Analytical Essays on Marketing

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Abstract

This dissertation consists of two essays on retail pricing strategies, each of which is described below.

The first essay, "Signalling and Commitment Using Retail Prices," investigates how a store can signal the prices of its unadvertised products using its advertised prices. A model of a two product retail market is presented in which stores advertise the price of one product and customers do not know the price of the other product before selecting which store to visit. In a model with complete information, only the commitment function is possible and stores with different marginal costs charge different prices for each product. When customers do not know each store's marginal cost type, an opportunity arises for each store to signal its cost type using its advertised prices. In such a model, additional equilibria exist. In particular, stores with different costs may charge the same advertised price while continuing to charge different prices for the unadvertised product. Data from competing drycleaning stores support the model predictions. A number of additional properties of the equilibria are discussed and possible extensions to the model are proposed.

The second essay, "Attracting Customers Using Efficient Discounting Strategies," addresses the discounting decisions faced by a retail store. When should a store offer discounts? How many products should be discounted? Which products should be discounted? How deep should the discounts be? This essay argues that

stores discount in order to attract switching customers. However, when discounting, the stores earns less profit from their loyal customers. Therefore, stores only discount if the profit earned from the switching customers exceeds the profit lost from the loyal customers. In deciding how many products to discount and which products to discount, stores determine how to offer utility to the switching customers while sacrificing the least profit from their loyal customers. When there are few loyal customers, so that the market exists predominantly of switching customers, the optimal strategy is to discount all products. If a substantial loyal segment is introduced, we find that discounting a narrow product range is most likely when the store has a clear preference for which products it would rather discount. As the store becomes more indifferent as to which product it would prefer to discount, the probability that it will discount a larger number of products increases. The findings also help to characterize which stores will discount, which products will be discounted and how deep the discounts will be.

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Table of Contents

Chapter 1

Signalling and Commitment Using Retail Prices

Introduction

A typical supermarket stocks approximately 15,000 products but advertises fewer than 100 of these products and prices.[1] Because customers know that stores must charge the advertised prices, advertising influences store choice by setting price expectations for the advertised products. Store choice may be further influenced if the advertised prices allow customers to form reliable inferences about the prices of the unadvertised products. There is widespread belief in both the industry[2] and the literature[3] that customers do use advertised prices to form expectations about the prices of other products. However, there is no theory explaining how this can occur. What prevents high priced stores from advertising the same prices as low priced stores, thus making it impossible for customers to form a meaningful inference?

This paper offers an explanation as to how and when stores can use their advertised prices to influence customers' price expectations for other products in the store. The underlying rationale is that stores can only afford to advertise low prices if the stores are relatively efficient. When these efficiencies affect many products in the store's product range,[4] then customers will find that the efficient stores also charge lower unadvertised prices.

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[1]To advertise more would incur unsustainable advertising costs and/or diminish the effectiveness of the advertising.

[2]This paper was motivated by discussions with executives of several supermarket chains.

[3]See Markin (1971 p468); Parsons and Price (1973 p127); Nagle (1987 p184); Feichtinger, Luhmer and Sorger (1988 p188); and Borenstein (1991 p357).

[4]Consider for example purchasing power or distribution efficiencies.

Feichtinger et al (1988) investigate the interaction of pricing and advertising strategies when different products have different influences on customers' overall price expectations for a store. The authors show that if lower prices are needed to enhance price expectations then price promotions should be accompanied by a relatively low level of advertising. Customers do not ignore their previous experience when interpreting the advertising. Rather, high price expectations reduce the impact of advertising so that low advertised prices are a more credible signal when price expectations are favorable. While these results are intuitively sound, they come about because price expectations are updated gradually, which may occur in a single product market. The results do not depend on only some prices having an impact on store selection.

The commitment role of advertising (fixing customers' price expectations for the advertised products) has been used to explain loss leader behavior and has received extensive attention in the literature. Empirical studies can be found in Walters (1988) and Walters and MacKenzie (1988), both of which report that some, but not all, loss leaders increase store profit by stimulating store traffic. Theoretical contributions have been made by Gerstner and Hess (1990), Hess and Gerstner (1987) and Lal and Matutes (1992). Gerstner and Hess (1990) discuss bait and switch pricing, in which retailers advertise low priced understocked brands and convince customers to purchase higher priced substitutes. Hess and Gerstner (1987) illustrate increased sales of complements, while Lal and Matutes (1992) portray additional sales of goods with independent demands.

The latter paper analyzes the pricing and advertising strategies of two firms when store choice relies on advertised prices and expectations of the unadvertised prices. In equilibrium both firms advertise the same good and commit themselves to offering a positive surplus to customers purchasing that product, while setting high prices for the unadvertised good. This result relies on an assumption of symmetry across goods and firms and is limited to the case where customer expectations about unadvertised prices are the same for all goods and firms. Customers have no need to update their expectations about unadvertised prices after seeing the advertised prices, as they recognize that it is optimal for stores to set the unadvertised prices at their customers' common reservation levels.

All of these theoretical contributions either assume perfect information or define customers' price expectations exogenously. Consequently, the literature lacks a formal investigation of the role that advertising plays in signalling the prices of unadvertised products. The model presented in this paper will investigate both commitment and price signalling effects. It will be shown that the unique pure strategy equilibrium in a complete information commitment model only exists in a portion of the parameter space in a more general incomplete information model. Price signalling results in the existence of two additional pure strategy equilibria and provides a further explanation for loss leader behavior.

The model requires that the price of one product is known by customers before they make their store selection decisions. Price knowledge may be gained without advertising exposure from previous experience or word of mouth. The term *salient* will be used to describe prices which are known by customers before they visit a store. Prices which are only learned upon arrival at a store will be described as *non-salient*.

A formal model of a retail market is presented in Section 1 and a complete information commitment model is analyzed in Section 2. The additional impact of price signalling is investigated in Section 3, and testable properties of the various equilibria are reviewed in section 4. Section 5 describes an empirical test of some of the model predictions and the paper concludes with a summary of the findings and a discussion of opportunities for further research.

I. A Model of a Retail Market

A model of a multiproduct market incorporating imperfect information, demand and supply sided heterogeneities is constructed. An economic representation of this market will demonstrate the influence that price has upon customer store selection, customer purchase quantities, marginal revenue and competitors' price responses. By reconciling these conflicting incentives equilibrium pricing strategies will be defined.

Consider two stores competing for heterogenous customers. The stores are differentiated and the customers are heterogenous on the same dimension, so that (with no additional information) different customers *a priori* prefer different stores. The differentiation and heterogeneity may reflect either location or consumer preferences. For example, consumers may place different levels of importance on different store attributes, such as product availability, cleanliness, and length of queues, while stores may vary in these attributes. For ease of exposition, it will be assumed that location is the basis of customer heterogeneity and store differentiation. Let the stores be located at the two ends of a linear city [0, 1] with a unit mass of customers distributed between them. Assume that the distribution of customers between the two stores is uniform.

The stores sell two products labelled *a* and *b*. The stores advertise the price of product *a*.[5] Without loss of generality the advertising cost for each store is set to zero. Customers use the advertised price of good *a* to predict

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[6]The exogeneity of the decision to advertise is justified if advertising is necessary for a store to be included in customers' consideration sets. Furthermore, when advertising is costless, both stores will prefer to advertise a price strictly less than their monopoly prices if advertising beneath the monopoly price will attract market share away from the competition.

the price of good b and then choose which store they will visit. They incur a constant marginal travelling cost equal to t. Assume that t is sufficiently high to ensure that customers visit only one store.

The products are homogenous between stores; thus the utility that customers enjoy when consuming a product does not depend on where the product was purchased. Within store demand for each product is independent but differs between the products. Customers face decreasing marginal utility for product b, resulting in downward sloping demand. Customers have unit demand for product a subject to a reservation price.[6] The store profit and customer utility functions are defined as:

$$\pi_j = N^j\left[q_a^j(p_a^j - c_a^j) + q_b^j(p_b^j - c_b^j)\right]$$

$$U_i^j = q_a^j(v_a - p_a^j) + q_b^j(v_b - q_b^j - p_b^j) - t\,|i-j|$$

where:

π_j	=	the profit earned by a store located at j, $j \in \{0,1\}$;
U_i^j	=	the utility derived by a customer located at i who shops at store j, $i \in [0,1]$;
N_j	=	the number of customers who shop at store j;
$p^j_{a(b)}$	=	the price charged for product a (b) by store j;
$q^j_{a(b)}$	=	the quantity of product a (b) purchased from store j by each customer;
$c^j_{a(b)}$	=	the cost of each unit of product a (b) to store j;
$v_{a(b)}$	=	the customers' value parameter for product a (b);
i	=	the location of customer i; and
j	=	the location of store j.

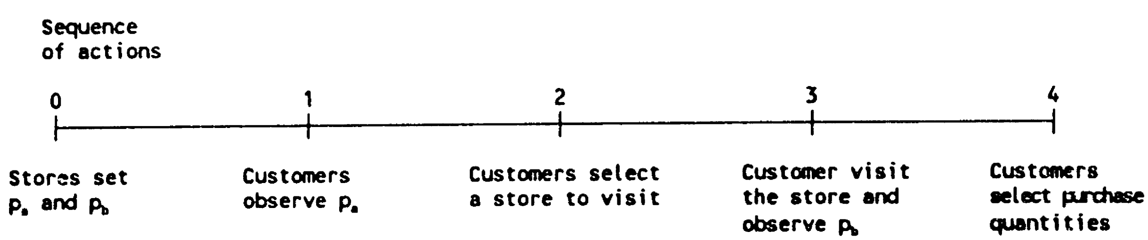
[6]Unit demand functions are common in the marketing and economics literature. Recent examples in the literature include: Narasimhan (1988), Lal and Matutes (1989), Gerstner and Hess (1990), Klemperer (1992) and Lattin and Ortmeyer (1991). The use of a unit demand in this instance greatly simplifies the analysis. See, however, note 9 below.

Unit demand for product *a* requires that for each customer: q_a equals one if $v_a > p_a$, and equals zero otherwise.

The stores may be of a high or a low cost type. The marginal costs for each good are perfectly correlated, so a low cost store enjoys cost efficiencies over a high cost store when selling both product *a* and product *b*. The correlation between marginal product costs is reflected by:

$$C^j \in \{\underline{C}, \overline{C}\};$$
$$\underline{C} = (\underline{c}_a, \underline{c}_b);$$
$$\overline{C} = (\overline{c}_a, \overline{c}_b);$$
$$\overline{c}_a > \underline{c}_a \quad \textit{and} \quad \overline{c}_b > \underline{c}_b$$

The market begins with the stores simultaneously setting prices for both goods and advertising the price of good *a*. Customers observe the advertised prices and select a store to visit. On arrival at the chosen store they observe the posted price of good *b* and decide on purchase quantities for each product. In accordance with FTC regulations, stores cannot change the price of product *a* after it has been advertised and must charge all customers the same prices.



II. Commitment

It is clear that retailers enjoy a limited local monopoly over ustcmers once they have arrived at the store (Bliss, 1988; and Gerstner and Hess, 1990). The extent of this monopoly is determined by the location of competing stores and by any additional switching costs. Stores may attract customers and earn monopoly profits from the sale of unadvertised products by committing themselves to charging low prices for a sample of advertised products. In order to investigate the role of commitment, it is initially assumed that customers know each store's cost type before they decide where to shop. Attention will be focused on the more interesting market in which one store faces high marginal cost and the competing store is a low cost type.

Heterogeneities between customers influence their store selection but do not affect their product selections. On arrival at a store, all customers behave in the same manner. If $v_a > p_a$ they will each select one unit of product *a*. For simplicity, it will be assumed that $v_b > c_b$,[7] and that all customers prefer to participate in the market, rather than choosing not to shop at all. Decreasing marginal utility for product *b* results in downward sloping individual customer demand for that product:

$$q_b = \frac{v_b - p_b}{2}.$$

Customers do not know the price of product *b* when they make their store selection decision. Therefore *N* is independent of p_b, and stores are concerned only with maximizing their in-store monopoly profits when setting the price of product *b*. The resulting first order condition is:

[7] If $v_b < c_b$ then the market for product *b* is not viable and no sales will take place.

$$\frac{d\pi}{dp_b} = (p_b - c_b)\frac{\partial q_b}{\partial p_b} + q_b = 0$$

substituting for q_b:

$$p_b = \frac{v_b + c_b}{2}.$$

As expected, the price of product b increases with c_b; q_b together with both store profit and customer utility decrease with c_b:[8]

$$\pi = N\left[p_a - c_a + \frac{1}{8}(v_b - c_b)^2\right]$$

$$U = v_a - p_a + \frac{1}{16}(v_b - c_b)^2 - t|I - J|.$$

A decrease in the price of product a affects store profit in two ways. The revenue received from each customer's unit purchase of product a decreases (the *marginal revenue* effect),[9] and more customers visit the store because the store has committed itself to charging less for product a (the *commitment*

[8]Unnecessary notation is discarded to ease interpretation.

[9]If a downward sloping demand replaces the unit demand for product a, a decrease in the price of product a will have the additional consequence of increasing the quantity of product a purchased by each customer. However, total expenditure by each customer will still decrease as long as marginal revenue is greater than zero. In the region of an optimal price, marginal revenue will always exceed zero as the marginal gain from customers switching to the store is always positive (axcept in a degenerate case where one of the stores has zero market share), and at an optimum price these two marginal effects are equal. Therefore, introduction of a downward sloping demand for product a gives an analogous result.

effect).[10] These observations allow the prediction of an optimal price relationship.

Finding 1:

Low cost stores will set a lower price for product a, $p_Y > p_X$,[11] and earn greater profits than high cost stores, $\pi_L(.) > \pi_H(.)$ for all p_a.

In Figure 1, $\pi_L(.)$ represents the profit earned by a low cost store as the advertised price varies, while $\pi_H(.)$ represents the profit earned by a high cost store. The prices p_X and p_Y are the prices at which the profit functions $\pi_L(.)$ and $\pi_H(.)$ (respectively) are maximized.

The intuition for this Finding[12] is that low cost stores are more willing to reduce the price of product *a* in order to attract customers because low cost stores earn more profit from each customer. Recall that the optimal price for *high* cost stores is found at the point where the marginal revenue effect of a price change equals the commitment effect. At this price, a *low* cost store earns more profit from each customer's purchases of each product, which increases the size of the commitment effect.[13] The low cost store also attracts more customers which increases the magnitude of the marginal revenue effect (see note 14), however, given the linear demand for product *b*, the increase in the commitment effect outweighs the change in the marginal

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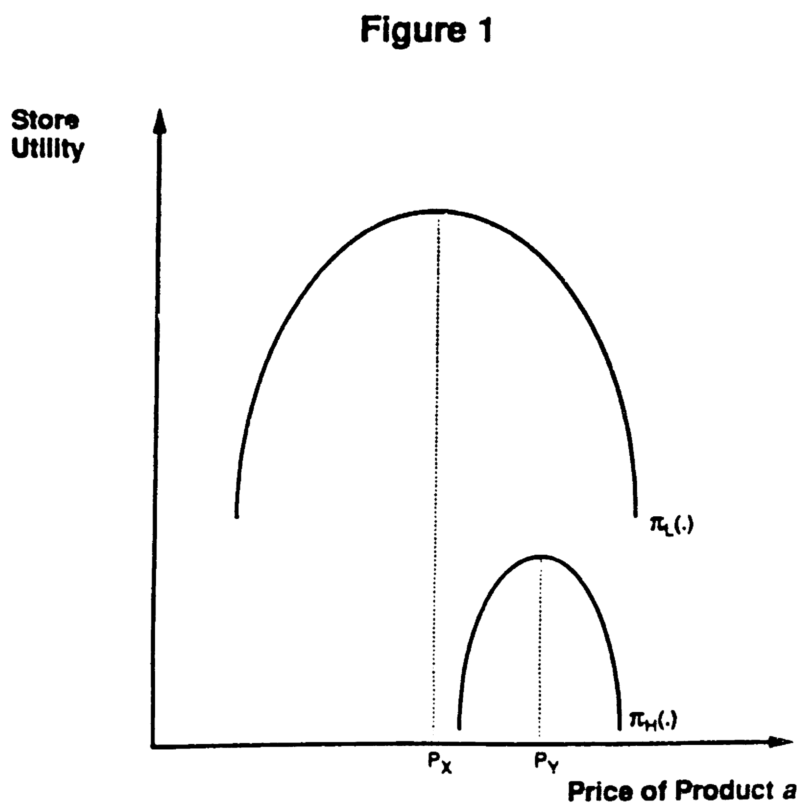
[10]A price increase results in the same marginal effects operating in opposite directions.

[11]Defined with respect to Figure 1.

[12]A formal proof is available from the author.

[13]Each customer contributes more profit so the effect of each customer switching to or from the store is increased. This characterizes the behavior of the second derivatives.



revenue effect. It is possible to create demand functions for product *b* under which $p_Y < p_X$, but only if customer utility for product *b* is very price sensitive.

When the price of product *a* is reduced, the size of the marginal revenue effect increases[14] and the size of the commitment effect decreases (lower profit from each customer). At the optimum low cost p_a, the marginal revenue effect again equals the commitment effect. Low cost stores earn greater profits than high cost stores when they charge the same price for product *a* because at all prices (including the high cost firm's optimal price) they attract

[14]More customers visit the store so the effect of a change in the revenue received from each customer's unit purchase of product *a* is increased.

additional customers and they face lower marginal costs.

The unique pure strategy optimal price for each store can be found by defining each store's market share as a function of the relative prices, deriving the first order conditions and equating the resulting reaction functions (see Appendix 1):

$$p_H = t + \frac{2\overline{c_a}}{3} + \frac{\underline{c_a}}{3} - \frac{z}{16} - \frac{1}{8}(v_b - \overline{c_b})^2$$

$$p_L = t + \frac{\overline{c_a}}{3} + \frac{2\underline{c_a}}{3} - \frac{z}{16} - \frac{1}{8}(v_b - \overline{c_b})^2$$

where:

p_H = the high cost store's equilibrium price for product a;

p_L = the low cost store's equilibrium price for product a; and

z = $(v_b - \underline{c_b})^2 - (v_b - \overline{c_b})^2$.

III. Signalling

When customers are uncertain of a store's cost type an opportunity arises for the store to signal its cost type, and the prices of its unadvertised products, using its advertised prices. To investigate price signalling, uncertainty will be introduced into the model by disregarding the rather unrealistic assumption that each store's cost type is common knowledge. It will now be assumed that while each store knows its own cost type, neither the competing store nor the customers can observe a store's type. The stores and customers believe that each type is equally likely and is independent between stores.

As in the commitment model, the actual price of product b is not known by customers prior to their store selection decision, so stores are concerned only with maximizing their in-store monopoly profits:

$$p_b = \frac{v_b + c_b}{2}.$$

Customers rationally predict p_b and, therefore, prefer to purchase product b from a low cost store.[15] The advertised price of product a provides the only information from which they are able to infer each store's cost type.

It will be assumed that customers adopt the *intuitive criterion* to eliminate unrealistic beliefs. Under this criterion customers believe that a store will only deviate from an equilibrium price if it is possible for the store to profit from the deviation. Store types which cannot gain from an observed deviation, even if the deviation resulted in the most favorable customer beliefs (that the store is

[15]Recall that customer welfare is a decreasing function of c_b.

a low cost type), are thougnt not to be the deviating store.[16] For some deviations the intuitive criterion may eliminate both cost types or may not eliminate either type. If so, assume that customers continue to maintain equal beliefs about the deviating store's type, unless (given those equal beliefs) only one cost type can gain by pricing either above or below the equilibrium price. When only one cost type can benefit from such a deviation, then a store deviating in that direction (above or below the equilibrium price) is believed to be the type for which such a deviation maybe profitable. For example, when the intuitive criterion is of no help,[17] if a store deviates by pricing below a proposed equilibrium price, and only the low cost type ever has an incentive to deviate in that direction when customers have equal beliefs about the store's type, customers will believe that the deviating store is a low cost type.

An additional profit implication may now be observed upon a change in the price of product *a*. Because customers prefer to purchase product *b* from a low cost store, if a price change results in a revision of customers' beliefs as to the store's cost type, the rate at which customers switch to or from a competing store may vary (the *signalling* effect). While a price change may result in a change of customer beliefs, a change in customer beliefs also alters the optimal price that each store would like to charge:

Finding 2:

As customers become more confident that a store is a low cost type, the store (whether a high or low cost store) would like to charge higher prices for product a, $p_X < p_Y < p_Z$,[18] *and earn greater total profits,* $\pi_j(.,1) > \pi_j(.,\frac{1}{2}) > \pi_j(.,0)$ *for all* p_a.

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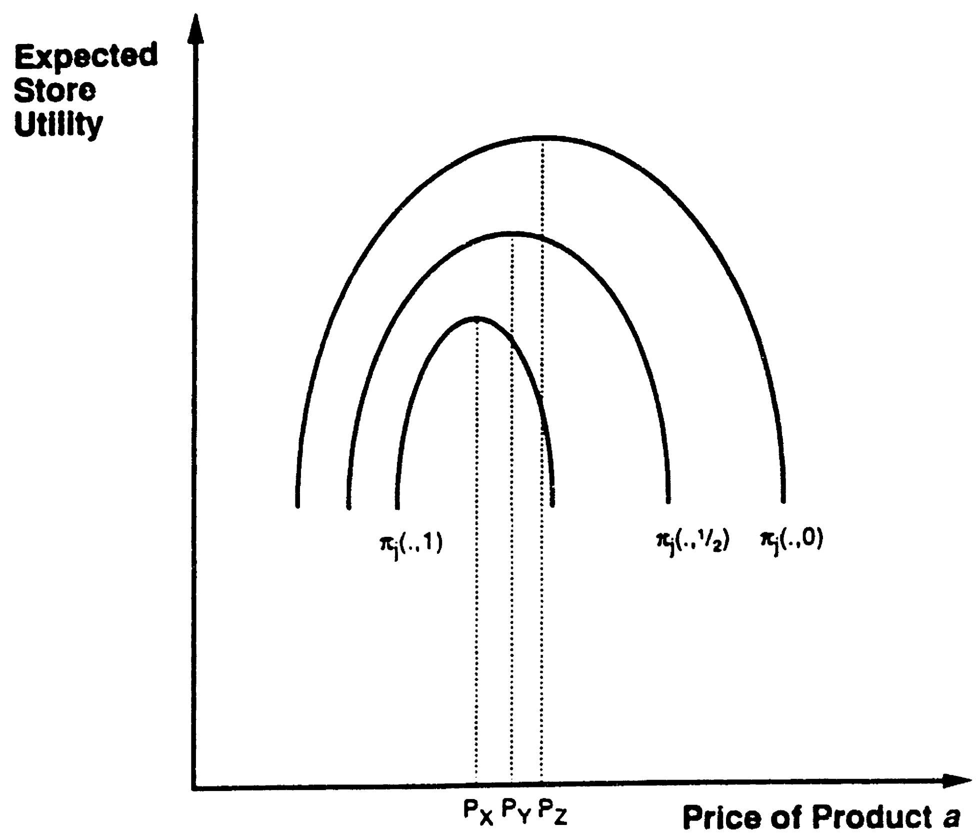
[16]See Fudenberg and Tirole (1991, pp487-488).

[17]Because it eliminates both cost types or does not eliminate either type.

[18]Defined with respect to Figure 2.

Figure 2



$\pi_j(.,1)$ represents the profit earned by a store of cost type j as its advertised price varies when customers believe that the store is a high cost type with probability 1. $\pi_j(.,½)$ and $\pi_j(.,0)$ are defined analogously; $\pi_j(.,½)$ represents the expected profit earned by a store of cost type j when customers maintain equal beliefs about the store's cost type,[19] and $\pi_j(.,0)$ represents the expected profit earned by a store of cost type j when customers believe that the store faces low marginal costs. The prices p_X, p_Y and p_Z are the prices at which the profit functions $\pi_j(.,1)$, $\pi_j(.,½)$ and $\pi_j(.,0)$ (respectively) are maximized.

When customers believe that a store is more likely to be low cost, customers require less of a reduction in the price of product *a* in order to attract them into the store. More specifically, as beliefs become more favorable market share grows, which increases the potential marginal revenue effect of increasing price. Stores react by increasing the price of product *a*, which reduces market share. As a result, the potential marginal revenue effect falls (there are fewer customers), and the commitment and switching effects grow (each customer is worth more to the store), until the marginal revenue effect equals the combined marginal switching and commitment effects.[20]

The utility relationships summarized in Figures 1 and 2 will be used to illustrate conditions for the existence of two types of pure strategy equilibria in which both stores have positive market share and customers purchase positive quantities of both goods.[21] The first is a fully *separating* equilibrium, with stores of different types charging asymmetric prices for both goods. Customers are able to correctly infer each store's cost type from the advertised prices before selecting which store to visit. In the second, stores of different types advertise and charge the same price for product *a*, while continuing to

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[19]Changing the prior beliefs about each store's cost type will change the position of this curve.

[20]A formal proof for Finding 2 is available from the author.

[21]Attention is restricted to equilibria in which v_a exceeds p_a for each store.

charge different prices for product b.[22] Under this partially *pooling* equilibrium, customers learn no additional information about the stores' cost types from the advertised prices.

Under a separating equilibrium a high cost store must expect to earn more when it is recognized to be a high cost type than it would expect to earn by pretending to be a low cost type. This requires that a low cost store be willing to charge a price for product a which will discourage a high cost store from hiding its true identity. Figures 3 and 4 demonstrate the two circumstances in which this result is sustained.

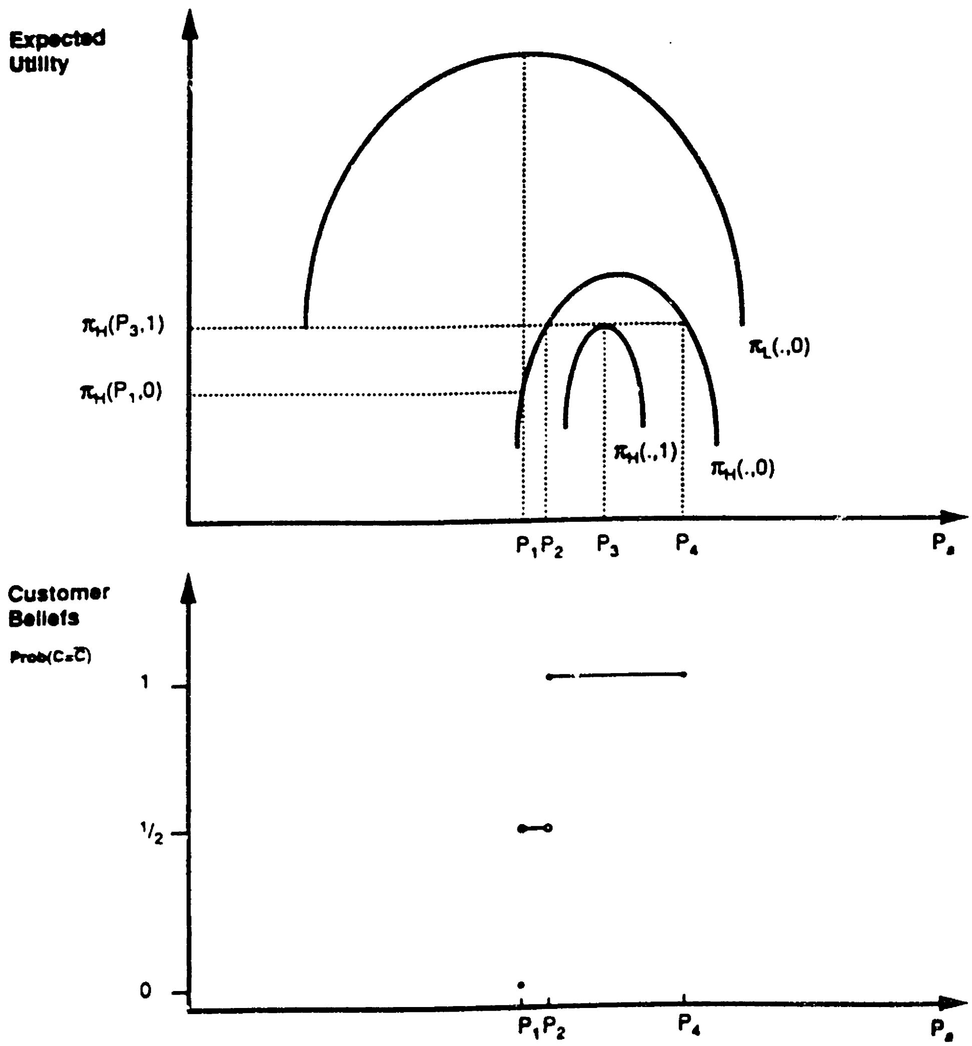
In Figure 3, p_1 and p_3 represent the prices which maximize $\pi_L(.,0)$ and $\pi_H(.,1)$ respectively, while p_2 and p_4 are the prices at which the profit function $\pi_H(.,0)$ equals $\pi_H(p_3,1)$. In Figure 4, p_{14} and p_{16} maximize $\pi_L(.,½)$ and $\pi_H(.,1)$ respectively; p_{12} and p_{16} are the prices at which the profit function $\pi_H(.,0)$ equals $\pi_H(p_{16},1)$; p_{13} is the smaller of the two prices at which $\pi_H(.,½)$ equals $\pi_H(p_{16},1)$; and p_{11} is the smaller of the two prices at which $\pi_L(.,0)$ equals $\pi_L(p_{14},½)$.

In Figure 3, a high cost store would rather charge p_3 and be recognized as a high cost store, than charge p_1 and pretend to be low cost: $\pi_H(p_3,1) > \pi_H(p_1,0)$. In Figure 4, the profit maximizing price for a store correctly recognized to be low cost occurs within the price region in which high cost stores would be willing to deviate and imitate a low cost store $[p_{12}, p_{16}]$. If a store charges above p_{12}, customers are no longer certain that the store is a low cost type. The maximum a low cost store expects to earn under these less favorable beliefs (when pricing above p_{12}), is less than what it expects to earn

[22]The stores always maximize their in-store monopoly profits when setting the price of product b, see discussion above.

FIGURE 3

Existence of a Cheap Separating Equilibrium



when it charges p_{12} and is perceived to be a low cost store: $\pi_L(p_{14}, \frac{1}{2}) < \pi_L(p_{12}, 0)$. Therefore, a fully separating equilibrium exists if $\pi_H(p_3, 1) > \pi_H(p_1, 0)$ (Figure 3) or, when this condition is not satisfied, if $\pi_L(p_{14}, \frac{1}{2}) < \pi_L(p_{12}, 0)$ (Figure 4).

To facilitate later discussion, the separating equilibrium corresponding to

FIGURE 4

Existence of a Costly Separating Equilibrium

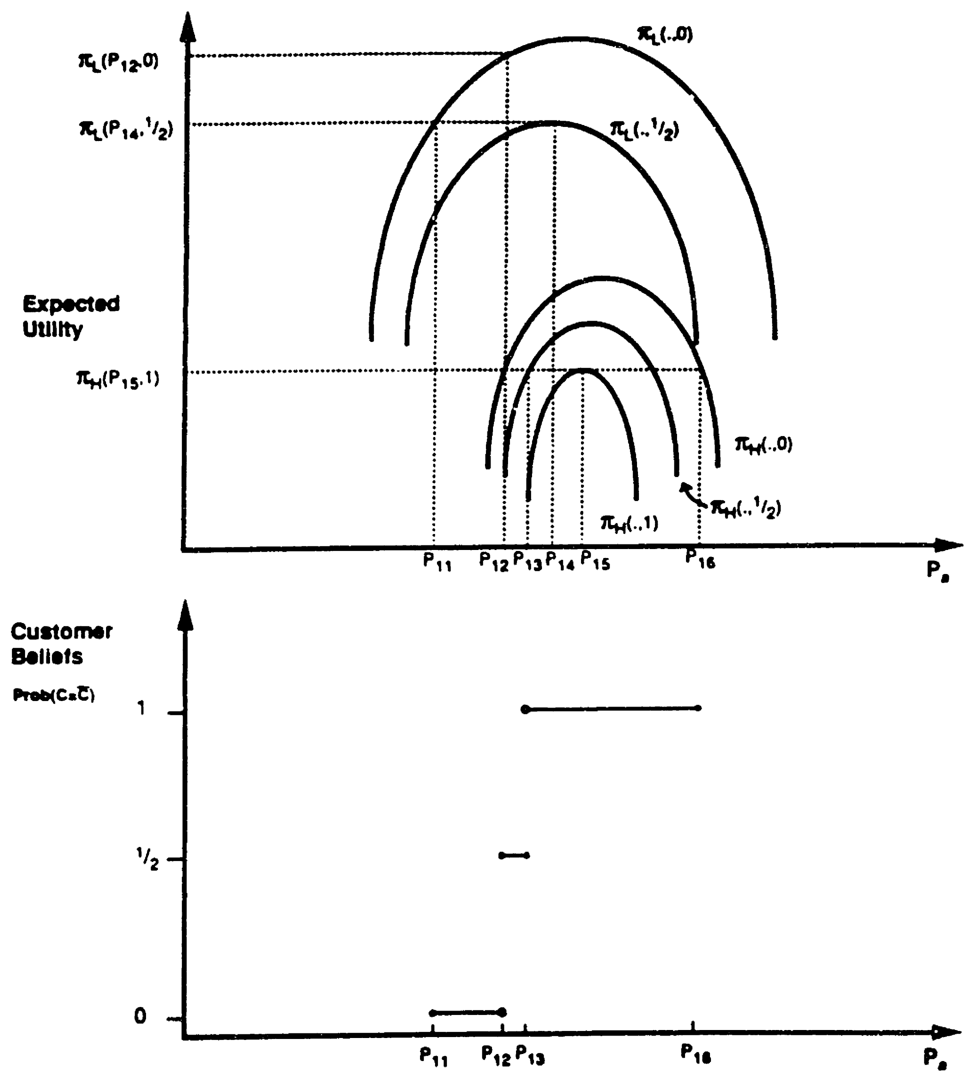


Figure 4 will be described as the *costly* separating equilibrium. The separating equilibrium depicted in Figure 3 will be referred to as the *cheap* separating equilibrium.

Proposition 1:

A cheap separating equilibrium exists iff:

$$\frac{1}{2}(\overline{c}_a + \underline{c}_a)^2 + \frac{z}{32}\left[5(\overline{c}_a - \underline{c}_a) - 4t + \frac{3z}{8}\right] > 0 \qquad (1)$$

$$t + \overline{c}_a + \frac{5z}{2^7} - \frac{1}{8}(v_b - \overline{c}_b)^2 - \frac{3R}{8} > 0 \qquad (2)$$

where:

$$R = \left[\frac{z^2}{2^8} + tz\right]^{\frac{1}{2}}$$

$$z = \left[(v_b - \underline{c}_b)^2 - (v_b - \overline{c}_b)^2\right]$$

When condition 1 is not satisfied, a costly separating equilibrium exists iff condition 2 is satisfied and:

$$\frac{z}{32}\left[t + \frac{z}{32} + \frac{R}{8}\right] - \left[\frac{1}{2}(\overline{c}_a - \underline{c}_a) + \frac{z}{16} - \frac{R}{4}\right]^2 > 0 \qquad (3)$$

If condition 1 is satisfied then $\pi_H(p_3, 1) > \pi_H(p_1, 0)$. Condition 2 ensures that it is possible to find a positive equilibrium price for low cost stores[23] ($p_{12} > 0$). Satisfaction of condition 3 gives $\pi_L(p_{14}, \frac{1}{2}) < \pi_L(p_{12}, 0)$. Derivations and equilibrium prices may be found in Appendix 2.

Care should be exercised when comparing Figures 3 and 4 as the expected utility functions are generally not the same. A store's expected utility depends on the prices that a competing store is expected to charge. In the cheap separating equilibrium, low cost stores are able to set their prices at the profit

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[23]At which high cost stores earn less than their equilibrium profits.

maximizing price for a store recognized to be low cost. In the costly separating equilibrium, low cost stores are restricted to a price of p_{12}, to ensure that customers are able to correctly recognize their cost type. The difference between these low cost equilibrium prices results in different profit maximizing prices for separating high cost stores; in general $p_3 \neq p_{16}$. An exception arises if $\pi_H(p_3, 1) = \pi_H(p_1, 0)$. When this occurs both types of separating equilibria predict the same equilibrium prices.

Under the pooling equilibrium both stores charge and advertise the same price for product *a*, so customers rely on their *a priori* beliefs and believe that each store is equally likely to be a high or a low cost type. From Finding 1 it is obvious that when stores with different costs face the same customer beliefs, the low cost stores will wish to charge a lower price for product *a* than the price charged by high cost stores.[24] Using this result, it can be shown that the unique pooling price for product *a* is at the low cost optimum, p_{24}. Consider Figure 5:

In Figure 5, p_{24}, p_{25} and p_{26} maximize $\pi_L(.,½)$, $\pi_H(.,1)$ and $\pi_H(.,½)$ respectively; p_{23} and p_{28} are the prices at which the profit function $\pi_H(.,0)$ equals $\pi_H(p_{24}, ½)$; and p_{22} and p_{27} are the prices at which $\pi_L(.,0)$ equals $\pi_L(p_{24}, ½)$.

A price in the region $[p_{23}, p_{24})$ cannot be an equilibrium as, if customers believe that both cost types are equally likely, both store types have an incentive to deviate by charging a higher price. Under the belief refinements adopted, when both stores wish to increase price given equal beliefs, equal beliefs are maintained. Similarly, for prices above p_{26}, both store types have

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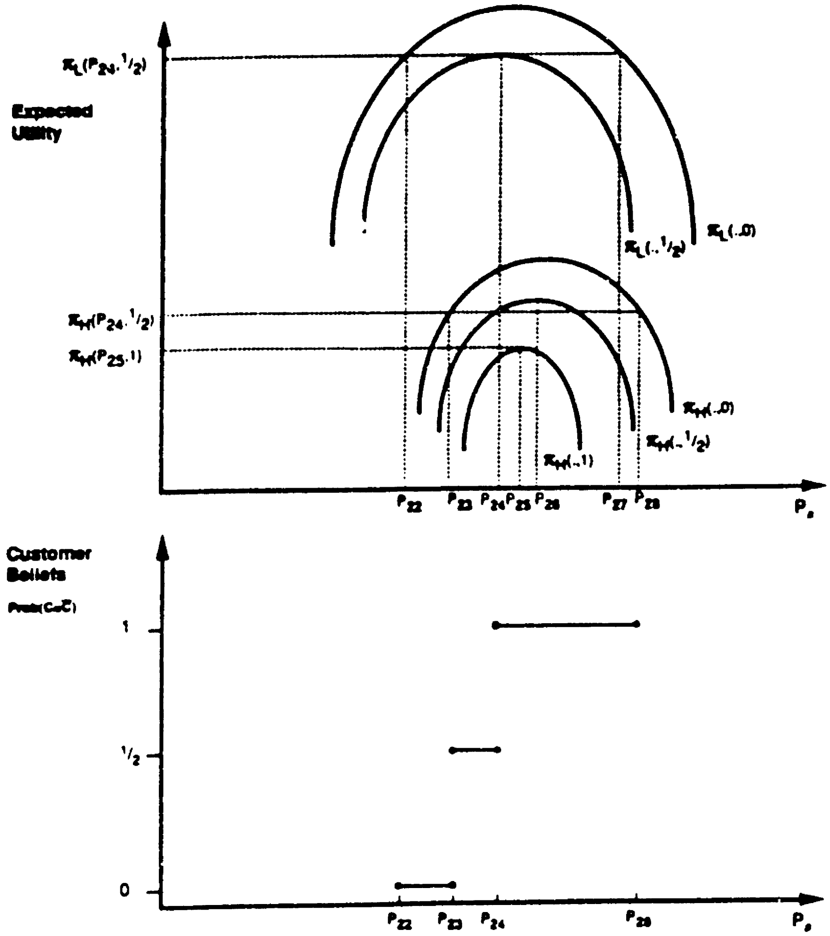
[24]This result occurs because low cost stores earn more profit from each customer and therefore incur greater marginal profit losses for each customer that switches to a competing store.

FIGURE 5

Existence of a Pooling Equilibrium



an incentive to deviate by reducing their prices, so prices in this region cannot be equilibria. For prices in the region $(p_{24}, p_{28}]$, if customers believe each cost type is equally likely then the different cost types would like to deviate in different directions. High cost stores would like to charge higher prices for product *a* while low cost stores would rather charge lower prices. In these circumstances, the belief refinements prescribe that stores charging beneath the proposed equilibrium price are believed to be low cost stores, while stores charging above the proposed equilibrium price are thought to be high cost stores. Clearly, if low cost stores prefer to deviate when equal beliefs are maintained, they will also prefer to deviate if, upon deviation, customers believe that they are low cost stores.

At p_{24}, if customers believe that each cost type is equally likely then neither store type wishes to deviate below the equilibrium price and only high cost stores prefer to price above p_{24}. Therefore, at prices below p_{24} equal customer beliefs are maintained, while customers believe that stores pricing above p_{24} are high cost stores.

Beliefs for larger price deviations outside the locum of p_{24} are determined by the intuitive criterion. Only low cost stores expect to gain from deviating below p_{23} when deviating stores are believed to be low cost types. Under the same favorable beliefs, only high cost stores prefer to deviate from p_{24} and charge above p_{27}. In Appendix 4 it is shown that $p_{22} < p_{23}$, and that $p_{27} < p_{28}$.

For a pooling equilibrium to exist, p_{23} must be less than zero,[25] preventing a low cost store from pricing at or below this point. This requires that the profits earned from the sale of product *b* are large enough to make the high cost stores willing to *give away* product *a* in order to attract customers by appearing to be a low cost store. Furthermore, a high cost store must prefer the profit it expects to earn by charging p_{24}, and hiding its identity, to the maximum profit it expects when its identity as a high cost type is revealed: $\pi_H(p_{24}, \frac{1}{2}) > \pi_H(p_{26}, 1)$.

Proposition 2:

A pooling equilibrium exists iff:

[26]Arbitrage opportunities may limit the price of product *a* to a level strictly greater than zero.

$$t + \frac{1}{2}(\overline{c_a} + \underline{c_a}) - \frac{3z}{64} - \frac{1}{8}(v_b - \overline{c_b})^2 - \frac{Q}{8} < 0 \tag{4}$$

$$\frac{tz}{32} - \left[\frac{1}{2}(\overline{c_a} - \underline{c_a}) + \frac{5z}{64}\right]^2 > 0 \tag{5}$$

where:

$$Q = \left[\frac{9z^2}{2^6} + 16(\overline{c_a} - \underline{c_a})^2 + 2tz + 3z(\overline{c_a} - \underline{c_a})\right]^{\frac{1}{2}}$$

Satisfaction of condition 4 ensures that $p_{23} < 0$, while condition 5 gives $\pi_H(p_{24}, \frac{1}{2}) > \pi_H(p_{25}, 1)$. Derivations and equilibrium prices may be found in Appendix 3.

IV. Testable Properties of the Equilibria

In the complete information commitment model, a unique pure strategy equilibrium exists in which stores predict their competitors' prices and set profit-maximizing prices in response. This results in high cost stores charging higher prices than low cost stores for both products.

When the more realistic assumption that customers and competing stores are uncertain of store cost types is adopted, an opportunity arises for each store to signal its cost type to the customers through its advertised prices. A *cheap* equilibrium corresponding to the pure commitment equilibrium in the complete information model may exist. Both stores prefer to fully reveal their cost types when setting their advertised prices. However, this equilibrium only exists in a limited region of the parameter space. Outside this region two further pure strategy equilibria are possible. A *costly* separating equilibrium may exist in which low cost stores reduce their advertised prices to protect their low cost image. In addition, there is potential for a pooling equilibrium in which stores of different types charge the same advertised price while continuing to charge different unadvertised prices.

Consider the behavior of the various equilibrium conditions as t varies in Figure 6.[26] The point t_4 represents the maximum of $\pi_L(p_{12}, 0) - \pi_L(p_{14}, \frac{1}{2})$, while the points t_1, t_2, t_3, t_5, and t^* represent the values for t at which the respective conditions are binding.

The following results are established in Appendix 5:

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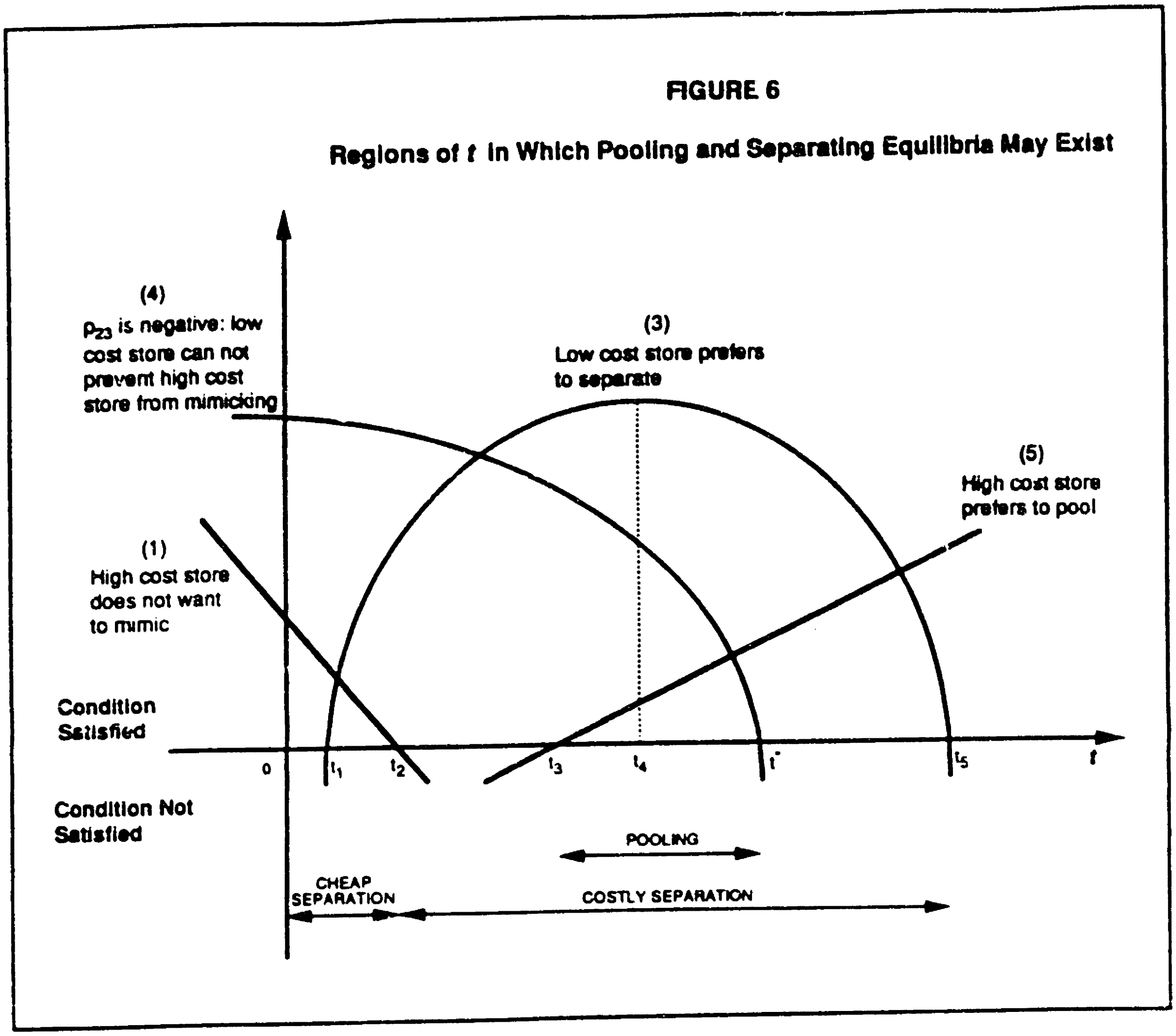
[26]The location of condition 2 ($p_{12} > 0$) in t space relative to the other conditions depends upon the values of the other parameters. For this reason condition 2 is excluded from Figure 6.



$$0 < t_1 < t_2 < t_3 < t_4 < t_5.$$

Existence of a pooling equilibrium requires that $t^* > t_3$ and that $t \in [t_3, t^*]$. A cheap separating equilibrium is possible when $t < t_2$,[27] while a costly separating equilibrium may exist in the region between t_2 and t_5. The possible existence of a separating equilibrium below the region of t in which a pooling equilibrium may exist can be understood when it is recalled that the decision to enforce a separating equilibrium is made by the low cost stores. As the value of t decreases, the number of customers who switch when low cost

[27]Subject to the condition that t must be sufficiently high to prevent customers from visiting both stores.

stores underprice their higher priced competitors increases, which makes separating more attractive to low cost stores.

Both the commitment and the separating equilibria predict that advertised prices are increasing functions of travelling cost. This result is in contrast to what would be expected in a monopoly market where a monopolist extracts the consumer surplus. As t increases the consumer surplus is reduced, forcing a profit maximizing monopolist to reduce price in order to maintain customer participation. In the competitive market, optimal prices are found when the marginal revenue effect equals the commitment and signalling effects. An increase in t makes it more expensive for customers to switch stores, reducing the size of the commitment and signalling effects. By increasing the price of product a, the marginal revenue effect is reduced (market share decreases) and the commitment and signalling effects are increased (each customer is worth more to the store), resulting in a return to optimality.

In the pooling equilibrium, stores of different types charge the same price for product a. In the commitment model and the cheap separating equilibrium the difference between product a prices at stores of different cost types is independent of t.[28] Price differences for product a in a costly separating equilibrium are a positive function of t within the region that a costly separating equilibrium is possible, $t \in [t_2, t_6]$ (see Appendix 6). Therefore the difference in price charged for product a between separating stores of different cost types is a continuous monotonically increasing function of t.[29] The commitment and various signalling equilibria all predict that unadvertised prices are independent

[28]This result may not hold when each consumer has a downward sloping demand for product a.

[29]When $\pi_H(p_3, 1) = \pi_H(p_1, 0)$ the cheap and costly separating equilibria predict the same equilibrium prices, so no discontinuity in price variation is observed at t_2.

of t.

The model predicts asymmetric negative cross-category price elasticity of aggregate demand (cf. Mulhern and Leone, 1991). An increase in the price of product a reduces market share which in turn reduces aggregate sales of product b. However, the cross price elasticity occurs in only one direction: the price of product b does not influence the aggregate demand for product b.

V. Empirical Tests of Hypotheses

Both the commitment and incomplete information models provide the following results:

Result 1: Price differences between stores for products whose prices are not known by customers prior to visiting a store are independent of the distance between stores.

Result 2: Stores charging different advertised prices will not charge the same advertised price.

The incomplete information model also predicts monotonically increasing price differences between competing stores as the distance between the stores increases. The commitment model does not give this result when each customer has unit demand for the salient product. However, if downward sloping demands are introduced, the commitment model may give a similar result:

Result 3: Price differences between competing stores for products whose prices are known by customers before visiting a store increase as the distance between the stores increases.

The introduction of uncertainty and the opportunity to signal cost types gives a further result which is not provided by the full information commitment model:

Result 4: Stores charging different prices for products whose prices are not known prior to visiting a store may pool and charge the same prices for products whose prices are known.

Data collected from the Boston drycleaning market was used to test these predictions. The data included individual store prices and the distance from each store to its closest competitor. To ensure that the price comparisons were between competing stores, any store whose nearest neighbor faces closer competition from a third store was excluded from the sample. Where multiple stores set identical prices as a result of either a franchising, co-operative or single owner relationship, these stores and all stores for which one of these stores is a closest neighbor were also excluded from the sample. The remaining dataset contained twenty six pairs of competing stores.

The price charged to launder mens' shirts was believed to be one of the most salient cleaning services due to the relatively high frequency with which this service is used and its price is advertised. Mens' woollen suits and sweaters are used to represent services for which prices are not known by customers prior to visiting the store. Where the price charged to dryclean a sweater varied between different types of sweaters, the minimum possible price was collected.

Inspection of the correlations between shirt, suit and sweater price differences (see Table 1) indicates significant ($\alpha = 0.05$) similarity in suit and sweater pricing strategies and a separate pricing strategy for shirts. This is consistent with the joint categorization of suits and sweaters as non-price salient and the separate description of shirts as price salient. Alternatively, this pattern of correlations maybe explained by variance in cost differences for drycleaning and laundering services. However, this explanation is not consistent with the combination of results 3 and 4.[30]

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[30]The intuition does not require perfect correlation between cost types for each product. The extent of correlation between the products' cost types will determine how accurately the advertised prices signal the unadvertised price levels and will therefore influence the resulting parameter conditions.

Correlations Between Price Differences

	$p_b^1 - p_b^0$	$p_c^1 - p_c^0$
$p_a^1 - p_a^0$	.1080	.0563
$p_b^1 - p_b^0$		.4115[1]

n = 26

[1] Significant at 95% level.

p_a^i = the price charged by store i to launder mens' shirts

p_b^i = the price charged by store i to dryclean a suit

p_c^i = the price charged by store i to dryclean a sweater

None of the stores in the final sample appeared to perform their cleaning services on the shop premises. Therefore, variance in marginal costs between stores may result from supply agreements negotiated with central drycleaning facilities or variance in transportation arrangements between the store and the facility. Evidence of variance in marginal costs is apparent in the variance in the price differences for cleaning suits and sweaters.[31]

To investigate the first result, differences in the prices that the competing store charge to dryclean suits and sweaters were correlated with the distance that the stores are apart. It was not possible to reject the null hypothesis that there is no correlation between store location and these price differences. This

[31]The equilibria all predict that differences in unadvertised prices are proportional to the cost differences between the competing stores.

finding is consistent with the prediction that price variation for products whose prices are not known by customers prior to visiting a store[32] is independent of the distance between competing stores.

Expected price differences for laundering mens' shirts vary under the different equilibria:

Cheap Separating

$$p_a^1 - p_a^0 = \Delta\left(2v_b - 2\underline{c}_b + \frac{1}{2}\right) - \Delta^2$$

where:

$$\Delta = \overline{c}_b - \underline{c}_b$$

Costly Separating

$$p_a^1 - p_a^0 = \frac{1}{4}\left[2t\Delta(v_b - \underline{c}_b) - t\Delta^2 + \frac{\left(2\Delta(v_b - \underline{c}_b) - \Delta^2\right)^2}{2^8}\right]^{\frac{1}{2}} - \frac{6\Delta}{64}(v_b - \underline{c}_b) + \frac{3\Delta^2}{64}$$

Pooling

$$p_a^1 - p_a^0 = 0$$

The price differences as a function of t under the two separating equilibria are portrayed in Figure 7. The discontinuity in slope at point t_2 is due to the change from the cheap to the costly separating equilibrium at this point.

To investigate the consistency of the dry cleaning data with this result, the association between price differences for laundering mens' shirts and t is approximated using a linear relationship. In this instance t, which represents the constant marginal cost of travelling between stores, is measured by the

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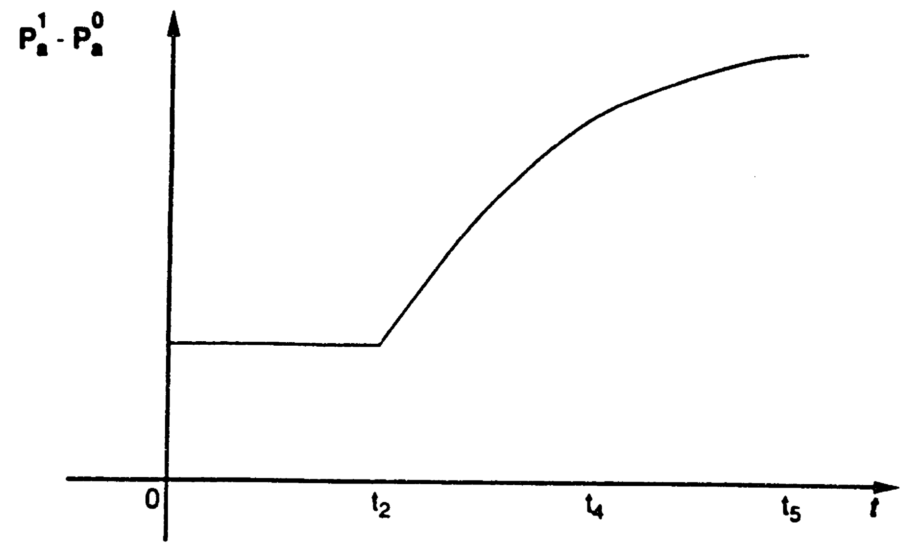
[32]Represented in this instance by the price charged to dryclean a suit or a sweater.

Figure 7

Expected Advertised Price Differences As A Function Of Store Dispersion



distance the stores are apart.[33] The models predict that advertised price differences also vary as a function of the cost differences between stores for the unadvertised product, Δ. To approximate this variance, the difference in prices charged to dryclean suits is included as a covariate.[34]

Observed Relationship Between Advertised Price Differences and Store Dispersity

Model:

$$p_a^1 - p_a^0 = \alpha + \beta_1 \Delta + \beta_2 t$$

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[33]The model assumes that the distance between stores is fixed but the cost of travelling may vary. Equivalent results are obtained if it is assumed that the cost of travelling is fixed but the distance between stores may vary.

[34]See note 32.

Independent Variables	Coefficient	Significance
t	0.0014	0.072
Δ	0.0317	0.168
constant	0.10	
R^2	0.10	
n	26	

where:

t = the geographic distance between stores by road; and

Δ = $p_b^I - p_b^O$.

The sign of the t coefficient is positive (as expected) and is approaching significance. Its non-significance (or marginal significance) may result from the linear approximation of a non linear relationship, a paucity of costly separating equilibria or the relatively few degrees of freedom in the dataset. The model makes a number of simplifying assumptions (including unit demand for the salient product) and does not attempted to incorporate every market feature. Quality differences between stores and additional competitive pressure from stores outside the pair of closest neighbors introduces additional variance into the data which is not explained by the model. Notwithstanding, the positive and near significant t coefficient is generally consistent with the third result.

Three of the 26 pairs of competing stores charge the same price for all three cleaning services, suggesting that the stores in each pair face the same marginal costs. Of the remaining 23 pairs of stores, three pairs charge different prices to dryclean both sweaters and suits but charge the same price to launder mens' shirts. The price differences for cleaning suits and sweaters indicates that the stores in each pair are different cost types and the stores are therefore pooling when setting the price of mens' shirts. These three stores are evidence that pooling may occur, and provide support for the fourth result. None of the stores charging different prices to launder mens' shirts charge the same price to dryclean suits. This finding is consistent with the second result.

Conclusions

A complete information commitment model and an incomplete information model incorporating both commitment and signalling have been developed. In each model customers know the price of one product but not the price of a second product when deciding which store to visit. In the complete information commitment model, a unique pure strategy equilibrium exists in which stores predict their competitors' prices and set profit-maximizing prices in response. This results in high cost stores charging higher prices than low cost stores for both products.

When customers are uncertain of store cost types, the opportunity for stores to signal their types using the advertised prices results in a variety of equilibria. An equilibrium corresponding to the pure commitment equilibrium may exist with stores of different types again charging different profit maximizing prices for both products. Both stores prefer to fully reveal their cost types when setting the price of the advertised product. It is clear that low cost stores will never attempt to convince customers that they are high cost types as the profit that they can earn from doing so is less than what they can earn if customers believe that they are efficient.[35] However, under appropriate parameter conditions, high cost stores will prefer to sacrifice marginal revenue from their unit sales of product *a* in order to attract more customers by pretending to be low cost types. Therefore, the equilibrium corresponding to the commitment outcome only exists in a limited region of the parameter space in the signalling model.

Outside this region, high cost stores will set advertised prices which imitate the prices charged by a low cost store. This may result in two

[36]See Finding 2.

alternative outcomes. First, low cost stores may choose to reduce their advertised prices to discourage high cost stores from pretending to be low cost. Second, if low cost stores are prevented from pricing low enough to accurately signal their cost type, a pooling equilibrium may result. In the pooling equilibrium stores of different cost types charge the same price for product *a* while continuing to charge different prices for product *b*. In such an outcome, the advertised prices provide customers with no information about the prices of the unadvertised products.

Tests of the model predictions were conducted using data from the Boston drycleaning industry. The data is consistent with the joint model predictions that unadvertised prices are independent of the relative location of competing stores. The data is also generally consistent with the equilibrium implications of price signalling: there was some evidence both of pooling between stores of different types on the salient prices and of increased differences in salient prices when stores are more dispersed.

These results clarify the role that price signalling plays in a multiproduct retail market. Signalling may result in an additional incentive for low cost stores to reduce their advertised prices in order to ensure that customers that customers can distinguish between store types. In this respect, signalling provides an additional explanation for loss leader behavior. Furthermore, the seemingly familiar equilibrium in which stores charge the same advertised price but different prices for the unadvertised products requires a signalling explanation. This result cannot be explained by the commitment literature.

Signalling through advertised price levels is only one of a number of alternative mechanisms to signal unadvertised price levels. Advertising an

Everyday Low Price strategy is an alternative as is dissipative advertising[36] or a guarantee to match competitors' prices. However, price signalling using advertised prices is a relatively efficient signal. Low cost stores prefer to charge lower advertised prices (Finding 1) and are better able to sustain the marginal revenue loss when price is lowered as they earn higher profits than high cost stores from any additional customers who come to the store. Indeed, in a cheap separating equilibrium low cost stores can credibly signal their cost type without cost.

The models presented in this paper overlook a variety of market features which influence retail pricing strategies. In order to both motivate future research and review the limitations in the present research, a number of these omitted market features are discussed.

Stores differ in the quality of the products that they sell and/or in the quality of the shopping experience that they offer, while customers differ in their propensity to pay for product quality and shopping pleasure. Where a high quality shopping environment can be maintained by increasing variable cost, and customers have different propensities to pay for an improvement in quality, an asymmetric pricing equilibrium may result. One group of stores may choose to incur the additional marginal cost of maintaining high quality, while other stores are able to attract less quality conscious customers by offering a lower price.

The selection of which product to advertise is exogenously determined for the stores in the current model. Lal and Matutes (1992) consider which of two products stores in a competitive duopoly would choose to advertise. They found multiple equilibria, in which firms advertising either the high or the low

[36]See Milgrom and Roberts (1986).

reservation product, with customers enjoying the same overall surplus and stores earning the same profit under either strategy. In general, customers prefer stores that advertise the product for which they face the greatest danger of being exploited once they arrive at the store. By advertising products which have high price variance or low price elasticity, stores commit themselves to a fixed price and are unable to set monopoly prices.

Advertising's commitment role may explain why a higher priced store would prefer to advertise and reveal its undesirable high prices rather than choosing not to advertise at all. Consider the asymmetric shopping quality equilibrium discussed above. While some customers may be willing to pay more for increased quality or service, the extent of this willingness is limited. If customers believe that the cost of obtaining increased quality exceeds the marginal value derived they will prefer to shop at lower priced lower quality stores. Advertising allows stores to commit to not charging more for quality than the additional benefit customers expect to enjoy.

Retailing and purchase decisions are repeated over many periods. Extending the model to include multiple periods would allow customer loyalty and repeat purchasing behavior to be considered. Stores may choose to disguise their true cost type by initially setting low prices for both salient and non salient products in an effort to attract customers back to the store in subsequent periods. This would suggest that stores may charge less than the monopoly price for their unadvertised products.

Adding features to the present model may enrich the legitimacy of the equilibria but is likely to confuse the intuition and detract from the clarity of the results. Indeed, a strength of the results is their generality and the paucity of assumptions upon which they rely.

Appendix 1

Equilibrium Prices Under the Complete Information Commitment Model

Given the alternative competitor cost types, first find the relative market shares by identifying the customer who is indifferent between shopping at either store. Let l equal the market share for a low cost store located at 0 and $1-l$ equal the market share for a high cost store located at 1:

$$v_a - p_H + \frac{1}{16}(v_b - \overline{c_b})^2 - t(1-l) = v_a - p_L + \frac{1}{16}(v_b - \underline{c_b})^2 - tl$$

$$l = \frac{1}{2} + \frac{z}{32t} + \frac{p_H - p_L}{2t}$$

Profit for each store may be defined as a function of p_H and p_L, giving a first order condition and reaction function for p_H and p_L. Solving the resulting simultaneous equations gives the equilibrium prices:

$$\pi_L(0) = \left[\frac{1}{2} + \frac{z}{32t} + \frac{p_H - p_L}{2t}\right] \cdot \left[p_L - \underline{c_a} + \frac{1}{8}(v_b - \underline{c_b})^2\right]$$

$$\frac{\partial \pi_L(0)}{\partial p_L} = \left[\frac{1}{2} + \frac{z}{32t} + \frac{p_H - p_L}{2t}\right] - \frac{1}{2t}\left[p_L - \underline{c_a} + \frac{1}{8}(v_b - \underline{c_b})^2\right] = 0$$

$$p_L = \frac{t}{2} + \frac{p_H}{2} + \frac{\underline{c_a}}{2} + \frac{z}{32} - \frac{1}{16}(v_b - \underline{c_b})^2$$

$$\pi_H(0) = \left[\frac{1}{2} - \frac{z}{32t} + \frac{p_L - p_H}{2t}\right] \cdot \left[p_H - \overline{c_a} + \frac{1}{8}(v_b - \overline{c_b})^2\right]$$

$$\frac{\partial \pi_H(0)}{\partial p_H} = \left[\frac{1}{2} - \frac{z}{32t} + \frac{p_L - p_H}{2t}\right] - \frac{1}{2t}\left[p_H - \overline{c_a} + \frac{1}{8}(v_b - \overline{c_b})^2\right] = 0$$

$$p_H = \frac{t}{2} + \frac{p_L}{2} + \frac{\overline{c_a}}{2} - \frac{z}{32} - \frac{1}{16}(v_b - \overline{c_b})^2$$

Appendix 2

Conditions for the Existence of Separating Equilibria

The approach adopted is to first identify the equilibrium prices for high and low cost stores under the cheap separating equilibrium (p_3 and p_1 respectively). To determine the existence condition for a cheap separating equilibrium, the profit earned by a high cost store when charging its equilibrium price p_3 is compared with the profit it would expect to earn if it deviated and charged p_1. The second existence condition required for a costly separating equilibrium is derived by first calculating p_{12}, p_{14}, and p_{15}. The low cost store's incentive to deviate is then examined by comparing the profit earned by a low cost store when charging p_{12} and p_{14}.

Find p_1 and p_3:

First find the expected market share, N_L, for a low cost separating stores under a cheap separating equilibrium. Let p^* represent the optimal price charged by a low cost store. The store predicts that if its competitor is a high cost store it will charge p_3, while if it is a low cost store it will charge p_1. Given the alternative competitor cost types, find the relative market shares by identifying the customer who is indifferent between shopping at either store (let l equal the low cost store's market share):

high cost competing store:

$$v_a - p_3 + \frac{1}{16}(v_b - \overline{c_b})^2 - t(1-l) = v_a - p^* + \frac{1}{16}(v_b - \underline{c_b})^2 - tl$$

$$l = \frac{1}{2} + \frac{z}{32t} + \frac{p_3 - p^*}{2t}$$

low cost competing store:

$$v_a - p_1 + \frac{1}{16}(v_b - \underline{c_b})^2 - t(1-l) = v_a - p^* + \frac{1}{16}(v_b - \underline{c_b})^2 - tl$$

$$l = \frac{1}{2} + \frac{p_1 - p^*}{2t}$$

Note that l increases as the competitor's price increases, decreases as the store's own price increases and includes an adjustment for customer beliefs about each store's cost type. The store believes that the other store is equally likely to be a high or low cost type, so the store's expected market share (N_L) is:

$$N_L = \frac{1}{2} + \frac{z}{64t} + \frac{p_3 + p_1}{4t} - \frac{p^*}{2t}$$

Expected profit for the low cost store may be defined as a function of p^*, p_1 and p_3, giving a first order condition and reaction function for p^*:

$$\pi_L(0) = \left[\frac{1}{2}+\frac{z}{64t}+\frac{p_3+p_1}{4t}-\frac{p^*}{2t}\right]\cdot\left[p^*-\underline{c}_s+\frac{1}{8}(v_b-\underline{c}_b)^2\right]$$

$$\frac{\partial\pi_L(0)}{\partial p^*} = \left[\frac{1}{2}+\frac{z}{64t}+\frac{p_3+p_1}{4t}-\frac{p^*}{2t}\right]-\frac{1}{2t}\left[p^*-\underline{c}_s+\frac{1}{8}(v_b-\underline{c}_b)^2\right] = 0$$

$$p^* = \frac{t}{2}+\frac{p_3}{4}+\frac{p_1}{4}+\frac{\underline{c}_s}{2}+\frac{z}{64}-\frac{1}{16}(v_b-\underline{c}_b)^2$$

in equilibrium $p_1 = p^*$:

$$p_1 = \frac{2t}{3}+\frac{p_3}{3}+\frac{2\underline{c}_s}{3}+\frac{z}{48}-\frac{1}{12}(v_b-\underline{c}_b)^2$$

By a similar analysis p_3 may be derived:

$$p_3 = \frac{2t}{3}+\frac{p_1}{3}+\frac{2\overline{c}_s}{3}-\frac{z}{48}-\frac{1}{12}(v_b-\overline{c}_b)^2$$

Solving the simultaneous equation gives:

$$p_1 = t+\frac{3\underline{c}_s}{4}+\frac{\overline{c}_s}{4}-\frac{5z}{2^6}-\frac{1}{8}(v_b-\overline{c}_b)^2$$

$$p_3 = t+\frac{\underline{c}_s}{4}+\frac{3\overline{c}_s}{4}-\frac{3z}{2^6}-\frac{1}{8}(v_b-\overline{c}_b)^2$$

Existence Condition for a Cheap Separating Equilibrium

If a high cost store deviates and charges p_1 it will attract market share equal to N_L, and earn expected profit $\pi_H(p_1,0)$. Comparing this profit with the expected equilibrium profit when a high cost store charges p_3 gives the existence condition for a cheap separating equilibrium:

$$\pi_H(p_1,0) = \left[\frac{1}{2}+\frac{z}{64t}+\frac{p_3-p_1}{4t}\right]\cdot\left[p_1-\overline{c}_s+\frac{1}{8}(v_b-\overline{c}_b)^2\right]$$

$$\pi_H(p_3,1) = \left[\frac{1}{2}-\frac{z}{64t}+\frac{p_1-p_3}{4t}\right]\cdot\left[p_3-\overline{c}_s+\frac{1}{8}(v_b-\overline{c}_b)^2\right]$$

$$\pi_H(p_3,1) > \pi_H(p_1,0) \;\Rightarrow\; \frac{1}{2}(\overline{c}_a-\underline{c}_a)^2+\frac{z}{32}\left[5(\overline{c}_a-\underline{c}_a)-4t+\frac{3z}{8}\right]>0$$

Find p_{12}, and p_{15}:

To derive p_{15}, consider the price charged by a store that is correctly recognized to be high cost, when it expects that its competitor will charge p_{15} if it is also high cost and p_{12} otherwise:

$$\pi_H(1) = \left[\frac{1}{2}-\frac{z}{64t}-\frac{p^*}{2t}+\frac{p_{12}+p_{15}}{4t}\right]\cdot\left[p^*-\overline{c}_a+\frac{1}{8}(v_b-\overline{c}_b)^2\right]$$

$$\frac{\partial\pi_H(1)}{\partial p^*} = \left[\frac{1}{2}-\frac{z}{64t}-\frac{p^*}{2t}+\frac{p_{12}+p_{15}}{4t}\right]-\frac{1}{2t}\left[p^*-\overline{c}_a+\frac{1}{8}(v_b-\overline{c}_b)^2\right]$$

$$p^* = \frac{t}{2}+\frac{p_{12}}{4}+\frac{p_{15}}{4}+\frac{\overline{c}_a}{2}-\frac{z}{64}-\frac{1}{16}(v_b-\overline{c}_b)^2$$

Letting $p^* = p_{15}$:

$$p_{15} = \frac{2t}{3}+\frac{p_{12}}{3}+\frac{2\overline{c}_a}{3}-\frac{z}{48}-\frac{1}{12}(v_b-\overline{c}_b)^2$$

Find p_{12} by equating $\pi_H(p_{15},1)$ with $\pi_H(0)$, and substituting for p_{15}:

$$\left[\frac{1}{2}-\frac{z}{64t}+\frac{p_{12}-p_{15}}{4t}\right]\cdot\left[p_{15}-\overline{c}_a+\frac{1}{8}(v_b-\overline{c}_b)^2\right] =$$

$$\left[\frac{1}{2}+\frac{z}{64t}+\frac{p_{15}-p_{12}}{4t}\right]\cdot\left[p_{12}-\overline{c}_a+\frac{1}{8}(v_b-\overline{c}_b)^2\right]$$

$$-p_{12}^2+p_{12}\left[2t+2\overline{c}_a+\frac{5z}{64}-\frac{1}{4}(v_b-\overline{c}_b)^2\right]+\frac{9zt}{64}$$

$$+\left[t+\overline{c}_a+\frac{z}{16}-\frac{1}{8}(v_b-\overline{c}_b)^2\right]\cdot\left[-t-\overline{c}_a-\frac{z}{64}+\frac{1}{8}(v_b-\overline{c}_b)^2\right] = 0$$

Applying the quadratic formula, simplifying and taking the smaller solution:

$$
\begin{aligned}
p_{12} &= t+\overline{c}_a+\frac{5z}{2^7}-\frac{1}{8}(v_b-\overline{c}_b)^2 - \frac{3}{8}\left[\frac{z^2}{2^8}+tz\right]^{\frac{1}{2}} \\
&= t+\overline{c}_a+\frac{5z}{2^7}-\frac{1}{8}(v_b-\overline{c}_b)^2 - \frac{3R}{8}
\end{aligned}
$$

where:

$$
R = \left[\frac{z^2}{2^8}+tz\right]^{\frac{1}{2}}
$$

Substituting p_{12} into the definition of p_{15}:

$$
p_{15} = t+\overline{c}_a-\frac{z}{2^7}-\frac{1}{8}(v_b-\overline{c}_b)^2-\frac{R}{8}
$$

Find p_{14}:

Consider the market share of a store pricing at p_{14} when the customer is uncertain (has equal beliefs) about the store's type:

high cost competing store:

$$
v_a-p_{15}+\frac{1}{16}(v_b-\overline{c}_b)^2-t(1-l) = v_a-p_{14}+\frac{1}{32}(v_b-\underline{c}_b)^2+\frac{1}{32}(v_b-\overline{c}_b)^2-tl
$$

$$
l = \frac{1}{2}+\frac{z}{64t}+\frac{p_{15}-p_{14}}{2t}
$$

low cost competing store:

$$
v_a-p_{12}+\frac{1}{16}(v_b-\underline{c}_b)^2-t(1-l) = v_a-p_{14}+\frac{1}{32}(v_b-\underline{c}_b)^2+\frac{1}{32}(v_b-\overline{c}_b)^2-tl
$$

$$
l = \frac{1}{2}-\frac{z}{64t}+\frac{p_{12}-p_{14}}{2t}
$$

This results in the following expected profit, first order condition and optimal price for the low cost deviating store:

$$\pi_L(^1/_2) = \left[\frac{1}{2} - \frac{p_{14}}{2t} + \frac{p_{15}+p_{12}}{4t}\right]\cdot\left[p_{14} - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right]$$

$$\frac{\partial \pi_L(^1/_2)}{\partial p_{14}} = \left[\frac{1}{2} - \frac{p_{14}}{2t} + \frac{p_{15}+p_{12}}{4t}\right] - \frac{1}{2t}\left[p_{14} - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right] = 0$$

$$p_{14} = t + \frac{1}{2}(\overline{c}_a + \underline{c}_a) - \frac{7z}{2^7} - \frac{1}{8}(v_b - \overline{c}_b)^2 - \frac{R}{8}$$

Second Existence Condition for a Costly Separating Equilibrium

If a low cost store deviates and charges p_{14} it will expect to earn $\pi_L(p_{14}, ½)$. Comparing this profit with the expected equilibrium profit when a low cost store charges p_{12} gives the second existence condition for a costly separating equilibrium:

$$\pi_H(p_{12},0) = \left[\frac{1}{2} + \frac{z}{64t} + \frac{p_{15}-p_{12}}{4t}\right]\cdot\left[p_{12} - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right]$$

$$\pi_H(p_{14},^1/_2) = \left[\frac{1}{2} - \frac{p_{14}}{2t} + \frac{p_{15}+p_{12}}{4t}\right]\cdot\left[p_{14} - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right]$$

$$\pi_H(p_{12},0) > \pi_H(p_{14},^1/_2) \Leftrightarrow \frac{z}{32}\left[t + \frac{R}{8} + \frac{z}{2^7}\right] - \left[\frac{1}{2}(\overline{c}_a - \underline{c}_a) + \frac{z}{16} - \frac{R}{4}\right]^2 > 0$$

Appendix 3

Conditions for the Existence of a Pooling Equilibrium

First identify the equilibrium pooling price, p_{24}. This result is used to calculate p_{25}, the optimum deviating price for a high cost store. To derive the existence condition describing when high cost stores are willing to pool, compare the profit a high cost store expects to earn when charging the equilibrium pooling price p_{24} with the profit it would expect to earn if it deviated and charged p_{25}. The second existence condition is found by calculating p_{23} and determining when it is less than zero.

Find p_{24}:

The expected market share, I, for a store charging p^*, when customers have equal beliefs about each store's type, and the competing store is charging p_{24}, can be shown to be:

$$I = \frac{1}{2} + \frac{p_{24} - p^*}{2t}$$

This results in the following expected profit and profit maximizing price for a low cost store:

$$\pi_L(^1/_2) = \left[\frac{1}{2} + \frac{p_{24} - p^*}{2t}\right] \cdot \left[p^* - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right]$$

$$\frac{\partial \pi_L(^1/_2)}{\partial p^*} = \left[\frac{1}{2} + \frac{p_{24} - p^*}{2t}\right] - \frac{1}{2t}\left[p^* - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right] = 0$$

$$p^* = \frac{t}{2} + \frac{p_{24}}{2} + \frac{\underline{c}_a}{2} - \frac{1}{16}(v_b - \underline{c}_b)^2$$

In equilibrium $p_{24} = p^*$:

$$p_{24} = t + \underline{c}_a - \frac{1}{8}(v_b - \underline{c}_b)^2$$

Find p_{25}:

If a high cost store deviates and charges p_{25} it will be correctly perceived to be a high cost store:

$$\pi_H(1) = \left[\frac{1}{2} - \frac{z}{64t} + \frac{p_{24} - p_{25}}{2t}\right]\cdot\left[p_{25} - \overline{c}_a + \frac{1}{8}(v_b - \overline{c}_b)^2\right]$$

$$\frac{\partial \pi_L(1)}{\partial p_{25}} = \left[\frac{1}{2} - \frac{z}{64t} + \frac{p_{24} - p_{25}}{2t}\right] - \frac{1}{2t}\left[p_{25} - \overline{c}_a + \frac{1}{8}(v_b - \overline{c}_b)^2\right] = 0$$

$$p_{25} = t + \frac{1}{2}(\overline{c}_a + \underline{c}_a) - \frac{5z}{64} - \frac{1}{8}(v_b - \overline{c}_b)^2$$

When are high cost stores willing to pool?

Compare a high costs store's pooling profit with its optimal deviating profit:

$$\pi_H(p_{24}, {}^1\!/_2) > \pi_H(p_{25}, 1) \;\Leftrightarrow\; \frac{tz}{32} - \left[\frac{1}{2}(\overline{c}_a - \underline{c}_a) + \frac{5z}{2^8}\right]^2 > 0$$

Find p_{23}:

To find p_{23}, equate $\pi_H(½, p_{24})$ with $\pi_H(0, p_{23})$, and substitute for p_{24}:

$$\frac{1}{2}\left[p_{24} - \overline{c}_a + \frac{1}{8}(v_b - \overline{c}_b)^2\right] = \left[\frac{1}{2} + \frac{z}{64t} + \frac{p_{24} - p_{23}}{2t}\right]\cdot\left[p_{23} - \overline{c}_a + \frac{1}{8}(v_b - \overline{c}_b)^2\right]$$

$$-p_{23}^2 + p_{23}\left[2t + \overline{c}_a + \underline{c}_a - \frac{3z}{32} - \frac{1}{4}(v_b - \overline{c}_b)^2\right] + t\left[\frac{1}{8}(v_b - \underline{c}_b)^2 - t - \underline{c}_a\right]$$
$$+ \left[t + \underline{c}_a + \frac{z}{32} - \frac{1}{8}(v_b - \underline{c}_b)^2\right]\cdot\left[-\overline{c}_a + \frac{1}{8}(v_b - \overline{c}_b)^2\right] = 0$$

Applying the quadratic formula, simplifying and taking the smaller solution:

$$p_{23} = t + \frac{1}{2}(\overline{c}_a + \underline{c}_a) - \frac{3z}{2^6} - \frac{1}{8}(v_b - \overline{c}_b)^2$$

$$- \frac{1}{8}\left[\frac{9z^2}{2^6} + 16(\overline{c}_a - \underline{c}_a)^2 + 2tz + 3z(\overline{c}_a - \underline{c}_a)\right]^{\frac{1}{2}}$$

Pooling requires that p_{23} is less than zero:

$$p_{23} = t + \frac{1}{2}(\overline{c}_a + \underline{c}_a) - \frac{3z}{2^6} - \frac{1}{8}(v_b - \overline{c}_b)^2 - \frac{Q}{8} < 0.$$

where:

$$Q = \left[\frac{9z^2}{2^6} + 16(\overline{c}_a - \underline{c}_a)^2 + 2tz + 3z(\overline{c}_a - \underline{c}_a)\right]^{\frac{1}{2}}$$

Appendix 4

Proof that the Intuitive Criterion Results in Monotonic Beliefs Under a Pooling Equilibrium

From the derivation of p_{23} in Appendix 3 it clear that p_{28} is simply the larger solution to the quadratic equation that results from equating $\pi_H(p_{24}, ½)$ and $\pi_H(p_{23}, 0)$:

$$p_{23} = t + \frac{1}{2}(\overline{c}_a + \underline{c}_a) - \frac{3z}{2^6} - \frac{1}{8}(v_b - \overline{c}_b)^2 - \frac{Q}{8}$$

$$p_{28} = t + \frac{1}{2}(\overline{c}_a + \underline{c}_a) - \frac{3z}{2^6} - \frac{1}{8}(v_b - \overline{c}_b)^2 + \frac{Q}{8}$$

To find p_{22} and p_{27}, equate $\pi_L(p_{24}, ½)$ with $\pi_L(0)$, and substitute for p_{24}:

$$\frac{1}{2}\left[p_{24} - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right] = \left[\frac{1}{2} + \frac{z}{64t} + \frac{p_{24} - p^*}{2t}\right] \cdot \left[p^* - \underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right]$$

$$-p^{*2} + p^*\left[2t + 2\underline{c}_a + \frac{z}{32} - \frac{1}{4}(v_b - \underline{c}_b)^2\right] + t\left[\frac{1}{8}(v_b - \underline{c}_a)^2 - t - \underline{c}_a\right]$$
$$+ \left[t + \underline{c}_a + \frac{z}{32} - \frac{1}{8}(v_b - \underline{c}_b)^2\right] \cdot \left[-\underline{c}_a + \frac{1}{8}(v_b - \underline{c}_b)^2\right] = 0$$

Applying the quadratic formula and simplifying:

$$p^* = t + \underline{c}_a + \frac{z}{2^6} - \frac{1}{8}(v_b - \underline{c}_b)^2 \pm \frac{1}{8}\left[\frac{z^2}{2^6} + 2tz\right]^{\frac{1}{2}}$$

or:

$$p_{22} = t + \underline{c}_a + \frac{z}{2^6} - \frac{1}{8}(v_b - \underline{c}_b)^2 - \left[\frac{z^2}{2^6} + 2tz\right]^{\frac{1}{2}}$$

$$p_{27} = t + \underline{c}_a + \frac{z}{2^6} - \frac{1}{b}(v_b - \underline{c}_b)^2 + \left[\frac{z^2}{2^6} + 2tz\right]^{\frac{1}{2}}$$

Monotonic beliefs require that $p_{22} < p_{23}$, and $p_{27} < p_{28}$:

$$p_{23} - p_{22} = \frac{z}{16} + \frac{1}{2}(\overline{c}_a - \underline{c}_a) + \frac{1}{8}(S - Q)$$

$$p_{28} - p_{27} = \frac{z}{16} + \frac{1}{2}(\overline{c}_a - \underline{c}_a) + \frac{1}{8}(Q - S)$$

where:

$$S = \left[\frac{z^2}{2^6} + 2tz\right]^{\frac{1}{2}}$$

$$Q = \left[\frac{9z^2}{2^6} + 16(\overline{c}_a - \underline{c}_a)^2 + 2zt + 3z(\overline{c}_a - \underline{c}_a)\right]^{\frac{1}{2}}$$

Note that $Q > S$, so by inspection it is clear that $p_{27} < p_{28}$. To derive a contradiction, assume, $p_{22} > p_{23}$. Therefore:

$$\frac{Q}{8} > \frac{z}{16} + \frac{1}{2}(\overline{c}_a - \underline{c}_a) + \frac{S}{8}$$

Squaring both sides:

$$\frac{9z^2}{2^6} + 16(\overline{c}_a - \underline{c}_a)^2 + 2zt + 3z(\overline{c}_a - \underline{c}_a) > \frac{17z^2}{2^6} + 16(\overline{c}_a - \underline{c}_a)^2 + 2tz + 4z(\overline{c}_a - \underline{c}_a)$$
$$+ S\left[z + 8(\overline{c}_a - \underline{c}_a)\right]$$

$$0 > \frac{z^2}{8} + z(\overline{c}_a - \underline{c}_a) + S\left[z + 8(\overline{c}_a - \underline{c}_a)\right]$$

Contradiction, which proves $p_{22} < p_{23}$.

Appendix 5

Behavior of the Equilibrium Conditions

$0 < t_1$:

Consider the slope of $\pi_L(p_{12},0) - \pi_L(p_{14},½)$:

$$\frac{\partial\left[\pi_L(p_{12},0) - \pi_L(p_{14},{}^1/_2)\right]}{\partial t} = -\frac{z}{2^4} + \frac{z}{8R}(\bar{c}_a - \underline{c}_a) + \frac{9z^2}{2^9 R}$$

$$\frac{\partial^2\left[\pi_L(p_{12},0) - \pi_L(p_{14},{}^1/_2)\right]}{\partial t^2} < 0$$

where:

$$R = \left[\frac{z^2}{2^8} + tz\right]^{\frac{1}{2}}$$

Find the maximum t_4 by setting the slope equal to zero:

$$t_4 = \frac{5z}{16} + \frac{16}{z}(\bar{c}_a - \underline{c}_a)^2 + \frac{9}{2}(\bar{c}_a - \underline{c}_a) > 0$$

At $t = 0$:

$$R = \frac{z}{2^4}$$

$$\pi_L(p_{12},0) - \pi_L(p_{14},{}^1/_2) = -\frac{z^2}{2^{10}} - \frac{1}{4}(\bar{c}_a - \underline{c}_a)^2 - \frac{3z}{2^6}(\bar{c}_a - \underline{c}_a) < 0$$

$$\frac{\partial\left[\pi_L(p_{12},0) - \pi_L(p_{14},{}^1/_2)\right]}{\partial t} = \frac{z}{4} + 2(\bar{c}_a - \underline{c}_a) > 0$$

At $t = 0$, $\pi_L(p_{12},0) - \pi_L(p_{14},½)$ is strictly less than zero and increasing. Therefore $t_1 > 0$.

$t_1 < t_2$:

At t_2, $\pi_H(p_3,1) - \pi_H(p_1,0)$ equals zero, therefore:

$$t_2 = \frac{3z}{32} + \frac{5}{4}(\bar{c}_a - \underline{c}_a) + \frac{4}{z}(\bar{c}_a - \underline{c}_a)^2 > 0$$

$\pi_L(p_{12}, 0) - \pi_L(p_{14}, ½)$ is greater than zero iff:

$$\frac{z}{2^5}\left[t + \frac{z}{2^5} + \frac{R}{8}\right] > \left[\frac{1}{2}(\overline{c}_a - \underline{c}_b) + \frac{z}{16} - \frac{R}{4}\right]^2$$

Simplifying:

$$R\left[(\overline{c}_a - \underline{c}_b) + \frac{9z}{2^6}\right] > \frac{tz}{8} + \frac{13z^2}{2^{10}} + (\overline{c}_a - \underline{c}_b)^2 + \frac{z}{4}(\overline{c}_a + \underline{c}_b)$$

Squaring both sides, simplifying and substituting for t_2:

$$\frac{175z^4}{2^{17}} + \frac{265z^3}{2^{13}}(\overline{c}_a - \underline{c}_b) + \frac{297z^2}{2^{10}}(\overline{c}_a - \underline{c}_b)^2 + \frac{37z}{2^5}(\overline{c}_a - \underline{c}_b)^3 + \frac{7}{4}(\overline{c}_a - \underline{c}_b)^4 > 0$$

Therefore at t_2, $\pi_L(p_{12}, 0) - \pi_L(p_{14}, ½)$ is greater than zero, so $t_1 < t_2$.

$t_2 < t_3$:

At t_3, $\pi_H(p_{24}, ½) - \pi_H(p_{25}, 1)$ equals zero, therefore:

$$t_3 = \frac{25z}{2^7} + \frac{5}{2}(\overline{c}_a - \underline{c}_b) + \frac{8}{z}(\overline{c}_a - \underline{c}_b)^2 > 0$$

Subtracting t_2 from t_3 equals:

$$t_3 - t_2 = \frac{13z}{2^7} + \frac{5}{4}(\overline{c}_a - \underline{c}_b) + \frac{4}{z}(\overline{c}_a - \underline{c}_b)^2 > 0.$$

$t_3 < t_4$:

Subtract t_3 from t_4:

$$t_4 - t_3 = \frac{15z}{2^7} + 2(\overline{c}_a - \underline{c}_a) + \frac{8}{z}(\overline{c}_a - \underline{c}_a)^2 > 0$$

$t_4 < t_5$:

This result is obvious from the behavior of $\pi_L(p_{1,2}, 0) - \pi_L(p_{1,4}, \frac{1}{2})$.

Appendix 6

Price Variation in a Costly Separating Equilibrium is Positive and Increasing Over t

$$p_{15} - p_{12} = \frac{R}{4} - \frac{3z}{2^6}$$

$$\frac{\partial\left[p_{15} - p_{12}\right]}{\partial t} = \frac{z}{8R} > 0$$

$$\frac{\partial^2\left[p_{15} - p_{12}\right]}{\partial t^2} < 0$$

where:

$$R = \left[\frac{z^2}{2^8} + tz\right]^{\frac{1}{2}}$$

Price variance, $p_{15} - p_{12}$, is positive iff:

$$R > \frac{3z}{16}$$

Squaring both sides:

$$\frac{z^2}{2^8} + tz > \frac{9z^2}{2^8}$$

$$t > \frac{z}{32}$$

Note that for a costly separating equilibrium to exist $\pi_H(p_{12}, 0) > \pi_H(p_{14}, \tfrac{1}{2})$, which requires that:

$$t > \frac{25z}{2^7} + \frac{5}{2^7}(\overline{c_a} - \underline{c_a}) + \frac{1}{8z}(\overline{c_a} - \underline{c_a})^2 > \frac{z}{32}$$

Therefore, price variation in a costly equilibrium is a positive concave function of t in the region in which a pooling equilibrium exists.

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Chapter 2

Attracting Customers Using Efficient Discounting Strategies

I. Introduction

Data collected from weekly advertising by two competing supermarket chains in Boston over a 5 week period revealed 37 instances in which the chains advertised identical products (same brand and size) in the same week. Comparison of the advertised prices with an additional sample of 89 unadvertised prices revealed that one chain discounts at a significantly deeper level.[37] However, the deep discounting store advertised only half as many products as its shallow discounting competitor. This result is at first surprising but is not limited to the Boston grocery market. Walmart, K Mart and a number of other leading retailers have recently begun to offer shallow discounts on all of the products in their stores, while many of their competitors continue to offer deeper, short term discounts on a varying sample of advertised products. Are some of these stores making mistakes? How can such different strategies be optimal when the stores are selling to the same market? These questions may be restated in terms of the pricing decisions facing a retailer. Should a store offer discounts? If a store does discount, how many products should be discounted? Which products should be discounted? How deep should the discounts be?

This paper offers an answer to these questions by investigating the relationship between customer utility and store profit. A store must discount in order to attract switching customers to the store.[38] However, when discounting, the store earns less profit from sales to store loyal customers.

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[37]The deep discounting store's advertised prices were on average 13.6% lower than its competitor while its unadvertised prices were a mere 7.3% lower. The difference is significant ($\alpha=0.05$).

[38]Empirical evidence that a meaningful proportion of shoppers switch between stores is provided by Keng and Ehrenberg (1984) and Bucklin and Lattin (1991).

Therefore, when deciding whether to discount in order to attract the switching customers, stores must weigh the loss of profits from their loyal consumers with the benefits of the anticipated increase in store traffic. In deciding how many products to discount, which products to discount and the size of the discounts, the store finds the way to offer utility to the switching customers that sacrifices the least profit from the store loyal customers. When store costs or the utility of the loyal or switching customers vary, the optimal discounting strategies change. By focussing on these changes it is possible to characterize the circumstances in which different discounting strategies are optimal.

The typical textbook in retailing management does contain some common wisdom as to which products should be discounted. There is general consensus that discounted products should be well known branded merchandise, for which the target customers have strong demand, high purchase frequency and high price knowledge.[39] This literature offers little guidance as to the circumstances in which it is optimal to discount or how deep the discounts should be. There are no insights describing how many products should be discounted and little or no justification for the common wisdom that is reported.

Academic research addressing these questions is also sparse. One line of research has focussed on single product firms in an attempt to explain the frequency and depth of manufacturer trade promotions.[40] However, the retailer's problem differs from that of the manufacturer. The retailer seeks to

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[39]See Bolen (1978 p211); James, Walker and Etzel (1981 p256); and Arnold, Capella and Smith (1983 p435).

[40]See for example: Rao (1991); Raju, Srinivasan and Lal (1990); and Narasimhan (1988) in the marketing literature; and Varian (1980), Salop and Stiglitz (1977) and Butters (1977) in the economics literature.

maximize overall store profit across many products while the manufacturer is generally concerned with the profit of a single product. Consequently, the manufacturer must only decide whether to discount, and if so, the depth of the discount that will be offered. The manufacturer need not consider how many products to discount or select which products they will be. Nor is the manufacturer concerned with the potential profit that switching customers contribute from their purchases of other products at the store.

Lattin and Ortmeyer (1991) offer an explanation for the coexistence of stores adopting an EDLP (everyday low prices) strategy[41] and stores that offer deep discounts on a varying sample of products. They argue that under an EDLP strategy, retailers do not incur the fixed costs associated with deep discounting, while deep discounting results in lower variable costs because discounts can be timed to coincide with manufacturers' trade deals. EDLP retailers target a segment of customers who have low price sensitivity while the deep discounting retailers sell to a majority of the consumers who are highly price sensitive. Coughlan and Vilcassim (1989) argue that everyday low pricing is an equilibrium for both firms in a duopoly market, but that as a result, the stores make zero economic rent. In response, they explore institutional ways that retail profits may be increased.

It should be noted that the practice of offering short term discounts on a varying sample of products is a dynamic pricing problem, while the intuition presented in this paper is a static explanation of retail pricing. Notwithstanding, the findings in this paper offer insights into the dynamic problem by predicting which products may be subject to short term discounts and how deep the discounts will be. The paper also helps to explain why some stores may prefer to offer deep discounts on a limited sample of products while

[41]Shallow discounts on all of the products in the store.

their counterparts offer shallow discounts on all of their products.

Lal and Matutes (1992) were the first to address the issue of how many products and which products retailers prefer to discount. They present a model of a two product duopoly market in which customers differ in their loyalty for each store due to their geographic distribution between the two stores. They find that stores are indifferent between discounting either or both products. It will be shown that this finding results from an assumption that customers have unit demand for each product. The rate at which stores lose profit and customers gain utility as price is reduced is therefore constant between the products. The results do not survive the introduction of a more general demand function that allows marginal utility and marginal profit to vary across products and price levels (a feature of almost all continuous demand functions). This paper demonstrates that when marginal surplus and store profit are not held constant, trade-offs between customer utility and store profit determine the optimal discounting strategy.

It will be shown that store preferences for which products to discount depend on the relative levels of marginal profit and marginal surplus for each product. We find that the products most likely to be discounted are those for which the store faces low marginal costs and for which the switching customers have stronger demand than the loyal customers. A store which is indifferent as to which product it would rather discount is more likely to offer shallow discounts on a broader variety of products than a store with a clear preference for which product it would rather discount. The findings also show that stores are more likely to offer discounts in order to attract switching customers when there are many switching customers, when the size of the discount required to attract the switching customers is small and when the switching customers contribute profit from their purchases of other products at the store. The size of the discount depends upon how many products are discounted, how willing the switching customers are to visit the store and the strength of the switching customers' demand for the discounted products.

The underlying intuition is introduced in Section II using a two product market in which all customers are switchers. In order to attract customers the optimal strategy for the store is to discount both products. When a segment of store loyal customers is added in Section III, stores must consider the relative efficiency of discounting each product and it is shown that stores may prefer to discount one, both or neither of their products. Section IV describes the circumstances under which each strategy is optimal and the paper concludes with a summary of findings and a discussion of how robust the model is to relaxation of the simplifying assumptions.

II. A Retail Market Without Store Loyal Customers

To highlight the role of loyal customers in determining which products stores prefer to discount, optimal discounting strategies will be derived in a model without loyal customers. Consider a monopolist store in a market with a single segment of consumers of unit mass. The customers have alternative sources from which they can purchase the products and must be offered a surplus of at least Q in order to attract them into the store.

The store would like to charge monopoly prices, which maximize the store's profit given that the customers come to the store. However, it will be assumed that at these (monopoly) prices the customers will earn strictly less utility than Q and will therefore prefer not to come to the store. The store may attract the customers by reducing the price of one or both of its products. A decision to lower a price below the monopoly price level will be interpreted as a decision to *discount* the product. When a store discounts a product below its monopoly price the price is advertised and known to customers before they decide whether to visit the store. To avoid unnecessary complexity the cost of advertising is set to zero. If a price is not discounted, the customers expect that the store will charge the monopoly price.

We focus on the pricing decisions for two products. The demand for each product is independent, downward sloping and maybe derived from the following utility function:[42]

[42]The over-parameterization of the utility function simplifies investigation of how the optimal discounting strategy is influenced by changes in the utility function. Standardizing and removing parameters would hinder the calculation of ceteris paribus comparative statics.

$$U = e_1 q_1 (v_1 - a_1 p_1 - q_1) + e_2 q_2 (v_2 - a_2 p_2 - q_2) \tag{1}$$

such that the optimal purchase quantity is:

$$q_i = \frac{v_i - a_i p_i}{2} \tag{2}$$

where:

q_i = quantity of product i purchased by each customer, $i \in \{1,2\}$;
p_i = price charged for product i by the store;
v_i = customers' value or need for product i;
a_i = price sensitivity parameter for product i; and
e_i = the relevance of product i in determining which store customers visit.

The parameters v_i and a_i may be jointly interpreted as reflecting the strength of customer demand for product i. At a given price, customers will purchase greater quantities of product i and enjoy higher levels of utility when v_i is high and/or a_i is low.[43] The introduction of parameters, reflecting how much influence the utility earned from each product has on the customers' store choice (e_i) was prompted by research indicating that the level of influence may vary between products. Dickson and Sawyer (1990) found that, for some products, shoppers rarely compare the price of competing brands, indeed they often do not even check the price of the items that they have selected. Consequently for these products, customer price knowledge is very low and the prices of these products are not expected to affect which store customers visit.

Store profit is defined as:

$$\pi = N[q_1(p_1 - c_1) + q_2(p_2 - c_2) + L] \tag{3}$$

where:

N = the number of customers who visit the store, $N \in \{0,1\}$;

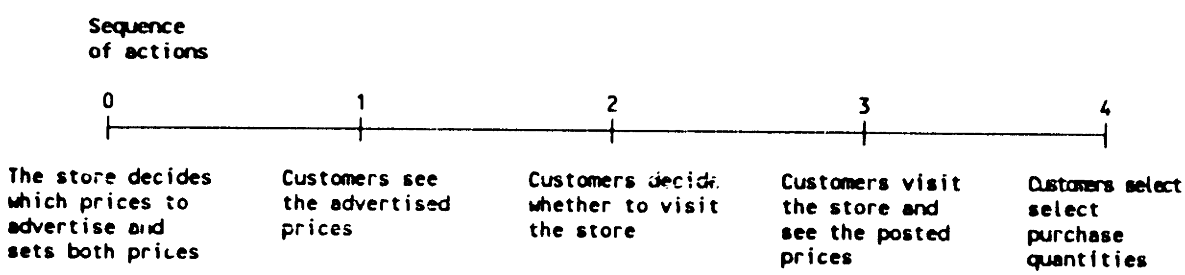
---

[43]See equation 2.

c_i = cost of each unit of product i to the store; and
L = the profit contributed by each customer from sales of other products.[44]

The customers' store selection decision requires that: N equals zero when $U < Q$ and equals one[45] otherwise. Q can implicitly be used to measure the extent of competition in the market. When competition is intense, perhaps because the cost of searching for alternatives at competing stores is low, Q is expected to be high. We focus on the more interesting case in which the utility earned when the store charges its monopoly prices is strictly less than Q (the store must discount in order to attract customers).

It is assumed that the store's marginal costs are assumed to be sufficiently low so that it is feasible for the store to offer customers utility of at least Q and still make positive profits. The market begins with the store deciding which prices to discount and simultaneously setting prices for both goods. Customers observe the discounted (advertised) price(s) and decide whether they will visit the store. If the customers do visit the store, upon arrival they observe the posted prices of the unadvertised products and select purchase quantities for both products.



[44]This may be assumed to equal zero for much of the following discussion. The parameter recognizes that customers may purchase products which do not influence their store selection decision (products for which e_i equals 0).

[45]Recall that customers are of unit mass.

To maximize profit, the store solves the following Lagrangian:

$$\max_{p_1,p_2} \quad \pi(p_1,p_2) = N\left[q_1(p_1 - c_1) + q_2(p_2 - c_2) + L\right] \qquad (4)$$
$$s.t. \quad U(p_1,p_2) \geq Q$$

which gives the following Kuhn-Tucker conditions:

$$\frac{\partial \pi}{\partial p_1} + \frac{\partial U}{\partial p_1}\lambda = 0 \qquad (5)$$

$$\frac{\partial \pi}{\partial p_2} + \frac{\partial U}{\partial p_2}\lambda = 0 \qquad (6)$$

$$\lambda\left[U(p_1,p_2) - Q\right] = 0 \qquad (7)$$

where: λ = the Lagrangian multiplier.

These conditions provide the intuition for Lemma 1:

Lemma 1

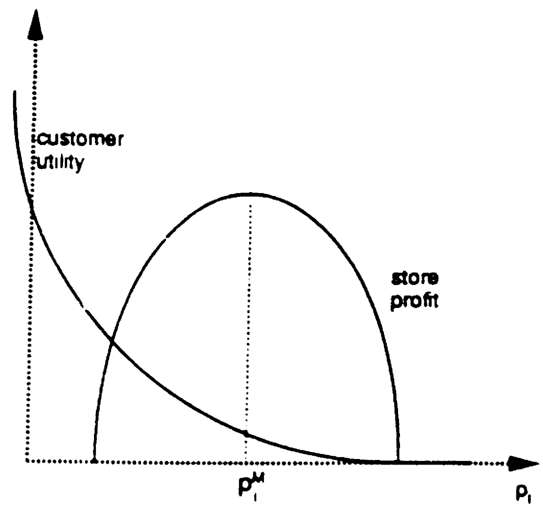
In a two product market in which a monopolist store must discount in order to attract customers, the monopolist will discount both products below the monopoly price.

A formal proof of this Lemma can be found in Appendix 1. To understand the intuition, consider Figure 1. If the store is charging the monopoly price (p_i^M) for one product it must be charging strictly less than the monopoly price for the other product to ensure that customers visit the store.[46] At the monopoly price, marginal profit equals zero while marginal utility is strictly negative. Therefore a reduction in price by ϵ will result in the customer receiving a positive increase in surplus with little loss in store profit. The increase in

[46]Recall that c_i is sufficiently low so that there are prices at which the monopolist can make positive profits.

Figure 1

Relationship Between Store Profit and Customer Utility



surplus will allow the store to increase the price of its other product. At prices below the monopoly price marginal profit is positive so the store will earn strictly higher profits.

This is a different result than what was learned from the previous work of Lal and Matutes (1992). It demonstrates how the shapes of the customer utility and store profit functions influence the optimal discounting strategy. The result is not limited to the two product case, but applies to any number of products for which demand is downward sloping. Because discounting is always inexpensive in the neighborhood of the undiscounted price, stores will prefer to discount all of the products that the target (switching) customers are willing to buy. The result is also not limited to markets in which there are no loyal customers. When store loyal customers are introduced, the store will consider which products the switching customers would buy if the products were not discounted (and the store charged its profit maximizing monopoly price). If the switching customers are willing to buy a product at its undiscounted price, then Lemma 1 asserts that stores that discount will prefer to discount this product. As a result, all of the products purchased by the switching customers will be discounted and the only products not discounted are products which the store prefers to sell to just the loyal customers (at a price higher that what the switching customers would be willing to pay).[47]

[47]Switching customers may purchase undiscounted products if there are cost or advertising effectiveness constraints which limit the number of products that it is profitable to advertise. Products which do not influence store choice (products for which e_i equals zero) may also be purchased by the switching customers at an undiscounted price.

III. A Retail Market With Store Loyal and Switching Customers

Consider the introduction of a second segment of customers who always visit the store. Allow these *store loyal* customers to have a mass of Y and the switching customers to have a mass of X. We assume that the loyal customers have stronger demand than the switching customers so that the price at which the switchers no longer enjoy positive surplus is strictly less than the price at which the loyal customers will no longer purchase, i.e.:[48]

$$\frac{v_i^y}{a_i^y} > \frac{v_i^x}{a_i^x} \qquad (8)$$

If brand switching customers also tend to be store switchers, this assumption is consistent with previous empirical research which has shown that brand switching customers tend to be more price sensitive than brand loyal customers.[49] Furthermore, the differing demands offers an explanation for why the switching customers may not visit while the loyal customers always participate. Customers with weaker demand enjoy lower utility at a given price and so are less likely to be willing to incur the effort to shop at the store and/or to sacrifice the opportunity to spend their income elsewhere. Indeed, the decision to come to the store could be endogenized for both customer segments by assuming that they face the same participation constraint but the switching segment has weaker demand.

In Figure 2, π_S and π_L represent the profit earned by the store from the

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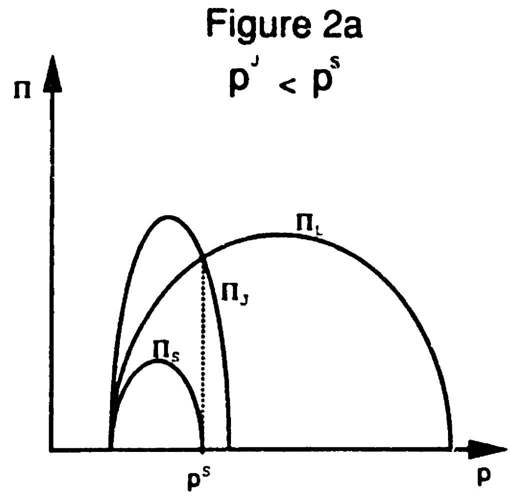
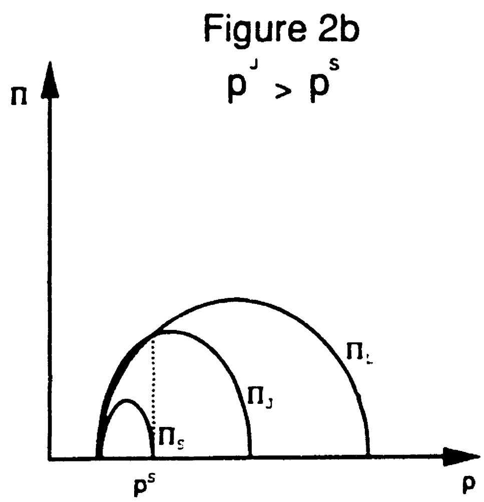
[48]See equation 2, letting $v_i = v_i^j$, where $j \in \{x,y\}$ and a_i^j is defined analogously.

[49]See, for example, Narasimhan (1984a) and Jolson, Wiener and Rosecky (1985).





switching and loyal customer segments (respectively).[50] The joint profit function is depicted by π_J, assuming that the switching customers eventually purchase negative quantities. The joint monopoly price, p_i^J, maximizes π_J. The monopoly price when selling to just the loyal segment, p_i^L, maximizes π_L and the

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[50]The profit functions in Figure 2 assume that $L^x = L^y = 0$. If either of these assumptions were relaxed, the relevant profit function would be shifted up on the y-axis (profit) but would not change with respect to the x-axis (price) and, as a result, would not affect the discussion.

price at which switching customers no longer purchase positive quantities, p_i^S, is found when π_S returns to zero. Of course negative purchase quantities are not possible, so when $p_i > p_i^S$ the store profit function follows the curve of π_L. For prices below p_i^S the store profit function follows the curve of π_J.

When $p_i^J < p_i^S$ the profit function is a bimodal function of the price of product i (Figure 2a). When $p_i^J > p_i^S$ the profit function is unimodal (Figure 2b). In the bimodal case the store's profit function has two maxima. When setting the undiscounted prices (if any) the store must evaluate whether it prefers to sell the undiscounted product to both the loyal and the switching customers (charging p_i^J) or to sell just to the loyal customers (charging p_i^L).[51] As already discussed at the end of the previous Section, Lemma 1 asserts that discounting both products is strictly more profitable than discounting just one product if the switching customers buy the undiscounted product. Therefore, if a store prefers to sell an undiscounted product to both customer segments, it is optimal for the store to discount both products.[52]

When selling to just the loyal segment, the profit maximizing price, p_i^L, is:

$$p_i^L = \underset{p_i}{\operatorname{argmax}} \quad Y\left[\frac{(v_i^Y - a_i^Y p_i)(p_i - c_i)}{2}\right]$$

$$= \frac{c_i}{2} + \frac{v_i^Y}{2a_i^Y} \tag{9}$$

To maximize profit, the store solves the same Lagrangian[53] that it faced in Section II (equation 4). However the resulting Kuhn Tucker conditions may

[51]This issue only arises when the store is discounting just one product. If neither product is discounted switching customers do not visit the store, while if both products are discounted there are no undiscounted prices to set.

[52]Or, possibly, not to discount at all.

[53]Where $U = U^*$.

now be satisfied in four ways:

Discounting neither product:

$$p_1 = p_1^L \qquad p_2 = p_2^L \qquad U^x(p_1^L, p_2^L) = 0$$
$$\frac{\partial \pi}{\partial p_1} = \frac{\partial \pi}{\partial p_2} = \frac{\partial U^x}{\partial p_1} = \frac{\partial U^x}{\partial p_2} = 0 \tag{10}$$

Discounting only product 1:

$$p_1 = \tilde{p}_1 \qquad p_2 = p_2^L \qquad U^x(\tilde{p}_1, p_2^L) = Q$$
$$\frac{\partial \pi}{\partial p_1} + \frac{\partial U^x}{\partial p_1}\lambda = \frac{\partial \pi}{\partial p_2} = \frac{\partial U^x}{\partial p_2} = 0 \tag{11}$$

Discounting only product 2:

$$p_1 = p_1^L \qquad p_2 = \tilde{p}_2 \qquad U^x(p_1^L, \tilde{p}_2) = Q$$
$$\frac{\partial \pi}{\partial p_2} + \frac{\partial U^x}{\partial p_2}\lambda = \frac{\partial \pi}{\partial p_1} = \frac{\partial U^x}{\partial p_1} = 0 \tag{12}$$

Discounting both products:

$$p_1 = p_1^* \qquad p_2 = p_2^* \qquad U^x(p_1^*, p_2^*) = Q$$
$$\frac{\partial \pi}{\partial p_1} + \frac{\partial U^x}{\partial p_1}\lambda = \frac{\partial \pi}{\partial p_2} + \frac{\partial U^x}{\partial p_2}\lambda = 0 \tag{13}$$

The optimal strategy will be determined by which of the four outcomes earns the store the greatest profit. At any point in parameter space the optimum will be unique,[54] however, in different parameter spaces different outcomes will be preferred. The boundaries of the parameter space in which each outcome is preferred (and is therefore the optimal outcome) can be characterized by the following four conditions:

[54]Except, at most, on sets of zero measure.

(0) The store prefers to discount product 1 rather than discount product 2:

$$\pi(\tilde{p}_1, p_2^L) - \pi(p_1^L, \tilde{p}_2) \geq 0 \tag{14}$$

(1) The store prefers to discount product 1 rather than discount neither:

$$\pi(\tilde{p}_1, p_2^L) - \pi(p_1^L, p_2^L) > 0 \tag{15}$$

(2) The store prefers to discount both products rather than discount neither:

$$\pi(p_1^*, p_2^*) - \pi(p_1^L, p_2^L) > 0 \tag{16}$$

(3) The store prefers to discount both products rather than discount product 1:

$$\pi(p_1^*, p_2^*) - \pi(\tilde{p}_1, p_2^L) > 0 \tag{17}$$

Complete parameter representations of these conditions are provided in Appendix 2. Without loss of generality, we will label the products so that condition 0 is always satisfied.[55] The store will prefer to discount both products only when it earns more from doing so than what it would earn from discounting product 1 alone or from discounting neither product. So the store will discount both products only when both condition 2 and condition 3 are satisfied and when doing so will charge p_i^* for each product (respectively). If condition 3 is not satisfied but condition 1 is, then the store prefers to discount only product 1 and the store will charge $\tilde{p}_1$ and p_2^L. If neither condition 1 nor condition 2 are satisfied the store will not discount either product and will charge monopoly prices for both, p_i^L. Because switching consumers earn less

[55]Conditions 1 and 3 are stated in a manner which assumes that condition 0 is satisfied.

surplus as prices are increased,[56] when discounting only one product stores must offer a deeper discount on that product than when both products are discounted: $\bar{p}_1 < p_i^* < p_i^L$. The various outcomes correspond to not discounting, deep discounting a limited product range and shallow discounting many products.

In Appendix 3 it will be demonstrated that each optimum is feasible and that each region of feasible parameter space maps to a unique optimum.[57] The parameter spaces in which each strategy is optimal will be investigated in Section IV by describing how changes in the model parameters affect satisfaction of conditions 0 to 3.

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[56]This assumes that the switching customers enjoy positive surplus from product *i*, or $p_i < p_i^S$. When this condition is not satisfied switching consumers do not purchase any of product *i* and their surplus is therefore not affected by *local* changes in the price of that product.

[57]Except, at most, on sets of zero measure. Note that both $\{\bar{1},2,\bar{3}\}$ and $\{1,\bar{2},3\}$ are intransitive and are therefore empty sets (where $\bar{1}$ equals the complement of condition 1).

IV. Which Discounting Strategy is Optimal?

We have seen that stores maybe willing to discount one or all of their products in order to attract switching customers into the store. In doing so they sacrifice a portion of their potential monopoly profits from sales to loyal customers but gain additional profits from sales to the switching customers. The amount of additional profit earned from sales to the switching segment is determined by the utility function of that segment, the store's profit function and the size of that segment. The extent to which monopoly profits are sacrificed from the switching segment is also influenced by these factors together with the utility function and size of the loyal segment. Consequently, the willingness of stores to discount one product or both products in order to attract the switching customers is affected by the store's profit function, the utility functions of both the switching and the loyal customers, and the relative sizes of the switching and loyal segments. The circumstances that favor each discounting strategy are examined in turn.

IV.i. Should a Store Offer Discounts?

A store should offer discounts to attract the switching customers only when the additional profit earned from these customers exceeds the profit lost from the loyal customers. Proposition 1 is derived from comparative statics on conditions 1 and 2 (summarized in Appendix 4).

Proposition 1

A monopolist store will offer discounts in order to attract switching customers when, ceteris paribus:

a. *The number of switching customers is large and/or the number of loyal customers is small (X is large and/or Y is small);*

b. *The store has low marginal costs (c_i are small);*

c. *The utility that must be offered to the switching customers in order to attract them to the store is small (Q is small);*

d. *Switching customers contribute a lot of profit from purchases of undiscounted products (L^x is large);*

e. *The relevance of the products in determining which store the switching customers visit is high (e_i^x are large);*

f. *Switching customers have strong demand (price sensitivity, a_i^x, is small and/or the value of consumption, v_i^x, is large); and*

g. *Loyal customers have weak demand (a_i^y is large and/or v_i^y is small).*

The intuition for and implications of each of these effects will be considered in turn.

The number of switching customers is large and/or the number of loyal customers is small (X is large and/or Y is small): When there are many switching customers and few loyal customers the increase in store traffic when discounting is large while the resulting loss of monopoly profit affects fewer loyal customers. The relative size of the loyal segment could be influenced by the location or age of the store. Older, more established stores maybe expected to have a larger loyal segment than new entrants into the market. If this is correct we would expect to see a higher willingness to discount amongst newer stores than amongst stores which have been in the market for some time.

The store has low marginal costs (c_i are small): When marginal costs are low, stores prefer to charge lower prices[58] because they benefit more from an increase in demand. As a result, stores with lower marginal costs are more

[58]Note: the derivatives of both p_i^L and p_i^J with respect to c_i are strictly positive.

likely to discount both because their losses from the loyal customers are offset by greater profit from the resulting increase in volume and because the undiscounted monopoly price is lower. We would expect stores which have distribution, labor or purchasing power efficiencies to be more willing to discount than their less efficient counterparts.

The utility that must be offered to the switching customers in order to attract them to the store is small (Q is small): The profit lost from the loyal customers when discounting increases as the size of the required discount increases. Therefore, as Q increases, discounting becomes more expensive and less attractive.[59] As already discussed, Q may be interpreted as a measure of the intensity of competition between competing stores. The more intense the competition, the more utility that must be offered in order to attract the switching customers and the higher the level of Q. This finding suggests that stores are less motivated to compete for switching customers when the intensity of competition is high.

Switching customers contribute a lot of profit from purchases of undiscounted products (L^x is large): While the results presented by Dickson and Sawyer (1990) suggest that the prices of some products are not expected to affect which stores customers visit,[60] purchases of these products do contribute to store profit. The benefits of attracting switching customers to the store increase when the switching customers contribute profit from their purchases of additional undiscounted products (L^x is large). Therefore, we would expect to see more discounting by stores at which customers purchase undiscounted products which have little influence on their store selection decision.

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[59]The factors influencing the size of the required discount are investigated in Section IV.iv.

[60]This phenomenon is captured in the model by the introduction of the e_i' parameters, see earlier discussion.

The relevance of the products in determining which store the switching customers visit is high (e_i^* are large): Products which have a lot of influence over which store a customer visits may be described as *salient* to that customer. Highly salient items could be products: that have been advertised more frequently (by retailers or manufacturers), for which the purchase frequency and/or customer involvement in the purchase decision are high and about which customers have the highest price knowledge. When products in a store are highly salient to the switching customers,[61] stores do not need to lower their prices as far in order to attract the switching customers. As a result, attracting the switching customers is less costly (in terms of monopoly profit lost from the loyal customers) and these stores are more likely to offer discounts.

Switching customers have strong demand and/or loyal customers have weak demand: If price sensitivity amongst switching customers is low and/or these customers place high value on each unit of consumption, then a reduction in price increases the utility of the switching customers at a faster rate. As a result, less of a discount is required in order to attract them into the store. However, if demand amongst the loyal customers is also strong, the loss of monopoly profit when discounting will be large. Therefore, unless there is evidence that the strength of demand differs between loyal and switching customers (for at least some products), demand parameters do not directly explain which stores will discount.[62]

IV.ii. Which Products Should a Store Discount?

If a store decides that it is going to discount in order to attract the switching customers and it prefers to do so by deep discounting a narrow

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[61]Or *all* customers (both switching and loyal customers).

[62]These parameters may indirectly explain which stores discount by modifying the effects of other parameters on this decision.

product range,[63] which products should it choose to discount? Proposition 2 is derived from comparative statics on condition 0 (summarized in Appendix 4).

Proposition 2

A monopolist store will prefer to discount products for which, ceteris paribus:

a. *The store has low marginal costs (c_i is small);*

b. *The relevance of the product in determining which store switching customers visit is high (e_i^X is large);*

c. *Loyal customers have weak demand (a_i^Y is large and/or v_i^Y is small); and*

d. *Switching customers have strong demand (a_i^X is small and/or v_i^X is large).*

Intuition and interpretation are again provided for each of the results.

The store has low marginal costs (c_i is small): Recall that when marginal costs are low the store gains more profit from an increase in volume than when marginal costs are high. If demand is downward sloping, volume increases when prices are discounted. As a result, stores prefer to discount products with relatively low marginal cost because the gain in profit from the increase in volume is amplified. We would expect to see stores discounting the products for which they have special marginal cost advantages. This finding offers theoretical support for the empirical finding that retailers will pass a

[63] The store must choose whether to deep discount a narrow product range or shallow discount a broader range of products. The factors influencing this decision will be investigated in the following sub-section (IV.iii).

portion of manufacturers' trade discounts on to their customers.[64] Indeed, this result shows how manufacturers can use trade discounts to influence retailers' discounting strategies.

The relevance of the product in determining which store customers visit is high (e^s_i is large): When a product is salient to the switching customers, so that the utility that they derive greatly influences which store they visit, smaller discounts are sufficient to attract the switching customers. Because discounting is costly (in terms of profit lost from the loyal customers) we would expect stores to discount the products for which the switching customers have high salience. As already discussed, these may be the frequently purchased, frequently advertised, high involvement and/or high price knowledge products.

As a preliminary test of whether advertised products are more frequently purchased than unadvertised products, the 37 products advertised by both competing chains in the comparison discussed at the start of this paper were divided into *edible* and *non-edible* products. These products were then compared with separate lists of the 200 highest volume (number of packages sold) edible and non-edible consumer package goods in the Boston marketplace in 1992 (The Griffin Report of Food Marketing, 1992).[65] Seven of the 24 mutually advertised edible products were on the list of the 200 highest volume edible products while three of the 13 advertised non-edible products were on the list of the highest volume non-edible products. If the chains stock more than 2,513 brand-size alternatives, then the proportion of edible advertised products appearing on the list of most frequently purchased products is

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[64]See Bucklin and Lattin (1989); and Chevalier and Curhan (1976).

[65]Results were tabulated from scan data supplied by major food retailers in the Boston marketplace.

significantly higher ($\alpha=0.05$) than would be expected by chance.[66] The corresponding number of non-edible products is 2168. Because supermarkets typically stock over 20,000 different brand size alternatives, these results are consistent with the prediction that stores are more likely to advertise products which customers purchase frequently. Chevalier and Curhan (1976) also present empirical evidence which is consistent with this hypothesis.

Loyal customers have weak demand and/or switching customers have strong demand: When demand for a product is strong, customers purchase larger quantities and earn greater utility at a given price. For this reason, switching customers can be attracted with smaller discounts when the switching customers have strong demand for the discounted product, while, discounting is less expensive (in terms of profit lost from loyal customers) when loyal customers have weak demand for the discounted product. Therefore stores prefer to discount products for which switching customers have strong demand and/or loyal customers have weak demand. There is some empirical evidence that deal prone or brand switching customers typically come from middle income households.[67] If this also holds true for store loyal customers, we would expect products which are targeted at these households to be advertised more frequently. This would suggest that higher priced luxury product categories are less likely to be discounted as are low quality brands within a category (such as generic store brands).

IV.iii. How Many Products Should a Store Discount?

Given that it has decided to discount, the store must decide upon the most efficient manner in which to do so. Both the model and current practice

[66]The more products that a store stocks, the less likely that any single product will appear by chance in a sample of 200. The test was conducted by comparing the difference between proportions assuming a *Bernoulli process*).

[67]See, for example, Narasimhan (1984a, 1984b) and Nielsen (1985).

suggest that in some situations deep discounting a narrow range of products is optimal, while in other circumstances the store is better to offer shallow discounts on a broader range of products. The comparative statics on condition 3 (summarized in Appendix 4) were used to derive Proposition 3. Recall that the products have been labelled to ensure that when discounting a single product the store will discount product 1.

Proposition 3

A monopolist store will prefer to offer shallow discounts on both products rather than deep discounting a single product when, ceteris paribus:

a. *The marginal cost of product 1 is high and/or the marginal cost of product 2 is low (c_1 is high and/or c_2 is low);*

b. *The relevance of product 2 in determining which store customers visit is high (e_2^x is large);*

c. *Loyal customers have strong demand for product 1 (a_1^y is small and/or v_1^y is large);*

d. *Loyal customers have weak demand for product 2 (a_2^y is large and/or v_2^y is small); and*

e. *Switching customers have strong demand for product 2 (a_2^x is small and/or v_2^x is large).*

Recall the results of Section IV.ii describing which products stores will select to discount when deep discounting. The store prefers to discount products which strongly influence the customers' store selection decisions and for which the store has low marginal costs, switching customers have strong demand and/or loyal customers have weak demand. Labelling the products to ensure that condition 0 is satisfied necessarily ensures that product 1 satisfies these characteristics better than product 2. Proposition 3 states that the store is more likely to prefer to shallow discount both products when product 2 also satisfies these criteria. Consequently, Proposition 3 may be interpreted as finding that stores will prefer to shallow discount if both products perform similarly when evaluated against the criteria in Proposition 2. Deep discounting is likely when the store has a clear preference for deep discounting one of the products over the other, while shallow discounting is more likely when the store is indifferent as to which product it would rather deep discount.

This finding provides an explanation for why one store may choose to shallow discount while its counterpart, facing the same customer segments and customer demands, may choose to deep discount. If a store has similar marginal cost efficiencies for each of its products it is more likely to shallow discount than a store which enjoys special marginal cost advantages over a subsample of its products. Variance in marginal costs across products may result from different distribution practices or varying bargaining power. If the bargaining power between a store and its supplier reflects the relative size of each party, large store chains will enjoy considerable bargaining power with all suppliers while the bargaining power available to smaller firms will vary with the size of each supplier. Moreover, when all products are distributed through a common distribution system, the variance in distribution and warehousing costs will be less than if some products are delivered by the manufacturer directly to each store while others are distributed through the retailer's warehouses. If these arguments are correct we would expect shallow discounting and EDLP strategies to be more common amongst larger chains (greater bargaining power) and chains using a common distribution system for all of their products. Consistent with these predictions, Walmart and K Mart, two of America's three largest retailers (Fortune 1992), have all adopted EDLP pricing strategies.

By a similar argument, if the marginal costs faced by two stores are approximately equal when averaged across products, but they vary more in one store, then it is possible that one store will charge lower advertised prices while its counterpart charges lower unadvertised prices. Such an outcome is reported in Lattin and Ortmeyer (1991) as a feature of markets in which stores asymmetrically choose EDLP or deep discounting strategies. The store with low variance in its marginal costs prefers to advertise many products at a shallow discount, while the stores with high variance will offer deep discounts on the products with the smallest relative marginal cost. The unadvertised prices will be higher at the high variance store because it faces relatively high marginal costs on these products.

This argument may initially appear to be inconsistent with the findings in the *Signalling and Commitment Using Retail Prices* essay in Chapter 1. In that essay, it is shown that if there is positive correlation in marginal costs across products, then stores charging lower advertised prices may also be expected to charge lower unadvertised prices. The apparent inconsistency arises from a breach of the assumption that marginal costs are positively correlated across products. When the average marginal cost is held constant, a relatively low marginal cost for one product implies a relatively high marginal cost for another product which represents *negative* correlation in marginal costs. If marginal costs are positively correlated across products the results in the current essay are consistent with price signalling: the stores that are most likely to discount (stores with low marginal costs) also prefer to charge lower undiscounted prices.

IV.iv. How Deep Will the Discount(s) Be?

Because the store would prefer to charge higher prices and only discounts in order to attract the switching customers, the store will discount no more than is necessary to ensure that the switching customers visit. What level of discount is necessary? This depends on how much utility must be guaranteed to the switching customers to ensure that they visit, how many products are discounted and the rate at which the utility enjoyed by the switching customers increases as prices are reduced.

Proposition 4

A monopolist store will offer deeper discounts when, ceteris paribus:

a. *The utility that must be offered to the switching customers in order to attract them to the store is large (Q is large);*

b. *The store discounts fewer products; and*

c. *Switching customers have strong demand for the discounted products (a_i^* is small and/or v_i^* is large).*

Proofs to Proposition 4 are obvious from the earlier discussion and for this reason are not presented formally. The intuition for the results relies upon switching customers earning more utility when prices are lower. Therefore, the more utility that switching customers demand to attract them to the store, the deeper the required discounts(s). If the amount of utility demanded by the switching customers reflects the intensity of competition in the market, we would expect to see deeper discounts in more competitive markets. This result is consistent with the finding in the first essay that the intensity of competition between stores (or the distance the stores are apart) influences the prices of the discounted (advertised) products but does not influence the undiscounted prices.

We would also expect to see deeper discounts when the range of products being discounted is small. To guarantee the same overall level of utility, the utility enjoyed from consumption of each discounted product must be higher when fewer products are being discounted. Furthermore, the stronger the customer demand for a product the less that the price must be reduced in order to offer an equivalent level of utility.

V. Conclusions

A model of a two product retail market has been presented to demonstrate how the optimal discounting strategy is influenced by trade-offs between customer utility and store profit. These trade-offs, which are overlooked in the only previous analytical research on this topic, are capable of providing insights into many of the discounting decisions confronting a retail store, including: when should a store offer discounts; how many products should be discounted; which products should be discounted; and how deep should the discounts be?

Because discounting is always expensive in the region of the optimal undiscounted price, stores will prefer to discount all of the products that the target (switching) customers are willing to buy. As a result, unless exogenous market features limit the number or type of products that it is feasible to discount (advertise), the only products that it is not optimal to discount are products which the store prefers to sell solely to the loyal customers (at a price higher than what the switching customers would be willing to pay). The results show that stores are more likely to offer discounts when there are many switching customers or few loyal customers and when marginal costs are low. Discounting is also more attractive to the store if loyal customers have weaker demand for the discounted products than switching customers, or when switching customers purchase many other products at the store and the utility that they derive from the discounted products greatly influences their store selection decision.

When deciding which products to discount, the store must compare the rates at which switching customers gain utility and the store loses profit as the price is reduced. Products most likely to be discounted are those products for which stores have special marginal cost advantages and products for which the demand amongst loyal customers is weak while demand amongst the switching customers is strong and/or salient. These results are consistent with and provide an explanation for the common wisdom contained in the retailing management literature. The decision as to which products to discount is also obviously a function of how many products the store would like to discount. We find that discounting a narrow product range is most likely when the store has a clear preference for discounting one of the products over the other. As the store becomes more indifferent as to which product it would rather discount, the probability that it will discount a larger number of products increases.

Because discounting is expensive, the store will not discount more than is necessary in order to attract the switching customers. As a result, the depth of the discount(s) offered depends upon the number of products discounted, the amount of utility that must be offered in order to attract the switching customers and the strength of demand amongst the switching customers. The stronger the switchers demand, the smaller the required utility and the more products that are discounted then the smaller the resulting discount(s).

These results allow us to provide a possible explanation for the questions posed in the introduction: how can different discounting strategies be optimal for different stores? The locations, ages of the stores and other factors may result in the stores attracting loyal customer segments of different sizes or with different demands. Distribution and purchasing power variations may cause marginal costs to vary both in the average across products and in the variance between products. The stores may offer different product ranges so that the profit earned from customers' purchases of undiscounted products varies. The results show that store differences such as these are capable of explaining why some stores may adopt a shallow discounting or EDLP strategy while others offer deep discounts on a small range of products or do not discount at all.

A number of limitations resulting from modelling simplifications are worthy of discussion. The model assumes that the switching segment is homogenous: either all of the segment visits the store or none of the customers in the segment visit. Clearly, in practice, this is unlikely to happen. However, as long as the distribution of customers is sufficiently bimodal the findings will not meaningfully change. A cost of advertising could be introduced to make discounting costly to the store. The findings would survive the introduction of such a cost with the implication that the larger the cost of advertising the less attractive it is to discount. If the advertising cost was variable and increased as the number of advertised products increased then deep discounting a few products would become more attractive compared to shallow discounting multiple products.

The introduction of additional products is consistent with the current intuition, so the findings also survive the introduction of additional products. Although the number of equilibrium conditions required to define the optimal strategy increases dramatically, the role of each parameter in determining the optimal discounting strategy will remain the same.

For ease of computation the findings are derived using a linear customer utility function. The comparative statics on c_i, X, Y, Q and L^j are robust to almost any reasonable utility functions. The findings concerning the parameters in the switching customers' utility function rely on the signs of the derivatives of U_i^x and q_i^x with respect to each parameter. The signs of the derivatives of q_i^y and dq_i^y/dp_i govern the results for the loyal customers' utility function. These signs are generally predictable from each parameter's interpretation, which suggests that the findings are robust to a broad variety of more general demand functions.

Discounting just one product can only be an optimal strategy if store loyal customers have stronger demand than switching customers for at least one product. If this is not the case, switching customers will be willing to buy both products at their undiscounted prices, which, in accordance with Lemma 1, implies that discounting both products will be strictly preferred to discounting just one product. Arguing that switching customers have stronger demand than loyal customers for both products would not be consistent with the empirical research describing deal prone behavior and would not be consistent with the explanation offered to justify why only one segment of customers is willing to switch.

Introduction of a competing store would allow the effects of competition to be incorporated explicitly. In a two firm Stackelberg market, the Stackelberg leader must decide whether it prefers to target just its loyal customers or to discount and attract the switching customers. To attract the target customers the Stackelberg leader must ensure that it offers enough utility to both lure the switching customers to the store and to dissuade the Stackelberg follower from also discounting. The follower faces the same decisions as the monopolist store in the present model, with Q endogenously determined by the leaders discounting decision. The intuition remains the same for both stores - a store will only discount if it earns more profit from attracting the switching customers than what it loses from its loyal customers. When deciding *how* to discount, the store finds the way to offer utility to the switching customers that sacrifices the least profit from the loyal customers.

The findings also help to explain optimal discounting strategies when competing firms choose their discounting strategies simultaneously. Assuming a locational model in which the switching customers were distributed between the stores, then (given appropriate assumptions) a pure strategy equilibrium would exist in which stores simultaneously choose a discounting strategy while expecting each other to behave optimally. In deciding how to discount in order to attract switching customers at least cost, the store will face essentially the same efficiency trade offs as in the current model. A decision not to discount at all may be optimal if a store has a large segment of loyal customers.

Demands for different products are not always independent. In the case of complements, such as cheese and crackers or salsa and chips, an increase in demand for one of the products will have a positive effect on demand for the other. Other products, such as competing brands of the same good, represent substitutes - an increase in demand for one brand maybe expected to reduce demand for its competitors. For a variety of reasons, the positive cross price elasticities exhibited by complements makes it more attractive to discount complements than substitutes. The level of discount required is smaller and less expensive; the overall increase in demand for both customer segments offsets the loss of monopoly profits from the loyal customers; and the monopoly prices when selling just to the loyal customers are lower for complements than they are for substitutes.

There are a number of additional reasons a store may choose to offer discounts. The retailing management literature notes that seasonal merchandise, over-bought merchandise and damaged merchandise are all often subject to discounts,[68] although discounts for these products are perhaps better explained as a change in the optimal price following a reduction in customer demand. Previous research has shown that advertised prices may be used to signal both the quality of the products offered by the store[69] and the

[68]A general discussion is presented in Arnold, Cappella and Smith (1983 p434-435).

[69]A number of papers have investigated price signalling of quality. See Chu and Chu (1990) for a recent review.

prices of other unadvertised products sold by the store.[70] Investigating which product(s) it is optimal to discount when the objective of discounting is not just to commit to offering surplus, but also to signal information about quality or unadvertised prices, may offer additional insights to the current findings. The choice of products to advertise may also be influenced by the degree of asymmetric knowledge in the market. The prices of fruit and vegetables vary frequently as a result of supply fluctuations which are generally observable only by the retailer. By advertising the prices of these products, retailers are able to resolve customer uncertainty and (if the customer is risk averse) make visiting the store more attractive.

Currently, it is rare for research investigating customer purchasing behavior to discriminate between customer segments, perhaps because the importance of doing so was not previously apparent. This research highlights the importance of separately estimating the purchasing behavior of switching and loyal customers. A number of the normative implications in the paper assume that the store can separately assess the strength of demand for each product in each customer segment. The findings also contain a large number of empirically testable hypotheses. Future empirical research is planned to more fully investigate the consistency of each of the hypotheses with current retail practice.

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[70]See Simester (1993).

Appendix 1

Proof of Lemma 1

> **Lemma 1**: In a two product market in which a monopolist store must discount in order to attract any customers, the monopolist will discount *both* products below the monopoly price.

Proof: In order to demonstrate inconsistency, assume that the monopolist discounts only one product (product 1) so that $\partial\pi/\partial p_1 > 0$. The customers' downward sloping demand implies that $\partial U/\partial p_i < 0$ for all $p_i <$ some $\hat{p}$ where the monopoly price $p_i^M < \hat{p}$. For the first Kuhn Tucker condition, equation (5), to be satisfied, $\lambda > 0$. As a result the second Kuhn Tucker condition, equation (6), is only satisfied when $\partial\pi/\partial p_2 > 0$. But this is *inconsistent* with the store charging its monopoly price for product 2. For both Kuhn Tucker conditions to be satisfied the store must charge strictly less than the monopoly price for both products.

Appendix 2

Parameter Representations of the Equilibrium Conditions:

(0) The store prefers to discount product 1 rather than discount product 2:

$$\begin{aligned}&\left[X(v_1^x - a_1^x \bar{p}_1) + Y(v_1^y - a_1^y \bar{p}_1)\right](\bar{p}_1 - c_1) + Y(v_2^y - a_2^y p_2^L)(p_2^L - c_2)\\ &- \left[X(v_2^x - a_2^x \bar{p}_2) + Y(v_2^y - a_2^y \bar{p}_2)\right](\bar{p}_2 - c_2) - Y(v_1^y - a_1^y p_1^L)(p_1^L - c_1) > 0\end{aligned} \tag{18}$$

where:

$$\bar{p}_i = \frac{v_i^x - 2\sqrt{Q}}{a_i^x} \tag{19}$$

$$p_i^L = \frac{c_i}{2} + \frac{v_i^y}{2a_i^y} \tag{20}$$

(1) The store prefers to discount product 1 rather than discount neither product:

$$\left[X(v_1^x - a_1^x \bar{p}_1) + Y(v_1^y - a_1^y \bar{p}_1)\right](\bar{p}_1 - c_1) + XL^x - Y(v_1^y - a_1^y p_1^L)(p_1^L - c_1) < 0 \tag{21}$$

(2) The store prefers to discount both products rather than discount neither product:

$$\begin{aligned}&\left[X(v_1^x - a_1^x p_1^*) + Y(v_1^y - a_1^y p_1^*)\right](p_1^* - c_1) - Y(v_1^y - a_1^y p_1^L)(p_1^L - c_1) + XL^x\\ &+ \left[X(v_2^x - a_2^x p_2^*) + Y(v_2^y - a_2^y p_2^*)\right](p_2^* - c_2) - Y(v_2^y - a_2^y p_2^L)(p_2^L - c_1) < 0\end{aligned} \tag{22}$$

where:

$$Q = \frac{a_1^x(v_1^x - a_1^x p_1^*)^2}{4} + \frac{a_2^x(v_2^x - a_2^x p_2^*)^2}{4}$$

(3) The store prefers to discount both products rather than discount product 1:

$$\begin{aligned}&\left[X(v_1^x - a_1^x p_1^*) + Y(v_1^y - a_1^y p_1^*)\right](p_1^* - c_1) - \left[X(v_1^x - a_1^x \bar{p}_1) + Y(v_1^y - a_1^y \bar{p}_1)\right](\bar{p}_1 - c_1)\\ &+ \left[X(v_2^x - a_2^x p_2^*) + Y(v_2^y - a_2^y p_2^*)\right](p_2^* - c_2) - Y(v_2^y - a_2^y p_2^L)(p_2^L - c_1) < 0\end{aligned} \tag{24}$$

Appendix 3

Mapping from the Equilibrium Conditions to the Equilibria

Assume that the products are labelled to ensure that condition (0) is satisfied.

Table 1

Condition 1 $\pi_1 > \pi_0$	Condition 2 $\pi_{1,2} > \pi_0$	Condition 3 $\pi_{1,2} > \pi_1$	Outcome
×	×	×	Discount neither product
✓	×	×	Advertise only product one
✓	✓	×	Advertise only product one
✓	✓	✓	Advertise both products
×	✓	✓	Advertise both products
×	×	✓	Discount neither product
×	✓	×	Empty set
✓	×	✓	Empty set

✓ condition is satisfied
× condition is not satisfied

π_0 profit earned from discounting neither product
π_1 profit earned from discounting just product 1
$\pi_{1,2}$ profit earned from discounting both products

In order to demonstrate both that the *six* feasible regions of parameter space are indeed non empty and (by implication) to prove existence of each equilibrium discounting strategy, examples of parameter values satisfying the various equilibrium conditions are presented in Table 3. The parameter values also all satisfy the model assumptions, namely (for $i \in \{1,2\}$):

$$v_i^Y > v_i^X > p_i^*$$

$$v_i^Y > p_i^L > p_i^J > p_i^* > \bar{p}_i > 0$$

$$\pi(\bar{p}_1, p_2^L) > \pi(p_1^L, \bar{p}_2)$$

Table 2

Y	L^x	v_2^x	v_2^y	Condition 1 $\pi_1 > \pi_0$	Condition 2 $\pi_{1,2} > \pi_0$	Condition 3 $\pi_{1,2} > \pi_1$
10	0	15	35	×	×	×
5	0	15	35	✓	×	×
5	1000	15	35	✓	✓	×
20	1000	15.5	31.4	✓	✓	✓
20	833	15.5	31.4	×	✓	✓
20	0	15.5	31.4	×	×	✓

In all of these examples the remaining parameter values were set as follows ($i \in \{1,2\}$ and $j \in \{x,y\}$):

$$a_i^j = e_i^j = X = 1$$
$$L^y = c_i = 0$$
$$Q = 15 \qquad v_1^x = 55 \qquad v_1^y = 110.$$

Appendix 4

Proofs to Propositions 1-3

Deriving the comparative statics to prove Propositions 1-3 requires the straight-forward differentiation of the equilibrium conditions in Appendix 2 with respect to each parameter. The comparative statics are summarized in Table 3.

Table 3

Parameter	Condition 0 $\pi_1 > \pi_2$	Condition 1 $\pi_1 > \pi_0$	Condition 2 $\pi_{1,2} > \pi_0$	Condition 3 $\pi_{1,2} > \pi_1$
X	?	+	+	?
Y	?	-	-	?
Q	?	-	-	?
L^x	0	+	+	0
c_1	-	-	-	+
c_2	+	0	-	-
a_1^x	-	-	-	?
a_1^y	+	+	+	-
a_2^x	+	0	-	-
a_2^y	-	0	+	+
v_1^x	+	+	+	?
v_1^y	-	-	-	+
v_2^x	-	0	+	+
v_2^y	+	0	-	-
e_1^x	+	+	+	?
e_1^y	0	0	0	0
e_2^x	-	0	+	+
e_2^y	0	0	0	0

\+ condition more likely to be satisfied if parameter is large
\- condition less likely to be satisfied if parameter is large
0 parameter does not affect likelihood of satisfying condition
? sign depends upon parameter conditions

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