We Can't Neglect the Transition Region

Transition region and laminar region have similar length

\[ h(x) \sim x^{-1/2} \]
\[ h(x) \sim x^{-0.2} \]

The two-step, laminar-then-turbulent model is incorrect!

Summary of the Correlation

For smooth, sharp-edged, flat plates with zero pressure gradient and either uniform wall temperature (UWT) or uniform heat flux (UHF)

Combining formula

\[ \text{Nu}(Re, Pr) = \left[ \text{Nu}_{\text{lam}}^6 + \left( \text{Nu}_{\text{trans}}^{10} + \text{Nu}_{\text{turb}}^{-5} \right)^{-1/2} \right]^{1/5} \]

Eq. (9)

Laminar region

\[ \text{Nu}_{\text{lam}}(Re, Pr) = \frac{0.332 Re^{1/2}Pr^{1/3}}{Pr = 10^5} \]

UWT
\[ \frac{0.453 Re^{1/2}Pr^{1/3}}{Pr = 10^5} \]

UHF

With an unheated starting length of \( x_1 \) (UHF or UWT), use

\[ \text{Nu}_{\text{lam}}(Re, Pr) \cdot \left[ 1 - \left( x_1 / x \right)^{3/4} \right]^{-1/3} \]

Transition region

\[ \text{Nu}_{\text{trans}}(Re, Pr) = \text{Nu}_{\text{trans}}(Re, Pr) \cdot \left( Re / Re_1 \right)^c \]

\[ c = 0.9922 \log_{10} Re_1 - 3.013 \text{ for } Re_1 < 5 \times 10^5 \]

Turbulent region (UHF and UWT)

\[ \text{Nu}_{\text{turb}}(Re, Pr) = \text{Re} \cdot \text{Pr}(C_f/2) \]

Eq. (6)

\[ C_f(Re) = \frac{1 + 12.7 (Pr^{2/3} - 1) / (Pr^{2/3})}{\ln(0.06 Re)} \]

For gases only, the following equation has similar accuracy

\[ \text{Nu}_{\text{turb}}(Re, Pr) = 0.0296 \text{Re}^{0.8} \text{Pr}^{0.6} \text{for gases} \]

Data from Multiple Independent Experiments

0.7 < Pr ≤ 257 4,000 ≤ Re ≤ 4,300,000

Free-stream turbulence levels up to 5%

Fully turbulent air data fit to std. dev. of ±5.5%

Classical Colburn analogy (1933)

Not recommended: Colburn’s St = \( (C_f/2) Pr^{2/3} \) was based on b.l. data for air and does not support a wide range of Pr. Colburn’s suggestion to use it for laminar flow compared a UWT formula to misplotted UHF data.

Similarity solution for UHF laminar b.l.

This result (Fage & Falkner, 1931; Imai, 1958) is not widely known

\[ \text{Nu}_{\text{lam}} = 0.4587 \text{Re}^{1/2} \text{Pr}^{1/3} \]

Similarity solution

but close to integral-method (replace 0.4587 by 0.4535). Pre-1950, wall boundary conditions often overlooked (Colburn 1933; Jakob & Dow 1946)