Application of Option Valuation Techniques in Valuing Petroleum Leases

by

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Submitted to the Department of Civil and Environmental Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Civil Engineering

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

February 1994

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Abstract

Financial appraisal of an oil and gas project is an important issue for an oil company making an investment of capital, labor, and technology in the project. Governments also use valuations to establish the presale value of the reserves and to study the effect of policy changes on the revenues it expects to receive from the sale of the leases. The conventional methods of discounted cash flow analysis present several weaknesses in determining the fair value of such a project. An approach using financial option theory along with a model of market equilibrium of the petroleum reserves is used in determining the value of leases of petroleum properties. This approach is first tested by valuing an oversimplified description of an actual OCS lease. A sensitivity analysis of the different parameters used in the valuation is performed in order to determine those variables that are most sensitive in the valuation. The political risk faced by oil companies investing internationally is also viewed with an option's approach. This valuation method is then used in valuing an offshore petroleum project in Kazakhstan on the northeastern coast of the Caspian Sea. This joint exploration and development of the Tengiz oil field is being undertaken by the U.S. oil company Chevron and Tengizneftegas, which is Kazakhstan's national oil company. The thesis will study the validity of using this approach in both the cases and will examine the many theoretical and practical issues involved in extending financial option pricing theory to valuing real options.

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Acknowledgments

I would like to thank my thesis supervisor Dr. Massood Samii for his patience, guidance and encouragement during the course of this thesis. His numerous comments, suggestions and corrections has greatly improved the substance and presentation of this thesis.

My thanks to all my friends and colleagues at the Volpe Center and especially the TIMS Group for the enriching experience of working with them during my stay at MIT. It was a great privilege to have known and worked with all of you.

I owe a substantial debt to all my friends who have cheered me on and have provided a willing ear and given me valuable advise during my education at MIT. A special thanks to Jalaj and Hasit Dani, Milos and Jana Hauskrecht, Francesco Duina, Gaurav Rewari, Venugopal Nair, Sougata Dutta, Anand Shroff, Safala Prabhu, and Murali Vemulapati for being such wonderful people.

On the personal side, my greatest debt and gratitude go to my parents and my sister Gulrez whose belief in me, encouragement and love have been the driving forces throughout my education and life. I dedicate this thesis to my parents Mushira, and Khuzem Shikari.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 General

Petroleum continues to be the lifeblood that feeds the arteries of the world's economic, industrial and social development. Crude oil and its companion product, natural gas, supply almost 67 percent of all our energy requirements. The oil industry in size and structure is bigger and in operations more international than any other industry in the world. Oil is a distinct necessity and is indispensable for the operation of industries, the movement of commerce, the facilitation of transportation and is also an essential factor for national defense. Oil however is not uniformly distributed around the world and the various countries of the world can be divided into two divisions, namely 'oil haves' and 'oil have-nots'. There is a high degree of interdependence between these two types of nations. The great bulk of the world oil resources are owned by a few small lesser developed countries. The economies of these nations depend almost fully on their oil revenues and their economic and petroleum policies are governed by their need to maximize the benefits from these revenues. The goal of these less-developed countries or commonly referred to as developing countries is to improve their socio-economic condition, have a higher standard of living, and be part of the modern world. These countries thus rely on the export of primary commodities such as petroleum as their vehicle of development. However, many of these countries lack the technology, capital, skilled labor and market outlets necessary to develop their resources. They require the technical expertise of western oil companies to extract the oil for them. These nations thus give concessions or leases to international oil companies to explore and develop the reserves. A petroleum lease is a contractual arrangement between the owner of a prospective oil reservoir (often a government) and the firms that will explore, develop and produce the oil. It is interesting and important to study the nature and structure of the oil industry, its evolution over time and the changing relationship among its major players, namely the oil producing countries, the oil consuming countries and the international oil companies. It will then be easier to understand the process of valuating such a lease and the complexity involved in the process.

Historical Perspective

The history of private foreign investment in petroleum dates back to the turn of the century. Britain and Germany were anxious to gain access to the promising oil reserves of the Middle East. The first petroleum 'concession' - the term used to describe the traditional agreements under which foreign firms had extensive and exclusive rights to exploit the natural resources of a specified area- was granted by the Shah of Persia to William D'Arcy in 1901. The scramble by the European powers to win concessions in the Middle East was mirrored by the activities of American companies in Mexico and Latin America, Russians in Romania, the Dutch in the East Indies, and British firms in India and Burma. Following the close of World War I, American oil interests, supported by the U.S. Department of State, attempted to gain a foothold in the Middle East. The interwar period saw many petroleum concessions granted, as virtually all of the known and suspected oil rich areas of the world were divided into territorial assignments held by the international oil companies. Concessions covering Saudi Arabia, Iran, Iraq, Bahrain, Kuwait, Abu Dhabi, and Qatar were granted during these years. The largest oil firms, the international majors as they came to be known were Exxon, Gulf, Texaco, Mobil, Standard Oil of California(Socal),British Petroleum(BP), Royal Dutch Shell, and Compagnie Francaise de Petroles(CFP) controlled between 95 and 100 percent of the Middle East holdings. The political relationship behind the agreements was analogous to that of colony and colonizer, small territory and great power. The Middle East was more an amalgam of fiercely independent, divided sheikdoms than a region of sovereign states and most of the political control was assumed by Europeans, particularly Britain.

The relationship between the consumer and producer countries institutionalized a flow of trade and a symbiotic relationship that netted inexpensive crude for the companies and previously undreamed riches for the local rulers. The international oil companies had the technology, capital, skilled labor, and market outlets to develop the resources while the Middle Eastern countries having virtually no domestic demand, no necessary inputs to produce oil, and in the face of political and economic instability had little negotiating strength in their dealings with the oil companies.

The Changing Relationship

The concession relationship largely insulated the oil companies from political interference by the leaders of the host governments. However over time the bargaining power of the host governments steadily increased and they have used this strength to increase their revenues and managerial control of such projects. Various countries now introduced income-tax for oil producing firms. The idea of 50-50 profit sharing was adopted in Saudi Arabia in 1950 and soon became standardized throughout the region. The nationalization of Iranian oil was the first real political challenge to the oil companies in the Middle East. Governments of oil producing nations began cooperating with each other as a means of furthering their common interests and goals and the Organization of Petroleum Exporting Countries (OPEC) was formed in 1960. The goal of OPEC was initially modest: restoration of prices which had existed before the reductions posted by the international companies.

Thus the bargaining power of oil companies was the greatest when they enjoyed unchallenged control over the vital inputs - capital and skills needed to search for, produce and market oil. The availability of capital from other sources, the large number of independent oil companies vying for access to crude, the existence of highly specialized companies that provide engineering and other services for petroleum development projects to both the host governments as well as oil firms, for a fee, and the emergence of national oil companies have all contributed to the decline in the market position and pcwer wielded by the international oil companies. With respect to a particular agreement or relationship between a host state and a foreign oil company, the firm's position is thus strongest at the outset. It is at this point that the host country is most dependent on the oil firm to supply the missing or scarce resources needed for production. At this stage however the geological and economic risks are the highest for the firm. As the project proceeds further, there is a tendency for the bargaining strength to shift in the favor of the host government. At this stage, even though most of geological and technological risks have been removed, the firm faces political risk because of the increased bargaining leverage of the host government. All these factors need to be taken into consideration when the oil firm bids for a project to explore and develop reserves in a foreign nation.

1.2 The Problem and its Significance

The appraisal of an oil and gas development project from a financial point of view is an important issue for the oil company making an investment of capital, labor, and technology in the project. Governments also use valuations to establish presale reservation prices and to study the effect of policy changes on revenues it expects to receive from lease sales. Because the bidding process involves billions of dollars, it is important to obtain accurate valuations. The fundamental purpose of any such financial evaluation is to determine whether a project is financially sound and worthwhile and, if it is not, whether and how it can be made so. The valuation of a petroleum lease is a complex capital budgeting exercise that requires the evaluation of a series of decisions which will be made in the light of unfolding information about geological, technological, economic and political risks.

In the conventional discounted cash flow (DCF) analysis, expected future cash flows from the project are estimated, then discounted back to the present and summed to give the net present value (NPV) of the project. In order to determine the cash flows from such a project it would be necessary to specify statistical distributions (not necessarily independent) for the exploration costs, quantities of oil reserves and oil prices and development and operating costs. To complicate matters further the analyst must also make assumptions about the timing of the exploration and development, as well as the rate of extraction. The rate of extraction is also known as the rate of depletion of the resource and it determines the timing and scale of the investment. The analyst must decide at what point the present value of an additional barrel left in the ground exceeds the price it fetches in the market. Since the seminal work of Hotelling(1931) on 'The Economics of Exhaustible Resources', numerous studies and models have been presented on this subject. Once these decisions have been made, the analyst must use the prices, costs, quantities, and timing decisions and determine the time path of the expected cash flows from the project. As typically applied, this DCF approach involves multivariate Monte Carlo simulations. Another important variable needed for this approach is the proper discount rate or opportunity cost of capital. This is the rate at which the future cash flows from the project are to be discounted to their present value and it must reflect the risk associated the project. Since a typical oil development project has several stages, the risk associated with each stage must be dealt with separately and cash flows discounted at their respective rates. Hence a risk analysis of a project entails determining the risks associated with each stage of the project and determining the way these factors are correlated. Many of these risks may be diversified away and it is essential to determine a fair rate of return for the project. Hence summarizing, the weaknesses of this type of approach are:

- The proper timing for investment is not evident. Hence the timing of the flows are typically arbitrary and subject to error.
- different companies and governments may have different assessments of future statistical distributions, and thus the expected path of hydrocarbon prices. This would not necessarily conform to the aggregate expectations held by the capital markets and it would lead to divergent valuations.
- Determining the correct set of discount rates is also a difficult task, in the presence of the statistical structure of the cash flows and the subjectivity of

USGS Estimate	High Industry Bid	Percentage Difference
535.00	44.06	91.76
1813.00	1761.00	2.87
25.00	916.00	3564
4499.00	2119.79	52.88
5939.00	5791.00	2.49
1195.00	1609.03	34.60
612.00	264.08	56.84
530.00	529.54	0.08

Table 1.1: USGS and Industry Lease Evaluation Comparison ($/acre)

the risks involved in the project. Besides different stages of the project have different degrees of risks and thus the timing of the investment will have a direct influence on the net value of the project.

- Multivariate discounted cash flow analysis are often complex and costly.

- Information about the tract is also relativity sparse at the time of bidding and thus the geological and cost distributions can vary from company to company, giving large discrepancies in the respective valuations.

Exacerbating these practical difficulties of evaluating oil leases by the NPV method is the apparent sensitivity of the estimates to relatively minor changes in assumption. This fact can be seen in Table 1 which gives the initial and revised lease estimates of the United States Geological Survey (USGS) and the high bids from the industry for eight tracts leased during the Outer Continental Shelf (OCS) Lease Sale Number 35. The estimates of USGS and the industry are widely divergent and the percentage difference between the industry bid and the USGS estimate as seen below varies from as low as 0.08% to as high as 3500%.

New methods to value petroleum leases include extending financial option pricing theory in valuing leases. As described below, the option valuation technique shows how it is not subject to the first four of the drawbacks listed above. Because it is a purely financial valuation tool, however, the problems and uncertainty associated with improper and inaccurate geological and cost assessments will remain.

1.3 Research Goals and Methodology

The goal of this thesis is to use option valuation theory to develop a new approach to value leases for offshore petroleum. An option is a security that involves the right but not the obligation to buy or sell an asset within a specified period of time. Various contractual claims to real assets have been found to display option-like characteristics which suggests that the Black-Scholes [3] analysis would also be useful in valuing such claims. An option pricing model proposed by Paddock et al [22], which is presented in the thesis demonstrates how to integrate an explicit model of equilibrium in the market for developed petroleum reserves with option-pricing theory to derive the value this real option. This integration is necessary because real options require a deeper understanding of equilibrium in the market for the underlying asset (developed petroleum reserves) than valuing options on financial assets. As mentioned before, any approach to valuing leases on exhaustible natural resources would require a rule specifying when and if a firm should explore and develop a particular leased property. Deriving such an optimal rule is often difficult, especially when using conventional discounted cash flow techniques. Several economic models [26] have been developed to determine the optimal time for investment and production of petroleum reserves. The option valuation approach however leads to a straightforward form for this rule which depends only upon observable variables. Using this analysis we are able to study the effects of exploration and development costs and lags, and relinquishment requirements on exploration and development investment-timing decisions.

This option pricing technique is first tested by evaluating an OCS lease and comparing the result with estimates obtained by the USGS and industry bids. USGS uses a complicated model and discounted cash flow methods to obtain their estimate. This method is then used to evaluate an international offshore petroleum lease by considering the Tenghiz field in Kazakhstan which is being developed by Chevron. By using these examples the study will examine the many theoretical and practical issues involved in extending financial option pricing theory to valuing real options.

1.4 Organization of the Study

After the introduction, Chapter 2 describes the various stages of an oil and gas exploration and development program. Valuing a petroleum lease involves valuing the cash flows from a three stage process namely exploration, development, and extraction. This chapter will explore each of these three stages, the various variables associated with them as well as the risks involved in them. This chapter also explains the various types of leases used in the petroleum industry and the distribution of risk between the lessor and lessee for each one of them.

Chapter 3 will present a theoretical option pricing model which uses option pricing techniques along with a model of market equilibrium for the underlying asset (petroleum reserves) to value petroleum leases. This chapter will also give a detailed description of the fundamental variables that go into this model and will emphasize the advantages of this method to price petroleum leases over the conventional discounted cash flow methods. It will also address several theoretical and practical problems in using this approach in valuing real assets that are not present in applying option pricing theory to financial assets.

Chapter 4 will show how such a valuation technique can be applied to value petroleum leases by considering an offshore petroleum lease as an example. The lease considered is a simplified description of the OCS Lease Sale No. 139 which leased a total of 5,213 blocks in the Central Gulf of Mexico in May 1992. I will also perform a sensitivity analysis on the variables which are inputs in the valuation equation and explain the implications of this analysis.

In Chapter 5, I apply the options methodology in evaluating an offshore petroleum lease in which political risk plays an important role in the valuation. The project considered is the development of the Tenghiz field in the republic of Kazakhstan. This is a joint venture undertaken by Chevron and the republic.

In Chapter 6, I analyze political risk using an option theory approach and provide a way for firms operating internationally to evaluate this risk.

Finally, Chapter 7 summarizes the whole study,its usefulness and its limitations as well as the possible objections of using this approach for valuing petroleum leases.

Chapter 2

Risk and Return in Petroleum Exploitation

Investments in petroleum exploitation are inherently risky due to the variability of possible outcomes. In order to determine the net present value of a risky investment such as a petroleum exploitation project the oil company and the government participating in the venture must firstly estimate the cash flows from the project and then estimate the expected rate of return or discount rate for bearing the project risk.

Modern financial theory quantifies the risk associated with a risky investment by measuring the variance or standard deviation of the project outcomes. Project risk can be broken down into two parts. There is the "unique" or project specific risk and there is the industry-wide risk also called the "systematic risk". For example, an investment in drilling oil has an expected rate of return that is dependent on the value of oil found. This value is itself dependent on the price of oil when extracted and the actual quantity of oil recovered. The "quantity of oil" risk is purely a geological risk and is specific to the project itself while the "oil-price" risk is dependent on the market and economy as a whole. Thus the oil-price risk is a systematic risk as the market price of oil is independent of the particular project (as even very large oil discoveries seldom have any significant effect on the price of oil).

Typically investors can diversify and reduce the project-specific risk[12]. Since large international oil companies undertake numerous petroleum exploitation projects worldwide, much of the project-specific risk is diversified away and international oil companies would not demand a premium return on individual projects when discounting their overall cash flows. These companies still bear the market or industry risk for which they would demand a premium on their discount rates. From the petroleum producing countries point of view this will be quite different. Many of these countries are developing countries and have economies that are wholly dependent on the petroleum revenues that are generated from petroleum production. These countries also have pre-commitments on spending a large portion of their oil revenues and hence behave like highly leveraged corporations. Also, for several reasons these countries hold almost undiversified portfolios in which most of their investment is in the local petroleum industry. Since a large portion of these country's assets are invested in only one industry, their measure of risk is set by the variance of the oil stream itself rather than the covariance with a diversified portfolio of assets. Discount rates and risk factors are thus different from the two investors' perspectives. This is because a developing, oil producing country holds significant industry and project specific risks as well as the market or systematic risk. It can be argued therefore that these countries should generally have higher discount rates than the oil companies which hold largely diversified assets. In the next section, I enumerate the factors affecting the discount rates of both the oil company and the oil producing country. Contractual agreements between the oil company and the oil producing country serve as a mechanism of distributing the risk between the two parties. Different contract forms produce incentives and disincentives for the two parties and I shall describe some of these contract forms and how the risk is borne and allocated in each by the parties involved in the project. Finally, a brief discussion is also made on the efficiency of the contractual agreements in allocating risks to the party best capable of handling it and means by which the contracting parties can increase contract efficiency.

2.1 Discount Rates for Private Oil Companies

From the CAPM, the formulation of the discount rate D is given as $D = R_f + \beta(R_m - R_f)$ where R_f is the risk-free rate, R_m is the expected return on the market portfolio and β is a measure of the covariance between the market rate and the return on the particular asset in question. The premium over the risk-free rate is the return for bearing the risk which is the undiversifiable variability of the project outcome. The risk measure beta does not reflect the volatility of the asset itself, but the volatility which that asset contributes to the market portfolio. If the returns of the project fluctuate identically with the market the asset neither adds to nor subtracts from the risk of holding the market portfolio; beta is unity and the risk premium of the asset is the same as the market risk premium. If the asset's return are positively correlated and fluctuate more widely than the market it adds to the market risk and its beta is greater than unity; if negatively correlated it diminishes the risk and beta is less than zero. Even though international oil companies would not face non-systematic risk as on the whole they hold diversified portfolios, they would need to determine the type and amount of risk they bear in each individual project. Project risk is something which cannot be destroyed and it exists for all the investing parties. This risk can however definitely be transferred from one party to the other and in Section 2.3 below I explain how these risks can be transferred between to parties in exchange of return.

2.2 Discount Rates for Oil Producing Countries

As mentioned above because of certain restrictions which governments of lesser-developed oil-producing countries face the discount rates for these countries differ from the discount rates of the oil companies. Some of these factors and the manner in which they effect the rate of return are described below:

1. Diversification. This is probably the most important difference between the less-developed country and the oil company. With the exception of only Kuwait and Brunei, the income of most of these countries is derived from the oil revenues.

2. Lesser developed countries as leveraged corporations. These countries also have a high levels of debt in the form of claims on their gross oil revenues. Many of these oil producing countries hold smaller portfolios of assets and revenues from oil production forms a significant portion of the national income. Also, greater the income from oil, greater not less is the dependence of these nations on petroleum revenues; i.e. the nation becomes more oil-dependent. Since petroleum is an exhaustible resource it is expected that the risk-adjusted rate to be even higher from the perspective of this country. This adjustment of the cost of capital is also known as the "national" or "social" rate of discount.

3. Political Instability. This is an additional source of risk, resulting from the inherent instability of most of these governments

2.3 Different types of Contractual Agreements

2.3.1 Service Contract

A service contract stipulates a fixed negotiated payment for services rendered. In its pure form the oil company would negotiate a fixed cash payment for exploring, developing and extracting the oil. However, in the case of oil exploration and development, where the scope of the activity and the scale of investment required is usually unknown or subject to enormous variability at the time the contract is negotiated the term "service contract" is extended to include arrangements which provides the company a pre-specified rate of return on its total investment. This return would cover costs and allow a reasonable profit, regardless of the outcome of the project. Since the oil company receives a fixed rate of return on its investment it is not subject to either geologic or oil price risk. Both these risks are borne wholly by the host country. The only risk which the company bears is the host country's ability and willingness to "pay" especially when the oil discovery is small. Likewise, there are no extra returns for the oil company in the event of a particularly successful oil discovery. The cash return will be larger in absolute size the larger the investment made, such as in the case of a large oil find which involves a greater investment in development and extraction costs. Sometimes the government may decide to shift part of the risk to the company by entering a "risk service contract" where the company bears the cost of exploration and is then granted a service contract for development. The geological risk associated with exploration is thus transferred from the host country onto the oil company. the company will be able to recover its exploration costs only if economical levels of reserves are discovered. As in the case of a service contract the oil company is not exposed to oil price risk as its revenues depend only on the quantity of oil discovered and the investment made.

2.3.2 Toll or Fee per Barrel Contract

In this type of contract form the oil company is paid a fixed fee or toll per barrel of oil produced. The investment of exploration and development is made by the oil company which thus bears 100 percent of the geological and "quantity risk". Thus, in order to attain a minimum return on its investment the company will have to meet certain production levels. However since the negotiated fee is based on the physical volume of oil produced and not on the market value of oil, the oil company does not bear any oil price risk. The cash flows for the oil company are also shifted to the future when production begins and it the oil company faces increased political risk of events taking place during the life of the project which could jeopardize its cash flows. The host country simply bears the oil price risk and part of the quantity risk as it shifts the risk associated with petroleum exploration and development costs wholly onto the oil company.

2.3.3 Production Sharing Contract

In a production sharing contract, more of the risks associated with petroleum exploitation is borne by the oil company. As the name suggests, the oil company receives a share of the oil produced or its monetary equivalent market value. Thus, in addition to the geological, political and part of the quantity risk, the oil company also bears part of the price risk as its revenues depend on the future market price of the oil. it is expected that oil companies would demand a higher rate of return on capital in such contracts as a significant proportion of the project risk is borne by the oil company.

2.3.4 Bids and Bonuses System

There are various types of bids and bonuses contracts which may be applied by Governments. The commonest of them is the bonus-bidding system which is widely applied in the United States and in Canada. Under this type of system leases are awarded to the highest bidder in a competitive bidding system. The highest bidder in a sealed bid system wins the lease and the right to extract petroleum. This system in which the highest bidder gets the right to develop the land ensures that the expected economic rents from exploiting the petroleum reserves accrue to the host country. As the oil company bids and receives the right to develop and produce the oil it bears 100 percent of the risks of petroleum exploitation. The host country bears no risk and this system works efficiently from the country's point of view when there is effective competition among the investors, which ensures that the economic rents wholly transfer to the country. However, in face of collusion amongst investors, insufficient competition, geologic, exploration and extraction uncertainties, investors tend to put a high risk premium on the bids they submit and the country does not receive the full economic rent form the petroleum reserves. This bid and bonus system is used extensively for leasing offshore petroleum tracts in the Outer Continental shelf by the Mineral Management Service (MMS) of the United

States government.

2.3.5 Royalties and Production taxes

For a long time, royalties have been the traditional way to compensate landlords for the exploitation of all types of natural resources. In some countries like the U.S. and Canada, royalties are not regarded as a tax but rather as the reward to the landlord for giving exploitation rights to an investor. For many years, a flat rate of 12.5 percent was employed, but recently considerable variations in rates have been common.

As a fiscal device, royalties have some obvious advantages to the Governments. Revenue starts on the first producing day of the project and usually continue for the life of the project. The calculation and estimation of these royalties is far easier than any other fiscal device. In this system, most of the risk is borne by the oil company while the government bears part of oil price and quantity risk as revenues (in the form of royalties) on oil produced depend upon the quantity and price of oil produced. However, the government does not bear any of the investment cost risk and any development cost overruns would not affect the country's revenue. Such types of agreements are also beneficial to Governments as most of the geological and technological risks are transferred to the investing oil company and the Government need not worry about development cost overruns as this would not directly affect the Government's revenues. The Government still shares the risk of the reserve quantity and oil prices, as if the reserve is small or the oil prices fall, the revenues will correspondingly reduce as well.

2.3.6 North Sea Type Special Taxes Contracts

In this type of contract, employed by many of the North Sea producing countries like Denmark, the Netherlands, Norway and the United Kingdom, the country exercises a special tax structure related to profits of the oil company. Thus the country extracts a high-tax rate when the oil fields are very profitable and a lower rate when fields are less profitable. With this type of tax structure a large portion of both the quantity and price risk is borne by the country. This system works well in areas where there is large variation in field profitability and it is difficult for any one party to wholly bear both the quantity and the oil price risk.

2.3.7 Windfall Profit or Excise Duty Contracts

This type of contract is similar to the North Sea contract in which the country exercises a tax based on the difference between a base-price and the market value of oil for a particular period. The basic idea is that oil companies would incur windfall gains from large oil-price increases and these gains are legitimately extracted by the host country. From a risk-sharing point of view, this type of tax leaves the exploration and development risks with the investor. The oil price risk is shared by both the parties when the market price exceeds the base price. The reduction in the tax to zero when the market price falls to the base level gives "downside" protection to the investor compared to a straight production tax. There are several problems associated with this type of contract, one being the determination of the base price. In fixing a base price, fields with greater exploration costs would become uneconomic to exploit while others would earn more than adequate returns.

2.4 Evaluation and Desirability of Different Contracts

Each of the contracts described above differ from each other in the types and timings of cash flows and the manner in which the project risks are allocated between the oil producing country and the oil company. The contract forms described above differ along several dimensions. Among the most important are:

(a) the allocation of geological, costs and oil price risks.

(b) the incentives they create for the party managing the project.

(c) the extent to which they introduce contracting risks, i.e., risks of non-performance by one or both parties.

A contract is said to be efficient when any change in the contract agreement produces a benefit for one of the contracting parties at the cost of making the other party worse off. The terms of a contract thus move towards this Pareto model of efficiency, by improving contract efficiency to attain this state. In case of efficient allocation of project risks, the risk should be allocated to the party most capable of handling it. For example one party may be more exposed to geological or quantity risks than the other and is may be at a comparative disadvantage in bearing that risk. Both parties can improve their position by agreeing on an appropriate distribution of risks among themselves. With managerial incentives the same holds since oil companies are used in part to maintain the managerial control over the project especially when technology and experience are in scarce supply. This implies that the government is not in a position to fully specify all actions of the oil company. In these cases the prime assurance to the government that the project is managed appropriately is to structure the contract whereby it is in the company's own interest to manage the project as the government would have done if it had comparable information and experience. Finally, reduction of the contracting risks is also beneficial to both the parties. It may seem that the government may profit short-term from unilateral action, in the future, the oil company recognizing the likelihood of such action would demand compensation in the form of reduced investment or a higher promised share. These three aspects effecting contract efficiency are now examined more closely for each specific type of contract described above.

Chapter 3

Option Valuation of Real Assets

This chapter presents the option valuation approach in the valuation of real assets in general and the case of offshore petroleum leases in particular. The option valuation methodology described in this chapter draws heavily and closely follows the methodology developed by Paddock, Siegel and Smith.[1] This model combines the option pricing techniques used in the valuation of financial options with a model of market equilibrium of the underlying asset and explains why this needs to be so. It is explained as how valuation of a petroleum lease can be modeled as valuing cash flows from a three-stage process of exploration, development and extraction. The stages form a set of nested options and each of these three stages has distinct characteristics relevant in applying the option valuation (OV) approach in evaluating petroleum leases. Certain plausible assumptions need to be made for each of these stages and these are described in the sections below.

[1] See "Option Valuation of Claims on Real Assets: The Case of Offshore Petroleum Leases", by James L. Paddock, Daniel R. Siegel, and James L. Smith, 1988.

3.1 Characteristics of the Stages

3.1.1 The Exploration Stage

The exploration stage in a petroleum project may be modeled from the perspective of an oil company as an option to make exploration expenditures and in return receive undeveloped reserves of petroleum. Since the nature of this option to explore and receive the undeveloped reserves is similar to a financial option which gives its owner the right but not the obligation to pay the exercise price in exchange of the stock, the value of this option to a petroleum leaseholder may be calculated using financial option theory[2]. Just as a stock option has an expiration date, the lease agreement has a relinquishment date which stipulates that the owner of the lease must give up the lease if he does not explore and develop the property by a certain date. Thus, just as the owner of a stock option loses the right of exercising the option after the expiration date of the stock option, so does the owner of a petroleum lease lose his claim on the lease at the relinquishment date. However there are important differences, regarding uncertainties in the exploration process and costs which differentiate options on financial assets and contingent claims to petroleum leases.

The primary uncertainty in the exploration stage is the "quantity risk" or in other words the uncertainty regarding the quantity of hydrocarbon (oil and natural gas) reserves. The quantity of hydrocarbon reserves is a determining factor in the total value of the petroleum lease as well as the exercise cost of the lease. Much of this uncertainty is resolved only after the exploration has been undertaken. This is different from options on financial assets where the value of the asset at exercise is a function of only the price of the asset at the exercise date. Also, because offshore development costs are primarily driven by factors related to the size of the reserves (economies of scale), exploration also resolves the uncertainty about development costs. Thus, the exploration stage may be

[2]See Appendix A.

looked as the option to spend the expected exploration costs $\bar{E}$, and receive the expected value of undeveloped reserves,

$$C^{\star}(V) = \int QC(V, T - t; X(Q))dF(Q) \tag{3.1}$$

where:

Q = quantity of recoverable hydrocarbons in the tract (possibly zero).

$X(Q)$ = per unit development cost (in real dollars), a function of quantity.

V = *current value of a unit of developed hydrocarbon reserves.*

$F(Q)$ = probability distribution of the quantity of hydrocarbon reserves.

$C(V, T - t; X(Q))$ = *current* per unit value of exploration option i.e. value of option given the current per unit value of a developed reserve and per unit development (exercise) cost.

t = current date

T = relinquishment date of the lease

The current value of the reserves obtained after exploration are represented as the expected value of the undeveloped reserves, because the quantity risk is almost entirely technological and geological. Since technological and geological risk mainly relate to a particular project, they are a part of the non-systematic or idiosyncratic risk and they require no risk premium.

3.1.2 The Development Stage

The completion of the exploration stage, gives important new information of the petroleum tract to the leaseholder. As mentioned above, exploration removes much of the uncertainty regarding the quantity of hydrocarbon reserves. This information on the reserve quantity also gives the holder of the petroleum lease an estimate of the magnitude of development costs with a reasonable amount of accuracy. The leaseholder now holds the option to pay the development costs and install production capacity. Therefore the ownership of undeveloped reserves is the option to obtain the developed reserves, Q by paying the development costs. The value of this per unit of the undeveloped reserves is $C(V, T - t; X(Q))$. An important assumption being made in both the exploration and development stages of the petroleum exploitation project is that the exploration and development expenditures are treated as being paid in one instantaneous lump-sum at the time of exercise of these options.

3.1.3 The Extraction Stage

Once the leaseholder has exercised his development option, he owns the reserves. He now has the option to extract the reserves if he chooses. The process of determining the value of the developed reserve requires several assumptions to be made about the oil quality, future extraction rates and costs, tax and royalties to be paid, and the hydrocarbon prices. Fortunately, however, a firm can determine the market value of a unit of developed reserve by observing the value placed on that quantity in competitive secondary markets. Such markets do exist, so a firm can now within reasonable tolerance, determine the market value of a given quantity and type of developed reserve. This market value reflects the value of reserves with similar extraction rates and operating costs, quality of hydrocarbons and the tax regime under which the tract is being valued. Of these the extraction rates and operating costs for a particular tract are the most difficult to predict, ex ante. Fortunately, extraction rates and operating costs do not vary as much as exploration and development costs for different tracts. Hence the option valuation technique uses the market information about the value of the developed hydrocarbons in an explicit and straightforward manner.

3.2 Valuation

3.2.1 Valuing the Development Option

It can be seen from Section 3.1 that a claim on a petroleum lease may be modeled as a compound option, where the exploration option is an option on the development option. Since there exists markets for trading developed reserves of petroleum, the value of the extraction option is already incorporated in the current market value of a developed reserve. The valuation of compound financial options has been explored by Geske[7], however there are important modifications in applying this to real assets. Valuing stock options merely require a stochastic process generating stock prices. In the Black-Scholes[3] world, the continuously compounded rate of return of stock prices is assumed to follow a lognormal distribution. it is not necessary to understand equilibrium in the market for the stock itself. In order to understand why we must consider equilibrium in the market for real assets we will briefly need to examine the theory of valuing call options. The detailed explanation for valuing financial options is described in Appendix A of the thesis.

Because an undeveloped reserve is analogous to a stock call option, by simple arbitrage arguments it can be proved that:

$$C(V, X) \geq V - Xe^{-r\tau} \geq V - X \tag{3.2}$$

where V is the current value of the stock price in the case of a stock option or the current value of the developed reserve in the case of petroleum reserves , r is one plus the risk free rate of interest and X is the exercise price. As can be seen from the equation, the owner of the option receives an amount equal to $V - X$ if the option is exercised right away and an amount equal to $V - Xe^{-r\tau}$ if he waits until the expiration date; τ time units away. As $C(V, X)$, the price of the option, is greater than $V - Xe^{-r\tau}$, in the event of no payments in the form of dividends, it will always be optimal to wait until the expiration date before exercising the option, and in the event the owner wishes to liquidate his position before the expiration date he would be better off selling the option rather than exercising it. This is the general result for financial stock options and there seems to be a easy solution for determining the complex problem of when to develop the reserves: wait for the last possible date allowed by the lease owner before developing. The alternative would be to sell the lease itself.

However this result leads us to a puzzling conclusion. In the case there were no relinquishment requirements on holders of petroleum leases, then according to (3.2), no company would ever develop its reserves. This is clearly not a practical equilibrium. The reason, of course, is that if no one develops his reserves, there is no supply of the produced reserves or extracted petroleum. Herein lies the distinction between financial assets and claims to physical assets. When a stock option is exercised, there is no net addition to the supply of the underlying stock. However when an undeveloped reserve is developed, the net supply of the developed reserves increases. The result (3.2) would hold unambiguously if the value of a developed reserve is expected to increase at an equilibrium "risk adjusted rate of interest". Therefore, it seems likely that in the case of equilibrium in the market for developed reserves, the value of these reserves will be expected to increase at less than the risk adjusted rate of interest. This will be verified in the equations below. Thus, if this is true, then the development decision rule will be different from that which is implicit in (3.2) (i.e., wait until the last possible moment).

In equilibrium in the market of petroleum reserves, the expected net present value of holding these reserves is comprised of the payouts of the producing reserves in the form of the sale of crude and the capital gains of the remaining reserves left in the ground. Let B_t denote the total number of units of petroleum in a developed reserve at time t, V_t denote the value of a unit of developed reserve at time t, and R_t denote the instantaneous per unit time net payoff from holding the reserve again at time t. Then assuming the standard Ito stochastic process[3] generating returns on the developed reserves, we have:

$$R_t dt / B_t V_t = \alpha_v^* dt + \sigma_v dz_v \tag{3.3}$$

where α_v^* is the expected rate of return to the owner, σ_v is the instantaneous per unit time standard deviation of the rate of return, and dz_v is an increment to a standard Wiener (diffusion) process. Thus if the owner of a developed reserve is to be compensated for the opportunity cost of investing in the developed reserve, α_v^* must equal the expected rate of return on a stock with risk $\sigma_v dz_v$. The assumption that the total rate of return follows a diffusion process is as plausible as the assumption that stock prices also follow such a process. The product $B_t V_t$ represents the market value of an asset whose owners expect to be compensated for their investment.

The net payoff as mentioned above comes from two sources: (1) the profits from the producing reserves and (2) the capital gains from holding the reserves. In order to determine the profits from the producing reserves, an assumption regarding the rate of extraction of the developed reserves also needs to be made. It is assumed that the production yields from extracting reserves follows an exponential decline curve[4]. Thus if *gamma* represents, the rate of extraction of the reserves, the quantity of reserves at time t will be represented as:

$$B_t = B_{t-1} e^{-\gamma t} \tag{3.4}$$

From Equation (3.4) above, the rate of extraction would be:

$$dB_t = -\gamma B_t dt \tag{3.5}$$

[3]Based on the efficient market hypothesis rates of returns of the underlying asset are assumed to follow a continuous-time random walk.

[4]This is a standard assumption in the literature of petroleum extraction (refer to Adelman and Jacoby [1979]) and reflects geological constraints on the extraction rate.

The net payoff from holding these reserves over the small interval of time d_t can be represented as the sum of the extraction profits and the capital gains.

$$R_t dt = [\gamma B_t P_t dt] + [(1 - \gamma dt) B_t (V_t + dV_t) \ - B_t V_t] \tag{3.6}$$

where P_t denotes the after-tax operating profit from selling a unit of petroleum. Substituting Equation (3.6) into Equation (3.3) yields the stochastic process for the value of the developed reserve[5].

$$\frac{dV}{V} = (\alpha_v{}^\star - \delta_t)dt + \sigma_v dz_v$$

$$\frac{dV}{V} = \alpha_v dt + \sigma_v dz_v \tag{3.7}$$

where

$$\alpha_v = \alpha_v{}^\star - \delta_t$$

and

$$\delta_t = \gamma[P_t - V_t]/V_t$$

δ_t is called as the payout rate of the producing developed reserve, and α_v is the expected rate of capital gain.

It is clear from Equation (3.7) that in equilibrium no lease holder or owner of

[5]Note that $dtdV_t = 0$. $\alpha_v{}^\star$ and α_v can vary with time.

developed reserves would want to hold non-producing developed reserves[6]. The expected rate of return of holding a non-producing developed reserve, α_v is less than the required rate of return of an asset with risk, $\alpha_v{}^*$, on an asset with risk $\sigma_v dz_v$. The rate of return shortfall is the payout rate δ_t of the producing reserves.

There are two techniques to derive the European call option pricing formula. Both these derivations have been presented by Black and Scholes [3] in their classic paper. The first way in which this formula is derived is known as the "replication derivation" and it relies on the fact that the returns from an option position can be exactly replicated by a dynamic portfolio consisting of the underlying security and a short position in risk free bonds. From this approach it can be concluded that there will definitely be a riskless profit or "free lunch" if the option is not priced exactly according to this formula by either buying or selling the option and taking up a complimentary position in the underlying asset and bonds. This approach is purely an arbitrage one and is independent of considerations to capital market equilibrium. The second derivation which is called the "equilibrium derivation", evolves from the requirement that the option earns an equilibrium rate of return commensurate with the risk of holding that option as an asset.

It has already been shown above that options on non-producing developed petroleum reserves earn a rate of return lower than that necessary to induce investors to hold it. In this case, even if the replication strategy is feasible, it is inefficient. A consequence of this is that the correct option pricing formula is different than the Black-Scholes formula. Unlike the Black- Scholes formula this case requires as an input the difference between the expected rate of return of the underlying asset and the expected rate of return which would be required

[6]In general it can be assumed that the after-tax profits from selling the reserves, P, would exceed the value of the developed reserve in the ground, V, because one would prefer a barrel of oil above the ground to a barrel of oil below the ground due to extraction costs and time to extraction. This assumption may not hold true if the storage costs of a barrel of oil above the ground is greater than the sum of both the extraction costs and the time value costs.

to induce hold of the asset. The reason why the Black Scholes formula needs modification when the owner of an asset gets below the expected rate of return is demonstrated as shown:

Consider the asset with price C, following the Ito process:

$$\frac{dV}{V} = \alpha_v dt + \sigma_v dz_v \tag{3.8}$$

Now suppose that the equilibrium expected rate of return necessary to compensate investors for bearing the risk of holding the asset is $\alpha > \alpha_V$. The formula for the price of a call option written on this asset will be correct if the following two conditions are met.

1) At its expiration date, the option has a value of:

$$Max[0, V_t - X] \tag{3.9}$$

2) The formula implies that the option earns a rate of return consistent with capital market equilibrium. Merton [15] has shown in his derivation of the Black Scholes formula that the payoff of a call option may be replicated by holding a continuously adjusted, self-financing portfolio which contains the asset and risk free bonds. This suggests that in order to prevent any arbitrage opportunities, the price of the option C must equal the cost of the replicating portfolio. From Merton's derivation the cost of the replicating portfolio is precisely given by the Black-Scholes formula:

$$C(V_0, X, T-t) = V_0 N(d_1) - X e^{-r(T-t)} N(d_2) \tag{3.10}$$

where

$$d_1 = [\ln(V_0/X) + (r + \sigma_v^2/2)(T-t)]/\sigma_v \sqrt{T-t}$$

$$d_2 = d_1 - \sigma_v \sqrt{T-t}$$

where $T - t$ is the time to maturity, X is the exercise price or development cost of the reserves, r is the risk free interest rate, and $N(.)$ is the standard cumulative density function.

The option price as given in the formula above satisfies the first condition, in that it has the correct payoff at the expiration date. However it fails to satisfy the second condition. This is easily verified by a particular case by setting $X = 0$ in the equation above. Then $C(V, 0, T - t) = V$, and

$$\frac{1}{dt}E[\frac{dC}{C}] = \frac{1}{dt}E[\frac{dV}{V}] = \alpha_v < \alpha \qquad (3.11)$$

The replicating portfolio, and thus the option, earns a below the equilibrium rate of return. It is can easily be shown that the option on an asset earns a below-equilibrium rate of return. Since an option can be replicated by a portfolio of bonds and the underlying asset, we can write

$$C(V_0, X, T-t) = \omega_1(V, T-t) + \omega_2(V, T-t)B(T-t) \qquad (3.12)$$

where $B(T-t)$ is the price of a default free bond maturing in time $T-t$, $\omega_1 > 0$ is the quantity of the underlying asset and $\omega_2 < 0$ is the quantity of bonds held in the replicating portfolio. As the bonds earn an equilibrium rate of return, the option will earn an equilibrium rate of return only if the underlying asset earns an equilibrium rate of return. Since we have shown that the underlying asset earns a below-market rate of return and is held in positive quantity it follows that the option will also earn a below market rate of return.

In order to determine the value of an asset which earns a below the market rate of return, it is clear that the replication strategy followed by Merton will not work. One approach would be to follow Constantinides [5] and impose the condition that the option earns a rate of return commensurate with its risk. This would require specifying a particular model of market equilibrium. Another more general version of this argument was proposed by McDonald and Siegel [13]. In their derivation of Merton's formula they replaced V by V^*, where

$$V_t^* = V_t e^{-(\alpha-\alpha_v)(T-t)} \tag{3.13}$$

V^* unlike V, earns an equilibrium rate of return[7].

Thus the derivation of Merton of the Black-Scholes formula yields the same equation as before with only V replaced with V^*.

$$C(V_0, X, T-t; \delta) = V_0 e^{-\delta(T-t)} N(d_1^*) - X e^{-r(T-t)} N(d_2^*) \tag{3.14}$$

where

$$d_1^* = [\ln(V_0/X) + (r - \delta + \sigma_v^2/2)(T-t)]/\sigma_v\sqrt{T-t}$$

$$d_2^* = d_1^* - \sigma_v\sqrt{T-t}$$

where $\delta = \alpha - \alpha_v$. Also at time $t = T$, $V_T = V^*{}_T$, it is clear that the above equation satisfies the first condition viz; at maturity the option has a value of $\text{Max}[0, V_T - X]$. Further, V^* earns an equilibrium rate of return and so the second condition is also satisfied. Therefore the option pricing formula given in Equation (3.12) above is the price of the call option. Even though there is no

[7]Using Ito's Lemma, it can be shown that V_t^* follows the Ito process
$\frac{dV^*}{V^*} = \alpha dt + \sigma_v dz_v$
Thus, V^* has the same risk as V, but earns rate of return α.

underlying asset with price V^*, this argument identifies the option value which satisfies the second condition.

To verify this argument, consider the special case where the exercise price of the option D is equal to 0. Then $C(V, 0, T-t; \delta) = Ve^{-}\delta(T-t) = V^*$. The risk of the option is then same as $V^*{}_t$, and it appreciates at the same rate α^*. For the case when the rate of return shortfall, δ equals zero, the equation (3.12) reduces to (3.9)[8]. Thus, it is clear that $C(V_0, X, T-t; 0) = C(V_0, X, T-t)$. Also, it is easy to show that $\frac{dC}{d\delta} < 0$. Therefore $C(V_0, X, T-t; \delta) < C(V_0, X, T-t)$ if $\delta > 0$, which shows that the equilibrium option price is lower than that obtained in Equation (3.9). The replication strategy is too expensive because it involves holding some of the asset which by assumption earns a subnormal rate of return. The equation (3.9) is also the price of an European call option which pays a dividend at the proportional rate δ. Formally the rate of return shortfall, δ, is like a dividend, in that it is a portion of the total required rate of return of the underlying asset which does not wholly accrue to the option holder and thus affects the option price.

It may also be noted that V^* can also be interpreted as the price that would be paid today for time T delivery of the underlying asset. Thus if $F = V^*e^{r(T-t)}$ is the forward price of the asset and (3.12) can be seen as equivalent to Black's (3.9) formula for the price of a commodity option, with $Fe^{-r(T-t)}$ substituted for V^*. From this it can be said that if a futures market existed for the asset, then the replication and equilibrium prices are the same when the futures contract is used in the replication strategy.

3.2.2 Valuing the Exploration Option

After determining the technique of valuing the development option on a tract of explored (undeveloped)reserves, the value of the option to explore the unex-

[8]This is why Black and Scholes could use both the arbitrage approach and the equilibrium method to arrive at their derivation of the call formula. For a non-dividend paying stock, $delta = 0$.

plored tract, which is modeled as the option to make the expected exploration expenditures, E and receive the expected value of the undeveloped reserves $C^*(V)$ as defined in Equation (3.1). In valuing the unexplored tract, various complications arise out of the properties of the development option and the optimal development timing. In order to simplify the problem, it is assumed that it is optimal to begin development immediately after successful exploration has taken place. The appropriateness of this assumption is discussed below. Using this simplifying assumption the development option can be collapsed into the exploration option. Thus the combined exploration and development option on the petroleum exploitation project may be modeled as the option to spend exploration and development expenditures (finding costs) on a tract and in return obtain the developed reserves. Assuming that the development option is always exercised immediately immediately after exploration, the above option will have a value given by,

$$C(V, X, T-t; \delta) = V - X \tag{3.15}$$

Then combining (3.1) and (3.13), it is clear that exercising the exploration option requires paying E and receiving

$$C^*(V) = \int Q[V - X(Q)]dF(Q) = V \int QdF(Q) - \int QX(Q)dF(Q) \tag{3.16}$$

Alternately we can view exercising the exploration option as paying;

$$\left(\int QX(Q)dF(Q)\right) + \bar{E} = \bar{X} + \bar{E}$$

and receiving,

$$V \int Q dF(Q) = V\bar{Q},$$

where $\bar{X}$ and $\bar{Q}$ are the expected total development cost and the expected reserve quantity, respectively. Using the homogeneity of the valuation discussed above, the value of the unexplored tract can be represented as

$$\bar{Q}W(V, T - t, \bar{S}; \delta) \tag{3.17}$$

where

$$\bar{S} = (\bar{X} + \bar{E})/\bar{Q}$$

and $W(V, T - t, \bar{S}; \delta)$ is the current value of an option to receive a unit of developed reserves by paying the per unit combined exploration and development cost $\bar{S}$.

3.2.3 Exploration and Development Lags

The above analysis and the option valuation equation assumes that exploration and development occur immediately and that these expenditures are made in a one lump-sum basis i.e. the owner of the reserves receives the developed reserves immediately upon beginning exploration and therefore development of the unexplored tract. In fact, there exists a lag between the expenditure of exploration and development expenditures and receiving the developed reserve. Let t' denote the total length of this lag between the time the lease owner exercises his option to the time he actually receives the underlying asset. The value of a claim at time t to receive a a unit of developed reserve at time $t + t'$ is simply the present value;

$$V'_t = e^{-\alpha_v^* t'} E_t[V_{t+t'}] = e^{-\alpha_v^* t'} V_t e^{(\alpha_v^* - \delta)} = e^{-\delta t'} V_t \tag{3.18}$$

Thus in order to account for the exploration and development lags, the value of the V^* in the option valuation equation need only be replaced by V'^*, given by:

$$V_t'^* = e^{-\delta t'} V_t^* = e^{-\delta(T-t+t')} V_t \tag{3.19}$$

3.3 Comparison between the Option Valuation and Discounted Cash Flow Technique

One of the most important features of models used to price stock options is the small number of input parameters required. These same advantages are present in the valuation of petroleum tracts by the option valuation approach. The parameters needed to value the leases by this approach are listed below:

- Value of developed reserve discounted by the development lag.
- Variance of rate of change of the value of the developed reserve.
- The per unit exploration and development cost of the developed reserve.
- Relinquishment requirement; namely the time to expiration of the lease.
- Riskless rate of interest.
- Net production revenue less depletion; rate of return shortfall.

The way in which each of these parameters are estimated are described in the following two chapters when this option approach is applied to two specific cases: an offshore petroleum lease in the Central Gulf of Mexico and a joint venture petroleum exploitation project in Kazakhstan. As in the case of stock options, the most important parameter not required in the valuation is the risk-adjusted rate of return or the expected future prices of the petroleum or developed reserves. Therefore as in the case of stock options one need not calculate the systematic risk of the asset while determining its value. Comparing these information requirements to those needed in the formula using the discounted cash flow method, it can be concluded that the option valuation approach reduces the information requirements needed to value a petroleum lease.

The discounted cash flow method also requires the analyst to make assumptions about the expected future oil prices based on the time of petroleum reserve extraction. The option valuation approach not only lets the market place a price on the developed reserves, by finding market prices of developed reserves similar to those that the firm would acquire after exploration and development but also gives a good indication in determining the optimal time of extracting these reserves.

3.4 Some Inferences from the Model

The option pricing model, described above, like any model provides an extremely simplified and unrealistic description of an actual Outer Continental Shelf (OCS) lease or any other kind of petroleum exploitation contract. In general leases which give the right to explore, develop and extract oil are contractually complicated and more explicit models are required in order to use this technique in valuing leases. However, this approach has its usefulness when the decision to develop the lease depends critically upon oil prices. If the development decision is virtually "automatic", "way in the money", to use options terminology, then the option approach in valuing such leases provides no significant value to the valuation. As development is most price-sensitive and least automatic in the case of offshore leases, the case of the OCS leases is considered first in determining the usefulness and appropriateness of applying option-pricing theory to the valuation to the valuation of real assets.

Chapter 4

Option Valuation Approach to OCS Lease Sales

As an illustration of the Option Valuation (OV) methodology in valuing petroleum leases, estimates for the market value of federal lease tracts are derived for selected offshore petroleum tracts. For the purpose of valuation, the federal lease sale No. 139 in the Central Gulf of Mexico is selected and the estimates obtained by this method is compared with the estimates reached by the U.S. Geological Survey (USGS) and with the high and average bids from the industry. The tract specific data which is required for the valuation is obtained from the USGS.

4.1 Outer Continental Shelf Petroleum Program

The Outer Continental Shelf (OCS) Oil and Gas Leasing and Production Program is administered by the Minerals Management Service (MMS) of the Department of the Interior (DOI). In order to provide access to possible new sources of oil and gas offshore, the DOI, through the MMS, leases out tracts in the Outer Continental Shelf. Since the inception of the program in 1954 to the end of 1986, 86 lease sales have been held. The total bonus amount paid by the industry for all the leased tracts from 1954 to the fiscal year 1984 was nearly $51 billion and royalties amounted to more than $23.8 billion. About 6.7 billion barrels of crude oil and condensate and 66.6 trillion cubic feet of natural gas was produced in this interval. The OCS program is divided into four main regions, Alaska, Atlantic, Gulf of Mexico and Pacific. (see Figure 4.1)

The lease considered in this study is the lease sale in the Gulf of Mexico region. The Gulf of Mexico OCS region includes approximately 159.2 million acres (35.9 million acres in the Western Gulf, 47.7 in the Central Gulf, and 75.6 in the Eastern Gulf) and covering approximately 580,000 square miles, with a maximum water depth of about 12000 feet. Since 1954, this region has offered over 566 million acres in 62 lease sales. As a result of these sales, the MMS has received over $42.9 billion in bonus bids. The Gulf of Mexico accounts for 82 percent of the total acreage leased by all the four OCS regions.

4.2 Different Types of Bidding Systems for OCS

There are several different types of contracts allowed in the OCS Oil and Gas leasing and production program. Some of them are briefly described here.

4.2.1 The Present Bonus Bid System

In this system, the bidders are permitted to do geophysical exploration for a nominal fee. The geophysical work is often done by a consortia of oil companies, each of whose members is required not to disclose any of the data. After a sealed bid offer from each firm is held, the exclusive exploratory drilling and extraction rights for each tract is awarded. The lessee is required to pay a fixed royalty of

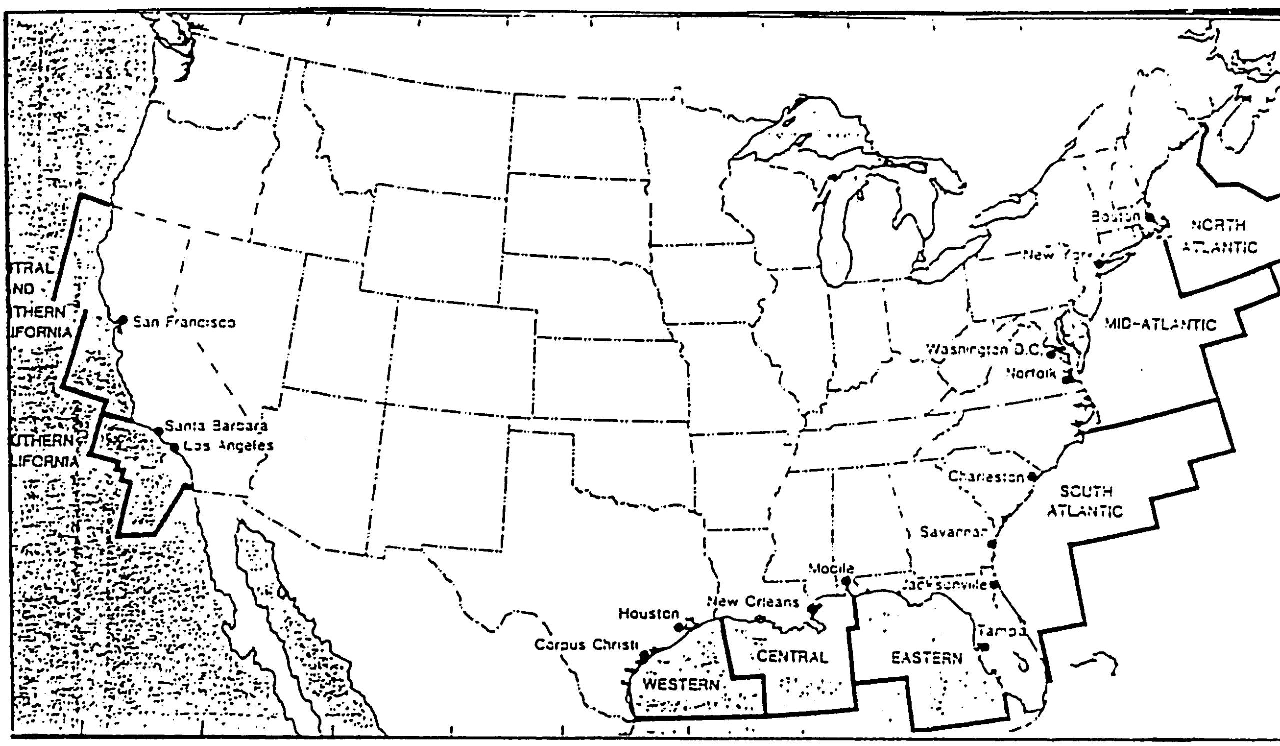


Figure 4-1: Federal Offshore Petroleum-Outer Continental Shelf Region

$16\frac{2}{3}\%$[1] on each unit of oil and gas produced, pay a nominal yearly rental, and abide by the USGS safety and environmental regulations. In case the company decides to abandon the development of the tract, the tract reverts back to the government. This policy is known as bonus bidding.

4.2.2 Work Obligation Permitting Bidding

There are many variants of this type of contract, but it basically involves potential bidders to submitting their development and production plans for a particular tract and the government choosing the most aggressive and best-considered plan.

4.2.3 Royalty Bidding

This kind of bidding involves, competitive bidding on the share of the actual gross revenues associated with the resources. This is generally done on a percentage of market value. As compared to bonus bidding, royalty bidding involves a transfer of the risk associated with the uncertainty existing prior to exploratory drilling from the developer to the government.

4.2.4 Percentage of Excess Profit Bidding

In this considerably more attractive form of bidding, which has never been used in U.S. petroleum leasing, the developer pays the government only the bid percentage of the differential between the market price and the resource cost.

[1] Only tracts having a royalty payment of $16\frac{2}{3}\%$ or one-sixth tracts are considered as the developed reserve values are most appropriate for this type of tract.

4.3 Tract and Sale Data

Federal lease sale No. 139 was held on May 1992, and covered the region in the Central Gulf of Mexico. (see Figure 4.2) A total of 5213 blocks in 28.1 million acres of waters of 4m to more than 3,425 m, 3 to 220 miles off Texas, Louisiana, Mississippi, and Alabama were awarded in the sale. MMS estimated the sale area's recoverable reserves at 25 million barrels of oil and 5 billion cubic feet (bcf) of gas. The blocks in waters less than 400m of water; 1149 of them were offered for a lease period of 5 years, for waters between 400 and 900 m a lease term of 8 years and a 10 year lease period for blocks in waters of 900 to 3000 m. The following variables need to be determined as they are inputs in the valuation equation.

(a) mean and variance for quantities of recoverable oil reserves.

(b) mean and variance for quantities of recoverable condensate reserves.

(c) mean and variance for quantities of recoverable gas reserves.

(d) probability that the tract is dry.

(e) expected exploration cost.

(f) expected development cost.

(g) USGS estimate of tract value.

The mean and variance of the quantities of recoverable resources are conditional on the tract not being dry. Therefore the statistical distribution of oil and gas reserves for a given tract is joint lognormal based on the underlying assumption that the tract is wet.

4.4 Inputs to the Valuation Equation

In order to valuate the OCS Lease using the option valuation technique described in Chapter 3, the following parameters need to be determined as inputs

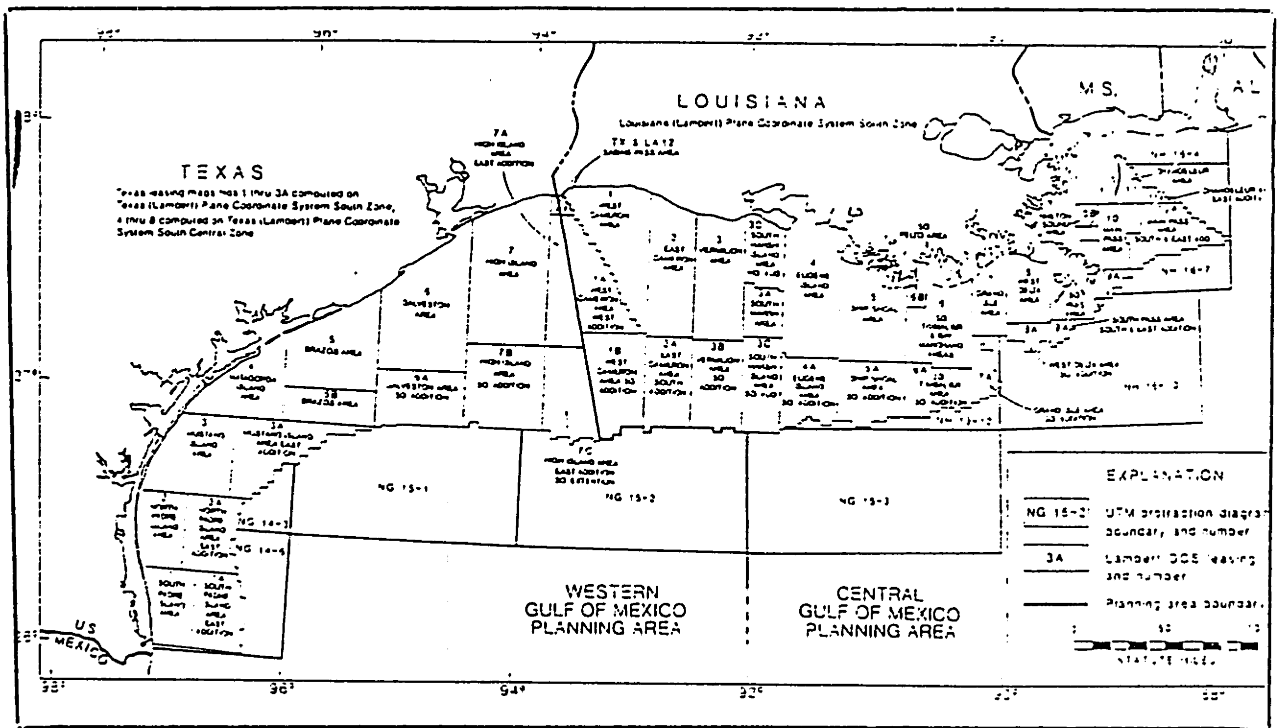


Figure 4-2: Gulf of Mexico OCS Region leasing map

to the valuation equation.

4.4.1 Developed Reserve Value

A necessary input in the option valuation equation to obtain the market value of the developed reserves of the oil and gas on the date of the lease sale. There are several companies whose assets comprise wholly of developed petroleum reserves and whose stock trade openly in stock exchanges. An example of such companies are the London and Scottish Marine Oil Company, Ltd., whose stock trades on the London Stock Exchange and the Permian Basin Royalty Trust whose stock trades on the New York Stock Exchange. The value of developed reserves can be calculated from the stock value of these oil production stocks since they comprise of claims to these assets. Another rough estimate of the reserves can be made by following the simplifying assumption that oil developed reserve prices are approximately one third of crude oil prices. The value of a barrel of crude oil in May 1992; the time of lease bidding was $17.31 which gave the approximate value of the developed reserve to be around $6 per barrel. Valuation of gas reserves are a bit more problematic. Strictly on a Btu-equilibrating scale basis[2], the value of an tcf(thousand cubic feet) of gas (at burner tip) would be approximately one sixth the value of the developed oil reserve, approximately $1. From Table 4.1, it can be seen that there are reasons to doubt such a direct relation between the in-situ values of the two fuels. This table gives the average wellhead value of crude oil and natural gas in the United States for the years 1981 to 1991. The prices for natural gas vary from 6% to 16% of the value of a barrel of crude. The reasons for this is that there are several factors effecting the value of natural gas including the preference of natural gas as a fuel in many applications as well as the different methods of extracting, transporting and storing the fuel, and different tax considerations.

[2]1 barrel of crude oil is approximately equal to 6MBtu(Million British thermal units). Since the energy value of thousand cubic feet of natural gas is 1MBtu,the value of natural gas on a pure Btu equilibrating scale should be 1/6th the value of oil.

United States Average Wellhead Value of Crude Oil and Natural Gas

Year	Average Wellhead Value per Barrel of Crude Oil ($/barrel)	Average Wellhead Value per thousand cubic feet of Natural Gas ($/tcf)	Natural Gas as Percentage of Value of Crude Oil
1981	$31.77	$1.98	6.23%
1982	$28.52	$2.46	3.63%
1983	$26.19	$2.59	9.39%
1984	$25.38	$2.66	10.28%
1985	$24.09	$2.51	10.42%
1986	$12.51	$1.94	15.51%
1987	$15.40	$1.67	10.84%
1988	$12.58	$1.69	13.43%
1989	$15.74	$1.69	10.74%
1990	$20.03	$1.71	8.54%
1991	$16.50	$1.61	9.76%

Source: U.S. Energy Information Administration, Natural Gas Annuals

Table 4.1: United States Average Wellhead Value of Crude Oil and Natural Gas

An important point to be remembered here is that the market value of the reserve must be the one that has the same hydrocarbon quality, cost structure, and tax regime as that of the tract being valued. This information is not available here for each tract that was leased, however, firms using this method for valuation would have access to that kind of detailed information and would use a more accurate estimate of the current developed oil and gas reserve values.

4.4.2 Variance of Reserve Value

The variance of the rate of return of the developed market reserve is also required as an input in the valuation equation. One way in which this parameter can be obtained is to observe the variance of the market values of the developed reserves. Unfortunately, even though developed reserves are traded in the capital markets, market value data is not available at regular enough intervals to estimate this variance accurately. However, since the simplifying assumption has been made that developed reserve prices are approximately one third of crude oil prices, the variance of the rate of change of crude prices would roughly serve as a proxy for the variance of the rate of change of developed reserve prices. The basis of this assumption is the observed high correlation between crude oil price and developed reserve prices. In order to calculate this, monthly data for the 11 year period from 1981 till the middle of 1992 are taken, see Appendix C, and the annualized variance of the total world crude prices is calculated to be $\sigma^2 = 0.10574$, This implies that the standard deviation of the developed reserves is $\sigma = 0.32517$.

4.4.3 Expected Exploration and Development Expenditures

Exploration and development expenditures are defined as the expenditures incurred in finding oil and natural gas, excluding the cost of the oil property or lease itself. The cost of drilling exploratory wells comprises a large proportion of the total exploration expenditures. These include the cumulative cost of drilling and equipping dry holes, oil and gas wells. Other exploration costs include the cost of geological and geophysical studies performed to determine the magnitude and depth of the reserves as well as direct overhead expenses of oil companies. About 90 percent of the exploration expenditures are "intangible" and can be expensed for tax purposes. Intangible costs are those items which are expensed and have no salvage value after use or completion of the project and generally possess no physical identity. Examples of such intangible costs are labor, power, fuel, freight and hauling, water and repair expenses. This class of expenses are usually written off as expense items in the income account during the year in which they incur. The remaining 10 percent of the costs are "tangible" costs, and represent physical property and assets which have a salvage value at the end of their life cycles. Examples of such items are the drilling derrick, pipes, cranes and other smaller equipment. These costs are capitalized and retired through annual depreciation charges. Because such a high proportion of the exploration costs can be expensed, the exploration costs are simply multiplied by $1 - \tau$; where τ is the corporate income tax rate to give the actual exploration expenditures. Development expenses are the expenses of drilling and equipping the oil and gas wells, lease and support equipment and development overheads incurred in developing reserves. Approximately 50 percent of the development expenditures in the Gulf of Mexico can be expensed as they are intangibles [22]. The remainder of the expenses are tangible and depreciated. Therefore the after-tax development costs will be about $[1 - (\tau)(0.50)]$ of the actual development expenditures.

Throughout the 1980s there existed a wide gap between U.S. and foreign finding costs. However, because of improved technology and prospect highgrading the cost differential between U.S. and foreign finding costs has been nonexistent in 1991 [24]. The average estimate of the U.S. onshore and offshore finding costs for the period between 1978 and 1991 was $4.97 per barrel of oil equivalent.

The value of offshore petroleum finding costs for the same period was \$6.42. In the absence of more specific information this value of the total exploration and development cost is used as an input in the option valuation. Table 4.2 gives the average total exploration and development cost per barrel of oil equivalent.

Oil companies and contractors bidding for petroleum tracts in the Outer Continental Shelf and the Minerals Management Service of the U.S. Department of Energy have access to more specific information on the expected exploration and development costs for a particular tract of land. The exploration and development cost for the tract is dependent on the expected value and variance of the quantity of reserves in the tract as well as tract specific geological information and the estimated depth of the reserves.

The USGS prepares its estimate of the total exploration and development expenditures for each tract. It is necessary for the purpose of valuation to determine these costs as a function of the reserve size. Assuming a deterministic form of this equation [22] as:

$$D_j = A_j[6Q_{oj} + Q_{gj}]^{\beta} \tag{4.1}$$

where Q_{oj} and Q_{gj} are quantities of oil and gas recoverable reserves on the j th tract, and A_j is a tract specific scaling parameter that would vary from tract-to-tract depending upon the water depth and drilling depth. The term in the square bracket would thus represent the total volume in tcf, of both oil and gas reserves in tract j, using the conversion: 1 barrel = 6tcf. The factor β, is an economy wide scale parameter, set equal to 2/3 which is consistent with studies done on the development costs in the Gulf of Mexico. In order to obtain the parameters, A_j, the following fitting procedure can be used. First, we take the expectation of a second order Taylor Expansion of (4.1), yields,

$$\bar{D_j} = A_j[6\bar{Q_{oj}} + \bar{Q_{gj}}]^{\beta} + (18\sigma_{oj}^2 + \sigma_{gj}^2/2 + 6\sigma_{ogj})A_j\beta(\beta-1)[6\bar{Q_{oj}} + \bar{Q_{gj}}]^{\beta-2} \tag{4.2}$$

where σ_{oj}^2 and σ_{gj}^2 are variances of the oil and gas quantities and σ_{ogj} is the covariance between them, the bars represent the expected values of these quantities. An arbitrary assumption can be made that, $\sigma_{ogj} = 0.5\sigma_{oj}\sigma_{gj}$ and the equation (4.2) can be solved for the equilibrating value of A_j for each tract, using the tract-specific means of the distributions for D_j, Q_{oj}, and Q_{gj}, as estimated by USGS.

4.4.4 Relinquishment Requirements

As of 1984, the DOI began a 5 year OCS Oil and Gas Leasing Program, to cover the period from mid-1986 through mid-1991. As per OCS leasing guidelines, tracts in waters less than 400m would have five year leases, tracts in waters between 400m and 900m would have eight year leases and those in waters between 900m and 3000m would have ten year leases.

4.4.5 Riskless Rate

The riskless interest rate appears in the valuation equation as it is the certainty equivalent of the rate of return of the underlying asset. Because investors are interested in the after-tax rate of return, the appropriate rate of interest taken is the rate of return on tax-free bonds or the after-tax return of treasury securities. The five, eight and ten years maturity U.S. Government securities in May 1992 had yields of 6.88%, 7.34% and 7.58% respectively[3]. Ideally tracts having the same relinquishment date should be valued separately with the appropriate yield rate. Since data on the individual tracts is not available, in this study a pre-tax rate of 7%[4] is assumed as the risk free rate of return. Assuming an effective tax rate of 46%, the after-tax rate works out to be 3.78%.

[3]The eight year yield was interpolated from the seven and ten year maturities.
[4]Five year Treasury instruments had a yield of 6.88% in May, 1992 (the time the lease was bidded).

Oil Exploration and Development Cost
(in Dollars per Barrel Equivalent)

Year	U.S.	Foriegn Average	Worldwide
1978	6.64	4.15	5.14
1979	11.74	6.66	9.78
1980	10.57	6.17	8.46
1981	11.68	5.59	8.53
1982	9.86	8.66	9.44
1983	9.08	5.25	7.20
1984	6.80	3.94	5.60
1985	9.41	5.34	7.48
1986	6.53	4.57	5.52
1987	4.25	3.57	3.88
1988	3.96	4.82	4.94
1989	3.16	4.56	4.68
1990	3.69	3.57	4.10
1991	3.88	6.12	6.07

Average Finding Cost (dollars per barrel):

U.S.: 7.23
Foriegn Average: 5.21
Worldwide: 6.49

Standard Deviation of Finding Cost (dollars per barrel):

U.S.: 3.129
Foriegn Average: 1.387
Worldwide: 1.980

Source: Petroleum Outlook, Greenwich, CT.

Table 4.2: Oil Exploration and Development Cost (1978-1991)

4.4.6 Delta

It can be recalled from Chapter 3, that δ is the payout rate of the producing developed reserve, and is given by:

$$\delta_t = \gamma[P_t - V_t]/V_t \tag{4.3}$$

where, γ is the extraction rate of the producing reserve, P_t is the after tax operating profit from selling a unit of petroleum, and V_t is the per unit value of the developed reserve, both at time t. The after-tax operating profit from production is

$$\begin{aligned} P_t &= M_t - OC_t - \tau(M_t - OC_t - DA_t) \\ &= M_t(1-x) - 0.46M_t(1-x-y) \end{aligned} \tag{4.4}$$

where,

M_t = market price of crude oil, per barrel

OC_t = operating cost, per barrel

DA_t = depreciation allowance, per barrel

$x = OC_t/M_t$,assumed as 0.30 and constant over time, and

$y = DA_t/M_t$,assumed as 0.20 and constant over time.

τ = corporate tax rate, taken to be 48% in the study.

With an extraction rate $\gamma = 0.10$ which is consistent with data taken from that period, the value of δ obtained by solving equation (4.4) is 0.041.

Assuming that combined exploration and development takes place in about a year. Therefore, given this value of δ, $\hat{V}/V = e^{-\delta t} = e^{-0.041} = 0.9598$. The present value of the developed reserves are therefore, \$5.54 per barrel of oil and \$1.54 per tcf gas. These are the values which are used in valuation below.

4.5 Comparing the Results

As mentioned above, the OCS lease sale number 139 is evaluated using the option pricing model described in Chapter 3. The inputs to the valuation equation as well as the value of the lease is tabulated in Table 4.3. The estimate for the quantity of oil and gas reserves is obtained from the Gulf of Mexico Update: May 1988-July 1989 by determining the estimated oil and gas reserves in each of the tracts. For OCS Lease Sale No.139, these reserves are estimated to be around 25 million barrels of oil and 5 billion cubic feet of natural gas. The total after-tax value of the exploration and development expenditures are taken to be $3.88 per barrel of oil equivalent. The value of this OCS petroleum lease in the central Gulf of Mexico as calculated by the option pricing formula is $51.6 million. The total high bids for the 151 tracts leased in this sale was $56.9 million.

4.6 Sensitivity Analysis

On way to verify the appropriateness of the option valuation method in valuing petroleum leases as well as examine the effect of the variables driving this value is to perform a sensitivity analysis on these variables. The value of the OCS lease above has been calculated based on imperfect and uncertain information on several of the input parameters of the option valuation equation. Though the option valuation approach to valuing leases eliminates the need to estimate reserve values at the time of extraction and thus the expected rate of return, many of the other uncertainties with respect to geological and other risks remain. It is thus interesting to study how sensitive the lease value is to each of these factors. In order to study the effect of each factor independently, each of the input variables is analyzed separately keeping all other variables constant. The results of the analysis and its interpretation is presented in the section below.

Valuation of OCS Lease Sale No:139

Inputs to the Valuation equation:

a) Value of the Developed Oil Reserve: (Vo) (dollars per barrel)		$5.54
Value of the Developed Condendate Reserve: (Vg) (dollars per thousand cubic ·		$1.58
b) Estimated Quantity of Developed Oil Reserves:(millions of barrels)		25
Estimated Quantity of Developed Gas Reserves:(billions of cubic feet)		5
Total Expected Value of Developed Reserve(V):		$146,400,000.00
c) Variance of the Underlying Asset (Developed Reserve):		0.10574
d) Relinquishment Date of Reserves: (years)		5
e) Riskless Interest Rate		7.00%
Tax-Rate		46%
After-tax Riskless Rate:		3.78%
f) Expected Finding Cost per barrel of Oil equivalent:		$3.88
Total Expected Finding Cost (D):		$97,000,000.00
g) Calculation of Delta:		
Market Value of Crude Oil Per Barrel: M	$17.31	
Operating Cost/Market Value of Crude:	0.30	
Depreciation Allowance/Market Value of Crude:	0.20	
Tax rate:	46%	
Extraction Rate of Oil and Gas Reserves:	0.10	
After-tax Operating Profit: P	$8.14	
Delta:		0.041
Value of OCS Lease No:139		$51,637,713.21

Table 4.3: Valuation of OCS Lease Sale No. 139

4.6.1 Sensitivity to Value of Developed Reserve

The total value of the developed reserve is the product of the value per unit of these reserves at the inception of the lease and the estimated quantity of the developed reserves. Figure 4.3 shows how the lease value varies as a function of the current developed reserve price. The value of the developed reserve is varied from $3 per barrel of oil equivalent to $12 per barrel. The corresponding value of the lease varies from approximately $15 million to $170 million. Since it is assumed that developed reserve returns are lognormally distributed, and the variance of these returns is estimated to be 0.1057, the 95 percent interval for developed reserve values is $8.70 and $32. Thus, it can be concluded that with a 95% confidence level, the value of the petroleum lease will lie between $15 million and $145 million.

Figure 4.4 illustrates how the petroleum lease value changes with the quantity of the developed reserves. For the OCS Lease No. 139, the estimated quantity of oil reserves is estimated to be 25 million barrels while the estimated quantity of natural gas is estimated to be 5 billion cubic feet. Assuming, a fixed value of the price of developed oil and natural gas reserves of $5.54 per barrel of oil and $1.58 per thousand cubic feet of natural gas it can be seen that the value of the lease varies from a high of $265 million for 60 million barrels of oil equivalents found to a low of zero if the developed reserves were only $5 million barrels. Thus with a 25% estimation error in the quantity of oil on the lower side, the value of the lease decreases by 36%; with 50% estimation error, the value falls by 76%. If on the other hand the quantity of reserves are greater than estimated, the value of the lease increases by approximately 50% for a 25% increase in reserve quantity and further increases to 86% of its current value when there is a 50% error in estimation.

4.6.2 Sensitivity to Volatility

Figure 4.5 indicates the sensitivity of the lease value to the volatility of developed reserve prices. The value of the developed reserve is calculated for various levels of volatility of crude oil prices, from zero volatility to 75% volatility. The value of the OCS lease is then calculated for three different values of the total developed reserve value. It is seen that for a reserve value of $50 million which is less than the exercise cost of $97 million, the value of the lease for zero volatility is zero. This is because if there is no possibility of price movement of the underlying reserve (implied by zero volatility) then the option has zero value as the exercise price is greater than the current value of the developed reserves. For the other case, when the value of the developed reserve is taken as $150 million, the value of the petroleum lease with zero volatility is $53 million which is exactly $V - K$ where $V = 150$ and $K = 97$. With increasing volatility, however, the value of the lease increases and at a 75% volatility level this value reaches approximately $80 million. The third case when the developed reserve value is taken as $250 million similar to the second case.

4.6.3 Sensitivity to Exploration and Development Costs

Greater the exercise price of the lease, lower is the value of the lease. In the case of the stylized OCS lease considered above, the exercise costs are assumed to be comprised of only the total exploration and development expenditures. In reality all costs associated with the lease must be accounted for and included in the exercise cost. A sensitivity analysis is performed to study the effect of changing exercise costs to the value of petroleum lease. Once again, three different scenarios are considered, for total developed reserves equal to $50, $150 and $250 million. Keeping all other variables constant, Figure 4.6 illustrates the effect of changing exercise costs to the value of the lease.

4.6.4 Sensitivity to Risk Free Interest Rate

Interest rates are not constant. The option valuation technique implies a flat term structure of interest rates through the duration of the option. Figure 4.7 shows how the value of the developed reserve varies with different interest rates. The rate considered is the after-tax interest rate as that is the rate used in the valuation. A tax rate of 48% is also assumed for the purpose of the sensitivity analysis.

4.6.5 Sensitivity to Delta

Delta as described in Chapter 3 is defined as the payout rate of the developed reserves. This value of δ is dependent on the rate at which the reserves are extracted. Figure 4.8 shows how this value of δ varies with different reserve extraction rates. an extraction rate of 10% is assumed in the valuation of the lease which is consistent with other leases in the Outer Continental Shelf. It can be seen that the value of δ varies from nearly zero for a 2.5% rate of extraction to approximately 0.4 for a 100% extraction rate (implying instantaneous removal of all reserves. Figure 4.9 then illustrates as to how the lease value changes with different values of the extraction rate. Thus with a very small extraction rate, the value of the reserves is almost $70 million which exponentially decreases to zero for the case of 100% extraction rate.

Figure 4.3 Sensitivity of Lease Value to Value of Developed Reserve

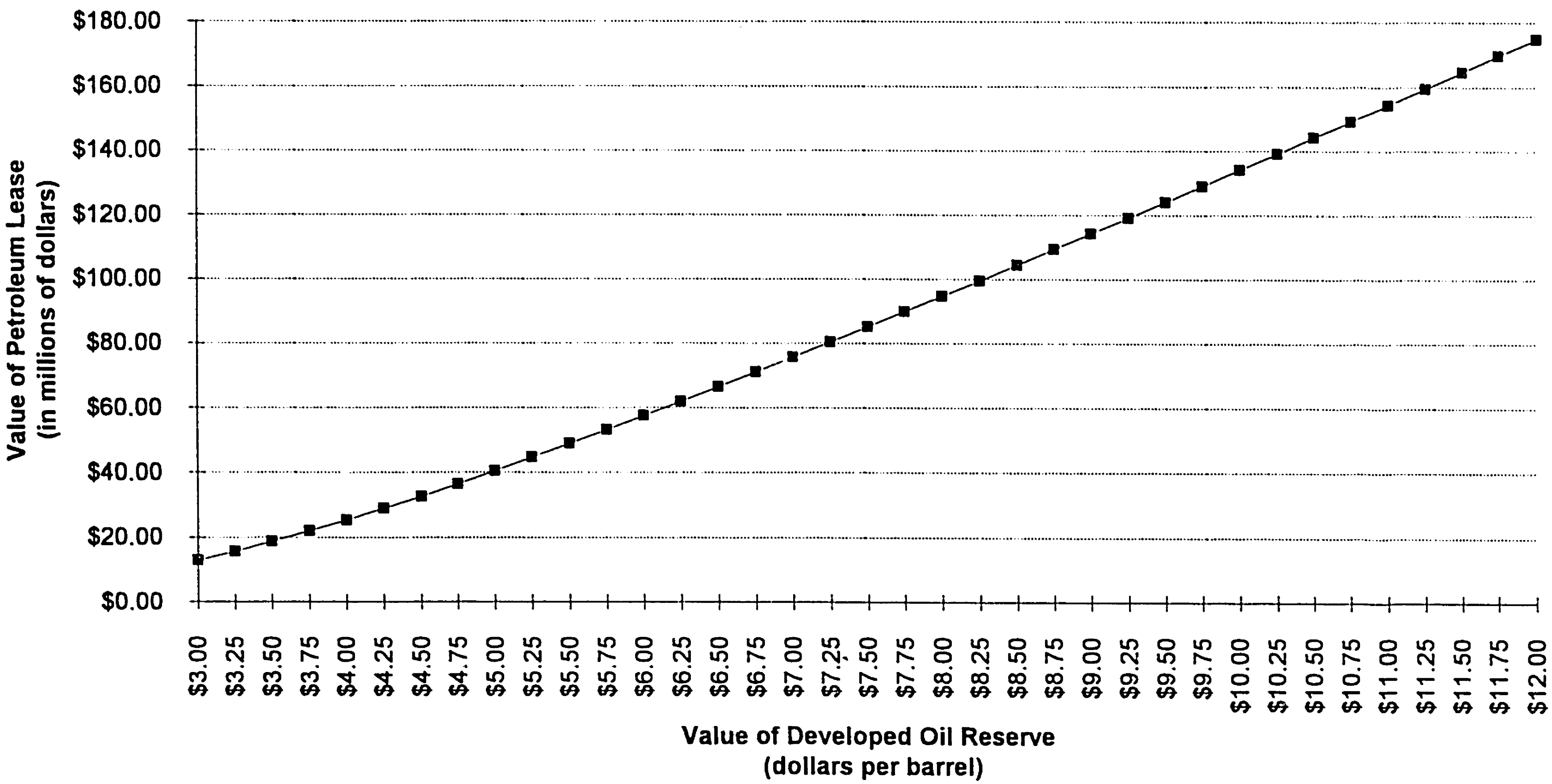


Figure 4.4 Sensitivity of Lease Value to Quantity of Estimated Reserves

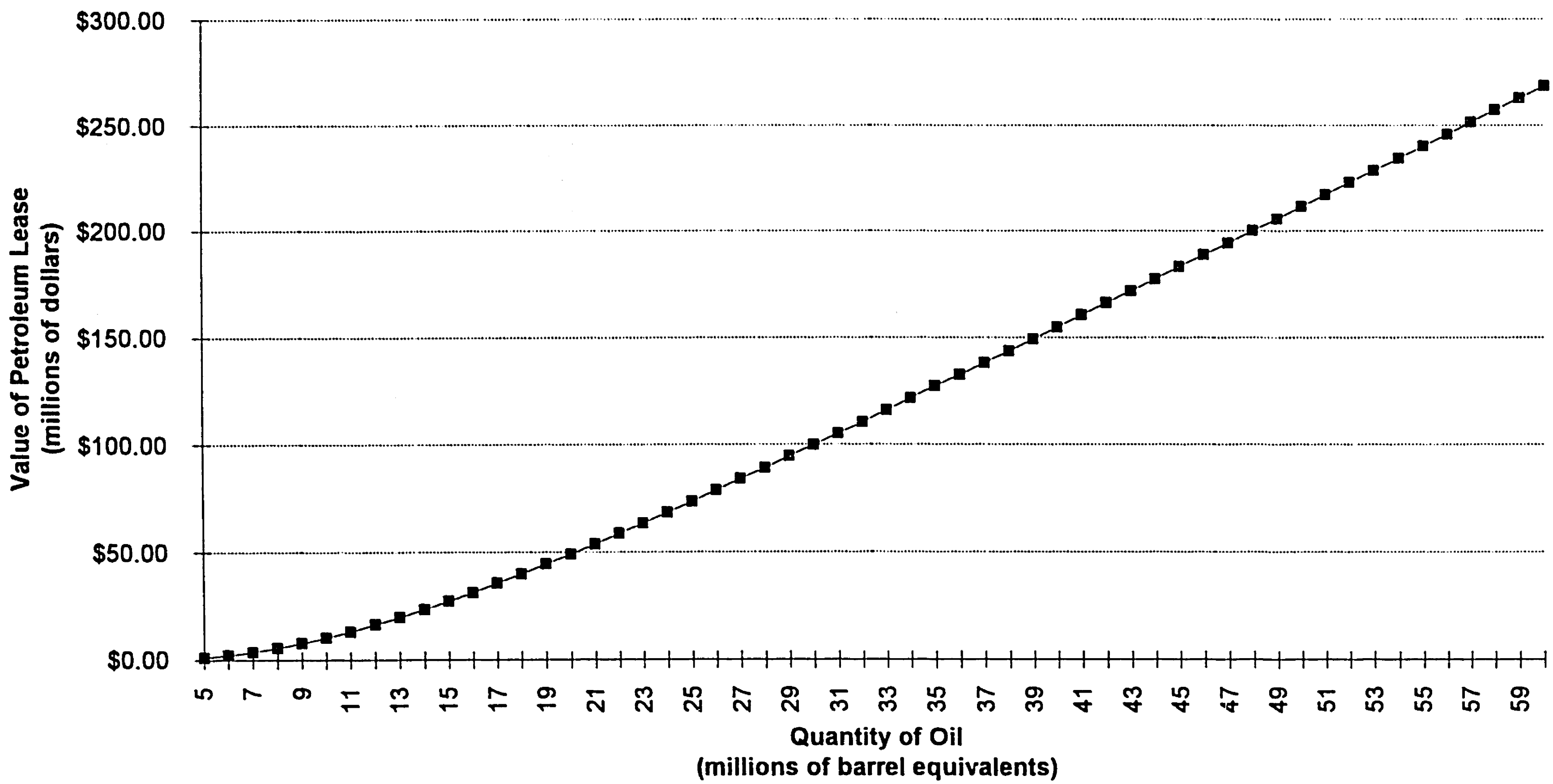


Figure 4.5 Sensitivity to Variance of Developed Reserve Prices

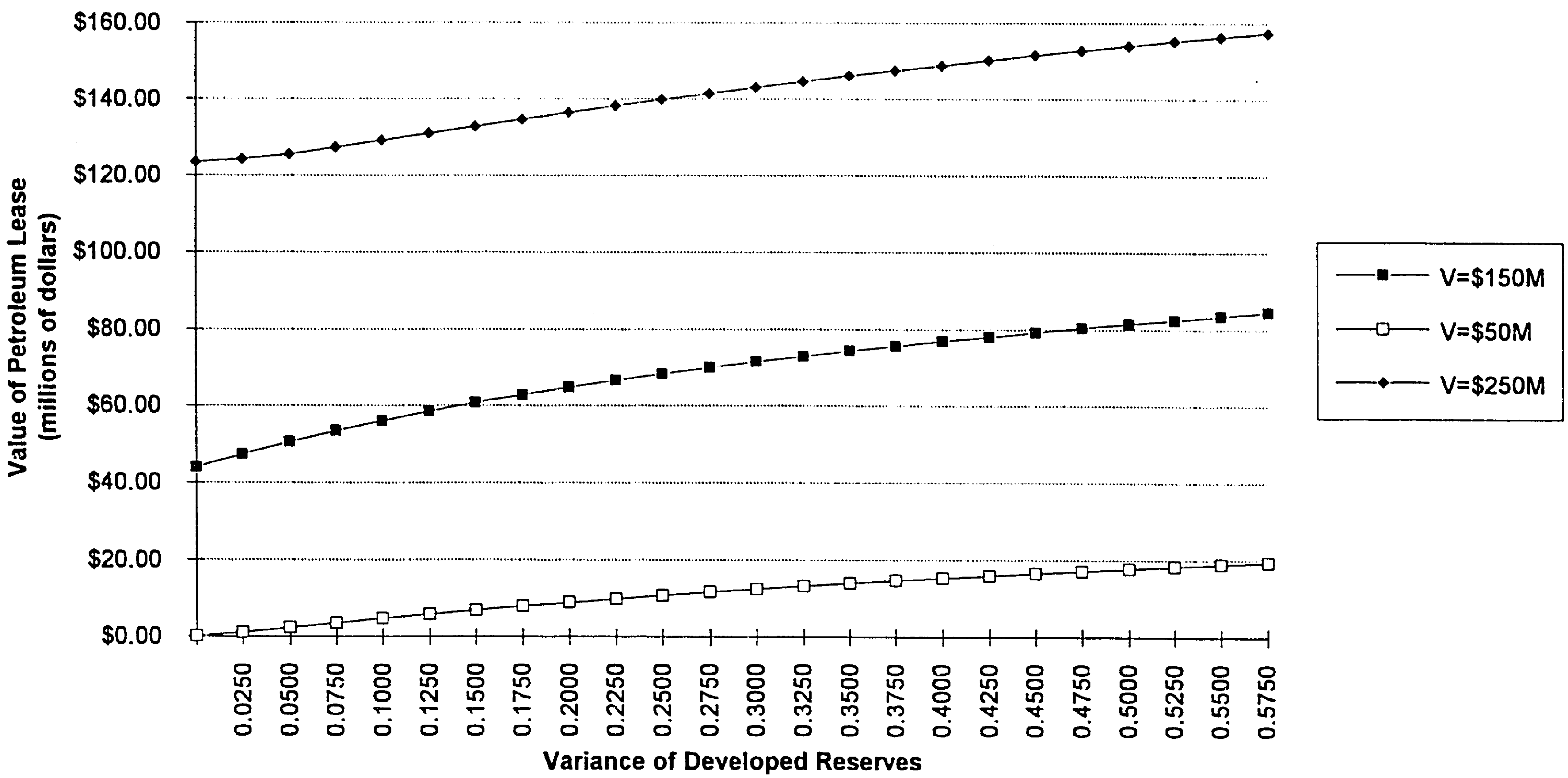


Figure 4.6 Sensitivity of Lease Value to Finding Expenditures

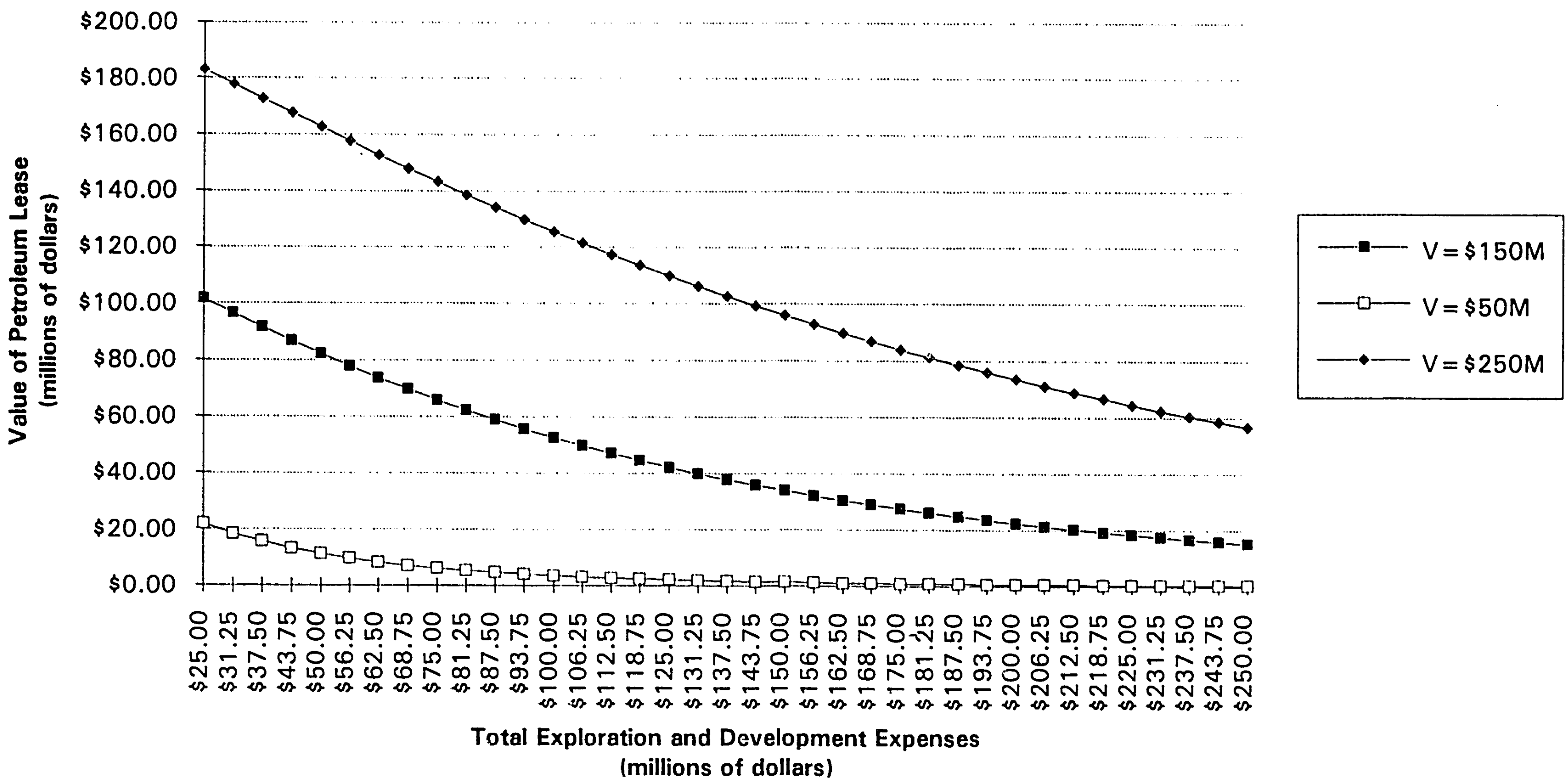


Figure 4.7 Sensitivity of Lease Value to Interest Rates

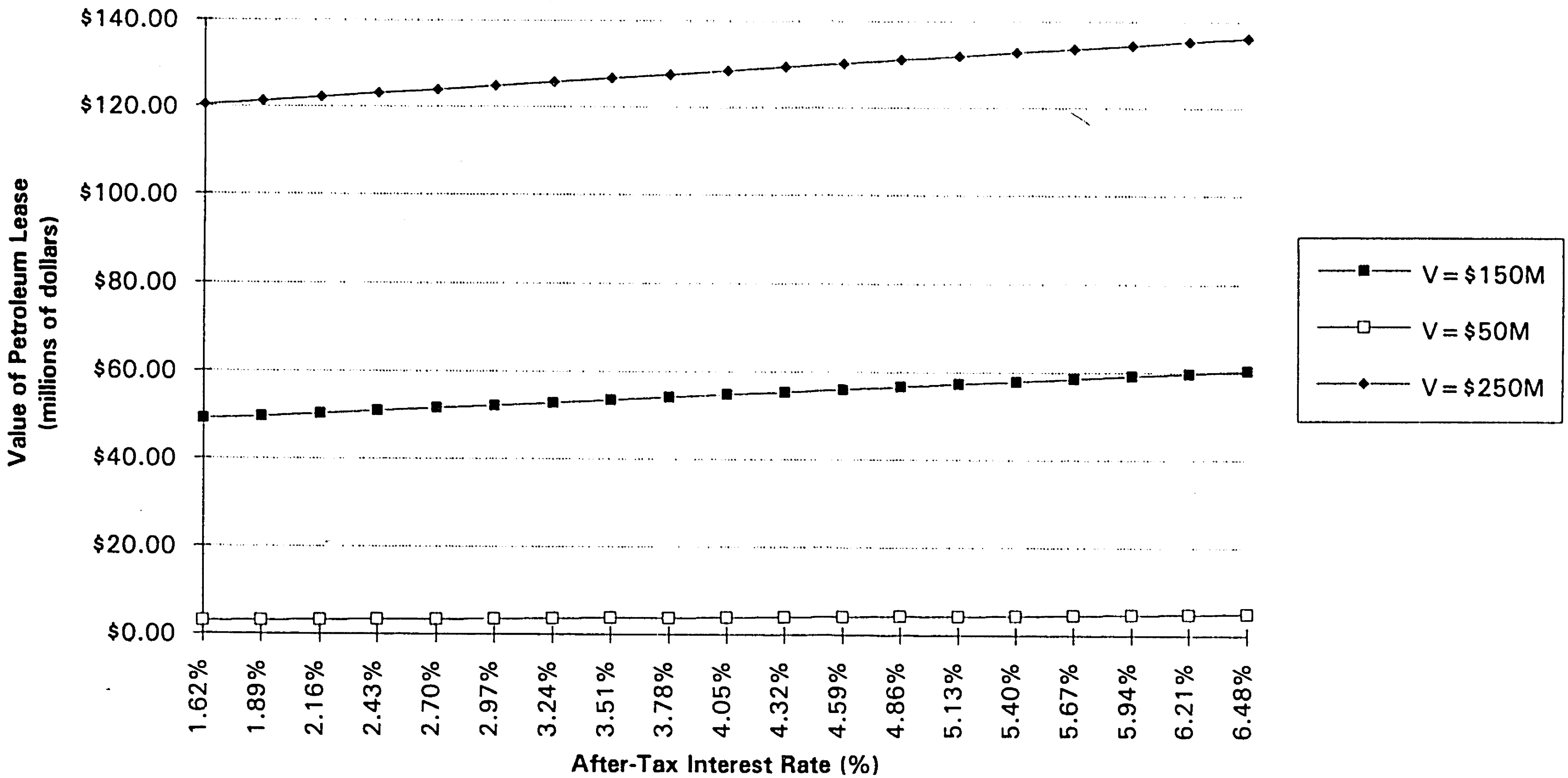


Figure 4.8 Sensitivity of Delta (Payout Rate) to Extraction Rate

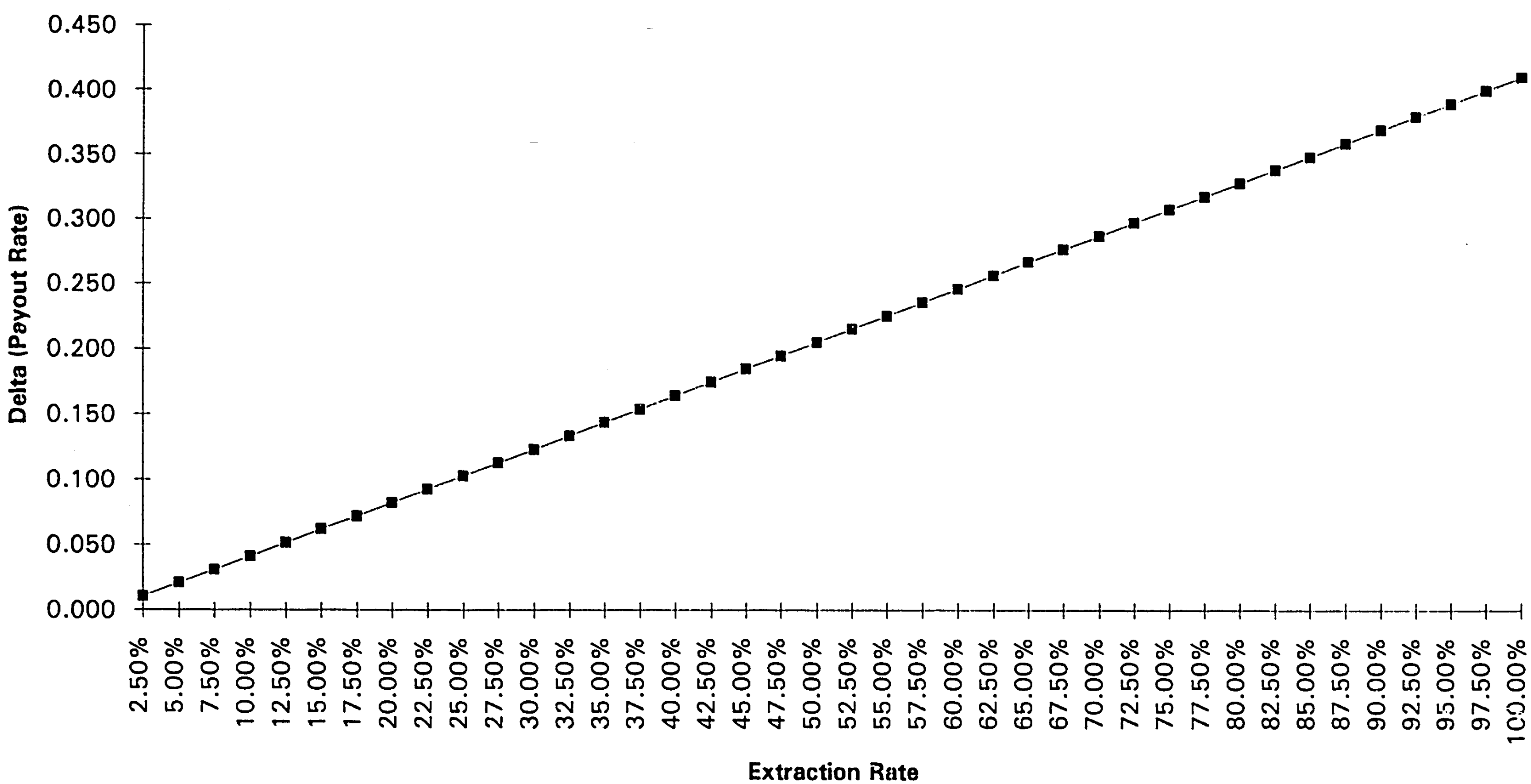
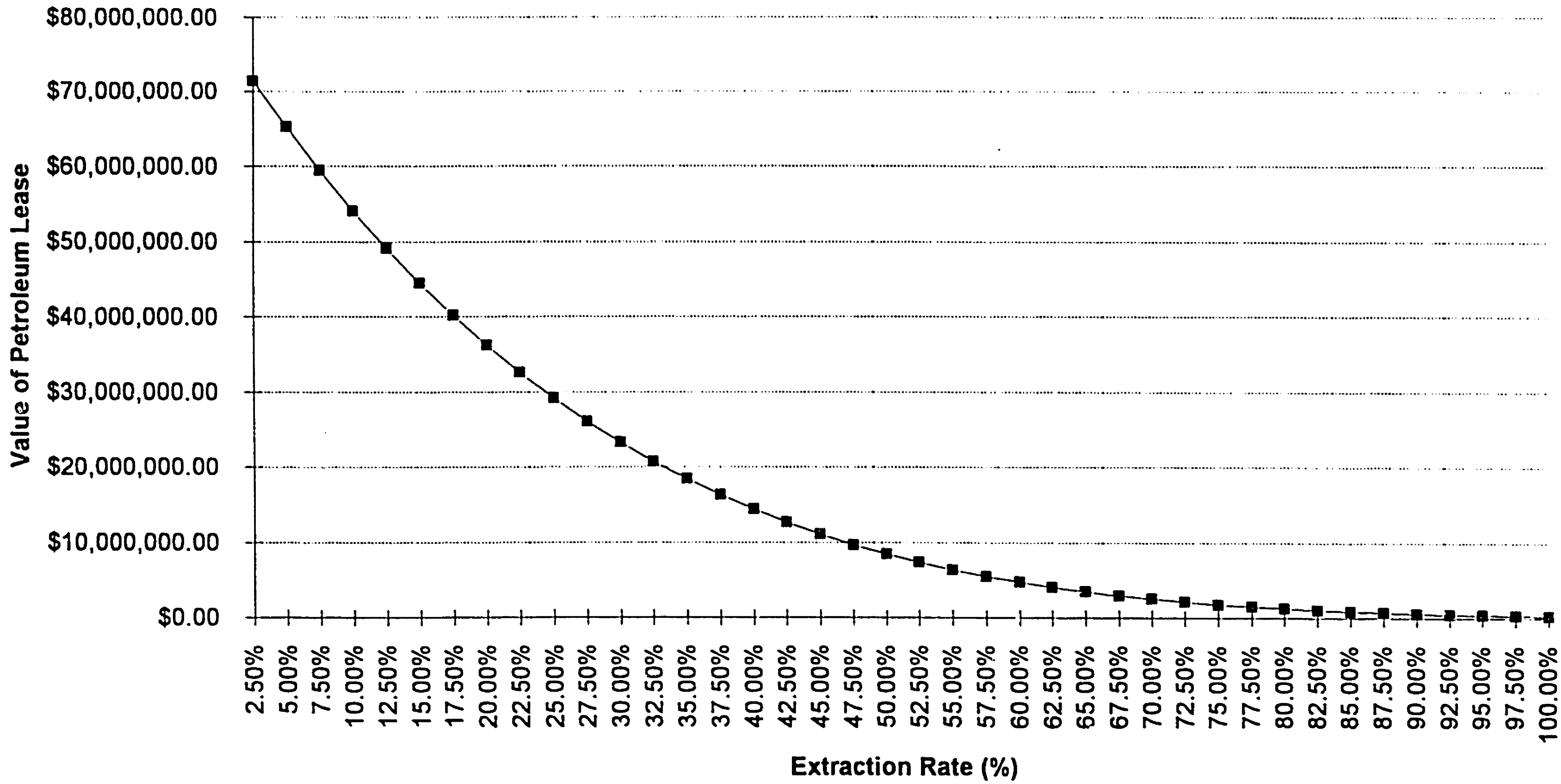


Figure 4.9 Sensitivity of Lease Value to Extraction Rates



Chapter 5

Option Valuation of a Joint Petroleum Venture-Tenghiz Project in Kazakhstan

After examining the technique of applying option valuation methodology in valuing a stylized lease in the Outer Continental Shelf of the United States, I shall extend the application of this technique in valuing petroleum exploration and development ventures by U.S. oil companies overseas. In particular the joint venture between Chevron Corporation and the former Soviet republic of Kazakhstan to develop the Tenghiz oil field is considered for valuation. Further assumptions need to be made when applying option valuation theory to evaluate overseas joint ventures. The next section gives a brief background on Kazakhstan and important aspects of its current and future economic and fiscal policies[8]. After this I describe the contractual details of the venture between Chevron and Kazakhstan and explain the option value inherent in the contract. Section 5.3 then discusses additional factors and assumptions which need to be made in applying option valuation theory in valuating this joint venture. The various inputs to the option valuation equation[1] are then quantified in Section 5.4 and a value calculated for this project. Finally, I conclude this chapter with a discussion on the appropriateness of using this technique in valuing such petroleum projects, the reasons of bias and the attractiveness and limitations of using this technique in these cases.

5.1 Background

Kazakhstan has a land area the size of Western Europe and stretches from the Caspian Sea to China (Figure 5.1). It is one of the major republics of the former Soviet Union and ranks second in size (2.7 million square kilometers) after the Russian Federation, third in output (4 percent of the net material product (NMP) of the U.S.S.R. and fourth in population (approximately 17 million people). The population comprises of roughly 40 percent ethnic Kazakhs, 40 percent Slavs (mainly ethnic Russians) and 100 other ethnic and national groups. Kazakhstan has about one-fifth of the combined arable land of the former U.S.S.R. and thus a significant proportion of its NMP (37 percent0 in 1991 comprised of agricultural products. It also possesses substantial mineral resources and has developed large-scale mining and processing activities. In a 1990 estimate, it had 90 percent of the total U.S.S.R. reserves of chrome, and nearly 50 percent of the reserves of lead, wolfram, copper and zinc, 19 percent of the total U.S.S.R. coal production and 7 percent of its petroleum production. The country also has an adequate infrastructure, well developed cultural and health facilities and major secondary and higher education institutions. Currently the public sector dominates all the economic activity. Out of an estimated stock of fixed assets worth over 200 billion rubles in 1991, about 90 percent was state owned, while the rest was equally divided between cooperatives and the private sector. Kazakhstan's national parliament enacted the

[1] Refer to Chapter 3

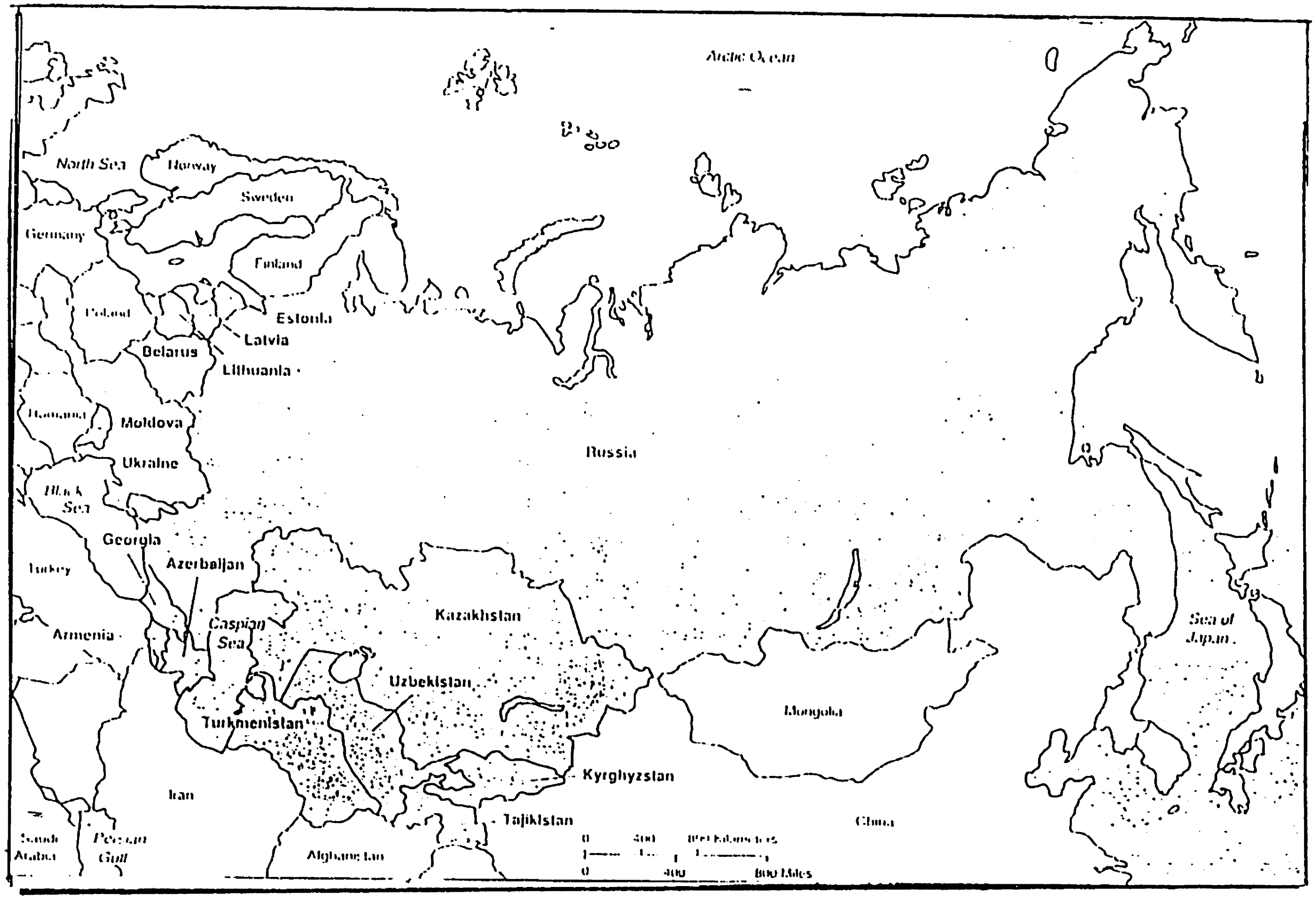


Figure 5.1 Geographical Map of Soviet Republics

law declaring independence from the Soviet Union on December 16, 1991. In the first presidential elections Nursultan Nazabayev was elected President. The policy under his regime has been to endorse economic reforms and transform to a market-based economic system. The strategy pursued by the newly formed government is based on five key principles: (1) to allow price liberalization to play an essential role in the allocation of resources and the generation of a supply response; (2) to promote the development of a private sector through an extensive privatization program and liberal investment conditions; (3) to aim at price stability through tight financial policies within the context of a common monetary and exchange rate policy with other members of the ruble area; (4) to endeavor to maintain free trade, in part through coordinated efforts with other republics of the former U.S.S.R.; and (5) to contain the impact of the transition upon selected groups of the population through creation of an adequate social safety net. There have been several important price and structural reforms which have been implemented in the new republic since 1992. The government has let the prices of most goods and services be set freely according to market conditions. Following a long period of stable and centrally fixed prices, average retail prices rose by 84 percent in 1991. The economic reform called for a two- to threefold increase in most of the prices that remained under control. Even though there have been large price increases, they have not been sufficient to close the wide deviations with respect to either border prices or long-run incremental costs of production. This is notable in the case of crude oil where even though the pre-tax wholesale price of crude has increased fivefold to 350 rubles per ton[2], it still remains far below the current world price for comparable crudes. There has also been an accelerated privatization program which called for upto 50 percent of assets in the industrial sector, 40 percent in agriculture and all assets in housing and the services sector to be privatized. During 1991, the government sold 380 enterprises, mainly in the services sector with gross

[2]Prior to the introduction of the dual exchange rate in Russia, the interbank rate is Kazakhstan was about 30 rubles per U.S. dollar; since then it has been below the rate arising from the foreign exchange auctions held by the Central Bank of Russia.

assets valued at about 1 billion rubles. During this initial stage no foreign participation was allowed in the privatization, except for nationals of other states of the former Soviet Union. Since then however, foreign participation in some of these programs has been considered. There has also been a massive reform in Kazakhstan's fiscal policy. On the revenue side, the 1992 budget was based on important changes in the tax system which included new taxes and increases in certain tax rates. Among the indirect taxes, there was introduced a combination of excise taxes and a value added tax (VAT) at a rate of 28 percent which represents a major fiscal reform.

5.2 Details of the Venture

The joint venture between Chevron Corporation and the Republic of Kazakhstan was agreed to in May 1992. The joint venture company formed by the venture is called Tengizchevroil, or TCO and it is owned jointly by Tengizneftegas, a subsidiary of Kazakhstan's national oil company, and the Chevron Overseas Company, which is a subsidiary of Chevron. The 40- year long joint venture, TCO, which begun operations in January 1993 involves exploration, development and production in an area of about 4000 square kilometers in the pre-Caspian basin and includes both the Tenghiz and the Korolev oil fields. Once the venture is profitable, Kazakhstan will receive 80 percent of the profits and Chevron the remaining 20 percent. Under the financial terms of the 40-year venture, TCO will pay the Government of Kazakhstan bonuses, taxes and royalties and $50 million for regional development.

It is estimated that the project will involve an initial investment over the first three years of $1.5 billion by the joint venture partners. The ultimate investment over the 40-year period could total $20 billion. The option value inherent in this project is that the joint partnership TCO has been given the option to explore and develop these two oil fields by making an initial investment of $1.5

billion. This is the value these firms invested and could be looked as the call price of the option. It has also been estimated that a production level of at least 250,000 barrels per day would be reached within 5 years of the project. Here, the simplifying assumption is made that all exploration and development expenditures are completed during this five year period. All other expenditures after this are taken to be lifting and day-to-day operating costs as well as the cost of transporting this oil. Thus for the purpose of this evaluation the option inherent in this joint venture is assumed to have a call price of $1.5 billion and a relinquishment requirement of 5-years.

5.3 Assumptions

The option pricing model[3] assumes that the exercise price of the option is known with certainty. However, in this case of option pricing there is a large uncertainty regarding the exercise costs. These costs may themselves be stochastic processes which also depend on the value of the quantity of reserves discovered. An extension to Black-Scholes formula given by Fischer may be used in this case for an option with a stochastic exercise price. For the purpose of this valuation, the exercise price is assumed fixed and independent of the reserve size. A sensitivity analysis is performed to indicate how the value of the lease would change with different finding costs. There are also uncertainties about the quantity of hydrocarbons in the field. Since development costs are a function of the quantity of reserves, it is expected that uncertainty about the reserve quantity would create the same affect on option value as stochastic unit development costs would. In our case, however, since development costs are assumed to be independent of the quantity of reserves, the expected value of the reserve quantity is used in the option valuation equation. There are also certain additional risks which an international oil company operating in the republic of Kazakhstan would need to take into consideration. Primary among them are tax uncertainties or changes in ownership rights and in the contract. These risks can together be grouped as political risks and are discussed further in Chapter 6. Another risk which they would likely to face is the exchange rate risk. A great portion of this kind of risk is dependent on the contractual arrangements between Chevron and Kazakhstan. This risk can be reduced for Chevron if it is allowed to export whole or part of its oil share at market prices. Other possible sources of risk for Chevron are the fact that it is one of the first countries to carry out a joint venture of this scale in Kazakhstan and it is operating under absolutely unknown conditions. A host of possible issues such as labor unrests, transporting and refining the petroleum difficulties can arise and all of these need to be evaluated before the start of the project.

5.4 Inputs to the Valuation Equation

The inputs to the option valuation equation are as described below:

5.4.1 Estimate of the Petroleum Reserves

The Tenghiz oil field is located in the pre-Caspian basin. The pre-Caspian basin is one of the largest depressions in the world, spanning an area of more than 500,000 square kilometers.(Figure 5.2) It is bordered to the north and the west by elevated structural elements of the southeastern Russian platform, and to the east, southeast, and south by Hercynides fold belts. During the submergence over the period of hundreds of millions of years from the upper Proterozoic- lower Paleozoic to the present, the pre-Caspian basin has accumulated an enormous sequence of sediments ranging in thickness from 5 km in the marginal zones to 20 to 24 km in the central region. With its in-place reserves estimated at 25 billion barrels of oil and 46 trillion cubic feet of gas, the field by most estimates is one of the supergiants of the 1980s when it was discovered. Chevron estimates that about six billion to nine billion barrels of these are recoverable. It is estimated[3] that within 5 years of signing the agreement, production would reach 250,000 barrels per day. The project's program calls for an initial investment of $1.5 billion spread over 3 years and $20 billion during the life of the project. TCO expects that the Tenghiz field will reach its peak production of approximately 700,000 barrels of oil a day by 2010. Currently the field produces about 65,000 barrels a day.

5.4.2 Developed Reserve Value

The value of a barrel of OPEC crude oil in the international market on May 1992 was $17.03. The wholesale value of this oil in Kazakhstan was very different from these market levels during the same period. The pre-tax wholesale price of crude oil in Kazakhstan was increased fivefold from its pre-reform (pre-1991) level and fixed at 350 rubles per ton[4]. Prior to the introduction of the dual exchange market in Russia, the interbank rate in Kazakhstan was about 30 rubles per U.S. dollar. Thus the value of a barrel of crude oil sold in the wholesale market in Kazakhstan works out to be just $1.60. As can be seen from above, this value is well below the current world or OPEC price for comparable crudes. Assuming as before that developed reserves prices were roughly a third of crude oil prices the value of developed reserves in Kazakhstan would be $0.55 per barrel[5]. Since there is no detailed contractual information available regarding the amount of hydrocarbons Chevron can sell for market prices (approximately $18 per barrel of crude) It is initially assumed for the purpose of the valuation that the joint venture partners (Chevron and Tengizneftegas) would be required to sell all developed petroleum reserves to the republic of Kazakhstan at the wholesale prices mentioned above. If the above assumption that petroleum reserves would be sold to the republic and payment for the same was to be received in rubles,

[3]Oil and Natural Gas Journal, October 19, 1992 pp 26

[4]1 barrel of crude oil is equivalent to 0.137 metric tons.

[5]Compare to the value $5.77 in the case of the OCS petroleum lease.

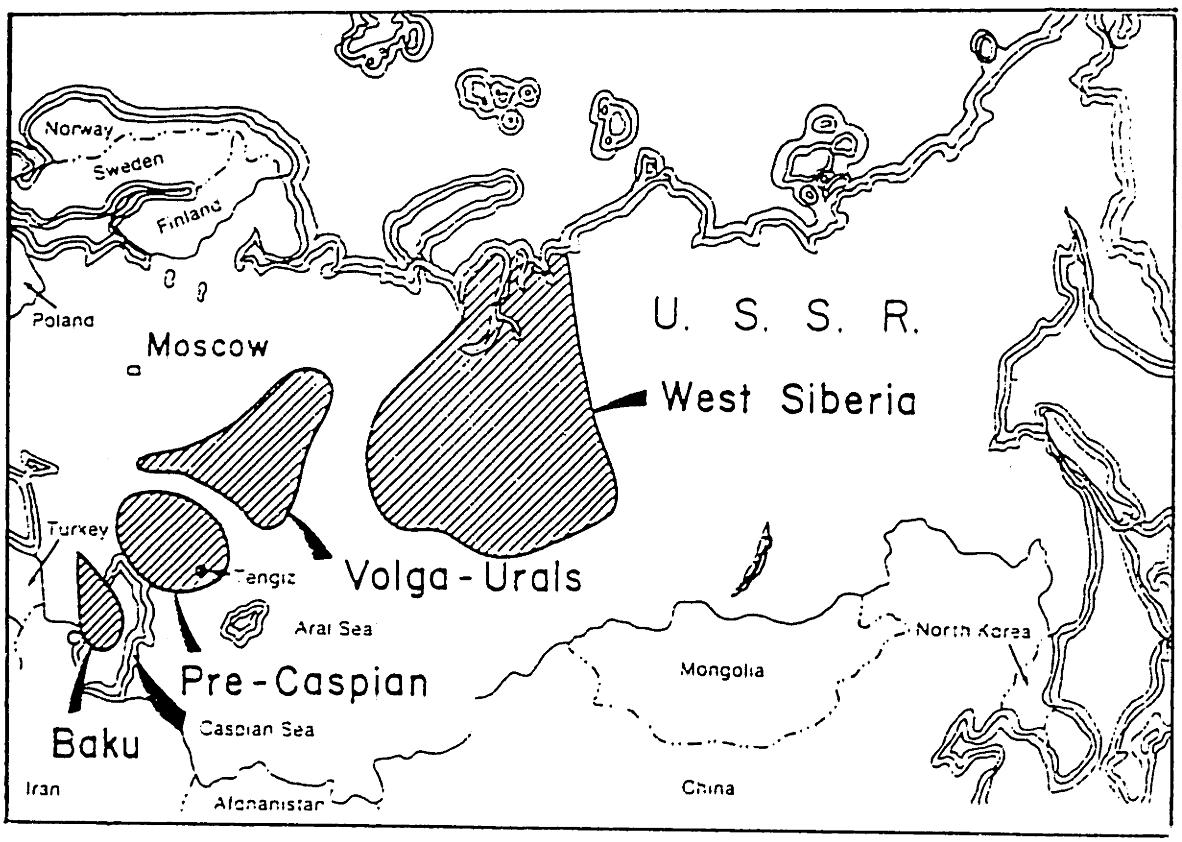


Figure 5-2: Basins in which early Soviet oil exploration was conducted, showing location of the Tenghiz field

Chevron would be faced not only with the usual risks associated with petroleum exploitation but also with currency risk. Since in recent times the ruble has proved extremely volatile, this would be an important risk factor which Chevron would need to calculate. The company would need a way to hedge this exposure to ruble fluctuations which could potentially prove to be extremely expensive. Thus in order to study different scenarios, where Chevron would be allowed to sell part or whole of its share of reserves at market prices a sensitivity analysis (see Figure 5.3) is performed to observe how the value of this joint venture project varies with different developed reserve prices.

5.4.3 Variance of Reserve Prices

It is difficult to obtain an accurate estimate of the volatility of the reserve prices since there exists no historical data on crude oil prices in Kazakhstan. Prices in Kazakhstan and the U.S.S.R. in general were stable and centrally fixed prior to 1991 and did not bear any correlation with free market prices. Even under the current reforms, the prices in the energy sector are regulated by the state. The pre-reform prices of oil, gas condensate and natural gas have been multiplied by an adjustment coefficient of five. This means that current rates have increased fivefold to more correctly represent freer markets but they are still controlled by the government. For the purpose of this valuation, the volatility of OPEC crude oil prices is used and a sensitivity analysis performed (Figure 5.5). The option valuation theory assumes constant volatility throughout the life of the option. This assumption may not very well hold true in here as it is quite likely that there could be further reforms to the price structure of petroleum in Kazakhstan in the next few years which could cause dramatic shifts in the price volatility. Several factors responsible for this change are possible- changes in labor and production costs, changes in world selling prices of crude, changes in general economic prospects and market sentiment which may alter the volatility of the crude prices and changes within Kazakhstan as it moves towards a market economy. Since all of these factors are uncertain, it is not possible to predict future volatility with any certainty. Thus in the absence of any information, a value of 0.1275[6] is taken as an estimate of the annual variance of the crude oil prices, which is equivalent to a standard deviation of 35.70%. Again while determining the volatility of developed reserves, no consideration has been given to volatility of exchange rate movements of the ruble. In the unlikely scenario where Chevron's entire returns are in rubles, the exchange rate volatility of the ruble would also need to be estimated and incorporated in the volatility used in the option valuation equation.

5.4.4 Expected Development and Exploration Costs

Exploration and development costs in Kazakhstan would in general be expected to much below the exploration and development costs in the United States. This is one of the biggest factors driving overseas investment in oil and gas exploration. There are several reasons to account for this difference. One reason is the larger size of the fields in Kazakhstan compared to the U.S. This is responsible for economies of scale which cause lowering of the finding costs. Secondly, since many of these fields are recent discoveries, the petroleum is found closer to the surface and thus extracted more cheaply. Another plausible reason for increased overseas investment is that the U.S. is a mature producing area and many areas in the Outer Continental Shelf which have large estimated reserves are not available for leasing due to a Congressionally-imposed moratoria. Environmental regulations have also been responsible for the increased cost of doing business in some areas where drilling is permitted. It is difficult to obtain an estimate of the finding costs in Kazakhstan. Based on finding costs in other OPEC countries, like Kuwait and Saudi Arabia, a fairly good estimate of the finding costs can be estimated. For the purpose of this valuation, the equivalent finding cost per barrel of oil is taken as $0.50. A sensitivity analysis on this

[6] Appendix C: Volatility estimates of OPEC crude oil prices

value is also performed (Figure 5.6).

5.4.5 Relinquishment Requirements

The total length of the project is taken to be 40 years. However,from the contract above, it can be seen that production is expected to begin after 5 years and reach a value of 250,000 b/d. This production is then going to peak to 700,000 b/d by the year 2010. In order to evaluate this project using the option valuation theory, the time to relinquishment of the option is taken to be 5 years.

5.4.6 Riskless Rate

Interest rates for deposits held in commercial banks in Kazakhstan for long term investments were 10 percent per annum at the end of 1991.

5.4.7 Tax Rate

One of the biggest uncertainties in joint ventures with the former republics as well as Russia in the area of petroleum projects is the tax uncertainty. Tax issues play a key role in defining the economic viability of a project, and large changes in tax structure causes the entire project to be re-evaluated. In face of uncertainties in the tax structure international partners would tend to delay their investment if the anticipated tax increases are liable to wipe out future profits. There are several important changes in the tax system since the January 1992 price reforms. These have included new taxes as well as increases in certain tax rates, and reflected an expected positive impact of the price liberalization on indirect taxes. In Kazakhstan, there are compulsory payments to the state of all foreign exchange proceeds of resident corporations. The surrender requires that 70 percent of all state ordered export proceeds are given to the state. For production beyond the state orders, also called as the "free output", in excess of 5 percent of the state orders, 50 percent accrues to the state. Since the development project is a joint venture between Chevron and the republic, Chevron would face an effective tax rate of 50 percent. This value of the tax rate is used in the valuation below.

5.4.8 Delta

It can be recalled from Chapter 3 and Chapter 4, that δ is the payout rate of the producing developed reserve, and is given by:

$$\delta_t = \gamma[P_t - V_t]/V_t \tag{5.1}$$

where, γ is the extraction rate of the producing reserve, P_t is the after tax operating profit from selling a unit of petroleum, and V_t is the per unit value of the developed reserve, both at time t.

The extraction rate γ in this case may be taken as roughly 3 percent which is consistent with the information available on the existing production of the estimated reserves. Keeping all other factors consistent as in the earlier case, the value of δ calculated from equation (4.4) is 0.0105

Assuming that combined exploration and development takes place in about a year. Therefore, given this value of δ, $\hat{V}/V = e^{-\delta t} = e^{-0.0105*3} = 0.9690$. The present value of the developed reserves is therefore, \$0.53 per barrel of oil. This value is used in valuation below (Table 5.1).

5.5 Value of the Project

The value of the petroleum project from the perspective of TCO and calculated in Table 5.1 below is \$1.8 billion for the initial exploration and development stage. This is the price of the option to undertake this project. As mentioned above, TCO initially has to make an investment of $1.5 billion if the first three years of the project which is analogous to the paying the value of the option to explore and develop the field.

Valuation of Joint Venture in Kazakhstan		
Inputs to the Valuation equation:		
Value of the Developed Oil Reserve: (Vo) (dollars per barrel)		$0.53
Expected Quantity of Developed Oil Reserves:(millions of barrels)		9000
Total Expected Value of Developed Reserve:		$4,750,000,000.00
Variance of the Underlying Asset(Developed Reserve):		0.1275
Relinquishment Date of Reserves: (years)		5
Riskless Interest Rate		10.00%
Tax-Rate		50%
After-tax Riskless Rate:		5.00%
Expected Finding Cost per barrel of oil equivalent:		$0.50
Total Expected Finding Cost:		$4,500,000,000.00
Calculation of Delta:		
Market Value of Crude Oil Per Barrel: M	$0.53	
Operating Cost/Market Value of Crude:	0.3	
Depreciation Allowance/Market Value of Crude:	0.2	
Tax rate:	50%	
Extraction Rate of Oil and Gas Reserves:	0.03	
After-tax Operating Profit: P	0.2385	
Delta:		0.0105
Value of Project as per Option Valuation:		$1,300,000,000.0

Table 5.1: Valuation of Petroleum Joint Venture in Kazakhstan

Figure 5.3 Sensitivity of Project Value to Value of Reserve

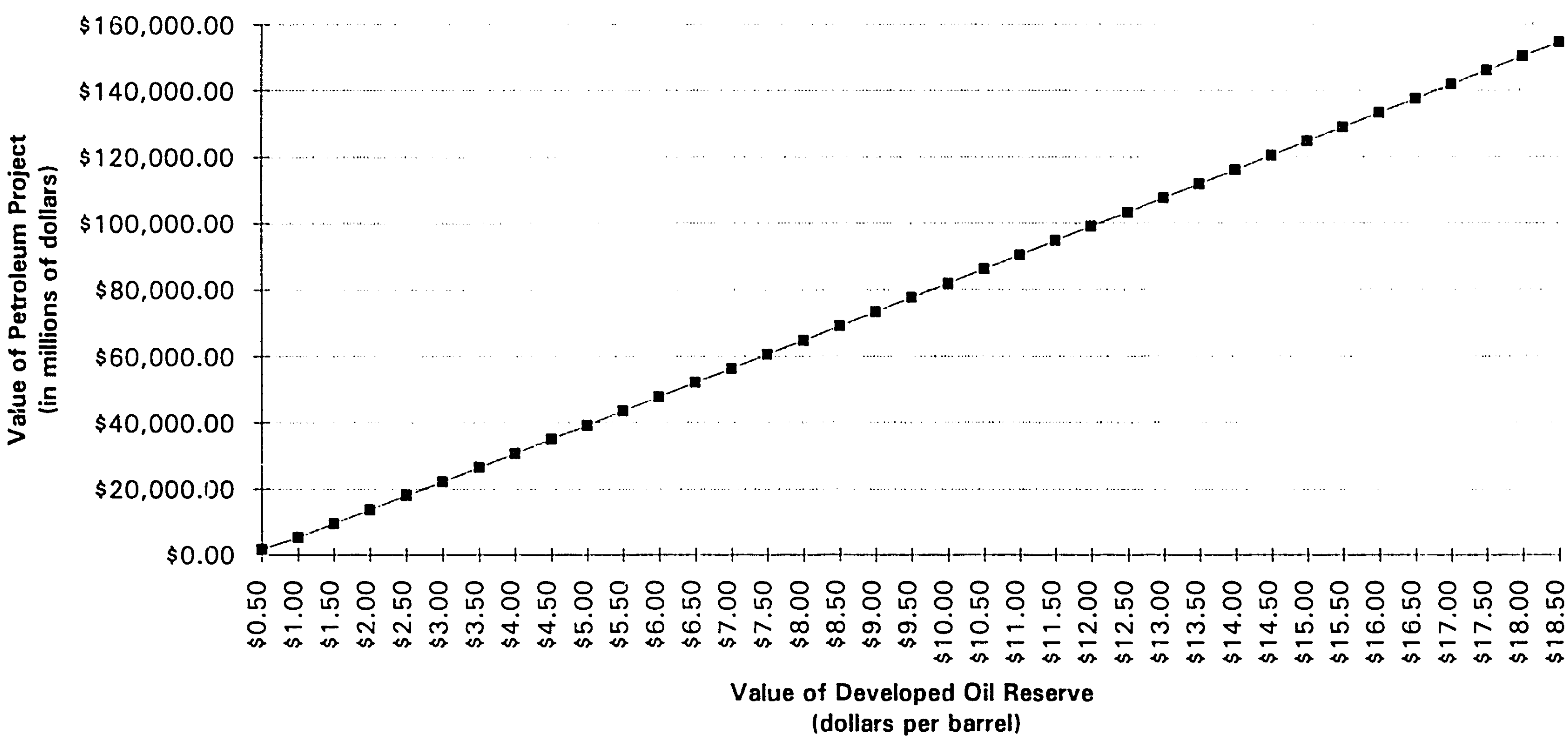


Figure 5.4 Sensitivity of Project to Quantity of Estimated Reserves

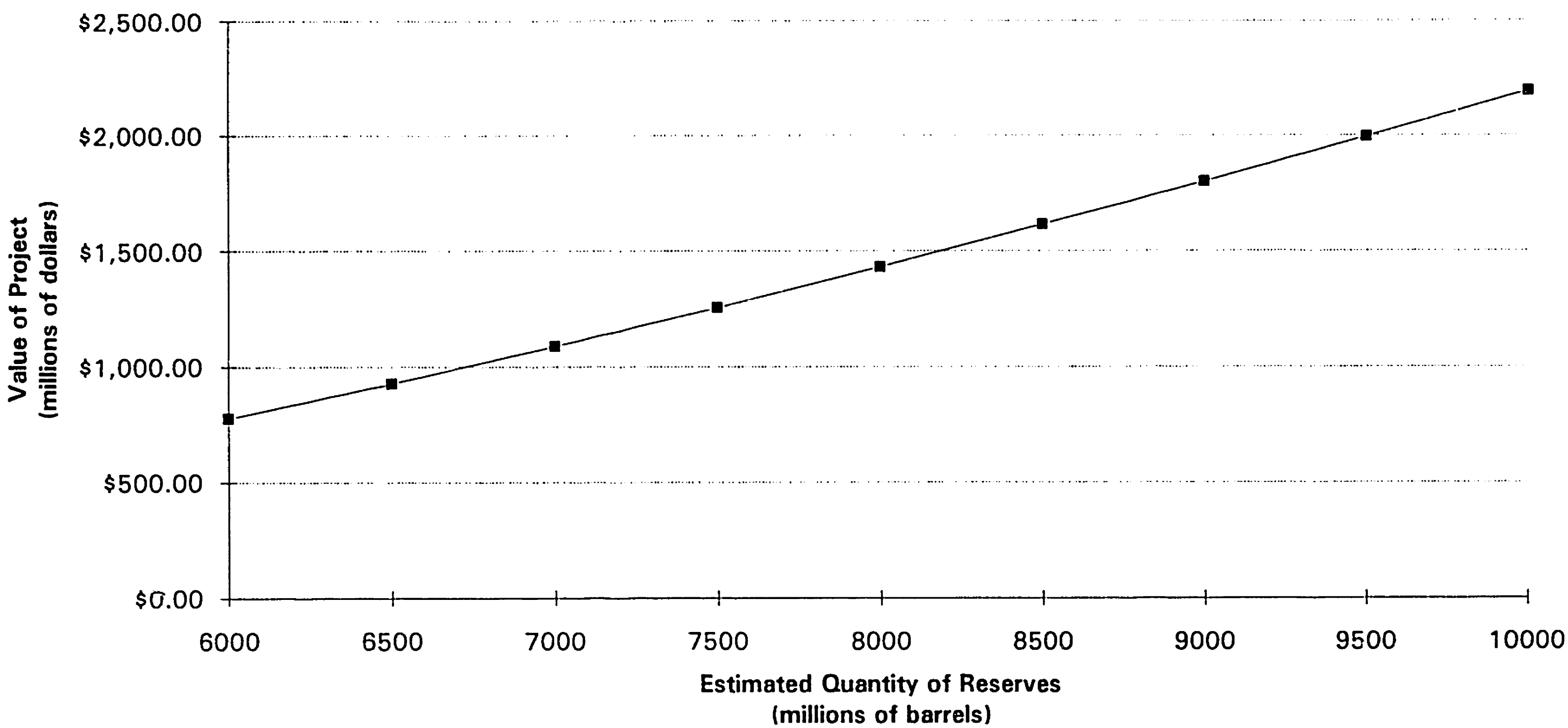


Figure 5.5 Sensitivity to Variance of Oil Prices

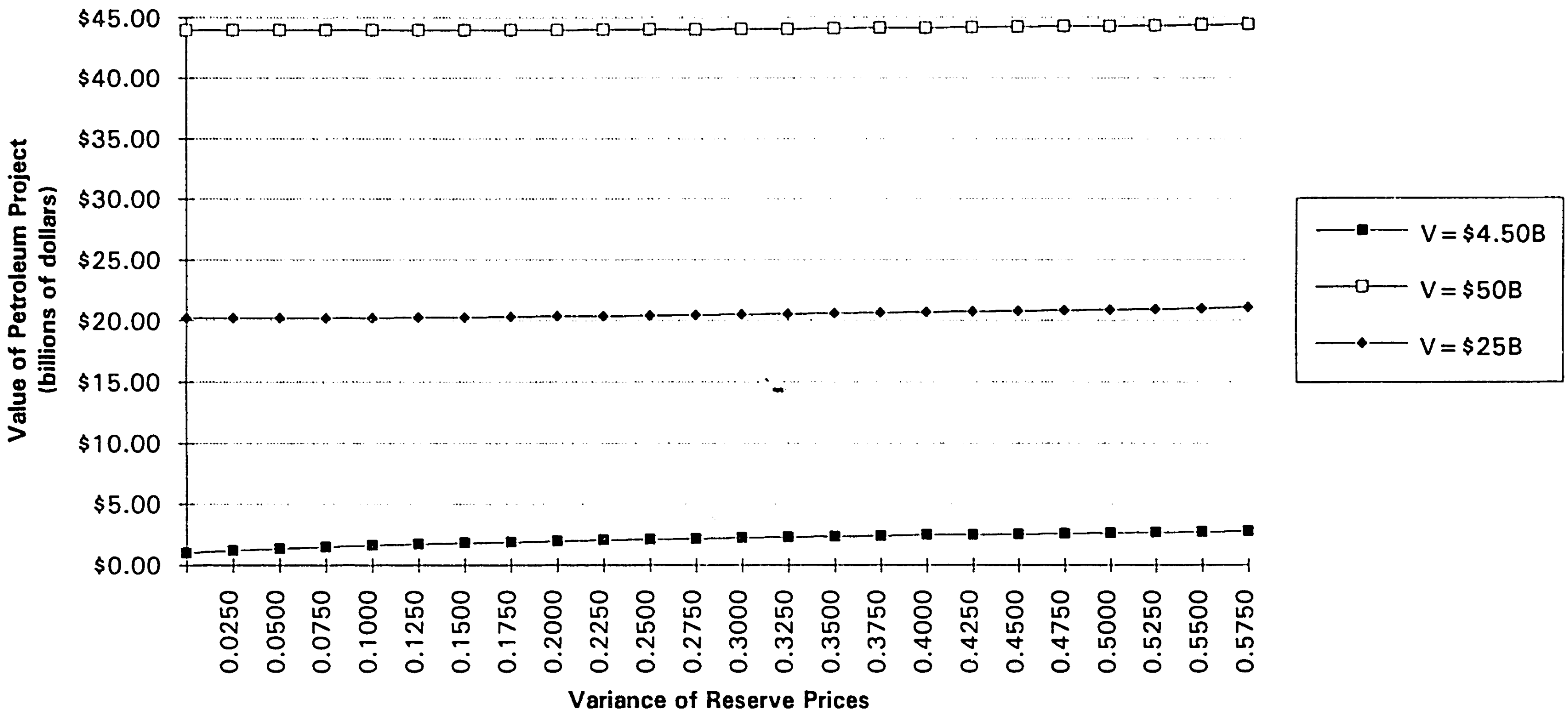
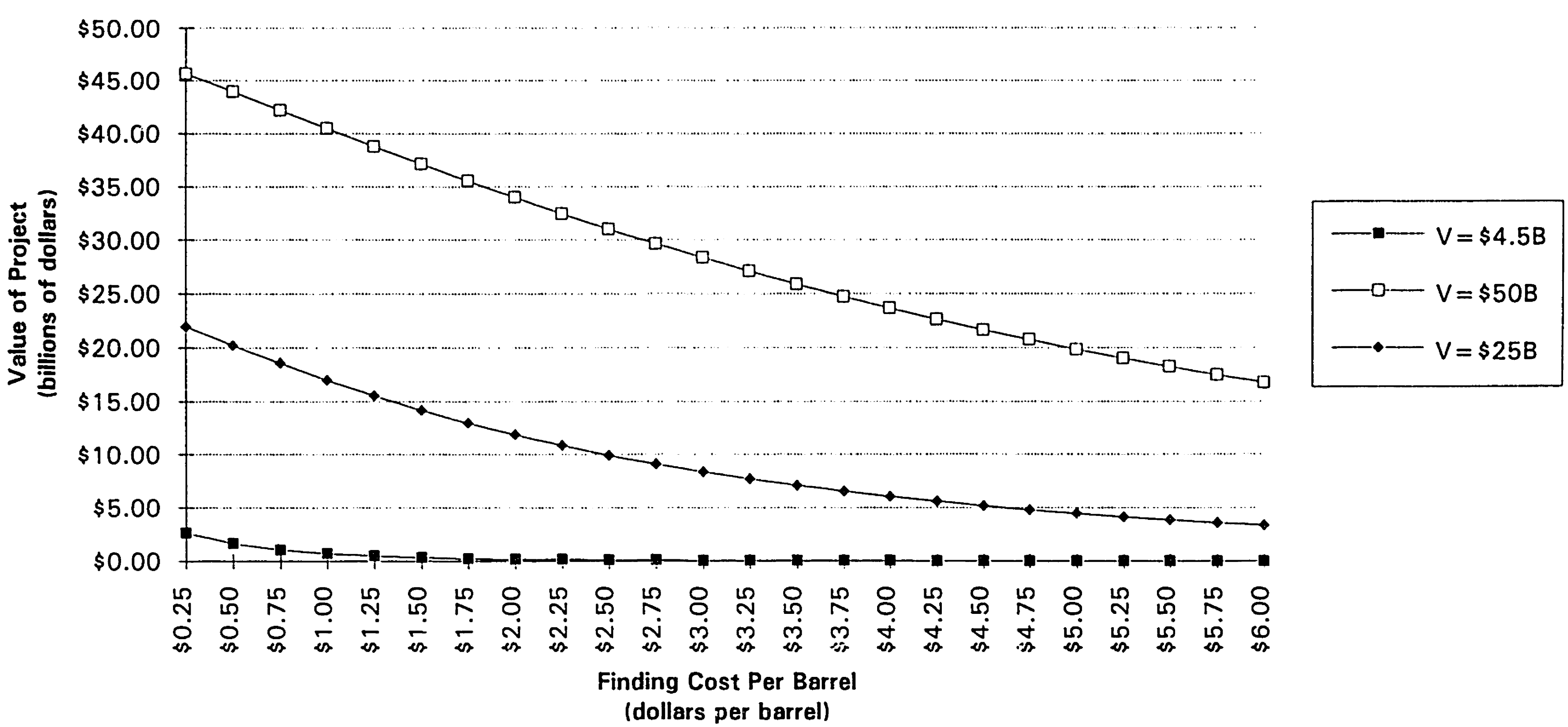


Figure 5.6 Sensitivity of Project to Finding Expenditures



Chapter 6

Evaluating Political Risk in the Petroleum Industry-An Options Approach

6.1 Introduction

In addition to the traditional economic and business risks, corporate decision makers have begun to recognize a new category of risks that are political in nature. This category of risk has been briefly mentioned in Chapter 2 and aspects of it are described in more detail here. Political risks may affect companies operating both domestically and internationally. Foreign government interference in corporate operations was the exception rather than the rule throughout the 1950's and most of the 1960s. However, by the late 1960s political conditions began to have a greater impact on the interests of firms with investments abroad. Domestic political environment also affects company affairs, however companies tend to be more familiar and comfortable with the domestic environment. Executives understand the political processes at home, and know how to influence political outcomes, and see the future as fairly predictable and stable.

Foreign political conditions, on the other hand, are less well understood and therefore seen as inherently more risky and less predictable.

Because each country constitutes a distinct business environment and political risks are usually identified with foreign investments, transnational corporations (TNCs) - firms that operate in a number of countries - are particularly concerned on the impact of political changes on corporate interests. In the natural resource sector in particular, TNCs are essential to the performance of the national economy, of many developing countries which are totally dependent on earnings from exports and tax receipts for government revenues. By the very nature of their operations, the activities of TNCs are inseparable from issues of modernization, economic power, employment, and distribution of wealth. Foreign firms also affect the foreign policy and international concerns of states in terms of issues of trade, balance of payments, hard currency earnings and the global distribution of wealth and consumption of resources.

To a large degree, host governments' assertiveness of control over their respective national resources has centered on the issue of natural resources. Virtually every major oil production agreement abroad has been subjected to renegotiation or unilateral alternation in favor of the host country. Foreign equity in crude has all but disappeared, exposing the oil firms to risks of supply shortages and interruptions and price instabilities. Although the nonconcession modes of agreement often imply a reduced capital investment on the part of the companies, which translates into a reduced amount of economic exposure to risk, the uncertainty and vagaries of the political economy of petroleum creates a high risk environment. Many of the modern service agreements moreover require that the oil companies assume the burden of initial capital outlays involving millions of dollars. The political risk that the government will alter the terms of the agreement after a commercial find has been made has proved to be a painful reality to several companies involved. In this chapter, I discuss the various kinds of political risks a firm faces while undertaking an international project. Specifically, I focus on which of these risks would affect Chevron in its joint venture project in Kazakhstan described in the last chapter. Lastly, I present a method to view the expropriation risk of oil projects by foreign governments using an option valuation approach. Using this approach also helps a oil company to understand why and when a host government would be likely to expropriate the project for pure economic reasons and ways by which they could prevent this unilateral action.

6.2 What is Political Risk?

A problem arises regarding how to distinguish between political risk and normal commercial or business risks. Any potential change that stems from political acts or decisions on the part of the government and other political actors would be termed as political risk. For example a change in tax laws constitutes a political risk because, even though the result may be reduced profits, the source of the changes are political. Political risks are not undifferentiated, homogeneous occurrences. The risks may vary from country to country, between different sectors of the economy, company involved, and the particulars of each project. Within the economy, political risks can be dichotomized as micro or macro. Macro political risks are aimed at all foreign firms, while micro risks are targeted towards particular sectors of the economy, types of firms, or even individual companies. Risks may also be classified as transfer, operational, administrative/statutory, ownership, or contractual. Operational risks are those that entail the possibility that effective control over operations may be wrested from the foreign company and vested in the host government. Political risk can be caused by a variety of government action ranging from complete expropriation to restrictions in hiring practices or a different tax structure or environmental regulation. In the following section, I will classify the different types of political risks that a firm is exposed to and later on use option theory to make an assessment of some of these risks. As will be seen, as international companies begin to take some of these risks into consideration while evaluating a project, the option approach loses its attractiveness. This is because the approach assumes that there is no risk of default by the parties entering into an option contract. In the case of political expropriation or unilateral changes in tax structure by the host government, this is no longer true, and the project needs to be properly discounted to take into account these risks.

6.3 Expropriation of Oil Properties

There are three terms used to describe the taking over of foreign property by a government - confiscation, nationalization, and expropriation. Confiscation refers to the taking of property without compensation, often for punitive reasons, and is most common at times of war. The other two terms are frequently used interchangeably and the most meaningful distinction between the two is that expropriation refers to the taking over of one or more properties within a single area of economic activity, whereas nationalization refers to the government's taking of all properties within the area. Some of the reasons that may explain a government's decision on the expropriation of foreign property are enumerated below.

6.3.1 Ideology

This is asserted as perhaps the primary explanation for government take-over of foreign property and investment. Ideologies of Marxism and socialism are two widely held ones that advocate state ownership as the means of production. With the fall of the Communist regime in the U.S.S.R. and the breaking up of its republics this seems an unlikely event to take place. The government of Kazakhstan in particular is anxious for rapid economic development and foreign investment in its move towards a market economy and it seems unlikely that it would fall back on its agreement with Chevron. The repercussions to future investments in Kazakhstan would be severe in case this was to happen.

6.3.2 Political Situation

The internal political situation within a host country may also be the cause of expropriation. A crisis or instability, such as social or labor unrest, pending elections, overthrow of the government can threaten the security of the foreign investment. The expropriation of a foreign corporation can attest to the patriotism of the government or can divert public attention from other difficulties facing the country. Russia and many of the former republics of the U.S.S.R. are currently facing volatile political conditions. Kazakhstan seems to be more stable than others, but the probability of the government being overthrown and replaced by a more one more conservative towards foreign investors is yet quite high. In evaluating the project, Chevron needs to consider this risk and appropriately discount for it.

6.3.3 Economic Situation

The economic climate of a country may also bring about expropriation in some cases. An unfavorable economic climate -inflation, unemployment, stagnation, foreign exchange shortage- can be the cause of expropriation if it is perceived to be the cause of the unstable political condition. This is another factor which may force the government of Kazakhstan to perform some unilateral action against Chevron. An unhealthy internal environment because of unemployment, inflation may cause popular opinion against foreign investors which may create pressures on the government to take action against its foreign investors. A way in which Chevron and other foreign investors can reduce this risk is to take an active role in the socio-economic development of the region. In this case, Chevron is expected to spend approximately $50 million on regional development which may be viewed as an expenditure to "hedge" against internal political action.

6.3.4 Host Country/Mother Country

Another reason which may cause the expropriation of a foreign investment is the relationship the host country holds with the mother country of the foreign company. A company sometimes may pay for the soured relationship between its mother country and the country in which it is investing. In this case, the United States is supportive of the development in Kazakhstan and this reason to expropriate the project is also quite unlikely.

6.3.5 Bargaining Power

Another generally accepted fact to explain expropriation, is the bargaining power between the host country and the foreign corporations. Historically, the Lesser Developed Countries (LDCs) have been in a weak bargaining position in their efforts to attract foreign investment. Foreign companies were the sole source of scarce resources -capital, technical know-how, managerial ability and marketing networks- and were heavily dependent on foreign corporations to fulfil these needs. As governments of LDC's gain expertise in these areas, and competition amongst foreign firms increase, the bargaining power shifts to the host country. In the case of Kazakhstan, it is heavily dependent on the United States and the other industrialized countries for technology, capital and managerial skills. The bargaining power currently and perhaps for the next decade lies with the foreign investors in Kazakhstan. This point is in favor for Chevron and it would not need to discount its project cash flows to account for this risk.

6.4 An Option Approach in Assessing Expropriation Risk

In this section, I use option methodology as a tool in determining and accessing the expropriation risk faced by a oil company investing in a petroleum exploitation project. When the company is awarded the right to explore, develop and extract oil from a particular tract of land, the company pays the government the price of the lease. The government receives this bid amount and in addition receives the implicit right to expropriate this project from the firm at any point in time. The firm is thus short a position of this call option where the underlying asset is the project itself. The firm must therefore include in its evaluation of the project the value of this option which it gives to the government of the country in which it is investing. From option pricing theory it can easily be seen that an option on a project that makes no payout to its investors and with an infinite duration has a price which is equal to the full value of the project. The value of this implicit call option which the government receives is denoted by C. A government expropriating a project faces several exercising costs. Some of these costs include the compensation paid to the contractor when expropriating the project. If this compensation is not fair, the government will also bear other less obvious costs such as sanctions by the company's parent country or reduced future investment and technological transfer and from other countries. Let the total expropriation cost to a government be denoted by X. Finally, companies pays back its investors in the form of dividend payments. For example a company investing in a foreign country repatriates its earnings home to its shareholders. Let the present value of all these dividend payments be denoted by D. The value of the project is simply the present value of the project itself and is dependent on several factors including the price of crude and the quantity of reserves. Following usual notation, this may be denoted by V. Using simple arbitrage relationships, it can be proved that the value of this implicit option is greater than $V - X - D$. Thus if the value of the option is less than this amount, the government would have an incentive to exercise its option. I shall now describe the various ways by which the value of this option changes and the times when a government for pure economic reasons would expropriate a project.

As mentioned above, the value of this option is dependent on the exercise cost which the government will need to pay for expropriating the project. Some of these costs may be the direct costs the country pays as compensation to the investor for expropriating the project. However it is important to make an estimate of the indirect costs of expropriation which a government will bear by making such a decision. These cots include and are not limited to costs of reduced future investments, technologically and possible sanctions against it by the company's parent country. In the case of domestic investment the country does not need to consider these costs as the contracting company is operating locally. It can thus be reasonable to argue that if these exercise costs are low or are perceived to be low, there is a high probability of unilateral government action against the company. The value of these costs also are time-dependent. Less developed countries are more dependent on foreign investment and technology during the initial phases of the project. Thus there is very small likelihood of government expropriation during this stage. As the project progresses and production begins, the government's dependence on the oil company reduces and so does its expropriation costs.

Another important variable in the value of the option is the value of the project itself. In the initial stages of the project, the company and the government faces large geological and development cost risk which are the risks of the magnitude of reserve size as well as the cost required in developing and extracting these reserves. The exploration stage reduces a significant portion of both these risks which results in an increase in the value of the project. Governments are thus more likely to exercise their expropriation option when these risks have been eliminated and the project has a greater value. Second government exercise might be prompted by asset market characteristics such as market disequilibrium. If a government recognizes that the above average returns now will be followed by below average expected returns, then it would probably exercise its option now. If a firm's profitability, like product sales, also has a seasonal pattern, and the government's share of the project depends on the firm's profitability, the government may wish to exercise it's option as the value of the project will decline through maturity. Optimal exercise of the option will thus probably occur near the peak of the firm's growth period.

The last input to this option approach is the value of the dividend payments made by the company to its investors. As mentioned above, for a non-dividend paying stock the value of a perpetual call option is equal to the value of the underlying asset. This explains an investor's preference towards dividends payments, because if the government can exercise it's option without paying the full value to the firm and the investor's, then the dividend stream is worth more to investor than the unrealized capital. On the other hand a government fearing a large repatriation of the firm's investment in the form of dividend payments to its investors will consider expropriation in order to stop this repatriation of capital. To the extent that capital repatriation of the capital is the only reason that a government would expropriate a project, as long as there is a safe level of repatriation of capital, there would be no danger of expropriation of the project.

Chapter 7

Summary and Conclusion

This thesis has used an extension of option valuation theory in the valuation of contingent claims on offshore petroleum leases. The model presented in this thesis draws heavily from the model developed by Paddock et al[22]. Many of the theoretical and practical difficulties associated with using discounted cash flow (DCF) analysis is discussed. The theoretical complexity in using the conventional discounted cash flow technique arises because of the fact that in order to perform a DCF analysis, a risk adjusted cost of capital needs to be determined. It is very difficult to account for all the factors affecting the risk of a project. Performing a financial evaluation on a petroleum lease is therefore a complex task. Part of the complexity arises from the numerous uncertainties and diverse risks which affect lease value. These risks and uncertainties affect both, the cash flows and the discount rates at which these flows are discounted. Understanding and accounting for these risks is critical for analyzing the economic feasibility of the project.

In Chapter2, I have discussed many of these risks and uncertainties. Some of these factors discussed include the future course of oil prices, future development and operating costs, quantity of the recoverable reserves, and political risks of tax and/or regulatory changes which could all adversely affect future cash flows and the value of the project. I have also given a brief review of the various different fiscal devices open to the Government of a oil producing country intent on extracting economic rents from its petroleum reserves. Each of these instruments have been regarded in the context of the objectives of both the investing oil company and the Government of the host country. The manner in which each of these contract forms distributes the total risk of the project is also analyzed. The concept of efficient allocation of risks is discussed wherein risks are borne between the contracting parties by the party best capable of handling it.

The option valuation model[22] discussed in Chapter 3 combines the option pricing techniques used in the valuation of financial assets along with a model of market equilibrium to value real assets such as petroleum leases. An important distinction between real assets and financial assets lies in the fact that when a financial stock option is exercised there is no net change in the volume of the underlying stock, whereas the exercising of a real option increases the net supply of the real asset. The option approach in valuing petroleum leases and projects has several advantages over the conventional discounted cash flow approach in valuing these leases. First, it requires less data as it uses market information more efficiently. Second, it is far easier to compute and thus less subject to error. Third it gives some indication about the optimal timing of the investment. Finally, by using this approach both in valuing leases as well as evaluating political risks associated with a project it serves to be a valuable insight in explaining both government behavior and company policy. Both the applications presented in this thesis also show great promise in the use of this technique in evaluating petroleum leases.

As in the case of stock options, the most important parameter not required to be estimated is the risk-adjusted rate of return or the expected future price distribution of the underlying asset. Therefore in the case of stock options one need not know the systematic risk of the underlying asset. Comparing these information requirements to those needed in the formula using the discounted cash flow method, the option valuation approach reduces the information requirements needed to determine the value of a petroleum lease. The option valuation theory uses market prices of the underlying asset to determine the inherent risk of the asset. This is because current market prices reflect all future events and risks as well as some of the inherent political risks. Thus by using this value in the estimation the option valuation theory eases the burden of estimation of risks to the market. In fact, besides the variance of the prices of the developed reserves, no other parameter needs to be estimated in applying the option valuation to stylized leases.

An important inference from the model used in this thesis to value leases in the Outer Continental Shelf and the joint venture in Kazakhstan is that the model provides an extremely simplified and not fully realistic description of actual OCS leases or joint ventures. However, this approach still maintains its utility especially when the decision to develop the lease depends critically upon the future price of crude oil. Also, it is seen that when the option is way in the money, i.e. development is virtually automatic, this approach provides no significant value in the valuing of the project. The option pricing approach also gives an indication when, if any, early exercise of a lease would occur.

This option valuation approach is then applied to determine a value of an OCS lease sale and a joint venture in the republic of Kazakhstan between the government and Chevron Corporation. Among the several variables that formed an input to the valuation equation, the developed reserve value is one of the most important determinant of option value. The value of this parameter was approximated by taking 1/3th the value of a barrel of crude oil. However, it would be useful to obtain this value from more public sources. As mentioned in Chapter 4, this value could be obtained by studying and analyzing the stock price of companies whose assets solely comprise of these developed reserves. An important consideration in this case is controlling for variables such as differing tax regimes, price controls, cost structures and oil quality. Evaluating the value of the lease sale in the Outer Continental Shelf, it was seen that the value of the lease as calculated by the option approach lies very close with the total of the high bids received in the sale. The OCS lease best approximates a stylized lease to which option valuation methodology can be applied, and as seen the result obtained is close to market value of the lease. In the case of valuing the joint venture project between Chevron and the republic of Kazakhstan, several uncertainties arise due to exchange rate risks, tax uncertainties and other political risks. In face of these additional risks, the option valuation approach loses its inherent attractiveness as Chevron would need to evaluate these risks and properly discount for them.

Another area where this approach could be extended is a more complex specification of the development decision. In this model, the development expenditure is lumped instantaneously with the exploration expenditure. In reality, companies have latitude in their development decision. It may, for example be optimal to develop slowly to take advantage of new information. This gives the company another option of discontinuing development if it becomes unprofitable to do so.

Even though the option valuation approach eliminates much of the uncertainty in determining the value of a petroleum project, many uncertainties remain. An example of one such risk is the political risk of the project being expropriated by the government of being subject to sudden adverse changes in tax structure. In Chapter 6, I have provided a way to view this political risk with an option approach. Viewing this risk by using option methodology provides a qualitative means of understanding why and when expropriation is likely and ways in which the investing companies can frame company policy to avoid it.

Finally, even though this approach has been applied specifically in valuing petroleum leases, with suitable modifications this model can be used in valuing contingent claims on other real assets which show option like characteristics.

Appendix A

Option Pricing Theory

A.1 Introduction

Option markets exist for a wide variety of instruments and in order to explain the basic definitions of the terms involved, reference will be made to the oldest and largest of these markets, namely, options on common stocks.

> **A *call* option is a contract giving its owner the right to *buy* a fixed number of shares of a specified common stock at a fixed price at any time on or before a given date.**

The act of making this transaction is known as *exercising* the option and the specified stock is known as the underlying security. The fixed price is known as the *exercise* or *striking price*, and the given date is the maturity or *expiration date*. The individual who creates or issues a call is termed as the *writer* or seller of the call and the individual who purchases the call is termed as the *holder* or buyer. The market price of the call is known as the premium or the *call price.*

An important point to be noted is that the buyer of a call has the right but not the obligation to exercise this option in exchange for the security, so the exchange will only take place if the buyer feels that it is in his best interest to do so. If the buyer does not exercise his right on or before the expiration date the call expires worthless and the buyer no longer retains the right to exercise the option. Another alternative for the buyer of the call is to sell the call at its concurrent market price, thereby cancelling his position.

To represent the contractual implications of a call on its expiration date mathematically let,

K = the striking price

S^* = market price of the underlying security on the expiration date, and

C^* = the value of the call (to one share) on its expiration date.

Then at the expiration date;

$$C^* = \begin{cases} S^* - K & \text{if } S^* > K \\ 0 & \text{if } S^* < K \end{cases}$$

In contrast to a call,

> **A *put* option is a contract giving its owner the right to *sell* a fixed number of shares of a specified common stock at a fixed price at any time on or before a given date.**

Again this exchange will only take place at the choice of the owner of the contract. If P^* is the value of a put (to one share) on its expiration date, then

$$P^* = \begin{cases} 0 & \text{if } S^* > K \\ K - S^* & \text{if } S^* < K \end{cases}$$

Thus as described above there are two types of options - puts and calls. All option contracts of the same type written on the same underlying stock constitute a *class* of options. Thus call and put options on the same underlying stock are considered separate classes. Within a given class of options, all contracts having the same expiration date and striking price constitute an option *series*.

It is simple to determine the value of an option on its expiration date. The process of determining the value of an option at any time before its expiration is much more difficult. Now let,

S = current market price of the underlying security,

$C =$ the current value of an associated call, and

$P =$ the current value of an associated put.

Now since *American* puts and calls can be exercised at any date before or upto the expiration date, the current value, C, of a call must be at least equal to $\max[0, S - K]$, and the current value of a put must be at least equal to $\max[0, K - S]$. The values $\max[0, S - K]$ and $\max[0, K - S]$ are referred to as the exercise value or *parity* value of a call and put respectfully. However since an option may be more valuable retained than exercised, its value may be more than the parity value. The difference $C - max[0, S - K]$ for a call and $P - max[0, K - S]$ are known as the *premium over parity* of an option. The market price of an option is thus equal to its parity value plus its premium over parity.

There are some other terms which are applied to options before their expiration dates. If $S > K$, a call is said to be *in-the-money*; if $S = K$, it is *at-the-money* and finally if $S < K$ the call is said to be *out-of-the-money*. For a put the definitions are reversed and it is said to be in-the-money if $S < K$. In general it can be said that an option is in the money if its parity value is positive. The terms *deep-in-the-money* and *deep-out-of-the-money* are used to describe options whose parity values are very high and low respectively. It is important to note that the option markets are a *zero-sum* game. That is for example when the option buyer profits, the option writer loses, and vice versa.

There are four elementary types of positions that can be taken:

1. Uncovered
2. Hedge
3. Spread
4. Combination

There are six uncovered or naked positions- long stock, short stock, buy call, write call, buy put, and write put. Hedges, spreads, and combinations are types of covered positions, in which one or more securities protect the returns of one or more other securities, all related to the same underlying stock.

A *hedge* combines an option with its underlying stock in such a way that either the stock protects the option against loss or the option protects the stock against loss.

In other words a hedge may either be a long position in stock protected by writing call(s) or purchasing put(s) or a *reverse hedge* where a short position in stock is protected by buying call(s) or writing put(s).

A *spread* combines options or different series but of the same class, where some are bought and others are written.

A *combination* combines options of different types on the same underlying stock so that they are either both bought or both written.

The most common of combination combines a put and a call on the same underlying stock, with the same price and the same expiration date. This combination is known as the *straddle.*

A.2 Determinants of Option Value

After seeing the various option positions and knowing the value of the option on the expiration date, it is seen that the value of the option depends upon only two factors at that time. However at any time before expiration there are several other variables which play an important role in determining option value.

The following is a list of the six fundamental determinants of option value:

1. Current stock price (S)
2. Striking Price (K)
3. Time to Expiration (t)
4. Stock Volatility
5. Interest Rates

6. Cash Dividends

It is apparent that the first two items will influence option prices even before the expiration date. The higher the stock price, higher will be the value of a call and lower the value of a put. Similarly, higher the striking price, lower will be the call value and higher the put value.

Another important determinant of option value is the *volatility*. Volatility can be thought of as a measure of the dispersion of possible future stock prices. The higher the volatility of a stock, the greater the likelihood that the stock will do either very well or very poorly. These effects are offset for the owner of a stock but not of a call. The owner of a call takes the full dollar benefit of stock movements in his favor but is protected from most of the dollar loss from any unfavorable movement of the stock price. Consequently , the higher the volatility of the stock over the lifetime of the call, higher will be the value of the call.

The higher the *interest rate*, the lower will be the present value of the striking price the buyer has contracted to pay in the event of exercise. Thus a higher interest rate has the effect of lowering the striking price and it may be concluded that higher the interest rate, higher will be the call price.

The *time to expiration* measures the amount of time remaining in the life of an option. One effect of a longer time to expiration works in the same manner as a higher volatility. Simply put, there is a greater chance that there may be a big movement in the price of even a very low volatility stock in a longer time period. Hence the call and put premium tend to be higher, the more time remaining to expiration. However time to expiration also affects other variables such as cash dividends paid to the owners of the underlying stock which affects calls and puts differently and hence the net effect of a longer time to expiration is ambiguous and depends on the particular stock itself.

The last determinant of option value as already mentioned above are the *cash dividends*. This affects the value of the call in two ways. Ordinary dividends can be considered as partial liquidation of the firm with a resulting lowering in the stock price. From common arbitrage arguments it can easily be shown that for no arbitrage opportunities to occur, the stock price must fall by exactly the amount of the cash dividend when the stock opens for trading on the ex-dividend date. The owner of the stock gets both the cash dividend as well as the stock appreciation. However the owner of the option does not get the payoff from the dividends and hence any such payoff that affects the price of the stock must be reflected in the call/put price. As cash dividends lowers the stock price immediately after the payoff date, greater the cash dividends paid, lower will be the value of a call and higher will be the value of a put.

Some institutional factors that would affect option value would be tax regulations, margin requirements, and transaction costs. The influence of these factors however may be small.

A.3 General Arbitrage Relationships Amongst Options

Option values must possess certain properties so that no arbitrage opportunities exist. Riskless arbitrage opportunities are situations that require no initial investment but that yield a positive amount immediately and only non-negative amounts in the future under all possible circumstances. Hence the results provide a set of necessary conditions that should be satisfied under any option pricing theory. In other words if any of these relations are violated there will exist opportunities in the market for making arbitrage profits and if we can safely assume that markets are efficient, then prices will soon adjust to remove such opportunities. The following arbitrage relations are merely listed and not proved but one may easily do so in all cases by assuming that if the relation was not satisfied, opportunities for making riskless profits would exist. For further reading on arbitrage relations for options, refer to the book "Options Markets" by John Cox and Mark Rubenstein.

Following are the arbitrage restrictions for **call** options:

Proposition 1 The value of a call is never less than the larger of

- zero

- the stock price minus the striking price
- the stock price minus the present value of the striking price minus the present value of the maximum dividends that will be paid during the remaining life of the option.

and the value of a call is never greater than the price of its underlying stock.

$$S > C > max[0, S - K, S - Kr^{-t} - D] \tag{A.1}$$

Proposition 2

The value of a call can never be less than the value of an otherwise identical call with a higher striking price.

$$C(K_1) > C(K_2) \quad \text{if } K_2 > K_1 \tag{A.2}$$

The difference in the values of two otherwise identical calls is never greater than the difference in their striking prices:

$$K_2 - K_1 > C(K_1) - C(K_2) \quad \text{if } K_2 > K_1 \tag{A.3}$$

Of the three otherwise identical calls with striking prices $K_3 > K_2 > K_1$, the value of the middle call is never greater than a weighted average of the values of the extreme calls, where the weights are $(K_3 - K_2)/(K_3 - K_1)$ for the first call and $(K_2 - K_1)/(K_3 - K_1)$ for the third call.

$$C(K_2) < \left(\frac{K_3 - K_2}{K_3 - K_1}\right) C(K_1) + \left(\frac{K_2 - K_1}{K_3 - K_1}\right) C(K_3) \tag{A.4}$$

Proposition 3

The value of a call can never be less than the value of an otherwise identical call with a shorter time to expiration.

$$C(t_2) > C(t_1) \quad \text{if } t_2 > t_1 \tag{A.5}$$

This may not necessarily hold true for European calls as they can only be exercised at the expiration date.

Proposition 4

The value of a call must be greater than $S - K$ at any time other than the expiration date or just before an ex-dividend date.

Proposition 5

There are three arbitrage restrictions that relate to the optimal timing of exercise.

1. A call should never be exercised at any time other than the expiration date or just before an ex-dividend date.

2. If the present value of the maximum dividends to be paid during the remaining life of a call will at all times be less than the concurrent present value of the interest that can be earned on the striking price during the remaining life of the call, then the call should not be exercised before the expiration date.

3. If at any time it is optimal to exercise a call, then it is never optimal to leave unexercised an otherwise identical call that has either a lower striking price or a shorter time to expiration.

For puts the arbitrage restrictions are as follows:

Proposition 1 The value of a put is never less than the larger of:

- zero
- the striking price minus the stock price.
- the present value of the striking price minus the stock price plus the present value of the minimum dividends that will be paid during the remaining life of the option.

and the value of a call is never greater than the price of its underlying stock.

$$K > P > max[0, K - S, D + Kr^{-t} - S] \tag{A.6}$$

Proposition 2

The value of a put can never be less than the value of an otherwise identical call with a lower striking price.

$$P(K_1) > P(K_2) \quad \text{if } K_2 > K_1 \tag{A.7}$$

The difference in the values of two otherwise identical puts is never greater than the difference in their striking prices:

$$K_2 - K_1 > P(K_1) - P(K_2) \quad \text{if } K_2 > K_1 \tag{A.8}$$

Of three otherwise identical puts with striking prices $K_3 > K_2 > K_1$, the value of the middle put is never greater than a weighted average of the values of the extreme puts, where the weights are $(K_3 - K_2)/(K_3 - K_1)$ for the first put and $(K_2 - K_1)/(K_3 - K_1)$ for the third put.

$$P(K_2) < \left(\frac{K_3 - K_2}{K_3 - K_1}\right) P(K_1) + \left(\frac{K_2 - K_1}{K_3 - K_1}\right) P(K_3) \tag{A.9}$$

Proposition 3

The value of a put can never be less than the value of an otherwise identical put with a shorter time to expiration.

$$P(t_2) > P(t_1) \quad \text{if } t_2 > t_1 \tag{A.10}$$

Proposition 4

There are three arbitrage restrictions that relate to the optimal timing of exercise.

(a) If throughout a period ending with time t' until expiration, the present value of the maximum dividends to be paid during the remainder of this period will at all times be greater than the concurrent present value of the interest that can be earned on the striking price during the remainder of the period, then the put should never be exercised before the end of the period.

(b) If at any time it is optimal to exercise a put, then it is never optimal to leave unexercised an otherwise identical put that has either a higher striking price or a shorter time to expiration.

A.4 An Option Pricing Formula

The variables on which the value of an option depends on have already been stated above. The theory of option pricing has had a long and illustrious history. In 1973 it undertook a revolutionary change when Fischer Black and Myron Scholes presented the first completely satisfactory equilibrium option pricing model.[1] At the same time, Robert Merton, Professor of Finance at the Massachusetts Institute of Technology, extended their model in several ways. The mathematical model developed by Black-Scholes was very elaborate and complicated. William Sharpe, Professor of Finance at Stanford University, used the replication technique to derive the same result.

A.4.1 The Binomial Model

In this approach to developing an option pricing formula it is assumed that the stock follows a multiplicative binomial process over discrete time periods. The movement of the stock will then be as follows over each individual time period. If the stock price is S at the beginning of the first period, then it can have two possible values at the beginning of the next period, uS or dS. The rate of return of the stock over each period can have two possible values: $u - 1$, with

[1] Refer to their article, "The Pricing of options and Corporate Liabilities", which appeared in the May-June 1973 issue of the *Journal of Political Economy*, pp 637-659.

a probability of q, or $d-1$ with a probability of $(1-q)$.[2] Let r be the interest rate for that period which is assumed to be constant and positive. Also it is required that $d < r < u$. If this inequality does not hold true, riskless profit can be made involving the security and riskless borrowing and lending. Also perfect capital markets are assumed wherein there are no taxes, transaction costs, or margin requirements. Hence individuals are allowed to sell short any security and receive full use of the proceeds. Further it is assumed that the markets are competitive; i.e. a single individual may buy or sell any amount of a security without affecting its price.

In order to value a call, starting with the simplest situation: with its expiration date only one period away. Then if C is the current call price, C_u is its value at the end of the period if the stock price goes to uS and C_d is its value if the stock price goes down to dS. Since there is only one period remaining till expiration, at the end of this period, the call value is known.

Now if a portfolio is constructed containing Δ shares of stock and the dollar amount B in riskless bonds. This portfolio will cost $S\Delta + B$.

Since Δ and B are selected in any way which we want, we could choose them to equate the end-of-the-period values of the portfolio and the call for each possible outcome. This would require,

$$\begin{aligned} uS\Delta + rB &= C_u \\ dS\Delta + rB &= C_d \end{aligned} \tag{A.11}$$

Solving the above two simultaneous equations will lead to,

$$\Delta = \frac{C_u - C_d}{(u-d)S}$$

[2]It is assumed here that for the moment, the stock does not pay any dividend.

$$B = \frac{uC_d - dC_u}{(u-d)S} \tag{A.12}$$

With Δ and B chosen in this particular way, we have constructed what is known as an equivalent portfolio. Using arbitrage arguments again it can be said that the current value of the call C cannot be less than the current value of the equivalent portfolio $S\Delta + B$. Therefore,

$$\begin{aligned} C &= S\Delta + B \\ &= \frac{C_u - C_d}{u-d} + \frac{uC_d - dC_u}{(u-d)r} \\ &= \left[\left(\frac{r-d}{u-d} \right) C_u + \left(\frac{u-r}{u-d} \right) C_d \right] / r \end{aligned} \tag{A.13}$$

if this value is greater than $S - K$, and it not, $C = S - K$.

The above equation can be simplified by defining $p \equiv (r-d)/(u-d)$, so that $1 - p = (u-r)/(u-d)$ and eqn (A.13) can be written as

$$C = \left[pC_u + (1-p)C_d \right] / r$$

Hence it can be seen that the exact value of a call one period prior to expiration in terms of S, K, u, d, and r. A notable feature in the formula is that the probability q does not enter in the formula. This means that even if different investors have different subjective probabilities of an upward and downward movement of the stock, they would still agree on the relationship of C to S and r. Secondly the value of the call does not depend upon the investors attitude towards risk. The only assumption that was made was that the investors preferred more wealth to less wealth and would take advantage of any arbitrage opportunities available. The formula would be same whether the investors were risk-neutral or risk-averse.

The above simple formula can now easily be expanded to the case for a call with two periods remaining till expiration. In keeping with the binomial model the stock can take on only three possible values after two periods:

For the call, $C_u u$ stands for the value of the call two periods from the current time if the stock price moves upward each period; $C_u d$ and $C_d d$ have analogous definitions. At the end of the current period there will be only one period left till expiration and the problem is identical to the first simple case. Thus,

$$\begin{aligned} C_u &= [pC_u u + (1-p)C_u d]/r \\ C_d &= [pC_u d + (1-p)C_d d]/r \end{aligned} \qquad \text{(A.14)}$$

Again over here a portfolio of $S\Delta + B$ is chosen in stocks and bonds. The new value of Δ and B can be obtained by replacing with the new values of C_u and C_d. Even with two periods remaining until expiration we can say that riskless arbitrage opportunities will exist if the current price of the call is not equal to the new value of the portfolio or $S - K$, whichever is greater. The important difference to be noted that at the end of the first period, the call may still be mispriced with respective to the market and we would not be assured a riskless profit at the end of the first period. However if the portfolio is so adjusted at the end of every period, and held till the expiration we would still be assured of a profit as any mispricing which exists during the life of the call would have to disappear at the expiration date. Consequently it may be said that even with two periods left before expiration, a strategy could be followed that would guarantee riskless profits with no net investment if the current market price of a call differs from the maximum of $S\Delta + B$ and $S - K$. Hence the larger of these two values is the price of the call.

Combining the equations (A.14) and (A.15) the following is obtained:

$$
\begin{aligned}
C &= [p^2 C_u u + 2p(1-p)C_u d + (1-p)^2 C_d d]/r^2 \\
&= p^2 max[0, u^2 S - K] + 2p(1-p) max[0, duS - K] + \\
&\quad (1-p)^2 max[0, d^2 - K]/r^2
\end{aligned} \tag{A.15}
$$

It can easily be shown that the above equation value is always greater than $S - K$ if as assumed, r is greater than one, hence this equation gives the exact value of a call.

Using this recursive procedure, the value of a call with any number periods to go before expiration can be worked out and a general valuation formula for n periods to go before expiration may be written down as:

$$
C = \sum_{j=0}^{n} \left(\frac{n!}{j!(n-j)!} \right) p^j (1-p)^n - j max[0, u^j d^{n-j} S - K]/r^n \tag{A.16}
$$

With a little bit of adjustment, the formula can be written as:

$$
C = S\Phi[a; n, p'] - Kr^{-n}\Phi[a; n, p] \tag{A.17}
$$

where

$$
\begin{aligned}
p &\equiv (r-d)/(u-d) \\
p' &\equiv (u/r)p \\
a &\equiv \text{the smallest nonnegative integer greater than} \log(k/SD^n/\log(u/d)
\end{aligned}
$$

The above formula is the development of the Sharpe's binomial method for the pricing of financial options.

A.5 The Black-Scholes Formula

The binomial formula which was derived by Sharpe, breaks up the period till expiration into periods. There is a natural tendency to think of a trading period as some length of calendar time, perhaps a day. With this in mind there are two objections to the binomial formula, firstly that prices in a day from now on may take on many more than just two possible values. Furthermore, the market is not open for trading only once a day, but instead trading takes place almost continuously. These objections are valid ones, however the option pricing formula developed earlier has the flexibility to meet them. Although it may be natural to think of a trading period to be one day, there is nothing that prevents us to have a much shorter time period. Thus you may consider a shorter time period- say an hour or even a minute. By doing so, both objections are met, trading would take place far more frequently, and the stock price would take on hundreds of possible values till the end of the day.

However if the interval of time is small, an adjustment would have to be made to keep the probability small that the stock price will change by a large amount over a minute. It would be incorrect to assume that the stock price would have the same percentage up and down movements for one minute as it did for one day. In order to make this more precise, suppose that h represents the elapsed time between successive stock price changes. That is, if t is the fixed length of time till expiration, and n is the number of periods of length h prior to expiration, then

$$h \equiv t/n \tag{A.18}$$

As trading takes place more and more often, h gets closer and closer to zero. The interval-dependent variables $r, u, andd$ must be adjusted in such a way that empirically realistic results are obtained as h becomes smaller and smaller or alternatively as $n \to \infty$.

As r means one plus the riskless interest rate over a fixed interval of time; namely the time left till expiration, let $\hat{r}$ refer to the one plus the interest rate over a period (trading interval) of length h. The value of $\hat{r}$ depends on the number of subintervals, n into which t is divided. Over the n periods until expiration, the total return is $\hat{r}^n$ where $n = t/h$. Now it is important that $\hat{r}$ depends upon n in a particular manner such that the total return $\hat{r}^n$ over the fixed time t remains the same. This follows from the fact that the total return over a fixed interval of time t must have nothing to do with the way in which the interval is divided into.

If this is to be the case, we have,

$$\hat{r}^n = \hat{r}^t \tag{A.19}$$

Therefore, $\hat{r} = r^{t/n}$. The terms u and d must also be defined in terms of n. At this point, it may be assumed that the stock follows either a continuous or a jump stochastic process. In the first situation, very small random changes in the stock price occur in each and every small time interval. The stock price will fluctuate incessantly, but its path can be drawn without lifting pen from paper. In contrast, in the second case, the stock price will move in a smooth deterministic way, but will occasionally experience sudden discontinuous changes. It may be recalled that over each period the stock price would experience a one plus rate of return of u with a probability q and d with a probability $1 - q$. It will be easier to work with $\log u$ and $\log d$ which are the continuously compounded compounded rate of return on the stock over each period.

Consider a typical period of five moves, say u, d, u, u, d. Then, $S^* = uduudS$, $S^*/S = u^3d^2$, and $\log(S^*/S) = 3\log u + 2\log d$. More generally, over n periods

$$log(S^*/S = jlogu + (n-j)logd = jlog(u/d) + nlogd \tag{A.20}$$

where j is the random number of upward moves occurring during the n periods to expiration. THerefore the expected value of $\log(S^*/S)$ is

$$E[\log(S^*/S] = E(j)[\log(u/d) + n \log d \qquad \text{(A.21)}$$

and its variance is given by,

$$Var[\log(S^*/S] = Var(j)[\log(u/d)]^2 \qquad \text{(A.22)}$$

As each of the n possible upward moves has probability q. Thus $E(j) = nq$. Also, since the variance of each period is

$$q(1-q)^2 + (1-q)(0-q)^2 = q(1-q) \qquad \text{(A.23)}$$

then $Var(j) = nq(1-q)$. Combining all of this, we have

$$\begin{aligned} E[\log(S^*/S] &= [q[\log(u/d) + \log d]n \equiv \hat{\mu}n \\ Var[\log(S^*/S] &= [q(1-q)[\log(u/d)]^2]n \equiv \hat{\sigma^2}n \end{aligned} \qquad \text{(A.24)}$$

As $n \to \infty$, a reasonable conclusion would not be reached if either $\hat{\mu}n$ or $\hat{\sigma^2}n$ went to zero or infinity. Since t is a fixed length of time, in searching for a realistic result, appropriate adjustments must be made to $u, d, \text{and} q$. In doing so we must make at least the mean and variance of the compounded rate of return of the assumed stock price coincide with that of the actual stock price as $n \to \infty$. Suppose the actual empirical values of $\hat{\mu}n$ and $\hat{\sigma^2}n$ are labelled as μt and $\sigma^2 t$. Then we would want to choose $u, d \text{and} q$ so that,

$$\begin{aligned} [q[\log(u/d) + \log d]n &\to \mu t \\ q(1-q)[\log(u/d)]^2 n &\to \sigma^2 tasn \to \infty \end{aligned}$$

It can easily be shown that this is accomplished by letting

$$
\begin{aligned}
u &= e^{\sigma\sqrt{t/n}}, \\
d &= e^{-\sigma\sqrt{t/n}}, \text{ and} \\
q &= \frac{1}{2} + \frac{1}{2}(\mu/\sigma)\sqrt{t/n}
\end{aligned}
\qquad \text{(A.25)}
$$

The Black and Scholes formula was also derived by assuming continuous trading and the assumption of a lognormal distribution for stock prices. The formula derived by Black and Scholes and rewritten in terms of the notation above, is,

$$C = SN(x) - Kr^{-t}N(x - \sigma\sqrt{t}) \qquad \text{(A.26)}$$

where

$$x \equiv \frac{\log(S/Kr^{-t})}{\sigma\sqrt{t}} + \frac{1}{2}\sigma\sqrt{t}$$

Following is one interpretation of the formula. If the call is exercised on the expiration date, the call is received, but in return the striking price of the call will have to be paid. This exchange will only take place if the call finishes in the money. The first term in the above formula, $SN(x)$ is the present value of receiving the stock, the second term, $-Kr^{-t}N(x - \sigma\sqrt{t})$, is the present value of paying the striking price if and only if $S^* > K$. Just as we would expect from this interpretation,if S is very large relative to K, then $N(x) \approx N(x - \sigma\sqrt{t}) \approx 1$, and $C \approx S - K^r - t$. The first term in the Black-Scholes formula, $SN(x)$ is the amount invested in the stock; the second term, $Kr^{-t}N(x - \sigma\sqrt{t})$, is the amount borrowed.

It can also be confirmed that the binomial formula described above converges to the Black-Scholes formula when t is divided into more and more subintervals, and $\hat{r}, u, d, \text{and} q$ are chosen in the way described above - that is the multiplicative binomial probability distribution of stock prices goes to the lognormal distribution.

Appendix B

Definition of some Terms used in the Petroleum Industry

- **Acreage** Land under lease.
- **Acre-Foot** a unit of volume used in calculating reserves
- **API Density** density of oil, in degrees; expressed in numbers form 6 (very heavy) to 60 (very light).
- **Bit** a cutting tool used in drilling
- **Blowout** an uncontrolled flow of fluid from the well.
- **BPD** barrels per day.
- **Bright Spot** intense seismic reflection off a gas reservoir.
- **Casing** pipe put down into a well and cemented to the sides of the well.
- **Cement** minerals that grow between clastic grains to solidify a sedimentary rock; also refers to slurry that binds the casing to the well walls.
- **confirmation Well** a well discovered after the discovery well to prove the location of a new petroleum deposit.
- **Coning** the drawing of water up into an oil reservoir caused by too rapid oil production.

- **Crude Oil** a mixture of liquid molecules formed primarily of carbon and hydrogen.
- **Derrick** the steel tower on a drilling rig.
- **Development Drilling** drilling in the extent of a known field.
- **Discovery Well** a well that locates a new petroleum deposit.
- **Dry Gas** natural gas without condensate liquids.
- **farm In** a lease obtained from another company for drilling in return for a royalty.
- **Farm Out** a lease given to another company for drilling.
- **Field** several gas and/or oil pools that are related to a certain geologic feature.
- **Gas** light hydrocarbons that occur as a gas under surface conditions (natural gas).
- **Hydrocarbons** molecules formed primarily by carbon and hydrogen atoms.
- **In-Fill Drilling** drilling between producing wells in an oil field to yield more petroleum at a faster rate.
- **Log** a record of rock properties in a well
- **Mcf** 1000 cubic feet, a standard measure for natural gas.
- **Mineral Rights** the legal ownership of oil and gas below a parcel of land.
- **Operator** the person of company who contracts to drill a well or maintain a producing lease.
- **Producer** a well that will flow commercial amounts of oil and gas.
- **Rank Wildcat** an exploratory well that is drilled far away from any area of known production.
- **Relief Well** a well drilled close to a blowout well in order to pump heavy drilling mud into the uncontrolled well in the subsurface.

- **Reserves** the amount of oil and/or gas that is expected to be produced.
- **Royalties** the percentage of the revenue from oil and gas production that is paid to the mineral rights owner.
- **Section** one square mile of land.
- **Spacing** the government regulation of land area which must surround one well.
- **Stepping Out** drilling to the sides of a discovery well to determine the limits of the reservoir.
- **Surface Rights** the legal ownership of the surface of a parcel of land.
- **Tight Hole** a well about which a company is keeping secret as much information as possible.
- **Trap** a high area on the reservoir rock where oil and/or gas accumulates.
- **Water Table** the subsurface level below which the pores in the soil or rock saturated with water.
- **Well Sitter** a geologist on the drilling site.
- **Wildcat Well** an exploratory well.

Appendix C

Volatility of Oil Prices

An important variable in the option valuation equation is the volatility of oil prices. This variable has to be calculated from the oil prices and is done as follows:

The Black-Scholes option valuation formula is based on the assumption that stock prices are lognormally distributed. This means that the natural logarithm of the price relative over any period has a normal distribution, with mean and variance proportional to the length of the period. The tables on the following pages calculate the annual volatility of crude oil prices. The volatility is calculated separately for OPEC crude oil, non-OPEC oil and then total world oil. The crude oil prices are monthly data and have been taken from the Department of Interior's Weekly Petroleum Status Report. The period taken was the 12 year period from September 1981 to July 1993 which is fairly indicative of the current volatility of oil prices.

Variance of OPEC Crude Oil Prices

Year	Sj	Rj=Sj/Sj-1	log Rj	log Rj - u	(log Rj - u)^2
		Variance of OPEC Crude Oil Prices			
Sep 81	$33.99				
Oct 81	$33.98	0.9997	-0.0003	0.0051	0.000026
Nov 81	$33.83	0.9956	-0.0044	0.0010	0.000001
Dec 81	$34.09	1.0077	0.0077	0.0130	0.000170
Jan 82	$34.27	1.0053	0.0053	0.0107	0.000113
Feb 82	$34.13	0.9959	-0.0041	0.0013	0.000002
Mar 82	$34.10	0.9991	-0.0009	0.0045	0.000020
Apr 82	$33.98	0.9965	-0.0035	0.0019	0.000003
May 82	$33.73	0.9926	-0.0074	-0.0020	0.000004
Jun 82	$33.70	0.9991	-0.0009	0.0045	0.000020
Jul 82	$33.47	0.9932	-0.0068	-0.0015	0.000002
Aug 82	$33.57	1.0030	0.0030	0.0084	0.000070
Sep 82	$33.56	0.9997	-0.0003	0.0051	0.000026
Oct 82	$33.56	1.0000	0.0000	0.0054	0.000029
Nov 82	$33.56	1.0000	0.0000	0.0054	0.000029
Dec 82	$33.54	0.9994	-0.0006	0.0048	0.000023
Jan 83	$33.54	1.0000	0.0000	0.0054	0.000029
Feb 83	$33.54	1.0000	0.0000	0.0054	0.000029
Mar 83	$33.55	1.0003	0.0003	0.0057	0.000032
Apr 83	$33.10	0.9866	-0.0135	-0.0081	0.000066
May 83	$28.87	0.8722	-0.1367	-0.1313	0.017251
Jun 83	$28.82	0.9983	-0.0017	0.0037	0.000013
Jul 83	$28.82	1.0000	0.0000	0.0054	0.000029
Aug 83	$28.87	1.0017	0.0017	0.0071	0.000051
Sep 83	$28.73	0.9952	-0.0049	0.0005	0.000000
Oct 83	$28.63	0.9965	-0.0035	0.0019	0.000004
Nov 83	$28.59	0.9986	-0.0014	0.0040	0.000016
Dec 83	$28.59	1.0000	0.0000	0.0054	0.000029

Variance of OPEC Crude Oil Prices

Jan 84	$28.59	1.0000	0.0000	0.0054	0.000029
Feb 84	$28.59	1.0000	0.0000	0.0054	0.000029
Mar 84	$28.59	1.0000	0.0000	0.0054	0.000029
Apr 84	$28.59	1.0000	0.0000	0.0054	0.000029
May 84	$28.59	1.0000	0.0000	0.0054	0.000029
Jun 84	$28.59	1.0000	0.0000	0.0054	0.000029
Jul 84	$28.59	1.0000	0.0000	0.0054	0.000029
Aug 84	$28.59	1.0000	0.0000	0.0054	0.000029
Sep 84	$28.59	1.0000	0.0000	0.0054	0.000029
Oct 84	$28.74	1.0052	0.0052	0.0106	0.000113
Nov 84	$28.58	0.9944	-0.0056	-0.0002	0.000000
Dec 84	$28.58	1.0000	0.0000	0.0054	0.000029
Jan 85	$28.51	0.9976	-0.0025	0.0029	0.000009
Feb 85	$28.51	1.0000	0.0000	0.0054	0.000029
Mar 85	$28.17	0.9881	-0.0120	-0.0066	0.000044
Apr 85	$27.97	0.9929	-0.0071	-0.0017	0.000003
May 85	$27.99	1.0007	0.0007	0.0061	0.000037
Jun 85	$27.99	1.0000	0.0000	0.0054	0.000029
Jul 85	$27.96	0.9989	-0.0011	0.0043	0.000019
Aug 85	$27.97	1.0004	0.0004	0.0057	0.000033
Sep 85	$27.92	0.9982	-0.0018	0.0036	0.000013
Oct 85	$27.92	1.0000	0.0000	0.0054	0.000029
Nov 85	$27.88	0.9986	-0.0014	0.0040	0.000016
Dec 85	$27.79	0.9968	-0.0032	0.0022	0.000005
Jan 86	$27.81	1.0007	0.0007	0.0061	0.000037
Feb 86	$27.76	0.9982	-0.0018	0.0036	0.000013
Mar 86	$27.35	0.9852	-0.0149	-0.0095	0.000090
Apr 86	$15.37	0.5620	-0.5763	-0.5709	0.325942
May 86	$12.69	0.8256	-0.1916	-0.1862	0.034677
Jun 86	$13.48	1.0623	0.0604	0.0658	0.004327
Jul 86	$12.16	0.9021	-0.1031	-0.0977	0.009539
Aug 86	$9.70	0.7977	-0.2260	-0.2206	0.048682
Sep 86	$12.60	1.2990	0.2616	0.2670	0.071266
Oct 86	$13.46	1.0683	0.0660	0.0714	0.005100
Nov 86	$13.66	1.0149	0.0147	0.0201	0.000405

Variance of OPEC Crude Oil Prices

Dec 86	$13.76	1.0073	0.0073	0.0127	0.000161
Jan 87	$13.50	0.9811	-0.0191	-0.0137	0.000187
Feb 87	$18.13	1.3430	0.2949	0.3003	0.090159
Mar 87	$17.34	0.9564	-0.0446	-0.0392	0.001534
Apr 87	$17.36	1.0012	0.0012	0.0065	0.000043
May 87	$17.38	1.0012	0.0012	0.0065	0.000043
Jun 87	$17.38	1.0000	0.0000	0.0054	0.000029
Jul 87	$17.41	1.0017	0.0017	0.0071	0.000051
Aug 87	$17.48	1.0040	0.0040	0.0094	0.000088
Sep 87	$17.45	0.9983	-0.0017	0.0037	0.000013
Oct 87	$17.40	0.9971	-0.0029	0.0025	0.000006
Nov 87	$17.38	0.9989	-0.0012	0.0042	0.000018
Dec 87	$17.36	0.9988	-0.0012	0.0042	0.000018
Jan 88	$16.77	0.9660	-0.0346	-0.0292	0.000852
Feb 88	$16.67	0.9940	-0.0060	-0.0006	0.000000
Mar 88	$15.60	0.9358	-0.0663	-0.0610	0.003715
Apr 88	$13.76	0.8821	-0.1255	-0.1201	0.014429
May 88	$15.59	1.1329	0.1248	0.1302	0.016948
Jun 88	$15.18	0.9738	-0.0266	-0.0212	0.000449
Jul 88	$14.17	0.9335	-0.0689	-0.0635	0.004028
Aug 88*	$13.75	0.9704	-0.0301	-0.0247	0.000610
Sep 88	$13.58	0.9876	-0.0124	-0.0071	0.000050
Oct 88	$12.23	0.9006	-0.1047	-0.0993	0.009865
Nov 88	$11.70	0.9567	-0.0443	-0.0389	0.001515
Dec 88	$9.86	0.8427	-0.1711	-0.1657	0.027462
Jan 89	$12.96	1.3144	0.2734	0.2788	0.077711
Feb 89	$15.61	1.2045	0.1860	0.1914	0.036645
Mar 89	$15.04	0.9635	-0.0372	-0.0318	0.001012
Apr 89	$16.54	1.0997	0.0951	0.1005	0.010091
May 89	$17.72	1.0713	0.0689	0.0743	0.005520
Jun 89	$16.91	0.9543	-0.0468	-0.0414	0.001714
Jul 89	$15.63	0.9243	-0.0787	-0.0733	0.005377
Aug 89	$16.59	1.0614	0.0596	0.0650	0.004224
Sep 89	$15.62	0.9415	-0.0602	-0.0549	0.003010
Oct 89	$16.30	1.0435	0.0426	0.0480	0.002304

Variance of OPEC Crude Oil Prices

Nov 89	$16.87	1.0350	0.0344	0.0398	0.001581
Dec 89*	$16.25	0.9632	-0.0374	-0.0321	0.001028
Jan 90	$17.72	1.0905	0.0866	0.0920	0.008462
Feb 90	$18.70	1.0553	0.0538	0.0592	0.003506
Mar 90	$17.92	0.9583	-0.0426	-0.0372	0.001385
Apr 90	$16.61	0.9269	-0.0759	-0.0705	0.004974
May 90	$13.67	0.8230	-0.1948	-0.1894	0.035878
Jun 90	$15.09	1.1039	0.0988	0.1042	0.010861
Jul 90	$13.23	0.8767	-0.1315	-0.1262	0.015916
Aug 90*	$15.00	1.1338	0.1256	0.1309	0.017148
Sep 90	$24.56	1.6373	0.4931	0.4985	0.248457
Oct 90	$27.72	1.1287	0.1210	0.1264	0.015982
Nov 90	$33.38	1.2042	0.1858	0.1912	0.036553
Dec 90	$28.92	0.8664	-0.1434	-0.1380	0.019054
Jan 91	$23.86	0.8250	-0.1923	-0.1869	0.034948
Feb 91	$23.20	0.9723	-0.0281	-0.0227	0.000514
Mar 91	$16.59	0.7151	-0.3354	-0.3300	0.108878
Apr 91	$15.65	0.9433	-0.0583	-0.0529	0.002803
May 91	$16.69	1.0665	0.0643	0.0697	0.004862
Jun 91	$16.46	0.9862	-0.0139	-0.0085	0.000072
Jul 91	$15.81	0.9605	-0.0403	-0.0349	0.001218
Aug 91	$17.07	1.0797	0.0767	0.0821	0.006735
Sep 91	$16.92	0.9912	-0.0088	-0.0034	0.000012
Oct 91*	$18.75	1.1082	0.1027	0.1081	0.011682
Nov 91	$19.79	1.0555	0.0540	0.0594	0.003525
Dec 91	$19.14	0.9672	-0.0334	-0.0280	0.000785
Jan 92	$16.24	0.8485	-0.1643	-0.1589	0.025255
Feb 92	$15.71	0.9674	-0.0332	-0.0278	0.000773
Mar 92	$16.47	1.0484	0.0472	0.0526	0.002770
Apr 92	$16.22	0.9848	-0.0153	-0.0099	0.000098
May 92	$17.03	1.0499	0.0487	0.0541	0.002929
Jun 92	$17.86	1.0487	0.0476	0.0530	0.002806
Jul 92	$19.42	1.0873	0.0837	0.0891	0.007943
Aug 92	$18.90	0.9732	-0.0271	-0.0218	0.000473
Sep 92	$18.40	0.9735	-0.0268	-0.0214	0.000459

Variance of OPEC Crude Oil Prices

Oct 92	$19.09	1.0375	0.0368	0.0422	0.001781
Nov 92	$19.16	1.0037	0.0037	0.0090	0.000082
Dec 92	$17.67	0.9222	-0.0810	-0.0756	0.005711
Jan 93	$16.52	0.9349	-0.0673	-0.0619	0.003833
Feb 93	$15.78	0.9552	-0.0458	-0.0404	0.001636
Mar 93	$16.71	1.0589	0.0573	0.0626	0.003925
Apr 93	$16.99	1.0168	0.0166	0.0220	0.000484
May 93	$16.87	0.9929	-0.0071	-0.0017	0.000003
Jun 93	$16.31	0.9668	-0.0338	-0.0284	0.000805
Jul 93	$15.82	0.9700	-0.0305	-0.0251	0.000631
		Σ	**-0.7647914**	Σ	**1.49793**
		μ=	**-0.0053859**		
Unbiased Volatility:			**0.01062**		
Estimate of Annual Variance:			**0.1275**		
Estimate of Annual Standard Deviation			**0.3570**		

Variance of non-OPEC Crude Oil Prices

		Variance of non-OPEC Crude Oil Prices			
Year	Sj	Rj=Sj/Sj-1	log Rj	log Rj - u	(log Rj - u)^2
Sep 81	$34.72				
Oct 81	$34.72	1.0000	0.0000	0.0051	0.000026
Nov 81	$34.39	0.9905	-0.0096	-0.0045	0.000020
Dec 81	$35.06	1.0195	0.0193	0.0244	0.000595
Jan 82	$34.73	0.9906	-0.0095	-0.0044	0.000019
Feb 82	$34.35	0.9891	-0.0110	-0.0059	0.000035
Mar 82	$34.02	0.9904	-0.0097	-0.0046	0.000021
Apr 82	$31.93	0.9386	-0.0634	-0.0583	0.003400
May 82	$31.89	0.9987	-0.0013	0.0038	0.000015
Jun 82	$31.82	0.9978	-0.0022	0.0029	0.000008
Jul 82	$31.93	1.0035	0.0035	0.0085	0.000073
Aug 82	$31.93	1.0000	0.0000	0.0051	0.000026
Sep 82	$31.93	1.0000	0.0000	0.0051	0.000026
Oct 82	$31.91	0.9994	-0.0006	0.0045	0.000020
Nov 82	$31.91	1.0000	0.0000	0.0051	0.000026
Dec 82	$31.84	0.9978	-0.0022	0.0029	0.000008
Jan 83	$31.77	0.9978	-0.0022	0.0029	0.000008
Feb 83	$31.72	0.9984	-0.0016	0.0035	0.000012
Mar 83	$31.35	0.9883	-0.0117	-0.0066	0.000044
Apr 83	$29.24	0.9327	-0.0697	-0.0646	0.004171
May 83	$28.20	0.9644	-0.0362	-0.0311	0.000968
Jun 83	$28.20	1.0000	0.0000	0.0051	0.000026
Jul 83	$28.21	1.0004	0.0004	0.0055	0.000030
Aug 83	$28.46	1.0089	0.0088	0.0139	0.000194
Sep 83	$28.66	1.0070	0.0070	0.0121	0.000146
Oct 83	$28.63	0.9990	-0.0010	0.0040	0.000016
Nov 83	$28.79	1.0056	0.0055	0.0106	0.000113
Dec 83	$28.83	1.0014	0.0014	0.0065	0.000042

Variance of non-OPEC Crude Oil Prices

Jan 84	$28.70	0.9955	-0.0045	0.0006	0.000000
Feb 84	$28.65	0.9983	-0.0017	0.0034	0.000011
Mar 84	$28.65	1.0000	0.0000	0.0051	0.000026
Apr 84	$28.72	1.0024	0.0024	0.0075	0.000057
May 84	$28.71	0.9997	-0.0003	0.0047	0.000023
Jun 84	$28.77	1.0021	0.0021	0.0072	0.000052
Jul 84	$28.77	1.0000	0.0000	0.0051	0.000026
Aug 84	$28.77	1.0000	0.0000	0.0051	0.000026
Sep 84	$28.57	0.9930	-0.0070	-0.0019	0.000004
Oct 84	$28.70	1.0046	0.0045	0.0096	0.000093
Nov 84	$28.18	0.9819	-0.0183	-0.0132	0.000174
Dec 84	$28.18	1.0000	0.0000	0.0051	0.000026
Jan 85	$28.16	0.9993	-0.0007	0.0044	0.000019
Feb 85	$28.16	1.0000	0.0000	0.0051	0.000026
Mar 85	$27.98	0.9936	-0.0064	-0.0013	0.000002
Apr 85	$27.88	0.9964	-0.0036	0.0015	0.000002
May 85	$27.36	0.9813	-0.0188	-0.0137	0.000189
Jun 85	$27.20	0.9942	-0.0059	-0.0008	0.000001
Jul 85	$26.31	0.9673	-0.0333	-0.0282	0.000794
Aug 85	$25.76	0.9791	-0.0211	-0.0160	0.000257
Sep 85	$25.87	1.0043	0.0043	0.0094	0.000088
Oct 85	$26.34	1.0182	0.0180	0.0231	0.000534
Nov 85	$26.62	1.0106	0.0106	0.0157	0.000246
Dec 85	$27.39	1.0289	0.0285	0.0336	0.001130
Jan 86	$26.60	0.9712	-0.0293	-0.0242	0.000584
Feb 86	$24.70	0.9286	-0.0741	-0.0690	0.004763
Mar 86	$19.94	0.8073	-0.2141	-0.2090	0.043673
Apr 86	$14.19	0.7116	-0.3402	-0.3351	0.112289
May 86	$12.27	0.8647	-0.1454	-0.1403	0.019680
Jun 86	$13.05	1.0636	0.0616	0.0667	0.004452
Jul 86	$11.77	0.9019	-0.1032	-0.0981	0.009631
Aug 86	$9.49	0.8063	-0.2153	-0.2102	0.044192
Sep 86	$11.93	1.2571	0.2288	0.2339	0.054715
Oct 86	$12.83	1.0754	0.0727	0.0778	0.006057
Nov 86	$12.81	0.9984	-0.0016	0.0035	0.000012

Variance of non-OPEC Crude Oil Prices

Dec 86	$13.45	1.0500	0.0488	0.0538	0.002900
Jan 87	$13.94	1.0364	0.0358	0.0409	0.001671
Feb 87	$17.54	1.2582	0.2297	0.2348	0.055139
Mar 87	$17.03	0.9709	-0.0295	-0.0244	0.000596
Apr 87	$17.45	1.0247	0.0244	0.0295	0.000868
May 87	$17.42	0.9983	-0.0017	0.0034	0.000011
Jun 87	$18.15	1.0419	0.0411	0.0461	0.002130
Jul 87	$18.10	0.9972	-0.0028	0.0023	0.000005
Aug 87	$19.01	1.0503	0.0491	0.0541	0.002932
Sep 87	$18.02	0.9479	-0.0535	-0.0484	0.002341
Oct 87	$17.61	0.9772	-0.0230	-0.0179	0.000321
Nov 87	$17.84	1.0131	0.0130	0.0181	0.000327
Dec 87	$17.04	0.9552	-0.0459	-0.0408	0.001663
Jan 88	$16.21	0.9513	-0.0499	-0.0448	0.002011
Feb 88	$15.61	0.9630	-0.0377	-0.0326	0.001064
Mar 88	$15.16	0.9712	-0.0293	-0.0242	0.000584
Apr 88	$14.06	0.9274	-0.0753	-0.0702	0.004932
May 88	$15.44	1.0982	0.0936	0.0987	0.009746
Jun 88	$15.51	1.0045	0.0045	0.0096	0.000093
Jul 88	$15.20	0.9800	-0.0202	-0.0151	0.000228
Aug 88*	$14.60	0.9605	-0.0403	-0.0352	0.001238
Sep 88	$14.00	0.9589	-0.0420	-0.0369	0.001359
Oct 88	$12.87	0.9193	-0.0842	-0.0791	0.006251
Nov 88	$12.21	0.9487	-0.0526	-0.0475	0.002261
Dec 88	$11.16	0.9140	-0.0899	-0.0848	0.007195
Jan 89	$13.53	1.2124	0.1926	0.1977	0.039073
Feb 89	$15.90	1.1752	0.1614	0.1665	0.027724
Mar 89	$15.64	0.9836	-0.0165	-0.0114	0.000130
Apr 89	$17.04	1.0895	0.0857	0.0908	0.008250
May 89	$18.08	1.0610	0.0592	0.0643	0.004139
Jun 89	$17.86	0.9878	-0.0122	-0.0071	0.000051
Jul 89	$16.75	0.9378	-0.0642	-0.0591	0.003489
Aug 89	$17.48	1.0436	0.0427	0.0478	0.002281
Sep 89	$16.29	0.9319	-0.0705	-0.0654	0.004279
Oct 89	$16.97	1.0417	0.0409	0.0460	0.002115

Variance of non-OPEC Crude Oil Prices

Nov 89	$17.71	1.0436	0.0427	0.0478	0.002283
Dec 89*	$18.00	1.0164	0.0162	0.0213	0.000455
Jan 90	$18.31	1.0172	0.0171	0.0222	0.000492
Feb 90	$19.57	1.0688	0.0666	0.0716	0.005133
Mar 90	$18.96	0.9688	-0.0317	-0.0266	0.000706
Apr 90	$17.78	0.9378	-0.0643	-0.0592	0.003500
May 90	$14.84	0.8346	-0.1807	-0.1757	0.030854
Jun 90	$15.83	1.0667	0.0646	0.0697	0.004855
Jul 90	$13.99	0.8838	-0.1236	-0.1185	0.014035
Aug 90*	$15.25	1.0901	0.0862	0.0913	0.008342
Sep 90	$25.49	1.6715	0.5137	0.5188	0.269156
Oct 90	$29.25	1.1475	0.1376	0.1427	0.020360
Nov 90	$34.08	1.1651	0.1528	0.1579	0.024941
Dec 90	$30.65	0.8994	-0.1061	-0.1010	0.010197
Jan 91	$24.98	0.8150	-0.2046	-0.1995	0.039785
Feb 91	$23.41	0.9371	-0.0649	-0.0598	0.003578
Mar 91	$18.68	0.7979	-0.2257	-0.2206	0.048671
Apr 91	$17.31	0.9267	-0.0762	-0.0711	0.005051
May 91	$17.75	1.0254	0.0251	0.0302	0.000912
Jun 91	$17.21	0.9696	-0.0309	-0.0258	0.000666
Jul 91	$16.52	0.9599	-0.0409	-0.0358	0.001283
Aug 91	$18.16	1.0993	0.0946	0.0997	0.009949
Sep 91	$17.93	0.9873	-0.0127	-0.0077	0.000059
Oct 91*	$19.20	1.0708	0.0684	0.0735	0.005407
Nov 91	$20.58	1.0719	0.0694	0.0745	0.005551
Dec 91	$20.02	0.9728	-0.0276	-0.0225	0.000506
Jan 92	$17.24	0.8611	-0.1495	-0.1444	0.020853
Feb 92	$16.89	0.9797	-0.0205	-0.0154	0.000238
Mar 92	$17.54	1.0385	0.0378	0.0429	0.001837
Apr 92	$17.60	1.0034	0.0034	0.0085	0.000072
May 92	$16.63	0.9449	-0.0567	-0.0516	0.002662
Jun 92	$17.74	1.0667	0.0646	0.0697	0.004859
Jul 92	$17.98	1.0135	0.0134	0.0185	0.000343
Aug 92	$17.91	0.9961	-0.0039	0.0012	0.000001
Sep 92	$19.25	1.0748	0.0722	0.0772	0.005967

Variance of non-OPEC Crude Oil Prices

Oct 92	$19.66	1.0213	0.0211	0.0262	0.000685
Nov 92	$19.98	1.0163	0.0161	0.0212	0.000451
Dec 92	$18.76	0.9389	-0.0630	-0.0579	0.003353
Jan 93	$17.60	0.9382	-0.0638	-0.0587	0.003450
Feb 93	$16.63	0.9449	-0.0567	-0.0516	0.002662
Mar 93	$17.74	1.0667	0.0646	0.0697	0.004859
Apr 93	$17.98	1.0135	0.0134	0.0185	0.000343
May 93	$17.91	0.9961	-0.0039	0.0012	0.000001
Jun 93	$17.47	0.9754	-0.0249	-0.0198	0.000391
Jul 93	$16.84	0.9639	-0.0367	-0.0316	0.001001
		Σ	**-0.7236**	Σ	**1.084091**
		μ=	**-0.0051**		
Unbiased Volatility:			**0.0077**		
Estimate of Annual Variance:			**0.0923**		
Estimate of Annual Standard Deviation			**0.3037**		

Variance of Total World Crude Oil Prices

Year	Sj	Rj=Sj/Sj-1	log Rj	log Rj - u	(log Rj - u)^2
Sep 81	$34.13				
Oct 81	$34.13	1.0000	0.0000	0.0053	0.000028
Nov 81	$33.96	0.9950	-0.0050	0.0003	0.000000
Dec 81	$34.31	1.0103	0.0103	0.0155	0.000240
Jan 82	$34.38	1.0020	0.0020	0.0073	0.000053
Feb 82	$34.18	0.9942	-0.0058	-0.0006	0.000000
Mar 82	$34.08	0.9971	-0.0029	0.0023	0.000005
Apr 82	$33.48	0.9824	-0.0178	-0.0125	0.000157
May 82	$33.28	0.9940	-0.0060	-0.0007	0.000001
Jun 82	$33.25	0.9991	-0.0009	0.0044	0.000019
Jul 82	$33.05	0.9940	-0.0060	-0.0008	0.000001
Aug 82	$33.13	1.0024	0.0024	0.0077	0.000059
Sep 82	$33.11	0.9994	-0.0006	0.0046	0.000022
Oct 82	$33.11	1.0000	0.0000	0.0053	0.000028
Nov 82	$33.11	1.0000	0.0000	0.0053	0.000028
Dec 82	$33.03	0.9976	-0.0024	0.0028	0.000008
Jan 83	$33.01	0.9994	-0.0006	0.0046	0.000022
Feb 83	$33.00	0.9997	-0.0003	0.0049	0.000024
Mar 83	$32.89	0.9967	-0.0033	0.0019	0.000004
Apr 83	$31.95	0.9714	-0.0290	-0.0237	0.000564
May 83	$28.62	0.8958	-0.1101	-0.1048	0.010986
Jun 83	$28.59	0.9990	-0.0010	0.0042	0.000018
Jul 83	$28.59	1.0000	0.0000	0.0053	0.000028
Aug 83	$28.72	1.0045	0.0045	0.0098	0.00C096
Sep 83	$28.71	0.9997	-0.0003	0.0049	0.000024
Oct 83	$28.63	0.9972	-0.0028	0.0025	0.000006
Nov 83	$28.65	1.0007	0.0007	0.0060	0.000035
Dec 83	$28.67	1.0007	0.0007	0.0059	0.000035

Variance of Total World Crude Oil Prices

Jan 84	$28.63	0.9986	-0.0014	0.0039	0.000015
Feb 84	$28.61	0.9993	-0.0007	0.0046	0.000021
Mar 84	$28.61	1.0000	0.0000	0.0053	0.000028
Apr 84	$28.63	1.0007	0.0007	0.0060	0.000035
May 84	$28.62	0.9997	-0.0003	0.0049	0.000024
Jun 84	$28.65	1.0010	0.0010	0.0063	0.000040
Jul 84	$28.65	1.0000	0.0000	0.0053	0.000028
Aug 84	$28.65	1.0000	0.0000	0.0053	0.000028
Sep 84	$28.58	0.9976	-0.0024	0.0028	0.000008
Oct 84	$28.73	1.0052	0.0052	0.0105	0.000110
Nov 84	$28.43	0.9896	-0.0105	-0.0052	0.000028
Dec 84	$28.43	1.0000	0.0000	0.0053	0.000028
Jan 85	$28.38	0.9982	-0.0018	0.0035	0.000012
Feb 85	$28.38	1.0000	0.0000	0.0053	0.000028
Mar 85	$28.10	0.9901	-0.0099	-0.0047	0.000022
Apr 85	$27.94	0.9943	-0.0057	-0.0005	0.000000
May 85	$27.76	0.9936	-0.0065	-0.0012	0.000001
Jun 85	$27.69	0.9975	-0.0025	0.0027	0.000007
Jul 85	$27.35	0.9877	-0.0124	-0.0071	0.000050
Aug 85	$27.15	0.9927	-0.0073	-0.0021	0.000004
Sep 85	$27.06	0.9967	-0.0033	0.0019	0.000004
Oct 85	$27.26	1.0074	0.0074	0.0126	0.000159
Nov 85	$27.36	1.0037	0.0037	0.0089	0.000079
Dec 85	$27.63	1.0099	0.0098	0.0151	0.000227
Jan 86	$27.31	0.9884	-0.0116	-0.0064	0.000041
Feb 86	$26.56	0.9725	-0.0278	-0.0226	0.000511
Mar 86	$24.48	0.9217	-0.0815	-0.0763	0.005821
Apr 86	$14.93	0.6099	-0.4945	-0.4892	0.239348
May 86	$12.53	0.8392	-0.1752	-0.1700	0.028898
Jun 86	$13.33	1.0638	0.0619	0.0671	0.004508
Jul 86	$12.00	0.9002	-0.1051	-0.0999	0.009972
Aug 86	$9.62	0.8017	-0.2211	-0.2158	0.046574
Sep 86	$12.34	1.2827	0.2490	0.2543	0.064645
Oct 86	$13.24	1.0729	0.0704	0.0756	0.005723
Nov 86	$13.36	1.0091	0.0090	0.0143	0.000204

Variance of Total World Crude Oil Prices

Dec 86	$13.63	1.0202	0.0200	0.0253	0.000638
Jan 87	$13.69	1.0044	0.0044	0.0096	0.000093
Feb 87	$17.88	1.3061	0.2670	0.2723	0.074130
Mar 87	$17.21	0.9625	-0.0382	-0.0329	0.001085
Apr 87	$17.40	1.0110	0.0110	0.0162	0.000263
May 87	$17.40	1.0000	0.0000	0.0053	0.000028
Jun 87	$17.69	1.0167	0.0165	0.0218	0.000474
Jul 87	$17.71	1.0011	0.0011	0.0064	0.000041
Aug 87	$18.03	1.0181	0.0179	0.0232	0.000536
Sep 87	$17.65	0.9789	-0.0213	-0.0160	0.000258
Oct 87	$17.47	0.9898	-0.0103	-0.0050	0.000025
Nov 87	$17.54	1.0040	0.0040	0.0093	0.000086
Dec 87	$17.25	0.9835	-0.0167	-0.0114	0.000130
Jan 88	$16.57	0.9606	-0.0402	-0.0350	0.001223
Feb 88	$16.29	0.9831	-0.0170	-0.0118	0.000139
Mar 88	$15.42	0.9466	-0.0549	-0.0496	0.002464
Apr 88	$13.88	0.9001	-0.1052	-0.1000	0.009993
May 88	$15.53	1.1189	0.1123	0.1176	0.013824
Jun 88	$15.31	0.9858	-0.0143	-0.0090	0.000081
Jul 88	$14.90	0.9732	-0.0271	-0.0219	0.000479
Aug 88*	$14.40	0.9664	-0.0341	-0.0289	0.000834
Sep 88	$13.67	0.9493	-0.0520	-0.0468	0.002188
Oct 88	$12.45	0.9108	-0.0935	-0.0882	0.007785
Nov 88	$11.87	0.9534	-0.0477	-0.0425	0.001802
Dec 88	$10.31	0.8686	-0.1409	-0.1356	0.018400
Jan 89	$13.15	1.2755	0.2433	0.2486	0.061782
Feb 89	$15.70	1.1939	0.1772	0.1825	0.033303
Mar 89	$15.26	0.9720	-0.0284	-0.0232	0.000537
Apr 89	$16.72	1.0957	0.0914	0.0966	0.009336
May 89	$17.85	1.0676	0.0654	0.0706	0.004991
Jun 89	$17.25	0.9664	-0.0342	-0.0289	0.000837
Jul 89	$16.02	0.9287	-0.0740	-0.0687	0.004723
Aug 89	$16.90	1.0549	0.0535	0.0587	0.003449
Sep 89	$15.85	0.9379	-0.0641	-0.0589	0.003468
Oct 89	$16.53	1.0429	0.0420	0.0473	0.002233

Variance of Total World Crude Oil Prices

Nov 89	$17.15	1.0375	0.0368	0.0421	0.001770
Dec 89*	$17.50	1.0204	0.0202	0.0255	0.000648
Jan 90	$17.91	1.0234	0.0232	0.0284	0.000807
Feb 90	$18.98	1.0597	0.0580	0.0633	0.004004
Mar 90	$18.26	0.9621	-0.0387	-0.0334	0.001117
Apr 90	$16.99	0.9304	-0.0721	-0.0668	0.004467
May 90	$14.04	0.8264	-0.1907	-0.1855	0.034396
Jun 90	$15.33	1.0919	0.0879	0.0932	0.008678
Jul 90	$13.47	0.8787	-0.1293	-0.1241	0.015399
Aug 90*	$17.00	1.2621	0.2327	0.2380	0.056644
Sep 90	$24.86	1.4624	0.3800	0.3853	0.148455
Oct 90	$28.28	1.1376	0.1289	0.1341	0.017995
Nov 90	$33.94	1.2001	0.1824	0.1877	0.035228
Dec 90	$29.50	0.8692	-0.1402	-0.1350	0.018212
Jan 91	$24.24	0.8217	-0.1964	-0.1911	0.036532
Feb 91	$23.27	0.9600	-0.0408	-0.0356	0.001266
Mar 91	$17.29	0.7430	-0.2970	-0.2918	0.085138
Apr 91	$16.20	0.9370	-0.0651	-0.0599	0.003584
May 91	$17.04	1.0519	0.0506	0.0558	0.003114
Jun 91	$16.72	0.9812	-0.0190	-0.0137	0.000188
Jul 91	$16.05	0.9599	-0.0409	-0.0356	0.001271
Aug 91	$17.31	1.0785	0.0756	0.0808	0.006533
Sep 91	$17.04	0.9844	-0.0157	-0.0105	0.000110
Oct 91*	$18.47	1.0839	0.0806	0.0858	0.007368
Nov 91	$20.05	1.0855	0.0821	0.0873	0.007627
Dec 91	$19.44	0.9696	-0.0309	-0.0256	0.000658
Jan 92	$16.58	0.8529	-0.1591	-0.1539	0.023680
Feb 92	$16.12	0.9723	-0.0281	-0.0229	0.000524
Mar 92	$16.85	1.0453	0.0443	0.0495	0.002454
Apr 92	$16.41	0.9739	-0.0265	-0.0212	0.000450
May 92	$17.31	1.0548	0.0534	0.0586	0.003439
Jun 92	$18.14	1.0479	0.0468	0.0521	0.002713
Jul 92	$19.69	1.0854	0.0820	0.0872	0.007611
Aug 92	$19.14	0.9721	-0.0283	-0.0231	0.000533
Sep 92	$18.70	0.9770	-0.0233	-0.0180	0.000324

Variance of Total World Crude Oil Prices

Oct 92	$19.29	1.0316	0.0311	0.0363	0.001319
Nov 92	$19.44	1.0078	0.0077	0.0130	0.000169
Dec 92	$18.04	0.9280	-0.0747	-0.0695	0.004829
Jan 93	$16.89	0.9363	-0.0659	-0.0606	0.003675
Feb 93	$16.07	0.9515	-0.0498	-0.0445	0.001982
Mar 93	$17.07	1.0622	0.0604	0.0656	0.004306
Apr 93	$17.33	1.0152	0.0151	0.0204	0.000415
May 93	$17.24	0.9948	-0.0052	0.0000	0.000000
Jun 93	$16.74	0.9710	-0.0294	-0.0242	0.000585
Jul 93	$16.19	0.9671	-0.0334	-0.0282	0.000793
		Σ	**-0.74578**	Σ	**1.242388**
		μ=	**-0.00525**		
Unbiased Volatility:			**0.00881**		
Estimate of Annual Variance:			**0.10574**		
Estimate of Annual Standard Deviation			**0.32517**		

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