Stress line generation for structurally performative architectural design

Stress Line Generation for Structurally Performative Architectural Design

Abstract

Principal stress lines, which are pairs of orthogonal curves that indicate trajectories of internal forces and therefore idealized paths of material continuity, naturally encode the optimal topology for any structure for a given set of boundary conditions. Although stress line analysis has the potential to offer a direct, and geometrically-provocative approach to optimization that can synthesize both design and structural objectives, its application in design has generally been limited due to the lack of standardization and parameterization of the process for generating and interpreting stress lines. Addressing these barriers that limit the application of the stress line methods, this paper proposes a new implementation framework that will enable designers to take advantage of stress line analysis to inform conceptual structural design. Central to the premise of this research is a new conception of structurally-inspired design exploration that does not impose a singular solution, but instead allows for the exploration of a diverse high-performance design space in order to balance the combination of structural and architectural design objectives.

1. Introduction

Design motivated by geometry and form

In architectural design, structural performance, efficiency, and expressiveness are often goals that can best be achieved when integrated into generative processes in conceptual design. Such efforts, while varied in specific methodologies, attempt to capitalize on the critical relationship between architectural geometry and structural behavior, and can lead to innovation in each: efficient structures often entail complex geometric solutions that can be formally compelling. However, the realization of structural-led exploration has generally been unmet due to a lack of advanced design tools that can effectively synthesize architectural and structural considerations (Mueller and Ochsendorf 2013).

One prominent exception is in the field of surface structures, which have received significant attention from researchers in recent years. The advent of interactive, dynamic, and computational methods for solving three-dimensional equilibrium structures, such as particle-spring systems (Kilian 2005) implemented in parametric tools like Kangaroo (Piker 2013), has combined with advances in digital fabrication to enable designers to generate structurally-inspired forms that are highly curvilinear and complex. However, the focus has centered primarily on global geometries. In comparison, little design-oriented research has been conducted to create high-performance topologies for these complex surfaces.

Since the final structural performance of any structure is influenced considerably by its topology, which also has major impact on visual appearance and constructibility, there is a need for more knowledge and tools that enable designers to create efficient and elegant topologies for surface structures. This paper proposes such a method, based on the theory of principal stress lines, an increasingly popular concept that nevertheless lacks sufficient investigation in the architectural design realm.

A simple shell geometry discretized with a stress-line-based topology is presented in Figure 1 as a motivating example. Analysis has found the stress-line-inspired topology to be more efficient than the grid-based topology that was used initially to form-find the geometry.

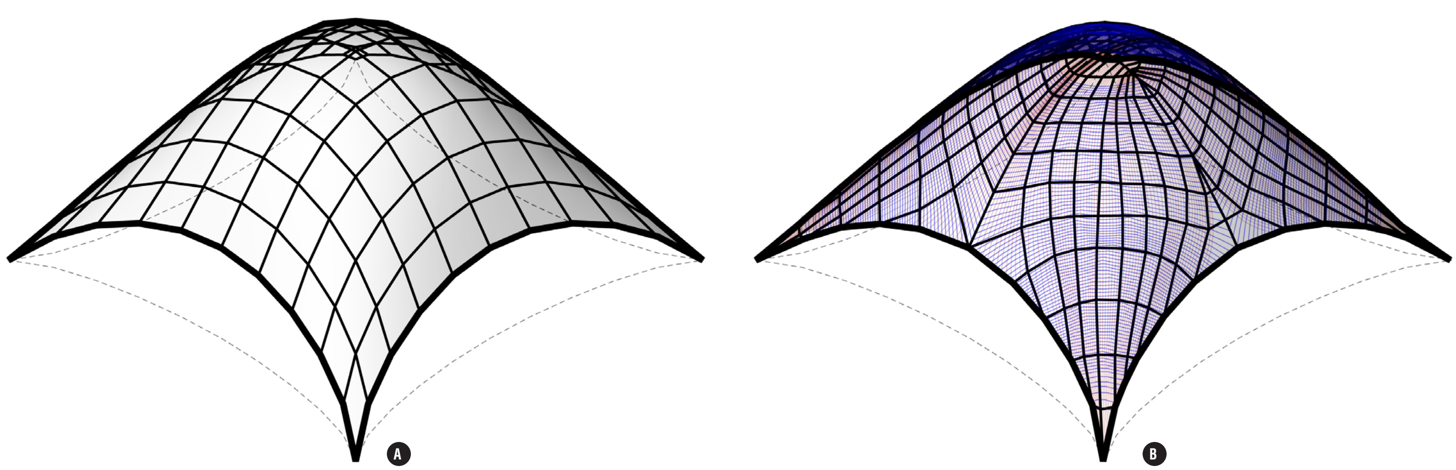


Figure 1 Motivating example of a grid shell with topology based on: A) original mesh grid, and B) principal stress lines

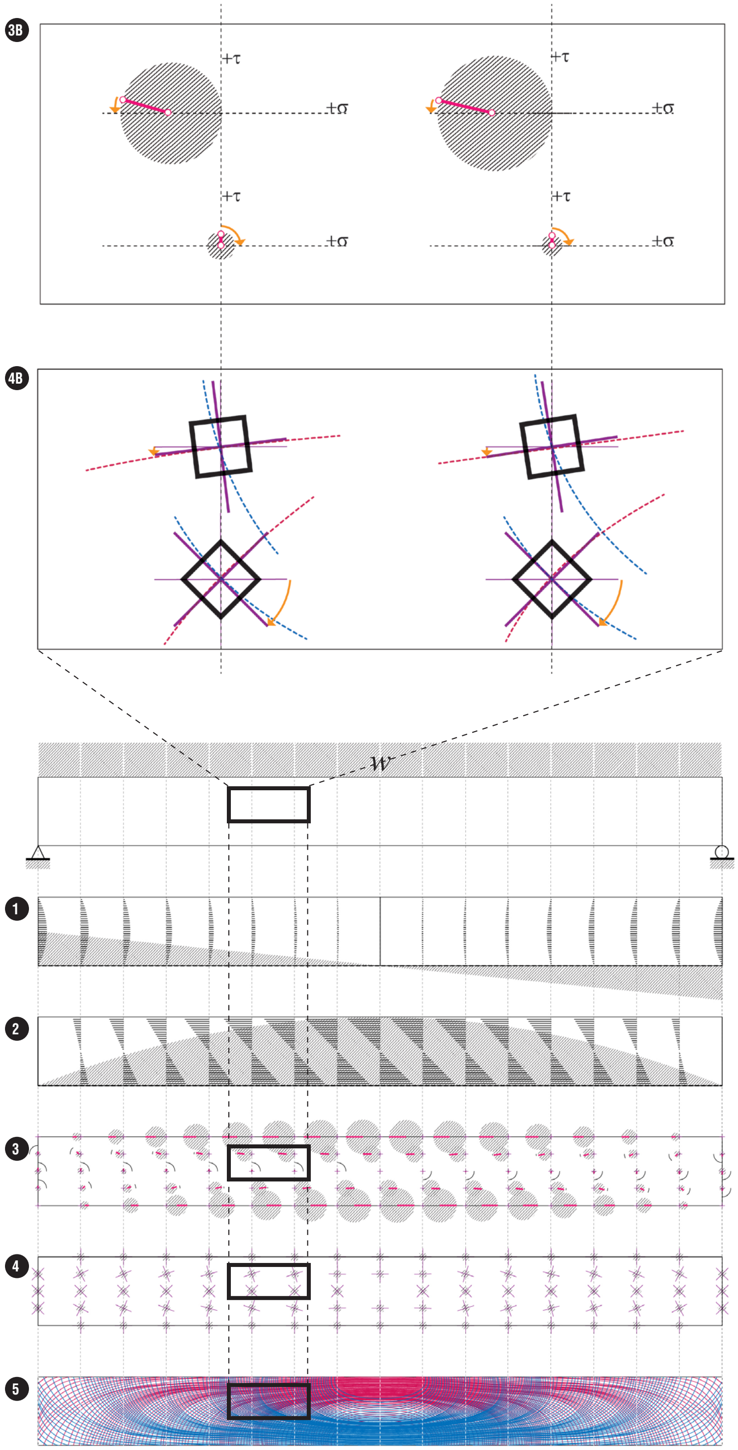


Figure 2
Simply-supported beam under uniformly-distributed load
1) Shear stress distribution over cross Section, and shear diagram
2) Bending stress distribution over cross section, and moment diagram
3) Mohr's circle construction for selected points across beam's span
3B) Detailed Mohr's circle construction for selected beam area
4) Reoriented plane with principal stress directions
4B) Detailed reorientation of plane element and initial stress line projections for selected beam area
5) Principal stress line field for simply-supported beam

2. Background on stress line

2.1 Principle stress direction

To understand how principal stress lines are constructed, a review of principal stress directions is needed. First, any structural continuum can be decomposed infinitely into infinitesimal cubical elements to describe the state of stress for each point. In the case of a two-dimensional structural continuum, the state of stress can uniquely be represented by two normal stress components and one shear stress component.

Rotating the element, the state of stress will remain unchanged, but the stress components will correspond to the new orientation. Stress transformation allows the orientation of planes with special stress properties to be obtained. Specifically, the orientation of normal stress components corresponding to the planar orientation in which the shear stress is zero and normal stresses are maximum is known as principal stress directions (Hibbeler 2011).

2.1.1 Principal stress lines and properties

When a sufficient number of principal stress planes are determined for a collection of points across the structural body, the principal stress line field is constructed by connecting these projections. Structural designers are interested in principal stress lines because they provide a visualization of the natural force flow of an applied load on the system, which shows the lines of desirable material continuity for a given design domain.

As Figure 2 illustrates, there is a striking regularity and order to the patterns created by principal stress lines. Mathematically, the required transformation orientation is expressed in the following equation:

$$\tan 2\theta = \frac{2\,\tau_{xy}}{(\sigma_x - \sigma_y)}$$

Therefore, the solution has two roots that are set at 90° apart, which establishes the visually distinctive quality of stress lines as two orthogonally intersecting families of curves.

Given a design domain for a finite system of isotropic material operating within the elastic range, the following properties must also hold, as documented by Chen and Li (2010):

1) Principal stress directions are neither affected by changes to material stiffness, nor the rescaling of the applied forces,

2) Principal stress field are affected by attributes of the design domain, such as the location and degree of fixities of the loading and support conditions respectively, and the geometry of the continuum structure, and

3) The optimal shape for a design domain is contained in the principal stress field.

The consistent regularity of principal stress fields, with their potent capacity to suggest optimal topology as a function of geometry regardless of materiality, make the principal stress line technique a compelling methodology for constructing an all-encompassing framework for topology finding.

2.1.2 Topology optimization and convergence

Although the numerical production of stress-line-based topology

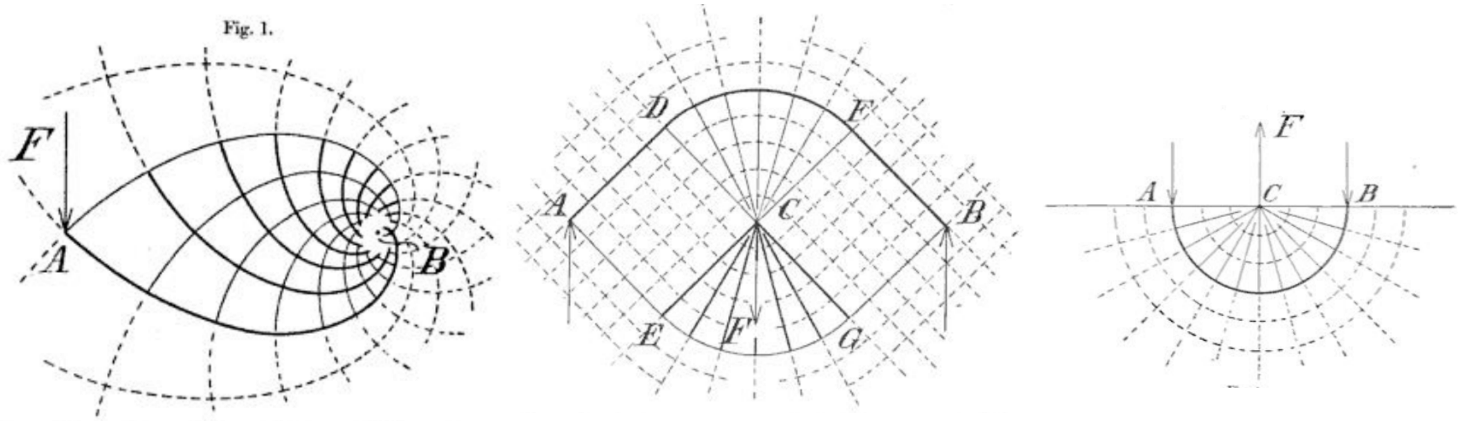


Figure 3 Michell's analytical formulation of optimum trusses for various cases (1904).

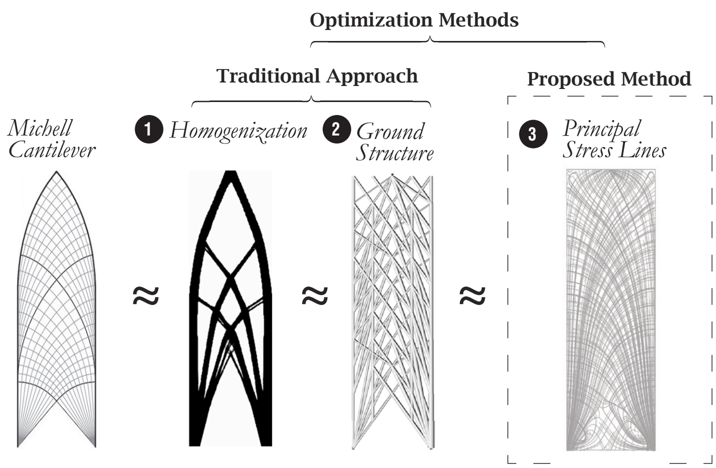


Figure 4 Convergence of optimization results

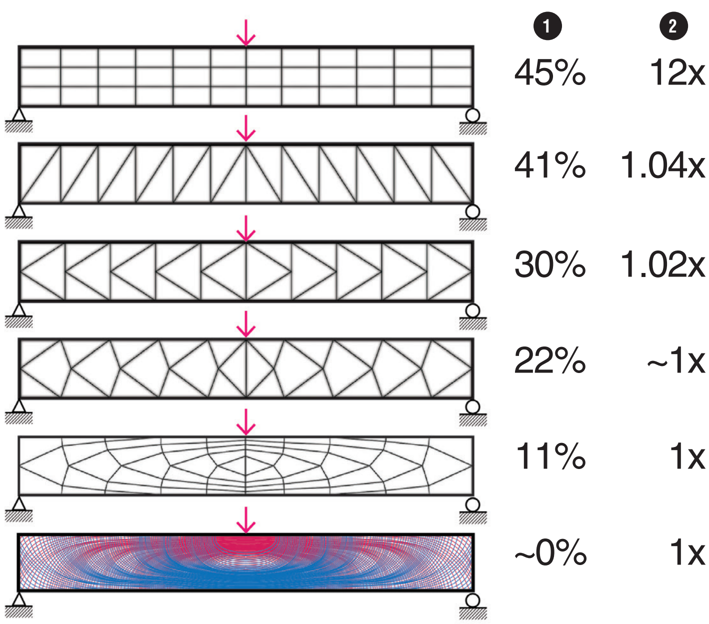


Figure 5 Simply-supported trusses' stress lines conformity and performances: 1) Average angle deviation 2) Material quantity

constitutes a relatively new inquiry, the research traces its roots to classical studies on structural optimization by Michell. Michell formulated the analytical derivation for several well-known optimal truss structures (1904). While Michell was not explicitly considering principal stress trajectories, his results closely resemble the principal stress lines for the design domain he examined, as Figure 3 and 4 show.

The knowledge of Michell's theory, however, does not easily deliver hints on the optimal geometric layout for any design domain (Sokół 2011). Solutions are often infeasible to obtain, such that the considerable research that has since been developed relates largely to numerical methods that seek to reveal the optimum structure through highly computationally intensive procedures. These methods include ground structure and homogeneous methods, which are commonly afflicted with issues like disconnected structures and gray areas that render results unusable (Lu and Kota 2006).

When meaningful results are produced, a convergence is typically evident between the suggested optimal and the principal stress lines of the given design domain. These recurring resemblances suggests the potential value of a more direct approach for obtaining optimal structural topologies that uses principal stress analysis.

Figure 5 illustrates the potential efficacy of a stress-line-based method of topological-finding: the performance of a simply supported truss improves as it becomes more similar to the principal stress lines of its design domain.

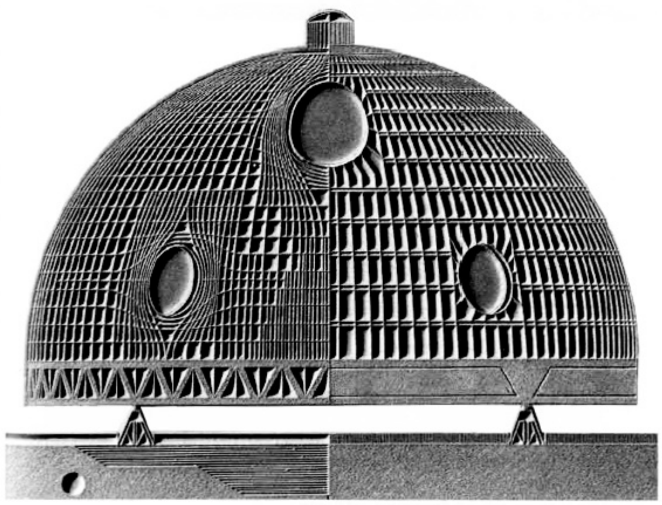
2.2 Potential design applications

2.2.1 Engineering and architectural precedents

Despite the research gap on the topic of stress-line-inspired design, a number of important practitioners have adapted the theory of stress lines to various creative applications.

In architecture, it is widely documented that Nervi's structural designs are frequently influenced by the idea of 'force flow,' with Gatti Wol Mill being the most notable implementation (Halpern, Billington, and Adriaenssens 2013). Similarly, Catalano has

Figure 6 Stress-line-inspired architectural precedents:
Above: Gatti Wool Factory floor system (Halpern, Billington, and Adriaenssens 2013)
Below: Catalano's spherical design proposal (Allen 2010)



capitalized the concept in many of his design proposals (Allen 2010). In recent years, interests in stress lines have reemerged across as advances in digital fabrication technologies have enabled the rapid production of complex curvilinear forms (Block and Rippmann 2013). These examples, however, are exceptional cases led by expert practitioners. To the rest of the engineering and design communities, the valuable guidance potential of stress lines remains unmet due to the lack of parameterization, standardization, and evaluation on the process for generating, interpreting, and analyzing stress lines.

2.2.2 Problem and objective

The research is important because conventional tools available to designers for generating stress lines are generally concerned with the visualization of stress flow, as opposed to the manifestation of force flow under pragmatic design constraints intended for materialization.

In tools integrated with parametrized design interfaces, such as Millipede (Panagiotis 2014) and Karamba (Preisinger 2015), few functionalities are available to incorporate designer inputs. Even when adjustments informed by structural considerations are possible, there is a lack of documentation on the structural implication of these settings. More problematic still is the lack of discretization of the various stages of stress line interpolation, as these tools merely deliver an end result, thereby depriving the designer the capacity to alter geometric characteristics of the stress lines that are intrinsic to the method at the various stages of generation. Furthermore, there is no guarantee that the produced stress lines will lead to usable structural patterns (see Figure 7).

Acknowledging that few design applications of stress line methods in design have emerged due to the problems identified, this paper proposes a consolidated framework for understanding how stress lines can be adapted in design. Particularly, the research is motivated to make stress lines generation a transparent process that is highly configurable by designers. The design application of stress lines is strongest when designers can actively participate in its generation.

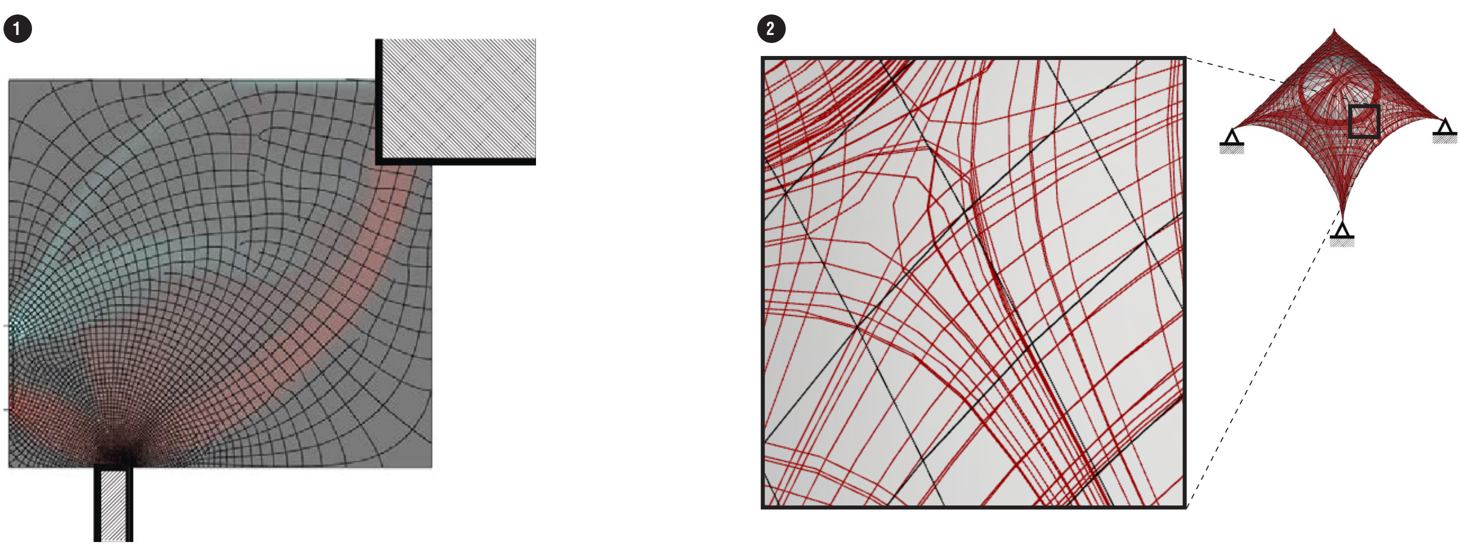


Figure 7 Common problems found in stress lines produced by popular designer-oriented structural analysis tool: 1) Discontinuous lines, and 2) Poor resolution, overlaps, and undesirable intersections

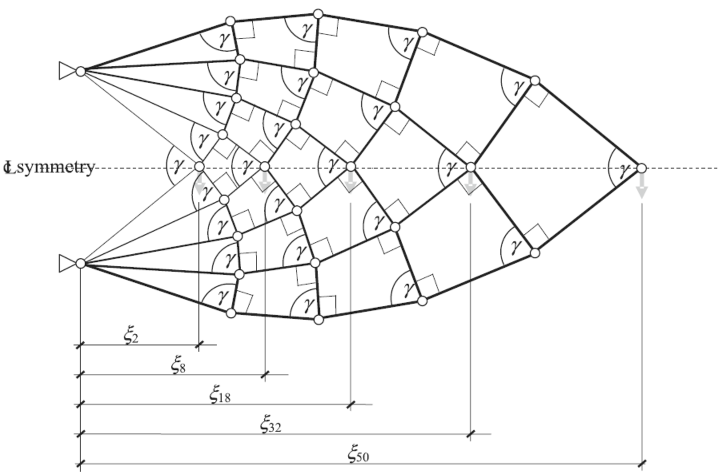


Figure 8 Geometric characterization of Michell cantilever (Mazurek, Baker and Tort 2011)

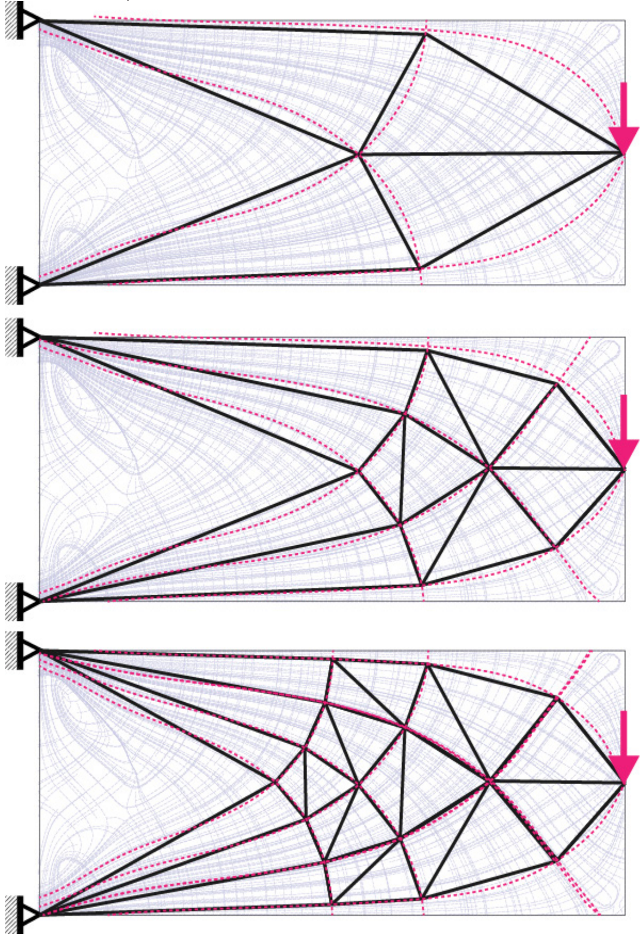


Figure 9 Modified implementation of Chen and Li's method (2010)

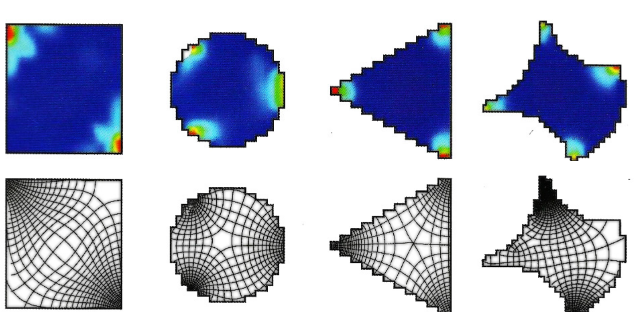


Figure 10 Panagiotis and Kaijima's method to scale stress line by patterns (2014)

3. Stress line construction: theory

3.1 Process overview

3.1.1 Direct stress line construction

Structural patterns indicative of the internal stress trajectories of forces can be obtained in several approaches. These methods fall under two categories: 1) Direct, or 2) Iterative.

Direct method can be analytical or graphical: the optimum layout is determined either mathematically by satisfying theoretical constraints, or from geometric descriptions that seek to characterize the analytical formulations. Consequently, direct methods are prescribed, and do not require the use finite element analysis (FEA). An example of direct analytical approach is Michell's optimal truss derivation (1904), whereas Mazurek et al.'s graphical characterization of the same cantilever is an example of a direct geometrical approach (2011), as shown in Figure 8.

3.1.2 Iterative stress line interpolation

The problem with the direct approach is that existing formulations are derived only for limited cases. Thus, most stress line construction methods are instead numerical and iterative. The overall process is as followed: given an initial point, or seed, the principal stress directions for that point are found. A line is drawn along these directions, and its end point becomes the starting point for the subsequent iteration. This process repeats until the stress line reaches the design boundaries. With the conclusion of the stress line for one seed, the drawing of a new stress line corresponding to the next seed in the sequence begins (Panagiotis and Kaijima 2014). The number of seeding points are calibrated to create a uniformly dense base stress line field.

Not surprisingly, the quality of stress lines from iterative approaches varies widely: the results depend on the parameters in the different stages of production, and on the methods used to calculate the stress direction. Conventionally, stress directions in the iterative approach are calculated numerically with FEA. Directions, however, can also be calculated exactly by transforming analytically-determined stress components.

While analytical calculations offer precision, in cases involving complex structural designs, it may not be convenient or possible to derive the theoretical derivations. To maximize application potential, this paper develops a method for constructing stress lines using a common FEA tool, while addressing the problems that are inherent to the iterative numerical approach, which include 1) low stress line resolution 2) poor stress direction interpolation, and 3) stress line discontinuities (Halpern, Billington, and Adriaenssens 2013).

3.1.3 Post-processing research

Following the initial construction of a base stress line field, procedures are then applied to process select stress lines for materialization. Some relevant research has been presented recently in these areas.

On post-processing, Michalatos and Kaijima (2014) implemented a periodic global reparameterziation algorithm that enables the scaling of the principal stress vector field according to an input scalar field, such as a projected image, which may encode performance values. The approach, which is shown in Figure 9, allows the density of the resultant field to vary roughly according to

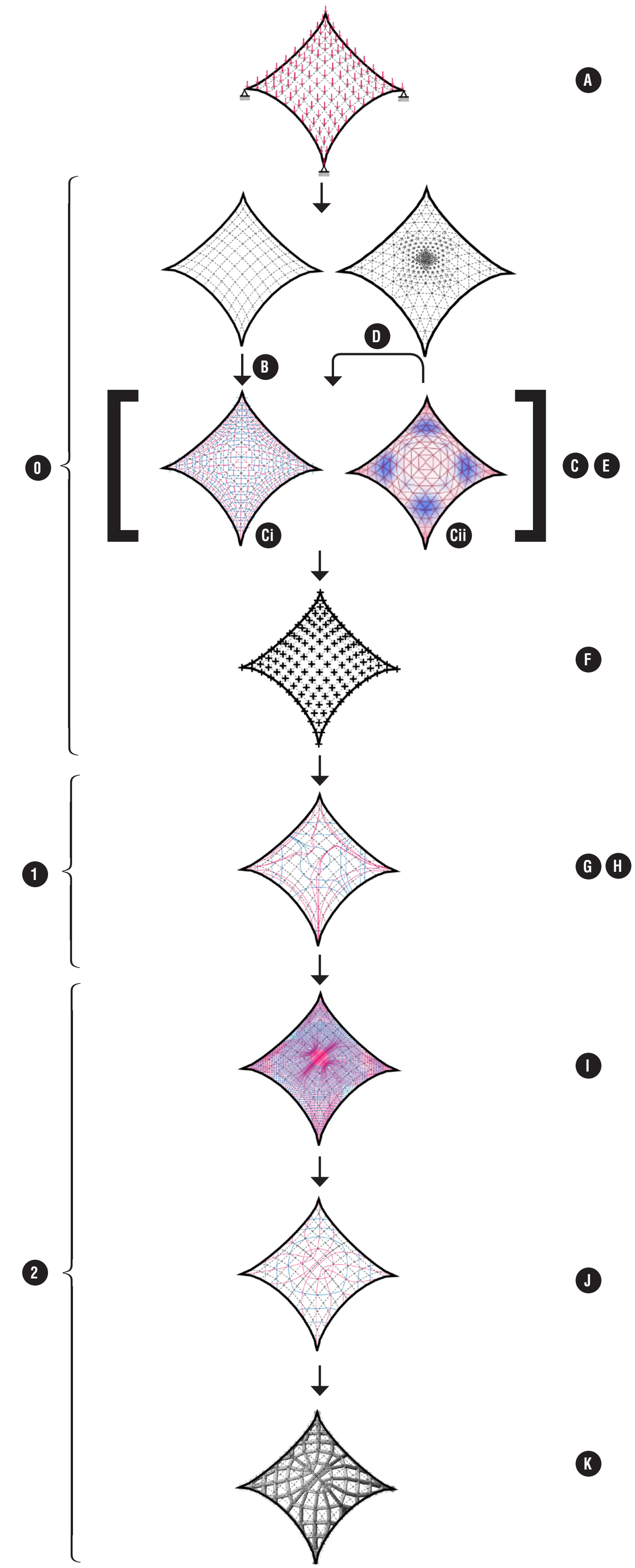


Figure 11 Proposed stress line process: 0) Preparation, 1) Generation, and 2) Post-processing; Subroutines: A) Parallel shell domain specification, B) Initial surface meshing, C) FEA structural analysis, which obtains Ci) Principal stress directions, and Cii) Various utilization metrics, D) Remeshing, E) FEA reanalysis, F) Seeding, G) Interpolation, H) Tracing, I) Uniformly-spaced base stress line field, J) Selection, K) Materialization

stress magnitude.

Working on the problem of strategic stress line selection, Chen and Li (2006) devised a simple algorithm that incrementally builds a stress line-based structure by co-opting a new set of stress line curvatures to reduce the approximation error in each iteration, thus giving the designer some control over the subdivision of the stress line-based structure (see Figure 10).

These approaches allude to the participation of the designers, who are empowered to affect the initial stress line field according to multiple considerations. More importantly, they do not directly materialize the initially constructed stress line field, but instead use it as a reference on which potential discretized elements are based.

3.2 Proposed process outline

Although a number of interesting research results related to stress lines have emerged recently, the fundamental procedure dictating the drawing of stress lines has remained largely unchanged, even when there is significant room for further development. Research on stress line methods is commonly compartmentalized into two approaches: a brief procedure on seeding and drawing, followed by a disproportionately elaborate post-processing procedure. In addition, there is little quantitative documentation on the impact of process variation to structural performance.

This paper develops a stress line computation and materialization framework that expands the fundamental procedure, and develops the possibilities that are intrinsic to each stage of the process. Specifically, the production of stress-line-inspired topology is divided into three stages: initialization, generation, and selection. Figure 11 illustrates the breakdown.

The purpose of initialization is to construct an appropriate mesh topology to characterize the design domain investigated, to conduct the initial analysis from which structural data are obtained to form the basis of the stress line construction, and to create an appropriate seeding plan.

In generation, this paper explores different methods for interpolating stress trajectories, and significantly expands on the general tracing algorithm to include rule-based corrections that can help reduce the numerical noise that is often present in stress line interpolation.

Finally, the implementation concludes with strategies for processing and selecting the stress lines based on performance criteria.

As an overall objective, the proposed framework seeks to minimize the reliance on FEA, by adopting a number of geometric criteria.

3.3 Scope and case studies

Although stress-line-based solutions have theoretical application potential in all structural systems, the implementation demonstrated in this paper focuses on both planar and form-found 2.5D membrane structures. Since members in these systems are subjected only to in-plane stresses, their normal stresses would be constant across their cross section depth contributed primarily by axial forces with negligible bending. Particularly, the proposed framework is implemented on five main structural types: 1-2) planar cantilever and simply supported beams under a point load, and 3-5) regular form-found grid shells with 3-, 4-, and 5- supports (see Figure 12).

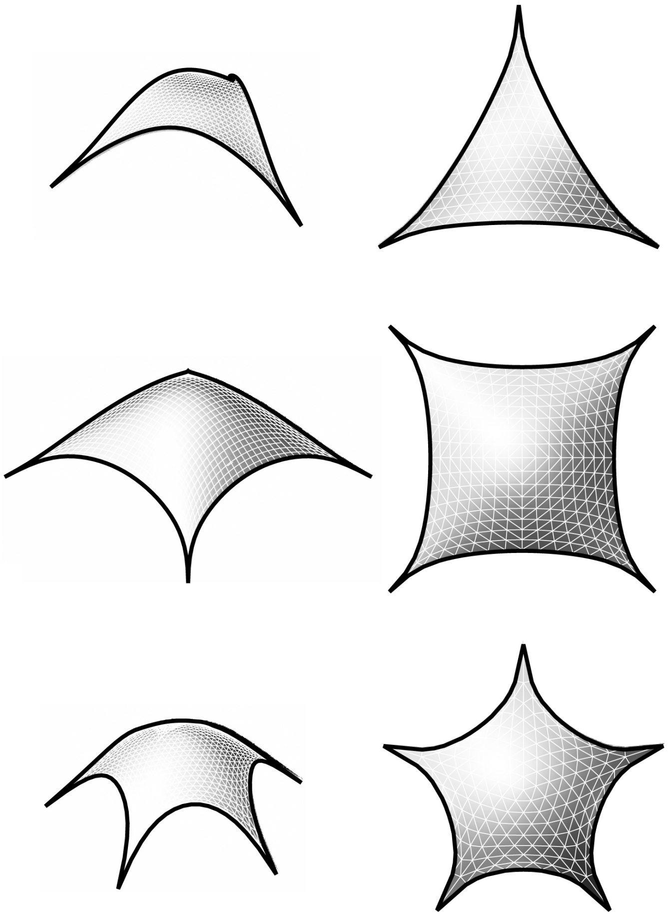


Figure 12 Grid shells explored in this paper

3.4 Commercial Computational Tools

To maximize the design potential of the proposed framework, the research presented on this paper is developed using popular 3D modeling tool Rhinoceros 3D (McNeel 2015), within the parametric visual programming language environment of Grasshopper 3D (Rutten 2013). Structural analyses were conducted using the plug-in Karamba, whereas the various surfaces used to implement the proposed stress line based framework were initially form-found using Kangaroo Physics.

3.5 Principal stress direction conformity

A central metric that is developed on this paper, which is used both to assess, and steer the development of stress-line-based results, is *angle conformity*. Since the research is based on the concept that materialization of stress lines can lead to efficient structures, it is important that there is a consistent method for measuring the closeness of a given topology to the base stress line field of the given design domain. Essentially, the orientation of each member is compared against its closest set of principal stress directions in a parallel shell analysis using a highly refined mesh to obtain an average value that suggests the angle approximation error, as shown in Figure 13.

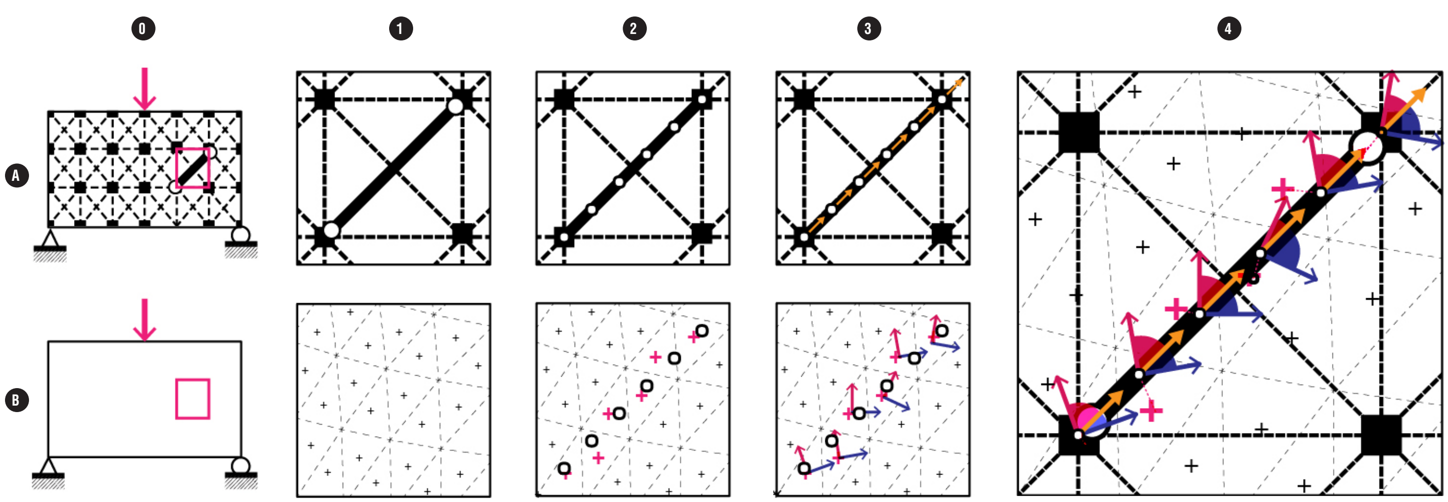


Figure 13 Measuring the conformity of a frame structure to the principal stress directions of the same design domain: 0B) Conduct parallel shell analysis for 0A's design domain; 1A) Identify discretized member to be analyzed; 2A) Divide member according to predetermined resolution; 2B) Identify closest finite elements; 3A) Obtain member's tangent vector; 3B) Obtain principal stress directions; 4) Compare angle deviation to both stress directions, obtain average for each directions, and choose the minimum set.

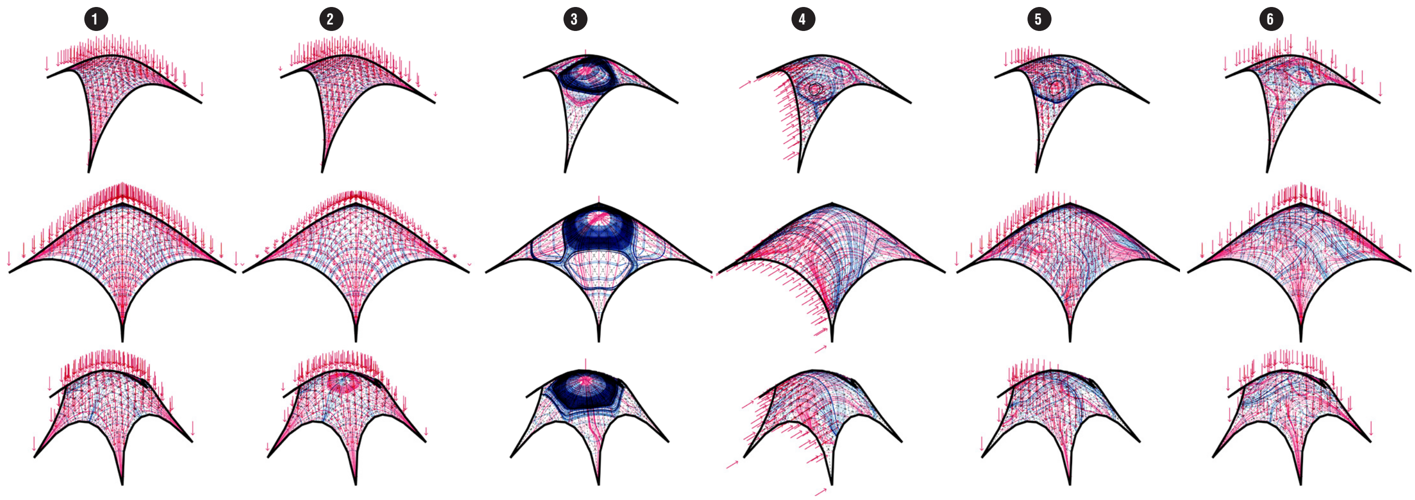


Figure 14 Stress lines for different loading conditions:
1) Equal point load on all nodes, 2) Self-weight, 3) Central point load, 4) Lateral load, 5) Asymmetrically vertical loads, and 6) Vertical loads on random nodes

4. Implementation

4.1 Preparation

4.1.1 Design domain specification

The analytical information required for the construction of the base stress line field is provided by the parallel shell analysis that is conducted initially. Generally, both the support conditions and the mesh geometry of the shell should mirror the actual constraints as closely as possible. For 2.5D shell cases, the best results are obtained when the structure is analyzed with the loading condition used to form-find the shell structure as seen in Figure 14.

4.1.2 Structural analysis

The proposed methods are mostly based on three types of analytical data: principal stress directions, principal stress magnitudes and overall member element utilization.

4.1.3 Seeding

Seeding presents an opportunity for the designer to incorporate both spatial and structural objectives that are determined by the nodal configurations. In the absence of particular constraints, the objective of seeding is to determine a collection of starting points from which a uniformly spaced principal stress field can be constructed for later processing and selection.

Conventionally, the designer would sample curvatures that are characteristics of the design domain, such as the design boundaries,

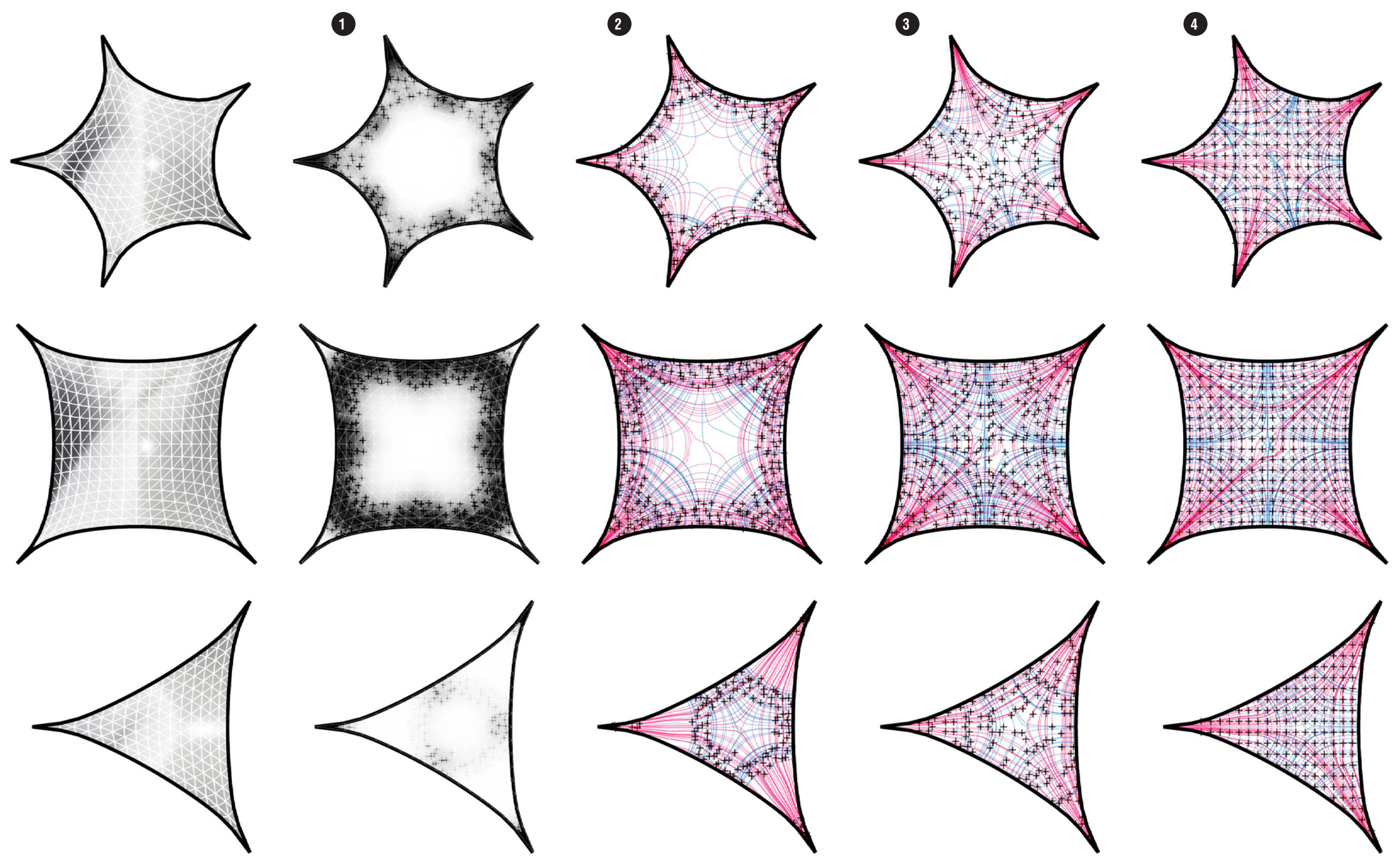


Figure 15 Seeding plan examples: 1) shell element utilization, 2) Seeding by Utilization, 3) random seeding, 4) uniform plan-based seeding

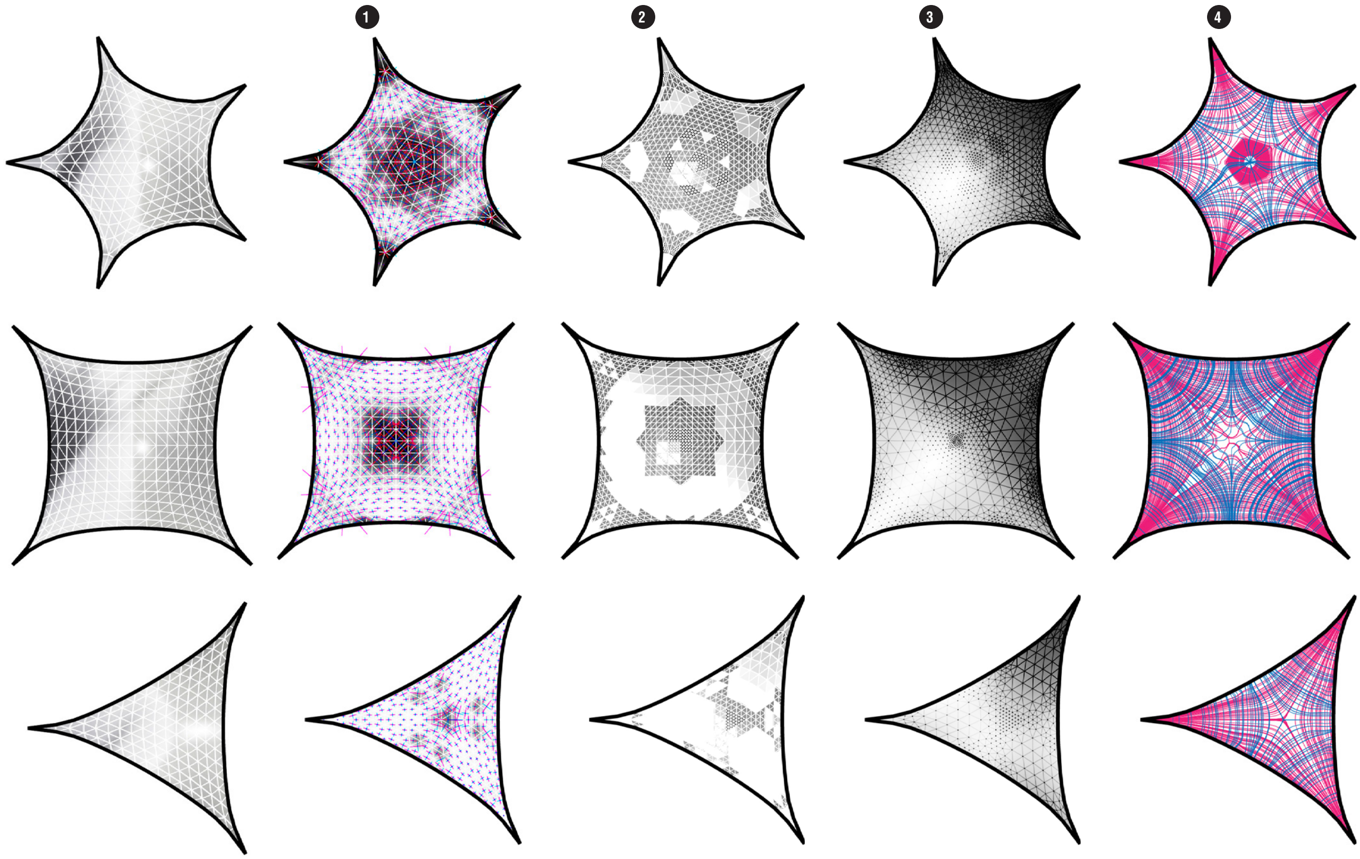


Figure 16 Remeshing procedure: 1) variation in principal stress directions 2) Subdivision 3) Relaxed Delaunay mesh 4) Refined stress lines

to generate the initial seeding points (Chen and Li 2008). While the approach can feasibly generate a uniformly distributed grid for most 2-D planar cases with simple loading conditions, the simplistic approach risks omission of large swaths of the principal stress lines at the minimum direction in more complex 2.5-D and 3-D shell cases where circumferential stresses are present. Thus, the entire area contained within the design domain should be considered as potential starting points.

This paper identifies two seeding strategies: *Guided* and *Arbitrary*. Guided strategies relate the seeding plan to analytical values, such as the magnitude of stresses. Conversely, arbitrary methods may be random, or based on other regular patterns associated with the input mesh that do not necessarily correspond to performance. These approaches are compared in Figure 15.

Known nodal constraints, which might correspond pragmatically to internal programmatic, light and mechanical constraints, can be incorporated into the seeding plan.

4.1.4 Mesh subdivision by stress direction

Since the quality of the principal stress direction results are related to the overall density of the shell mesh used in the initial analysis, further improvements to the results will require additional mesh subdivision. This paper proposes the uses of strategic mesh subdivision to reveal greater details where the stress lines are changing in direction the most, such that improvement to the accuracy of the stress lines can be realized without excessive increases to computational demand.

Figure 16 illustrates the basic process: principal stress directions for each mesh element are compared to those of its adjacent cells within a predetermined radius to produce a neighborhood angle deviation value for each cell that is then normalized and projected to a targeted subdivision range. Each cell is then subdivided accordingly to obtain a new set of vertices that are then triangulated with a Delaunay algorithm and relaxed using particle spring simulation to create the refined mesh (1934). Analyses have confirmed that the angle conformity for stress lines are higher in the strategically densified mesh than in the original mesh.

4.2 Generation

4.2.1 Interpolation method

Once a seeding plan is determined, two additional inputs are required to draw the principal stress lines: principal stress directions, and segment length. Figure 17 compares the interpolation methods this section discusses. In a conventional tracing algorithm, the principal stress directions are obtained by finding the mesh element that is closest to the starting point in the current iteration (Michalatos and Kaijima 2014). Referred to here as a first-order approach, this method can generate biased line results because the principal stress direction information is generally a value averaged for the center of the respective finite element.

The most common method devised to address this bias is by linear interpolation. In a third-order approximation method, the three data points forming a triangle element that contain the targeted point are identified; and the stress direction data for the data point is then calculated by interpolation (Chen and Li 2008). Although the third-order interpolation leads to approximation that is more accurate than that of first-order, the criteria for selecting the reference points are not always consistent, especially in irregular meshes.

To achieve consistency and accuracy of respectively the interpolation and approximation methods, a new (1+n)-order approximation method is proposed: for every starting point, the nearest finite element is identified, in addition to n finite elements with centers that are located within a per-determined radius from the starting point. The data corresponding these points are extracted, and weighted according to proximity. Figure 18.1 compares the results using the various approaches, and confirms the improved variation in the (1+n)-order approach.

4.2.2 Line Length

Generally, bias in stress line results are reduced by decreasing the segment length (Halpern, Billington, and Adriaenssens 2013). The lower bound of the segment length, however, is limited by the interpolation method chosen. For the first-order approach, the limit is set at the average mesh edge length, since a lower length segment would result in the same finite element data being used in numerous iterations. With higher-order approaches, finer segment lengths can be used to produce smoother, and more accurate stress lines (see Figure 18.2).

4.2.3 Sequential iterative tracing

Conventional tracing algorithms prescribe only the termination condition of the tracing procedure, which is met when a stress line reaches the design boundaries. Thus, a considerable number of decisions in the structure of the algorithm are left to the discretion of the designer: points may be traced sequentially or concurrently, and the two principal stress directions may also be traced independently, or simultaneously in the same iteration.

To addresses problems with this approach, this paper proposes a sequential tracing procedure subjected to rule-based corrections, where the pair of stress lines for each seeding point are traced independently in succession of the stress lines for other seeding points. The entire tracing algorithm is structured as a two-layered loop. Whereas the outer loop is responsible for supervising and adjusting the seeding plan when necessary, and tracking the information associated to each stress line (such as direction: maximum or minimum), the inner layer loop is tasked with the tracing of the stress lines and the application of rule-based corrections. The proposed approach allows stress lines undergoing tracing to respond to both existing and emergent conditions.

The general algorithm for each iteration i of a given seeding point Pj is as followed: receive the starting point for each of the four length segments of the current iteration, interpolate the stress directions associated with the principal direction for each of the four points, detect location-based geometric conditions for the starting point that warrant corrections, make relevant adjustments to the stress directions, and project the four points according to their directional vector to create the four new segments. Prior to the acceptance of the new length segments, the new lines are examined against the previously constructed stress lines, to determine whether the emergent stress line qualify for several termination rules, which are explained in the following section. The arrival of any four points to the design boundary will trigger a termination tracker that removes the respective point from future iterations. For points eligible for continuation, the projected point becomes the starting point of the subsequent iteration, as illustrated in Figure 19.

Rule-based corrections

Iterative stress line tracing is prone to numerical issues that lead to biased, and even unusable results. By correcting numerical noise as it emerges in the tracing process, a sequenced iterative approach ensures that the stress lines drawn will explicitly

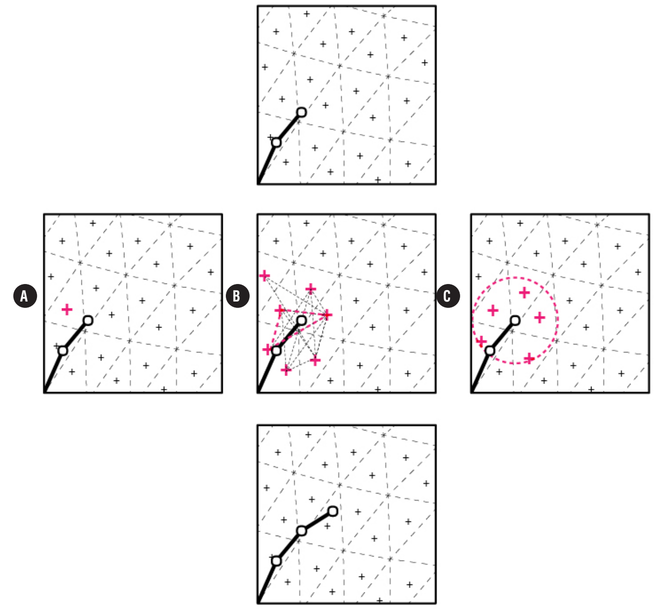


Figure 17 Interpolation procedure: 1) Input starting point of length segment in current iteration; Interpolation by approximation order of A) 1, B) 3, and C) 1+N; 2) Calculate end point of traced line segment

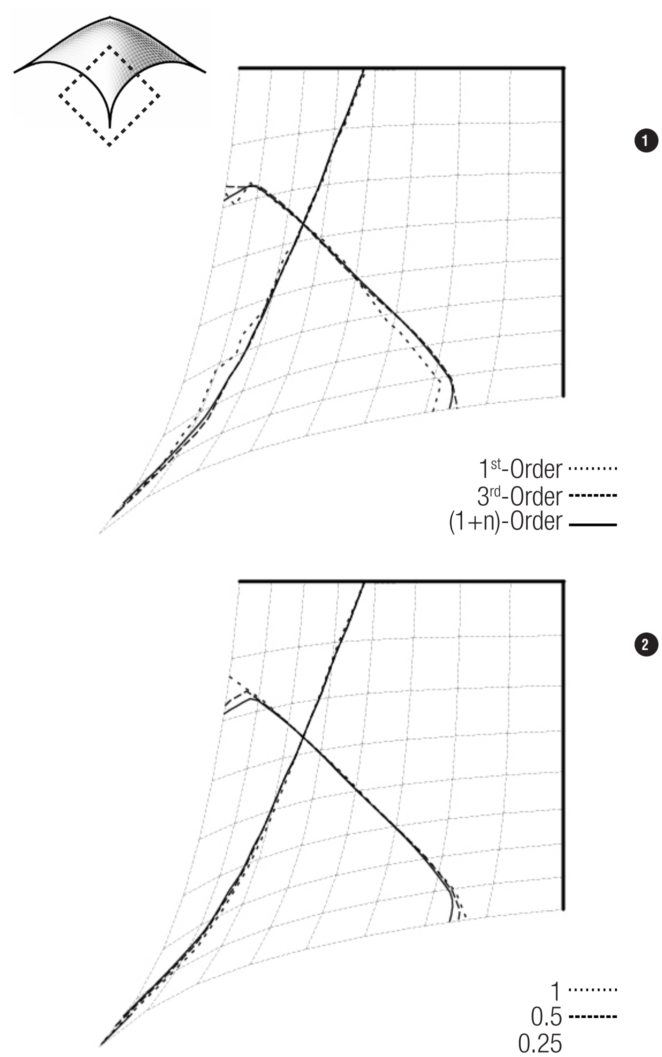


Figure 18 Variation to stress line results by:
1) Approximation order, and 2) Segment length with (1+N)-Order

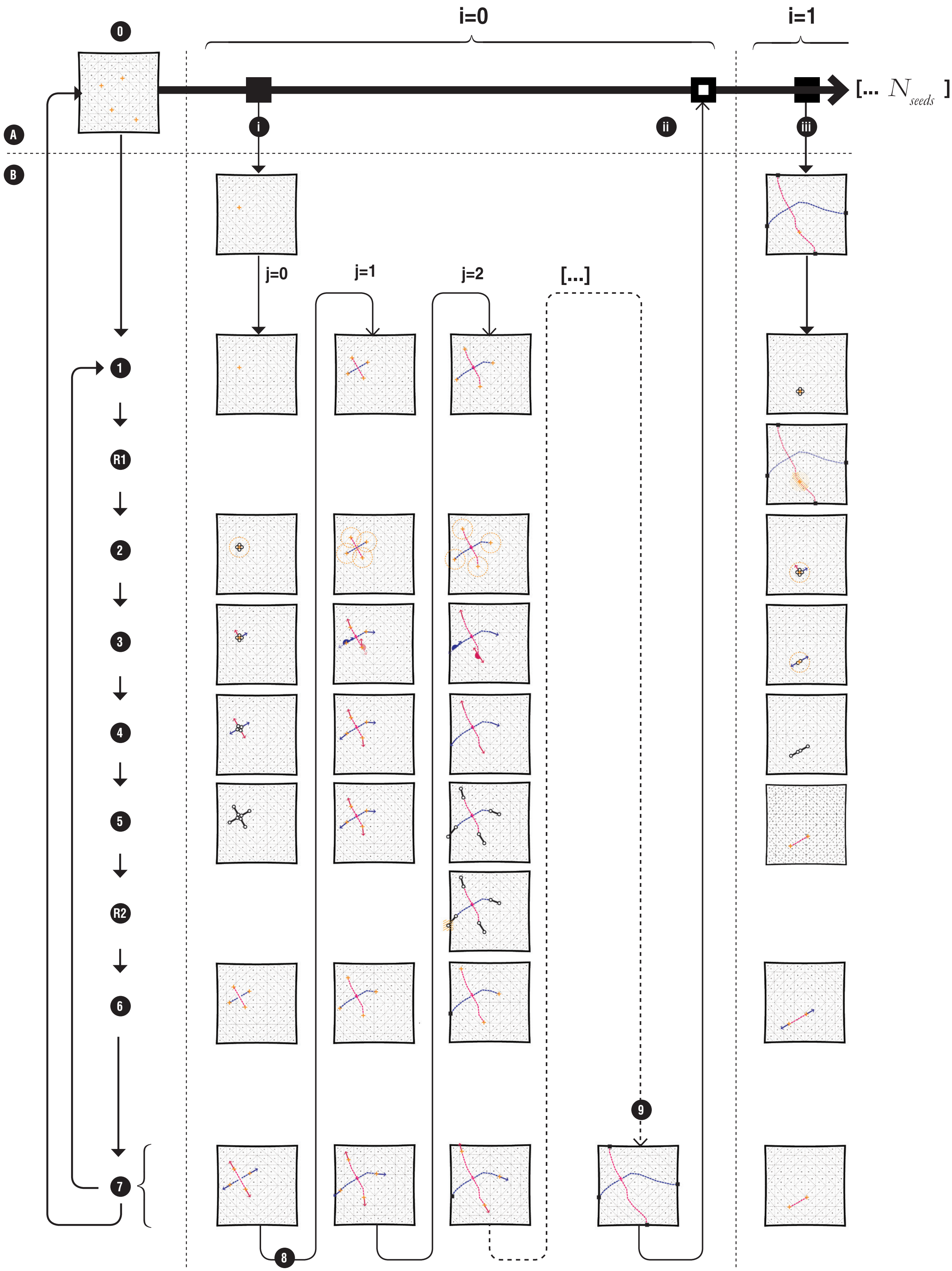


Figure 19 Diagram of proposed tracing algorithm: it features an A) Outer loop, and an B) Inner loop, which begins with 0) the input of a seeding plan. The main outer engine i) sequentially inputs a seeding point into the inner loop, and ii) eventaully receives fully-traced stress lines from it, which is followed by c) the input of an additional seed - a process that repeats until all seeds are traced. The inner loop's subroutines are as follows: 0) Receive starting point; R1) Recognize shape for drawing rules 1; 2) Interpolate; 3) Calculate Vector; 4) Correct flipped vectors ; 5) Draw segments; R2) Recognize shapes for termination rules; 6) Append curves; 7) Output results; 8) Reiterate, and 9) Terminate stress lines when they meet the design boundaries.

meet constructibility and spatial requirements. Preliminary implementation of the paper's rule-based tracing approach are identified here:

Detection of Circumferential Stress: As shown in Figure 20, conventional tracers generally loop indefinitely when a stress line reaches an area with circumferential stresses. This rule, which is described in Figure 21.1, detects the presence of circumferential stresses and closes the stress lines accordingly.

Enforce Offset: When the starting point of currently traced stress line approaches a defined distance from an existing stress lines, the rules, which is depicted in Figure 22.2, ensure that the stress lines for each direction will maintain a sufficient offset, and will not intersect with each other due to approximation biases.

Bypass Seeding: To eliminate the production of redundant and overlapping stress lines, which increases post-processing burden, the detection of a prior principal stress line nearby at the initial tracing iteration for a seeding point will lead to termination for the respective principal stress direction, as Figure 22.3 shows.

In Figure 19, stress lines results from the preliminary implementation of the proposed tracing algorithm are compared with the results generated using Karamba's built-in stress line feature.

4.3 Post-processing

4.3.1 Density and architectural implication

Although a Michell structure can be constructed for a variety of boundary conditions with varying numbers of members, the theoretically-optimized Michell solution has an infinite number of infinitely small bars with infinitely low stress. It is also known that additional gains in structural efficiency achieved by increasing the number of members in the Michell structure will plateau in as the Michell structure densifies (Mazurek, Baker and Tort 2011).

These conclusions also applies to the stress line-based structure: results for the simple gridshell case with 4 supports are shown in

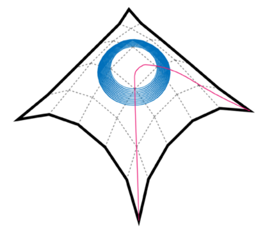


Figure 20 Circumferential stress error in commercial tool

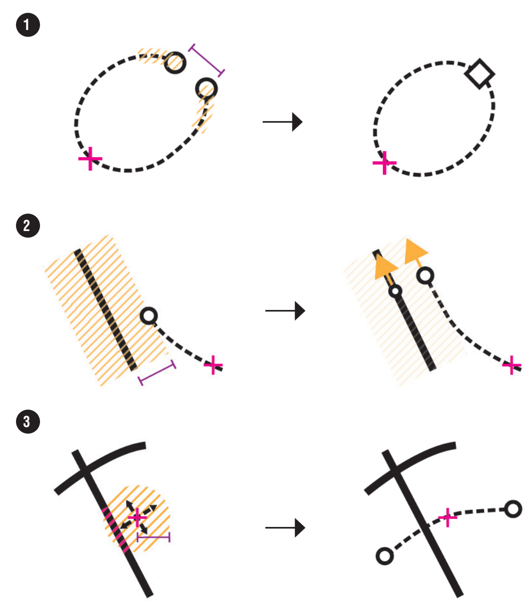


Figure 21 Diagrammatic representation of correction rules' parameters

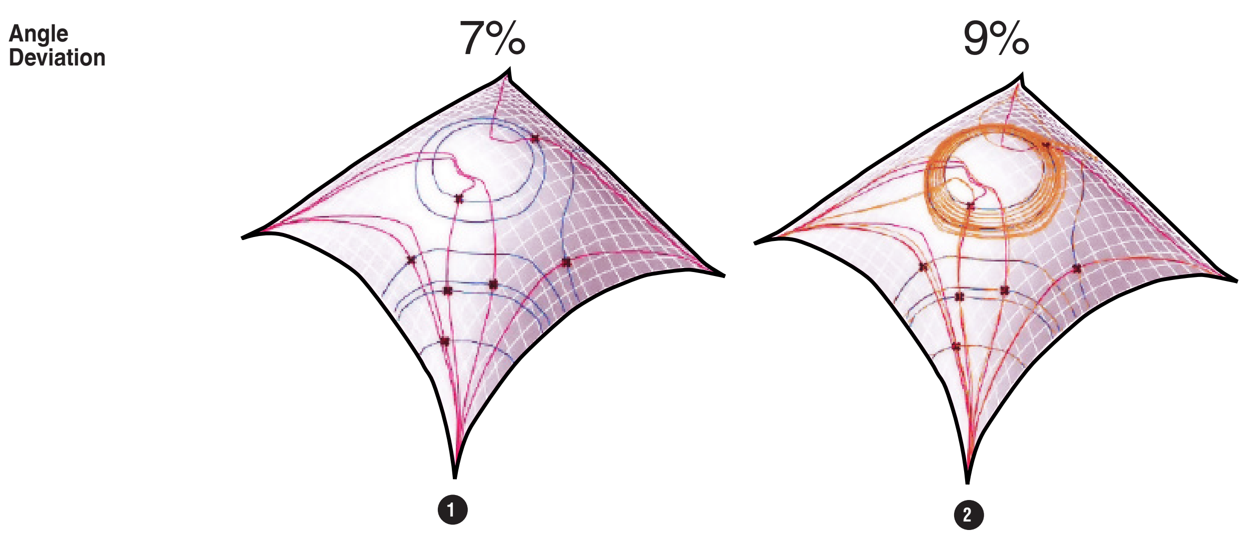
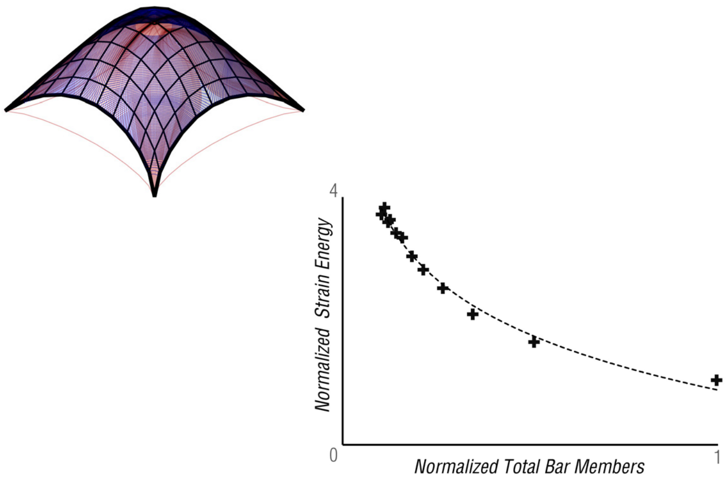


Figure 22 Visual and angle deviation results of stress lines traced with sample points using: 1) Proposed algorithm 2) Karamba



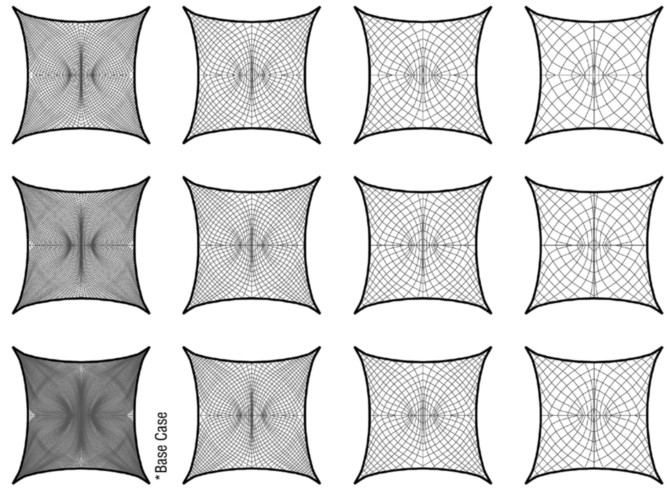


Figure 23 Stress line density and performance

Figure 20. The ideally optimized stress-line structure, which has an infinite number of members, is neither usable in a practical design context nor significantly improved compared to a lower density. Hence the targeted density of the stress-line base structure is ultimately a decision based on constructibility and design concerns.

This paper proposes a method that seeks to select the stress lines that most contribute to structural performance.

4.3.2 Problem formulation

In structural optimization, efficiency is often defined in terms of structural volume or stiffness. Commonly, the sum of force multiplied by the length for each member of the system is used as an indicator for structural volume (Mueller, and Ochsendorf 2013). However, the method's application is limited to axial-only solutions, in which members are experiencing equal stress across their cross section. Since both the curvature of the stress lines, and the mesh geometry from which the shell analysis is derived, are both approximations of an idealized funicular experience, the discretized structural systems derived from stress lines will inevitably be subjected to bending stresses.

To measure structural efficiency, this paper uses the minimization of strain energy as the objective function, which is sometimes referred to as the minimization of compliance, or the maximization of stiffness (Rozvany, Bendsøe and Kirsch 1995; Achtziger 1997). The strain energy of a system, which is calculated by multiplying stiffness by deformation squared, can account for structural efficiency when the members of all compared cases are sized to achieve constant total volume.

4.3.3 Comparative analysis procedure

Following any modifications to the stress line-based structure, the members' section areas are rescaled according to a base case prior to ensure that the total structural volume remains constant for all cases (see Figure 24). The magnitude of external loads applied to each node are also redistributed according to the percentage of the shell's surface geometry that each node is supporting for the updated topology - an estimate obtained through the tributary area method. The load values are normalized such that the total applied force remains constant in all selection cases. With the load values adjusted and the structural members globally resized, an initial analysis is conducted to obtain the stress values required

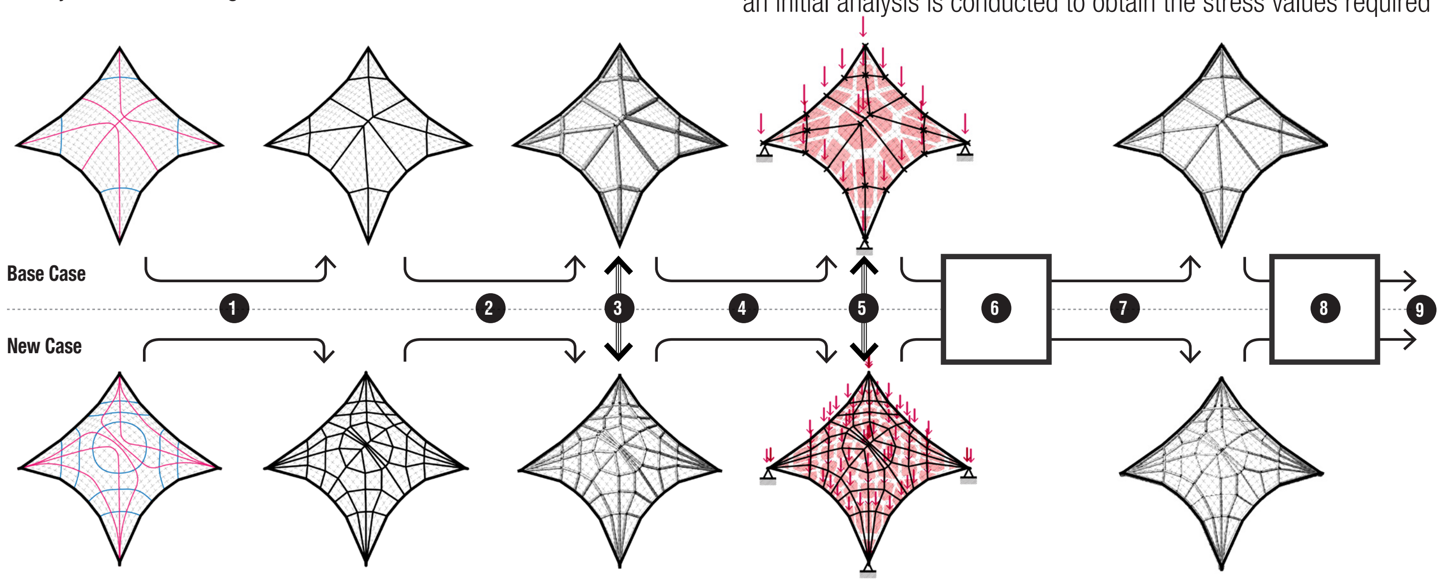


Figure 24 Comparative analysis procedure: 1) Assemble linearized frame structure; 2) Adjust constant diameter to ensure 3) equal total volume for all cases; 4) Calculate tributary area and assign loads; 5) Normalize load values to ensure that the total applied forces are equal for all cases; 6) Analyze; 7) Resize members according to stress ratio; 8) Final analysis; 9) Obtain strain energy value for comparison.

to determine the stress ratio for each member, which is calculated by dividing the maximum stress of each member by the maximum stress of any member in the entire system. Individual members' cross section areas are then rescaled according to their stress ratio, in order to ensure that the structural material within a system is distributed according to stress requirement. A final FEA is conducted to obtain the strain energy data used for comparison. Similar to the concept of "fully stressed design," the two-iteration approach ensures that each topology is compared using their reasonable optimal sizing.

4.3.4 Selection by reduction in angle deviation

As illustrated in Figure 25, the proposed method is an iterative algorithm that incrementally selects n number of stress lines in each iteration for materialization. Selection is heuristic, and based on fitness objectives that can act as a proxy for structural performance. In the proposed algorithm, approximation error by angle deviation is used as the metric for performance.

As highly complex geometries that are merely visualizations of the force flow for a given design domain, the stress lines are not directly materializable. The process proposed here extracts only the coordinate position of the intersection point of the selected set of stress lines, while maintaining the same topological connectivity. The geometric simplification maximizes the constructibility of the materialized stress lines, and ensures that the resultant structure can be modeled as 3D frame structures with mostly axial behavior.

The process begins with an initial starting point input from the user. In each iteration, a random subset of n- stress lines intersecting the input points, or lines are identified and evaluated according to the approximation error of the resultant combined discretized and linearized structure. The combination minimizing the objective is selected, and used as the input lines for the subsequent iteration, which in turn searches for the n stress lines intersecting the new input lines. The approach alternates between the minimum and maximum principal stress directions in each iteration, thus ensuring uniform growth to the stress line density. Figure 26 confirms the approaches ability to generate considerable improvement to structural performance within only a few iterations.

The conceptual benefit of an additive process that constructs a stress line-based topology from a blank state is that it gives the designer the opportunity to monitor the trade-off between structural performance and the incremental increase in the density of the stress-line-based structure, such that the designer may precisely identify the optimum density particular to the design constraints. An additive approach is also the least exhaustive computationally, as the geometry only gradually increases complexity in each iteration.

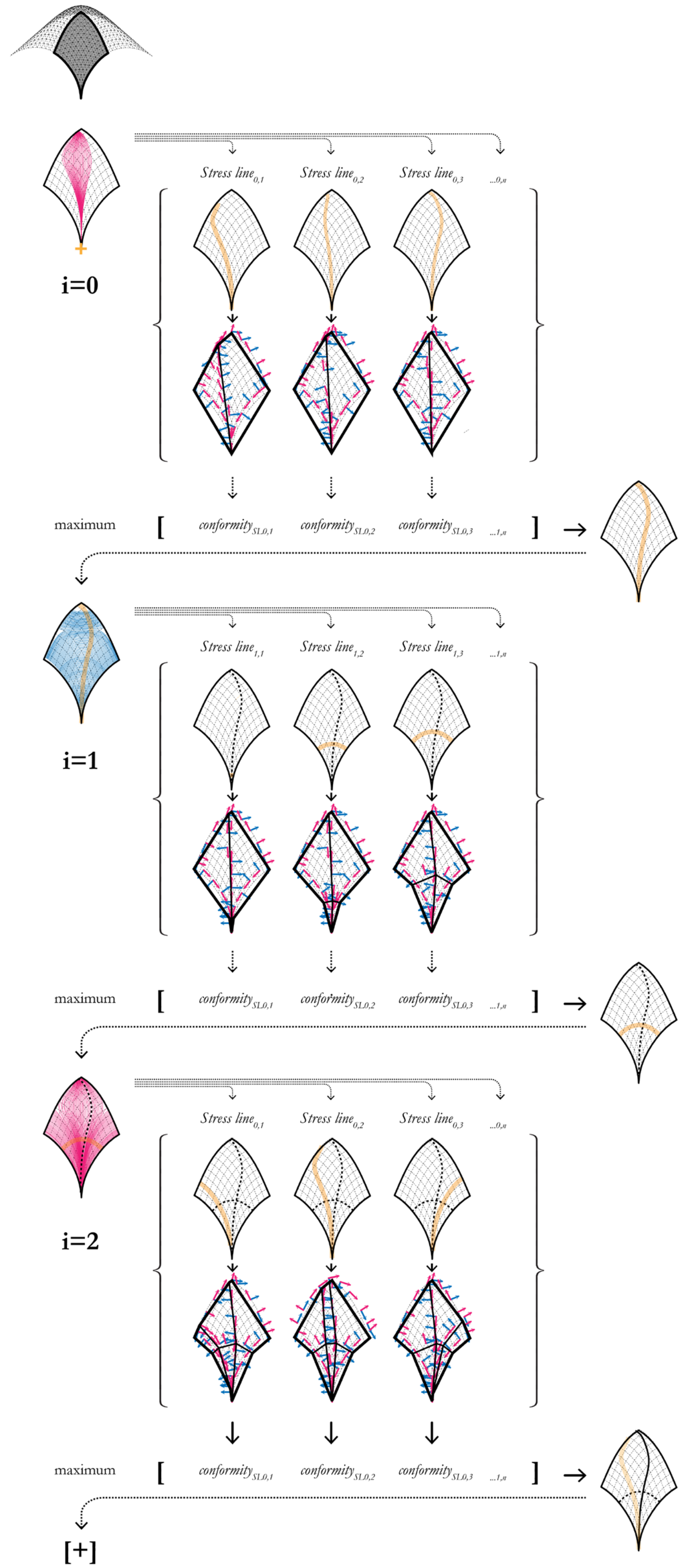
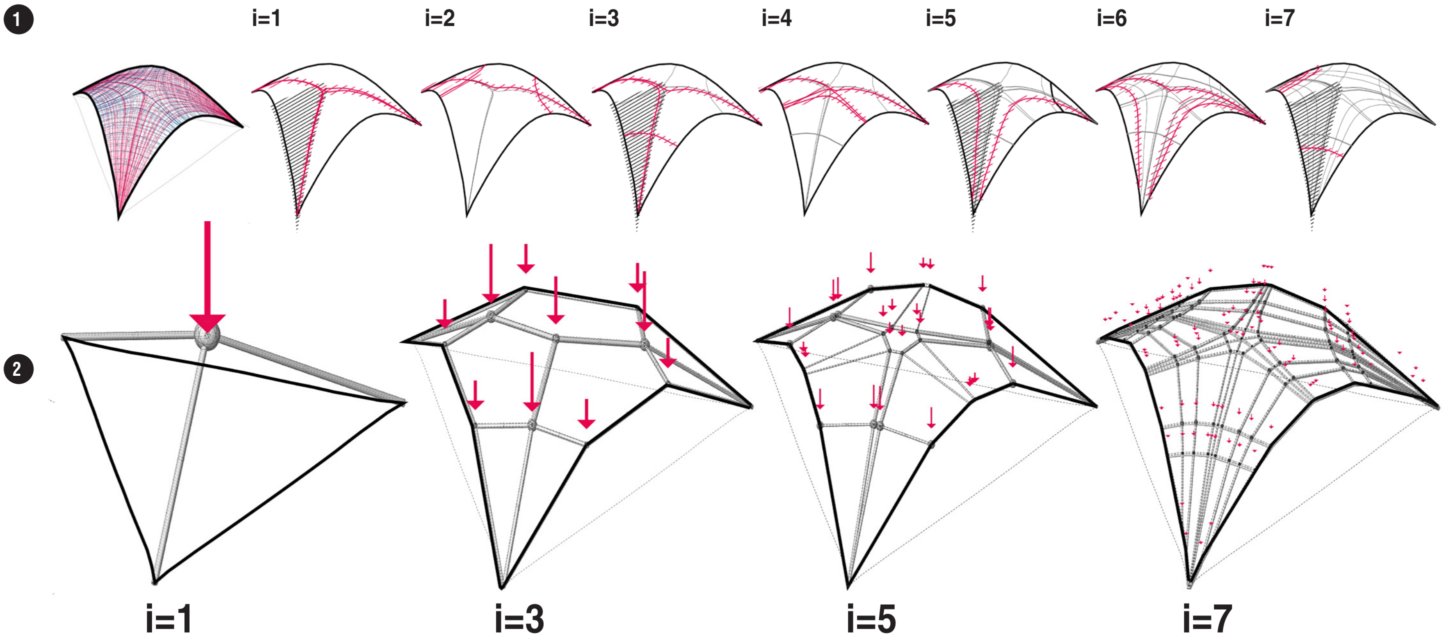


Figure 25 Incremental stress line selection procedure



	i=1	i=3	i=5	i=7
Angle Deviation (%)	22	11	10	8.9
		Base Case		
Total Length	40	127.5	198	389.40
%Length of Base Case	0.333	1	2	4
Force × Length	5686.4	4500	4100	3738
***e* = Moment/Force**	0.0037	0.10	0.07	0.05
Strain Energy	0.021	0.019	0.017	0.014
Total Force	200			

Figure 26 Structural performance of stress-line-based designs developed with proposed selection method:
1) Direct principal stress lines selection results, and 2) Discretized and linearized structure extracted from stress lines

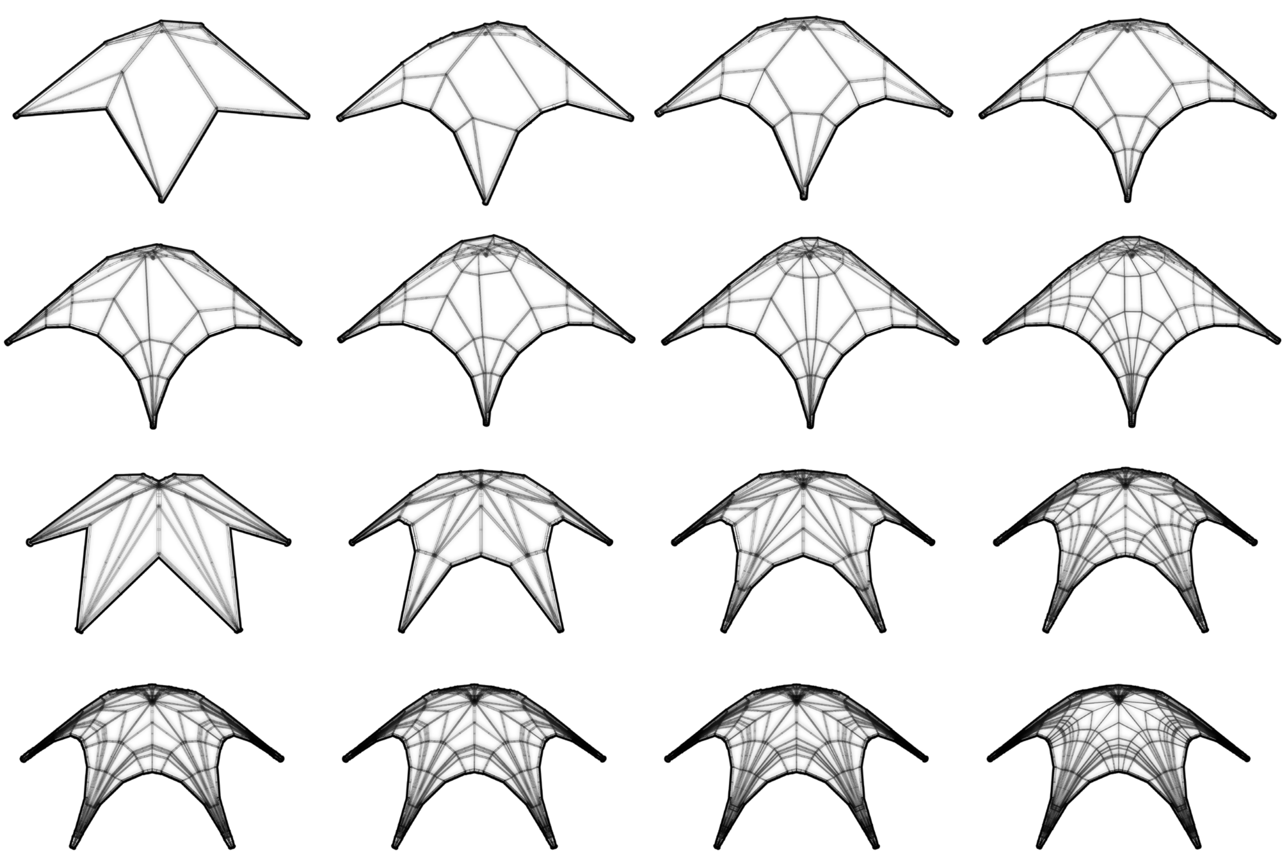


Figure 27 Stress line selection results for grid shell case with 4- and 5- supports.

5. Conclusion

5.1 Summary and potential impact

This paper aims to bring clarity to a developing field within conceptual structural design that is often misunderstood and applied arbitrarily. Specifically, this is achieved by focusing on the fundamental areas in stress line generation that remain open in related research, such as methods to improve interpolation and the iterative algorithm tracing the stress lines. The basic outline of the stress line method has been reinterpreted, and the many sub-processes involved in stress line generation have been rigorously codified, so that several previously unknown opportunities and issues in the different stages of stress line generation have been discovered.

5.2 Future work

The presented research has numerous applications that directly expand conceptual structural design for architecture at multiple scales. For example, the methods presented in this paper could be used to inform designs of buildings and bridges realized in a number of materials; new digital fabrication techniques could help make geometrically complex stress-line-inspired designs achievable. One specific application currently under investigation by the authors is the use of stress line research to address limitations of current additive manufacturing (or 3D printing) technologies. Because conventionally, material is deposited in layers parallel to the horizontal printing bed, printed specimens are anisotropic, with strength and stiffness that varies significantly depending on the orientation of the applied forces. This problem can potentially be solved by adding materials along three-dimensional paths based on stress lines, both at prototype and end-use scales, resulting in high-performing, geometrically novel structures (Mueller, Irani, and Jenett 2014).

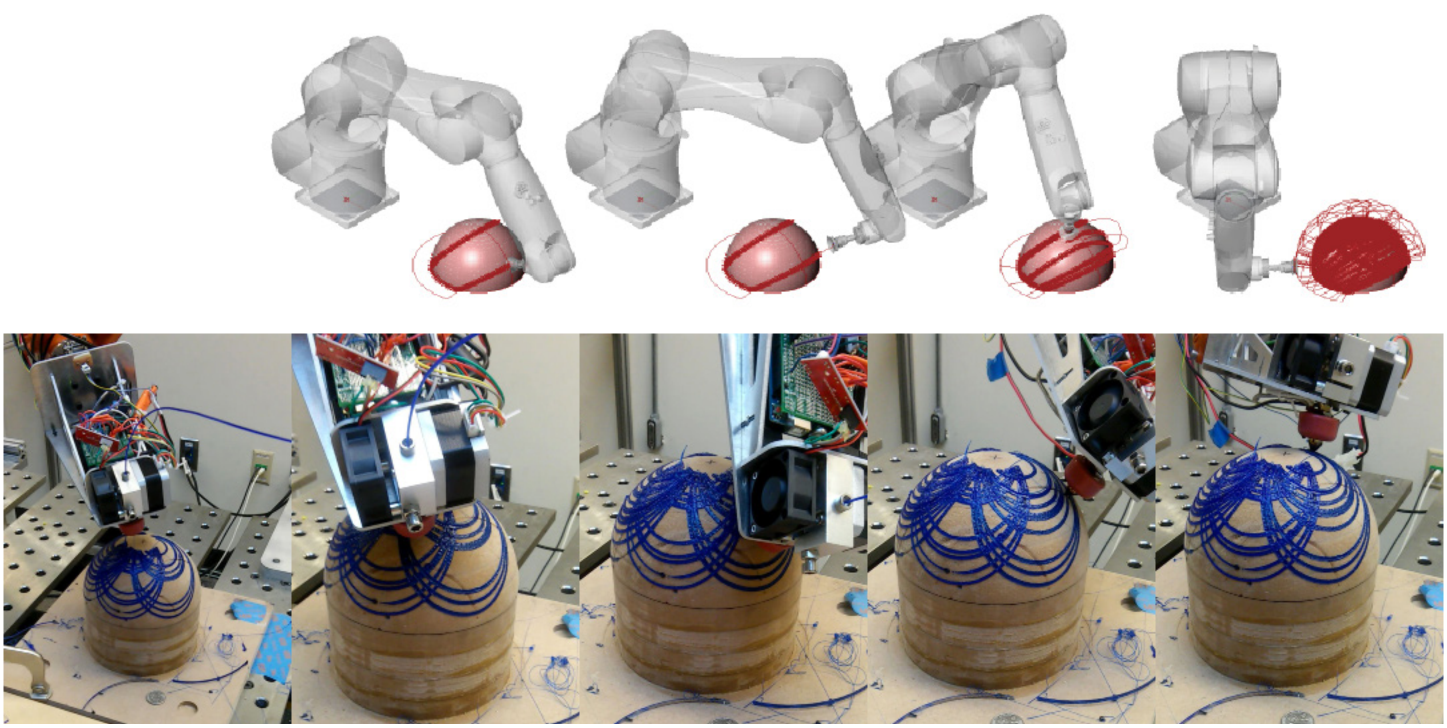


Figure 28 Recent developments from *Stress Line Additive Manufacturing*

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