On the Cleanness of Cuspidal Character Sheaves

ON THE CLEANNESS OF CUSPIDAL CHARACTER SHEAVES

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Abstract. We prove the cleanness of cuspidal character sheaves in arbitrary characteristic in the few cases where it was previously unknown.

1. Statement of results

1.1. Let $\mathbf{k}$ be an algebraically closed field of characteristic exponent $p \ge 1$. Let G be a connected reductive algebraic group over $\mathbf{k}$ with adjoint group G_{ad}. It is known that, if A is a cuspidal character sheaf on G, then $A = IC(\bar{\Sigma}, \mathcal{E})[\dim \Sigma]$ where Σ is the inverse image under $G \to G_{ad}$ of a single conjugacy class in G_{ad}, $\mathcal{E}$ is an irreducible local system on Σ equivariant under the conjugation G-action and IC denotes the intersection cohomology complex. (For any subset γ of G we denote by $\bar{\gamma}$ the closure of γ in G.) We say that A is clean if $A|_{\bar{\Sigma}-\Sigma} = 0$. This paper is concerned with the following result.

Theorem 1.2. *Any cuspidal character sheaf of G is clean.*

By arguments in [L2, IV, §17] it is enough to prove the theorem in the case where G is almost simple, simply connected. In this case the theorem is proved in [L2,V,23.1] under the following assumption on p: if $p = 5$ then G is not of type E_8; if $p = 3$ then G is not of type E_7, E_8, F_4, G_2; if $p = 2$ then G is not of type E_6, E_7, E_8, F_4, G_2. In the case where $p = 5$ and G is of type E_8 or $p = 3$ and G is of type E_7, E_8, F_4, G_2 or $p = 2$ and G is of type E_6, E_7, G_2, there are some cuspidal character sheaves on G for which the arguments of [L2, V, §23] do not apply, but Ostrik [Os] found a simple proof for the cleanness of these cuspidal character sheaves. The proof of the theorem in the remaining case ($p = 2$ and G of type E_8 or F_4) is completed in 3.8; in the rest of this section we place ourselves in this case.

Note that a portion of our proof relies on computer calculations (via the reference to [L4] in 2.4(a) and the references to [L3], [L5]).

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1.3. For any complex of $\bar{\mathbf{Q}}_l$-sheaves K on G let $\mathcal{H}^i K$ be the i-th cohomology sheaf of K and let ${}^pH^i K$ be the i-th perverse cohomology sheaf of K. If M is a perverse sheaf on G and A is a simple perverse sheaf on G let $(A : M)$ be the number of times that A appears in a Jordan-Hölder series of M. We write "G-local system" instead of "G-equivariant $\bar{\mathbf{Q}}_l$-local system for the conjugation action of G". We set $\Delta = \dim G$.

The next two properties are stated for future reference.

(a) *Let A be a cuspidal character sheaf on G and let X be a noncuspidal character sheaf on G. Then $H^*_c(G, A \otimes X) = 0$.*
(See [L2, II, 7.2].)

(b) *Let γ be a unipotent class in G and let $\mathcal{L}$ be an irreducible noncuspidal G-local system on γ. Then there exists a noncuspidal character sheaf X of G such that* $\text{supp}(X) \cap G_u \subset \bar{\gamma}$ *and $X|_\gamma = \mathcal{L}[d]$ for some $d \in \mathbf{Z}$.*
(See [L1, 6.5].)

1.4. From the results already quoted we see that if G' is the centralizer of a semisimple element $\neq 1$ of G, then any cuspidal character sheaf of G' is clean. Using [L2, II, 7.11] we then see that any cuspidal character sheaf of G with non-unipotent support is clean. Thus it is enough to prove the cleanness of cuspidal character sheaves with support contained in G_u, the unipotent variety of G. For any $i \in \mathbf{N}$ we denote by γ_i a distinguished unipotent class in G of codimension i (assuming that such class exists); note that γ_i is unique if it exists. According to Spaltenstein [Sp1, p.336], γ_i carries an irreducible cuspidal local system precisely when $i \in I$ where $I = \{10, 20, 22, 40\}$ (type E_8) and $I = \{4, 6, 8, 12\}$ (type F_4); this cuspidal local system (necessarily of rank 1) is unique (up to isomorphism) and denoted by $\mathcal{E}_i$ except if $i = 10$ (type E_8) and $i = 4$ (type F_4) when there are two non-isomorphic irreducible cuspidal local systems on γ_i denoted by $\mathcal{E}_i, \mathcal{E}'_i$. We can then form the four admissible (see [L2, I, (7.1.10)]) complexes $A_i = IC(\bar{\gamma}_i, \mathcal{E}_i)[\dim \gamma_i]$ ($i \in I$) on G and the admissible complex $A'_i = IC(\bar{\gamma}_i, \mathcal{E}'_i)[\dim \gamma_i]$ (where $i = 10$ for type E_8, $i = 4$ for type F_4). According to Shoji [Sh2] these five admissible complexes are character sheaves on G; they are precisely the character sheaves on G with support contained in G_u. From [L2, II, 7.9] we see that A_{40} is clean (type E_8) and A_{12} is clean (type F_4). According to Ostrik [Os], A_{10}, A'_{10}, A_{22} are clean (type E_8) and A_4, A'_4, A_6 are clean (type F_4). Moreover, from [Os] it follows that

(a) *if G is of type E_8 and $i \in \mathbf{Z}$ then $\mathcal{H}^i(A_{20})|_{\gamma_{22}}$ does not contain $\mathcal{E}_{22}$ as a summand.*

1.5. We show:

(a) *Let $i = 20$ (type E_8), $i = 8$ (type F_4). Let $A = A_i$. Let γ be a unipotent class of G. Then $\oplus_j \mathcal{H}^j A|_\gamma$ does not contain any irreducible noncuspidal G-local system as a direct summand.*

Assume that this is not true and let γ be a unipotent class of minimum dimension such that $\oplus_j \mathcal{H}^j A|_\gamma$ contains an irreducible noncuspidal G-local system, say $\mathcal{L}$, as a direct summand. We can assume that $\mathcal{H}^{i_0} A|_\gamma$ contains $\mathcal{L}$ as a direct summand

and that for $j > i_0$, $\mathcal{H}^j A|_\gamma$ is a direct sum of irreducible cuspidal G-local systems on γ. Clearly,

(b) *for any unipotent class $\gamma' \subset \bar\gamma - \gamma$, $\oplus_j \mathcal{H}^j A|_{\gamma'}$ is a direct sum of irreducible cuspidal G-local systems.*

We can assume that $\gamma \subset \bar\gamma_i$; if $\gamma = \gamma_i$ the result is obvious so that we may assume that $\gamma \subset \bar\gamma_i - \gamma_i$. By 1.3(b) we can find a a noncuspidal character sheaf X of G such that $\text{supp}(X) \cap G_u \subset \bar\gamma$ and $X|_\gamma = \mathcal{L}^*[d]$ for some $d \in \mathbf{Z}$. By 1.3(a) we have $H_c^*(G, A \otimes X) = 0$. Since $\text{supp}(A) \subset G_u$ it follows that $H_c^*(G_u, A \otimes X) = 0$. Since $\text{supp}(X) \cap G_u \subset \bar\gamma$ it follows that $H_c^*(\bar\gamma, A \otimes X) = 0$.

We show that $H_c^*(\bar\gamma - \gamma, A \otimes X) = 0$. It is enough to show that for any unipotent class $\gamma' \subset \bar\gamma - \gamma$ we have $H_c^*(\gamma', A \otimes X) = 0$. Using (b) we see that it is enough to show that for any irreducible cuspidal G-local system $\mathcal{E}'$ on γ' we have $H_c^*(\gamma', \mathcal{E}' \otimes X) = 0$. We can find a cuspidal character sheaf A' on G such that $\text{supp}(A') = \bar\gamma'$, $A'|_{\gamma'} = \mathcal{E}'[\dim \gamma']$. Then A' must be A_{40} or A_{22} (for type E_8) and A_{12} (for type F_4); in particular A' is clean. Hence

$$H_c^*(\gamma', \mathcal{E}' \otimes X) = H_c^*(\bar\gamma', A' \otimes X) = H_c^*(G, A' \otimes X)$$

and this is 0 by 1.3(a).

From $H_c^*(\bar\gamma, A \otimes X) = 0$, $H_c^*(\bar\gamma - \gamma, A \otimes X) = 0$ we deduce that $H_c^*(\gamma, A \otimes X) = 0$ that is $H_c^*(\gamma, A \otimes \mathcal{L}^*) = 0$. Let $\delta = \dim \gamma$. We have $H_c^{2\delta + i_0}(\gamma, A \otimes \mathcal{L}^*) = 0$. We have a spectral sequence with $E_2^{r,s} = H_c^r(\gamma, \mathcal{H}^s(A) \otimes \mathcal{L}^*)$ which converges to $H_c^{r+s}(\gamma, A \otimes \mathcal{L}^*)$.

We show that $E_2^{r,s} = 0$ if $s > i_0$. It is enough to show that $H_c^*(\gamma, \mathcal{E}'' \otimes \mathcal{L}^*) = 0$ for any irreducible cuspidal G-local system $\mathcal{E}''$ on γ. We can find a cuspidal character sheaf A'' on G such that $\text{supp} A'' = \bar\gamma$, $A''|_\gamma = \mathcal{E}''[\delta]$. Since $\gamma \subset \bar\gamma_i - \gamma_i$ we see that A'' must be A_{40} or A_{22} (type E_8) or A_{12} (type F_4) so that A'' is clean. Hence

$$H_c^*(\gamma, \mathcal{E}'' \otimes \mathcal{L}^*) = H_c^*(\bar\gamma, A'' \otimes X) = H_c^*(G, A'' \otimes X)$$

and this is 0 by 1.3(a).

We have also $E_2^{r,s} = 0$ if $r > 2\delta$. It follows that

$$E_2^{2\delta, i_0} = E_3^{2\delta, i_0} = \cdots = E_\infty^{2\delta, i_0}.$$

But $E_\infty^{2\delta, i_0}$ is a subquotient of $H^{2\delta + i_0}(\gamma, A \otimes \mathcal{L}^*)$ hence it is zero. It follows that $0 = E_2^{2\delta, i_0} = H_c^{2\delta}(\gamma, \mathcal{H}^{i_0}(A) \otimes \mathcal{L}^*)$. Since $\mathcal{L}$ is a direct summand of $\mathcal{H}^{i_0}(A)$ it follows that $H_c^{2\delta}(\gamma, \mathcal{L} \otimes \mathcal{L}^*) = 0$. This is clearly a contradiction. Thus (a) is proved.

1.6. We show:

(a) *Let A be a cuspidal character sheaf on G such that* $\text{supp}(A) = \bar\gamma$, *$\gamma$ a unipotent class in G; let $\mathcal{E}$ be an irreducible G-local system on γ such that $A|_\gamma = \mathcal{E}[\delta]$, $\delta = \dim \gamma$. Let Y be a noncuspidal character sheaf of G. Then $\oplus_j \mathcal{H}^j Y|_\gamma$ does not contain $\mathcal{E}^*$ as a direct summand.*

Assume that $\oplus_j \mathcal{H}^j Y|_\gamma$ contains $\mathcal{E}$ as a direct summand. We can find i_0 such that $\mathcal{H}^{i_0} Y|_\gamma$ contains $\mathcal{E}$ as a direct summand but $\mathcal{H}^j Y|_\gamma$ does not contain $\mathcal{E}$ as a direct summand if $j > i_0$. We have $H_c^{2\delta}(\gamma, \mathcal{E} \otimes \mathcal{H}^{i_0} Y) \neq 0$. By 1.3(a) we have $H_c^*(G, A \otimes Y) = 0$ hence $H_c^*(\bar\gamma, A \otimes Y) = 0$. We show that

(b) $H_c^*(\bar\gamma-\gamma, A\otimes Y)=0$.
If A is clean then (b) is obvious. Thus to prove (b) we may assume that $A=A_{20}$ (type E_8) and $A=A_8$ (type F_4). It is enough to show that for any unipotent class $\gamma'\subset\bar\gamma-\gamma$ we have $H_c^*(\gamma', A\otimes Y)=0$. It is enough to show that $H_c^*(\gamma',\mathcal{H}^j(A)\otimes Y)=0$ for any j. If $\mathcal{H}^jA|_{\gamma'}=0$, this is obvious. Thus we may assume that $\mathcal{H}^jA|_{\gamma'}\ne 0$. By 1.5(a), $\mathcal{H}^jA|_{\gamma'}$ is a direct sum of (at least one) copies of irreducible cuspidal G-local systems on γ'. It follows that $\gamma'=\gamma_{40}$ (type E_8) and $\gamma'=\gamma_{12}$ (type F_4); we use that in type E_8 we have $\gamma'\ne\gamma_{22}$; see 1.4(a). It is then enough to show that $H_c^*(\gamma',\mathcal{E}'\otimes Y)=0$ where $\mathcal{E}'$ is $\mathcal{E}_{40}$ (type E_8) and $\mathcal{E}'$ is $\mathcal{E}_{12}$ (type F_4). Let $A'=A_{40}$ (type E_8) and $A'=A_{12}$ (type F_4). Since A' is clean we have

$H_c^*(\gamma',\mathcal{E}'\otimes Y)=H_c^*(\bar\gamma', A'\otimes Y)=H_c^*(G, A'\otimes Y)$

and this is 0 by 1.3(a).

Using (b) and $H_c^*(\bar\gamma, A\otimes Y)=0$ we deduce that $H_c^*(\gamma, A\otimes Y)=0$ hence $H_c^*(\gamma,\mathcal{E}\otimes Y)=0$. Thus $H_c^{2\delta+i_0}(\gamma,\mathcal{E}\otimes Y)=0$. We have a spectral sequence with $E_2^{r,s}=H_c^r(\gamma,\mathcal{E}\otimes\mathcal{H}^sY)$ which converges to $H_c^{r+s}(\gamma,\mathcal{E}\otimes Y)$. We show that

$E_2^{r,s}=0$ if $s>i_0$.

It is enough to show that $H_c^*(\gamma,\mathcal{E}\otimes\mathcal{L})=0$ for any noncuspidal irreducible G-local system $\mathcal{L}$ on γ. This follows by applying the argument in line 8 and the ones following it in the proof of [L2, II, 7.8] to $\Sigma=\gamma$ (a distinguished unipotent class) and to $\mathcal{F}=\mathcal{E}\otimes\mathcal{L}$ (an irreducible G-local system on γ not isomorphic to $\bar{\mathbf{Q}}_l$).

We have also $E_2^{r,s}=0$ if $r>2\delta$. It follows that $E_2^{2\delta,i_0}=E_3^{2\delta,i_0}=\dots=E_\infty^{2\delta,i_0}$. But $E_\infty^{2\delta,i_0}$ is a subquotient of $H^{2\delta+i_0}(\gamma,\mathcal{E}\otimes Y)$ hence it is zero. It follows that $0=E_2^{2\delta,i_0}=H_c^{2\delta}(\gamma,\mathcal{E}\otimes\mathcal{H}^{i_0}Y)$. This contradicts $H_c^{2\delta}(\gamma,\mathcal{E}\otimes\mathcal{H}^{i_0}Y)\ne 0$. This proves (a).

Note that a property like (a) appeared (in good characteristic) in the work of Shoji [Sh1] and Beynon-Spaltenstein [BS].

2. Preliminaries to the proof

2.1. Let $\mathcal{B}$ be the variety of Borel subgroups of G. Let $\mathbf{W}$ be a set indexing the set of orbits of G acting on $\mathcal{B}\times\mathcal{B}$ by $g:(B,B')\mapsto(gBg^{-1},gB'g^{-1})$. For $w\in\mathbf{W}$ we write $\mathcal{O}_w$ for the corresponding G-orbit in $\mathcal{B}\times\mathcal{B}$. Define $\underline{l}:\mathbf{W}\to\mathbf{N}$ by $\underline{l}(w)=\dim\mathcal{O}_w-\dim\mathcal{B}$. Then $\mathbf{W}$ has a natural structure of (finite) Coxeter group with length function $\underline{l}$ (see for example [L3,0.2]); it is the Weyl group of G.

For $w\in\mathbf{W}$ let $\mathfrak{B}_w=\{(g,B)\in G\times\mathcal{B};(B,gBg^{-1})\in\mathcal{O}_w\}$. Define $\pi_w:\mathfrak{B}_w\to G$ by $\pi_w(g,B)=g$. Let $K_w=\pi_{w!}\bar{\mathbf{Q}}_l$, a complex of sheaves on G. Let

$\mathfrak{B}_{\le w}=\{(g,B)\in G\times\mathcal{B};(B,gBg^{-1})\in\cup_{y\le w}\mathcal{O}_y\}$,

$\mathfrak{B}_{<w}=\{(g,B)\in G\times\mathcal{B};(B,gBg^{-1})\in\cup_{y<w}\mathcal{O}_y\}$.

Define $\pi_{\le w}:\mathfrak{B}_{\le w}\to G$ by $\pi_{\le w}(g,B)=g$. Define $\pi_{<w}:\mathfrak{B}_{<w}\to G$ by $\pi_{<w}(g,B)=g$. Let $K_{\le w}=\pi_{\le w!}(IC(\mathfrak{B}_{\le w},\bar{\mathbf{Q}}_l))$, a complex of sheaves on G (here $\bar{\mathbf{Q}}_l$ is viewed as a local system on the open dense smooth subvariety $\mathfrak{B}_w$ of $\mathfrak{B}_{\le w}$). Let $K_{<w}=\pi_{<w!}(IC(\mathfrak{B}_{\le w},\bar{\mathbf{Q}}_l))$, a complex of sheaves on G.

2.2. We show:

(a) *Let* $y \in \mathbf{W}$. *We have* ${}^pH^jK_y = 0$ *if* $j < \Delta + \underline{l}(y)$.
We can assume that the result holds when G is replaced by a Levi subgroup of a proper parabolic subgroup of G. We can also assume that G is semisimple. We first prove (a) for y such that y has minimal length in its conjugacy class. If y is elliptic and it has minimal length in its conjugacy class in $\mathbf{W}$ then, according to [L5, 0.3(c)], π_y is affine and using [BBD, 4.1.1] we have ${}^pH^jK_y[\Delta + \underline{l}(y)]) = 0$ if $j < 0$ hence ${}^pH^{j+\Delta+\underline{l}(y)}K_y = 0$ if $j < 0$ so that (a) holds for y. If y is non-elliptic and it has minimal length in its conjugacy class in $\mathbf{W}$ then, according to [GP, 3.2.7], y is contained in the subgroup $\mathbf{W}'$ of $\mathbf{W}$ generated by a proper subset of the set of simple reflections of $\mathbf{W}$. Then $\mathbf{W}'$ can be viewed as the Weyl group of a Levi subgroup L of a proper parabolic subgroup P of G. Define $K_{y,L}$ in terms of y, L in the same way as K_y was defined in terms of y, G. For any j we have

(b) $\mathrm{ind}_P^G({}^pH^jK_{y,L}) = {}^pH^{j+\Delta-\Delta'}K_y$.
where ind_P^G is as in [L2, 4.1] and $\Delta' = \dim L$. (This is proved along the same lines as [L2, I, 4.8(a)].) If $j < \Delta + \underline{l}(y)$ we have $j' < \Delta' + \underline{l}$ where $j' = j - \Delta + \Delta'$ hence ${}^pH^{j'}K_{y,L} = 0$ so that $\mathrm{ind}_P^G({}^pH^{j'}K_{y,L}) = 0$ and

$0 = {}^pH^{j'+\Delta-\Delta'}K_y = {}^pH^jK_y$
so that (a) holds for y.

We now prove (a) for any $y \in \mathbf{W}$ by induction on $\underline{l}(y)$. If $\underline{l}(y) = 0$ then $y = 1$ has minimal length in its conjugacy class and (a) holds. Now assume that $\underline{l}(y) > 0$ and that the result is known for y' such that $\underline{l}(y') < \underline{l}(y)$. By [GP, 3.2.9] we can find a sequence $y = y_0, y_1, \dots, y_t$ in $\mathbf{W}$ such that $\underline{l}(y_0) \ge \underline{l}(y_1) \ge \dots \ge \underline{l}(y_t)$, y_t has minimal length in its conjugacy class and for any $i \in [0, t-1]$ we have $y_{i+1} = s_iy_is_i$ for some simple reflection s_i. Since (a) is already known for y_t it is enough to verify the following statement:

(c) *if* $i \in [0, t-1]$ *and (a) holds for* $y = y_{i+1}$ *then (a) holds for* $y = y_i$.
If $\underline{l}(y_i) = \underline{l}(y_{i+1})$ then, by an argument similar to that in [L3, 5.3], we see that there exists an isomorphism $\mathfrak{B}_{y_i} \xrightarrow{\sim} \mathfrak{B}_{y_{i+1}}$ commuting with the G-actions and commuting with $\pi_{y_i}, \pi_{y_{i+1}}$; hence $K_{y_i} = K_{y_{i+1}}$ and (c) follows in this case. Thus we can assume that $\underline{l}(y_i) > \underline{l}(y_{i+1})$ so that $\underline{l}(y_i) = \underline{l}(y_{i+1}) + 2$. We set $z = y_i, z' = y_{i+1}, s = s_i$. For $(g, B) \in \mathfrak{B}_z$ we can find uniquely B_1, B_2 in $\mathcal{B}$ such that $(B, B_1) \in \mathcal{O}_s$, $(B_1, B_2) \in \mathcal{O}_{z'}$, $(B_2, gBg^{-1}) \in \mathcal{O}_s$. Adapting an idea in [DL, §1], we define a partition $\mathfrak{B}_z = \mathfrak{B}_z^1 \cup \mathfrak{B}_z^2$ by

$$\mathfrak{B}_z^1 = \{(g, B) \in \mathfrak{B}_z; B_2 = gB_1g^{-1}\}, \mathfrak{B}_z^2 = \{(g, B) \in \mathfrak{B}_z; B_2 \ne gB_1g^{-1}\}.$$

Let $\pi_z^1 : \mathfrak{B}_z^1 \to G$, $\pi_z^2 : \mathfrak{B}_z^1 \to G$ be the restrictions of π_z. Let $K_z^1 = \pi_{z!}^1\bar{\mathbf{Q}}_l$, $K_z^2 = \pi_{z!}^2\bar{\mathbf{Q}}_l$. It is enough to show that ${}^pH^jK_z^1 = 0$ and ${}^pH^jK_z^2 = 0$ if $j < \Delta + \underline{l}(z)$. Now $(g, B) \mapsto (g, B_1)$ is a morphism $\mathfrak{B}_z^1 \to \mathfrak{B}_{z'}$, in fact an affine line bundle. It follows that $K_z^1 = K_{z'}[-2]$. Thus ${}^pH^jK_z^1 = {}^pH^{j-2}K_{z'}$. This is 0 for $j < \Delta + \underline{l}(z)$ since $j - 2 < \Delta + \underline{l}(z')$. Now $(g, B) \mapsto (g, B_2)$ is a morphism $\mathfrak{B}_z^2 \to \mathfrak{B}_{sz'}$, in fact a line bundle with the zero-section removed. It follows that for any j we have an exact sequence of perverse sheaves on G:

$${}^pH^{j-1}K_{sz} \to {}^pH^jK_z^2 \to {}^pH^j(K_{sz}[-2]).$$

Since $\underline{l}(sz')=\underline{l}(z)-1$ we know that (a) holds for sz. If $j<\Delta+\underline{l}(z)$ then $j-1<\Delta+\underline{l}(sz')$ hence ${}^pH^{j-1}K_{sz'}=0$ and ${}^pH^j(K_{sz'}[-2])={}^pH^{j-2}K_{sz'}=0$; the exact sequence above then shows that ${}^pH^jK^2_z=0$. This completes the inductive proof of (c) hence that of (a). (A somewhat similar strategy was employed in [OR] to prove a vanishing property for the cohomology of the varieties X_w of [DL]; I thank X.He for pointing out the reference [OR] to me.)

2.3. We show:

(a) *Let $y\in\mathbf{W}$ and let A be a character sheaf on G such that $(A:\oplus_j{}^pH^jK_{y'})=0$ for any $y'\in\mathbf{W}$, $y'<y$. Then $(A:{}^pH^jK_y)=0$ for any $j\ne\Delta+\underline{l}(y)$. Moreover, if $j=\Delta+\underline{l}(y)$, there exists a (necessarily unique) subobject ${}^pH^jK_y^A$ of ${}^pH^jK_y$ such that ${}^pH^jK_y/{}^pH^jK_y^A$ is semisimple, A-isotypic and $(A:{}^pH^jK_y^A)=0$.*

From our assumption we deduce (as in [L2, III, 12.7]) that $(A:\oplus_j{}^pH^jK_{<y})=0$. Hence the obvious morphism $\phi_j:{}^pH^jK_y\to{}^pH^jK_{\le y}$ satisfies $(A:\ker\phi_j)=0$, $(A:\text{coker}\phi_j)=0$. In particular, $(A:{}^pH^jK_{\le y})=(A:{}^pH^jK_y)$ for any j. Since $\pi_{\le y}$ is proper, ${}^pH^jK_{\le y}$ is semisimple, see [BBD]. Hence there is a unique direct sum decomposition of perverse sheaves ${}^pH^jK_{\le y}={}^pH^jK_{\le y,A}\oplus M$ such that ${}^pH^jK_{\le y,A}$ is semisimple, A-isotypic and $(A:M)=0$. Let

$u:{}^pH^jK_{\le y,A}\oplus M\to{}^pH^jK_{\le y,A}$

be the first projection. The composition

${}^pH^jK_y\xrightarrow{\phi_j}{}^pH^jK_{\le y,A}\oplus M\xrightarrow{u}{}^pH^jK_{\le y,A}$

is surjective (the image of ϕ_j contains ${}^pH^jK_{\le y,A}$ since $(A:\text{coker}\phi_j)=0$). Let ${}^pH^jK_y^A$ be the kernel of this composition. Then ${}^pH^jK_y/{}^pH^jK_y^A\cong{}^pH^jK_{\le y,A}$ hence ${}^pH^jK_y/{}^pH^jK_y^A$ is semisimple, A-isotypic. Moreover

$$(A:{}^pH^jK_y^A)=(A:{}^pH^jK_y)-(A:{}^pH^jK_y^A{}^pH^jK_y)$$
$$=(A:{}^pH^jK_{\le y})-(A:{}^pH^jK_{\le y,A})=(A:M)=0.$$

By the Lefschetz hard theorem [BBD, 5.4.10] we have for any j':

${}^pH^{-j'}(K_{\le y}[\Delta+\underline{l}(y)])\cong{}^pH^{j'}(K_{\le y}[\Delta+\underline{l}(y)])$

hence for any j, ${}^pH^jK_{\le y}\cong{}^pH^{2\Delta+2\underline{l}(y)-j}K_{\le y}$. It follows that

${}^pH^jK_{\le y,A}\cong{}^pH^{2\Delta+2\underline{l}(y)-j}K_{\le y,A}$

so that

(b) ${}^pH^jK_y/{}^pH^jK_y^A\cong{}^pH^{2\Delta+2\underline{l}(y)-j}K_y/{}^pH^{2\Delta+2\underline{l}(y)-j}K_y^A$.

Using 2.2(a) we have ${}^pH^jK_y=0$ if $j<\Delta+\underline{l}(y)$. Hence ${}^pH^jK_y/{}^pH^jK_y^A=0$ if $j<\Delta+\underline{l}(y)$. Using (b) we deduce ${}^pH^jK_y/{}^pH^jK_y^A=0$ if $j>\Delta+\underline{l}(y)$. Thus ${}^pH^jK_y/{}^pH^jK_y^A=0$ if $j\ne\Delta+\underline{l}(y)$. Since $(A:{}^pH^jK_y^A)=0$ it follows that $(A:{}^pH^jK_y)=0$ if $j\ne\Delta+\underline{l}(y)$. This completes the proof of (a).

2.4. In this subsection we assume that G is adjoint. Let w be an elliptic element of $\mathbf{W}$ which has minimal length in its conjugacy class C. We assume that the unipotent class $\gamma=\Phi(C)$ in G (Φ as in [L3, 4.1]) is distinguished and that det$(1-$

w) is a power of p (the determinant is taken in the reflection representation of $\mathbf{W}$). According to [L4, 0.2],

(a) *the variety* $\pi_w^{-1}(\gamma)$ *is a single* G*-orbit for the* G*-action* $x : (g, B) \mapsto (xgx^{-1}, xBx^{-1})$ *on* $\mathfrak{B}_w$.

We show:

(b) $K_w[2\underline{l}(w)]|_\gamma \cong \oplus_{\mathcal{E}} \mathcal{E}^{\oplus \mathrm{rk}(\mathcal{E})}$ *where* $\mathcal{E}$ *runs over all irreducible* G*-local systems on* γ *(up to isomorphism).*

Let $(g, B) \in \pi_w^{-1}(\gamma)$ and let $Z_G(g)$ be the centralizer of g. According to [L3, 4.4(b)] we have

(c) $\dim Z_G(g) = \underline{l}(w)$.

We have a commutative diagram

$$\begin{array}{ccc} G & \xrightarrow{\beta} & G/Z_G(g) \\ \alpha \big\downarrow & & \alpha' \big\downarrow \\ \pi_w^{-1}(\gamma) & \xrightarrow{\sigma} & \gamma \end{array}$$

where β is the obvious map, $\alpha(x) = (xgx^{-1}, xBx^{-1})$, $\alpha'(x) = xgx^{-1}$, $\sigma(g', B') = g'$. Now α is surjective by (a); it is also injective since by [L3, 5.2] the isotropy groups of the G-action on $\mathfrak{B}_w$ are trivial (we use our assumption on $\det(1 - w)$). Thus α is a bijective morphism so that $\alpha_! \bar{\mathbf{Q}}_l = \bar{\mathbf{Q}}_l$. Hence

$$K_w[2\underline{l}(w)]|_\gamma = \sigma_! \bar{\mathbf{Q}}_l[2\underline{l}(w)] = \sigma_! \alpha_! \bar{\mathbf{Q}}_l[2\underline{l}(w)] = \alpha'_! \beta_! \bar{\mathbf{Q}}_l[2\underline{l}(w)].$$

We now factorize β as follows:

$G \xrightarrow{\beta_1} G/Z_G(g)^0 \xrightarrow{\beta_2} G/Z_G(g)$.

Since all fibres of β_1 are isomorphic to $Z_G(g)^0$ (an affine space of dimension $\underline{l}(w)$, see (c)), we have $\beta_! \bar{\mathbf{Q}}_l \cong \bar{\mathbf{Q}}_l[-2\underline{l}(w)]$. Thus

$$K_w[2\underline{l}(w)]|_\gamma \cong \alpha'_! \beta_{2!} \bar{\mathbf{Q}}_l = (\alpha' \beta_2)_! \bar{\mathbf{Q}}_l.$$

Now $\alpha' \beta_2$ is a principal covering with (finite) group $Z_G(g)/Z_G(g)^0$; (b) follows. (The proof above has some resemblance to the proof of [L2, IV, 21.11].)

3. Completion of the proof

3.1. In this section (except in 3.10) we assume that $p = 2$ and that G is of type E_8 or F_4. We have the following result:

(a) *Let* $y \in \mathbf{W}$ *be an elliptic element of minimal length in its conjugacy class and let* $i \in I$ *be such that* $\pi_y^{-1}(\gamma_i) \neq \emptyset$. *Then* $\underline{l}(y) \geq i$.

Indeed, from [L3, 5.7(iii)] we have $\dim \gamma_i \geq \Delta - \underline{l}(y)$ and it remains to use that $\dim \gamma_i = \Delta - i$.

3.2. Let $i\ne i'$ in I. Then

(a) $\oplus_j\mathcal{H}^jA_{i'}|_{\gamma_i}$ *does not contain* $\mathcal{E}_i$ *as a direct summand except possibly when* $i=40,i'=20$ *(type* E_8*) and* $i=12,i'=8$ *(type* F_4*).*

If $i'\ne 20$ (type E_8) and $i'\ne 8$ (type F_4) this follows from the cleanness of $A_{i'}$. If $i'=20,i\ne 40$ (type E_8) and $i'=8,i\ne 12$ (type F_4) this follows from the fact that $\gamma_i\not\subset\bar\gamma_{i'}$ except when $i'=20,i=22$ (type E_8) when the result follows from 1.4(a).

Note that if $i'=10$ (type E_8) and $i'=4$ (type F_4) then

(b) $\oplus_j\mathcal{H}^jA'_{i'}|_{\gamma_i}=0$

by the cleanness of $A'_{i'}$. If $i=10$ (type E_8) and $i=4$ (type F_4) then

(c) $\oplus_j\mathcal{H}^jA_{i'}|_{\gamma_i}=0$

since $\gamma_i\not\subset\bar\gamma_{i'}$.

3.3. We show:

(a) *Assume that* $i\in I$, $y\in\mathbf{W}$, $\underline{l}(y)<i$. *Assume also that* $i\ne 40$ *(type* E_8*) and* $i\ne 12$ *(type* F_4*). Then* $(A_i:\oplus_j{}^pH^jK_y)=0$. *If, in addition,* $i=10$ *for type* E_8 *and* $i=4$ *for type* F_4 *then* $(A'_i:\oplus_j{}^pH^jK_y)=0$.

Assume that the first assertion of (a) is false. Then we can find $y'\in\mathbf{W}$ such that $\underline{l}(y')<i$, $(A_i:\oplus_j{}^pH^jK_{y'})\ne 0$ and $(A_i:\oplus_j{}^pH^jK_{y''})=0$ for any $y''\in\mathbf{W}$ with $y''<y'$. Using 2.3(a) we see that $(A_i:{}^pH^jK_{y'})=0$ for any $j\ne\Delta+\underline{l}(y')$ hence $(A_i:{}^pH^jK_{y'})\ne 0$ for $j=\Delta+\underline{l}(y')$. It follows that $\sum_j(-1)^j(A_i:{}^pH^jK_{y'})\ne 0$. Using [L2, I, 6.5] we deduce that $\sum_j(-1)^j(A_i:{}^pH^jK_{y'_1})\ne 0$ for any $y'_1\in\mathbf{W}$ that is conjugate to y'. If y' is not elliptic then some y'_1 in the conjugacy class of y' is contained in the subgroup $\mathbf{W}'$ of $\mathbf{W}$ generated by a proper subset of the set of simple reflections of $\mathbf{W}$. Then $\mathbf{W}'$ can be viewed as the Weyl group of a Levi subgroup L of a proper parabolic subgroup P of G. Define $K_{y'_1,L}$ in terms of y'_1,L in the same way as K_y was defined in terms of y,G. We have $(A_i:\oplus_j{}^pH^jK_{y'_1})\ne 0$. From this and from the equality 2.2(b) (for y'_1 instead of y) we deduce that $(A_i:\text{ind}_P^G({}^pH^jK_{y'_1,L}))\ne 0$ for some j. Hence $(A_i:\text{ind}_P^G(\tilde A))\ne 0$ for some character sheaf $\tilde A$ on L; this contradicts the fact that A_i is a cuspidal character sheaf. We see that y' is elliptic. If the conjugacy class of y' contains an element y'_2 such that $\underline{l}(y'_2)<\underline{l}(y')$ then using again [L2, I, 6.5], we deduce from $\sum_j(-1)^j(A_i:{}^pH^jK_{y'})\ne 0$ that $\sum_j(-1)^j(A_i:{}^pH^jK_{y'_2})\ne 0$ hence $(A_i:{}^pH^jK_{y'_2})\ne 0$, contradicting the choice of y'. We see that y' has minimal length in its conjugacy class.

For any G-equivariant perverse sheaf M on G we set $\chi_i(M)=\sum_j(-1)^j(\mathcal{E}_i:\mathcal{H}^jM|_{\gamma_i})$ where $(\mathcal{E}_i:?)$ denotes multiplicity in a G-local system. For any noncuspidal character sheaf X on G we have $\chi_i(X)=0$, see 1.6(a). For any cuspidal character sheaf X on G with nonunipotent support we have clearly $\chi_i(X)=0$.

If $i'\in I-\{i\}$ then $\chi_i(A_{i'})=0$ by 3.2. Also, if $i'=10$ (type E_8) and $i'=4$ (type F_4) and $i'\ne i$ then $\chi_i(A'_{i'})=0$ by 3.2.

From the definition we have $\chi_i(A_i)\ne 0$. Since $(A_i:{}^pH^jK_{y'})=0$ for any $j\ne\Delta+\underline{l}(y')$ and $(A_i:{}^pH^jK_{y'})\ne 0$ for $j=\Delta+\underline{l}(y')$ it follows that $\chi_i({}^pH^jK_{y'})\ne 0$ for

$j \ne \Delta+\underline{l}(y')$ and $\chi_i({}^pH^jK_{y'})=0$ for $j=\Delta+\underline{l}(y')$. Hence $\sum_j(-1)^j\chi_i({}^pH^jK_{y'})\ne 0$. Hence $\sum_j(-1)^j(\mathcal{E}_i:\mathcal{H}^jK_{y'}|_{\gamma_i})\ne 0$. It follows that $K_{y'}|_{\gamma_i}\ne 0$ so that $\pi_{y'}^{-1}(\gamma_i)\ne\emptyset$. Using 3.1(a) we deduce that $\underline{l}(y')\ge i$. This contradicts $\underline{l}(y')<i$ and proves the first assertion of (a). The proof of the second assertion of (a) is entirely similar,

3.4. We now prove a weaker version of 3.3(a) assuming that $i=40$ (type E_8) and $i=12$ (type F_4).

(a) *If* $y\in\mathbf{W}$, $\underline{l}(y)<20$ *(type* E_8*) and* $\underline{l}(y)<8$ *(type* F_4*) then* $(A_i:\oplus_j{}^pH^jK_y)=0$.

We go through the proof of 3.3(a). The first two paragraphs remain unchanged. In the third pragraph, the sentence

"If $i'\in I-\{i\}$ then $\chi_i(A_{i'})=0$ by 3.2."

must be modified as follows:

"If $i'\in I-\{i\}$ and $i'\ne 20$ (type E_8) and $i'\ne 8$ (type F_4) then $\chi_i(A_{i'})=0$ by 3.2. Moreover, if $i'=20$ (type E_8) and $i'=8$ (type F_4) then by 3.3(a), $(A_{i'}:{}^pH^jK_{y'})=0$ for any j, since $\underline{l}(y')<20$ (type E_8) and $\underline{l}(y')<8$ (type F_4)". Then the fourth paragraph remains unchanged and (a) is proved.

3.5. For any $i\in I$ we consider the conjugacy class C_i of $\mathbf{W}$ whose elements have the following characteristic polynomial in the reflection representation $\mathcal{R}$ of $\mathbf{W}$:

(type E_8): q^8-q^4+1 (if $i=10$), $(q^4-q^2+1)^2$ (if $i=20$), $(q^2-q+1)^2(q^4-q^2+1)$ (if $i=22$), $(q^2-q+1)^4$ (if $i=40$);

(type F_4): (q^4-q^2+1) (if $i=4$), q^4+1 (if $i=6$), $(q^2-q+1)^2$ (if $i=8$), $(q^2+1)^2$ (if $i=12$).

We choose an element w_i of minimal length in C_i. Then $\underline{l}(w_i)=i$. Note that w_i is elliptic and $\det(1-w_i,\mathcal{R})$ is 1 (type E_8) and a power of 2 (type F_4).

Let Φ be the (injective) map from the set of elliptic conjugacy classes in $\mathbf{W}$ to the set of unipotent classes in G defined in [L3, 4.1]. We have $\Phi(C_i)=\gamma_i$.

Note that the correspondence between C_i and (the characteristic zero analogue of) γ_i appeared in another context in the (partly conjectural) tables of Spaltenstein [Sp2].

3.6. In this subsection we set $i=20, i'=40$ (type E_8) and $i=8, i'=12$ (type F_4). Let $w=w_i$. We have the following results.

(a) *If* $j\ne\Delta+i$ *then* $(A_i:{}^pH^jK_w)=0$ *and* $(A_{i'}:{}^pH^jK_w)=0$.

(b) *If* $j=\Delta+i$ *then* $(A_i:{}^pH^jK_w)=1$*; there exists a unique subobject* Z *of* ${}^pH^jK_w$ *such that* $(A_i:Z)=0$ *and* ${}^pH^jK_w/Z\cong A_i$ *and there exists a unique subobject* Z' *of* ${}^pH^jK_w$ *such that* $(A_{i'}:Z')=0$ *and* ${}^pH^jK_w/Z'$ *is semisimple,* $A_{i'}$*-isotypic.*

(a) follows from 2.3(a) applied with $y=w$ and with A equal to A_i or $A_{i'}$. (The assumptions of 2.3(a) are satisfied by 3.3(a), 3.4(a).) As in the proof of 3.3(a) we see that for any character sheaf A' not isomorphic to A_i we have $\chi_i(A')=0$. From 2.4(b) we see that $\sum_j(-1)^j\chi_i({}^pH^{j+2i}K_w)=1$ (we use that $\mathcal{E}_i$ has rank 1). Hence $\sum_j(-1)^j(A_i:{}^pH^jK_w)\chi_i(A_i)=1$ that is $(-1)^{\Delta+i}(A_i:{}^pH^{\Delta+i}K_w)\chi_i(A_i)=1$.

Since $\chi_i(A_i)=\pm 1$ it follows that $(A_i:{}^pH^{\Delta+i}K_w)=1$ proving the first assertion of (b). The remaining assertions of (b) follow from 2.3(a) applied with $y=w$ and with A equal to A_i or $A_{i'}$.

3.7. In the setup of 3.6 we show:

(a) *for any j, $\oplus_k\mathcal{H}^k({}^pH^jK_w)|_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand.*

Assume first that $j\ne\Delta+i$. It is enough to show that for any character sheaf X such that $(X:{}^pH^jK_w)\ne 0$, $\oplus_k\mathcal{H}^kX_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. If X is noncuspidal this follows from 1.6(a); if X is cuspidal, it must be different from A_i or $A_{i'}$ (see 3.6(a)) and the result follows from the cleanness of cuspidal character sheaves other than A_i.

Assume now that for some k, $\mathcal{H}^k({}^pH^{\Delta+i}K_w)|_{\gamma_{i'}}$ contains $\mathcal{E}_{i'}$ as a direct summand. This, and the previous paragraph, imply that for some k, $\mathcal{H}^k(K_w)|_{\gamma_{i'}}$ contains $\mathcal{E}_{i'}$ as a direct summand. In particular $K_w|_{\gamma_{i'}}\ne 0$ so that $\pi_w^{-1}(\gamma_{i'})\ne\emptyset$. Using 3.1(a) we deduce that $\underline{l}(w)\ge i'$ that is, $i\ge i'$. This contradiction proves (a).

3.8. We preserve the setup of 3.6. We have ${}^pH^{\Delta+i}K_w=Z+Z'$ since

${}^pH^{\Delta+i}K_w/(Z+Z')$

is both A_i-isotypic and $A_{i'}$-isotypic. (It is a quotient of ${}^pH^{\Delta+i}K_w/Z$ which is A_i-isotypic and a quotient of ${}^pH^{\Delta+i}K_w/Z'$ which is $A_{i'}$-isotypic.) As in the proof of 3.7(a) we see that all composition factors X of $Z\cap Z'$ (which are necessarily not isomorphic to A_i or $A_{i'}$) satisfy the condition that $\oplus_k\mathcal{H}^k(X)|_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. It follows that $\oplus_k\mathcal{H}^k(Z\cap Z')_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. Using this and 3.7(a) we deduce that $\oplus_k\mathcal{H}^k({}^pH^{\Delta+i}K_w/(Z\cap Z'))_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand. Since ${}^pH^{\Delta+i}K_w=Z+Z'$, the natural map ${}^pH^{\Delta+i}K_w/(Z\cap Z')\to({}^pH^{\Delta+i}K_w/Z)\oplus({}^pH^{\Delta+i}K_w/Z')$ is an isomorphism. It follows that

(a) $\oplus_k\mathcal{H}^k({}^pH^{\Delta+i}K_w/Z)_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand;

(b) $\oplus_k\mathcal{H}^k({}^pH^{\Delta+i}K_w/Z')_{\gamma_{i'}}$ does not contain $\mathcal{E}_{i'}$ as a direct summand.

Since ${}^pH^{\Delta+i}K_w/Z\cong A_i$, we see that (a) (together with 1.5(a)) proves the cleanness of A_i thus completing the proof of Theorem 1.2. Since ${}^pH^{\Delta+i}K_w/Z'$ is a direct sum of copies of $A_{i'}$ and $\oplus_k\mathcal{H}^k(A_{i'})_{\gamma_{i'}}=\mathcal{E}_{i'}$ we see that (b) implies ${}^pH^{\Delta+i}K_w/Z'=0$. This, together with 3.6(a),(b) implies that

(c) $(A_{i'}:{}^pH^jK_w)=0$ *for any j.*

3.9. In view of the cleanness of G, we can restate 3.2(a) in a stronger form:

(a) *Let $i\ne i'$ in I. Then $\oplus_j\mathcal{H}^jA_{i'}|_{\gamma_i}$ does not contain $\mathcal{E}_i$ as a direct summand.*

Using this the proof of 3.3(a) applies in greater generality and yields the following result.

(b) *Assume that $i\in I$, $y\in\mathbf{W}$, $\underline{l}(y)<i$. Then $(A_i:\oplus_j{}^pH^jK_y)=0$. If, in addition, $i=10$ for type E_8 and $i=4$ for type F_4 then $(A'_i:\oplus_j{}^pH^jK_y)=0$.*

From (b), 2.3(a) and 2.4(b) we deduce as in 3.6 the following result for any $i\in I$:

(c) *If $j\ne\Delta+i$ then $(A_i:{}^pH^jK_{w_i})=0$; if $j=\Delta+i$ then $(A_i:{}^pH^jK_{w_i})=1$ and there exists a unique subobject Z of ${}^pH^jK_{w_i}$ such that $(A_i:Z)=0$ and*

${}^pH^jK_{w_i}/Z \cong A_i$.
The same result holds for $i = 10$ (type E_8) and $i = 4$ (type F_4) if A_i is replaced by A'_i.

3.10. Note that, once Theorem 1.2 is known, the parity property [L2, III, (15.13.1)] can be established for a reductive group in any characteristic as in [L2]. (Incidentally, note that 3.9(c) establishes the parity property for the character sheaves A_i for $p = 2$, type E_8 or F_4.) Using this we see that essentially the same proof as in [L2] establishes [L2, V, Theorems 23.1, 24.4, 25.2, 25.6] (but not [L2, V, Theorem 24.8]) for a reductive group in any characteristic.

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