HEAT REQUIREMENT FOR CHEMICALLY SELF-HEATING CONTAINERS

by

GERALD STEPHEN ADOLPH

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Signature of Author .... Signature redacted
Department of Chemical Engineering

Certified by........................ Signature redacted
Thesis Supervisor

Accepted by........................ Signature redacted
Chairperson, Departmental Committee on Theses

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ABSTRACT

Self Heating Containers

Gerald S. Adolph

Advisor: Prof. James H. Porter

This paper investigates the heat requirement for maintaining a cylindrical container and contents at some given temperature. First a detailed analysis is done followed by a specific example. The aim is to present a general procedure for screening potential chemical heating systems or container design parameters. In order to facilitate this a computer program is presented to compute the required heat and various key temperatures.

SELF HEATING CONTAINERS

The idea of self heating containers is one which has intrigued the commercial and scientific world. The applications of chemically generated heat ranges from food containers to the limits of one's imagination. For example, in 1973 another M.I.T. student wrote a master's thesis on chemically heating an ocean diver's wet suit. Numerous patents exist using double-walled containers with various heat generating compounds such as aluminum, copper sulfate, potassium chlorate, calcium sulfate, calcium oxide, and even beeswax. The reactions employed include oxidation-reduction, fusion, combustion, hydration and many more. The main problem involved is finding a suitable chemical reaction for your particular situation. Those in the food or drug business will be considerably more concerned about toxicity than the shaving cream manufacturer.

The entrepreneur who initiates efforts to produce a self heating container will probably follow the logical sequence-

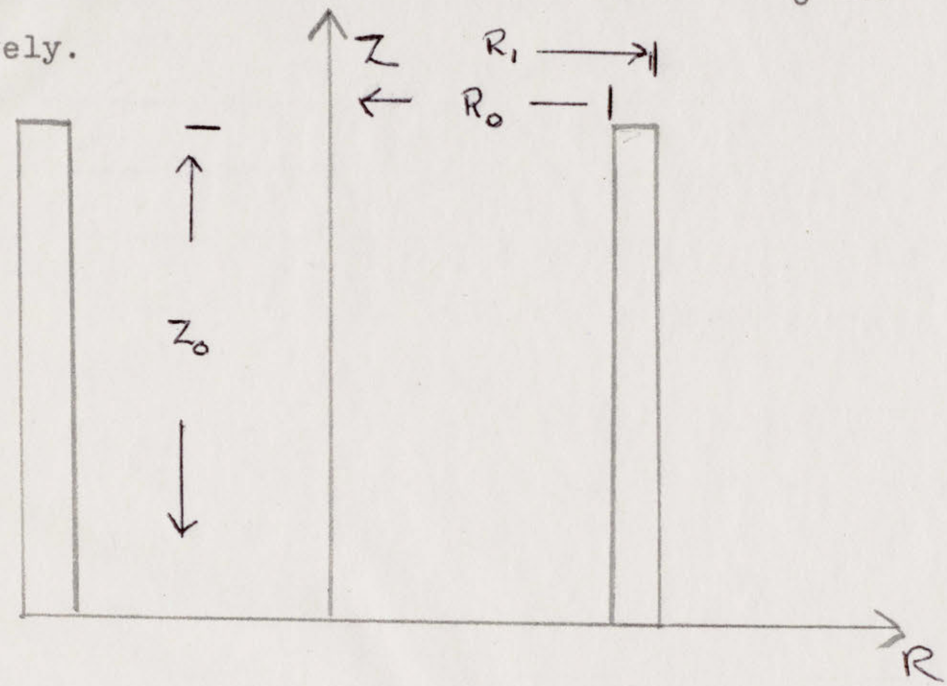
1) Identify tenperature requirements
2) Identify a suitable chemical system
   (based on heat required, toxicity, timing, triggering and economics)
3) Design the container

The temperature desired is generally specified by the product and its' uses whereas the design of the container is specified by the product, uses, chemical system, and physical limitations (i.e. vending machine size).

I became interested in formulating a generalized procedure for calculating the heat needed to maintain a given system at some desired temperature. By varying the the chemical systems, physical demensions or constants, heat supplied or temperature required, one could construct an optimal system and narrow doen possible choices thus saving research and development costs.

A. The Material To Be Heated

Consider the open cylindrical double-walled container of height Z_0, inner radius R_0, outer radius R_1. We will denote the widths of the inner and outer walls as W_0 and W_1 respectively.



Let T_A be the ambient temperature which is assumed to be constant. Also we will assume the bottom of the cup is very well insulated and that heat loss is negligible. Constructing a heat balance on a differential volume element of the substance to be maintained will yeild the following equation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial Z^2}=\frac{\rho \bar{C}p}{Rs}\;\frac{T}{t}$$

where r= radial position

Z= horizontal position

T= substance temperature= T(r,z)

ρ = density

Cp= heat capacity

t= time   h= heat transfer coefficient

k= thermal conductivity

h= heat transfer cor

Since we are interested in maintaining a temperature rather than achieving one, we may drop the time dependent term which describes the heat accumulation. This gives us a steady state equation -

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial Z^2} = 0$$

In specifying the boundary conditions we want T to be finite at r=0. Similarly $\frac{\partial T}{\partial Z} = 0$ at Z=0. At the surface of the material we have cooling of the form $k_m \frac{T}{Z} = h_m (T-T\ )$ at $Z=Z_o$ and on the side we would look for conductive heat transfer. The model is now as follows:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial Z^2} = 0$$

subject to $\frac{T}{Z} = 0$ at Z=0

T is finite at r=0

$k_m \frac{T}{Z} = h_m (T-T\ )$ at $Z=Z_o$

$-k_m \frac{T}{r} = Q_w$ at $r=R_o$

Q_w = generated heat which passes through the wall into the fluid

For mathematical convenience, let's define $\overline{T}$ (r,Z) such that

$$\overline{T}(r,Z) = T(r,Z) - T_A$$

Now we have

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{T}}{\partial r}\right) + \frac{\partial^2 \overline{T}}{\partial Z^2} = 0$$

such that $\frac{\partial \overline{T}}{\partial Z} = 0$, Z=0 $\qquad -k_m \frac{\partial \overline{T}}{\partial r} = Q_w$, $r=R_o$

$\overline{T}$ finite, r=0 $\qquad k_m \frac{\partial \overline{T}}{\partial Z} = h_m \overline{T}$

In order to place limits on our solution we will consider the best case and the worst case. The worst case is that of the well-mixed fluid. This is valid since the temperature gradient within the fluid and the and the convective cooling at the surface will cause considerable convection currents and result in a mixed fluid of uniform themperature. It is this temperature we seek to control.

The best solution is that of the stagnant fluid (not likely) or the solid or near solid substance. In this instance there will be a temperature profile which will specify the coolest location within the substance. It is this location's temperature we wish to control.

1) Well-Mixed Case

This is easy to solve, since the uniform temperature resulting from well-mixing, allows us treat the material as a bulk entity with T independent of r and Z. It is obvious that the heat we must supply is equal to that which is lost.

Heat In = Heat Out

$Q_W = h_m A_m \overline{T}$ where A_m is the surface area of the material.

so, $Q_W = h_m \pi R_o^2 \overline{T}$

Given some specific optimal temperature $\overline{T}$ is determined, which means Q_W is also determined.

2) The Stagnant Case

The stagnant case requires us to use the original equation to compute the temperature profile and heat loss.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \overline{T}}{\partial r}\right) + \frac{\partial^2 \overline{T}}{\partial Z^2} = 0$$

$$\frac{\partial T}{\partial Z} = 0, \quad Z=0$$

$$k_m \frac{\partial \overline{T}}{\partial Z} = h\overline{T}, \quad Z=Z_o$$

$\overline{T}$ is fininte, r=0

$$-k_m \frac{\partial \overline{T}}{\partial r} = Q_W \text{ at } r = R_o$$

Assume some product solution plus a parcular solution.

The General Product form is $\overline{T}(r,Z) = R(r)\, M(Z)$.

Substitution into the Differential Equation yields

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{M''}{M} = 0.$$

Separating variables gives

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} = \frac{-M''}{M} = L^2$$

where L^2 is the constant of separation L 0

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} - L^2 = 0$$

This is a modified Bessel's Equation of order zero with the Standard solution

$$R(r) = C_1 I_o(Lr) + C_2 K_o(Lr)$$

$$\frac{M''}{M} + L^2 = 0$$

$$M(Z) = C_3 \sin(LZ) + C_4 \cos(LZ)$$

$C_3=0$ since $\frac{\partial \overline{T}}{\partial Z} = 0$ at $Z=0$ (B.C. 1)

$C_2 = 0$ since $K_o(Lr)$ is not finite at $r=0$ (B.C. 3)

Therefore, $\overline{T}(r,Z)$ $I_o(Lr)\cos(LZ)$ [1]

Using B.C. $k_m \frac{\partial \overline{T}}{\partial Z} = L\overline{T}$, $Z=Z_o$

$$k_m \sum_{n=1}^{\infty} L_n I_o(L_n r)\sin(L_n Z_o) = h_m \sum_{n=1}^{\infty} I_o(Lr)\cos(LZ)$$

for the equation to be true, each

$$k_m L_n \sin(L_n Z_o) = h_m \cos(L_n Z_o).$$

This gives us the eigenvalue equation to determine L

$$\tan(L_n Z_o) = \frac{\left(\frac{h_m}{k_m} Z_o\right)}{L_n Z_o}$$

From the last B.C. $-k_m \frac{\overline{T}}{r} = Q_w$

1. At this point we aren't really sure whether or not we are dealing with discrete or continuous values however, these types of heat transfer problems generally yield discrete values.

$$\overline{T}(r,Z) = \sum_{n=1}^{\infty} A_n I_o(L_n r)\cos(L_n Z)$$

$$-k_m \sum_{n=1}^{\infty} A_n L_n I_o'(L_n R_o)\cos(L_n Z) = Q_w$$

Multiply by $\cos(L_m Z)$ and integrate from 0 to Z_o

$$\int_0^{Z_o} -k_m \sum_{n=1}^{\infty} A_n L_n I_o'(L_n R_o)\cos(L_n Z)\cos(L_m Z)\, dZ$$

$$= \int_0^{Z_o} Q_w \cos(L_m Z)\, dZ$$

Due to orthogonality

$$-k_m A_n L_n I_o' \quad \cos(L_n Z)\cos(L_m Z)\, dZ = 0 \quad \text{for } n \neq m$$

Therefore, all terms in the sum vanish except n = m.

For n=m

$$\int_0^{Z_o} C_n \cos^2(L_n Z)\, dZ = \frac{C_n}{2}\int_0^{Z_o} (1-\cos(2L_n Z))\, dZ$$

$$= \frac{C_n}{2}\left(Z - \frac{\sin(2L_n Z)}{2L_n}\right)\Bigg]_0^{Z_o} = \frac{C_n}{2}\left(Z_o - \frac{\sin(2L_n Z_o)}{2L_n}\right)$$

$$= \frac{C_n Z_o}{2}\left(1 - \frac{\sin(2L_n Z_o)}{2L_n Z_o}\right)$$

For our case $\quad C_n = -A_n k_m L_n I_o'(L_n R_o)$

Therefore, for n=1,2,3,...

$$A_n = -\frac{2 \quad Q_w \cos(L_n Z)\, dZ}{Z_o L_n k_m I_o'(L_n R_o)\left(1 - \frac{\sin(2L_n Z_o)}{2L_n Z_o}\right)}$$

Since we're looking for a constant Q_w

$$A_n = -\frac{4Q_w \sin(L_n Z_o)}{k_m I_o'(L_n Z_o) L_n (2L_n Z_o - \sin(2L_n Z_o))}$$

A particular solution to the D.E. is $T=T_A$ (or $\overline{T}=0$)

Let $A_n^* = A_n/Q_w$

Therefore, the total solution is

$$T(r,Z) = T_A + Q_w \sum_{n=1}^{E} A_n^* I_o(L_n r)\cos(L_n Z)$$

where E is number of eigenvalues

We can now specify either T at some point to find the necessary Q_w or vice-versa. We can either use the minimum T according to the function, the centerline T, or in the case of food containers we may wish to use some T at the surface ($Z=Z_o$) since this is what will most likely be in contact with the mouth.

B. The Heating Element

Having found Q_w, the heat to be supplied to the contents, we now need to know Q, the total heat to be generated.

$Q = Q_w + Q_A + Q_1$ where Q_1 = heat absorbed

Q_2 = heat lost to atmosphere

Again referring to the steady state condition $\rightarrow Q_1 = 0$

A heat balance on the chemically reacting system yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial Z^2} + \frac{Q}{k} = \frac{\rho C_p}{k}\frac{\partial T}{\partial t}$$

Adding to the steady state condition, is the obviously desirable assumption that the width of this heating region will be significantly less than the height of the cup. If insulated at the top and bottom the equation simplifies to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{Q}{k_h} = 0 \qquad R_o \quad r \quad R_1 - W_o \qquad W_o = \text{outer wall thickness}$$

With the general solution

$$T = A + B \ln r - \frac{Qr^2}{4k_h}$$

Where k_h = thermal conductivity of the heating substance. In order to determine the constants A and B we must apply the apporpriate boundary conditions.

$T = T_2$ at $r = R_1 - W_o$

$T = T_1$ at $r = R_o + W_1$ $\qquad W_1$ = inner wall thickness

We can determine T_1 since we know the desired Q_w and

$$Q_w = \frac{k_1(T_1 - T(R_o,Z))}{W_1} \qquad k_1 = \text{inner wall conductivity}$$

hence, $T_1 = \frac{W_1 Q_w}{R_1} + T(R_o,Z)$ $\qquad k_o$ = outer wall conductivity

Combining the two wall temperatures

$$T_1 - T_2 = B \ln \frac{a}{b} - (a^2 - b^2) \frac{Q}{4k} \qquad \text{where } a = R_o + W_1$$

$$b = R_1 - W_o$$

From the original solution

$$\frac{dT}{dr} = \frac{B}{r} - \frac{rQ}{2k}$$

At r = b $\quad \frac{dT}{dr} = \frac{T_2 - T_A}{W_o} = \frac{B}{b} - \frac{bQ}{2k_o}$

solving for B gives

$$B = \frac{b(T_2 - T_A)}{W_o} + \frac{b^2 Q}{2k_o}$$

$$T_1 - T_2 = \left(\frac{b(T_2 - T_A)}{W_o} + \frac{b^2 Q}{2k_o}\right) \ln \frac{a}{b} - (a^2 - b^2) \frac{Q}{4k_o}$$

$$\frac{T_1 - T_2}{\ln \frac{a}{b}} - \frac{b(T_2 - T_A)}{W_o} = \frac{b^2 Q}{2k_o} - \frac{(a^2 - b^2)}{\ln \frac{a}{b}} \frac{Q}{4k_o}$$

$$T_2 = - \left(\left(\frac{1}{\ln \frac{a}{b}} + \frac{b}{w_o}\right)^{-1} \left(\frac{-T_1}{\ln \frac{a}{b}} - \frac{bT_A}{w_o} + \frac{b^2 Q}{2k} + \frac{(b^2 - a^2)Q}{4k_o \ln \frac{a}{b}}\right)\right)$$

Realizing that $Q = Q_w + Q_o$ and $Q_o = k_o \left(\frac{T_2 - T_A}{w_o}\right)$ and utilizing much algebra

$$T_2 = - \left(\left(1 - \frac{w_o}{b \ln \frac{a}{b}}\right)\left(\frac{(b^2 - a^2)Q_w}{4k_o} - T_1\right) + \left(w_o + b \ln \frac{a}{b}\right)\left(\frac{T_A}{w_o} + \frac{bQ_w}{2k_o}\right)\right.$$

$$\left. + T_A\left(\frac{(b^2 - a^2)}{4w_o} + \frac{b}{2}\right)\right)\left(1 + \frac{(b^2 - a^2)}{4w_o} + \frac{b}{2}\right)^{-1}$$

Having computed T_2 we can now compute Q_o and subsequently Q, the total heat to be generated. The variable "a" is specified by the desired container inner dimensions, based on the amount of material to be enclosed. The variable "b" is generally specified by machine tolerances (i.e. vending, handling, packing). The trick is to find a chemical which will generate an appropriate amount of heat within that set volume.

C. An Example

Given the preceeding information, an actual example would be quite tedious to derive by hand. In formulating a computer program (See Appendix 1) to do the calculations, I divided the problem into two parts. First is the solution of the eigen-value equation and secondly, the calculation of Q_w . In formulating the eigenvalues, one must be careful about the numerical technique used. Methods which rely on derivatives, (i.e. Newton Raphson), or the use of zero values for the variables, will cause the computer to do peculiar things since the function c/x goes to infinity at the origin and tan (x) is discontinuous and singular at regular intervals.

Let us take the problem of designing a cup for some hot liquid. For simplicity we will choose water since it is the basis for many common beverages (i.e. coffee and tea).

For water in a 9 oz. cup:

$h(\text{to air}) \cong 1.22 \ \frac{\text{BTU}}{\text{hr ft}^2\,°\text{F}}$

$k = 0.381 \ \frac{\text{BTU}}{\text{hr ft }°\text{F}}$

d(diameter) = .21 ft.

z_0 (height)= .29167 ft.

Based on subjective tests, coffee is "hot" at 150° F. We will use this as a target temperature for calculating the heat requirement (Q_w). Assume the room has an ambient temperature of about 70° F. We also must specify some radial and vertical position that we wish to control. The calculations are easily done by hand for the well mixed case and the results of this and the computer analysis of the stagnant case is summarized in Table 1. Since we are speaking of an edible item we will take a relatively low toxic material for our example. Calcium oxide generates approximately 327 calories per gram upon hydration and is the topic of at least two patents, (See Appendix II). Given the analysis of section B the calculation of T_2 and consequently Q is straightforward. If the situation arose where one had complete freedom with "b", the proper course would be to specify some desired Q (probably the minimum which is $Q=Q_w$). For mathematical simplicity we will use this case assuming good heat transfer through the inner wall and good outer insulation. Table 2 summarizes this data.

TABLE 1

Case	Temperature Control Location	Heat Required BTU/minute	Surface	Middle	Bottom	Wall
1	surface	2.076×10^2	150	80.2	151.5	83.4
2	middle	1.62×10^3	697.2	150	708.6	174.7
3	bottom	-2.05×10^2	140	80.0	150	83.2
4	wall	1.24×10^3	549.3	131.1	558.3	150
Well Mixed	entire body	13.52	150	150	150	150

TABLE 2

Case	Heating Volume Required
1	5.64×10^{-3}
2	4.41×10^{-2}
3	5.58×10^{-3}
4	3.37×10^{-2}
Well mixed	6.15×10^{-5}

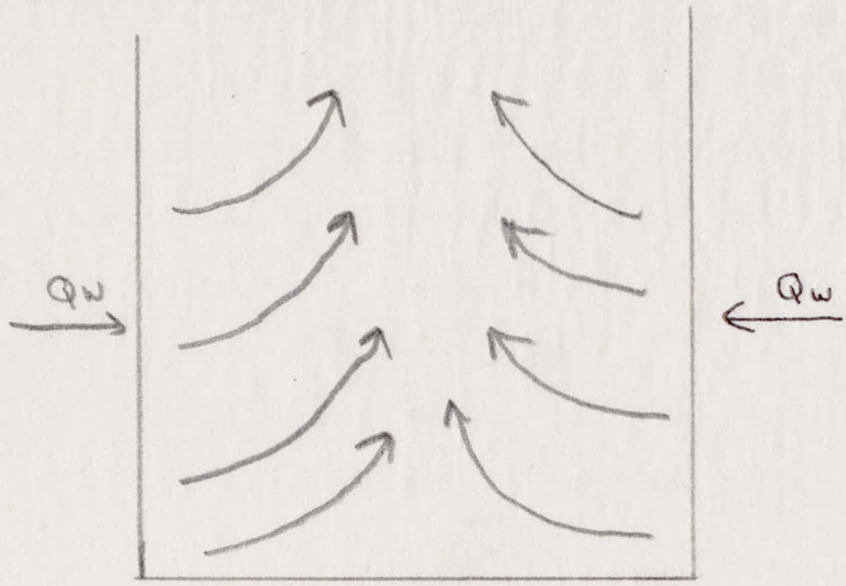
D. Interpretation of Data

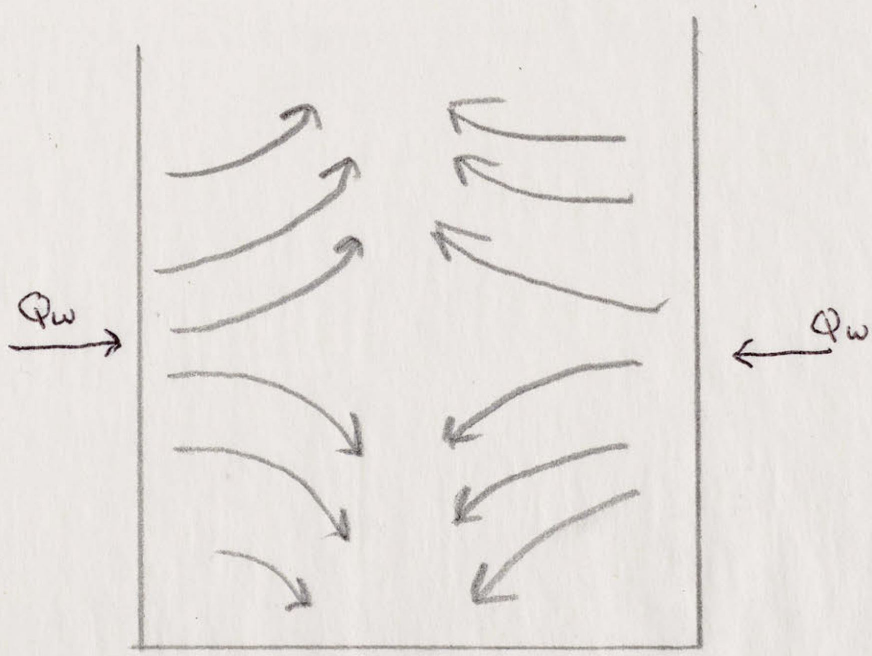
For all cases, as expected, the temperature decreased with radius however the distribution with respect to vertical position is quite curious. One would have expected the temperature to be coolest on the surface and hottest at the bottom with some even distribution in between. Instead, in most cases, we found a "U" shaped distribution with th middle being the cool spot. I can think of two ways to explain this. The first is obvious - the model is wrong. We have ignored in this stagnant model, the convection currents that would be present in a fluid of this type whereas in calculating h_m we used the density, viscosity, thermal expansion coefficient - factors which would indicate considerable mixing. If this were an inviscid fluid we would probably have gotten much different results. However this was intended to give us an upper limit on the design parameters.

The other possible explanation is that of heat flows. Since we know that heat rises we may have had a flow pattern such as that shown in figure 2. This would have resulted in the bottom being the cold spot and heat flows from warm to cold giving us a combined flow such as that shown in figure 3. This would give a minimum temperature somewhere in the center of the container.

The appearance of a negative heat requirement when the temperature control is at the bottom is a mathematical consequence of operating in the region of the origin. I would not put much faith in the actual number but the negative sign tells us that the middle of the bottom is a hot spot ant that to maintain it at 150° F would require the removal of heat. This would cause all of the temperatures above it to be cooler than desired, which is not unexpected.

Based solely upon the stagnant model, case 1 with the temperature at the surface being controlled would be the best choice. It will give the temperature profile closest to the optimal with the important spot, the one being consumed first, at the desired temperature. Since the well mixed model is probably better I would predict a heat requirement somewhere in the range of 40-75 BTU, and I would design the cup on the basis of this first approximation. Once designed, the prototype can be tested in a real situation to find the empirical parameters.





E. CONCLUSION

Based on this narrow analysis and the example, I feel that self-heating containers are technically feasible and not very difficult to design. There are, of course, other areas which must be investigated:

1) The time dependance should be considered.
2) Especially for a fluid, a mass and momentum transfer analysis is necessary; first to consider the convective currents within the container and their effect on the temperature distribution, and secondly there is significant heat loss at the surface due to vaporization of the fluid. We have ignored this.
3) Clearly chemical investigation must be done to find appropriate heating elements (time, amount of heat, cost, toxicity).
4) There must be considerable thought given to the control and triggering of this heating reaction.
5) Finally, an analysis is required to decide whether the heat source should be in the wall the base or both.

APPENDIX I

COMPUTER SYMBOLS

EPSIL____ Tolerance for accuracy of tangent function solution

DELTA----- Increment for increasing (decreasing) x in solution technique

C ------- h^*z_0/k (from eigenvalue equation)

TK-------- Thermal conductivity

Z_0 -------- Height of container

RCONT----- radial position of control temperature point

ZCONT----- Vertical position of control point

TOPT------ Optimal temperature

Tamb------ ambient temperature

L--------- Eigenvalues

A--------- Constant A_n^* from stagnant temperature profile

TESTR,TESTA----Test radial and vertical positions for finding key temperatures

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PAGE   1

// JOB T

LOG DRIVE   CART SPEC   CART AVAIL   PHY DRIVE
  0000        12FA        12FA         0000

V2 M12   ACTUAL  8K   CONFIG  8K

// FOR
*IOCS(TYPEWRITER,KEYBOARD,DISK,CARD,1132PRINTER)
*ONE WORD INTEGERS
*LIST ALL
C     THIS PROGRAM WILL CALCULATE THE HEAT REQUIRED TO MAINTAIN A GIVEN
C     CYLINDRICAL CONTAINER ,FILLED WITH SOME MATERIAL AT A PARTICULAR
C           TEMPERATURE
      REAL L(50)
      DIMENSION X(50),A(50),XZ(50),XX(50),DIV(50),TESTR(5),TESTZ(5)
      DO 90 J=1,4
      N=1
      READ (2,5) EPSIL,DELTA,C,TK,ZC,RCONT,ZCONT,TOPT,TAMB
 5    FORMAT (F10.5,F10.5)
      WRITE(3,88) J,RCONT,ZCONT
      WRITE (3,50)
      TOT=0.0
      SUM=0.
C     THIS SECTION OF THE PROGRAM CALCULATES THE SIGNIFICANT ROOTS
C     TO THE EQUATION 'X*TAN(X)=CONSTANT'
      DO 75 N=1,50
      X(N)=N*3.1
      FUNC=SIN(X(N))/COS(X(N))-C/X(N)
      IF (ABS(FUNC)-EPSIL) 40,40,15
 15   IF (FUNC) 20,40,30
 20   X(N)=X(N)+DELTA
      FUNC=SIN(X(N))/COS(X(N))-C/X(N)
      IF(ABS(FUNC)-EPSIL)40,40,24
 24   CONTINUE
      IF (FUNC) 20,40,25
 25   DELTA=DELTA/2
 30   X(N)=X(N)-DELTA
      FUNC=SIN(X(N))/COS(X(N))-C/X(N)
      IF(ABS(FUNC)-EPSIL)40,40,34
 34   IF (FUNC) 35,40,30
 35   DELTA=DELTA/2
      GO TO 20
 40   L(N)=X(N)/ZC
      CALL BESI (X(N),1,BI,IER)
      A(N)=4*SIN(X(N))/(L(N)*TK*BI*(2*X(N)-SIN(2*(X(N)))))
      DELTA=0.1
      XZ(N)=L(N)*ZCONT
      XX(N)=L(N)*RCONT
      CALL BESI (XX(N),0,BI,IER)
      DIV(N)=A(N)*BI*COS(XZ(N))
      SUM=SUM+ABS(A(N))
      TOT=TOT+DIV(N)
```

```
      WRITE (3,55) L(N),N
      IF (ABS(A(N)/SUM)-.0001) 80,80,75
75    CONTINUE
C     NOW USING THE EIGENVALUES CALCULATED ABOVE,THIS SECTION WILL FIND THE
C     DESIRED HEAT TO BE SUPPLIED TO THE MATERIAL
80    QW=(TOPT-TAMB)/TOT
      WRITE (3,85) QW
C     FINALLY WE WILL EVALUATE THE TEMPERATURES AT SOME OTHER KEY POINTS
      DO 89 I=1,4
      READ (2,1) TESTR(I),TESTZ(I)
 1    FORMAT (F10.5,F10.5)
      TTOT=0.
      DO 2 K=1,N
      V=L(K)*TESTR(I)
      S=L(K)*TESTZ(I)
      CALL BESI (V,0,VB,IER)
      TTOT=TTOT+A(K)*VB*COS(S)
 2     CONTINUE
      TEMPT=TAMB+ABS(QW*TTOT)
      WRITE (3,3) TESTR(I),TESTZ(I),TEMPT
 3    FORMAT (1X,//,'  FOR R =',F10.5,'  + Z =',F10.5,'    TEMPERATURE ='
     1,E15.7///)
89    CONTINUE
50    FORMAT (1X,'      EIGENVALUES      ','          N          ',//)
55    FORMAT (1X,/E15.7,5X,I7)
85    FORMAT (1X,'  HEAT REQUIRED IS ',E15.7,'BTU PER HR.'/)
88    FORMAT (1X,'THIS IS CASE',I5,'WITH TEMP CONTROL AT',F10.5,4X,F10.5
     1/)
90    CONTINUE
      CALL EXIT
      END
 VARIABLE ALLOCATIONS
     X(R )=0062-0000      A(R )=00C6-0064    XZ(R )=012A-00C8    XX(R )=018E-012C   DIV(R )=01F2-0190 TESTR(R )=01FC-01F4
 TESTZ(R )=0206-01FE      L(R )=026A-0208 EPSIL(R )=026C      DELTA(R )=026E          C(R )=0270          TK(R )=0272
    Z0(R )=0274       RCONT(R )=0276     ZCONT(R )=0278       TOPT(R )=027A       TAMB(R )=027C         TOT(R )=027E
   SUM(R )=0280        FUNC(R )=0282        BI(R )=0284         QW(R )=0286       TTOT(R )=0288           V(R )=028A
     S(R )=028C          VB(R )=028E     TEMPT(R )=0290          J(I )=029C          N(I )=029D         IER(I )=029E
     I(I )=029F           K(I )=02A0

 STATEMENT ALLOCATIONS
 5    =02B4  1    =02B7  3    =02BA  50   =02D8  55   =02EF  85   =02F5  88   =030B  15   =03B2  20   =03B9  24   =03F6
 25   =03FD  30   =0404  34   =0441  35   =0448  40   =0451  75   =0529  80   =0532  2    =0590  89   =05B5  90   =05BE

 FEATURES SUPPORTED
  ONE WORD INTEGERS
  IOCS

 CALLED SUBPROGRAMS
  FSIN    FCOS    FABS    BESI    FADD    FADDX   FSUB    FMPY    FMPYX   FDIV    FDIVX   FLD     FLDX    FSTO    FSTOX
  FSBR    FDVR    FLOAT   TYPEZ   CARDZ   PRNTZ   SRED    SWRT    SCOMP   SFIO    SIOFX   SIOF    SIOI    SUBSC   SDFIO

 REAL CONSTANTS
  .000000E 00=02A6    .310000E 01=02A8    .100000E 00=02AA    .100000E-03=02AC
```

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INTEGER CONSTANTS
     1=02AE       4=02AF       2=02B0       3=02B1       50=02B2       0=02B3

CORE REQUIREMENTS FOR
 COMMON        0  VARIABLES     678  PROGRAM     802

END OF COMPILATION

// XEQ

THIS IS CASE     1WITH TEMP CONTROL AT    0.00000          0.29170

        EIGENVALUES             N


  0.1168994E 02                 1

  0.2203607E 02                 2

  0.3264599E 02                 3

  0.4333361E 02                 4
   HEAT REQUIRED IS    0.1245756E 05BTU PER HR.


 FOR R =    0.00000  + Z =    0.14580   TEMPERATURE =  0.8020434E 02



 FOR R =    0.00000  + Z =    0.29170   TEMPERATURE =  0.1500000E 03



 FOR R =    0.00000  + Z =    0.00000   TEMPERATURE =  0.1514588E 03



 FOR R =    0.10500  + Z =    0.14580   TEMPERATURE =  0.8335227E 02


THIS IS CASE     2WITH TEMP CONTROL AT    0.00000          0.14580

        EIGENVALUES             N
```

```
0.1168994E 02          1
0.2203607E 02          2
0.3264599E 02          3
0.4333361E 02          4
 HEAT REQUIRED IS   0.9766485E 05BTU PER HR.

FOR R =   0.00000  + Z =   0.14580   TEMPERATURE =  0.1500000E 03

FOR R =   0.00000  + Z =   0.29170   TEMPERATURE =  0.6971844E 03

FOR R =   0.00000  + Z =   0.00000   TEMPERATURE =  0.7086210E 03

FOR R =   0.10500  + Z =   0.14580   TEMPERATURE =  0.1746792E 03

THIS IS CASE    3WITH TEMP CONTROL AT   0.00000        0.01000

     EIGENVALUES            N

0.1168994E 02          1
0.2203607E 02          2
0.3264599E 02          3
0.4333361E 02          4
 HEAT REQUIRED IS  -0.1231665E 05BTU PER HR.

FOR R =   0.00000  + Z =   0.14580   TEMPERATURE =  0.8008892E 02
```

FOR R = 0.00000 + Z = 0.29170 TEMPERATURE = 0.1490951E 03

FOR R = 0.00000 + Z = 0.00000 TEMPERATURE = 0.1505374E 03

FOR R = 0.10500 + Z = 0.14580 TEMPERATURE = 0.8320124E 02

THIS IS CASE 4WITH TEMP CONTROL AT 0.10500 0.14580

EIGENVALUES	N
0.1168994E 02	1
0.2203607E 02	2
0.3264599E 02	3
0.4333361E 02	4

HEAT REQUIRED IS 0.7463934E 05BTU PER HR.

FOR R = 0.00000 + Z = 0.14580 TEMPERATURE = 0.1311391E 03

FOR R = 0.00000 + Z = 0.29170 TEMPERATURE = 0.5493190E 03

FOR R = 0.00000 + Z = 0.00000 TEMPERATURE = 0.5580594E 03

(52)

```
FOR R =   0.10500  + Z =   0.14580   TEMPERATURE =  0.1500000E 03

// *ENDJOB
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(92)

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APPENDIX II

Summary of Patents

2,876,634 3,766,975	These relate to double walled containers in which the hot liquid gives up heat to (a) beeswax, and (b) water. As the hot beverage cools, the reservoir starts returning heat to the system. In the case of beeswax, this is enhanced by the latent heat of fusion as the beeswax resolidifies.
3,079,911 3,213,932	Both of these systems use double walled containers in which the heating systems can be in the base. The first of the two patents uses oxidation- reduction system comprising. aluminum powder, copper sulfate, potassium chlorate, and calcium sulfate activated by water. The second uses calcium oxide and water for heating and calcium chloride for cooling.
3,101,707	This is another oxidation-reduction system using aluminum powder, copper sulfate, potassium chlorate, etc., which actually produces steam.
3,871,357	This is another calcium oxide-water system. In this patent, a cutting blade is used to remotely slice open a package containing part of the heating system.
3,903,011	Calcium oxide, calcium chloride and a thickener provide the heat.

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