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— A unification of force and matter by quantum information*

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Four revolutions in physics and the second quantum revolution – a unification of force and matter by quantum information

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Abstract

Newton's mechanical revolution unifies the motion of planets in the sky and falling of apple on earth. Maxwell's electromagnetic revolution unifies electricity, magnetism, and light. Einstein's relativity revolution unifies space with time, and gravity with space-time distortion. The quantum revolution unifies particle with waves, and energy with frequency. Each of those revolution changes our world view. In this article, we will describe a revolution that is happening now: the second quantum revolution which unifies matter/space with information. In other words, the new world view suggests that elementary particles (the bosonic force particles and fermionic matter particles) all originated from quantum information (qubits): they are collective excitations of an entangled qubit ocean that corresponds to our space. The beautiful geometric Yang-Mills gauge theory and the strange Fermi statistics of matter particles now have a common algebraic quantum informational origin.

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Symmetry is beautiful and rich.

Quantum entanglement is even more beautiful and richer.

I. FOUR REVOLUTIONS IN PHYSICS

We have a strong desire to understand everything from a single or very few origins. Driven by such a desire, physics theories were developed through the cycle of discoveries, unification, more discoveries, bigger unification. Here, we would like review the development of physics and its four revolutions¹. We will see that the history of physics can be summarized into three stages: 1) all matter is formed by particles; 2) the discovery of wave-like matter; 3) particle-like matter = wave-like matter. It appears that we are now entering into the fourth stage: matter and space = information (qubits), where qubits emerge as the origin of everything [1–8]. In the current standard model for the elementary particles, we introduce a gauge field for each of the electromagnetic, weak, and strong interactions. We introduce an anti-commuting field for each of the fermionic matter particles. In this fourth stage, it appears that we do not need to introduce so many different gauge fields and anti-commuting fields to describe the bosonic force particles and fermionic matter particles. All we need are qubits which form a qubit ocean that corresponds to our space. All elementary bosonic force particles and fermionic matter particles can appear as the collective excitations in such a qubit ocean. In other words, all elementary particles can be unified by quantum information (qubits).

¹ Here we do not discuss the revolution for thermodynamical and statistical physics.

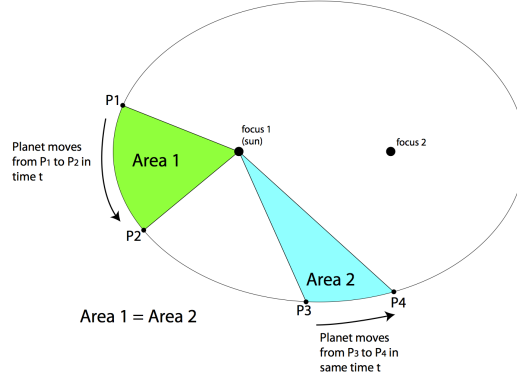


FIG. 1: Kepler's Laws of Planetary Motion: 1) The orbit of a planet is an ellipse with the Sun at one of the two foci. 2) A line segment joining a planet and the Sun sweeps out equal areas during equal intervals of time. 3) The square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit.

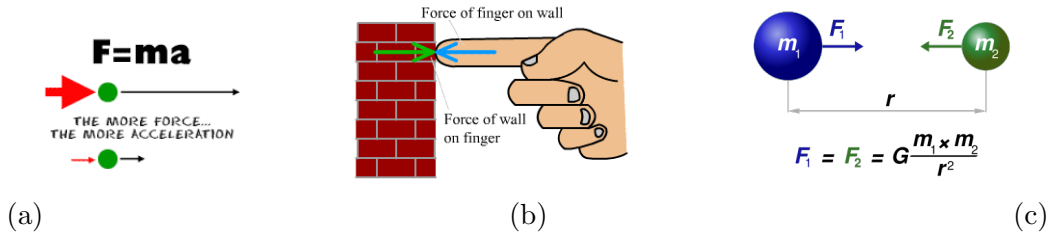


FIG. 2: Newton laws: (a) the more force the more acceleration, no force no acceleration. (b) action force = reaction force. (c) Newton's universal gravitation: $F = G \frac{m_1 m_2}{r^2}$, where $G = 6.674 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$.

A. Mechanical revolution

Although the down pull by the earth was realized even before human civilization, such a phenomena did not arose any curiosity. On the other hand the planet motion in the sky has arose a lot of curiosity and led to many imaginary fantasies. However, only after Kepler found that planets move in a certain particular way described by a mathematical formula (see Fig. 1), people started to wonder: Why are planets so rational? Why do they move in such a peculiar and precise way. This motivated Newton to develop his theory of gravity and his laws of mechanical motion (see Fig. 2). Newton's theory not only explains the planets motion, it also explains the down-pull that we feel on earth. The planets motion in the sky and the apple falling on earth look very different (see Fig. 3), however, Newton's theory unifies the two seemingly unrelated phenomena. This is the first revolution in physics – the mechanical revolution.

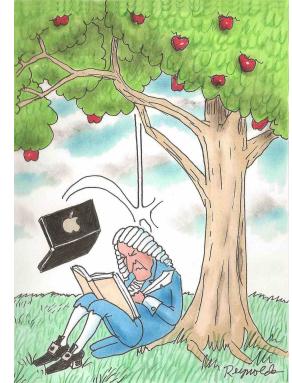
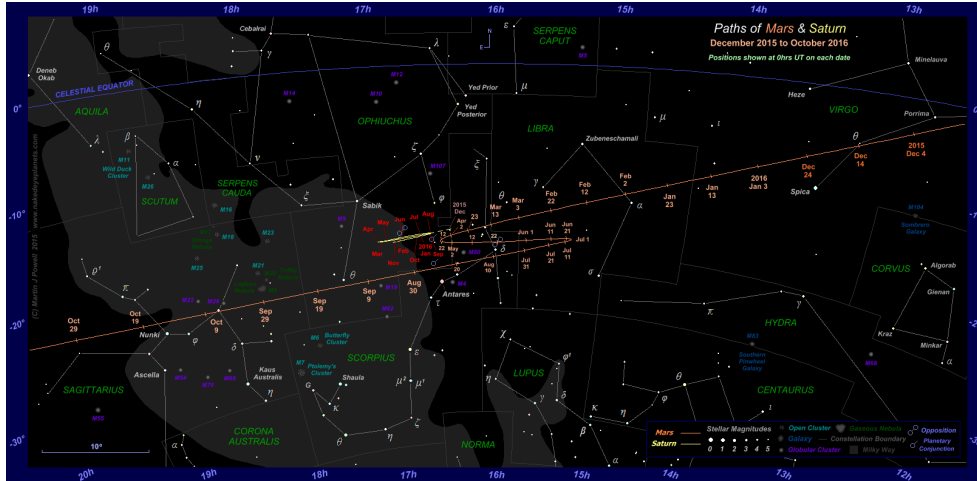


FIG. 3: The perceived trajectories of planets (Mar and Saturn) in the sky. The falling of apple on earth and the motion of planet in the sky are unified by Newton theory.

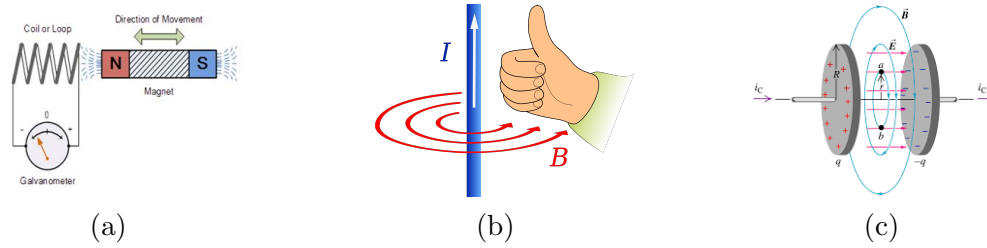


FIG. 4: (a) Changing magnetic field can generate an electric field around it, that drives an electric current in a coil. (b) Electric current I in a wire can generate a magnetic field B around it. (c) A changing electric field E (just like electric current) can generate a magnetic field B around it.

Mechanical revolution

All matter are formed by particles, which obey Newton's laws. Interactions are instantaneous over distance.

After Newton, we view all matter as formed by particles, and use Newton's laws for particles to understand the motion of all matter. The success and the completeness of Newton's theory gave us a sense that we understood everything.

B. Electromagnetic revolution

But, later we discovered that two other seemingly unrelated phenomena, electricity and magnetism, can generate each other (see Fig. 4). Our curiosity about the electricity and magnetism leads to another giant leap in science, which is summarized by Maxwell equations. Maxwell theory unifies electricity and magnetism and reveals that light is merely an

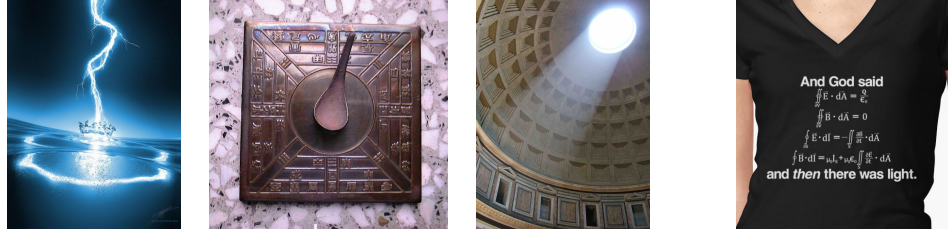


FIG. 5: Three very different phenomena, electricity, magnetism, and light, are unified by Maxwell theory.

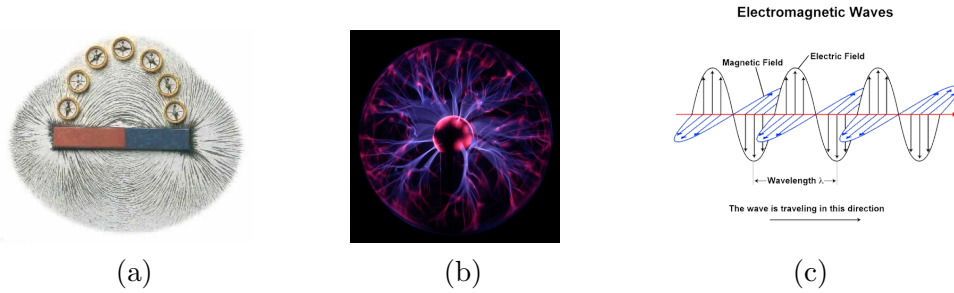


FIG. 6: (a) Magnetic field revealed by iron powder. (b) Electric field revealed by glowing plasma. (c) They form a new kind of matter: light – a wave-like matter

electromagnetic wave (see Fig. 5). We gain a much deeper understanding of light, which is so familiar and yet so unexpectedly rich and complex in its internal structure. This can be viewed as the second revolution – electromagnetic revolution.

Electromagnetic revolution

The discovery of a new form of matter – wave-like matter: electromagnetic waves, which obey Maxwell equation. Wave-like matter causes interaction.

However, the true essence of Maxwell theory is the discovery of a new form of matter – wave-like (or field-like) matter (see Fig. 6), the electromagnetic wave. The motion of this wave-like matter is governed by Maxwell equation, which is very different from the particle-like matter governed by Newton equation $F = ma$. Thus, the sense that Newton theory describes everything is incorrect. Newton theory does not apply to wave-like matter, which requires a new theory – Maxwell theory.

Unlike the particle-like matter, the new wave-like matter is closely related to a kind of interaction – electromagnetic interaction. In fact, the electromagnetic interaction can be viewed as an effect of the newly discovered wave-like matter.

C. Relativity revolution

After realizing the connection between the interaction and wave-like matter, one naturally ask: does gravitational interaction also corresponds to a wave-like matter? The answer is yes.

First, people realized that Newton equation and Maxwell equation have different symmetries under the transformations between two frames moving against each other. In other words, Newton equation $F = ma$ is invariant under Galileo transformation, while Maxwell equation is invariant under Lorentz transformation (see Fig. 7). Certainly, only one of the above two transformation is correct. If one believes that physical law should be the same in different frames, then the above observation implies that Newton equation and Maxwell equation are incompatible, and one of them must be wrong. If Galileo transformation is correct, then the Maxwell theory is wrong and needs to be modified. If Lorentz transformation is correct, then the Newton theory is wrong and needs to be modified. Michelson-Morley experiment showed that the speed of light is the same in all the frames, which implied the Galileo transformation to be wrong. So Einstein choose to believe in Maxwell equation. He modified Newton equation and developed the theory of special relativity. Thus, Newton theory is not only incomplete, it is also incorrect.

Einstein has gone further. Motivated the equivalence of gravitational force and the force experienced in an accelerating frame (see Fig. 8), Einstein also developed the theory of general relativity.[9] Einstein theory unifies several seeming unrelated concepts, such as space and time, as well as interaction and geometry. Since the gravity is viewed as a distortion of space and since the distortion can propagate, Einstein discovered the second wave-like matter – gravitational wave (see Fig. 9). This is another revolution in physics – relativity revolution.

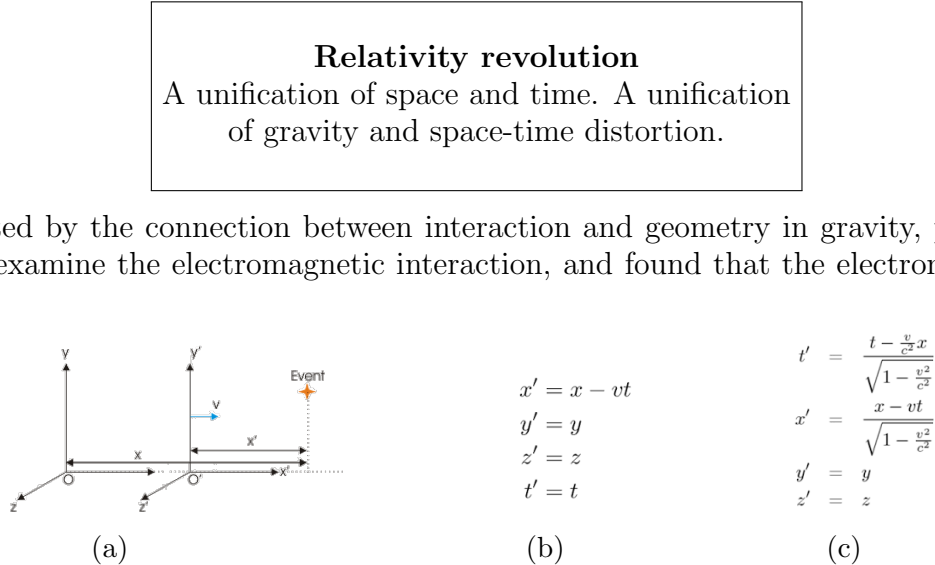


FIG. 7: (a) A rest frame and a moving frame with velocity v . An event is recorded with coordinates (x, y, z, t) in the rest frame and with (x', y', z', t') in the moving frame. There are two opinions on how (x, y, z, t) and (x', y', z', t') are related: (b) Galilean transformation or (c) Lorentz transformation where c is the speed of light. In our world, the Lorentz transformation is correct.

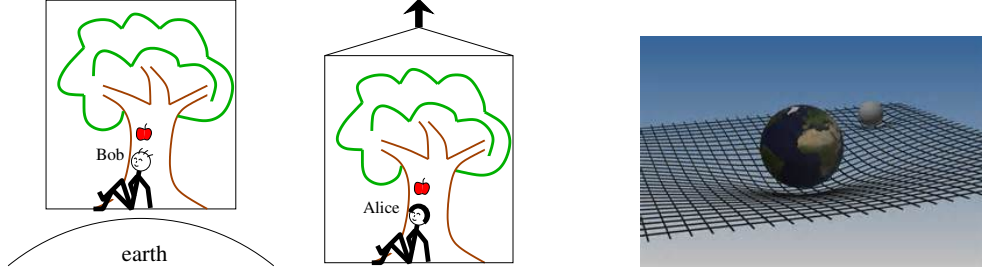


FIG. 8: The equivalence of the gravitational force of the earth and the force experienced in an accelerating elevator, leads to a geometric way to understand gravity: gravity = distortion in space. In other words, the “gravitational force” in an accelerating elevator is related to a geometric feature: the transformation between the coordinates in a still elevator and in the accelerating elevator.

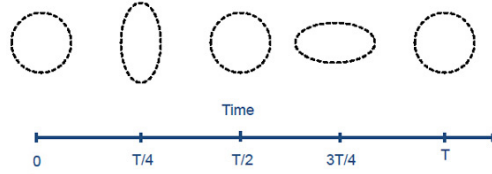


FIG. 9: Gravitational wave is a propagating distortion of space: a circle is distorted by a gravitational wave.

teraction is also connected to geometry. Einstein’s general relativity views gravity as a distortion of space, which can be viewed as a distortion of local directions of space (see Fig. 10). Motivated by such a picture, in 1918, Weyl proposed that the unit that we used to measure physical quantities is relative and is defined only locally. A distortion of the unit system can be described by a vector field which is called gauge field. Weyl proposed that such a vector field (the gauge field) is the vector potential that describes the electromagnetism. Although the above particular proposal turns out to be incorrect, the Weyl’s idea is correct. In 1925, the complex quantum amplitude was discovered. If we assume the complex phase is relative, then a distortion of unit system that measure local complex phase can also be described by a vector field. Such a vector field is indeed the vector potential

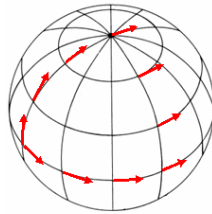


FIG. 10: A curved space can be viewed as a distortion of local directions of the space: parallel moving a local direction (represented by an arrow) around a loop in a curved space, the direction of the arrow does not come back. Such a twist in local direction corresponds to a curvature in space.

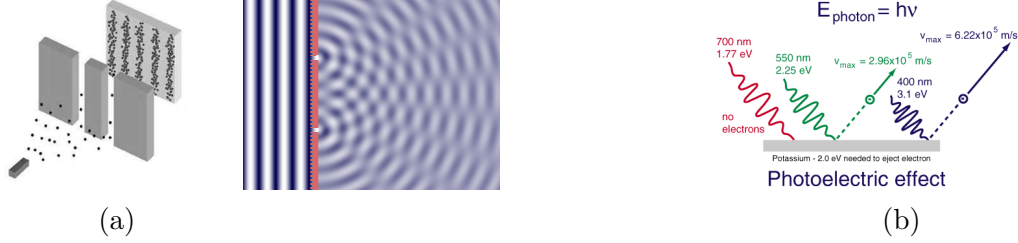


FIG. 11: (a) An electron beam passing through a double-slit can generate an interference pattern, indicating that electrons are also waves. (b) Using light to eject electrons from a metal (the photoelectric effect) shows that the higher the light wave frequency (the shorter the wave length), the higher the energy of the ejected electron. This reveals that a light wave of frequency f can be viewed a beam of particles of energy $E = hf$, where $h = 6.62607004 \times 10^{-34} \frac{\text{m}^2\text{kg}}{\text{s}}$.

that describes the electromagnetism. This leads to a unified way to understand gravity and electromagnetism: gravity arises from the relativity of spacial directions at different spacial points, while electromagnetism arises from the relativity of complex quantum phases at different spatial points. Further more, Nordström, Möglichkeit, Kaluza, and Klein showed that both gravity and electromagnetism can be understood as a distortion of space-time provided that we think the space-time as five dimensional with one dimension compactified into a small circle.[10–12] This can be viewed as an unification of gravity and electromagnetism. Those theories are so beautiful. Since that time, the geometric way to view our world has dominated theoretical physics.

D. Quantum revolution

However, such a geometric view of world was immediately challenged by new discoveries from microscopic world.² The experiments in microscopic world tell us that not only Newton theory is incorrect, even its relativity modification is incorrect. This is because Newton theory and its relativistic modification are theories for particle-like matter. But through experiments on very tiny things, such as electrons, people found that the particles are not really particles. They also behave like waves at the same time. Similarly, experiments also reveal that the light waves behave like a beam of particles (photons) at the same time (see Fig. 11). So the real matter in our world is not what we thought it was. The matter is neither particle nor wave, and both particle and wave. So the Newton theory (and its relativistic modification) for particle-like matter and the Maxwell/Einstein theories for wave-like matter cannot be the correct theories for matter. We need a new theory for the new form of existence: particle-wave-like matter. The new theory is the quantum theory that explains the microscopic world. The quantum theory unifies the particle-like matter and wave-like matter.

² Many people have ignored such challenges and the geometric view of world becomes the main stream.

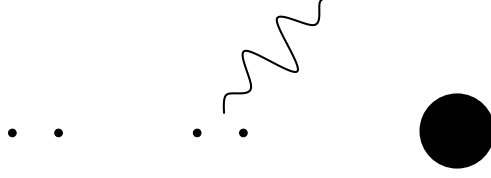


FIG. 12: To observe two points of distance l apart, we need to send in light of wave length $\lambda < l$. The corresponding photon has an energy $E = hc/\lambda$. If l is less than the Planck length $l < l_P$, then the photon will make a back hole of size larger than l . The black hole will swallow the two points, and we can never measure the separation of two points of distance less than l_P . What cannot be measured cannot exist. So the notion of “two points less than l_P apart” has no physical meaning and does not exist.

Quantum revolution

There is no particle-like matter nor wave-like matter. All the matter in our world is particle-wave-like matter.

From the above, we see that quantum theory reveals the true existence in our world to be quite different from the classical notion of existence in our mind. What exist in our world are not particles or waves, but somethings that are both particle and wave. Such a picture is beyond our wildest imagination, but reflects the truth about our world and is the essence of quantum theory. To understand the new notion of existence more clearly, let us consider another example. This time it is about a bit (represented by spin-1/2). A bit has two possible states of classical existence: $|1\rangle = |\uparrow\rangle$ and $|0\rangle = |\downarrow\rangle$. However, quantum theory also allows a new kind of existence $|\uparrow\rangle + |\downarrow\rangle$. One may say that $|\uparrow\rangle + |\downarrow\rangle$ is also a classical existence since $|\uparrow\rangle + |\downarrow\rangle = |\rightarrow\rangle$ that describes a spin in x -direction. So let us consider a third example of two bits. Then there will be four possible states of classical existence: $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$. Quantum theory allows a new kind of existence $|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle$. Such a quantum existence is entangled and has no classical analogues.

Although the geometric way to understand our world is a main stream in physics, here we will take a position that the geometric understanding is not good enough and will try to advocate a very different non-geometric understanding of our world. Why the geometric understanding is not good enough? First the geometric understanding is not self-consistent. It contradicts with quantum theory. The consideration based quantum mechanics and Einstein gravity indicates that two points separated by a distance less than the Planck length

$$l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.616199 \times 10^{-35} \text{m} \quad (1)$$

cannot exist as a physical reality (see Fig. 12). Thus the foundation of the geometric approach – manifold – simply does not exist in our universe, since manifold contains points with arbitrary small separation. This suggests that geometry is an emergent phenomenon that appears only at long distances. So we cannot use geometry and manifold as a foundation to understand fundamental physical problems.

Second, Maxwell theory of light and Einstein theory of gravity predict light waves and gravitational waves. But the theories fail to tell us what is waving? Maxwell theory and Einstein theory are built on top of geometry. They fail to answer what is the origin of the apparent geometry that we see. In other words, Maxwell theory and Einstein theory are incomplete, and they should be regarded as effective theories at long distances.

Since geometry does not exist in our world, this is why we say the geometric view of world is challenged by quantum theory. The quantum theory tell us such a point of view to be wrong at length scales of order Planck length. So the quantum theory represents the most dramatic revolution in physics.

II. IT FROM QUBIT, NOT BIT – A SECOND QUANTUM REVOLUTION

After realizing that even the notion of existence is changed by quantum theory, it is no longer surprising to see that quantum theory also blurs the distinction between information and matter. In fact, it implies that information is matter, and matter is information [13]. This is because the frequency is an attribute of information. Quantum theory tells us that frequency is energy $E = hf$, and relativity tells us that energy is mass $m = E/c^2$. Both energy and mass are attributes of matter. So matter = information. This represents a new way to view our world.

The essence of quantum theory
The energy-frequency relation $E = hf$ implies
that **matter = information**.

The above point of view of “matter = information” is similar to Wheeler’s “it from bit”, which represents a deep desire to unify matter and information. In fact, such an unification has happened before at a small scale. We introduced electric and magnetic field to informationally (or pictorially) describe electric and magnetic interaction. But later, electric/magnetic field became real matter with energy and momentum, and even a particle associated with it.

However, in our world, “it” are very complicated. (1) Most “it” are fermions, while “bit” are bosonic. Can fermionic “it” come from bosonic “bit”? (2) Most “it” also carry spin-1/2. Can spin-1/2 arises from “bit”? (3) All “it” interact via a special kind of interaction – gauge interaction. Can “bit” produce gauge interaction? Can “bit” produce waves that satisfy Maxwell equation? Can “bit” produce photon?

In other words, to understand the concrete meaning of “matter from information” or “it from bit”, we note that matter are described by Maxwell equation (photons), Yang-Mills equation (gluons and W/Z bosons), as well as Dirac and Weyl equations (electrons, quarks, neutrinos). The statement “matter = information” means that those wave equations can all come from qubits. In other words, we know that elementary particles (*i.e.* matter) are described by gauge fields and anti-commuting fields in a quantum field theory. Here we try to say that all those very different quantum fields can arise from qubits. Is this possible?

All the waves and fields mentioned above are waves and fields in space. The discovery of gravitational wave strongly suggested that the space is a deformable dynamical medium. In fact, the discovery of electromagnetic wave and the Casimir effect already strongly suggested

that the space is a deformable dynamical medium. As a dynamical medium, it is not surprising that the deformation of space give rise to various waves. But the dynamical medium that describe our space must be very special, since it should give rise to waves satisfying Einstein equation (gravitational wave), Maxwell equation (electromagnetic wave), Dirac equation (electron wave), *etc* . But what is the microscopic structure of the space? What kind of microscopic structure can, at the same time, give rise to waves that satisfy Maxwell equation, Dirac/Weyl equation, and Einstein equation?

Let us view the above questions from another angle. Modern science has made many discoveries and has also unified many seemingly unrelated discoveries into a few simple structures. Those simple structures are so beautiful and we regard them as wonders of our universe. They are also very myterious since we do not understand where do they come from and why do they have to be the way they are. At moment, the most fundamental mysteries and/or wonders in our universe can be summarized by the following short list:

Eight wonders:

- (1) Locality.
- (2) Identical particles.
- (3) Gauge interactions.[14–16]
- (4) Fermi statistics.[17, 18]
- (5) Tiny masses of fermions ($\sim 10^{-20}$ of the Planck mass).[2, 19, 20]
- (6) Chiral fermions.[6, 7, 21, 22]
- (7) Lorentz invariance.[23]
- (8) Gravity.[9]

In the current physical theory of nature (such as the standard model), we take the above properties for granted and do not ask where do they come from. We put those wonderful properties into our theory by hand, for example, by introducing one field for each kind of interactions or elementary particles.

However, here we would like to question where do those wonderful and mysterious properties come from? Following the trend of science history, we wish to have a single unified understanding of all of the above mysteries. Or more precisely, we wish that we can start from a single structure to obtain all of the above wonderful properties.

The simplest element in quantum theory is qubit $|0\rangle$ and $|1\rangle$ (or $|\downarrow\rangle$ and $|\uparrow\rangle$). Qubit is also the simplest element in quantum information. Since our space is a dynamical medium, the simplest choice is to assume the space to be an ocean of qubits. We will give such an ocean a formal name “qubit ether”. Then the matter, *i.e.* the elementary particles, are simply the waves, “bubbles” and other defects in the qubit ocean (or quibt ether). This is how “it from qubit” or “matter = information”.

Qubit, having only two states $|\downarrow\rangle$ and $|\uparrow\rangle$, is very simple. We may view the many-qubit state with all qubits in $|\downarrow\rangle$ as the quantum state that correspond to the empty space (the vacuum). Then the many-qubit state with a few qubits in $|\uparrow\rangle$ correspond to a space with a few spin-0 particles described by a scaler field. Thus, it is easy to see that a scaler field can emerge from qubit ether as a density wave of up-qubits. Such a wave satisfy the Euler equation, but not Maxwell equation or Yang-Mills equation. So the above particular qubit ether is not the one that correspond to our space. It has a wrong microscopic structure and cannot carry waves satisfying Maxwell equation and Yang-Mills equation. But this line of thinking may be correct. We just need to find a qubit ether with a different microscopic structure.

However, for a long time, we do not know how waves satisfying Maxwell equation or

Yang-Mills equation can emerge from any qubit ether. The anti-commuting wave that satisfy Dirac/Weyl equation seems even more impossible. So, even though quantum theory strongly suggests “matter = information”, trying to obtain all elementary particles from an ocean of simple qubits is regarded as impossible by many and has never become an active research effort.

So the key to understand “matter = information” is to identify the microscopic structure of the qubit ether (which can be viewed as space). The microscopic structure of our space must be very rich, since our space not only can carry gravitational wave and electromagnetic wave, it can also carry electron wave, quark wave, gluon wave, and the waves that correspond to all elementary particles. Is such a qubit ether possible?

In condensed matter physics, the discovery of fractional quantum Hall states[24] bring us into a new world of highly entangled many-body systems. When the strong entanglement becomes long range entanglement[25], the systems will possess a new kind of order – topological order[26, 27], and represent new states of matter. We find that the waves (the excitations) in topologically ordered qubit states can be very strange: they can be waves that satisfy Maxwell equation, Yang-Mills equation, or Dirac/Weyl equation. So the impossible become possible: all elementary particles (the bosonic force particles and fermionic matter particles) can emerge from long range entangled qubit ether and be unified by quantum information [2–8, 28, 29].

We would like to stress that the above picture is “it from qubit”, which is very different from Wheeler’s “it from bit”. As we have explained, our observed elementary particles can only emerge from long range entangled qubit ether. The requirement of quantum entanglement implies that “it cannot from bit”. In fact “it from entangled qubits”.

III. A STRING-NET LIQUID OF QUBITS AND A UNIFICATION OF GAUGE INTERACTIONS AND FERMI STATISTICS

In this section, we will consider a particular entangled qubit ocean – a string liquid of qubits. Such entangled qubit ocean support new kind of waves and their corresponding particles. We find that the new waves and the emergent statistics are so profound, that they may change our view of universe. Let us start by explaining a basic notion – “principle of emergence”.

A. Principle of emergence

Typically, one thinks the properties of a material should be determined by the components that form the material. However, this simple intuition is incorrect, since all the materials are made of same components: electrons, protons and neutrons. So we cannot use the richness of the components to understand the richness of the materials. In fact, the various properties of different materials originate from various ways in which the particles are organized. Different orders (the organizations of particles) give rise to different physical properties of a material. It is the richness of the orders that gives rise to the richness of material world.

Let us use the origin of mechanical properties and the origin of waves to explain, in a more concrete way, how orders determine the physics properties of a material. We know that a deformation in a material can propagate just like the ripple on the surface of water. The propagating deformation corresponds to a wave traveling through the material. Since

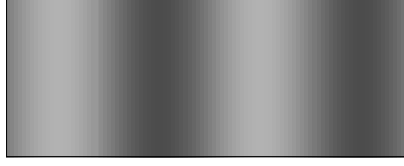


FIG. 13: Liquids only have a compression wave – a wave of density fluctuations.

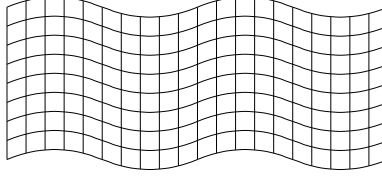


FIG. 14: Drawing a grid on a solid helps us to see the deformation of the solid. The vector u^i in (3) is the displacement of a vertex in the grid. In addition to the compression wave (*i.e.* the density wave), a solid also supports transverse wave (wave of shear deformation) as shown in the above figure.

liquids can resist only compression deformation, so liquids can only support a single kind of wave – compression wave (see Fig. 13). (Compression wave is also called longitudinal wave.) Mathematically the motion of the compression wave is governed by the Euler equation

$$\frac{\partial^2 \rho}{\partial t^2} - v^2 \frac{\partial^2 \rho}{\partial x^2} = 0, \quad (2)$$

where ρ is the density of the liquid.

Solid can resist both compression and shear deformations. As a result, solids can support both compression wave and transverse wave. The transverse wave correspond to the propagation of shear deformations. In fact there are two transverse waves corresponding to two directions of shear deformations. The propagation of the compression wave and the two transverse waves in solids are described by the elasticity equation

$$\frac{\partial^2 u^i}{\partial t^2} - T_j^{ikl} \frac{\partial^2 u^j}{\partial x^k \partial x^l} = 0 \quad (3)$$

where the vector field $u^i(\mathbf{x}, t)$ describes the local displacement of the solid.

We would like to point out that the elasticity equation and the Euler equations not only describe the propagation of waves, they actually describe all small deformations in solids and liquids. Thus, the two equations represent a complete mathematical description of the mechanical properties of solids and liquids.

But why do solids and liquids behave so differently? What makes a solid to have a shape and a liquid to have no shape? What are the origins of elasticity equation and Euler equations? The answer to those questions has to wait until the discovery of atoms in 19th century. Since then, we realized that both solids and liquids are formed by collections of atoms. The main difference between the solids and liquids is that the atoms are organized very differently. In liquids, the positions of atoms fluctuate randomly (see Fig. 15a), while in solids, atoms organize into a regular fixed array (see Fig. 15b).³ It is the different

³ The solids here should be more accurately referred as crystals.



FIG. 15: (a) Particles in liquids do not have fixed relative positions. They fluctuate freely and have a random but uniform distribution. (b) Particles in solids form a fixed regular lattice.

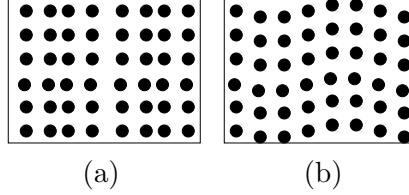


FIG. 16: The atomic picture of (a) the compression wave and (b) the transverse wave in a crystal.

organizations of atoms that lead to the different mechanical properties of liquids and solids. In other words, it is the different organizations of atoms that make liquids to be able to flow freely and solids to be able to retain its shape.

How can different organizations of atoms affect mechanical properties of materials? In solids, both the compression deformation (see Fig. 16a) and the shear deformation (see Fig. 16b) lead to real physical changes of the atomic configurations. Such changes cost energies. As a result, solids can resist both kinds of deformations and can retain their shapes. This is why we have both the compression wave and the transverse wave in solids.

In contrast, a shear deformation of atoms in liquids does not result in a new configuration since the atoms still have uniformly random positions. So the shear deformation is a do-nothing operation for liquids. Only the compression deformation which changes the density of the atoms results in a new atomic configuration and costs energies. As a result, liquids can only resist compression and have only compression wave. Since shear deformations do not cost any energy for liquids, liquids can flow freely.

We see that the properties of the propagating wave are entirely determined by how the atoms are organized in the materials. Different organizations lead to different kinds of waves and different kinds of mechanical laws. Such a point of view of different kinds of waves/laws originated from different organizations of particles is a central theme in condensed matter physics. This point of view is called the principle of emergence.

B. String-net liquid of qubits unifies light and electrons

The elasticity equation and the Euler equation are two very important equations. They lay the foundation of many branches of science such as mechanical engineering, aerodynamic engineering, *etc.* But, we have a more important equation, Maxwell equation, that describes light waves in vacuum. When Maxwell equation was first introduced, people firmly believed that any wave must corresponds to motion of something. So people want to find out what is the origin of the Maxwell equation? The motion of what gives rise electromagnetic wave?

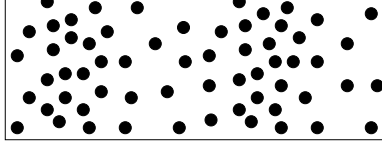


FIG. 17: The atomic picture of the compression wave in liquids.

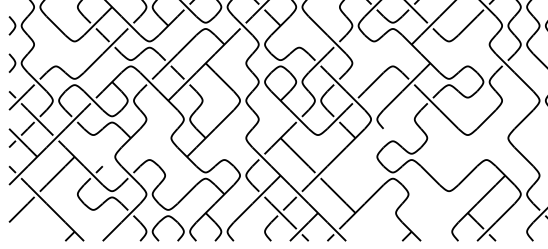


FIG. 18: A quantum ether: The fluctuation of oriented strings give rise to electromagnetic waves (or light). The ends of strings give rise to electrons. Note that oriented strings have directions which should be described by curves with arrow. For ease of drawing, the arrows on the curves are omitted in the above plot.

First, one may wonder: can Maxwell equation comes from a certain symmetry breaking order? Based on Landau symmetry-breaking theory, the different symmetry breaking orders can indeed lead to different waves satisfying different wave equations. So maybe a certain symmetry breaking order can give rise to a wave that satisfy Maxwell equation. But people have been searching for ether – a medium that supports light wave – for over 100 years, and could not find any symmetry breaking states that can give rise to waves satisfying the Maxwell equation. This is one of the reasons why people give up the idea of ether as the origin of light and Maxwell equation.

However, the discovery of topological order[26, 27] suggests that Landau symmetry-breaking theory does not describe all possible organizations of bosons/spins. This gives us a new hope: Maxwell equation may arise from a new kind of organizations of bosons/spins that have non-trivial topological orders.

In addition to the Maxwell equation, there is an even stranger equation, Dirac equation, that describes wave of electrons (and other fermions). Electrons have Fermi statistics. They are fundamentally different from the quanta of other familiar waves, such as photons and phonons, since those quanta all have Bose statistics. To describe the electron wave, the amplitude of the wave must be anti-commuting Grassmann numbers, so that the wave quanta will have Fermi statistics. Since electrons are so strange, few people regard electrons and the electron waves as collective motions of something. People accept without questioning that electrons are fundamental particles, one of the building blocks of all that exist.

However, from a condensed matter physics point of view, all low energy excitations are collective motion of something. If we try to regard photons as collective modes, why can't we regard electrons as collective modes as well? So maybe, Dirac equation and the associated fermions can also arise from a new kind of organizations of bosons/spins that have non-trivial topological orders.

A recent study provides an positive answer to the above questions.[3, 30, 31] We find that if bosons/spins form large oriented strings and if those strings form a quantum liquid



FIG. 19: The fluctuating strings in a string liquid.



FIG. 20: A “density” wave of oriented strings in a string liquid. The wave propagates in x -direction. The “density” vector \mathbf{E} points in y -direction. For ease of drawing, the arrows on the oriented strings are omitted in the above plot.

state, then the collective motion of the such organized bosons/spins will correspond to waves described by Maxwell equation and Dirac equation. The strings in the string liquid are free to join and cross each other. As a result, the strings look more like a network (see Fig. 18). For this reason, the string liquid is actually a liquid of string-nets, which is called string-net condensed state.

But why the waving of strings produces waves described by the Maxwell equation? We know that the particles in a liquid have a random but uniform distribution. A deformation of such a distribution corresponds a density fluctuation, which can be described by a scalar field $\rho(\mathbf{x}, t)$. Thus the waves in a liquid is described by the scalar field $\rho(\mathbf{x}, t)$ which satisfy the Euler equation (2). Similarly, the strings in a string-net liquid also have a random but uniform distribution (see Fig. 19). A deformation of string-net liquid corresponds to a change of the density of the strings (see Fig. 20). However, since strings have an orientation, the “density” fluctuations are described by a vector field $\mathbf{E}(\mathbf{x}, t)$, which indicates there are more strings in the \mathbf{E} direction on average. The oriented strings can be regarded as flux lines. The vector field $\mathbf{E}(\mathbf{x}, t)$ describes the smeared average flux. Since strings are continuous (*i.e.* they cannot end), the flux is conserved: $\partial \cdot \mathbf{E}(\mathbf{x}, t) = 0$. The vector density $\mathbf{E}(\mathbf{x}, t)$ of strings cannot change in the direction along the strings (*i.e.* along the $\mathbf{E}(\mathbf{x}, t)$ direction). $\mathbf{E}(\mathbf{x}, t)$ can change only in the direction perpendicular to $\mathbf{E}(\mathbf{x}, t)$. Since the direction of the propagation is the same as the direction in which $\mathbf{E}(\mathbf{x}, t)$ varies, thus the waves described by $\mathbf{E}(\mathbf{x}, t)$ must be transverse waves: $\mathbf{E}(\mathbf{x}, t)$ is always perpendicular to the direction of the propagation. Therefore, the waves in the string liquid have a very special property: the waves have only transverse modes and no longitudinal mode. This is exactly the property of the light waves described by the Maxwell equation. We see that “density” fluctuations of strings (which are described by a transverse vector field) naturally give rise to the light (or electromagnetic) waves and the Maxwell equation.[2, 3, 31–34]

It is interesting to compare solid, liquid, and string-net liquid. We know that the particles

in a solid organized into a regular lattice pattern. The waving of such organized particles produces a compression wave and two transverse waves. The particles in a liquid have a more random organization. As a result, the waves in liquids lost two transverse modes and contain only a single compression mode. The particles in a string-net liquid also have a random organization, but in a different way. The particles first form string-nets and string-nets then form a random liquid state. Due to this different kind of randomness, the waves in string-net condensed state lost the compression mode and contain two transverse modes. Such a wave (having only two transverse modes) is exactly the electromagnetic wave.

To understand how electrons appear from string-nets, we would like to point out that if we only want photons and no other particles, the strings must be closed strings with no ends. The fluctuations of closed strings produce only photons. If strings have open ends, those open ends can move around and just behave like independent particles. Those particles are not photons. In fact, the ends of strings are nothing but electrons.

How do we know that ends of strings behave like electrons? First, since the waving of string-nets is an electromagnetic wave, a deformation of string-nets correspond to an electromagnetic field. So we can study how an end of a string interacts with a deformation of string-nets. We find that such an interaction is just like the interaction between a charged electron and an electromagnetic field. Also electrons have a subtle but very important property – Fermi statistics, which is a property that exists only in quantum theory. Amazingly, the ends of strings can reproduce this subtle quantum property of Fermi statistics.[30, 35] Actually, string-net liquids explain why Fermi statistics should exist.

We see that qubits that organize into string-net liquid naturally explain both light and electrons (gauge interactions and Fermi statistics). In other words, string-net theory provides a way to unify light and electrons.[3, 31] So, the fact that our vacuum contains both light and electrons may not be a mere accident. It may actually suggest that the vacuum is indeed a string-net liquid.

C. More general string-net liquid and emergence of Yang-Mills gauge theory

Here, we would like to point out that there are many different kinds of string-net liquids. The strings in different liquids may have different numbers of types. The strings may also join in different ways. For a general string-net liquid, the waving of the strings may not correspond to light and the ends of strings may not be electrons. Only one kind of string-net liquids give rise to light and electrons. On the other hand, the fact that there are many different kinds of string-net liquids allows us to explain more than just light and electrons. We can design a particular type of string-net liquids which not only gives rise to electrons and photons, but also gives rise to quarks and gluons.[2, 30] The waving of such type of string-nets corresponds to photons (light) and gluons. The ends of different types of strings correspond to electrons and quarks. It would be interesting to see if it is possible to design a string-net liquid that produces all elementary particles! If this is possible, the ether formed by such string-nets can provide an origin of all elementary particles.⁴

We like to stress that the string-nets are formed by qubits. So in the string-net picture, both the Maxwell equation and Dirac equation, emerge from *local* qubit model, as long as

⁴ So far we can use string-net to produce almost all elementary particles, except for the graviton that is responsible for the gravity. In particular, we can even produce the chiral coupling between the $SU(2)$ gauge boson and the fermions from the qubit ocean [6, 7].

the qubits from a long-range entangled state (*i.e.* a string-net liquid). In other words, light and electrons are unified by the long-range entanglement of qubits!

The electric field and the magnetic field in the Maxwell equation are called gauge fields. The field in the Dirac equation are Grassmann-number valued field.⁵ For a long time, we thought that we have to use gauge fields to describe light waves that have only two transverse modes, and we thought that we have to use Grassmann-number valued fields to describe electrons and quarks that have Fermi statistics. So gauge fields and Grassmann-number valued fields become the fundamental build blocks of quantum field theory that describe our world. The string-net liquids demonstrate we do not have to introduce gauge fields and Grassmann-number valued fields to describe photons, gluons, electrons, and quarks. It demonstrates how gauge fields and Grassmann fields emerge from local qubit models that contain only complex scalar fields at the cut-off scale.

Our attempt to understand light has long and evolving history. We first thought light to be a beam of particles. After Maxwell, we understand light as electromagnetic waves. After Einstein’s theory of general relativity, where gravity is viewed as curvature in space-time, Weyl and others try to view electromagnetic field as curvatures in the “unit system” that we used to measure complex phases. It leads to the notion of gauge theory. The general relativity and the gauge theory are two corner stones of modern physics. They provide a unified understanding of all four interactions in terms of a beautiful mathematical framework: all interactions can be understood geometrically as curvatures in space-time and in “unit systems” (or more precisely, as curvatures in the tangent bundle and other vector bundles in space-time).

Later, people in high-energy physics and in condensed matter physics have found another way in which gauge field can emerge:[36–39] one first cut a particle (such as an electron) into two partons by writing the field of the particle as the product of the two fields of the two partons. Then one introduces a gauge field to glue the two partons back to the original particle. Such a “glue-picture” of gauge fields (instead of the fiber bundle picture of gauge fields) allow us to understand the emergence of gauge fields in models that originally contain no gauge field at the cut-off scale.

A string picture represent the third way to understand gauge theory. String operators appear in the Wilson-loop characterization[40] of gauge theory. The Hamiltonian and the duality description of lattice gauge theory also reveal string structures.[41–44]. Lattice gauge theories are not local bosonic models and the strings are unbreakable in lattice gauge theories. String-net theory points out that even breakable strings can give rise to gauge fields.[45] So we do not really need strings. Qubits themselves are capable of generating gauge fields and the associated Maxwell equation. This phenomenon was discovered in several qubit models[1, 28, 34, 38, 46] before realizing their connection to the string-net liquids.[32] Since gauge field can emerge from local qubit models, the string picture evolves into the entanglement picture – the fourth way to understand gauge field: gauge fields are fluctuations of long-range entanglement. I feel that the entanglement picture capture the essence of gauge theory. Despite the beauty of the geometric picture, the essence of gauge theory is not the curved fiber bundles. In fact, we can view gauge theory as a theory for long-range entanglement, although the gauge theory is discovered long before the notion of long-range entanglement. The evolution of our understanding of light and gauge interaction: particle beam \rightarrow wave \rightarrow electromagnetic wave \rightarrow curvature in fiber bundle \rightarrow

⁵ Grassmann numbers are anti-commuting numbers.

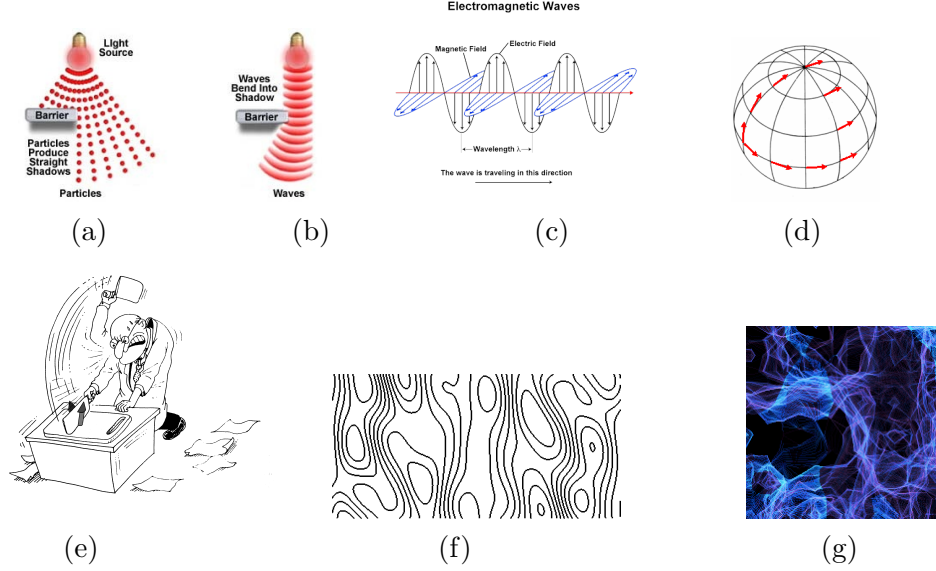


FIG. 21: The evolution of our understanding of light (and gauge interaction): (a) particle beam, (b) wave, (c) electromagnetic wave, (d) curvature in fiber bundle, (e) glue of partons, (f) wave in string-net liquid, (g) wave in long-range entanglement of many qubits.

glue of partons \rightarrow wave in string-net liquid \rightarrow wave in long-range entanglement, represents 200 year’s effort of human race to unveil the mystery of universe (see Fig. 21).

Viewing gauge field (and the associated gauge bosons) as fluctuations of long-range entanglement has an added bonus: we can understand the origin of Fermi statistics in the same way: fermions emerge as defects of long-range entanglement, even though the original model is purely bosonic. Previously, there are two ways to obtain emergent fermions from purely bosonic model: by binding gauge charge and gauge flux in $(2+1)\text{D}$, [47, 48] and by binding the charge and the monopole in a $U(1)$ gauge theory in $(3+1)\text{D}$. [49–53] But those approaches only work in $(2+1)\text{D}$ or only for $U(1)$ gauge field. Using long-range entanglement and their string-net realization, we can obtain the simultaneous emergence of both gauge bosons and fermions in *any* dimensions and for any gauge group. [2, 30, 31, 35] This result gives us hope that maybe every elementary particles are emergent and can be unified using local qubit models. Thus, long-range entanglement offer us a new option to view our world: maybe our vacuum is a long-range entangled state. It is the pattern of the long-range entanglement in the vacuum that determines the content and the structures of observed elementary particles. Such a picture has an experimental prediction that will be described in the next section III D.

We like to point out that the string-net unification of gauge bosons and fermions is very different from the superstring theory for gauge bosons and fermions. In the string-net theory, gauge bosons and fermions come from the qubits that form the space, and “string-net” is simply the name that describe how qubits are organized in the ground state. So string-net is not a thing, but a pattern of qubits. In the string-net theory, the gauge bosons are waves of collective fluctuations of the string-nets, and a fermion corresponds to one end of string. In contrast, gauge bosons and fermions come from strings in the superstring theory. Both gauge bosons and fermions correspond to small pieces of strings. Different vibrations of the small pieces of strings give rise to different kind of particles. The fermions in the superstring

theory are put in by hand through the introduction of Grassmann fields.

D. A falsifiable prediction of string-net unification of gauge interactions and Fermi statistics

In the string-net unification of light and electrons,[3, 31] we assume that the space is formed by a collection of qubits and the qubits form a string-net condensed state. Light waves are collective motions of the string-nets, and an electron corresponds to one end of string. Such a string-net unification of light and electrons has a falsifiable prediction: *all fermionic excitations must carry some gauge charges*. [30, 35]

The $U(1) \times SU(2) \times SU(3)$ standard model for elementary particles contains fermionic excitations (such as neutrons and neutrinos) that do not carry any $U(1) \times SU(2) \times SU(3)$ gauge charge. So according to the string-net theory, the $U(1) \times SU(2) \times SU(3)$ standard model is incomplete. According to the string-net theory, our universe not only have $U(1) \times SU(2) \times SU(3)$ gauge theory, it must also contain other gauge theories. Those additional gauge theories may have a gauge group of Z_2 or other discrete groups. Those extra discrete gauge theories will lead to new cosmic strings which will appear in very early universe.

IV. A NEW CHAPTER IN PHYSICS

Our world is rich and complex. When we discover the inner working of our world and try to describe it, we often find that we need to invent new mathematical language describe our understanding and insight. For example, when Newton discovered his law of mechanics, the proper mathematical language was not invented yet. Newton (and Leibniz) had to develop calculus in order to formulate the law of mechanics. For a long time, we tried to use theory of mechanics and calculus to understand everything in our world.

As another example, when Einstein discovered the general equivalence principle to describe gravity, he needed a mathematical language to describe his theory. In this case, the needed mathematics, Riemannian geometry, had been developed, which led to the theory of general relativity. Following the idea of general relativity, we developed the gauge theory. Both general relativity and gauge theory can be described by the mathematics of fiber bundles. Those advances led to a beautiful geometric understanding of our world based on quantum field theory, and we tried to understand everything in our world in term of quantum field theory.

Now, I feel that we are at another turning point. In a study of quantum matter, we find that long-range entanglement can give rise to many new quantum phases. So long-range entanglement is a natural phenomenon that can happen in our world. They greatly expand our understanding of possible quantum phases, and bring the research of quantum matter to a whole new level. To gain a systematic understanding of new quantum phases and long-range entanglement, we like to know what mathematical language should we use to describe long-range entanglement? The answer is not totally clear. But early studies suggest that tensor category [30, 54–60] and group cohomology [61, 62] should be a part of the mathematical frame work that describes long-range entanglement. The further progresses in this direction will lead to a comprehensive understanding of long-range entanglement and topological quantum matter.

However, what is really exciting in the study of quantum matter is that it might lead

to a whole new point of view of our world. This is because long-range entanglement can give rise to both gauge interactions and Fermi statistics. In contrast, the geometric point of view can only lead to gauge interactions. So maybe we should not use geometric pictures, based on fields and fiber bundles, to understand our world. Maybe we should use entanglement pictures to understand our world. This way, we can get both gauge interactions and fermions from a single origin – qubits. We may live in a truly quantum world. So, quantum entanglement represents a new chapter in physics.

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