Towards Resilient Plug-and-Play Microgrids

by

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Abstract

Microgrids have the potential to increase renewable energy penetration, reduce costs, and improve reliability of the electric grid. However, today’s microgrids are unreliable, lack true modularity, and operate with rudimentary control systems. This thesis research makes contributions in the areas of microgrid modeling and simulation; microgrid testing and model validation; and advanced control design and tools in microgrids. These contributions are a step toward design, commissioning, and operation of resilient plug-and-play (pnp) microgrids, which will pave the way towards a more sustainable and electric energy abundant future for all.

Thesis Supervisor: James L. Kirtley

Title: Professor of Electrical Engineering
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To the people of my homeland, the people of Southern Cameroons, a free and dignified people: The words of Rev. Dr. Martin Luther King Jr. ring ever so true: “… the arc of the moral universe is long, but it bends toward justice”.

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Chapter 1

Introduction

1.1 The importance of microgrids

A microgrid is a group of interconnected loads and distributed energy resources (DERs) within clearly defined electrical boundaries that acts as a single controllable entity with respect to the grid and where microgrid can connect and disconnect from the grid to enable it to operate in both grid-connected or island-mode as defined by US DOE Energy Microgrid Exchange Group [1]. A simple microgrid comprising battery storage, diesel genset, PV, and wind turbine is illustrated in Fig. 1.

![Fig. 1. Illustration of a simple microgrid](image.png)

One might pose a valid question: why are microgrids so appealing and why is so much attention being focused on them? There is no unique answer but a collection of trends and data points that together paint a more coherent picture and can shed additional light on this topic.
1.1 The importance of microgrids

As the Industry 4.0 [2] is dawning on us Digitalization, Decarbonization, and Decentralization (D3) are three key trends fueling the electric grid revolution. Convergence of exponential reduction of cost of PV, proliferation of digital technologies, ubiquitous communication networks, and the desire to empower grid consumers and unlock grids potential flexibility and adaptive architecture are main drivers behind D3. In addition, climate change and the desire for improved energy security and resiliency are driving further the decentralization. Renewable energy sources such as wind, photovoltaic (PV), biomass, and pumped hydro are rapidly growing sources of electricity. In the U.S alone, they contribute more than 15% of net electricity generation [3]. Globally, they account for 24%, and this number is expected to keep growing.

Microgrids provide a path towards: decentralized and more resilient grid, more layered control architecture on the distribution network, and integration of renewables and other energy sources on the distribution network. They can also integrate combined heat and power (CHP), paving the way toward more complex energy networks, which also create additional value to businesses and communities [4].

1.1.1 Improving electric reliability and resilience

In 2012, Hurricane Sandy caused the deaths of many, and widespread destruction of property. In the US alone, it caused power outages to over 8 million customers across 21 states for days and weeks [5]. This and other major disasters have further motivated the development of microgrids because they can allow continued electricity access to pockets or neighborhoods within the disaster-affected regions while the main grid is unavailable [6]. Such microgrids can serve critical loads such as hospital life support systems, sterilization of food, water, and medical equipment, provide power for communications, power for help and rescue crews and equipment, and provide some minimum comfort during the disaster and subsequent recovery period.

1.1.2 Lowering energy costs or creating revenue

One emerging application of grid-connected microgrids is to provide ancillary services to the grid [7]. From the viewpoint of the grid or utility operator, a microgrid can appear as a dispatchable load or energy source with fine granularity. Examples of auxiliary services include load/energy curtailment, power dispatch, frequency and voltage support. Such services can offset electricity costs and even generate net revenue for the microgrid owner.
1.1.3 **Enabling electricity access**

In areas with weak or non-existent grid, microgrids provide a natural way to grow decentralized power networks, from bottom up, by building independent microgrids and networking them together [8]. Indeed, it is estimated that more than 1 billion people in the developing world require access to electricity [9]. Many of these people live in remote and often enclaved areas that have no electric grid infrastructure and little chance of building one anytime soon. Small-scale microgrids can be deployed to address these needs in the short to medium term. Compared to traditional grids that require massive capital investments in transmission lines and large scale power generators, such small-scale power systems require lower capital costs.

1.2 **Plug-and-play microgrids and their challenges**

While microgrids hold significant promise, their wider proliferation has been hindered by large design cost, unreasonable controller costs, non-recurring engineering costs, cost of commissioning and life cycle maintenance to name a few. Many of these problems can be alleviated with interface standardization, modular approach, more intelligent control, in one word with plug-and-play approach. Indeed, the ideal microgrid should have a true plug-and-play characteristic. It should allow normal, seamless integration of any new loads or sources without the need for complex re-engineering. Some challenges stand in the way of achieving this objective. Broadly, most of these challenges can be organized into four main categories as follows:

1.2.1 **Stability**

Compared to traditional power systems, microgrids are different in that: (i) they tend to have lower levels of rotating inertia since they are mostly based on power-electronics interfaced DERs [7]; (ii) the power coupling and distribution systems tend to be less inductive and more resistive [10]; (iii) the energy sources, many of which are renewable, may be variable or intermittent.

Together, these challenges result in small signal and large signal stability issues that require advanced design and testing tools, as well as more intelligent controls in order to predict and overcome them.
1.2 Plug-and-play microgrids and their challenges

1.2.2 Communications and interoperability
Current microgrid models usually employ a communication network that connects individual microgrid resources from different vendors to each other and/or to a centralized microgrid controller. There is a large number of different communications protocols, each with its own set of different configuration options and flavors [11]. In addition, each DER implements vendor-specific and sometimes obscure control algorithms. Furthermore, there is a need to develop or improve on clear specification of microgrid controller, and DER functionality, as well as information exchange model. These challenges can hamper the smooth development, deployment, operation, and expansion of microgrids.

Recently, efforts are being made to standardize microgrid control specifications. For example IEEE 2030.7 [12] defines functional specifications of a microgrid controller, such as dispatch, transitioning between island and grid-connection, etc., that should be common among vendors.

Another standard, IEEE 2030.8 [13] specifies testing requirements of microgrid controllers. The controllers are tested while carrying out the functions defined in IEEE 2030.7 such as dispatch, islanding, power ramp up/down, etc., while key measurements are taken and compared with the pre-set requirements.

The Sunspec Alliance [14] is a consortium of DER equipment manufactures and software developers that aims to achieve open interoperable specification of information models for stakeholders in order to achieve plug-and-play microgrids. One application of this initiative is the Sunspec Modbus Specification, which adds a layer of abstraction on top of the native Modbus register specification. It allows the Sunspec user to focus on the functions of each register by using intuitive easy-to-remember register names, rather than trying to remember or replicate difficult native Modbus registers and addresses.

Even though they are far from resolving the interoperability issues, such efforts are a great step in the direction of enabling plug-and-play microgrids.

1.2.3 Microgrid testing and validation
Historically, the layered control architecture of the traditional power grid comprises three main levels that occur on different time scales [15] illustrated in Fig. 2.
(i) Primary controls: Occur on the time scale of seconds. They concern short-term stability and power balance dynamics. The power-frequency and reactive power-voltage droops are primary control mechanisms. Primary control creates instantaneous frequency errors.

(ii) Secondary controls: They occur on the time scale of minutes. Their function is to restore frequency to its nominal value and balance generation between different power generation areas. A slow communication network called Supervisory Control and Data Acquisition (SCADA) is often used to enforce this control layer.

(iii) The tertiary controls concern long term planning on the time scale of hours. It implements algorithms such as power flow optimization, dispatch, and unit commitment.

Compared to the traditional power system, microgrid dynamics occur on much faster timescales as illustrated in Fig. 3 that require new approaches in microgrid design, control and testing. These faster time scales occur because the microgrids are dominated by power electronics-interfaced DERs usually powered by variable energy sources. The amount of rotating inertia in microgrids is significantly lower, and therefore faster dynamics are more common than in large traditional grids.

Whereas in traditional power grid expansion and interconnection studies, a simple power flow analysis might be sufficient, microgrids require a completely new set of tools such as high bandwidth controller hardware-in-the-loop (cHIL) for microgrid, sometimes referred to as
1.3 Thesis contributions

microgrid digital twin, in order to accurately model, design, simulate, deploy, operate, and extend them.

Microgrid dynamics and dominant time scales

1.2.4 Regulatory and institutional

As microgrids are growing in numbers, the push for more comprehensive regulatory framework is increasing. Rights and responsibilities of microgrid developers, owners, and operators are very opaque and undefined. Legal frameworks are not well defined in areas from interconnection requirements to electricity market participation.

1.3 Thesis contributions

These four major challenges are creating significant barriers toward faster adoption of microgrids. In this thesis, we tackle the first three and provide new results, insights, designs, and new tools that are paving the way towards more flexible, cheaper, and smarter plug-and-play microgrids.

The three key contributions of this thesis are as follows:
1. **Microgrid modeling and simulation:** This set of contributions addresses the need for high-fidelity microgrid component models and system level models.

2. **Microgrid testing and model validation:** This set of contributions addresses a gap in model validation of high bandwidth DER and microgrid models. A real microgrid testbed is constructed and used for validating and improving DER and microgrid models.

3. **Advanced controls and tools** in microgrids: In this set of contributions, we propose and demonstrate a new cascaded state feedback adaptive controller for 3 phase inverters. A framework for model identification and control optimization of dynamic microgrids is developed.

### 1.4 Thesis scope and organization

Modern microgrids are true cyber-physical systems. They comprise power and energy processing devices, and control and communication layer (digital controls). We attempt to present the contributions of this thesis in a similar way.

In the earlier chapters of this thesis, we focus on modeling of microgrid DER components. An analysis of wound-rotor synchronous generators uncovers unstable behavior that can cause challenges in islanded microgrids if not properly understood and addressed (Chapter 2). A detailed natural gas engine model is developed (Chapter 3) for microgrid simulation.

In Chapter 4, a microgrid model consisting of genset, inverter, and load is constructed and used to develop a framework for model identification and controller optimization on controller hardware-in-the-loop (cHIL) platform.

A cascaded state-feedback adaptive controller is developed in Chapter 5 for inverter control. By automatically adjusting the controller gains online, and in real time, the proposed controller enables good inverter performance even with variations in the inverter parameters. This helps minimize stability issues when plugging an inverter into a microgrid with different characteristics from the design condition.

In the first part of Chapter 6, a prototyping platform for hardware-in-the-loop integration of microgrid device controllers is presented. The concept of *Digital Twin* is explored and applied to a complex microgrid model consisting of diverse DER models, and physical DER controllers.
1.4 Thesis scope and organization

In the second part of the chapter, a simple physical low-voltage microgrid testbed is constructed, consisting of synchronous-machine and inverter-based DERs. The testbed is used to test and validate individual microgrid components, DER models, and overall microgrid simulation.

A general conclusion of the thesis, appendices, and bibliography complete this work.
Chapter 2

Unstable Equilibrium Points in Standalone Synchronous Generator

2.1 Introduction

Synchronous generators with wound rotors can be considered the backbone of the power system and have been studied by engineers for many decades. Detailed models of this machine have been developed over time, in particular the two-axis model [16] which is widely applied in numerical simulations. Reduced order models of the machine have been developed which are suitable for large traditional grids. However, with increasing interest in microgrids, higher order machine models are more appropriate because microgrids can experience relatively larger disturbances, and have fewer actuators to mitigate their effects than traditional power systems.

In order to develop suitable controllers (prime mover and excitation), it is necessary to understand the machine dynamics. Going from the large signal nonlinear model in Krause [17] supplying a standalone and passive load, we find that the small signal model with electrical states in flux linkages per second can have a right half-plane (RHP) pole. Further investigation of the nonlinear model via phase plane analysis and time domain simulation reveals that there are many unstable equilibrium points in the standalone condition. One would expect, a priori, for the machine to reject slight disturbances when supplying a passive, standalone load in the open loop.

1 A version of this chapter was published in E Fonkwe et al., “Unstable Equilibrium Points in Standalone Synchronous Generator”, IEEE Energy Conversion Congress and Exposition (ECCE) 2018, pg 5786-5790
2.1 Introduction

In section 2.2 of this chapter, we recall the model and associated equations. In section 2.3, we explore the eigenvalues as we vary the load at different terminal voltages. We also examine the trajectories of the nonlinear model for different initial conditions. We conclude the chapter in section 2.4. Unless otherwise stated, the following nomenclature in Table 1 defines all variables and parameters used in this chapter. The machine parameters provided are for a 4MVA, 13.8kV rms line-line synchronous machine.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_s$</td>
<td>Stator winding resistance ($\Omega$)</td>
<td>0.427 $\Omega$</td>
</tr>
<tr>
<td>$L_{ls}$</td>
<td>Stator leakage inductance (H)</td>
<td>0.006 H</td>
</tr>
<tr>
<td>$L_{md}$</td>
<td>d-axis magnetizing inductance (H)</td>
<td>0.297 H</td>
</tr>
<tr>
<td>$L_{mq}$</td>
<td>q-axis magnetizing inductance (H)</td>
<td>0.217 H</td>
</tr>
<tr>
<td>$r_{fd}$</td>
<td>Field winding resistance ($\Omega$)</td>
<td>0.098 $\Omega$</td>
</tr>
<tr>
<td>$L_{f0d}$</td>
<td>Field winding leakage inductance (H)</td>
<td>0.064 H</td>
</tr>
<tr>
<td>$r_{kd}$</td>
<td>d-axis damper winding resistance ($\Omega$)</td>
<td>13.455 $\Omega$</td>
</tr>
<tr>
<td>$r_{kq}$</td>
<td>q-axis damper winding resistance ($\Omega$)</td>
<td>1.212 $\Omega$</td>
</tr>
<tr>
<td>$L_{kd}$</td>
<td>d-axis damper winding leakage inductance (H)</td>
<td>0.472 H</td>
</tr>
<tr>
<td>$L_{kq}$</td>
<td>q-axis damper winding leakage inductance (H)</td>
<td>0.030 H</td>
</tr>
<tr>
<td>$J$</td>
<td>Machine moment of inertia (kg.m$^2$)</td>
<td>78.08 kg.m$^2$</td>
</tr>
<tr>
<td>$V_{g,rms}$</td>
<td>Grid line-to-neutral rms voltage (V)</td>
<td>7967 V</td>
</tr>
<tr>
<td>$f_g$</td>
<td>Grid frequency (Hz)</td>
<td>60 Hz</td>
</tr>
<tr>
<td>poles</td>
<td>Number of machine poles</td>
<td>4</td>
</tr>
<tr>
<td>$\Psi_{qs}, \Psi_{ds}$</td>
<td>q-axis and d-axis stator flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>$\Psi_{kq}$</td>
<td>Stator-referred q-axis damper winding flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>(\Psi_{rd})</td>
<td>Stator-referred field winding flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>(\Psi_{kd})</td>
<td>Stator-referred d-axis damper winding flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>(V'<em>{qs}, V'</em>{ds})</td>
<td>q-axis and d-axis grid voltage in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>(r_x, L_x)</td>
<td>Line resistance and inductance (Ω; H)</td>
<td></td>
</tr>
<tr>
<td>(0.1 \text{ mΩ}; 0.1 \text{ mH})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\omega_r)</td>
<td>Rotor electrical speed (electrical rad/s)</td>
<td></td>
</tr>
<tr>
<td>(\omega_b)</td>
<td>Base rotor electrical speed (electrical rad/s)</td>
<td></td>
</tr>
<tr>
<td>(377 \text{ rad/s})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(V'<em>{qs}, V'</em>{sd})</td>
<td>q-axis and d-axis machine terminal voltage in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>Power angle (electrical rad)</td>
<td></td>
</tr>
<tr>
<td>(T_e)</td>
<td>Electromagnetic torque (Nm)</td>
<td></td>
</tr>
<tr>
<td>(T_{mech})</td>
<td>Mechanical torque (Nm)</td>
<td></td>
</tr>
<tr>
<td>(i_{qs}, i_{ds})</td>
<td>q-axis and d-axis stator currents in rotor reference frame (A)</td>
<td></td>
</tr>
<tr>
<td>(v_{fd})</td>
<td>Stator-referred field voltage (V)</td>
<td></td>
</tr>
<tr>
<td>(X_{q}, X_{d})</td>
<td>q-axis and d-axis stator winding reactance (Ω)</td>
<td></td>
</tr>
<tr>
<td>(X_{mq}, X_{md})</td>
<td>q-axis and d-axis stator-referred magnetizing reactance (Ω)</td>
<td></td>
</tr>
<tr>
<td>(X_{ls})</td>
<td>Stator winding leakage reactance (Ω)</td>
<td></td>
</tr>
<tr>
<td>(X_{kq}, X_{kd})</td>
<td>q-axis and d-axis damper winding reactance (Ω)</td>
<td></td>
</tr>
<tr>
<td>(X_{fd})</td>
<td>Field winding reactance (Ω)</td>
<td></td>
</tr>
<tr>
<td>(X_{lkq}, X_{lkd})</td>
<td>q-axis and d-axis damper winding leakage reactance (Ω)</td>
<td></td>
</tr>
<tr>
<td>(X_{fld})</td>
<td>Field winding leakage reactance (Ω)</td>
<td></td>
</tr>
</tbody>
</table>

Subscript to represent the linearization (operating) point of a state

Prefix to represent small signal notation
2.2 Machine model

To be consistent with [17], we use flux linkages per second (i.e. flux linkages multiplied by $\omega_b$) as the electrical states. A circuit model of the system is shown in Fig. 4, and the state equations that describe it are given in equation (2.1). Note that by closing the switch, the machine is connected to infinite bus. Opening the switch is standalone connected to passive load.

where:

$$\dot{\omega}_r = \left(\frac{\text{poles}}{2}\right) \frac{1}{J} \left(T_{mech} - T_e\right)$$

$$\delta = \omega_r$$

$$T_e = \left(\frac{3}{2}\right) \left(\frac{\text{poles}}{2}\right) \left(\frac{1}{\omega_b}\right) \left(\psi_{qs} \dot{i}_{qs} - \psi_{ds} \dot{i}_{ds}\right)$$

Parameters $a_k, b_k, d_k, e_k, f_k, g_k, h_k, m_k, n_k, \theta_k, p_k, q_k, r_k$ are defined in Appendix A. The modeling approach is that adopted in [18], where the inductance and resistance of the line (and load, if in standalone) are lumped into the machine inductance and resistance. The operating point is determined by the desired $P_{load}, Q_{load}$, and $v_r$ which determines the series R-L parameters of the passive load. Initial conditions are determined by computing the required $v_{jq}, T_e(0)$, and solving for flux linkages per second from equation (2.1) after setting the time derivatives to 0.
2.3 Eigenvalue and trajectory analysis

2.3.1 Small signal analysis

The small signal linearized state space model is stated in equation (2.3), where the parameters $s_{kk}$, $t_{kk}$, $w_{kk}$, $x_{kk}$, $y_{kk}$ are defined in Appendix A. Note that in the standalone case, we remove the power angle state from equation (2.3), as it is unnecessary to fully describe the system.

\[
\begin{bmatrix}
\Delta \psi'_{qs} \\
\Delta \psi'_{ds} \\
\Delta \psi'_{qs} \\
\Delta \psi'_{fd} \\
\Delta \psi'_{kd} \\
\Delta \dot{\omega}_r \\
\Delta \delta
\end{bmatrix} =
\begin{bmatrix}
a_k & -\omega_{r(0)} & b_k & 0 & 0 & -\psi'_{ds(0)} & -\psi'_{ds(0)}\omega_b \\
\omega_{r(0)} & d_k & 0 & e_k & f_k & \psi'_{qs(0)} & \psi'_{qs(0)}\omega_b \\
g_k & 0 & h_k & 0 & 0 & 0 & 0 \\
0 & m_k & 0 & n_k & \theta_k & 0 & 0 \\
0 & p_k & 0 & q_k & r_k & 0 & 0 \\
s_{kk} & t_{kk} & u_{kk} & w_{kk} & x_{kk} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \psi'_{qs} \\
\Delta \psi'_{ds} \\
\Delta \psi'_{qs} \\
\Delta \psi'_{fd} \\
\Delta \psi'_{kd} \\
\Delta \omega_r \\
\Delta \delta
\end{bmatrix}
\]

(2.3)

We are interested in the open loop poles of the linearized standalone system under varying load conditions. We vary the load between 0.1MW – 4MW; 0.1MVAr – 2MVAr (lagging) at terminal voltages between 10kVrms and 26kVrms line-line. In Fig. 5 we show the locations of the dominant pole with changes in $P_{load}$ and $v_t$ for a reactive power of 0.1MVAr (lagging) and terminal voltage of 13.8kV rms line-line. The red vertical line separates the left-half plane (LHP) from the RHP of the $s$-plane. Decreasing $P_{load}$ and/or increasing $v_t$ causes the dominant pole to move away from the unstable region.

In Fig. 6, we show the linearized small signal stability of the standalone system as a function of $P_{load}$, $Q_{load}$, and $v_t$. All Hurwitz stable operating points are plotted in yellow, and blue if unstable; it can be seen that the small signal stable region grows with increasing terminal voltage. All
2.3 Eigenvalue and trajectory analysis

equilibrium points considered in Fig. 6 are stable and attractive when the machine is grid-tied - we do not show more results due to space constraints.

![Eigenvalue and trajectory analysis](image1)

**Fig. 5. Variations of dominant pole location with varying load power and terminal voltage**

**Fig. 6. Small signal stable regions (in yellow) and unstable regions (blue) under varying load and terminal voltage conditions for standalone system**

From participation factor (ppf) analysis [19], it appears that out of all the machine states, the speed and field flux are dominant, including those cases of small signal instability that have been analyzed. Fig. 7 shows the absolute (de-normalized) ppf for the speed and field flux states ($\omega_r$, and $\psi_{fd}'$), where the color code is as follows:

(i) Red semi-surface and yellow semi-surface represent ppf values for speed and field flux (respectively), where the system is small signal unstable
(ii) Green semi-surface and blue semi-surface represent ppf values for speed and field flux (respectively), where the system is small signal stable.

It can be seen that the speed state is mostly dominant in the open semi-surface showing instability for the voltage levels considered.

![Image of participation factors of rotor speed and field flux states](image)

*Fig. 7. Participation factors of rotor speed and field flux states*

### 2.3.2 Nonlinear system trajectories

To supplement the Hurwitz-unstable observations, we employ phase plane analysis of the nonlinear system. In order to easily visualize the phase portrait, we reduce the model order from 6 to 2 states which are $\omega_r$, and $\psi'_{fd}$. The choice is motivated by the following reasons: (i) from experience [17], stator quantities are fast and can be considered algebraic. This eliminates $\psi'_{qs}$ and $\psi'_{ds}$ as states; (ii) damper winding states are assumed to be faster than the field, removing $\psi'_{kq}$ and $\psi'_{kd}$ as states. (iii) We have just shown from participation factor analysis in Fig. 7 that $\omega_r$, and $\psi'_{fd}$ are dominant.

So we have a differential algebraic system that can be written as equation (2.4).
2.3 Eigenvalue and trajectory analysis

\[
\begin{align*}
\dot{\psi}_f = m_k \psi_f' + n_k \psi_f' + \theta_k \psi_f' + \omega_k \psi_f' ; \\
\dot{\omega}_r = \left( \frac{\text{poles}}{2J} \right) (T_{\text{mech}} - T_r) \\
\end{align*}
\]

We consider one Hurwitz-stable equilibrium point at $P=1\text{MW}; Q=0.1\text{MVAr}$ (lagging) for $v_t = 13.8\text{kVrms Line-Line}$ (see Fig. 6(b)). The trajectories for a number of initial conditions about this equilibrium point are overlaid on the phase plane, as shown in Fig. 8. Starting points are circles and end points are squares. The vertical and horizontal axes are $\omega_r$ and $\psi_{fd}$ respectively. The equilibrium point is stable and attractive.

However, consider an operating point in the Hurwitz-unstable regions of Fig. 6, for example the operating point: $P=2\text{MW}; Q=1\text{MVAr}$ (lagging) for $v_t = 13.8\text{kVrms Line-Line}$. A phase portrait is shown in Fig. 9 where it can be seen that the equilibrium point (represented by circle close to center of the figure) is unstable. Small deviations around that point cause the system to deviate significantly from the steady state speed. Depending on the initial condition, we can see that the system settles to another attractive equilibrium point.

In the phase portraits (Fig. 8 and Fig. 9), the vectors of the phase plane (shown as red arrows) are obtained from a reduced order model of the nonlinear system, whereas the trajectories (solid lines) are those of the full order nonlinear system.

Fig. 10 is a time domain simulation of the full order non-linear system corresponding to the black trajectory of Fig. 8. Speed is perturbed at time $t=0$, and we can see that the equilibrium point is stable and all the states return to the original pre-disturbance values.

Conversely Fig. 11, is a time domain simulation of the full order non-linear system corresponding to the black trajectory of Fig. 9. We see that for a small perturbation about the equilibrium point (unstable), the states are repelled towards a new equilibrium point.
Fig. 8. Phase portrait for attractive equilibrium point of nonlinear system

Fig. 9. Phase portrait for unstable equilibrium point of nonlinear system
2.3 Eigenvalue and trajectory analysis

2.3.3 Nonlinear system equilibrium points

The equilibrium points, which are the roots of equation (2.1), can be computed using a standard nonlinear solver. For example, starting from the unstable operating point: \( P=2\text{MW}; Q=1\text{MVAr} \) (lagging) for \( v_l=13.8\text{kVrms}\ Line-Line \), with inputs \( T_{\text{mech}}=10.67k\text{Nm}; v_{\text{fd}}=19.69\text{V} \), we can identify 2 equilibrium points which are:

Fig. 10. Time-domain full-order nonlinear system states for the black trajectory from Fig. 8 (stable equilibrium point)

Fig. 11. Time domain full-order nonlinear system states for the black trajectory from Fig. 9 (unstable equilibrium point)
Unstable Equilibrium Points in Standalone Synchronous Generator

\[
\begin{bmatrix}
\psi_{qs}(0) & \psi_{ds}(0) & \psi_{kq}(0) & \psi_{kd}(0) & \omega_r(0)
\end{bmatrix}_{\text{equilibrium 1 (unstable)}} = \begin{bmatrix}
10^-4 
-0.8590V & 0.5378V & -0.5749V & 1.4800V & 0.9916V & 0.037rad.s^{-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\psi_{qs}(0) & \psi_{ds}(0) & \psi_{kq}(0) & \psi_{kd}(0) & \omega_r(0)
\end{bmatrix}_{\text{equilibrium 2 (stable)}} = \begin{bmatrix}
10^-4 
-0.7558V & 1.8647V & -0.5058V & 2.4545V & 1.9661V & 0.0096rad.s^{-1}
\end{bmatrix}
\] (2.5)

We think it is possible that there could be more equilibria. The equilibrium point that the solver returns depends on the initial conditions of the nonlinear solver.

2.4 Conclusion

A linearized small signal analysis of the standalone synchronous generator with two damper windings revealed many Hurwitz-unstable equilibrium points, which is supported by phase portrait analysis of the nonlinear system. These results can enable more suitable design of torque (prime mover) and excitation controllers. As future work, it could prove interesting to search for more stable equilibrium points in the vicinity of the unstable ones.
Chapter 3

Improved Natural Gas Engine Model for Microgrid Simulation

3.1 Introduction

Natural gas is the primary source of U.S. electricity generation, accounting for about 34% of total generation in kWh [20]. It mainly consists of methane (CH₄) with varying amounts of other hydrocarbons such as ethane, propane, butane, and pentane. The amount of energy in kWh from natural gas is expected to grow as more coal-fired plants are being replaced by natural gas units.

In the US alone, it is estimated that 82% of current microgrid installed capacity is based on fossil fuels of which diesel accounts for about 60%, while natural gas accounts for 40% and rising [21]. Diesel has traditionally been the main backup option due to its widespread availability, and engine reliability. However, compared to diesel, natural gas has a number of advantages:

1. It is more environmentally-friendly: Per unit of thermal energy produced, diesel emits roughly 38% more CO₂ than natural gas [22]. Even though this number can be mitigated somewhat by considering a more complete life cycle analysis of methane production, transportation, and conversion, it is easy to motivate increasing usage of natural gas in future microgrids at the expense of diesel.

2. Diesel prices are more easily affected by natural disasters: For example, one impact of Hurricane Harvey in August 2017 was a sharp rise in the price of diesel and other refined
3.2 Internal combustion (I.C) engine models

fuels [23]. This was mainly due to the shutdown of most of the refineries in the affected areas, and the massive flooding of road transportation networks. In addition, even when fuel delivery is possible in the aftermath of disaster, relief forces (such as the National Guard) can commandeer commercial fuel trucks to power their relief efforts. On a side note, solar photovoltaic (PV) energy and battery storage quickly run out during some major disasters such as hurricanes, due to insufficient sunlight for possibly several days.

3. On the other hand, natural gas enjoys a widespread underground network of pipelines from diverse natural gas producers at different geographical locations. As a result, it is less likely that a single major disaster will halt gas delivery to the entire affected area.

These reasons motivate interest in studying natural gas gensets for microgrid applications.

In this chapter, we develop a mean value dynamic gas engine model combining some first principles and phenomenological relationships. We propose improved fitting functions for engine parameters such as thermal efficiency, and volumetric efficiency. In the case where engine data is not available for a given simulation, or a generic engine is to be simulated, we also propose equations to predict major engine parameters from a few specifications such as engine nominal output power.

3.2 Internal combustion (I.C) engine models

IEEE has proposed several standard gas turbine models for power systems studies [24]. However, such standard models are not available for reciprocating internal combustion (I.C) engines, which make up the vast majority of genset units. This may be expected because I.C gensets have not traditionally been considered as major power systems sources.

In many studies that consider diesel gensets, the engine dynamics are often represented by a fixed transport delay [25][26] as shown in Fig. 12. The governor and valve (actuator) are usually represented by a series of time constants ($T_1-T_3$, and $T_4-T_6$, respectively) with little, if any, justification provided for the values of these constants. Such simplified models may have been motivated by the fact that the traditional power grid normally sees only minor disturbances in frequency. However, in a realistic microgrid, the genset is expected to operate often through more
extreme events such as black-starting, greater frequency excursions, and larger load transients. Therefore, the traditional simplified models may be inadequate.

Look-up table (LUT)-based models [27], can provide more model fidelity since they contain nonlinear torque-speed mapping data. However, this modeling approach obscures much of the engine physics from the modeling application.

Unlike diesel engines that operate by compression ignition (C.I), gas engines usually operate with a spark ignition (S.I) mechanism. Therefore, reducing the engine dynamics to a simple transport delay is not sufficient because intake manifold dynamics must be considered [28].

Natural gas engines have a ‘lean burn’ feature that can be exploited to increase engine efficiency. Lean burning refers to increasing the air-fuel ratio above the stoichiometric value, $\lambda_{eq}$. The stoichiometric air-fuel ratio is the mass of air that is required to completely burn a given mass of fuel with no excess air or excess fuel remaining. Higher air fuel ratios make the engine run cooler (and therefore more efficiently) up to a point, beyond which the probability of engine knock goes up significantly. Lean burning also helps reduce NO$_x$ emissions (due to lower combustion temperature) which is a desirable attribute. Often, the normalized or specific air-fuel ratio, $\lambda_{sp}$, is used instead of the absolute air-fuel ratio. The specific air-fuel ratio is the absolute air-fuel ratio divided by the stoichiometric air-fuel ratio.

Different fuel types are used in natural gas engines. Table 2 gives details about the characteristics of some commonly used fuels.
Table 2. Different fuel types for gas engine

<table>
<thead>
<tr>
<th>Fuel Type</th>
<th>Composition</th>
<th>Lower Heating Value (J/kg)</th>
<th>Molar Mass of Fuel (kg/mol)</th>
<th>Stoichiometric air-fuel ratio (grams of air to grams of fuel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural Gas</td>
<td>Methane (mostly)</td>
<td>$40.71 \times 10^6$</td>
<td>$19.5 \times 10^{-3}$</td>
<td>17.2</td>
</tr>
<tr>
<td>Biogas[29]</td>
<td>Methane: 60% Carbon dioxide: 38% Nitrogen: 2%</td>
<td>$18.57 \times 10^6$</td>
<td>$20.83 \times 10^{-3}$</td>
<td>7.98</td>
</tr>
<tr>
<td>Coke Gas[30][31]</td>
<td>Hydrogen: 70% Methane: 30%</td>
<td>$36.72 \times 10^6$</td>
<td>$6.22 \times 10^{-3}$</td>
<td>21.13</td>
</tr>
<tr>
<td>Wood Gas[32]</td>
<td>Nitrogen: 50.9% Carbon Monoxide: 27% Hydrogen: 14% Carbon dioxide: 4.5% Methane: 3%; Oxygen: 0.6%</td>
<td>$5.7 \times 10^6$</td>
<td>$24.66 \times 10^{-3}$</td>
<td>2.245</td>
</tr>
</tbody>
</table>

3.3 Natural gas engine modeling

The proposed natural gas engine model is based on a mean-value approach that combines first principles (e.g. mass flow over a throttle, filling/emptying of a manifold) and phenomenological relationships (e.g. volumetric efficiency, thermal efficiency). Mean value models are designed to accurately simulate average engine torque without more complicated cylinder-to-cylinder simulations [28][33]. Mean value models are generally suitable for model-based controls. The 4-stroke engine type is considered. Even though 2-stroke S.I. engines generally have a higher power-
to-weight ratio compared to 4-stroke engines, they tend to be less fuel efficient, and emit more hydrocarbons, primarily due to a higher occurrence of engine misfire [34]. The extra weight of 4-stroke engines may not be as much of a concern as the thermal efficiency and emissions, particularly for gensets that are generally used in stationary or quasi-stationary applications. It is easy to see why 2-stroke genset applications are uncommon [35].

Many gas engine models in the literature consider engine speed as an engine state, and generally assume a constant load torque or a known load torque profile [33]. This is not realistic for power system applications because the load is electromagnetic in origin and varying.

Lean burn engine models generally ignore effects of varying specific air-fuel ratio, $\lambda_{sp}$, on shaft torque [28], yet increasing $\lambda_{sp}$ is one of the main benefits that we seek to achieve by using natural gas engines. The proposed model considers the effects of varying $\lambda_{sp}$ on shaft torque variation using a statistical approach.

A requirement for accurate natural gas engine modeling is knowledge of certain key engine parameters such as intake manifold volume, throttle area, and engine cylinder count [28]: However, such parameters may not be readily available especially considering a desired engine rating in a specific microgrid simulation.

Furthermore, in the absence of a specific engine datasheet, power systems specialists may not be well-versed in selecting geometric engine parameters that affect steady state and dynamical behaviors of the system. Our proposed model attempts to generate realistic physical engine parameters from a few specifications, such as nominal shaft power and speed.

Finally, the proposed model should be simple enough to be simulated on a real-time simulation platform.

Unless otherwise stated, symbols used in this chapter are provided in the nomenclature in Table 3.

### Table 3. Nomenclature for natural gas engine model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}_{ai}$</td>
<td>Input air mass flow rate over throttle (kg/s)</td>
</tr>
<tr>
<td>$\dot{m}_{fi}$</td>
<td>Input fuel mass flow rate over throttle (kg/s)</td>
</tr>
</tbody>
</table>
### 3.3 Natural gas engine modeling

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}_i$</td>
<td>Charge flow rate over throttle (into intake manifold) (kg/s)</td>
</tr>
<tr>
<td>$\dot{m}_o$</td>
<td>Charge flow rate out intake manifold (into cylinders) (kg/s)</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Rate of change of charge inside intake manifold (kg/s)</td>
</tr>
<tr>
<td>$\eta_{volumetric}$</td>
<td>Volumetric efficiency at any pressure</td>
</tr>
<tr>
<td>$\eta_{volumetric,ambient}$</td>
<td>Ambient volumetric efficiency (ambient condition refers to pressure at throttle inlet)</td>
</tr>
<tr>
<td>$\eta_{rpm}$</td>
<td>Engine speed (rpm) at peak volumetric efficiency</td>
</tr>
<tr>
<td>$\eta_{therm}$</td>
<td>Indicated thermal efficiency</td>
</tr>
<tr>
<td>$A_{eq}(\alpha)$</td>
<td>Throttle area function (m$^2$) as a function of throttle opening angle $\alpha$</td>
</tr>
<tr>
<td>$B$</td>
<td>Bore of engine cylinder (m)</td>
</tr>
<tr>
<td>$c_d$</td>
<td>Throttle discharge coefficient</td>
</tr>
<tr>
<td>$D_{cj}$</td>
<td>Diameter of crank main journal (m)</td>
</tr>
<tr>
<td>$D_{cm}$</td>
<td>Mean equivalent crank diameter (m)</td>
</tr>
<tr>
<td>$D_{cp}$</td>
<td>Diameter of crank pin journal (m)</td>
</tr>
<tr>
<td>$d_{valve}$</td>
<td>Butterfly valve semi-minor axis (m)</td>
</tr>
<tr>
<td>$f$</td>
<td>Fuel fraction at intake manifold</td>
</tr>
<tr>
<td>$H$</td>
<td>Heating value of fuel (J/kg)</td>
</tr>
<tr>
<td>$k_{eq}$</td>
<td>Equivalent ratio of specific heats of air-fuel mixture</td>
</tr>
<tr>
<td>$k_{eq,a1}$</td>
<td>Constant part of $k_{eq}$</td>
</tr>
<tr>
<td>$k_{eq,a2}$</td>
<td>Constant that characterizes temperature-dependence of $k_{eq}$ (K$^{-1}$)</td>
</tr>
<tr>
<td>$k_{eq,a7}$</td>
<td>Constant that characterizes $\lambda_{sp}$-dependence of $k_{eq}$</td>
</tr>
<tr>
<td>$m$</td>
<td>Total mass of fuel and air inside intake manifold (kg)</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Air mass inside intake manifold (kg)</td>
</tr>
<tr>
<td>$M_a$</td>
<td>Molar mass of air (kg/mol)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Mass of air inside intake manifold (kg)</td>
</tr>
<tr>
<td>$m_{cj}$</td>
<td>Number of crank main journals</td>
</tr>
<tr>
<td>$M_{eq}$</td>
<td>Equivalent molecular mass of air-fuel mixture over throttle (intake manifold inlet) (kg/mol)</td>
</tr>
<tr>
<td>$M_{eqm}$</td>
<td>Equivalent molecular mass of air-fuel mixture in the intake manifold (kg/mol)</td>
</tr>
<tr>
<td>$m_f$</td>
<td>Fuel mass inside intake manifold (kg)</td>
</tr>
<tr>
<td>$M_f$</td>
<td>Molar mass of fuel (kg/mol)</td>
</tr>
<tr>
<td>$n_{cp}$</td>
<td>Number of crank pin journals</td>
</tr>
<tr>
<td>$n_{cyl}$</td>
<td>Number of cylinders</td>
</tr>
<tr>
<td>$n_{rpm}$</td>
<td>Engine speed (rpm)</td>
</tr>
<tr>
<td>$n_{rpm}$</td>
<td>Crankshaft rotational speed (rpm)</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Partial pressure of fuel in intake manifold (Pa)</td>
</tr>
<tr>
<td>$P_{loss}$</td>
<td>Engine power losses (W)</td>
</tr>
<tr>
<td>$P_m$</td>
<td>Intake manifold absolute pressure (Pa)</td>
</tr>
<tr>
<td>$P_{mef}$</td>
<td>Frictional mean effective pressure (Pa)</td>
</tr>
<tr>
<td>$P_o$</td>
<td>Throttle inlet absolute pressure (compressor output pressure, if compressor is used) (Pa)</td>
</tr>
<tr>
<td>$P_{shaft}$</td>
<td>Engine power available at the shaft (W)</td>
</tr>
<tr>
<td>$P_{therm}$</td>
<td>Thermal power developed by engine (W)</td>
</tr>
<tr>
<td>$R$</td>
<td>Ideal gas constant (J/(mol. K))</td>
</tr>
<tr>
<td>$R_{valve}$</td>
<td>Butterfly valve semi-major axis (m)</td>
</tr>
<tr>
<td>$S$</td>
<td>Stroke of engine cylinder (m)</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Throttle inlet temperature (compressor output temperature, if compressor is used) (K)</td>
</tr>
<tr>
<td>$T_{shaft}$</td>
<td>Engine torque available at the shaft (Nm)</td>
</tr>
<tr>
<td>$V$</td>
<td>Manifold + port passage volume (m$^3$)</td>
</tr>
<tr>
<td>$V_d$</td>
<td>Total displacement of all engine cylinders (m$^3$)</td>
</tr>
</tbody>
</table>
### 3.3 Natural gas engine modeling

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>Spark advance angle (angle before top dead center) (for a theoretical 4-stroke engine, it can take values between -90° and 0°)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Absolute air-fuel ratio over throttle (also at intake manifold inlet)</td>
</tr>
<tr>
<td>$\lambda_{eq}$</td>
<td>Stoichiometric air-fuel ratio over throttle (also at intake manifold inlet)</td>
</tr>
<tr>
<td>$\lambda_{sp}$</td>
<td>Specific air-fuel ratio over throttle (also at intake manifold inlet)</td>
</tr>
<tr>
<td>$\lambda_{spb}$</td>
<td>Specific air-fuel ratio of cylinder charge (also intake manifold outlet)</td>
</tr>
<tr>
<td>$\tau_d$</td>
<td>Engine delay (s)</td>
</tr>
</tbody>
</table>

A flow diagram of the main gas engine model blocks and controls is shown in Fig. 13.

![Flow diagram of the main gas engine model blocks and controls](image)

**Fig. 13.** Conceptual view of main gas engine components and controls

Some engines such as GE Jenbacher Type 6 [36], use a multi-stage turbo charger and a multi-stage intercooler. The turbo charger compresses the air-fuel mixture to a high pressure. The compressor output is at high pressure and high temperature, $P_o$ and $T_i$. To prevent the risk of detonation downstream in the manifold or engine cylinders, the intercooler which is located between the compressor and throttle cools the input charge to temperature of $T_0$. Throttle valve has an adjustable plate that changes the effective throttle area and therefore the amount of charge flow into the manifold. Intake manifold then distributes the charge into the cylinders where combustion and torque production take place.

In the proposed model, the dynamics of mixer, compressor, and cooler are not considered (along with their control systems). Instead, their outputs ($\lambda_{sp}$, $P_o$, $T_0$, mass flow rate) are available as engine model inputs. If desired, more dynamics can be included to replace these inputs. The major model inputs and outputs are shown in Fig. 14.
3.3.1 Air-fuel mixer

3.3.1.1 Mixer function

The role of the air-fuel mixer (and its control system) is to set the correct proportions of air and fuel in the air-fuel mixture at the throttle inlet as well as in the intake manifold. The control objective could be for engine performance (rich burn mixture) or to minimize NOx emissions (lean burn mixture) and save fuel [37].

3.3.1.2 Mixer model

The proportion of air relative to fuel is the air-fuel ratio which can be expressed as (3.1):

$$
\lambda = \frac{\dot{m}_{ai}}{\dot{m}_{fi}} = \frac{\dot{m}_i - \dot{m}_{fi}}{\dot{m}_{fi}} = \lambda_{sp} \lambda_{eq}
$$

(3.1)

It is often more convenient to express the absolute air-fuel ratio $\lambda$, as a relative quantity called the specific air-fuel ratio $\lambda_{sp} = \lambda / \lambda_{eq}$ (absolute air-fuel ratio divided by the stoichiometric air-fuel ratio). The specific air-fuel ratio $\lambda_{sp}$ is a model input. In other words, the model assumes that air-fuel mixing over the throttle is complete and instantaneous. If desired, more dynamics can be included to replace this input.

3.3.1.3 Dependence of NO\textsubscript{x} emissions on $\lambda$

Dependence of NO\textsubscript{x} emissions on $\lambda$ for a specific gasoline engine is shown in Fig. 15, reproduced from [38].
3.3 Natural gas engine modeling

In the absence of more engine-specific NO\textsubscript{x} charts, we experimented with several function fits to the emissions data from Fig. 15. We used spline, Weibull integral, and Gaussian functions to fit the data as faithfully as possible. Even though spline fit had the least root mean square error (RMSE) when only the data range of the regressor was considered, the behavior of the fit outside of this range was poor (Fig. 16). Compared to the spline fit, Weibull integral produces an RMSE of 20.369, while the Gaussian fit produces an RMSE of 95.296. However, whereas Weibull integral requires 9 parameters for the fit (equation (3.2)), Gaussian needs only 3 (equation (3.3)). In addition, Gaussian shows no bias for extreme low or high values of λ\textsubscript{sp} unlike Weibull integral.

---

**Fig. 15.** Dependence of NO\textsubscript{x} emissions on air-fuel ratio for a gasoline engine reproduced from [38]

**Fig. 16.** Poor behavior of spline fit (dashed blue) outside of range of regressor
Fig. 17. Comparison of different function fits. NO\textsubscript{x} data points taken from reference [38]

\[
\text{NO\textsubscript{mole fraction ppm}} = \left[ 1750 \left(1 - e^{-\left(\frac{\text{spb}}{x_1}\right)^4}\right) - 1750 \left(1 - e^{-\left(\frac{\text{spb}}{x_2}\right)^4}\right) + 8.5 \right] \left[1 - \tanh\left(2\frac{\text{spb}}{x_3} - 5\right)\right] \tag{3.2}
\]

Where:

\[
\begin{align*}
x_1 &= 1.01272 \\
x_2 &= 16.58216 \\
x_3 &= 1.220723 \\
x_4 &= 1
\end{align*}
\]

\[
\text{NO\textsubscript{mole fraction ppm}} = a \exp\left(-\frac{(\text{spb}-b)^2}{c}\right) \tag{3.3}
\]

Where:

\[
\begin{align*}
a &= 2402 \text{ ppm} \\
b &= 1.118 \\
c &= 0.164
\end{align*}
\]

3.3.2 Throttle body model

3.3.2.1 Throttle function

The role of the throttle is to change the amount of the input charge mixture flowing into the intake manifold. The input charge is the mixture of air and fuel. The throttle does not change the composition of the charge mixture but simply its amount that is proportional to the throttle area.
3.3 Natural gas engine modeling

function. For traditional throttles like the butterfly throttle valve, the area function is a nonlinear function of the throttle angle.

3.3.2.2 Throttle geometry
A simple butterfly throttle body is shown in Fig. 18 where the shaded portion is the throttle area. The mass flow through the throttle is controlled by changing the angle $\alpha$ of the throttle plate shown in red. Viewed from the throttle cross-section, shown in the figure on the right, the area function is computed simply by subtracting the area of an ellipse from that of a circle. The ellipse semi-minor axis is $d_{\text{valve}} = R_{\text{valve}} \cos \alpha$ and the semi-major axis is $R_{\text{valve}}$. Therefore the throttle area function is:

$$A_{eq}(\alpha) = \pi R_{\text{valve}}^2 - \pi d_{\text{valve}} R_{\text{valve}} = \pi R_{\text{valve}}^3 \left(1 - \cos \alpha \right)$$

(3.4)

The throttle area equation in (3.4) neglects non-idealities such as throttle leakage, and effects of throttle shaft.

Fig. 18. Simple butterfly throttle body

3.3.2.3 Throttle mass flow
The charge flow is modeled as a one-dimensional steady, isentropic, compressible flow as in [33]. The throttle mass flow can be subdivided into 2 regimes: unchoked (or sub-sonic) flow and choked (or sonic) flow as shown in equation (3.5):
Improved Natural Gas Engine Model for Microgrid Simulation

\[
\dot{m}_i = \begin{cases} 
    c_j A_e q_i (\alpha) \frac{P_m}{RT_i} \frac{1}{M_{eq}} \left( \frac{2}{k_{eq}} \right) \frac{1}{k_{eq}} \left( 1 - \frac{P_m}{P_0} \right)^{\frac{1}{k_{eq}}} & \text{if } \frac{P_m}{P_0} > \left( \frac{2}{k_{eq} + 1} \right) \\
    c_j A_e q_i (\alpha) \frac{P_m}{RT_i} \frac{1}{M_{eq}} \left( \frac{2}{k_{eq} + 1} \right) & \text{if } \frac{P_m}{P_0} \leq \left( \frac{2}{k_{eq} + 1} \right) \end{cases}
\]

(3.5)

The expression for the equivalent air-fuel molar mass over the throttle is given in equation (3.6):

\[ M_{eq} = \frac{m_a M_a + m_f M_f}{m_a + m_f} = \frac{\lambda_{eq} M_a + M_f}{\lambda_{eq} + 1} \]

(3.6)

It is assumed in (3.6) that air-fuel mixing is complete and instantaneous.

If the variations in throttle inlet temperature and specific air fuel ratio are small, the specific heat ratio is approximately constant, such as in previous engine models [28]. A more complete treatment of specific heat ratio dependence on charge temperature and specific air-fuel ratio is shown in [39]. We approximate this dependence with a simplified equation (3.7):

\[ k_{eq} = k_{eq,a1} + k_{eq,a2} T_0 + \frac{k_{eq,a7}}{\lambda_{eq}} \]

(3.7)

3.3.3 **Intake manifold**

3.3.3.1 **Intake manifold function**

The role of the intake manifold is to equally distribute the air-fuel charge to the engine cylinders for combustion and torque production.

3.3.3.2 **Intake manifold model**

The intake manifold is modeled with two states which are manifold pressure \( P_m \) and fuel fraction \( f \) [28]. Assuming the air and fuel mix completely and instantly in the manifold, the absolute pressure therein is essentially governed by the filling and emptying dynamics of a volume as shown by equation (3.8):

\[
\frac{dP_m}{dt} = \frac{RT_m}{VM_a} \left( \dot{m}_i - \dot{m}_f \right) + \frac{RT_m}{VM_f} \dot{m}_f - \frac{RT_m}{VM_{eqm}} \dot{m}_o
\]

(3.8)
3.3 Natural gas engine modeling

Where:

- $M_{eqm}$ is the equivalent molar weight of the air-fuel mixture given by $(1 - f)M_a + fM_f$

- $\dot{m}_o$ is the speed density formula for a 4-stroke engine and is given by equation (3.9):

$$\dot{m}_o = \frac{n_{rpm} V_d \eta_{vol} P_m M_{eqm}}{120 RT_m}$$  \hspace{1cm} (3.9)

The fuel fraction in the manifold can be written as:

$$f = \frac{m_f}{m}$$  \hspace{1cm} (3.10)

Differentiating both sides results in equation (3.11):

$$\dot{f} = \frac{\dot{m}_f - f\dot{m}_o}{m} - \frac{\dot{m}_f}{m}$$  \hspace{1cm} (3.11)

From conservation of mass, $\dot{m} = \dot{m}_i - \dot{m}_o$, and assuming that the rate of change of the equivalent molar mass $M_{eqm}$ is negligible, equation (3.11) becomes:

$$\dot{f} = \frac{RT_m}{VM_{eqm} P_m} \left[ \dot{m}_i - f\dot{m}_i \right]$$  \hspace{1cm} (3.12)

Where $M_{eqm} = (1 - f)M_a + fM_f$.

In previous work [28], it appears the authors assumed $M_{eqm} \approx M_a$ in equation (3.12), which leads to erroneous steady state values.

### 3.3.3.3 Volumetric Efficiency

The volumetric efficiency relates the volume of fluid displaced by a plunger to its swept volume. Whereas it is considered constant in some studies [28], it is mainly a function of engine speed (rpm) and manifold pressure, [40]. In [40], the authors propose the following function for the ambient volumetric efficiency, where an ambient pressure of 1 bar is considered:

$$\eta_{vol,amb} = a + b \cdot n_{rpm} + c \cdot n_{rpm}^2 + d \cdot P_m$$  \hspace{1cm} (3.13)
Where \( a, b, c, d \) are constants from a polynomial function fit.

Considering a general throttle inlet pressure of \( P_o \), we modify equation (3.13) as follows:

\[
\eta_{vol} = a_{vol} + b_{vol} \cdot n_{rpm} + c_{vol} \cdot n_{rpm}^2 + d_{vol} \cdot \left( \frac{P_m}{P_o} \right)
\]  

(3.14)

We obtained volumetric efficiency data of an experimental gasoline engine from [40]. The polynomial fit parameters in equation (3.14) were calculated as \((a_{vol}, b_{vol}, c_{vol}, d_{vol}) = (-0.1712, 5.75 \times 10^{-5}, -9.584 \times 10^{-9}, 0.9352)\). It is a good fit with RMSE values of: 0.011, 0.0047, 0.0044, 0.0073, and 0.0104 for \( P_m = 0.5 \) bars, 0.6 bars, 0.7 bars, 0.8 bars, and 0.9 bars respectively. Data for \( \eta_{vol} \) and the fitted polynomial functions are plotted in Fig. 19. The solid blue vertical line denotes the end of the data. As expected, while the fit is good within the data provided, it can behave badly outside this range, for example by producing negative values of \( \eta_{vol} \).

![Fig. 19. Volumetric efficiency data and polynomial fits](image)

Through trial-and-error, we derive a volumetric efficiency function of the form shown in equation (3.15).

\[
\eta_{vol} = \left[ 1 + a_{vol} \left( \frac{P_m}{P_0} - 1 \right) + b_{vol} \left( \frac{P_m}{P_0} \right)^2 \right] \exp \left[-\left( \frac{\eta_{vol} - \eta_{vol}^{fit}}{1000} \right)^2 \right]
\]

Where:

\[
(a_{vol}, b_{vol}, c_{vol}) = \text{fit constants} = (-0.8421, 1.368, 6.689)
\]  

(3.15)
3.3 Natural gas engine modeling

For a given manifold pressure $P_m$, the rpm at which the peak volumetric efficiency occurs is factored into the fit as a parameter. This is especially useful if only limited engine information is available. The fit also ensures that we obtain a volumetric efficiency of 100% for $P_m = P_o$. Compared to the original data from [40], it produces RMSE values of: 0.0133, 0.0134, 0.0185, 0.0075, 0.0377 for $P_m = 0.5\, \text{bars}$, $0.6\, \text{bars}$, $0.7\, \text{bars}$, $0.8\, \text{bars}$, $0.9\, \text{bars}$ respectively. Furthermore, the proposed function shows good behavior beyond the range of the regressor (no predicted negative values of $\eta_{\text{vol}}$) as shown in Fig. 20.

![Volumetric efficiency with proposed function fit](image)

**Fig. 20. Volumetric efficiency with proposed function fit**

3.3.4 Cylinders and torque production

3.3.4.1 Thermal power

The proportion of thermal energy from the combustion that is available to do work is represented by the thermal efficiency $\eta_{\text{therm}}$. The steady state thermal power is given by:

$$P_{\text{therm}} = H \eta_{\text{therm}} \dot{m}_f$$

(3.16)

Thermal losses occur through heat transfer [41]. Thermal efficiency is also affected by $\lambda_{spb}$, and spark advanced angle $\theta$. In [28], dependence of $\eta_{\text{therm}}$ on $\lambda_{spb}$ and $\theta$ is approximated by a polynomial function fit of the form:

$$\eta_{\text{therm}}(\lambda_{spb}, \theta) = \left( s_0 + s_1 \lambda_{spb} + s_2 \lambda_{spb}^2 \right) + \left( s_3 + s_4 \lambda_{spb} + s_5 \lambda_{spb}^2 \right) \theta + \left( s_6 + s_7 \lambda_{spb} + s_8 \lambda_{spb}^2 \right) \theta^2$$

(3.17)
Where $s_0$ to $s_8$ are parameters of the function fit. As we have already seen, the polynomial fits result in low RMSE when compared to the efficiency data. However, the behavior of the fit outside of the data range can be problematic. We propose a function fit of the form in equation (3.18).

$$
\eta_{therm} = \left[a_{therm} + b_{therm}\theta_{norm} + c_{therm}\theta_{norm}^2\right]\exp\left(\frac{\lambda_{sp} + d_{therm} + e_{therm}\theta_{norm}}{f_{therm} + g_{therm}\theta_{norm}}\right)^2
$$  

(3.18)

Where:

$\theta_{norm} = \text{normalized spark angle (i.e. } \theta_{norm} = \theta/90^\circ; \text{ therefore } \theta_{norm} \text{ can take values between -1 and 0)}$

$(a_{therm}, b_{therm}, c_{therm}, d_{therm}, e_{therm}, f_{therm}, g_{therm}) = (0.3377, -0.6737, -1, -1.275, 0.3614, 0.7526, 1.755)$.

Data for $\eta_{therm}$ as a function of $\theta$ and $\lambda_{spb}$ and the function fits are shown in Fig. 21.

The entire combustion process is approximated by a variable transport delay $\tau_d$ [28]. More specifically, $\tau_d$ is the delay between the fuel flow out of the intake manifold and the generation of torque by all the cylinders. For a 4-stroke engine, it is approximated by equation (3.19):

$$
\tau_d = \frac{60}{n_{rpm}} \left(1 + \frac{1}{n_{cyl}}\right) + \frac{45}{6n_{rpm}}
$$  

(3.19)
3.3 Natural gas engine modeling

3.3.4.2 Friction losses

The engine friction is mainly attributed to 3 sources [38]:

(i) Pumping friction which is the work done by the pistons on the cylinder gases during intake and exhaust strokes.

(ii) Rubbing friction which occurs due to the relative motion between adjacent engine components. It is further influenced by the lubricant characteristics such as temperature, flow rate, and fluidity.

(iii) Friction due to other accessories such as fans, alternator, pumps, etc.

The term ‘friction’ in engine modeling is generally used in a more loose fashion than in Mechanics, because we consider engine losses due to auxiliary components such as fans, alternators, etc.

Since the experimental conditions to determine the individual contributions of each of the sources of friction are quite different from actual engine operating conditions, and since friction components act upon others, some studies such as [42] have suggested that an experimental determination of total engine friction is more realistic and more accurate. The authors establish an estimate for the frictional mean effective pressure (FMEP) $P_{mf}$ (in Pa) which is shown to be proportional to a dimensionless constant as in equation (3.20):

$$P_{mf} = K_f \left( n_{rpm} \right) \frac{\sqrt{SD_{cm}}}{B} \times 1 \times 10^6$$

(3.20)

Where

$$K_f \left( n_{rpm} \right) = \left( k_{fa} \left( \frac{n_{rpm}}{1000} \right)^2 + k_{fb} \right)$$

(3.21)

The constants $(k_{fa}, k_{fb}) = (3.37 \times 10^{-3}, 0.194)$

$P_{mf}$ is transformed into a power loss by the relationship in equation (3.22) for a 4-stroke engine:

$$P_{loss} = \frac{P_{mf} V_d}{120} n_{rpm}$$

(3.22)

The parameter $D_{cm}$ is the mean crank diameter. It is computed from geometrical parameters of the crankshaft. The function of the crankshaft is to translate the linear reciprocating motion of the
Improved Natural Gas Engine Model for Microgrid Simulation

pistons into rotational motion. Therefore it plays a key role in the frictional losses. A 2-dimensional illustration of an inline crankshaft for a 4-cylinder engine is shown in Fig. 22 below. The crankpin journals are where the piston rods connect to the crankshaft. They are displaced from the main axis of the crankshaft. The counter weights help in balancing the torque produced by each cylinder. The connecting webs connect different main axis of the crankshaft to the pin journals.

![Fig. 22. 4-cylinder inline crankshaft](image)

The parameter $D_{cm}$ is computed as follows ([42]):

$$D_{cm} = \frac{K_C \left( \sum_{j=1}^{m_j} D_{cj} + \sum_{l=1}^{n_{cp}} D_{cp} \right)}{m_j + n_{cp}}$$

(3.23)

Where $K_C$ is the coefficient of $D_{cm}$ depending on the number of cylinders (we selected a constant value of 1 for our proposed gas engine model).

3.3.4.3 Shaft torque

With knowledge of the thermal power and frictional losses, the available shaft torque is then computed as:

$$T_{shaft} = \frac{P_{therm} - P_{loss}}{n_{rpm} \left( \frac{\pi}{30} \right)}$$

(3.24)

One challenge with equation (3.24) is that it appears to behave badly for very small values of $n_{rpm}$. In an actual engine, a starter motor is often used to prime the engine up to a minimum speed (e.g.
200rpm) after which the fuel flow is engaged. Therefore, in practical terms, equation (3.24) should behave properly.

### 3.4 Engine steady state performance

The parameters in Table 8 (Appendix B) were used to compute the steady state engine performance of a theoretical 20-cylinder gas engine at wide open throttle (WOT). We considered sufficient WOT condition for a selection of $A_{eq}$ that resulted in $P_m > 80\%$ of $P_o$ for all the values of $n_{rpm}$ and $\lambda_{sp}$ considered. The plots of $T_{shaft}$ and $P_{shaft}$ vs $n_{rpm}$ for different $\lambda_{sp}$ are shown in Fig. 23. The dynamic response of the engine to a step change of $A_{eq}$ from 0.01m$^2$ to 0.0205m$^2$, for $n_{rpm} = 1000rpm$, $\lambda_{sp} = 1.2$, $P_o = 0.3MPa$, is shown in Fig. 24. We see therein that the steady-state shaft power and torque match with the steady-state values predicted by the power and torque characteristics (solid green, and dotted green lines, respectively) of Fig. 23.

![Fig. 23. Steady state gas engine power and torque vs rpm at WOT](image)

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3.5 Generating parameters for a generic gas engine

In many microgrid simulations, incomplete or no engine data is available. Furthermore, in preliminary studies, it may be desired to simulate a gas engine with very few specifications such as the rated shaft power $P_{shaft}$, and yet hope to use reasonable engine parameters. Given that most power systems specialists may not be versed with detailed engine parameters, there is a need for a relatively simple way to predict these parameters from a set of few specifications such as $P_{shaft}$. This is the objective we aim to achieve in this section.

We examined available online data from more than 100 gas engines (see Table 9 to Table 12 in the Appendix B). In Fig. 25 to Fig. 30, we plot some engine parameters vs $P_{shaft}$, and volume of one cylinder. Where possible, function fits are computed to match the data as closely as possible. The following predictors were obtained for some of the major engine parameters.

3.5.1 $V_d$ predictor

Total engine displacement is predicted from the data (Fig. 25) as follows:

$$V_d = a(P_{shaft} + b)^c$$  \hspace{1cm} (3.25)

Where $V_d$ is in liters, and $P_{shaft}$ is the rated engine power in kW. The fit parameters $(a, b, c) = (0.0130, 17.5248, 1.2026)$. 

---

Fig. 24. Engine response to step change of $A_{eq}$ from 0.01$m^2$ to 0.0205$m^2$ for $\lambda_{sp} = 1.2$, $n_{rpm} = 1000$rpm; $P_o = 0.3$MPa
The intake manifold and port passage volume \( V \) is usually between 1 to 1.5 times \( V_d \). We picked a factor of 1.25.

### 3.5.2 \( n_{cyl} \) predictor

From Fig. 26, number of cylinders is estimated as follows:

\[
    n_{cyl} = \frac{P_{shaft}}{aP_{shaft} + b}
\]  

(3.26)

Where \( P_{shaft} \) is rated engine power in kW. The fit parameters \((a, b) = (0.0870, 2.8692)\). \( n_{cyl} \) is rounded up to the next larger even number. Most engines have an even number of cylinders to minimize balancing issues.

### 3.5.3 Bore and Stroke predictor

From Fig. 27, the bore-to-stroke ratio is predicted as:

\[
    \left( \frac{B}{S} \right)^* = a \exp\left(-P_{shaft}b\right) + c
\]

(3.27)

Where the fit parameters \((a, b, c) = (0.1087, 0.0002765, 0.8003)\). Assuming \( n_{cyl} \) and \( V_d \) have already been determined (equations (3.26) and (3.25)), cylinder bore is then computed as in equation (3.28):

\[
    B = \sqrt[3]{\frac{4V_d}{n_{cyl}\pi}}
\]

(3.28)

Where \( B \) is in m, and \( V_d \) in m\(^3\).

The stroke is then easily found from combining equations (3.27) and (3.28):

\[
    S = \frac{B}{\left( \frac{B}{S} \right)^*}
\]

(3.29)

### 3.5.4 Main crank journal diameter \( D_{cj} \) predictor

From Fig. 29, \( D_{cj} \) is predicted as:
\[ D_{cj} = a \log \left( \frac{V_d}{n_{cyl}} + b \right) + c \]  \hspace{1cm} (3.30)

Where \( D_{cj} \) is in inches, \( V_d \) is in liters, and the fit parameters \((a, b, c) = (0.5031, 1.4678, 1.6583)\).

In addition, we make the following assumptions regarding the crankshaft:

1. The number of crank pin journals (or rod journals) \( m_{cp} \) is equal to the number of cylinders \( n_{cyl} \).
2. The number of main journals \( n_{cj} \) is less than the number of pin (or rod) journals by one.
3. Given a main journal diameter \( D_{cj} \), the rod journal diameter \( D_{cp} \) is 0.8 times the main journal diameter. This approach was used instead of attempting a fit, as the limited data available would have made this exercise futile (see Fig. 30).

No clear reasonable fit could be made for the starter data from Fig. 28.

3.5.5 \( A_{eq} \) throttle area predictor

Without specific throttle data, we predict \( A_{eq} \) as follows:

1. Assume wide open throttle (WOT) condition. \( P_m \) is set to 95\% of \( P_o \). This also results in the condition of unchoked flow of input charge over the throttle (equation (3.5)).
2. Assuming known spark angle \( \theta \) and specific air-fuel ratio \( \lambda_{sp} \), estimate thermal efficiency \( \eta_{therm} \) (equation (3.18)) and \( P_{loss} \) at selected nominal speed (equation (3.22)). This gives us thermal power \( P_{therm} \).
3. From \( P_{therm} \), estimate fuel mass flow rate \( \dot{m}_f \) (equation (3.16)) which in steady state is equivalent to \( \dot{f}m_o \).
4. In steady state, fuel fraction \( f \) can be expressed as:

\[ f(0) = \frac{1}{\lambda_{sp}A_{eq} + 1} \]  \hspace{1cm} (3.31)

5. From steps (3), and (4), charge flow rate \( \dot{m}_o \) can be calculated.
6. From equation (3.9), volumetric efficiency \( \eta_{vol} \) can now be calculated. The parameter \( n_{rpm}^* \) can now be computed for consistency from equation (3.15)
7. Since in steady state, assuming mass flow conservation, $\dot{m}_i = \dot{m}_o$, we can now determine area function $A_{eq}$ as:

\[
A_{eq} = \frac{\dot{m}_o}{C_d \left( \frac{P_0}{RT_0} \right)^{\frac{1}{k}} \left( \frac{P}{P_0} \right)^{\frac{1}{k_{eq}}} \left( \frac{2k_{eq}}{k_{eq} - 1} \right) \left( 1 - \frac{P}{P_0} \right)^{\frac{k_{eq} - 1}{k_{eq}}} } \]

(3.32)
Fig. 25. Total engine displacement vs rated engine power

Fig. 26. kW/cylinder vs rated engine power

Fig. 27. Engine bore-on-stroke ratio vs rated engine power

Fig. 28. Starter motor rating vs rated engine power

Fig. 29. Main journal diameter vs volume of one cylinder

Fig. 30. Pin journal diameter vs volume of one cylinder [43]
3.6 Conclusion

In this chapter, we developed an improved mean-value natural gas engine model suitable for microgrid simulation. The proposed model considers first principles (such as mass flows) as well as phenomenological relationships. Compared to previous works, improved fitting functions are developed for important model parameters such as volumetric efficiency, and thermal efficiency. We also propose a set of predictors of important engine parameters from a few specifications such as engine nominal shaft power, and nominal speed. Steady state and dynamic engine plots are provided.
Chapter 4

Model Identification of Dynamic Microgrids and Controller Optimization with High Fidelity Hardware-in-the-Loop Platform

4.1 Introduction

Traditionally, modeling and control requirements for DERs (i.e. solar inverters, battery storage, wind turbines) and loads assume a stiff grid [44]. However, microgrids are exhibiting more complex dynamic behavior due to the increased penetration of power electronics interfaced sources [7]. Hence, there is a need for high-fidelity modeling tools to enable rapid development and deployment of advanced control strategies.

Offline simulation tools (e.g. MATLAB/Simulink) are typically slow and make a number of modeling assumptions. On the other hand, high-bandwidth controller hardware-in-the-loop (cHIL) enables more realistic simulations and allows direct interface to real device controllers – a feature which may be difficult to replicate on offline platforms due to non-real-time constraints. This chapter demonstrates the capability of cHIL to accurately measure and extract frequency domain

---

2 A version of this chapter was published in E. Fonkwe et al., “Model Identification of Dynamic Microgrids and Controller Optimization with High Fidelity Hardware-in-the-Loop Platform”, IEEE 17th Workshop on Control and Modeling for Power Electronics (COMPEL), 2016
models of microgrids, such as the one shown in Fig. 31. We describe a framework for model identification and model order reduction (MOR) of highly dynamic microgrids based on a high fidelity hardware-in-the-loop (HIL) platform. An impedance analyzer, built into the real-time simulation, is used to estimate the frequency response of a three-phase synchronous generator connected in three different configurations: (1) Machine connected to a grid via an impedance (weak and strong grids are considered); (2) Machine supplying a standalone load. The HIL platform produces frequency response models that closely match the theoretical expectations. (3) The synchronous generator and a solar PV inverter operating in power and voltage control modes are connected to a standalone load. (4) HIL platform is also used to measure inverter output impedance. Estimated frequency response models are discussed. Section 4.2 develops the theoretical small signal models for the synchronous generator in the different configurations; section 4.3 compares the analytical results and those from the HIL platform, and uses model identification and model order reduction (MOR) for transfer function estimation; section 4.4 concludes the chapter. Unless otherwise mentioned, all symbols used in this chapter are defined in Table 4.

![Schematic diagram of a small microgrid testbed](image)

Table 4. Nomenclature for chapter 4

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_s$</td>
<td>Stator resistance ($\Omega$)</td>
<td>0.1 $\Omega$</td>
</tr>
</tbody>
</table>
**Model Identification of Dynamic Microgrids and Controller Optimization with High Fidelity Hardware-in-the-Loop Platform**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{ls}$</td>
<td>Stator leakage inductance (H)</td>
<td>$0.004527 , H$</td>
</tr>
<tr>
<td>$L_{md}$</td>
<td>Direct-axis magnetizing inductance (H)</td>
<td>$0.1086 , H$</td>
</tr>
<tr>
<td>$L_{mq}$</td>
<td>Quadrature-axis magnetizing inductance (H)</td>
<td>$0.1086 , H$</td>
</tr>
<tr>
<td>$R_{fd}$</td>
<td>Field winding resistance ($\Omega$)</td>
<td>$1.208 , \Omega$</td>
</tr>
<tr>
<td>$L_{fd}$</td>
<td>Field winding leakage inductance (H)</td>
<td>$0.01132 , H$</td>
</tr>
<tr>
<td>$R_{kd}$</td>
<td>Damper winding direct-axis resistance ($\Omega$)</td>
<td>$3.142 , \Omega$</td>
</tr>
<tr>
<td>$R_{kq}$</td>
<td>Damper winding quadrature-axis resistance ($\Omega$)</td>
<td>$4.772 , \Omega$</td>
</tr>
<tr>
<td>$L_{kdd}$</td>
<td>Damper winding direct-axis leakage inductance (H)</td>
<td>$0.007334 , H$</td>
</tr>
<tr>
<td>$L_{kqq}$</td>
<td>Damper winding quadrature-axis leakage inductance (H)</td>
<td>$0.01015 , H$</td>
</tr>
<tr>
<td>$J$</td>
<td>Machine moment of inertia ($\text{kg.m}^2$)</td>
<td>$25 , \text{kg.m}^2$</td>
</tr>
<tr>
<td>$v_{g,rms}$</td>
<td>Line-to-neutral rms voltage (V)</td>
<td>$277 , \text{V}$</td>
</tr>
<tr>
<td>$f_g$</td>
<td>Grid frequency (Hz)</td>
<td>$60 , \text{Hz}$</td>
</tr>
<tr>
<td>poles</td>
<td>Number of machine poles</td>
<td>4</td>
</tr>
<tr>
<td>$\psi_{qs}$</td>
<td>q-axis stator flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>$\psi_{ds}$</td>
<td>d-axis stator flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>$\psi_{kq}$</td>
<td>q-axis damper winding flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>$\psi_{fd}$</td>
<td>d-axis field winding flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>$\psi_{kd}$</td>
<td>d-axis damper winding flux linkage per second in rotor reference frame (V)</td>
<td></td>
</tr>
<tr>
<td>$v'_{qs}$</td>
<td>q-axis machine terminal voltage in rotor reference frame, for a machine connected to an ideal grid</td>
<td></td>
</tr>
<tr>
<td>$v'_{ds}$</td>
<td>d-axis machine terminal voltage in rotor reference frame, for a machine connected to an ideal grid</td>
<td></td>
</tr>
</tbody>
</table>
### 4.1 Introduction

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_\text{s}$, $L_x$</td>
<td>Load/grid impedance ($\Omega; \text{H}$)</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Rotor electrical speed (electrical rad/s)</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Base rotor electrical speed (electrical rad/s)</td>
</tr>
<tr>
<td>$v_{\text{s},abc}$</td>
<td>Machine terminal voltage in $abc$ frame</td>
</tr>
<tr>
<td>$v'_{\text{s},qd0}$</td>
<td>Machine $qd0$ terminal voltage in rotor reference frame for a machine connected to a weak grid</td>
</tr>
<tr>
<td>$v^\ast_{\text{s},qd0}$</td>
<td>Machine $qd0$ terminal voltage in reference frame tied to the grid for a machine connected to a weak grid</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Power angle (electrical rad)</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Electromagnetic torque (Nm)</td>
</tr>
<tr>
<td>$T_{\text{mech}}$</td>
<td>Mechanical torque (Nm)</td>
</tr>
<tr>
<td>$(0)$</td>
<td>Subscript to represent the linearization (operating) point of a state</td>
</tr>
<tr>
<td>$\Delta$</td>
<td>Prefix to represent small signal notation</td>
</tr>
<tr>
<td>$i_{\text{r}qs}$</td>
<td>q-axis stator current in rotor reference frame (A)</td>
</tr>
<tr>
<td>$i_{\text{r}ds}$</td>
<td>d-axis stator current in rotor reference frame (A)</td>
</tr>
<tr>
<td>$i_{\text{r}kd}$</td>
<td>q-axis damper winding current in rotor reference frame (A)</td>
</tr>
<tr>
<td>$i_{\text{r}fd}$</td>
<td>d-axis field winding current in rotor reference frame (A)</td>
</tr>
<tr>
<td>$i_{\text{r}kd}$</td>
<td>d-axis damper winding current in rotor reference frame (A)</td>
</tr>
<tr>
<td>$X_q$</td>
<td>q-axis stator winding impedance ($\Omega$)</td>
</tr>
<tr>
<td>$X_{mq}$</td>
<td>q-axis stator-referred magnetizing impedance ($\Omega$)</td>
</tr>
<tr>
<td>$X_d$</td>
<td>d-axis impedance ($\Omega$)</td>
</tr>
<tr>
<td>$X_{md}$</td>
<td>d-axis stator-referred magnetizing impedance ($\Omega$)</td>
</tr>
<tr>
<td>$X_{ls}$</td>
<td>Stator winding leakage impedance ($\Omega$)</td>
</tr>
</tbody>
</table>
4.2 Small signal modeling of three-phase synchronous generator

In this section the small signal models of the synchronous machine are obtained under different conditions. In one set of conditions, the machine is connected to an ideal grid via an impedance. Different grid strengths are considered by varying the impedance at the point of common coupling. In another case, the machine supplies an isolated series $R$-$L$ load. A generic modeling approach which is based on the schematic in Fig. 32 is adopted to accommodate for these different conditions.

In the standalone case, the amplitude of the grid voltage $v_{g,abc}$ is set to 0, while assuming a constant frequency. In all modeling cases, the impedance $R_s$, $L_s$ can be lumped in to the machine stator resistance and leakage inductance $R_s$ and $L_s$ respectively in order to compute the machine currents (this is illustrated by the blue enclosure in Fig. 32). The modeling approach can be summarized in 2 steps:

1. Add $R_x$, and $L_x$ to $R_s$ and $L_s$ respectively. The model becomes a machine with stator resistance and leakage inductance $(R_s + R_x)$ and $(L_s + L_x)$ respectively that is connected to an infinite bus (strong grid).

2. Compute the machine fluxes and currents, and then solve for the machine terminal voltage $v_{x,abc}$. 

---

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{kq}$</td>
<td>q-axis damper winding impedance (Ω)</td>
</tr>
<tr>
<td>$X_{kd}$</td>
<td>d-axis damper winding impedance (Ω)</td>
</tr>
<tr>
<td>$X_{fd}$</td>
<td>d-axis field winding impedance (Ω)</td>
</tr>
</tbody>
</table>
4.2 Small signal modeling of three-phase synchronous generator

4.2.1 Large signal synchronous machine model

The state space equations of a 3-phase synchronous machine with 2 damper windings can be written as:

\[
\begin{bmatrix}
\psi'_{qs} \\
\psi'_{ds} \\
\psi'_{kq} \\
\psi'_{fd} \\
\psi'_{kd}
\end{bmatrix} =
\begin{bmatrix}
a & -\omega & b & 0 & 0 \\
\omega & c & 0 & d & e \\
f & 0 & g & 0 & 0 \\
0 & h & 0 & i & j \\
0 & k & 0 & l & m
\end{bmatrix}
\begin{bmatrix}
\psi'_{qs} \\
\psi'_{ds} \\
\psi'_{kq} \\
\psi'_{fd} \\
\psi'_{kd}
\end{bmatrix} +
\begin{bmatrix}
\omega_b & 0 & 0 \\
0 & \omega_b & 0 \\
0 & 0 & 0 \\
0 & 0 & \omega_b \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
v'_{qs} \\
v'_{ds} \\
v'_{kq} \\
v'_{fd} \\
v'_{kd}
\end{bmatrix}
\]

(4.1)

where the electrical states are flux linkages per second [17]. The constants \(a, b, c, d, e, f, g, h, i, j, k, l, m\) are defined in Appendix C. The electro-mechanical equations are given as:

\[
\begin{align*}
\dot{\omega} &= \frac{T_{mech} - T_e}{J} \left( \frac{2}{\text{poles}} \right) \\
\dot{\delta} &= \omega_b \\
T_e &= \left( \frac{3}{2} \right) \left( \frac{\text{poles}}{2} \right) \left( \frac{1}{\omega_b} \right) (\psi'_{qs}i'_{qs} - \psi'_{ds}i'_{ds})
\end{align*}
\]

(4.2)

The relationships between the flux linkages per second and currents are summarized in equation (4.3):

\[
\begin{bmatrix}
\psi'_{qs} \\
\psi'_{ds} \\
\psi'_{kq} \\
\psi'_{fd} \\
\psi'_{kd}
\end{bmatrix} =
\begin{bmatrix}
-X_q & 0 & 0 & X_{mq} & 0 & 0 \\
0 & -X_d & 0 & 0 & X_{md} & X_{md} \\
0 & 0 & X_{tr} & 0 & 0 & 0 \\
-X_{mq} & 0 & 0 & X_{kq} & 0 & 0 \\
0 & -X_{md} & 0 & 0 & X_{fd} & X_{fd} \\
0 & 0 & -X_{md} & 0 & 0 & X_{kd}
\end{bmatrix}
\begin{bmatrix}
i'_{qs} \\
i'_{ds} \\
i'_{kq} \\
i'_{fd} \\
i'_{kd}
\end{bmatrix}
\]

(4.3)

Of particular interest, for the purposes of this chapter, are the expressions for \(i'_{qs}\) and \(i'_{ds}\) which can be written as the set of equations (4.4):

\[
\begin{align*}
i'_{qs} &= d_i\psi'_{qs} + e_i\psi'_{kq} \\
i'_{ds} &= a_i\psi'_{ds} + b_i\psi'_{kd} + c_i\psi'_{fd}
\end{align*}
\]

(4.4)

The constants \(a_i, b_i, c_i, d_i, e_i\) are defined in Appendix C.
Having knowledge of the expressions for the machine currents, it is desired to compute the machine terminal voltage \( v_{x,abc} \) as shown in Fig. 32 for a machine connected to an ideal voltage source via an impedance. The terminal voltage can be expressed as in equation (4.5) where the emboldened terms denote matrix representations.

\[
v_{x,abc} = R_x i_{x,abc} + \frac{d}{dt} \lambda_{x,abc} + v_{g,abc}
\]

(4.5)

Where \( \lambda_{x,abc} \) is the flux linking \( L_x \).

Applying an \( (abc) \)-to-\( (qd0) \) transform to both sides of equation (4.5) yields equation (4.6), where the reference frame is tied to the rotor:

\[
v'_{x,qd0} = R_x i'_{qd0} + K \left( \frac{d}{dt} K^{-1} \right) \lambda'_{x,qd0} + \frac{d}{dt} \lambda'_{x,qd0} + v'_{g,qd0}
\]

(4.6)

Where the transformation \( K \) is defined in Appendix C.

Assuming that the impedance \( R_x, L_x \) is linear, diagonal, and balanced, and after some manipulations on equation (4.6), the terminal voltage \( qd \) expressions can be written as (4.7):

\[
\begin{bmatrix}
\psi'_{qd}
\psi'_{qd}
\end{bmatrix} =
\begin{bmatrix}
T_1 & \omega T_2 & T_3 & \omega T_4 & \omega T_5
\omega T_6 & T_7 & \omega T_8 & T_9 & T_{10}
\end{bmatrix}
\begin{bmatrix}
\psi'_{qs}
\psi'_{ds}
\psi'_{kq}
\psi'_{kd}
\end{bmatrix} +
\begin{bmatrix}
T_{11} & 0 & 0
0 & T_{12} & T_{13}
\end{bmatrix}
\begin{bmatrix}
\psi'_{qs}
\psi'_{ds}
\psi'_{kd}
\end{bmatrix}
\]

(4.7)

Where the constants \( T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}, T_{12}, T_{13} \) are defined in Appendix C.

Equation (4.7) can also be expressed in a reference frame tied to the ideal grid voltage source by employing the transform:

\[
v'^g_{x,qd0} = K^g v'_{x,qd0}
\]

(4.8)

Where \( K^g \) is defined in the Appendix C.
4.2 Small signal modeling of three-phase synchronous generator

In many cases, the excitation system is designed to regulate the machine terminal voltage to a specified absolute value [45], for example by using the peak value of the fundamental instantaneous terminal voltage which can be expressed as in equation (4.9):

\[ v_i = \sqrt{(v'_{x,q})^2 + (v'_{x,d})^2} \]  

(4.9)

4.2.2 Small signal synchronous machine model

Equation (4.1) is non-linear as the machine speed is multiplied with other states. By perturbing equation (4.1) and eliminating steady state and non-linear terms, a linearized small-signal model can be written as in equation (4.10), where the constants \( A_i, B_i, C_i, D_i, E_i \), are defined in Appendix C.

\[
\begin{bmatrix}
\Delta \psi'_{qs} \\
\Delta \psi'_{ds} \\
\Delta \psi'_{kq} \\
\Delta \psi'_{fd} \\
\Delta \dot{\omega}_i \\
\Delta \delta 
\end{bmatrix} =
\begin{bmatrix}
a & -\omega_{r(0)} & b & 0 & 0 & -\psi'_{dr(0)} & -\psi'_{dr(0)} \omega_b \\
\omega_{r(0)} & c & 0 & d & e & \psi'_{qr(0)} & \psi'_{qr(0)} \omega_b \\
f & 0 & g & 0 & 0 & 0 & 0 \\
0 & h & 0 & i & j & 0 & 0 \\
0 & k & 0 & l & m & 0 & 0 \\
A_i & B_i & C_i & D_i & E_i & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta \psi'_{qs} \\
\Delta \psi'_{ds} \\
\Delta \psi'_{kq} \\
\Delta \psi'_{fd} \\
\Delta \omega_i \\
\Delta \delta
\end{bmatrix} +
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & k_{mech} & 0 & 0 & 0 & 0 \\
\omega_b & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\Delta v_{fd} \\
\Delta T_{mech}
\end{bmatrix}
\]

(4.10)

Similarly, equation (4.7) is non-linear as the rotor speed is multiplied with other states. A linearized small signal expression is written in equation (4.11), where \( T_{mech} \) is defined in Appendix C.

-70-
\[
\begin{bmatrix}
\Delta v_{x,q}^r \\
\Delta v_{x,d}^r
\end{bmatrix}
= \begin{bmatrix}
T_1 & \omega_r T_6 \\
\omega_r T_2 & T_7 \\
T_3 & \omega_r T_8 \\
\omega_r T_4 & T_9 \\
\omega_r T_5 & T_10 \\
T_{14} & \psi^{r}_{kq(0)} T_8 \\
-T_{11} v_{ds(0)} & T_{12} v_{qg(0)}
\end{bmatrix}
\begin{bmatrix}
\Delta \psi_{qs}^r \\
\Delta \psi_{qs}^l \\
\Delta \psi_{ds}^r \\
\Delta \psi_{ds}^l \\
\Delta \psi_{kd}^r \\
\Delta \psi_{kd}^l \\
\Delta \omega_r \\
\Delta \delta
\end{bmatrix}
+ \begin{bmatrix}
0 \\
T_{13}
\end{bmatrix}
\Delta v_{jd}^r
\] (4.11)

Equation (4.11) can also be expressed in a reference frame tied to the ideal grid voltage source as:

\[
\Delta v_{x,qd0}^q = \begin{bmatrix}
\cos \delta_{(0)} & \sin \delta_{(0)} \\
-\sin \delta_{(0)} & \cos \delta_{(0)}
\end{bmatrix}
\begin{bmatrix}
v_{x,d(0)}^r \\
v_{x,q(0)}^r
\end{bmatrix}
\begin{bmatrix}
\Delta \psi_{qs}^r \\
\Delta \psi_{qs}^l \\
\Delta \psi_{ds}^r \\
\Delta \psi_{ds}^l \\
\Delta \omega_r \\
\Delta \delta
\end{bmatrix}
\] (4.12)

Finally, equation (4.9), after linearization, can be written as equation (4.13).

\[
\Delta v_i \approx \frac{v_{x,q(0)}^r}{v_{r(0)}} \Delta v_{x,q} + \frac{v_{x,d(0)}^r}{v_{r(0)}} \Delta v_{x,d}
\] (4.13)

For constant speed scenarios, the equations (4.10) and (4.11) can be used except that the angular velocity and power angle states must be omitted.

In order to ascertain the validity of the small signal machine models developed above, they must be compared with the large signal model for small perturbations in the input around a few operating points.

4.2.2.1 Small signal model validation: Grid-connected machine

The small signal model is tested for different values of the impedance \( R_x \), \( L_x \) and compared with the large signal model. Fig. 33 shows a time domain comparison between the large signal and small signal terminal voltage quantities, and rotor electrical speed for 2 successive step changes of 0.03% in field voltage. For this case, \( R_x = 0.5 \, \Omega \) and \( L_x = 0.0021 \, H \) for the operating point corresponding to a grid-injected power of \( P = 200kW \), \( Q = 100kVAR \) inductive. The relative error between the two models is less than 0.01%, thus validating the small signal model.
4.2 Small signal modeling of three-phase synchronous generator

4.2.2.2 Small signal model validation: Standalone

In this configuration, the grid voltage rms $v_{g,\text{rms}}$ is set to 0. The load is selected as a series $R-L$ branch with values $R_x = 100\Omega$ and $L_x = 0.0021\,\text{H}$. Fig. 34 compares the terminal voltage quantities, and rotor electrical speed for the large and small signal models; the relative errors are less than 0.3%.

Fig. 33. Comparison of large and small signal models for small perturbations of $v_{fd}$ for machine connected to a weak grid

Fig. 34. Comparison of large and small signal models for small perturbations of $v_{fd}$ for machine connected in standalone
4.3 Comparison between theoretical models and HIL simulation platform

Several commercial real-time simulation platforms are available, such as dSPACE [46], RTDS [47], OPAL-RT [48]. For this section, we use a Typhoon HIL real-time simulator [49]. The setup is shown in Fig. 35. Typhoon HIL 602 real-time simulation platform with physical device controller. The encircled part is a TI F28377 controller for sending gate commands to an inverter model, as well as closed-loop control of the inverter which is running on the platform; ‘2’ is the real-time simulator, while the digital and analog inputs/outputs are marked by ‘3’.

The real-time HIL platform is effectively utilized as an impedance analyzer. It is configured to measure a designated output (machine states) while a designated input ($v_{fd}$) is being perturbed. A 1μs fixed time step is used. The results of the HIL platform are compared against the linearized models developed in section 4.2.2. The machine runs in open loop while the field voltage $v_{fd}$ is perturbed with sinusoids having frequencies between 1Hz and 1kHz; the values of the various states are recorded; the frequency response is extracted with an implementation of the Goertzel algorithm [50]. The Goertzel algorithm is an efficient FFT technique that extracts the magnitude and phase of a small set of frequency components; finally, the machine HIL frequency response is compared with the small signal models. Approximate transfer functions are extracted with the vector fitting technique in [51].

![Fig. 35. Typhoon HIL 602 real-time simulation platform with physical device controller](image)

4.3.1 HIL vs small signal: Grid-connected machine

The results from the HIL platform are compared with the small signal models for a few selected states in Fig. 36 and Fig. 37. A good match can be seen for both models at most frequencies;
4.3 Comparison between theoretical models and HIL simulation platform

however \( \omega_r \) (Fig. 37 (b)) does not match the small signal model for perturbations greater than approximately 70Hz. The mismatch which occurs at relatively low dB values is due to errors induced by numerical precision. Vector fitting is used to obtain 5th order approximate transfer functions which are included in the figures. More channels are shown in Fig. 38 for different values of \( R_s, L_s \), and it can be seen that the magnitudes increase as the grid becomes weaker.

![Fig. 36. Theoretical vs HIL Bode plots for \( v_{gs} \) and \( \omega_r \) for machine connected to a weak grid](image)

![Fig. 37. Theoretical vs HIL Bode plots for \( v_{gs} \) and \( \omega_r \) for machine connected to a weak grid](image)

![Fig. 38. Bode plots for grid-connected mode with different impedance values (s.g \( \equiv \) strong grid; w.g \( \equiv \) weak grid)](image)
4.3.2 **HIL vs small signal: Standalone**

For this scenario, the machine rotor is assumed to possess a constant angular velocity ($120\pi$ electrical rad/s). The load is a series $R$-$L$ branch with $R_x = 100\Omega$ and $L_x = 5mH$. The operating point corresponds to a load active power of 200kW. Fig. 39 shows the frequency responses for a couple of selected channels. A close match between the theoretical model and HIL model is observed.

![Frequency Responses](image)

*Fig. 39. Bode plots of field-voltage-to-machine $v_{rqx}$ and $v_{rdx}$ channels at constant rotor speed (theoretical small signal in solid blue; HIL results in red dotted) in standalone*

4.3.3 **Model identification of microgrid**

The HIL platform is configured to simulate a small microgrid consisting of a synchronous generator, a solar photovoltaic (PV) array connected to a neutral point clamped (NPC) inverter, and a linear load, as shown in Fig. 31. The inverter is controlled by a real physical TI F28377 DSP. Some waveforms obtained from the HIL platform are shown in Fig. 40.
4.3 Comparison between theoretical models and HIL simulation platform

For this model identification scenario, the load is selected as a series R-L branch with a rating of 83.4kW, lagging power factor of 0.88 at a nominal line-to-line voltage of 480V. The NPC inverter can operate in unity power factor (upf) mode or it can be configured to maintain a constant voltage at the point of common coupling through reactive power control.

In the first case where the inverter is configured to operate in upf, the inverter active power is varied from 5% to 14% of the rated load power and the machine terminal voltage control channels are extracted as shown in Fig. 41 and Fig. 42. It was not possible for the inverter to inject active power beyond 14% in upf due to an overvoltage protection in the inverter controller. In both figures, whereas the data points are represented by dots, the solid continuous lines represent the theoretical linearized small signal models of the constant speed machine operating in standalone, with adjustments made to the delivered active power.
In Fig. 43 and Fig. 44, the constant speed machine frequency response is extracted when the inverter operates in constant terminal voltage mode. The injected solar PV power is varied from 5% to 35% in increments while the inverter is controlled to supply reactive power to keep the line-to-line rms voltage at 480V at the point of common coupling.

Transfer functions are extracted through vector fitting and then the gain ($G_m$) and phase ($P_m$) margins are computed for the different inverter modes and for different penetration of solar PV as shown in Fig. 45 and Fig. 46 (gain margins that are not shown are infinite). The ‘reference model’ refers to the constant speed machine standalone equations without any inverter (as derived in Section 4.2). The reference model has infinite gain margin for all operating points, and for all control channels; phase margin of 180° for $v_{x,q}$ channel; phase margin of 121.9° for $v_{x,d}$ channel; and a phase margin of 116.1° for $v_{x,t}$ channel. Comparing the data in Fig. 45 and Fig. 46 to the reference model values, it can be concluded that:

1. For the $v_{x,q}$ channel, the addition of the inverter worsens the gain and phase margins. The margins worsen further as the inverter penetration increases. The margins in voltage control mode are better than in upf mode; therefore when connected, the inverter should operate in voltage control mode as much as possible.
4.3 Comparison between theoretical models and HIL simulation platform

2. For the $v_{x,d}$ channel, the addition of the inverter improves the phase margins. The improvement is better when the inverter operates in voltage control mode. The phase margin drops as the inverter penetration increases. The gain margin is not affected.

3. For the $v_{x,f}$ channel, the addition of the inverter worsens the gain and phase margins in upf mode, and these margins worsen further as inverter penetration increases. In voltage control mode, the gain margin is mostly unaffected, while the phase margins worsen as the inverter penetration increases.

From these observations, it can be concluded that the inverter should operate as much as possible in the voltage control mode in order to minimize impact on machine voltage stability margins. The extracted models can further be used to properly design controllers, as briefly discussed subsequently.
Suppose it is desired to tune the excitation system for the synchronous machine connected to a weak grid. For this purpose, a DC4B excitation system is considered [45] which is modeled in the Typhoon HIL platform as shown in Fig. 47. The parameters of the exciter are shown in Table 5. The controller parameters, $K_p$ and $K_i$, are tuned using Matlab’s “pidtool” and the following values selected: $K_i = 0.14811; K_p = 0$. Settling time: 17.6s; Phase margin = 88.4°; overshoot = 0%. Fig. 48(a) shows the closed loop step response for a 1% reference change, while Fig. 48(b) shows the percent relative error which confirms the accuracy of the modeling platform for closed loop response.
In another experiment, the load rejection of the system is examined by observing the terminal voltage with a step increase in the load. The synchronous machine is operating in closed-loop with $K_i = 1$, while the $K_p$ is varied (Fig. 49 and Fig. 50). It can be seen that as the gain is increased, the settling time reduces, but the overshoot increases, as would be expected. Furthermore, it can be seen that the undershoot is less severe when the inverter is in operation (Fig. 50) than when it is not (Fig. 49), suggesting that the inverter improves the load rejection of the system.

![Diagram](image)

**Fig. 47. DC4B exciter model**

**Table 5. Parameters for DC4B exciter**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_r$</td>
<td>0s (voltage transducer disabled)</td>
</tr>
<tr>
<td>$K_a$</td>
<td>1</td>
</tr>
<tr>
<td>$T_a$</td>
<td>0.001s</td>
</tr>
<tr>
<td>$T_e$</td>
<td>0.1s</td>
</tr>
<tr>
<td>$K_e$</td>
<td>1</td>
</tr>
<tr>
<td>$K_f$</td>
<td>0 (feedback stabilization disabled)</td>
</tr>
<tr>
<td>$K_d$</td>
<td>0</td>
</tr>
</tbody>
</table>
HIL platform is leveraged to measure the output impedance of the inverter in the $qd$ reference frame, in upf operation. As shown in Fig. 51, the set-up injects $q$ or $d$ axes disturbances in to the grid voltage. $Q_{(o)}$ and $D_{(o)}$ are operating points of $q$ and $d$ axis voltages while the ‘disturbance’ subscript indicates input disturbances. The current $i_{L2}$ is measured and transformed into the $qd$ reference frame. From these measurements, the output admittances $i_{q,d}/v_{q,d}$ are computed. Impedance can be obtained through a simple inversion of the admittance.

An approximate theoretical model is obtained by replacing the inverter with an equivalent incremental resistor and then computing the state space representation, as shown in equation (4.14)
where $R_x = R_{L2} + R_C$, and $R_y = R_{L1} + R_C$. As an example, the $q$-axis admittance frequency response is plotted in Fig. 52 and shows a close match with the theoretical expectation in the low and high frequency regions. Differences can be explained as the action of control loops that were not modeled.

\[
\begin{align*}
\frac{di_{L2q}}{dt} &= \left[ \begin{array}{cccccc} \frac{-R_x}{L_2} & -\omega & \frac{R_C}{L_2} & 0 & -\frac{1}{L_2} & 0 \\
\omega & \frac{-R_x}{L_2} & 0 & \frac{R_C}{L_2} & 0 & -\frac{1}{L_2} \\
\frac{R_C}{L_1} & 0 & -\frac{R_x}{L_1} & -\omega & \frac{1}{L_1} & 0 \\
0 & \frac{R_C}{L_1} & \omega & -\frac{R_x}{L_1} & 0 & \frac{1}{L_1} \\
\frac{1}{C} & 0 & -\frac{1}{C} & 0 & 0 & -\omega \\
0 & \frac{1}{C} & 0 & -\frac{1}{C} & \omega & 0 \end{array} \right] \\
\frac{dv_{Cq}}{dt} &= \left[ \begin{array}{c} -\frac{1}{L_2} \\
0 \\
\frac{1}{L_2} \\
0 \\
0 \\
0 \end{array} \right]
\end{align*}
\]

\[
\begin{align*}
\frac{di_{L2d}}{dt} &= \left[ \begin{array}{cccccc} \frac{-R_x}{L_2} & -\omega & \frac{R_C}{L_2} & 0 & -\frac{1}{L_2} & 0 \\
\omega & \frac{-R_x}{L_2} & 0 & \frac{R_C}{L_2} & 0 & -\frac{1}{L_2} \\
\frac{R_C}{L_1} & 0 & -\frac{R_x}{L_1} & -\omega & \frac{1}{L_1} & 0 \\
0 & \frac{R_C}{L_1} & \omega & -\frac{R_x}{L_1} & 0 & \frac{1}{L_1} \\
\frac{1}{C} & 0 & -\frac{1}{C} & 0 & 0 & -\omega \\
0 & \frac{1}{C} & 0 & -\frac{1}{C} & \omega & 0 \end{array} \right] \\
\frac{dv_{Cd}}{dt} &= \left[ \begin{array}{c} \frac{1}{L_2} \\
0 \\
0 \\
\frac{1}{L_2} \\
0 \\
0 \end{array} \right]
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} i_{L2q} \ i_{L2d} \ i_{L1q} \ i_{L1d} \ v_{Cq} \ v_{Cd} \end{bmatrix} &= \left[ \begin{array}{cccccc} \frac{-R_x}{L_2} & -\omega & \frac{R_C}{L_2} & 0 & -\frac{1}{L_2} & 0 \\
\omega & \frac{-R_x}{L_2} & 0 & \frac{R_C}{L_2} & 0 & -\frac{1}{L_2} \\
\frac{R_C}{L_1} & 0 & -\frac{R_x}{L_1} & -\omega & \frac{1}{L_1} & 0 \\
0 & \frac{R_C}{L_1} & \omega & -\frac{R_x}{L_1} & 0 & \frac{1}{L_1} \\
\frac{1}{C} & 0 & -\frac{1}{C} & 0 & 0 & -\omega \\
0 & \frac{1}{C} & 0 & -\frac{1}{C} & \omega & 0 \end{array} \right] \begin{bmatrix} i_{L2q} \ i_{L2d} \ i_{L1q} \ i_{L1d} \ v_{Cq} \ v_{Cd} \end{bmatrix} + \left[ \begin{array}{c} 0 \\
0 \\
0 \\
0 \\
0 \\
0 \end{array} \right] \end{align*}
\]
4.4 Conclusion

The main objective of this chapter was to examine the usefulness of HIL platform as a tool for model identification and model order reduction of complex dynamic micro-grids. After developing accurate synchronous machine small signal models, it was shown that the HIL-derived results of a synchronous machine connected in different configurations closely match the theoretical small signal models. The platform was used in measuring inverter output impedance. The effects of increasing inverter penetration were analyzed, and it was shown that voltage-control operation of the inverter improved stability margins. Therefore, the model identification framework provides deeper insights into the dynamics of the system under consideration, and this can be extended to more complex systems.
Chapter 5

A Cascaded State Feedback Adaptive Control of Three Phase Inverter with LCL-Filter

5.1 Introduction

Microgrids of the future are expected to be dominated by inverter-interfaced distributed energy resources (DER). Unlike traditional synchronous machine-based sources, inverters must shape the current and voltage into sinusoidal waveforms with minimal total demand distortion (TDD). This requires high performance, high bandwidth controllers. It is typical to find control bandwidths in the range 500Hz to 5kHz.

In a 3 phase inverter with LCL-filter, one of the more common control strategies is cascaded control, where the overall controller is cascaded in a single-input single-output (SISO) fashion as follows: (i) a fast inner current control loop; (ii) a slower outer capacitor voltage control loop; (iii) a slower outer power control loop. Compared to single loop control, cascaded control can simplify controller design of complex processes and provide high performance (command tracking, disturbance rejection, etc.) of the closed loop system [52]. However, traditional fixed gain controls can suffer from degraded performance and unstable behavior when the plant parameters deviate from their nominal values. Adaptive controllers on the other hand can adjust their parameters online and in real time, in order to satisfy certain system objectives. Adaptive control can provide
5.1 Introduction

equivalent nominal performance to fixed gain control, and superior performance when the plant parameters deviate from their nominal values [53].

In [54], a state feedback adaptive controller is applied to a standalone inverter with LC-filter. The plant is modeled in the synchronous reference frame. The authors make some assumptions on the size of the filter capacitor $C_f$ and they obtain the adaptive control laws considering a total of 4 unknown system parameters. The adaptive controller is shown to provide superior performance (compared to a static gain controller) under linear and non-linear load conditions. However, no mention is made of parametric convergence.

An output tracking model reference adaptive controller (MRAC) for inverter with LCL-filter is proposed in [55]. The problem formulation requires the plant input-to-output transfer function to have a relative degree of 1, which motivates the authors’ choice of the control variable as the inverter-side current. In this control method, the inverter is modeled in the stationary reference frame which prevents the cross-coupling that occurs in rotating reference frame models. The controller shows good performance even considering large variations in grid-side inductance.

In this chapter, we propose a cascaded state feedback adaptive control of a three-phase inverter with LCL-filter. The cascaded control concept simplifies overall controller design and ensures good command tracking/performance. The state feedback results in improved transient response via optimal feedback gains for multiple-input multiple-output systems (MIMO) than SISO control design. The adaptive control is based on the adaptive laws developed in [56] for a plant with unknown system matrix $A$, and unknown input matrix $B$. It ensures correct command tracking in the presence of large variations in the LCL filter parameters. With sufficient persistent excitation, accurate parameter estimation can be obtained.

Compared to output-tracking MRAC [55], state feedback adaptive control has a couple of advantages:

1. It may not require enforcing conditions on the structure of the problem such as the relative degree and minimum phase condition. This is because it does not use a transfer function formulation. As such, any of the inverter states (inverter-side currents, filter capacitor voltages, or grid-side currents) can be controlled adaptively if a matching condition exists.
2. The nominal plant controller (state feedback gain) is computed for a MIMO system, which may be better optimized than the state feedback gain computed for a SISO system in the case of traditional output-tracking MRAC.

3. In some ways, state space representation is a more intuitive problem formulation than transfer functions. This is because state space parameters may be much more physically meaningful than coefficients of the complex variable (Laplace variable) in a transfer function.

On the other hand, a major drawback of state feedback adaptive control is that it requires all states to be accessible which, in practical terms, may increase design complexity and costs. Also, the requirement of a matching condition may restrict the types of problems it can be directly applied to.

In a cascaded system, there are normally transient errors between the command signal computed by the outer loop, and the output of the adjacent inner loop. This is not normally a problem for traditional feedback control. However, a direct application of the MRAC in [56] to the outer loops, without additional modifications, may result in instability since the adaptive laws that were developed did not consider such errors. Another contribution of this chapter is accounting for these transient input errors by extending the work in [57] to the MIMO case with unknown $B$-matrix. This enables overall cascaded system to show good performance and parameter convergence. Simulation results are used to evaluate the overall control system.

### 5.2 Plant model

The three-phase inverter with LCL filter is shown in Fig. 53.

![Fig. 53. Three phase inverter with LCL filter](image)
5.2 Plant model

The state space equation in synchronous \((qd)\) reference frame of the average inverter model can be written as follows (zero sequence is neglected):

\[
\begin{bmatrix}
\frac{di_{Lq}}{dt} \\
\frac{di_{Ld}}{dt} \\
\frac{di_{L2q}}{dt} \\
\frac{di_{L2d}}{dt} \\
\frac{dv_{Cfq}}{dt} \\
\frac{dv_{Cfd}}{dt}
\end{bmatrix} =
\begin{bmatrix}
\frac{r_{L1}}{L_1} & -\omega & 0 & 0 & -\frac{1}{L_1} & 0 \\
\omega & -\frac{r_{L1}}{L_1} & 0 & 0 & 0 & -\frac{1}{L_1} \\
0 & 0 & -\frac{r_{L2}}{L_2} & -\omega & \frac{1}{L_2} & 0 \\
0 & 0 & \omega & -\frac{r_{L2}}{L_2} & 0 & \frac{1}{L_2} \\
\frac{1}{C_f} & 0 & -\frac{1}{C_f} & 0 & 0 & -\omega \\
0 & \frac{1}{C_f} & 0 & -\frac{1}{C_f} & \omega & 0
\end{bmatrix}
\begin{bmatrix}
i_{L1q} \\
i_{L1d} \\
i_{L2q} \\
i_{L2d} \\
v_{Cfq} \\
v_{Cfd}
\end{bmatrix}
+ 
\begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
v_{invq} \\
v_{invd}
\end{bmatrix}
+ 
\begin{bmatrix}
-\frac{1}{L_1} \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
v_{gq} \\
v_{gd}
\end{bmatrix}
\]

(5.1)

Where the control inputs are:

\[
\begin{align*}
v_{invq} &= \frac{V_{dc}d_q}{2} \\
v_{invd} &= \frac{V_{dc}d_d}{2}
\end{align*}
\]

(5.2)

Where \(d_q\) and \(d_d\) are the \(q\) and \(d\) axes modulation indices respectively. They can take values between -1 and 1. The other inputs \(v_{gq}\), and \(v_{gd}\) are the inverter terminal \(q\), and \(d\) axes voltages respectively. They are considered as disturbances.

Equation (5.1) can be re-written in cascaded form as the set of equations in (5.3):
When formulated as equation (5.3), the plant can be seen as consisting of three loops or cascades. The control inputs for the innermost loop \( (i_{L1q}) \) are \( v_{invq} \) and \( v_{invd} \). The control inputs for the first outer loop \( (v_{Cfq}) \) are \( i_{L1q} \) and \( i_{L1d} \), while the control inputs for the second outer loop \( (i_{L2q}) \) are \( v_{Cfq} \) and \( v_{Cfd} \). The other inputs are considered as disturbances.

### 5.3 Statement of the control problem

Consider the state-space MIMO system:

\[
\dot{x} = Ax + Bu
\]  
(5.4)

Where: \( x(t) \) is the state vector \( (x \in \mathbb{R}^{nx1}) \), \( A \) is the system matrix \( (A \in \mathbb{R}^{nxn}) \), \( B \) is the input matrix \( (B \in \mathbb{R}^{nxm}) \), and \( u(t) \) is the control input \( (u \in \mathbb{R}^{nx1}) \).

The problem is to determine the input \( u = \theta x \) such that the closed loop system is stable and the state \( x \) is brought to 0 (or follows a reference input). In MRAC, a known stable closed-loop reference model with desired characteristics is selected. It can be expressed as:

\[
\dot{x}_m = A_m x_m + B_m r
\]  
(5.5)
5.3 Statement of the control problem

Where $x_m$ is the closed-loop reference model state vector ($x_m \in \mathbb{R}^{n_m}$), $A_m$ is the closed-loop reference model system matrix ($A_m \in \mathbb{R}^{n_m}$), $B_m$ is the closed-loop reference model input matrix ($B_m \in \mathbb{R}^{n_m}$), and $r(t)$ is the closed-loop reference model control input ($r \in \mathbb{R}^{n_1}$).

5.3.1 Matching condition

One of the main assumptions of state-feedback MRAC [53] is that there exists an unknown $\theta^*$ such that:

$$A + B\theta^* = A_m$$

(5.6)

Equation (5.6) is the matching condition, where $\theta^* \in \mathbb{R}^{n_m}$. The adaptive control law computes an estimate of $\theta^*$ which forces the plant $A$ to behave like the closed-loop reference model $A_m$.

Considering the complete plant model in equation (5.1), the structure of the input matrix $B$ is such that it is not possible to obtain an exact matching condition in general. It is only possible in the particular case where there is no uncertainty in the parameters $r_{L2}$, $L_2$, $C_f$, which is scenario with limited usefulness.

If the plant is re-written in the cascaded form of equation (5.3), it is immediately clear that an exact matching condition exists for every stage. This is one of the motivations for the cascaded adaptive control concept in this chapter.

5.3.2 Case of known $B$-matrix

In [53], a control input $u = \theta x$ is determined for the simplest case when the input matrix $B$ is known, leading to a globally stable system. The adaptive control law is derived as in equation (5.7).

$$\dot{\hat{\theta}} = \hat{\theta} = -\Gamma B^T P e x^T$$

(5.7)

Where:

$\hat{\theta}$ is an estimate of $\theta^*$

$\tilde{\theta}$ is the parameter estimation error, $\tilde{\theta} = \hat{\theta} - \theta^*$

$\Gamma$ is a symmetric $m \times m$ diagonal matrix that modifies the rate of adaptation

$e$ is the state error vector, $e = x - x_m$
A Cascaded State Feedback Adaptive Control of Three Phase Inverter with LCL-Filter

$P$ is a Lyapunov $n \times n$ matrix, which is the solution of equation (5.8):

$$A_m^T P + PA_m = -Q < 0$$  \hspace{1cm} (5.8)

Where $Q$ is any $n \times n$ symmetric positive definite matrix.

The Lyapunov function is proposed as:

$$V = e^T P e + \text{Trace} \left( \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \right)$$  \hspace{1cm} (5.9)

Whose time derivative is:

$$\dot{V} = -e^T Q e \leq 0$$  \hspace{1cm} (5.10)

Since $\dot{e}$ is bounded, it follows that $\lim_{t \to \infty} e(t) = 0$ (Barbalat’s Lemma) which implies uniform stability in the large of the equilibrium point.

5.3.3 Case of unknown $B$-matrix

When $B$ is not known, the adaptive law obtained in [53] results in only local stability.

In [56], a globally stabilizing controller for the case of unknown $B$ is obtained, subject to certain constraints on the structure of $B$. It is assumed that the $B$-matrix is expressed.

$$B = B_m \Psi^r$$  \hspace{1cm} (5.11)

Where $\Psi^r$ is an unknown sign-definite matrix such that there exists a sign-definite matrix $M$ and a known symmetric positive-definite matrix $\Gamma_0$ satisfying the following equation:

$$\Gamma_0 \Psi^r + \Psi^{rT} \Gamma_0 = M$$  \hspace{1cm} (5.12)

Assuming the control input can be written for the tracking case as:

$$u = \dot{\theta}(t)x + N(t)r$$  \hspace{1cm} (5.13)

The error model is:

$$e = x - x_m \Rightarrow \dot{e} = A_m e + B_m \Psi^r \tilde{\theta}(t)x + B_m \Psi^r \tilde{\phi}(t)r$$  \hspace{1cm} (5.14)

Where:
5.3 Statement of the control problem

\[
\begin{aligned}
\dot{\theta}(t) &= \theta(t) - \dot{\theta} \\
\dot{\phi}(t) &= N(t) - \Psi^{-1}
\end{aligned}
\]  

(5.15)

The adaptive laws are chosen as:

\[
\begin{aligned}
\dot{\hat{\theta}} &= \dot{\theta} = -\Gamma^{-1}B^T_m P e x^T \\
\dot{\hat{\phi}} &= \dot{N} = -\Gamma^{-1}B^T_m P e r^T
\end{aligned}
\]  

(5.16)

A Lyapunov candidate function is selected as:

\[
V = e^T P e + Trace \left[ \dot{\hat{\theta}}^T \left( \Psi^{TT} \Gamma \right) \dot{\hat{\theta}} \right] + Trace \left[ \dot{\hat{\phi}}^T \left( \Psi^{TT} \Gamma \right) \dot{\hat{\phi}} \right]
\]

(5.17)

The time derivative is:

\[
\dot{V} = e^T \left[ A^T_m P + PA_m \right] e \equiv -e^T Q e \leq 0
\]

(5.18)

Applying Barbalat’s lemma, \( \lim_{t \to \infty} e(t) = 0 \). Therefore, the controller is uniformly stable in the large.

5.3.4 Cascaded adaptive control

A conceptual illustration of the cascaded adaptive control concept proposed in this chapter is shown in Fig. 54, where \( K_{1b}, K_{2b}, K_{3b} \) are \([2x2]\) integral gains designed through linear quadratic integrator (LQI) which we discuss in the section 5.4. In Fig. 54, the gain matrices \( \theta_1(t), \theta_2(t), \theta_3(t), N_1(t), N_2(t), \) and \( N_3(t) \) are adjusted adaptively.

![Fig. 54. Cascaded adaptive control concept](image)

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However, a direct implementation of the adaptive laws in equation (5.16) may result in unstable behavior because there exist transient input errors for the outer loops which were not considered in the synthesis of the adaptive laws. These input errors can be expressed as:

\[
\begin{align*}
\Delta u_1 &= i_{Lq,qd}(t) - i_{Lq,qd,ref}(t) \\
\Delta u_2 &= v_{Cq,qd}(t) - v_{Cq,qd,ref}(t)
\end{align*}
\]  

(5.19)

Unless the control inputs are saturated, and assuming the cascaded system is stable, the following conditions should hold:

\[
\begin{align*}
\lim_{t \to \infty} \Delta u_1(t) &= 0 \\
\lim_{t \to \infty} \Delta u_2(t) &= 0
\end{align*}
\]  

(5.20)

In [57], the problem of adaptive control for a first order plant (scalar system) considering saturated input is considered. We briefly recall only some of the salient points. The scalar plant is described by the equation:

\[
\dot{x}(t) = ax(t) + bu(t)
\]  

(5.21)

The closed-loop reference model is defined as in equation (5.22):

\[
\dot{x}_m(t) = a_{m}x_m(t) + b_{m}r(t)
\]  

(5.22)

The adaptive controller is selected as:

\[
\begin{align*}
\theta(t) &= \theta(t)x(t) + k(t)r(t) \\
\dot{\theta}(t) &= v(t) \\
u(t) &= \begin{cases} 
0 & \text{if } |v(t)| \leq u_0 \\
 u_0 \text{sgn}(v(t)) & \text{if } |v(t)| > u_0
\end{cases}
\end{align*}
\]  

(5.23)

Where \( u_0 \) is the upper bound on the magnitude of \( u(t) \). In equation (5.23) \( v(t) \) is the desired input of the plant while \( u(t) \) is the actual or real input. Equation (5.23) mathematically captures differences between desired and actual plant inputs, which is suitable for application to the cascaded control problem.

Combining equations (5.21) and (5.23), the closed loop system can be written as:

\[
\dot{x}(t) = (a + b\theta(t))x(t) + b\Delta u(t) + bk(t)r(t)
\]  

(5.24)
5.3 Statement of the control problem

Where:

\[ \Delta u(t) = u(t) - v(t) \]  \hspace{1cm} (5.25)

The following errors are defined:

\[
\begin{cases}
    e(t) = x(t) - x_m(t) \\
    \phi(t) = \theta(t) - \theta^* \\
    \psi(t) = k(t) - k^*
\end{cases}
\]  \hspace{1cm} (5.26)

Where:

\[
\begin{align*}
    \theta^* &= \frac{a_m - a}{b} \\
    k^* &= \frac{b_m}{b}
\end{align*}
\]  \hspace{1cm} (5.27)

The error derivative is:

\[ \dot{e}(t) = a_m e(t) + b \phi(t)x(t) + b \psi(t)r(t) + b \Delta u(t) \]  \hspace{1cm} (5.28)

To remove the effect of \( \Delta u(t) \), an auxiliary signal \( e_{\Delta}(t) \) is introduced. It is defined by its derivative which is written as:

\[ \dot{e}_{\Delta}(t) = a_m e_{\Delta}(t) + k_{\Delta}(t) \Delta u(t) \]  \hspace{1cm} (5.29)

Where \( k_{\Delta}(t) \) is an estimate of the input scalar \( b \).

Then a new error signal \( e_u(t) \) is defined as: \( e_u(t) = e(t) - e_{\Delta}(t) \). Its time derivative is:

\[ \dot{e}_u(t) = a_m e_u(t) + b \phi(t)x(t) + b \psi(t)r(t) + \lambda(t) \Delta u(t) \]  \hspace{1cm} (5.30)

Where \( \lambda(t) = b - k_{\Delta}(t) \).

The adaptive laws are chosen as:

\[
\begin{align*}
    \dot{\phi}(t) &= -\gamma \text{sgn}(b)e_u \dot{x} \\
    \dot{\psi}(t) &= -\gamma \text{sgn}(b)e_u \dot{r} \\
    \dot{\lambda}(t) &= -\gamma e_u \Delta u(t)
\end{align*}
\]  \hspace{1cm} (5.31)
Where $\gamma$ is a gain that can be chosen to adjust the speed of adaptation.

A Lyapunov candidate function is selected as:

$$V = \frac{1}{2} \left[ e_a^2 + \|b\| \left( \frac{\phi^2}{\gamma} + \frac{\psi^2}{\gamma} + \frac{\lambda^2}{\gamma} \right) \right]$$

(5.32)

From where it can be shown that $\dot{V} = a_a e_a^2 \leq 0$. Applying Barbalat’s lemma allows conclusion of uniform stability in the large.

We extend this analysis to the MIMO case by re-writing the plant as follows:

Where: $x \in \mathbb{R}^{m \times 1}$, $A \in \mathbb{R}^{m \times m}$, $B \in \mathbb{R}^{m \times m}$, $v \in \mathbb{R}^{m \times 1}$, $\Delta u \in \mathbb{R}^{m \times 1}$, and:

$$\begin{align*}
v(t) &= \Theta(t)x(t) + N(t)r(t) \\
\Delta u(t) &= u(t) - v(t)
\end{align*}$$

(5.33)

For the case of 2 inputs, $u(t)$ can be written as:

$$u(t) = \begin{bmatrix} v_q(t) \\ v_d(t) \end{bmatrix} \quad \begin{array}{ll}
\text{if } |v_q(t)| \leq u_{0q}(t) \text{ and } |v_d(t)| \leq u_{0d}(t) \\
\text{if } |v_q(t)| > u_{0q}(t) \text{ and } |v_d(t)| \leq u_{0d}(t) \\
\text{if } |v_q(t)| \leq u_{0q}(t) \text{ and } |v_d(t)| > u_{0d}(t) \\
\text{if } |v_q(t)| > u_{0q}(t) \text{ and } |v_d(t)| > u_{0d}(t)
\end{array}$$

(5.34)

Where the positive quantities $u_{0q}(t)$ and $u_{0d}(t)$ are the magnitude limits of the corresponding control input. Considering the same reference model (equation (5.5)), and making the same assumptions on the structure of the unknown input matrix (equations (5.11) and (5.12)), the time derivative of the error vector $e(t) = x(t) - x_m(t)$ can be written as:

$$\dot{e}(t) = A \dot{x}(t) + B \Psi \dot{\theta}(t) x(t) + B \Psi \dot{\phi}(t) r(t) + B \Delta u(t)$$

(5.35)

We define an auxiliary vector $e_{\Delta}(t)$ whose time derivative is:

$$\dot{e}_{\Delta}(t) = A \dot{e}_{\Delta}(t) + B \Delta u(t)$$

(5.36)
5.3 Statement of the control problem

Where:

\[ B_\Delta(t) = B_m \Omega(t) \quad (5.37) \]

Now a new error vector is defined as \( e_u(t) = e(t) - e_m(t) \), whose time derivative is:

\[ \dot{e}_u(t) = A_m e_u(t) + B_m \Psi^r \hat{\theta}(t)x(t) + B_m \Psi^r \hat{\phi}(t)r(t) + B_m \hat{\Omega}(t) \Delta u \quad (5.38) \]

Where:

\[
\begin{align*}
\hat{\theta}(t) &= \theta(t) - \theta^* \\
\hat{\phi}(t) &= N(t) - \Psi^{-1} \\
\hat{\Omega}(t) &= \Omega^* - \Omega(t)
\end{align*}
\quad (5.39)
\]

We choose the following adaptive laws:

\[
\begin{align*}
\dot{\hat{\theta}}(t) &= \hat{\theta}(t) = -\Gamma^{-1} B_m^T Pe_u(t)x^T \\
\dot{\hat{\phi}}(t) &= \hat{\phi}(t) = -\Gamma^{-1} B_m^T Pe_u(t)r^T \\
\dot{\hat{\Omega}}(t) &= -\hat{\Omega}(t) = -\Gamma^{-1} B_m^T Pe_u(t) \Delta u^T
\end{align*}
\quad (5.40)
\]

A Lyapunov candidate function is selected as:

\[ V = e_u^T Pe_u + \text{Trace} \left[ \hat{\theta}^T \left( \Psi^r \Gamma \right) \hat{\theta} \right] + \text{Trace} \left[ \hat{\phi}^T \left( \Psi^r \Gamma \right) \hat{\phi} \right] + \text{Trace} \left[ \hat{\Omega}^T \Gamma \hat{\Omega} \right] \quad (5.41) \]

The time derivative can be computed as:

\[ \dot{V} = e_u^T \left[ A_m^T P + PA_m \right] e_u \leq 0 \quad (5.42) \]

From Barbalat’s lemma, \( \lim_{t \to \infty} e(t) = 0 \). Therefore, the controller is uniformly stable in the large.

5.3.5 Disturbance rejection

The adaptive laws in equation (5.40) do not consider the disturbance inputs. In order for the error between the reference model and the plant to tend asymptotically to 0 and in order to get parameter convergence, we must account for the disturbances.
The system is cascaded into 3 stages as shown in equation (5.3). Since the disturbance signals are measured, their effect is removed by adding them into the corresponding control input. In other words, we modify the control inputs as follows:

\[
\begin{bmatrix}
    V_{invq}^* \\
    V_{invd}^*
\end{bmatrix} =
\begin{bmatrix}
    V_{invq} + V_{Cfq} \\
    V_{invd} + V_{Cfd}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    I_{L1q, ref}^* \\
    I_{L1d, ref}^*
\end{bmatrix} =
\begin{bmatrix}
    I_{L1q, ref} + I_{L1q} \\
    I_{L1d, ref} + I_{L1d}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    V_{Cfq, ref}^* \\
    V_{Cfd, ref}^*
\end{bmatrix} =
\begin{bmatrix}
    V_{Cfq, ref} + V_{gq} \\
    V_{Cfd, ref} + V_{gd}
\end{bmatrix}
\]

\[ (5.43) \]

5.3.6 Parameter convergence and numerical issues

In theory, if the adaptive laws result in a globally stable system, if the disturbances are accounted for, and if the inputs are sufficiently persistently exciting, the parameters should converge to their true values. After running several simulations of the system, we found that the values of the adaptation gains should not be too large, otherwise the simulation might suffer from numerical precision and numerical instability issues.

We also saw that the numerical behavior of the simulated system improves if we modify the cascaded structure of Fig. 54 slightly as shown in Fig. 55 below. Here, the reference model instead of the plant is driving the reference input to a given cascaded stage in order to achieve command tracking.

Fig. 55. Modified cascaded adaptive control concept
5.4 Nominal state feedback gains

So far, we have derived the adaptive laws for the cascaded system with known reference models. In this section, we discuss how these reference models can be obtained.

5.4.1 Classical state feedback

In classical state feedback control, the feedback gains for a given plant can be computed using linear quadratic regulator (LQR) or eigenvalue (pole) placement.

LQR for state space systems is a well-known result in the literature. Under the assumption that the pair \((A, B)\) is stabilizable, LQR computes an optimal gain matrix \(K\) that minimizes the quadratic cost function:

\[
J(u) = \int_{0}^{\infty} (x^T Q x + u^T R u + 2x^T N u) dt
\]

(5.44)

Where \(Q, R, N\) are cost matrices such that \(R>0\), and \(Q-NR^{-1}N^T\geq 0\)

The feedback gain \(K\) which minimizes \(J\) is:

\[
K = R^{-1}(B^T S + N^T)
\]

(5.45)

Where \(S\) is the solution of the Riccati equation:

\[
A^T S + SA - (SB + N)R^{-1}(B^T S + N^T) + Q = 0
\]

(5.46)

From where the reference model \(A_m\) can be computed as:

\[
A_m = A - BK
\]

(5.47)

The cost matrices \(Q, R, N\) often need to be tuned iteratively until satisfactory closed loop performance is obtained.

For improved command tracking, it is common to use the linear quadratic integrator (LQI). LQI is simply the LQR of the following augmented plant:
On the other hand, pole placement computes the feedback gains that place the closed loop poles at desired locations. Closed loop performance is must be verified and then the target eigenvalues adjusted iteratively.

5.4.2 Cascaded state feedback

Whereas classical state feedback control produces gains for optimal feedback (LQR) or pole placement for a given subsystem, these gains are not necessarily optimal or even stable for the overall cascaded system. This is the problem of global stabilization by partial state feedback. It is addressed for example in [58] where some stabilizing state feedback gains are proposed for a class of triangular systems described by:

\[
\begin{align*}
\dot{z} &= q(z, x_i) \\
\dot{x}_i &= x_2 + f_i(z, x_1, \ldots, x_r), \quad 1 \leq i \leq r
\end{align*}
\]  

(5.49)

However, in the model under consideration (equation (5.1)), the cross-coupling terms \(\omega\) appear because of the \(qd\) transform. To eliminate them, the values of \(L_1, L_2, C_f\) must be known exactly, which defeats the purpose of adaptive control, since they are assumed to be unknown. Furthermore, every state is coupled to at least one other. Therefore, it is not possible to apply the solutions in [58].

We propose the following reference model design method combining both LQR and pole placement strategies.

5.4.2.1 Innermost control loop (\(i_{L1qd current control loop}\) design steps

- Step 1: Design LQI gains for a desired bandwidth, settling time, and overshoot requirement for the system in equation (5.50):

\[
\begin{align*}
A_{\text{augmented}} &= \begin{bmatrix} A & 0 \\ \cdots & \cdots \\ -C & 0 \end{bmatrix} \\
B_{\text{augmented}} &= \begin{bmatrix} B \\ \cdots \\ 0 \end{bmatrix}
\end{align*}
\]  

(5.48)
5.4 Nominal state feedback gains

\[
\begin{bmatrix}
\frac{di_{Lq}}{dt} \\
\frac{di_{Ld}}{dt}
\end{bmatrix} = \begin{bmatrix}
-\frac{r_{L1}}{L_i} & -\omega \\
\omega & -\frac{r_{L1}}{L_i}
\end{bmatrix} \begin{bmatrix}
i_{L1q} \\
i_{L1d}
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{1}{L_i}
\end{bmatrix} \begin{bmatrix}
v_{invq} \\
v_{invd}
\end{bmatrix}
\]

(5.50)

This is equivalent to designing LQR gains for the augmented system in equation:

\[
\begin{bmatrix}
\frac{df_{Lq}}{dt} \\
\frac{df_{Ld}}{dt}
\end{bmatrix} = \begin{bmatrix}
-\frac{r_{L1}}{L_i} & -\omega & 0 & 0 \\
\omega & -\frac{r_{L1}}{L_i} & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
i_{L1q} \\
i_{L1d} \\
v_{invq} \\
v_{invd}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\frac{1}{L_i}
\end{bmatrix} \begin{bmatrix}
v_{invq} \\
v_{invd}
\end{bmatrix}
\]

(5.51)

Where \(f_{Lq}, f_{Ld}\) are the integral (augmented) states.

LQR on the augmented system produces the feedback matrix \(K\) such that \(u = -Kx\) where:

\[
\begin{align*}
K &= \begin{bmatrix} \Theta \mid K_{ib} \end{bmatrix} \\
\Theta &= \begin{bmatrix} \theta_{t1} & \theta_{t2} \\ \theta_{21} & \theta_{22} \end{bmatrix} \\
K_{ib} &= \begin{bmatrix} k_{ib,11} & k_{ib,12} \\ k_{ib,21} & k_{ib,22} \end{bmatrix}
\end{align*}
\]

(5.52)

In equation (5.52), \(\Theta\) are the gains of the unaugmented system (5.50) (which would be obtained by the traditional LQR), whereas the gains \(K_{ib}\) are the gains of the integral states of the augmented system (5.51).

From numerical simulations, it is desirable to enforce \(\theta_{t2} = \theta_{21} = 0\), which helps to achieve more accurate parameter estimation, since it will remove any need for adaptation on the \(\omega\) term in the closed loop system.

- Step 2: Check the stability of the overall closed loop system in equation (5.53):

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\[
\begin{bmatrix}
\frac{di_{1q}}{dt} \\
\frac{di_{1d}}{dt} \\
\frac{di_{2q}}{dt} \\
\frac{di_{2d}}{dt} \\
\frac{dv_{Cf}}{dt} \\
\frac{dv_{Cd}}{dt} \\
\frac{df_{1q}}{dt} \\
\frac{df_{1d}}{dt}
\end{bmatrix} =
\begin{bmatrix}
\frac{-r_{L1}}{L_1} - \frac{\theta_{11}}{L_1} & -\omega - \frac{\theta_{12}}{L_1} & 0 & 0 & -\frac{1}{L_1} & 0 & -\frac{k_{ib,11}}{L_1} & -\frac{k_{ib,22}}{L_1} \\
\omega - \frac{\theta_{21}}{L_1} & \frac{-r_{L1}}{L_1} - \frac{\theta_{22}}{L_1} & 0 & 0 & 0 & 0 & -\frac{k_{ib,21}}{L_1} & -\frac{k_{ib,22}}{L_1} \\
0 & 0 & -\frac{r_{L2}}{L_2} & -\omega & \frac{1}{L_2} & 0 & 0 & 0 \\
0 & 0 & \omega & -\frac{r_{L2}}{L_2} & 0 & \frac{1}{L_2} & 0 & 0 \\
\frac{1}{C_f} & 0 & -\frac{1}{C_f} & 0 & 0 & -\omega & 0 & 0 \\
0 & \frac{1}{C_f} & 0 & \frac{1}{C_f} & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
i_{L1q} \\
i_{L1d} \\
i_{2q} \\
i_{2d} \\
v_{Cf} \\
v_{Cd} \\
f_{L1q} \\
f_{L1d}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
i_{L1q,ref} \\
i_{L1d,ref}
\end{bmatrix}
\] (5.53)

- Step 3: Readjust the LQR cost matrices (\(Q, R, N\)) until the system in equation (5.53) shows satisfactory performance.

5.4.2.2 1st outer control loop (\(v_{Cfd}\) voltage control loop) design steps

- Step 1: Select closed loop poles for the augmented system:

\[
\begin{bmatrix}
\frac{dv_{Cf}}{dt} \\
\frac{dv_{Cd}}{dt} \\
\frac{df_{Cf}}{dt} \\
\frac{df_{Cd}}{dt}
\end{bmatrix} =
\begin{bmatrix}
0 & -\omega & 0 & 0 \\
\omega & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\frac{1}{C_f} \\
0 \\
0 & \frac{1}{C_f} \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
i_{L1q} \\
i_{L1d}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
v_{Cf} \\
v_{Cd} \\
f_{L1q} & 1 \text{ 0}
\end{bmatrix}
\] (5.54)

Where \(f_{Cf}, f_{Cd}\) are the augmented integral states. The poles should be slower than those of the system (5.51) but faster than the slow poles of the system (5.53). We considered a range of 2 to 6 times slower. Compute the feedback gain matrix \(K_2\) with pole placement, where:
In equation (5.55), $\Theta_2$ are the gains of the unaugmented system, whereas the gains $K_{2b}$ are the gains of the integral states of the augmented system (5.54).

Similarly, it is desirable to enforce $m_{12} = m_{21} = 0$, which helps to achieve more accurate parameter estimation by removing the need for adaptation on the $\omega$ term in the closed loop system.

- Step 2: Verify the stability of the overall closed loop system in equation (5.56):

- Step 3: Readjust the pole placement targets in step 1 until satisfactory performance of the system in equation (5.56).
5.4.2.3 2\textsuperscript{nd} outer control loop (ii_{2qd} current or power control loop) design steps

- Step 1: Select closed loop poles for the augmented system:

\[
\begin{bmatrix}
\frac{d i_{L2q}}{dt} \\
\frac{d i_{L2d}}{dt} \\
\frac{df_{L2q}}{dt} \\
\frac{df_{L2d}}{dt}
\end{bmatrix} = \begin{bmatrix}
-\frac{r_{L2}}{L_2} & -\omega & 0 & 0 \\
\omega & -\frac{r_{L2}}{L_2} & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\frac{1}{L_2} & 0 \\
0 & \frac{1}{L_2} \\
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
v_{Cfq} \\
v_{Cfd}
\end{bmatrix}
\]

(5.57)

Where \( f_{L2q}, f_{L2d} \) are the augmented integral states. The poles should be slower than those of the system (5.51) but faster than the slow poles of the system (5.56). We considered a range of 2 to 6 times slower. Compute the feedback gain matrix \( K_3 \) with pole placement, where:

\[
K_3 = \begin{bmatrix} \Theta_3 & K_{3b} \end{bmatrix}
\]

\[
\Theta_3 = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{bmatrix}
\]

\[
K_{3b} = \begin{bmatrix} k_{3b,11} & k_{3b,12} \\ k_{3b,21} & k_{3b,22} \end{bmatrix}
\]

(5.58)

Similarly, it is desirable to enforce \( n_{12} = n_{21} = 0 \), which helps to achieve more accurate parameter estimation by removing the need for adaptation on the \( \omega \) term in the closed loop system.

- Step 2: Verify the stability of the overall closed loop system in equation (5.59):
5.5 Simulation (average model)

- Step 3: Readjust the pole placement targets in step 1 until satisfactory performance of the system in equation (5.59).

5.5 Simulation (average model)

We consider simulation results for an inverter with the following parameters: $L_1 = 0.47\text{mH}; L_2 = 0.47\text{mH}; C_f = 40\text{uF}; V_{gq} = 100\text{V}; V_{gd} = 0\text{V}; \omega = 377 \text{rad/s}$. Tracking tests and parameter estimation tests are considered.

Regarding the command tracking, three sets of tracking tests are considered as follows:

(1) All 3 cascaded control loops are closed. This type of control approach can be applied to a grid-tied inverter in grid-following (or power injection) mode.
(2) Innermost loop ($i_{L1qd}$) and 1st outer loop ($v_{Cfqd}$) closed, with 2nd outer loop ($i_{L2qd}$) open. This control structure can be applied to an inverter in grid-forming mode (or voltage control mode).

(3) Innermost loop ($i_{L1qd}$) closed and all other loops open. This control structure can also be used for grid-tied inverter operation in grid-following (or power control) mode; however it will be less accurate than mode (1) because of active and reactive power mismatch due to other components of the LCL filter.

Finally, the parameter estimation tests show the capability of the proposed control strategy to estimate the parameters under simultaneous large variations of the LCL filter parameters.

5.5.1 Tracking problem: All 3 cascaded loops closed

All 3 cascaded control loops are closed. This control structure can be used, for example, in a grid-following or power control application. The outermost loop current references $i_{L2qd,ref}$ are each switching between +5A and -5A at 1Hz, with a 90° phase difference between both references, and then we observe the tracking performance of the 2nd outer loop (Fig. 56), the 1st outer loop (Fig. 57), and the innermost loop (Fig. 58). It can be seen that good tracking is obtained even under changes in nominal plant parameters by 100%.
5.5 Simulation (average model)

5.5.2 Tracking problem: Innermost loop ($i_{L_{1qd}}$) and 1st outer loop ($v_{Cfqd}$) closed, with 2nd outer loop ($i_{L_{2qd}}$) open

The innermost loop ($i_{L_{1qd}}$) and 1st outer loop ($v_{Cfqd}$) are closed. The 2nd outer loop ($i_{L_{2qd}}$) is open. This control structure can be used, for example, in a grid-forming or voltage control application. The voltage references $v_{Cfqd,ref}$ are each switching between +100V and -100V each with a frequency of 1Hz, with a 90° phase difference between both references, and then we observe the tracking
performance of the 1st outer loop (Fig. 59), and the innermost loop (Fig. 60). It can be seen that good tracking is obtained.

![Graph](image1)

**Fig. 59.** $v_{Cfqd}$ tracking performance

![Graph](image2)

**Fig. 60.** $i_{L1qd}$ inner loop tracking performance

5.5.3 Tracking problem: Innermost loop only ($i_{L1qd}$) closed. All other loops open

This control method can be used for example in current control or grid-following. However due to the rest of the LCL filter components, active and reactive power control is less accurate than $i_{L2qd}$
control. $i_{L1qdref}$ is switching between -5A and 5A at a rate of 1Hz with a 90° phase difference between both references. Fig. 61, shows that good tracking is obtained.

![Fig. 61. $i_{L1qd}$ inner loop tracking](image)

### 5.5.4 Parameter convergence

The complete 3-loop cascaded system is simulated with varying LCL parameters. Reference currents are set as: $i_{L2,qref} = 10A$; $i_{L2d,ref} = 10A$. $L_1$ is changed from nominal value $(0.47mH)$ to twice nominal value and back with a 0.1Hz frequency (Fig. 62). $C_f$ is changed from $40\mu F$ to $50\mu F$ and back with a 0.1Hz frequency (Fig. 63). $L_2$ is changed from nominal value $(0.47mH)$ to twice nominal value and back with a 0.1Hz frequency (Fig. 64). It can be seen that in all these cases, the proposed cascaded adaptive control system shows parametric convergence while correctly tracking reference currents.
Fig. 62. Parameter convergence with step changes in $L_1$

Fig. 63. Parameter convergence with step changes in $C_f$
In this chapter, a cascaded adaptive controller for 3 phase LCL filter is proposed. The inverter and controller are simulated and show good tracking performance and parameter convergence under changing plant conditions.

One idea to explore for future work is the derivation of the reference model. Linear matrix inequalities (LMI) could potentially improve the performance of the nominal cascaded system.
Chapter 6

Testing and Validation of a High-Fidelity Controller Hardware-in-the-Loop Microgrid Testbed

6.1 Introduction

Microgrids are challenging to design, deploy, test, and maintain. The lack of appropriate engineering tools, industry standards, and field experience to develop, operate, and assess microgrids and DER control technologies dampens the enthusiasm for adaptation by utility companies. Whereas equipment manufacturers can test their individual DER controllers and solutions, they may not have the capability to test the interaction of their system with equipment from other vendors in a common environment. This leads to a perceived risk that their equipment may not operate correctly in actual field conditions.

Furthermore, the power systems industry does not yet have openly available, widely-adopted benchmarks adequate to test features or interoperability of microgrid and DER controllers, especially for networked power systems. The current approach to integrate and test controllers

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3 Portions of this chapter relating to Banshee distribution network are published in: Reynaldo Salcedo ; Edward Corbett; Christopher Smith; Erik Limpaecher; Raajiv Rekha; John Nowocin ; Georg Lauss; Edwin Fonkwe ; Murilo Almeida ; Peter Gartner ; Scott Manson ; Bharath Nayak ; Ivan Celanovic ; Christian Dufour ; M.Omar Faruque ; Karl Schoder ; Ron Brandl ; Panos Kotsampopoulos ; Thrug Ham Ha ; Ali Davoudi ; Ali Dehkordi ; Kai Strunz , “Banshee distribution network benchmark and prototyping platform for hardware-in-the-loop integration of microgrid and device controllers”, IET Journal of Engineering, 2019, pp1-10, DOI: 10.1049/joe.2018.5174.
6.1 Introduction

pushes project development risk to the final commissioning stage where equipment is deployed in
the field, rather than reducing integration risk through staged testing and verification during the
design process using low-cost prototyping.

This situation is changing as IEEE standards P2030.7 and P2030.8 [12], [13] specify microgrid
controller functions and testing. In addition, a revision of IEEE standard 1547 addressing
microgrid interconnection issues more squarely has been proposed.

Real-time simulation (RTS) technologies capable of interfacing system emulators with hardware
devices facilitate development of power systems prototyping platforms to systematically assess
control functionalities of vendors’ product lines [59][60]. When RTS is combined with actual
device controllers, this concept is known as controller hardware-in-the-loop (cHIL). Compared to
cHIL (Fig. 66), traditional offline simulation (Fig. 65) is less expensive, but less realistic because
it can only simulate device controllers. Full power testing (Fig. 66) is obviously most accurate,
but also the most expensive testing method. Sometimes, the concept of power hardware-in-the-
loop (PHIL) is used. In PHIL, a mathematical model of a DER or other device is running on a RTS
system and the outputs are amplified by a real world power amplifier (e.g. an inverter). It is costlier
and more accurate than cHIL but also less expensive than full power testing.

![Diagram](image_url)

Fig. 65. Offline simulation tradeoff

Fig. 66. cHIL tradeoff

Fig. 67. Full power test tradeoff
When cHIL is combined with realistic microgrid modeling, design, operation and maintenance, and microgrid extension, this concept is referred to as Microgrid Digital Twin.

In this Section 6.2 of this chapter, we describe a microgrid digital twin platform applied to a real-life, reconfigurable test feeder referred to as ‘Banshee’ [61]. In section 6.3, we describe a physical low-voltage microgrid testbed for model validation of DER and microgrid models. A concluding section closes the chapter.

6.2 Microgrid Digital Twin Platform for Testing and Benchmarking Microgrids and Device Controllers

This section describes a microgrid digital twin platform applied to a real-life, reconfigurable test feeder referred to as “Banshee”. It presents challenges typically found in a community microgrid, small island, and industrial facility. Banshee resembles emerging microgrids around the world, making it a comprehensive benchmark to evaluate microgrid performance. Presently, industry leaders use the system to demonstrate advanced controls and functionalities [62].

6.2.1 Reasons for microgrid digital twin

- Microgrid and DER controllers vary extensively from vendor to vendor in capabilities, maturity levels, and project-specific integration work. Possible incompatibilities and lack of standardized integration procedures raise concerns for engineers and project developers. Digital twin helps to evaluate the gamut of system conditions, guides debugging of control algorithms, demonstrates controllers’ marketed capabilities, and supports revision of electrical feasibility. Thus, microgrid adoption can significantly accelerate by facilitating realistic demonstrations, enabling risk-reduction testing, and pre-commissioning system integration and testing.

- High non-recurring engineering cost because of the project-specific integration and interoperability testing can be alleviated by use of digital twin.

- Digital twin can reduce risk of damage to expensive equipment during deployment and integration testing.
6.2 Microgrid Digital Twin Platform for Testing and Benchmarking Microgrids and Device Controllers

6.2.2 Banshee Distribution Network Topology

The Banshee distribution system is a real-life, reconfigurable, small industrial facility serviced by three non-dedicated utility radial feeders. A one-line diagram of the system is shown in Fig. 111 in Appendix D. It consists of three adjacent feeders with limited connectivity through normally open switches, each capable of carrying the site’s critical load. The overall electrical demand of the feeders ranges from 5MW to 14MW. The system ratings include medium voltages of 13.8kVrms and 4.16kVrms, and low voltages of 480Vrms and 208Vrms line-line. There are 18 aggregated loads continuously supplied by the feeders. These loads are categorized as critical, priority, and interruptible. Table 6 provides a summary of all elements in the system.

<table>
<thead>
<tr>
<th>Component</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feeder protection relays</td>
<td>3</td>
</tr>
<tr>
<td>Power inverter controllers</td>
<td>2</td>
</tr>
<tr>
<td>Motor/generator controllers</td>
<td>2</td>
</tr>
<tr>
<td>Diesel generator</td>
<td>1</td>
</tr>
<tr>
<td>CHP generator</td>
<td>1</td>
</tr>
<tr>
<td>PV system</td>
<td>1</td>
</tr>
<tr>
<td>BESS</td>
<td>1</td>
</tr>
<tr>
<td>Distribution transformer</td>
<td>22</td>
</tr>
<tr>
<td>Medium- and low-voltage line (&gt;500 ft)</td>
<td>19</td>
</tr>
<tr>
<td>Induction motors with compressor loads</td>
<td>2</td>
</tr>
<tr>
<td>Low-voltage loads</td>
<td>18</td>
</tr>
<tr>
<td>Circuit breakers</td>
<td>55</td>
</tr>
<tr>
<td>Virtual protective relays</td>
<td>50</td>
</tr>
</tbody>
</table>

The loads follow time-varying electrical demand profiles. Furthermore, there are two large induction motors rated 200 horsepower with compressor loads. Critical loads highlight strict requirements of continuous electrical service, power quality, and reliability. Priority loads are ideally always electrically served, but in the case of contingencies or islanding operations with lack of generation reserve, the loads may be disconnected. Interruptible loads are not necessarily required during contingencies or islanded conditions. Providing a weight of importance to the loads facilitates the evaluation of microgrid controllers and their functionalities.
Rotating generation assets consist of a 4MVA diesel generator and a 3.5MVA natural gas-fired combined heat and power (CHP) system operating at 13.8kV. The genset controllers operate and protect these units. The configurations of the controllers match the engine types of diesel and natural gas-fired generators. These controllers switch the operation of the gensets from grid-forming to grid-following depending on the state of the breaker at the point of interconnection (POI). A voltage and frequency (V/F) droop control with 4 percent linear droop is used in grid-forming mode. These controllers receive dispatch set-point commands from the microgrid controllers without operator intervention.

Banshee includes a 3MVA PV array and a 2.5MVA battery-based energy storage system (ESS). Two inverter module controllers capable of four-quadrant operations and grid-mode transition techniques operate these assets. The PV follows a time-varying irradiance profile that matches a predefined test sequence. The microgrid controller fully operates the ESS for power factor correction, peak shaving and smoothing, and possibly power export. Among other things, Banshee facilitates the evaluation of the microgrid controller’s ability to perform smart load shedding prior and during islanded conditions.

Banshee is reconfigurable via a number of circuit breakers that also act as load-shedding disconnects. Microgrid controllers can interface and command these breakers via hardware or simulated relays. However, circuit breakers for the diesel generator and natural gas CHP solely actuate via the embedded logic of the genset controller.

Three hardware feeder protection relays monitor the point of interconnection (POI) between Banshee and the utility grid. Virtual relays provide protection against internal system faults and facilitate telemetry to the microgrid controllers. Protection elements include synchronism check (ANSI 25), phase instantaneous overcurrent (ANSI 50P), ac inverse-time overcurrent (ANSI 51), phase undervoltage (27P), and phase overvoltage (ANSI 59P).

6.2.3 Platform Setup and Integrated Components
An illustration of the Banshee digital twin is shown in Fig. 68. Layer 1 represents the distribution management system, market signal source, forecasting engine, and microgrid controller, all communicating using Modbus TCP or IEC 61850 messaging via a firewall in Layer 2. Layer 2 shows the cyber aspect of the platform including a firewall to manage communications traffic, and
various Ethernet gateways to enable translation from Serial or Controller Area Network (CAN) bus into Modbus TCP.

Layer 3 consists of commercial device controllers. There are two genset controllers to control a diesel generator and a natural gas-fired CHP plant. Two power controllers operate a battery energy storage system (BESS) and a photovoltaic (PV) system. Three feeder protection relays actuate circuit breakers at the points of coupling with the utility.

Layer 4 denotes custom-made interface circuitry (HIL Connect) to close the control loop between simulated equipment and their corresponding hardware controllers through scaling and mapping of generated signals. Depending on the controller, this may be necessary in order to minimize the amount of modification involved in making it part of the cHIL platform, providing almost identical real-world signals the controller would be expected to see. For example, in Fig. 69, a genset controller is interfaced to the real-time simulator via the HIL Connect that provides amplified analog channels (current and voltage), as well as digital I/O.

Layer 5 represents the test feeder and virtual components implemented in the Typhoon HIL digital real-time simulator.

In addition to device integration testing, the cHIL platform includes integration with a simplified supervisory distribution management system (DMS) following user instructions or test sequence. The DMS instructs the microgrid controller to adjust optimization objectives through commands such as import limits, export request, power factor correction, and disconnect requests.
6.2.4 Modeling of Banshee components

6.2.4.1 Circuit breaker and protective relay model

The circuit breakers are represented as controlled three-phase switches with integrated measurement probes acting as ideal potential transformers and current transformers, each with an individually predefined mechanical delay for open and close operations. Virtual or hardware relays
use measured values of phase currents and terminal voltages to compute trip conditions. Once trip conditions are satisfied or the microgrid controller requests the circuit breaker to open, a disconnect command is issued to the breaker causing switches of each phase to open at their current zero crossing.

The relay model emulates commonly used protection elements such as instantaneous overcurrent, ac inverse-time overcurrent, undervoltage, overvoltage, and synchronism check. Similar to commercial units, the model facilitates system telemetry, circuit breaker operations for load shedding or reconfiguration, and multiple protection group settings. External controllers may actuate the relays via Modbus TCP using dedicated IP addresses.

The synchronism check is available for dead-bus conditions and two energised circuit breaker terminals, where the voltage of one or both terminals is near zero without faults present in the circuit. Once terminal voltages, slip frequency, and angle difference fall within appropriate settings, the synchronism check will indicate that it is permissible to close the circuit breaker contacts.

### 6.2.4.2 Diesel genset model

The diesel genset model consists of an engine model (DEGOV) [25] and a wound rotor synchronous generator model with 2-damper windings (7-states). In the governor, the innermost control loop is a speed control loop that attempts to keep engine speed at 1800rpm, whereas the outer loop (which is implemented in a real genset controller and therefore not modelled) controls power or frequency. A modelled DC1A exciter [45] controls genset terminal voltage, whereas the external genset controller can be configured to control reactive power, power factor, and also voltage.

### 6.2.4.3 CHP model

The CHP model consists of a natural gas reciprocating engine prime mover, a synchronous machine, heat recovery system, boiler, and associated controls. An illustration of the CHP system is shown in Fig. 70. The gas engine model is based on the model developed in [28] with adjustments in parameters to scale up the engine power.. Details include valve assembly, throttle body, intake manifold, and engine block. The governor and exciter control system configurations are similar to the diesel genset controls explained above. The thermal loop considers a notional aggregation of a thermal mass that represents time-varying building heat load, a boiler that
maintains temperature at a defined set point, and CHP heat that considers mixing and piping transport delays. The model assumes a heat exchanger can draw 20 percent of engine waste heat to preheat the condensation return to the boiler.

6.2.4.4 Inverter model

The PV and BESS inverters are modeled in cHIL with 2-level switching models that are piece-wise linear. IGBT switching devices are considered ideal (either ON or OFF). In the Banshee model, they are switching at 5kHz. The inverter filter is of the LCL type. An illustration of the inverter model is shown in Fig. 71. The physical inverter controllers interface with the inverter models via high bandwidth analog and digital I/O.

6.2.4.5 Load model

Most of the loads are implemented as constant admittance (R-L loads). Some are implemented as constant power. These are current source feedback systems as shown in Fig. 72. The power
setpoints can be adjusted on-the-fly through the digital twin SCADA system. A few loads are induction machines using the 5-states squirrel cage induction machine model.

![Diagram](image)

*Fig. 72. Constant power load model*

### 6.2.5 Simulation results and power flow analysis

Banshee is simulated on four Typhoon HIL 603 units with a time step of 4µs for the electrical system and 20-nanosecond digital sampling for the inverter controllers. This high bandwidth facilitates detailed studies of switching harmonics and their influences on the system. Fig. 73 shows the inverter output currents before and after filtering for a 300-kilowatt step load during grid-forming mode. The zoomed-in overlay of the waveforms shows the effect of the inverter switching frequency to the current injected into the grid.

Banshee digital twin simulation results in sinusoidal steady state are compared with a power flow analysis. In Fig. 74, breaker voltage magnitudes of digital twin are compared with power-flow assuming both constant admittance and constant power loads. In Fig. 75, breaker current magnitudes of digital twin and power-flow analysis are compared. In both cases, it can be seen that there is a close match between simulation and theoretical expectations.
Testing and Validation of a High-Fidelity Controller Hardware-in-the-Loop Microgrid Testbed

Fig. 73. Inverter DER switching waveforms

Fig. 74. Circuit breaker voltage magnitudes (in pu) from cHIL (Typhoon HIL) and power-flow analysis (Matpower)
6.3 Model Validation of a Real-Time Microgrid Emulator System and its Components

One fundamental assumption that is made in cHIL and digital twin applications such as Banshee [61] is that the DER models and other emulated components are accurate. This requires that the models being used be validated not just in steady state but also dynamically. However, it is difficult to find overlay of model and real waveforms at different time scales from sub-cycle time scale [63]–[65] to higher time scales, for component level and system level considerations. This is the problem of model validation.

On the level of individual DERs, model validation is important because it sets bounds on the amount of confidence in particular models and simulation strategies. Depending on the time scale and simulation conditions, different models of the same DER or phenomenon could be used.

Ideally, DER models would be reflective of reality within the frequency bandwidth capabilities of cHIL being considered (typically between 1 kHz to 250 kHz for state-of-the-art). In turn, this provides more confidence in more complex microgrid simulations that may be too difficult or too expensive to implement on a laboratory scale.

In this section, we construct and test a low voltage microgrid validation platform which we use to compare dynamic response measurements from actual microgrid components (including DERs,
Testing and Validation of a High-Fidelity Controller Hardware-in-the-Loop Microgrid Testbed

switches) with data obtained from the cHIL models. Differences are noted and potential fixes suggested. The proposed microgrid validation platform, which we call Microgrid X can be used to test advanced DER control algorithms, DER and microgrid models.

6.3.1 Microgrid X description

The proposed physical microgrid is a simple low-voltage (208 Vrms L-L) system that consists of one genset unit, one PV inverter, one ESS inverter, and dynamic loads. A one-line diagram is shown in Fig. 76. A three phase implementation plan of Microgrid X is shown in Fig. 112 and Fig. 113 in Appendix D. A more detailed description of the major components is provided in Table 7.

Photos of the microgrid implementation are shown in Fig. 77 (front), Fig. 78 (rear), Fig. 79 (genset unit), and Fig. 80 (auxiliary power).

![One-line diagram of Microgrid X](image)

*Fig. 76. One-line diagram of Microgrid X*

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Rating</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>BUS_01 to BUS_07</td>
<td>Bus bars</td>
<td>100A</td>
<td>8</td>
</tr>
<tr>
<td>Grid</td>
<td>3 phase step down auto-transformer</td>
<td>208Vrms L-N/3kVA</td>
<td>1</td>
</tr>
<tr>
<td>KBx-KB6</td>
<td>Three phase circuit breakers</td>
<td>10A</td>
<td>8</td>
</tr>
</tbody>
</table>
### 6.3 Model Validation of a Real-Time Microgrid Emulator System and its Components

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>Power (kW)</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBx-CB6</td>
<td>Three phase contactors with aux. contact</td>
<td>20A</td>
<td>8</td>
</tr>
<tr>
<td>M1-M6</td>
<td>Panel metering</td>
<td>N/A</td>
<td>6</td>
</tr>
<tr>
<td>GENSET CART</td>
<td>PMDC Motor-Generator system and instrumentation</td>
<td>2.2kW/208Vrms L-N</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Generator augmented to 2-bearing system</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV Inverter</td>
<td>PV system emulator + inverter</td>
<td>2kW</td>
<td>1</td>
</tr>
<tr>
<td>ESS Inverter</td>
<td>Energy Storage System emulator + inverter</td>
<td>2kW</td>
<td>1</td>
</tr>
<tr>
<td>Exciter supply</td>
<td>Exciter system power supply</td>
<td>300W</td>
<td>1</td>
</tr>
<tr>
<td>Armature supply</td>
<td>PMDC Motor power supply</td>
<td>2kW</td>
<td>1</td>
</tr>
<tr>
<td>Load emulator</td>
<td>Active load emulator inverter</td>
<td>2kW</td>
<td>1</td>
</tr>
</tbody>
</table>
6.3.2 Genset unit setup

The genset unit consists of a permanent magnet dc (PMDC) motor prime mover and a wound rotor synchronous generator. The PMDC motor is rated at 2.2kW, while the generator (LSA 40) is rated at 10kW, though it is only used up to 2kW in the microgrid. A torque sensor and speed sensor are mounted to the system (Fig. 79).

While designing the genset unit setup, we faced a difficulty in properly coupling the motor and generator. This difficulty arose because the generator was provided with only one bearing, located on the non-driven end. The drive-end required a S.A.E type 5 transmission coupling which is standard on some engines. In order to solve the problem, we designed a flange bearing mount for the driven end of the generator. The design schematic and actual implementation are shown in Fig. 81 and Fig. 82 respectively. To facilitate coupling, a custom shaft extender was designed (Fig. 83) and implemented (Fig. 84).
6.3 Model Validation of a Real-Time Microgrid Emulator System and its Components

6.3.3 Inverter unit test rig

A rig for unit testing of the inverters is shown in Fig. 85 and Fig. 86. Inverter control, and data acquisition are accomplished with the Typhoon HIL 604.

6.3.4 Tandem microgrid control, data acquisition, and model validation

For cHIL is one of the central pieces of Microgrid X. It performs the following functions:
• Data acquisition (DAQ) from the physical Microgrid X and its components via a HIL Connect which performs signal isolation, scaling, and level-shifting for analog as well as digital signals.
• Closed loop control of the physical Microgrid X DERs and contactors
• Real-time simulation of Microgrid X model
• Closed loop control of simulated Microgrid X
6.3 Model Validation of a Real-Time Microgrid Emulator System and its Components

6.3.5 Characterization of contactor feedback

The phenomenon of contact bounce is well known in the literature [66]–[68]. Contact bounce and associated electrical arcs cause contact wear and eventual failure. Contact delay is the time delay between the command to close (or open) the contact and the moment of closure (or opening). In
digital signal processing, contact bounce and delay can lead to erroneous input information, and erratic behavior of embedded applications.

In Microgrid X, we use electromechanical switches called contactors that interface each DER or load to the rest of the microgrid. The contactor application is shown in Fig. 88. The state (opened or closed) of the 4th contact is used to determine individual DER, and overall microgrid control strategies; for example switching inverter control between grid-following and grid-forming modes depending on state of utility interface contactor. The feedback contact is mounted on the same mechanical base as the power contacts. Without proper consideration for contact delay and bounce, the DER may not operate properly in particular during transient events such as islanding or utility connection.

![Fig. 88. Typical contactor application](image)

A switch bounce model suitable for offline simulation (SIMULINK) that considers magnetic force characteristics of the coil, and other physical parameters of the switch is derived in [68]. Such models require contactor construction parameters, which may not be easily available. In addition, simplified contactor models are required to ensure that constraints of low latency and fast real-time simulation are met.

From empirical observations of the feedback contact in the experimental setup, we propose a statistical model that considers contact turn on and turn off delays, and contact bounce.

6.3.5.1 Procedure

A 24Vdc signal is connected to the positive polarity of the feedback contact. The negative polarity of the contact is connected to 24Vdc digital input of the HIL DAQ. The return of the 24Vdc source is also connected to the return of the digital ground of the HIL DAQ. The same 24Vdc and its
6.3 Model Validation of a Real-Time Microgrid Emulator System and its Components

return are also connected to phases A and B of the power contacts of the contactor, and monitored as an analog signal in the HIL DAQ. This is as a control mechanism to verify that no loss of 24V power occurs unduly. The 10V digital outputs from the HIL are amplified to 24Vdc in the HIL Connect and applied to the contactor coil. A 2Hz digital signal with a 50% duty is generated from HIL and used to drive the contactor CB2 coil. Waveform captures are triggered in HIL DAQ and post-processed to estimate the statistical parameters.

6.3.5.2 Observations

Fig. 89 shows contactor response as we continuously excite and de-excite the coil. A more detailed view of the interval between the blue circles is plotted in Fig. 90 and shows the initial turn-off delay and contact bounce during closure. We define the interval marked $T_1$ as the contact initial turn-off delay; we define interval $T_2$ as the contact bounce time during closure. Similar definitions apply for contactor opening.

![Fig. 89. Contactor excitation and feedback with 2Hz, 50% duty digital signal](image)
We consider contactor data from 40 open commands, and 40 close commands. The following quantities are analyzed:

- Contact turn-off delay
- Contact turn-on delay
- Contact bounce time during closure
- Contact bounce time during opening
- Number of contact bounces during closure
- Number of contact bounces during opening

The distributions of the above parameters for the different realizations are plotted on histograms from Fig. 90 to Fig. 96, and estimates are computed for a normal distribution. Contact bounce time during opening appears more or less constant.
6.3 Model Validation of a Real-Time Microgrid Emulator System and its Components

Fig. 91. Distribution of number of contactor feedback bounces during closure. Mean=14.6, Std=3.14

Fig. 92. Distribution of number of contactor feedback bounces during opening. Mean=12.75, Std=6.25

Fig. 93. Distribution of contact turn-on delay time. Mean=29.57ms, Std=0.05ms

Fig. 94. Distribution of contact turn-off delay time. Mean=71.08ms; Std=0.73ms

Fig. 95. Distribution of contact bounce time during closure. Mean=2.08ms; Std=0.42ms

Fig. 96. Distribution of contact bounce time during opening. Mean=0.003ms; Std=0.01ms
6.3.6 Characterization of genset unit

Since we have limited test data on the PMDC and generator components of the genset unit, it is important to first characterize them properly before engaging in more complex experiments and analysis.

6.3.6.1 Determination of PMDC torque characteristic

The goal is to determine the torque characteristic of the prime mover, which is a function of the form:

\[ T_{PMDC} = k_{PMDC} \cdot i_{PMDC} \]  

(5.60)

Where:

\( T_{PMDC} \) = PMDC torque (Nm)

\( k_{PMDC} \) = PMDC torque constant (Nm/A)

\( i_{PMDC} \) = PMDC armature current (A)

**Procedure**

The genset is spun up to different speeds and allowed to settle. Then the PMDC power is shut off and the genset begins coasting to a rest while speed and current data are recorded (Fig. 97).

**Observations**

Due to noisy nature of data, function fits are applied (Fig. 98) and used to extract the relationships of the form:

\[ \frac{d\omega}{dt} = \frac{1}{J} \left( a\omega^2 + b\omega + c \right) \]  

(5.61)

The coefficients fitted from equation (5.61) are used to predict the torque at the origin of each of the coasting characteristics of Fig. 97, whereupon a torque-current characteristic for the PMDC is generated (Fig. 100).

6.3.6.2 Determination of PMDC armature resistance

A plot of PMDC terminal voltage vs current is shown in Fig. 101 from which PMDC winding resistance is estimated as 1.5Ω.
6.3.6.3 PMDC loss model

With knowledge of PMDC torque characteristic (equation (5.60)) and armature resistance, a simple PMDC loss model is developed and plotted in Fig. 102. The loss model has some inaccuracies when the PMDC losses are less than 1.5W.
Testing and Validation of a High-Fidelity Controller Hardware-in-the-Loop Microgrid Testbed

Fig. 97. Coasting of genset unit to determine torque characteristic of PMDC motor drive

Fig. 98. Fitting functions to noisy coasting data

Fig. 99. Measurement of angular acceleration vs angular velocity

Fig. 100. Experimentally determined torque characteristic of PMDC motor

Fig. 101. Experimental determination of PMDC winding resistance

Fig. 102. PMDC loss distribution at low input power
6.3.6.4 Generator harmonics

The generator produces harmonics which are not accounted for in the simulation model of the synchronous machine (Fig. 103). These terminal voltage harmonics are analyzed for loaded and unloaded conditions (Fig. 104).

![Fig. 103. Genset current, terminal voltage (L-N), and terminal voltage (L-L)](image)

![Fig. 104. Terminal voltage harmonics under loaded and unloaded conditions](image)

6.3.6.5 Generator transient response

Load steps are applied to the generator and the responses are analyzed (Fig. 105 and Fig. 106).
6.3.7  **Inverter open loop steady state and transient model validation**

Three phase inverter model is evaluated in open-loop for steady state and transient (Fig. 107 and Fig. 108). It is found that the cHIL model agrees well with the theoretical averaged model. However both cHIL and averaged models show more oscillation during the transient than the real inverter prototype. The real inverter is more lossy than the models due to core losses and switching losses. Spectral analysis of inverter-side current show more energy in the inverter models at higher frequencies than in the real inverter (Fig. 109). However the load-side current shows close spectral match with the simulated inverter model (Fig. 110).

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Fig. 107. Real inverter currents vs HIL model, and theoretical average model
6.4 Conclusion

This first part of this chapter makes a case for the importance of Digital Twin to facilitate design and testing of microgrids and DER controllers. Banshee, a real-life power system that serves as a microgrid and DER controller integration test system is discussed and simulated. Banshee supplements existing test feeders by providing the means to evaluate microgrid controller functionalities that involve reconfiguration of networked circuits, blackstart, management of multiple points of coupling with the utility service, active resynchronization of multiple islands,
and other features defined in IEEE 2030.7. We highlight the benefits of cHIL to evaluate controllers and perform compliance testing of standards. Power flow simulations are setup to confirm the validity of the real-time simulations of the Banshee distribution network benchmark.

In the second part, we propose a simple physical low voltage microgrid testbed called *Microgrid X*. The testbed is part of a tandem microgrid model validation system. It is used to verify, validate, and improve models of microgrid DERs and other components. It can also be used to verify the validity of a complete microgrid by comparing the results of the physical microgrid testbed to those of the real-time simulated version.
Conclusion and Future Work

In this thesis, we presented contributions towards achieving plug-and-play microgrids. The importance of microgrids was motivated in the introductory chapter, as well as their challenges. The following couple of chapters focused on modeling of microgrid DER components. More specifically, in Chapter 2, we presented an analysis of wound-rotor synchronous generators that uncovered unstable behavior, which can cause challenges in islanded microgrids if not properly understood and addressed. We also developed a detailed natural gas engine model for microgrid simulation in Chapter 3. In Chapter 4, a microgrid model consisting of genset, inverter, and load was constructed and used to develop a framework for model identification and controller optimization on controller hardware-in-the-loop (cHIL) platform.

We developed a cascaded state-feedback adaptive controller in Chapter 5 for inverter control. By automatically adjusting the controller gains online, and in real time, the proposed controller enables good inverter performance even with variations in the inverter/microgrid parameters.

In Chapter 6, a prototyping platform for hardware-in-the-loop integration of microgrid device controllers was presented. We explored the Microgrid Digital Twin concept. Furthermore, a simple physical low-voltage microgrid testbed was constructed, consisting of synchronous-machine and inverter-based DERs.

As future work, more DER and Microgrid model validation are required in order to build more and more confidence in microgrid digital twin, thus inching us even closer to true plug-and-play microgrids.
Appendix A: Parameters for Chapter 2

The parameters mentioned in equation (2.1) are defined in the set of equations below:

\[ a_k = \frac{\omega_b r_s}{X_{ls}} \left( \frac{X_{aq}}{X_{ls}} - 1 \right); \]
\[ b_k = \frac{\omega_b r_s X_{aq}}{X_{ls} X_{lkq}}; \]
\[ d_k = \frac{\omega_b r_s}{X_{ls}} \left( \frac{X_{ad}}{X_{ls}} - 1 \right); \]
\[ e_k = \frac{\omega_b r_s X_{ad}}{X_{ls} X_{fjd}}; \]
\[ f_k = \frac{\omega_b r_s X_{ad}}{X_{ls}^2 X_{fjd}}; \]
\[ g_k = \frac{\omega_b r_{kq} X_{aq}}{X_{ls} X_{lkq}}; \]
\[ h_k = \frac{\omega_b r_{kq}}{X_{ls} X_{fjd}} \left( \frac{X_{aq}}{X_{lkq}} - 1 \right); \]
\[ m_k = \frac{\omega_b r_{kq} X_{ad}}{X_{ls} X_{fjd}}; \]
\[ n_k = \frac{\omega_b r_{kq}}{X_{ls} X_{fjd}} \left( \frac{X_{ad}}{X_{lkq}} - 1 \right); \]
\[ \theta_k = \frac{\omega_b r_{fjd} X_{ad}}{X_{fjd} X_{lkd}}; \]
\[ p_k = \frac{\omega_b r_{fjd} X_{ad}}{X_{fjd} X_{lkd}}; \]
\[ q_k = \frac{\omega_b r_{fjd} X_{ad}}{X_{fjd} X_{lkd}}; \]
\[ r_k = \frac{\omega_b r_{lkd}}{X_{lkd}} \left( \frac{X_{ad}}{X_{lkd}} - 1 \right); \]
\[ X_{aq} = \left( \frac{1}{X_{mq}} + \frac{1}{X_{ls}} + \frac{1}{X_{lkq}} \right)^{-1}; \]
\[ X_{ad} = \left( \frac{1}{X_{md}} + \frac{1}{X_{ls}} + \frac{1}{X_{fjd}} + \frac{1}{X_{lkd}} \right)^{-1}. \]

(5.62)

The parameters mentioned in equation (2.3) are stated as follows:

\[ s_{kk} = \alpha \left( \frac{a_k - d_k}{r_b \omega_b} \right) \psi_{d(0)}' + \left( -\frac{e_k}{r_b \omega_b} \right) \psi_{f(0)}' + \left( -\frac{f_k}{r_b \omega_b} \right) \psi_{f(0)}' \]
\[ t_{kk} = \alpha \left( \frac{a_k - d_k}{r_b \omega_b} \right) \psi_{q(0)}' + \left( \frac{b_k}{r_b \omega_b} \right) \psi_{q(0)}' \]
\[ u_{kk} = \alpha \left( \frac{b_k}{r_b \omega_b} \psi_{q(0)}' \right); \]
\[ v_{kk} = \alpha \left( -\frac{e_k}{r_b \omega_b} \psi_{q(0)}' \right); \]
\[ x_{kk} = \alpha \left( -\frac{f_k}{r_b \omega_b} \psi_{q(0)}' \right); \]
\[ \alpha = -\left( \frac{1}{J} \right) \left( \frac{3}{2} \right) \left( \frac{poles}{2} \right)^2 \left( \frac{1}{\omega_b} \right). \]

(5.63)
Appendix B: Parameters and Data for Chapter 3

Table 8. Parameters for steady state gas engine verification

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### Appendix B: Parameters and Data for Chapter 3

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## Table 9. Engine data (part 1)

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(KUBOTA, CUMMINS, CATERPILLAR, GE Jenbacher)
### Table 10. Engine data (part 2)

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Appendix C: Parameters for Chapter 4

From equation (4.1), the following symbols are defined:

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a = \frac{\omega_0 r}{X_{ls}} \left( \frac{X_{sq}}{X_{ls}} - 1 \right); \quad b = \frac{\omega_0 r}{X_{ls}} \frac{X_{mq}}{X_{ls} X_{ikq}}; \quad c = \frac{\omega_0 r}{X_{ls}} \left( \frac{X_{ad}}{X_{ls}} - 1 \right); \quad d = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{fjd}}
\]

\[
e = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{ikq}^*}; \quad f = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{ikq}^*}; \quad g = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{ikq}^*} - 1; \quad h = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{fjd}}
\]

\[
i = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{fjd}^*} \left( \frac{X_{ad}}{X_{ls} X_{ikq}^*} - 1 \right); \quad j = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{fjd}^*}; \quad k = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{ikq}^*}; \quad l = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{fjd}^*}
\]

\[
m = \frac{\omega_0 r}{X_{ls}} \frac{X_{ad}}{X_{ls} X_{fjd}^*} \left( \frac{X_{ad}}{X_{ls} X_{ikq}^*} - 1 \right)
\]

From equation (4.4), the following symbols are defined:

\[
a_i = \frac{1}{X_{ls}} \left( \frac{X_{ad}}{X_{ls}} - 1 \right); \quad b_i = \frac{X_{ad} X_{ikq}}{X_{ls} X_{fjd}^*}; \quad c_i = \frac{X_{ad}}{X_{ls} X_{fjd}^*}
\]

\[
d_i = \frac{1}{X_{ls}} \left( \frac{X_{mq}}{X_{ls}} - 1 \right); \quad e_i = \frac{X_{mq}}{X_{ls} X_{ikq}^*}
\]

From equations (4.7), (4.10), and (4.11), the following symbols are defined:

\[
T_1 = R_x d_1 + L_x A_3; \quad T_2 = T_6 = L_x (a_i - d_i); \quad T_3 = R_x e_1 + L_x B_3
\]

\[
T_4 = L_x c_1; \quad T_5 = L_x b_1; \quad T_7 = R_x a_1 + L_x B_2
\]

\[
T_8 = -L_x e_1; \quad T_9 = R_x c_1 + L_x C_2; \quad T_{10} = R_x b_1 + L_x D_2
\]

\[
T_{11} = 1 + L_x C_3; \quad T_{12} = 1 + L_x a_c \omega_b; \quad T_{13} = c_1 L_x \omega_b
\]

\[
T_{14} = T \psi_{pr} ; T_{15} = T \psi_{pl}(0); \quad A_3 = d_1 a + e_1 f; \quad B_3 = d_1 b + e_1 g
\]

\[
C_3 = d_1 \omega_b; \quad B_2 = a_c + b_1 k + c_1 h; \quad C_2 = a_i d + b_1 l + c_1 i
\]

\[
D_2 = a_i e + b_1 m + c_1 j
\]

From equations (4.6) and (4.8), the following symbols are defined:
Appendix C: Parameters for Chapter 4

\[
K = \frac{2}{3} \begin{bmatrix}
\cos \theta_r & \cos \left(\theta_r - \frac{2\pi}{3}\right) & \cos \left(\theta_r + \frac{2\pi}{3}\right) \\
\sin \theta_r & \sin \left(\theta_r - \frac{2\pi}{3}\right) & \sin \left(\theta_r + \frac{2\pi}{3}\right) \\
0.5 & 0.5 & 0.5 \\
\end{bmatrix}
\]

\[
'K' = \begin{bmatrix}
\cos \delta & \sin \delta & 0 \\
-\sin \delta & \cos \delta & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\] (67)
Appendix D: Additional Figures for Chapter 6

Fig. 111. One-line diagram of the Banshee Distribution Network
Appendix D: Additional Figures for Chapter 6

Fig. 112. Implementation details of Microgrid X (part 1/2)
Fig. 113. Implementation details of Microgrid X (part 2/2)
Bibliography


[58] Z.-P. Jiang, “Comments on ‘A remark on partial-state feedback stabilization of cascade


