Time-to-contact Statistics as a Proxy for Accident Probabilities

by R'mani Haulcy

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Author: ____

Department of Electrical Engineering and Computer Science
May 2, 2019

Certified by: ____

Berthold K.P. Horn
Professor of Computer Science and Engineering
Thesis Supervisor

Accepted by: ____

Leslie A. Kolodziejski
Professor of Electrical Engineering and Computer Science
Chair, Department Committee on Graduate Students

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Abstract

Accidents are relatively rare, and this makes it difficult to study the impact of traffic system changes or vehicle control changes on accident rates. One potential solution to this problem is the use of time-to-contact (TTC) statistics as a proxy for accident probabilities. Low TTC can be used as a measure of potential danger. Simulations were performed to explore whether inverse TTC can serve as a good proxy of accident probability. The resulting data was then analyzed to investigate how inverse TTC varies with the mixture of vehicles with bilateral control as opposed to car-following control. Previously, it was found that a relatively high mixture ratio is needed to prevent phantom traffic jams. The results in this paper show that there is a benefit to mixing bilateral control cars into general traffic, even at relatively low mixture ratios. Simulations were also performed to see how acceleration and jerk vary with the mixture of vehicles with bilateral control so that passenger comfort could be quantified. The results show that bilateral control improves passenger comfort.

Thesis Supervisor: Berthold Horn
Title: Professor of Computer Science and Engineering

Acknowledgements

I would like to thank my advisor, Professor Berthold Horn, for reviewing this thesis and for providing invaluable feedback and guidance in a calm, patient and supportive manner. I would also like to thank Liang Wang for all of his assistance with the development of the simulator.

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Chapter 1

Introduction

1.1 Motivation

The study of autonomous vehicles has become increasingly popular in the last few years. The integration of autonomous vehicles into society is inevitable, with several companies competing to be the first to release a fully-functioning autonomous vehicle and other companies, such as General Motors, announcing plans to launch their autonomous vehicles this year. While surveys show that citizens respond positively to the idea of an autonomous vehicle, studies also show that a lot of people are uncomfortable with the idea of riding in a fully-autonomous vehicle [13]. Alleviating citizens' concerns about safety and increasing their trust in autonomous vehicles is arguably one of the biggest hurdles to the successful integration of autonomous vehicles into society.

A solution to this challenge requires developing a metric for assessing the safety of autonomous vehicles. This thesis proposes using inverse time-to-contact (TTC) as a proxy for accident probability. TTC is the distance to the vehicle ahead divided by the relative velocity. It is the amount of time it would take for a collision to occur if nothing were to change. TTC is described in greater detail in [15, 6]. Low TTC can be used as a measure of potential danger, which means that TTC can be used to determine the safety of an autonomous vehicle.

The verification of the safety of an autonomous vehicle is one important step. Even after safety is verified, it is important to ensure that the vehicle drives in a way that makes the passenger feel comfortable. While certain speeds and accelerations may be verified as safe, those speeds may still make the passenger perceive the ride as being unsafe. As a result, acceleration and jerk are recorded in addition to TTC and analyzed to assess passenger comfort.

The vehicles described in this thesis use bilateral control. Bilateral control is a method in which the state of the following vehicle and the leading vehicle is considered as opposed to only considering the state of the leading vehicle (e.g. traditional car-following cruise control methods). Bilateral control has been shown to suppress traffic instabilities [5] and can be implemented as an advanced form of cruise control using additional sensors [16, 7]. Bilateral control does not require full autonomy. It can be thought of as a modification of adaptive cruise control and can be integrated into vehicles driven by human drivers today.

There are potential societal and personal benefits of suppressing traffic flow instabilities and improving passenger comfort when many vehicles use bilateral control. The safety verification of vehicles equipped with bilateral control and the improved passenger comfort could justify installing bilateral control in vehicles since there is then also a clear advantage to individual vehicle owners.

1.2 Related Work

Many researchers have attempted to verify the safety of autonomous vehicles. One logical approach that researchers have proposed involves demonstrating the safety of autonomous vehicles by testing them on real roads and observing their performance. However, the results in [8] show that the scarcity of traffic accidents compared to the number of miles driven leads to the need for an impractically large number of miles to verify the safety of an autonomous vehicle.

Another approach involves studying the effect of platooning systems on safety. Some researchers have shown that platooning systems enhance safety [3] while others have formally verified the safety of platooning systems [12, 9] and systems in which all of the vehicles are using some form of adaptive cruise control [12]. Researchers have also proposed building intelligent intersections and roads and have verified the safety of such systems [10].

All of the approaches mentioned above assume that all of the vehicles on the road are autonomous vehicles. However, it is harder to verify the safety of autonomous vehicles when human drivers are also on the road. The approach proposed in this thesis differs from the rest because the vehicles being simulated are not all autonomous vehicles. Inverse TTC is being proposed as a proxy for accident probability (and hence TTC is being proposed as a measure of safety) for roads in which there is a mixture of human drivers and autonomous vehicles (i.e. no platoons). This makes the approach described in this thesis more immediately relevant to the society we are currently living in and much easier to implement.

Researchers have also focused on various aspects of passenger comfort. Some researchers have focused on designing controllers that prioritize the passenger's comfort by limiting the speed of the autonomous vehicle to that of human drivers [17]. Others have focused on passenger comfort as it relates to path planning and navigation and have solutions in which paths are chosen with the passenger's preferences and comfort in mind [4]. In this thesis, acceleration and jerk are used as a proxy for passenger comfort and are analyzed to explore the correlation between bilateral control and passenger comfort.

1.3 Objective

The goals of this thesis are: (1) to show that TTC is a good measure of accident probability and that accident probability decreases as the number of vehicles with bilateral control increases, and (2) to show that bilateral control improves passenger comfort by analyzing the changes in jerk and acceleration for different traffic scenarios. Three different traffic scenarios were simulated: (1) pure car-following traffic, (2) pure bilateral control traffic and (3) mixed traffic of a given mixing ratio. The resulting data was analyzed to investigate the relationship between the inverse TTC and the mixture ratio of vehicles with bilateral control as opposed to car-following control, as well as the relationship between bilateral control and passenger comfort.

The remainder of this thesis is organized in the following way: Chapter 2 describes the simulator that was used for data collection and Chapter 3 provides analysis of the inverse TTC simulation results in MATLAB. Chapter 4 analyzes the acceleration and jerk simulation results in MATLAB and Chapter 5 compares the improvement in performance for bilateral control and car-following vehicles separately. Chapter 6 concludes this thesis and discusses future work.

Chapter 2

Simulation

A simulator was written in Java and run using the Java IDE IntelliJ IDEA. Three different traffic scenarios were considered: (1) pure car-following traffic, (2) pure bilateral control traffic, and (3) mixed traffic of a given mixture ratio.

Simulations were run for each of the three traffic scenarios mentioned above so that the performance of vehicles with bilateral control could be compared to that of car-following vehicles. Simulating traffic with a mixture of both car-following and bilateral control cars also allowed for the analysis of the behavior of traffic as the percentage of cars with bilateral control increased. The inverse TTC examined in each scenario was used to determine how safe each scenario was and how the degree of safety changed as the number of vehicles with bilateral control changed.

Parameter	Value
kd	0.1
kv	0.2
kc	0.01
v_des ($\frac{m}{s}$)	30
max_v ($\frac{m}{s}$)	44
min_v ($\frac{m}{s}$)	0
reaction_time (s)	1
max_a ($\frac{m}{s^2}$)	3
min_a ($\frac{m}{s^2}$)	-3
Threshold: 1/TTC	10

Table 2.1: Parameter values used during simulation.

2.1 Observations

2.1.1 Initial Setup

Before the simulator was run, a control window was launched and different parameter values were chosen. A list of the different parameters, along with their default values, can be seen in Table 2.1 and Figure 2.1 shows an image of the control window. A total of 30 cars were used for each simulation and each car used bilateral control or car-following. Each simulation was run for 60 seconds and the same parameter values were used for all three traffic scenarios.

2.1.2 Pure Car-Following Traffic

30 vehicles all using car-following were simulated. A screen shot of the simulation after 60 seconds can be seen in Figure 2.2. The traffic flows smoothly initially but a phantom traffic jam eventually forms. This traffic jam can be seen in

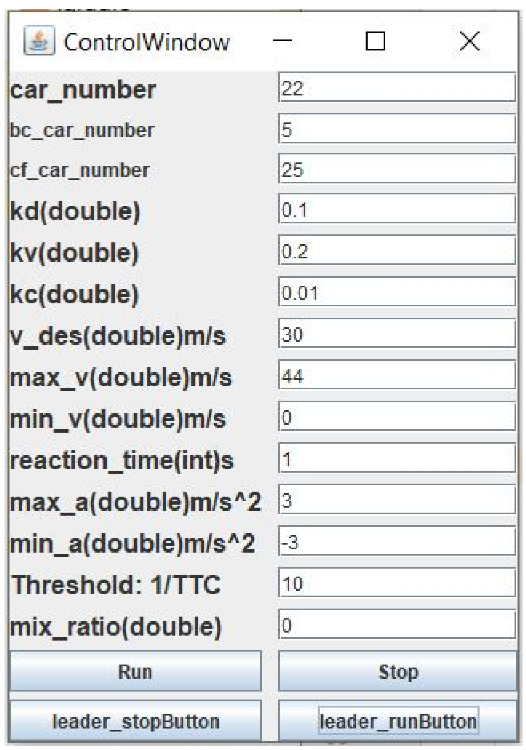


Figure 2.1: Control Window used to set parameter values before running simulations.

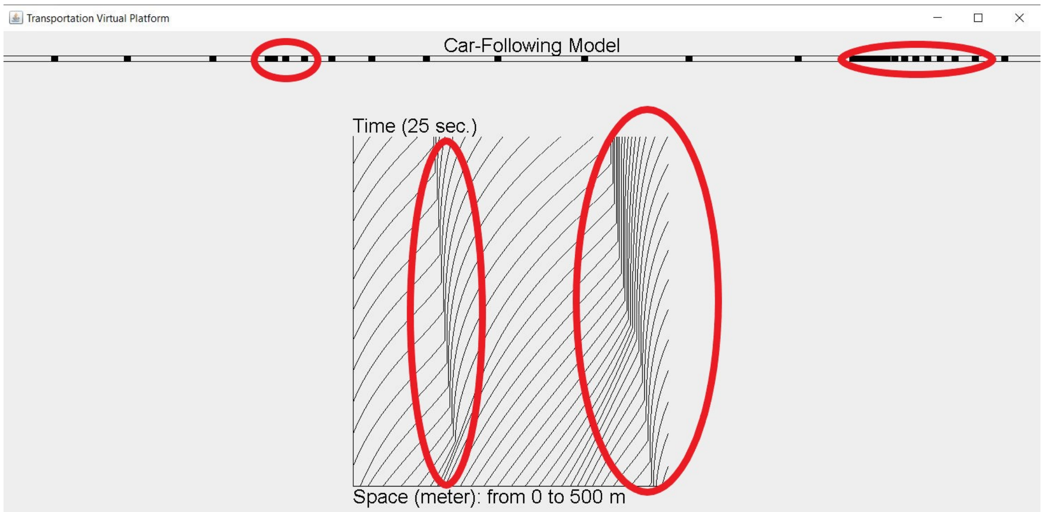


Figure 2.2: Pure car-following simulation.

Figure 2.2 in the circled spaces on the road. The circled areas on the plot are also indicative of a traffic jam forming. This behavior is expected because research has shown that traffic jams naturally form for car-following models [11].

2.1.3 Pure Bilateral-Control Traffic

30 vehicles all using bilateral control were simulated. A screen shot of the simulation after 60 seconds can be seen in Figure 2.3. The traffic flows smoothly for the entire 60 seconds and no phantom traffic jams form. The cars remain equidistant from each other and no ripples develop in the plot. This behavior is expected because research has shown that bilateral control suppresses traffic instabilities [5].

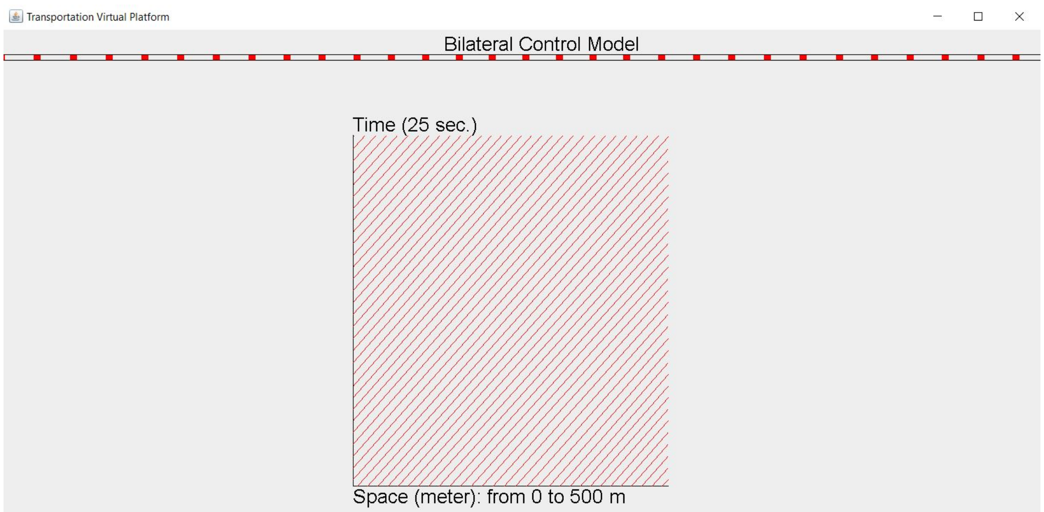


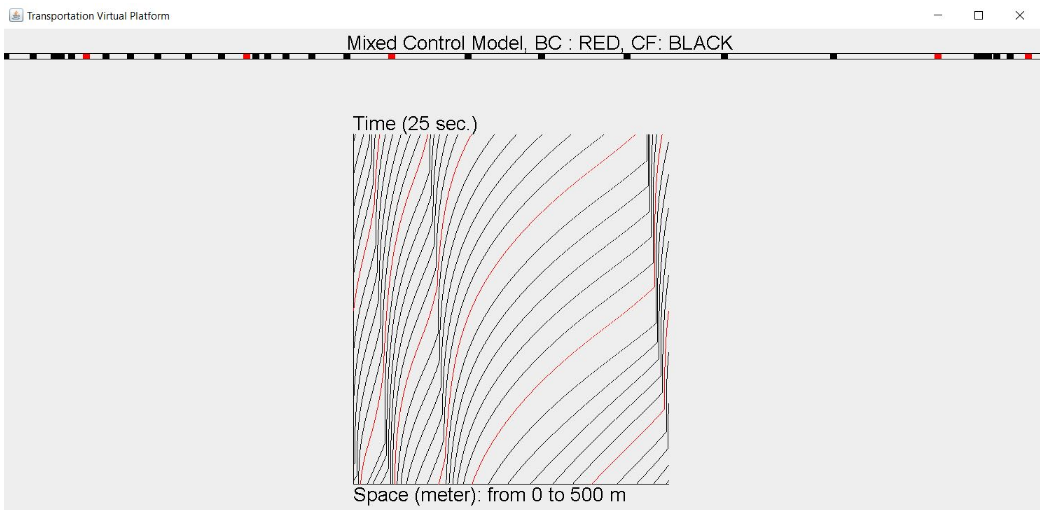
Figure 2.3: Pure bilateral control simulation.

2.1.4 Mixed Traffic

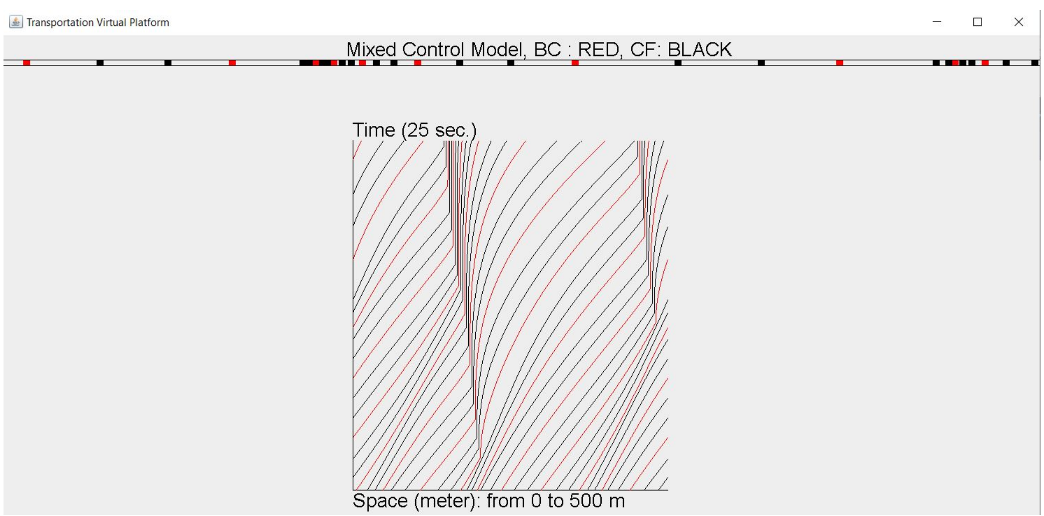
30 vehicles with various mixture ratios were simulated. Screen shots of the 60 second simulations for three different mixture ratios can be seen in Figures 2.4 and 2.5. These figures show that traffic flow improves as the number of bilateral control cars increases. The decrease in the number of vertical lines in the plots as the number of bilateral control cars increases illustrates this improvement in traffic flow. When there are more bilateral control cars than car-following cars (Figure 2.5), traffic flows smoothly for the entire 60 seconds.

2.2 Implications of Observed Behavior

The aforementioned simulations show that mixing bilateral control cars into general traffic significantly improves traffic flow and reduces the number of phantom

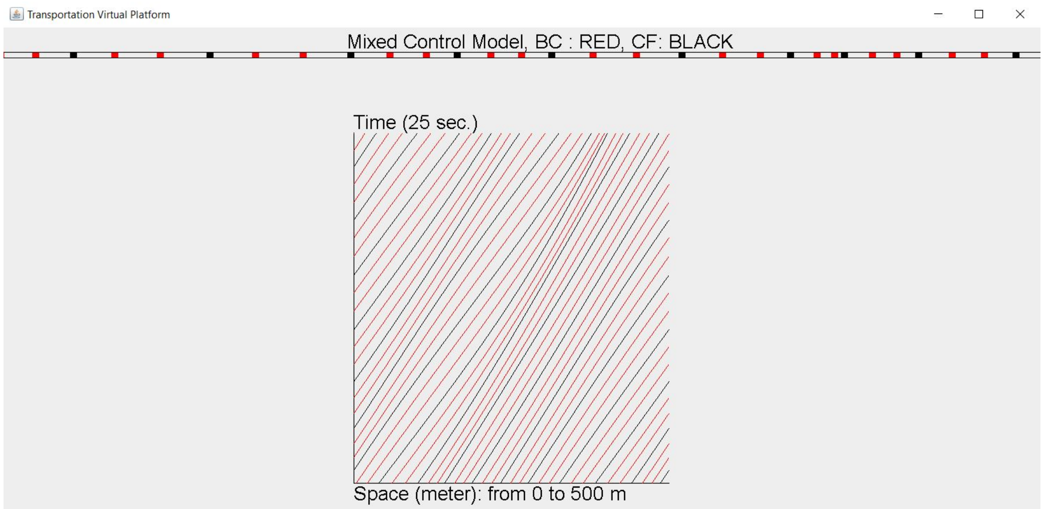


(a) 5 bilateral control cars and 25 car-following cars.



(b) 10 bilateral control cars and 20 car-following cars.

Figure 2.4: Mixed simulation.



(a) 20 bilateral control cars and 10 car-following cars.

Figure 2.5: Mixed simulation (cont).

traffic jams that form, even before the traffic consists entirely of vehicles with bilateral control. These results suggest that the analysis of the recorded TTC will show that bilateral control vehicles are “safer” than car-following vehicles and that safety increases as the number of bilateral control vehicles increases. This hypothesis will be explored in the next chapter.

Chapter 3

Analysis: Inverse TTC

Inverse TTC was calculated during simulation and the values were saved in a text file. That file was used to analyze the inverse TTC values for each traffic scenario. The analysis consisted of making histograms of the inverse TTC in MATLAB and observing how often there were inverse TTC values outside of a specified “safe” range of values. This “safe” range was chosen to be a following distance of 2 seconds or more in accordance with the “two-second rule” recommended by the Road Safety Authority (RSA) [1]. This means that inverse TTC values of 0.50 or less correspond to a safe following distance.

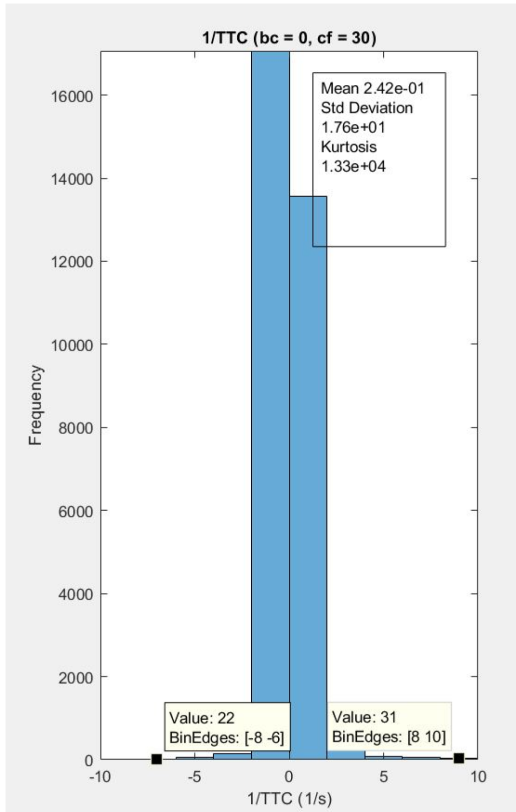


Figure 3.1: Histogram of inverse TTC for pure car-following traffic.

3.1 Observations

3.1.1 Pure Car-Following Traffic

A histogram of the inverse TTC for pure car-following traffic can be seen in Figure 3.1. This figure clearly shows that there are moments when the cars were not within a safe following distance of two seconds or more (e.g. the marked data points show that there are inverse TTC values greater than 0.5). This means that there were several potentially dangerous situations during the simulation.

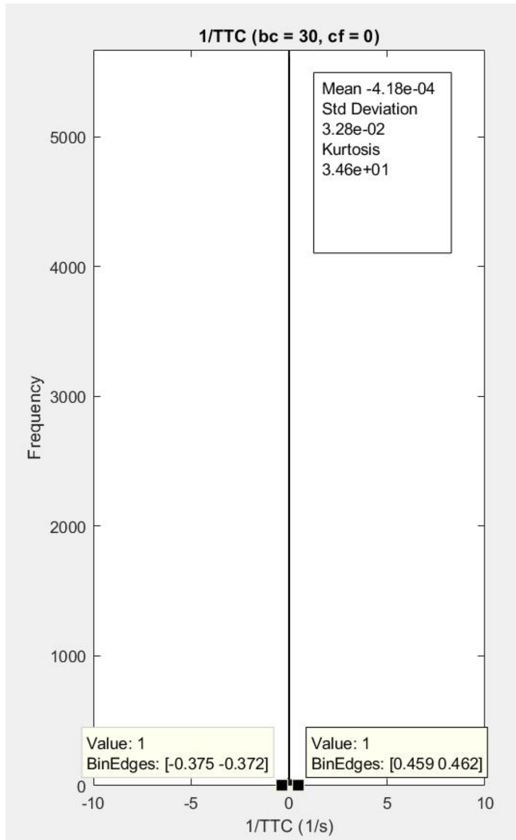


Figure 3.2: Histogram of inverse TTC for pure bilateral control traffic.

3.1.2 Pure Bilateral-Control Traffic

A histogram of the inverse TTC for pure bilateral control traffic can be seen in Figure 3.2. The marked data points in the figure show that the absolute value of the inverse TTC never exceeded 0.462. This means that all of the vehicles maintained a safe following distance and there were no potentially dangerous situations during the simulation.

3.1.3 Mixed Traffic

Histograms of the inverse TTC for mixed traffic with various mixture ratios can be seen in Figure 3.3. As the number of bilateral control cars increases, the range of inverse TTC values decreases. There is a significant reduction in the range of inverse TTC values when the number of bilateral control vehicles increases from 5 to 10. Once the number of bilateral control cars reaches 20 and above, all of the inverse TTC values are within the "safe" range of 0.5 or less.

3.2 Discussion

The simulation and MATLAB results show that introducing vehicles with bilateral control capabilities into traffic increases safety by (1) reducing the rate in which phantom traffic jams form and (2) reducing accident probability by maintaining a safe following distance of 2 seconds or more. Simulating and analyzing mixed traffic of different mixture ratios shows that bilateral control vehicles improve safety and reduce accident probability even when human drivers remain on the road.

The simulation results and the results of graphical analysis are consistent, which means that the suppression of traffic instabilities is reflected in the inverse TTC values that are recorded. This correlation suggests that inverse TTC can serve as a quantitative measure of accident probability and safety.

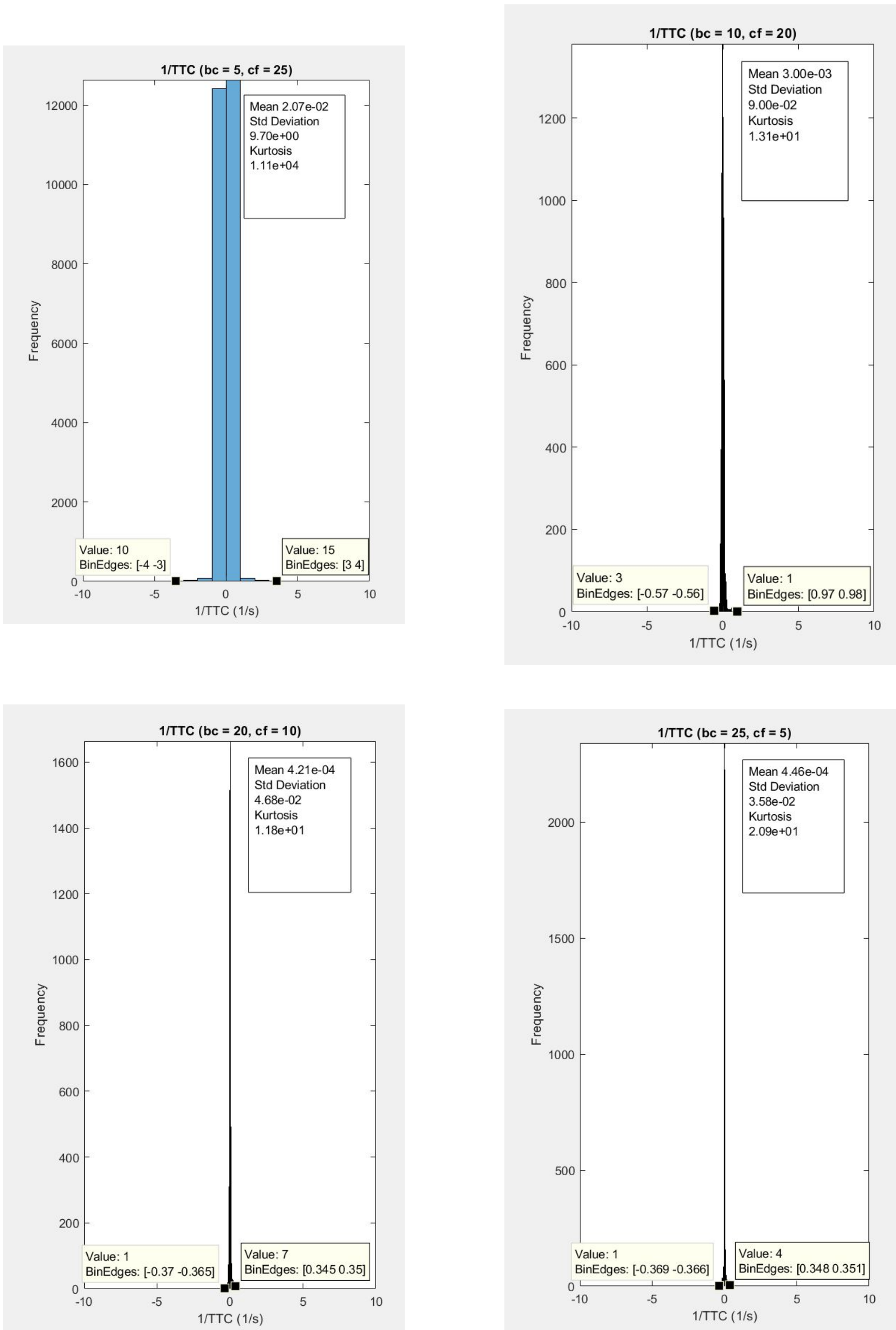


Figure 3.3: Histograms of inverse TTC for mixed traffic of various mixture ratios.

Chapter 4

Analysis: Acceleration and Jerk

Acceleration and jerk were calculated during simulation and the values were saved in a text file. That file was used to analyze the acceleration and jerk values for each traffic scenario. The analysis consisted of making histograms of the acceleration and jerk in MATLAB and observing how the range of acceleration and jerk values changed as the number of vehicles with bilateral control changed. It is difficult to choose one set of acceleration and jerk values that satisfies all passengers because there are a variety of different factors that can influence whether a passenger feels comfortable in a vehicle, including the type of activity the passenger is performing while riding in the vehicle [2]. However, it has been shown that smaller acceleration and jerk values are better for passenger comfort overall [14]. Therefore, a decrease in acceleration and jerk is correlated with an improvement in passenger comfort in this thesis.

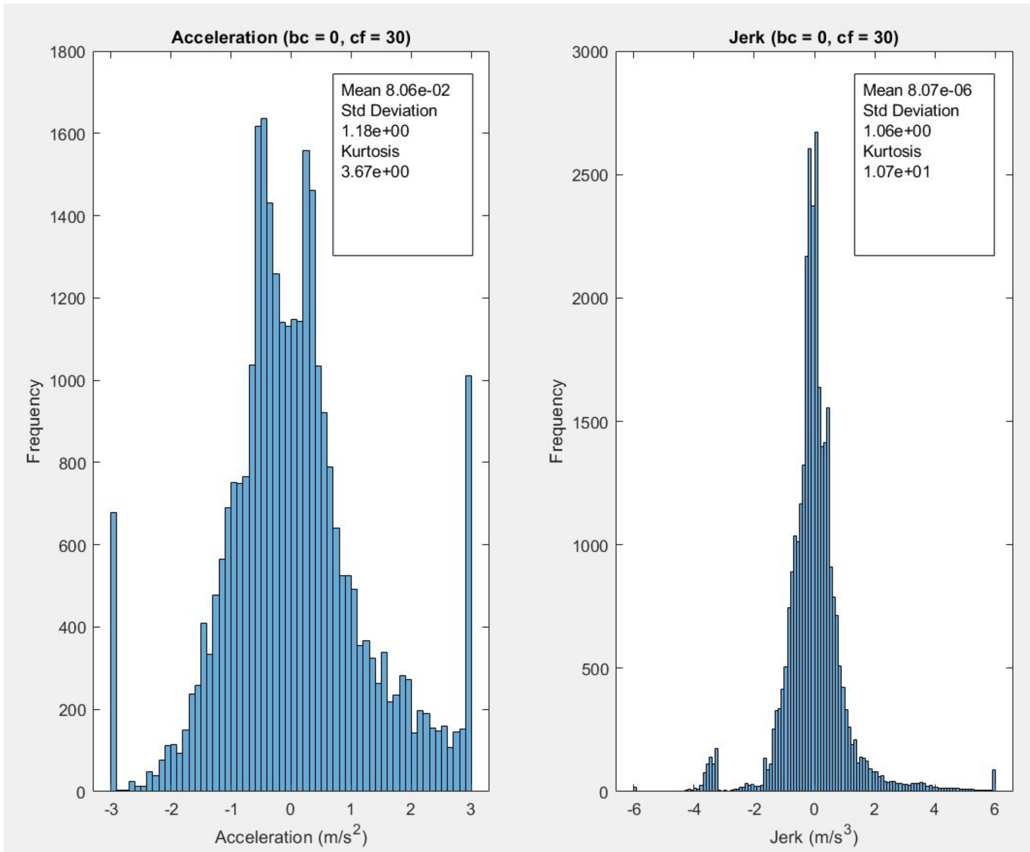


Figure 4.1: Histogram of acceleration and jerk for pure car-following traffic.

4.1 Observations

4.1.1 Pure Car-Following Traffic

A histogram of the acceleration and jerk for pure car-following traffic can be seen in Figure 4.1. This figure shows that there are a wide range of acceleration and jerk values (up to 3 m/s^2 for acceleration and 6 m/s^3 for jerk). This means that there were some potentially uncomfortable situations during the simulation.

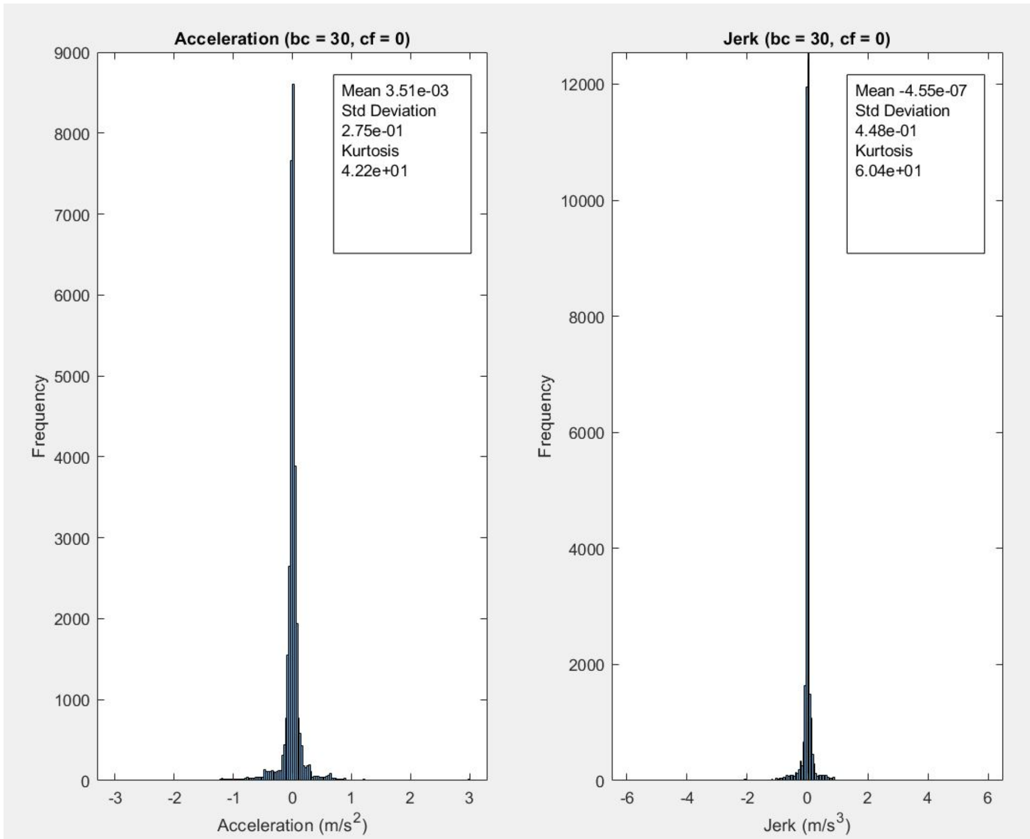


Figure 4.2: Histogram of acceleration and jerk for pure bilateral control traffic.

4.1.2 Pure Bilateral Control Traffic

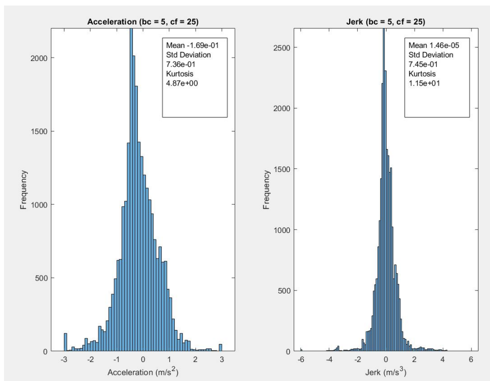
A histogram of the acceleration and jerk for pure bilateral control traffic can be seen in Figure 4.2. This figure shows that the range of acceleration and jerk values decreases significantly when traffic consists entirely of vehicles with bilateral control. The maximum acceleration was approximately 1 m/s^2 and the maximum jerk was approximately 2 m/s^3 compared to the 3 m/s^2 and 6 m/s^3 mentioned above for acceleration and jerk, respectively.

4.1.3 Mixed Traffic

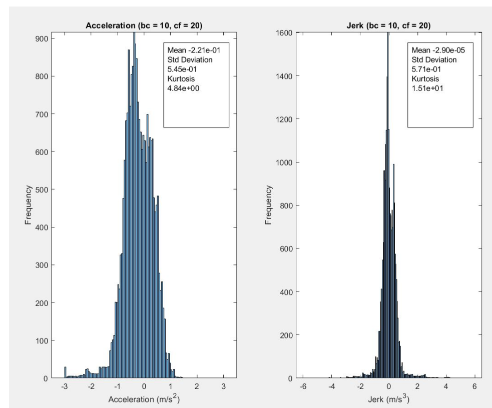
Histograms of the acceleration and jerk for mixed traffic with various mixture ratios can be seen in Figure 4.3. There is a significant reduction in the range of acceleration and jerk values when the number of bilateral control vehicles increases from 10 to 20. As the number of bilateral control cars increases, the range of acceleration and jerk values decreases.

4.2 Discussion

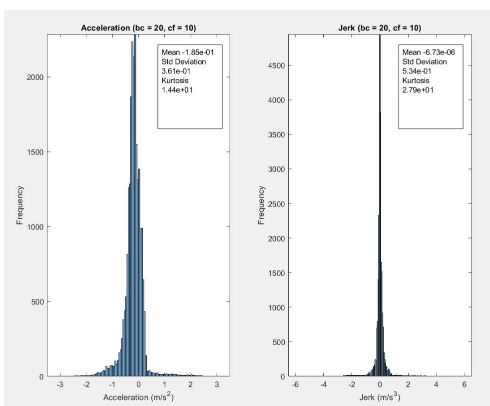
The MATLAB results show that introducing vehicles with bilateral control capabilities into traffic improves passenger comfort by reducing the range of acceleration and jerk values. Simulating and analyzing mixed traffic of different mixture ratios shows that bilateral control vehicles improve passenger comfort even when human drivers remain on the road.



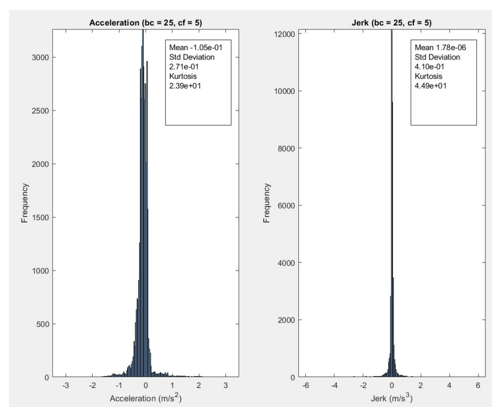
(a) bc = 5, cf = 25



(b) bc = 10, cf = 20



(c) bc = 20, cf = 10



(d) bc = 25, cf = 10

Figure 4.3: Histograms of acceleration and jerk for mixed traffic of various mixture ratios.

Chapter 5

Analysis: Improvement Comparison

The figures from the previous chapters were used to create 2 separate sets of histograms (one for bilateral control vehicles and one for car-following vehicles) in which the inverse TTC, acceleration and jerk values for the bilateral control vehicles were plotted separately from the values associated with the car-following vehicles. The improvement in the values as the number of bilateral control vehicles increased was compared for each type of vehicle to see whether the bilateral control vehicles improved more than the car-following vehicles did, or vice versa. Histograms were generated in MATLAB in a similar way to the histograms in the previous chapters.

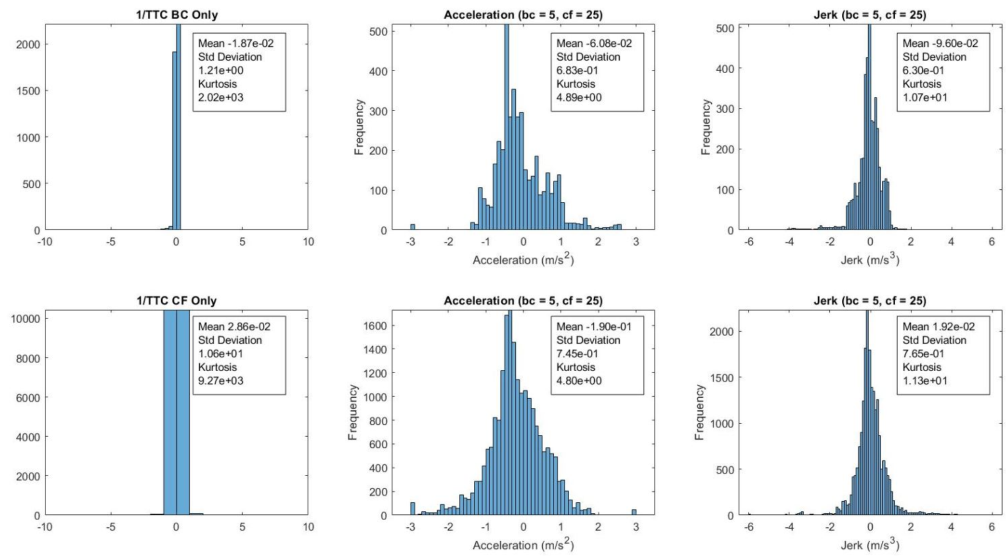
5.1 Mixed Traffic

Histograms of the inverse TTC, acceleration and jerk for mixed traffic with various mixture ratios can be seen in Figures 5.1 and 5.2. There appears to be a larger improvement (i.e. the range of inverse TTC values decreases more as well as the range of acceleration and jerk values) for the car-following vehicles as opposed to the bilateral control vehicles as the number of bilateral control vehicles increases from 5 to 10. As the number of bilateral control vehicles increases beyond 10, the inverse TTC, acceleration and jerk value ranges seem to decrease (i.e. performance improves) at a similar rate for both types of vehicles.

5.2 Discussion

Overall, the histograms in Figures 5.1 and 5.2 seem to show that there is a greater improvement in performance for car-following vehicles as opposed to bilateral control vehicles when the mixture ratio is smaller (i.e. 5 to 10 bilateral control vehicles) and that there is a not a significant difference in the improvement for each type of vehicle when the mixture ratio is higher (i.e. 20 to 25 bilateral control vehicles).

However, it is important to remember that a different number of each type of vehicle is being plotted and those plots are being compared (i.e. histograms of values associated with 5 bilateral control vehicles versus histograms of values associated with 25 car-following vehicles). Therefore, it is difficult to know whether the differences in the improvement between Figures 5.1a and 5.1b are



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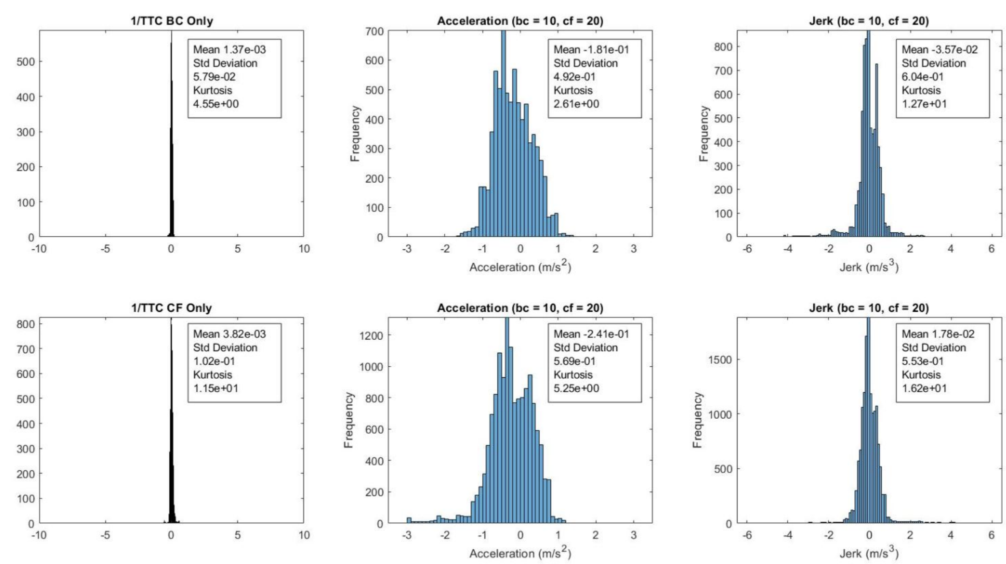
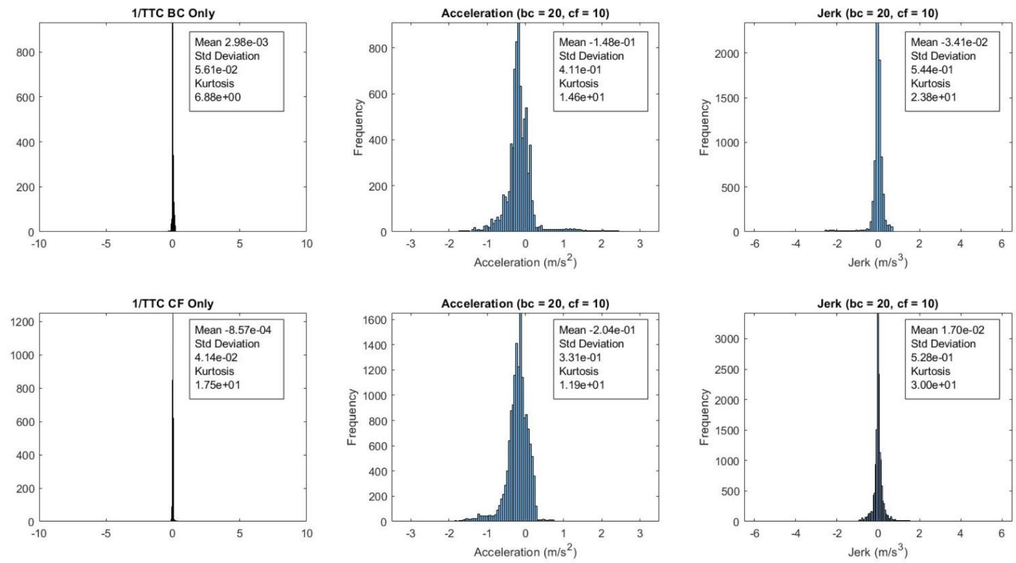
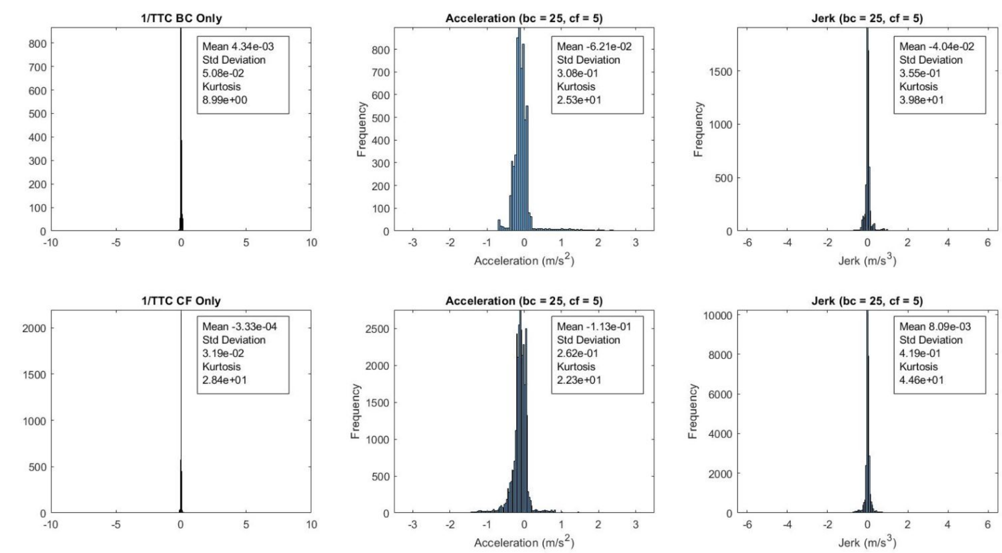


(b) bc = 10, cf = 20

Figure 5.1: Histograms of inverse TTC, acceleration and jerk for mixed traffic of various mixture ratios with separate plots for bilateral control vehicles and car-following vehicles.



(a) bc = 20, cf = 10



(b) bc = 25, cf = 5

Figure 5.2: More histograms of inverse TTC, acceleration and jerk for mixed traffic of various mixture ratios with separate plots for bilateral control vehicles and car-following vehicles.

a result of the type of vehicle or a result of the difference in the amount of data being plotted for each type of vehicle. As a result, the figures in this chapter can not be used to conclude for certain whether one type of vehicle improves more than the other.

Chapter 6

Conclusion

The results presented in the previous chapters show that there are potential societal and personal benefits of suppressing traffic flow instabilities and improving passenger comfort when some vehicles use bilateral control. It is not necessary for all vehicles to use bilateral control in order for there to be benefits. Installing bilateral control in individual vehicles has the potential to save inordinate amounts of fuel that is wasted as a result of phantom traffic jams while also saving people time by reducing the time of travel to a destination. It also has the potential to improve passenger comfort and improve the health of our planet by reducing CO2 emissions [5].

6.1 Future Work

In order to make guarantees about freedom from collisions for vehicles with bilateral control, the basic linear models used in bilateral control can be modified. Attractive forces can be replaced with repulsive forces and this modification can be simulated and analyzed similarly to what has been presented in this thesis. Simulation and analysis of the new controller will allow for the exploration of the effect of the modification of the models on the inverse TTC and, thus, the safety of the controller. This work has the potential to improve the safety of bilateral control and further support the installation of bilateral control in individual vehicles.

Appendix A

Additional Information

A.1 Additional Figures

Some additional mixed simulation figures can be seen in Figures A.1 and A.2. Additional histograms for inverse TTC, acceleration and jerk can be seen in Figures A.3, A.4 and A.5.

A.2 Additional Resources

More information about bilateral control and the suppression of traffic flow instabilities can be found at https://people.csail.mit.edu/bkph/Traffic_Flow_Animation. The original simulator created by Dr. Liang Wang can also be found and downloaded at this link.

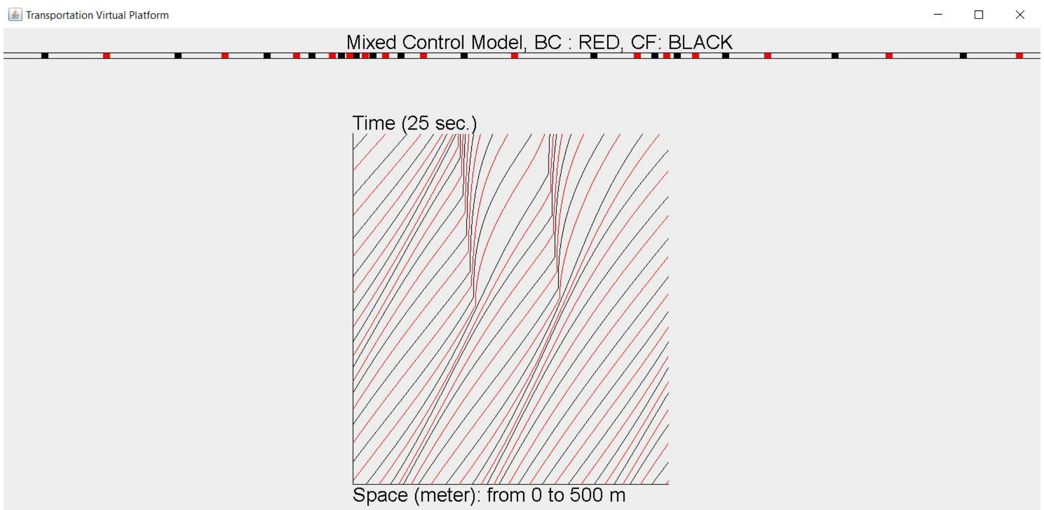


Figure A.1: 15 bilateral control cars and 15 car-following cars.

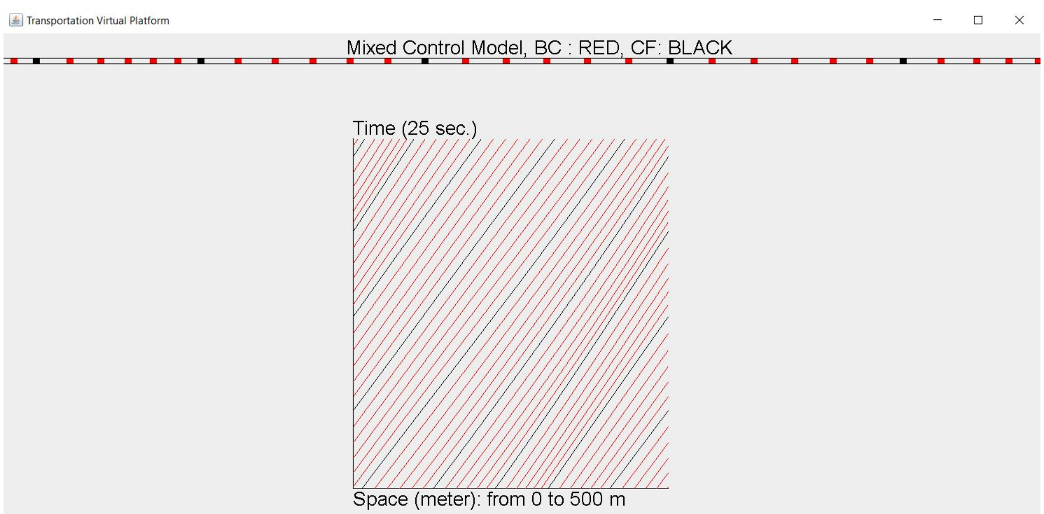


Figure A.2: 25 bilateral control cars and 5 car-following cars.

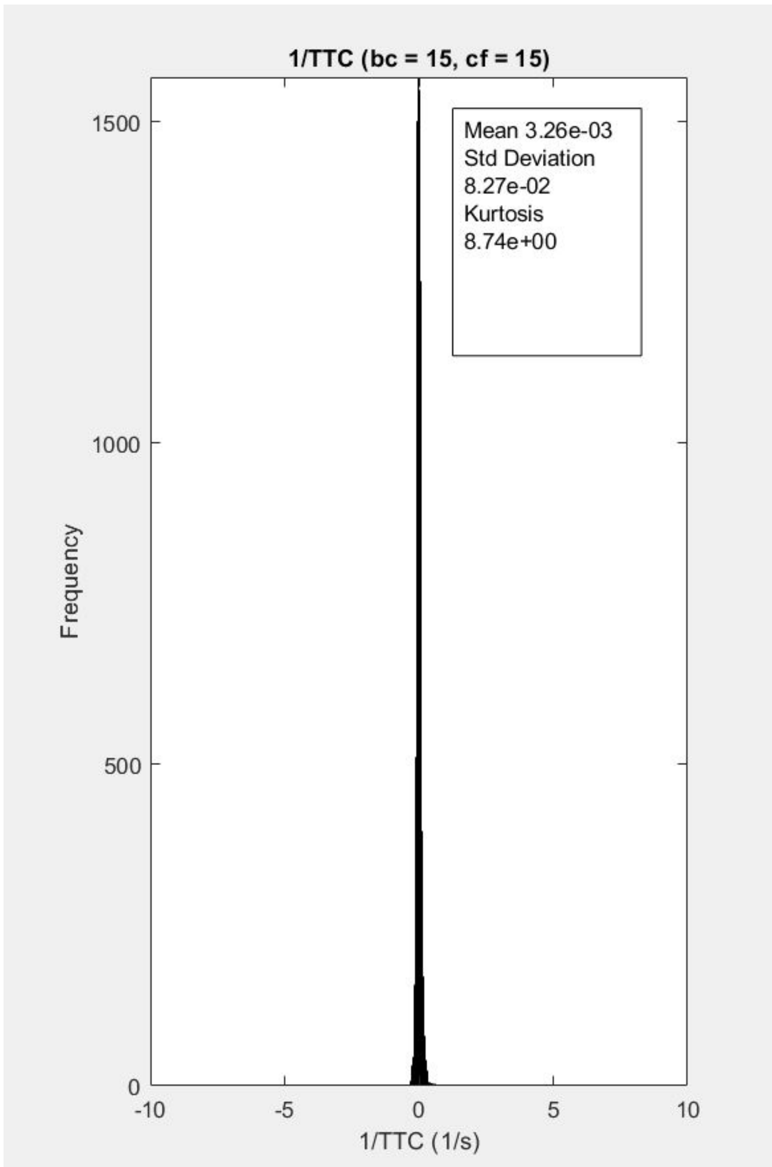


Figure A.3: Histogram of inverse TTC for 15 bilateral control vehicles and 15 car-following vehicles.

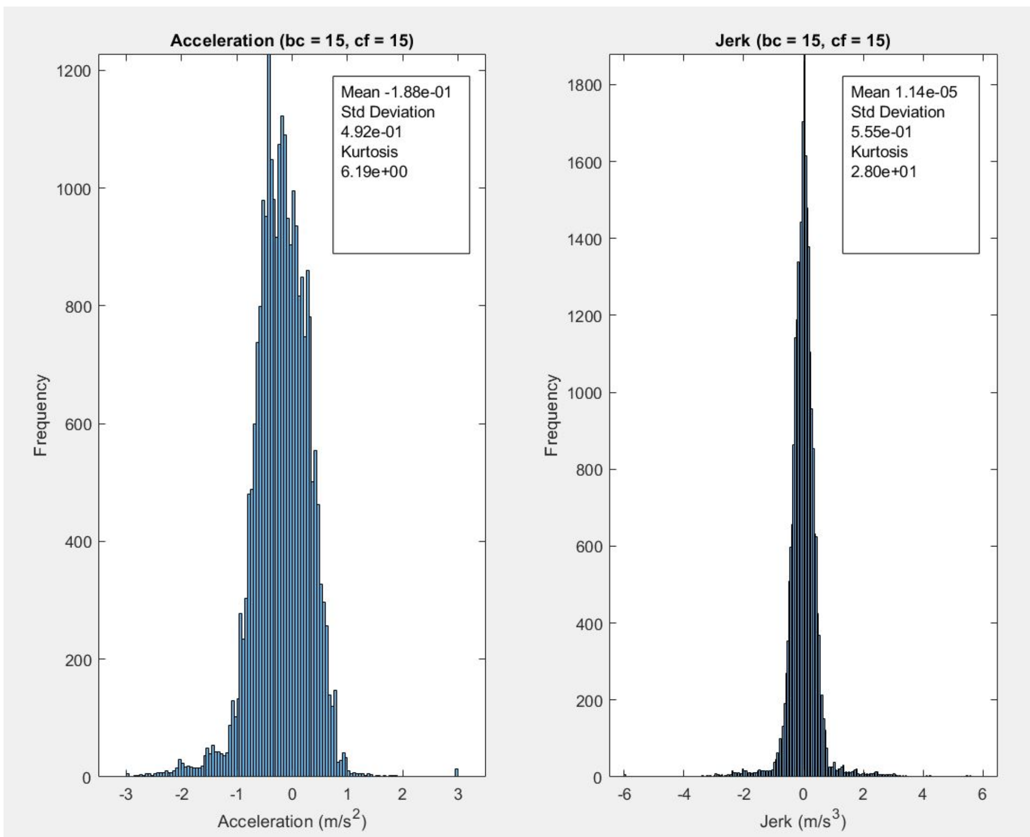


Figure A.4: Histogram of acceleration and jerk for 15 bilateral control vehicles and 15 car-following vehicles.

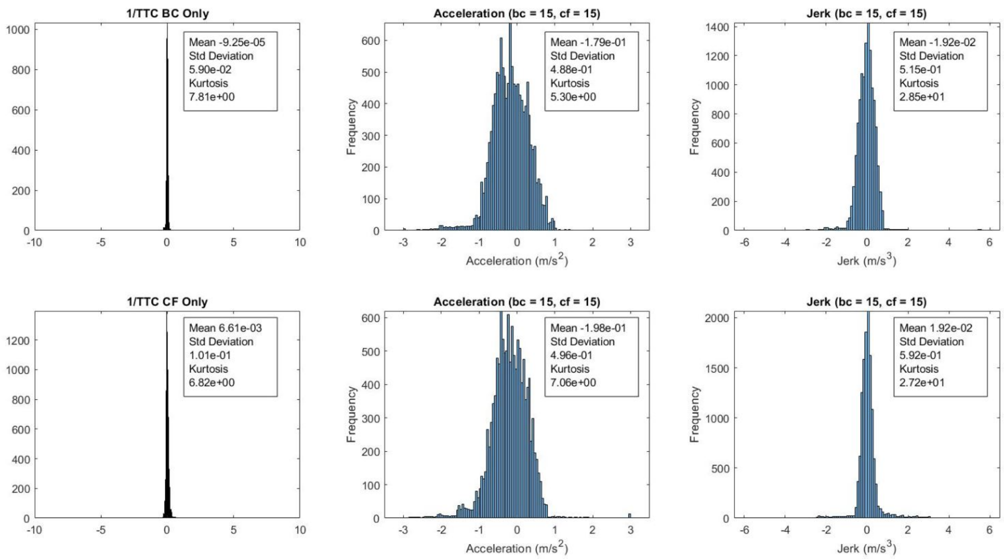


Figure A.5: Histogram of inverse TTC, acceleration and jerk with separate plots for 15 bilateral control vehicles and 15 car-following vehicles.

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