STABILITY CALCULATIONS FOR ROTATING GAS FLOWS

by

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ABSTRACT

A numerical method is developed for solution of the basic equations governing small perturbations of an inviscid compressible fluid in rapid rotation. Under the assumption of small wave-like perturbations, acoustic and inertial modes are investigated for different degrees of compressibility, as measured by the nondimensional speed of sound. A swirling Poiseuille flow is found to be unstable for a sufficiently strong axial flow as measured by the ratio, ε, of maximum axial velocity to the peripheral velocity of the cylinder. Stability limits $\varepsilon=\varepsilon_B$ for the flow are investigated, by considering the dependence of ε_B on wavenumber k, for incompressible and compressible flows, respectively. By comparing stability properties of two different nonaxisymmetric mode types, it is found that higher modes and wavenumbers are more stable, and that compressibility can have a stabilizing effect. In addition, a few special cases with a single axial jet are considered. For the range of wavenumbers investigated, no instability was found. The program developed can also be used to investigate the stability of a free vortex.

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LIST OF SYMBOLS

z, r, ϕ	cylindrical coordinates
L	radius of outer cylinder
ROX	radius of inner cylinder
a	velocity of sound
γ'	ratio of specific heats
m	azimuthal wavenumber
k	axial wavenumber
ω	complex frequency = $\omega_r + i\omega_i$
Ω	frequency of rigid body rotation
ε	non-dimensional flow parameter
u_z^*	axial velocity
u_r^*	radial velocity
u_ϕ^*	azimuthal velocity
ρ	density
p^*	pressure
W	basic axial velocity
V	basic azimuthal velocity
T	basic temperature
T_o	temperature of inner boundary
ρ_o	basic density
ρ_{oo}	density at inner boundary
p_o	basic pressure

f	axial flow parameter
ε^*	non-dimensional axial flow parameter
J_m	Bessel function of the first kind, of order m
K_m	modified Bessel function of second kind, of order m
J	Richardson number
Y	disturbance vector
A	operator matrix

CHAPTER 1

INTRODUCTION

A. Rationale

The stability of rotating flows is of great importance in many fluid mechanical problems, such as in vortex ring stability, for flows in gas centrifuges, flows in gas-turbines and swirling flows in nozzles, and also in a variety of metereological phenomena such as tornadoes.

With better known stability properties of compressible rotating flows, ways may be found to improve designs of high speed rotating machinery so as to increase their efficiency and stability of operation.

B. Outline of Problem

The work presented here is an investigation of the stability of rotating, inviscid, compressible flows with arbitrary axisymmetric distributions of axial and swirl velocities. Linear theory has been used to obtain an equation for wave-like perturbations, and an eigenvalue problem is formulated. The solution of this gives the frequencies and mode shapes of the free oscillations possible in the system. The flow is assumed to be contained inside a rigid cylindrical annulus, with arbitrary diameter of the inner cylinder. Another possible flow-configuration is that of a free vortex.

C. Previous Work

1. Free Modes

The frequencies of modes for rigidly rotating fluids in the absence of axial flow have been discussed by many authors.

A. Acoustic Modes

The non-rotating acoustic frequencies, in a compressible fluid, are related to the zeroes of the Bessel functions derivatives.

$$\frac{d}{dx} J_m(x) = 0 \qquad (1.01)$$

with solutions x_n, corresponding to

$$\omega_r = a\sqrt{x_n^2 + k^2} \qquad (1.02)$$

This has been shown by Rayleigh.

B. Inertial Modes

Greenspan (1968) has given analytical results concerning inertial modes, in an incompressible flow enclosed in a cylindrical container.

The solutions of the transcendental equations (1.03)

$$x \frac{d}{dx} J_{|m|}(x) + m\left(1 + \frac{x^2}{k^2}\right)^{1/2} J_{|m|}(x) = 0$$

$$x \frac{d}{dx} J_{|m|}(x) - m\left(1 + \frac{x^2}{k^2}\right)^{1/2} J_{|m|}(x) = 0 \qquad (1.04)$$

give two sets of values, x_n, with each one corresponding to the frequency for the two sets of inertial modes

$$\omega_r = m + 2\left(1 + \frac{x_n^2}{k^2}\right)^{-1/2}$$

AND $$\omega_r = m - 2\left(1 + \frac{x_n^2}{k^2}\right)^{-1/2} \qquad (1.05)$$

$$\omega_i = 0$$

C. Case of $\gamma' = 1$

When the ratio of specific heats γ' is equal to unity, it is possible to obtain analytic solutions,as shown by Gans (1974). The frequencies for the free oscillations can be found from the eigenvalue relation

$$\frac{m}{-\omega_r+m}(-\omega_r+m+2)\,\Phi\left\{\frac{1}{2}\left[a^2\frac{4+(\omega_r+m)^2}{(-\omega_r+m)^2}k^2+\frac{2-\omega_r+m}{-\omega_r+m}\left[1+(-\omega_r+m)^2-2[-\omega_r+m]\right]\right], m+1; -\frac{1}{2a^2}\right\} - \frac{(-\omega_r+m)}{a^2(m+1)}\,\Phi\left\{\frac{1}{2}\left[a^2\frac{4+(\omega_r+m)^2}{(-\omega_r+m)^2}k^2+\frac{2-\omega_r+m}{-\omega_r+m}\left[1+(-\omega_r+m)^2-2[-\omega_r+m]\right]\right]+1, m+2; -\frac{1}{2a^2}\right) = 0 \quad (1.06)$$

where Φi is a hypergeometric function in the notation of Erdelyi et al.(1953)

In the limit, when the speed of sound goes to zero ($a \to 0$) the relation was shown to become

$$4-(\omega - m)^2=0$$

or

$$\omega = m \pm 2 \quad (1.07)$$

2. General Stability Criteria for Rotating Flows

With a basic velocity $(W(r),0,V(r))$, Howard and Gupta (1962) have derived sufficient stability conditions for rotating incompressible flows.

In the case of axisymmetric perturbations ($m = 0$) they proved that the flow is stable if the equivalent Richardson number everywhere is greater than one quarter :

$$J(r) = \frac{\frac{d}{dr}(r^2 V)}{r^3(\frac{d}{dr}W)^2} \geq \frac{1}{4} \quad (1.08)$$

For the general case, allowing nonaxisymmetric perturbations ($m \neq 0$), Howard and Gupta showed that the sufficient condition condition of stability is:

$$\frac{k^2}{r^3}\frac{d}{dr}(r^2V^2) - (2km \mp r^2)V\cdot\frac{dW}{dr} - \frac{1}{4}\left[k\frac{dW}{dr} + m\frac{d}{dr}\left(\frac{V}{r}\right)\right] \geq 0 \tag{1.09}$$

When rigid body rotation is written $V = \Omega r$ and Poiseuille flow $W = \epsilon\Omega(1-r^2)$ it is possible to find stability conditions for in the different cases.

Equation (1.08) for axisymmetric modes gives that the Richardson number

$$J(r) = \frac{1}{\epsilon^2 r^2} \geq \frac{1}{4}$$

which is equivalent to stability for small axisymmetric perturbations for a Poiseuille flow if

$$\epsilon \leq 2 \tag{1.10}$$

For nonaxisymmetric perturbations condition (1.09) gives

$$4(k + m\epsilon) - k\epsilon^2 r^2 \geq 0$$

which can be written as

$$\epsilon \leq \frac{2}{k^2 r^2}\left(m + \sqrt{k^2 r^2 + m^2}\right) \leq \frac{2}{k^2}\left(m + |m|\sqrt{1 + \frac{k^2}{m^2}}\right)$$

Hence the stability condition for small perturbations becomes

$$\epsilon \leq \frac{k}{|m|} - \frac{1}{4}\frac{k^3}{|m|^3} + O(k^5) \; ; \quad \text{FOR } m < 0 \tag{1.11}$$

AND

$$\epsilon \leq \frac{2}{k^2}\left(2|m| + \frac{1}{2}\frac{k^2}{|m|} - \frac{1}{8}\frac{k^4}{|m|^3} + O(k^5)\right) \; ; \quad \text{FOR } m > 0 \tag{1.12}$$

Conditions (1.11) and (1.12) indicate that perturbations with negative azimuthal wavenumbers ($m < 0$) are most unstable and that such disturbances represent waves with retrograde propagation relative to the rigid-body rotation.

Pedly (1968) gives a necessary and sufficient condition for the stability of incompressible rotating Poiseuille flow.

By setting

$$W(r) = \epsilon\,(1-r^2) = 2\epsilon^* f$$

with

$$f = \frac{1}{2}\frac{\epsilon}{\epsilon^*}(1-r^2)$$

he showed that the instability condition becomes

$$\beta\left(\beta - \frac{1}{r}\frac{df}{dr}\right) < 0$$

where

$$\beta = \frac{k}{\epsilon^* m}$$

This gives

$$\frac{k}{m}\left(\frac{k}{m} + \epsilon\right) < 0 \tag{1.13}$$

for negative wavenumbers the criterion for instability becomes

$$\epsilon > \frac{k}{|m|} \tag{1.14}$$

D. Present Work

Closed form analytical results are available only for special cases such as incompressible flow or when the ratio between specific heats equals unity. The aim of the present work is that of stability investigation of any nonviscous compressible rotating flows with as general basic velocities and temperature distributions as possible.

A numerical method has been used in order to solve for the eigenvalues and the eigenmodes for the small wave-like perturbations. For different axial and azimuthal wavenumbers the computer program yields a set of complex frequencies which depend on the compressibility of the flow as measured by the speed of sound of the fluid. The imaginary part of the frequency determines the exponential growth rates of the disturbances. In the case of pure rigid body rotation, the numerical results give zero growth, as would be expected. For sufficiently large axial flows, however, growing oscillations may arise. Of special interest is how the stability-boundary varies with wavenumber and sound speed. As will be seen, from the present results, compressibility may tend to stabilize the flow. Rigid body rotation with a superposed Poiseuille flow has been investigated, but the computer program allows the specification of any radial distribution of azimuthal or axial velocity. In the calculations the temperature is constant, but in general the temperature can be any function of the distance from the center.

CHAPTER 2

ANALYTIC FORMULATION

A. Basic State and Linearization

Basic equations for the inviscid compressible flow are written in terms of velocity $\bar{u}^* = (u_z^*, u_r^*, u_\phi^*)$, density ρ^*, pressure P* and entropy S*, in cylindrical coordinates (z, r, ϕ) as follows:

Momentum,

1).

$$\frac{\partial u_z^*}{\partial t} + \bar{u}^* \cdot \nabla u_z^* = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial z}, \qquad (2.01)$$

$$\frac{\partial u_r^*}{\partial t} + \bar{u}^* \cdot \nabla u_r^* - \frac{(u_\phi^*)^2}{r} = -\frac{1}{\rho^*}\frac{\partial p^*}{\partial r}, \qquad (2.02)$$

$$\frac{\partial u_\phi^*}{\partial t} + \bar{u}^* \cdot \nabla u_\phi^* + \frac{u_r^* u_\phi^*}{r} = -\frac{1}{\rho^* r}\frac{\partial p^*}{\partial \phi}, \qquad (2.03)$$

Continuity,

2).

$$\frac{\partial \rho^*}{\partial t} + \bar{u}^* \cdot \nabla \rho^* + \rho^* \nabla \cdot \bar{u}^* = 0 \qquad (2.04)$$

3). The condition of constant entropy,

$$\frac{\partial S^*}{\partial t} + \bar{u}^* \cdot \nabla S^* = 0$$

For a ideal fluid this gives the relation

$$\frac{Dp^*}{Dt} = \left(\frac{\partial p^*}{\partial \rho^*}\right)_{s^*} \frac{D\rho^*}{Dt} = a^{*2}\left(\frac{\partial \rho^*}{\partial t} + \bar{u}^* \cdot \nabla \rho^*\right) = -a^2 \rho^* \nabla \cdot \bar{u}$$

or

$$\frac{\partial p^*}{\partial t} + \bar{u}^* \cdot \nabla p^* + a^2 \rho^* \nabla \cdot \bar{u}^* = 0 \qquad (2.05)$$

The operators which have been used are

$$\nabla \begin{Bmatrix} u_z^* \\ u_r^* \\ u_\phi^* \\ \rho^* \\ p^* \end{Bmatrix} = \left(\frac{\partial}{\partial z}, \frac{\partial}{\partial r}, \frac{1}{r}\frac{\partial}{\partial \phi}\right) \begin{Bmatrix} u_z^* \\ u_r^* \\ u_\phi^* \\ \rho^* \\ p^* \end{Bmatrix}$$

and

$$\nabla \cdot \bar{u}^* = \left(\frac{\partial}{\partial z}, \frac{\partial}{\partial r} + \frac{1}{r}, \frac{1}{r}\frac{\partial}{\partial \phi}\right) \cdot \bar{u}^*$$

The steady-state solutions of these equations used as a basic flow are assumed to be axisymmetric and independent of Z, namely

$$\bar{u}^* = (W(r), 0, V(r)),$$

$$\rho^* = \rho_0(r),$$

$$p^* = P_0(r),$$

with P_o and S_o being related by $\frac{1}{\rho_0}\frac{dP_0}{dr} = \frac{V^2}{r}$

Under the assumption of small wave-like perturbations, the disturbance quantities may be written as

$$u_z^* = W(r) + w(r) \exp\{i(kz + m\phi - \omega t)\},$$

$$u_r^* = u(r) \exp\{i(kz + m\phi - \omega t)\},$$

$$u_\phi^* = V(r) + v(r) \exp\{i(kz + m\phi - \omega t)\},$$

$$\rho^* = \rho_0(r) + \rho(r)\, \exp\{ i(kz + m\phi - \omega t)\} ,$$

$$p^* = P_0(r) + p(r)\, \exp\{ i(kz + m\phi - \omega t)\}$$

(2.07)

For stability of the flow it is necessary that the perturbations have amplitudes that do not increase with time. For infinitesimal disturbances it is not only necessary but also sufficient that each complex eigenfrequency has a non-positive imaginary part ($\omega_i \leq 0$). However, for greater initial perturbations the flow could still be unstable and further, it is not necessarily true that in an unstable region the time-growing perturbations will cause the laminar flow to become turbulent. It might very well turn out, that the disturbances will change the laminar flow in such a way that it becomes a new laminar but possibly unsteady flow which is more stable.

The complex amplitudes of the perturbations and permissible values of the characteristic frequency $\omega = \omega_r + i\omega_i$ are determined from the eigenvalue problem for the linear system of differential equations.

This system becomes

$$i\gamma w(r) + \frac{dW(r)}{dr} \cdot u(r) = -\frac{ik}{\rho_0} p(r) ,$$

$$i\gamma u(r) - 2\frac{V(r)}{r} v(r) = -\frac{1}{\rho_0}\frac{d}{dr} p(r) + \frac{V^2}{\rho_0 r}\rho(r) ,$$

$$\left(\frac{d}{dr} + \frac{1}{r}\right) V(r) \cdot u(r) + i\gamma v(r) = -\frac{i}{\rho_0}\frac{m}{r} p(r) ,$$

$$i k\, w(r) + \left(\frac{1}{\rho_0}\frac{d\rho_0}{dr} + \frac{d}{dr} + \frac{1}{r}\right)u(r) + i\,\frac{m}{r}v(r) = -i\,\frac{\gamma}{\rho_0}\,\rho(r)$$

$$i k\, w(r) + \left(\frac{V^2(r)}{r}\,\frac{1}{a^2} + \frac{d}{dr} + \frac{1}{r}\right)u(r) + i\,\frac{m}{r}v(r) = -\frac{i\gamma}{\rho_0}\frac{p(r)}{a^2}$$

(2.08)

where
$$\gamma = \left(-\omega + k\,W(r) + \frac{m}{r}\,V(r)\right) \qquad (2.09)$$

For a complete system of eigenfunctions any initial values of velocity, density and pressure can be expanded in series of these eigenfunctions, to produce a general solution of the initial value problem.

The above system of equations may be written in short-hand notation as

$$A \circ Y = 0$$

with the disturbance vector

$$Y^T = [\,w(r),\ u(r),\ v(r),\ p(r),\ \rho(r)\,]$$

A is an operator matrix defined by

$$A = \begin{bmatrix} i\gamma & DW & 0 & i\frac{k}{\rho_0} & 0 \\ 0 & i\gamma & -2\frac{V}{r} & \frac{1}{\rho_0}D & -\frac{V^2}{\rho_0 r} \\ 0 & DV + \frac{V}{r} & i\gamma & i\frac{m}{\rho_0 r} & 0 \\ ik & +\frac{1}{\rho_0}D\rho_0 + \frac{1}{r} + D & i\frac{m}{r} & 0 & i\frac{\gamma}{\rho_0} \\ ik & \frac{V^2}{a^2 r} + \frac{1}{r} + D & i\frac{m}{r} & i\frac{\gamma}{\rho_0 a^2} & 0 \end{bmatrix} \qquad (2.10)$$

and

$$D = \frac{d}{dr}$$

From the equation of state, the following relation

$$\frac{1}{\rho_0}\frac{d\rho_0}{dr} = \frac{V^2}{r}\frac{\gamma'}{a^2} - \frac{1}{T}\frac{dT}{dr} \tag{2.11}$$

is derived, which gives an expression for the density

$$\rho_0 = \rho_{00} \exp\left\{ \int_{r_0}^{r} \left[\frac{V^2}{r}\frac{\gamma'}{a^2} - \frac{1}{T}\frac{dT}{dr} \right] dr \right\} \tag{2.12}$$

with ρ_{00} as the density at the inner boundary $(r = r_0)$.

By this the pressure will be given by:

$$P_0 = \rho_0 \frac{a^2}{\gamma'} \tag{2.13}$$

B. Reduction to a Single Equation

It is possible to show that the disturbance density and velocities in axial and azimuthal directions can be expressed in terms of the radial perturbation velocity and the pressure, respectively.

Hence

$$\rho = -i\frac{\rho_0}{\gamma}\left(\frac{1}{a^2}\frac{V^2}{r}(1-\gamma') + \frac{DT}{T}\right)u + \frac{p}{a^2} \tag{2.14}$$

$$w = \left(i\, DW\, u - k\frac{p}{\rho_0}\right)\frac{1}{\gamma} \tag{2.15}$$

and

$$v = \left(i\left(DV + \frac{V}{r}\right)u - \frac{m}{r}\frac{p}{\rho_0}\right)\frac{1}{\gamma} \tag{2.16}$$

where the perturbation pressure also is related to u by:

$$p = i\left(\left(\frac{m}{r}\left(DV + \frac{V}{r}\right) + kDW - \frac{\gamma}{r}\left(\frac{V^2}{a^2}+1\right)\right)u - \gamma Du\right)\frac{\rho_0}{\left(k^2 + \frac{m^2}{r^2} - \frac{\gamma^2}{a^2}\right)} \tag{2.17}$$

It is thus possible to calculate all the disturbance quantities once the component of the disturbance velocity in the radial direction is known.

By eliminating all but one of the linear equations in the system, we get the following second order differential equation for the radial perturbation velocity u :

$$\frac{d^2u}{dr^2} + \left(\frac{1}{\rho_0}\frac{d\rho_0}{dr} + \frac{1}{S}\frac{dS}{dr} + \frac{1}{r}\right)\frac{du}{dr} - \left\{\frac{\alpha}{\gamma}\left(\frac{1}{\rho_0}\cdot\frac{d\rho_0}{dr} + \frac{1}{S}\frac{dS}{dr}\right) + \frac{1}{\gamma}\frac{d\alpha}{dr} + \frac{\beta}{\gamma^2 S} - \frac{\eta\alpha}{\gamma^2}\right\}u = 0 \tag{2.18}$$

where $$S = \left(k^2 + \frac{m^2}{r^2} - \frac{\gamma^2}{a^2}\right)^{-1} \tag{2.19}$$

$$\alpha = \left[\frac{m}{r}\left(\frac{d}{dr} + \frac{1}{r}\right)V + k\frac{dW}{dr} - \frac{\gamma}{r}\left(\frac{V^2}{a^2} + 1\right)\right] \tag{2.20}$$

$$\beta = \frac{1}{a^2}\frac{V^4}{r^2} - \frac{V^2}{r}\frac{1}{\rho_0}\frac{d\rho_0}{dr} + \gamma^2 - 2\frac{V}{r}\left(\frac{d}{dr} + \frac{1}{r}\right)V \tag{2.21}$$

and $$\eta = \frac{\gamma}{a^2}\frac{V^2}{r} - 2\frac{m}{r}\frac{V}{r} \tag{2.22}$$

C. Non-Dimensionalization

With the frequency of rigid body rotation, Ω , as the reference frequency and the radius of the outer cylinder, L , as the reference length and the density at the inner-boundary ρ_{00} as the corresponding reference density; time, pressure and velocity are $[\Omega^{-1}]$, $[\rho_{00}(L\Omega)^2]$ and $[L\Omega]$, respectively.

D. Boundary Conditions

For the second order equation (2.18) two boundary conditions need to be specified.

In the case of two concentric cylinders as boundaries for the fluid, the radial velocity is zero at the walls i.e.

$$u(1) = u(Rox) = 0 , \tag{2.23}$$

(Rox = radius of inner cylinder)

with no inner cylinder we have the following condition for the radial velocity:

$$u^*(z,0,\phi) = -u^*(z,0,\phi+\pi) \tag{2.24}$$

This arises because of continuity at the origin and because the positive r-direction is oppositely directed at points π radians apart.

From (2.07) the expression for u^* is

$$u^*(z,r,\phi) = u(r)\ \exp\{i(kz+m\phi\ -\omega t)\}$$

thus, the condition becomes either

$$-1 = \exp\{\ im\pi\ \}$$

or $$u(0) = 0$$

which implies that the radial velocity can be different from zero only when the azimuthal wavenumber is an uneven quantity; in fact, the series expansions at the origin(Appendix 1) shows that this can occur only for m =-1 or m +1.

Another boundary condition is required in the case of a free vortex. With the assumption of an outer potential flow, the following condition:

$$\nabla^2 \Phi = 0 \tag{2.25}$$

is valid in the region outside vortex and where Φ denotes the velocity potential.

In a cylindrical coordinate system this becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\Phi + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0 \tag{2.26}$$

In a similar manner to (2.07), the velocity potential can be written

$$\Phi = \Psi(r) \cdot \exp\{i(kz + m\phi - \omega t)\} \tag{2.27}$$

(2.27) and (2.26) give the following equation:

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} - \left(k^2 + \frac{m^2}{r^2}\right)\Psi = 0 \tag{2.28}$$

with solutions in form of modified Bessel functions of the second kind,

$$\Psi(r) = B \cdot K_{|m|}(|k| r) \tag{2.29}$$

Once the velocity potential is known, it is possible to determine the disturbance velocity components

$$u_r = \Phi_r = B\,|k| \cdot K'_{|m|}(|k| r) \exp\{i(kz + m\phi - \omega t)\} \tag{2.30}$$

and

$$u_\phi = \frac{1}{r}\Phi_\phi = B\,\frac{im}{r} K_{|m|}(|k| \cdot r) \exp\{i(kz + m\phi - \omega t)\} \tag{2.31}$$

The ratio between them becomes

$$\frac{u_r}{u_\phi} = -i \frac{|k| \cdot r}{m} \cdot \frac{K'_{|m|}(|k| \cdot r)}{K_{|m|}(|k| \cdot r)}$$

which can be rewritten by use of recurrence formulas, to give

$$\frac{u_r}{u_\phi} = i\left(|m| + |k| \cdot \frac{K_{|m|-1}}{K_{|m|}}\right)\frac{1}{m} \tag{2.32}$$

as the proper outer boundary-condition for a free vortex.

CHAPTER 3

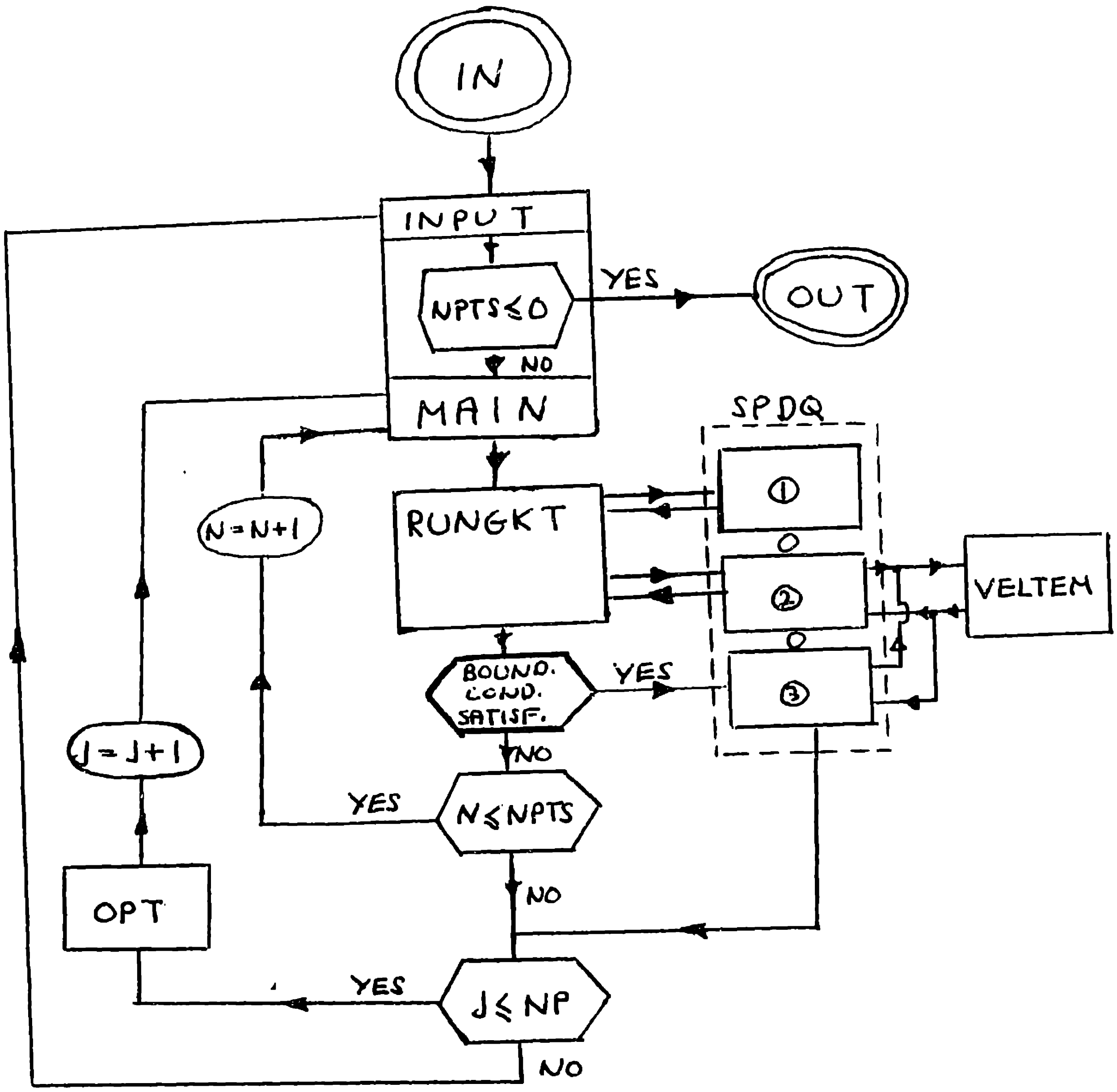
METHOD OF SOLUTION

A. Numerical Techniques

In the numerical procedure, the disturbance-velocity is calculated step by step by means of a fourth order Runge-Kutta method starting from the inner boundary (Collatz 1960) The differential equation (2.18) for the disturbance velocity may have one, or sometimes two, regular singular points in the region of interest, and special care must be exercized in the numerical integration through these points. The eigenvalue is found by varying the complex frequency until the outer boundary-condition is satisfied. In the case of a rigid cylinder the boundary condition (2.23) is that of zero radial velocity. For a free vortex, however, the appropriate boundary condition is found by matching radial and azimuthal velocity to an outer potential flow. In order to estimate new values of the frequency, in the iteration procedure, the method of Newton Raphson has been used with complex variables.

The program consists of a main part and a number of subroutines, one of them called "RUNGKT" containing the numerical integration procedure.

The block diagram below will serve as a short description of how the program works. An extended explanation with flow charts of all subroutines can be found in appendix 3 .



(3.01)

B. Singular Points

For the differential equation (2.18) it is seen that, at the origin (r=0) a singularity arises and a series expansion for small r is needed in order to be able to start the numerical procedure for solving the equation. This has been done by means of the method of Frobenius (Appendix 1), where it is also shown that there must be two different expansions, depending on whether or not the perturbation is axisymmetric. There is another regular singular point, whenever

$$0 \leq \frac{m^2}{\frac{\gamma^2}{a^2} - k^2} \leq 1 \qquad (3.02)$$

the singularity is located at

$$r = \sqrt{\frac{m^2}{\frac{\gamma^2}{a^2} - k^2}} \qquad (3.03)$$

This is obviously only possible when

$$\frac{\gamma^2}{a^2} \geq k^2 \qquad (3.04)$$

which means that the singularity arises when the propagation velocity of the disturbance is equal to the sound velocity. The critical radius will be called the sonic radius or sonic point. A third singularity arises when

$$\gamma = 0 \qquad (3.05)$$

From (2.09) it is seen that this occurs when the frequency is

$$\omega = \omega_r = k W(r) + \frac{m}{r} V(r) \qquad (3.06)$$

which is a purely real quantity.

One way to avoid some of the numerical problems associated with this singularity, is to add a small non zero imaginary part to the frequency.

CHAPTER 4

RESULTS

A. Modes in Pure Rigid Body Rotation

For an inviscid compressible rotating flow, several modes have been investigated for different degrees of compressibility. In a general case, when viscosity must be taken into account, the viscous damping will be greater for perturbations with higher wavenumber. In the following stability investigation a small axial wavenumber $k= \pi/10$ in most of the calculations was chosen. From the stability criteria (1.10), (1.11) and (1.12), it becomes clear that nonaxisymmetric modes with negative wavenumbers $(m<0)$ are expected to be most unstable. This implies wave which propagates against the direction of rigid body rotation (retrograde modes). Disturbances with $m=-1$ will first be studied and then a comparison with results for other values of m will be made. The ratio between specific heats is chosen equal to 1.09 (corresponding to freon 114,A). For purely rigid body rotation, with no inner cylinder, the mode frequencies as a function of sound speed, can be seen from Figure 1 . According to the behavior in the limit, when $a\rightarrow\infty$, it is clear that there exist two different kinds of modes. Both of them are found to be neutrally stable with the imaginary part of frequency vanishing, as would be expected. In the limit, when $a\rightarrow\infty$, it is seen that some of the modes have frequencies roughly proportional to α, and they are called acoustic modes.

Concerning the acoustic modes, Rayleigh has shown that the frequencies, for a cylindrical wave guide is given by (1.02) in absence of rotation. In the limit when the speed of sound approaches infinity, this value will agree with that given from a rotating fluid, because the speed of rotation is much less than that of sound.

The calculated values of ω/a, with $\alpha = 10^4$, is compared with their asymptotic limit values (1.02)

Mode NR	Numerical Results		Asymptotic Values	
	ω_r/a	ω_i/a	ω_r/a	ω_i/a
1	1.86724	$0.3 \cdot 10^{-7}$	1.86761	0
2	5.34050	$0.5 \cdot 10^{-7}$	5.34070	0
3	8.54193	0.0	8.54193	0
4	11.7098	$0.1 \cdot 10^{-9}$	11.7102	0
5	14.8672	$0.1 \cdot 10^{-9}$	14.8670	0

These values are in very good agreement which proves that the numerical errors, due to the singularity in the sonic point, must be negligible.

Another set of limiting value occurs when speed of sound vanishes. Then, the different modes have a tendency to go to one of two values at $\omega_r = 1$ and $\omega_r = -3$. This is in agreement with the results that Gans (1974) obtained when he investigated a compressible flow, with specific heat ratio $\gamma' = 1$. He found that the eigenvalue relation (1.06) gives the relation (1.07), in the limit when $a \to 0$, have the values $\omega_r = m \pm 2$. Figure 2 and 3 show the frequency as a function of sound speed, for small values of sound. Further will the mode form for $a = 10^4$ and $a = 1$ be given on figures 4,5,6.

The other kind of modes is called inertial and have a much smaller dependence on the sound speed (Figure 7). The two sets of modes are located near each other with the values of the frequencies less or greater than -1. Their limiting values, when $a \to \infty$, have been shown by Greenspan (1968). From two transcendental equations (1.03, 1.04) he obtains two sets of solutions which corresponds to the two inertial mode frequencies (1.05).

The numerical values of ω, with $a = 10^4$, is compared with their asymptotic values:

Mode NR	Numerical Results		Asymptotic Values	
	ω_r	ω_i	ω_r	ω_i
1	-0.82193	0	-0.82119	0
2	-0.90639	0	-0.90660	0
1	-1.15187	0	-1.15170	0
2	-1.08583	0	-1.08564	0

From the stability discussions in Chapter 3, it is seen that the case when $\gamma' = o$, represents a case (3.05) when any solutions are impossible to obtain. For pure rigid body rotation, the frequency becomes $\omega = m$, (3.06). This is the same value, about which the inertial modes are located. It is also seen that higher modes have values closer to m and it seems to be so, that $\gamma = o$ implicates the limit point for higher inertial modes.

By adding a small quantity to the azimuthal velocity, the sensitivity of the stability properties to small departures from rigid body rotation can be studied. Figure 10 shows, that the mode form changes much, but that the frequency is almost constant with vanishing imaginary part.

The effect of varying γ' has also been studied for the first inertial mode. The following eigenvalues were found:

Ratio of Specific Heats	ω_r	ω_i
1.25	-0.75842	10^{-9}
1.15	-0.76497	10^{-8}
1.11	-0.76788	10^{-8}
1.09	-0.76394	0
1.07	-0.77094	10^{-8}
1.03	-0.77417	10^{-8}

The modes are stable, as would be expected, and 10% change in γ' gives only a 2% change in W_r.

B. Instabilities for a Swirling Poiseuille flow

For a flow of purely rigid body rotation, the different modes were found to be stable. Some of these modes could, however, become unstable by superposing an axial flow. The axial velocity distribution is chosen to be that of Poiseuille:

$$W(r) = \Omega \cdot \epsilon \ (1 - r^2)$$

where ϵ becomes a parameter indicating the strength of the axial flow.

For a fluid with m = -1, the change of the first inertial mode is studied, when ε increases from zero to one.

The first case concerns an incompressible fluid ($a = 10^4$) with three different values or the axial wavenumber. From Figure 11 it is seen that for $\epsilon > \epsilon_B = k$ the imaginary part of the frequency becomes positive, indicating instability. From stability considerations for an incompressible rotating fluid (1.10), (1.11) and (1.14), it is seen that the stability condition is $\epsilon \leq \frac{k}{|m|} - \frac{1}{2}\frac{k^3}{|m|^3}$ which is in excellent agreement with the numerical results (Figure 13).

For a fluid with the same azimuthal wavenumber (m = -1), but which is compressible with sound speed equal to unity, stability investigations has been performed. ε has been increased from zero to one for four different values of the wavenumber;

$k = 0.05, 0.10, 0.31416$ and 0.45. From Figure 12 it is seen that the values of ε $(=\varepsilon_B)$, for which the imaginary part of the frequency is positive, is for all k greater than the corresponding values for the incompressible case.This indicates that the fluid has become more stable with decreasing a. This is of course more significant for lower wavenumbers. In Figure 13 it is shown that with $k = 0.05$ the critical ε increases to 0.7 when $a = 1$,which is an increase of fourteen times. For $k = 0.45$ the increase is only 10%. A fourth order polynomial through the four given values , indicates that ε_B has minimum 0.35 for $k = 0.185$. Hence a compressible flow of this kind ($m = -1$) will be stable for all wavenumbers when ε is less than 0.35. For $\varepsilon > 0.35$, perturbations with $k = 0.185$ will first become unstable.

There are some indications that ε_B is less for modes of higher order. Thus, for $a = 1$ and $k = \pi/10$ it is found that ε_B is 0.42 for the first mode but only 0.35 for the second mode. But it is also seen, that for $\varepsilon = 1$, ω_i will be greater for the first mode (0.54 and 0.30 respectively) and this indicates greater amount of instability.

The stability boundary for $m = -2$ compared with that of $m = -1$, can be seen in (Figure 13).Curves for incompressible fluid shows good agreement with the stability criteria (1.11). For a compressible flow the condition becomes that for $\varepsilon < 0.3$, the flow is stable for all k, but when ε increases,modes with wavenumber k = 0,38 will first become unstable. It is seen from **Figure** 13 that the m = -2 mode is more stable for smaller wavenumbers, but that it will become less stable for k > 0.23 The stability for a viscous fluid will depend upon the stability for small wave numbers, because viscous damping has greater effect on higher wavenumbers.

CHAPTER 5

CONCLUSIONS

A. Discussion of Results

The numerical method of solving this eigenvalue problem was found to yield many useful results. The computer program was written in such a way as to provide solutions under very general conditions. Not only can the radial disturbance velocity be obtained, but also all the disturbance velocities and density and pressure perturbations. This is of value when the outer boundary condition is other than that of a rigid cylinder. For pure rigid body rotation the great difference between acoustic and inertial modes could be shown by comparing their eigenvalues for different degrees of compressibility. The computed results showed excellent agreement with available analytical results for special cases. All modes were found to be stable for pure rigid body rotation, but instability would arise when a sufficiently strong axial Poiseuille flow was superimposed. Stability limits could be obtained for different wavenumbers and the stability conditions for incompressible flows was found to be in agreement with known stability criteria. The stability properties for a compressible rotating fluid proved to be quite different from that of a incompressible fluid. A stability increase due to compressibility was found for all axial wavenumbers. Important was the considerable stabilization of perturbations with small axial wavenumbers, because an incompressible fluid is most unstable in this range. By comparing results with two different azimuthal wavenumbers, m, it was shown that higher azimuthal wavenumbers will be more stable for small axial wavenumber k and less stable for greater k. The range for which all axial wavenumbers are stable was shown to be $\varepsilon < 0.35$ for $m = -1$ and $\varepsilon < 0.30$ for $m = -2$ with greater axial flow, instability occurs for a certain axial wavenumber, which was found to be $k = 0.185$ for $m = -1$ and $k = 0.38$ for $m = -2$. By considering a viscous fluid, the viscous damping is expected to have its greatest effect for greater wavenumbers. From this one may conclude that compressibility will have a stabilizing effect.

B. Suggestions for future investigations.

An axial flow, of Poiseuille type, can have a destabilizing effect on a rotating fluid. Hence it would be expected that other kinds of axial flows will show similar behavior. In the case of a free jet in the interior, the expected instability has not yet been demonstrated. One needs theoretical guidance in this area in order to understand which wavenumbers are most likely to be unstable.

Another interesting application to free vorticies, for which Widnall et.al (1974) have used the computer program in the study of the stability of vortex rings. The program is especially useful in this problem, because an arbitrary vorticity distribution can be specified. In the case of a free vortex, an outer boundary condition must be specified under the assumption of an outer potential flow as shown in chapter 2. With a minor modifications of the program, it should also be possible to investigate other boundary conditions, such as would arise in the study of the interaction between a fluid and a flexible cylinder .

APPENDIX 1

Frobenius method of expansion has been used at the origin (Jeffreys,1956)

A. Nonaxisymmetric Modes

The differential equation considered is

$$\frac{d^2u}{dr^2} = A3\,\frac{du}{dr} + A4\,u,$$

where the coefficients are

$$A3 = -\left(\frac{1}{\rho_0}\frac{d\rho_0}{dr} + \frac{1}{S}\frac{dS}{dr} + \frac{1}{r}\right),$$

and $A4 = \left(\frac{\alpha}{\gamma}\left(\frac{1}{\rho_0}\frac{d\rho_0}{dr} + \frac{1}{S}\frac{dS}{dr}\right) + \frac{1}{\gamma}\frac{d\alpha}{dr} + \frac{\beta}{\gamma^2 S} - \frac{\eta\alpha}{\gamma^2}\right).$

For rigid body rotation, the azimuthal velocity is

$$V = \Omega r$$

and for Poiseuille flow ,the axial velocity is

$$W = \epsilon\Omega(1-r^2).$$

γ from (2.09) can now be written as

$$\gamma = -\omega + k\epsilon\Omega(1-r^2) + m\Omega,$$

and its derivative becomes

$$D\gamma = -2k\epsilon\Omega r,$$

Further,the expression(2.11) becomes

$$\frac{D\rho_0}{\rho_0} = \frac{\Omega^2\gamma'}{q^2}r - \frac{DT_0}{T_0}.$$

From (2.18) comes the value of s, which will be

$$S = \frac{r^2}{m^2}\left(1 + \frac{r^2}{m^2}\left(k^2 - \frac{\gamma^2}{q^2}\right)\right)^{-1},$$

and its derivative becomes

$$DS = -S^2\left(-2\frac{m^2}{r^3} + \frac{\gamma^2}{q^2}\frac{DT}{T} - 2\frac{\gamma D\gamma}{q^2}\right),$$

and from that one gets the following expression:

$$\frac{DS}{S} = \frac{2}{r} - \frac{2}{m^2}\left(k^2 - \frac{\gamma^2}{q^2}\right)r - \frac{\gamma^2}{m^2 q^2}\frac{DT}{T}\cdot r^2$$

The expression for α from (2.20) can be written as

$$\alpha = (2m\Omega - \gamma)\frac{1}{r} - \left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\right)r$$

and its derivative becomes

$$D\alpha = (\gamma - 2m\Omega)\frac{1}{r^2} - \frac{\gamma\Omega^2}{a^2} + \frac{\Omega^2\gamma}{a^2}\frac{DT_0}{T_0}r + 2k\epsilon\frac{\Omega^3}{a^2}r^2$$

which gives

$$\alpha\left[\frac{D\rho_0}{\rho_0} + \frac{DS}{S}\right] = 2(2m\Omega-\gamma)\frac{1}{r^2} - (2m\Omega-\gamma)\frac{DT}{T_0}\cdot\frac{1}{r} +$$
$$\cdot\left[\frac{2k}{m^2}(\gamma k - 2m\Omega(k+\epsilon m)) + \left((2m\Omega-\gamma)(\Omega^2\gamma' + 2\frac{\gamma^2}{m^2} - 2\gamma\Omega^2\right)\frac{1}{a^2}\right] +$$
$$+\left[2k\epsilon\Omega + \left[\gamma\Omega^2 - (2m\Omega-\gamma)\frac{\gamma^2}{m^2}\right)\frac{1}{a^2}\right]\frac{DT_0}{T_0}r + O(r^2)$$

From (2.21) comes the value of β which will be

$$\beta = (\gamma^2 - 4\Omega^2) + \Omega^2\frac{DT_0}{T_0}r + (1-\gamma')\frac{\Omega^4}{a^2}r^2$$

and hence one gets

$$\frac{\beta}{\gamma S} = \frac{m^2}{\gamma}(\gamma^2 - 4\Omega^2)\frac{1}{r^2} + \frac{\Omega^2 m^2}{\gamma}\frac{DT}{T}\frac{1}{r} +$$
$$+\frac{1}{\gamma}\left[(\gamma^2 - 4\Omega^2)k^2 + (1-\gamma')\frac{m^2\Omega^4}{a^2} - (\gamma^2 - 4\Omega^2)\frac{\gamma^2}{a^2}\right]$$
$$+\frac{\Omega^2}{\gamma}\left(k^2 - \frac{\gamma^2}{a^2}\right)\frac{DT}{T}r$$

The quantity η is given by (2.22) and becomes

$$\eta = \frac{\gamma\Omega^2}{a^2}r - 2m\Omega\frac{1}{r}$$

and the following quantity will be obtained

$$-\frac{\eta\alpha}{\gamma} = \frac{2m\Omega}{\gamma}(2m\Omega - \gamma)\frac{1}{r^2} -$$

$$-\frac{1}{\gamma}\left[4km\epsilon\Omega^2 + 2m\gamma\frac{\Omega^3}{a^2} + \frac{\gamma\Omega^2}{a^2}(2m\Omega - \gamma)\right)$$

The coefficients in the differential equation will be expressed in a series valid for small r (near the origin)

$$A3 = -\left(\frac{3}{r} + B + B1 r + B2 r^2 + B3 r^3 + B4 r^4\right) + O(r^5)$$

and

$$A4 = (m^2-1)\frac{1}{r^2} + C1\frac{1}{r} + C2 + C3 r + C4 r^2 + C5 r^3 + O(r^4)$$

where the terms in the expansions are

$$B = -\frac{DT_o}{T_o} \quad ,$$

$$B1 = \left(\frac{\Omega^2 \gamma'}{a^2} - \frac{2}{m^2}\left(k^2 - \frac{\gamma^2}{a^2}\right)\right) ,$$

$$B2 = -\frac{\gamma^2}{m^2 a^2} \cdot \frac{DT_o}{T_o} \quad ,$$

$$B3 = \left(\frac{2}{m^4}\left(k^2 - \frac{\gamma^2}{a^2}\right)^2 - \frac{4\gamma k \epsilon \Omega}{m^2 a^2}\right) ,$$

$$B4 = \frac{\gamma^2}{m^4 a^2}\left(k^2 - \frac{\gamma^2}{a^2}\right) \frac{DT_o}{T_o} \quad ,$$

and

$$C1 = \frac{1}{\gamma^2}(m\Omega - \gamma)^2 \frac{DT_o}{T_o} \quad ,$$

$$C2 = \left((2m\Omega - \gamma)\left(\frac{\Omega^2\gamma'}{a^2} - \frac{2}{m^2}\left(k^2 - \frac{\gamma^2}{a^2}\right)\right) - \right.$$

$$-\left[2\left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\right) - \frac{\gamma\Omega^2}{a^2} + \frac{m^2\Omega^4}{\gamma a^2}(1-\gamma) + \frac{(\gamma^2 - 4\Omega^2)}{\gamma}\left(k^2 - \frac{\gamma^2}{a^2}\right) + \right.$$

$$\left. + \frac{2m\Omega}{\gamma}\left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\right) + \frac{\Omega^2}{a^2}(2m\Omega - \gamma)\right]\frac{1}{\gamma} \quad ,$$

$$C3 = \left(-(2m\Omega - \gamma)\frac{\gamma^2}{m^2 a^2} + \left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\right) + \frac{\Omega^2 k^2}{\gamma}\right)\frac{1}{\gamma}\frac{DT_o}{T_o} \quad ,$$

$$C4 = \Big((2m\Omega-\gamma)\Big(\frac{2}{m^4}\Big(k^2-\frac{\gamma^2}{a^2}\Big)^2 - \frac{4\gamma k\epsilon\Omega}{m^2a^2}\Big) - \Big(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\Big)\Big(\frac{\Omega^2\gamma'}{a^2} - \frac{2}{m^2}\Big(k^2-\frac{\gamma^2}{a^2}\Big)\Big) + \frac{2k\epsilon\Omega^3}{a^2} + \frac{\Omega^4}{\gamma a^2}(1-\gamma')\Big(k^2-\frac{\gamma^2}{a^2}\Big) + \frac{\Omega^2}{a^2}\Big(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\Big)\Big]\frac{1}{\gamma},$$

$$C5 = \Big((2m\Omega-\gamma)\Big(k^2-\frac{\gamma^2}{a^2}\Big)\frac{\gamma^2}{m^4a^2} + \frac{\gamma^2}{m^2a^2}\Big(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\Big)\Big]\frac{1}{\gamma}\,\frac{DT_0}{T_0}.$$

Further,it is assumed that the solution will be of the form (Frobenius method)

$$u = \sum_{n=0}^{\infty} A_n r^{n+\alpha^*}$$

and the derivatives become

$$Du = \sum_{n=0}^{\infty} A_n (n+\alpha^*) r^{n+\alpha^*-1}$$

$$D^2u = \sum_{n=0}^{\infty} A_n (n+\alpha^*)(n+\alpha^*-1) r^{n+\alpha^*-2}$$

The differential equation becomes

$$\sum_{n=0}^{\infty} A_n (n+\alpha^*)(n+\alpha^*-1) r^{n+\alpha^*} = -(3 + Br + B1r^2 + B2r^3 + B3r^4 + B4r^5)\sum_{n=0}^{\infty} A_n(n+\alpha^*)r^{n+\alpha^*} + ((m^2-1) + C1r + C2r^2 + C3r^3 + C4r^4 + C5r^5)\sum_{n=0}^{\infty} A_n r^{n+\alpha^*}$$

From the condition that terms of the same order of magnitude equals zero

$$O(r^{\alpha}): \quad \alpha^*(\alpha^*-1) = -3\alpha^* + (m^2-1)$$

which gives two solutions $\alpha_1^* = m-1$
$\alpha_2^* = -m-1$

The condition on u to be finite at the origin, makes only non negative solutions possible

$$\alpha^* = |m| - 1$$

and then u will be written as

$$u = A_0 r^{|m|-1}\Big(1 + \sum_{n=1}^{\infty} a_n r^n\Big)$$

B. Axisymmetric Modes

When the azimuthal wave number is equal to zero (m=0), the value of S becomes

$$S = \left(k^2 - \frac{\gamma^2}{a^2}\right)$$

and hence the following expression will be

$$\frac{DS}{S} = -\frac{\gamma^2}{a^2}\left(\frac{1}{k^2 - \frac{\gamma^2}{a^2}}\right)\frac{DT_o}{T_o} - \frac{4\gamma k\epsilon\Omega}{a^2}\,\frac{1}{\left(k^2 - \frac{\gamma^2}{a^2}\right)}\,r$$

which gives

$$\alpha\left[\frac{D\rho_o}{\rho_o} + \frac{DS}{S}\right] = \gamma\left[\frac{k^2}{k^2 - \frac{\gamma^2}{a^2}}\right]\frac{DT_o}{T_o}\frac{1}{r} + \frac{\gamma\Omega}{a^2}\left[\Omega\gamma' - \frac{4\gamma k\epsilon}{k^2 - \frac{\gamma^2}{a^2}}\right]r$$

$$+\left(\frac{k^2}{k^2 - \frac{\gamma^2}{a^2}}\right)\left(2k\epsilon\Omega - \frac{\gamma\Omega^2}{a^2}\right)\frac{DT}{T_o}\,r + \left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\right)\frac{\Omega}{a^2}\left(\Omega\gamma' - \frac{4\gamma k\epsilon}{\left(k^2 - \frac{\gamma^2}{a^2}\right)}\right)r^2$$

The derivative of α becomes

$$D\alpha = \frac{\gamma}{r^2} - \frac{\gamma\Omega^2}{a^2} + \frac{\Omega^2\gamma}{a^2}\frac{DT_o}{T_o}\,r + 2k\epsilon\frac{\Omega^3}{a^2}\,r^2$$

and further, the quantities below will assume the form

$$\frac{\beta}{\gamma S} = \frac{1}{\gamma}\left(\left(\gamma^2 - 4\Omega^2\right)\left(k^2 - \frac{\gamma^2}{a^2}\right)\right) + \frac{\Omega^2}{\gamma}\left(k^2 - \frac{\gamma^2}{a^2}\right)\frac{DT}{T}\,r$$

and

$$-\frac{\eta\alpha}{\gamma} = \frac{\Omega^2\gamma}{a^2} + \frac{\Omega^2}{a^2}\left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2}\right)r^2$$

Then it is possible to write the coefficients in the differential equation as

$$A3 = -\left[\frac{1}{r} + B + B1\,r + B2\,r^2 + B3\,r^3 + B4\,r^4\right] + O(r^5)$$

and

$$A4 = \left[\frac{1}{r^2} + C1\frac{1}{r} + C2 + C3\cdot r + C4\,r^2 + C5\,r^3\right] + O(r^4)$$

The terms in the expansions are

$$B = \left[\frac{k^2}{k^2 - \frac{\gamma^2}{a^2}} \right] \frac{DT}{T},$$

$$B1 = \frac{\Omega}{a^2}\left[\Omega \gamma' - \frac{4\gamma k \epsilon}{k^2 - \frac{\gamma^2}{a^2}} \right],$$

$$B2 = B3 = B4 = 0$$

and further

$$C1 = B,$$

$$C2 = \frac{\Omega}{a^2}\left[\Omega\gamma' - \frac{4\gamma k\epsilon}{k^2 - \frac{\gamma^2}{a^2}} \right] + (\gamma^2 - 4\Omega^2)\left(k^2 - \frac{\gamma^2}{a^2}\right),$$

$$C3 = \left(\left(\frac{k^2}{k^2 - \frac{\gamma^2}{a^2}} \right)\left(2k\epsilon\Omega + \frac{\gamma\Omega^2}{a^2} \right) + \frac{\Omega^2}{a^2} \right) \frac{DT}{T},$$

$$C4 = \frac{\Omega^2}{a^2}\left(\left(2k\epsilon + \frac{\gamma\Omega}{a^2} \right)\left(\Omega\gamma' - \frac{4\gamma k\epsilon}{k^2 - \frac{\gamma^2}{a^2}} \right) + \right.$$

$$\left. + \Omega\left(4k\epsilon + \frac{\gamma\Omega}{a^2} \right) \right],$$

$$C5 = 0$$

The equation becomes

$$\sum_{n=0}^{\infty} A_n (n+\alpha^*)(n+\alpha^*-1)\, r^{n+\alpha^*} = -(1 + Br + B1 r^2 + B2 r^3 + B3 r^4 +$$

$$+ B5 r^5) \cdot \sum_{n=0}^{\infty} A_n (n+\alpha^*)\, r^{n+\alpha^*} + (1 + C1\, r + C2\, r^2 +$$

$$+ C3\, r^3 + C4\, r^4) \sum_{n=0}^{\infty} A_n r^{n+\alpha^*}$$

Comparing terms of the same order of magnitude gives

$$O(r^{\alpha^*}) : \alpha^*(\alpha^* - 1) = -\alpha^* + 1$$

This gives two values $\begin{cases} \alpha_1^* = 1 \\ \alpha_2^* = -1 \end{cases}$

but only $\alpha^* = 1$

is possible, because u must be finite at the origin.

Hence, the expression of u becomes;

$$u = A_o r \left(1 + \sum_{n=1}^{\infty} a_n r^n\right) .$$

A_o is chosen in such a way that the function will have the same order of magnitude, regardless of r. This simplifies the choice of numerical limit for the boundary conditions.

Further, the terms in the expansion becomes

$$a_1 = \frac{(C1 - \alpha^* B)}{(2\alpha^* + 3)}$$

$$a_2 = \frac{(C1 - (1+\alpha^*)B)}{(4\alpha^* + 8)} a_1 + \frac{(C2 - \alpha^* B)}{(4\alpha^* + 8)}$$

$$a_3 = \frac{(C1 - (2+\alpha^*)B)}{(6\alpha^* + 15)} a_2 + \frac{(C2 - (1+\alpha^*)B1)}{(6\alpha^* + 15)} a_1 + \frac{(C3 - \alpha^* B2)}{(6\alpha^* + 15)}$$

$$a_4 = \frac{(C1 - (3+\alpha^*)B)}{(8\alpha^* + 24)} a_3 + \frac{(C2 - (2+\alpha^*)B1)}{(8\alpha^* + 24)} a_2 + \frac{(C3 - (1+\alpha^*)B2)}{(8\alpha^* + 24)} a_1 + \frac{(C4 - \alpha^* B3)}{(8\alpha^* + 24)}$$

$$a_5 = \frac{(C1 - (4+\alpha^*)B)}{(10\alpha^* + 35)} a_4 + \frac{(C2 - (3+\alpha^*)B1)}{(10\alpha^* + 35)} a_3 + \frac{(C3 - (2+\alpha^*)B2)}{(10\alpha^* + 35)} a_2 + \frac{(C4 - (1+\alpha^*)B3)}{(10\alpha^* + 35)} a_1 + \frac{(C5 - \alpha^* B4)}{(10\alpha^* + 35)}$$

From the series expansion of u it is seen that for $m = \pm 1$, the function will assume the value A_o at the origin (r=0).

In the special case of constant temperature and an incompressible flow, it is seen that in the nonaxisymmetric case the coefficients become $A3 = \frac{3}{r} - \left[\frac{2}{m^2}k^2\right] r + \left[\frac{2}{m^4} k^4\right] r^3$

and $$A4 = \frac{m^2-1}{r^2} + C1\frac{1}{r} + C2 + C3\,r + C4\,r^2 + C5\,r^3 + O(r^4)$$

For large m, A3 becomes $A3 = \frac{3}{r}$

In the axisymmetric case , A3 and A4 becomes

$$A3 = \frac{1}{r}$$

and $$A4 = \frac{1}{r^2} + (\gamma^2 - 4\Omega^2)k^2$$

When $m \neq 0$, the differential equation becomes

$$\frac{d^2u}{dr^2} + \frac{3}{r}\frac{du}{dr} - \left[\frac{m^2-1}{r^2} + Z(r)\right]u = 0$$

The transformation

$$u = \frac{\Phi}{r}$$

gives

$$Du = \frac{D\Phi}{r} - \frac{\Phi}{r^2}$$

$$D^2u = \frac{1}{r}D^2\Phi - \frac{2}{r^2}D\Phi + \frac{2}{r}\Phi$$

and the equation becomes

$$D^2\Phi + \frac{1}{r}D\Phi + \left[Z(r) - \frac{m^2}{r^2}\right]\Phi = 0$$

The solution becomes

$$\Phi = J_m(\sqrt{Z}\,r)$$

and u will be

$$u = \frac{1}{r}J_m(\sqrt{Z}\,r)$$

This is obviously not valid for $m = \pm 1$.

APPENDIX 2

Frobenius expansion at the second singular point.

The differential equation is

$$D^2u = -\left[\frac{D\rho_0}{\rho_0} + \frac{DS}{S} + \frac{1}{r}\right]Du + \left[\frac{\alpha}{\gamma}\left[\frac{D\rho_0}{\rho_0} + \frac{DS}{S}\right] + \frac{D\alpha}{\gamma} + \frac{\beta}{\gamma^2 S} + \frac{\eta\alpha}{\gamma^2}\right]u$$

The expression for S is

$$S = \left(k^2 + \frac{m^2}{r^2} - \frac{\gamma^2}{a^2}\right)^{-1} = \frac{r^2}{m^2}\left(1 - \frac{r^2}{m^2}\left(\frac{\gamma^2}{a^2} - k^2\right)\right)^{-1}$$

or

$$S = -\frac{r_1^2 r^2}{m^2(r+r_1)} \cdot \frac{1}{(r-r_1)}$$

and from this expression it is seen that the singular point is located at $r = r_1$, where :

$$r_1 = \sqrt{\frac{m^2}{\frac{\gamma^2}{a^2} - k^2}}$$

This singularity occurs when the frequency is :

$$\omega \gtrless m \pm a\sqrt{m^2 + k^2}$$

The coefficients in the differential equation can be expanded in series.

The first coefficient becomes

$$A3 = \frac{1}{(r-r_1)}\left(G + HO\,(r-r_1)\right)$$

where G and H0 are :

$$G = \left(\frac{2m^2}{r^3} + \frac{2\gamma D\gamma}{a^2} - \frac{\gamma^2}{a^2}\frac{DT}{T}\right)\frac{r_1^2}{m^2} \cdot \frac{r^2}{(r+r_1)} ,$$

and

$$HO = -\left(\frac{V^2\gamma'}{ra^2} - \frac{DT}{T} + \frac{1}{r}\right)$$

- The second coefficient becomes:

$$A4 = (G1 + H1(r - r_1)) \frac{1}{(r - r_1)}$$

where G1 and H1 are:

$$G1 = -\frac{\alpha}{\gamma}\left(\frac{2m^2}{r^3} + \frac{2\gamma D\gamma}{a^2} - \frac{\gamma^2}{a^2}\frac{DT}{T}\right) \cdot \frac{r_1^2}{m^2}\frac{r^2}{(r+r_1)}$$

and

$$H1 = \frac{\alpha}{\gamma}\left(\frac{D\rho_0}{\rho_0}\right) + \frac{D\alpha}{\gamma} + \frac{\beta}{\gamma^2 S} - \frac{\eta\alpha}{\gamma^2}$$

But as G, H0, G1 and H1 are functions of r it is necessary to expand them also and we get:

$$A3 = \left[G\Big|_{r=r_1} + (DG + H0)\Big|_{r=r_1}(r - r_1) + \left(\tfrac{1}{2}D^2G + DH0\right)\Big|_{r=r_1}(r-r_1)^2\right]\frac{1}{(r - r_1)}$$

and

$$A4 = \left[G1\Big|_{r=r_1} + (DG1 + H1)\Big|_{r=r_1}(r - r_1)\right]\frac{1}{(r - r_1)}$$

The expressions above contain first and second derivative of G, and they are denoted by DG and D^2G.

DG and D^2G becomes:

$$DG = \frac{r_1^2}{m^2}\left[D(GG)\frac{r^2}{(r+r_1)} + GG\left(\frac{2r}{(r+r_1)} - \frac{r^2}{(r+r_1)^2}\right)\right]$$

and

$$D^2G = \frac{r_1^2}{m^2}\left[D^2(GG)\frac{r^2}{(r+r_1)} + 2D(GG)\left(\frac{2r}{(r+r_1)} - \frac{r^2}{(r+r_1)^2}\right) + GG\left[\frac{2}{(r+r_1)} - \frac{4}{(r+r_1)^2} + \frac{2r}{(r+r_1)^3}\right]\right]$$

With GG means the expression:

$$GG = \frac{2m^2}{r^3} + \frac{2\gamma D\gamma}{a^2} - \frac{\gamma^2}{a^2}\frac{DT}{T}$$

and by DGG and D^2GG denotes the first and the second derivative of GG and they becomes:

$$D(GG) = \left(-\frac{6m^2}{r^4} + \frac{1}{a^2}\left(2(D\gamma)^2 + 2\gamma D^2\gamma - 4\gamma D\gamma\frac{DT}{T} + \gamma\frac{DT}{T^2} + \dots\right)\right)$$

and

$$D^2(GG) = \left(+\frac{24m^2}{r^5} + \frac{1}{a^2}\left[6D\gamma D^2\gamma + 2\gamma D^3\gamma - 6\left((D\gamma)^2 + \gamma D^2\gamma\right)\frac{DT}{T} - \right.\right.$$

$$\left.\left. - \left[4\gamma D\gamma - D\gamma + \frac{2\gamma}{T}\right]\frac{DT}{T^2} + \dots\right]\right)$$

The expression for γ is: $\gamma = -\omega + kW + \frac{m}{r} V$

If $D\gamma$, $D^2\gamma$ and $D^3\gamma$ are the first, second and third derivative of , their expressions become:

$$D\gamma = kDW + m\left[\frac{DV}{r} - \frac{V}{r^2}\right]$$

$$D^2\gamma = kD^2W + m\left[\frac{D^2V}{r} - 2\frac{DV}{r^2} + \frac{2V}{r^3}\right]$$

and $D^3\gamma = kD^3W + m\left[\frac{D^3V}{r} - \frac{3D^2V}{r^2} + 6\frac{DV}{r^3} - \frac{6V}{r^4}\right]$

The derivatives of H0 and G1 are:

and $DH0 = -\left[\frac{V\gamma'}{ra^2}\left[2DV - \frac{V}{r} - V\frac{DT}{T}\right] + \frac{DT}{T^2} - \frac{D^2T}{T} - \frac{1}{r^2}\right]$

$$DG1 = -\frac{\alpha}{\gamma}DG - \left(\frac{D\alpha}{\gamma} - \frac{\alpha}{\gamma^2}D\gamma\right)G$$

The Frobenius expansion gives:

$$u = \sum_{n=0}^{\infty} A_n \epsilon^{n+\alpha^*}, \quad \epsilon = r - r_1$$

The differential equation becomes:

$$\sum_{n=0}^{\infty} A_n (n+\alpha^*)(n+\alpha^*-1)\,\epsilon^{n+\alpha^*-2} =$$

$$= \left(G + (DG + H0)\epsilon + (D^2G + DH)\epsilon^2\right)\sum_{n=0}^{\infty} A_n(n+\alpha^*)\,\epsilon^{n+\alpha^*-2} +$$

$$+ \left(G1 + (DG1 + H1)\epsilon\right)\sum_{n=0}^{\infty} A_n\,\epsilon^{n+\alpha^*-1}$$

and comparson of the same order of magnitude gives

$$\mathcal{O}(\epsilon^{\alpha^*-2}) \quad : \quad \alpha^*(\alpha^* - 1 - G) = 0$$

which have two possible values of α^*

AND
$$\begin{cases} \alpha_1^* = 0 \\ \alpha_2^* = 1 + G \end{cases}$$

Near the location $r = r_1$, the value of u can be given by a series expansion

$$u = C\sum_{j=0}^{\infty} a_j \epsilon^{j+\alpha_2^*} + B\sum_{j=0}^{\infty} b_j \epsilon^{j+\alpha_1^*}$$

in a small parameter $\epsilon = r - r_1$.

The constants C and B can be chosen in order to make the function and its derivative in agreement with the known properties at one point. The values of a_j are given by a recurrence formula, With a_0 chosen equal to unity and a_1 as

$$a_1 = \frac{(DG + H_0)\alpha_2^* + G_1}{(1+\alpha_2^*)(\alpha_2^* - G)} \cdot a_0$$

and

the recurrence formula will give a_j, $(j \geq 2)$:

$$a_j = \frac{(DG + H_0)(j-1+\alpha_2^*) + G_1}{(j+\alpha_2^*)(j+\alpha_2^*-1-G)}\, a_{j-1} + \frac{(D^2G + DH)(j-2+\alpha_2^*) + (DG_1 + H_1)}{(j+\alpha_2^*)(j+\alpha_2^*-1-G)}\, C$$

b_j will be given in the same way as for a_j, but with α_1^* instead of α_2^* in the formula.

In the case of rigid body rotation and no axial flow, the value of G becomes

$$G = \frac{2m^2}{r^3} \cdot \frac{r_1^2}{m^2}\left(\frac{r^2}{r+r_1}\right) = \frac{2r_1^2}{r(r+r_1)}$$

When $r = r_1$ it is seen that $G=1$, which makes it impossible to get b_j for $j \geq 2$,

u will now become

$$u = C\sum_{j=0}^{\infty} a_j \ln|\epsilon| \cdot \epsilon^{j+\alpha_2^*} + B\sum_{j=0}^{\infty} b_j \epsilon^{j+\alpha_1^*}$$

where $b_2=1$ and $b_j=0$ $(j < 2)$

APPENDIX 3

DESCRIPTION OF COMPUTER PROGRAM

The computer program consists of a main program and four subroutines. The subroutine "VELTEM" prescribes the basic velocity and temperature distributions. For example, the radial dependence of the axial flow can be either of poiseuille type or in the form of a number of jets. The jets can be wall jets and as many as three axial jets, at a radial distance "RO", in the interior region. A given input variable is the amount of flow "FF" in the first jet and in addition to that, the flow "FF" and "FB" for the free and boundary jets, can be prescribed. All jets can be more or less concentrated. "E" and "EB" are the widths of the free and boundary jets respectively. Other input values are the ratio of specific heats γ' denoted by "GHAM" and the sound velocity a as "AC". Further "EM" and "XK" denote the wavenumbers in azimuthal and axial directions, respectively. In order to describe the amount of poiseuille flow, there is initially given a parameter "EPS" which is the ratio of the centerline axial velocity to the peripheral velocity. For each sets of parameters, there is an initial guess of the value of the complex frequency ω_r, which in the program is written as "FREAM", with the real part, ω_r as "FREQ" and the imaginary part, ω_i as "AMP". The subroutine "RUNGKT" is used to integrate the second order differential equation (2.18), for **u**. The numerical method used is a fourth order Runge-Kutta procedure with complex arithmetic in double precision (Collatz 1960). The second part of the subroutine "SPDQ" is used in order to get the values for the needed coefficients. The values of the perturbation velocity and its derivative are denoted by " U" and " DU ", respectively, and are prescribed for 50 equidistant radial points between zero and one. The step length in the numerical procedure is "HH" which has been given the value of 0.01. Near the sonic point, where applicable, the steplength is reduced by a factor of 50 from the ordinary step length, in order to come as close as possible to the singular point, thereby reducing the influence of numerical error. Under the assumption of continuity, the values of velocity, u, and its first derivative, at the nearest point after the singularity are obtained by independent linear extrapolation.

In the case of an innercylinder, with radius "ROX", the numerical procedure can start directly from the inner boundary, with the assumption that the derivative "DU" will have a finite starting value, here chosen to be 0.63 + 0.29i.

Without inner cylinder, because of the singularity at the origin, it is necessary to start with a series expansion for u and its derivative out to a small distance, of 0.005, from the origin. The first part of the subroutine "SPDQ" will provide the proper series expansion, whether the perturbation is axisymmetric or not. In order to satisfy the outer boundary condition, within a small error, which in this case was chosen to be $\frac{1}{2}10^{-5}$, the value of the function u and, in the case of a free vortex, some of the other perturbation quantities, have been taken into account.

With the parameter "IS" two different kinds of outer boundary conditions can be specified. A value less than two refers to a rotating fluid inside a concentric rigid cylinder, and the proper boundary condition is zero radial velocity. This condition is considered satisfied when $|u|$ calculated at $r = 1$ is less than $\frac{1}{2}10^{-5}$. If "IS" is greater than two, the flow condition is that of a free vortex, and the third part of subroutine "SPDQ" can be used to get the proper values of the perturbation quantities in order to see whether the boundary condition is satisfied.

The subroutine "OPT" provides a new value of the complex frequency which is found by the use of the Newton-Raphson method in complex variables.

For this, two earlier values of the frequency are needed. Therefore, a second frequency is selected by adding a small quantity to the initial guess of the frequency. The magnitude of this small quantity, which in the program, is named "FSTEP", was chosen as $10^{-6} + 10^{-6}i$.

The maximum number of iterations is specified each time as the value of the quantity "NP". However if it is not possible, with this number of iterations to satisfy the boundary conditions within specified error limit, the text "Can Not Converge" is written out, and the iteration procedure is terminated. After that the next two input cards give new initial values. However if the next cards are blank, the program will terminate.

When the outer boundary condition is satisfied to the desired accuracy the calculation is terminated. The third part of the subroutine "SPDQ" is used not only to calculate the other perturbation quantities, but also to write them out together with the basic values of velocity, density and pressure for fifty values of the radius. Further, it is possible to examine a sequence of cases when some of the parameters; k, a, FF, or ROX is changed by a small amount in each step. In order to estimate a proper starting value in each new case, the frequencies for the earlier cases have been taken into account in such a way that a linear or second order polynomial has been used. In the second iteration, the parameter is increased by one twentieth of its original value and then it is sufficient to use the previously obtained value of the frequency as a starting value. The maximum number of iterations is limited by "NPTS", after which the calculation is terminated, unless the boundary condition has been reached to within the desired tolerance in a previous iteration, in which case the calculation will be interupted earlier.

In the subroutine "OPT" where the new frequency is given, the maximum frequency change is limited by a quantity "STEG". "STEG" is two ,or twice the nondimensional speed of sound, depending on whether the mode that is looked for is of the inertial or acoustical kind. This has been done in order to avoid loosing the track of the eigenvalue. If the amplitude of the radial perturbation velocity at the outer boundary fails to decrease in an iteration, the maximum frequency change is decreased by a factor of 1.5.

In order to get a good initial value of the frequency, it is important to know the behavior of the perturbation quantity for a range of frequencies. This can be done by writing out this quantity for a number of different values of the frequency. If both "NPTS" and "NP" are given the value unity, then a number of "MM" values will be written out with the frequency increased with the amount of "DFREQ" each time. Finally, the parameter "DE" can be used to study the influence of small deviation from the state of rigid body rotation.

In order to remember the date for calculations, it is possible to provide this information on the first data card where day, month, and year is given by "ID", "IM", "IY" resp. This is only a practical detail, but which can prove to be useful.

The next two cards hold the needed input values for

GHAM = Ratio of specific heats

EM = Azimutal wavenumber (m)

XK = Axial wavenumber (k)

AC = The speed of sound (a)

ROX = Radius of inner cylinder

FF = Flux in one of the jets

EPS = Parameter indicating amount of axial Poiseuille flow (ε)

FREQ = Real part of frequency (ω_r)

AMP = Imaginary part of frequency (ω_r)

NPTS = Number of iterations with change of some parameter

NP = Number of iterations in order to find the frequency

DXK = Change in axial wavenumber

DAC = Change in speed of sound

DROX = Change in radius of inner-cylinder

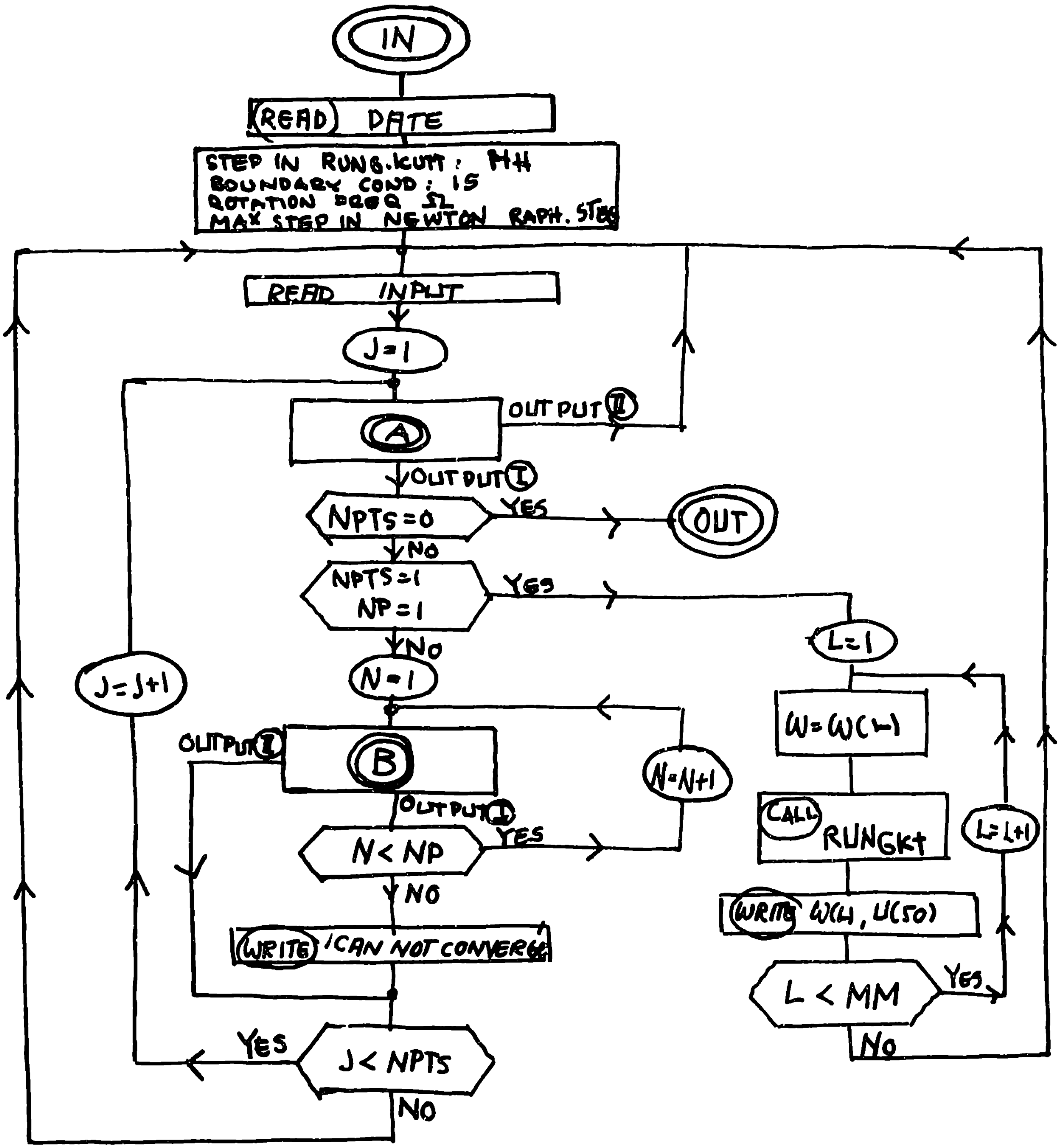
DFF = Change in jet flow

DPS = Change of parameter indicating amount of axial Poiseuille flow

DFREQ = Change in real part of the frequency

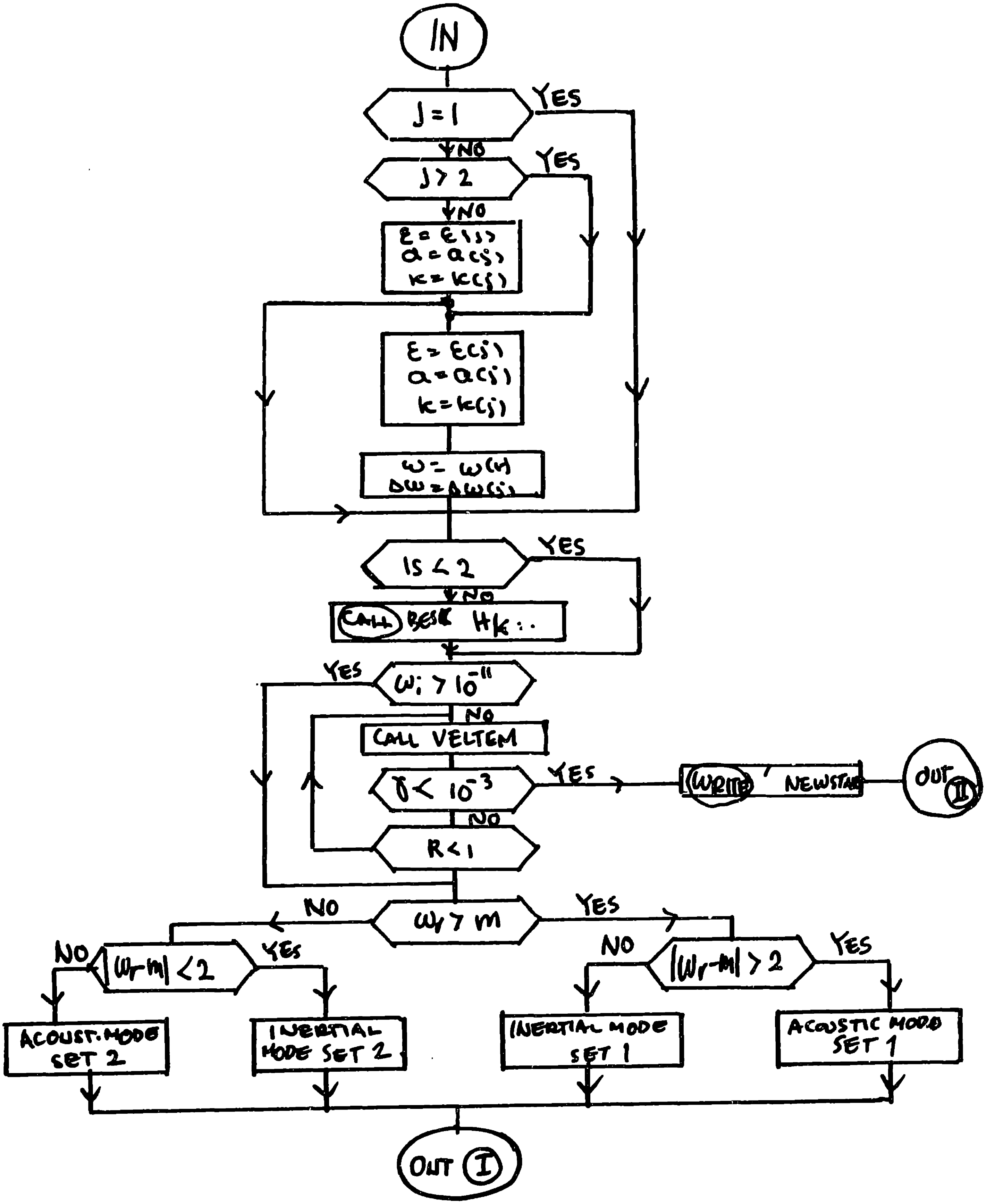
MM = Number of change DFREQ

DE = Parameter indicating disturbance of rigid body rotation


IN
READ DATE
STEP IN RUNG.KUTT: HH
BOUNDARY COND: IS
ROTATION FREQ Ω
MAX STEP IN NEWTON RAPH. STEP
READ INPUT
J=1
A
OUTPUT II
OUTPUT I
NPTS=0
YES
OUT
NO
NPTS=1
NP=1
YES
NO
N=1
L=1
W=W(L)
OUTPUT II
B
N=N+1
OUTPUT I
N<NP
YES
CALL RUNGKT
L=L+1
NO
WRITE W(L), U(50)
WRITE 'CAN NOT CONVERGE
L<MM
YES
NO
J=J+1
YES
J<NPTS
NO

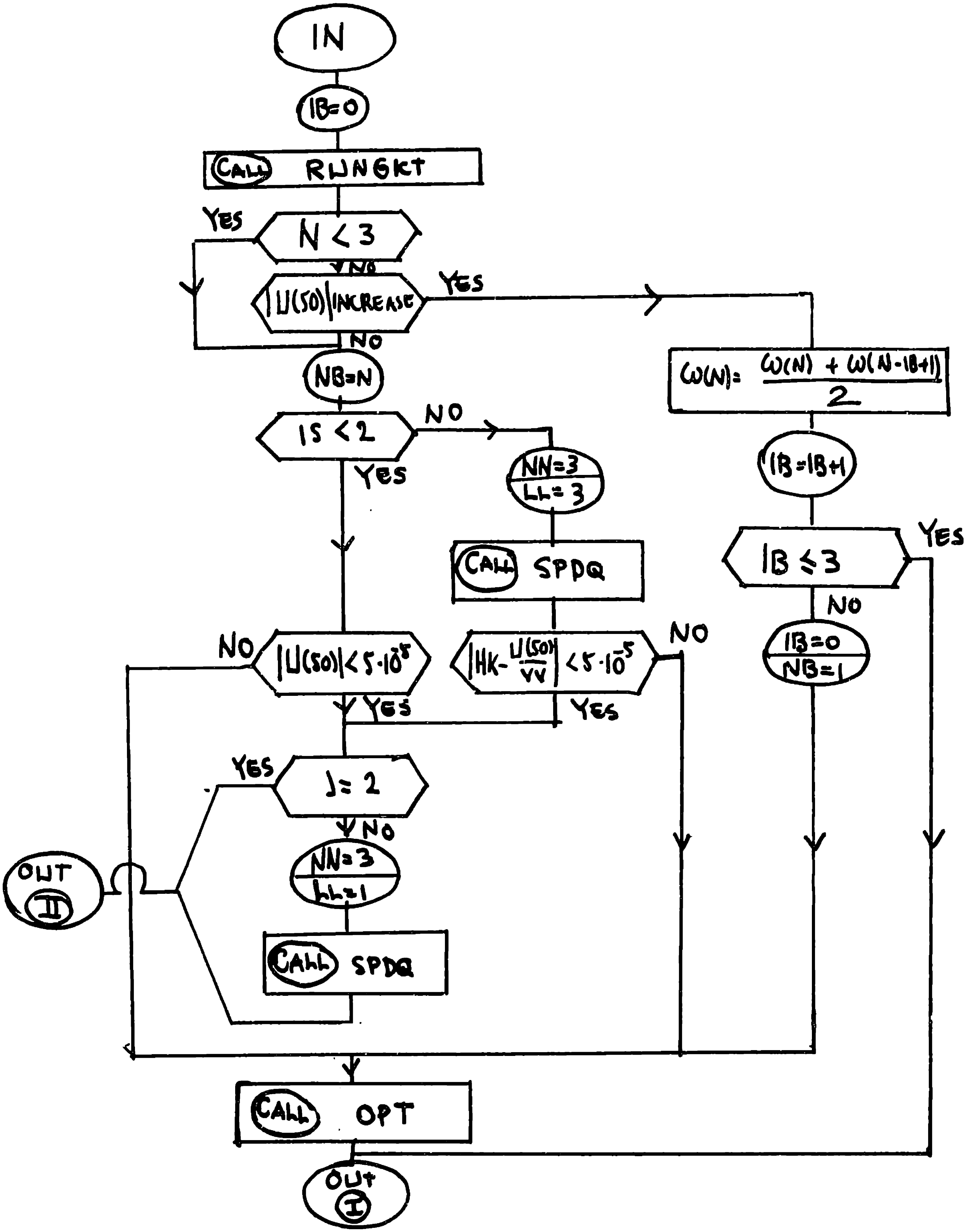
Flow Chart 1.

A



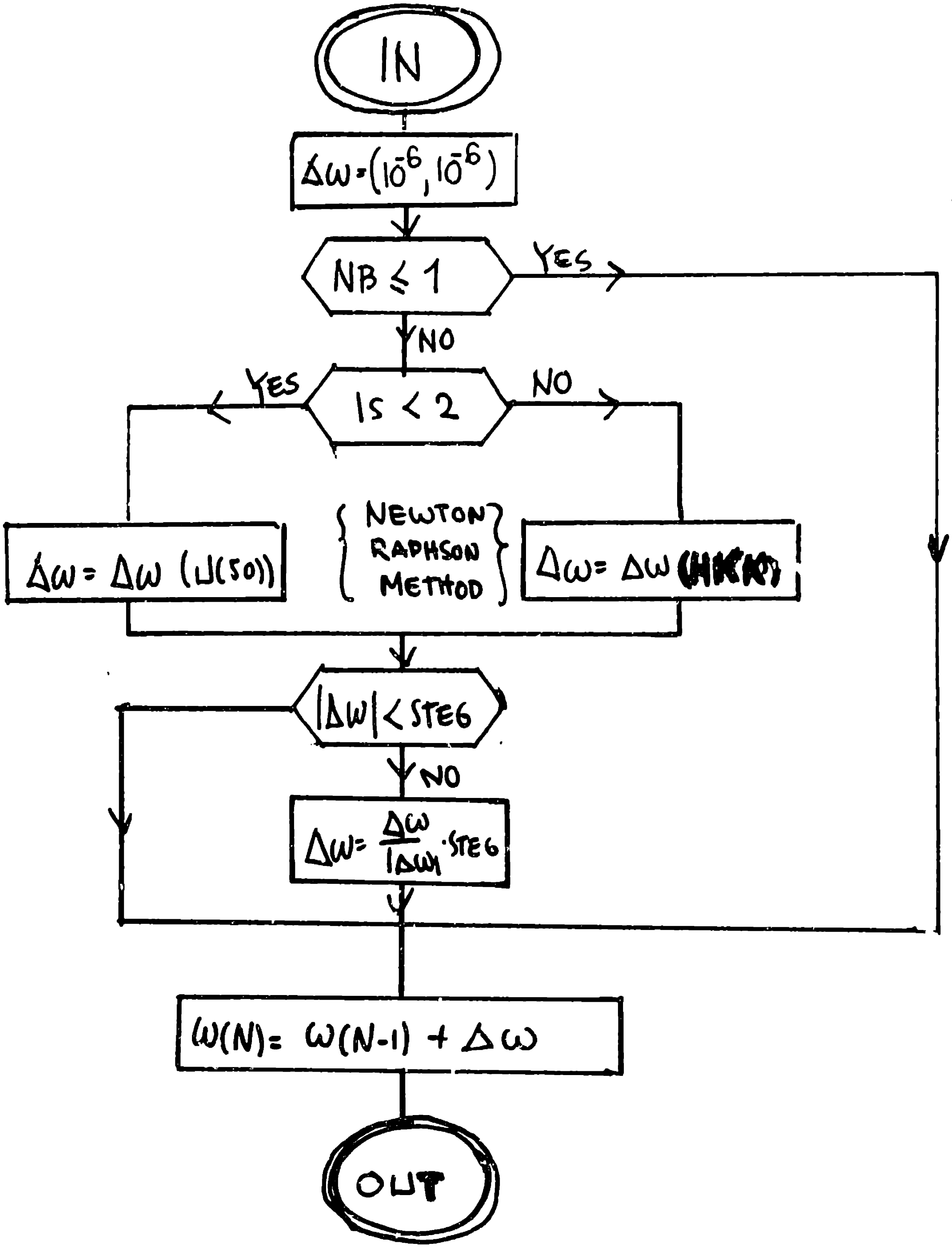
Flow Chart 2.

B

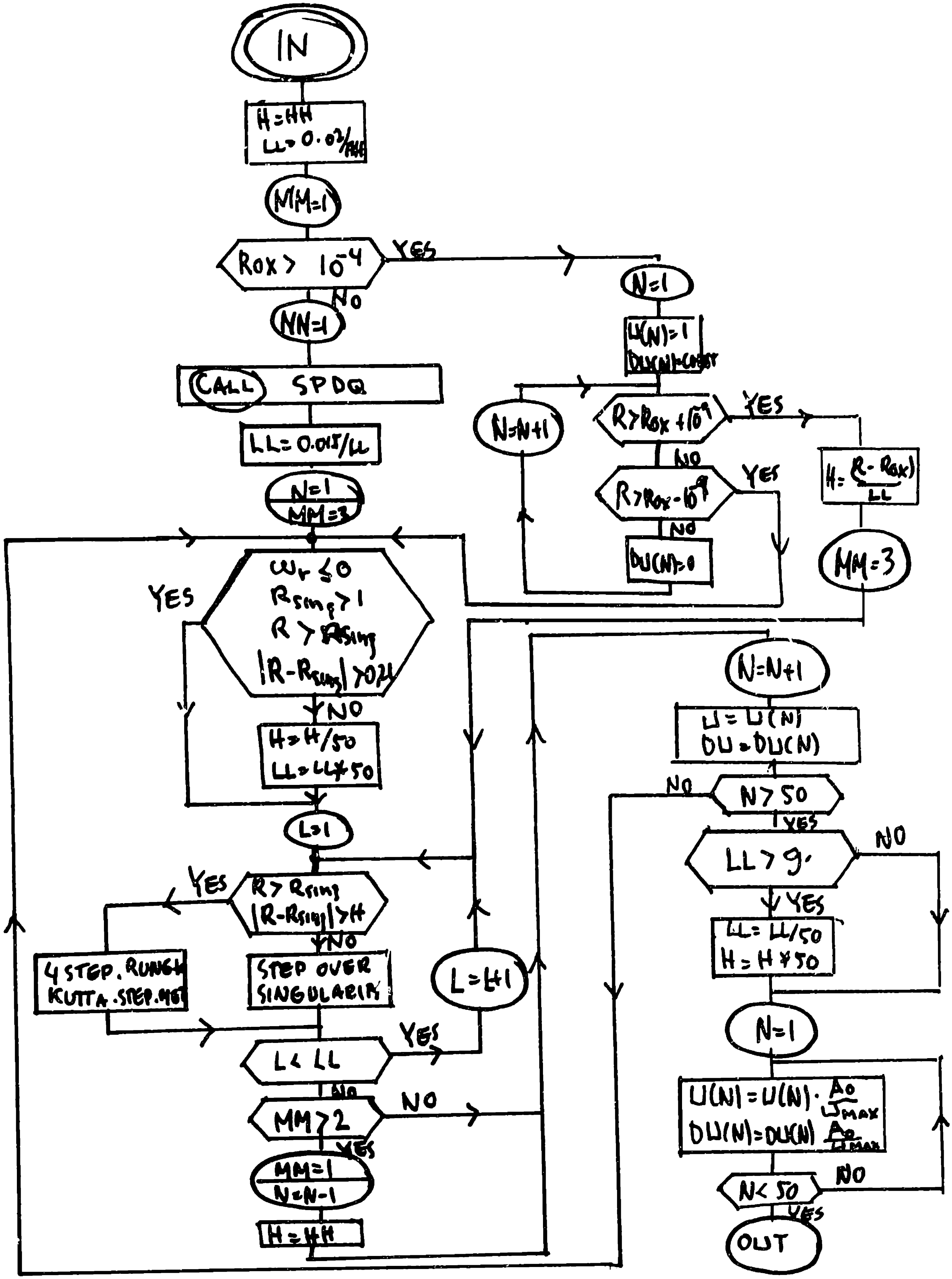


Flow Chart 3.

OPT

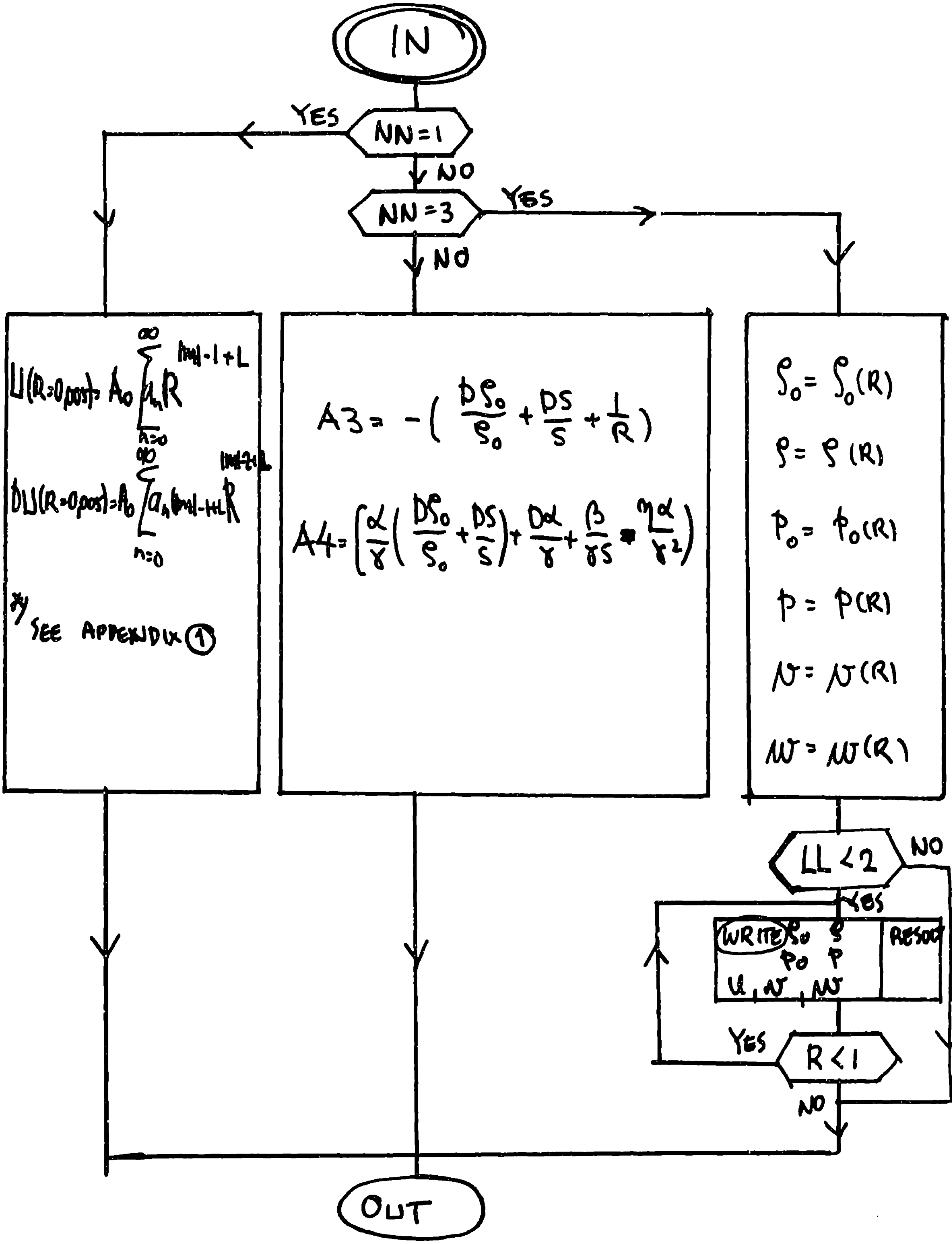


Flow Chart 4.

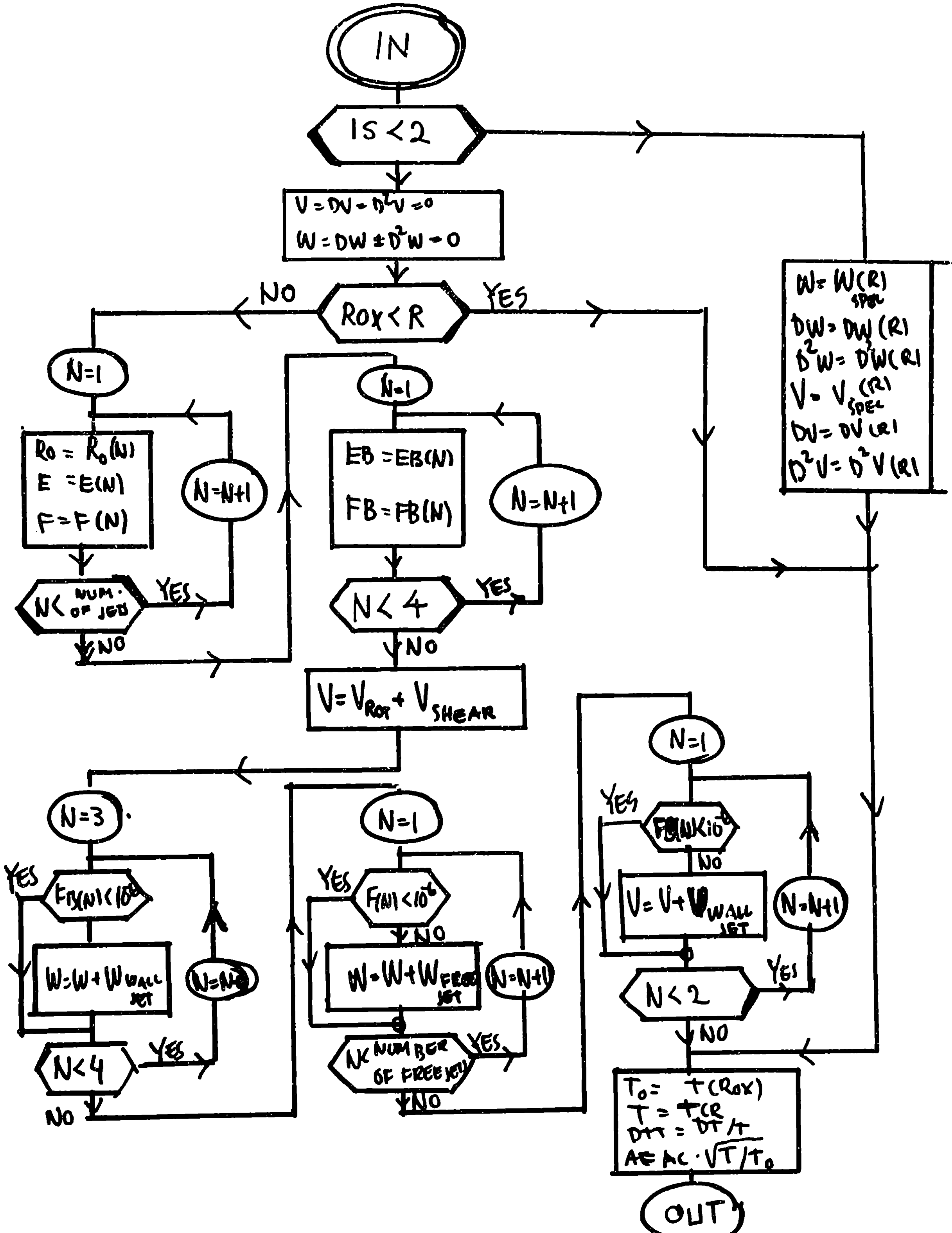

IN
H=HH
MM=1
Rox > 10^-4
YES
NO
NN=1
CALL SPDQ
LL=0.015/LL
N=1
MM=3
N=1
U(N)=1
N=N+1
R>Rox+10^-9
YES
NO
R>Rox-10^-9
YES
NO
DU(N)=0
H= (R-Rox)/LL
MM=3
YES
R > Rsing
NO
H=H/50
LL=LL*50
L=1
YES
R>Rsing
|R-Rsing|>H
NO
4 STEP. RUNGE KUTTA .STEP.MET
STEP OVER SINGULARITY
L=L+1
YES
L<LL
NO
NO
MM>2
YES
MM=1
N=N-1
H=HH
N=N+1
U=U(N)
DU=DU(N)
NO
N>50
YES
NO
LL>9.
YES
LL=LL/50
H=H*50
N=1
NO
N<50
YES
OUT

Flow Chart 5.

SPDG



Flow Chart 6.


IN
IS<2
V=DV=D²V=0
W=DW=D²W=0
NO
YES
ROX<R
N=1
Ro = Ro(N)
E = E(N)
F = F(N)
N=N+1
N< NUM. OF JETS
EB = EB(N)
FB = FB(N)
N<4
V = V ROT + V SHEAR
W = W(R) SPEC
DW = DW(R)
D²W = D²W(R)
V = V(R) SPEC
DV = DV(R)
D²V = D²V(R)
N=3
N=1
W=W+W WALL JET
W=W+W FREE JET
N< NUMBER OF FREE JET
V = V+W WALL JET
N<2
OUT

Flow Chart 7.

```
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 U,DU,FREAM,GAM,Z1,UEND,UE1,FREAO,DFREA,FREAN
     1 ,S,DEL,PP,WIR,HK,VV,HKD,HKK,HK1,AB,BA,AAO
     2 ,DSS,ALPHA,DALPH,VEETA,EETA
      DIMENSION U(50),DU(50),FREAM(2),UEND(2),FREAN(30),WIR(40),HKK(2)
      COMMCN U,DU,FREAM,GAM,EM,XCMEG,EPS,XK,GHAM,AC,A,ROX,R,V,W,FF,VV
     1 IS,DGAM,S,DSS,DRHC,ALPHA,DALPH,VEETA,EETA,AAO,DE
      READ(5,79C0) ID,IM,IY
   10 READ(5,8000) GHAM,EM,XK,AC,ROX,FF,EPS,FREQ,AMP
      REAC(5,8005) NPTS,NP,DXK,DAC,DROX,DFF,DPS,DFREQ,MM,DE
      HH=0.C1DC
      IF(NPTS .EQ. 0) GO TO 400
      IS=1
      XOMEG=1.DO
      STEG=2.DO
      Z1 = DCMPLX(0.DO,1.CO)
      NPN=NPTS+NP
      FREAC=DCMPLX(0.DO,0.CO)
      FREAM(1)=DCMPLX(C.CO,0.DO)
      FREAM(2)=FREQ+AMP*Z1
      UEND(1)=DCMPLX(0.DC,0.DO)
      HKK(1)=DCMPLX(C.DO,C.DO)
      EPSC=EPS
      ACC=AC
      FFO=FF
      ROXC=ROX
      XKO=XK
      DO 2CC J=1,NPTS
      IF(J.EQ.1) GO TO 17
      IF(J.GT.2) GO TO 11
      EPS=EPSO+DPS/20.DO
      AC=ACO+DAC/20.DO
      FF=FFC+DFF/20.DO
      ROX=ROXO+DROX/20.DO
      XK=XKO+DXK/20.DO
      L=19
      GO TO 17
   11 EPS=EPSO+(J-2)*DPS
      AC=ACC+(J-2)*DAC
      FF=FFC+(J-2)*DFF
      ROX=ROXO+(J-2)*DROX
      XK=XKO+(J-2)*DXK
      IF(J.GT.3) GO TO 12
      FREAM(2)=FREAN(J-1)+(FREAN(J-1)-FREAN(J-2))*DFLOAT(L)
      GO TC 17
   12 IF(J-5) 13,14,15
   13 ET=1.CO
      TV=ET+19.DO
      TR=TV+20.DO
      GO TC 16
   14 ET=19.DO
      TV=ET+20.0
      TR=TV+20.DO
```

```
      GO TO 16
   15 ET=20.D0
      TV=ET+20.0
      TR=TV+20.D0
   16 AB=(FREAN(J-2)/ET-FREAN(J-1)/TV)/(ET-TV)+FREAN(J-3)/(ET*TV)
      BA=FREAN(J-2)/ET-AB*ET-FREAN(J-3)/ET
      FREAM(2)=AB*TR**2+BA*TR+FREAN(J-3)
      FREAC=FREAM(2)-FREAN(J-1)
   17 CONTINUE
      IF(IS.LE.2) GO TO 21
      CALL BESK(XK,0,BK0,ITER)
      CALL BESK(XK,1,BK1,ITER)
      HK=Z1*(1.D0+XK*BK0/BK1)/EM
   21 IF(AMP.GT.0.01D-9) GO TO 26
      KN=0
   22 DO 25 N=1,50
      R=0.02D0*DFLOAT(N)
      CALL VELTEM(DV,DDV,DW,DDW,DTT)
      GAM=-FREAM(2)+XK*W+EM*V/R
      ABSGA=CDABS(GAM)
      IF(ABSGA-1.D-3) 23,23,25
   23 ABFRE=CDABS(FREAM(2))
      KN=KN+N
      IF(KN.LE.100) GO TO 25
      WRITE(6,8008) GAM,FREAM(2),R
      GO TO 200
   25 CONTINUE
   26 WRITE(6,8009) ID,IM,IY
      WRITE(6,8010) GHAM,ROX,XOMEG,EPS,EM,XK,AC
      IF(NPN.LE.2) GO TO 76
      RF=FREQ-EM
      JJ=1
      IF(RF.GT.0.D0) GO TO 27
      JJ=2
   27 RF=DABS(RF)-0.5D0
      IF(RF.GT.0.D0) GO TO 28
      WRITE(6,8011) JJ,FF,HH,DE
      GO TO 29
   28 WRITE(6,8012) JJ,FF,HH,DE
      STEG=STEG*AC
      FREAM(2)=FREAM(2)*AC
      DFREQ=DFREQ*AC
   29 IB=0
      UEND(2)=DCMPLX(1.D0,1.D0)
      WRITE(6,8015)
   35 DO 75 N=1,NP
      CALL RUNGKT(HH)
   45 UEND(2)=U(50)
      CDU=CDABS(U(50))
   49 DFREA=FREAM(2)-FREAC
      IF(N.LT.3) GO TO 50
      IF(CDU.LT.ABSU) GO TO 50
      UEND(2)=UEND(1)
```

```
      UEND(1)=UE1
      WRITE(6,8020) N,FREAM(2),DFREA,STEG,U(50),CDU
      STEG=STEG/1.5D0
      FREAM(1)=FREAO
      FREAM(2)=(FREAM(2)+FREAM(1))/2.D0
      IB=IB+1
      IF(IB.LE.3) GO TO 72
      IB=0
      NB=1
      FREAM(2)=FREAO
      GO TO 70
   50 WRITE(6,8022)
      NB=N
      IF(IS.LT.2) GO TO 60
      LL=3
      NN=3
      CALL SPDQ (NN,LL,A3,A4)
      WRITE(6,8022)
   55 WRITE(6,8023) HK
      HKD=HK-U(50)/VV
      HKK(2)=HKD
      WRITE(6,8024) U(50),VV,HKD
      AHK=CDABS(HKD)
      WRITE(6,8025) N,FREAM(2),DFREA,STEG,HK,AHK
      IF(AHK-5.D-5) 65,65,70
   60 WRITE(6,8025) N,FREAM(2),DFREA,STEG,U(50),CDU
      ABSU=CDU
      IF(ABSU-5.D-5) 65,65,70
   65 LL=1
      FREAN(J)=FREAM(2)
      IF(J.EQ.2) GO TO 200
      NN=3
      CALL SPDQ(NN,LL,A3,A4)
      GO TO 200
   70 CALL OPT(NB,IS,FREAM,UEND,HKK,STEG)
      IF(IS.LT.2) GO TO 71
      HK1=HKK(1)
      HKK(1)=HKK(2)
      GO TO 72
   71 UE1=UEND(1)
      UEND(1)=UEND(2)
   72 FREAO=FREAM(1)
   75 CONTINUE
      WRITE(6,8050)
      GO TO 10
   76 CONTINUE
      FREAM(2)=FREAM(2)*AC
      DFREQ=DFREQ*AC
      FREAM(2)=FREAM(2)-DFREQ
      DO 79 L=1,MM
      FREAM(2)=FREAM(2)+DFREQ
      GAM=-FREAM(2)+XK*XOMEG*EPS*(1.D0-R**2)+EM*XOMEG
      ABSGA=CDABS(GAM)
```

```
      IF(ABSGA.LE.1.D-3) GO TO 79
      CALL RUNGKT(HH)
      CDU=CDABS(U(50))
      WRITE(6,8040) FREAM(2),CDU
   79 CONTINUE
  200 CONTINUE
      GO TO 10
  400 CONTINUE
      STOP
 7900 FORMAT(2I2,I4)
 8000 FORMAT(D5.3,D5.1,6D10.5,D10.7)
 8005 FORMAT(2I5,6D10.5,I3,D7.4)
 8008 FORMAT(1H ,'GAM=',2D14.6,2X,'FREAM=',2D14.6,2X,'R=',D14.6,2X,
     1 'TRY NEW STARTINGPOINT')
 8009 FORMAT(1H1,'DAY =',I2,1X,'MONTH =',I2,1X,'YEAR =',I4/)
 8010 FORMAT(1H ,1X,'GHAM=CP/CV =',D11.4,2X,'RADIUS OF INNERCYLINDER ='
     1 ,D11.4,2X,'ANGULAR VELOCITY =',D10.2,1X,'ROSSBY NUMBER EPS =',
     2 D12.4//,' ANGULAR WAVE-NUMBER M =',D10.2,' *** AXIAL WAVE-NUMBER
     3 K =',D12.5,' *** SOUND VELOCITY AC =',D12.4//)
 8011 FORMAT(1H ,'INERTIAL MODE SET=',I2,5X,
     1 'FLUX IN JET ONE = ',D10.4,1X,'HH = ',D10.4,1X,'DE = ',D11.4/)
 8012 FORMAT(1H ,'ACOUSTIC MODE SET=',I2,5X,
     1 'FLUX IN JET ONE = ',D10.4,1X,'HH = ',D10.4,1X,'DE = ',D11.4/)
 8015 FORMAT(1H ,1X,'N',2X,'FREQ=W(REAL)',2X,'AMP=W(IMAG)',3X,'DFREQ',
     1 8X,'DAMP',9X,'STEG',4X,'B',4X,'UREAL',8X,'UIMAG',9X,'ABS(U)'/)
 8018 FORMAT(1H ,30X,'WIR = ',2D13.6,1X,'|U| = ',D13.6)
 8020 FORMAT(1H ,I2,1X,2D13.6,1X,2D13.6,1X,D9.3,'B',1X,2D13.6,1X,
     1 D13.6,'B')
 8022 FORMAT(/)
 8023 FORMAT(1H ,1X,'HK = ',2D13.6)
 8024 FORMAT(1H ,1X,'U = ',2D13.6,1X,'VV = ',2D13.6,1X,'HK-U/VV =',
     1 2D13.6)
 8025 FORMAT(1H ,I2,1X,2D13.6,1X,2D13.6,1X,D9.3,2X,2D13.6,1X,D13.6)
 8030 FORMAT(//)
 8040 FORMAT (1H ,1X,'W(REAL) =',D13.6,1X,'W(IMAG) =',D13.6,1X,
     1 'ABS(U) =',D13.6)
 8050 FORMAT(1H0,'CAN NOT CONVERGE')
      END
```

```
      SUBROUTINE OPT(NB,IS,FREAM,UEND,HKK,STEG)
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 FREAM,UEND,Z1,FSTEP,DFREA,HKK
      DIMENSION FREAM(2),UEND(2),HKK(2)
      Z1 = DCMPLX(0.D0,1.D0)
      FSTEP=DCMPLX(1.D-6,1.D-6)
      IF(NB-1) 10,10,20
   10 FREAM(1)=FREAM(2)
      FREAM(2)=FREAM(1)+FSTEP
      GO TO 400
   20 IF(IS.GT.2) GO TO 25
      DFREA=(FREAM(2)-FREAM(1))*UEND(2)/(UEND(1)-UEND(2))
      GO TO 27
   25 DFREA=(FREAM(2)-FREAM(1))*HKK(2)/(HKK(1)-HKK(2))
   27 CDAFR=CDABS(DFREA)
      IF(CDAFR.GT.STEG) GO TO 30
      IF(STEG.GT.0.5D-3) GO TO 40
      STEG=1.D-3
      GO TO 40
   30 DFREA=STEG*DFREA/CDAFR
   40 FREAM(1)=FREAM(2)
      FREAM(2)=FREAM(1)+DFREA
  400 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RUNGKT(HH)
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 U,DU,FREAM,GAM,U0,V0,U1,V1,HK,UU,DUU,DDUU,DDU,A3,A4,
     1 G,G1,H1,AA,S1,S2,EETA,ALPHA,DALPH,VEETA,S,DSS,E1,E,U12,V12,AA0
     2 ,DG1,GG,DGG,DDGG,DG,DDG,S3,S4,SN1,SN2,SSS,BB,B,C,A0,B0,U11,V11
      DIMENSION U(50),DU(50),HK(4),FREAM(2),DUU(4),DDUU(4),DDU(50)
     1 ,AA(10),S1(2),S2(2),S3(2),S4(2),BB(10)
      COMMON U,DU,FREAM,GAM,EM,XOMEG,EPS,XK,GHAM,AC,A,ROX,R,V,W,FF,VV
     1 IS,DGAM,S,DSS,DRHO,ALPHA,DALPH,VEETA,EETA,AA0,DE
      U0=(0.D0,0.D0)
      U11=(0.D0,0.D0)
      V0=(0.D0,0.D0)
      V11=(0.D0,0.D0)
      H=HH
      LL=0.021D0/HH
      MM=1
      N=0
      NL=0
      IF(ROX.GT.1.D-4) GO TO 4
      NN=1
      CALL SPDQ(NN,LL,A3,A4)
      N=1
      H=0.015D0/DFLOAT(LL)
      MM=3
      GO TO 8
    4 AA0=DCMPLX(1.D0,0.26D0)
    5 N=N+1
      NL=N
      R=0.02D0*DFLOAT(N)
      RR=R-ROX
      U(N)=DCMPLX(0.D0,0.D0)
      DU(N)=DCMPLX(0.63D0,0.29D0)
      IF(RR.GE.1.D-9) GO TO 7
      IF(RR.GE.-1.D-9) GO TO 8
      DU(N)=DCMPLX(0.D0,0.D0)
      GO TO 5
    7 R=ROX
      H=RR/DFLOAT(LL)
      MM=3
      GO TO 9
    8 R=0.02D0*DFLOAT(N)
      W2=FREAM(2)
      CALL VELTEM(DV,DDV,DW,DDW,DTT)
      GAM=-FREAM(2)+XK*W+EM*V/R
      G2=(GAM/A)**2-XK**2
      IF(G2.LE.0.D0) GO TO 9
      R1=DSQRT((EM**2)/G2)
      IF(R1.GT.1.D0) GO TO 9
      R2=DABS(R-R1)
      IF(R.GT.R1) GO TO 9
      IF(R2.GT.0.021D0) GO TO 9
      H=HH/50.D0
      LL=LL*50
```

```
    9 U1=U(N)
      V1=DU(N)*H
      DO 29 L=1,LL
      U12=U1-U0-U11
      V12=V1-V0-V11
      U11=U1-U0
      V11=V1-V0
      R0=R
      U0=U1
      V0=V1
      M=0
      IF(LL.LE.9) GO TO 10
      CALL VELTEM(DV,DDV,DW,DDW,DTT)
      GAM=-FREAM(2)+XK*W+EM*V/R
      G2=GAM**2
      R1=DSQRT((EM**2)/((G2/(A**2))-XK**2))
      R2=DABS(R-R1)
      IF(R2-H) 16,16,10
   10 NN=2
      CALL SPDQ(NN,LL,A3,A4)
   11 M=M+1
      HK(M)=(H**2)*(A3*(V1/H)+A4*U1)/2.D0
      IF(M-3) 12,14,15
   12 R=R0+H/2.D0
      U1=U0+V0/2.D0+HK(1)/4.D0
      V1=V0+HK(M)
      IF(M-1) 10,10,11
   14 R=R0+H
      U1=U0+V0+HK(M)
      V1=V0+2.D0*HK(M)
      GO TO 10
   15 U1=U0+V0+(HK(1)+HK(2)+HK(3))/3.D0
      V1=V0+(HK(1)+2.D0*HK(2)+2.D0*HK(3)+HK(4))/3.D0
      IF(LL.LE.9) GO TO 29
      GO TO 29
   16 IF(R-R1) 17,10,10
   17 CONTINUE
      U1=U1+U11+U12
      V1=V1+V11+V12
      R=R0+H
   29 CONTINUE
      IF (MM.LE.2) GO TO 30
      MM=1
      N=N-1
      H=HH
   30 N=N+1
      U(N)=U1
      DU(N)=V1/H
      R=0.02D0*DFLOAT(N)
      IF(LL-9) 40,40,38
   38 LL=LL/50
      H=H*50.D0
   40 IF(N-50) 8,400,400
```

```
  400 CONTINUE
      IF(RCX.GT.1.D-4) GO TO 405
      ME=EM
      IF(ME-1) 405,500,405
  405 N=1
  410 N=N+1
      R=0.02D0*DFLOAT(N)
      UD=CDABS(U(N))-CDABS(U(N-1))
      IF(UD-0.D-9) 420,450,450
  420 IF(R-ROX) 460,460,425
  425 AA0=AA0/U(N-1)
      GO TO 460
  450 IF(N-50) 410,455,455
  455 AA0=AA0/U(50)
  460 DO 470 N=1,50
      U(N)=AA0*U(N)
  470 DU(N)=AA0*DU(N)
  500 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SPDQ(NN,LL,A3,A4)
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 U,DU,FREAM,GAM,G2,GE,B11,B1,B2,B3,B4,C1,C2,C3,C4,C5,
     1 A3,A4,C,D1,D2,D3,D4,E2,E3,V1,
     2 AA0,AA1,AA2,AA3,AA4,AA5,
     3 EETA,ALPHA,DALPH,VEETA,S,DSS,Z1,DEL,VV,WW,PP,RRHO
     4 ,EPSIO,DELTA,DEPSI
      DIMENSION U(50),DU(50),FREAM(2)
      COMMON U,DU,FREAM,GAM,EM,XOMEG,EPS,XK,GHAM,AC,A,ROX,R,V,W,FF,VV
     1 IS,DGAM,S,DSS,DRHO,ALPHA,DALPH,VEETA,EETA,AA0,DE
      GO TO (100,200,300), NN
  100 ALF = DABS(EM)-1.D0
      XK2=XK**2
      EM2=EM**2
      EM4=EM2**2
      O2=XOMEG**2
      O3=O2*XOMEG
      O4=O2**2
      GHG=1.D0-GHAM
      R=0.005D0
      CALL VELTEM(DV,DDV,DW,DDW,DTT)
      A2=(1.D0/A)**2
      GAM=-FREAM(2)+XK*W+EM*V/R
      G2=GAM**2
      GE=1.D0/GAM
      EEM=DABS(EM)
      IF(EEM.LE.0.01D0) GO TO 101
      C=(XK2-G2*A2)/EM2
      B=-DTT
      B1=O2*A2*GHAM-2.D0*C
      B2=-(G2*A2*DTT)/EM2
      B3=2.D0*C**2-4.D0*GAM*XK*EPS*XOMEG*A2/EM2
      B4=G2*A2*C*DTT/EM2
      D1=2.D0*EM*XOMEG-GAM
      D2=2.D0*XK*EPS*XOMEG+GAM*O2*A2
      D3=2.D0*EM*XOMEG
      D4=GAM*O2*A2
      E2=-D4
      E3=-E2*DTT
      E4=2.D0*XK*EPS*O3*A2
      V1=G2-4.D0*O2
      V2=O2*DTT
      V3=O4*A2*GHG
      C1=(B*D1+EM2*GE*V2)*GE
      C2=(B1*D1-2.D0*D2+E2+EM2*GE*(V3+V1*C)-GE*(D3*D2+D4*D1))*GE
      C3=(B2*D1-B*D2+E3+EM2*GE*V2*C)*GE
      C4=(B3*D1-B1*D2+E4+EM2*GE*V3*C+D4*D2)*GE
      C5=(B4*D1-B2*D2)*GE
      GO TO 109
  101 B=(XK2/(XK2-G2*A2))*DTT
      B1=XOMEG*A2*(XOMEG*GHAM-4.D0*GAM*XK*EPS/(XK2-G2*A2))
      B2=0
      B3=0
```

```
      B4=0
      C1=B
      C2=B1+(G2-4.D0*O2)*(XK2-G2*A2)
      C3=B*(2.D0*XK*EPS*XOMEG+GAM*O2*A2)+O2*A2*DTT
      C4=B1*XOMEG*(2.D0*XK*EPS+GAM*XOMEG*A2)+
     1 O2*A2*(4.D0*XK*EPS*XOMEG+GAM*O2*A2)
      C5=0
      ALF=1.D0
  109 CONTINUE
      AA0=DCMPLX(1.D0,0.26D0)
      C1=C1-ALF*B
      AA1=C1/(2.D0*ALF+3.D0)
      C1=C1-B
      C2=C2-ALF*B1
      AA2=(C1*AA1+C2)/(4.D0*ALF+8.D0)
      C1=C1-B
      C2=C2-B1
      C3=C3-ALF*B2
      AA3=(C1*AA2+C2*AA1+C3)/(6.D0*ALF+15.D0)
      C1=C1-B
      C2=C2-B1
      C3=C3-B2
      C4=C4-ALF*B3
      AA4=(C1*AA3+C2*AA2+C3*AA1+C4)/(8.D0*ALF+24.D0)
      C1=C1-B
      C2=C2-B1
      C3=C3-B2
      C4=C4-B3
      C5=C5-ALF*B4
      AA5=(C1*AA4+C2*AA3+C3*AA2+C4*AA1+C5)/(10.D0*ALF+35.D0)
      U(1)=AA0*(1.D0+(AA1+(AA2+(AA3+(AA4+AA5*R)*R)*R)*R)*R)*(R**ALF)
      DU(1)=AA0*(ALF+((ALF+1.D0)*AA1+((ALF+2.D0)*AA2+((ALF+3.D0)*AA3+
     1 ((ALF+4.D0)*AA4+(ALF+5.D0)*AA5*R)*R)*R)*R)*R)*(R**(ALF-1.D0))
  110 CONTINUE
      GO TO 400
  200 CALL VELTEM(DV,DDV,DW,DDW,DTT)
      GAM=-FREAM(2)+XK*W+EM*V/R
      DGAM=XK*DW+EM*(R*DV-V)/(R**2)
      S=1.D0/(XK**2+(EM/R)**2-(GAM/A)**2)
      DSS=(2.D0*(EM**2)/(R**3)+2.D0*GAM*DGAM/(A**2)-DTT*(GAM/A)**2)*S
      DRHO=(GHAM/R)*(V/A)**2-DTT
      ALPHA=EM*(R*DV+V)/(R**2)+XK*DW-GAM*((V/A)**2+1.D0)/R
      DALPH=-2.D0*EM*V/(R**3)+EM*DDV/R+XK*DDW+
     1 ((GAM/R-DGAM)*((V/A)**2+1.D0)-GAM*V*(2.D0*DV-V*DTT)/(A**2))/R
      VEETA=((V**2)/(A*R))**2-(V**2)*DRHO/R+GAM**2-2.D0*V*(R*DV+V)
     1 /(R**2)
      EETA=(GAM/R)*(V/A)**2-2.D0*EM*V/(R**2)
      A3=-(DRHO+DSS+1.D0/R)
      A4=(ALPHA*(DRHO+DSS)+DALPH+VEETA/(GAM*S)-(EETA*ALPHA)/GAM)/GAM
      GO TO 400
  300 CONTINUE
      IF(LL.GT.2) GO TO 313
      WRITE(6,8310)
```

```
      WRITE(6,8320)
      ABSUU=0.0D0
      M=3
      MMM=0
      DO 310 N=1,50
      R=0.02D0*DFLOAT(N)
      IF(N.LE.1) GO TO 305
      ABSUU=ABSU
  305 ABSU=CDABS(U(N))
      MM=M
      IF(ABSUU.LE.ABSU) GO TO 306
      M=1
      GO TO 307
  306 M=3
  307 IF(M.GE.MM) GO TO 308
      MMM=MMM+1
  308 CALL VELTEM(DV,DDV,DW,DDW,DTT)
      TTO=(AC/A)**2
  310 WRITE(6,8325) R,U(N),ABSU,DU(N),V,W,TTO
      WRITE(6,8310)
      IF(RCX.GT.0.0001D0) GO TO 311
      WRITE(6,8360) AAO,MMM
      WRITE(6,8370) ROX,RCX
      GO TO 312
  311 CONTINUE
      WRITE(6,8365) ROX,AAO,MMM
  312 WRITE(6,8375)
  313 Z1=DCMPLX(0.D0,1.D0)
      CC=1.0D0
      SUM=0.0D0
      SUMM=SUM
      XSUMM=1.D0
      H=0.010D0
      DO 320 L=1,50
      DO 315 LLL=1,2
      M=2*(L-1)+LLL-1
      R=H/2.D0+H*DFLOAT(M)
      CALL VELTEM(DV,DDV,DW,DDW,DTT)
      SUM=SUM+GHAM*(H/R)*(V/A)**2
      IF(R-ROX) 314,314,315
  314 SUMM=SUM
      XSUMM=DEXP(SUMM)
  315 XSUM=DEXP(SUM)
      RHO=CC*((AC/A)**2)*XSUM/XSUMM
      R=0.02D0*DFLOAT(L)
      IF(ROX.LT.R) GO TO 316
      RHO=0.D0
  316 GAM=-FREAM(2)+XK*W+EM*V/R
      S=1.D0/(XK**2+(EM/R)**2-(GAM/A)**2)
      DRHO=(GHAM/R)*(V/A)**2-DTT
      DEL=EM*(R*DV+V)/(R**2)+XK*DW-GAM*((V/A)**2+1.D0)/R
      PP=Z1*S*(DEL*U(L)-GAM*DU(L))
      WW=(Z1*DW*U(L)-XK*PP)/GAM
```

```
      VV=(Z1*(R*DV+V)*U(L)-EM*PP)/(GAM*R)
      P=(RHC/GHAM)*A**2
      RRHO=-Z1*((V/A)**2-R*DRHO)*U(L)/(R*GAM)+PP/(A**2)
      RRHC=RRHO*RHO
      PP=PP*RHO
      IF(LL.GT.2) GO TO 320
      WRITE(6,8400) R,RHO,RRHO,P,PP,VV,WW
  320 CONTINUE
  400 CONTINUE
      RETURN
 8310 FORMAT(//)
 8320 FORMAT(1H ,3X,'R',10X,'U-REAL',7X,'U-IMAG',9X,'ABS(U)',9X,
     1 'DU-REAL',6X,'DU-IMAG',8X,'V',11X,'W',11X,'T/TO'//)
 8325 FORMAT(1H ,1X,D8.2,1X,'*',2D13.6,1X,'*',D13.6,1X,'*',2D13.6,
     1 1X,'*',D12.5,1X,'*',D10.3,1X,'*',D10.3)
 8360 FORMAT(1H ,'U,VV,WW,PP,RRHO,ARE MULTIPLIED BY THE SAME CONSTANT
     1 =>  U(0.000D+00) = (',2D10.3,')','*** MODE NR =',I3//)
 8365 FORMAT(1H ,'U,VV,WW,PP,RRHO,ARE MULTIPLIED BY THE SAME CONSTANT
     1 => DU(',D9.3,') = (',2D10.3,')','*** MODE NR =',I3//)
 8370 FORMAT(1H ,1X,'RHO(',D9.3,') = RHOO',3X,'T(',D9.3,') = TO'/)
 8375 FORMAT(1H ,'R',9X,'RHO/RHOC',10X,'RRHC/RHCO',7X,'P/RHOO',14X,
     1 'PP/RHOO',15X,'VV',19X,'WW'//)
 8400 FORMAT(1H ,D8.2,1X,D12.6,1X,2D10.3,1X,D12.6,1X,2D10.3,1X,2D10.3
     1 ,1X,2D10.3)
      END
```

```
      SUBRCUTINE VELTEM(DV,DDV,DW,DDW,DTT)
      IMPLICIT REAL*8(A-H,O-Z)
      COMPLEX*16 U,DU,FREAM,GAM
     1 ,VV,S,DSS,ALPHA,DALPH,VEETA,EETA,AAO
      DIMENSION U(50),DU(50),FREAM(2),UEND(2),FREAN(15),
     1 E(3),EB(4),F(3),FB(4),RO(3)
      COMMON U,DU,FREAM,GAM,EM,XCMEG,EPS,XK,GHAM,AC,A,ROX,R,V,W,FF,VV
     1 IS,DGAM,S,DSS,DRHO,ALPHA,DALPH,VEETA,EETA,AAO,DE
      V=0.DC
      DV=0.DO
      DDV=C.DO
      W=0.DC
      DW=0.CO
      DDW=C.DO
      IF (ROX.GT.R) GO TO 50
      PIE=3.14159DO
      RO(1)=DSQRT(0.145DO)
      RO(2)=0.85DO
      RO(3)=DSQRT(0.52DO)
      E(1)=4.D-2
      E(2)=4.D-2
      E(3)=4.D-2
      EB(1)=4.D-2
      EB(2)=4.C-2
      EB(3)=4.D-2
      EB(4)=4.D-2
      F(1)=0.DO
      F(2)=FF
      F(3)=C.DO
      FB(1)=0.DO
      FB(2)=0.DO
      FB(3)=0.CO
      FB(4)=0.CO
      V=XOMEG*R+DE*(DATAN(1.DO/(R*(R-1.01DO)))/PIE+0.5DO)
      ED=(R*(R-1.01DO))**2
      DV=XCMEG-DE*(2.DO*R-1.01DO)/(ED*PIE)
      DDV=-DE*2.DO*(1.DO-(2.DO*R-1.01DO)*R*(R-1.01DO)*1.01DO/ED)/
     1 (ED*PIE)
      ROO=RCX
      DO 10 N=3,4
      IF (FB(N).LE.0.0000C1DO) GC TO 10
      EK=(EB(N))**2
      VE1=((R-ROO)**2)/EK
      IF (VE1.GT.18.DO) GC TO 10
      V1=DEXP(-VE1)
      VE2=(ROO**2)/EK
      IF (VE2.GT.18.DO) GC TO 2
      V2=DEXP(-V2)
      GO TC 3
    2 V2=0.DO
    3 VE3=((1.DO-ROO)**2)/EK
      IF (VE3.GT.18.DO) GC TO 4
      V3=DEXP(-VE3)
```

```
      GO TO 5
    4 V3=0.D0
    5 VP=2.D0*FB(N)*V1/(PIE*(EK*(V2-V3)+2.D0*ROO*DSQRT(EK*PIE)))
      V=V+VP
      DVP=-2.D0*VP*(R-ROO)/EK
      DV=DV+DVP
      DDV=DDV-2.D0*(DVP*(R-ROO)+VP)/EK
      ROO=1.D0
   10 CONTINUE
      DO 20 N=1,3
      IF (F(N).LE.0.000001D0) GO TO 20
      EK=(E(N)/2.D0)**2
      WE1=((R-RO(N))**2)/EK
      IF (WE1.GT.18.D0) GO TO 20
      W1=DEXP(-WE1)
      WE2=(RO(N)**2)/EK
      IF (WE2.GT.18.D0) GO TO 12
      W2=DEXP(-WE2)
      GO TO 13
   12 W2=0.D0
   13 WE3=((1.D0-RO(N))**2)/EK
      IF (WE3.GT.18.D0) GO TO 14
      W3=DEXP(-WE3)
      GO TO 15
   14 W3=0.D0
   15 WP=F(N)*W1/(PIE*(EK*(W2-W3)+2.D0*RO(N)*DSQRT(EK*PIE)))
      W=W+WP
      DWP=-2.D0*WP*(R-RO(N))/EK
      DW=DW+DWP
      DDW=DDW-2.D0*(DWP*(R-RO(N))+WP)/EK
   20 CONTINUE
      COO=1.D0
      ROO=ROX
      DO 45 N=1,2
      IF(ROO.LE.EB(1)) GO TO 40
   32 IF (FB(N).LE.0.000001D0) GO TO 40
      EK=(EB(N)/1.87D0)**2
      WE4=((R-ROO)**2)/EK
      IF (WE4.GT.18.D0) GO TO 40
      W4=DEXP(-WE4)
      WE5=(ROO**2)/EK
      IF (WE5.GT.18.D0) GO TO 34
      W5=DEXP(-WE5)
      GO TO 35
   34 W5=0.D0
   35 WE6=((1.D0-ROO)**2)/EK
      IF (WE6.GT.18.D0) GO TO 36
      W6=DEXP(-WE6)
      GO TO 37
   36 W6=0.D0
   37 WR=FB(N)*W4/(PIE*EK*(COO*DSQRT(PIE*EK)+W5-W6))
      WP=(R-ROO)*WR
      W=W+WP
```

```
      DW=DW+WR*(1.D0-2.D0*((R-ROO)**2)/EK)
      DDW=DDW+WP*2.D0*(1.C0+2.D0*((R-ROO)**2)/EK)/EK
   40 COO=0.5D0
      ROO=1.D0
   45 CONTINUE
   50 CONTINUE
      B=((ROX)**3)*(1.C0-(ROX)**2)/(1.D0-(ROX)**3)
      W=W+XOMEG*EPS*(1.D0+B-B/(R**3)-R**2)
      DW=DW+XOMEG*EPS*(3.C0*B/(R**4)-2.D0*R)
      DDW=DDW-XCMEG*EPS*(12.D0*B/(R**5)+2.D0)
      GO TO 70
   60 W=0.D0
      DW=C.D0
      DDW=0.D0
      V=((R**2-3.D0)*R**2+3.D0)*R
      DV=((5.DC*R**2-9.DC)*R**2+3.D0)
      DDV=(10.D0*R**2-9.D0)*R*2.D0
   70 TO=273.DC
      T=273.D0
      DT=0.D0
      DTT=DT/T
      A=AC*DSQRT(T/TO)
      RETURN
      END
```

DAY =10 MONTH = 1 YEAR =1974

GHAM=CP/CV = 0.10900+01 RADIUS OF INNERCYLINDER = 0.0 ANGULAR VELOCITY = 0.10D+01 ROSSBY NUMBER EPS = 0.0

ANGULAR WAVE-NUMBER M = -0.10D+01 *** AXIAL WAVE-NUMBER K = 0.31416D+00 *** SOUND VELOCITY AC = 0.10000+01

INERTIAL MODE SET= 1 FLUX IN JET ONE = 0.0 HH = 0.1000D-01 DE = 0.0

N	FREQ=W(REAL)	AMP=W(IMAG)	DFREQ	DAMP	STEG	A	UREAL	UIMAG	ABS(U)
1	-0.769400D+00	0.0	-0.769400D+00	0.0	0.200D+01		-0.549301D-05	-0.142818D-05	0.567564D-05

R	U-REAL	U-IMAG	ABS(U)	DU-REAL	DU-IMAG	V	W	T/TC
0.20D-01	0.100000D+01	0.260000D+00	0.103325D+01	-0.101041D+00	-0.262707D-01	0.23000D-01	0.0	0.100D+01
0.40D-01	0.997363D+00	0.259314D+00	0.103052D+01	-0.169525D+00	-0.440765D-01	0.43000D-01	0.0	0.100D+01
0.60D-01	0.993163D+00	0.258222D+00	0.102618D+01	-0.250730D+00	-0.651897D-01	0.60000D-01	0.0	0.100D+01
0.80D-01	0.987332D+00	0.256706D+00	0.102016D+01	-0.332223D+00	-0.863780D-01	0.90000D-01	0.0	0.100D+01
0.10D+00	0.979881D+00	0.254769D+00	0.101246D+01	-0.412734D+00	-0.107311D+00	0.10000D+00	0.0	0.100D+01
0.12D+00	0.970833D+00	0.252417D+00	0.100311D+01	-0.491788D+00	-0.127865D+00	0.12000D+00	0.0	0.100D+01
0.14D+00	0.960221D+00	0.249657D+00	0.992146D+00	-0.569047D+00	-0.147952D+00	0.14000D+00	0.0	0.100D+01
0.16D+00	0.948085D+00	0.246502D+00	0.979606D+00	-0.644215D+00	-0.167496D+00	0.16000D+00	0.0	0.100D+01
0.18D+00	0.934468D+00	0.242962D+00	0.965537D+00	-0.717015D+00	-0.186424D+00	0.18000D+00	0.0	0.100D+01
0.20D+00	0.919422D+00	0.239050D+00	0.949990D+00	-0.787187D+00	-0.204669D+00	0.23000D+00	0.0	0.100D+01
0.22D+00	0.903060D+00	0.234780D+00	0.933022D+00	-0.854485D+00	-0.222166D+00	0.22000D+00	0.0	0.100D+01
0.24D+00	0.885263D+00	0.230168D+00	0.914696D+00	-0.918679D+00	-0.238857D+00	0.24000D+00	0.0	0.100D+01
0.26D+00	0.866275D+00	0.225231D+00	0.895076D+00	-0.979555D+00	-0.254684D+00	0.26000D+00	0.0	0.100D+01
0.28D+00	0.846104D+00	0.219987D+00	0.874235D+00	-0.103691D+01	-0.269598D+00	0.28000D+00	0.0	0.100D+01
0.30D+00	0.824823D+00	0.214454D+00	0.852246D+00	-0.109058D+01	-0.283550D+00	0.39000D+00	0.0	0.100D+01
0.32D+00	0.802507D+00	0.208652D+00	0.829188D+00	-0.114039D+01	-0.296500D+00	0.32000D+00	0.0	0.100D+01
0.34D+00	0.779234D+00	0.202601D+00	0.805142D+00	-0.118619D+01	-0.308411D+00	0.34000D+00	0.0	0.100D+01
0.36D+00	0.755087D+00	0.196322D+00	0.780191D+00	-0.122788D+01	-0.319249D+00	0.36000D+00	0.0	0.100D+01
0.38D+00	0.730147D+00	0.189838D+00	0.754423D+00	-0.126534D+01	-0.328989D+00	0.38000D+00	0.0	0.100D+01
0.40D+00	0.704501D+00	0.183170D+00	0.727924D+00	-0.129850D+01	-0.337610D+00	0.40000D+00	0.0	0.100D+01
0.42D+00	0.679236D+00	0.176341D+00	0.700786D+00	-0.132729D+01	-0.345094D+00	0.42000D+00	0.0	0.100D+01
0.44D+00	0.651439D+00	0.169374D+00	0.673098D+00	-0.135166D+01	-0.351432D+00	0.44000D+00	0.0	0.100D+01
0.46D+00	0.624159D+00	0.162292D+00	0.644952D+00	-0.137161D+01	-0.356619D+00	0.46000D+00	0.0	0.100D+01
0.48D+00	0.596605D+00	0.155117D+00	0.616440D+00	-0.138713D+01	-0.360653D+00	0.48000D+00	0.0	0.100D+01
0.50D+00	0.566744D+00	0.147873D+00	0.587653D+00	-0.139823D+01	-0.363540D+00	0.53000D+00	0.0	0.100D+01
0.52D+00	0.540704D+00	0.140583D+00	0.558681D+00	-0.140457D+01	-0.365292D+00	0.52000D+00	0.0	0.100D+01
0.54D+00	0.512574D+00	0.133269D+00	0.529615D+00	-0.140739D+01	-0.365922D+00	0.54000D+00	0.0	0.100D+01
0.56D+00	0.484437D+00	0.125954D+00	0.500543D+00	-0.140558D+01	-0.365451D+00	0.56000D+00	0.0	0.100D+01
0.58D+00	0.456378D+00	0.118658D+00	0.471552D+00	-0.139963D+01	-0.363904D+00	0.58000D+00	0.0	0.100D+01
0.60D+00	0.428479D+00	0.111404D+00	0.442725D+00	-0.138965D+01	-0.361310D+00	0.60000D+00	0.0	0.100D+01
0.62D+00	0.400818D+00	0.104213D+00	0.414144D+00	-0.137578D+01	-0.357703D+00	0.62000D+00	0.0	0.100D+01
0.64D+00	0.373473D+00	0.971029D-01	0.385890D+00	-0.135815D+01	-0.353120D+00	0.64000D+00	0.0	0.100D+01
0.66D+00	0.346516D+00	0.900941D-01	0.358037D+00	-0.133693D+01	-0.347602D+00	0.66000D+00	0.0	0.100D+01
0.68D+00	0.320018D+00	0.832047D-01	0.330658D+00	-0.131229D+01	-0.341195D+00	0.68000D+00	0.0	0.100D+01
0.70D+00	0.294046D+00	0.764520D-01	0.303822D+00	-0.128441D+01	-0.333945D+00	0.70000D+00	0.0	0.100D+01
0.72D+00	0.268662D+00	0.698522D-01	0.277595D+00	-0.125348D+01	-0.325904D+00	0.72000D+00	0.0	0.100D+01
0.74D+00	0.243926D+00	0.634207D-01	0.252036D+00	-0.121971D+01	-0.317125D+00	0.74000D+00	0.0	0.100D+01
0.76D+00	0.219891D+00	0.571717D-01	0.227202D+00	-0.118332D+01	-0.307662D+00	0.76000D+00	0.0	0.100D+01
0.78D+00	0.196609D+00	0.511184D-01	0.203146D+00	-0.114451D+01	-0.297573D+00	0.78000D+00	0.0	0.100D+01
0.80D+00	0.174125D+00	0.452726D-01	0.179915D+00	-0.110353D+01	-0.286917D+00	0.80000D+00	0.0	0.100D+01
0.82D+00	0.152481D+00	0.396451D-01	0.157551D+00	-0.106058D+01	-0.275752D+00	0.82000D+00	0.0	0.100D+01
0.84D+00	0.131713D+00	0.342455D-01	0.136093D+00	-0.101592D+01	-0.264140D+00	0.84000D+00	0.0	0.100D+01
0.86D+00	0.111854D+00	0.290821D-01	0.115573D+00	-0.969772D+00	-0.252141D+00	0.86000D+00	0.0	0.100D+01
0.88D+00	0.929309D-01	0.241620D-01	0.960206D-01	-0.922370D+00	-0.239816D+00	0.88000D+00	0.0	0.100D+01
0.90D+00	0.749662D-01	0.194912D-01	0.774586D-01	-0.873949D+00	-0.227227D+00	0.90000D+00	0.0	0.100D+01
0.92D+00	0.579782D-01	0.150743D-01	0.599058D-01	-0.824740D+00	-0.214433D+00	0.92000D+00	0.0	0.100D+01
0.94D+00	0.419803D-01	0.109149D-01	0.433760D-01	-0.774974D+00	-0.201493D+00	0.94000D+00	0.0	0.100D+01
0.96D+00	0.269814D-01	0.701517D-02	0.278785D-01	-0.724873D+00	-0.188467D+00	0.96000D+00	0.0	0.100D+01
0.98D+00	0.129861D-01	0.337639D-02	0.134179D-01	-0.674657D+00	-0.175411D+00	0.98000D+00	0.0	0.100D+01
0.10D+01	-0.549301D-05	-0.142818D-05	0.567564D-05	-0.624536D+00	-0.162380D+00	0.10000D+01	0.0	0.100D+01

U,VV,WW,PP,RRHO,ARE MULTIPLIED BY THE SAME CONSTANT => U(0.000D+00) = (0.100D+01 0.650D-16)*** MODE NR = 1

RHO(0.0) = RHOO T(0.0) = TO

R	RHO/RHOO	RRHO/RHOO	F/RHOO	PP/RHOO	VV	WW
0.20D-01	0.100022D+01	-0.437D-02 0.168D-01	0.917631D+00	0.920D-02-0.354D-01	0.259D+00-0.995D+00	-0.149D+00 0.573D+00
0.40D-01	0.100087D+01	0.251D-01-0.967D-01	0.918232D+00	0.184D-01-0.707D-01	0.258D+00-0.994D+00	0.546D-01-0.210D+00
0.60D-01	0.100196D+01	0.355D-01-0.136D+00	0.919233D+00	0.275D-01-0.106D+00	0.256D+00-0.984D+00	0.585D-01-0.225D+00
0.80D-01	0.100349D+01	0.462D-01-0.178D+00	0.920637D+00	0.365D-01-0.140D+00	0.252D+00-0.971D+00	0.680D-01-0.262D+00
0.10D+00	0.100546D+01	0.570D-01-0.219D+00	0.922445D+00	0.454D-01-0.175D+00	0.248D+00-0.954D+00	0.786D-01-0.302D+00
0.12D+00	0.100788D+01	0.676D-01-0.260D+00	0.924660D+00	0.542D-01-0.209D+00	0.243D+00-0.933D+00	0.895D-01-0.344D+00
0.14D+00	0.101074D+01	0.780D-01-0.300D+00	0.927284D+00	0.629D-01-0.242D+00	0.236D+00-0.908D+00	0.100D+00-0.386D+00
0.16D+00	0.101405D+01	0.883D-01-0.340D+00	0.930321D+00	0.713D-01-0.274D+00	0.229D+00-0.881D+00	0.111D+00-0.426D+00
0.18D+00	0.101781D+01	0.983D-01-0.378D+00	0.933775D+00	0.796D-01-0.306D+00	0.221D+00-0.849D+00	0.121D+00-0.466D+00
0.20D+00	0.102204D+01	0.108D+00-0.415D+00	0.937651D+00	0.876D-01-0.337D+00	0.212D+00-0.815D+00	0.131D+00-0.504D+00
0.22D+00	0.102673D+01	0.117D+00-0.451D+00	0.941953D+00	0.954D-01-0.367D+00	0.202D+00-0.778D+00	0.140D+00-0.540D+00
0.24D+00	0.103189D+01	0.126D+00-0.486D+00	0.946688D+00	0.103D+00-0.396D+00	0.192D+00-0.738D+00	0.149D+00-0.574D+00
0.26D+00	0.103753D+01	0.135D+00-0.519D+00	0.951862D+00	0.110D+00-0.423D+00	0.181D+00-0.695D+00	0.158D+00-0.606D+00
0.28D+00	0.104365D+01	0.143D+00-0.551D+00	0.957481D+00	0.117D+00-0.450D+00	0.169D+00-0.650D+00	0.165D+00-0.636D+00
0.30D+00	0.105027D+01	0.151D+00-0.581D+00	0.963553D+00	0.123D+00-0.475D+00	0.157D+00-0.603D+00	0.173D+00-0.664D+00
0.32D+00	0.105739D+01	0.158D+00-0.609D+00	0.970087D+00	0.130D+00-0.499D+00	0.144D+00-0.554D+00	0.179D+00-0.689D+00
0.34D+00	0.106503D+01	0.165D+00-0.635D+00	0.977091D+00	0.135D+00-0.521D+00	0.131D+00-0.503D+00	0.185D+00-0.712D+00
0.36D+00	0.107319D+01	0.171D+00-0.659D+00	0.984575D+00	0.141D+00-0.542D+00	0.117D+00-0.452D+00	0.190D+00-0.732D+00
0.38D+00	0.108188D+01	0.177D+00-0.682D+00	0.992548D+00	0.146D+00-0.561D+00	0.104D+00-0.399D+00	0.195D+00-0.749D+00
0.40D+00	0.109111D+01	0.183D+00-0.702D+00	0.100102D+01	0.150D+00-0.578D+00	0.896D-01-0.345D+00	0.198D+00-0.763D+00
0.42D+00	0.110091D+01	0.187D+00-0.720D+00	0.101001D+01	0.154D+00-0.594D+00	0.755D-01-0.290D+00	0.202D+00-0.775D+00
0.44D+00	0.111128D+01	0.191D+00-0.736D+00	0.101952D+01	0.158D+00-0.608D+00	0.613D-01-0.236D+00	0.204D+00-0.784D+00
0.46D+00	0.112223D+01	0.195D+00-0.750D+00	0.102957D+01	0.161D+00-0.620D+00	0.470D-01-0.181D+00	0.206D+00-0.790D+00
0.48D+00	0.113379D+01	0.198D+00-0.761D+00	0.104018D+01	0.164D+00-0.631D+00	0.328D-01-0.126D+00	0.206D+00-0.794D+00
0.50D+00	0.114597D+01	0.200D+00-0.770D+00	0.105135D+01	0.166D+00-0.639D+00	0.188D-01-0.723D-01	0.207D+00-0.795D+00
0.52D+00	0.115878D+01	0.202D+00-0.776D+00	0.106310D+01	0.168D+00-0.646D+00	0.492D-02-0.189D-01	0.206D+00-0.792D+00
0.54D+00	0.117225D+01	0.203D+00-0.780D+00	0.107546D+01	0.169D+00-0.651D+00	-0.868D-02 0.334D-01	0.205D+00-0.788D+00
0.56D+00	0.118639D+01	0.203D+00-0.782D+00	0.108843D+01	0.170D+00-0.653D+00	-0.220D-01 0.845D-01	0.203D+00-0.780D+00
0.58D+00	0.120122D+01	0.203D+00-0.781D+00	0.110204D+01	0.170D+00-0.654D+00	-0.349D-01 0.134D+00	0.200D+00-0.770D+00
0.60D+00	0.121677D+01	0.202D+00-0.777D+00	0.111630D+01	0.170D+00-0.653D+00	-0.474D-01 0.182D+00	0.197D+00-0.758D+00
0.62D+00	0.123306D+01	0.201D+00-0.771D+00	0.113125D+01	0.169D+00-0.649D+00	-0.594D-01 0.228D+00	0.193D+00-0.743D+00
0.64D+00	0.125011D+01	0.198D+00-0.763D+00	0.114689D+01	0.167D+00-0.644D+00	-0.709D-01 0.273D+00	0.189D+00-0.726D+00
0.66D+00	0.126795D+01	0.195D+00-0.752D+00	0.116326D+01	0.165D+00-0.636D+00	-0.818D-01 0.314D+00	0.184D+00-0.707D+00
0.68D+00	0.128661D+01	0.192D+00-0.738D+00	0.118037D+01	0.163D+00-0.627D+00	-0.920D-01 0.354D+00	0.178D+00-0.686D+00
0.70D+00	0.130611D+01	0.188D+00-0.722D+00	0.119826D+01	0.160D+00-0.616D+00	-0.102D+00 0.391D+00	0.172D+00-0.663D+00
0.72D+00	0.132648D+01	0.183D+00-0.704D+00	0.121695D+01	0.157D+00-0.602D+00	-0.111D+00 0.425D+00	0.166D+00-0.639D+00
0.74D+00	0.134776D+01	0.178D+00-0.683D+00	0.123647D+01	0.153D+00-0.587D+00	-0.119D+00 0.457D+00	0.159D+00-0.611D+00
0.76D+00	0.136997D+01	0.172D+00-0.660D+00	0.125686D+01	0.148D+00-0.570D+00	-0.126D+00 0.486D+00	0.152D+00-0.583D+00
0.78D+00	0.139316D+01	0.165D+00-0.635D+00	0.127813D+01	0.143D+00-0.551D+00	-0.133D+00 0.511D+00	0.144D+00-0.554D+00
0.80D+00	0.141737D+01	0.158D+00-0.608D+00	0.130034D+01	0.138D+00-0.530D+00	-0.139D+00 0.534D+00	0.136D+00-0.523D+00
0.82D+00	0.144262D+01	0.150D+00-0.578D+00	0.132350D+01	0.132D+00-0.507D+00	-0.144D+00 0.554D+00	0.128D+00-0.492D+00
0.84D+00	0.146896D+01	0.142D+00-0.547D+00	0.134767D+01	0.125D+00-0.482D+00	-0.148D+00 0.571D+00	0.119D+00-0.459D+00
0.86D+00	0.149643D+01	0.133D+00-0.513D+00	0.137287D+01	0.119D+00-0.455D+00	-0.152D+00 0.584D+00	0.111D+00-0.426D+00
0.88D+00	0.152503D+01	0.124D+00-0.478D+00	0.139916D+01	0.111D+00-0.428D+00	-0.155D+00 0.595D+00	0.102D+00-0.392D+00
0.90D+00	0.155496D+01	0.115D+00-0.440D+00	0.142657D+01	0.104D+00-0.399D+00	-0.157D+00 0.602D+00	0.931D-01-0.358D+00
0.92D+00	0.158612D+01	0.104D+00-0.402D+00	0.145515D+01	0.957D-01-0.368D+00	-0.158D+00 0.607D+00	0.842D-01-0.324D+00
0.94D+00	0.161863D+01	0.939D-01-0.361D+00	0.148495D+01	0.873D-01-0.336D+00	-0.158D+00 0.608D+00	0.752D-01-0.289D+00
0.96D+00	0.165247D+01	0.830D-01-0.319D+00	0.151603D+01	0.786D-01-0.302D+00	-0.158D+00 0.607D+00	0.663D-01-0.255D+00
0.98D+00	0.168779D+01	0.718D-01-0.276D+00	0.154843D+01	0.696D-01-0.268D+00	-0.157D+00 0.603D+00	0.575D-01-0.221D+00
0.10D+01	0.172461D+01	0.603D-01-0.232D+00	0.158221D+01	0.603D-01-0.232D+00	-0.155D+00 0.596D+00	0.487D-01-0.187D+00

DAY =10 MONTH = 1 YEAR =1974

GHAM=CP/CV = 0.10500D+01 RADIUS OF INNERCYLINDER = 0.30000D+00 ANGULAR VELOCITY = 0.100D+01 ROSSBY NUMBER EPS = 0.0

ANGULAR WAVE-NUMBER M = -0.100D+01 *** AXIAL WAVE-NUMBER K = 0.31416D+00 *** SOUND VELOCITY AC = 0.10000D+01

INERTIAL MODE SET= 1 FLUX IN JET ONE = 0.0 HH = 0.10000D-01 DE = 0.0

N	FREQ=W(REAL)	AMP=W(IMAG)	CFREQ	CAMP	STEG	B	UREAL	UIMAG	ABS(U)
1	-0.765400D+00	0.0	-0.765400D+00	0.0	0.2000+01		0.264523D+00	0.687760D-01	0.273318D+00
2	-0.765395D+00	0.100000D-05	0.100000D-05	0.100000D-05	0.2000+01		0.264527D+00	0.687825D-01	0.273323D+00
3	-0.820790D+00	-0.162457D-06	-0.513510D-01	-0.116246D-05	0.2000+01		-0.414364D-01	-0.107730D-01	0.428081D-01
4	-0.813831D+00	0.304807D-07	0.655859D-02	0.152337D-06	0.2000+01		0.352309D-02	0.916214D-03	0.364027D-02
5	-0.814376D+00	0.208496D-09	-0.545392D-03	-0.306722D-07	0.2000+01		0.176877D-04	0.460024D-05	0.182762D-04

R	U-REAL	U-IMAG	ABS(U)	DU-REAL	DU-IMAG	V	W	T/TO
0.200-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.400-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.600-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.800-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.100+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.120+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.140+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.160+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.180+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.200+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.220+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.240+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.260+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.280+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.100D+01
0.300+00	0.0	0.0	0.0	0.125326D+02	0.325647D+01	0.300000D+00	0.0	0.100D+01
0.320+00	0.226628D+00	0.585233D-01	0.234163D+00	0.102121D+02	0.265513D+01	0.320000D+00	0.0	0.100D+01
0.340+00	0.411306D+00	0.106935D+00	0.424980D+00	0.831666D+01	0.216233D+01	0.340000D+00	0.0	0.100D+01
0.360+00	0.561422D+00	0.145970D+00	0.580088D+00	0.674149D+01	0.175279D+01	0.360000D+00	0.0	0.100D+01
0.380+00	0.682605D+00	0.177477D+00	0.705299D+00	0.541294D+01	0.140736D+01	0.380000D+00	0.0	0.100D+01
0.400+00	0.779230D+00	0.202600D+00	0.805137D+00	0.427837D+01	0.111238D+01	0.400000D+00	0.0	0.100D+01
0.420+00	0.854775D+00	0.222242D+00	0.883154D+00	0.329547D+01	0.857663D+00	0.420000D+00	0.0	0.100D+01
0.440+00	0.912057D+00	0.237135D+00	0.942380D+00	0.244798D+01	0.636470D+00	0.440000D+00	0.0	0.100D+01
0.460+00	0.953400D+00	0.247884D+00	0.985058D+00	0.170261D+01	0.442680D+00	0.460000D+00	0.0	0.100D+01
0.480+00	0.980761D+00	0.254998D+00	0.101337D+01	0.104732D+01	0.272303D+00	0.480000D+00	0.0	0.100D+01
0.500+00	0.995810D+00	0.258911D+00	0.102892D+01	0.465668D+00	0.122114D+00	0.500000D+00	0.0	0.100D+01
0.520+00	0.100000D+01	0.260000D+00	0.103325D+01	-0.400118D-01	-0.104031D-01	0.520000D+00	0.0	0.100D+01
0.540+00	0.994611D+00	0.258599D+00	0.102768D+01	-0.485372D+00	-0.127237D+00	0.540000D+00	0.0	0.100D+01
0.560+00	0.980786D+00	0.255004D+00	0.101339D+01	-0.884553D+00	-0.229984D+00	0.560000D+00	0.0	0.100D+01
0.580+00	0.959557D+00	0.249485D+00	0.991455D+00	-0.123056D+01	-0.319945D+00	0.580000D+00	0.0	0.100D+01
0.600+00	0.931864D+00	0.242285D+00	0.962846D+00	-0.153154D+01	-0.398200D+00	0.600000D+00	0.0	0.100D+01
0.620+00	0.898572D+00	0.233625D+00	0.928447D+00	-0.179100D+01	-0.465661D+00	0.620000D+00	0.0	0.100D+01
0.640+00	0.860480D+00	0.223725D+00	0.889089D+00	-0.201197D+01	-0.523112D+00	0.640000D+00	0.0	0.100D+01
0.660+00	0.818332D+00	0.212766D+00	0.845539D+00	-0.219709D+01	-0.571244D+00	0.660000D+00	0.0	0.100D+01
0.680+00	0.772019D+00	0.200933D+00	0.798514D+00	-0.234876D+01	-0.610678D+00	0.680000D+00	0.0	0.100D+01
0.700+00	0.724590D+00	0.188393D+00	0.748681D+00	-0.246915D+01	-0.641979D+00	0.700000D+00	0.0	0.100D+01
0.720+00	0.674248D+00	0.175305D+00	0.696665D+00	-0.256030D+01	-0.665677D+00	0.720000D+00	0.0	0.100D+01
0.740+00	0.622360D+00	0.161814D+00	0.643052D+00	-0.262412D+01	-0.682271D+00	0.740000D+00	0.0	0.100D+01
0.760+00	0.569454D+00	0.148058D+00	0.588386D+00	-0.266245D+01	-0.692238D+00	0.760000D+00	0.0	0.100D+01
0.780+00	0.516020D+00	0.134165D+00	0.533176D+00	-0.267709D+01	-0.696043D+00	0.780000D+00	0.0	0.100D+01
0.800+00	0.462516D+00	0.120254D+00	0.477894D+00	-0.266576D+01	-0.694138D+00	0.800000D+00	0.0	0.100D+01
0.820+00	0.409365D+00	0.106435D+00	0.422975D+00	-0.264219D+01	-0.686965D+00	0.820000D+00	0.0	0.100D+01
0.840+00	0.356953D+00	0.928077D-01	0.368820D+00	-0.259605D+01	-0.674972D+00	0.840000D+00	0.0	0.100D+01
0.860+00	0.305635D+00	0.794652D-01	0.315797D+00	-0.253300D+01	-0.658581D+00	0.860000D+00	0.0	0.100D+01
0.880+00	0.255734D+00	0.664909D-01	0.264237D+00	-0.245470D+01	-0.638222D+00	0.880000D+00	0.0	0.100D+01
0.900+00	0.207538D+00	0.539599D-01	0.214438D+00	-0.236277D+01	-0.614319D+00	0.900000D+00	0.0	0.100D+01
0.920+00	0.161304D+00	0.419390D-01	0.166667D+00	-0.225880D+01	-0.587287D+00	0.920000D+00	0.0	0.100D+01
0.940+00	0.117256D+00	0.304866D-01	0.121154D+00	-0.214437D+01	-0.557536D+00	0.940000D+00	0.0	0.100D+01
0.960+00	0.755886D-01	0.196530D-01	0.781017D-01	-0.202102D+01	-0.525465D+00	0.960000D+00	0.0	0.100D+01
0.980+00	0.364646D-01	0.948080D-02	0.376770D-01	-0.189026D+01	-0.491468D+00	0.980000D+00	0.0	0.100D+01
0.100+01	0.176877D-04	0.460024D-05	0.182762D-04	-0.175356D+01	-0.455925D+00	0.100000D+01	0.0	0.100D+01

U,VV,WW,PP,RRHO,ARE MULTIPLIED BY THE SAME CONSTANT =) CU(0.300D+00) = (0.184D+02-0.329D+01)*** MODE NR = 1

R	RHO/RHO0	RRHO/RHOO		P/RHOO	PP/RHOO		VV		WW	
0.200D-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.400D-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.600D-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.800D-01	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.100D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.120D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.140D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.160D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.180D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.200D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.220D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.240D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.260D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.280D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.300D+00	0.0	0.0	0.0	0.0	0.0	0.0	0.103D+01	-0.396D+01	-0.971D-01	0.373D+00
0.320D+00	0.100670D+01	-0.255D-02	0.554D-02	0.922652D+00	-0.123D-01	0.474D-01	0.513D+00	-0.351D+01	-0.226D-01	0.870D-01
0.340D+00	0.101465D+01	0.415D-01	-0.160D+00	0.920321D+00	0.227D-01	-0.872D-01	0.657D+00	-0.330D+01	0.411D-01	-0.158D+00
0.360D+00	0.102182D+01	0.833D-01	-0.320D+00	0.927446D+00	0.559D-01	-0.215D+00	0.803D+00	-0.309D+01	0.100D+00	-0.385D+00
0.380D+00	0.103009D+01	0.123D+00	-0.472D+00	0.945038D+00	0.874D-01	-0.336D+00	0.750D+00	-0.289D+01	0.155D+00	-0.594D+00
0.400D+00	0.103885D+01	0.160D+00	-0.615D+00	0.953107D+00	0.117D+00	-0.451D+00	0.697D+00	-0.268D+01	0.205D+00	-0.787D+00
0.420D+00	0.104821D+01	0.195D+00	-0.745D+00	0.961664D+00	0.145D+00	-0.559D+00	0.644D+00	-0.248D+01	0.251D+00	-0.964D+00
0.440D+00	0.105805D+01	0.227D+00	-0.875D+00	0.970721D+00	0.172D+00	-0.660D+00	0.590D+00	-0.227D+01	0.293D+00	-0.113D+01
0.460D+00	0.106852D+01	0.258D+00	-0.991D+00	0.980291D+00	0.196D+00	-0.755D+00	0.536D+00	-0.206D+01	0.330D+00	-0.127D+01
0.480D+00	0.107952D+01	0.286D+00	-0.110D+01	0.990387D+00	0.219D+00	-0.843D+00	0.481D+00	-0.185D+01	0.364D+00	-0.140D+01
0.500D+00	0.109111D+01	0.311D+00	-0.120D+01	0.100102D+01	0.240D+00	-0.924D+00	0.426D+00	-0.164D+01	0.394D+00	-0.152D+01
0.520D+00	0.110331D+01	0.334D+00	-0.129D+01	0.101221D+01	0.259D+00	-0.998D+00	0.371D+00	-0.143D+01	0.420D+00	-0.161D+01
0.540D+00	0.111614D+01	0.355D+00	-0.136D+01	0.102398D+01	0.277D+00	-0.106D+01	0.315D+00	-0.121D+01	0.442D+00	-0.170D+01
0.560D+00	0.112960D+01	0.373D+00	-0.143D+01	0.103633D+01	0.292D+00	-0.112D+01	0.260D+00	-0.100D+01	0.460D+00	-0.177D+01
0.580D+00	0.114372D+01	0.389D+00	-0.149D+01	0.104929D+01	0.305D+00	-0.117D+01	0.205D+00	-0.790D+00	0.474D+00	-0.182D+01
0.600D+00	0.115853D+01	0.401D+00	-0.154D+01	0.106287D+01	0.317D+00	-0.122D+01	0.151D+00	-0.582D+00	0.485D+00	-0.186D+01
0.620D+00	0.117404D+01	0.411D+00	-0.158D+01	0.107710D+01	0.326D+00	-0.125D+01	0.984D-01	-0.379D+00	0.491D+00	-0.189D+01
0.640D+00	0.119027D+01	0.418D+00	-0.161D+01	0.109199D+01	0.333D+00	-0.128D+01	0.467D-01	-0.180D+00	0.495D+00	-0.190D+01
0.660D+00	0.120726D+01	0.42[illegible]D+00	-0.164D+01	0.110756D+01	0.338D+00	-0.130D+01	-0.339D-02	0.130D-01	0.494D+00	-0.190D+01
0.680D+00	0.122502D+01	0.424D+00	-0.163D+01	0.112387D+01	0.341D+00	-0.131D+01	-0.517D-01	0.199D+00	0.491D+00	-0.189D+01
0.700D+00	0.124359D+01	0.424D+00	-0.163D+01	0.114091D+01	0.342D+00	-0.132D+01	-0.979D-01	0.377D+00	0.484D+00	-0.186D+01
0.720D+00	0.126295D+01	0.420D+00	-0.162D+01	0.115870D+01	0.341D+00	-0.131D+01	-0.142D+00	0.545D+00	0.474D+00	-0.182D+01
0.740D+00	0.128324D+01	0.414D+00	-0.159D+01	0.117729D+01	0.337D+00	-0.130D+01	-0.183D+00	0.704D+00	0.462D+00	-0.178D+01
0.760D+00	0.130440D+01	0.406D+00	-0.156D+01	0.119670D+01	0.332D+00	-0.128D+01	-0.222D+00	0.853D+00	0.447D+00	-0.172D+01
0.780D+00	0.132648D+01	0.393D+00	-0.151D+01	0.121695D+01	0.324D+00	-0.125D+01	-0.257D+00	0.990D+00	0.429D+00	-0.165D+01
0.800D+00	0.134952D+01	0.379D+00	-0.146D+01	0.123805D+01	0.315D+00	-0.121D+01	-0.290D+00	0.111D+01	0.409D+00	-0.157D+01
0.820D+00	0.137356D+01	0.363D+00	-0.140D+01	0.126015D+01	0.304D+00	-0.117D+01	-0.319D+00	0.123D+01	0.387D+00	-0.149D+01
0.840D+00	0.139864D+01	0.344D+00	-0.132D+01	0.128316D+01	0.290D+00	-0.112D+01	-0.345D+00	0.133D+01	0.363D+00	-0.140D+01
0.860D+00	0.142480D+01	0.323D+00	-0.124D+01	0.130716D+01	0.275D+00	-0.106D+01	-0.368D+00	0.141D+01	0.337D+00	-0.130D+01
0.880D+00	0.145209D+01	[illegible]	-0.116D+01	0.133218D+01	0.258D+00	-0.994D+00	-0.387D+00	0.149D+01	0.311D+00	-0.119D+01
0.900D+00	0.148054D+01	0.275D+00	-0.106D+01	0.135828D+01	0.240D+00	-0.923D+00	-0.402D+00	0.155D+01	0.283D+00	-0.109D+01
0.920D+00	0.151019D+01	0.249D+00	-0.956D+00	0.138550D+01	0.220D+00	-0.846D+00	-0.414D+00	0.159D+01	0.254D+00	-0.976D+00
0.940D+00	0.154112D+01	0.220D+00	-0.847D+00	0.141388D+01	0.198D+00	-0.763D+00	-0.423D+00	0.163D+01	0.224D+00	-0.863D+00
0.960D+00	0.157337D+01	0.190D+00	-0.732D+00	0.144346D+01	0.176D+00	-0.676D+00	-0.426D+00	0.164D+01	0.194D+00	-0.748D+00
0.980D+00	0.160[illegible]D+01	0.159D+00	-0.612D+00	0.147421D+01	0.152D+00	-0.584D+00	-0.429D+00	0.165D+01	0.164D+00	-0.632D+00
0.100D+01	0.164[illegible]D+01	0.127D+00	-0.488D+00	0.150647D+01	0.127D+00	-0.494D+00	-0.428D+00	0.164D+01	0.134D+00	-0.517D+00

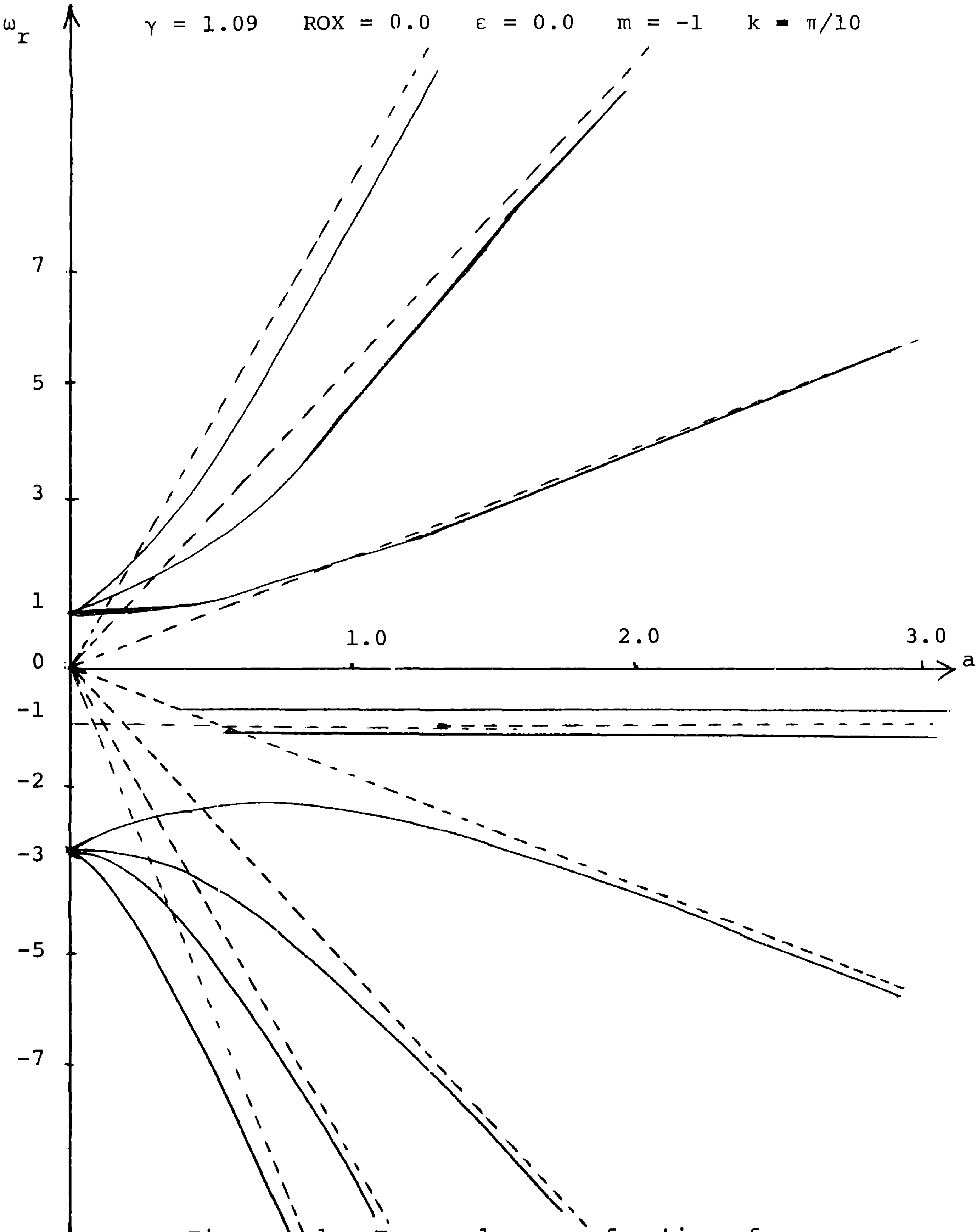


Figure 1. Free modes as a function of a.

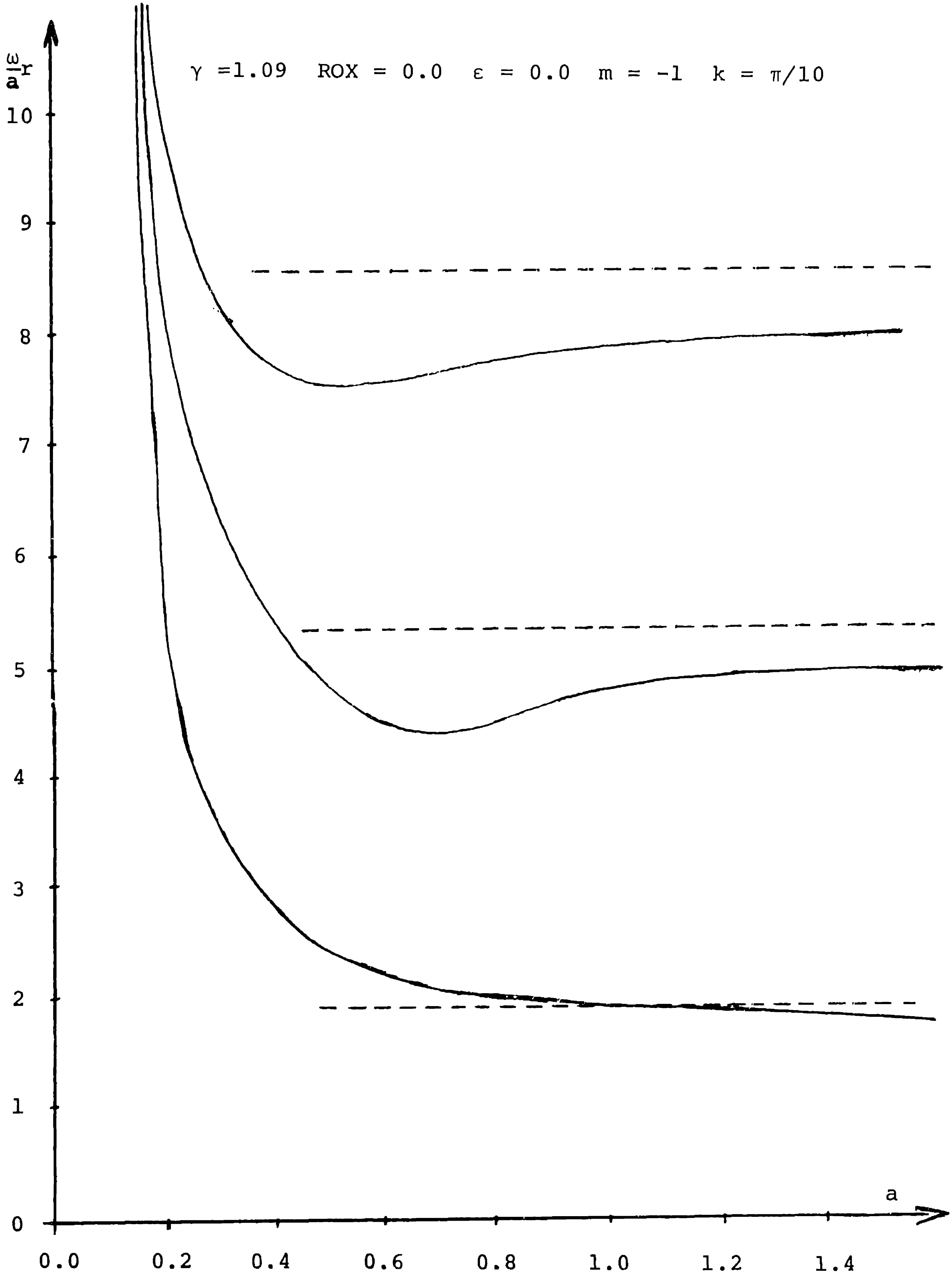


Figure 2. Acoustic modes as a function of a.

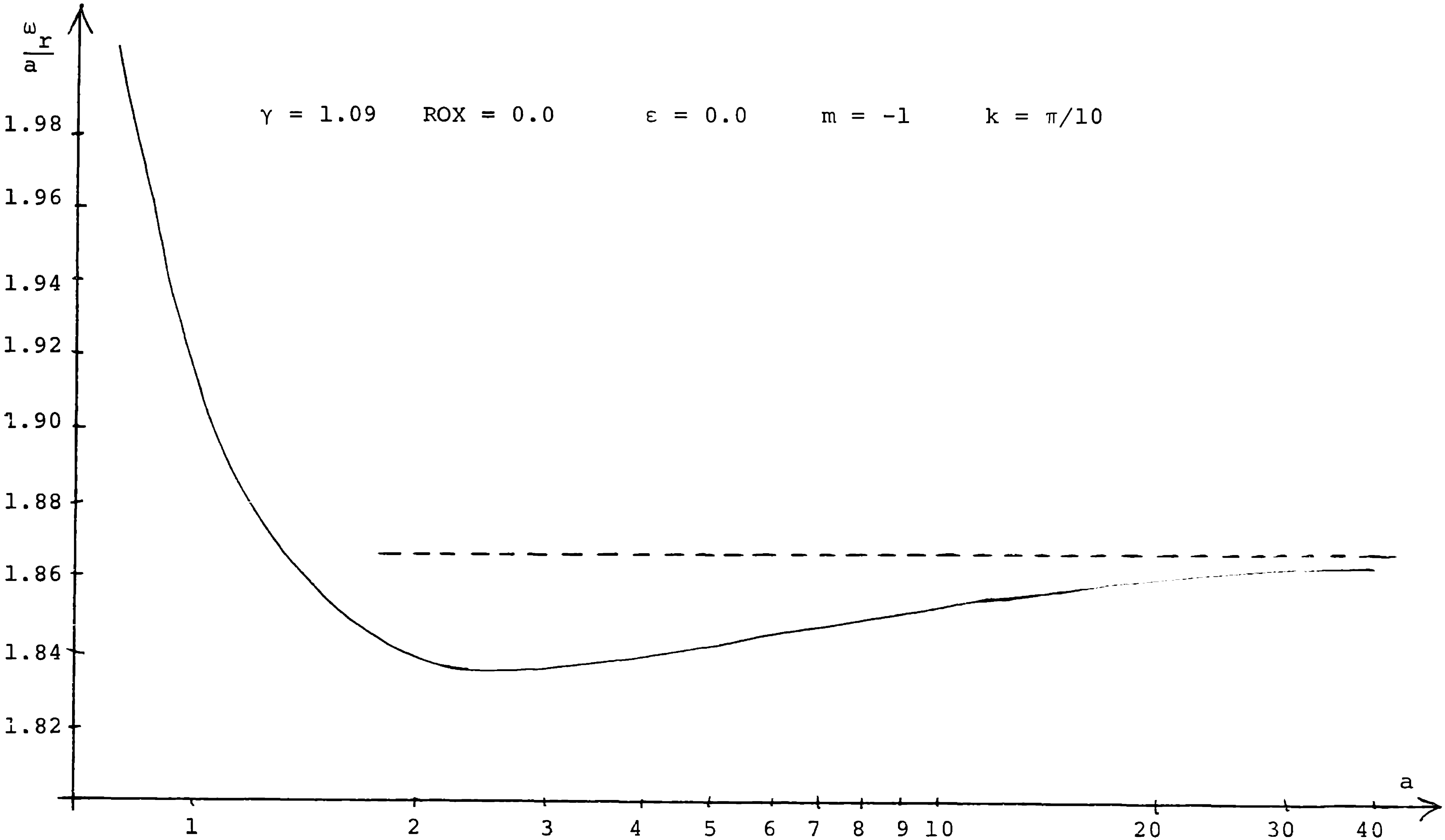


Figure 3. First acoustic mode as a function of a.

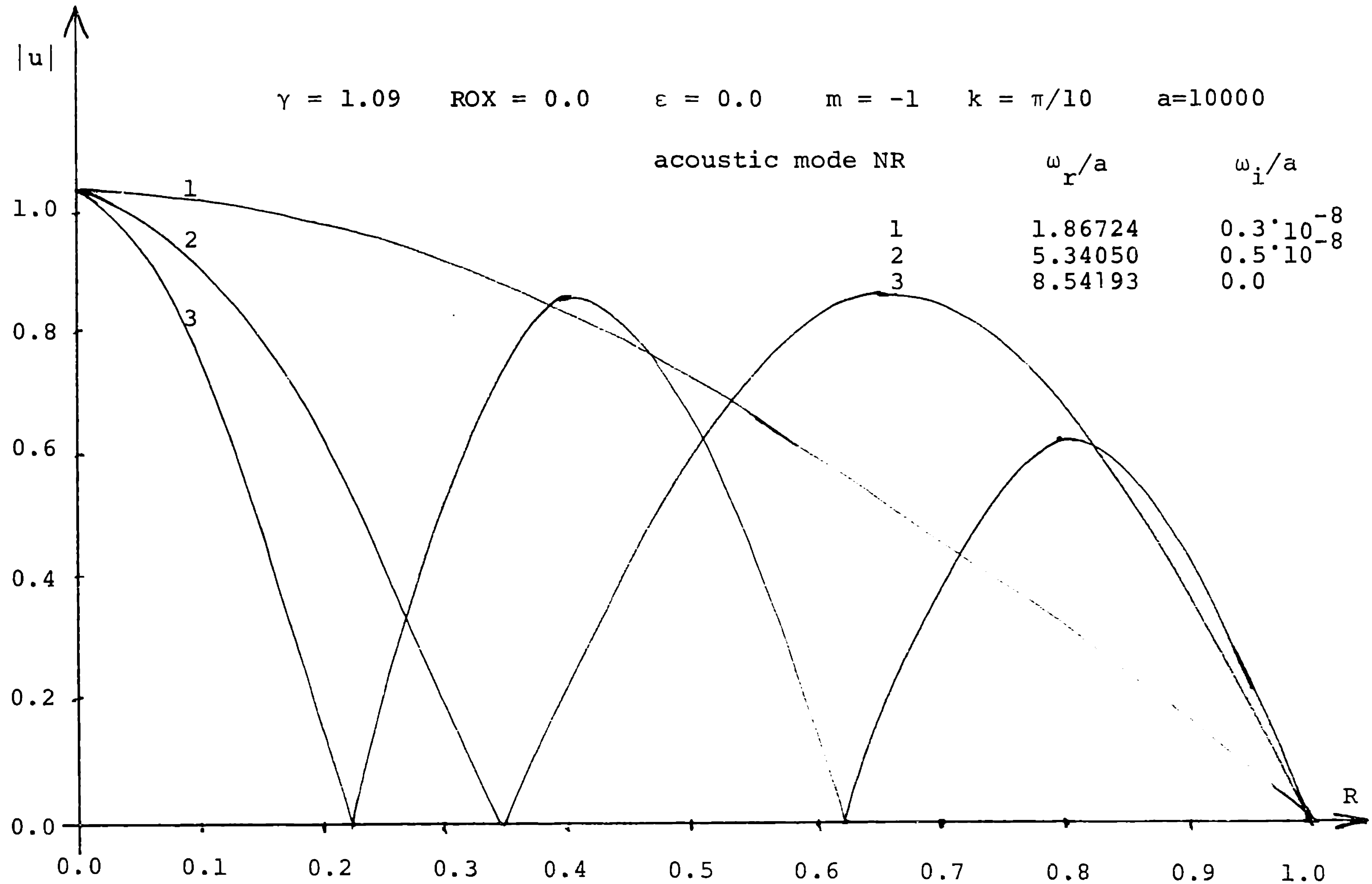


Figure 4. Mode form for first, second and third acoustic mode .

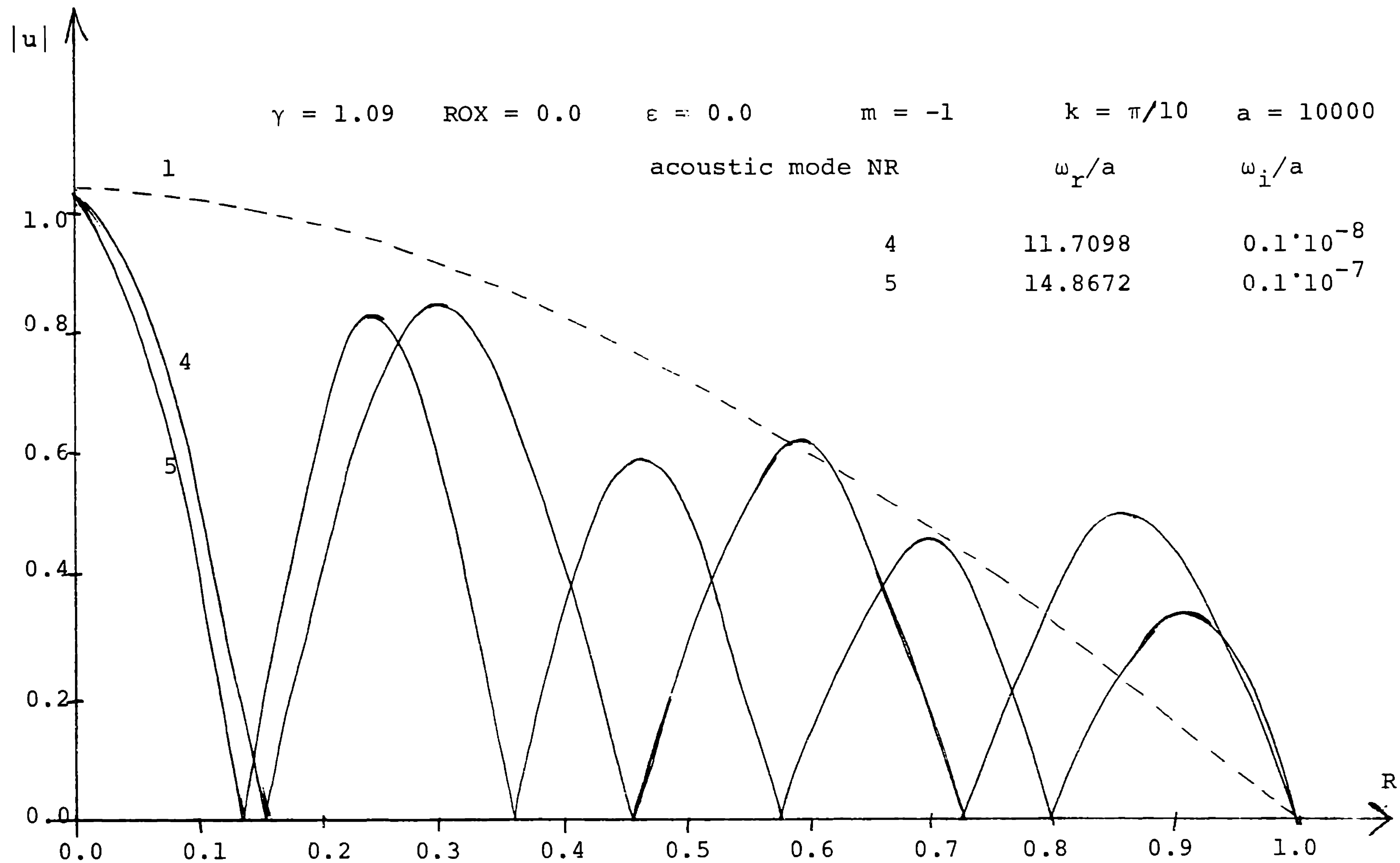


Figure 5. Mode form for fourth and fifth acoustic mode .

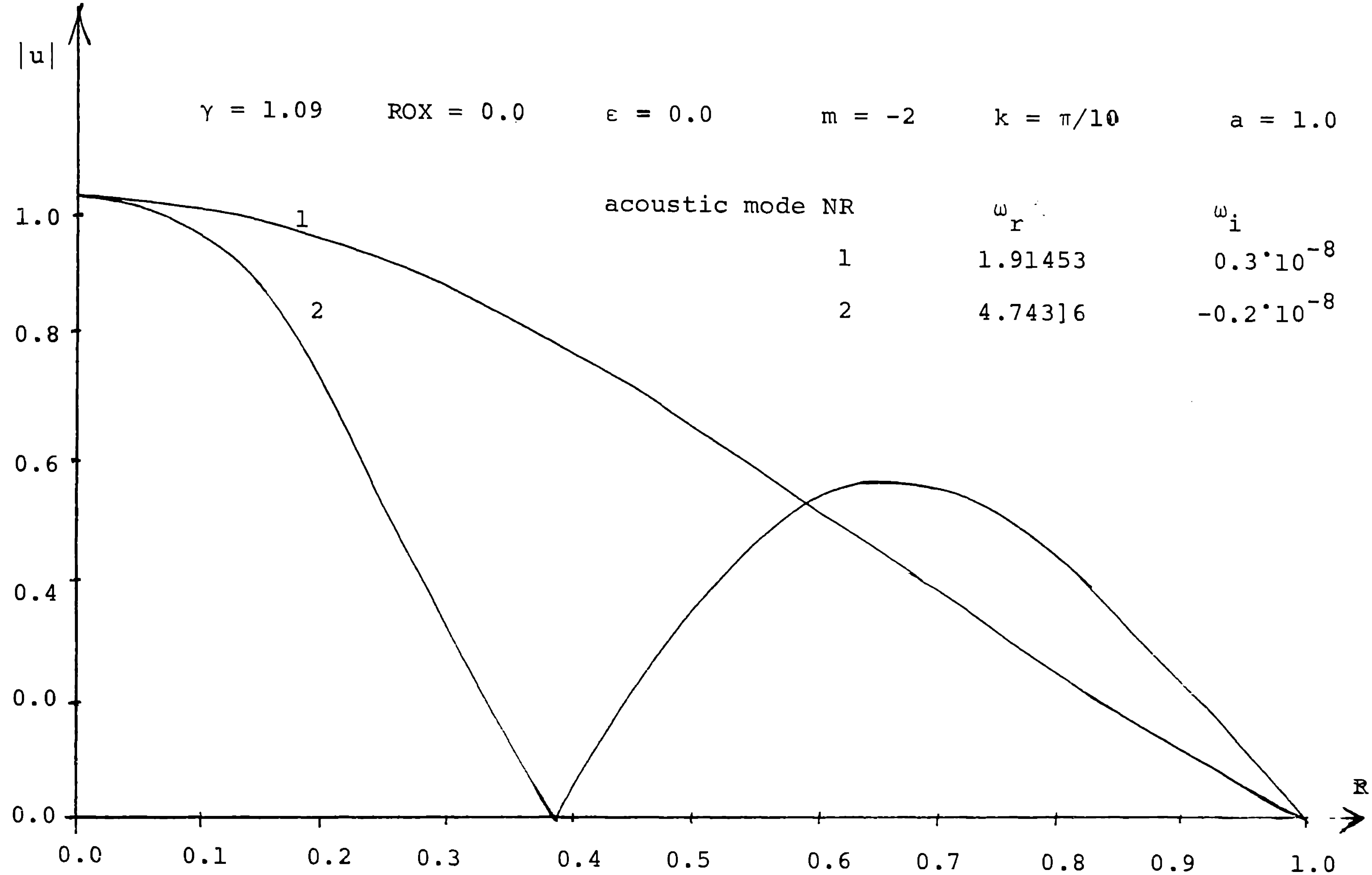


Figure 6.. Mode form for first and second acoustic mode .

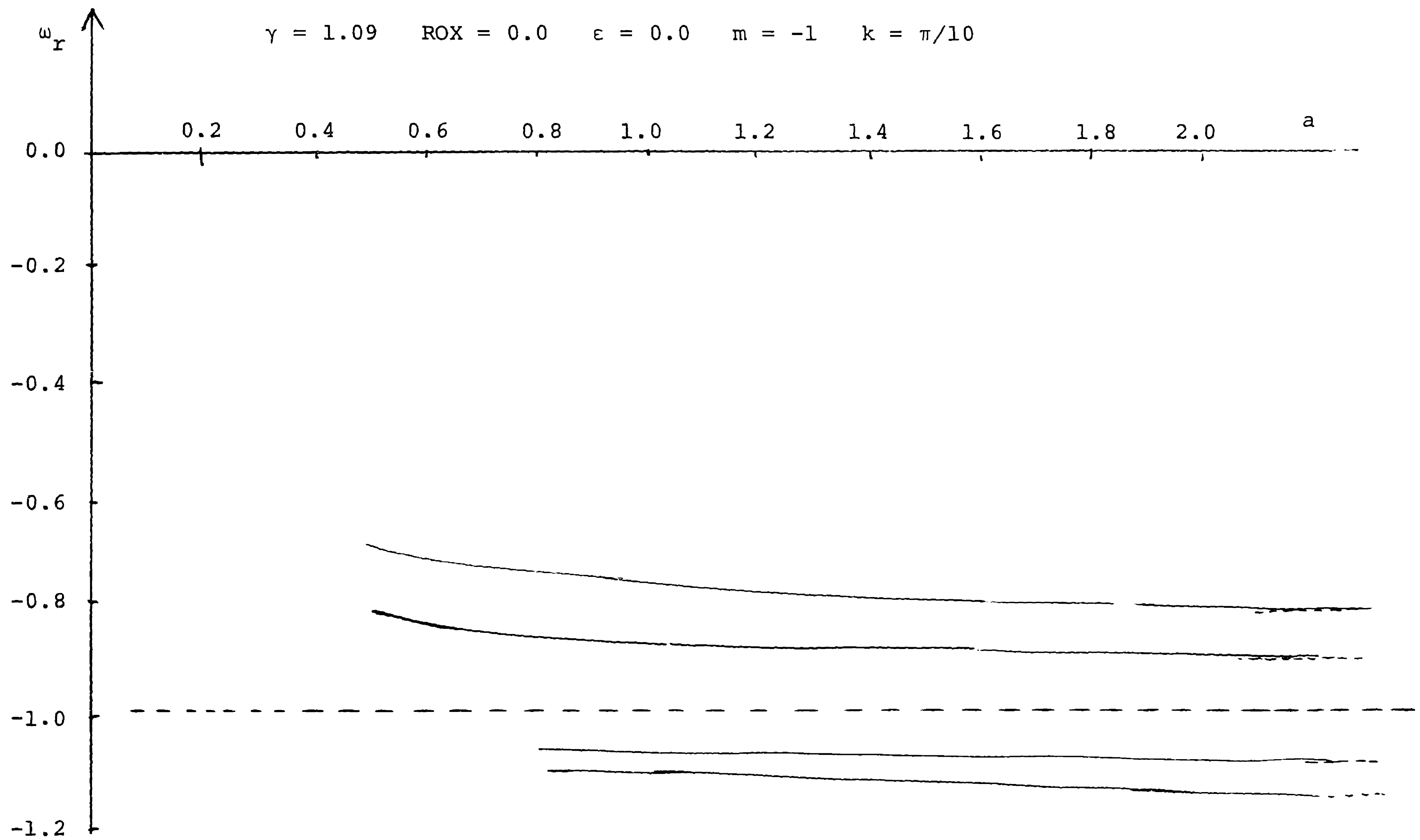


Figure 7. Inertial modes as a function of a.

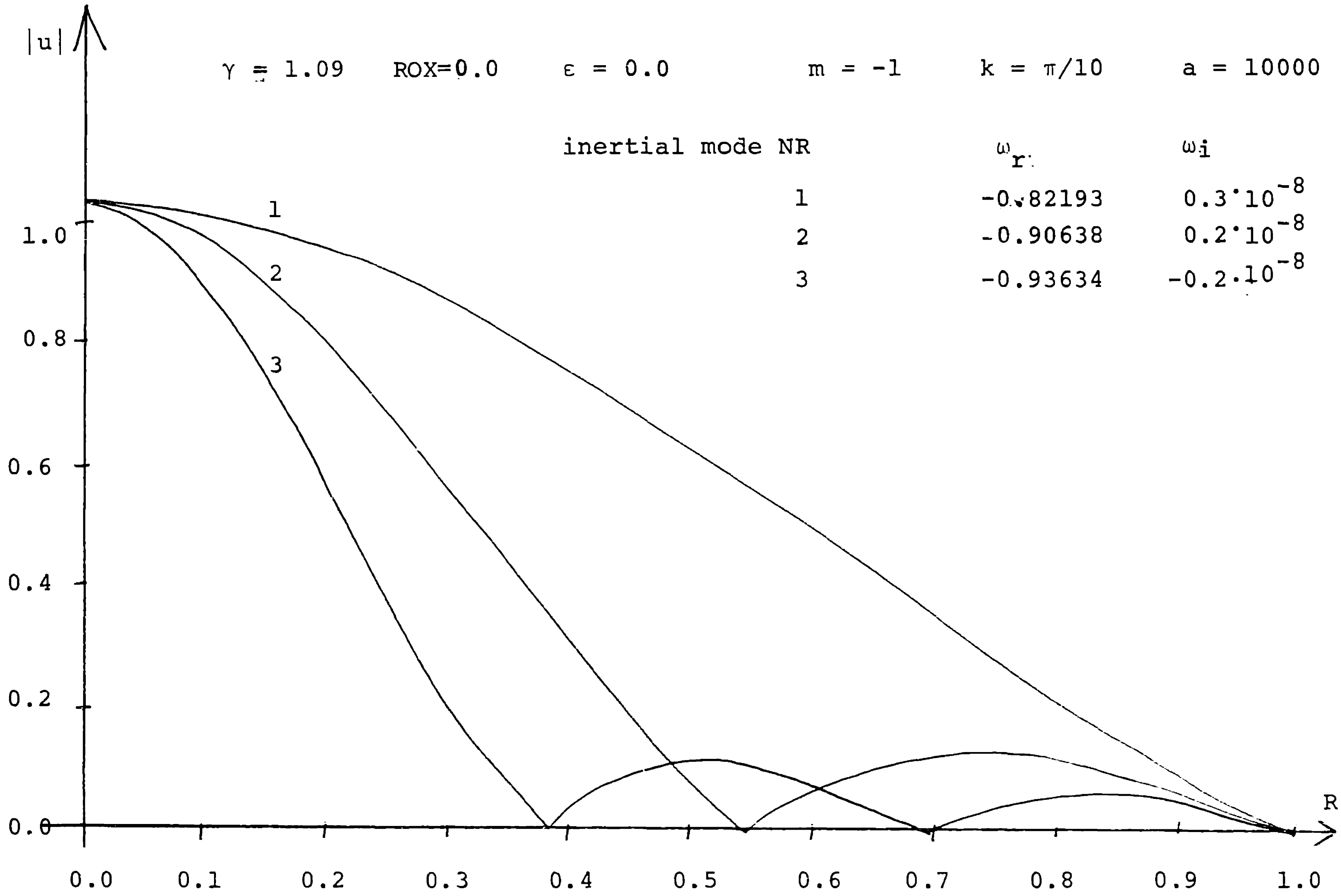


Figure 8. Mode form for first, second and third inertial modes .

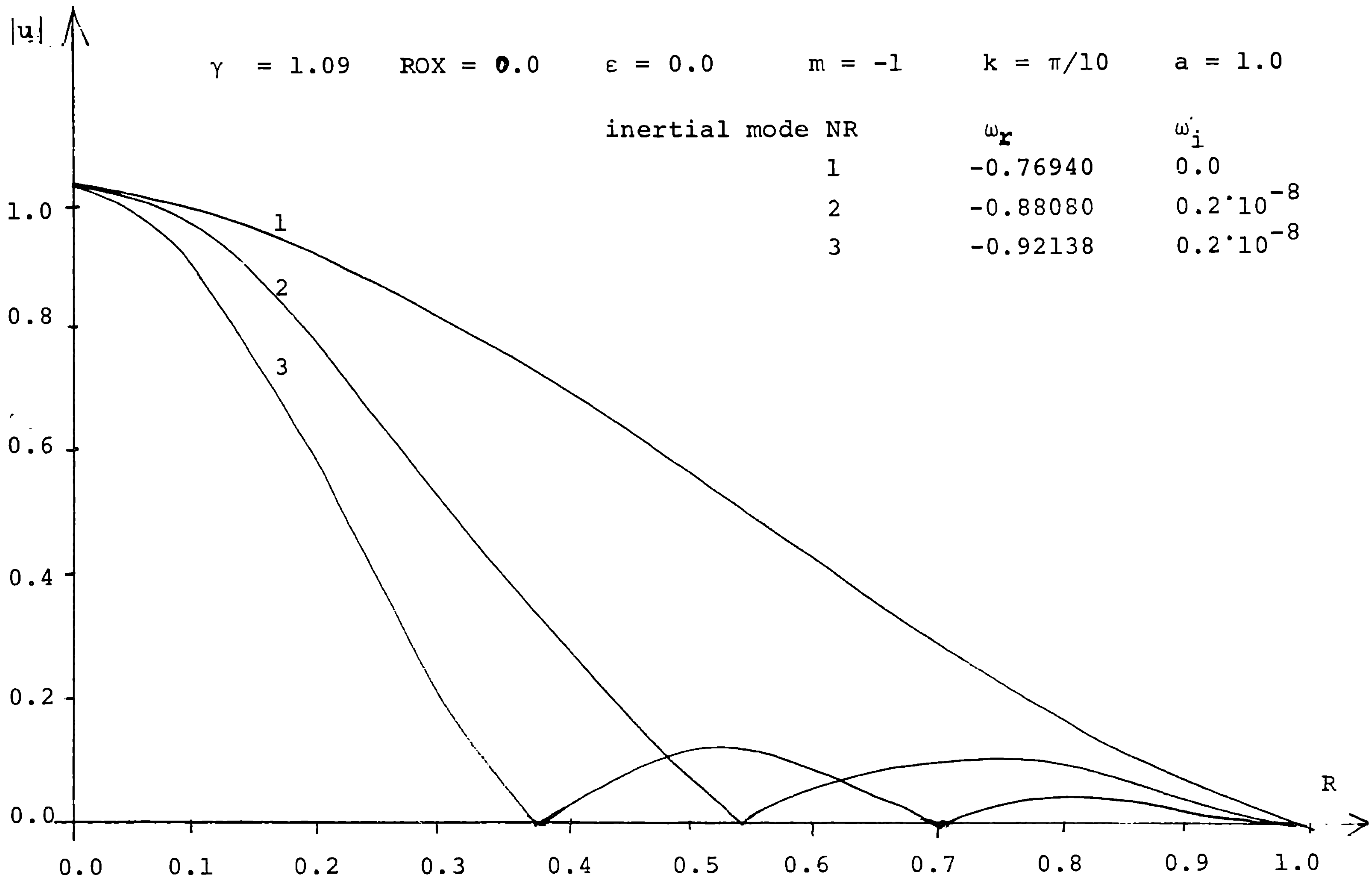


Figure 9. Mode form for first, second and third inertial mode .

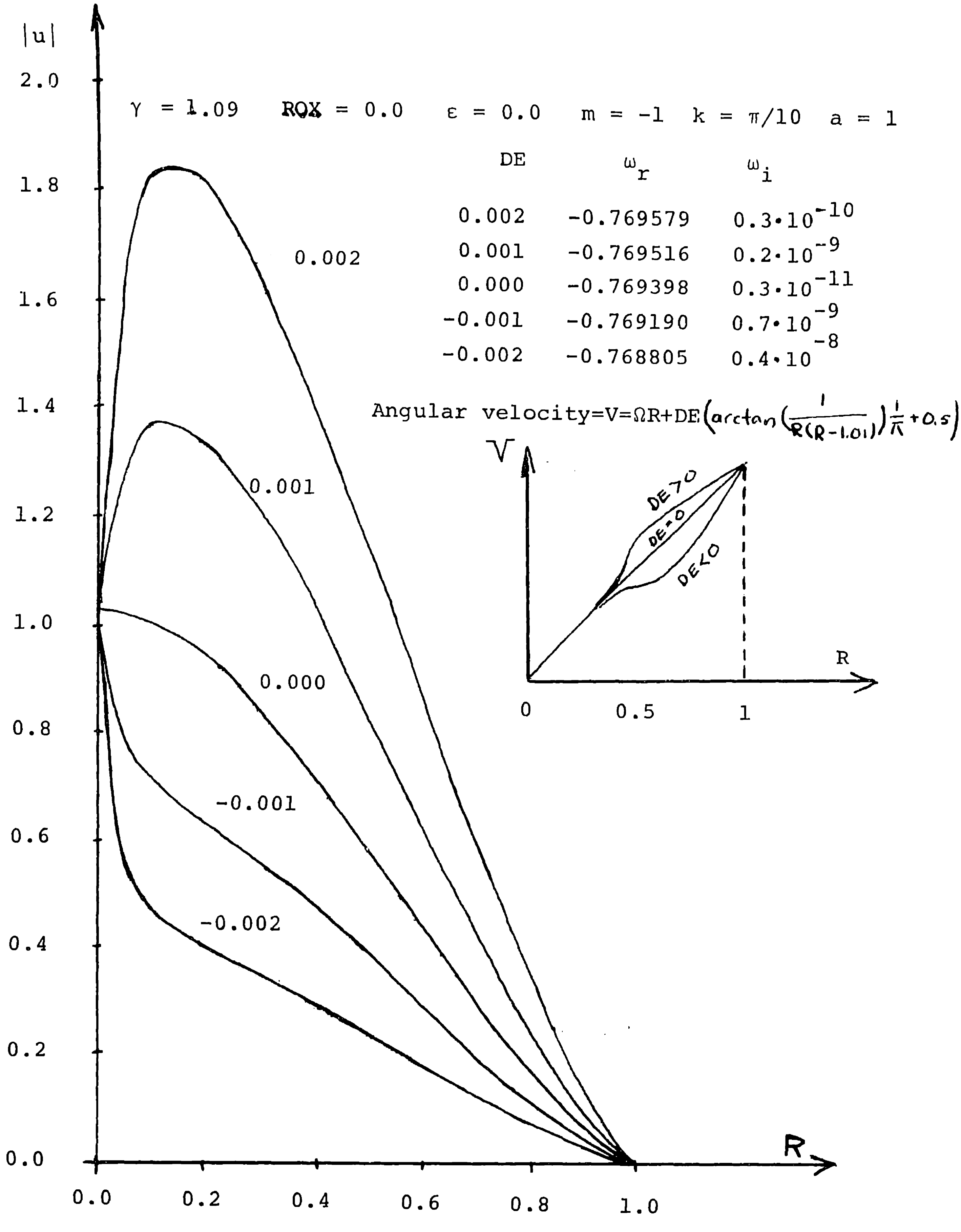


Figure 10. Mode form for first inertial mode.

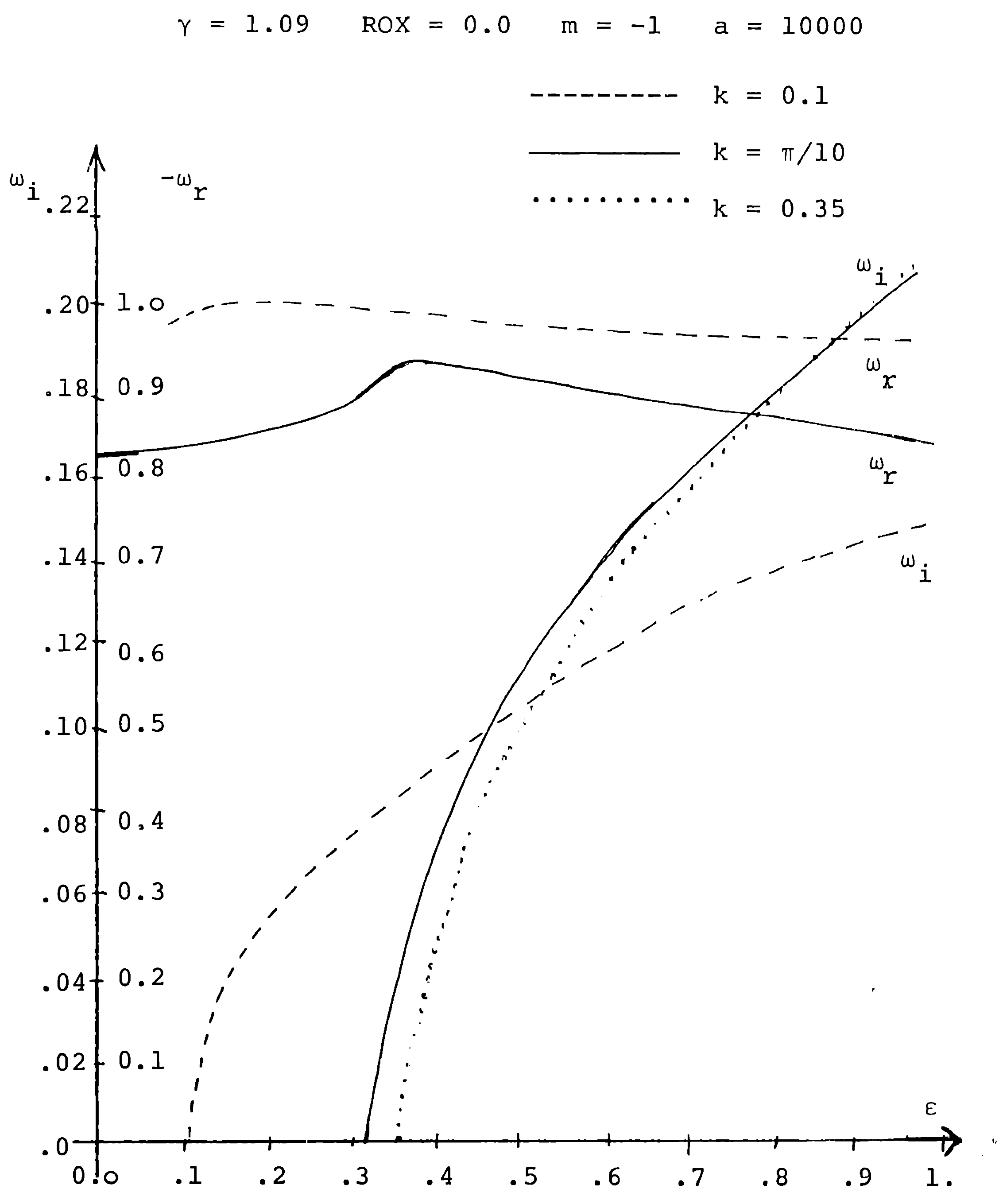


Figure 11. Eigenvalue as a function of a .

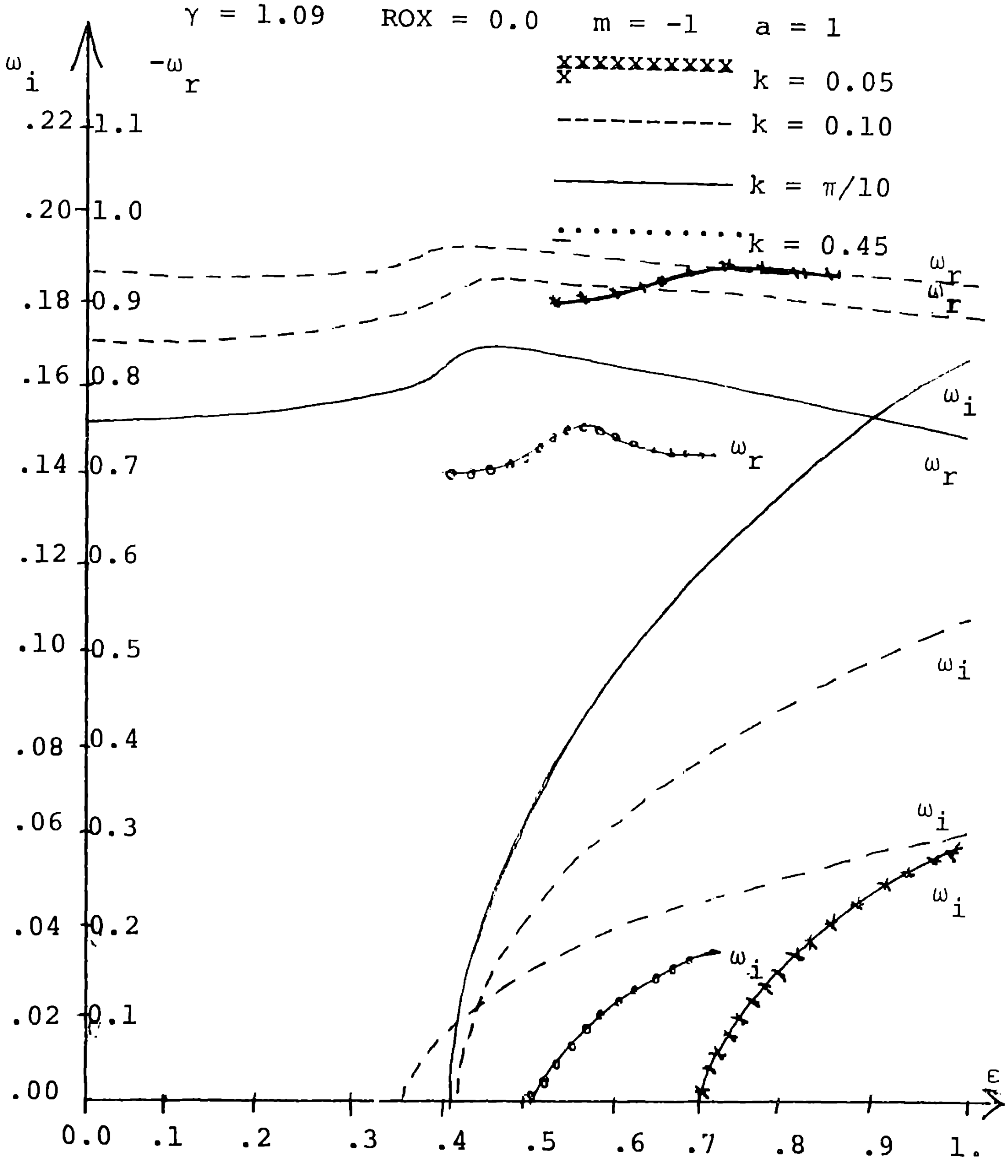


Figure 12. Eigenvalue as a function of a .

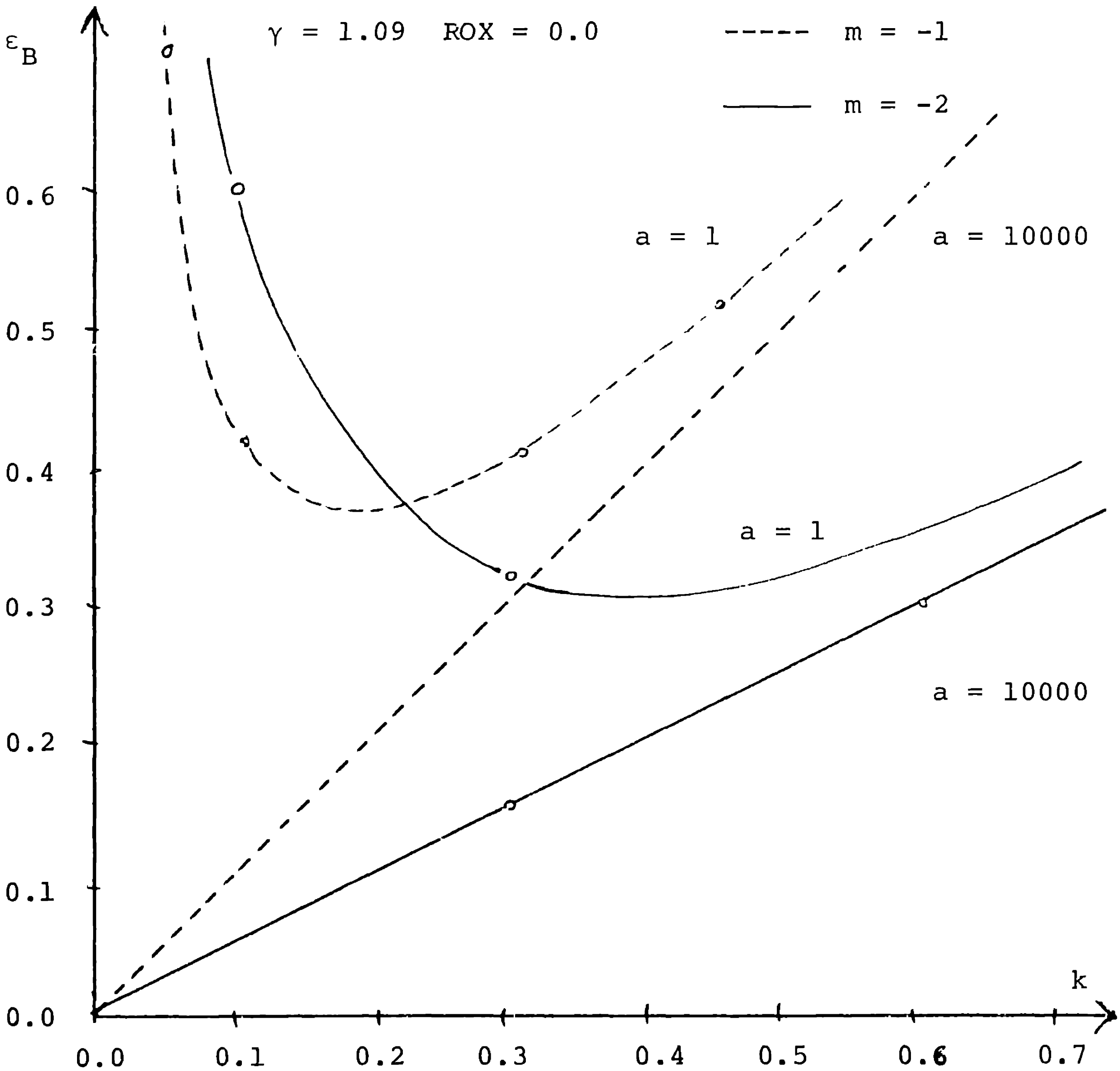


Figure 13. Stability boundary as a function of k.

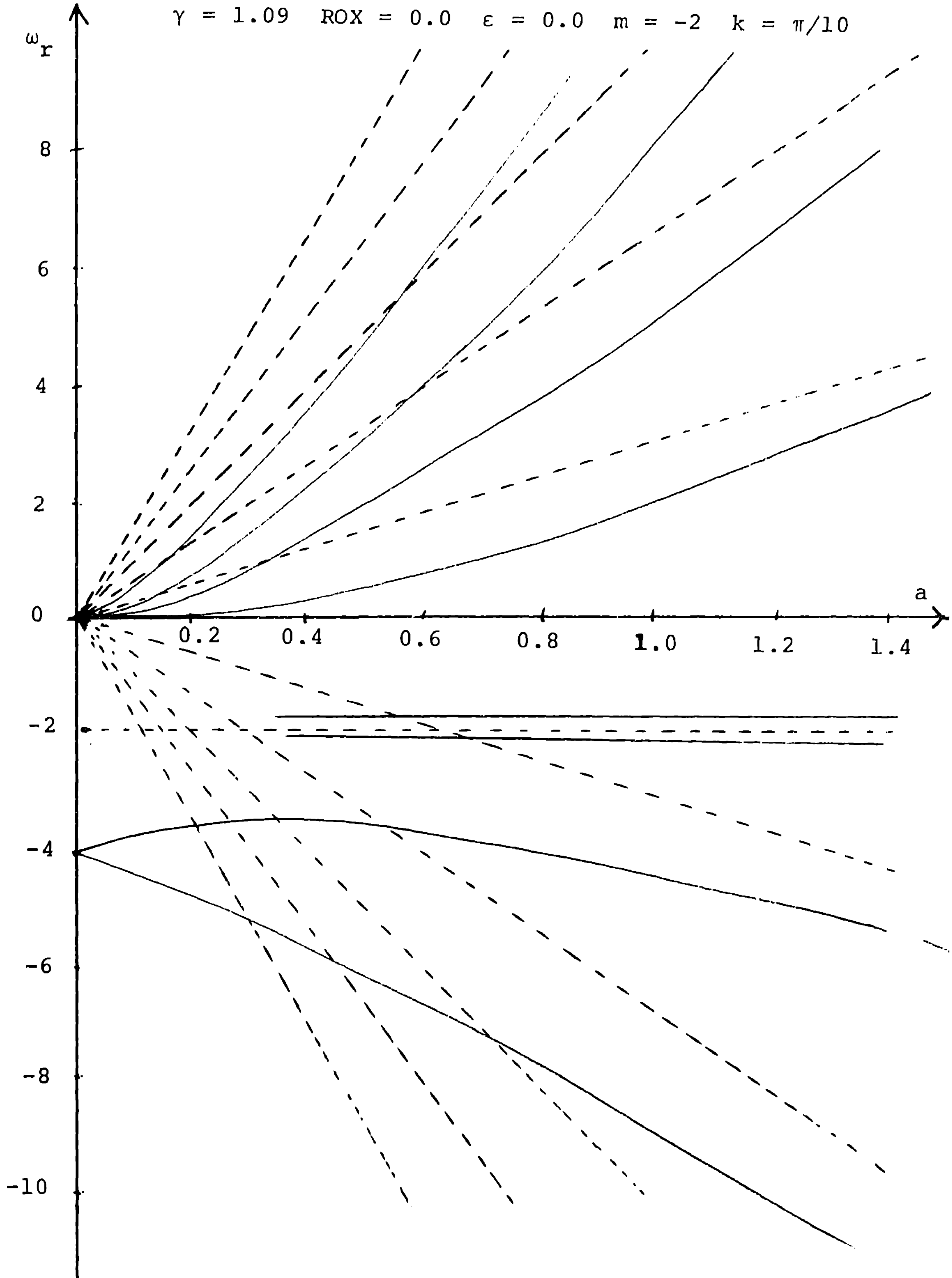


Figure 14. Free modes as a function of a.

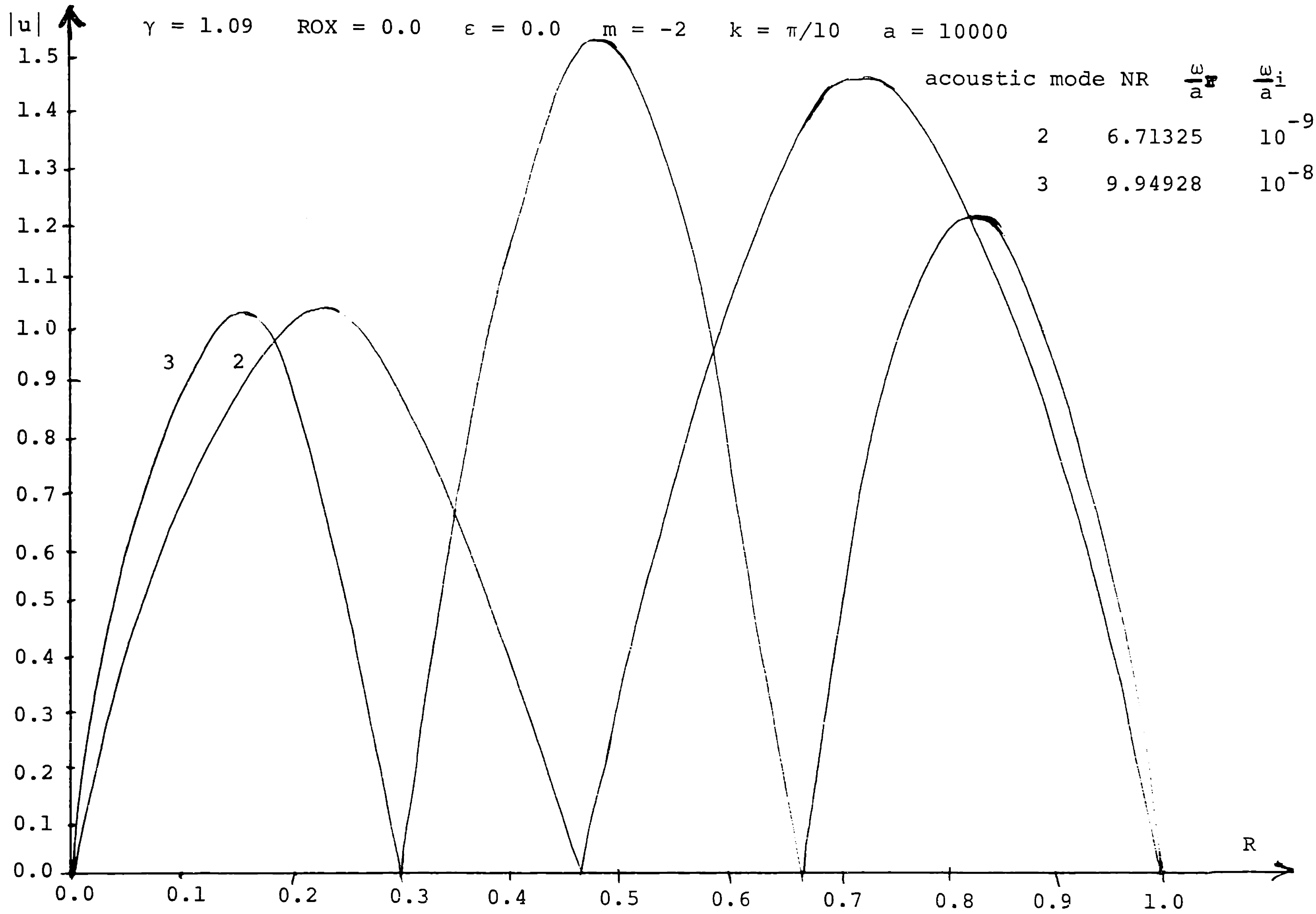


Figure 15. Mode form for second and third acoustic mode.

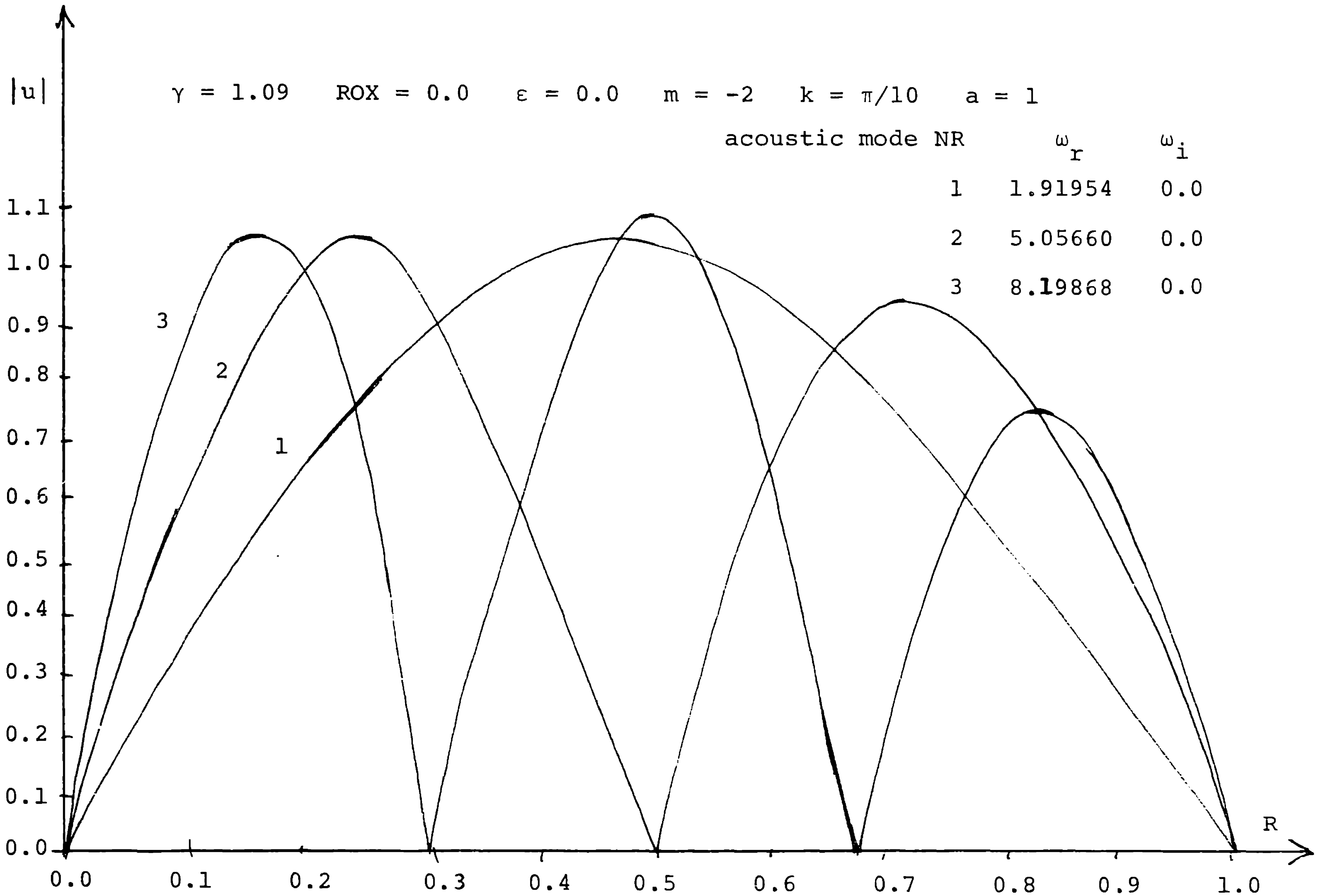


Figure 16. Mode form for first, second and third acoustic mode.

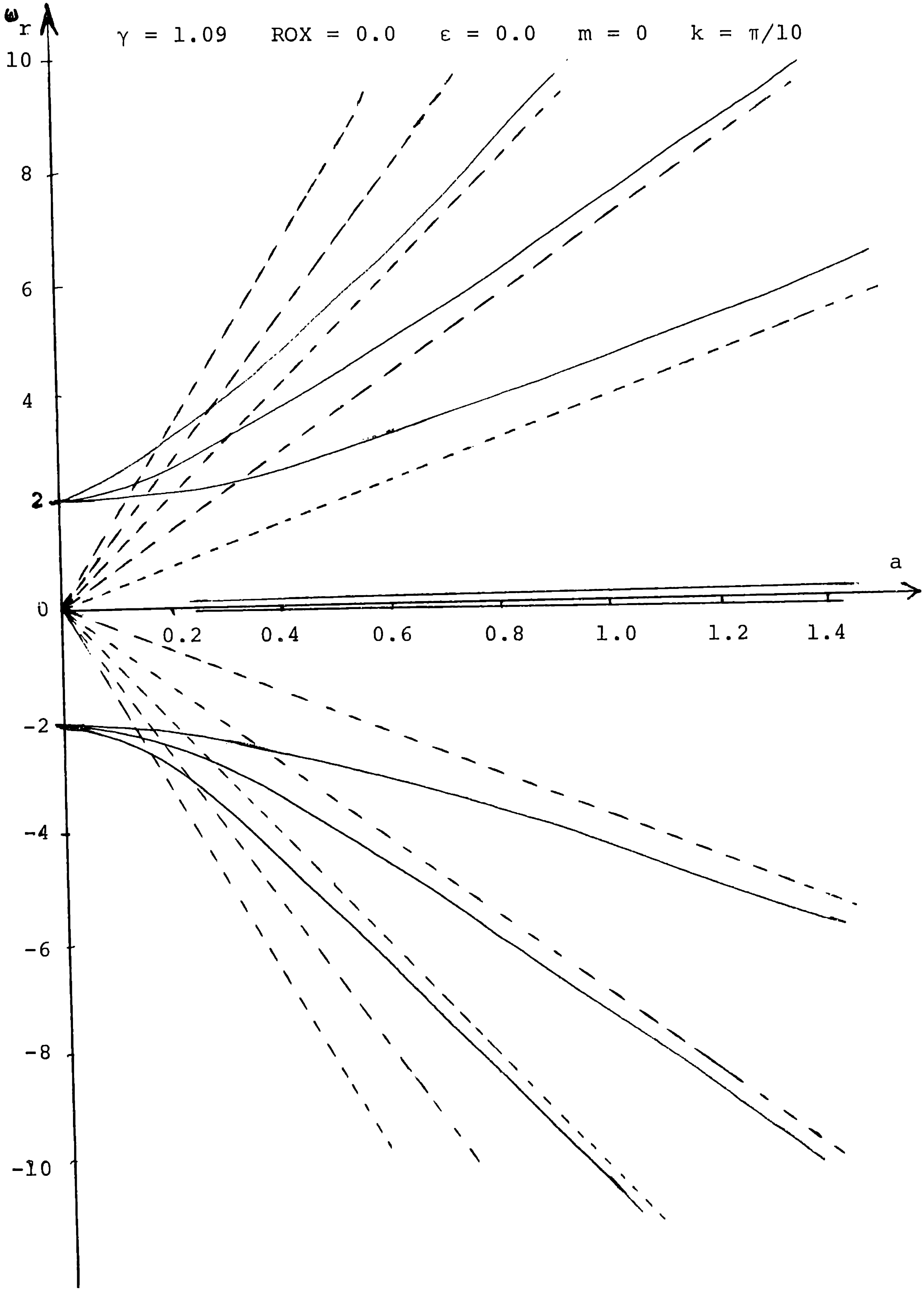


Figure 17. Free modes as a function of a.

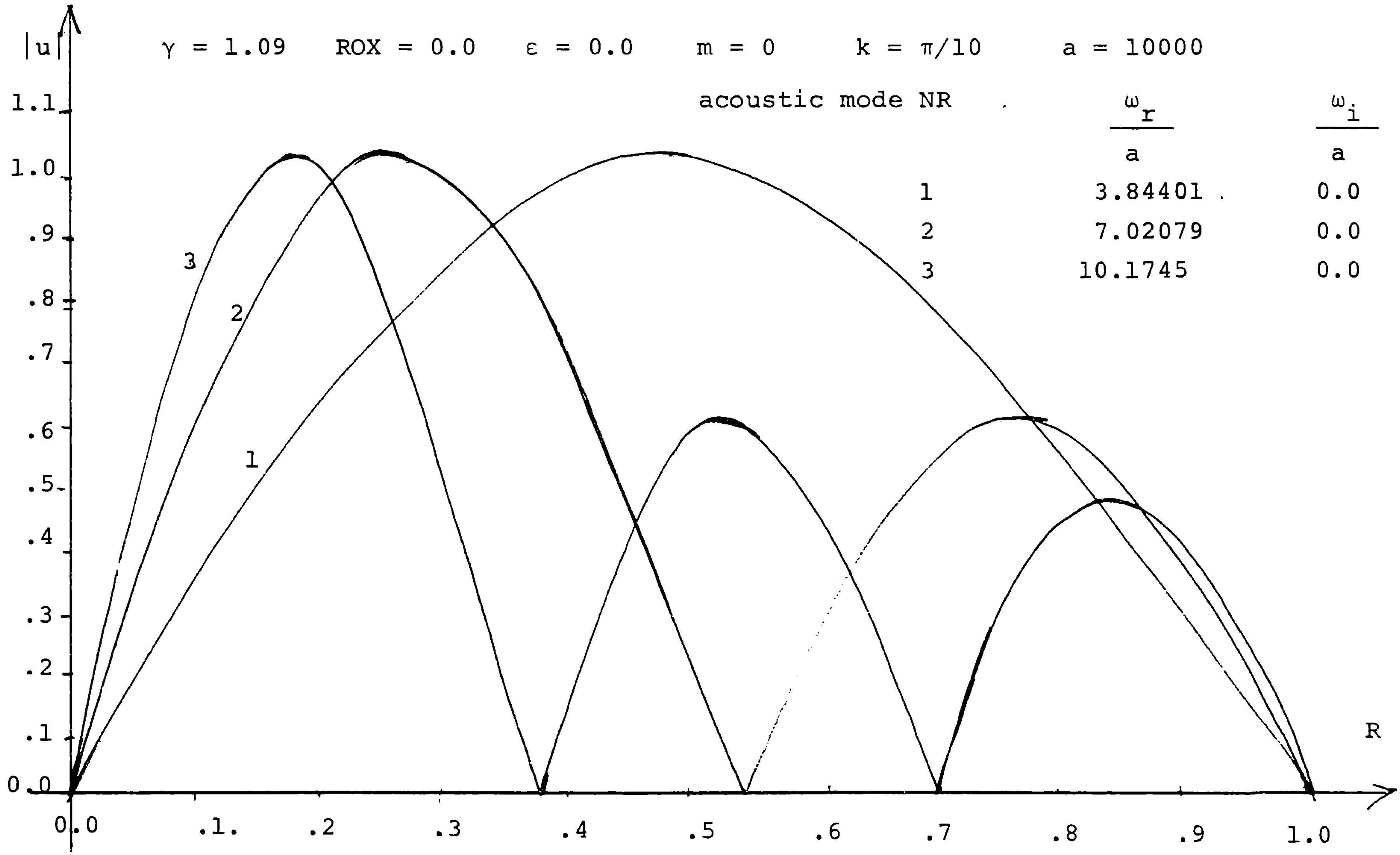


Figure 18. Mode form for first, second and third acoustic mode .

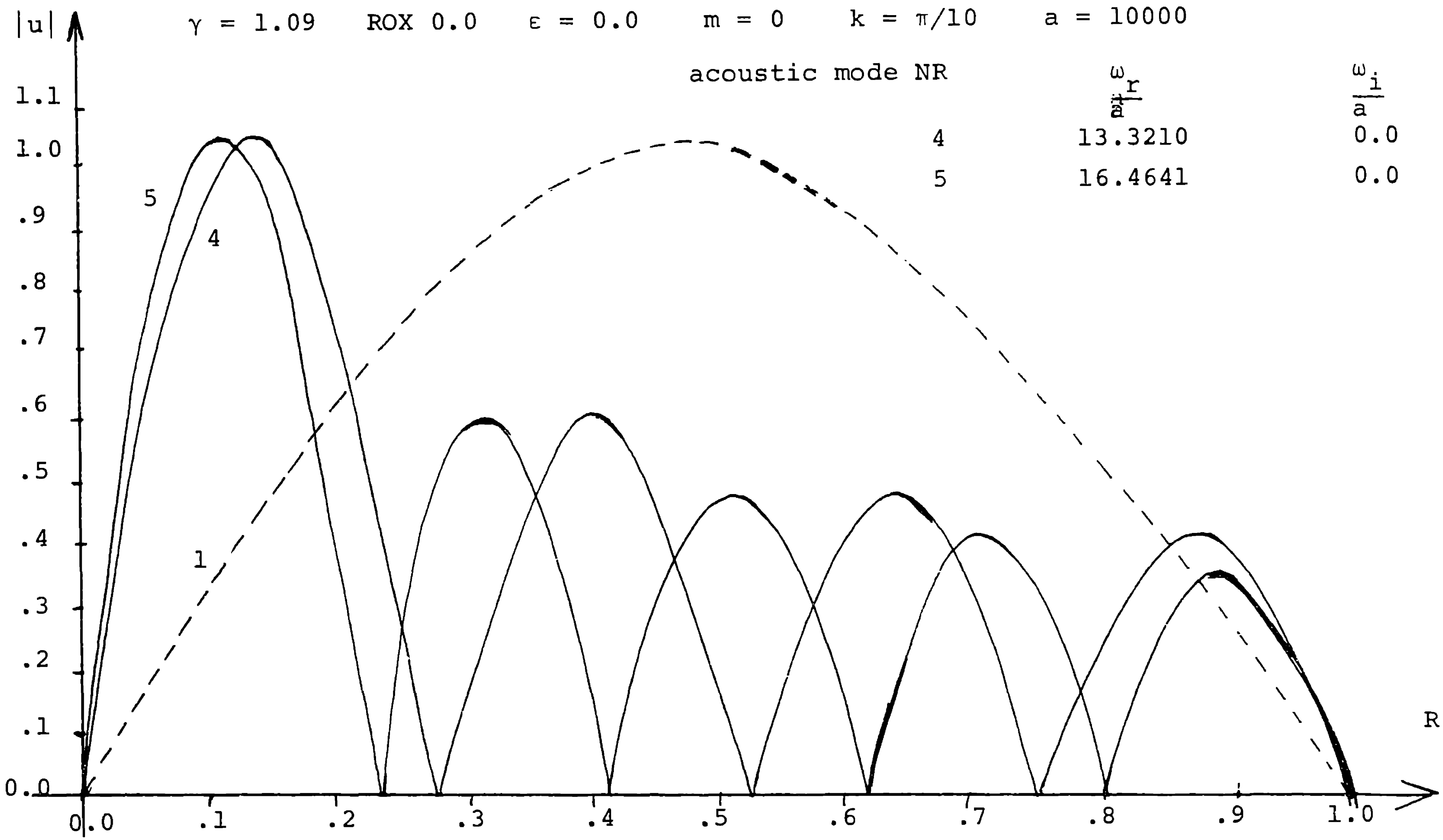


Figure 19. Mode form for fourth and fifth acoustic mode .

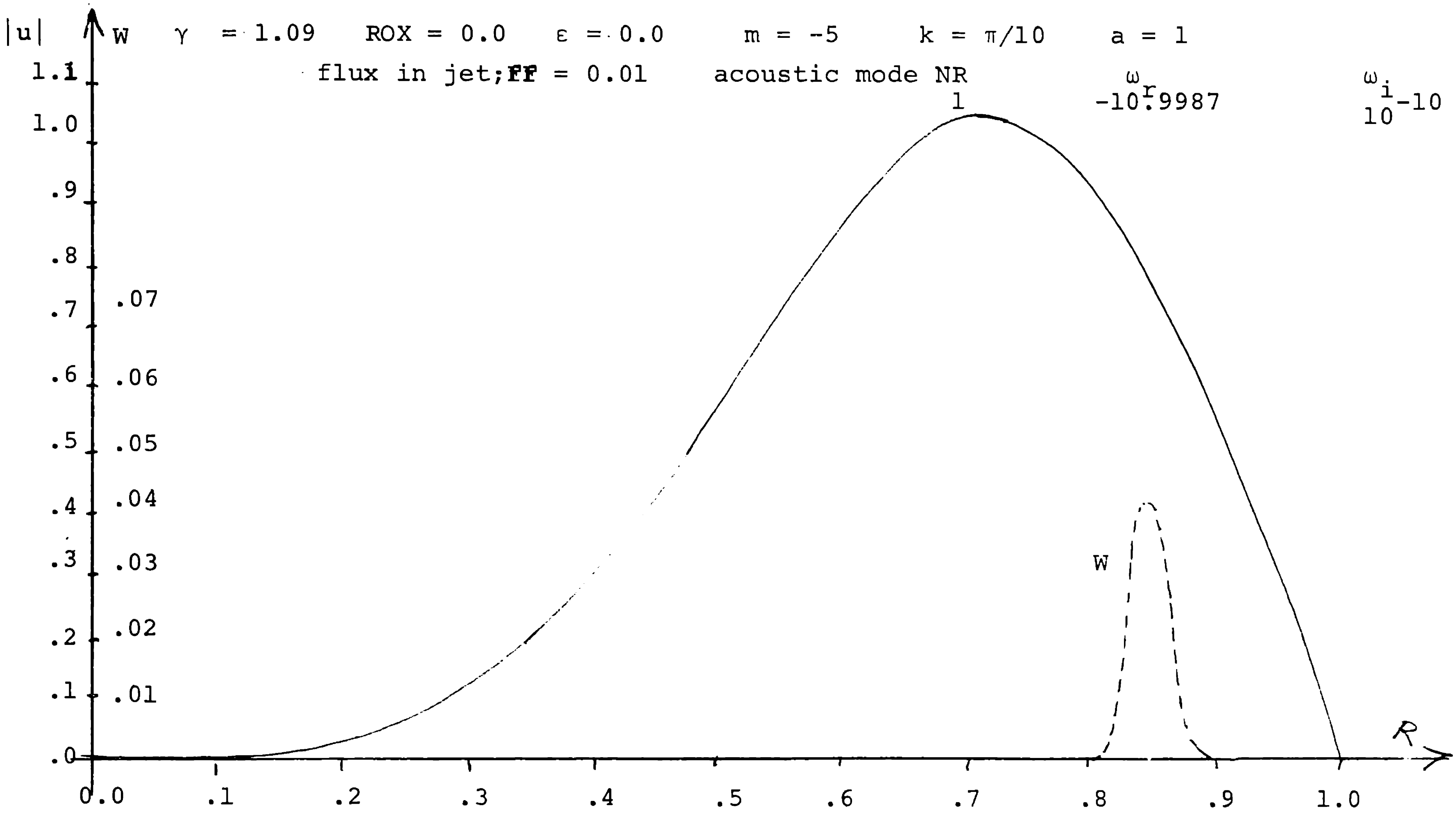


Figure 20. Mode form for first acoustic mode .

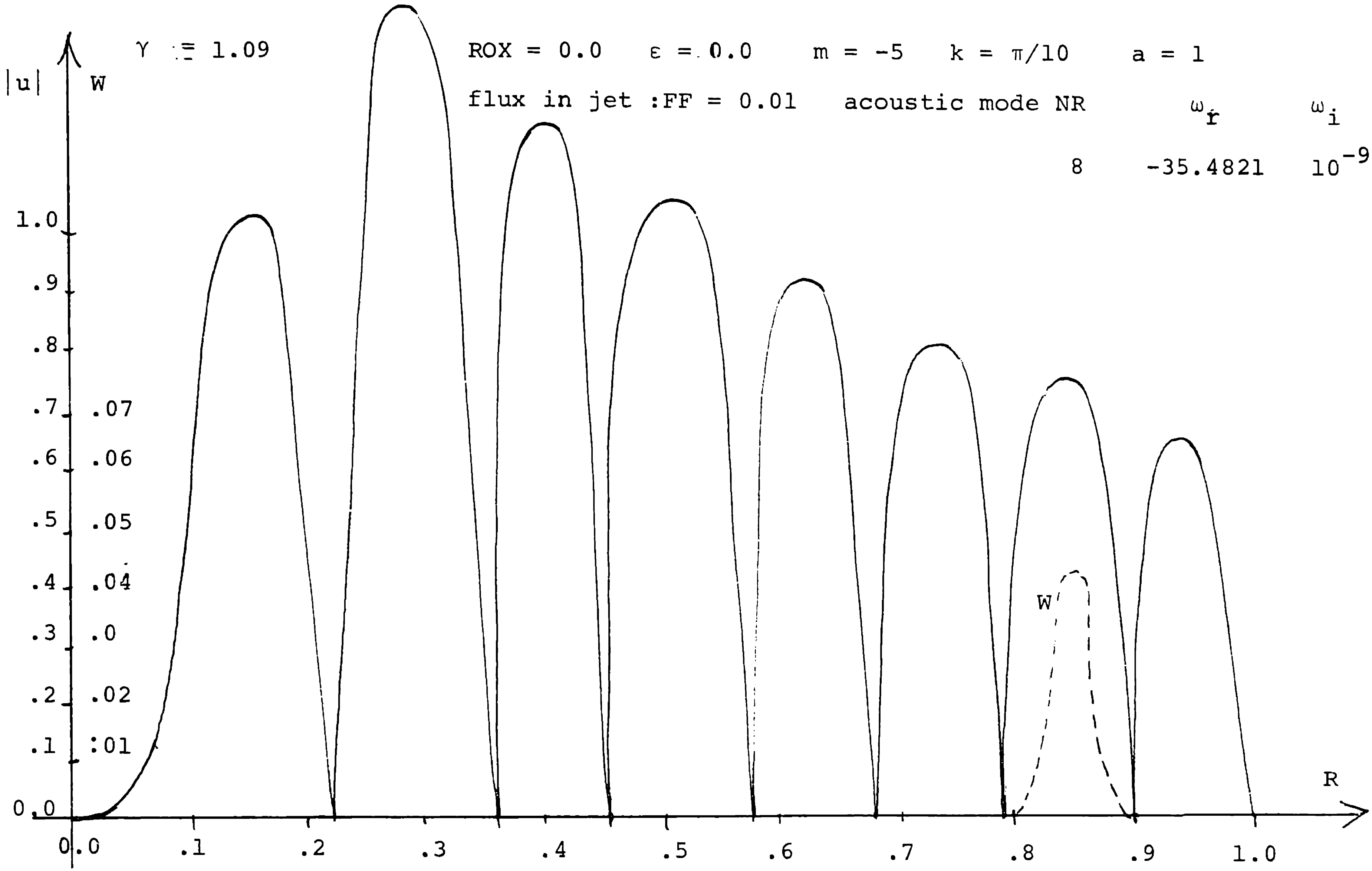


Figure 21. Mode form for first acoustic mode .

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