\left(1 - N\left(\frac{q}{\sigma_x}\right)\right)\sqrt{2\pi}\exp\left(0.5\left(\frac{q}{\sigma_x}\right)^2\right)}{\sigma_x}
\end{aligned}$$

where $N()$ denotes the standard normal distribution function. Note that the right-hand side of the equation is a function of q, since all other variables on the right-hand side are constants. Define the right-hand side as $H(q)$ and rewrite this equation as

$$b = H(q).$$

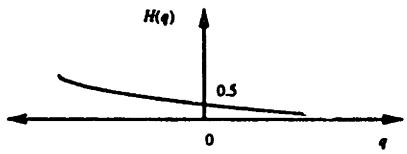
A separating equilibrium, if it exists, consists of a value of q such that this equality holds.

Note that b, as defined earlier, is a function of the correlation coefficient r for given standard deviations σ_1 and σ_2. Since σ_1 and σ_2 are assumed to be constants, whereas r varies between -1 and 1, the value of q for which this equality holds will depend on r. Furthermore, note that once we obtain such a q for a certain r, and hence a certain b, P_1 and P_2 are determined from

$$\begin{aligned} P_1 &= bE(x|x < q) \\ P_2 &= (b-1)E(x|x > q) \end{aligned}$$

since the conditional expectations are functions of q only.

Numerical analysis on a computer of the function $H(q)$ indicated that it has the following shape (we normalized $\sigma_x = 1$).



By definition,

$$\begin{aligned} b &= \frac{\text{cov}\,(V_1, x)}{\text{var}\,(x)} \\ &= \frac{(\sigma_1)^2 - r\sigma_1\sigma_2}{(\sigma_x)^2}. \end{aligned}$$

First, suppose $\frac{\sigma_1}{\sigma_2} \leq r \leq 1$; then, $b \leq 0$ and hence there is no value of q that satisfies $b = H(q)$ (since $H(q) > 0$). Therefore, we do not get a separating equilibrium. Now suppose $-1 \leq r < \frac{\sigma_1}{\sigma_2}$. Then $0 < b < 0.5$ and hence there exists a positive value of q such that $b = H(q)$. Therefore, we do get a separating equilibrium. The positivity of q, which

implies $P_1 > P_2$ in this separating equilibrium was expected. Since asset 2 is riskier than asset 1, the buyer is only willing to buy it at a lower price than asset 1.

Note that at this separating equilibrium, both P_1 and P_2 are negative. To see this, we plug the expressions derived above for the conditional expectations of x, i.e., $E(x|x < q)$ and $E(x|x > q)$ into the equations for P_1 and P_2.

$$\begin{aligned} P_1 &= bE(x|x < q) \\ &= \frac{-b\sigma_x \exp\left(-0.5\left(\frac{q}{\sigma_x}\right)^2\right)}{\text{prob}\,(x < q)\sqrt{2\pi}} \end{aligned}$$

Since $0 < b < 0.5, -b < 0$ and hence $P_1 < 0$. Likewise, it can be shown that $P_2 < 0$.

The negativity of P_1 and P_2 may appear somewhat surprising since both assets have a mean of zero and buyers/sellers are risk neutral. However, there is a simple explanation: recall that this separating equilibrium is obtained only when $-1 \leq r < \frac{\sigma_1}{\sigma_2}$, i.e., in situations in which the two assets are not perfectly positively correlated; under such circumstances, the seller will decide to sell asset 1 (asset 2) when its value is low because the value of asset 2 (asset 1) is high; to protect himself from the seller's right to choose, the buyer requires a price below the asset's mean (zero), i.e., a negative price. Finally, note that when the two assets are equally risky, i.e., when $\sigma_1 = \sigma_2$,

$$\begin{aligned} b &= \frac{((\sigma_1)^2 - r\sigma_1\sigma_2)}{(\sigma_x)^2} \\ &= \frac{((\sigma_1)^2 - r\sigma_1\sigma_2)}{((\sigma_1)^2 + (\sigma_2)^2 - 2r\sigma_1\sigma_2)} \\ &= \frac{((\sigma_1)^2 - r(\sigma_1)^2)}{((\sigma_1)^2 + (\sigma_1)^2 - 2r(\sigma_1)^2)} \end{aligned}$$

$$= \frac{((\sigma_1)^2 - r(\sigma_1)^2)}{2((\sigma_1)^2 - r(\sigma_1)^2)} = 0.5.$$

Since $H(q) = 0.5$ when $q = 0$, we get $b = H(q)$ when $q = 0$ and hence a separating equilibrium with $P_1 = P_2$. This is not surprising since the two assets are equally risky. Nevertheless, note that P_1 and P_2 are still negative for the above mentioned reason. We summarise these results in Lemma 1:

Lemma 1:

(a) For $-1 \leq r < \frac{\sigma_1}{\sigma_2}$ there exists a separating equilibrium with $q > 0$ and $P_2 < P_1 < 0$; in the special case $\sigma_1 = \sigma_2$, this separating equilibrium holds with $q = 0$ and $P_2 = P_1 < 0$.

(b) For $\frac{\sigma_1}{\sigma_2} \leq r \leq 1$ no separating equilibrium exists.

We now investigate the existence of a pooling equilibrium. A pooling equilibrium consists of prices P_1 and P_2 such that the seller is always forced to sell one of the two assets. One possible pooling equilibrium is one in which only asset 1 (the least risky asset) is sold at price $P_1 = 0$, i.e., at a price equal to its mean value. To support this equilibrium we set $P_2 = -\infty$; this ensures that the seller never sells asset 2. Note that since the seller is forced to always sell asset 1, the buyer is willing to buy it at a zero price (equal to its mean); this price is higher than the price of asset 1 in the separating equilibrium described earlier because in that equilibrium the seller had the right to choose and there were circumstances in which he would choose one rather than the other asset; in this pooling equilibrium the seller retains the right to choose but is forced to always choose asset 1 through the choice of P_2.

Similarly, it is possible to have a pooling equilibrium in which the seller always sells asset 2 at a price $P_2 = 0$, provided $P_1 = -\infty$. The reasoning is identical to the one provided for the other pooling equilibrium. Note, however, that if this game is repeated many times and its equilibrium is attained through a learning process, the pooling equilibrium in which asset 1 is always sold is more likely to hold; the reason is that in the early rounds of the game, the buyers will be much more willing to buy asset 1 since it entails a lower level of risk than asset 2.

We summarize these results in Lemma 2.

Lemma 2: The plausible pooling equilibrium is one in which asset 1 is always sold at a price equal to zero; if a seller deviates by offering asset 2, the buyer buys at $P_2 = -\infty$.

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