Age of Information and Mobility

by

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Abstract

Age of information is a recently proposed metric that measures the freshness of information at a destination receiving data from an information source. It has become popular in the networking and queuing community, especially for studying delivery of real time status updates. In this thesis, we explore applications of AoI to mobile and adhoc networks. More specifically, we look at two problems - 1) Age optimal information collection and dissemination from locations arranged on a graph, using a mobile agent that travels between them, and 2) Age-based transmission schemes for a group of mobile agents which need to continuously exchange information while moving around in a cell partitioned network. We also derive expressions for age metrics for discrete time queuing systems under various service disciplines, and service and arrival time distributions.

Thesis Supervisor: Eytan Modiano
Title: Professor of Aeronautics and Astronautics

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Contents

List of Figures

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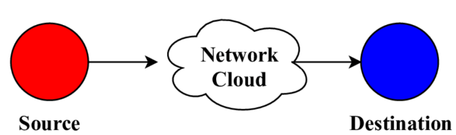
Chapter 1

Introduction

Consider a system in which a source generates updates that traverse a network to reach the destination. The goal of the system is to ensure that the destination gets fresh information. Age of information (AoI), a destination centric metric of information freshness, was first introduced in [1]. It measures the time that has elapsed since the last received fresh update was generated at the source. Over the past few years, a rapidly growing body of work has analyzed AoI for various queuing systems [1–10] and wireless networks [11–17].



Throughout this work, we assume a discrete-time slotted system. Depending on the setting, we consider active sources which can generate fresh packets in every time-slot, as well as uncontrollable sources which generate packets according to some random process. The age process $A(t)$ at any destination increases by 1 in every time-slot in which it does not

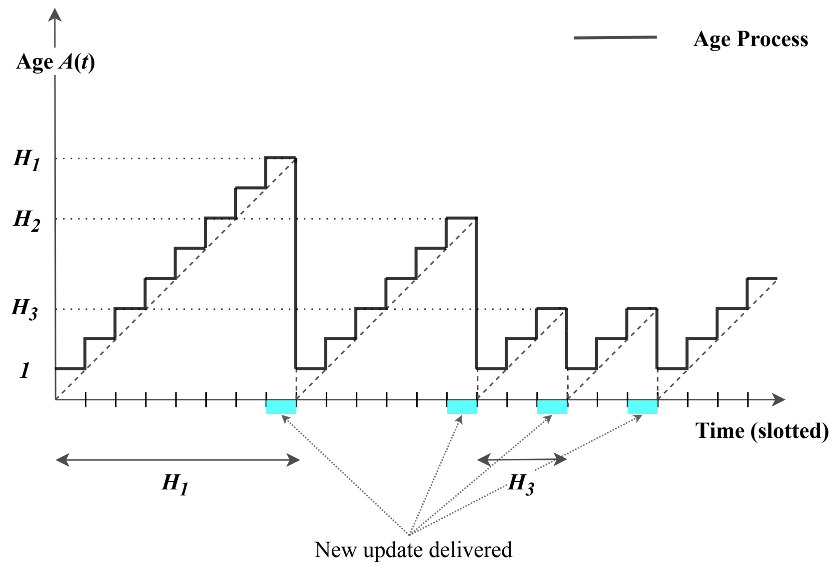


Figure 1-1: Age $A(t)$ evolution with time t for an active source with inter-update intervals H_1, H_2, H_3, and so on.

receive a useful update. For every useful update, the age drops to the amount of the time that has elapsed since the delivered update was generated at the source.

We track the age process $A(t)$ as the value of AoI at the beginning of every time-slot. Assume that the ith packet is generated at time t_i. Then, $A(t)$ satisfies the following recursion

$$A(t+1) = \begin{cases} A(t) + 1, & \text{if no delivery at time } t \\ \min\{t - t_i, A(t)\} + 1, & \text{if } i \text{ is delivered.} \end{cases}$$

Note that for an active source, the age always drops to 1 upon delivery of an update, since only the most recent update is delivered. See Figures 1-1 and 1-2 for examples of age evolution for active and uncontrollable sources.

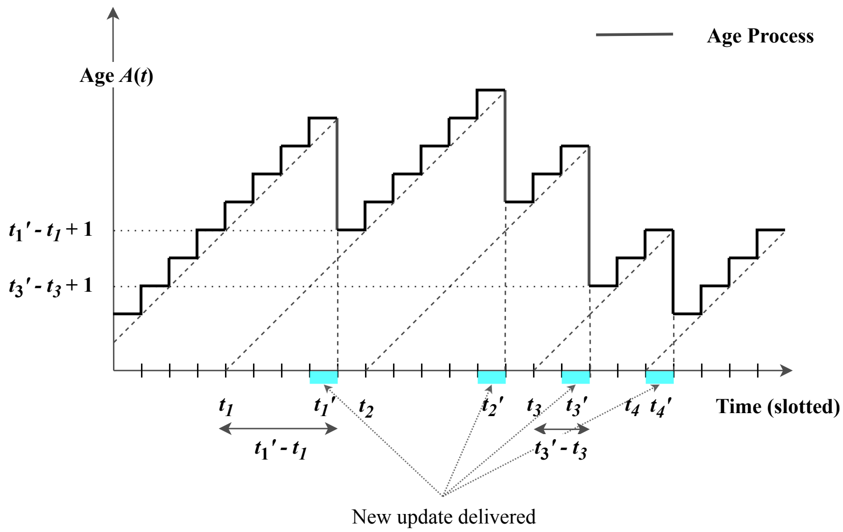


Figure 1-2: Age $A(t)$ evolution with time t for an uncontrollable source with packet generation times t_1, t_2, t_3, ... and packet delivery times t_1', t_2', t_3', ...

1.1 Motivation for Age of Information

Age of information is extremely useful as a metric for freshness in networks where the main concern is real time delivery of status updates. There has been a rapidly growing body of literature applying AoI as a metric of interest to various networking applications, e.g. cache updating [18], networked control systems [19, 20], networks with real-time traffic such as wireless sensor networks or real time processing in augmented and virtual reality systems, [12–17, 21, 22], and vehicular networks [11].

AoI is a distinct notion from throughput and delay, the more traditional QoS metrics used in networking literature. and leads to different scheduling policies as compared to schemes that attempt to minimize throughput and delay. We will see this in detail later.

1.2 Metrics for Age of Information

It is common in AoI literature to look at both peak age and average age. The peak age A^{p} for an age process $A(t)$ is the time average of its peak values. Observe that the age process $A(t)$ reaches peak in the time-slot that it receives a useful packet delivery, since the age drops in the next time-slot. Thus, the expression for peak age is given by

$$A^{\mathrm{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A(t)\mathbb{1}_{\{\text{update delivered at time } t\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{\text{update delivered at time } t\}}}.$$

The average age A^{ave} is just the time average of the entire age process, and is given by the following expression

$$A^{\mathrm{ave}} \triangleq \limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} A(t).$$

In the next section we discuss the difference between AoI for discrete time systems as we have introduced above and AoI for continuous time system, as is common in literature.

1.3 AoI for discrete time Queues

As discussed earlier, age of information (AoI) has been analyzed for a large variety of queuing systems over the past few years. Here, we provide a brief survey of these results. AoI was first studied for the first come first serve (FCFS) M/M/1, M/D/1, and D/M/1 queues in [1]. AoI for M/M/2 and M/M/∞ was studied in [2,3], in order to demonstrate the advantage of having parallel servers. In [8], age was analyzed for parallel last come first serve (LCFS) servers, with preemptive service. Age analysis for queues with packet deadlines, in which a packet deletes itself after its deadline expiration, is considered in [9, 10, 23]. In [24], age has been analyzed under packet transmission errors. In [4], AoI for the LCFS queue with Poisson arrivals and Gamma distributed service was considered. In [5, 6], the LCFS queue scheduling discipline, with preemptive service, is shown to be age optimal, when the service times are exponentially distributed.

More recently, a complete characterization of age distribution for FCFS and LCFS queues, with and without preemption, was done in [7, 25]. In [26], it is proved that a heavy tailed service minimizes age for LCFS queue under preemptive service and the G/G/∞ queue.

As is evident from the literature, AoI has thus far been analyzed in detail for continuous time queuing models. Discrete time queuing systems often arise in practice, especially in wireless networks [15], and are the focus of this thesis. In [15], the authors derived peak and average age expressions for the discrete time FCFS G/Geo/1 queue. The result lead to the derivation of separation principle in scheduling and rate control for age minimization in wireless networks.

In Chapter 2, we analyze age metrics for various discrete time queuing models using the results and analytical tools developed in [7, 25, 26]. We first consider the FCFS Geo/G/1 queue, with and without vacations. When taking vacations, we note that taking deterministic vacations, is the best resort towards minimizing age. We then derive peak and average age expressions for the discrete time LCFS G/G/1 with preemptive service. We build upon proof techniques from earlier results [15, 26, 27], and find that observations from the continuous time scenario carry forward to the discrete time scenario.

In the next two sections, we introduce the two problems that are the main focus of this thesis and discuss related works in literature.

1.4 Mobility on a Graph

Many emerging applications depend on the collection and delivery of status updates between a set of ground terminals and a central terminal using mobile agents. Examples include: measuring traffic at road intersections [28], temperature, and pollution in cities [29], ocean monitoring using underwater autonomous vehicles [30], and surveillance using UAVs [31]. All of these applications depend upon regular status updates, that are communicated in a timely manner, so as to keep the central terminal and the ground terminals updated with fresh information.

Age of Information (AoI), the metric described earlier, captures timeliness of received information [1,15,32]. Unlike packet delay, AoI measures the lag in obtaining information at the destination node, and is therefore suited for applications involving gathering or dissemination of time sensitive updates. Age of information, at a destination, is defined as the time that elapsed since the last received information update was generated at the source. AoI, upon reception of a new update packet, drops to the time elapsed since generation of the packet, and grows linearly otherwise.

We consider the problem of AoI minimization in gathering and dissemination of information updates, between a set of ground terminals and a central terminal. The information updates can be as small as a single packet containing temperature information or a high fidelity image or a video file. The ground terminals are equipped with low power transmitters, and a mobile agent is used to gather and disseminate information.

The age or freshness of information gathered and disseminated depends on the trajectory of the mobile agent, whose mobility is constrained to a *mobility graph*

$G = (V, E)$. The mobile agent can move from ground terminal i to ground terminal j only if $(i, j) \in E$. This model can be used to capture the fact that the agent may not be able to move between any arbitrary locations due to topological limitations. We discuss the system model and our results in detail in Chapter 3.

1.4.1 Related Work

The problem of persistent monitoring in dynamic environments has been considered in [33–35] using tools from optimal control. These works focus on minimizing uncertainty when source locations are time varying, rather than timely monitoring over a fixed set of locations. There is also a large body of work focused on planning trajectories for a mobile agent to optimize traditional performance metrics in wireless sensor networks; primarily throughput, delay and network lifetime; by leveraging variants of the Travelling Salesman Problem (TSP) [36–40]. We observe connections to this line of work, when we establish the optimality of a Hamiltonian cycle trajectory in a symmetric setting.

Optimal sampling trajectories for signals using mobile agents have been considered in [41] and [42]. However, these works deal with sampling rates and perfect reconstruction of time invariant fields rather than freshness of information for sources generating real-time updates.

Closer to our work in this thesis are [43] and [44], in which some approximation trajectories to minimize maximum latency on metric graphs were proposed. In [45], the authors consider trajectory planning for a mobile agent to minimize AoI. They obtain the best permutation of nodes for the mobile agent to visit in sequence, given Euclidian distances between the nodes. In our work, mobility is constrained by a general graph G, and we seek the optimal trajectory over the space of all trajectories allowed on this graph G, not just permutations of nodes.

In the following section, we introduce a setting with multiple mobile nodes in an ad-hoc wireless network.

1.5 AoI in Mobile Ad-Hoc Networks

Consider a situation in which mobile nodes need to be aware of each other's state information (position, velocity, etc.) or global information of the environment to make decisions. These decisions crucially depend on the dynamic nature of the environment, for example, mobile robots cooperating to perform a task over some region, or self-driving cars moving safely across a city without colliding, or mobile nodes traversing a time-changing environment and delivering information to a base station.

To achieve such a goal, it is crucial to have continuous status updates between mobile nodes and it is plausible that the overall system performance depends on how frequently nodes/destinations receive fresh information about the dynamic phenomena. Age of Information is a metric that helps us capture precisely this idea of fresh information, and guides the design of communication protocols that achieve better system performance. In Chapter 4, we discuss the scaling of AoI in a cell partitioned network with multiple mobile agents under i.i.d. mobility.

1.5.1 Related Work

The motivation for this work comes from the well established line of work on capacity and delay scaling and trade-off analysis in adhoc wireless networks with and without mobility. We provide a brief survey of the major works along these lines.

For a complete survey, see [46]. While results in these settings typically have simplified modelling assumptions, the general scaling and achievability results provide key insight into designing better practical wireless networks and evaluating their performance.

The study of capacity scaling in ad-hoc wireless networks begins with the seminal work by Gupta and Kumar in [47], where they develop the protocol model for analyzing wireless networks. In this work, they assume n identical randomly located nodes, each capable of transmitting at a fixed rate and using a fixed range, forming a wireless network. They derive fundamental capacity limits in this unicast network model and develop simple schemes to achieve order optimal throughput rates per node. However, their model consists of fixed nodes and does not take mobility into account.

Grossglauser and Tse extend the model to include mobility in [48] and [49]. They come to the conclusion that mobility drastically changes throughput scaling in ad-hoc wireless networks, and it is in fact possible to have constant throughput per node even as the number of nodes grows very large. They develop a simple two-hop relaying scheme to achieve this throughput.

Throughput and delay scaling, and the tradeoff between them were analyzed in [50–53] under simple mobility models, like i.i.d. mobility. Similar results were derived for Brownian mobility in [54]. We will focus on the model developed in [51] - using cell partitioned networks with i.i.d. or Markov mobility.

Observe that all the works that we have discussed till now only dealt with unicast networks, i.e. n nodes are divided into $n/2$ source-destination pairs, each with their own traffic. Capacity and delay scaling in a cell partitioned broadcast mobile network, where all nodes receive traffic from a set of sources, was analyzed in [55]. The authors observed that there is nearly no capacity-delay tradeoff in such networks, i.e. one can achieve near order optimal throughput and delay simultaneously. This motivates our study of age of information in mobile ad-hoc networks. We consider a broadcast scenario similar to [55], a cell partitioned network similar to [51], and try to find AoI optimal packet forwarding schemes along with AoI scaling with the network size.

There has been prior work on AoI scaling in wireless networks, primarily [56], where the authors consider fixed nodes under the unicast scenario similar to the Gupta-Kumar model and analyze AoI scaling. [57] discusses the age of gossip messages in a mobile ad-hoc network using spatial mean field analysis, while [11] discusses AoI in vehicular networks as a useful metric for analyzing performance. We discuss our system model and results in detail in Chapter 4.

1.6 Outline and Contributions

The remainder of this thesis is organized as follows

- **Chapter 2** describes age of information for discrete time queues. We establish a general relationship between AoI for a discrete time slotted system with AoI in a corresponding continuous time queuing system. We derive closed form expressions for discrete time AoI for various arrival and service time distributions, and queuing disciplines, extending results on continuous time AoI.

- **Chapter 3** describes the mobility on a graph problem. First, we introduce the information gathering problem. We consider the design of trajectories for the mobile agent to minimize peak and average age. We consider the space of randomized trajectories, in which the mobile agent traverses edges according to a random walk on the mobility graph G. We show that a randomized trajectory is in fact peak age optimal, and that it can be obtained in polynomial time using the Metropolis-Hastings algorithm. We then prove that solving for the average age optimal trajectory is NP-hard, in a symmetric setting, and propose a heuristic randomized trajectory that is simultaneously peak age optimal and factor-$8\mathcal{H}$ average age optimal, where $\mathcal{H}$ is the mixing time of the randomized trajectory on G. The factor $\mathcal{H}$ can scale with the graph size, especially if the graph is not well connected. Thus, we propose an age-based trajectory, in which the mobile agent uses the current AoI to determine its motion, and show that it is factor-2 optimal in a symmetric setting. We then introduce the information dissemination problem. Here, the central terminal sends updates for each ground terminal via the mobile agent. The mobile agent queues these update packets in a first-come-first-serve (FCFS) queue, and delivers them to the respective ground terminal when the mobile agent reaches it. We, now, not only have to design the trajectory of the mobile agent, but also determine the optimal rate at which the central terminal generates information updates for each ground terminal. We show that the peak age optimal randomized trajectory of the information gathering problem, along with a simple update generation rate, is at most a factor-$O(\mathcal{H})$ optimal, in both peak and average age.

- **Chapter 4** considers the problem of characterizing AoI in a mobile wireless ad-hoc network setting. We discuss the cell partitioned model with i.i.d. mobility. We then introduce the single-source model and provide an optimal broadcast policy. We also demonstrate scaling of AoI with the size of the network in three different regimes. We then develop a heuristic policy for the multiple source setting and provide numerical results comparing different forwarding policies.

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Chapter 2

AoI for Discrete Time Queues

2.1 Relating Continuous and Discrete AoI

As discussed earlier, we track the age process $A(t)$ as the value of AoI at the beginning of every time-slot. Assume that the ith packet is generated at time Y_i. Then, $A(t)$ satisfies the following recursion

$$A(t+1) = \begin{cases} A(t) + 1, & \text{if no service at time } t \\ \min\{t - Y_i, A(t)\} + 1, & \text{if } i \text{ is served.} \end{cases}$$

Both peak and average age are defined as usual. The peak age A^{p} is the time average of age values at time instants when there is useful packet delivery. The average age A^{ave} is the time-average of the entire age process $A(t)$. Note that when a useful packet delivery occurs in time-slot t, then $A(t+1) \leq A(t)$. Thus,

$$A^{\mathrm{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A(t)\mathbb{1}_{\{A(t+1)\leq A(t)\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{A(t+1)\leq A(t)\}}}, \text{ and} \tag{2.1}$$

$$A^{\mathrm{ave}} \triangleq \limsup_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} A(t). \tag{2.2}$$

Now, consider a continuous time queue corresponding to the original discrete time system. In this queue, we assume the interval $[t, t+1)$ to correspond to the t^{th} time-slot in our discrete system. Packets arriving in our original discrete system at time-slot τ arrive at time $t = \tau$ in the new system, while packets departing at time-slot τ in the discrete system actually depart at time $t = \tau + 1$. For this continuous time system, we define the age process $A_{\mathrm{cont.}}(t)$ as a continuous time process that increases linearly at a rate of 1, until it receives a fresh update, and then drops to the age of the received update. That is,

$$A_{\mathrm{cont.}}(t) = t - Y_i,$$

where Y_i is the time at which packet i finished processing and is the freshest packet to have finished processing. Let the age peaks of this process be $A_1, A_2, ...$ Then, we can define peak and average age for this continuous process as follows -

$$A^{\mathrm{p}}_{\mathrm{cont.}} \triangleq \limsup_{N\to\infty} \frac{\sum_{n=1}^{t=N} A_n}{N}, \text{ and} \tag{2.3}$$

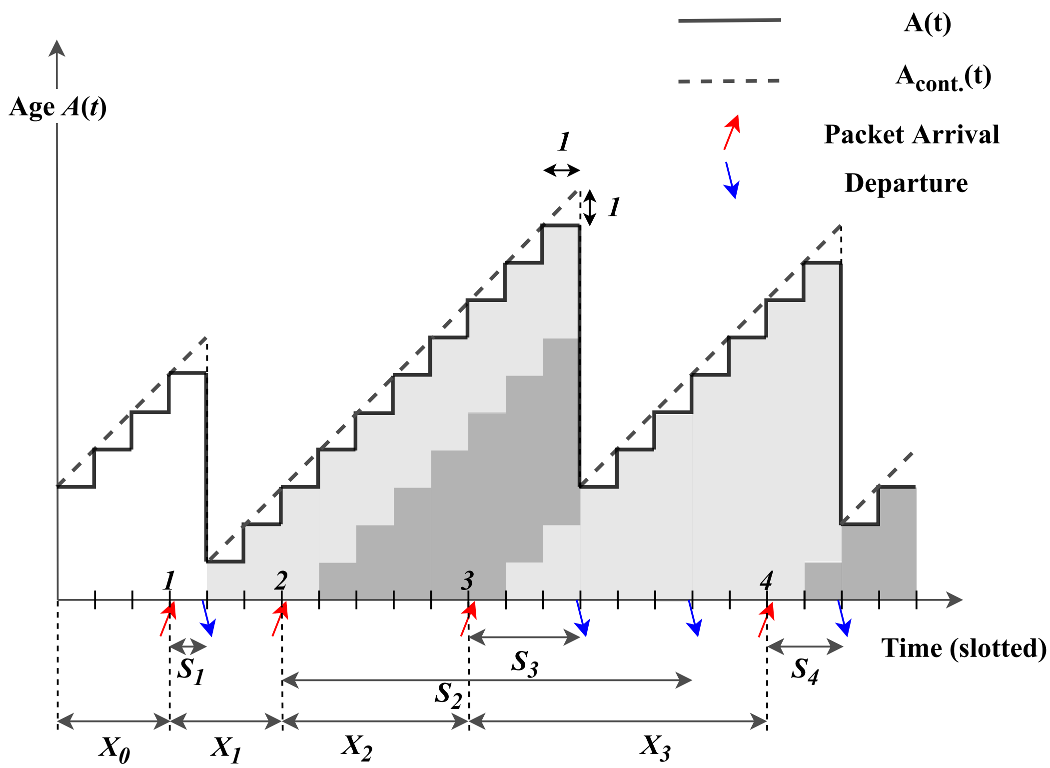

Figure 2-1: Age $A(t)$ evolution over time t along with the corresponding continuous age process. X_i are inter-arrival times, S_i are service times for packet i.

$$A^{\text{ave}}_{\text{cont.}} \triangleq \limsup_{T\to\infty} \frac{1}{T} \int_{t=1}^{T} A_{\text{cont.}}(t). \tag{2.4}$$

We now relate the expressions of peak and average age in the discrete time scenario to the corresponding continuous time scenario.

Theorem 1. *The peak and average age for any discrete time queue, assuming that the peak and average age of the corresponding continuous time age process* *are well defined, is given by*

$$A^{p} = A^{p}_{cont.} - 1, \tag{2.5}$$

and

$$A^{ave} = A^{ave}_{cont.} - \frac{1}{2}. \tag{2.6}$$

Proof. An explanation for this result is easy to see via a graphical argument. Observe in Figure 2-1 that the peaks of $A(t)$ and $A_{\text{cont.}}(t)$ always differ by 1. This is true regardless of the service or arrival time distributions and the queuing discipline. Thus if the value $A^{\text{p}}_{\text{cont.}}$ is well defined, A^{p} is also well defined and is given by Equation 2.5.

Similarly, consider any time interval $[t, t+1)$. The discrete age process stays constant in this time interval at $A(t)$ and the area under the curve is just $A(t)$. The continuous age process $A_{\text{cont.}}(t)$ increases from $A(t)$ to $A(t)+1$ in any such interval, and the area under it is given by $A(t) + \frac{1}{2}$ since it has an added triangle of area $\frac{1}{2}$. This implies that

$$\int_{t=1}^{T+1} A_{\text{cont.}}(t) = \sum_{t=1}^{T} \int_{t}^{t+1} A_{\text{cont.}}(t) = \sum_{t=1}^{T} (A(t) + \frac{1}{2}). \tag{2.7}$$

Assuming that the continuous time average age $A^{\text{ave}}_{\text{cont.}}$ is well defined, we can divide Equation 2.7 by $T+1$ and take the limit supremum as T goes to ∞ to get Equation 2.6. This completes our proof. □

Theorem 1 tells us that for the analysis for AoI in discrete time queues, it is sufficient to analyze the AoI of a corresponding continuous time queuing system with the same service and arrival distributions, and the same queuing discipline. As discussed in Section 1.3, analysis for continuous time queuing systems has already been done in a variety of settings and can thus be directly applied to the discrete time setting. We discuss closed form expressions for AoI in a few discrete time systems in the following sections.

2.2 Geo/G/1 Queue

Consider a discrete time Geo/G/1 queue, where an arrival occurs at time t with probability λ in an i.i.d. fashion, while the service times S are generally distributed with mean $\mathbb{E}[S] = 1/\mu$. The arrival process is thus i.i.d. geometric with parameter λ, and both the arrival interval as well as service times are **positive integers**. According to our convention, if the service time of a packet is 1, then it gets served in the same time-slot as the one in which it was generated.

We obtain expressions for peak and average age for this discrete time Geo/G/1 queue. Observe that this corresponds to the FCFS G/G/1 setting analyzed in [25], but with the same arrival time distribution (i.i.d. geometric) and service time distribution, albeit with support on positive integers.

Theorem 2. *The peak and average age for the discrete time Geo/G/1 queue are given by*

$$A^{p} = \frac{1}{\lambda} + \frac{1}{\mu} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} - 1, \tag{2.8}$$

and

$$A^{ave} = \frac{1}{\mu} + \frac{(1-\lambda)(1-\rho)}{\lambda L_S(1-\lambda)} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)}, \tag{2.9}$$

where $L_S(x) \triangleq \mathbb{E}[x^S]$ is the probability generating function of S and $\rho = \frac{\lambda}{\mu} < 1$.

Proof. Using Theorem 1, we can use the expression for peak age derived for the corresponding continuous time FCFS queue from [25]

$$A^{\mathrm{p}} = A^{\mathrm{p}}_{\mathrm{cont.}} - 1 = \mathbb{E}\left[T + X\right] - 1, \tag{2.10}$$

where T denotes the time an update sends in the queue and X is the inter-arrival time between two updates. From [58, Chapter 4.6.1], for a Geo/G/1 queue we have

$$\mathbb{E}[T] = \frac{\lambda \mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{1}{\mu}, \tag{2.11}$$

where S denotes the service time. Substituting this and $\mathbb{E}\left[X\right] = \frac{1}{\lambda}$ in (2.10), we obtain the expression for peak age.

For average age, we again find the average age of the corresponding continuous time system $A^{\mathrm{ave}}_{\mathrm{cont.}}$, and then use Theorem 1 to get the final result. For the derivation of $A^{\mathrm{ave}}_{\mathrm{cont.}}$ see Appendix A.1. $\square$

We observe that the peak age expression for a Geo/G/1 queue is near identical to that of the M/G/1 queue derived in [26] with an additional term $\frac{-\rho}{2(1-\rho)} - \frac{1}{2}$ added due to the discretization. We use the probability generating function for analyzing average age due to the discrete nature of the service distribution.

2.3 Geo/G/1 Queue with Vacations

Consider a discrete time Geo/G/1 queue with vacations, where an arrival occurs at time t with probability λ in an i.i.d. fashion, while the service times S are generally distributed positive integers with mean $\mathbb{E}\left[S\right] = 1/\mu$. When the queue is empty, the server takes i.i.d. vacations V that are generally distributed with mean $\mathbb{E}\left[V\right]$, until a new arrival enters the queue. Geo/G/1 queues with vacations were used to find age optimal random walks for information dissemination on graphs in [27]. M/M/1 queues with vacations were also used to study the age of updates in a simple relay network in [59]. We obtain an expression for peak age and bounds for average age in the FCFS discrete time Geo/G/1 queue with vacations. Since the arrivals in any time time-slot are i.i.d. Bernoulli, from now on we refer to this queue as a Ber/G/1 queue with vacations.

Theorem 3. *The peak age for the discrete time Ber/G/1 queue with vacations is given by*

$$A^p = \frac{1}{\lambda} + \frac{1}{\mu} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{3}{2}, \tag{2.12}$$

and the average age is upper bounded by the peak age

$$A^{ave} \leq A^p + \frac{1}{2}. \tag{2.13}$$

Proof. As usual, the peak age for the corresponding continuous time FCFS queue is given by [25]

$$A^{\mathrm{p}}_{\mathrm{cont.}} = \mathbb{E}\left[T + X\right]. \tag{2.14}$$

Given that vacation times are distributed i.i.d according to random variable V, using a residual time argument one can show that [60]

$$\mathbb{E}[T] = \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{1}{\mu} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{1}{2}, \tag{2.15}$$

where S denotes the service time. We will prove this in detail in Section 3.3. Substituting this and $\mathbb{E}[X] = \frac{1}{\lambda}$ in (2.14), we obtain the expression for peak age using Theorem 1.

For the corresponding continuous time FCFS queue, the average age $A^{\text{ave}}_{\text{cont.}}$ is given by [1]

$$A^{\text{ave}}_{\text{cont.}} = \frac{\mathbb{E}[X_n^2]/2 + \mathbb{E}[X_n T_n]}{\mathbb{E}[X_n]} = \frac{1}{\lambda} - \frac{1}{2} + \lambda \mathbb{E}[X_n T_n], \tag{2.16}$$

where $\frac{1}{\lambda} = \mathbb{E}[X_n]$ and a packet arrives in every time-slot with probability λ, $X_1, X_2, ...$ are i.i.d. packet inter-arrival times and $T_1, T_2, ...$ are corresponding times spent in the system by each packet. Observe that X_n and T_n are negatively correlated (see [25] for a proof). Intuitively, a smaller inter-arrival time means more congestion and more time spent in the system. Thus,

$$A^{\text{ave}} \leq \frac{1}{\lambda} - \frac{1}{2} + \lambda \mathbb{E}[X_n] \mathbb{E}[T_n] = \mathbb{E}[X_n] + \mathbb{E}[T_n] - \frac{1}{2} = A^{\text{p}} + \frac{1}{2}, \tag{2.17}$$

where the last equality is due to Theorem 1. Thus, the average age is upper bounded by the peak age of the system up to a constant. □

We observe that the peak age for a Ber/G/1 queue with vacations splits into two terms - the peak age for a Ber/G/1 queue without vacations, as derived in the previous section, and a term that depends only on the vacations. From Figure 2-2, we also observe numerically that the lighter the tail of the vacation distribution, better the age. We see that deterministic vacations minimize average age, given a fixed value of $\mathbb{E}[V]$.

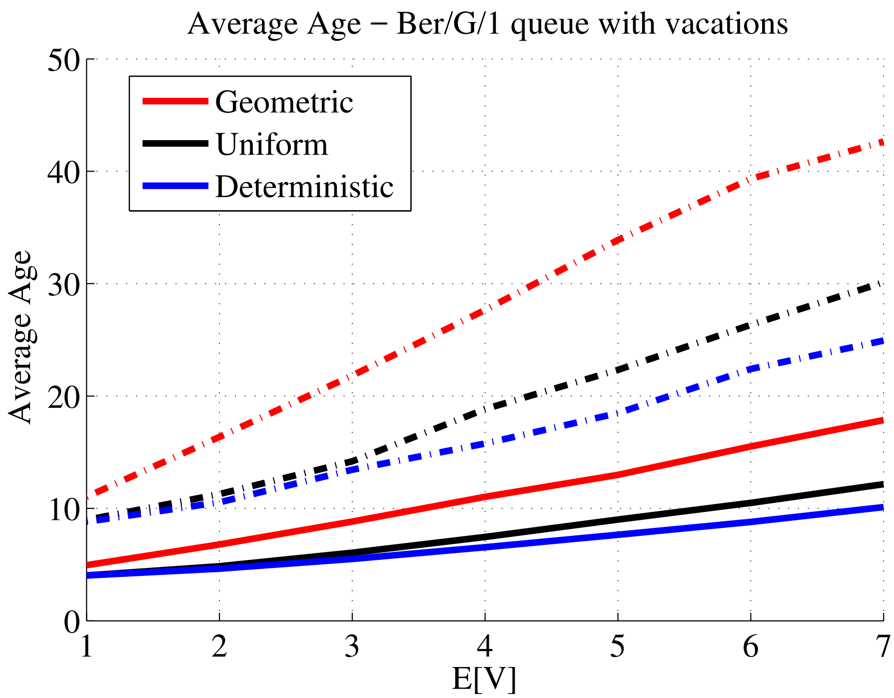

Figure 2-2: Average age for geometrically distributed server times with probability $\mu = 0.75$. We compare vacations with geometric, uniform bounded, and deterministic distributions having the same mean as it varies from 1 to 7. Solid lines represent arrival probability $\lambda = 0.3$ and dashed lines represent $\lambda = 0.6$.

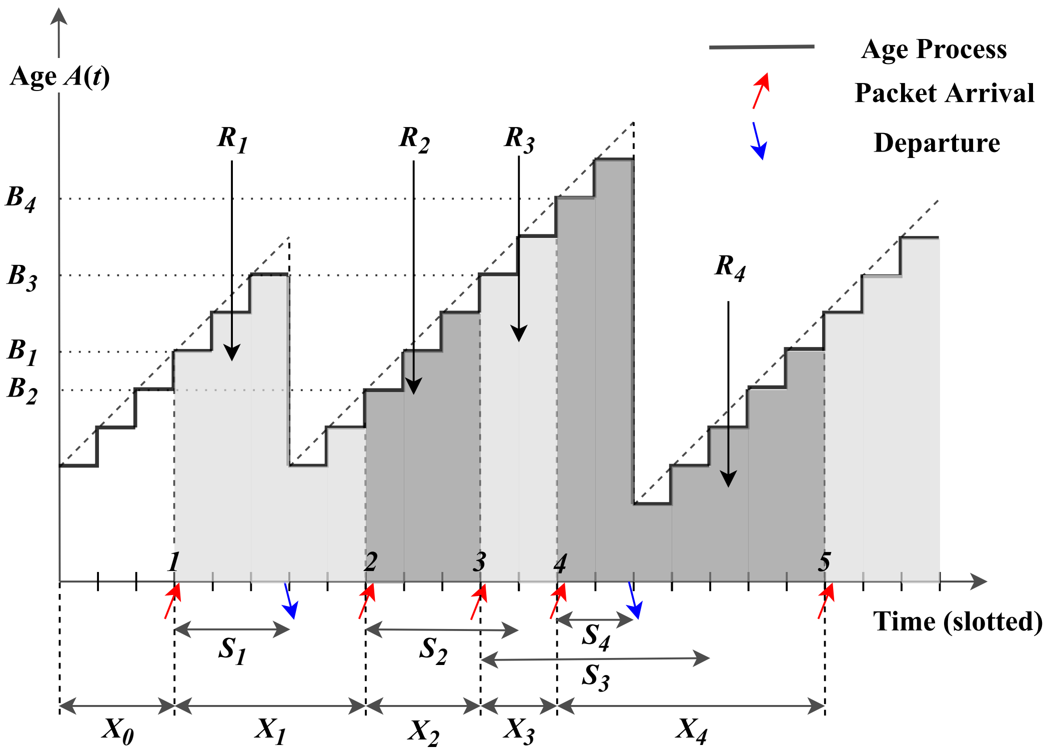

Figure 2-3: Age $A(t)$ evolution in time t for an LCFS queue with preemption.

2.4 LCFS Queues

Consider a discrete time LCFS G/G/1 queue with preemptive service, in which a newly arrived packet gets priority for service immediately. We assume that packets arrive at the beginning of a time-slot and leave at the end of a time-slot. Update packets are generated according to a renewal process, with inter-generation times distributed according to p_X. The service times are distributed according to p_S, i.i.d. across packets.

Using Theorem 1 and results from [26], we can directly obtain closed form expressions for peak and average age for general inter-generation and service time distributions.

Let X_i denote the inter-generation time between the ith and $(i+1)$th update packet. Due to preemption, not all packets get serviced on time to contribute to age reduction. We illustrate this in Figure 2-3. Observe that packets 2 and 3 arrive before packet 4. However, packet 2 is preempted by packet 3, which is subsequently preempted by packet 4. Thus, packet 4 is serviced before 2 and 3. Service of packet 2 and 3 (not shown in figure) does not contribute to age curve $A(t)$ because they contain stale information.

Theorem 4. *For the discrete time LCFS G/G/1 queue, the peak and average age are given by*

$$A^{p}_{G/G/1} = \frac{\mathbb{E}\left[X\right]}{\mathbb{P}\left[S \leq X\right]} + \frac{\mathbb{E}\left[S\mathbb{I}_{S\leq X}\right]}{\mathbb{P}\left[S \leq X\right]} - 1,$$

and

$$A^{ave}_{G/G/1} = \frac{1}{2}\frac{\mathbb{E}\left[X^2\right]}{\mathbb{E}\left[X\right]} + \frac{\mathbb{E}\left[\min\left(X, S\right)\right]}{\mathbb{P}\left[S \leq X\right]} - \frac{1}{2},$$

where X and S denotes the independent inter-generation and service time random variables, respectively.

Proof. The proof follows directly from Theorem 1 and the age analysis of continuous time LCFS G/G/1 queues in [26]. For a more direct proof of this result, see [61]. □

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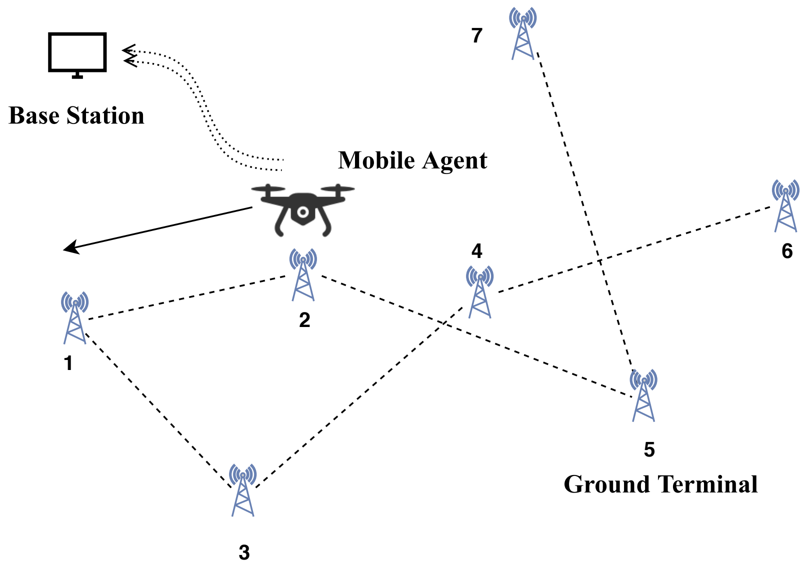
Chapter 3

Mobility on a Graph

3.1 System Model

We consider a central terminal that needs to communicate with a set of ground terminals V. The ground terminals are equipped with low power, low range radio communication devices, and cannot directly communicate with the central terminal, or with each other. An autonomous mobile agent m, is used as a relay between the central terminal and the ground terminals, while moving across the geographical region where the ground terminals are spread.

The mobility of the agent is constrained by a *mobility graph* $G = (V, E)$, where m can travel from ground terminal i to ground terminal j only if $(i, j) \in E$. The graph G, thus, constraints the set of allowable moves. We consider a time-slotted system, with slot duration normalized to unity. In the duration of a time-slot, the mobile agent stays at a ground terminal to gather or disseminate information, and moves to any of its neighbours in G for the next time-slot. The mobility graph can be constructed from the limitations of a slot duration, distances between ground



terminals, and speed of the mobile agent.

We consider two problems: *information gathering* and *information dissemination*. In the information gathering problem, every time the mobile agent reaches a ground terminal $i \in V$, the ground terminal sends a fresh update to the mobile agent, which is immediately relayed to the central terminal. The age $A_i(t)$, at the central terminal, for the ground terminal i drops to 1. When the mobile agent is not at the ground terminal i, the age $A_i(t)$ increases linearly. See Figure 3-1. The evolution of $A_i(t)$ in the information gathering problem can be written as:

$$A_i(t+1) = \begin{cases} A_i(t) + 1, & \text{if } m(t) \neq i \\ 1, & \text{if } m(t) = i \end{cases} \tag{3.1}$$

where $m(t)$ denotes the location of the mobile agent at time t. Note that the age evolution depends on the trajectory that the mobile agent follows on the mobility graph G.

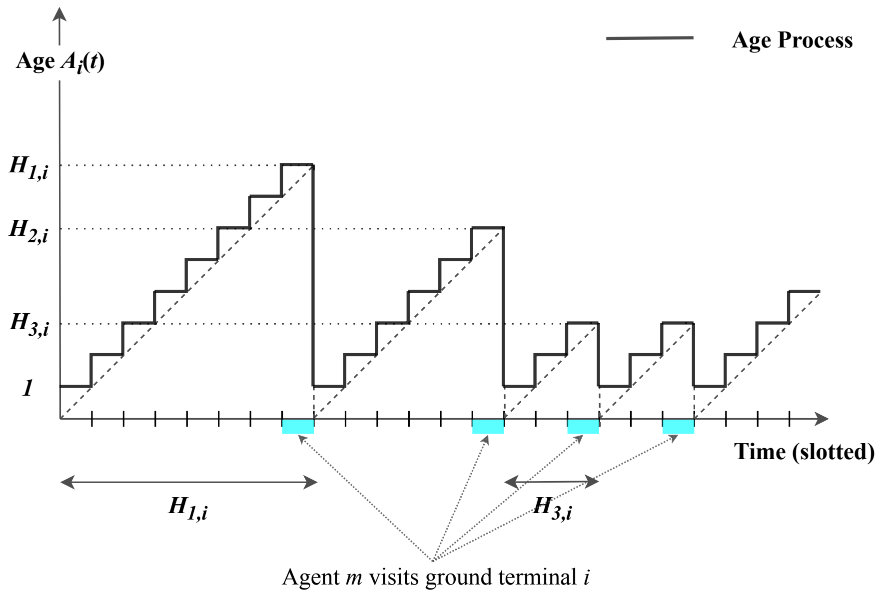


Figure 3-1: Information gathering problem: time evolution of age $A_i(t)$; $H_{k,i}$ is the k^{th} return time to terminal i.

In the information dissemination problem, the central terminal generates updates for each ground terminal. The generated updates are then transmitted to the mobile agent. The mobile agent queues updates received from the central terminal in a set of V FCFS queues, one for each ground terminal. The mobile agent delivers the head-of-line update in queue i, to ground terminal i, when it reaches i. The central terminal has no control over the FCFS queues on the mobile agent, however, it can control the update generation rate λ_i, for each ground terminal i.

The age $A_i(t)$, at the ground terminal i, increases by 1 every time the mobile agent is not at i, or when it is at i but has no updates to deliver. Otherwise, a successful delivery of the head-of-line update occurs in time slot t, and the age $A_i(t)$ drops to the age of the head-of-line update in queue i. See Figure 3-2. This evolution

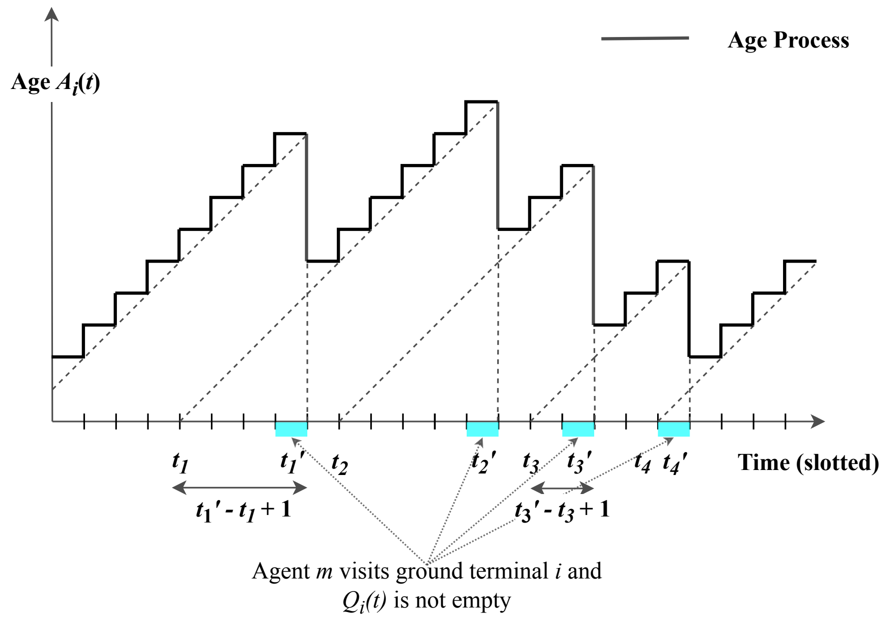


Figure 3-2: Information dissemination problem: time evolution of age $A_i(t)$; t_k, t_k' are the generation and reception times of the k^{th} status update for terminal i.

of age $A_i(t)$ can be written as:

$$A_i(t+1) = \begin{cases} A_i(t)+1, & \text{if } m(t) \neq i \\ A_i(t)+1, & \text{if } m(t) = i \text{ and } \mathcal{Q}_i(t) = \emptyset \\ t - G_i(t) + 1, & \text{if } m(t) = i \text{ and } \mathcal{Q}_i(t) \neq \emptyset \end{cases}, \tag{3.2}$$

where $G_i(t)$ is the time of generation of the head of line packet in queue i, at time t, and $\mathcal{Q}_i(t)$ denotes the set of packets in the mobile agent's queue i at time t.

3.1.1 Age Metrics

AoI is an evolving function of time. We consider two time average metrics of AoI. Average age, for ground terminal i, is defined as the time averaged area under the age curve:

$$A_i^{\text{ave}} \triangleq \limsup_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} A_i(t). \tag{3.3}$$

In Figures 3-1 and 3-2, we see that the age $A_i(t)$ peaks before a fresh update is delivered. In the information gathering case, a fresh update is delivered every time the mobile agent visits i, i.e. $m(t) = i$. Whereas, in the information dissemination case, a fresh update is delivered whenever $m(t) = i$ and the queue $\mathcal{Q}_i(t) \neq \emptyset$. The peak age A_i^{p}, for ground terminal i, defined as an average of all the peaks in the age evolution curve $A_i(t)$, can be written as

$$A_i^{\text{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A_i(t)\mathbb{1}_{\{m(t)=i\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i\}}}, \tag{3.4}$$

in the information gathering case and

$$A_i^{\text{p}} \triangleq \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A_i(t)\mathbb{1}_{\{m(t)=i,\mathcal{Q}_i(t)\neq\emptyset\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i,\mathcal{Q}_i(t)\neq\emptyset\}}}, \tag{3.5}$$

in the information dissemination case.

We define the network peak and average age to be

$$A^{\text{p}} = \sum_{i\in V} w_i A_i^{\text{p}} \quad \text{and} \quad A^{\text{ave}} = \sum_{i\in V} w_i A_i^{\text{ave}}, \tag{3.6}$$

where $w_i > 0$ are weights representing the relative importance of a ground terminal i. Our goal is to minimize network peak and average age.

3.1.2 Trajectory Space

We use $\mathbb{T}$ to denote a reasonably large space of trajectories:

$$\mathbb{T} = \{ \text{ Trajectory } \mathcal{T} \mid f_i(\mathcal{T}) \text{ exists and is positive } \forall\, i \in V \ \},$$

where $f_i(\mathcal{T})$ denotes the fraction of time-slots, the trajectory $\mathcal{T}$, is at ground terminal i:

$$f_i(\mathcal{T}) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{1}_{\{m(t)=i\}}. \tag{3.7}$$

For a trajectory $\mathcal{T} \in \mathbb{T}$, the limit (3.7) exists and is positive for all $i \in V$. This requirement is to ensure that the peak and average age are both finite and well defined.

Peak and average age depend on the trajectory $\mathcal{T} \in \mathbb{T}$. We use $A^{\mathrm{p}}(\mathcal{T})$ and $A^{\mathrm{ave}}(\mathcal{T})$ to denote network peak and average age, respectively, for $\mathcal{T} \in \mathbb{T}$. In the following two sections, we introduce the problem of finding trajectories that minimize network peak and average age, and try to find solutions to these problems.

3.2 Information Gathering

In this section, we consider the problem of age optimal information gathering with active sources. We define optimal peak and average age to be

$$A^{\mathrm{p}*}_{\mathcal{G}} = \min_{\mathcal{T}\in\mathbb{T}} A^{\mathrm{p}}(\mathcal{T}), \quad \text{and} \quad A^{\mathrm{ave}*}_{\mathcal{G}} = \min_{\mathcal{T}\in\mathbb{T}} A^{\mathrm{ave}}(\mathcal{T}), \tag{3.8}$$

where $\mathbb{T}$ denotes the space of all trajectories for the mobile agent.

We first consider randomized trajectories, where the mobile agent moves according to a random walk on the mobility graph. We shall show that for peak age optimality, such randomized trajectories suffices. We then show that the average age optimization is NP-hard, and propose a heuristic randomized trajectory. In Section 3.2.5, we propose an age-based trajectory for better average age performance.

3.2.1 Randomized Trajectories

We start by defining the class of randomized trajectories:

Definition A trajectory $m(t)$, on mobility graph G, is said to be a *randomized trajectory* if $m(t)$ is an irreducible Markov chain defined by a transition probability matrix $\mathbf{P}$:

$$\mathbb{P}m(t+1) = j|m(t) = i = P_{i,j}, \tag{3.9}$$

for all t and $i, j \in V$, where $P_{i,j} = 0$ for $(i, j) \notin E$.

For convenience, we shall refer to $m(t)$, defined above, as the randomized trajectory $\mathbf{P}$, where $\mathbf{P}$ to denote the matrix with entries $P_{i,j}$. Note that $P_{i,j}$ is the probability that the mobile agent, when at ground terminal i, moves to ground terminal j for the next time slot. The constraint: $P_{i,j} = 0$ for $(i, j) \notin E$, ensures that the randomized trajectory adheres to the mobility constraints defined by G.

We assume in the definition of a randomized trajectory $\mathbf{P}$, that $m(t)$ is an irreducible Markov chain over the state space V. This is desired, since the mobile agent has to traverse through all the nodes, repeatedly, for a positive fraction of time, or otherwise the resulting peak and average age would be unbounded.

For any randomized trajectory $\mathbf{P}$, we obtain explicit expressions for network peak and average age. We use the notation $A^{\mathrm{p}}(\mathbf{P})$ and $A^{\mathrm{ave}}(\mathbf{P})$ to show explicit dependence of peak and average age on the randomized trajectory $\mathbf{P}$.

Theorem 5. *The network peak and average age for a randomized trajectory* $\mathbf{P}$ *is given by*

$$A^{p}(\mathbf{P}) = \sum_{i \in V} \frac{w_i}{\pi_i}, \quad \text{and} \quad A^{ave}(\mathbf{P}) = \sum_{i \in V} \frac{w_i z_{ii}}{\pi_i}, \tag{3.10}$$

where π *is the unique stationary distribution obtained by solving* $\pi \mathbf{P} = \pi$ *and* z_{ii} *are diagonal elements of the matrix* $Z \triangleq (I - \mathbf{P} + \Pi)^{-1}$*, where* Π *is an* $n \times n$ *matrix with entries* $\Pi_{i,j} \triangleq \pi_j, \ \forall i, j \in V$*.*

Proof. The key step in proving the result above is to observe that the peak age of the ground terminal i, namely A_i^{p}, depends only on the mean of return times to terminal i; see Figure 3-1. Whereas, the average age A_i^{ave} for i depends on both, the mean and the variance, of return times to terminal i.

Given a randomized trajectory $\mathbf{P}$, the mean of return times to terminal i is given by $\frac{1}{\pi_i}$, while the second moment of the return times is given by $\frac{-1}{\pi_i} + \frac{2 z_{ii}}{\pi_i^2}$; see [62]. Using this fact, we are able to obtain the explicit expressions for peak and average age. Let A_i^p be the peak age for ground terminal i. We define $H_{k,i}$ to be the k^{th} return time to ground terminal i. Then, the k^{th} age peak for $A_i(t)$ has a value of $H_{k,i}$. Let K be the total number of returns to i over a time-horizon T. Then, the expected peak age of ground terminal i is given by

$$A_i^p = \lim_{T\to\infty} \mathbb{E}\left[\frac{\sum_{t=1}^{t=T} A_i(t)\mathbb{1}_{\{m(t)=i\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i\}}}\right] = \lim_{K\to\infty} \mathbb{E}\left[\frac{1}{K}\sum_{k=1}^{t=K} H_{k,i}\right]. \tag{3.11}$$

Note that return times to a ground terminal i are i.i.d. random variables given a randomized trajectory $\mathbf{P}$. So, we can use the law of large numbers to get

$$A_i^p = \mathbb{E}[H_{1,i}] = \frac{1}{\pi_i}, \tag{3.12}$$

where π_i is the stationary distribution for Markov chain $\mathbf{P}$. The last equality follows from the fact that the expected return time to a state i for an irreducible Markov chain is given by the inverse of its stationary probability. Thus, the network age is given by

$$A^{\text{p}} = \sum_{i\in V} w_i A_i^p = \sum_{i\in V} \frac{w_i}{\pi_i}. \tag{3.13}$$

For average age, we define a renewal-reward process using $H_{k,i}$ as our i.i.d. renewal intervals and sum of age $A_i(t)$ during each interval as our reward. Let $T_{k,i} = \sum_{l=1}^{k-1} H_{l,i}$ be the starting time of the kth renewal. The total reward in between two visits to ground terminal i is the sum of the i^{th} age process $A_i(t)$ across all time-slots during that interval.

Note that, for the k^{th} renewal interval, $A_i(t)$ grows from 1 to $H_{k,i}$ over the $H_{k,i}$ time-slots. Thus, the total reward for the k^{th} renewal interval is given by -

$$\sum_{t=T_{k,i}}^{t=T_{k,i}+H_{k,i}} A_i(t) = \sum_{a=1}^{H_{k,i}} a = \frac{H_{k,i}^2 + H_{k,i}}{2}. \tag{3.14}$$

Note that this reward is also i.i.d. across renewals as it depends only on $H_{k,i}$. Thus, by application of the elementary renewal theorem for renewal-reward processes we get

$$A_i^{\text{ave}} = \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} A_i(t)\right] = \frac{\mathbb{E}[H_{1,i}^2 + H_{1,i}]}{2\mathbb{E}[H_{1,i}]}. \tag{3.15}$$

For irreducible Markov chains, we know the following results hold [62]:

$$\mathbb{E}[H_{1,i}] = \frac{1}{\pi_i}, \forall i \in V \text{ and} \tag{3.16}$$

$$\mathbb{E}[H_{1,i}^2] = \frac{-1}{\pi_i} + \frac{2z_{ii}}{\pi_i^2}, \tag{3.17}$$

for all $i \in V$, where z_{ii} is the i^{th} diagonal element of the matrix $Z = (I - P + \Pi)^{-1}$, with Π being a matrix in which all rows are the stationary distribution vector π: $\Pi_{i,j} = \pi_j$ for all $i, j \in V$.

Substituting (3.16) and (3.17) in (3.15), we get

$$A_i^{\text{ave}} = \frac{z_{ii}}{\pi_i}, \tag{3.18}$$

for all $i \in V$, and therefore,

$$A^{\text{ave}} = \sum_{i\in V} w_i A_i^{\text{ave}} = \sum_{i\in V} \frac{w_i z_{ii}}{\pi_i}. \tag{3.19}$$

□

3.2.2 Peak Age Minimization

We first formulate the peak age minimization problem over the space of randomized trajectories. We shall see that a peak age optimal randomized trajectory suffices for optimality over the space of all trajectories.

Using the results in Theorem 5, we can write the peak age minimization problem over the space of randomized trajectories as:

$$\begin{aligned}
\underset{\mathbf{P},\pi}{\text{Minimize}} \quad & \sum_{i\in V} \frac{w_i}{\pi_i}, \\
\text{subject to} \quad & P_{i,j} \geq 0,\ \forall(i,j), \quad \text{and} \quad \mathbf{P1} = \mathbf{1}, \\
& \pi\mathbf{P} = \pi,\ \mathbf{1}^T\pi = 1,\ \text{and}\ \pi_i \geq 0\ \forall i \\
& P_{i,j} = 0,\ \forall(i,j) \notin E, \\
& \mathbf{P}\ \text{is irreducible.}
\end{aligned} \tag{3.20}$$

Note that $\mathbf{P}$ characterizes a randomized trajectory, while π is the unique stationary distribution associated with it.

This problem is difficult to solve because the irreducibility constraint cannot be expressed in a simple, solvable manner. Further, relaxing the irreducibility constraint can yield a trivial solution like $\mathbf{P} = I$, which are neither irreducible nor anywhere close to optimal.

However, the problem (3.20) can be transformed to finding an irreducible $\mathbf{P}$, with a given stationary distribution. This is a simpler problem and can be solved using the Metropolis-Hastings algorithm.

Lemma 1. *Let* $\pi_i^* \triangleq \frac{\sqrt{w_i}}{\sum_{j\in V}\sqrt{w_j}}$*, for all* $i \in V$*, to be a distribution on* V*, and a randomized trajectory* $\mathbf{P}$ *satisfy* $\pi^*\mathbf{P} = \pi^*$*. Then,* $(\pi^*, \mathbf{P})$ *solves* (3.20)*.*

Proof. Suppose we could choose any stationary distribution π on V. Then to minimize the network peak age, we would need to solve the following optimization problem

$$\begin{aligned} \underset{\pi}{\text{Minimize}} \quad & \sum_{i \in V} \frac{w_i}{\pi_i}, \\ \text{subject to} \quad & \sum_i \pi_i = 1, \\ & \pi_i \geq 0, \forall i \in V. \end{aligned} \tag{3.21}$$

Using KKT conditions for the optimization problem (3.21), it is straightforward to see that

$$\pi_i^* = \frac{\sqrt{w_i}}{\sum_i \sqrt{w_i}}, \forall i \in V. \tag{3.22}$$

Clearly, if we could find a randomized trajectory $\mathbf{P}$ that achieves this stationary distribution π^*, then it would be peak age optimal. Thus, any randomized trajectory $\mathbf{P}$ that satisfies $\pi^* = \pi^* \mathbf{P}$ is peak age optimal. □

Observe that the expression above implies that the fraction of time spent at a node is proportional to the square root of its weight. This is similar to the "square root principle" first derived in peer-to-peer settings in [63]. Similar square root based scheduling results have been derived for minimizing age in single hop networks [18,64]

Lemma 1 implies that a randomized trajectory $\mathbf{P}$, that satisfies $\pi^* \mathbf{P} = \pi^*$, is a peak age optimal, over the space of all randomized trajectories. We now construct one such randomized trajectory: for π^* given in Lemma 1, define a Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$:

$$P_{i,j}^{\text{mh}} = \begin{cases} P_{i,j}^{\text{rw}} \min(1, \frac{\pi_j^* P_{j,i}^{\text{rw}}}{\pi_i^* P_{i,j}^{\text{rw}}}), & \text{if } i \neq j \text{ and } (i,j) \in E \\ 1 - \sum_{j: j \neq i} P_{i,j}^{\text{mh}}, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}, \tag{3.23}$$

where

$$P^{\text{rw}}_{i,j} = \begin{cases} \frac{1}{d_i}, & \text{if } i \neq j \text{ and } (i,j) \in E \\ 0, & \text{otherwise} \end{cases}, \ \forall i,j \in V, \tag{3.24}$$

and d_i equals the out degree of terminal i in the mobility graph G. It is known that such a randomized trajectory $\mathbf{P}^{\text{mh}}$ satisfies $\pi^*\mathbf{P} = \pi^*$ [62]. We, therefore, have the following result.

Theorem 6. *The Metropolis-Hastings randomized trajectory* $\mathbf{P}^{mh}$ *solves* (3.20)*, i.e. it is peak age optimal over the space of all randomized trajectories.*

We considered randomized trajectories, where the mobile agent moves from terminal i to j with probability $P_{i,j}$. We now show that, for peak age optimality, such a randomization suffices.

Theorem 7. *The Metropolis-Hastings randomized trajectory* $\mathbf{P}^{mh}$ *is peak age optimal over the space of all trajectories* $\mathbb{T}$*, namely* $A^{p*}(\mathbf{P}^{mh}) = A^{p*}_{\mathcal{G}}$.

Proof. We establish a more general result. Namely, any randomized trajectory which satisfies $\pi^*\mathbf{P} = \pi^*$, where $\pi^*_i = \frac{\sqrt{w_i}}{\sum\limits_{j\in V}\sqrt{w_j}}$, is peak age optimal over the space of all trajectories:

$$A^{\text{p}*}(\mathbf{P}) = A^{\text{p}*}_{\mathcal{G}}.$$

To prove this, it suffices to argue that the peak age for any trajectory is lower bounded by $\sum_{i\in V} \frac{w_i}{\pi^*_i}$, where π^* is as given in Theorem 6.

Let $H_{k,i}$ to be the k^{th} return time to node i. If K is the total number of returns to ground terminal i over a time horizon T, then the peak age A_i^{p} is given by

$$A_i^{\text{p}} = \limsup_{T\to\infty} \frac{\sum_{t=1}^{t=T} A_i(t)\mathbb{1}_{\{m(t)=i\}}}{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i\}}} = \limsup_{K\to\infty} \frac{1}{K}\sum_{k=1}^{k=K} H_{k,i}. \tag{3.25}$$

Now, the fraction of time-slots in which the mobile agent is at ground terminal i, is given by

$$f_i = \lim_{T\to\infty} \frac{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i\}}}{T} = \lim_{K\to\infty} \frac{K}{\sum_{k=1}^{k=K} H_{k,i}} = \frac{1}{A_i^{\text{p}}}, \tag{3.26}$$

and therefore, $A^{\text{p}} = \sum_{i\in V} w_i A_i^{\text{p}} = \sum_{i\in V} \frac{w_i}{f_i}$. Note that f_i, being the fraction of time-slots the mobile agent is at terminal i, is a distribution over V. Thus, A^{p} can be lower bounded by

$$A^{\text{p}} = \sum_{i\in V} w_i A_i^{p} \geq \min_{\{f_i\geq 0,\ \sum_i f_i=1\}} \sum_{i\in V} \frac{w_i}{f_i} = \sum_{i\in V} \frac{w_i}{\pi_i^*}, \tag{3.27}$$

where the last equality is obtained by solving the optimization problem, just as in the proof of Lemma 1. □

Thus, we are able to obtain a peak age optimal trajectory, namely $\mathbf{P}^{\text{mh}}$. Further, the matrix $\mathbf{P}^{\text{mh}}$ can be computed in polynomial time; in $O(|V|^2)$ time. Therefore, the peak age minimization problem is solved in polynomial time. For details on how to derive the Metropolis-Hastings Markov chain and a nice geometric interpretation, see [65] and [66].

3.2.3 Average Age Minimization

We now consider the average age minimization problem. We first argue that in the symmetric setting, namely $w_i = 1 \ \forall \ i \in V$,[1] the average age minimization problem is NP-hard

Theorem 8. *The problem of finding an average age optimal trajectory is NP-hard in the symmetric setting of* $w_i = 1 \ \forall \ i \in V$.

Proof. To prove NP-hardness, we establish equivalence between the average age minimization problem and the Hamiltonian cycle problem, in the symmetric setting. We know that more connected the graph, lower is its network average age. Therefore, the average age for G = (V, E) is lower bounded by the average age for the complete graph K(V), given by $\frac{|V|(|V|+1)}{2}$. This lower bound can be obtained by using Theorem 9 and setting $w_i = 1, \ \forall i$.

If the graph is Hamiltonian, we can achieve this average age lower bound by setting the trajectory equal to a Hamiltonian cycle. This is because in a cyclical trajectory, the agent visits every terminal exactly once in every $|V|$ time-slots. Further, if the graph is not Hamiltonian, the optimal average age is strictly greater than $\frac{|V|(|V|+1)}{2}$. This is because in the absence of a cycle on graph G, the agent cannot visit every terminal exactly once every $|V|$ time-slots. Therefore, if an algorithm were to solve the average age problem then the same algorithm could be used to determine whether the graph G is Hamiltonian or not; which is the Hamiltonian cycle problem. Since the Hamiltonian cycle problem is NP-complete, the average age minimization

[1] The weights w_i only measure relative significance of ground terminals. Thus, setting $w_i = 1 \ \forall \ i \in V$ is equivalent to setting $w_i = w_j \ \forall \ i, j \in V$.

problem must be NP-hard. □

Since solving the average age minimization problem is hard, we derive a lower bound on average age. Intuitively, if the mobility graph is better connected then it should yield a lower age. This is because a better connected mobility graph imposes fewer restrictions on mobility. The following result obtains a lower bound on network average age by comparing it with the network average age of a complete graph.

Theorem 9. *For any trajectory $\mathcal{T} \in \mathbb{T}$, the network average age is lower bounded by*

$$A^{ave}(\mathcal{T}) \geq \frac{1}{2} \sum_{i \in V} \left(\frac{w_i}{\pi_i^*} + w_i \right), \tag{3.28}$$

where $\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j \in V} \sqrt{w_j}}$ for all $i \in V$.

Proof. Let $H_{k,i}$ be the k^{th} return time to ground terminal i, and K be the total number of returns to i over a time-horizon T. Then the average age A_i^{ave} is given by (see proof of Theorem 1):

$$A_i^{\text{ave}} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{t=T} A_i(t) = \lim_{K \to \infty} \frac{\sum\limits_{k=1}^{k=K} (H_{k,i}^2 + H_{k,i})}{2 \sum\limits_{k=1}^{k=K} H_{k,i}}. \tag{3.29}$$

Define the empirical first and second moment of return times be $\hat{H}_i \triangleq \frac{1}{K} \sum\limits_{k=1}^{k=K} H_{k,i}$ and $\hat{H}_i^{(2)} \triangleq \frac{1}{K} \sum\limits_{k=1}^{k=K} H_{k,i}^2$, respectively. Further, define $\hat{\text{Var}}_i \triangleq \hat{H}_i^{(2)} - \hat{H}_i^2$ to be the empirical variance of return times. From (3.29), we have

$$A_i^{\text{ave}} = \frac{1}{2} + \lim_{K\to\infty} \frac{\hat{H}_i^{(2)}}{2\hat{H}_i} = \frac{1}{2} + \lim_{K\to\infty} \frac{\left(\hat{H}_i\right)^2 + \hat{\text{Var}}_i}{2\hat{H}_i}. \tag{3.30}$$

Using Cauchy-Schwarz inequality, we can obtain $\hat{\text{Var}}_i \geq 0$. Applying this to (3.30), we get

$$A_i^{\text{ave}} \geq \frac{1}{2} + \lim_{K\to\infty} \frac{\hat{H}_i}{2}, \tag{3.31}$$

Let f_i be the fraction of time-slots in which the mobile agent is at ground terminal i. Then,

$$f_i = \lim_{T\to\infty} \frac{\sum_{t=1}^{t=T} \mathbb{1}_{\{m(t)=i\}}}{T} = \lim_{K\to\infty} \frac{K}{\sum_{k=1}^{k=K} H_{k,i}} = \frac{1}{\lim_{K\to\infty} \hat{H}_i}, \tag{3.32}$$

since f_i is well defined and positive for all trajectories in $\mathbb{T}$. Substituting (3.32) in (3.31) we get $A_i^{\text{ave}} \geq \frac{1}{2} + \frac{1}{2f_i}$, for all i, and

$$A^{\text{ave}} = \sum_{i\in V} w_i A_i^{\text{ave}} \geq \frac{1}{2}\sum_{i\in V} w_i + \frac{1}{2}\sum_{i\in V} \frac{w_i}{f_i}. \tag{3.33}$$

Note that f_i, being the fraction of time-slots the mobile agent is at terminal i, is a distribution over V. Thus, the average age in (3.33) can be lower bounded by

$$\begin{aligned} A^{\text{ave}} &\geq \frac{1}{2}\sum_{i\in V} w_i + \frac{1}{2} \min_{\{f_i\geq 0,\ \sum_i f_i=1\}} \sum_{i\in V} \frac{w_i}{f_i}, \\ &= \frac{1}{2}\sum_{i\in V} w_i + \frac{1}{2}\sum_{i\in V} \frac{w_i}{\pi_i^*}, \end{aligned}$$

which proves the result. □

Note that the term $\sum_{i\in V}\frac{w_i}{\pi_i^*}$ is nothing but the optimal peak age $A_{\mathcal{G}}^{\mathrm{p}*}$; see Theorem 7. Furthermore, the lower bound in Theorem 9 is independent of the trajectory $\mathcal{T}$. Therefore, we get

$$A_{\mathcal{G}}^{\mathrm{ave}*} = \min_{\mathcal{T}\in\mathbb{T}} A^{\mathrm{ave}}(\mathcal{T}) \geq A_{\mathrm{LB}}^{\mathrm{ave}} = \frac{1}{2}A_{\mathcal{G}}^{\mathrm{p}*} + \frac{1}{2}\sum_{i\in V} w_i, \tag{3.34}$$

where $\mathbb{T}$ is the space of all trajectories. It must be noted that a similar result was derived in the case of link scheduling for age minimization in [15]. The similarity of the result is rooted in the fact that the information gathering problem in the complete graph case is equivalent to the link scheduling problem in [15], in which at most one link can be activated simultaneously.

3.2.4 A Heuristic Randomized Trajectory

Motivated by the peak age optimality results of the previous section, we restrict ourselves to the space of randomized trajectories, and propose a heuristic, called the *fastest-mixing randomized trajectory*, and prove an average age performance bound for it.

Using the results in Theorem 5, the average age minimization problem over the space of randomized trajectories can be written as

$$
\begin{aligned}
\underset{\mathbf{P},\pi,\mathbf{Z}}{\text{Minimize}} \quad & \sum_{i \in V} \frac{w_i z_{ii}}{\pi_i}, \\
\text{subject to} \quad & P_{i,j} \geq 0,\ \forall\ (i,j), \quad \text{and} \quad \mathbf{P1} = \mathbf{1}, \\
& \pi \mathbf{P} = \pi,\ \mathbf{1}^T \pi = 1,\ \text{and}\ \pi_i \geq 0\ \forall i \\
& P_{i,j} = 0,\ \forall (i,j) \notin E, \\
& \mathbf{P} \text{ is irreducible}, \\
& \Pi_{i,j} = \pi_j\ \forall\ (i,j), \\
& Z = (I - \mathbf{P} + \Pi)^{-1}.
\end{aligned} \tag{3.35}
$$

Here, $\mathbf{P}$ is the randomized trajectory and π the unique stationary distribution corresponding to $\mathbf{P}$. Solving (3.35) can be computationally complex. Not only do we have the irreducibility constraint, but also a non-linear constraint in $Z = (I - \mathbf{P} + \Pi)^{-1}$.

We next upper bound the network average age, for any randomized trajectory $\mathbf{P}$ of the mobile agent. We first define mixing time for a randomized trajectory.

To do this, we first discuss the notion of stopping rules and stopping times in a Markov chain. A stopping rule is a rule that observes the walk on a Markov chain and, at each step, decides whether or not to stop the walk based on the walk so far. Stopping rules can make probabilistic decisions and therefore the time at which the walk stops, called the stopping time, is a random variable.

Mixing Time [67] The hitting time from state distribution σ_1 to σ_2 on a Markov chain is the minimum expected stopping time over all stopping rules that, beginning at σ_1, stop in the exact distribution of σ_2. In other words, it is the expected number of steps that the optimal stopping rule takes to move from σ_1 to σ_2. This is denoted by $\mathcal{H}(\sigma_1, \sigma_2)$. The mixing time $\mathcal{H}$ of a Markov chain $\mathbf{P}$ is then defined as

$$\mathcal{H} \triangleq \sup_{\sigma \in \mathbf{\Delta}(V)} \mathcal{H}(\sigma, \pi), \tag{3.36}$$

where $\mathbf{\Delta}(V)$ is the collection of all distributions on V and π is the stationary distribution of $\mathbf{P}$. In other words, it is the expected time taken to reach stationarity using the optimal stopping rule and starting at the worst initial distribution.

Lemma 2. *The network average age for a randomized trajectory* $\mathbf{P}$ *is upper bounded by*

$$A^{ave}(\mathbf{P}) = \sum_{i \in V} \frac{w_i z_{ii}}{\pi_i} \leq 4\mathcal{H} A^{p}(\mathbf{P}) + \sum_{i \in V} w_i, \tag{3.37}$$

where $\mathcal{H}$ *denotes the mixing time of the randomized trajectory* $\mathbf{P}$*.*

Proof. First, we define the quantity $\mathcal{Z} \triangleq \max_i \sum_j |z_{ij} - \pi_j|$, called the discrepancy of the randomized trajectory $\mathbf{P}$. This definition implies that $z_{ii} \leq \mathcal{Z} + \pi_i, \ \forall i \in V$. Thus, we get the following upper bound:

$$\sum_{i \in V} \frac{w_i z_{ii}}{\pi_i} \leq \sum_{i \in V} \left(\frac{w_i \mathcal{Z}}{\pi_i} + w_i \right). \tag{3.38}$$

However, from [68] we know that $\mathcal{Z} \leq 4\mathcal{H}$, where $\mathcal{H}$ is the mixing time of the randomized trajectory $\mathbf{P}$. Thus, we have the required result

$$\sum_{i \in V} \frac{w_i z_{ii}}{\pi_i} \leq \sum_{i \in V} \left(\frac{4 w_i \mathcal{H}}{\pi_i} + w_i \right) = 4\mathcal{H} A^{\mathrm{p}}(\mathbf{P}) + \sum_{i \in V} w_i,$$

where the last equality follows from Theorem 5. □

We use this relation and suggest the following heuristic for minimizing age: *Find the fastest mixing randomized trajectory* $\mathbf{P}$ *on the mobility graph* G *that minimizes peak age.*

From the proof of Theorem 7, we know that for a randomized trajectory $\mathbf{P}$ to be peak age optimal all we need is $\pi_i \propto \sqrt{w_i}$, where π is the stationary distribution of $\mathbf{P}$. It, therefore, suffices to find $\mathbf{P}$ that satisfies $\pi_i \propto \sqrt{w_i}$, and simultaneously minimizes the mixing time $\mathcal{H}$. We call this the *fastest-mixing randomized trajectory*, and use $\mathbf{P}^*$ to denote it. The following result provides a way to obtain $\mathbf{P}^*$ by solving a convex program.

Theorem 10. *The fastest mixing randomized trajectory can be found by solving the following convex optimization problem:*

$$\begin{aligned} \underset{\mathbf{P}}{\text{Minimize}} \quad & \mu(\mathbf{P}) = ||\mathbf{P} - \Pi^*||_2, \\ \text{subject to} \quad & P_{i,j} \geq 0, \ \forall (i,j), \\ & \mathbf{P1} = \mathbf{1}, \\ & \pi^* \mathbf{P} = \pi^*, \quad \Pi^*_{i,j} = \pi^*_i \ \forall \ i,j \in V, \\ & P_{i,j} = 0, \forall (i,j) \notin E. \end{aligned} \tag{3.39}$$

Here $||A||_2$ *denotes the spectral norm of matrix* A *and* $\pi^*_i = \frac{\sqrt{w_i}}{\sum_{j \in V} \sqrt{w_j}}, \ \forall i \in V$.

Proof. From [65], we know that the fastest mixing, reversible Markov chain on a graph $G(V, E)$ having the stationary distribution π can be found by formulating the following convex program:

$$\begin{aligned} \underset{\mathbf{P}}{\text{Minimize}} \quad & ||D^{1/2}\mathbf{P}D^{-1/2} - qq^T||_2, \\ \text{subject to} \quad & P_{i,j} \geq 0, \ \forall (i,j) \\ & \mathbf{P1} = \mathbf{1}, \\ & \pi^*\mathbf{P} = \mathbf{P}^T\pi^*, \\ & P_{i,j} = 0, \forall (i,j) \notin E. \end{aligned} \tag{3.40}$$

Here $D = \text{diag}(\pi^*)$ and $q = (\sqrt{\pi_1^*}, \sqrt{\pi_2^*}, ..., \sqrt{\pi_n^*})$. Note that we do not require reversibility, so we can replace the detailed balance constraint $\pi^*\mathbf{P} = \mathbf{P}^T\pi^*$ with the global balance constraint $\pi^*\mathbf{P} = \pi^*$. Also, left and right multiplying $(D^{1/2}\mathbf{P}D^{-1/2} - qq^T)$ by matrices $D^{-1/2}$ and $D^{1/2}$, respectively, does not change the spectral norm; since $\mathbf{P}$ has the same eigen-values as $D^{1/2}\mathbf{P}D^{-1/2}$ and qq^T has the same eigen-values as $D^{-1/2}qq^TD^{1/2}$ [65]. Further, observe that $D^{-1/2}qq^TD^{1/2} = qq^T = \Pi^*$, where $\Pi^*_{i,j} = \pi_i^* \ \forall \ i,j \in V$. Thus, the optimization problem reduces to (3.39). This proves the required result. □

This convex program (3.39) finds a randomized trajectory $\mathbf{P}$ on G that is closest to the stationary randomized walk Π^*, in the spectral norm sense. Also, P^* is peak age optimal on graph G, since it satisfies $\pi_i^* \propto \sqrt{w_i}$. Note that, the problem (3.39) can be solved in polynomial time by converting it to a semi-definite program [65].

We now bound the average age performance of the fastest-mixing randomized trajectory.

Theorem 11. *The network average age of the fastest-mixing randomized trajectory is at most* $8\mathcal{H}$*-factor away from the optimal average age:*

$$\frac{A^{\text{ave}}(\mathbf{P}^*)}{A_{\mathcal{G}}^{ave*}} \leq 8\mathcal{H}, \tag{3.41}$$

where $\mathcal{H}$ *is the mixing time of* P^*.

Proof. Note that the peak age for the fastest-mixing randomized trajectory $\mathbf{P}^*$ is given by $A^{\text{p}}(\mathbf{P}^*) = \sum_{i\in V} \frac{w_i}{\pi_i^*}$, since $\pi^*\mathbf{P}^* = \pi^*$. From Theorem 9, a lower bound on average age is given by

$$A_{\text{LB}}^{\text{ave}} = \sum_{i\in V} \frac{1}{2}\left(\frac{w_i}{\pi_i^*} + w_i\right) = \frac{1}{2}A^{\text{p}}(\mathbf{P}^*) + \frac{1}{2}\sum_{i\in V} w_i. \tag{3.42}$$

To prove the result, it suffices to argue that $A^{\text{ave}}(\mathbf{P}^*)/A_{\text{LB}} \leq 8\mathcal{H}$. From (3.42) and Lemma 2, we get

$$\frac{A^{\text{ave}}(\mathbf{P}^*)}{A_{\text{LB}}^{\text{ave}}} \leq \frac{4\mathcal{H}A^{\text{p}}(\mathbf{P}^*) + \sum_{i\in V} w_i}{\frac{1}{2}A^{\text{p}}(\mathbf{P}^*) + \frac{1}{2}\sum_{i\in V} w_i}, \tag{3.43}$$

$$\leq 8\mathcal{H}, \tag{3.44}$$

since $\mathcal{H}$ is always greater than or equal to 1. □

To see the usefulness of the fastest-mixing randomized trajectory, and Theorem 11, consider a random geometric graph $\mathcal{G}(n, r)$. The graph consists of n nodes spread over a unit square with a link between every two nodes that are within a distance r. If v is the physical speed of the mobile agent, then r must equal $v\tau$, where τ is the slot duration. We know that mixing time of $\mathcal{G}(n, r)$ is $O\left(\frac{\log n}{r^2}\right)$, and therefore, the fastest-mixing randomized trajectory would be at most $O\left(\frac{\log n}{v_{max}^2 \tau^2}\right)$ factor optimal. For highly connected graphs, such as Dirac graphs in which the degree of each node is at least $|V|/2$, we have constant factor of optimality; since the mixing times are $O(1)$. [69] establishes a connection between the existence of long paths in graphs and their mixing times and that it is hard to find even constant factor approximations to the problem of finding the longest path on a general graph.

3.2.5 Age-based Trajectories

In the last two sub-sections, we proposed two randomized trajectories, namely $\mathbf{P}^{\text{mh}}$ and $\mathbf{P}^*$. Both were peak age optimal, while the latter was also factor-$\mathcal{H}$ average age optimal. We also noted that solving the average age problem is generally hard. We now propose an age-based trajectory which can be constant factor age optimal.

Age-based trajectory In every time slot, agent m moves to the location that has the highest weighted function of $A_i(t)$. Specifically, if $m(t) = i$ then

$$m(t+1) = \arg \max_{j:(i,j)\in E} w_j g\left(A_j(t)\right), \tag{3.45}$$

for all $i, j \in V$ and time t, where $g(\cdot)$ is an increasing function. We assume that ties are broken in order of vertex indices.

Examples of functions include $g(a) = a$ and $g(a) = a + a^2$. The idea for an age-based trajectory comes from results on age optimal scheduling [12, 64] that develop index based methods which are constant factor optimal. In the symmetric setting,

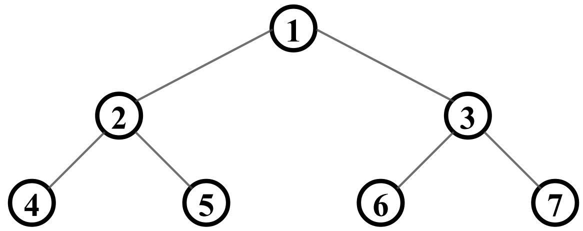


Figure 3-3: Mobility graph restricted to a binary tree.

where $w_i = 1 \; \forall \; i \in V$, the function $g(\cdot)$ does not matter and the agent moves greedily to the neighbouring node with the highest age.

In fact, we observe that the age-based trajectory is a repeated depth-first traversal of the mobility graph G. This can be verified easily when the mobility graph is a tree. Consider the tree in Figure 3-3, and assume that we start at the root node 1 with age for all nodes being zero. The trajectory of the agent following the rule described above would be $1 \to 2 \to 4 \to 2 \to 5 \to 2 \to 1 \to 3 \to 6 \to 3 \to 7 \to 3 \to 1$... This is precisely the depth-first traversal of the tree graph.

In the symmetric setting, where $w_i = 1 \; \forall \; i \in V$, we now prove that the age-based trajectory is factor-2 optimal.

Theorem 12. *In the symmetric setting $w_i = 1 \; \forall \; i \in V$, the network average age A^{ave} for the age-based trajectory is bounded by*

$$\frac{A^{ave}}{A_{\mathcal{G}}^{ave*}} \leq \frac{2|V|+1}{|V|+1} \leq 2, \tag{3.46}$$

for any increasing function $g(\cdot)$.

Proof. The number of steps taken to cover every vertex of a graph by performing a depth first search (DFS) traversal is upper bounded by $2|V|$, since every vertex is visited at least once and the sum total of visits after the first visit to all nodes is at most $|V|$. This is because every repeated visit to a vertex means that at least one new vertex was visited. Thus, every location gets visited at least once in every $2|V|$ time-slots. This implies that the average age of every terminal can be upper bounded by $\frac{(2|V|+1)}{2}$.

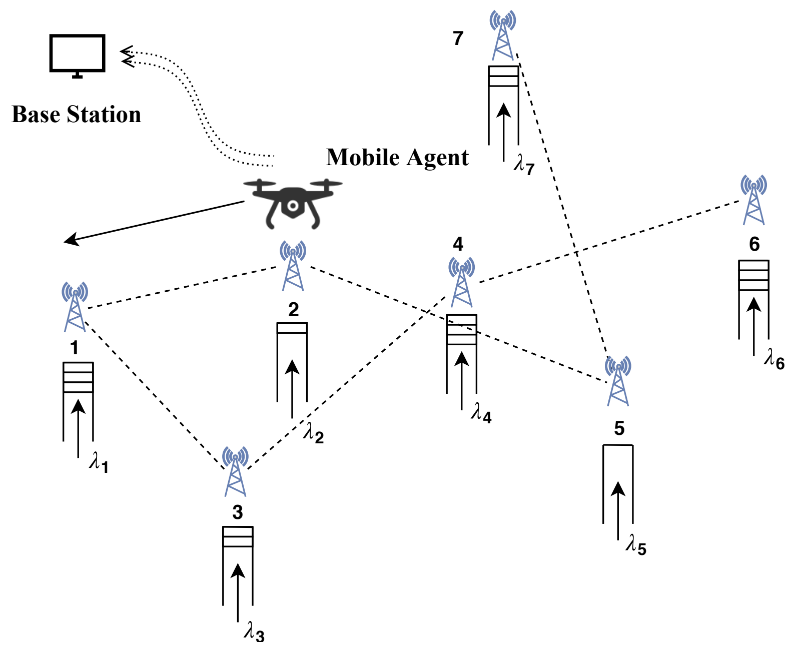
However, from our earlier discussion, we know that the average age of any terminal is lower bounded by $\frac{(|V|+1)}{2}$ if all the weights are 1. Combining the upper and lower bounds, we have the required result. □

This age-based policy can be implemented in an online fashion if the mobile agent has access to age $A_i(t)$ of the neighboring terminals. The complexity of implementing this trajectory is then at most linear in the time-horizon and $|V|$. However, it also suggests a polynomial time "offline" algorithm that does not need knowledge of ages or computation in every time-slot to achieve the same result -

1. Assume that the agent always starts at a fixed node v. Compute a depth-first traversal on the graph $G(V, E)$ starting at node v.
2. Compute the shortest path from the last visited node in the dfs traversal to v.
3. Append the path from step 2 to the dfs traversal in step 1. Follow this trajectory plan iteratively.

Note that both the dfs traversal and the shortest path can be computed beforehand in polynomial time. Using exactly the same arguments as for the greedy algorithm, this trajectory plan also achieves factor 2 optimality in the equal weight setting.

The utility of index-based policies is in situations where we include unreliable packet deliveries or time-varying weights and mobility graphs in our model. While



Markov chain based analysis works only for fixed graphs known beforehand, the age-based trajectory can be easily modified for use in dynamic settings. Showing performance guarantees in such settings is also an interesting line of future work.

3.3 Information Dissemination

We now consider the information dissemination problem. The central terminal generates updates for every ground terminal i, at rate λ_i, according to a Bernoulli process. The generated updates for the ground terminal i are sent to the mobile agent, which get queued in the ith FCFS queue. The mobile agent follows a trajectory $\mathcal{T}$, and delivers the head-of-line update in queue i to terminal i, when it reaches it. The FCFS queue assumption is motivated by uncontrollable MAC layer queues implemented in practice, where the generated updates get queued for transmission [11, 15].

Our objective is to minimize the network peak age and average age over the space of update generation rates $\boldsymbol{\lambda}$ and all trajectories $\mathbb{T}$:

$$A_{\mathcal{D}}^{\text{p}*} = \min_{\mathcal{T} \in \mathbb{T}, \boldsymbol{\lambda}} \sum_{i \in V} w_i A_i^{\text{p}}, \quad \text{and} \quad A_{\mathcal{D}}^{\text{ave}*} = \min_{\mathcal{T} \in \mathbb{T}, \boldsymbol{\lambda}} \sum_{i \in V} w_i A_i^{\text{ave}}, \tag{3.47}$$

where A_i^{p} denotes peak age and A_i^{ave} denotes the average age of terminal i. Their evolution is given by (3.2). For convenience, we have omitted their explicit dependence on $\mathcal{T} \in \mathbb{T}$ and $\boldsymbol{\lambda}$.

Motivated by the results for the information gathering problem, we consider randomized trajectories. Note that an arriving update in queue i has service time equal to the inter-visit times to ground terminal i, provided the update arrived when the queue i was not-empty; $\mathcal{Q}_i(t) \neq \emptyset$. However, when an update arrives to an empty queue i, the time to delivery is not the inter-visit time, and depends on the location of the mobile agent at the time of arrival.

Since the analysis of age for such a queueing system may be difficult, we provide an upper bound, by comparing the the ith queue with a discrete time Ber/G/1 queue with vacations: whenever the ith queue is empty pretend that it goes on a vacation, with vacation times having the same distribution as inter-visit time; otherwise the service times for the queue are just inter-visit times. Clearly, the age process of such a FCFS queue is an upper bound for the age process $A_i(t)$. Thus, we upper bound the peak age A_i^{p} and average age A_i^{ave}, by the peak and average age of this Ber/G/1 queue with vacations. We first analyze peak and average age of a Ber/G/1 queue with vacations.

3.3.1 Age for Ber/G/1 Queue with Vacations

Consider a discrete time FCFS Ber/G/1 queue with vacations, where an arrival occurs with probability λ, the service times S are generally distributed with mean $\mathbb{E}[S] = 1/\mu$, and the vacation times V are also generally distributed.

We obtain an expression for the peak age of a discrete time Ber/G/1 queue with vacations, and a bound on average age using Theorem 1.

Lemma 3. *The peak age for a discrete time FCFS Ber/G/1 queue with vacations is given by*

$$A^p = \frac{1}{\lambda} + \frac{1}{\mu} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{\mathbb{E}[V^2]}{2\mathbb{E}[V]} - \frac{3}{2}, \tag{3.48}$$

where $\rho = \frac{\lambda}{\mu}$, while the average age is upper-bounded by peak age, namely $A^{ave} \leq A^p + \frac{1}{2}$.

Proof. The continuous peak age for the FCFS queue is given by

$$A^{\mathrm{p}}_{\mathrm{cont.}} = \mathbb{E}[T + X], \tag{3.49}$$

where T denotes the time an update spends in the queue and X is the inter-arrival time between two updates. Given that vacation times are distributed i.i.d according to random variable V, we have

$$\mathbb{E}[T] = \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{1}{\mu} + \frac{\mathbb{E}[V^2]}{2\mathbb{E}[V]} - \frac{1}{2}, \tag{3.50}$$

where S denotes the service time distribution. Substituting this and $\mathbb{E}[X] = \frac{1}{\lambda}$ in (3.49), we obtain the expression for peak age.

Derivation of System Time

The proof is a discretized version of the proof for M/G/1 queues with vacations using residual service times as discussed in [70].

Let us define the residual service time for an update at time t, given by $R(t)$, as the amount of time remaining until the update currently at the head of the queue is complete, excluding the current time-slot. If the queue is empty, $R(t)$ equals zero.

From [70] we know that the expected waiting time in the queue can be found using the residual service times as follows

$$\mathbb{E}\left[T_Q\right] = \frac{\mathbb{E}\left[R\right]}{1-\rho}, \tag{3.51}$$

where $\rho = \frac{\lambda}{\mu}$, $\mathbb{E}\left[S\right] = \frac{1}{\mu}$ and $\mathbb{E}\left[R\right] = \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{t=T} R(t)\right]$. As in [70], $\mathbb{E}\left[R\right]$ can be computed using a graphical argument. Let service times for the mth packet be X_m, and let the kth vacation time be V_k. Let the total number of packets served be $M(T)$ and the total number of vacations be $L(T)$, over the entire time-horizon T. Then, we have

$$\frac{1}{T}\sum_{t=0}^{t=T} R(t) = \frac{1}{2}\frac{M(T)}{T}\frac{\sum_{m=1}^{m=M(T)}(X_m^2 - X_m)}{M(T)} + \frac{1}{2}\frac{L(T)}{T}\frac{\sum_{k=1}^{k=L(T)}(V_k^2 - V_k)}{L(T)}. \tag{3.52}$$

Using the strong law of large numbers and the fact that $\frac{M(T)}{T} \to \lambda$ and $\frac{L(T)}{T} \to \frac{(1-\rho)}{\mathbb{E}[V]}$, we get

$$\mathbb{E}\left[R\right] = \frac{\lambda(\mathbb{E}\left[S^2\right] - \mathbb{E}\left[S\right])}{2} + \frac{(1-\rho)(\mathbb{E}\left[V^2\right] - \mathbb{E}\left[V\right])}{2\mathbb{E}\left[V\right]}. \tag{3.53}$$

Combining (3.51) and (3.53), we get

$$\mathbb{E}\left[T_Q\right] = \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{1}{2}. \tag{3.54}$$

The total time spent in the system by a packet is given by the sum of its waiting time in the queue and its processing time, which implies

$$\mathbb{E}\left[T\right] = \mathbb{E}\left[S + T_Q\right] = \frac{1}{\mu} + \frac{\lambda\mathbb{E}[S^2] - \rho}{2(1-\rho)} + \frac{\mathbb{E}\left[V^2\right]}{2\mathbb{E}\left[V\right]} - \frac{1}{2}, \tag{3.55}$$

since $\mathbb{E}\left[S\right] = \frac{1}{\mu}$.

Average Age

Consider a $Ber/G/1$ queue with vacations that has i.i.d. packet inter-arrival times $X_1, X_2, ...$ Let T_n be the total time spent in the system by the n^{th} packet. Then, the continuous average age is given by [1]:

$$A^{\text{ave}}_{\text{cont.}} = \frac{1}{\lambda} - \frac{1}{2} + \lambda\mathbb{E}[X_n T_n], \tag{3.56}$$

where $\frac{1}{\lambda} = \mathbb{E}[X_n]$. To evaluate the term $\mathbb{E}[X_n T_n]$, we observe that larger inter-arrival times X_n between packets mean lesser wait times in the system T_n for individual packets. Thus, X_n and T_n are negatively correlated. Note that for negatively correlated random variables the following holds

$$\mathbb{E}\left[X_n T_n\right] \leq \mathbb{E}\left[X_n\right]\mathbb{E}\left[T_n\right]. \tag{3.57}$$

This implies

$$A^{\text{ave}} \leq \frac{1}{\lambda} - \frac{1}{2} + \lambda\mathbb{E}[X_n]\mathbb{E}\left[T_n\right] = \mathbb{E}\left[X_n\right] + \mathbb{E}\left[T_n\right] - \frac{1}{2} = A^{\text{p}} + \frac{1}{2}, \tag{3.58}$$

since $\mathbb{E}[X_n] = 1/\lambda$. □

3.3.2 Age Minimization Problem

Using Lemma 3, we now obtain an upper-bound on both, network peak and average age, for a given randomized trajectory $\mathbf{P}$ and update generation rates $\boldsymbol{\lambda}$.

Lemma 4. *For a randomized trajectory $\mathbf{P}$ and packet generation rates $\boldsymbol{\lambda}$, the peak and average age for a ground terminal i is upper-bounded by*

$$A_i^{UB} = \frac{1}{\pi_i}\left[1 + z_{ii} + \frac{1}{\rho_i} + \frac{z_{ii}\rho_i}{1-\rho_i}\right] - \frac{\rho_i}{1-\rho_i} - \frac{1}{2}, \tag{3.59}$$

for all $i \in V$, where π is the unique stationary distribution of $\mathbf{P}$, $Z = (I - \mathbf{P} + \Pi)^{-1}$, Π is a matrix with all rows equal to the stationary distribution vector π, and $\rho_i \triangleq \frac{\lambda_i}{\pi_i}$.

Proof. See Appendix A.2. □

We propose a policy, i.e. a randomized trajectory $\mathbf{P}$ and update generation rate $\boldsymbol{\lambda}$, that minimizes the age upper-bound $A^{\text{UB}} = \sum_{i \in V} w_i A_i^{\text{UB}}$:

Definition Separation Principle Policy

1. Mobile agent follows the randomized trajectory $\mathbf{P}^*$ obtained by solving (3.39).

2. Generate updates for the ground terminal i at rate

$$\lambda_i^* = \frac{\pi_i^*}{1 + \sqrt{z_{ii}^* - \pi_i^*}}, \tag{3.60}$$

where $\pi_i^* = \frac{\sqrt{w_i}}{\sum_{j \in V} w_j}$ and z_{ii} are diagonal elements of the matrix $Z = (I - \mathbf{P}^* + \Pi^*)^{-1}$.

We call it the separation principle policy for two reasons. Firstly, $\mathbf{P}^*$ is the fastest-mixing randomized trajectory, which we proposed for minimizing average age in the information gathering problem. Secondly, the update generation rate for the ground terminal i, depends only on z_{ii} and π_i, which are functions of the first and second moments of the return times to terminal i under trajectory $\mathbf{P}^*$:

$$\mathbb{E}\left[H_i\right] = \frac{1}{\pi_i} \text{ and } \mathbb{E}\left[H_i^2\right] = -\frac{1}{\pi_i} + \frac{2z_{ii}}{\pi_i},$$

where H_i denotes the return time to terminal i, starting from i, under the fastest mixing randomized trajectory $\mathbf{P}^*$. We now bound the performance of this separation principle policy.

Theorem 13. *The peak and average age of the separation principle policy is bounded by*

$$\frac{A^p}{A_{\mathcal{D}}^{p*}} \leq 4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2 \text{ and } \frac{A^{ave}}{A_{\mathcal{D}}^{ave*}} \leq 8\mathcal{H} + 8\sqrt{\mathcal{H}} + 4,$$

where $\mathcal{H}$ is the mixing time of the randomized trajectory $\mathbf{P}^*$.

Proof. We formulate the upper bound age minimization problem and use an approach similar to Lemma 2 and Theorem 7. We want to solve the upper bound age minimization problem, which can be stated as:

$$\begin{aligned} \underset{\mathbf{P},\boldsymbol{\rho}}{\text{Minimize}} \quad & \sum_{i\in V} w_i A_i^{\text{UB}}, \\ \text{subject to} \quad & P_{i,j} \geq 0,\ \forall (i,j), \\ & \mathbf{P1} = \mathbf{1}, \\ & P_{i,j} = 0,\ \forall (i,j) \notin E, \\ & \mathbf{P} \text{ is irreducible.} \end{aligned} \tag{3.61}$$

We first find the optimal packet generation rates given a random walk $\mathbf{P}$. Observe that the optimal queue utilization factors ρ_i can be solved for given any fixed irreducible random walk $\mathbf{P}$, i.e.

$$\rho_i^*(\mathbf{P}) = \arg\min_{\rho_i\in[0,1]} A_i^{\text{UB}}(\mathbf{P},\rho_i) = \frac{1}{1+\sqrt{z_{ii}-\pi_i}} \tag{3.62}$$

and

$$\min_{\rho_i\in[0,1]} A_i^{\text{UB}}(\mathbf{P},\rho_i) = A_i^{\text{UB}}(\mathbf{P},\rho_i^*) = \frac{z_{ii}-\pi_i+2\sqrt{z_{ii}-\pi_i}+2}{\pi_i}. \tag{3.63}$$

Thus, the upper bound age minimization problem reduces to

$$\begin{aligned}
\underset{\mathbf{P}}{\text{Minimize}} \quad & \sum_{i\in V} w_i\left(\frac{z_{ii}-\pi_i+2\sqrt{z_{ii}-\pi_i}+2}{\pi_i}\right), \\
\text{subject to} \quad & P_{i,j}\geq 0,\ \forall (i,j), \\
& \mathbf{P}\mathbf{1}=\mathbf{1}, \\
& P_{i,j}=0,\ \forall (i,j)\notin E, \\
& \mathbf{P} \text{ is irreducible.}
\end{aligned} \tag{3.64}$$

Now, we can relate the network age upper bound, given a random walk $\mathbf{P}$, to its mixing time $\mathcal{H}$. We assume optimal packet generation rates $\rho_i^*(\mathbf{P})$.

$$\begin{aligned}
\sum_{i\in V} w_i A_i^{\mathrm{UB}}(P,\rho_i^*(P)) &= \sum_{i\in V} w_i\left(\frac{z_{ii}-\pi_i+2\sqrt{z_{ii}-\pi_i}+2}{\pi_i}\right), \\
&\leq \sum_{i\in V} w_i\left(\frac{\mathcal{Z}+2\sqrt{\mathcal{Z}}+2}{\pi_i}\right), \\
&\leq \sum_{i\in V} w_i\left(\frac{4\mathcal{H}+4\sqrt{\mathcal{H}}+2}{\pi_i}\right),
\end{aligned}$$

where inequalities follow from the same argument as in the proof of Lemma 2. Setting $\mathbf{P}=\mathbf{P}^*$, we obtain

$$\sum_{i\in V} w_i A_i^{\mathrm{UB}}(\mathbf{P}^*,\rho_i^*(\mathbf{P}^*)) \leq \sum_{i\in V} w_i\left(\frac{4\mathcal{H}+4\sqrt{\mathcal{H}}+2}{\pi_i^*}\right), \tag{3.65}$$

where $\mathcal{H}$ is the mixing time of P^*. Note that $\sum_{i\in V}\frac{w_i}{\pi_i^*}$ is the optimal peak age in the information gathering problem, i.e. $A_{\mathcal{G}}^{\mathrm{p}*}=\sum_{i\in V}\frac{w_i}{\pi_i^*}$. This gives,

$$\frac{A^{\mathrm{UB}}(\mathbf{P}^*,\boldsymbol{\rho}^*)}{A_{\mathcal{G}}^{\mathrm{p}*}} \leq 4\mathcal{H}+4\sqrt{\mathcal{H}}+2. \tag{3.66}$$

Due to the presence of queues we have $A_{\mathcal{G}}^{\mathrm{p}*} \leq A_{\mathcal{D}}^{\mathrm{p}*}$. This, (3.66), and the fact that $A^{\mathrm{p}}(\mathbf{P}^*, \boldsymbol{\rho}^*) \leq A^{\mathrm{UB}}(\mathbf{P}^*, \boldsymbol{\rho}^*)$, yields the peak age bound on the separation principle policy:

$$\frac{A^{\mathrm{p}}(\mathbf{P}^*, \lambda^*)}{A_{\mathcal{D}}^{\mathrm{p}*}} \leq 4\mathcal{H} + 4\sqrt{\mathcal{H}} + 2,$$

since $\boldsymbol{\rho}^* = \boldsymbol{\lambda}^*$.

From the discussion following Theorem 9, we know that $2A_{\mathcal{G}}^{\mathrm{ave}*} \geq A_{\mathcal{D}}^{\mathrm{p}*}$. Also, $A_{\mathcal{G}}^{\mathrm{ave}*} \leq A_{\mathcal{D}}^{\mathrm{ave}*}$ and $A^{\mathrm{ave}}(\mathbf{P}^*, \boldsymbol{\rho}^*) \leq A^{\mathrm{UB}}(\mathbf{P}^*, \boldsymbol{\rho}^*)$. Combining these with (3.66) gives us

$$\frac{A^{\mathrm{ave}}(\mathbf{P}^*, \lambda^*)}{A_{\mathcal{D}}^{\mathrm{ave}*}} \leq 8\mathcal{H} + 8\sqrt{\mathcal{H}} + 4, \tag{3.67}$$

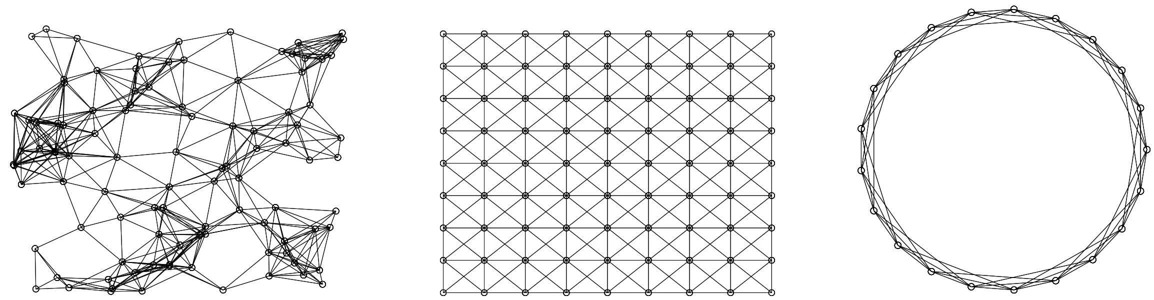
since $\boldsymbol{\rho}^* = \boldsymbol{\lambda}^*$. □

The separation principle policy is factor $O(\mathcal{H})$ peak age and average age optimal. It is worthwhile to note that a similar separation principle policy was established in a completely different setting of scheduling links for age minimization in [15]. Theorem 13 generalizes that result to a graph.

3.4 Simulation Results

We test the performance of our proposed trajectories on three different kinds of mobility graphs: random geometric graphs $\mathcal{G}(n, \frac{2}{\sqrt{n}})$,[2] grid graphs with diagonal edges, and 3-connected ring or cycle graphs; see Figure 3-4. We use n to denote the number of ground terminals, namely $n = |V|$. For the age-based policy, we set the function $g(a) = a^2 + a$, inspired by the index based policies in [15]. Link weights are picked uniformly at random from the interval $(1, 2]$ in an independent manner.

[2]Setting $r = \frac{2}{\sqrt{n}}$ for random geometric graphs ensures connectivity w.h.p.



(a) (b) (c)

Figure 3-4: (a) A random geometric graph with 100 nodes, (b) A grid graph with 81 nodes and diagonal edges, and (c) A 3-connected ring or cycle graph with 21 nodes.

We run our simulations for a total of 50000 time-slots, to get a good estimate of the peak and average age.

We consider the information gathering problem, and plot peak and average age for all the proposed trajectories of the mobile agent: the Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$, fastest mixing randomized trajectory $\mathbf{P}^{*}$, and age-based trajectory. Figure 3-5 plots peak age as a function of network size n for the random geometric graph $\mathcal{G}\left(n, 2/\sqrt{n}\right)$. We observe that the peak age for all the three proposed trajectories match. We know from Theorems 7 and 10 that that the two randomized trajectories, namely, the Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$ and the fastest mixing randomized trajectory $\mathbf{P}^{*}$, are both peak age optimal. Figure 3-5, therefore, suggests that even the age-based trajectory for the mobile agent is peak age optimal.

In Figure 3-6 we plot the average age performance of the proposed trajectories, as a function of network size n. Also plotted is the lower bound for average age derived in Theorem 9. We see that the age-based policy is nearly average age optimal, while the fastest mixing randomized trajectory $\mathbf{P}^{*}$ performs slightly better than the

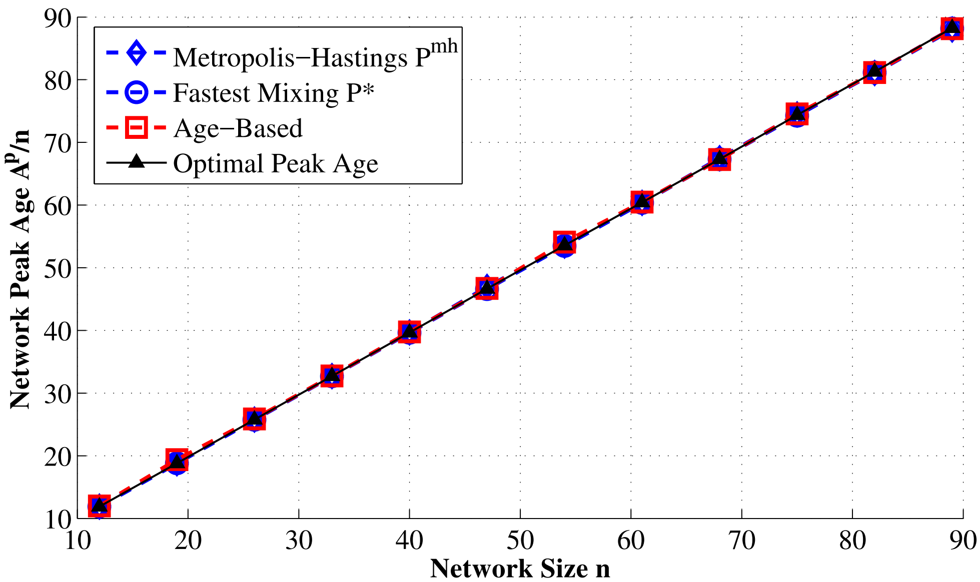


Figure 3-5: Information gathering problem in $\mathcal{G}(n, 2/\sqrt{n})$: network peak age as a function of network size n for several proposed trajectories of the mobile agent.

Metropolis-Hastings randomized trajectory $\mathbf{P}^{\text{mh}}$.

Theorem 11 proved that the fastest mixing randomized trajectory $\mathbf{P}^*$ is at least factor-$8\mathcal{H}$ optimal. Figure 3-6 validates this conclusion: for example, for $n = 90$ ground terminals, the average age for the fastest mixing randomized trajectory $\mathbf{P}^*$ is approximately a factor 3 away from the lower bound.

In Figures 3-7 and 3-8 we plot the average age performance for several proposed trajectories, as a function of the network size. The age-based policy, again outperforms the two randomized trajectories, and is nearly optimal. We observe that the average age for the fastest mixing randomized trajectory $\mathbf{P}^*$, namely $A^{\text{ave}}(\mathbf{P}^*)$, is much worse in the ring graph than in the grid graph. This is because the mixing time for the ring graph is much larger than for the grid graph.

In Figure 3-9, we simulate the performance of the separation principle policy

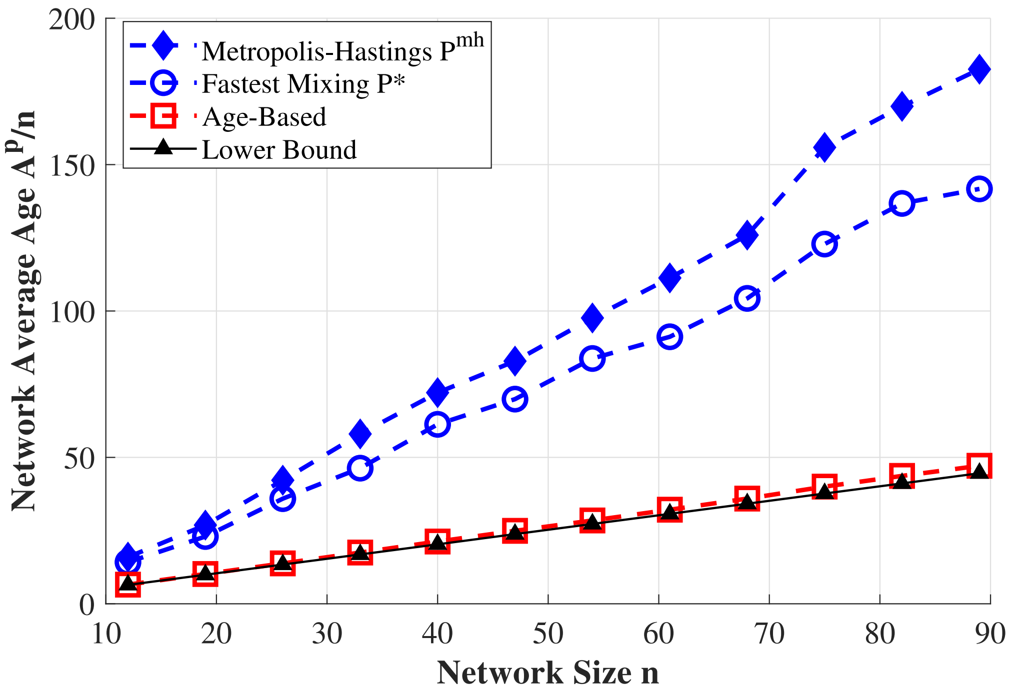


Figure 3-6: Information gathering problem in $\mathcal{G}(n, 2/\sqrt{n})$: network average age as a function of network size n for several proposed trajectories of the mobile agent. We average over 10 random graphs for each value of n.

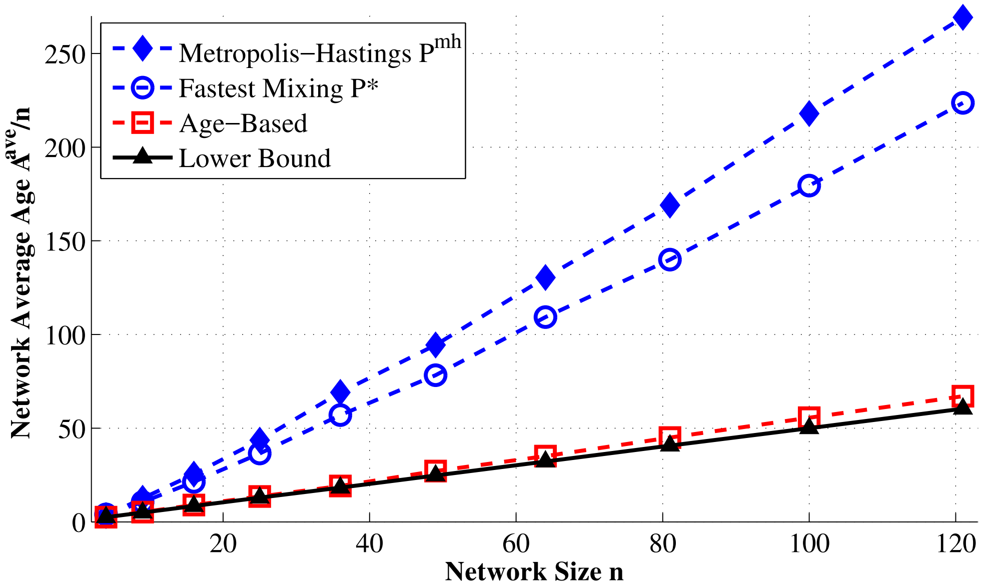


Figure 3-7: Information gathering problem in the Grid graph: network average age as a function of network size n for several proposed trajectories of the mobile agent.

for the information dissemination problem, for graph $\mathcal{G}(n, 2/\sqrt{n})$, and compare its age performance with the information gathering problem. We observe a significant deterioration of age, as a function of network size n, in the information dissemination case in comparison to the information gathering case. This, we note, is the cost of uncontrollable queues in the system on age performance.

3.5 Conclusions and Future Work

We considered the trajectory planning problem for a mobile agent, that traverses through a mobility graph G, to help timely exchange of information updates between a central terminal and a set of ground terminals V. In the information gathering problem, we showed that a randomized trajectory, namely the fastest-mixing

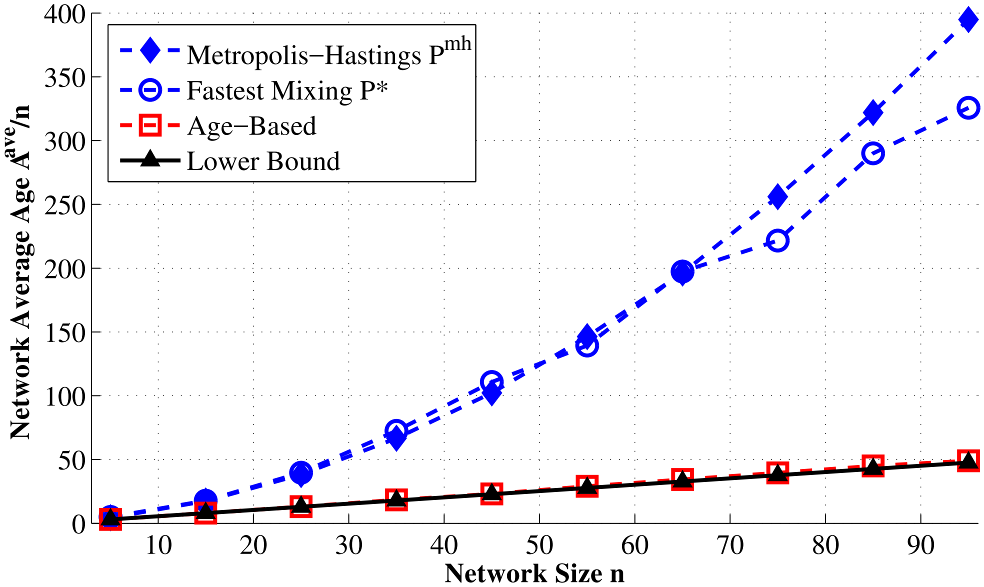


Figure 3-8: Information gathering problem in the Ring graph: network average age as a function of network size n for several proposed trajectories of the mobile agent.

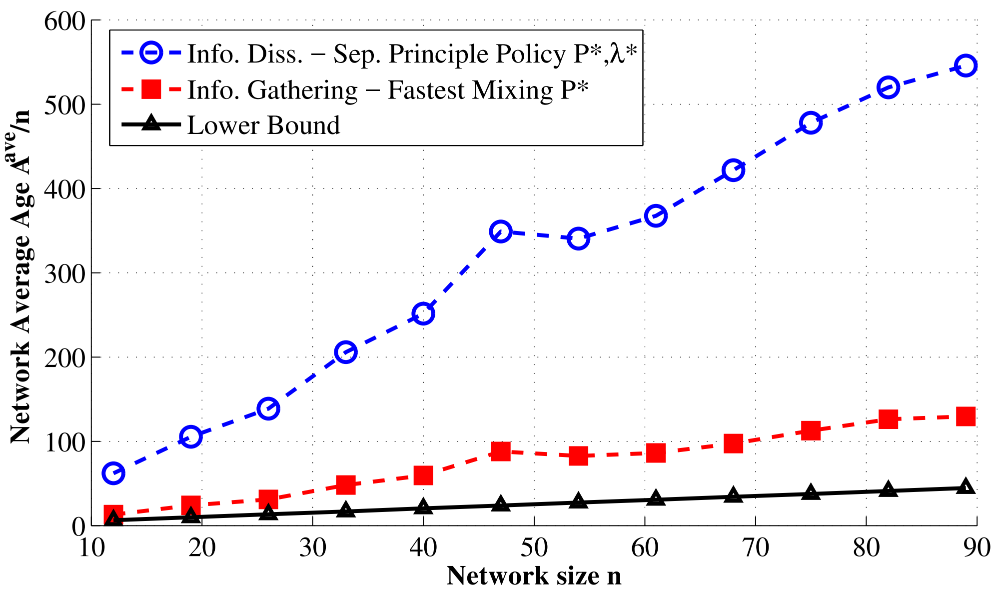


Figure 3-9: Network average age as a function of network size

randomized trajectory, is peak age optimal and factor-$\mathcal{H}$ average age optimal. We showed that obtaining an average age optimal trajectory can be NP-hard, while we constricted the peak age optimal trajectory in polynomial time. To improve the average age, we proposed an age-based policy, and showed it to be factor-2 average age optimal, in a symmetric setting. In the information dissemination problem, we proposed a separation principle policy, in which the mobile agent follows the fastest mixing randomized trajectory with a simple rate control. We proved that the separation principle policy is factor-$O(\mathcal{H})$ optimal, in both peak and average age.

We plan to extend our work with results for the age-based policy with general weight configurations. We also plan to look at graphs with edge-weights which represent distances or travel times between nodes, in contrast to our setting where we assume that the travel time between neighboring nodes is always equal to a single time-slot. Another interesting extension is to consider age optimal gathering or dissemination of information using multiple mobile agents in a similar graph based setting and understanding age of information in mobile ad-hoc networks in general. This motivates the second part of this thesis.

Chapter 4

AoI in Mobile Ad-hoc Networks

4.1 Single Source Model

We consider N mobile nodes moving across a torus of unit area divided into C cells of equal size.

i.i.d. Mobility - The nodes move in an i.i.d. manner across time-slots, i.e. any node is equally likely to be found in any of the cells at any given time-slot, independent of the positions of other nodes.

There is a source cell in which phenomenon of interest is taking place, and a destination cell which wants to keep track of what is happening at the source. Alternatively, all the mobile nodes want to keep track of what is happening at the source.

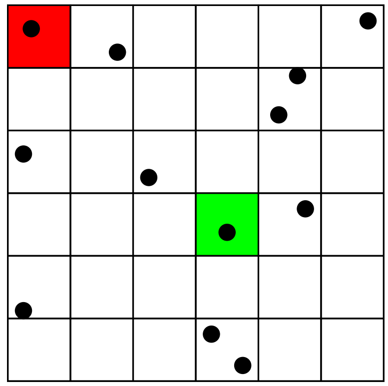


Figure 4-1: In this example $N = 12$ and $C = 36$. The red cell is the source, while the green cell is the destination, and the dots represent locations of mobile nodes at a particular time instant.

Cell Partitioned Network - Whenever a node enters the source cell, it samples the phenomenon happening there (receives a packet), for broadcasting in a later time-slot. Within each cell, if a node transmits a packet, every other node in that cell receives it correctly in the same time-slot. There is no interference between nodes transmitting in different cells. The destination receives packets from the mobile nodes, when they are in the destination cell.

A natural way to measure the freshness of information at the destination about the source is the Age of Information $A(t)$, defined as the number of time-slots ago the current information at the destination was generated at the source. Similarly, we define $A_i(t)$ as the number of time-slots ago the current information at node i was generated at the source. The expected long term average at the destination can then be expressed as

$$AoI = \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} A(t)\right]. \tag{4.1}$$

In this work, we find the optimal policy for transmission of packets in the above model that minimizes AoI at the destination. We also provide upper and lower bounds on expected long term average age of the system under different user node densities and the optimal policy.

4.1.1 Policy

The optimal packet broadcast policy in this setting is obvious. In every cell, we allow the node with the freshest information about the source to broadcast to every other node in the cell. The fact that this policy achieves minimum age at the destination can be proven by a sample path type argument. Consider a node and look at its age with respect to the source. In every time-slot, it receives the freshest information it can possibly receive, based on its location, by using the above policy. Thus, under this policy, any node delivering packets to the destination has the lowest possible age with respect to the source and so the destination also has minimum possible age with respect to the source. Observe that this result is independent of the mobility model of the underlying nodes. This is a crucial result that, as we will see, does not necessarily hold when we have multiple sources and destinations.

4.1.2 Age Analysis

We now want to analyze expected long term average age at the destination under the optimal policy given N agents, C cells and i.i.d. mobility. For this, we will use a result from [71] on age of information under random updates. Here, we state the result without proof. See [71] for details

Lemma 5. *Consider a source that randomly generates packets with i.i.d. packet inter-generation times* $X_1, X_2, \ldots$ *and a network cloud that delivers these packets* *to the destination with i.i.d. service times* $S_1, S_2, \ldots$. *Then, the following holds*

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} A(t)\right] \leq \frac{\mathbb{E}[X_1^2] + 2\mathbb{E}[X_1]\mathbb{E}[S_1]}{2\mathbb{E}[X_1]}. \tag{4.2}$$

Further, if packet generation times are geometrically distributed, the above equation reduces to

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} A(t)\right] \leq \mathbb{E}[X_1] + \mathbb{E}[S_1]. \tag{4.3}$$

Comparing the setting described in Lemma 5 to our model - the packet generation times correspond to time-slots in which some node visits the source cell and the network cloud is the set of N mobile nodes delivering packets to the destination.

Lemma 6. *The packet inter-generation times in our model are i.i.d. and geometrically distributed with parameter* $p = 1 - (1 - \frac{1}{C})^N$.

Proof. Consider the event E_t that some node visits the source cell at time-slot t. Then, by i.i.d. mobility, we have that

$$\mathbb{P}(E_t) = 1 - (1 - \frac{1}{C})^N, \forall t$$

and events E_t are independent for all t. Thus, inter-visit times to the source cell are geometrically distributed with the required parameter. These inter-visit times can also be viewed as hitting times to the source cell given N nodes with i.i.d.

mobility. □

We also note that for every packet picked up on a visit to the source cell, a packet which is at least as fresh is guaranteed to be delivered to the destination in a random time interval that is independent of corresponding inter-generation times. This will function as our service times analogue in Lemma 5.

For a packet s that is picked up by some node on a visit to the source cell at time-slot t, we define the following two time-intervals -

1. F_s = Flooding Time = the first time-instance since time-slot t until every mobile node has a copy of the packet picked up at time t or a fresher packet under the optimal policy, and

2. H_s = Hitting Time = the first time some node visits the destination, starting from time-slot $t + F_s + 1$.

Given these quantities, we have the following result

Lemma 7. *For a visit to the source cell that picks up packet s, a packet that is at least as fresh as the packet s, is guaranteed to be delivered to the destination in at-most $F_s + H_s$ time-slots after pickup, where F_s and H_s are random variables as defined above.*

Again, using independence across time-slots, these upper bound packet service times are i.i.d. and independent of corresponding packet generation intervals. Putting together the results from Lemmas 5, 6, and 7 we have an upper bound on the expected long term average age at the destination under the optimal policy.

Lemma 8. *Given the model described in section 4.1, with the optimal broadcast policy described in section 4.1.1, we have*

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} A(t)\right] \leq 2\mathbb{E}[H] + \mathbb{E}[F], \tag{4.4}$$

where H is the hitting time to the source/destination as described earlier, and F is the single packet flooding time in a system of N nodes and C cells with single packet broadcast allowed in every cell in every time-slot.

Proof. Observe that packets are picked up in i.i.d. intervals of time $H_1, H_2, H_3, ...$ distributed according to the hitting time required to reach the source. These packets are delivered in time that is upper bounded by i.i.d. intervals $F_1 + H'_1, F_2 + H'_2, F_3 + H'_3, ...$ where F_i is the time to flood the i^{th} packet and H'_i is the hitting time to the destination once every node has the i^{th} packet.

Using Lemma 6, $H_1, H'_1, H_2, H'_2, ...$ are i.i.d. geometric random variables. Now, we can apply the extension of Lemma 5 for geometric inter-arrivals to get the required result. □

We can also find an upper bound for the maximum age across all mobile nodes with respect to the source (the broadcast setting).

Lemma 9. *Given the model described in section 4.1, with the optimal broadcast policy described in section 4.1.1, we have -*

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} \max_{i\in[N]} A_i(t)\right] \leq \mathbb{E}[H] + \mathbb{E}[F], \tag{4.5}$$

where H is the hitting time to the source as defined earlier, and F is the single packet flooding time in a system of N nodes and C cells with single packet broadcast allowed in every cell in every time-slot.

Proof. The key idea is that we can drop the extra time required to hit the destination after the flooding process in our proof of Lemma 8. The result follows automatically. □

These results allow us to find upper bounds for age, provided expressions for hitting times and flooding times are available in such a setting. We will consider three different scaling regimes of N and C and derive upper bound on age for these regimes.

Constant Density - Consider the setting when N/C is a constant as N goes to infinity, i.e. the density of mobile nodes per cell remains constant. Such a setting has been considered in detail in [51, 55]. In both these works, the authors show that the expected flooding time for a single packet in the broadcast network is $O(\log(N))$.

On the other hand, using Lemma 6, hitting time is a geometric random variable with parameter $1-(1-1/C)^N$. Thus, we have

$$\mathbb{E}[H] = \frac{1}{1-(1-\frac{1}{C})^N} = \Theta(1), \text{ and}$$
$$\mathbb{E}[F] = O(\log(N)), \text{ as } N \to \infty$$

Combining the above with Lemma 8, we get the following result

Lemma 10. *For the single source model, when $N \to \infty$ and N/C is a constant, we have*

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} \max_{i\in[N]} A_i(t)\right] = O(\log N). \tag{4.6}$$

Sparse Regime - We consider the sparse regime from [55], where $C = \Theta(N^\alpha)$ and $\alpha > 1$. We call this the sparse regime since the the number of cells grows faster than the number of mobile nodes, so the density of mobile nodes goes to zero as $N \to \infty$. Using Lemma 6 to find the hitting time, and results from [55] to find a bound for the flooding time, we get

$$\mathbb{E}[H] = \frac{1}{1-(1-\frac{1}{C})^N} = O(N^{\alpha-1}), \text{ and}$$
$$\mathbb{E}[F] = O(N^{\alpha-1}\log(N)), \text{ as } N \to \infty$$

See Appendix A.3 for a proof of the hitting time order analysis. Combining the results above with Lemma 8, we get the following result

Lemma 11. *For the single source model, when $N \to \infty$ and $C = \Theta(N^\alpha)$ with $\alpha > 1$, we have*

$$\lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{t=T} \max_{i\in[N]} A_i(t)\right] = O(N^{\alpha-1}\log N). \tag{4.7}$$

Observe that this is consistent with the constant density case as $\alpha \to 1$.

Dense Regime - We also consider the Dense regime from [55], where $C = \Theta(N^\alpha)$ and $\alpha \in (0, 1)$. We call this the dense regime since the the number of cells grows slower than the number of mobile nodes, so the density of mobile nodes goes to infinity as $N \to \infty$. Using Lemma 6 to find the hitting time, and results from [55] to find a bound for the flooding time, we get

$$\mathbb{E}[H] = \frac{1}{1 - (1 - \frac{1}{C})^N} = O(1), \text{ and}$$
$$\mathbb{E}[F] = O(\log(\log(N))), \text{ as } N \to \infty$$

See Appendix A.3 for a proof of the hitting time order analysis. Combining the results above with Lemma 8, we get the following result

Lemma 12. *For the single source model, when $N \to \infty$ and $C = \Theta(N^\alpha)$ with $\alpha \in (0, 1)$, we have*

$$\lim_{T \to \infty} \mathbb{E}\left[\frac{1}{T} \sum_{t=1}^{t=T} \max_{i \in [N]} A_i(t)\right] = O(\log(\log(N))). \tag{4.8}$$

Observe that in all the three regimes that we discussed, our age upper bound is dominated in an order sense by the flooding time. That is, the flooding time is a good approximation bound for age. We confirm this with simulations in Section 4.3.

4.2 Multiple Sources

We extend the model from previous sections to a setting with multiple sources. Consider a cell partitioned communication model with C cells on a unit torus and

N agents moving across cells in an i.i.d manner. There are K source cells, which generate new status updates in every time-slot. If at any time-slot t, an agent i is in a source cell k, it receives a fresh packet from the source and its age $A_i^k(t+1)$ drops to 1. Apart from direct updates received when agents are close to data sources, they can communicate with other agents in the same cell. In every time-slot, a single packet can be broadcast in every cell, and every agent within the cell receives it correctly. The corresponding ages drop to the age of the broadcast packet.

Our goal is to find the time average age optimal broadcast policy in this mobile ad-hoc network setting. Note that this is an especially challenging task due to the changing connectivity between nodes as well as the multi-hop nature of communication. Minimizing AoI in a fixed multihop reliable network still remains an unsolved problem. So, we provide a simple greedy policy motivated by the symmetry in the our system and numerically compare its performance to other policies.

4.2.1 Policy

Local One-step Greedy Policy In every cell and in every time-slot, broadcast the packet the packet which leads to maximum immediate drop in total age in the next time-slot.

To prove that this policy is optimal, we need to show that the dynamic programming recursion in our problem simplifies to a series of one-step greedy maximization procedures. While we cannot prove this for general N or C, we provide an argument for why the greedy policy is indded optimal for the simplest multi-source setting when $N = 2, K = 2$, and $C = 4$.

We look at the simplest non-trivial setting of our model, i.e. $N = 2, K = 2$, and $C = 4$. Clearly, we can construct trajectories for which broadcasting packets

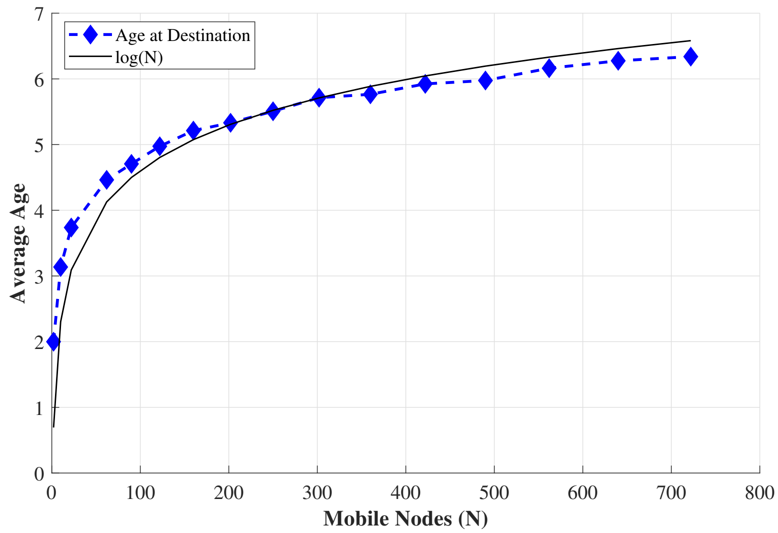


Figure 4-2: Average Age at the destination as a function of number of mobile nodes, for the constant density regime. We fix $N/C = 2.5$ nodes per cell

according to the greedy policy in every time-slot is not optimal. However, for every such "bad" trajectory, we argue that we can create a "mirror trajectory" for which the choosing the greedy action is better by a larger margin. Using the assumption of i.i.d. mobility, all future trajectories are equally likely. Thus, in expectation, broadcasting packets according to the greedy policy is expected average age optimal. Observe that the optimality of the greedy policy relies heavily on the i.i.d. mobility assumption, unlike the single-source case.

4.3 Simulation Results

From Figures 4-2, 4-3, and 4-4, we see that we can get good approximations for average age as a function of the system size for the three regimes as we discussed earlier. For Figure 4-2, we fix the density of mobile nodes to be 2.5 per cell and

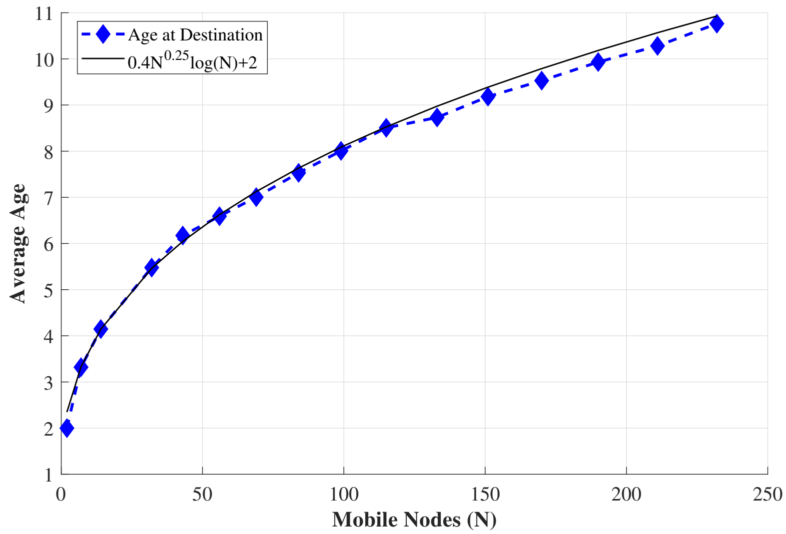


Figure 4-3: Average Age at the destination as a function of number of mobile nodes, for the sparse density regime. We fix $C = \Theta(N^{1.25})$

.

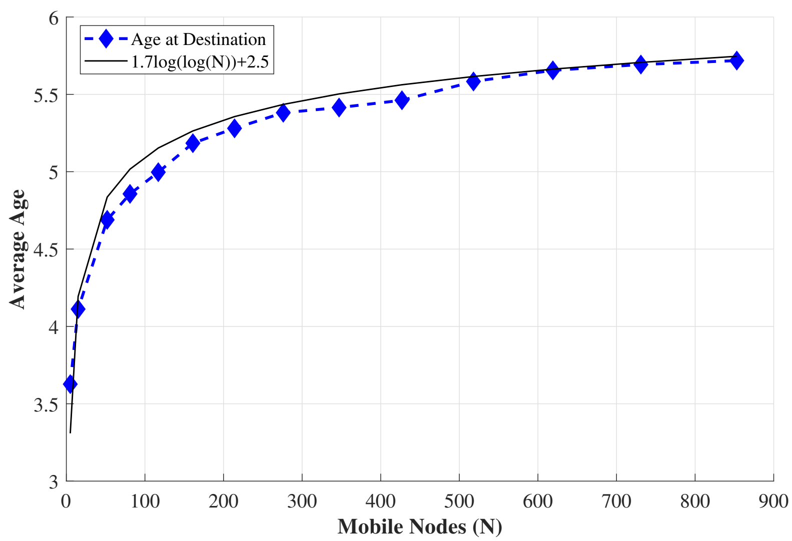


Figure 4-4: Average Age at the destination as a function of number of mobile nodes, for the dense regime. We fix $C = \Theta(N^{0.833})$

.

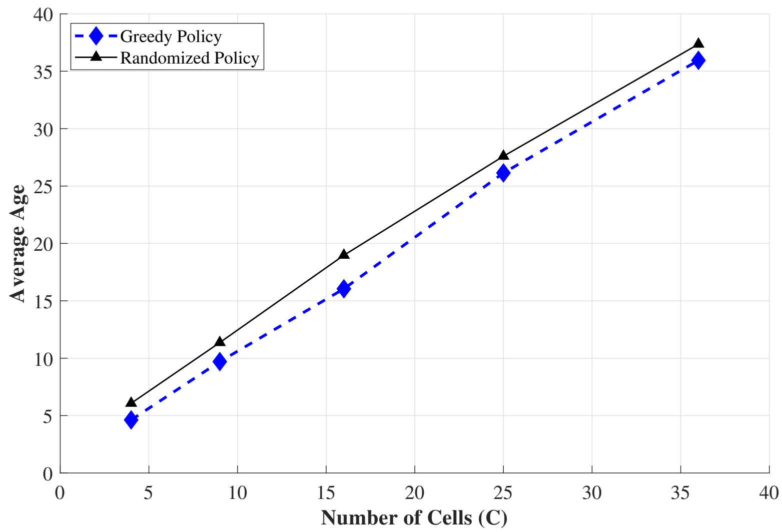


Figure 4-5: Network Average Age as a function of number of cells in the network for the 2 source case. We fix $N = 2$ and compare the greedy policy with a randomized policy.

observe that average age grows logarithmically with N, as expected from our age analysis. For Figure 4-3, we allow C to grow as fast as $N^{1.25}$. Thus, we are in the sparse regime, and observe that average age grows much faster, at a rate of $N^{0.25}\log(N)$. For Figure 4-4, we allow C to grow only as fast as $N^{0.833}$. Thus, we are in the dense regime, and observe that average age grows much slower - at a rate of $\log(\log(N))$.

In Figure 4-5, we compare the one step greedy policy derived for the multi-source setting with a randomized that picks and broadcasts a packet uniformly at random in each cell. We plot average age for 2 sources and 2 mobile nodes as the number of communication cells increases. As expected, the age increases with C for both policies, since the probability to hit the sources or land in a common cell decreases with the number of cells. Also, the greedy policy performs better than the

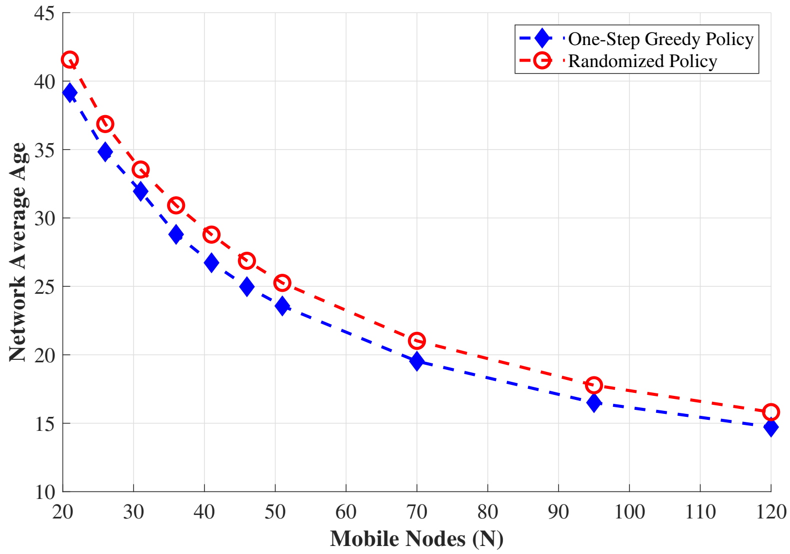


Figure 4-6: Network Average Age as a function of number of mobile nodes in the network. We fix the number of cells $C = 100$ and compare the greedy policy with a randomized policy.

randomized policy, however, the difference is not significant.

In Figure 4-6, we compare the one step greedy policy derived for the multi-source setting with a randomized that picks and broadcasts a packet uniformly at random in each cell. We plot average age by fixing the number of cells to be hundred and putting a source in each cell, as the number of mobile nodes increases. This can be thought of as a model for field sensing using multiple mobile agents. As expected, the age decreases with increasing N for both policies, since the probability to hit sources that haven't been visited, or landing in a common cell, increases with the number of nodes. Also, the greedy policy performs better than the randomized policy, however, the difference is not significant.

4.4 Conclusions and Future Work

In this chapter, we introduced a simple model to study age of information in mobile ad-hoc networks. We studied the scaling of AoI with the size of the network in three different regimes for the single source case, under the optimal packet forwarding policy. We also developed a heuristic one-step greedy policy for the multi-source case and provided numerical results to support our discussion.

The results presented in this chapter are preliminary, and a lot more work needs to be done in understanding age of information in mobile ad-hoc networks. The problem of minimizing weighted sum AoI in a multi-hop network with fixed reliable links remains unsolved, and so does the corresponding problem in multi-hop networks with wireless links and mobility. A better understanding of AoI scaling along with lower bounds for networks with mobility is also an interesting direction of future work.

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Appendix A

A.1 Proof for Average Age in Theorem 2

Consider a Geo/G/1 queue with i.i.d. geometric packet inter-arrival times $X_1, X_2, \ldots$ Let T_n be the total time spent in the system by the n^{th} packet. Then, the average age for the corresponding continuous time system is given by [1]

$$A^{\text{ave}}_{\text{cont.}} = \frac{\mathbb{E}[X_n^2]/2 + \mathbb{E}[X_n T_n]}{\mathbb{E}[X_n]} = \frac{1}{\gamma} - \frac{1}{2} + \gamma \mathbb{E}[X_n T_n], \tag{A.1}$$

where $\frac{1}{\gamma} = \mathbb{E}[X_n]$ and a packet arrives in every time-slot with probability γ. To evaluate the term $\mathbb{E}[X_n T_n]$, we use the following recursion -

$$T_n = \max\{T_{n-1} - X_n, 0\} + S_n, \tag{A.2}$$

where S_n is the service time of the n^{th} packet. Note that T_{n-1} and S_n are independent of X_n. Let $\mathbb{E}[S_n] = \frac{1}{\mu}$ and $\rho \triangleq \frac{\gamma}{\mu}$. Evaluating $\mathbb{E}[X_n T_n]$, we have

$$\begin{aligned} \mathbb{E}[X_n T_n] &= \mathbb{E}[X_n \max\{T_{n-1} - X_n, 0\}] + \mathbb{E}[S_n X_n], \\ &= \sum_{t=1}^{\infty} \mathbb{E}[X_n \max\{t - X_n, 0\}] \mathbb{P}(T = t) + \frac{\mathbb{E}[S]}{\gamma}, \end{aligned} \tag{A.3}$$

where $\mathbb{P}(T = t)$ is the probability mass function of the total time spent by a packet in the system. We need to evaluate the term $\mathbb{E}[X_n \max\{t - X_n, 0\}]$.

$$\begin{aligned}\mathbb{E}[X_n \max\{t - X_n, 0\}] &= \sum_{x=1}^{t} x(t-x)\mathbb{P}(X_n = x), \\ &= \sum_{x=1}^{t} x(t-x)\gamma(1-\gamma)^{x-1}, \\ = \frac{2(1-\gamma)^t - 2}{\gamma^2} &+ \frac{t(1-\gamma)^t - (1-\gamma)^t + t + 1}{\gamma}.\end{aligned} \tag{A.4}$$

Using (A.3) and (A.4), we now compute $\mathbb{E}[X_n T_n]$ as

$$\begin{aligned}\mathbb{E}[X_n T_n] = \frac{2\mathbb{E}[(1-\gamma)^T] - 2}{\gamma^2} + \frac{\mathbb{E}[T(1-\gamma)^T]}{\gamma} + \\ \frac{\mathbb{E}[T] + 1 - \mathbb{E}[(1-\gamma)^T]}{\gamma} + \frac{\mathbb{E}\left[S\right]}{\gamma}.\end{aligned} \tag{A.5}$$

We define $L_T(x) \triangleq \mathbb{E}[x^T]$. Then, $\mathbb{E}[(1-\gamma)^T] = L_T(1-\gamma)$, and $\mathbb{E}[T(1-\gamma)^T] = \frac{d}{dz}L_T(z)\big|_{z=1-\gamma}(1-\gamma)$. Also, from [58], we know that for a Geo/G/1 queue, the probability generating function of T is given by the following equation

$$L_T(z) = \frac{(1-\rho)(1-z)L_S(z)}{(1-z) - \gamma(1 - L_S(z))}. \tag{A.6}$$

Substituting $z = (1-\gamma)$ in the above expression we get

$$\begin{aligned}L_T(1-\gamma) &= 1 - \rho, \text{ and} \\ \frac{d}{dz}L_T(z)\bigg|_{z=1-\gamma} &= \frac{(1-\rho)}{\gamma}\left(\frac{1}{L_s(1-\gamma)} - 1\right).\end{aligned} \tag{A.7}$$

Putting all of these together along with the expression for $\mathbb{E}[T]$, we get

$$A^{\text{ave}}_{\text{cont.}} = \frac{1}{2} + \mathbb{E}[S] + \frac{(1-\gamma)(1-\rho)}{\gamma L_S(1-\gamma)} + \frac{\gamma\mathbb{E}[S^2]-\rho}{2(1-\gamma\mathbb{E}[S])}. \tag{A.8}$$

A.2 Proof of Lemma 4

Consider a randomized trajectory $\mathbf{P}$ and Bernoulli arrival rates $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots)$. From the arguments made in Chapter 3.3, we know that the peak age for the ground terminal i is upper-bounded by the peak age of a discrete time FCFS Ber/G/1 queue with vacations, for which the service times and vacation times have the same distribution as the inter-visit times $H_{1,i}$. Applying Lemma 3 we obtain

$$A^{\text{p}}_i \leq \frac{1}{\pi_i}\left[1 + z_{ii} + \frac{1}{\rho_i} + \frac{z_{ii}\rho_i}{1-\rho_i}\right] - \frac{\rho_i}{1-\rho_i} - 1 \triangleq A^{\text{UB}}_i, \tag{A.9}$$

where we have used the first and second moment of inter-visit times $H_{1,i}$ [62]:

$$\mathbb{E}[H_{1,i}] = \frac{1}{\pi_i},\ \mathbb{E}[H^2_{1,i}] = \frac{-1}{\pi_i} + \frac{2z_{ii}}{\pi_i^2}, \forall i \in V. \tag{A.10}$$

Similarly, we know that the average age for the ground terminal i is also upper-bounded by the average age for the FCFS Ber/G/1 queue with vacations. Using the fact that $A^{\text{ave}} \leq A^{\text{p}} + \frac{1}{2}$ for the Ber/G/1 queue with vacations (see Lemma 3), we also get $A^{\text{ave}}_i \leq A^{\text{UB}}_i$.

A.3 Hitting Time Order Analysis

From Lemma 6, we know that H is a geometric random variable with parameter $1-(1-1/C)^N$. Thus,

$$\mathbb{E}[H] = \frac{1}{1 - (1 - \frac{1}{C})^N}.$$

Now, if $C = \Theta(N^\alpha)$, then there exists some fixed $k > 0$ and $N_0 \geq 1$ possibly dependent on α such that

$$\mathbb{E}[H] \leq \frac{1}{1 - (1 - \frac{k}{N^\alpha})^N}, \forall N \geq N_0$$

For $\alpha \in (0, 1)$, $(1 - \frac{k}{N^\alpha})^N$ converges to zero as N goes to infinity. Thus,

$$\lim_{N \to \infty} \frac{1}{1 - (1 - \frac{k}{N^\alpha})^N} = 0,$$

which implies that

$$\mathbb{E}[H] = O(1), \forall \alpha \in (0, 1).$$

If $\alpha \geq 1$, observe that there exists some $c > 0$ such that for large enough N,

$$\frac{1}{1 - (1 - \frac{k}{N^\alpha})^N} \leq cN^{\alpha - 1}.$$

Thus,

$$\mathbb{E}[H] = O(N^{\alpha - 1}), \forall \alpha \geq 1.$$

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