Heat transfer in flat-plate boundary layers: a correlation for laminar, transitional, and turbulent flow

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We Can't Neglect the Transition Region

Transition region and laminar region have similar length

\[ h(x) = \begin{cases} \text{laminar} & x < x_l \\ \text{transition} & x_l < x < x_u \\ \text{turbulent} & x > x_u \end{cases} \]

The two step, laminar-then-turbulent model is incorrect!

Summary of the Correlation

For smooth, sharp-edged, flat plates with zero pressure gradient and either uniform wall temperature (UWT) or uniform heat flux (UHF)

### Combining formula

\[ \text{Nu}_x(Re, Pr) = \left( \frac{\text{Nu}_{\text{laminar}}^{10} + \text{Nu}_{\text{turbulent}}^{10}}{2} \right)^{1/10} \]  

**Eq. (9)**

#### Laminar region

\[ \text{Nu}_{\text{laminar}}(Re, Pr) = \frac{0.332 \, Re^{1/3} \, Pr^{1/3}}{0.453 \, Re^{1/3} \, Pr^{1/3}} \] for UWT

\[ \text{Nu}_{\text{laminar}}(Re, Pr) = \frac{0.332 \, Re^{1/3} \, Pr^{1/3}}{0.453 \, Re^{1/3} \, Pr^{1/3}} \] for UHF

With an unheated starting length of \( x_u \) (UHF or UWT), use

\[ \text{Nu}_{\text{laminar}}(Re, Pr) = \left( 1 - \left( \frac{x_u}{x_u} \right)^{2/3} \right)^{1/3} \]

#### Transition region

\[ \text{Nu}_{\text{transition}}(Re, Pr) = \text{Nu}_{\text{laminar}}(Re, Pr) \left( Re / Re_l \right)^c \]

\( c = 0.992 \, \log_{10} Re_l - 3.013 \) for \( Re < 5 \times 10^5 \)

#### Turbulent region (UHF and UWT)

\[ \text{Nu}_{\text{turbulent}}(Re, Pr) = \frac{Re \, Pr \, C_f(Re) / 2}{1 + 12 \, C_f(Re) / (Pr^{2/3} - 1) \, C_f(Re)^{1/2}} \]

**Eq. (6)**

\[ C_f(Re) = \frac{0.455}{\ln(0.6 \, Re^{2/3})} \]

For gases only, the following equation has similar accuracy

\[ \text{Nu}_{\text{turbulent}}(Re, Pr) = 0.0296 \, Re^{0.8} \, Pr^{0.6} \]

for gases

Typical Results (more data & fluids in paper)

**Data from Multiple Independent Experiments**

0.7 \( \leq Pr \leq 257 \)

4,000 \( \leq Re \leq 4,300,000 \)

free-stream turbulence levels up to 5%

**Fully turbulent air data fit to std. dev. of ±5.5%**

**Classical Colburn analogy (1933)**

Not recommended: Colburn's \( St = (C_f/2) \, Pr^{2/3} \) was based on b.l. data for air and does not support a wide range of \( Pr \). Colburn's suggestion to use it for laminar flow compared a UWT formula to misplotted UHF data.

**Similarity solution for UHF laminar b.l.**

This result (Fage & Falkner, 1931; Imai, 1958) is not widely known

\[ \text{Nu}_{\text{infty}} = 0.4587 \, Re^{1/3} \, Pr^{1/3} \]

similarity solution

but close to integral-method (replace 0.4587 by 0.4535). Pre-1950, wall boundary conditions often overlooked (Colburn 1933; Jakob & Dow 1946).