Essays on Uninsured Income Risk, Lumpy Investment and Aggregate Demand

by

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Abstract

This thesis consists of three chapters on uninsured income risk, lumpy investment and aggregate demand. The first chapter analyzes the non-linear response of durable spending to income shocks. Empirically, the average marginal propensity to spend (MPC) on durable goods increases with the size of income changes. I investigate whether a canonical model of lumpy durable investment with incomplete markets can replicate this fact. I first clarify analytically the source of non-linearity in this model, and I show that its sign depends on the relative strength of the extensive and intensive margins of durable adjustment. In numerical exercises, I find that the extensive margin dominates quantitatively, so that the model generates the form of non-linearity observed in the data. However, the magnitudes predicted by this canonical model are substantially lower than their empirical counterparts. I suggest various avenues to improve the quantitative performance of the model.

The second chapter investigates the general equilibrium implications of this form of non-linearity. I recognize that durable spending is strongly pro-cyclical, that workers employed in durable sectors have a more cyclical labor income than those employed in non-durable sectors, and that workers are imperfectly insured against these fluctuations. In turn, the average MPC on durables increases with income changes, so that this redistribution of labor incomes across sectors has aggregate effects. To formalize and quantify this mechansim, I develop a heterogeneous agent New Keynesian (HANK) model with multiple sectors and lumpy durable adjustment. There is no labor mobility between sectors and financial markets are incomplete, so that durable workers are more exposed to aggregate shocks. I first show analytically that the interaction between cyclical investment and redistribution amplifies the aggregate response of durable spending during booms and dampens it during recessions. I then quantify the importance of this mechanism using my structural model.

The third chapter focuses on the cyclical reallocation of workers across sectors or occupations. Specifically, I explore how uninsured income risk and liquidity frictions can hinder the efficient matching between workers and occupations. I investigate this question in a continuous-time Lucas-Prescott economy with incomplete markets. In this setting, uninsured income risk induces labor misallocation across occupations through two channels. First, it reduces workers' incentives to search (*ex ante*) for an occupation where they have a strong comparative advantage. Second, it induces excess separation (*ex post*) by forcing productive households to leave their occupation when their liquidity buffers are depleted. In general equilibrium, labor misallocation exacerbates endogeneously the effect of uninsured income risk, by depressing the value of equity that workers use as liquidity buffers.

Thesis Supervisor: Iván Werning
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Contents

Chapter 1

Lumpy Investment and the Non-Linear Response to Income Shocks*

Empirically, the average marginal propensity to spend on durable goods increases with the size of income shocks. In this paper, I explore whether a canonical model of lumpy durable investment with incomplete markets can replicate this fact. I first clarify the potential source of non-linearity in this model, by decomposing the investment response to income shocks into extensive and intensive margins. In numerical exercises, I confirm that this model predicts the correct form of non-linearity. However, I find that the magnitudes are substantially lower than those observed in the data. I suggest various avenues to improve the quantitative performance of the model.

1.1 Introduction

Marginal propensities to spend (MPCs) are commonly used as *sufficient statistics* to represent the response to aggregate shocks, and characterize optimal stabilization policies. A growing literature is interested in measuring these statistics and uncovering their determinants.[1] In this paper, I recognize that MPCs are not structural, invariant objects and I explore how they vary with the size of income shocks. Specifically, I am interested in non-linearities in the response of durable spending to income shocks.

My focus on durables is motivated by the existing evidence on the consumption re-

* I am indebted to Iván Werning, Ricardo Caballero and Marios Angeletos for their formidable guidance and support. I am grateful to Jonathan Parker and participants to the MIT Macro Lunch and MIT Macro Seminar for helpful discussions. All errors are my own.

[1] In particular, Kaplan and Violante (2014) and Kaplan and Violante (2014); Kaplan et al. (2018) identify an important role for convex and non-convex portfolio adjustment costs to explain the distribution of MPCs observed in the data.

sponse to transfers. In particular, Johnson et al. (2006) estimate negligible spending multipliers on durable goods in response to the 2001 earned income tax credit (EITC), while Souleles (1999) and Parker et al. (2013) estimate strong responses for durable expenditure following springtime tax refunds and the 2008 EITC, respectively. Parker et al. (2013) attribute the difference in MPCs across these episodes to the size of the corresponding transfers. Similarly, Fuster et al. (2018) and Christelis et al. (2019) provide survey evidence that the average MPC on durables *increases* with the size of (hypothetical) unexpected income changes. In contrast, the literature has consistently found that the average MPC on non-durables *decreases* with the size of income shocks,[2] though to a lesser extent. That is, non-linearities are potentially important for durable spending, much less so for non-durable spending.

The main contribution of this paper is to explore whether a canonical model of lumpy durable demand with incomplete markets can replicate this empirical evidence, and generate an increasing MPC on durables. I proceed in two steps. In the first step, I investigate the theoretical determinants of this non-linearity using this model of lumpy adjustment. Non-convex adjustment costs lead to a standard inaction region for durable adjustment. Durable adjustment occurs along two margins: an extensive margin, which controls the propensity of households to exit their inaction region and pay the adjustment cost; and an intensive margin, which determines their MPC conditional on adjustment. The contribution of each of these margins to aggregate non-linearities depends on two objects: the shape of the distribution of idiosyncratic characteristics at the edge of the inaction region; and the monotonicity and concavity of durable spending conditional on adjustment. Imposing restrictions that resemble those obtained in numerical simulations, I show that adjustment along the extensive margin *amplifies* expansionary income shocks as their magnitude increases: more and more households adjust their stock of durables. On the contrary, it *dampens* contractionary shocks. Lumpy and state-dependent adjustment plays a central role in this non-linear effect. Models of smooth or time-dependent adjustment, where only the intensive margin operates, would actually predict the *opposite* effect.[3,4]

In the second step, I calibrate my model and use it to quantify the importance of these aggregate non-linearities. I simulate the response of durable investment to income shocks, varying their sign and magnitude. I find that the average MPC on durable goods increases with income changes. For instance, for negative income shocks of the magnitude experi-

[2] A non-exhaustive (based on various approaches) list includes: Fagereng et al. (2018), Arellano et al. (2017) and Fuster et al. (2018).

[3] In this case, MPCs are declining in wealth when markets are incomplete: expansionary shocks are dampened as their size increases, and contractionary shocks are amplified.

[4] The empirical evidence favors state-dependent adjustment. See Berger and Vavra (2014, 2015) in the context of durable spending, and Caballero et al. (1997) in the context of capital adjustment, among others.

enced by durable workers during the Great Recession, the response of durable investment is roughly 10-15% lower than would be predicted without non-linearities. The opposite is true for positive shocks of the same magnitude. Decomposing the responses into extensive and intensive margins, I find that the former dominates quantitatively. In other words, lumpy adjustment entirely accounts for these aggregate non-linearities. Despite producing the correct form of amplification, the model generates non-linearities that are substantially lower than those observed in the data. I suggest various avenues to improve the quantitative performance of the model.

Related literature. My paper contributes to several strands of the literature on lumpy investment and the determinants of MPCs in heterogeneous agents economies.

First, my paper fits into an extensive literature on lumpy durable investment.[5] Durable adjustment is infrequent and discontinuous at the microeconomic level. In the closely related context of capital adjustment and price setting, these discontinuities tend to produce non-linearities at the macroeconomic level. In particular, Caballero et al. (1997) and Caballero and Engel (1999) find that firms' capital investment responds proportionately more to large shocks than smaller ones when adjustment is lumpy.[6] Building on this insight, I explore the aggregate non-linearities produced by a canonical model of lumpy durable investment with uninsured idiosyncratic risk and borrowing constraints. I find that expansionary income shocks are amplified as their size increases, while contractionary shocks are dampened. I trace this asymmetry back to the shape of the distribution of liquid assets around the durable adjustment thresholds, and the monotonicity of durable investment conditional on adjustment. I then explore the implications of this non-linearity in the presence of redistribution of labor income.

My quantitative model of durable demand builds directly on the one developed by Berger and Vavra (2015). My focus is complementary to theirs. Berger and Vavra (2015) find that their model of lumpy adjustment produces a response of durable investment to aggregate shocks that is pro-cyclical.[7] I show that this pro-cyclicality is consistent with the non-linearity that I document: an incremental income shock following a period of expansion is akin to a discrete aggregate shock in my model. I argue that the amplification is controlled in both cases by the same structural objects.

5 Seminal contributions on the subject include Arrow (1968), Nickell (1974), Pindyck (1988), Bar-Ilan and Blinder (1987) and Grossman and Laroque (1990). Again, see Bertola and Caballero (1990) for a review.

6 On price setting, Alvarez et al. (2017b) show that the degree of monetary non-neutrality in menu costs models is non-linear in the size of monetary shocks, and Alvarez et al. (2017a) find empirical support for this property.

7 See Bachmann et al. (2013) and Winberry (2019) in the context of capital investment, and Dotsey et al. (1999) and Burstein (2006) in the context of price setting.

Second, my paper is related to a growing literature on the determinants of MPCs. Heterogeneous agents, rational expectations models with a single liquid asset predict counterfactually low marginal propensities to spend (Heathcote et al. (2009) for a review). Recent contributions on the subject have emphasized the role of transaction costs. Kaplan and Violante (2014) introduce an illiquid, financial asset with non-convex adjustment costs to obtain a more realistic distribution of MPCs. Carroll et al. (2014), Kaplan et al. (2018) and Luetticke (2018) suppose convex adjustment costs instead. Auclert et al. (2018) highlight the role of transaction costs in matching the dynamic response of spending to unanticipated income shocks. These papers restrict their attention to non-durable spending. On the contrary, I am interested in durable spending, and I study the non-linear properties of its response to income shocks.[8]

Layout. I start by discussing the existing evidence on increasing MPCs on durables in Section 1.2. I then introduce in Section 1.3 the canonical model of durable demand with incomplete markets that I will use throughout this paper. I discuss the sources of non-linearity inherent to this model in Section 1.4. Section 1.5 describes the calibration of the model and the empirical targets. In Section 1.6, I quantify the degree of non-linearity in my calibrated model. Section 1.7 concludes. The appendix contains the proofs, and complementary quantitative results.

1.2 Evidence on Increasing MPCs on Durables

In this section, I briefly review supporting evidence on the response of durable spending to income shocks. This evidence is based on two main sources: actual responses to tax rebates and credits; and hypothetical responses based on consumer surveys.[9]

1.2.1 Earned Income Tax Rebates Credits

An extensive literature exploits springtime tax rebates or emergency tax credits to estimate MPCs. In the context of the 2001 EITC, Johnson et al. (2006) estimate negligible spending multipliers on durable goods. On the contrary, Souleles (1999) finds that the response of spending to springtime tax refunds is almost entirely driven by durable expenditure (and

[8] Kaplan and Violante (2014) also investigate how MPCs depend on the size of income shocks in their model of non-durable spending. They find that the average MPC on non-durables decreases with the size of income shocks, consistently with the empirical evidence.

[9] Another strand of the literature adopts structural approaches to identify MPCs (Blundell et al. (2008); Arellano et al. (2017)). These contributions focus on non-durable spending, and are mostly silent on non-linearities in the spending response to income shocks.

purchases of vehicles in particular), while Parker et al. (2013) document sizeable multipliers following the 2008 stimulus payment.[10] Parker et al. (2013) attribute the difference in MPCs across these episodes to the size of the corresponding transfers.[11] Table 1.2.1 lists the average transfer size and the average MPC on durable goods for these episodes. The average MPC is insignificant for a \$500 average transfer, but accounts for roughly half of the spending response for the range \$1,000 to \$2,500. On the contrary, the average MPC on non-durable decreases with the size of income shocks.

Table 1.2.1: Average Marginal Propensities to Spend (1 quarter)

	Johnson et al. (2006)	Parker et al. (2013)	Souleles (1999)
Type of transfer	Stimulus ('01)	Stimulus ('08)	Rebates (spring)
Average amount	\$480	\$1,000	\$2,500
$\overline{\text{MPC}}$ on durables	$\sim$ 0%	$\sim$ 50%	$\sim$ 55%
$\overline{\text{MPC}}$ on non-durables	$\sim$ 30%	$\sim$ 21%	$\sim$ 3%

1.2.2 Survey Evidence

Another strand of the literature is interested in consumers' reported MPCs based on surveys. An advantage of this approach is that it allows to elicit the *same* consumer's response across different (hypothetical) experiments. On the other hand, reported MPCs are potentially subject to various biases (Hainmueller et al. (2015); Karlan et al. (2016)).[12]

[10] Parker et al. (2013) find that their estimates of MPCs on durable goods are less precise than for non-durable goods. The corresponding figure in Table 1.2.1 corresponds to the average between the lower and upper bounds that they obtained (p. 2531).

[11] In particular (p. 2532): "For instance, some prior research finds that larger payments can skew the composition of spending towards durables, which is consistent with our findings given that the 2008 stimulus payments were on average about twice the size of the 2001 rebates."

[12] Parker and Souleles (2019) compare the effective MPCs estimated using the 2008 EITC (following Parker et al. (2013)) with those reported by households for the same episode (following Shapiro and Slemrod (2003)). They find that the two approaches deliver similar average MPCs. However, the distribution of MPCs obtained using actual consumption responses has a fater right tail compared to the ones reported in surveys. The average MPCs obtained by Fuster et al. (2018) in their survey are at the lower end of estimates in the literature.

Table 1.2.2: Average Marginal Propensities to Spend (1 quarter) – Fuster et al. (2018)

Amount	\$500	\$2,500	\$5,000
$\overline{\text{MPC}}$ on durables	1.9%	3.6%	5.0%
$\overline{\text{MPC}}$ non non-durables	5.7%	6.9%	8.4%

Fuster et al. (2018) use evidence from the Survey of Consumer Expectation (SCE) to study how participants would respond to one-time, unanticipated tax rebates of varying magnitudes, and how they would allocate this extra spending. Table 1.2.2 summarizes their findings. The average MPC increases with the size of the rebate, both for durable and non-durables. This effect is much more pronounced for durables: the average MPC on durable goods almost doubles when comparing a \$500 rebate and a \$2,500 rebate. Fuster et al. (2018) find that this increasing average MPC can be attributed to adjustment at *extensive* margin. The share of respondents who say they would spend more increases with the size of the income shock.

In a similar experiment involving Dutch households, Christelis et al. (2019) also document that the average MPC on durables increases with the size of income shocks.[13]

1.2.3 Taking Stock

Empirically, the average MPC on durables *increases* with the size of tax rebates. That is, the response of durable spending is *convex* in income changes. In Section 1.4.2, I show that a model of frictionless or time-dependent durable adjustment predicts that the average MPC *decreases* with the size of income changes. In this case, spending is *concave* in income changes due to a standard precautionary savings (Carroll and Kimball (1996)). To rationalize the empirical evidence, I explore whether a canonical model of lumpy durable investment with non-convex adjustment costs can produce an increasing average MPC on durables.

[13] Fuster et al. (2018) and Christelis et al. (2019) also document an asymmetry between the response of *total* spending to positive and negative income shocks. However, the *relative* response of durables is symmetric. Shea (1995) finds a similar asymmetry when estimating MPCs based on data from the Panel Study of Income Dynamics (PSID), while Jappelli and Pistaferri (2000) do not based on data from the Survey of Household Income and Wealth (SHIW). My analysis does not address this asymmetry in the response of total spending.

1.3 A Canonical Model of Lumpy Investment

I now introduce a canonical model of lumpy durable investment with incomplete markets. This model borrows from Berger and Vavra (2015), but recognizes that durables and non-durables are two different goods. That is, the price of durables and non-durables are allowed to differ. For concision, I only include here the main expressions. Appendix 1.A provides a full description of the full model.

1.3.1 Environment

Time is discrete, and there is no aggregate uncertainty.[14] Periods are indexed by $t \in \{0, 1, \ldots\}$. The two goods are indexed by $h \in \mathcal{H} \equiv \{c, d\}$, which denotes non-durables and durables, respectively.

Households. The economy is inhabited by a continuum of mass 1 of households. Households are characterized by three idiosyncratic states: their financial asset holdings (a), their holdings of durable goods (d), and their idiosyncratic labor supply shock (ζ).

Households consume durable and non-durable goods. Preferences are represented by

$$\mathbb{E}\left[\sum_{t\geq 0} \beta^t \frac{u\left(c_t, d_t\right)^{1-\sigma}}{1-\sigma}\right]$$

with discount factor $\beta \in (0, 1)$, and inverse elasticity of substitution $\sigma > 0$. Intratemporal preferences exhibit constant elasticity of substitutiton:

$$u\left(c, d\right) = \left[\vartheta^{\frac{1}{\nu}} c^{\frac{\nu-1}{\nu}} + (1-\vartheta)^{\frac{1}{\nu}} d^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$$

with share parameter $\vartheta \in (0, 1)$ and elasticity of substitution $\nu > 0$.

After observing their idiosyncratic labor income, households decide whether to adjust their stock of durable goods. Adjustment entails a non-convex cost Γ. Following Berger and Vavra (2015) and Kaplan et al. (2017), I assume that durable adjustment costs are proportional to the nominal value of the undepreciated stock of durable goods. If households do not adjust, they pay a maintenance cost, i.e. an investment required to repair or operate the existing stock of durables.[15] This maintenance corresponds to a share $\iota \in [0, 1]$ of the

[14] I focus on the effect of one-time, unanticipated but potentially persistent shocks.

[15] Maintenance is a standard feature of lumpy adjustment models (Bachmann et al. (2013); Berger and Vavra (2015)). It decreases the effective depreciation rate in the case of no adjustment. Fixing the depreciation rate, adjustment costs and idiosyncratic risk, a higher maintenance decreases the average frequency of

current depreciation of their stock of capital. Summing up, the non-convex adjustment costs are

$$\Gamma\left(d',d\right)=\begin{cases}(1-\delta)\,P^{d}\gamma d & \text{if } d'\neq(1-(1-\iota)\,\delta)\,d\\ 0 & \text{otherwise}\end{cases} \tag{1.3.1}$$

for some adjustment cost $\gamma \geq 0$, where d' denotes the new stock of durables and $\delta \in (0,1)$ denotes the depreciation rate. Here, $\mathbf{P} \equiv \left\{P^h\right\}_h$ denotes goods prices. I suppose that these adjustment costs take the form of services (real estate, moving, etc.) provided by the non-durable sector, while maintenance takes the form of investment goods (new windows, tires, etc.) purchased from the durable sector[16].

Households supply labor inelastically in their industry of employment. They are compensated in proportion to their idiosyncratic labor supply. The fiscal authority can potentially provide lump sum transfers to the households. Profits are redistributed symmetrically across households. Summing up, total individual (gross) incomes are

$$e_t\left(\zeta\right)=\zeta\mathcal{Y}+T_t+\pi, \tag{1.3.2}$$

where $\mathcal{Y}$ denotes the aggregate wage bill, T_t denotes tax rebates and π denotes an exogeneous source of income, e.g. aggregate profits claimed from firms. The idiosyncratic process for labor supply follows a Markov chain with transition kernel Σ on some set $S \subset \mathbb{R}_{++}$, with $\mathbb{E}\left[\zeta\right]=1$ and independence across households.

1.3.2 Households' Optimization

The households' problem can be formulated recursively. The durable adjustment choice solves:

$$\mathcal{V}_t\left(a,d,\zeta\right)=\max_{\mathcal{A}\in\{0,1\}}\left\{V_t\left(a,d,\zeta;\mathcal{A}\right)\right\}, \tag{1.3.3}$$

where $\mathcal{A} \in \{0,1\}$ denotes the adjustment decision and $V_t\left(\cdot;\mathcal{A}\right)$ denotes the continuation values associated to each adjustment option.

The value associated with no adjustment is

adjustment.

[16] This assumption is mostly innocuous. Spending on adjustment costs and maintenance are relatively inelastic to income changes in my calibrated model, compared to purchases of durable and non-durable goods.

$$V_t\left(a,d,\zeta;0\right)=\max_{\{c,a'\}}\frac{u\left(c,d^{\star}\right)^{1-\sigma}}{1-\sigma}+\beta\mathbb{E}_t\left[\mathcal{V}_{t+1}\left(a',d^{\star},\zeta'\right)\middle|\,\zeta\right] \tag{1.3.4}$$
$$\text{s.t. } P^c c+P^d\iota\frac{\delta}{1-(1-\iota)\,\delta}d^{\star}+a'\leq(1-\tau)\,e_t\left(\zeta\right)+(1+r)\,a$$
$$a'\geq 0,$$

with $d^{\star}\equiv(1-(1-\iota)\,\delta)\,d$. Here, $e_t(\zeta)$ denotes labor income, and r denotes the nominal interest rate, and τ denotes the tax rate on total income.

Similarly, the value associated with adjustment is

$$V_t\left(a,d,\zeta;1\right)=\max_{\{c,a',d'\}}\frac{u\left(c,d'\right)^{1-\sigma}}{1-\sigma}+\beta\mathbb{E}_t\left[\mathcal{V}_{t+1}\left(a',d',\zeta'\right)\middle|\,\zeta\right] \tag{1.3.5}$$
$$\text{s.t. } P^c_t c+P^d_t\left(d'-(1-\delta)\,d\right)+\Gamma\left(d',d\right)+a'$$
$$\leq(1-\tau)\,e_t\left(\zeta\right)+(1+r)\,a$$
$$a'\geq 0$$

1.4 Non-Linear Response to Income Shocks

I introduced a canonical model of lumpy durable demand with incomplete markets in the previous section. Durable adjustment is infrequent and discontinuous in this model. I am interested in the aggregate implications of these microeconomic discontinuities. In Section 1.4.1, I explore how lumpy adjustment at the micro level shapes non-linearities at the macro level. In particular, I identify the conditions under which this model replicates the empirical evidence, by producing an average MPC on durables that increases with the size of income changes. I discuss various related properties of this model in Section 1.4.2.

1.4.1 Lumpy Investment and Aggregate Non-Linearity

I assume that the economy is initially at its stationary equilibrium.[17] I consider an exogeneous one-time, unanticipated change in aggregate income in period $t=0$:

$$\mathcal{Y}_0=(1+\Delta)\,\mathcal{Y}$$

[17] State-contingency is an intrinsic property of models of lumpy investment (Bachmann et al. (2013); Berger and Vavra (2015); Winberry (2019)). Responses to aggregate shocks are typically larger following an expansion, compared to a contraction. See Section 1.6.4 for a discussion of the connection between state-contingency and non-linearity, and the interaction between redistribution and state-contingency.

for some $\Delta \in \mathbb{R}$, where $\mathcal{Y}$ denotes the aggregate wage bill at the stationary equilibrium. My focus is on the non-linearity of aggregate investment with respect to the size and sign of this income shock.

Aggregate durable investment in the first period as a function of the aggregate income shock is[18]

$$I(\Delta) \equiv \int \underbrace{\mathcal{A}(a,d,\zeta)}_{\text{Hazard}} \cdot \underbrace{(d^\star(a,d,\zeta) - (1-\delta)\,d)}_{\text{Gap to target}} \underbrace{d\Lambda(a - \zeta\Delta\mathcal{Y}, d, \zeta)}_{\text{Density}}$$
$$+ \int [1 - \mathcal{A}(a,d,\zeta)] \underbrace{\iota\delta d}_{\text{Maintenance}} \; d\Lambda(a - \zeta\Delta\mathcal{Y}, d, \zeta), \tag{1.4.1}$$

where $\mathcal{A}(\cdot)$ denotes the durable adjustment hazard and $d^\star(\cdot)$ denotes the adjustment target that solves (1.3.5). The first integral in (1.4.1) captures investment by households who pay the fixed adjustment cost, while the second integral captures maintenance by those who do not.

The model described in Section 1.3 generates a standard inaction region for durable adjustment. That is, the adjustment hazard takes the form of a step function. In my calibrated model, upward adjustment is the most relevant margin at the stationary distribution.[19] I focus on this case for illustration. The adjustment hazard satisfies

$$\mathcal{A}(a,d,\zeta) = \begin{cases} 1 & \text{if } a \geq \bar{a}(d,\zeta) \\ 0 & \text{otherwise} \end{cases} \tag{1.4.2}$$

for some threshold $\bar{a}(\cdot)$ that depends on durable holdings and idiosyncratic labor supply.

I am interested in the non-linear properties of the impulse response of aggregate durable investment to an income shock:[20]

$$\hat{I}(\Delta) \equiv \frac{I(\Delta)}{\mathcal{I}} - 1, \tag{1.4.3}$$

where $\mathcal{I}$ denotes aggregate investment at the stationary equilibrium.

Fixing the adjustment hazard, the impulse response given by (1.4.1) and (1.4.3) is de-

[18] The integration is over the idiosyncratic states (a, d, ζ).

[19] Downward changes account for roughly 1% of adjustments at the stationary equilibrium. They could potentially play a more important role in the presence of deleveraging shocks (Guerrieri and Lorenzoni (2017)).

[20] In the context of price setting, Caballero and Engel (2007) show that the aggregate response in the Caplin and Spulber (1987) model coincides with the average response across time at the firm-level, when the economy is stationary. In other words, my exercise captures the "average" degree of non-linearity over time in the household-level response of durable investment.

termined by adjustment along two margins: the *extensive* margin, i.e. the households' marginal propensity to adjust; and the *intensive* margin, i.e. their marginal propensity to invest conditional on adjustment. The following result decomposes the response of durable investment into these two margins. For the sake of exposition, I assume that the adjustment target $d^\star(\cdot, d, \zeta)$ is smooth and increasing, and that the distribution of liquid assets conditional on durable holdings and labor supply admits a smooth density $d\Lambda(a|d,\zeta)$.[21] I define $\Lambda^\star \equiv \text{marg}_{d,\zeta}\Lambda$. Finally, I consider discrete but plausibly small shocks.[22]

Proposition 1. *The response of durable investment to a positive income shock $\Delta > 0$ can be decomposed as follows:*

$$\hat{I}(\Delta) \equiv \underbrace{\Sigma_1(\Delta)}_{\text{Extensive margin}} + \underbrace{\Sigma_2(\Delta)}_{\text{Intensive margin}} + \underbrace{\varsigma(\Delta)}_{\text{Residual}} \tag{1.4.4}$$

with

$$\begin{aligned}\Sigma_1(\Delta) \equiv& \frac{1}{\mathcal{I}} \int \Bigg\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta) \\ &+ \kappa(d,\zeta) \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} \left(a + \zeta\Delta\mathcal{Y} - \bar{a}(d,\zeta)\right) d\Lambda(a|d,\zeta) \Bigg\} d\Lambda^\star \\ \Sigma_2(\Delta) \equiv& \frac{1}{\mathcal{I}} \int \mathcal{A}(a,d,\zeta) \left[d^\star(a+\zeta\Delta\mathcal{Y}, d,\zeta) - d^\star(a,d,\zeta)\right] d\Lambda(a,d,\zeta)\end{aligned}$$

for some $\kappa(d,\zeta) > 0$ and some residual $\varsigma(\Delta)$ that satisfies $\lim_{\Delta\to 0} \frac{\varsigma(\Delta)}{\Delta} = 0$. The analogous decomposition for a negative income shock $\Delta < 0$ is provided in Appendix 1.B.

Proof. See Appendix 1.B. □

The extensive margin consists of two terms. The first term captures *discontinuities* at the microeconomic level. Households who pay the fixed cost adjust their stock by a discrete amount.[23] In turn, the mass of households who adjust depends on the shape of the distribution of liquid assets. The second term reflects heterogeneity among households who pay the fixed cost after the income shock: those that were initially further away from

[21] For expositional purposes, I further assume that the conditional distribution $d\Lambda(\cdot|d,\zeta)$ has full support on $[0, a^\star]$ with $a^\star > \bar{A} \equiv \sup_{(d,\zeta)} \bar{a}(d,\zeta)$, so that some households adjust for each (d,ζ).

[22] Specifically, I assume that $\Delta \in \left[-\sup_{(d,\zeta)} \frac{1}{\zeta}\frac{a^\star - \bar{a}(d,\zeta)}{\mathcal{Y}}, \inf_{(d,\zeta)} \frac{1}{\zeta}\frac{\bar{a}(d,\zeta)}{\mathcal{Y}}\right]$ to avoid cases where either no household, or all households adjust. In the following examples, I assume some monotonicity properties around the durable adjustment threshold. These properties might not hold globally. I assume that Δ is sufficient small so that they apply.

[23] See Baumol (1952), Tobin (1956) and Scarf (1960) for seminal contributions the subject.

their adjustment threshold invest relatively less when adjusting their stock of durables. The intensive margin captures the decreasing propensity to invest conditional on adjustment, due to precautionary savings (Carroll and Kimball (1996); Bertola et al. (2005)). The residual plays a negligible role in my numerical simulations.

The following three examples clarify the contribution of each term in (1.4.4) to the impulse response of aggregate durable investment, and its non-linearity. For illustration, I focus on positive income shocks. The opposite case is symmetric. I collect all derivations in Appendix 1.B.4.

Example 1 (Extensive Margin I)**.** I first focus on adjustment at the extensive margin. Specifically, I illustrate the role of the shape of the distribution of liquid assets, i.e. the first term in $\Sigma_1(\Delta)$. I suppose that the adjustment target $d^\star(\cdot)$ is constant at some level $\bar{d} > 0$.[24] In this case, there is no adjustment at the intensive margin, households adjust to a common level, and the residual is zero:

$$\Sigma_2(\Delta) = 0 \quad , \quad \kappa(d,\zeta) = 0 \quad \text{and} \quad \varsigma(\Delta) = 0$$

The impulse response (1.4.4) satisfies

$$\hat{I}(\Delta) = \frac{1}{\mathcal{I}} \int \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta)\, d\Lambda^\star \tag{1.4.5}$$

The response of durable investment depends on the slope of the density of liquid assets around the thresholds. If $d\Lambda(\cdot|d,\zeta)$ is uniform, then $\hat{I}(\cdot)$ is linear in the size of the shock: there is neither amplification, nor dampening. Quantitatively, the relevant case is one where this density is *decreasing* around the thresholds (Section 1.6.1). Then, the impulse response of aggregate durable investment $\hat{I}(\cdot)$ is convex: more and more households exit their inaction region for expansionary shocks; less and less do so for contractionary shocks. In other words, positive income shocks are *amplified* as their size increases, and negative shocks are *dampened*. That is, the impulse response of aggregate durable investment $\hat{I}(\Delta)$ is *convex* in the income shock.

Figure 1.4.1 illustrates this non-linear amplification. The left panel depicts durable investment at the individual level (in blue), and the distribution of liquid assets (in red),[25,26] together with the durable adjustment threshold $\bar{a}(d,\zeta)$. I implicitly fix some level of

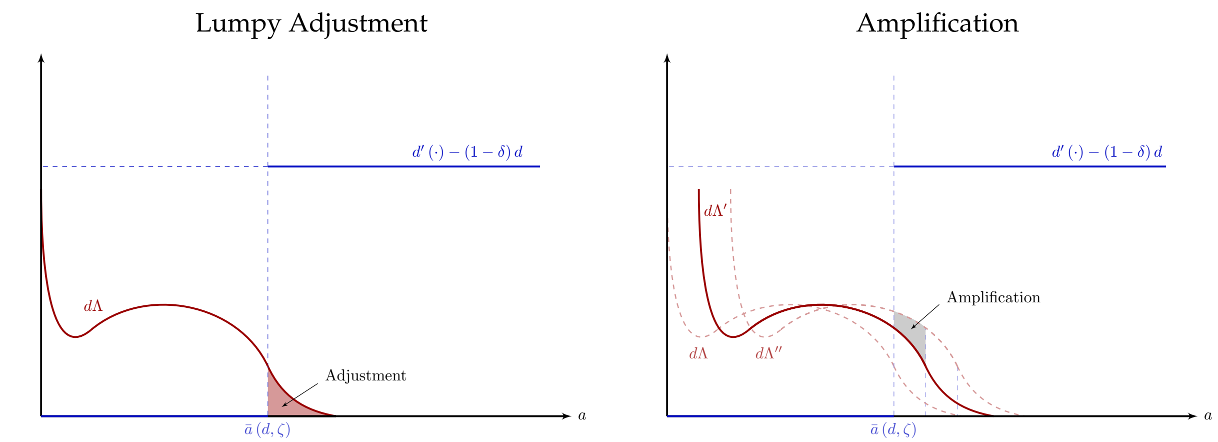
[24] That is, the durable adjustment cost satisfies $\Gamma_t(d',d) = \mathbf{1}_{\{d'=\bar{d}\}}\gamma + (1-\mathbf{1}_{\{d'=\bar{d}\}})M$ with $M \to +\infty$.

[25] The borrowing constraint $a \geq 0$ acts as a reflection barrier. Hence the mass at $a = 0$.

[26] The model is set in discrete time. Therefore, there is a positive mass of households at the stationary equilibrium outside of the inaction region. These households adjust in the current period, which depletes their stock of liquid assets. I discuss the role of discrete time at the end of this section.

durable holdings and labor supply. A positive income shock $\Delta > 0$ shifts the density of cash-on-hand to the right, as shown in the right panel. When the density of liquid assets is decreasing at the adjustment threshold, positive income shocks are amplified: more and more households adjust at the extensive margin. The opposite happens for negative shocks.

Figure 1.4.1: Extensive Margin I



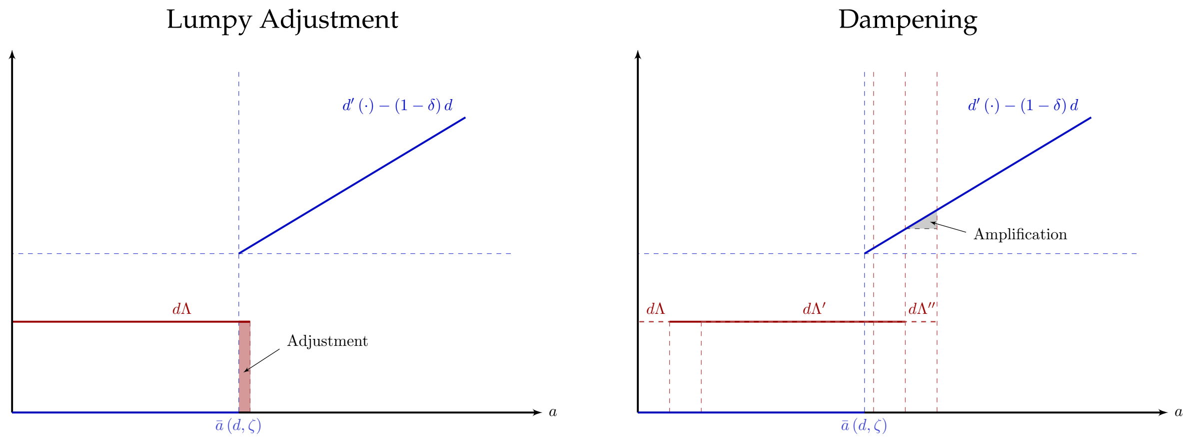
Example 2 (Extensive Margin II)**.** I focus again on adjustment at the extensive margin, but I now highlight the role of the adjustment target. For illustration, I assume that the adjustment target $d^{\star}(\cdot)$ is linear, with $\kappa(d,\zeta)$ denoting the corresponding slope. To abstract from the effect illustrated in the previous example, I suppose that the density $d\Lambda(a|d,\zeta)$ is uniform over $[0,a^{\star}]$ with $\bar{a}(\cdot) < a^{\star} < +\infty$. In this case, adjustment at the intensive margin is linear, and the residual is zero: $\varsigma(\Delta) = 0$. The impulse response (1.4.4) satisfies

$$\hat{I}(\Delta) = \theta\Delta + \left[\frac{1}{2}\frac{1}{\mathcal{I}}\frac{1}{a^{\star}}\int \kappa(d,\zeta)(\zeta\mathcal{Y})^2\, d\Lambda^{\star}\right]\Delta^2 \tag{1.4.6}$$

for some $\theta > 0$. Again, this form of adjustment at the extensive margin contributes to a *convex* impulse response of aggregate durable investment $\hat{I}(\cdot)$.

Figure 1.4.2 illustrates this case. As in the previous example, the left panel depicts durable investment at the individual level (in blue), and the distribution of liquid assets (in red), and the durable adjustment threshold. A positive income shock $\Delta > 0$ shifts the density of cash-on-hand to the right, as shown in the right panel. Positive income shocks are *amplified* as their size increases: households who decide to adjust do so increasing amounts. On the contrary, the effect of negative shocks is *dampened* as their magnitude increases.

Figure 1.4.2: Extensive Margin II



Example 3 (Intensive Margin)**.** Finally, I focus on the intensive margin, i.e. the one that would operate in a frictionless model. I assume that the adjustment target $d^\star(\cdot)$ is concave due to precautionary savings and binding borrowing constraints. By definition, the adjustment hazard and the thresholds satisfy $\mathcal{A}(a,d,\zeta) \equiv 1$ and $\bar{a}(d,\zeta) \equiv 0$ in this case. The residual is zero: $\varsigma(\Delta) = 0$. Then, the impulse response is

$$\hat{I}(\Delta) = \frac{1}{\mathcal{I}} \int \left[d^\star(a + \zeta\Delta\mathcal{Y}, d, \zeta) - d^\star(a,d,\zeta)\right] d\Lambda(a,d,\zeta)$$

The response of durable investment $\hat{I}(\Delta)$ is *concave*, i.e. it inherits the shape of the durable adjustment target. The effect of positive income shocks is *dampened* as their size increases, while the effect of negative shocks is *amplified*.

1.4.2 Discussion

Before exploring the general equilibrium implications of the aggregate non-linearities documented above, a few remarks are in order. First, I re-state the discussion of Section 1.4.1 in terms of measurable statistics with testable implications. Second, I discuss the role state-dependent adjustment in my model. Third, I consider a continuous time version of my model to identify which margins operate when aggregate disturbances have continuous paths, i.e. shocks “build up” smoothly. Finally, I briefly elaborate on the determinants of the slope of the distribution of financial assets around the durable adjustment threshold.

Marginal propensities to spend. In Examples 1 and 2, durable spending reacts proportionetaly more (less) to large positive (negative) shocks. This non-linearity can be formulated in terms of observable statistics. Specifically, let

$$\begin{aligned}\overline{\text{MPC}}^{d}(\Delta) &\equiv \int \frac{\partial}{\partial \Delta}\left[d'_{0}\left(a+\zeta\Delta\mathcal{Y}, d, \zeta\right)-(1-\delta)\, d\right] d\Lambda_{0} \\ &= \frac{d}{d\Delta}\mathcal{I}(\Delta) \end{aligned} \tag{1.4.7}$$

i.e. the average marginal propensity to spend (MPC) on durable goods.[27] In Examples 1 and 2, $\mathcal{I}(\cdot)$ is convex in income changes Δ. As a consequence, the average MPC on durable goods *increases* with income changes. As discussed in Section 1.2, this property is consistent with the evidence on the consumption response to tax rebates.

Time-dependent adjustment. Durable adjustment is lumpy and state-dependent (i.e. the adjustment hazard is endogeneous) in my model.[28] This property plays a central role in the non-linear response of durable spending in my setting. In models with frictionless or time-dependent adjustment (à la Calvo (1983)), or even purely non-durable consumption, only the intensive margin operates. In this case, the average MPC *decreases* with income changes due to precautionary savings, as illustrated in Example 3. I elaborate on this point in Appendix 1.B.2. The relative importance of these two effects, and the effective degree of non-linearity are quantitative questions. In Section 1.6, I use my calibrated model to implement the decomposition from Proposition 1. I find that adjustment at the extensive margin dominates. That is, the average MPC on durables increases with income changes.

Continuous time. My model is set in discrete time, following Berger and Vavra (2015).[29] In Appendix 1.B.3, I present a continuous time variant of my model where aggregate disturbances have continuous paths, i.e. shocks "build up" smoothly. This allows me to clarify the nature of the comparative statics in discrete time, and to identify which of the margins identified in Section 1.4.1 are specific to this comparative statics.

[27] Note that (1.4.7) corresponds to the *average* MPC on durables in each sector, which differs from individual MPCs. Individual MPC are not well-defined at the adjustment threshold (1.4.2).

[28] See Alvarez et al. (2017a) for a formal definition of state- and time-dependent adjustment.

[29] Historically, the literature has favored continuous time models and adopted a more reduced form approach. Durables were the only explicit state variable, and the evolution of their stock absent adjustment was specified exogeneously. See Bertola and Caballero (1990) for a review of this approach. A more recent strand of the literature has modelled *jointly* durables and financial assets accumulaton subject to uninsured idiosyncratic shocks. Most of these models are set in discrete time, including Berger and Vavra (2015) in the context of durables, Wong (2019) and Kaplan et al. (2017) in the context of housing, and Bachmann et al. (2013) and Winberry (2019) in the context of capital investment. Exceptions include Achdou et al. (2017) (extensions) and McKay and Wieland (2019).

Under the assumptions of Appendix 1.B.3, the flow of durable spending satisfies

$$i_t = \underbrace{\left(\bar{s} + \hat{m}\Delta\right)\Omega_0}_{\text{Impact}} - \underbrace{\mu\left(\bar{s} + \hat{m}\Delta\right)^2 \left(\exp\left(\delta t\right) - 1\right)\Omega_1}_{\text{Endogeneous distribution}} \tag{1.4.8}$$

for some $\Omega_0, \Omega_1 > 0$. Here, $\bar{s}, \hat{m} \geq 0$ control the savings rate, Δ denotes the (flow) income shock and μ denotes the slope of the density of the (conditional) stationary distribution of financial assets. Durable spending is given by two terms. The first one captures the first order impact of income changes: the larger the income shock, the larger the flow of households who exit their inaction region on impact $(t = 0)$. This term is linear in the size of the income shock. The second term captures the endogeneous response of the distribution of financial assets: the larger the income shock, the higher the stock of savings as time passes. When the slope of the density μ is negative, the flow of households who exit their inaction region increases over time. Consequently, this second term is non-linear in the size of the income shock, and operates for $t > 0$. This non-linear mechanism is the counterpart of the one discussed in discrte time (Example 1). The only difference is that it operates on impact $(t = 0)$ in discrete time, but takes time to build up in continuous time. Indeed, an income shock in discrete time is akin to an increase in Δ over a discrete interval $[0, t]$ in continuous time.

It should be noted that the continuous and discrete time cases differ in one important respect. The terms Ω_0, Ω_1 (Appendix 1.B.2) do not depend on the adjustment threshold $d^\star(\cdot, d)$ for $a > \bar{d}$ since households never effectively exit their inaction region in continuous time. In Section 1.6.1, I implement numerically the decomposition from Proposition 1. Not surprisingly, I find that the terms involving $\kappa(d, \zeta)$ and $\Sigma_2(\Delta)$, and the residual $\varsigma(\Delta)$ are small quantitatively. Instead, the extensive margin (setting $\kappa(d, \zeta) = 0$) dominates.

Shape of the density $d\Lambda(\cdot, d)$. The analysis of Section 1.4.1 identified a key role for the slope of the density of financial assets $\Lambda(\cdot|d, \zeta)$ around that adjustment threshold $\bar{a}(d, \zeta)$. Due to the importance of this object, I briefly elaborate on the determinants on this shape. For tractability, I build on the continuous time version of my model (Appendix 1.B.3).

Since I focus on the properties of the stationary distribution, I temporarily abstract from aggregate income shocks. The process for idiosyncratic states (a, D)[30] is defined by:

[30] In the following, I denote durable holdings by D instead of d to avoid any confusion with differential operators or integrands.

(i) a law of motion conditional on no adjustment,[31,32]

$$da(t) = s(a(t), D(t), \zeta(t))\, dt \tag{1.4.9}$$

$$dD(t) = -\delta D(t)\, dt \tag{1.4.10}$$

for some savings function $s(\cdot)$; (ii) an increasing adjustment threshold $\bar{a}(D)$ of the form (1.4.2); and (iii) an adjustment target $d^\star(\cdot)$ for durables. Financial assets after adjustment satisfy the following budget constraint

$$a(t) = \bar{a}(D) - P^d [d^\star(D) - D] \tag{1.4.11}$$

For illustration, I assume that the adjustment target $d^\star(\cdot)$ is constant at some level $\bar{d} > 0$, as in Example 1. Idiosyncratic income $\exp(\zeta(t))$ follows some diffusion process.

I am interested in the shape of the stationary distribution implied by the process described above. To gain some insights, I consider an initial distribution of idiosyncratic states Λ that is uniform on $\mathcal{S} \equiv \{(a, d) \,|a \leq \bar{a}(d)\}$, and I track its evolution over time while it converges to the stationary distribution. For illustration, I assume that the savings rate satisfies

$$s(a, D, \zeta) = \bar{s} + \hat{m} \exp(\zeta) \tag{1.4.12}$$

for some $\bar{s}, \hat{m} > 0$, i.e. households have a constant propensity to save out of transitory income shocks. I proceed heuristically in the text to keep the exposition brief. The evolution of this distribution is characterized more formally in Appendix 1.B.3.

The left panel of Figure 1.4.3 corresponds to the phase diagram associated to (1.4.9)-(1.4.10). Consider households who adjust over the interval $[0, t_0]$ with t_0 infinitesimally small. By assumption, these households share a common durable adjustment target $\bar{d} > 0$. They deplete their stock of financial assets (black arrow). After adjustment, households accumulate financial assets. I plot two possible paths associated to different realizations of the process $\{\zeta_t\}_{t \geq 0}$ (red curves). Eventually, households hit again the adjustment threshold $\bar{a}(\cdot)$ (blue line).

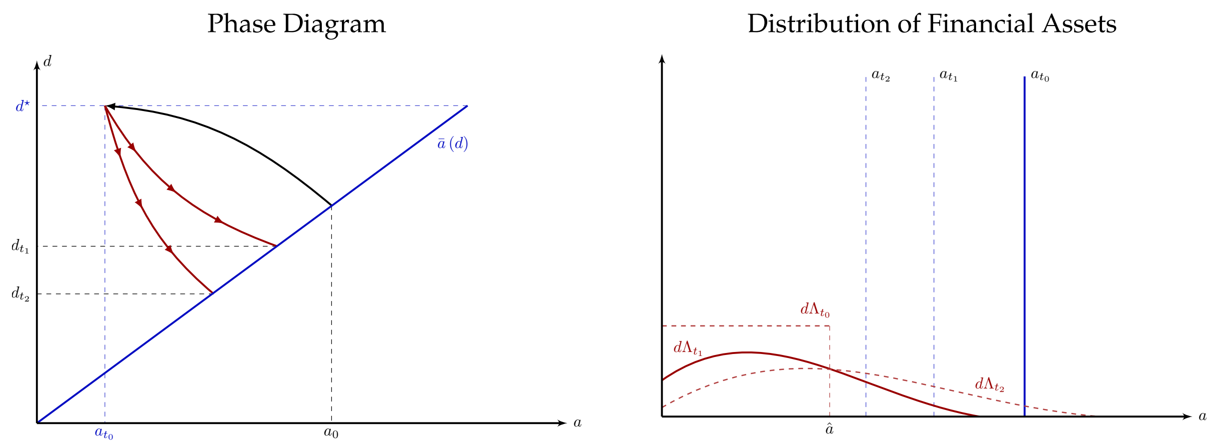
The right panel of Figure 1.4.3 depicts the evolution over time of two objects: the distribution of financial assets conditional on $d_t = \exp(-\delta t)\,\bar{d}$ and having adjusted over $[0, t_0]$; and the adjustment threshold $\bar{a}_t \equiv \bar{a}\left(\exp(-\delta t)\,\bar{d}\right)$. In period $t = t_0$, this distribution is

[31] I abstract from mass points at the borrowing constraint, by assuming below that savings are strictly positive.

[32] For simplicity, I abstract from maintenance ($\iota = 0$), and I assume that income shocks are independent over time.

uniform over $[0, \hat{a}]$ with $\hat{a} \equiv \sup_d \bar{a}(d) - P^d(\bar{d} - d)$, using (1.4.11) and by uniformity of the stationary distribution Λ. I consider two periods t_1 and t_2 with $t_0 < t_1 < t_2$. The mean and the variance of the distribution increase with time, from (1.4.9)–(1.4.10) and (1.4.12). On the contrary, the adjustment threshold decreases over time, since $\bar{a}(\cdot)$ is increasing, by assumption. At the top of the distribution of durables, i.e. for t small, the slope of the density is decreasing at the adjustment threshold, and the distance between the durable adjustment threshold $\bar{a}_t$ and the mode of the conditional distribution is large. As t increases, this distance shrinks.[33,34]

Figure 1.4.3: Density at the Adjustment Threshold



The predictions of my richer quantitative model are in-line with those described above (see Section 1.6.1). In particular, I document that: (i) the density of financial assets is decreasing around the adjustment threshold; but (ii) the distance between the adjustment threshold and the mode of the distribution is smaller at the bottom of the distribution of durable holdings.

1.5 Calibration

The remainder of the paper quantifies the mechanisms documented in Section 1.4. I first parametrize the model using a mix of external and internal calibration. Following Berger and Vavra (2015), I adopt a broad definition of durable goods that includes residential

[33] In theory, this distance could potentially become negative for sufficiently low holdings of durable holdings, i.e. the slope of the density could become positive. In my calibrated model, I find this to be the case only for a negligible share of households.

[34] In particular, the speed at which the threshold moves closer to the mode of the distribution is controlled by the savings rate, the durable depreciation rate, and the slope of the adjustment threshold.

investment and consumer durables. The calibration strategy follows Zorzi (2020a), so I do not discuss it here for concision. Table 1.5.1 describes the resulting parametrization.

1.6 Lumpy Investment and Non-Linearity

In this section, I assess the degree of aggregate non-linearities produced by my structural model. In Section 1.6.1, I implement the decomposition from Proposition 1 in my calibrated model. In Section 1.6.2, I quantify this non-linearity by simulating the partial equilibrium response of durable spending to persistent income shocks. In Section 1.6.3, I compare the resulting magnitudes with their empirical counterparts. Finally, I relate my findings to the literature on state-contingent responses with lumpy adjustment in Section 1.6.4.

1.6.1 Decomposition

I start by examining the two sources of aggregate non-linearities identified in Section 1.4.1: the extensive and intensive margins of durable adjustment. Specifically, I implement the decomposition from Proposition 1 for a one-time, transitory shock Δ.[35]

Figure 1.6.1 plots each of the three objects in (1.4.4) and (1.B.1). The total response (i.e. the sum of these three components) is non-linear: expansionary shocks are amplified as the size of the shock increases, while contractionary shocks are dampened. That is, the average MPC on durables increases with income changes. Adjustment at the extensive margin dominates quantitatively, and entirely accounts for this non-linear effect.[36] In other words, lumpy and state-contingent adjustment is responsible for this aggregate non-linearity, while precautionary savings plays a minor role. Figure 1.A.1 in Appendix 1.A.2 further decomposes the extensive margin of adjustment into each of the two terms in $\Sigma_1(\Delta)$. Consistently with the discussion on continuous time (Section 1.4.2), I find that the term capturing changes in the size of durable purchases conditional on adjustment is quantitatively small. However, it accounts for a non-negligible share of the non-linearity in the response of durable spending for expansionary shocks.

As illustrated by Examples 1 and 2, the extensive margin of adjustment is controlled by two objects: the slope of the density of the distribution of liquid assets around the adjustment thresholds; and the slope of the durable adjustment target. In turn, the intensive margin is shaped by the concavity of the durable investment target. Figure 1.A.2

[35] This transitory shock is annualized with a persistence that delivers a half-life of 6 quarters.

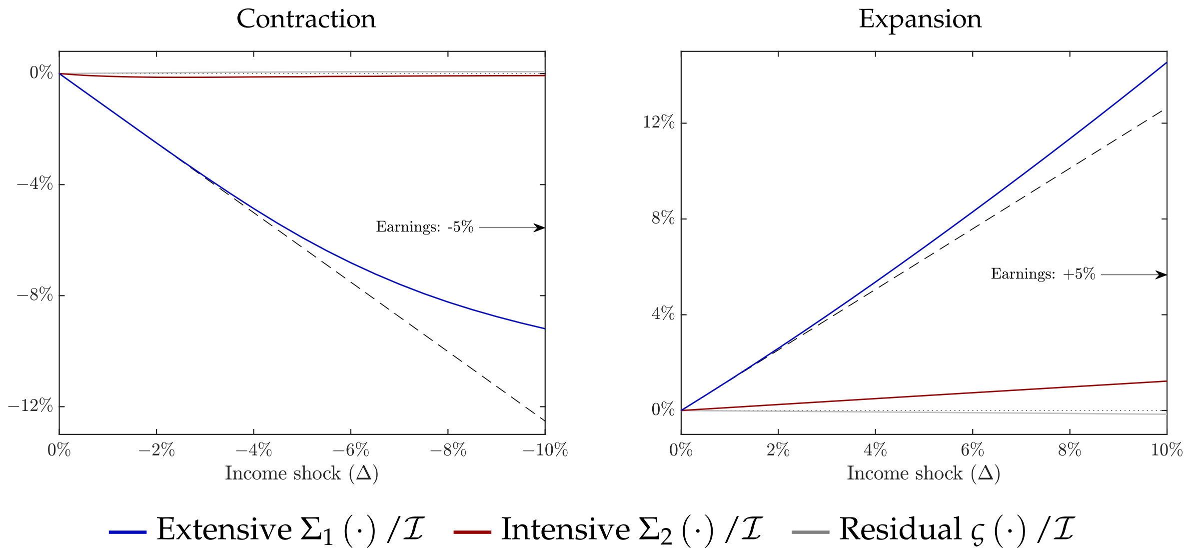
[36] This prediction is in-line with the findings of Fuster et al. (2018).

Table 1.5.1: Calibration

Parameter	Description	Calibration	Source / Target
Preferences			
β	Discount factor	0.985	Internal calibration
ν	Elasticity of substitution	1	See text
ϑ	Non-durable parameter	0.731	Internal calibration
σ	EIS (inverse)	4	See text
ε	Elasticity of substitution (inverse)	10	Kaplan et al. (2018)
Durable goods			
δ	Depreciation rate	0.018	Berger and Vavra (2015)
γ	Adjustment cost	0.025	Internal calibration
ι	Maintenance parameter	0.5	See text
Income process			
$\hat{\rho}$	Persistence	0.967	Floden and Lindé (2001)
$\hat{\sigma}$	Standard deviation	0.13	Floden and Lindé (2001)
Liquidity			
B	Ratio of bond supply to GDP	1.490	Internal calibration
Labor supply			
μ^d	Mass of households (durable)	0.187	CES
Production			
A^d	Relative productivity (durable)	0.490	Internal calibration
α	Decreasing returns	0.3	Berger and Vavra (2015)
Prices and policy			
λ	Calvo parameter	1	See text
φ	Taylor rule coefficient (inflation)	1.25	Kaplan et al. (2018)

(Appendix 1.A.2) plots these objects for various percentiles of the distribution of durable goods (fixing idiosyncratic labor supply at its median level). The density of liquid assets is typically *decreasing* around the adjustment threshold and the durable adjustment target is *increasing* but *concave*. As anticipated in Section 1.4.2, the adjustment threshold gets closer to the mode of the distribution of liquid assets as the stock of durables decreases.

Figure 1.6.1: Decomposition from Proposition 1



1.6.2 Persistent Income Shocks

I now consider the effect of an aggregate, persistent income shock. I abstract from redistribution for now. The wage bills satisfy: $\mathcal{Y}_t^h/\mathcal{Y}^h = \hat{\mathcal{Y}}_t$, with $\hat{\mathcal{Y}}_t = \psi_0 \rho^t$ for some persistence $\rho \in (0,1)$. I am interested in the non-linear properties of the aggregate response of durable spending as I vary ψ_0. I set ρ to obtain a half-life of 6 quarters and replicate the behavior of real filtered GDP since 1960.

Impulse responses. Figures 1.6.2 and 1.6.3 plot the cumulative response of spending over 4 quarters in terms of ψ_0, for expansionary and contractionary shocks.[37,38] The left panels correspond to durable investment, and the right panels to non-durable consumption. The dashed lines extrapolate the response associated to $\psi_0 = -0.01$ and $\psi_0 = 0.01$, respectively.

[37] I choose a horizon of 4 quarters for two reasons. First, the spending response to persistent income changes is typically hump-shaped in my calibration as discussed at the end of this section. A sufficiently long time horizon captures this delayed response. Second, averaging out over a longer horizons smoothes the impulse responses. The cumulative responses over two or more years are very similar.

[38] I annualize these cumulative responses by diving them by the number of quarters over which I cumulate.

I find that the response of durable spending to positive income shocks is amplified as their size increases. On the contrary, this response is dampened for negative income shocks. That is, durable spending is convex in income changes. This effect is economically significant. For instance, consider shocks of the magnitude experienced by durable workers during the Great Recession, i.e. a 10% persistent income change. Fixing this magnitude, the average MPC on durables is roughly 25% higher for expansionary shocks, compared to contractionary shocks. The response of non-durable consumption contrasts sharply with that of durable investment. Expansionary shocks are dampened due to precautionary savings, and contractionary shocks are amplified. In other words, lumpy and state-dependent durable adjustment not only undoes the effect of precautionary savings, but it also predicts that non-linearities actually operate in the opposite direction.

Timing. The analysis of Section 1.4 assumed that income shocks were fully transitory. Households are non-Ricardian in my model due to borrowing constraints, so the timing of income shocks is actually relevant and affects the profile of the impulse response. In particular, the degree of non-linearity needs not be uniform over time. Figures 1.6.2 and 1.6.3 effectively obscure these dynamic considerations by aggregating over a sufficiently long time horizon. Figure 1.A.4 in Appendix 1.A.2 plots the dynamic impulse responses of durable spending to (positive) income shocks of various sizes. These impulse responses are normalized by the size of these shocks. A clear pattern emerges: as the size of income shocks gets larger, the normalized impulse response increases on impact, and becomes more persistent. However, it typically decreases for a few quarters in the medium term, as households accumulate savings to finance larger purchases in the following periods. The cumulative response of durable spending increases with the size of income shocks.[39] That is, the average MPC on durables *increases* on impact and in the longer-term with the size of income changes, but *decreases* in the medium-term. Put it differently, the impulse response of durable spending (in level) is *convex* on impact and in the longer run, but *concave* for a few quarters after the shock.

[39] Note that the cumulative response of spending would necessarily be linear (for sufficiently long horizons) in the size of income shocks if households consumed a single good. Indeed, the increase in income is eventually spent. This need not be the case with multiple goods, as in my setting.

Figure 1.6.2: Impulse Response (4 quarters) – Contraction

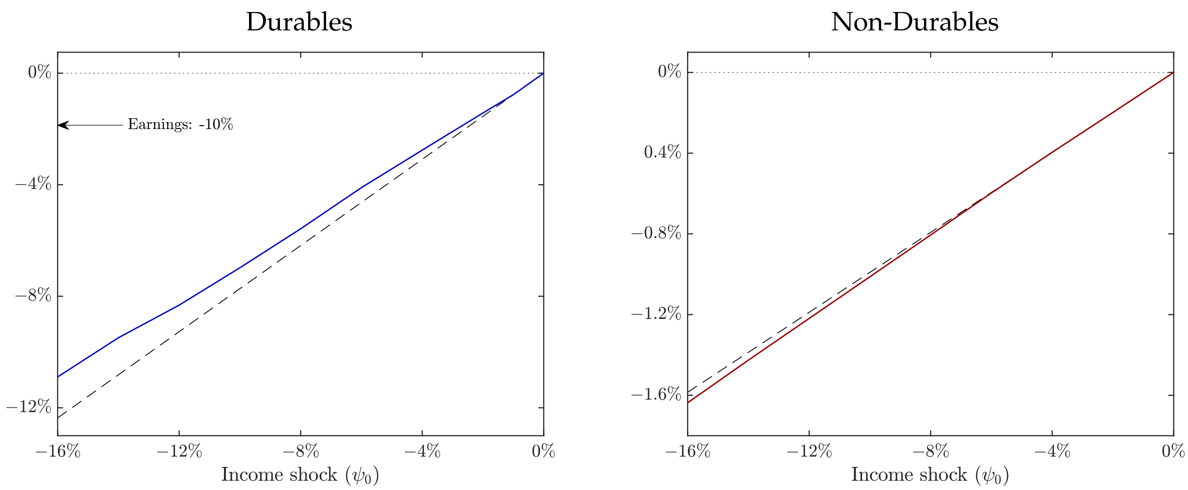


Figure 1.6.3: Impulse Response (4 quarters) – Expansion

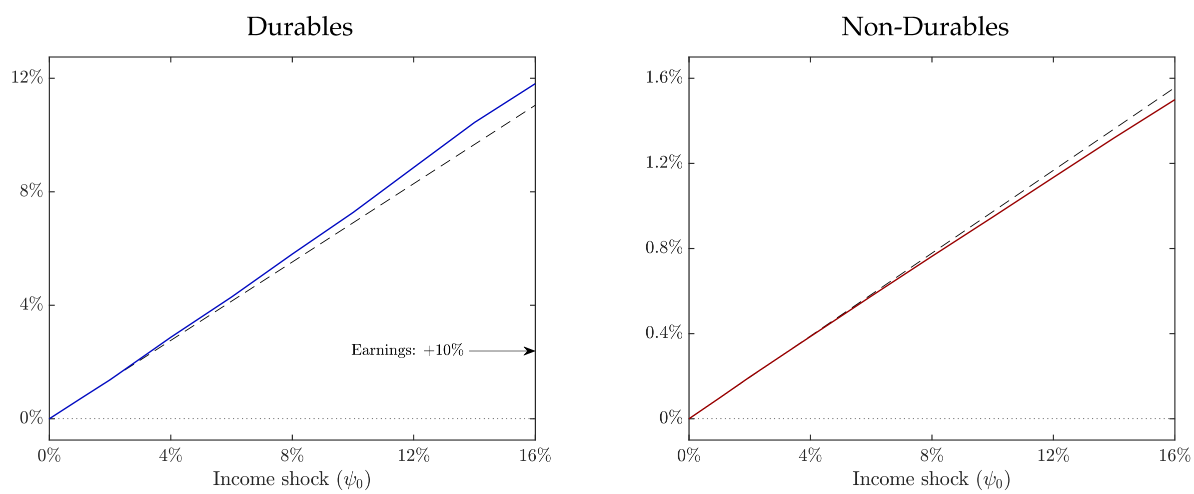
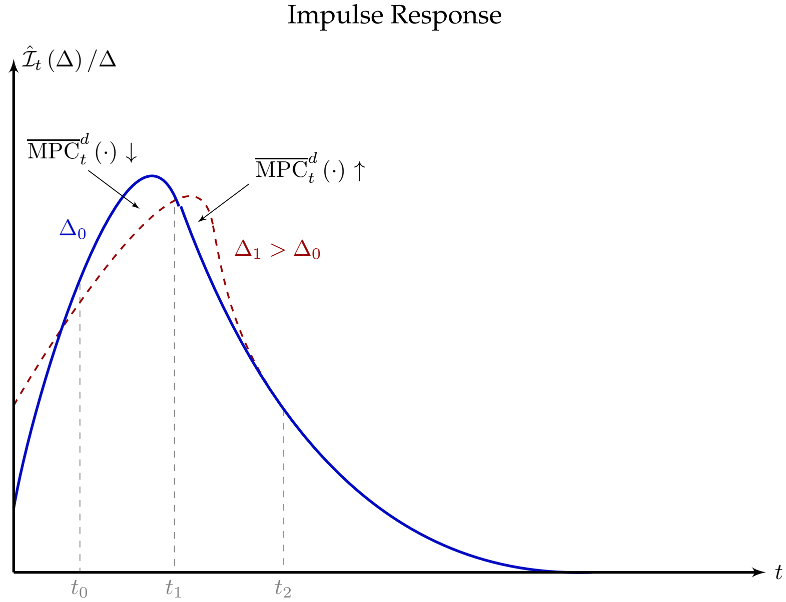


Figure 1.6.4: Persistent Income Shocks – Timing



For reference, Figure 1.6.4 schematizes this pattern and depicts the impulse response of durable investment $\hat{I}(\Delta)$ normalized by the size of the income change Δ for two possible values of this shock. The timing of the impulse response depends on the magnitude of the shock.

1.6.3 Magnitudes

In the previous sections, I confirmed that a canonical model of lumpy durable demand with incomplete markets produces the correct form of non-linearity: the average MPC on durables increases with the size of income changes. I now assess the importance of this non-linearity by comparing it to the empirical evidence. I then suggest avenues to improve the quantitative properties of the model.

Magnitudes. To assess the performance of the model, I compare the degree of non-linerity it produces with the one estimated by Fuster et al. (2018). Specifically, I conduct the same experiment as theirs using my model, by computing the average MPC on durables out of unanticipated, transitory income changes of the various sizes.[40] Table 1.6.1 reports these average MPCs. For comparison, I normalize the average MPC out of a $500 tax rebate to 1 in the data and in my model.[41] Not surprisingly, the model produces the correct form of

[40] Following Kaplan et al. (2018), I assume that the average quarterly labor income is $16,500.

[41] The average MPC on durables out of a $500 rebate is 19.4% in my model, while Fuster et al. (2018) obtain

non-linearity. However, the magnitudes are substantially lower than those observed in the data. Comparing a $5,000 rebate to a $500 rebate, the average MPC on durables increases roughly by 150% in the data, but only by 30% in the model. That is, this canonical model predicts non-linearities that are substantially smaller than those observed in the data.

Table 1.6.1: Average Marginal Propensities to Spend on Durables (1 quarter)

Amount	$500	$2,500	$5,000
Fuster et al. (2018)	100%	189%	263%
Model	100%	117%	131%

Taking stock. In ongoing work (Zorzi (2020b)), I develop a richer model of lumpy durable durable to improve its quantitative performance and better match the degree of non-linearity observed in the data. This richer model differs from the canonical model used in this paper in three respects.

First, I allow households to invest in multiple durable goods (consumer durables, vehicles and housing). This generalization is not only more realistic empirically. It also addresses a key shortcoming of the canonical model of lumpy durable demand that I use in this paper. By pooling all durable goods, this model requires that households pay a fixed cost (1.3.1) that is proportional to the nominal value of their *entire* stock of durables whenever they adjust this stock. The resulting fixed cost might be prohibitively high for the purchase of small- or medium-size durables. The evidence of Parker et al. (2013) suggests that vehicle purchases (medium-sized goods) are responsible for most of the non-linearity in durable demand. An excessively high fixed cost could artificially dampen the degree of non-linearity in the model. Figure 1.A.2 in Appendix 1.A.2 illustrates this point. When facing a high fixed cost of adjustment, households wait until they have accumulated large savings in financial assets before they adjust their stock of durables. Under a realistic calibration for the income process, the right tail of the distribution of financial assets flattens progressively. This effectively mutes the extensive margin of adjustment, which is the main source of non-linearity in my model (Sections 1.4.1 and 1.6.1).

Second, I introduce convex adjustment costs for financial assets. This addition serves two purposes. It allows to better match the average MPC on non-durables (Kaplan et al. (2018)). And it strenghtens the degree of non-linearity in the response of durable spending, by lowering the marginal benefit of savings for larger income shocks.

a response of 3.9%. Again, their estimates lie at the lower end of those obtained in the literature.

Finally, I suppose that households face observation costs (Bonomo et al. (2010); Alvarez et al. (2011, 2016)) on top of non-convex adjustment costs for durables. This feature effectively introduces a degree of time-dependency (Section 1.4.2) in the adjustment of durables. In itself, this addition reduces the degree of non-linearity produced by the model (Alvarez et al. (2018)). However, it helps address two another pathological properties of the canonical model I am using: the response of durable spending to income shocks is not sufficiently persistent; and the elasticity of durable spending to changes in the user cost is excessively high (House (2014); Winberry (2019); McKay and Wieland (2019)).

1.6.4 State-Contingency

I conclude this section by drawing a connection between the non-linearity that I focus on, and another property of models of lumpy adjustment: state-contingency.

Using a version of the income fluctuations problem (1.3.3)–(1.3.5) with aggregate risk, Berger and Vavra (2015) find that the response of durable investment to (income) changes is *pro-cylical*, i.e. the effect of shocks is amplified during expansions compared to contractions. For illustration, I compute the response of aggregate investment to a one-time, transitory income shock following a period of expansion or contraction. The initial boom or bust results from a persistent, unanticipated income change. Specifically, aggregate incomes satisfy $\mathcal{Y}_t/\mathcal{Y} = \hat{\mathcal{Y}}_t$, with $\hat{\mathcal{Y}}_t = \psi_0\rho^t$. The persistence ρ calibrated as in Section 1.6.2, and I vary the initial income shock ψ_0. A transitory, unanticipated income shock takes place after 6 quarters. It takes the form of an exogeneous \$1,000 transfer, i.e. the average 2008 stimulus payment. Table 1.6.2 reports the average (cumulative) MPC on durable goods over a year, for various magnitudes of the initial expansion or recession (ψ_0).[42] The cumulative response of aggregate durable spending is *larger* after a boom than a bust. The effect is relatively small in this example however, since the income shock is fully transitory, i.e. the present discounted value of income changes is small.

Table 1.6.2: Average Marginal Propensity to Spend on Durables (4 quarters)

Initial state (ψ_0)	-4%	-2%	0%	2%	4%
Average MPC (\$1,000)	0.2192	0.2225	0.2265	0.2292	0.2319

I now argue that this state-contingent amplification can be understood as a manifestation of the form of non-linearity that I am interested in. For expositional purposes, I

[42] It should be noted that average MPCs on durables is substantially smaller in my model (Table 1.6.2), than in the data (Table 1.2.1).

suppose there is a single source of aggregate disturbance $\{\xi_t\}_t$, which follows a Markov process of order 1. I am interested in the impulse response of aggregate durable investment, conditional on the state of the economy. Specifically, this response is parametrized by two aggregate states: the exogeneous disturbance (ξ_t) and the distribution of idiosyncratic states $(\mathbf{\Lambda}_t)$. The impulse response to an innovation z in ξ_t in period t is

$$\mathcal{R}\left(z;\xi_t,\mathbf{\Lambda}_t\right)=\mathcal{I}\left(\xi_t+z,\mathbf{\Lambda}_t\right)-\mathcal{I}\left(\xi_t,\mathbf{\Lambda}_t\right), \tag{1.6.1}$$

where $\mathcal{I}(\cdot)$ denotes aggregate durable investment (1.4.1). In the context of Section 1.6.2, $\{\xi_t\}_t$ corresponds to a persistent sequence of transfers. In this case, the impulse response $\mathcal{R}(\cdot)$ corresponds to the average MPC on durables following a transitory income shock.

There are three possible sources of state-dependency: the aggregate disturbance ξ_t itself; the (marginal) distribution of liquid assets $\text{marg}_a\mathbf{\Lambda}_t$; or the (marginal) distribution of durable holdings $\text{marg}_d\mathbf{\Lambda}_t$.[43] The distribution of durable holdings is unlikely to be responsible for the pro-cyclicality of the response of durable investment: durable holdings are already high at the peak of a boom, which should actually mitigate the response of durable expenditure. I thus focus on the two other states in the following.

By definition, dependence on the aggregate disturbance ξ_t corresponds to the non-linearity documented in Figures 1.6.2 and 1.6.3. Holding the distribution of idiosyncratic states at its stationary level $\mathbf{\Lambda}$ and fixing some innovation $z > 0$,

$$\mathcal{R}\left(z;\xi_t',\mathbf{\Lambda}\right)>\mathcal{R}\left(z;\xi_t,\mathbf{\Lambda}\right)\quad\forall\xi_t'>\xi_t\quad\Longleftrightarrow\quad\mathcal{I}\left(\cdot,\mathbf{\Lambda}\right)\text{ is strictly convex}$$

using (1.6.1).

Similarly, dependence on the (marginal) distribution of financial assets is intrinsically related to the form of non-linearity I am interested in. I illustrate this point using the continuous time version of my model presented in Appendix 1.B.3. Specifically, I am interested in the response to an (unanticipated) income shock Δ over an interval $[t,t']$ with $t' > t > 0$ after a sequence of (unanticipated) shocks $\Delta^\star$ over the interval $[0,t)$. These initial shocks shift the distribution of financial assets in period t. Under the assumptions of Appendix 1.B.3, the flow of durable investment in period t satisfies

$$i_t\left(\Delta\right)=\left(\bar{s}+\hat{m}\Delta\right)\Omega_0-\mu\left(\bar{s}+\hat{m}\Delta\right)\left(\bar{s}+\hat{m}\Delta^\star\right)\left(\exp\left(\delta t\right)-1\right)\Omega_1 \tag{1.6.2}$$

for some $\Omega_0,\Omega_1 > 0$. Here, μ denotes the slope of the density of financial assets at the

[43] Obviously, the *joint* distribution $\mathbf{\Lambda}_t$ is the relevant state variable. I focus on the marginal distributions for the sake of the argument.

adjustment threshold (stationary equilibrium), and $\bar{s}, \hat{m} \geq 0$ control the savings rate. In particular, durable spending (1.6.2) is linear in Δ, but contingent on the size of the initial income change and the duration over which it occurs $(\Delta^{\star}, t)$. Fixing some horizon t, the impulse response increases after a period of expansion $(\Delta^{\star} > 0)$, but decreases after a period of recession $(\Delta^{\star} < 0)$ whenever the slope $\mu^{\star}$ is negative (Section 1.6.1).

Note that (1.6.2) coincides with the dynamic impulse response to an income shock (1.4.8) when parametrized with $\Delta^{\star} = \Delta$. In particular, the degrees of non-linearity and state-contingency are determined by the same structural objects. The very reason why large shocks generate non-linear responses is that they induce changes the distribution of financial assets (Section 1.4.2), i.e. they endogeneously affect the aggregate state. In other words, the dynamic impulse response to an income shock is non-linear in the size of the shock Δ if and only if the impulse response to a shock at the same horizon exhibits state-contingency.

1.7 Conclusion

In this paper, I explore whether a canonical model of lumpy durable investment with incomplete markets can produce an average MPC on durable that increases with the size of income changes. I clarify the source of these macro non-linearities analytically, and I confirm in numerical exercises that the response of durable investment is non-linear in income changes. However, I find that the magnitudes are substantially lower than those observed in the data. I suggest various avenues to improve the quantitative performance of the model.

Appendix to Chapter 1

1.A Quantitative Appendix

In this Appendix, I describe the model in more details and I present complementary numerical results. Section 1.A.1 describes the full model. Section 1.A.2 presents additional results that are not reported in text. The approach used to simulate and calibrate the model is discussed in Zorzi (2020a).

1.A.1 Environment

For concision, Section 1.3 provided a partial description of the model. For reference, I now present the full model.

Timing. Periods are indexed by $t \in \{0, 1, \ldots\}$. There is no aggregate risk. I focus on one-time, unanticipated but persistent shocks. Each period effectively consists of two sub-periods, indexed by $t.0$ $(-)$ and $t.1$ $(+)$. Households make their adjustment decisions at $t.0$, and their consumption, saving and investment decisions at $t.1$. In period $t.0$, households are indexed by their financial asset holdings (a), their holdings of durable goods (d) and their idiosyncratic labor supply shock (ζ). In period $t.1$, households are in addition indexed by their adjustment choice $(\mathcal{A})$ during the previous sub-period.[44] The conditional distributions of idiosyncratic states within each sector, at the begining of each subperiod, are denoted by $\mathbf{\Lambda}_{t-1}^{-}$ and $\mathbf{\Lambda}_{t}^{+}$. All agents have perfect foresight, and all (aggregate) information is revealed at the begining of the first sub-period at $t = 0$.

Households. The households' value function in period $t.0$ satisfies:

$$\mathcal{V}_t\left(a, d, \zeta\right) = \max_{\mathcal{A} \in \{0,1\}} \left\{V_t\left(a, d, \zeta; \mathcal{A}\right)\right\} \tag{1.A.1}$$

[44] This additional state variable is for notational convenience. Financial assets are a sufficient statistics for this adjustment.

The adjustment choice satisfies:

$$\mathcal{A}_t(a,d,\zeta) \equiv \begin{cases} 1 & \text{if } V_t(a,d,\zeta;1) > V_t(a,d,\zeta;0) \\ 0 & \text{otherwise} \end{cases} \tag{1.A.2}$$

The continuation value functions in period $t.1$ associated to adjustment and no adjustment write:

$$\begin{aligned} V_t(a,d,\zeta;0) = \max_{\{c,a'\}} \ & \frac{u(c,d^\star)^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}(a',d^\star,\zeta')\middle|\zeta\right] \quad (1.A.3)\\ \text{s.t. } & P^c c + P^d \iota \frac{\delta}{1-(1-\iota)\delta} d^\star + a' \leq (1-\tau) e_t(\zeta) + (1+r) a \\ & a' \geq 0 \\ V_t(a,d,\zeta;1) = \max_{\{c,a',d'\}} \ & \frac{u(c,d')^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}(a',d',\zeta')\middle|\zeta\right] \quad (1.A.4)\\ \text{s.t. } & P^c c + P^d\left(d' - (1-\delta) d\right) + \gamma P^d (1-\delta) d + a' \\ & \leq (1-\tau) e_t(\zeta) + (1+r_t) a \\ & a' \geq 0 \end{aligned}$$

with $d^\star \equiv [1-(1-\iota)\delta]\, d$, where $e_t(\zeta) \equiv \zeta\mathcal{Y} + T_t + \pi$ denotes incomes. The distributions in period $t.0$ and $t.1$ evolve as follows:

$$\Lambda_t^+(a,d,\zeta,\mathcal{A}) = \Lambda_{t-1}^-(a,d,\zeta) \times \begin{cases} \mathcal{A}_t(a,d,\zeta) & \text{if } \mathcal{A} = 1 \\ 1-\mathcal{A}_t(a,d,\zeta) & \text{otherwise} \end{cases} \tag{1.A.5}$$

and

$$\Lambda_t^-(a',d',\zeta') = \sum_{\mathcal{A}} \sum_{\Omega_t(a',d';\mathcal{A})} \Lambda_t^+(a,d,\zeta,\mathcal{A})\, \Sigma\left(\log(\zeta')\,|\zeta\right) \tag{1.A.6}$$

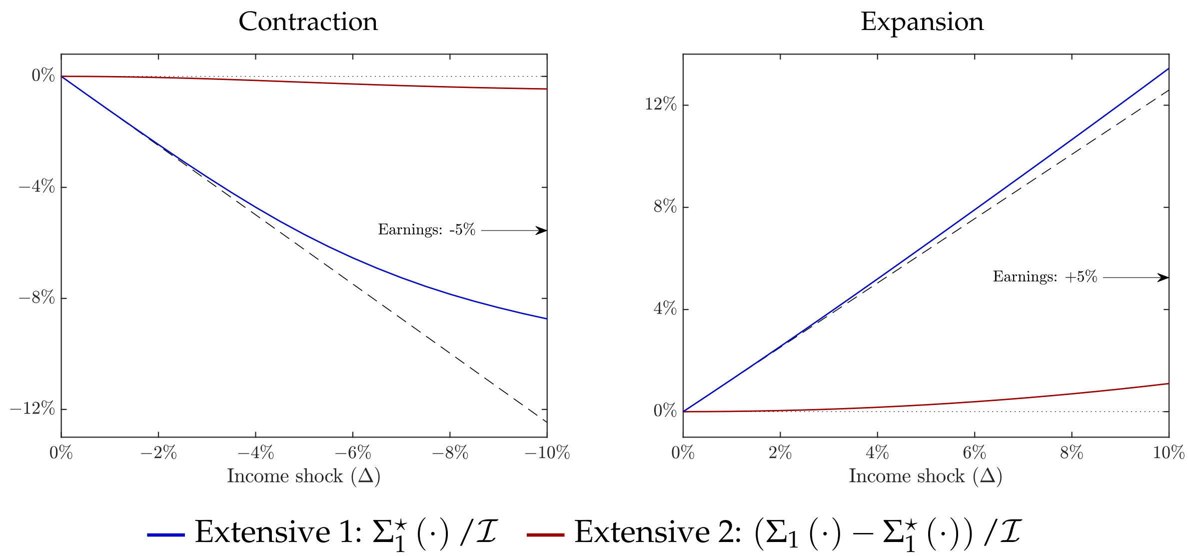
with $\Omega_t(a^\star,d^\star;\mathcal{A}) \equiv \{(a,d,\zeta)\,|a_t'(\cdot;\mathcal{A}) = a^\star, d_t'(\cdot;\mathcal{A}) = d^\star\}$, where Σ denotes the transition kernel characterizing the income process and a_t' and d_t' denote the solution to (1.A.1)–(1.A.4), with $d_t' \equiv d^\star$ when no adjustment. I define c_t similarly.

1.A.2 Complementary Numerical Results

I provide below some numerical results that were omitted from the main text.

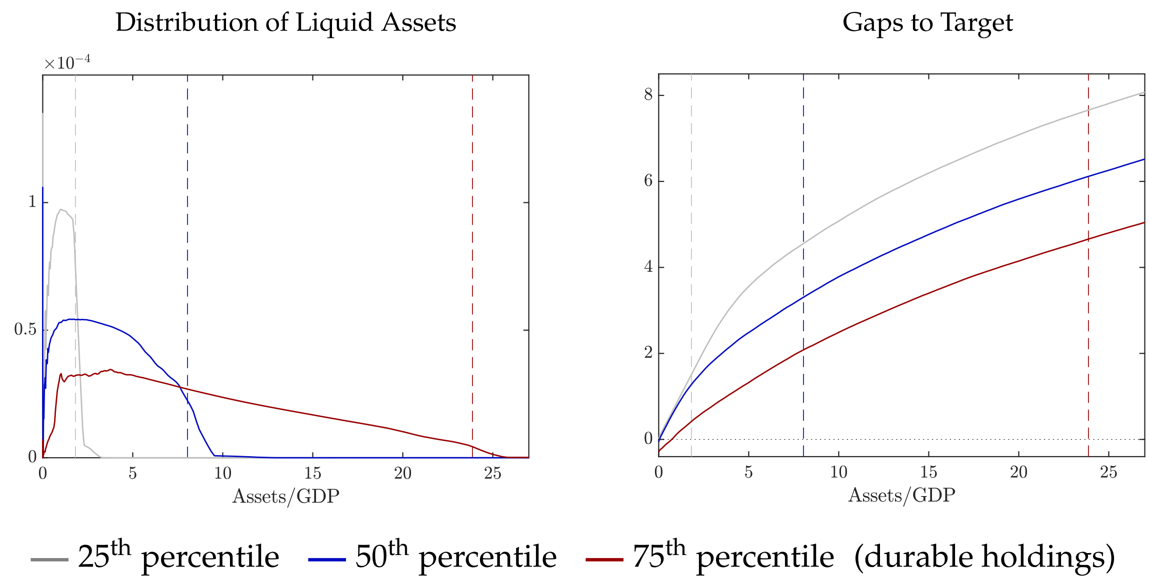
Extensive margin. In Section 1.6.1, I implement the decomposition from Proposition 1. In Figure 1.A.1, I further decompose the extensive margin of adjustment into the two terms in $\Sigma_1(\Delta)$ in (1.4.4) and (1.B.1). I denote by $\Sigma_1^{\star}(\cdot)$ the first term in $\Sigma_1(\cdot)$, i.e. with $\kappa(d, \zeta) = 0$.

Figure 1.A.1: Decomposition from Proposition 1



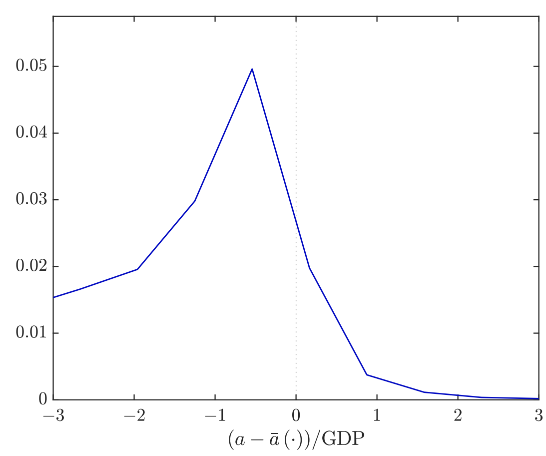
Sources of non-linearities. Figure 1.A.2 plots the density of the distribution of liquid assets (normalized by quarterly GDP) and the durable investment target for various percentiles of the distribution of durable goods (fixing idiosyncratic labor supply at its median level). The dashed lines denote the durable adjustment thresholds.

Figure 1.A.2: Sources of Non-Linearity (median labor supply)



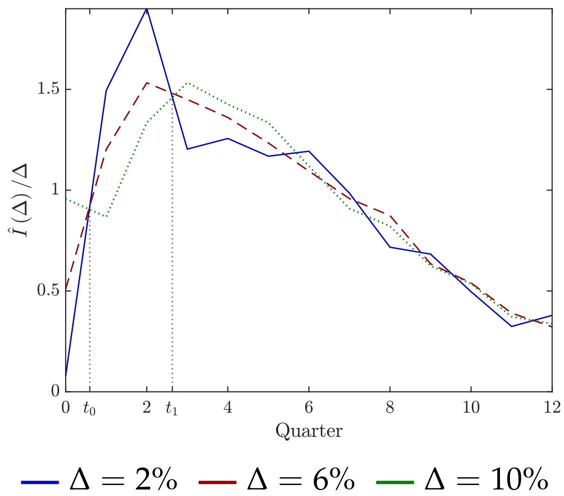
Distance to threshold. Figure 1.A.3 plots the distribution of distances to the adjustment threshold $a - \bar{a}(\cdot)$ (normalized by quarterly GDP) at the stationary equilibrium. A large fraction of households lies relatively close to their adjustment threshold.

Figure 1.A.3: Distribution of Distance to Adjustment threshold



Timing. Figure 1.A.4 plots the impulse response of durable spending to (positive) income changes of various sizes, normalized by the size of shocks. The response of durable spending (in level) is convex on impact in the size of income shocks, i.e. the normalized response is higher for larger income shocks. This impulse response is then concave for a few quarters (t_0 to t_1), and convex in the medium term (after t_1).

Figure 1.A.4: Impulse Responses to Earnings Shocks



1.B Omitted Proofs, Results and Derivations

1.B.1 Decomposition

I provide a decomposition of the response of durable investment in income shocks in Section 1.4.1. Expression (1.4.4) in Proposition 1 applies to positive income shocks. I first provide the analogous decomposition for negative income shocks, before proving these results.

Proposition 1 (cont'd). The response of durable investment to a negative income shock $\Delta < 0$ can be decomposed as follows:

$$\hat{I}(\Delta) \equiv \underbrace{\Sigma_1'(\Delta)}_{\text{Extensive margin}} + \underbrace{\Sigma_2'(\Delta)}_{\text{Intensive margin}} + \underbrace{\varsigma'(\Delta)}_{\text{Residual}} \tag{1.B.1}$$

with

$$\Sigma_1'(\Delta) \equiv \frac{1}{\mathcal{I}} \int \left\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta)\, d\right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta) + \kappa(d,\zeta) \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} (a-\bar{a}(d,\zeta))\, d\Lambda(a|d,\zeta) \right\} d\Lambda^\star$$

$$\Sigma_2'(\Delta) \equiv \frac{1}{\mathcal{I}} \int \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y},d,\zeta) - d^\star(a,d,\zeta)\right] d\Lambda(a|d,\zeta)\, d\Lambda^\star$$

for some $\kappa(d,\zeta) > 0$ and some residual $\varsigma'(\Delta)$ that satisfies $\lim_{\Delta\to 0}\frac{\varsigma'(\Delta)}{\Delta} = 0$.

Proof of Proposition 1. From (1.4.1)–(1.4.3),

$$\hat{I}(\Delta) \equiv \frac{1}{\mathcal{I}}\left[\int\int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a,d,\zeta)-(1-\delta)\,d\right)d\Lambda\left(a-\zeta\Delta\mathcal{Y}|\,d,\zeta\right)d\Lambda^{\star} + \int\int_{0}^{\bar{a}(d,\zeta)}\delta d\imath d\Lambda\left(a-\zeta\Delta\mathcal{Y},d,\zeta\right)d\Lambda^{\star}-\mathcal{I}\right]$$

where $\Lambda^{\star}\equiv \text{marg}_{d,\zeta}\Lambda$ denotes the marginal distribution of durable holdings and producitivty. By definition, aggregate durable investment at the stationary equilibrium satisfies: $\mathcal{I}\equiv I(0)$. Thus,

$$\hat{I}(\Delta) \equiv \frac{1}{\mathcal{I}}\left[\int\underbrace{\int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a,d,\zeta)-(1-\delta)\,d\right)d\hat{\Lambda}\left(a-\zeta\Delta\mathcal{Y}|\,d,\zeta\right)}_{\equiv\Omega_1}d\Lambda^{\star} + \int\underbrace{\int_{0}^{\bar{a}(d,\zeta)}\delta d\imath d\hat{\Lambda}\left(a-\zeta\Delta\mathcal{Y}|\,d,\zeta\right)}_{\equiv\Omega_2}d\Lambda^{\star}\right] \tag{1.B.2}$$

where $\hat{\Lambda}(a-\zeta\Delta\mathcal{Y}|\,d,\zeta)\equiv\Lambda(a-\zeta\Delta\mathcal{Y}|\,d,\zeta)-\Lambda(a|\,d,\zeta)$, with an abuse of notation.

Using the change of variable $a'\equiv a-\zeta\Delta\mathcal{Y}$, the first term in (1.B.2) is

$$\Omega_1 = \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{+\infty}\left(d^{\star}(a+\zeta\Delta\mathcal{Y},d,\zeta)-(1-\delta)\,d\right)d\Lambda(a|\,d,\zeta) - \int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a,d,\zeta)-(1-\delta)\,d\right)d\Lambda(a|\,d,\zeta)$$

Then,

$$\Omega_1 = \int_{\bar{a}(d,\zeta)}^{+\infty}\left(d^{\star}(a+\zeta\Delta\mathcal{Y},d,\zeta)-d^{\star}(a,d,\zeta)\right)d\Lambda(a|\,d,\zeta)\,d\Lambda^{\star} + \underbrace{\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)}\left(d^{\star}(a+\zeta\Delta\mathcal{Y},d,\zeta)-(1-\delta)\,d\right)d\Lambda(a|\,d,\zeta)}_{\equiv\hat{\Omega}_1} \tag{1.B.3}$$

The first integral in (1.B.3) corresponds to the *intensive* margin of adjustment in Proposition 1. The second integral contributes to the *extensive* margin, together with the term Ω_2 in (1.B.2).

This second integral can be decomposed as follows:

$$\begin{aligned}\hat{\Omega}_1 &= \int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right)-\bar{d}\left(d,\zeta\right)\right)d\Lambda\left(a|\,d,\zeta\right)\\&+\left(\bar{d}\left(d,\zeta\right)-\left(1-\delta\right)d\right)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|\,d,\zeta\right)\end{aligned}$$

where $\bar{d}\left(d,\zeta\right)\equiv d^\star\left(\bar{a}\left(d,\zeta\right),d,\zeta\right)$ denotes investment at the threshold. By assumption, $d^\star\left(\cdot\right)$ is smooth. Define $\kappa\left(d,\zeta\right)\equiv\frac{\partial}{\partial a}d^\star\left(a,d,\zeta\right)\Big|_{a=\bar{a}(d,\zeta)}>0$. Then,

$$\begin{aligned}\hat{\Omega}_1 \equiv&\kappa\left(d,\zeta\right)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)}\left(a+\zeta\Delta\mathcal{Y}-\bar{a}\left(d,\zeta\right)\right)d\Lambda\left(a|\,d,\zeta\right)+\varsigma\left(\Delta,d,\zeta\right)\\&+\left(\bar{d}\left(d,\zeta\right)-\left(1-\delta\right)d\right)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|\,d,\zeta\right)\end{aligned}\tag{1.B.4}$$

where $\varsigma\left(\Delta,d,\zeta\right)$ is defined residually.

Again, using a change of variable, the second term in (1.B.2) corresponds to

$$\begin{aligned}\Omega_2 &= \delta d\iota\left[\int_0^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|\,d,\zeta\right)-\int_0^{\bar{a}(d,\zeta)} d\Lambda\left(a|\,d,\zeta\right)\right]\\&= -\delta d\iota\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|\,d,\zeta\right)\end{aligned}\tag{1.B.5}$$

since $\Lambda\left(\cdot|\,d,\zeta\right)$ has positive support, from the household's income fluctuations problem (1.3.4)–(1.3.5).

Note that the previous expressions apply to both positive income shock $\Delta>0$ and negative income shocks $\Delta<0$. However, the attribution of each term to the extensive and intensive margins differs between these two cases. I thus treat them separately.

Case 1: $\Delta<0$. The expression (1.4.4) in the text follows by collecting the terms from (1.B.2)–(1.B.4) and (1.B.5), and by definition of the hazard (1.4.2) and the marginal distribution $\Lambda^\star$. Finally, $\varsigma\left(\Delta\right)$ is a second-order term, by Taylor's Theorem.

Case 2: $\Delta<0$. Note that

$$\begin{aligned}\Sigma_2\left(\Delta\right)=&\frac{1}{\mathcal{I}}\int\left\{\int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}\left[d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right)-d^\star\left(a,d,\zeta\right)\right]d\Lambda\left(a|\,d,\zeta\right)\right.\\&\left.+\int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty}\left[d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right)-d^\star\left(a,d,\zeta\right)\right]d\Lambda\left(a|\,d,\zeta\right)\right\}d\Lambda^\star\end{aligned}$$

Equivalently,

$$\Sigma_2(\Delta) = \frac{1}{\mathcal{I}} \int \left\{ \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} \left[d^\star(a+\zeta\Delta\mathcal{Y},d,\zeta) - \bar{d}(d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right.$$
$$+ \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[\bar{d}(d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta)$$
$$\left. + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y},d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right\} d\Lambda^\star \tag{1.B.6}$$

Then,

$$\Sigma_2(\Delta) = \frac{1}{\mathcal{I}} \int \left\{ \kappa(d,\zeta) \int_{\bar{a}(\cdot)}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} \zeta\Delta\mathcal{Y} d\Lambda(a|\, d,\zeta) \right.$$
$$\left. + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y},d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right\} d\Lambda^\star + \hat{\varsigma}(\Delta) \tag{1.B.7}$$

for some second-order term $\hat{\varsigma}(\Delta)$, by definition of $\kappa(d,\zeta)$ and $\bar{d}(d,\zeta)$. Therefore, summing up $\Sigma_1(\cdot)$ and $\Sigma_2(\cdot)$ in the expression (1.4.4) in the text:

$$\Sigma_1(\Delta) + \Sigma_2(\Delta) = \frac{1}{\mathcal{I}} \int \left\{ \left[\bar{d}(d,\zeta) - (1-\delta+\iota\delta) d \right] \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda(a|d,\zeta) \right.$$
$$+ \kappa(d,\zeta) \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} (a - \bar{a}(d,\zeta))\, d\Lambda(a|d,\zeta)$$
$$\left. + \int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{+\infty} \left[d^\star(a+\zeta\Delta\mathcal{Y},d,\zeta) - d^\star(a,d,\zeta) \right] d\Lambda(a|\, d,\zeta) \right\} d\Lambda^\star + \hat{\varsigma}(\Delta) \tag{1.B.8}$$

Finally, the expression (1.B.1) is obtained from (1.B.8), collecting the terms appropriately.

□

1.B.2 Time-Dependent Adjustment

State-dependent (or lumpy) adjustment is key for the the non-linear effect of income shocks discussed in Section 1.4.1. To illustrate this point, I consider an alternative formulation where adjustment is instead time-dependent à la Calvo (1983).

In this case, the households' problem is

$$V_t^h(a,d,\zeta;0) = \max_{\{c,a'\}} \frac{u(c,d^\star)^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}^h(a', d^\star, \zeta'; \mathcal{A}_{t+1}) \middle| \zeta \right] \tag{1.B.9}$$
$$\text{s.t. } P_t^c c + P_t^d \iota \frac{\delta}{1-(1-\iota)\delta} d^\star + a' \leq (1-\tau_t) e_t^h(\zeta) + (1+r_{t-1}) a$$
$$a' \geq 0$$

with $d^\star \equiv (1-(1-\iota)\delta)d$, and

$$V_t^h(a,d,\zeta;1) = \max_{\{c,a',d'\}} \frac{u(c,d')^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[V_{t+1}^h(a', d', \zeta'; \mathcal{A}_{t+1}) \middle| \zeta \right] \tag{1.B.10}$$
$$\text{s.t. } P_t^c c + P_t^d (d' - (1-\delta) d) + a'$$
$$\leq \left(1-\tau_t^h\right) e_t^h(\zeta) + (1+r_{t-1}) a$$
$$a' \geq 0$$

with $\{\mathcal{A}_t\}_{t\geq 0}$ following a Poisson process with intensity $\theta \in [0,1]$, with independence across households and sectors. Note that the case $\theta \equiv 1$, i.e. frictionless adjustment, coincides with the case $\Gamma_t(d',d) \equiv 0$ in the original formulation with lumpy adjustment.

Aggregate durable spending in period $t=0$ is

$$\begin{aligned} \mathcal{I}(\Delta) = & \theta \int \left[d^\star(a,d,\zeta) - (1-\delta) d \right] d\Lambda (a - \zeta \Delta \mathcal{Y}, d, \zeta) \\ & + (1-\delta) \iota \delta d d\Lambda (a - \zeta \Delta \mathcal{Y}, d, \zeta) \end{aligned} \tag{1.B.11}$$

where $d^\star(\cdot)$ solves the problem with adjustment (1.B.10) at $t=0$. Thus, the impulse response of durable spending is

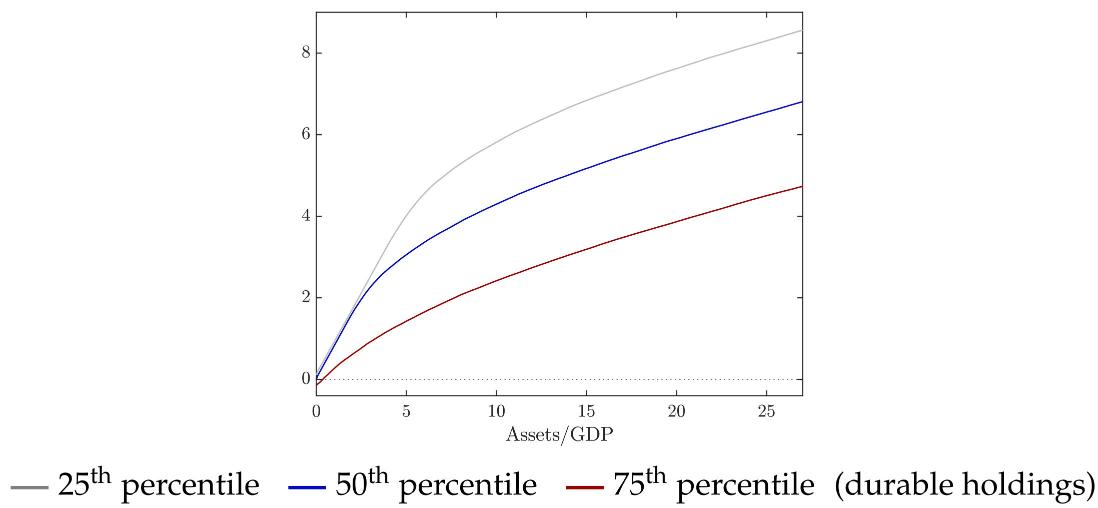
$$\hat{\mathcal{I}}(\Delta) = \frac{1}{\mathcal{I}} \theta \int \left[d^\star(a + \zeta \Delta \mathcal{Y}, d, \zeta) - d^\star(a + \zeta \Delta \mathcal{Y}, d, \zeta) \right] d\Lambda(a,d,\zeta) \tag{1.B.12}$$

since $\mathcal{I} \equiv \mathcal{I}(0)$. Therefore, the impulse response $\hat{\mathcal{I}}(\cdot)$ inherits the shape of durable spending $d^\star(\cdot; d, \zeta)$ as a function of liquid, financial assets.

Figure 1.B.1 plots durable spending $d^\star(\cdot)$ in terms of financial assets, for various levels of durable holdings and fixing productivity at its median level. Durable spending is concave due to precautionary savings (Carroll and Kimball (1996)). As a result, aggregate durable spending is *concave* in income changes. This stands is sharp contrast with the case of lumpy adjustment, where the durable spending is *convex* in income changes (see

Sections 1.4.1 and 1.6.1).

Figure 1.B.1: Durable Spending - Calvo (median labor supply)



1.B.3 Continuous Time

My model is set in discrete time. In this appendix, I consider instead a continuous time version. To simplify the expressions, I abstract from maintenance and uninsured idiosyncratic income risk in the following.

Dynamic system. For the sake of illustration, I assume that income shocks $\{\Delta_t\}_{t\geq 0}$ are perceived as transitory, even if I am interested in persistent sequences of income changes. As a consequence, households' optimal savings policy conditional on no adjustment is given by some time-invariant $s(a, D, \Delta)$.[45] The process for idiosyncratic states (a, D) is defined by: (i) a law of motion conditional on no adjustment,[46]

$$da(t) = s(a(t), D(t), \Delta(t))\, dt \tag{1.B.13}$$

$$dD(t) = -\delta D(t)\, dt; \tag{1.B.14}$$

(ii) an adjustment threshold $\bar{a}(D)$ of the form (1.4.2); and (iii) an adjustment target $d^{\star}(D)$ for durables.[47] Financial assets after adjustment satisfy the following budget constraint

[45] In the following, I denote durable holdings by D instead of d to avoid any confusion with differential operators or integrands.

[46] I abstract from mass points at the borrowing constraint, by assuming that there is no uninsured idiosyncratic income risk and that the savings rate is strictly positive (see Assumption 2 below).

[47] Formally, $\bar{a}(\cdot)$ is a jump reflecting barrier, and the adjustment target $d^{\star}(\cdot)$ corresponds to the reset point when hitting this barrier. Note that $a = \bar{a}(D)$ when hitting the barrier, so that $d^{\star}(\cdot)$ can be expressed in terms of D only.

(1.4.11). The economy is initially at its stationary equilibrium. I impose the following restrictions to simplify the analysis.

Assumption 1. *The adjustment threshold satisfies* $\bar{a}(\cdot) = \bar{a} > 0$.

Assumption 2. *The savings function satisfies*

$$s(a, D, \Delta) = \bar{s} + \hat{m}\Delta \tag{1.B.15}$$

for some $\bar{s} > 0$ *and some* $\hat{m} > 0$.

Assumption 3. *Households who adjust borrow up to their constraint,*

$$d^{\star}(D) = D + \frac{1}{P^d}\bar{a}(D)$$

The first restriction holds in my (discrete time) calibrated model. The second assumption stipulates the households have a constant marginal propensity to spend out of marginal, transitory income changes,[48] and that the savings rate does not depend on asset holdings. Finally, the third restriction allows me simplify the expressions when characterizing the evolution of the distribution of idiosyncratic states in the interior of the state-space ($a > 0$). This restriction also typically holds in my (discrete time) calibrated model.

Comparative statics. I consider a sequence of persistent (but unanticipated) income changes. That is, $\Delta_t \equiv \Delta \in \mathbb{R}$ for all $t \in [0, T]$ and some horizon $T > 0$.

Investment. Cumulative aggregate durable investment satisfies[49]

$$\begin{aligned} \mathcal{I}(t') - \mathcal{I}(t) \equiv & \int_0^{+\infty} \int_t^{t'} \left(d^{\star}(D) - D\right) d\Lambda_t\left(\hat{a}\left(\bar{a}, D; \hat{t}, t\right), \exp\left(\delta\left(\hat{t} - t\right)\right) D\right) \\ & + \varsigma\left(t', t\right) \end{aligned} \tag{1.B.16}$$

for some term $\varsigma(t', t)$ with $\lim_{s \to 0} \frac{\varsigma(t+s,t)}{s} = 0$. Here, I define the threshold $\hat{a}(\cdot)$ such that

$$a - \hat{a}\left(a, D; t', t\right) \equiv \int_t^{t'} s\left(\tilde{a}\left(\hat{t}\right), \exp\left(-\delta\left(\hat{t} - t'\right)\right) D, \Delta\right) d\hat{t}, \tag{1.B.17}$$

[48] However, spending need not be linear in income shocks (even conditional on no adjustment) when income changes over a discrete interval $[0, T]$. See the discussion at the end of this section.

[49] The second integral is over $\hat{t}$.

The process $\{\tilde{a}(\hat{t})\}_{\hat{t}\geq t}$[50] in (1.B.17) is characterized by

$$d\tilde{a}\left(\hat{t}\right) = s\left(\tilde{a}\left(\hat{t}\right), \exp\left(-\delta\left(\hat{t}-t'\right)\right) D, \Delta\right) d\hat{t} \tag{1.B.18}$$

with initial condition $\tilde{a}(0) \equiv \hat{a}(a, D; t', t)$. Figure 1.B.2 corresponds to the phase diagram associated to the system (1.B.13)–(1.B.14) and depicts the threshold $\hat{a}(\cdot)$ for various values of a.

Using the change of variables $D' \equiv \exp\left(\delta\left(\hat{t}-t\right)\right) D$ and $a \equiv a^{\star}\left(D; \hat{t}, t\right)$ with $a^{\star}\left(D; \hat{t}, t\right) \equiv \hat{a}\left(\bar{a}, \exp\left(-\delta\left(\hat{t}-t\right)\right) D; \hat{t}, t\right)$ and Assumptions 1 and 2,

$$\begin{aligned}\mathcal{I}\left(t'\right) - \mathcal{I}\left(t\right) = \int_0^{+\infty} \int_{a^{\star}(D;t',t)}^{\bar{a}} &\underbrace{\left(d^{\star}\left(\hat{D}\left(a,D\right)\right) - \hat{D}\left(a,D\right)\right)}_{\text{Gap to target}} \underbrace{d\Lambda_t\left(a,D\right)}_{\text{Density}} \\ &+ \varsigma\left(t', t\right)\end{aligned} \tag{1.B.19}$$

Here, $\hat{D}(a, D) \equiv \exp\left(-\delta\left(\hat{T}(a,d) - t\right)\right) D$, where $\hat{T}(a,d)$ denotes the stopping time associated to (1.B.13)–(1.B.14) and the barrier $\bar{a}(\cdot)$. By definition, $\hat{D}(\bar{a}, D) \equiv D$. Note that expression (1.B.19) is the continuous time counterpart of (1.4.1) in the text.

From (1.B.19), the durable investment flow satisfies

$$i_t \equiv \frac{d}{dt}\mathcal{I}\left(t\right) = -\int_0^{+\infty} \frac{\partial}{\partial t'} a^{\star}\left(D; t', t\right)\Big|_{t'=t} \left[d^{\star}\left(D\right) - D\right] \lambda_t\left(\bar{a}, D\right) dD \tag{1.B.20}$$

where λ_t denotes the density associated to Λ_t, using $a^{\star}(D; t, t) \equiv \bar{a}$ and $\hat{D}\left(a^{\star}(D; t, t), D\right) \equiv D$. Using (1.B.17)–(1.B.18) and Assumption 2,

$$\frac{\partial}{\partial t'} a^{\star}\left(D; t', t\right) = -\left(\bar{s} + \hat{m}\Delta\right) \tag{1.B.21}$$

Distribution. Using Assumptions 1 and 3, the joint distribution of idiosyncratic states evolves as follows[51]

$$\begin{aligned}\Lambda_{t'}\left(\mathcal{A}, \mathcal{D}\right) - \Lambda_t\left(\mathcal{A}, \mathcal{D}\right) \equiv &\int_{\mathcal{D}}^{\hat{d}(\mathcal{D};t'-t)} \int_0^{\hat{a}(\mathcal{A},D;t',t)} d\Lambda_t\left(a, D\right) \\ &- \int_0^{\mathcal{D}} \int_{\hat{a}(\mathcal{A},D;t',t)}^{\mathcal{A}} d\Lambda_t\left(a, D\right) \\ &+ \int_0^{\hat{D}(\mathcal{D})} \int_{\hat{a}(\bar{a},D;t',t)}^{\bar{a}} d\Lambda_t\left(a, D\right) + \hat{\varsigma}\left(\mathcal{A}, \mathcal{D}; t', t\right)\end{aligned} \tag{1.B.22}$$

[50] Implicitly, this process is indexed by (t, t') as well.
[51] I use $\mathcal{A}$ and $\mathcal{D}$ as arguments for the distribution Λ_t and a and D as integrands. This is a slight abuse of notation, since $\mathcal{A}$ is used in the text to denote that adjustment hazard.

when $0 \leq a < \bar{a}$, for some term $\hat{\varsigma}(\mathcal{A}, \mathcal{D}; t', t)$ with $\lim_{s \to 0} \frac{\hat{\varsigma}(\mathcal{A}, \mathcal{D}; t+s, t)}{s} = 0$. The three terms on the RHS of (1.B.22) capture households' inflows and outflows into the relevant region of the state space.

Figure 1.B.2: Density at Adjustment Threshold

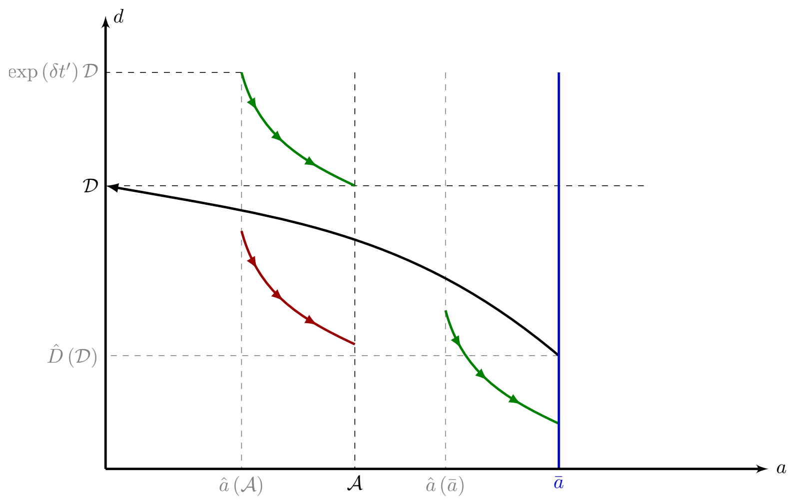


Figure 1.B.2 corresponds to the phase diagram associated to the system (1.B.13)–(1.B.14). Inflows in (1.B.22) are depicted in green, and outflows in red. The first term in (1.B.22) captures the inflows due to the depreciation of durables. This corresponds to households with an initial stock of durables below $\hat{d}(D; t' - t) \equiv \exp(\delta(t' - t)) D$ and sufficiently low financial assets. The second term captures the outflows due to accumulation of financial assets. Note that the reset level of financial assets is $a^\star(\cdot) = 0$ in either case, by Assumption 3. Therefore, outflows occur only when the adjustment target $d^\star(\cdot)$ exceeds $\mathcal{D}$. Hence $d^\star\left(\hat{D}(\mathcal{D})\right) \equiv \mathcal{D}$, and $\hat{D}(\mathcal{D}) \leq \mathcal{D}$ is well-defined by Assumption 3. The third term captures inflows from households whose initial holdings of financial assets exceeded $\mathcal{A}$ but was depleted after they adjusted their stock of durables. Inflows only occur when the adjustment target $d^\star(\cdot)$ is below $\mathcal{D}$.

From (1.B.22), the evolution of the density of idiosyncratic states is characterized by[52]

$$\begin{aligned}\partial_t \lambda_t(\mathcal{A}, \mathcal{D}) =& \partial_{\mathcal{D}} \left(\frac{\partial}{\partial t'} \hat{d}(\mathcal{D}; t' - t) \Big|_{t'=t} \lambda_t(\mathcal{A}, \mathcal{D}) \right) \\ & + \partial_{\mathcal{A}} \left(\frac{\partial}{\partial t'} \hat{a}(\mathcal{A}, D; t', t) \Big|_{t'=t} \lambda_t(\mathcal{A}, \mathcal{D}) \right) \end{aligned} \tag{1.B.23}$$

[52] Note that (1.B.23) coincides with the Fokker-Planck equation associated to the system (1.B.13)–(1.B.14), abstracting from the barrier $\bar{a}$. The reason is that the third term on the RHS of (1.B.22) does not contribute to the density for $a > 0$, by Assumption 3.

when $0 < \mathcal{A} < \bar{a}$ and for sufficiently small t, using the definition of $\hat{a}(\cdot)$ and $\hat{d}(\cdot)$. Using (1.B.17)–(1.B.18), the definitions of $a^{\star}(\cdot)$ and $\hat{d}(\cdot)$, and Assumption 2,

$$\frac{\partial}{\partial t'}\hat{d}\left(D;t',t\right)\bigg|_{t'=t} = \delta D \quad , \quad \frac{\partial}{\partial t'}a^{\star}\left(D;t',t\right)\bigg|_{t'=t} = -\left(\bar{s}+\hat{m}\Delta\right) \tag{1.B.24}$$

Finally, the evolution of investment is entirely characterized by (1.B.20)–(1.B.21) and (1.B.23)–(1.B.24).

Impulse response. I am interested in the response of cumulative investment, starting in period $t = 0$.[53]

From (1.B.20)–(1.B.21) and (1.B.23)–(1.B.24),

$$i_t\left(\Delta\right) \equiv \left(\bar{s}+\hat{m}\Delta\right)\int_0^{+\infty}\left[d^{\star}\left(D\right)-D\right]\lambda_t\left(\bar{a},D\right)dD \tag{1.B.25}$$

Note that

$$\frac{\partial}{\partial\Delta}i_t\left(\Delta\right)\bigg|_{t=0} = \hat{m}\int_0^{+\infty}\left[d^{\star}\left(D\right)-D\right]\lambda\left(\bar{a},D\right)dD, \tag{1.B.26}$$

For illustration, I suppose that the stationary distribution satisfies

$$\partial_a\lambda\left(\mathcal{A},\mathcal{D}\right) = \partial_a\lambda\left(\mathcal{A}',\mathcal{D}'\right) \equiv \mu < 0 \quad \text{and} \quad \lambda\left(\mathcal{A},\mathcal{D}\right) = \lambda\left(\mathcal{A},\mathcal{D}'\right)$$

for each $\mathcal{A},\mathcal{A}' \geq 0$ $\mathcal{D},\mathcal{D}' \geq 0$, i.e. the distribution has constant slope with respect to financial holdings and is uniform with respect to durable holdings. Then, λ_t inherits these properties, from (1.B.22) and using (1.B.17)–(1.B.18) and Assumption 2. Therefore,

$$\partial_t\lambda_t\left(\mathcal{A},\mathcal{D}\right) = \delta\lambda_t\left(\mathcal{A},\mathcal{D}\right) - \left(\bar{s}+\hat{m}\Delta\right)\mu$$

using (1.B.23)–(1.B.24) and Assumption 2. Then,

$$\lambda_t\left(\mathcal{A},\mathcal{D}\right) = \lambda\left(\mathcal{A},\mathcal{D}\right) - \mu\left(\bar{s}+\hat{m}\Delta\right)\left(\exp\left(\delta t\right)-1\right) \tag{1.B.27}$$

Finally, cumulative investment satisfies

$$i_t\left(\Delta\right) \equiv \left(\bar{s}+\hat{m}\Delta\right)\int_0^{+\infty}\left[d^{\star}\left(D\right)-D\right]\left[\lambda\left(\bar{a},\mathcal{D}\right)-\mu\left(\bar{s}+\hat{m}\Delta\right)\left(\exp\left(\delta t\right)-1\right)\right]dD \tag{1.B.28}$$

using (1.B.25) and (1.B.27). As in disctete time, the impulse response of investment is *convex* in income changes for any $t > 0$, whenever the density λ at the stationary equilib-

[53] That is, I parametrize the above expressions with 0 for t and t for t'.

rium is decreasing ($\mu < 0$). In other words, the average marginal propensity to spend on durables increases with income changes. I elaborate on these results in Section 1.4.1.

State-contingency. The form of non-linearity discussed above is intrinsically related to another property of models of lumpy adjustment: state-contingency.

I am interested in the response to an (unanticipated) income shock Δ over an interval $[t, t']$ with $t' > t$ after a sequence of (unanticipated) shocks $\Delta^\star$ over the interval $[0, t)$. By analogy with (1.B.20)–(1.B.21) and (1.B.27),

$$i_t\left(\Delta\right) = \left(\bar{s} + \hat{m}\Delta\right) \int_0^{+\infty} \left[d^\star\left(D\right) - D\right]\left[\lambda\left(\mathcal{A}, \mathcal{D}\right) - \mu\left(\bar{s} + \hat{m}\Delta^\star\right)\left(\exp\left(\delta t\right) - 1\right)\right] dD \quad (1.B.29)$$

Note that (1.B.28) and (1.B.29) coincide when $\Delta = \Delta^\star$, by definition. By construction, the slope of the density λ at the stationary equilibrium (μ) is responsible for both the non-linearity that I focus on, and the state-contingency of impulse responses. I elaborate on this point in Section 1.6.4. Also, note that the impulse response of investment is linear in Δ, but contingent on the size of the initial income change and the period over which it occurs $(\Delta^\star, t)$. In particular,

$$\frac{\partial^2}{\partial t \partial \Delta} i_t\left(\Delta\right) = -\delta \exp\left(\delta t\right) \mu \left(\hat{m}\right)^2 \Delta^\star \int_0^{+\infty} \left[d^\star\left(D\right) - D\right] dD$$

when $\bar{s} \to 0$, i.e. the economy before the income shocks is stationary. The impulse response increases over time after a period of expansion $(\Delta^\star > 0)$, but decreases after a period of recession $(\Delta^\star < 0)$, whenever the slope of the density of liquid assets at the adjustment threshold μ is negative. I elaborate on these results in Section 1.6.4.

1.B.4 Omitted Derivations

Derivations for Example 1. The impulse response satisfies:

$$\hat{I}\left(\Delta\right) = \frac{1}{\mathcal{I}} \int \left[\bar{d}\left(d, \zeta\right) - \left(1 - \delta + \iota\delta\right) d\right] \Psi\left(\Delta\right) d\Lambda^\star \quad (1.B.30)$$

with

$$\Psi\left(\Delta\right) \equiv \int_{\bar{a}(\cdot) - \zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda\left(a | d, \zeta\right) \quad (1.B.31)$$

Let

$$\eta\left(\Delta, \Delta'\right) \equiv \Psi\left(\Delta'\right) + \Psi\left(\Delta\right) - 2\Psi\left(\frac{\Delta + \Delta'}{2}\right) \quad (1.B.32)$$

with $\Delta' > \Delta$ without loss of generality.

From (1.B.31),

$$\begin{aligned}\eta\left(\Delta,\Delta'\right) &= 2\int_{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}}^{\bar{a}(\cdot)} d\Lambda\left(a|d,\zeta\right) + \int_{\bar{a}(\cdot)-\zeta\Delta'\mathcal{Y}}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) - 2\int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)} d\Lambda\left(a|d,\zeta\right) \\ &= \int_{\bar{a}(\cdot)-\zeta\Delta'\mathcal{Y}}^{\bar{a}(\cdot)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) - 2\int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right)\end{aligned}$$

Thus,

$$\eta\left(\Delta,\Delta'\right) = \int_{\bar{a}(\cdot)-\zeta\Delta'\mathcal{Y}}^{\bar{a}(\cdot)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}} d\Lambda\left(a|d,\zeta\right) - \int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\Lambda\left(a|d,\zeta\right)$$

Using the change of variable $a' \equiv a + \zeta\frac{\Delta'-\Delta}{2}\mathcal{Y}$ for the first integral,

$$\eta\left(\Delta,\Delta'\right) = \int_{\bar{a}(d,\zeta)-\zeta\frac{\Delta'+\Delta}{2}\mathcal{Y}}^{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}} d\hat{\Lambda}\left(a|\,d,\zeta\right) \tag{1.B.33}$$

where $\hat{\Lambda}\left(a|\,d,\zeta\right) \equiv \Lambda\left(a-\zeta\frac{\Delta'-\Delta}{2}\mathcal{Y}\middle|\,d,\zeta\right) - \Lambda\left(a|\,d,\zeta\right)$, with an abuse of notation. Then, $\eta\left(\Delta,\Delta'\right) > 0$ since the density $d\Lambda\left(a|d,\zeta\right)$ is decreasing at the adjustment threshold $\bar{a}\left(\cdot\right)$, by assumption. The convexity of the impulse response follows from (1.B.30)–(1.B.33).

Derivations for Example 2. From (1.4.4) and by definition of the adjustment threshold (1.4.2), the extensive margin satisfies:

$$\begin{aligned}\Sigma_1\left(\Delta\right) =& \frac{1}{\mathcal{I}}\frac{1}{a^\star}\int \Big\{ \left[\bar{d}\left(d,\zeta\right) - \left(1-\delta+\iota\delta\right)d\right]\zeta\Delta\mathcal{Y} \\ &+ \kappa\left(d,\zeta\right)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(a+\zeta\Delta\mathcal{Y}-\bar{a}\left(d,\zeta\right)\right)da \\ &+ \int_{\bar{a}(d,\zeta)}^{a^\star} \left(d^\star\left(a+\zeta\Delta\mathcal{Y},d,\zeta\right) - d^\star\left(a,d,\zeta\right)\right)da \Big\}\, d\Lambda^\star\end{aligned} \tag{1.B.34}$$

since the (conditional) distribution of liquid assets is uniform on $[0, a^\star]$ and shocks are sufficiently small, by assumption. By linearity of the adjustment target,

$$\Sigma_1\left(\Delta\right) = \theta\Delta + \frac{1}{\mathcal{I}}\frac{1}{a^\star}\int \kappa\left(d,\zeta\right)\int_{\bar{a}(d,\zeta)-\zeta\Delta\mathcal{Y}}^{\bar{a}(d,\zeta)} \left(a+\zeta\Delta\mathcal{Y}-\bar{a}\left(d,\zeta\right)\right)da\,d\Lambda^\star \tag{1.B.35}$$

where $\kappa(d,\zeta)$ denotes the slope of the adjustment target $d^{\star}(\cdot)$, with

$$\theta \equiv \frac{1}{\mathcal{I}}\frac{1}{a^{\star}}\int\left[\bar{d}(d,\zeta)-(1-\delta+\iota\delta)\,d+\kappa(d,\zeta)\,(a^{\star}-\bar{a}(d,\zeta))\right]\zeta\mathcal{Y}d\Lambda^{\star}$$

Integrating the second term in (1.B.35),

$$\Sigma_1(\Delta)=\theta\Delta+\left[\frac{1}{2}\frac{1}{\mathcal{I}}\frac{1}{a^{\star}}\int\kappa(d,\zeta)\,(\zeta\mathcal{Y})^2\,d\Lambda^{\star}\right]\Delta^2 \tag{1.B.36}$$

The convexity of the impulse response immediately follows from (1.B.36), since $\kappa(d,\zeta)>0$ by assumption.

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Chapter 2

Investment Dynamics and Cyclical Redistribution*

Demand for durable goods and residential investment is strongly pro-cyclical. Workers employed in durable industries are imperfectly insured against these fluctuations, leading to distributional consequences during booms and busts. This paper studies the interaction between cyclical durable demand and redistribution of labor income. I explore this feedback loop within a heterogeneous agent New Keynesian (HANK) model with multiple sectors and lumpy durable adjustment. Crucially, lumpy adjustment at the micro level generates non-linearities at the macro level: the average marginal propensity to spend on durable goods varies with the size of income shocks. As a result, sectoral redistribution of labor incomes has aggregate effects. I find that the interaction between cyclical investment and redistribution amplifies the aggregate response of durable spending during booms and dampens it during recessions. The lumpy nature of durable adjustment entirely accounts for this non-linear effect.

2.1 Introduction

Purchases of durable goods and residential investment are strongly pro-cyclical. Workers employed in the industries producing these goods are imperfectly insured against the fluctuations in durable expenditure, leading to distributional consequences during

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booms and busts (Guvenen et al. (2017)). Microeconomic evidence suggests that these income changes have a high pass-through to durable spending (Dynarski and Gruber (1997); Browning and Crossley (2009)). These two facts suggest the presence of a feedback loop that has not been explored in the literature: the cyclicality of durable expenditure induces a redistribution of labor income between sectors; in turn, this redistribution feeds back into the composition of spending and durable expenditure.

In this paper, I study this feedback loop and its role in the amplification of aggregate shocks. My starting point is a general equilibrium model of lumpy durable demand with incomplete markets (Berger and Vavra (2015)) and nominal rigidities. The main novelty lies in the presence of distributional effects. I recognize that consumption and investment goods are produced in different sectors. Redistribution stems from two features. First, durable spending is more elastic to aggregate shocks. This translates into a more cyclical demand for labor demand in durable industries. Second, labor mobility is restricted between sectors. Households employed in the durable sector fail to relocate and are disproportionately affected by these cyclical fluctuations.

Cyclical income inequality between sectors has aggregate effects in my model despite individuals having homogeneous preferences. The reason is that the response of aggregate durable spending to income shocks is non-linear in their size. This non-linearity is induced by infrequent and discontinuous adjustment of durables at the microeconomic level.[1,2] Specifically, the average marginal propensity to spend (MPC) on durable goods increases with income changes.[3] During expansions, households employed in the durable sector experience a disproportionate increase in income, and their MPCs on durables rise endogenously. As a result, cyclical income inequality amplifies the increase in durable spending during expansions. On the contrary, it dampens its decline during recessions.

The main contribution of this paper is to explore the interaction between cyclical income inequality and lumpy durable investment. Specifically, I embed a canonical model of lumpy durable investment in a multi-sector heterogeneous agent New Keynesian framework (HANK). Durable spending is strongly pro-cyclical, which effectively redistributes labor income between sectors in general equilibrium. Financial markets are incomplete, which limits risk sharing across households employed in different sectors.[4] To assess

[1] See Bertola and Caballero (1990) for a review on investment subject to lumpy adjustment. Fixed adjustment costs are essential to explain the micro behavior of durable investment (Eberly (1994); Attanasio (2000)), and its aggregate time series properties (Caballero (1990, 1993, 1994)).

[2] In particular, this form of discontinuities has been shown to produce aggregate non-linearities in the context of capital investment (Caballero et al. (1997); Caballero and Engel (1999)) and price setting (Alvarez et al. (2017a,b)).

[3] In a companion paper (Zorzi (2020)), I study the determinants of this non-linearity and I quantify its importance in this model.

[4] Participation in financial markets is limited (Mankiw and Zeldes (1991); Heaton and Lucas (2000)), and

the role of redistribution, I compare the aggregate response of durable investment in my model to that obtained in a counterfactual economy where fiscal policy undoes this redistribution and provides cross-sectoral insurance. I show that a cyclical income redistribution amplifies the effect of expansionary shocks and dampens contractionary shocks when the average MPC on durable investment is increasing with income changes.

An "earnings heterogeneity channel" (Auclert (2019); Werning (2015); Patterson (2019)) emerges endogenously in my setting. However, my model is set up so that there is no *ex ante* heterogeneity in the average MPC on durables across sectors. Redistribution of income between sectors is neutral for first order deviations from the steady state. Instead, I focus on non-linear amplification effects away from the stationary equilibrium. Redistribution drives a wedge across sectors in the households' propensity to adjust their stock of durables. In other words, MPCs on durable goods are heterogeneous *ex post* across sectors. This heterogeneity ensures that cyclical income inequality is non-neutral, and amplifies the response of durable spending during expansions and attenuates it during recessions. I show that state-dependent adjustment plays a crucial role in this form of non-linear amplification.

After having clarified the mechanisms analytically, I assess their quantitative importance in my structural model. Following Berger and Vavra (2015), I focus on productivity shocks. I find that income redistribution stimulates the response of durable spending by roughly 10% over the first year and a half after the occurrence of an expansionary shock, and increases the persistence of this response.

Related literature. My paper lies at the intersection of several strands of the literature on heterogeneous agents model, income redistribution, and multi-sector economies.

First, my paper fits into a growing literature on heterogeneous agent New Keynesian (HANK) models. My approach reconciles two separate strands of this literature: one interested in the role of durable spending and one focused on redistribution of labor income. Falling into the first category, Guerrieri and Lorenzoni (2017) explore the effect of deleveraging shocks in the presence of durable investment with convex adjustment costs. In a recent paper, McKay and Wieland (2019) quantify the effect of monetary policy and forward guidance in a model of lumpy durable investment. I introduce a role for cyclical income redistribution by assuming that labor markets are sector-specific. Households are either employed in the durable or non-durable sector and cannot relocate between them. Durable spending is strongly pro-cyclical, which induces a redistribution of labor income across sectors during booms and busts.

households fail to hedge against (sector-specific) aggregate risk (Massa and Simonov (2006)).

By introducing distributional concerns into a model of durable investment, I draw a connection to the existing literature on redistribution in heterogeneous agent models. Auclert (2019), Werning (2015) and Patterson (2019) explore the role of earnings heterogeneity for the marginal response to aggregate shocks in general equilibrium. My paper complements their work by exploring the non-linear effects of redistribution. In my model, redistribution is neutral for local deviations from the stationary equilibrium. This allows me to focus on higher-order effects. I find that redistribution amplifies expansionary shocks and dampens contractionary shocks. I show that lumpy investment is central to this non-linear amplification.

More broadly, my paper speaks to a literature on business cycle fluctuations in multi-sector economies. A first branch of this literature studies cyclical changes in the composition of spending and their implications for aggregate labor demand. In particular, Bils et al. (2013) and Jaimovich et al. (2019) investigate the interaction between cyclical spending on durable and luxury goods and the relatively low capital intensity in these sectors.[5] A second strand of this literature focuses on real rigidities and amplfication stemming from heterogeneous frequencies of price adjustment across sectors.[6] I set up my model to abstract from these considerations, and isolate the role of redistribution and the non-linearities it produces.

Layout. I start by introducing a multi-sector heterogeneous agent New Keynesian (HANK) model with lumpy investment in Section 2.2. I discuss the sources of non-linearity inherent to this model in Section 2.3, and explore the implications of income redistribution in this setting. I briefly review supporting evidence in Section 2.4. Section 2.5 describes the calibration of the model and the empirical targets. I explore the general equilibrium interaction between income redistribution and aggregate non-linearities in Section 2.6. Section 2.7 concludes. The appendix contains the proofs, and complementary quantitative results.

2.2 A Multi-Sector Model with Lumpy Investment

I introduce a multi-sector heterogeneous agent New Keynesian (HANK) model with lumpy durable investment. There are two sectors producing a consumption good and a durable investment good, respectively. Each household is employed in a given sector and is un-

[5] Alonso (2016) quantifies this effect in a heterogeneous agent model.
[6] References include Carvalho (2006), Nakamura and Steinsson (2010) and Pastén et al. (2018).

able to relocate betweem them.[7] The demand side of the economy builds on the canonical model of durable demand with incomplete markets from Berger and Vavra (2015). The supply side is standard. Prices are sticky à la Calvo (1983), which leads to sector-specific Phillips curves. I describe the environment below. For concision, I only include here the main expressions. Appendix 2.A provides a full description of the full model.

2.2.1 Environment

Time is discrete, and there is no aggregate uncertainty.[8] Periods are indexed by $t \in \{0, 1, \ldots\}$. The two goods are indexed by $h \in \mathcal{H} \equiv \{c, d\}$. Sector $h = c$ produces the non-durable consumption good, and sector $h = d$ produces the durable investment good.

Households. The economy is inhabited by a continuum of mass 1 of households. Each household is assigned permanently to a given sector h. Households are characterized by four idiosyncratic states: their financial asset holdings (a), their holdings of durable goods (d), their idiosyncratic labor supply shock (ζ), and the sector they are employed in (h). The mass of households in each sector is denoted by $\boldsymbol{\mu} \equiv \{\mu^h\}_h$.

Households consume durable and non-durable goods. Preferences are represented by

$$\mathbb{E}\left[\sum_{t\geq 0} \beta^t \frac{u\left(c_t, d_t\right)^{1-\sigma}}{1-\sigma}\right]$$

with discount factor $\beta \in (0, 1)$, and inverse elasticity of substitution $\sigma > 0$. Intratemporal preferences exhibit constant elasticity of substitutiton:

$$u\left(c, d\right) = \left[\vartheta^{\frac{1}{\nu}} c^{\frac{\nu-1}{\nu}} + (1-\vartheta)^{\frac{1}{\nu}} d^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$$

with share parameter $\vartheta \in (0, 1)$ and elasticity of substitution $\nu > 0$.

After observing their idiosyncratic labor income, households decide whether to adjust their stock of durable goods. Adjustment entails a non-convex cost Γ_t. Following Berger and Vavra (2015) and Kaplan et al. (2017), I assume that durable adjustment costs are proportional to the nominal value of the undepreciated stock of durable goods. If households do not adjust, they pay a maintenance cost, i.e. an investment required to repair or operate

[7] There is evidence of some cyclical reallocation between durable and non-durable sectors over the cycle (Loungani and Rogerson (1989)). In a separate paper (**?**), I allow for labor mobility between occupations in a heterogeneous agent, incomplete markets model.

[8] I focus on the effect of one-time, unanticipated but persistent shocks.

the existing stock of durables.[9] This maintenance corresponds to a share $\iota \in [0,1]$ of the current depreciation of their stock of capital. Summing up, the non-convex adjustment costs are

$$\Gamma_t\left(d',d\right) = \begin{cases} (1-\delta)\, P_t^d \gamma d & \text{if } d' \neq (1-(1-\iota)\,\delta)\, d \\ 0 & \text{otherwise} \end{cases}$$

for some adjustment cost $\gamma \geq 0$, where d' denotes the new stock of durables and $\delta \in (0,1)$ denotes the depreciation rate. Here, $\mathbf{P}_t \equiv \left\{P_t^h\right\}_h$ denotes goods prices. I suppose that these adjustment costs take the form of services (real estate, moving, etc.) provided by the non-durable sector, while maintenance takes the form of investment goods (new windows, tires, etc.) purchased from the durable sector[10].

Firms. Firms have access to technologies with decreasing returns:

$$y = F_t^h\left(l\right) \tag{2.2.1}$$

for some concave $F_t^h : [0,1] \to \mathbb{R}_+$ that allows for time-varying producitivity.[11] Here, l denotes firms' individual labor demand.

Nominal rigidities. Prices are flexible at the stationary equilibrium. Firms set prices to maximize profits, subject to technology and given wages. On the contrary, prices are sticky à la Calvo (1983) along the transition path. The idiosyncratic reset probability is denoted by $1-\lambda^h$ in each sector, with $\lambda^h \in [0,1]$. The (implicit) elasticity of substitution across varieties, within each sector, is denoted by $\varepsilon > 1$.

Households supply labor inelastically in their industry of employment. They are compensated in proportion to their idiosyncratic labor supply. Wages are flexible at the stationary equilibrium, but rigid along the transition path: $W_t^h = W^h$. Labor is demand-determined in this case. As a result, earnings in the durable sector are more elastic to aggregate shocks in general equilibrium. The fiscal authority can potentially use lump sum taxes to provide cross-sectoral insurance against these aggregate shocks. Profits are redistributed symmetrically across households. Summing up, total individual (gross) in-

[9] Maintenance is a standard feature of lumpy adjustment models (Bachmann et al. (2013); Berger and Vavra (2015)). It decreases the effective depreciation rate in the case of no adjustment. Fixing the depreciation rate, adjustment costs and idiosyncratic risk, a higher maintenance decreases the average frequency of adjustment.

[10] This assumption is mostly innocuous. Spending on adjustment costs and maintenance are relatively acyclical in my calibrated model, compared to purchases of durable and non-durable goods.

[11] Implicitly, I suppose that capital is firm-specific and fixed in the short-run.

comes are

$$e_t^h(\zeta) = \frac{1}{\mu^h}\zeta\left(\mathcal{Y}_t^h - T_t^h\right) + \pi_t, \tag{2.2.2}$$

where $\boldsymbol{\mathcal{Y}}_t \equiv \left\{\mathcal{Y}_t^h\right\}_h$ denotes the sector-specific wage bills, $\mathbf{T}_t \equiv \left\{T_t^h\right\}_h$ denotes lump sum taxes and π_t denotes aggregate profits claimed by households. The idiosyncratic process for labor supply follows a Markov chain with transition kernel Σ on some set $S \subset \mathbb{R}_{++}$, with $\mathbb{E}[\zeta] = 1$ and independence across households.

Policy. The government can potentially use lump sum taxes $\mathbf{T}_t$ to provide cross-sectoral insurance. It maintains an exogeneous and constant debt level $B > 0$ and sets linear taxes on income $\boldsymbol{\tau}_t$ to balance its flow budget. Monetary policy implements a standard Taylor rule:

$$i_t = \max\left\{r + \varphi^\pi\left(\Pi_t - 1\right), 0\right\}, \tag{2.2.3}$$

where Π_t denotes gross inflation[12] and r denotes the nominal interest rate at the stationary equilibrium.

2.2.2 Households' Optimization

The households' problem can be formulated recursively. The durable adjustment choice solves:

$$\mathcal{V}_t^h(a, d, \zeta) = \max_{\mathcal{A} \in \{0,1\}}\left\{V_t^h(a, d, \zeta; \mathcal{A})\right\}, \tag{2.2.4}$$

where $\mathcal{A} \in \{0, 1\}$ denotes the adjustment decision and $V_t^h(\cdot; \mathcal{A})$ denotes the continuation values associated to each adjustment option.

The value associated with no adjustment is

$$\begin{aligned} V_t^h(a, d, \zeta; 0) = \max_{\{c, a'\}} \; & \frac{u(c, d^\star)^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a', d^\star, \zeta')\middle|\,\zeta\right] \\ \text{s.t. } & P_t^c c + P_t^d \iota \frac{\delta}{1-(1-\iota)\delta} d^\star + a' \leq (1-\tau_t)\, e_t^h(\zeta) + (1 + r_{t-1})\, a \\ & a' \geq 0, \end{aligned} \tag{2.2.5}$$

with $d^\star \equiv (1 - (1-\iota)\delta)\, d$. Here, $e_t^h(\zeta)$ denotes labor income, and r_{t-1} denotes the nominal interest rate. Earnings are indexed by the idiosyncratic labor supply and the industry of employment.

12 See Appendix 2.A.1 for the definition of the price index.

Similarly, the value associated with adjustment is

$$V_t^h(a,d,\zeta;1) = \max_{\{c,a',d'\}} \frac{u(c,d')^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a',d',\zeta')\Big|\zeta\right] \tag{2.2.6}$$

$$\text{s.t. } P_t^c c + P_t^d\left(d' - (1-\delta)d\right) + \Gamma_t\left(d',d\right) + a' \leq \left(1-\tau_t^h\right)e_t^h(\zeta) + (1+r_{t-1})a$$

$$a' \geq 0$$

2.2.3 Earnings and Insurance

Households' gross earnings (2.2.2) consist of labor income, aggregate profits, and transfers from the government. Wage bills are given by

$$\mathcal{Y}_t^h \equiv W^h \hat{\mu}_t^h, \tag{2.2.7}$$

where demands for labor $\hat{\boldsymbol{\mu}}_t \equiv \left\{\hat{\mu}_t^h\right\}_h$ satisfy

$$F^h\left(\hat{\mu}_t^h\right) \equiv \Omega_t^h\left(\frac{1}{P_t^h}\right)^{-\varepsilon} Y_t^h \tag{2.2.8}$$

Here $\mathbf{Y}_t$ denotes aggregate demand for each good, and Ω_t^h denotes the productivity distorsion associated with price dispersion.[13] Aggregate profits are

$$\hat{\pi}_t \equiv \sum_h \left(P_t^h Y_t^h - \mathcal{Y}_t^h\right) \tag{2.2.9}$$

Households claim constant profits from firms: $\pi_t = \hat{\pi}$, where $\hat{\pi}$ denotes aggregate profits at the stationary equilibrium.[14] In the absence of insurance from fiscal policy, the strong pro-cyclical in the demand for durable investment Y_t^d translates into a high cyclicality of labor incomes $e_t^h(\zeta)$ in that sector, by (2.2.2) and (2.2.7)–(2.2.8). This leads to a redistribution of income between sectors.

I contrast two regimes to assess the role of redistribution. The first regime is one where fiscal policy is passive and labor income is endogenously redistributed between sectors.

[13] See Appendix 2.A.1 for the definition of this productivity distorsion.

[14] The comparative statics of interest is a persistent productivity shock. In the limit with rigid prices $\lambda^h \to 1$, this shock affects the allocation of revenues between wage bills and profits. Abstracting from heterogeneity in labor supply ζ, a productivity shock has no aggregate effect in this case if profits are rebated every period to households. For this reason, I assume that firms redistribute constant profits in each period, and save the rest by accumulating financial assets.

That is,

$$\mathbf{T}_t = 0 \tag{2.2.10}$$

for each period t.

The second regime (denoted by $\star$) corresponds to a counterfactual economy where fiscal policy undoes this redistribution of labor income using lump sum taxes. Specifically,

$$\frac{\mathcal{Y}_t^d - \mu^d T_t^{d,\star}}{\mathcal{Y}_t^c - \mu^c T_t^{c,\star}} = \frac{\mathcal{Y}^d}{\mathcal{Y}^c}, \tag{2.2.11}$$

with budget balance $\sum_h \mu^h T_t^{h,\star} \equiv 0$, where $\boldsymbol{\mathcal{Y}} \equiv \left\{\mathcal{Y}^h\right\}_h$ denotes wage bills at the stationary equilibrium. This second regime rules out distributional effects and effectively corresponds to the case considered in the literature on lumpy durable investment.

2.2.4 Market Clearing

Markets for goods clear:

$$Y_t^c = \sum_h \mu^h \int \left[c_t^h(a,d,\zeta) + \Gamma_t\left(d_t^{h,\prime}(a,d,\zeta), d\right)\right] d\Lambda_t^h \tag{2.2.12}$$

$$Y_t^d = \sum_h \mu^h \int \left[d_t^{h,\prime}(a,d,\zeta) - (1-\delta)\, d\right] d\Lambda_t^h, \tag{2.2.13}$$

where c_t^h and $d_t^{h,\prime}$ denote the solution to (2.2.5)–(2.2.6), with $d_t^{h,\prime} \equiv d^\star$ when there is no adjustment. Here, $\boldsymbol{\Lambda}_t \equiv \left\{\Lambda_t^h\right\}_h$ denotes the conditional distributions of idiosyncratic states within each sector.

At the stationary equilibrium, wages are flexible and the labor markets clear in the two sectors: $\boldsymbol{\mu} = \hat{\boldsymbol{\mu}}$. Then,[15]

$$Y_t^h = F^h\left(\mu^h\right), \tag{2.2.14}$$

where μ^h denotes the exogeneous mass of households located in sector h. However, the market clearing condition (2.2.14) typically does not hold along the transition path since wages are fixed. Labor demands are demand-determined:

$$\boldsymbol{\mu} = \mathbf{Z}_t \circ \hat{\boldsymbol{\mu}}_t \tag{2.2.15}$$

[15] There is no price dispersion within each sector at the stationary equilibrium, so that $\hat{\boldsymbol{\mu}}$ denotes both aggregate and individual labor demands at the steady state.

with $Z^h \equiv 1$ at the stationary equilibrium, and $0 \le Z_t^h < +\infty$ along the transition path, for each sector h and period t. That is, there is rationing of labor incomes in response to aggregate shocks.

Equilibrium. An equilibrium in my economy consists of sequences of policy functions for consumption and assets, distributions of idiosyncratic states, and prices, incomes and taxes such that households and firms optimize, the government balances its budget (and potentially provides insurance) and monetary policy implements a Taylor rule. I provide a formal definition of an equilibrium in Appendix 2.A.2. The economy is initially at its non-inflationary stationary equilibrium.

The comparative statics of interest is a one-time, unanticipated, and persistent aggregate productivity shock. I assume that this productivity shock is symmetric between the durable and non-durable sectors, so there is no redistribution of labor incomes in partial equilibrium. Instead, redistribution is *induced* in general equilibrium by the pro-cyclicality of durable investment. Financial markets are effectively incomplete with respect to this aggregate productivity shock.

2.3 Non-Linearity and Redistribution

I introduced a multi-sector model with lumpy investment in the previous section. Durable investment plays a dual role in my setting. First, its pro-cyclicality induces a redistribution of labor income between sectors. Second, its lumpiness implies that the average MPC on durable goods depends on the magnitude of income shocks. In this section, I discuss each of these roles, and I show that their interaction acts as a non-linear propagation channel. In Section 2.3.1, I discuss the source of aggregate non-linearities in my model. In Section 2.3.2, I clarify the role of durable investment as a source of labor income redistribution. I then explore the interaction between these two roles and its aggregate implications in Section 2.3.3. I draw a connection to the existing literatures on redistribution with heterogeneous agents in Section 2.3.4.

2.3.1 Lumpy Investment and Aggregate Non-Linearity

Durable adjustment is infrequent and discontinuous in my model. I am interested in the aggregate implications of these microeconomic discontinuities. For now, I abstract from redistribution, so I drop the sector index h.[16]

[16] For notational convenience, I abstract from the masses $\boldsymbol{\mu}$ in (2.2.2).

I consider an exogeneous one-time, unanticipated change in aggregate income in period $t = 0$:

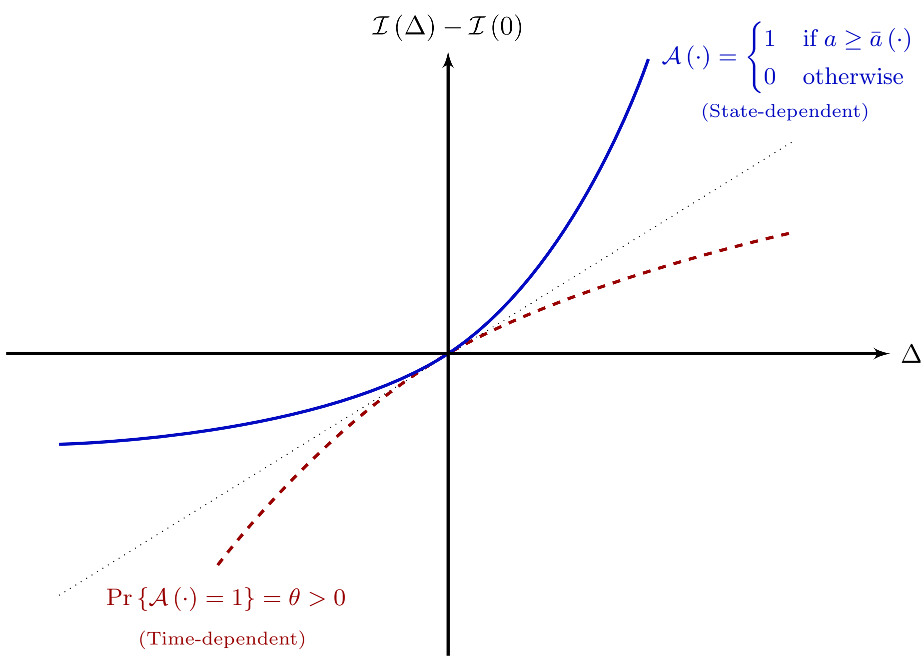
$$\mathcal{Y}_0 = (1 + \Delta)\,\mathcal{Y}$$

for some $\Delta \in \mathbb{R}$, where $\mathcal{Y}$ denotes the aggregate wage bill at the stationary equilibrium. My focus is on the non-linearity of aggregate investment with respect to the size and sign of this income shock. I let[17]

$$I(\Delta) \equiv \int \frac{\partial}{\partial \Delta}\left[d_0'(a + \zeta\Delta\mathcal{Y}, d, \zeta) - (1-\delta)\,d\right] d\Lambda_0\,,\ \overline{\text{MPC}}^d(\Delta) \equiv \frac{d}{d\Delta}\mathcal{I}(\Delta) \qquad (2.3.1)$$

denote aggregate durable investment in the first period and the associated average marginal propensity to spend on durables as functions of the aggregate income shock.

Figure 2.3.1: Response of Aggregate Durable Spending to Income Shocks



In a companion paper (Zorzi (2020)), I show that $\overline{\text{MPC}}^d$ *increases* with the size of income shocks in my model. Put it differently, the response of (aggregate) durable spending to income shocks $I(\Delta)$ is *convex*. I find that this non-linearity is entirely due to adjustment at the extensive margin. Both the behavior of $\overline{\text{MPC}}^d$ and the importance of the extensive margin are in-line with the empirical evidence (Section 2.4). However, the magnitudes predicted in my calibrated model are substantially lower than their empirical counterparts.

[17] The integration is over the idiosyncratic states (a, d, ζ).

Crucially, I find that lumpy durable adjustment together with non-convex adjustment costs, i.e. *state-dependent* adjustment, plays a central role in this form of non-linearity. In contrast, a model with convex adjustment costs or *time-dependent* adjustment predicts that $\overline{\text{MPC}}^d$ *decreases* with the size of income shocks due to a standard precautionary savings motive (Carroll and Kimball (1996)). That is, the response of (aggregate) durable spending to income shocks $I(\Delta)$ is *concave*. This prediction is counterfactual. Figure 2.3.1 contrasts these two cases. As discussed in Section 2.3.3, the exact form that this non-linearity takes plays an essential role in shaping the aggregate effect of income redistribution in my economy.

2.3.2 Redistribution in General Equilibrium

I now explore the role of durable investment for the redistribution of labor income in general equilibrium. Keeping with the rest of the paper, the comparative statics is a one-time, unexpected, and persistent aggregate productivity shock. I assume that this productivity shock is not targeted toward any particular sector, so that redistribution of labor income is entirely induced by endogeneous changes in households' spending. That is, $A_t^h / A^h = \hat{A}_t$ for each sector h, with

$$\log\left(\hat{A}_t\right) = \rho^A \log\left(\hat{A}_{t-1}\right) + \psi_t \tag{2.3.2}$$

for some persistence $\rho^A \in (0,1)$ and some innovation $\psi_0 \in \mathbb{R}$ in the first period, with $\psi_t \equiv 0$ for each consecutive period $t \geq 1$.

Following a productivity shock, the pro-cyclicality of durable spending induces a redistribution of labor income. I impose some restrictions to avoid introducing additional sources of redistribution. In particular, I assume that technologies and price stickiness satisfy a certain degree of symmetry across sectors.[18]

Assumption 4 (Technologies)**.** *Technologies are isoelastic and the elasticity is symmetric across sectors:* $F_t^h(l) = A_t^h l^\alpha$*, for some* $\alpha \in (0,1)$ *and some vector of productivities* $\mathbf{A}_t \equiv \{A_t^h\}_h$*. Productivities at the stationary equilibrium* $\mathbf{A}$ *are such that* $W^c = W^d$*, i.e. wages are symmetric across sectors.*

Assumption 5 (Price setting)**.** *Price stickiness is symmetric across sectors:* $\lambda^h = \lambda \in [0,1)$*.*

[18] This symmetry is somewhat restrictive empirically. In particular, the labor share is higher in the durable good sector, prices of durable goods tend to be more flexible (Bils and Klenow (2004)). These sources of heterogeneity and their role over the business cycle have been studied separately (Jaimovich et al. (2019); Pastén et al. (2018)). I choose to abstract from the sources of heterogeneity, to isolate the role of redistribution induced by cyclical durable investment.

Benchmark. To highlight the role of durable investment for the redistribution of labor income, I first establish a benchmark where the two goods are non-durable. That is, there is full depreciation: $\delta = 1$. The relative demand for the two goods is acyclical in this case, by homotheticity of the intratemporal preferences. Under Assumptions 4 and 5, there is no redistribution of labor income and thus no role for insurance from fiscal policy. I formalize this point in Proposition 2 (Appendix 2.B.1). This benchmark case clarifies that, in my setting, durability is *necessary* for income redistribution to take place.[19] Furthermore, it confirms that Assumptions 4 and 5 are as neutral as possible: the supply side does not induce any redistribution *per se*. I maintain these two assumptions throughout the paper.

Redistribution. Having established this benchmark, I now assume partial depreciation: $\delta \in (0,1)$. Durable spending is more cyclical than non-durable spending, which induces a redistribution of labor income across sectors. This redistribution depends on the elasticity of durable investment with respect to income and price changes. In particular, these elasticities are governed by: the elasticity of substitution between durables and non-durables ν; the adjustment cost γ; and the maintenance parameter ι.[20] In virtually every realistic calibration, durable investment is more cyclical than non-durable consumption. I assume that this is the case.

Assumption 6 (Monotonicity)**.** *The parameters characterizing the income fluctuations problem (2.2.4)–(2.2.6) are such that relative demand for durable invesment* $\left(d_t^h(\cdot) - (1-\delta)\, d\right)/c^h(\cdot)$ *and non-durable consumption* $c^h(\cdot)$ *increase with liquid assets.*

To understand the pro-cyclicality of relative spending on durable goods, consider a representative-agent version of the model presented in Section 2.2.[21] In this case,

$$\frac{d_t - (1-\delta)\, d_{t-1}}{c_t} = \frac{1-\vartheta}{\vartheta} \underbrace{\left(\frac{P_t^c}{P_t^d - \frac{(1-\delta) P_{t+1}^d}{1+r_t}} \right)^{\nu}}_{\text{User cost}} - \underbrace{(1-\delta)\, \frac{d_{t-1}}{c_t}}_{\text{Stock / flow}}$$

at optimum. A productivity shock of the form (2.3.2) affects the relative demand for the durable investment good through two channels. First, it raises incomes in general equilibrium. Non-durable consumption and the *stock* of durable consumption increase proportionately. Mechanically, the *flow* of durable investment is much more volatile. In the first

[19] This result is not specific to productivity shocks: the same obtains for monetary policy shocks, deleveraging shocks or symmetric tax rebates across sectors.

[20] For instance, durable demand is inelastic to income changes as $\gamma \to +\infty$. Similarly, the interest rate elasticity of durable demand is zero as $\nu \to 0$.

[21] Specifically, there is no idiosyncratic risk ($\Sigma \to_d 0$) and durable adjustment is frictionless ($\gamma = 0$).

place, this increase in incomes is induced by a fall in the real interest rate, when monetary policy is sufficiently responsive ($\varphi^{\pi} > 1$). This decrease in the households' *user cost* of durables tilts spending in favor of durable investment[22]. This second effect is modulated by the elasticity of substitution between durables and non-durables (ν).

2.3.3 Aggregate Amplification

In Sections 2.3.1 and 2.3.2, I clarified the dual role of lumpy durable investment in my model: its pro-cyclicality induces income redistribution; and micro discontinuities imply macro non-linearities. I now explore the interaction between these two properties, and the non-linear amplification it generates.

To assess the role of redistribution, I contrast the aggregate response of my economy under the two regimes (2.2.10)–(2.2.11) for fiscal policy. In the first regime, income redistribution takes place between sectors and fiscal policy provides no cross-sectoral insurance. In the second regime, fiscal policy undoes this redistribution and provides full (aggregate) insurance. For tractability, I focus on the general equilibrium response in period $t = 0$, keeping the sequence of labor incomes, aggregate profits, taxes, interest rates and sector-specific inflation $\mathbf{X}_t \equiv (\boldsymbol{\mathcal{Y}}_t, \pi_t, \tau_t, \mathbf{T}_t, r_t, \boldsymbol{\Pi}_t)$ fixed for each period $t > 1$. I also abstract from changes in the relative price of durable goods[23] by assuming that prices are rigid: $\lambda \equiv 1$. I suppose in the following that $\overline{\text{MPC}}^d$ increases with the size of income shocks, consistently with the empirical evidence. Again, I include the derivations in Appendix 2.B.2.

Example 4 (Amplification)**.** Consider an expansionary productivity shock: $\psi_0 < 0$.[24] Labor demand increases in both sectors, and so does spending on both goods in partial and general equilibrium, by Assumption 6. However, durable spending is more cyclical so that $\mathcal{Y}_0^d / \mathcal{Y}^d > \mathcal{Y}_0^c / \mathcal{Y}^c$, i.e. labor income is redistributed in favor of the durable sector.

Now, suppose that fiscal policy undoes this redistribution by providing aggregate insurance, using lump sum taxes (2.2.11),

$$T_0^d = -\frac{\mu^c}{\mu^d} T_0^c > 0$$

These transfers reduce the dispersion in the distribution of labor incomes. Assume that

[22] This second channel is particularly powerful in typical models of investment. Existing estimate suggest that this effect is overstated (Hall (1977); Shapiro (1986); McKay and Wieland (2019)).

[23] The relative price of durable goods is mostly acyclical in the data (Pistaferri (2016); Cantelmo and Melina (2018); McKay and Wieland (2019)).

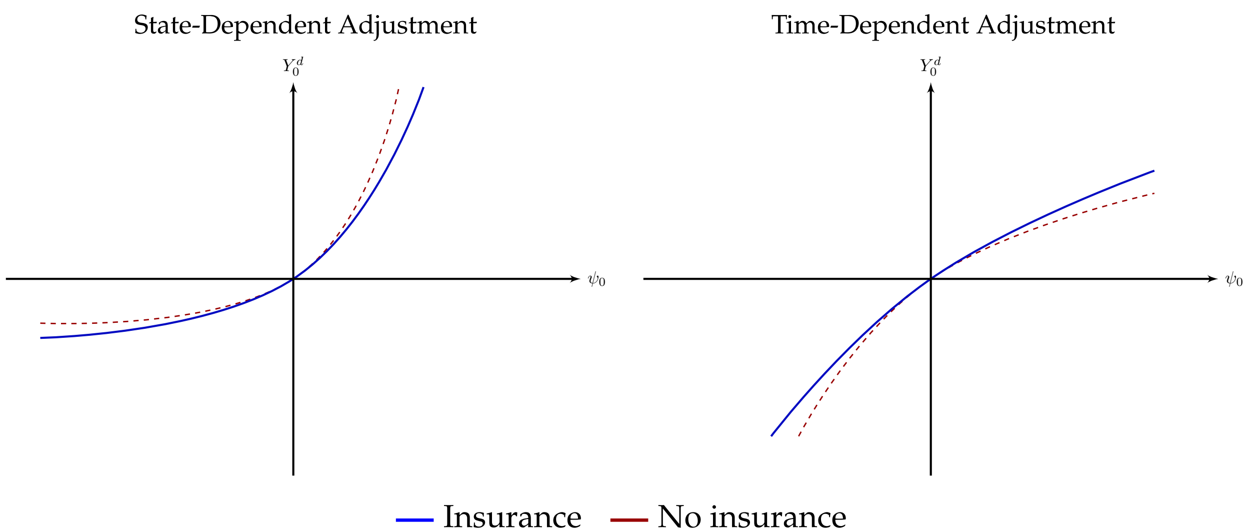
[24] Note that a *positive* productivity shock ($\psi_0 > 0$) is *contractionary* with rigid prices, since households claim constant profits from firms in my setting. The evidence on the effect of technology improvements on labor demand is mixed. References include Basu et al. (2006) and Alexopoulos (2011), among others.

aggregate durable spending by households employed in each sector

$$Y_0^{d,h} \equiv \int \left[d_0^{h,\prime}(a,d,\zeta) - (1-\delta)\, d \right] d\Lambda_0^h$$

is *convex* in (sectoral) incomes,[25] for each sector h. By Jensen's inequality, a lower dispersion due to fiscal policy depresses aggregate durable investment $Y_0^d \equiv \sum_h \mu^h Y_0^{d,h}$. Put it differently, endogeneous income redistribution between sectors *amplifies* the effect of an expansionary productivity shocks, compared to a benchmark with full aggregate insurance. Symmetrically, income redistribution *dampens* the effect of contractionary shocks.

Figure 2.3.2: Impulse Response of Aggregate Durable Spending



Lumpy and state-dependent adjustment is key to this non-linear amplification. Figure 2.3.2 contrasts the predictions of models with state- and time-dependent adjustment. As illustrated in Figure 2.3.1, the response of (aggregate) durable spending is concave in (sectoral) incomes with time-dependent adjustment. Therefore, endogeneous income redistribution depresses aggregate demand. As a result, cyclical income inequality *dampens* the response of durable spending to expansionary shocks, and *amplifies* the response to contractionary shocks. A realistic micro-foundation of the durable adjustment hazard is thus crucial to understand the effect of income redistribution on aggregate durable spending.

[25] Incomes are implicit to the time index.

2.3.4 Sufficient Statistics

The effect of income redistribution in my model can be understood through a sufficient statistics approach. To simplify the exposition, I suppose that the average income is symmetric across sectors at the stationary equilibrium. Furthermore, I focus on the first period ($t = 0$), by asumming that fiscal policy provides full (aggregate) insurance across sectors in the following periods ($t \geq 1$).[26]

Aggregate durable spending (2.2.13) can be decomoposed as follows:[27]

$$Y_0^d = \underbrace{\sum_h \mu^h \int \mathcal{I}_0 \left(a, d, \bar{e}_0 \left(\zeta\right)\right) d\Lambda^h}_{\text{(Aggregate) insurance}} + \underbrace{\sum_h \mu^h \int_0^{e_0^h - \bar{e}_0} \overline{\text{MPC}}_0^d \left(z\right) dz}_{\text{Redistribution}} \tag{2.3.3}$$

Here, $\bar{e}_0 \left(\zeta\right) \equiv \sum_h \mu^h e_0^h \left(\zeta\right)$ denotes the average income across sectors conditional a level of productivity, $e_0^h \equiv \int e_0^h \left(\zeta\right) d\Lambda^h$ denotes average income within a sector, $\bar{e}_0 \equiv \int \bar{e}_0 \left(\zeta\right) d\Lambda$ denotes the average income across sectors and $\mathcal{I}_0$ denotes investment at the household level as a function of current income.

The first term in (2.3.3) corresponds to aggregate durable spending when fiscal policy provides full (aggregate) insurance, i.e. (2.2.11) holds. In turn, the second term captures the effect of income redistribution when fiscal policy is passive, i.e. (2.2.10) holds. This term depends on two endogeneous objects: the extent of income redistribution between sectors $e_0^h - \bar{e}_0$; and the response of the average marginal propensity to spend on durables $\overline{\text{MPC}}_0^d$ to (sectoral) income changes. The behavior of these two objects was discussed in Sections 2.3.1 and 2.3.2. Incomes are more elastic to productivity shocks in the durable sector, and the average MPC on durables increases with the size of income changes. I now put these pieces together to understand how endogeneous income redistribution affects the response to aggregate shocks.

To assess the role of income redistribution, I start by comparing the first order responses of durable spending to a productivity shock under the two regimes of interest:[28]

$$\frac{d}{d\psi} \left(Y_0^d \left(0\right) - Y_0^{d,\star} \left(0\right)\right) = \mathbb{C}\text{ov} \left(\frac{d}{d\psi} \left(e_0^h \left(0\right) - \bar{e}_0 \left(0\right)\right), \overline{\text{MPC}}_0^d \left(0\right)\right)$$

where the covariance is over sectors h. My model is set up so that there is no *ex-ante*

[26] That is, incomes are symmetric across sectors for $t \geq 1$ conditional on a level of productivity (ζ).

[27] With full (aggregate) insurance for periods $t \geq 1$, households' current income (e) is a sufficient statistics for their productivity and sector of employment.

[28] I make the dependence of durable spending and incomes on the innovation in productivity (ψ) in (2.3.2) explicit. Again, the full (aggregate) insurance regime is denoted by $\star$.

heterogeneity in average MPCs on durables between sectors: $\overline{\text{MPC}}_0^{d,c} = \overline{\text{MPC}}_0^{d,d} \equiv \overline{\text{MPC}}_0^{d}$ at the stationary equilibrium.[29,30] In other words, the "earnings heterogeneity channel" studied by Auclert (2019), Werning (2015) and Patterson (2019) has no aggregate effects in my model for first order deviations from the stationary equilibrium. As a consequence, the response of durable spending does not depend on the degree of insurance from fiscal policy for *infinitesimal* aggregate shocks.

I now consider a discrete, expansionary shock ($\psi < 0$).[31] In this case, the second term in (2.3.3) is positive when fiscal policy is passive:

$$e_0^d - \bar{e}_0 = -\frac{\mu^c}{\mu^d}\left(e_0^c - \bar{e}_0\right) < 0 \implies \int_0^{e_0^d - e_0^\star} \overline{\text{MPC}}_0^d(z) > -\frac{\mu^c}{\mu^d}\int_0^{e_0^c - e_0^\star} \overline{\text{MPC}}_0^d(z)$$

since $\overline{\text{MPC}}_0^d$ is (strictly) increasing. Those who benefit from income redistribution during an expansion are willing to spend more on durables at the margin. Therefore, income redistribution *amplifies* the response of aggregate durable spending to expansionary shocks. The case of contractionary shock ($\psi > 0$) is symmetric. Those who suffer more from income redistribution during a contraction cut back less on durable spending. Therefore, income redistribution *dampens* the response of aggregate durable spending to contractionary shocks. Summing up:

$$Y_0^d(\psi) - Y_0^{d,\star}(\psi) \begin{cases} > 0 & \text{if } \psi < 0 \\ = & \text{if } \psi = 0 \\ < 0 & \text{otherwise} \end{cases}$$

Taken together, these observations explain why the response without insurance defines the *upper envelope* of the response with insurance in the left panel of Figure 2.3.2.

2.3.5 Timing and Redistribution

The analysis above assumed that income shocks were fully transitory. Households are non-Ricardian in my model due to borrowing constraints, so the timing of income shocks

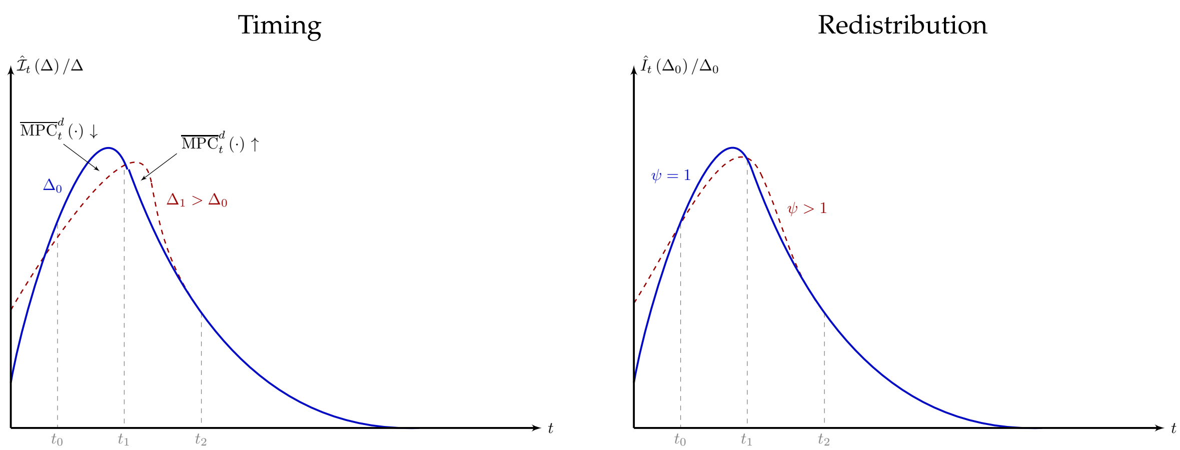
[29] The distribution of financial and durable holdings is symmetric across sectors at the stationary equilibrium, by Assumption 4.

[30] I focus on one-time, unanticipated shocks. If households employed in the durable sector anticipated aggregate risk, they could in theory accumulate higher precautionary savings. There is only limited empirical support for this prediction (Skinner (1988)), which can partly be attributed to heterogeneity in risk-aversion across sectors (Schulhofer-Wohl (2011)). Patterson (2019) finds that heterogeneity in MPCs across sectors plays a relatively minor role in the general equilibrium amplification of aggregate shocks.

[31] Again, a *positive* productivity shock ($\psi_0 > 0$) is *contractionary* with rigid prices.

is actually relevant and affects the profile of the impulse response. In particular, the degree of non-linearity needs not be uniform over time. In Zorzi (2020), I find that the avarage MPC on durables increases on impact as the size of income shocks gets larger, and the response becomes more persistent. However, the average MPC typically decreases for a few quarters in the medium term, as households accumulate savings to finance larger purchases in the following periods. The cumulative response of durable spending increases with the size of income shocks.[32] Put it differently, the impulse response of durable spending (in level) is *convex* on impact and in the longer run, but *concave* for a few quarters after the shock.

Figure 2.3.3: Persistent Income Shocks – Timing and Redistribution



For reference, the left panel of Figure 2.3.3 schematizes this pattern and depicts the impulse response of durable investment $\hat{I}(\Delta)$ normalized by the size of the income change Δ for two possible values of this shock. The timing of the impulse response depends on the magnitude of the shock. Accordingly, income redistribution should have an uneven effect across time. For illustration, suppose that sector-specific wage bills are given by:

$$\frac{\mathcal{Y}_t^d}{\mathcal{Y}^d} = \left(\frac{\mathcal{Y}_t}{\mathcal{Y}}\right)^\eta \quad \text{and} \quad \sum_h \mu^h \mathcal{Y}_t^h = \mathcal{Y}_t \tag{2.3.4}$$

for some elasticity $\eta \geq 1$,[33] where $\mathcal{Y}^{(h)}$ denote wage bills at the stationary equilibrium. That is, durable workers are more exposed to aggregate income shocks. As depicted in the right panel, income redistribution should *amplify* the response of durable spending

[32] Note that the cumulative response of spending would necessarily be linear (for sufficiently long horizons) in the size of income shocks if households consumed a single good. Indeed, the increase in income is eventually spent. This need not be the case with multiple goods, as in my setting.

[33] Guvenen et al. (2017) estimate an elasticity of $\eta \simeq 2$ for durable industries using administrative U.S. data.

on impact and increase its persistence, since the response of durable spending is convex in the size of income shocks over the corresponding range. On the contrary, income redistribution should *dampen* the response of durable spending for a few quarters after the occurrence of the shock. The general equilibrium responses I obtain in Section 2.6 confirm these predictions.

2.4 Supporting Evidence

In this section, I briefly review supporting evidence on the interaction between the cyclicality of durable spending and income redistribution.

2.4.1 Redistribution

Purchases of durable goods and residential investment are strongly pro-cyclical in the data (Kydland and Prescott (1982); Baxter (1996)). Hall (2005) and Bils et al. (2013) document a substantial pass-through to durable employment during booms and busts. Using administrative U.S. data, Guvenen et al. (2017) confirm that expansions and recessions have distributional consequences between workers employed in durable and non-durable sectors.[34]

I illustrate the importance of this sectoral income redistribution in two contexts: the Great Recession; and in response to well-identified, exogeneous shocks. To account for cross-sectoral labor mobility and movements in and out of the labor force, I use longitudinal data from the Panel Study of Income Dynamics (PSID).[35] The PSID is a representative panel survey of U.S. households conducted annually during 1968-1997, and bi-annually since then. The sample consists of male household's heads aged 24-65. To eliminate outliers, I exclude households with labor income lower than 5% of the annual average, and higher than the 95th percentile. I use (real) gross labor incomes as an empirical counterpart to labor earnings in my model.[36] I describe the data, the sample selection, and the specifications in Appendix 2.C.

The left panel of Figure 2.4.1 plots the time series of real spending during the Great

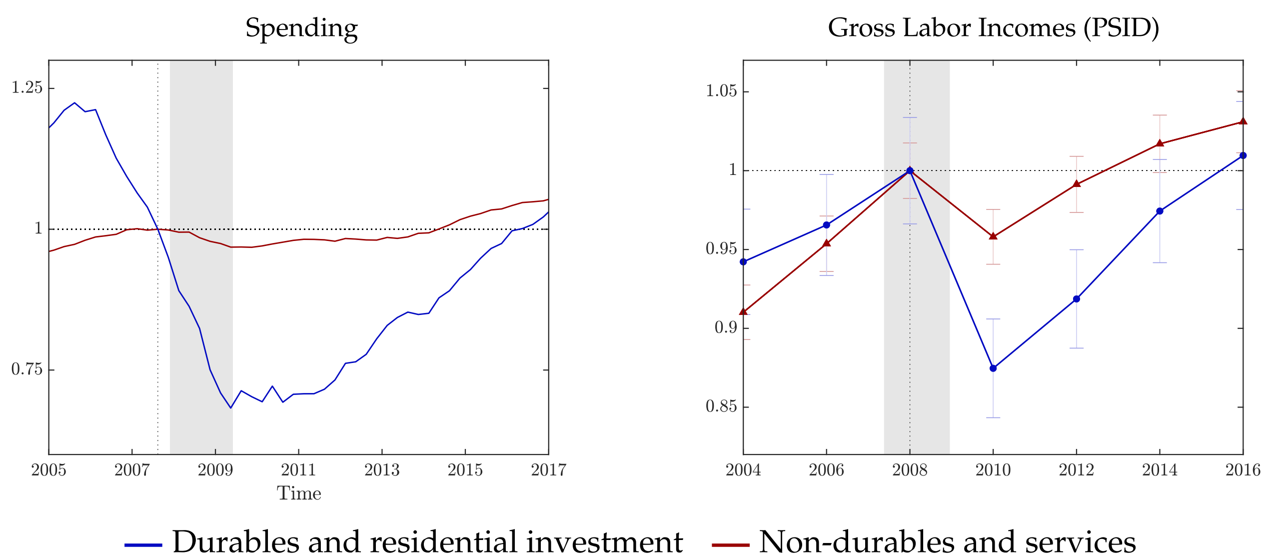
[34] They estimate a contemporaneous elasticity of individual income with respect to GDP of roughly 2 for durable industries.

[35] Other datasets commonly used for the study of income dynamics – such as the Current Population Survey (CPS), the Survey of Income and Program Participation (SIPP), or the National Longitudinal Survey of Youth (NLSY) – either have a shorter panel dimension, or their sample is not as representative of the U.S. population as the PSID's.

[36] I also report the responses for family income in Appendix 2.C.3 to account for unemployment insurance and intra-household risk-sharing. The conclusions are very similar in this case.

Recession for the two categories of interest: durables and residential investment; and non-durables and services. Durable spending fell by roughly 25% over this period, compared to 5% for non-durable spending. The right panel plots the time series of mean labor income for households employed in the corresponding sectors in 2008.[37] The recession led to a substantial redistribution of labor income between sectors. The pass-through was incomplete, however. Labor incomes in durable sectors decreased by 13% over two years, compared to a 4% decline in non-durable sectors.

Figure 2.4.1: Great Recession



For robustness, I estimate the sector-specific response of labor incomes to well identified, exogeneous shocks. The leading example in my paper is an unanticipated productivity shocks. Fluctuations in measured productivity are potentially endogeneous, however.[38] To address this concern, I focus instead on narratively-identified, exogeneous policy shocks. I use the series of exogeneous tax changes constructed by Romer and Romer (2010),[39] which are sufficiently persistent to be aggregated at the low-frequency of the PSID data.

The left panel of Figure 2.4.2 plots the response of spending on durables and non-durables following a one-standard deviation contractionary Romer and Romer (2010) tax

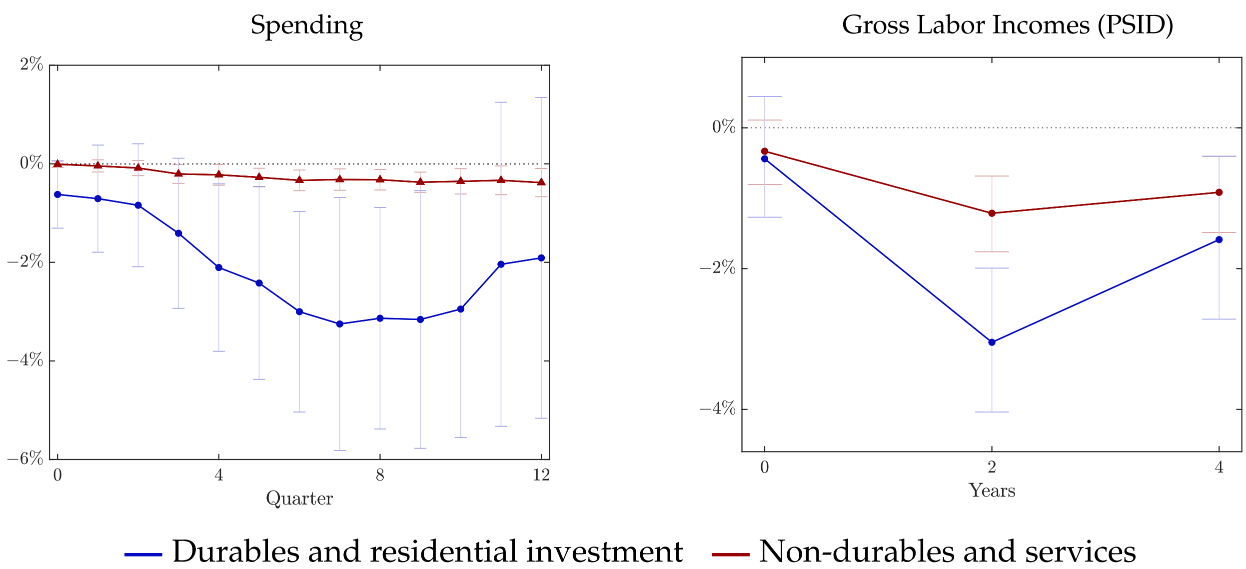
[37] Specifically, I allocate households to either the durable or non-durable sectors based on their industry of employment in 2008 (i.e. 2009 PSID wave). Fixing this cross-section, I compute mean labor income for each year.

[38] See Chari et al. (2007) and Buera and Moll (2015), among others. Productivity shocks identified via Structural Vector Auto-Regression (SVAR) might not be exogeneous either (Ramey (2011)).

[39] I prefer tax changes to government spending shocks (Ramey (2011)). Those are typically targeted toward a particular sector, which mechanically induces redistribution.

shock. Not suprisingly, durable spending is more elastic to these policy changes than non-durable spending.

Figure 2.4.2: Response to Exogeneous Tax Increase



To assess the importance of income redistribution in response to these policy changes, I specify the following moment condition:

$$X^{h,j}_{t+s} - X^{h,j}_{t-1} = \alpha^{h,j}_s + \psi^h_s s^\star_t + \mathbf{Z}^h_{t-1}\theta^h_s + \eta^{h,j}_{t,s} \quad \text{for each} \quad s \in \{0, 1, \ldots, S\} \tag{2.4.1}$$

together with the standard orthogonality condition. Here, $h \in \{c, d\}$ denotes the sector of employment in the previous period $t-1$, j indexed individuals, and s indexes the horizon of the impulse response. The variable X_t denotes labor income (in log), $s^\star_t$ corresponds to the external instrument of Romer and Romer (2010), and $\mathbf{Z}^h_t$ denotes the set of control variables. The coefficients of interest are $\{\hat{\psi}^h_s\}_{h,s}$, i.e. the cumulative impulse responses for each sector at various horizons. I specify (2.4.1) at the bi-annual frequency of the PSID. The set of control variables includes a sector-specific cubic time trends and 12 lags of the fiscal policy shock.

The right panel of Figure 2.4.2 plots these impulse responses, together with 90% confidence bands.[40] The pattern is similar as in the Great Recession: durable investment is strongly pro-cyclical, which leads to a substantial redistribution of labor income.

[40] Confidence intervals (90%) are bootstrapped (200 replications) to account for heteroskedasticity and serial correlation.

2.4.2 Increasing MPCs

In my model, the average MPC out of an income shock increases with the size of this shock (Section 2.3.1). This property ensures that cyclical income inequality has aggregate effects.

The literature estimates a wide range of MPCs on durables. In particular, Johnson et al. (2006) find negligible spending multipliers on durable goods in the context of the 2001 stimulus payment. On the contrary, Souleles (1999) estimates that the response of spending to springtime tax refunds is almost entirely driven by durable expenditure (and purchases of vehicles in particular), while Parker et al. (2013) document sizeable multipliers following the larger stimulus payment in 2008. Parker et al. (2013) attribute the difference in MPCs across these episodes to the size of the corresponding transfers. These findings are in-line with the survey evidence of Fuster et al. (2018) and Christelis et al. (2019), who document that the average MPC on durables increases with the size of (hypothetical) tax rebates.

In Zorzi (2020), I compare the predictions of my model with the existing empirical evidence. I find that the model generates the correct form of non-linearity. However, the magnitudes are substantially lower than those observed in the data. This small degree of non-linearity artificially dampens the general equilibrium effect of income redistribution in my structural model. As a consequence, the effect that I obtain in my general equilibrium exercise (Section 2.6) should be thought of as very conservative lower bounds on the actual effect.

2.5 Calibration

The remainder of the paper quantifies the mechanisms documented in Section 2.3. I first parametrize the model using a mix of external and internal calibration. Following Berger and Vavra (2015), I adopt a broad definition of durable goods that includes residential investment and consumer durables.

I calibrate five parameters internally: the discount factor (β), the preference parameter for non-durable goods (ϑ), the durable adjustment costs (γ), the exogeneous supply of liquidity (B), and the relative productivity in the durable sector $\left(A^d\right)$. I calibrate the remaining parameters externally, using standard values in the literature. I first review the external calibration, before discussing the targeted moments and the fitted parameters. Table 2.5.2 describes the parametrization. Data sources are listed in Appendix 2.C.1.

2.5.1 External Calibration

I set the inverse elasticity of intertemporal substitution to $\sigma = 4$. This value, while large compared to typical calibrations, is commonly used in models with durable goods (Guerrieri and Lorenzoni (2017); McKay and Wieland (2019)), which predict a high elasticity of durable investment to interest rate changes. I choose a unitary elasticity of substitution between durables and non-durables $\nu \to 1$, following Berger and Vavra (2015).[41] I choose a maintenance parameter of $\iota = 0.5$, which lies between the estimates of Berger and Vavra (2015) $(\iota = 0.8)$ and McKay and Wieland (2019) $(\iota = 0.35)$.

The stock of durable goods depreciates at roughly 2% $(\delta = 0.018)$. The income process (in log) follows an AR(1) process with persistence $\hat{\rho} = 0.975$ and standard deviation of innovations $\hat{\sigma} = 0.1$ to match the evidence of Floden and Lindé (2001). I set the mass of households in the durable sector to $\mu^d = 0.187$, based on data from the Current Employment Statistics (see Appendix 2.C.1). Based on Assumptions 4 and 5, I assume a certain degree of symmetry between sectors. Technologies are isoelastic and the elasticity is symmetric across sectors. I normalize productivity to 1 in the non-durable sector. Labor receives roughly 2/3 of revenues at the stationary equilibrium $(\alpha = 0.3)$. Similarly, price stickiness is symmetric across sectors. With imperfectly sticky prices $\left(\lambda^h < 1\right)$, the cyclicality of durable spending induces substantial changes in the relative price of durable goods. This prediction is not verified in the data, however.[42] Furthermore, models of lumpy investment typically predict an excessively high elasticity to changes in user cost.[43] For this reason, I assume that prices are fixed $\lambda^h = 1$ for my general equilibrium exercise (Section 2.6). Note that the elasticity of substitution across varieties $(\varepsilon = 10)$ and the coefficient in the monetary policy rule $(\varphi = 1.25)$ are irrelevant in this case.

2.5.2 Internal Calibration

The remaining parameters $\left(\beta, \vartheta, \gamma, B, A^d\right)$, together with the interest rate (r) and the price of the durable good $\left(P^d\right)$ at the stationary equilibrium, are the implicit solution to seven restrictions: two equilibrium conditions, and five empirical moments. The moments I target, their values and the sources I use are listed in Table 2.5.1.[44] Appendix 2.A.4 describes the calibration strategy in more details.

[41] The literature has typically used a unitary elasticity, based on the estimates of Ogaki and Reinhart (1998) and Piazzesi and Schneider (2007). However, values below unity are sometimes used to dampen the interest rate elasticity of durable demand (Barsky et al. (2016); McKay and Wieland (2019)).

[42] See Pistaferri (2016), Cantelmo and Melina (2018) and McKay and Wieland (2019), among others.

[43] House (2014) and Winberry (2019) make this point in the context of capital investment, and McKay and Wieland (2019) in the context of durable spending.

[44] In Table 2.5.1, "relative" refers to the ratio of the value for durable to that for non-durable.

I choose the discount factor β to target a real interest rate at the stationary equilibrium of 2.5%, which corresponds to the average value since 1970. I set the preference parameter for non-durable goods ϑ to obtain a ratio of investment to consumption of roughly 18%. I adjust the durable adjustment parameter γ to target an annual frequency of adjustment of 12%, as in Berger and Vavra (2015). I obtain $\gamma = 0.025$, which corresponds to an adjustment cost of 3.3% when expressed in terms of the *numéraire*.[45] The exogeneous supply of liquidity (B) targets a ratio of liquidity to annual GDP of 1.4, following McKay et al. (2016).[46] Finally, I choose the relative productivity in the durable goods sector A^d to target symmetric wages across sectors at the stationary equilibrium.

Table 2.5.1: Targeted Moments

Target	Value	Source
Real interest rate (A)	0.025	FRED and NIPA
Ratio of investment to consumption	0.18	NIPA
Frequency of durable adjustment (A)	0.10	Berger and Vavra (2015)
Liquidity supply to GDP (A)	1.4	McKay et al. (2016)
Relative wages	1	Normalization

2.6 General Equilibrium

In the previous sections, I discussed the source of non-linearities in my model and how it interacts with income redistribution in general equilibrium. I now explore the quantitative importance of this mechanism.

I simulate the response of my general equilibrium model to aggregate disturbances. Following Berger and Vavra (2015), I focus on productivity shocks. Income redistribution between sectors takes place endogeneously in my model, as durable investment responds more strongly than non-durable consumption.

The left panel of Figure 2.6.1 plots the general equilibrium responses of durable investment and non-durable consumption to an persistent, expansionary productivity shock of 5% with a half-life of 6 quarters. As expected, the impulse response of durable spending is

[45] This figure is slightly lower than the 5% typically used in the literature (Díaz and Luengo-Prado (2010); Berger and Vavra (2015)). However, the relative price of durables is larger than 1 in my model, which reduces the frequency of adjustment for any fixed cost.

[46] Liquidity includes: deposits, government-issued securities, corporate bonds and equitities and mutual fund shares.

Table 2.5.2: Calibration

Parameter	Description	Calibration	Source / Target
Preferences			
β	Discount factor	0.985	Internal calibration
ν	Elasticity of substitution	1	See text
ϑ	Non-durable parameter	0.731	Internal calibration
σ	EIS (inverse)	4	See text
ε	Elasticity of substitution (inverse)	10	Kaplan et al. (2018)
Durable goods			
δ	Depreciation rate	0.018	Berger and Vavra (2015)
γ	Adjustment cost	0.025	Internal calibration
ι	Maintenance parameter	0.5	See text
Income process			
$\hat{\rho}$	Persistence	0.967	Floden and Lindé (2001)
$\hat{\sigma}$	Standard deviation	0.13	Floden and Lindé (2001)
Liquidity			
B	Ratio of bond supply to GDP	1.490	Internal calibration
Labor supply			
μ^d	Mass of households (durable)	0.187	CES
Production			
A^d	Relative productivity (durable)	0.490	Internal calibration
α	Decreasing returns	0.3	Berger and Vavra (2015)
Prices and policy			
λ	Calvo parameter	1	See text
φ	Taylor rule coefficient (inflation)	1.25	Kaplan et al. (2018)

substantially larger than for non-durable consumption, and the response builds up gradually. The right panel plots the response of durable investment under the two regimes (2.2.10) and (2.2.11) for fiscal policy, i.e. with and without endogeneous labor income redistribution. As anticipated in Section 2.3.5, income redistribution has an uneven effect over time. I find that redistribution amplifies the cumulative response of durable spending by roughly 9% over the first year and a half after the shock, dampens this response by 21% over the next year, and then amplifies it again by 45% over the following two years.[47] That is, cyclical income inequality boosts the short-term response of durable spending during expansions and increases the persistence of this response. As noted in Section 2.4.2, my model predicts a degree of non-linearity in the response of aggregate durable spending to income shocks that is substantially lower than in the data. As a consequence, the magnitudes that I obtain as part of this general equilibrium exercise should be thought of as very conservative lower bounds on the true effects.

Figure 2.6.1: General Equilibrium

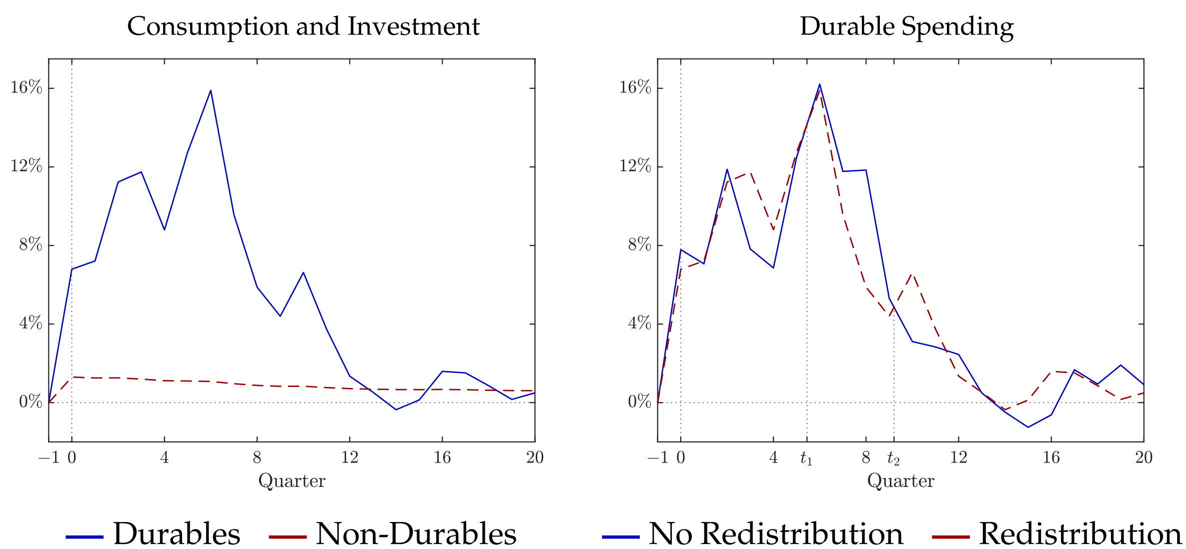


Figure 2.A.1 in Appendix 2.A.6 plots the response of non-durable consumption under the same two regimes for fiscal policy. There is no discernible amplification in this case, despite the complementarity between durables and non-durables in households' preferences. This finding confirms that lumpy and state-contingent adjustment is key to understand the role of cyclical income redistribution for the response of durable spending.

[47] The cumulative response over the full sample is larger with income redistribution, which is consistent with the analysis of Section 2.3.

2.7 Conclusion

In this paper, I study the implications of cyclical income inequality for the dynamics of aggregate durable spending. I explore this question using a multi-sector heterogeneous agent (HANK) model with lumpy durable investment. Durable demand plays a dual role in my setting. First, its pro-cyclicality induces a redistribution of labor income between durable and non-durable sectors. Second, lumpy durable adjustment at the micro level produces non-linearities at the macro level: the average MPC on durable goods increases with income changes. As a result, income redistribution has aggregate effects. It amplifies the response to expansionary shocks, and dampen that to contractionary shocks.

I first clarify these mechanisms analytically. I then simulate numerically the response of durable spending to productivity shocks in general equilibrium. I find that cyclical income inequality affects both the short-term response of durable spending and the persistence of this response.

Appendix to Chapter 2

2.A Quantitative Appendix

In this Appendix, I present the full model and describe the approach used to simulate and calibrate the model. Section 2.A.1 describes the full model. Section 2.A.2 provides a formal definition of an equilibrium. Section 2.A.3 provides the algorithm used to solve for the stationary equilibrium and the transition dynamics. I discuss the calibration strategy in Section 2.A.4. Finally, Section 2.A.5 provides details about the numerical implementation.

2.A.1 Environment

For concision, Section 2.2 provided a partial description of the environment. For reference, I now present the full model.

Timing. Periods are indexed by $t \in \{0, 1, \ldots\}$. There is no aggregate risk. I focus on one-time, unanticipated but persistent shocks. Each period effectively consists of two sub-periods, indexed by $t.0$ $(-)$ and $t.1$ $(+)$. Households make their adjustment decisions at $t.0$, and their consumption, saving and investment decisions at $t.1$. In period $t.0$, households are indexed by their financial asset holdings (a), their holdings of durable goods (d), their idiosyncratic labor supply shock (ζ) and the sector they are employed in (h). In period $t.1$, households are in addition indexed by their adjustment choice $(\mathcal{A})$ during the previous sub-period.[48] The conditional distributions of idiosyncratic states within each sector, at the begining of each subperiod, are denoted by $\mathbf{\Lambda}^-_{t-1}$ and $\mathbf{\Lambda}^+_t$. All agents have perfect foresight, and all (aggregate) information is revealed at the begining of the first sub-period at $t = 0$.

Households. The households' value function in period $t.0$ satisfies:

[48] This additional state variable is for notational convenience. Financial assets are a sufficient statistics for this adjustment.

$$\mathcal{V}_t^h(a,d,\zeta) = \max_{\mathcal{A}\in\{0,1\}} \left\{ V_t^h(a,d,\zeta;\mathcal{A}) \right\} \tag{2.A.1}$$

The adjustment choice satisfies:

$$\mathcal{A}_t^h(a,d,\zeta) \equiv \begin{cases} 1 & \text{if } V_t^h(a,d,\zeta;1) > V_t^h(a,d,\zeta;0) \\ 0 & \text{otherwise} \end{cases} \tag{2.A.2}$$

The continuation value functions in period $t.1$ associated to adjustment and no adjustment write:

$$\begin{aligned} V_t^h(a,d,\zeta;0) = \max_{\{c,a'\}} \; & \frac{u(c,d^\star)^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a',d^\star,\zeta') \middle| \zeta \right] && (2.A.3) \\ \text{s.t. } & P_t^c c + P_t^d \iota \frac{\delta}{1-(1-\iota)\delta} d^\star + a' \leq (1-\tau_t)\, e_t^h(\zeta) + (1+r_{t-1})\, a \\ & a' \geq 0 \\ V_t^h(a,d,\zeta;1) = \max_{\{c,a',d'\}} \; & \frac{u(c,d')^{1-\sigma}}{1-\sigma} + \beta\mathbb{E}_t\left[\mathcal{V}_{t+1}^h(a',d',\zeta') \middle| \zeta \right] && (2.A.4) \\ \text{s.t. } & P_t^c c + P_t^d\left(d' - (1-\delta)\, d\right) + \gamma P_t^d (1-\delta)\, d + a' \\ & \quad \leq (1-\tau_t)\, e_t^h(\zeta) + (1+r_{t-1})\, a \\ & a' \geq 0 \end{aligned}$$

with $d^\star \equiv [1-(1-\iota)\delta]\, d$, where $e_t^h(\zeta) \equiv \frac{1}{\mu^h}\zeta\left(\mathcal{Y}_t^h - T_t^h\right) + \pi_t$ denotes incomes and $\boldsymbol{\mu}$ denotes the mass of households in each sector. The distributions in period $t.0$ and $t.1$ evolve as follows:

$$\Lambda_t^{h,+}(a,d,\zeta,\mathcal{A}) = \Lambda_{t-1}^{h,-}(a,d,\zeta) \times \begin{cases} \mathcal{A}_t^h(a,d,\zeta) & \text{if } \mathcal{A} = 1 \\ 1 - \mathcal{A}_t^h(a,d,\zeta) & \text{otherwise} \end{cases} \tag{2.A.5}$$

and

$$\Lambda_t^{h,-}(a',d',\zeta') = \sum_{\mathcal{A}} \sum_{\Omega_t^h(a',d';\mathcal{A})} \Lambda_t^{h,+}(a,d,\zeta,\mathcal{A})\, \Sigma\left(\log(\zeta')\, |\zeta\right) \tag{2.A.6}$$

with $\Omega_t^h(a^\star,d^\star;\mathcal{A}) \equiv \left\{ (a,d,\zeta)\, | a_t^{h,\prime}(\cdot;\mathcal{A}) = a^\star, d_t^{h,\prime}(\cdot;\mathcal{A}) = d^\star \right\}$, where Σ denotes the transition kernel characterizing the income process and $a_t^{h,\prime}$ and $d_t^{h,\prime}$ denote the solution to (2.A.1)–(2.A.4), with $d_t^{h,\prime} \equiv d^\star$ when no adjustment. I define c_t^h similarly.

Firms. Prices are sticky along the transition path:

$$\left(P_t^h\right)^{1-\varepsilon} = \lambda^h \left(P_{t-1}^h\right)^{1-\varepsilon} + \left(1-\lambda^h\right)\left(P_t^{\star,h}\right)^{1-\varepsilon} \tag{2.A.7}$$

with initial condition $P_{-1}^h \equiv P^h$, i.e. prices at the stationary equilibrium. Reset prices $\mathbf{P}_t^\star$ satisfy:

$$P_t^{\star,h} = \left[\frac{1}{1-\alpha}\frac{\varepsilon}{\varepsilon-1}\frac{G_t^h}{H_t^h}\right]^{\frac{1-\alpha}{(1-\alpha)(1-\varepsilon)+\varepsilon}} \tag{2.A.8}$$

in each sector h, with

$$G_t^h = W^h\left[\left(\frac{1}{P_t^h}\right)^{-\varepsilon}\frac{Y_t^h}{A^h}\right]^{\frac{1}{1-\alpha}} + \lambda^h\frac{1}{1+r_t}G_{t+1}^h \tag{2.A.9}$$

$$H_t^h = \left(\frac{1}{P_t^h}\right)^{-\varepsilon} Y_t^h + \lambda^h\frac{1}{1+r_t}H_{t+1}^h \tag{2.A.10}$$

for each $t \in \{0, 1, \ldots, T-1\}$, and

$$G_T^h = \frac{1+r}{1+r-\lambda^h}W^h\left[\left(\frac{1}{P^h}\right)^{-\varepsilon}\frac{Y^h}{A^h}\right]^{\frac{1}{1-\alpha}}$$

$$H_T^h = \frac{1+r}{1+r-\lambda^h}\left(\frac{1}{P_h}\right)^{-\varepsilon} Y^h$$

for each sector h, where $\mathbf{W}$ denotes nominal nominal wages at the stationary equilibrium, and $(\mathbf{Y}, \mathbf{P}, r)$ denotes aggregate demands for each good, prices and the nominal interest rate at the steady state.

Policy. Depending on the regime of interest (insurance, or not), lump sum taxes satisfy:

$$\mathbf{T}_t = 0 \quad \text{or} \quad \frac{\mathcal{Y}_t^d - \mu^d T_t^d}{\mathcal{Y}_t^c - \mu^c T_t^c} = \frac{\mathcal{Y}^d}{\mathcal{Y}^c} \tag{2.A.11}$$

with $\sum_h \mu^h T_t^h \equiv 0$, where $\boldsymbol{\mathcal{Y}}$ denotes wages bills at the stationary equilibrium The government's flow budget constraint is

$$\tau_t \sum_h \mu^h \int e_t^h(\zeta)\, d\Lambda_t^h = -r_{t-1}B \tag{2.A.12}$$

Monetary policy implements a Taylor rule:

$$i_t = \max\left\{r + \varphi^\pi\left(\Pi_t - 1\right), 0\right\} \tag{2.A.13}$$

Here, $\Pi_t - 1 \equiv \Delta \log\left(\hat{P}_t\right)$ denotes the inflation rate, where

$$\hat{P}_t \equiv \left[\vartheta\left(P_t^c\right)^{1-\nu} + (1-\vartheta)\left(P_t^d\right)^{1-\nu}\right]^{\frac{1}{1-\nu}} \tag{2.A.14}$$

denotes the CES ideal price index.

Market clearing. Markets for goods clear:

$$Y_t^c = \sum_h \mu^h \int \left[c_t^h(a,d,\zeta) + \Gamma_t\left(d_t^{h,\prime}(a,d,\zeta), d\right)\right] d\Lambda_t^{h,+} \tag{2.A.15}$$

$$Y_t^d = \sum_h \mu^h \int \left[d_t^{h,\prime}(a,d,\zeta) - (1-\delta)\, d\right] d\Lambda_t^{h,+} \tag{2.A.16}$$

The TFP component associated to price dispersion is

$$\left(\Omega_t^h\right)^{\frac{1}{1-\alpha^h}} \equiv \lambda^h\left(\Omega_{t-1}^h\right)^{\frac{1}{1-\alpha^h}} + \left(1-\lambda^h\right)\left(P_t^{h,\star}\right)^{-\frac{\varepsilon}{1-\alpha^h}} \tag{2.A.17}$$

with $\Omega_{-1}^h \equiv \left(P^h\right)^{-\varepsilon}$. Demands for labor satisfy

$$\hat{\mu}_t^h \equiv \hat{\Omega}_t^h\left[\frac{1}{A^h}\left(\frac{1}{P_t^h}\right)^{-\varepsilon} Y_t^h\right]^{\frac{1}{1-\alpha}} \tag{2.A.18}$$

with $\hat{\Omega}_t^h \equiv \left(\Omega_t^h\right)^{\frac{1}{1-\alpha^h}}$. Wage bills and aggregate profits write

$$\mathcal{Y}_t^h = W^h \hat{\mu}_t^h \tag{2.A.19}$$

$$\hat{\pi}_t = \sum_h \left(P_t^h Y_t^h - \mathcal{Y}_t^h\right), \tag{2.A.20}$$

Households claim constant profits from firms: $\pi_t = \hat{\pi}$, where $\hat{\pi}$ denotes aggregate profits at the stationary equilibrium. In turn,

$$\boldsymbol{\mu} = \mathbf{Z}_t \circ \hat{\boldsymbol{\mu}}_t \tag{2.A.21}$$

with $Z^h \equiv 1$ at the stationary equilibrium, and $0 \leq Z_t^h < +\infty$ along the transition path, for each sector h and period t.

2.A.2 Definition of Equilibrium

Definition 1. *An* equilibrium *in my economy consists of a sequence policy functions for consumption and assets* $\{c_t(\cdot), a_t'(\cdot), d_t'(\cdot)\}_{t\geq 0}$*, a sequence of distributions of idiosyncratic states* $\{\boldsymbol{\Lambda}_{t-1}^-, \boldsymbol{\Lambda}_t^+\}_t$*, sequences for the nominal interest rates* $\{r_{t-1}\}_t$*, sectoral price indices* $\{\mathbf{P}_{t-1}\}_t$*, reset prices* $\{\mathbf{P}_t^\star\}_t$*, wages, wage bills and profits* $\{\mathbf{W}_t, \boldsymbol{\mathcal{Y}}_t, \pi_t\}_t$*, the TFP component associated to price dispersion* $\{\boldsymbol{\Omega}_{t-1}\}_t$ *and linear taxes* $\{\tau_t\}_t$ *such that: (a) the policy functions solve (2.A.1)–(2.A.4), given interest rates, prices and wage bills; (b) reset prices satisfy (2.A.8)–(2.A.10), given outputs and wages; (c) the distributions of idiosyncratic states evolve according to (2.A.5)–(2.A.6); (d) sectoral prices and the TFP component associated to price dispersion satisfy (2.A.7) and (2.A.17); (e) monetary policy implements the Taylor rule (2.A.13)–(2.A.14), given prices; (f) fiscal policy sets linear taxes to satisfy its flow budget constraint (2.A.12); and (g) the market clearing conditions for good (2.A.15)–(2.A.16) and labor (2.A.21) hold.*

2.A.3 Numerical Solution

Stationary Equilibrium

I solve numerically for the stationary equilibrium by iterating on the nominal interest rate r, the (relative) price of the durable good P^d, the vector of wage bills, profits[49] and linear taxes $\mathbf{X} \equiv (\boldsymbol{\mathcal{Y}}, \pi, \tau)$. The price of the non-durable good is used as a *numéraire* $P^c \equiv 1$ at the stationary equilibrium, and the TFP component associated to price dispersion satisfies $\Omega^h \equiv (P^h)^{-\varepsilon}$. I proceed in three steps.

Step 0 (Initial conditions)**.** I choose a set of initial conditions for (r, P^d) and $\mathbf{X}$. The procedure described in Section 2.A.4 provides a good guess.

[49] Profits are non-zero at the stationary equilibrium, due to decreasing returns.

Step 1 (Households)**.** Let

$$\hat{\mathcal{V}}^h(a,d,\zeta) \equiv \mathbb{E}\left[\mathcal{V}^h(a,d,\zeta') \middle| \zeta\right] \tag{2.A.22}$$

for each sector h. Starting from a guess $\hat{\mathcal{V}}_{-1}$ for this value function, I first solve the problems (2.A.3)–(2.A.4) to obtain policy functions (c_0, a_0', d_0'),[50] and the value functions associated to each adjustment choice. I obtain the adjustment flows $(\mathcal{A}_0)$ from (2.A.2), and the expected value functions $\hat{\mathcal{V}}_0^h$ from (2.A.1) and (2.A.22). If

$$\left\|\hat{\mathcal{V}}_0 - \hat{\mathcal{V}}_{-1}\right\|_{+\infty} < \epsilon^V$$

for some tolerance $\epsilon^V > 0$, I set $\hat{\boldsymbol{\mathcal{V}}} \equiv \hat{\boldsymbol{\mathcal{V}}}_0$ and $x \equiv x_0$, for each policy function $x \in \{c, a', d', \mathcal{A}\}$. Otherwise, I repeat this procedure after updating $\boldsymbol{\mathcal{V}}_{-1}$ with $\boldsymbol{\mathcal{V}}_0$.

Starting with a guess for the initial distribution $\boldsymbol{\Lambda}_{-1}^-$, I iterate on (2.A.5)–(2.A.6) using the policy functions obtained previously to obtain distributions $\boldsymbol{\Lambda}_0^-$ and $\boldsymbol{\Lambda}_0^+$. If

$$\left\|\boldsymbol{\Lambda}_0^- - \boldsymbol{\Lambda}_{-1}^-\right\|_{+\infty} < \epsilon^\Lambda$$

for some tolerance $\epsilon^\Lambda > 0$, I set $\boldsymbol{\Lambda}^- \equiv \boldsymbol{\Lambda}_0^-$ and $\boldsymbol{\Lambda}^+ \equiv \boldsymbol{\Lambda}_0^+$. Otherwise, I repeat this procedure after updating $\boldsymbol{\Lambda}_{-1}^-$ with $\boldsymbol{\Lambda}_0^-$.

Finally, aggregate demands $\mathbf{Y}$ for each good are obtained from (2.A.15)–(2.A.16) using the policy functions (c, d') and the distribution $\boldsymbol{\Lambda}^+$. Similarly, aggregate savings is defined as

$$S \equiv \sum_h \mu^h \int a'^{,h}(a,d,\zeta)\, d\Lambda^{h,+} \tag{2.A.23}$$

Step 2 (Firms and market clearing)**.** From (2.A.7)–(2.A.10), (2.A.18) and (2.A.21), I obtain the wages that insure labor market clearing in each sector:[51]

$$\frac{\bar{W}^h}{P^h} = A^h\left(1-\alpha^h\right)\frac{\varepsilon-1}{\varepsilon}\left(\mu^h\right)^{-\alpha}$$

Similarly, demand for labor in each sector satisfies

$$Y^h = A^h\left(\bar{\mu}^h\right)^{1-\alpha} \tag{2.A.24}$$

since $\Omega^h \equiv \left(P^h\right)^{-\varepsilon}$ at the stationary equilibrium.

[50] See Section 2.A.5 for details.

[51] To each set of initial conditions $\mathbf{X}$ corresponds a set of values for the same variables that satisfy the (stationary) equilibrium restrictions. I denote these variables with a "bar".

I then compute the associated wage bills and aggregate profits:

$$\bar{\mathcal{Y}}^h = \bar{W}^h \bar{\mu}^h$$
$$\bar{\pi} = \sum_h \left(P^h Y^h - \bar{\mathcal{Y}}^h \right)$$

Finally, I obtain the linear tax $\bar{\tau}$ from (2.A.12).

Step 3 (Convergence)**.** Finally, I check whether the vector of prices $\left(r, P^d\right)$ and wage bills, profits and linear taxes $\mathbf{X}$ insure market clearing. If,

$$|S| < \epsilon^S \quad \text{and} \quad \left|\mu^d - \bar{\mu}^d\right| < \epsilon^d$$

for some tolerances $\epsilon^S, \epsilon^d > 0$, with aggregate savings given by (2.A.23) and labor demand given by (2.A.24), then the policy functions $(c, a'$, $d', \mathcal{A})$, the distributions $\left(\mathbf{\Lambda}^-, \mathbf{\Lambda}^+\right)$, the vector of prices $\left(r, P^d\right)$, and wage bills, profits and linear taxes $\mathbf{X}$ form a (stationary) equilibrium.[52] Otherwise, I update the vector of prices $\left(r, P^d\right)$ to reduce excess demand and I repeat Step 1 onward. In this case, I also update the vector of wage bills, profits and linear taxes $\mathbf{X}$ using a weighted average of the initial guess $\mathbf{X}$, and the values computed above $\bar{\mathbf{X}}$.

Transition Dynamics

The comparative statics of interest is a one-time, unanticipated innovation in aggregate productivity. The shock is symmetric across sectors. Specifically,

$$\log\left(A_t^h\right) = \rho^A \log\left(A_t^h\right) + \psi_t$$

for each sector h, with $\psi_0 \in \mathbb{R}$ and $\psi_t \equiv 0$ for each period $t \geq 1$. I solve numerically for the transition dynamics by iterating on the sequence for $\mathbf{X}_t \equiv (r_t, \mathbf{P}_t, \boldsymbol{\mathcal{Y}}_t, \pi_t, \tau_t, \mathbf{T}_t)$, where $\mathbf{T}_t$ corresponds to the lump sum taxes that satisfy (2.A.11).[53] I proceed in four steps.

Step 0 (Initial and terminal conditions)**.** I set $\mathbf{X}_t = \mathbf{X}$, for each period t, as an initial guess, where $\mathbf{X}$ denotes the vector at the stationary equilibrium. Similarly, fix $\mathbf{\Lambda}_{-1}^- \equiv \mathbf{\Lambda}^-$ and $\mathcal{V}_{T+1} \equiv \mathcal{V}$ as initial and terminal conditions for the distribution of idiosyncratic states

[52] Note that the market clearing conditions for labor in the non-durable sector is implied by (2.A.21) and (2.A.23) when $S = 0$.

[53] In the case of ful (aggregate) insurance, I omit the "star" subscripts.

and the value function, where T denotes the horizon over which I compute the impulse responses.

Step 1 (Households)**.** First, I iterate backward on the functional equation (2.A.1)–(2.A.4), for each $t \in \{0, \ldots, T\}$, to obtain a sequence of policy functions $\{c_t, a'_t, d'_t, \mathcal{A}_t\}_t$. Then, I iterate forward on the transition kernel (2.A.5)–(2.A.6) using these policy functions, for $t \in \{0, \ldots, T\}$, to obtain a sequence of distributions $\{\mathbf{\Lambda}^-_{t-1}, \mathbf{\Lambda}^+_t\}_t$. Finally, I compute the sequence of aggregate demands for each goods $\{Y^c_t, Y^d_t\}_t$ using (2.A.15)–(2.A.16).

Step 2 (Firms and market clearing)**.** I iterate backward on (2.A.8)–(2.A.10), for $t \in \{0, \ldots, T\}$, to obtain the sequence of reset prices $\{\mathbf{P}^\star_t\}_t$. Then, I iterate forward on (2.A.7) and (2.A.17) to obtain a new sequence of sectoral price indices[54] $\{\bar{\mathbf{P}}_t\}_t$ and a sequence of TFP distortions $\{\mathbf{\Omega}_t\}_t$. Labor demands are computed from (2.A.18), using the sequence $\{Y^c_t, Y^d_t\}_t$ from Step 1 and the initial sequence of prices $\{\mathbf{P}_t\}_t$. Finally, I compute new sequences for the wage bills $\{\bar{\boldsymbol{\mathcal{Y}}}_t\}_t$ and aggreagte profits $\{\bar{\boldsymbol{\pi}}_t\}_t$ from (2.A.19)–(2.A.20).

Step 3 (Policy)**.** I obtain new sequences for lump sum taxes $\{\bar{\mathbf{T}}_t\}_t$ and linear taxes $\{\bar{\tau}_t\}_t$ from (2.A.11)–(2.A.12). Similarly, I compute the new sequence of nominal interest rate $\{\bar{r}_t\}_t$ set by the monetary policy rule (2.A.13), given the price index (2.A.14).

Step 4 (Convergence)**.** Finally, I check whether the sequence of endogeneous variables $\mathbf{X}_t \equiv (r_t, \mathbf{P}_t, \boldsymbol{\mathcal{Y}}_t, \pi_t, \tau_t, \mathbf{T}_t)$ insures market clearing. If,

$$\|\bar{x} - x\|_\infty < \epsilon^x$$

for some tolerance $\epsilon^x > 0$, for each sequence $x \in \{r, \mathbf{P}, \boldsymbol{\mathcal{Y}}, \pi, \tau, \mathbf{T}\}$, then the policy functions $\{c_t, a'_t, d'_t, \mathcal{A}_t\}_t$, the distributions $\{\mathbf{\Lambda}^-_{t-1}, \mathbf{\Lambda}^+_t\}_t$, and the prices and incomes $\{\mathbf{X}_t\}_t$ form an equilibrium. Otherwise, I update the sequence $\{\mathbf{X}_t\}_t$ using a weighted average of the initial sequences $\{\mathbf{X}_t\}_t$ and the new sequences $\{\bar{\mathbf{X}}_t\}_t$[55] and I repeat Step 1 onward.

2.A.4 Calibration Strategy

The internal calibration consists of solving for the vectors of parameters $(\beta, \vartheta, \gamma, B, A^d)$ — i.e. the discount factor, the preference parameter on non-durables, the (non-convex)

[54] Again, to each set of initial conditions $\mathbf{X}$ corresponds a set of values for the same variables that satisfy the equilibrium restrictions. I denote these variables with a "bar".

[55] To insure convergence of the algorithm, the corresponding weights a decreasing exponentially in the period t.

durable adjustment cost, the supply of bonds, and the relative productivity in the durable sector — and prices (r, P^d) — i.e. the real interest rate and the relative price of the durable good at the stationary equilibrium — that solve the following restrictions: the two market clearing conditions for labor (2.A.21); and the five empirical moments listed in Table 2.5.1. I proceed in two steps. First, I use the empirical targets and the restrictions in the model to pin down the parameters (A^d, B) and the prices (r, P^d). Then, I iterate over the remaining parameters to insure market clearing, and match the rest of the targets.

Step 1. The first empirical moment in Table 2.5.1 directly pins down r. From (2.A.18) and (2.A.21),

$$\mu_t^h \equiv \left(\frac{1}{A^h} Y_t^h \right)^{\frac{1}{1-\alpha}} \tag{2.A.25}$$

for each sector h, at the stationary equilibrium. The second moment in Table 2.5.1 and (2.A.25) thus pin down A^d. Similarly, from (2.A.8),

$$\frac{\bar{W}^h}{\bar{P}^h} = A^h \left(1 - \alpha^h\right) \frac{\varepsilon - 1}{\varepsilon} \left(\mu^h\right)^{-\alpha} \tag{2.A.26}$$

for each sector h. The last moment in Table 2.5.1 and (2.A.26) pin down $\bar{P}^d$, given the *numéraire* $\bar{P}^c \equiv 1$ and the vector of productivities $\mathbf{A}$ previously solved for. Aggregate output js defined as

$$\bar{Y} \equiv \sum_h \bar{P}^h \bar{Y}^h \quad \text{with} \quad \bar{Y}^h = A^h \left(\mu^h\right)^{1-\alpha} \tag{2.A.27}$$

for each sector h, by definition of the production technologies. The fourth moment in Table 2.5.1 and (2.A.27) pin down B, given the vectors of prices $\bar{\mathbf{P}}$ and productivities $\mathbf{A}$ obtained above.

Step 2. Finally, I iterate on the remaining parameters $(\beta, \vartheta, \gamma)$ to satisfy the rest of the restrictions. Specifically, I adjust β until aggregate savings (2.A.23) are zero. Similarly, I use ϑ to clear the market for durable goods (2.A.21), with labor demand given by (2.A.16) and (2.A.18). Finally, I vary γ until the aggregate frequency of adjustment

$$\mathcal{A}^T \equiv \sum_h \mu^h \int \mathcal{A}^h (a, d, \zeta)\, d\bar{\Lambda}^h$$

matches the third moment in Table 2.5.1.

2.A.5 Numerical Implementation

I describe below the computation of the impulse responses in Section **??**, and the numerical implementation of the algorithms described in Appendices 2.A.3 and 2.A.3.

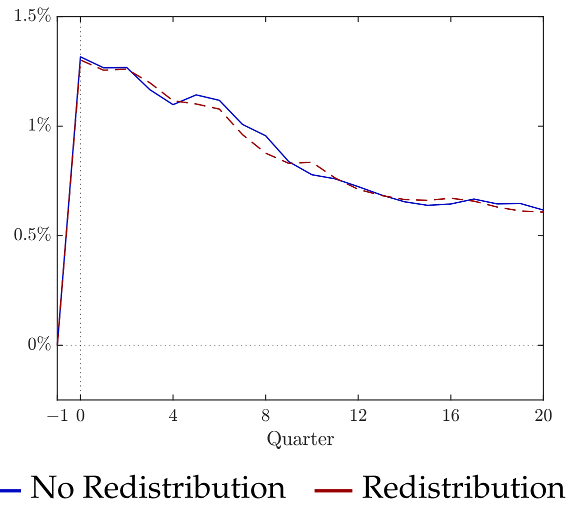
Grids and transition. The value functions are approximated on discrete grids for financial assets and durable goods, consisting of 100 points each. The distribution of financial assets and durable goods is discretized on grids consisting of 300 points. I interpolate policy functions linearly between grid points, and use a generalization of Young (2010)'s non-stochastic stimuation method with multiple assets when using the policy functions to iterate on the distribution of assets. For accuracy, the grid for financial assets is more dense in the neighborhood of the borrowing constraint. Finally, I discretize the income process on a 7-point grid using the method of Rouwenhorst (1995). I set $T = 100$ (quarters) when solving for the transition dynamics in general equilibrium (Section 2.6).

Numerical solution. The households' problems (2.A.3)–(2.A.4) are non-convex due to fixed adjustment costs. To solve for the optimal policy, I first perform a grid search on a fine grid to locate a candidate for the global maximum. I then use the gradient-free, simplex algorithm (Nelder-Meade) implemented in Matlab's fminsearch to solve for a local optimum, using this candidate as an initial condition. I use a tolerance of 10^{-5} to solve for the value function at the stationary equilibrium, and one of 10^{-17} for the stationary distribution (Step 1 in Appendix 2.A.3).

2.A.6 Complementary Numerical Results

In Section 2.6, I show that income redistribution affects the magnitude and the timing of the general equilibrium response of durable investment to productivity shocks. Figure 2.A.1 plots the corresponding responses for non-durable consumption. There is no discernible pattern in this case, which highlights the role of lumpy and state-contingent adjustment for the response of durable spending.

Figure 2.A.1: Non-Durable Spending



2.B Omitted Proofs, Results and Derivations

2.B.1 Benchmark Case

In Section 2.3.2, I discuss a benchmark where the two goods are non-durable: $\delta = 1$, i.e. full depreciation. In this case, no redistribution of labor income takes place across sectors and there is no role for insurance from fiscal policy. For concision, no formal statement is provided in the text. I do so in this section instead.

Proposition 2. *Let Assumptions 4-5 hold. Consider a persistent, unanticipated productivity shock of the form (2.3.2), for any $\psi_0 \in \mathbb{R}$. Then, taxes under the insurance regime (2.2.11) satisfy:*

$$\mathbf{T}^{\star} = 0$$

Consequently, the response of durable investment satisfies:

$$\frac{Y_0^d}{Y^d} = \frac{Y_0^{d,\star}}{Y^{d,\star}}$$

for each period $t \geq 0$, where $Y_0^{d,(\star)}$ denotes aggregate durable investment (2.2.13) under the regimes (2.2.10) and (2.2.11).

Proof of Proposition 2. First, I guess and verify that outputs in the economy without insurance ($\mathbf{T} \equiv 0$) satisfy:

$$\frac{Y_t^c}{Y^c} = \frac{Y_t^d}{Y^d} \tag{2.B.1}$$

By homotheticity,

$$c_t^h(a,d,\zeta) = c_t^\star e_t^h(a,d,\zeta) \quad \text{and} \quad d_t^{h,\prime}(a,d,\zeta) = d_t^\star e_t^h(a,d,\zeta) \tag{2.B.2}$$

with full depreciation. Here, $(c_t^\star, d_t^\star)$ denotes the (dual) cost-minimizing bundle that achieves $u(c,d') \geq 1$, and $e_t^h(a,d,\zeta)$ solves the following income fluctuations problem[56]:

$$\begin{aligned} V_t^h(a,\zeta) = \max_{\{e,a'\}} \; & \frac{e^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t\left[V_{t+1}^h(a',\zeta') \middle| \zeta \right] \\ \text{s.t. } & \hat{P}_t e + a' \leq (1-\tau_t)\, e_t^h(\zeta) + (1+r_{t-1})\, a \\ & a' \geq 0 \end{aligned}$$

where $\hat{P}_t$ denotes the CES ideal price index (2.A.14). In particular, $(c_t^\star, d_t^\star)$ is homogeneous of degree 0 in prices $\mathbf{P}_t$.

From the firm's price setting (2.A.8)–(2.A.10), the law of motion of price indices (2.A.7), the guess (2.B.1), and by Assumptions 4-5,

$$\frac{P_t^c}{P^c} = \frac{P_t^d}{P^d} \tag{2.B.3}$$

so that $(c_t^\star, d_t^\star) = (c^\star, d^\star)$, i.e. the relative demand is unchanged compared to the stationary equilibrium. Thus, the guess (2.B.1) is verified, from the market clearing conditions (2.A.15)–(2.A.16). The remaining equilibrium restrictions are satisfied.

Furthermore, note that:

$$\frac{\mathcal{Y}_t^c}{\bar{Y}^c} = \frac{\mathcal{Y}_t^d}{\bar{Y}^d} \tag{2.B.4}$$

since wages are rigid, using the definitions of labor demands (2.A.17)–(2.A.18), and the wage bills (2.A.19). Then,

$$T_t^\star = 0 \tag{2.B.5}$$

under the regime with insurance, for each period $t \geq 0$, by definition of taxes (2.A.11) and given the wage bills (2.B.4). □

[56] Note that the state variable d is redundant when $\delta = 1$, and households do not incur a fixed costs when adjusting their consumption of the good $h = d$.

2.B.2 Omitted Derivations

Derivations for Example 4. Consider an expansionary productivity shock $\psi_0 < 0$.[57] The decrease in productivity increases the demand for labor. In partial equilibrium, incomes increase proportionately across sectors. From (2.A.12), distortionary taxes decrease $\tau_0 < \tau$ which further contributes to an increase in liquid assets for households. Iterating on the market clearing conditions (2.2.12)–(2.2.13) and using Assumption 6,

$$\mathcal{Y}_0^d / \mathcal{Y}^d > \mathcal{Y}_0^c / \mathcal{Y}^c > 1 \tag{2.B.6}$$

in general equilibrium, since prices are rigid ($\lambda \equiv 1$).

Starting from this equilibrium, suppose now that fiscal policy provides full aggregate insurance. From (2.A.11) and (2.B.6), taxes satisfy:

$$T_0^d = -\frac{\mu^c}{\mu^d} T_0^c > 0 \tag{2.B.7}$$

Suppose that aggregate (sectoral) durable investment

$$Y_0^{d,h} \equiv \int \left[d_0^{h,\prime} \left(a, d, \zeta\right) - \left(1 - \delta\right) d \right] d\Lambda_0^h$$

is *convex* in (sectoral) incomes,[58] for each sector h. Then, the redistribution from fiscal policy (2.B.7) reduces aggregate durable investment in partial equilibrium, using (2.B.6). Aggregate non-durable consumption is unchanged, since the role of precautionary savings is negligible, by assumption. Therefore, incomes in durable sector sector while those in the non-durable sector are unchanged in partial equilibrium, from the market clearing conditions (2.2.12)–(2.2.13) and the definition of incomes (2.A.18)–(2.A.20). Again, iterating on the market clearing conditions (2.2.12)–(2.2.13) and using Assumption 6, aggregate durable investment is lower than in the economy without insurance ($\mathbf{T} \equiv 0$).

2.C Empirical Appendix

2.C.1 Aggregate Series

The series for the nominal interest rate, household consumption and investment expenditures, and employment are listed in Table 2.C.1.

[57] Again, note that a *positive* productivity shock ($\psi_0 > 0$) is *contractionary* in this setting.
[58] Incomes are implicit to the time index.

Table 2.C.1: Data Sources

Series	Seasonal Adjst	Source
Nominal Interest Rate		Fed Board H.15
Effective Fed funds rate	No	(line 1)
Deflator		NIPA 1.1.4.
PCE	Yes	(line 2)
Personal Consumption Expenditures		NIPA 1.1.3
Real	Yes	(lines 4-6)
Residential Investment		NIPA 1.1.3
Real	Yes	(lines 14)
Employment		CES (National)
All employees	Yes	(see text)
Population		NIPA 2.1
Total	No	(line 40)

I define durable spending as the sum of household's expenditures on durable goods and residential investment. Non-durable spending consists of the sum of households' expenditures on non-durable goods and services, minus housing and financial / insurance services. I define durable-employment as the sum of employment in: construction, durable manufacturing, wholesale trade and durable retail for durables, and repair and maintenance. Non-durable employment consits of the sum of employment in: non-durable manufacturing, wholesale trade and durable retail for durables, information, professional and business services, leisure and hospitality, and other services (except repair and maintenance). I exclude wholesale trade, retail trade (for data availability reasons) from either employment category. I exclude financial services and public administration.

2.C.2 PSID Data

In Section 2.4, I provide supporting evidence on the interaction between cyclical durable investment and redistribution. In particular, I confirm that labor income decreases proportionally more in durable sectors, compared to non-durable sectors, following a contractionary tax shock. To account for cross-sectoral labor mobility and movements in and out of the labor force, I use longitudinal data from the Panel Study of Income Dynamics (PSID).

Industry classification. Following Berger and Vavra (2015), I adopt a broad definition of durable goods when calibrating the quantitative model in Section 2.5. This definition includes both consumer durables, and residential investment. Consistently, I classify industries as either durable, or non-durable when using the PSID. Durable industries consists of construction, and durable manufacturing. Non-durable industries include non-durable manufacturing, and all services except public administration and the military, and finance[59].

Income data. My preferred measure of incomes corresponds to (pre-tax) labor income, deflated using a price index for total consumption expenditure. I also use family money income to account for unemployment insurance and intra-household risk sharing.

Sample selection. I use the bi-annual PSID waves from 1968 to 2015[60]. The sample consists of male household's heads aged 24-65. To eliminate outliers, I exclude households with labor income lower than 5% of the annual average, and higher than the 95th percentile. I use PSID longitudinal weights.

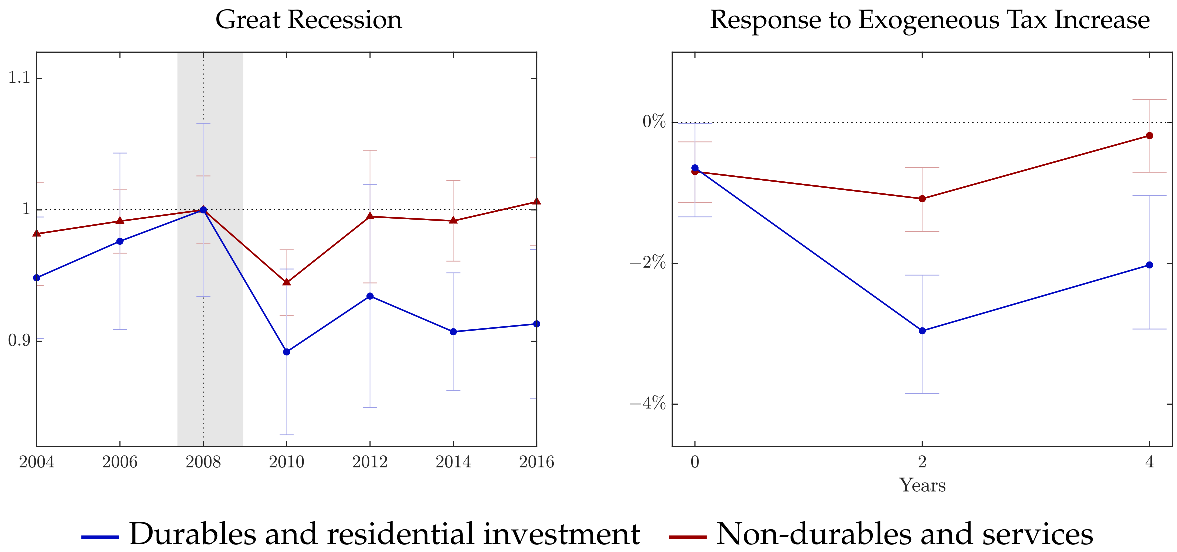
2.C.3 Complementary Empirical Evidence

The measure of income that I use in the text is (real) labor income from the PSID. For robustness, I also use family income in to account for unemployment insurance and intra-household risk sharing. Figure 2.C.1 plots the corresponding series. The pattern is very similar: aggregate shocks lead to a redistribution of income between durable and non-durable workers.

[59] Financial activities and real estate are inter-related, making their classification ambiguous. Public administration and the military spending / employment have no immediate counterpart in my model. Excluding these two industries is conservative with respect to the mechanism I am interested in since spending and employment are less cyclical for these two industries than for private industries.

[60] The industry classification in the PSID changed in 2017. I do not exploit this latest wage to avoid measurement issues.

Figure 2.C.1: Family Income (PSID)



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Chapter 3

Uninsured Income Risk and Labor Misallocation*

This paper explores how uninsured income risk can hinder the efficient matching between workers and firms. I consider a model with two-sided heterogeneity between workers and occupations. Workers are imperfectly insured against labor income risk due to incomplete markets and borrowing constraints. In this setting, uninsured income risk induces labor misallocation across occupations, by reducing the incentives to search for the efficient match (*ex ante*) and by forcing excess separation (*ex post*). In turn, labor misallocation reinforces endogeneously the effect of uninsured income risk, by depressing the value of equity used for self-insurance. I quantify these mechanisms, how they depend on the strength of assortative matching and the co-movements between occupations, and how they affect households differentially depending on the strength of their comparative advantage and across the wealth distribution. Finally, I assess the role of fiscal policy and social insurance in improving upon the *laissez-faire* allocation.

3.1 Introduction

In this paper, I explore how uninsured income risk can prevent an efficient assignment of workers to occupations. I investigate this question using a variant of the Lucas and Prescott (1974) model augmented with two-sided heterogeneity between workers and occupations and incomplete markets. In presence of liquidity constraints, workers are potentially unable to search for and remain in occupations where they have a stronger comparative advantage. In this case, labor is misallocated across occupations, and there

* I thank Iván Werning, Ricardo Caballero, Marios Angeletos and Arnaud Costinot for helpful suggestions. All errors are my own.

is room for social insurance to help the workers overcome the financial frictions they face.

The benchmark model builds on the continuous-time setup of Alvarez and Shimer (2011), and extends it along two dimensions. The first novelty is that households and occupations have permanent types, which allow for assortative matching. I am interested in two distinct (and complementary) motives for sorting: either the productivity of the match is indexed by the worker's and occupation's types *à la* Becker (1973);[1] or households and occupations differ in their risk tolerance and fundamental volatility, respectively.[2] Job search takes both place off- and on-the-job, which generates a job ladder.[3] The second novelty is that risk sharing is limited: worker- and occupation-specific income risk is uninsurable, and households face borrowing constraints.[4] I set up the model such that complete markets allocation is efficient. As a result, any inefficient in the allocation of labor across occupations can be directly attributed to imperfect risk sharing.

Incomplete markets hinder the efficient assignment of workers to occupations through two channels (Figure 3.1.1). First, imperfect risk sharing reduces workers' incentives to search (*ex ante*) for a match where they have a stronger comparative advantage.[5] In particular, households shorten the average duration of their search, and settle inefficiently for occupations that are expanding or perceived as safer. These two effects are more pronounced for workers with a strong comparative advantage. Second, uninsured income risk and borrowing constraints induce excess reallocation (*ex post*) across occupations. In presence of incomplete markets and borrowing constraints, households have an inefficiently high propensity to leave their occupation during a contraction. Workers sacrifice their comparative advantage for the liquidity benefits of moving to another occupation and improving their future income stream. This prediction is not uniform along the

[1] Empirically, the idenfication of this form of assortative matching is complicated by the presence of search frictions (Eeckhout (2018) for a review). Multi-dimensional heterogeneity plays an important role in explaining workers' sorting across occupations as observed in the data (Lindenlaub and Postel-Vinay (2017); Guvenen et al. (2020); Postel-Vinay and Lise (2020)). For tractability reasons, I focus on uni-dimensional heterogeneity, however.

[2] Empirically, risk tolerant households sort into riskier occupations (Guiso et al. (2002), Bonin et al. (2007), Guiso and Paiella (2008) and Schulhofer-Wohl (2011)).

[3] Job ladders are typically studied in search-theoretical frameworks (Burdett (1978); Moscarini and Postel-Vinay (2018)), as opposed to the Lucas and Prescott (1974) framework.

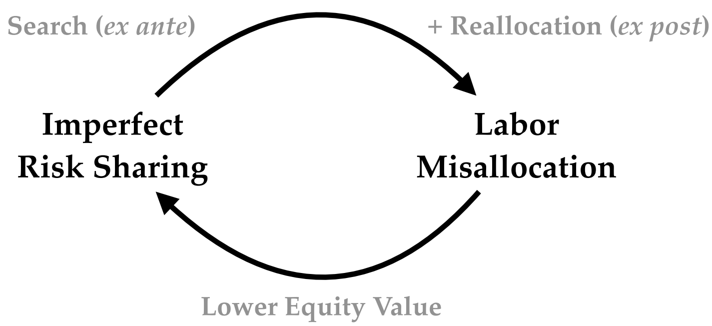
[4] The evidence suggests that liquidity frictions shorten the workers' job search (Chetty (2008)), reduce the quality of the resulting matches (Nekoei and Weber (2017)), and affect households' occupational choices by increasing the effective risk of some activities (Bianchi and Bobba (2012)).

[5] It's worth clarifying the terminology at this point. In the following, I suppose that there is heterogeneity among workers (θ) and firms ($\theta^\star$). The expected value (continuation value) of a match is log-supermodular in $(\theta, \theta^\star)$, and symmetric in θ at the lower value for $\theta^\star$. In the application with heterogeneous productivities, I interpret θ as skill and $\theta^\star$ as skill-intensity. In the application with heterogeneity in risk aversion and fundamental volatility, I interepret θ as risk tolerance and $\theta^\star$ as fundamental volatility. I refer to high θ households as having a (strong) *comparative advantage.*

wealth distribution when mobility entails a short-term liquidity cost. Workers with particularly low liquidty buffers are unable to afford this mobility cost and can on the contrary have an inefficiently low propensity to search for a new occupation.

These two channels interact and shape the workers' income profiles. Compared to the efficient allocation, workers with a strong comparative advantage fall short on climbing their job ladder. This reduces their earnings, and the liquidity buffers they accumulate. As a result, workers cycle excessively between occupations where they have a inefficiently low comparative advantage. In particular, transitory income shocks can have very persistent effects.

Figure 3.1.1: Channels



These effects are reinforced in general equilibrium (Figure 3.1.1). Labor misallocation depresses aggregate productivity,[6] which reduces the value of firms' equity that households use as liquidity buffer. This makes incomplete markets and borrowing constraints even more acute, and reinforces labor misallocation.

I first illustrate these mechanisms qualitatively in a benchmark model. I then develop a quantitative version of this model to assess their importance. I enrich my setup along three dimensions. First, I introduce more realistic labor markets, with wage rigidites, search frictions and unvoluntary unemployment *à la* Blanchard and Galí (2010), and convex labor adjustment costs within occupations. Second, I allow for occupation-specific mobility costs, which I calibrate to replicate the pattern of gross and net flows observed in U.S. data. Third, I introduce worker-specific idiosyncratic income shocks to better match the empirical distribution of earnings.

Finally, I assess the role of fiscal policy in improving upon the *laissez-faire* allocation. In particular, I explore how unemployment insurance and severance payments allow households to partially overcome the financial frictions that they face, and thus increase the

[6] The literature has mostly studied credit frictions on the *firms* side as a source of labor misallocation (Restuccia and Rogerson (2008); Hsieh and Klenow (2009); Baqaee and Farhi (2020)). Instead, and I focus on the role of financial frictions on the *households* side.

quality of workers-occupations matches. However, these social insurance programs also raise the value of unemployment and discourage labor supply. I use my structural model to solve for the optimal level and timing of unemployment insurance and severance payments that trade-off between these forces.

The present version of the paper is very preliminary. The full analysis is presented in Zorzi (2020b).

Related literature. My paper lies at the intersection of several strands of the literature on labor mobility, assortative matching, incomplete markets and social insurance programs.

First, my paper contributes to an important literature on labor reallocation across sectors and occupations over the business cycle, in the tradition of Lucas and Prescott (1974). Alvarez and Shimer (2011) develop a tractable, continuous-time version of this model, which I build on. They assume complete markets and homogeneous workers, while I focus precisely on incomplete markets and their implication for assortative matching. Following Alvarez and Veracierto (2000) and Alvarez and Veracierto (2001), I explore the role social insurance in the form of unemployment insurance and severance payments. The presence of occasionally-binding borrowing constraints in my setup creates a new motive for front-loaded severance payments: they allow workers to partially undo the financial frictions that they face in the short-term. Rogerson (2005) and Carrillo-Tudela and Visschers (2014) augment the Lucas and Prescott (1974) model with sector-specific human capital accumulation over the life cycle.[7] They assume that workers are risk neutral and abstract from their savings decision. Instead, I take worker's schedule of comparative advantage as exogeneous, and I study whether uninsured income risk and borowing constraints affects their assignment to occupations.

Second, my paper fits into a large literature on assortative matching and the role of labor market frictions in preventing an efficient assignment of workers to firms. Shimer and Smith (2000) study the conditions under which assortative matching is preserved in presence of search frictions. Moscarini (2001) and Teulings and Gautier (2004) explore how tight labor markets can disincentivize workers to search for occupations where they have a stronger comparative advantage.[8] In turn, Marimon and Zilibotti (1999) and Fuller et al. (2014) investigate how unemployment insurance can improve the quality of worker-firm matches by raising the value of search. Lise et al. (2016) quantify this mechanism in

[7] McLaughlin and Bils (2001) introduce assortative matching in a related multi-sector framework. Dvorkin and Monge-Nara (2019) allow for human capital accumulation in a dynamic, multi-sector general equilibrium model with labor mobility but focus on long-term growth (as opposed to cyclical fluctuations).

[8] A similar mechanism is present in Acemoglu and Shimer (2000), though their focus is not explicitly on assortative matching.

a calibrated model with on-the-job search. My paper revisits this question in the context of an heterogeneous agents model with occasionally-binding borrowing constraints.

Finally, my paper relates to a growing literature on labor supply and search in heterogeneous agent, incomplete markets economies. Chang and Kim (2006) allow for an extensive margin of labor supply in an Aiyagari (1994) model. Krusell et al. (2010) introduce a frictional labor market and quantify optimal unemployment insurance in this setting. Krusell et al. (2017) and Krusell et al. (2020) are interested in workers' gross flows between employment and unemployment. To the best of my knowledge, my paper is the first to embed the heterogeneous agents apparatus into a Lucas and Prescott (1974) model with occupation-specific risk.[9] Furthermore, my focus on assortative matching is novel.

Layout. I start by introducing a benchmark multi-occupations, Lucas and Prescott (1974) economy with assortative matching and incomplete markets in Section 3.2. I investigate how incomplete markets hinder the efficient assignment of workers to occupations in Section 3.3. Section 3.4 introduces a quantitative variant of this model. Section 3.5 discusses the next steps.

3.2 Benchmark Model

The environment consists of a muti-occupations, Lucas and Prescott (1974) economy augmented with two features: assortative matching, and incomplete markets. There is a continuum of *labor markets*, corresponding to various occupations. The product of these occupation is used as intermediary input in a final good sector. Occupations are subject to idiosyncratic demand shocks, while the final good sector is not. Mobility between the labor markets entails a liquidity cost. The economy is inhabited by a continuum of households who consume, save, supply labor in their occupation[10] and move between those. Households and occupations are indexed by permanent types, allowing for assortative matching. Households face incomplete markets and borrowing constraints, which is the only source of inefficiency in this economy. I introduce the environment in Section 3.2.1. I state the market clearing conditions in Section 3.2.2. Section 3.2.3 discusses some properties of the model.

[9] In separate work (Zorzi (2020a)), I study the role of income redistribution in a multi-sector, heterogeneous agents economy. However, I rule out labor mobility between sectors, and I abstract from unemployment.

[10] There is no unvoluntary unemployment in this benchmark model. I address this question the richer model presented in Section 3.4.

3.2.1 Environment

Time is continuous, and there is no aggregate uncertainty.[11] Periods are indexed by $t \geq 0$.

Occupations. A continuum of occupations produces intermediary inputs. Each occupation admits a representative firm. Occupations are characterized by three idiosyncratic states: their permanent type $(\theta^\star)$, their idiosyncratic demand shifter (φ), and the distribution of idiosyncratic states for households located in this occupation (Λ).[12] Firms' types $\theta^\star$ lie in some continuum Θ. In the following, I let $z^\star \equiv (\theta^\star, \varphi, \Lambda)$ denote the vector of occupations' idiosyncratic states, and π denote the associated measure.

The representative firm in each occupation has access to a linear technology $f : n \mapsto b^e n$ for some $b^e > 0$, where n denotes effective labor demand. Prices are flexible in these occupations and firms are competitive.

Final goods sector. A final goods producer combines the output of the various occupations:[13]

$$Q = \left[\int \exp(\varphi)^{\frac{1}{\nu}} y(z^\star)^{\frac{\nu-1}{\nu}} d\pi \right]^{\frac{\nu}{\nu-1}} \tag{3.2.1}$$

where φ denotes (stochastic) demand shifters and $y(z^\star)$ denotes demand for input produced by the occupations. The final good producer is competitive and its price is normalized to 1 (*numéraire*). The demand shifters follow a first-order autoregressive process, which is indexed by the occupation's type $\theta^\star$. Specifically,

$$d\varphi_t = -\eta(\varphi_t - \bar{\varphi}) dt + \sigma_{\theta^\star} B_t \tag{3.2.2}$$

for some (common) mean and persistence $\bar{\varphi} > 0$ and $\eta > 0$, and some type-specific volatility $\sigma_{\theta^\star} \geq 0$, where B_t denotes a (standard) Wiener process, with independence across occupations.

Households. The economy is inhabited by of a continuum of mass 1 of households. Each household is indexed by five idiosyncratic states: their permanent type (θ), their financial asset holdings (a), their mobility status (m), their mobility cost (λ), and the set of idiosyncratic states characterizing the occupation they are located in $(z^\star)$. Households' types θ

[11] I focus on the properties of the stationary equilibrium of this economy.

[12] Specifically, Λ corresponds to the distribution of (θ, a, m, λ) (defined below) in the relevant occupation.

[13] The exogeneous process for demand shifters is normalized so that $\int \exp(\varphi) d\pi \equiv 1$, i.e. $\bar{\varphi} = -\log \int \exp\left(\frac{1}{4}\frac{\sigma^2_{\theta^\star}}{\rho}\right) d\pi^\star$ using the notation introduced below, where $\pi^\star \equiv \text{marg}_{\theta^\star} \pi$.

lie in the continuum Θ. In the following, I let $z \equiv (\theta, a, m)$.

Preferences are indexed by the household's type θ and are represented by

$$\mathcal{U}_\theta \left(\{c_t\}_{t \geq 0} \right) \equiv U_0 \text{ with } U_t = \mathbb{E}_t \left[\int_{s \geq t} \rho f_\theta \left(c_s, U_s \right) ds \right] \tag{3.2.3}$$

for all $t \geq 0$, for some type-specific flow utility $f_\theta(\cdot)$ increasing and strictly concave in its first argument, and some common discount rate $\rho > 0$. Here, c_t denotes consumption of the final good. Households have access to riskless bonds (a) in zero net supply. They are subject to the following flow borrowing constraint

$$a \left(z, z^\star \right) \geq -\underline{\phi} \mathcal{Y} \left(z, z^\star \right) \tag{3.2.4}$$

for some borrowing parameter $\underline{\phi} \geq 0$, where $\mathcal{Y}(z, z^\star)$ denotes idiosyncratic labor income. The households' flow budget constraint is

$$da_t \left(z, z^\star \right) = \left[\mathcal{Y} \left(z, z^\star \right) + r a_t \left(z, z^\star \right) + T \left(z, z^\star \right) - c_t \left(z, z^\star \right) \right] dt \tag{3.2.5}$$

where r denotes the net interest rate,[14] and T denotes lump sum transfers that fiscal policy can potentially use to provide insurance across workers. Depending on the risk sharing regime of interest (see Section 3.3), households have in addition access to a complete set of contingent securities.

Households can have one of three mobility states $m \in \{0, 1, 2\}$.[15] When staying in their occupation $(m = 0)$, households make a discrete choice $\chi \in \{e, r\}$ between working at the prevailing wage or rest. When employed $(\chi = e)$, households supply inelastically 1 unit of labor. The productivity of this unit of labor is indexed by the household's type (θ) and the firm's $(\theta^\star)$. Specifically, the productivity of the match is $\zeta(\theta, \theta^\star)$, which allows for assortative matching. Each effective unit of labor is paid the wage $W(z^\star)$ that prevails in the corresponding occupation. When resting $(\chi = r)$ or moving between occupations $(m = 1)$, households instead produce the final good at home, with outputs $b^u > b^m(\theta) > 0$, respectively.

In each period, households make a discrete choice between remaining in their occupation $(m = 1)$ or move to another one $(m = 2)$. Households exit of this mobility state, i.e. they transition to a new occupation, is given by a stopping time that follows an exponential distribution with rate $\lambda > 0$, as in Alvarez and Shimer (2011). This rate is known by households before they make their mobility decisions, and follows itself a 2-point Markov

[14] I focus on stationary equilibria with no inflation for the final good.
[15] Time is continuous so that the exact timing of actions within each mobility state is irrelevant.

process on $\{\lambda_0, \lambda_1\}$, with $\lambda_0 > 0$ and $\lambda_1 = 0$.[16] I denote the transition rates between these two states by $s_0, s_1 > 0$, respectively. When exiting the mobility state, households draw an idiosyncratic state $z^\star$ from its stationary distribution, i.e. search is random. At that point, they have the choice between entering this occupation ($m = 2$) or keep searching for another one ($m = 1$). When entering an occupation, households enter an intermediary status ($m = 2$) in which they are not allowed to rest.[17] They exit this temporary status and become regular workers ($m = 0$) with a rate $s_2 > 0$.

Summing up, labor incomes satify:

$$\mathcal{Y}(z, z^\star) = \begin{cases} \zeta(\theta, \theta^\star) W(z^\star) & \text{if } m = 0, \chi = e \text{ or } m = 2 \\ b^u & \text{if } m = 0, \chi = r \\ b^m(\theta) & \text{if } m = 1 \end{cases} \tag{3.2.6}$$

Labor markets. Labor markets are frictionless in this benchmark environment. I allow for search frictions and unvoluntary empoyment within occupations in Section 3.4.

3.2.2 Equilibrium

Occupations. The good and labor market clearing conditions in each occupation are

$$y(z^\star) = b^e \int\int \mathbf{1}_{\{\chi(z)=e\}} \zeta(\theta, \theta^\star) \Lambda(dz, z^\star) \, d\pi \tag{3.2.7}$$

for some productivity $b^e > 0$.

Final good. The market clearing condition for the final goods is

$$\int c(z, z^\star) \Lambda(dz, z^\star) \, d\pi = Q + \int \left[b^u \mathbf{1}_{\{\chi(z)=u\}} + b^m \mathbf{1}_{\{m=1\}} \right] \Lambda(dz, z^\star) \, d\pi \tag{3.2.8}$$

Recursive equilibrium. A recursive equilibrium is defined as usual. It consists of value functions and policy functions for households, indexed by $(z, z^\star)$, for consumption, savings, labor supply and mobility such that households maximize (3.2.3) subject to (3.2.4)–(3.2.6), firms choose intermediary inputs optimally, and market clearing conditions (3.2.7)–(3.2.8) together with (3.2.1). In the following, I focus on the stationary equilbrium of this economy, i.e. aggregate quantities and prices are constant over time.

[16] That is, some households are unable to move across occupations even if they wanted to, for reasons outside the scope of this model. I discuss the role of this assumption in Section 3.2.3.

[17] That is, households cannot become unemployed and generate b^u until they have worked for a sufficient long period of time in this occupation.

3.2.3 Discussion

Before exploring how incomplete markets affect the matching betweeen workers and firms, I discuss some of the modelling choices.

Efficiency. The complete markets equilibrium is efficient, as in Alvarez and Shimer (2011). This property allows me to attribute any deviation from the first best allocation, i.e. labor misallocation across occupations, to imperfect risk sharing.

Assortative matching. I allow for two distinct sources of sorting. First, I let the productivity of the worker-occupation match depend on their types. Specifically, I assume that $\zeta(\theta, \theta^{\star})$ is log-supermodular as in Becker (1973). An extensive literature studies how such complementarity between workers and firms affects the allocation of labor across occupations, both in efficient environments[18] and in presence of search frictions.[19] I am interested in the role of imperfect risk in preventing workers to exploit efficiently their comparative advantage. Second, I allow households and occupations to differ in their risk tolerance and fundamental volatility, respectively. Specifically, I suppose that households' flow utility f_θ belong to the Epstein and Zin (1989) class, with type-specific risk aversions $\gamma_\theta > 0$ but a common (inverse) elasticity of substitution $\sigma > 0$.[20] The literature on the effect of risk aversion and liquidity frictions on labor supply and occupational choice is mostly empirical.[21,22] My framework allows me to explore whether incomplete markets and borrowing constraints discourage workers from seeking employment in risky occupations, by increasing the effective risk of being employed in those. When parametrized without assortative matching and complete markets, my model reduces down to Alvarez and Shimer (2011), with the exception that occupations are subject to demand shocks instead of productivity shocks.

[18] See Acemoglu and Autor (2011) and Costinot and Vogel (2015) for reviews of the literature.

[19] See Eeckhout (2018) for a survey.

[20] That is, is set $f_\theta(c, U) = \frac{\rho}{1-\sigma}\left(u^{\star}_\theta c^{1-\sigma} - ((1-\gamma_\theta)U)^{\frac{1-\sigma}{1-\gamma_\theta}}\right) / \left(((1-\gamma_\theta)U)^{\frac{\gamma_\theta-\sigma}{1-\gamma_\theta}}\right)$ (Duffie and Epstein (1992)), for some coefficients $\{u^{\star}_\theta\}_{\theta\in\Theta}$. I set these coefficients such that $f_\theta(c^{\star}, \mathcal{U}_\theta) = f_{\theta'}(c^{\star}, \mathcal{U}_{\theta'})$, for each pair of types (θ, θ'). Here, $c^{\star}$ denotes consumption in the complete markets, stationary equilibrium, and $\mathcal{U}_\theta$ solves (3.2.3).

[21] For empirical references, see footnote 2.

[22] A large literature investigates the role of preferences towards risk and borrowing constraints on entrepreneurial choice (Kihlstrom and Laffont (1979), Evans and Jovanovic (1989) and Vereshchagina and Hopenhayn (2009)). However, an occupational choice differs from an entrepreneurial choice in various respects. In particular, the degree of uninsurable income risk and the importance of financial frictions are potentially very different in these two contexts.

Mobility costs. Mobility across occupations entails two types of cost in my model. First, households face a short-term liquidity cost in the form of non-employment with benefits $b^m < b^u$ while transitioning (stochastically) between occupations at rate λ_0. Second, households incur an opportunity cost in the form of foregone earnings if they leave an occupation where they have a strong comparative advantage. In presence of incomplete markets and borrowing constraints, the timing of these costs is relevant. As I discuss in Section 3.3.4, two pairs (b^m, λ_0) with the same cost in present discounted value can have very different implications for the pattern of labor reallocation across occupations when risk sharing is imperfect. In particular, liquidity costs play an important role when households are unable to relocate in the short-term, i.e. when the transition rate s_0/s_1 is high. In this case, households drawn down their liquidity buffer after a contractionary shock before being able to relocate, making the liquidity cost potentially prohibitive.

Model reduction. As noted in Section 3.2.1, each occupation is indexed by two scalars, i.e. its type $(\theta^\star)$ and demand shifter (φ), and a distribution of households' idiosyncratic states (Λ). With a continuum of occupations, a (stationary) equilibrium involves a distribution of such distributions. The dimensionality of this object large. To circumvent this issue, the distributions Λ can be parametrized by a finite vector $\mathbf{\Lambda}^\star$.[23] In the following, I use Λ and $\mathbf{\Lambda}^\star$ interchangeably.

3.3 Risk Sharing and Efficiency

I am interested in the role of risk sharing (or the lack thereof) for the households' allocation across occupations. Specifically, I consider two risk sharing regimes and I compare their positive and normative implications. I start with a full insurance, complete markets benchmark.[24] I then allow for imperfect risk sharing. Specifically, I suppose that fiscal policy can provide partial insurance in the form of lump sum transfers.[25] I parametrize these transfers so they nest the two polar cases of full insurance and no insurance, and I perform a comparative statics with respect to this parameter.

[23] Algan et al. (2008) and Fernández-Villaverde et al. (2019) propose alternative approaches, among others.

[24] Formally, households contract on contingent securities in an *ex ante* period (with no endowments). Then, they are assigned randomly (and independently) across idiosyncratic states according to some measures $\pi^\star$. In turn, these idiosyncratic states evolve as described above. This yields a stationary distribution with measures π. Households form rational expectations when contracting, i.e. I am interested in the fixed point $\pi^\star = \pi$.

[25] In Section 3.4, I explicitly model unemployment insurance and severance payments in a model with frictional labor markets.

3.3.1 Assumptions

I specialize the environment presented in Section 3.2.1 in two respects. First, I focus exclusively on comparative advantage in this version of the paper. That is, I abstract from heterogeneity in risk tolerance and fundamental volatility across workers and occupations, respectively. For simplicity, I suppose that the inverse elasticity of intertemporal substitution (σ) and risk aversions (γ_θ) coincide, so that utility is separable:

$$U_0 = \mathbb{E}_0 \left[\int u(c_t)\, dt \right] \, , \; u(c) \equiv \frac{c^{1-\sigma}}{1-\sigma}$$

Assumption 7. $\gamma_\theta = \sigma > 0$*, for each households' type* $\theta \in \Theta$*, and* $\sigma_{\theta^\star} = \sigma^\star > 0$*, for each occupations' type* $\theta^\star \in \Theta$*.*

Second, I suppose that workers with a stronger comparative advantage in high $\theta^\star$ occupations are also more productive. Specifically, I assume that households are uniformly productive in the lowest $\theta^\star$ occupation. In other words, θ captures the workers' skill and $\theta^\star$ the occupations' skill-intensity. Skilled workers have both an *absolute* advantage, and a *comparative* advantage in high $\theta^\star$ occupations. Furthermore, I suppose that the schedule of benefits while moving $b^m(\theta)$ insures that the value of unemployment $\hat{V}(\theta)$ under complete markets (Section 3.3.2) is not indexed by the households' type. Put it differently, high skill workers have a higher cost of retraining when moving between occupations. This assumption rules out on-the-job search with complete markets.[26]

Assumption 8. $\zeta(\theta, \theta_0^\star) = \zeta(\theta', \theta_0^\star)$*, for each pair* (θ, θ')*, where* $\theta_0^\star \equiv \inf \Theta$*, and* $b^m(\theta)$ *is chosen such that* $\hat{V}(\theta) = \hat{V}_0$*, for each households' type* $\theta \in \Theta$*, with* $\hat{V}(\theta)$ *defined in (3.3.5).*

3.3.2 Complete Markets

This economy is frictionless, so that the complete markets equilibrium is efficient. I thus characterize it as the solution to the planner's problem.[27] Households' idiosyncratic states reduce to $z \equiv (\theta, m)$ and $z^\star$, with a slight abuse of notation.

Planner's problem. For reference, I state the planner's problem in Appendix 3.A. At optimum, the planner's problem reduces to solving for a drift $\boldsymbol{\mu}(z^\star)$ and a volatility $\boldsymbol{\sigma}(z^\star)$ for the occupations' idiosyncratic states, an associated distribution $g(z^\star)$, the type-specific

[26] As discussed in Section 3.3.3, on-the-job search potentially takes places with incomplete markets. Similarly, on-the-job search occurs in the richer model I introduce in Section 3.4, regardless of the degree of risk sharing.

[27] This approach parallels Alvarez and Shimer (2011).

mass of households $\hat{g}(\theta)$ moving between occupations, value functions for employment $\bar{V}(\theta, z^\star; \lambda)$, unemployment $\underline{V}(\theta, z^\star; \lambda)$ and mobility $\hat{V}(\theta)$,[28] occupation-specific cutoffs for rest $\underline{\theta}^r(z^\star)$, mobility $\underline{\theta}^m(z^\star)$ and entering an occupation $\underline{\theta}^e(z^\star)$, and occupation-specific outputs $y(z^\star)$ and aggregate output for the final good sector Q that satisfy the following conditions. First, the occupations' idiosyncratic states $(z^\star)$ evolve as follows:[29]

$$dz_t^\star \equiv \boldsymbol{\mu}(z_t^\star)\, dt + \boldsymbol{\sigma}(z_t^\star)\, \boldsymbol{B}_t^\star \tag{3.3.1}$$

where $\boldsymbol{B}_t^\star$ denotes a (standard) multivariate Wiener process, with independence across occupations. Then, the value of employment when allowed to move satisfies

$$\rho \bar{V}(\theta, z^\star; \lambda_0) = b^e \zeta(\theta, \theta^\star) \left(\frac{y(z^\star)}{Q}\right)^{-\frac{1}{\nu}} + \mathcal{A}\bar{V}(\theta, z^\star; \lambda_0) + s_1 \left[\bar{V}(\theta, z^\star; \lambda_1) - \bar{V}(\theta, z^\star; \lambda_0)\right] \tag{3.3.2}$$

Here, $\mathcal{A}f$ denotes the infinitesimal generator associated to a function $f(z^\star)$ and the process (3.3.1). The standard value-matching and smooth-pasting conditions hold along the cutoff for unemployment:[30]

$$\bar{V}(\underline{\theta}^r(z^\star), z^\star; \lambda_0) = \underline{V}(\underline{\theta}^r(z^\star), z^\star; \lambda_0) \text{ and } \partial_{z_j^\star}\bar{V}(\underline{\theta}^r(z^\star), z^\star; \lambda_0) = \partial_{z_j^\star}\underline{V}(\underline{\theta}^r(z^\star), z^\star; \lambda_0) \tag{3.3.3}$$

for each component $z_j^\star$ of $z^\star$, using Assumptions 7–8. The value of employment when not allowed to move satisfies

$$\rho \bar{V}(\theta, z^\star; \lambda_1) = b^e \zeta(\theta, \theta^\star) \left(\frac{y(z^\star)}{Q}\right)^{-\frac{1}{\nu}} + \mathcal{A}\bar{V}(\theta, z^\star; \lambda_1) + s_0 \left[\bar{V}(\theta, z^\star; \lambda_0) - \bar{V}(\theta, z^\star; \lambda_1)\right] \tag{3.3.4}$$

on the entire domain, i.e. without the boundary conditions (3.3.3). The value of unemployment satisfy similar restructions, so I omit them here for concision.[31] The value of

[28] Formally, these functions correspond to the marginal output of locating an additional worker in any activity.

[29] As a reminder, the occupations' idiosyncratic states $(z^\star)$ consist of their type $(\theta^\star)$, their idiosyncratic demand shock (φ), and the (discretized) distribution of households' idiosyncratic states within this occupation $(\boldsymbol{\Lambda}^\star)$. From the Martingale Representation Theorem, this vector follows a (multi-variate) Wiener process with some drift $\boldsymbol{\mu}(z^\star)$ and volatility $\boldsymbol{\sigma}(z^\star)$ at equilibrium.

[30] Note that $\underline{\theta}^m(z^\star) < \underline{\theta}^r(z^\star)$ by Assumptions 7–8, since the value of unemployment $\underline{V}(\cdot)$ is (strictly) decreasing in θ, while the value of mobility $\hat{V}(\cdot)$ is not. That is, there is no on-the-job search in this benchmark version of the model.

[31] In this case, there are two pairs of value-matching and smooth-pasting conditions, one for each of the two thresholds $\underline{\theta}^m(z^\star)$ and $\underline{\theta}^r(z^\star)$.

mobility satisfies

$$\rho \hat{V}(\theta) = b^m(\theta) + \lambda_0 \int \max\left\{V^\star(\theta, z^\star) - \hat{V}(\theta), 0\right\} dg \tag{3.3.5}$$

where the value of joining of new occupation is

$$\rho V^\star(\theta, z^\star) = b^e \zeta(\theta, \theta^\star)\left(\frac{y(z^\star)}{Q}\right)^{-\frac{1}{\nu}} + \mathcal{A}V^\star(\theta, z^\star) + s_2 \int \left[\bar{V}(\theta, z^\star) - V^\star(\theta, z^\star)\right] dg \tag{3.3.6}$$

The threshold for entering an occupation satisfies

$$V^\star\left(\underline{\theta}^e(z^\star), z^\star\right) = \hat{V}\left(\underline{\theta}^e(z^\star)\right) \tag{3.3.7}$$

In turn, the thresholds for unemployment $\underline{\theta}^r(z^\star)$ and mobility $\underline{\theta}^m(z^\star)$, together with (3.3.1), induce a drift $\boldsymbol{\mu}'(z^\star)$ and a volatility $\boldsymbol{\sigma}'(z^\star)$, a distribution of occupations' idiosyncratic states $g'(z^\star)$ a type-specific mass of households $\hat{g}'(\theta)$ moving between occupations.[32] The fixed point requirements are

$$\boldsymbol{\mu}'(z^\star) = \boldsymbol{\mu}(z^\star) \ , \ \boldsymbol{\sigma}'(z^\star) = \boldsymbol{\sigma}(z^\star) \ , \ g'(z^\star) = g(z^\star) \ , \ \hat{g}'(\theta) = \hat{g}(\theta) \tag{3.3.8}$$

Finally, occupation-specific outputs satisfy

$$y(z^\star) = b^e \int\int \mathbf{1}_{\{\theta \geq \underline{\theta}^r(z^\star)\}} \zeta(\theta, \theta^\star) \Lambda(dz, z^\star)\, dg, \tag{3.3.9}$$

and total output produced by the final good sector is defined as

$$Q \equiv \left[\int \exp(\varphi)^{\frac{1}{\nu}} y(z^\star)^{\frac{\nu-1}{\nu}} d\pi\right]^{\frac{\nu}{\nu-1}} \tag{3.3.10}$$

Labor allocation. This model is one of efficient allocation and reallocation of labor across occupations. As occupations are hit by demand shocks, the households with low comparative advantage in those occupations become unemployed ($\theta \leq \underline{\theta}^r(z^\star)$). Ultimately, those with the lowest comparative advantage ($\theta \leq \underline{\theta}^m(z^\star)$) leave the occupation altogether. They search for a new activity, and are allocated (stochastically) to one. This process repeats itself until households are assigned to an occcupation where they have a sufficiently strong comparative advantage. At this point, they have a lower propensity to leave this occupation as it is subject to subsequent demand shocks. Crucially, this efficient

[32] For the reason discussed in Section 3.2.3, the dimensionality of $g(z^\star)$ is large so that the associated flow equations are untractable. Instead, I approximate numerically these distributions by simulating the evolution of a large number of occupations.

allocation is supported by full risk sharing. The underlying payments insure that households have the proper incentives to search for a better occupation, and enough liquidity to remain in those as they are hit by demand shocks. From Section 3.3.3 on, I relax the assumption of complete markets and I explore how imperfect risk sharing affects these incentives and whether it contributes to labor misallocation across occupations.

Implementation. I supposed so far that households were able to contract (*ex ante*) on a complete set of contingent securities. Alternatively, the allocation characterized above can be implemented by lump sum transfers from fiscal policy $T(z, z^{\star})$.[33] These payments insure that households' total income (including tranfers) is constant over time, which allows them to perfectly smooth out consumption across periods and (idiosyncratic) states. Tax *differentials* satisfy

$$T(z, z^{\star}) - T(z', z^{\star,\prime}) = \mathcal{Y}(z', z^{\star,\prime}) - \mathcal{Y}(z, z^{\star}) \tag{3.3.11}$$

for each pairs of households' idiosyncratic states $(z, z^{\star})$ and $(z', z^{\star,\prime})$. In turn, incomes are given by (3.2.6) with

$$W(z^{\star}) = b^{e}\left(\frac{y(z^{\star})}{Q}\right)^{-\frac{1}{\nu}} \tag{3.3.12}$$

at equilibrium. Finally, the *level* of taxes is pinned down by

$$\int T(z, z^{\star})\, d\Lambda d\pi = 0 \tag{3.3.13}$$

These restrictions allow to solve for the contingent transfers $T(z, z^{\star})$. I use these transfers to parametrize the degree of risk sharing when markets are incomplete.

3.3.3 Incomplete Markets

I now assume that markets are incomplete. I investigate whether imperfect risk sharing can hinder the efficient allocation of labor across occupations. In presence of incomplete markets, the distribution of financial assets becomes relevant. That is, households' idiosyncratic states are $z \equiv (\theta, a, m)$ and $z^{\star}$.

Insurance. I allow fiscal policy to provide partial insurance in the form of lump sum

[33] Formally, these payments should be indexed by the households' permanent type and the entire history of shocks they are subject to, i.e. occupation-speific demand shocks, idiosyncratic mobility costs and their draws when moving between occupations. As is apparent from (3.3.11)–(3.3.13), households' *current* idiosyncratic states $(z, z^{\star})$ is a sufficient statistics for these payments.

transfers. However, these payments do not fully undo the financial frictions faced by the households. These transfers serve two purposes. First, they allow me to parametrize the degree of risk sharing in my economy, and explore how the efficiency of the allocation of labor changes as the market incompleteness becomes more acute. Second, they capture in reduced form various social insurance policies, i.e. unemployment insurance and severance payments. I explicitly model those from Section 3.4 on.

When assessing the role of imperfect risk sharing, I am interested in the channels through which it operates. In particular, I disentangle between uninsurable income changes that are specific to a household's *current* occupation or idiosyncratic to her. For this reason, I parametrize these transfers as follows:

$$T'(z, z^\star) = \psi \underbrace{\left[T(z, z^\star) - T^\star(\theta, z^\star)\right]}_{\text{Idiosyncratic insurance}} + \psi^\star \underbrace{T^\star(\theta, z^\star)}_{\text{Aggregate insurance}} - \bar{T} \tag{3.3.14}$$

for each pair of households' idiosyncratic states $(z, z^\star)$, given some loadings $\psi, \psi^\star \in [0, 1]$. The second term in (3.3.14),

$$T^\star(\theta, z^\star) \equiv \int_{\mathcal{S}(\theta)} T(z, z^\star) \Lambda(dz, z^\star) \tag{3.3.15}$$

with $\mathcal{S}(\theta) \equiv \{(z, z^\star) | z = (\theta, a, m) \text{ for some } a, m\}$, allows fisal policy to provide aggregate insurance, by insulating households against shocks affecting the occupations they work in (fixing a households' type). The first term, allows fiscal policy to insure households against the history of their idiosyncratic shocks. In particular, the model replicates the complete markets when setting $\psi = \psi^\star$. Finally, $\bar{T}$ insures that

$$\int T'(z, z^\star) \, d\Lambda d\pi = 0 \tag{3.3.16}$$

Equilibrium. A recursive equilibrium in this economy consists of a drift $\boldsymbol{\mu}(z^\star)$ and a volatility $\boldsymbol{\sigma}(z^\star)$ for the occupations' idiosyncratic states, a distribution of occupations' idiosyncratic states $g(z^\star)$, the type-specific mass of households $\hat{g}(\theta, a)$ moving between occupations, value functions for employment $\bar{V}(\theta, a, z^\star; \lambda)$, unemployment $\underline{V}(\theta, a, z^\star; \lambda)$ and mobility $\hat{V}(\theta, a)$,[34] occupation-specific cutoffs for rest $\underline{\theta}^r(a, z^\star)$ and mobility $\underline{\theta}^m(a, z^\star)$, sectoral output $y(z^\star)$ and aggregate ouput produced by the final good sector Q, and lump sum transfers $T'(z, z^\star)$ that satisfy the following conditions. Transfers are given by (3.3.14)–(3.3.16). The occupations' idiosyncratic states $(z^\star)$ still evolve as (3.3.1), re-

[34] Contrary to the complete markets case, these functions now correspond to the value functions (in level) associated to each activity.

gardless of the degree of risk sharing. The associated fixed point requirements (3.3.8) are unchanged, and so is the definition of occupation-specific output (3.3.9) and aggregate outpout produced by the final good sector (3.3.10). The only difference between these economies lies in households' decisions (and the distributions they induce). For the sake of notation, the expressions below implicitly assume that the cutoffs satisfy $\underline{\theta}^m(a, z^\star) < \underline{\theta}^r(a, z^\star)$ as in the complete markets allocation, i.e. there is no on-the-job search. This is not necessarily the case at equilibrium, as discussed in Section 3.3.4.[35] The value of employment when allowed to move satisfies

$$\rho \bar{V}(\theta, a, z^\star; \lambda_0) = \max_c \left\{ u(c) + \mathcal{A}_c^e \bar{V}(\theta, a, z^\star; \lambda_0) + s_1 \left[\bar{V}(\theta, a, z^\star; \lambda_1) - \bar{V}(\theta, a, z^\star; \lambda_0) \right] \right\} \tag{3.3.17}$$

Here, $\mathcal{A}_c^e f$ denotes the infinitesimal generator associated to a function $f(z^\star)$, the process (3.3.1) and the savings rate (3.2.5) with $\mathcal{Y}(z, z^\star) = b^e \zeta(\theta, \theta^\star) \left(\frac{y(z^\star)}{Q} \right)^{-\frac{1}{\nu}}$. Again, the standard value-matching and smooth-pasting conditions hold along the cutoff for unemployment:

$$\begin{aligned} &\bar{V}\left(\underline{\theta}^r(a, z^\star), a, z^\star; \lambda_0\right) = \underline{V}\left(\underline{\theta}^r(a, z^\star), a, z^\star; \lambda_0\right) \\ \text{and } &\partial_{z_j^\star} \bar{V}\left(\underline{\theta}^r(a, z^\star), a, z^\star; \lambda_0\right) = \partial_{z_j^\star} \underline{V}\left(\underline{\theta}^r(a, z^\star), a, z^\star; \lambda_0\right) \end{aligned} \tag{3.3.18}$$

for each component $z_j^\star$ of $z^\star$. In addition, the borrowing constraint (3.2.4) acts as a reflecting barrier:

$$\partial_a \bar{V}(\theta, a, z^\star; \lambda_0) \geq u'\left(\mathcal{Y}(z, z^\star) - r\underline{\phi}\mathcal{Y}(z, z^\star) + T(z, z^\star) \right) \tag{3.3.19}$$

The values of employment when not allowed to move satisfies

$$\rho \bar{V}(\theta, a, z^\star; \lambda_1) = \max_c \left\{ u(c) + \mathcal{A}_c^e \bar{V}(\theta, a, z^\star; \lambda_1) + s_0 \left[\bar{V}(\theta, a, z^\star; \lambda_0) - \bar{V}(\theta, a, z^\star; \lambda_1) \right] \right\} \tag{3.3.20}$$

Boundary conditions of the form (3.3.18) do not apply, while one of the form (3.3.19) does. The value of unemployment satisfy similar restrictions, so I omit them here for concision. Finally, the value of mobility satisfies

[35] Numerically, this requires to: (i) guess a sequence of cutoffs defining regions for unemployment and mobility; (ii) solve for the value functions, given this guess, and compare the values of unemployment and mobility to determine the regions over which unemployment and mobility are optimal; and (iii) update the guess until the regions coincide. The richer model introduced in Section 3.4 circumvents this issue by assuming that unemployment is unvoluntary.

$$\rho\hat{V}(\theta,a) = \max_c \left\{ u(c) + \mathcal{A}_c^m \hat{V}(\theta,a) + \lambda_0 \int \max\left\{ V^\star(\theta,a,z^\star) - \hat{V}(\theta,a), 0 \right\} dg \right\} \tag{3.3.21}$$

here $\mathcal{A}_c^m f$ denotes the infinitesimal generator associated to a function $f(z^\star)$, the process (3.3.1) and the savings rate (3.2.5) with $\mathcal{Y}(z,z^\star) = b^m(\theta)$. The value of joining a new occupation is

$$\rho V^\star(\theta,a,z^\star) = \max_c \left\{ u(c) + \mathcal{A}_c^e V^\star(\theta,a,z^\star) + s_2 \int \left[\bar{V}(\theta,a,z^\star) - V^\star(\theta,a,z^\star) \right] dg \right\} \tag{3.3.22}$$

The threshold for entering an occupation satisfies

$$V^\star\left(\underline{\theta}^e(a,z^\star), a, z^\star\right) = \hat{V}\left(\underline{\theta}^e(a,z^\star,a), a\right) \tag{3.3.23}$$

Again, a boundary condition of the form (3.3.19) applies.

Not surprisingly, (3.3.17)–(3.3.23) are equivalent to (3.3.2)–(3.3.7) when setting $\psi = \psi^\star = 1$, i.e. with complete markets. In this case, households are provided with enough liquidity to insure that their incentives to search across occupations and remain in them coincide with the planner's. Therefore, the equilibrium allocation is efficient. Away from this benchmark, households' incentives do not necessarily line up with the planner's. In particular, the cutoff for mobility $\underline{\theta}^m(a,z^\star)$ is now indexed by the households' holdings of financial assets. Imperfect risk sharing leads to an inefficient dispersion in the households' financial asset holdings (a). In turn, this creates room for labor misallocation across occupations.[36]

3.3.4 Labor Misallocation

To understand how incomplete markets and borrowing constraints affect the household's mobility decisions, I consider the following two partial equilibrium experiments.[37] The first one illustrates how incomplete markets affect households' incentives to search across occupations. The second one illustrates how it affects their incentives to leave an occupation when this activity is subject to demand shocks.

Inefficient search (*ex ante*)**.** The first experiment focuses on households' search. It consists of one-time, unanticipated mean preserving spread in the distribution of liquid financial

[36] I focus here on the mobility decision. However, incomplete markets also affect the labor supply decision, i.e. the choice between employment and unemployment, even absent mobility.

[37] In particular, I implicitly fix the drift $\boldsymbol{\mu}(z^\star)$ and volatility $\boldsymbol{\sigma}(z^\star)$, the aggregate distributions $g(z^\star)$ and $\hat{g}(\theta,a)$, and outputs $y(z^\star)$ and Q at their (general) equilibrium levels.

assets (Λ) among a random subset of households searching for an occupation $(m = 1)$ at $t = 0$.[38] Fiscal policy implements (3.3.14)–(3.3.16) with $\psi = \psi^\star = 1$ for all $t \geq 0$. In addition, it provides a lump sum payments at $t = \Delta > 0$ to undo the resulting dispersion in financial assets.[39] I am interested in the limit as $\Delta \to 0$.

Households who search are suddently heterogeneous in their asset holdings. For illustration, consider a household of type θ with assets $a = a_0 < 0$ who has the opportunity to exit the mobility state at $t = 0$. She draws an occupation $z^\star$ from its stationary distribution. With incomplete markets, the household transitions immediately back to mobility, i.e. keep searching, whenever $\theta < \underline{\theta}^e (a_0, z^\star)$. With complete markets instead, she keeps searching when $\theta < \underline{\theta}^e (0, z^\star)$.[40]

Households' search is inefficient whenever $\underline{\theta}^e (a, z^\star)$ is monotonic in a around $a = 0$. The shape of this cutoff typically depends on the state of an occupation $(z^\star)$. Inspecting (3.3.17)–(3.3.21), the lower financial asset holdings a, the higher the marginal utility of consumption. Effectively, the household becomes more impatient due to precautionary savings. When comparing employment and search, she prefers the option which provides more liquidity, i.e. the most front-loaded payoff. In turn, the liquidity provided by employment depends on the current state of the occupation $z^\star$.[41] When an occupation is expanding (contracting), its wage payoffs provide more (less) liquidity than search. Therefore, the cutoff for mobility $\underline{\theta}^e (a, z^\star)$ is expected to be *decreasing* in a when the occupation $z^\star$ is expanding, and *increasing* otherwise (at least locally around $a = 0$).

The left panel of Figure 3.1.1 illustrates this point, and depicts this cutoff for two occupations $z_1^\star$ (boom), and $z_0^\star$ (bust). Whenever the differentials $\underline{\Delta}^e (a, z) \equiv \underline{\theta}^e (a, z) - \underline{\theta}^e (0, z)$ are non zero, the assignment of workers to occupations is inefficient. Households with a low liquidity buffer are willing to enter an inefficiency large (small) share of booming (contracting) occupations. As time passes, households who search for a new occupation draw down their their liquidity buffer. This further affects their incentives to enter a new occupation.

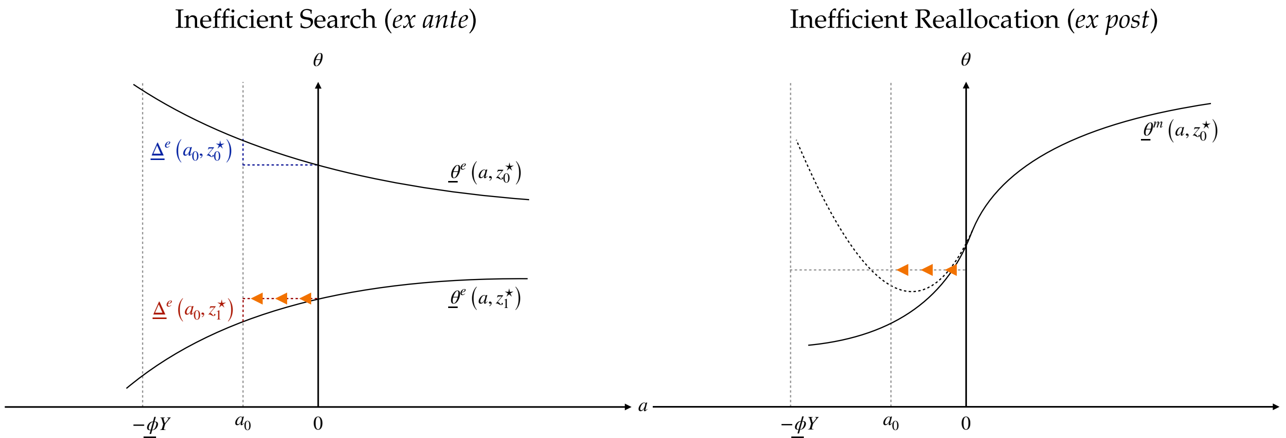
[38] I suppose that this subset has a mass of zero.

[39] Formally, I allow for a discontinuous measure of payments (T) in the flow budget constraint (3.2.5), so that asset holding are discontinuous at $t = \Delta$.

[40] By construction, the cutoff $\underline{\theta}^e (a, z^\star)$ coincides in the two cases, as $\Delta \to 0$.

[41] In the following, I compare occupations with different realizations of the demand shock φ, but with the same distributions of households' idiosyncratic states (Λ).

Figure 3.3.1: Labor Misallocation



Overall, imperfect risk sharing shortens households' average job search in this experiment whenever

$$\int \underbrace{\lambda_0 \int \mathbf{1}_{\{\theta \geq \underline{\theta}^e(a,z^\star)\}} dg}_{\text{Exit rate with IM}} \, d\hat{g}' > \int \underbrace{\lambda_0 \int \mathbf{1}_{\{\theta \geq \underline{\theta}^e(0,z^\star)\}} dg}_{\text{Exit rate with CM}} \, d\hat{g}$$

and increases its duration otherwise. Here, $\hat{g}$ and $\hat{g}'$ denotes the distribution before and after, respectively, the mean preserving spread in asset holdings, i.e. with perfect and imperfect risk sharing. Whether unemployment duration is inefficiently long or short depends on the exact primitives of my model. This is a quantitative question that I tackle in a richer framework from Section on 3.4. The empirical evidence provides some guidance: Chetty (2008) find that that liquidity frictions shorten the workers' job search, and Nekoei and Weber (2017) that they reduce the quality of the resulting matches.

Inefficient reallocation (ex post)**.** The second experiment focuses on household's decision to leave their occupation and search for a new one. Their trade-off is similar to the one discussed above, with one exception. Households already located in an occupation ($m = 1$) have access to rest ($\chi = r$), who those who search ($m = 0$) do not. This distinction creates a potentially rich pattern of labor reallocation across occupations.

This second experiment consists of one-time, unanticipated reduction in risk sharing from $\psi = \psi^\star = 1$ to $\psi' = \psi^{\star,\prime} = 0$ in some occupation $z^\star$ over a period $t \in [0, \Delta)$ for some $\Delta > 0$. Fiscal policy at $t \in [\Delta, +\infty)$ is as described in the previous experiment. Again, I am interested in the limit as $\Delta \to 0$.

Households are suddenly exposed to the demand shocks that hit their occupation.

For illustration, I consider a contractionary shock $(z_0^\star)$. Households who are allowed to move $(\lambda = \lambda_0)$ choose between two options: being unemployed with income b^u over a long period of time, or search for a new occupation with income b^m and an average transition time $1/\lambda_0$. Since $b^u > b^m$, moving entails a short-term liquidity cost. Households trade-off between this mobility cost and higher continuation value when assigned to a new occupation. Crucially, the associated cutoff for mobility $\underline{\theta}^m (a, z_0^\star)$ is potentially non-monotonic in its first argument. For sufficiently high holdings of financial assets (a), this cutoff is expected to be *increasing*: the lower the asset holdings, the higher the precautionary savings motive and the stronger the incentives to search for a better occupation. For sufficiently low financial asset holdings (a) however, this cutoff is potentially *decreasing*: the lower the asset holdings, the more prohibitive the liquidity cost of searching for a new occupation. That is, imperfect risk sharing is responsible for two forces. One induces *excess* reallocation in downturns among households with low liquidity buffers, as they have a higher marginal utility of improving their (expected) future earnings by moving to another occupation. The other commands *deficient* reallocation, as households are trapped in their activity and unable to pay the liquidity cost to transition to another one.

The right panel of Figure 3.3.1 illustrates this point. Again, I focus on a contractionary shock $(z_0^\star)$. I depict two possible cutoffs $\underline{\theta}^m (a, z_0^\star)$. As time passes, households draw down their their liquidity buffer, and they eventually have the possibility to move between occupations. Households' pattern of reallocation between occupations depends on the severity of the contraction $(z_0^\star)$, the payoffs when unemployed and moving between occupations (b^u, b^m), the expected duration of the transition between occupations $(1/\lambda_0)$, and the share of households who are allowed to move in a given period (s_0/s_1). Again, whether household's reallocation is efficient is a quantitative question that I tackle in a richer framework from Section on 3.4.

3.4 A Richer Model

In Section 3.3, I used my benchmark model to clarify how incomplete markets and borrowing constraints can hinder the efficient matching between workers and occupations. I now develop a richer setup to investigate the quantitative importance of the mechanisms I identified. I extend the model along several dimensions. First, I introduce nominal wage rigidities, unvoluntary unemployment and convex labor adjustment costs within occupations to obtain more realistic fluctuations in employment at the occupation level. Second, I introduce social insurance in the form of unemployment insurance and severance payments. This allows me to revisit the analysis of Alvarez and Veracierto (2000) and Alvarez and Veracierto (2001) in my setup with occasionally-binding borrowing constraints. Third, I let the mobility costs $\lambda_0 (\theta^\star)$ be indexed by the destination occupation, i.e. these costs are occupation-specific. This allows me target to the empirical distribution of employment across occupations and (aggregate) gross flows between sectors.[42] Fourth, I suppose that firm's technologies have decreasing returns. Households use firms' equity as a precautionary savings buffer. Finally, I introduce worker-specific uninsured idiosyncratic productivity shocks to better match the distribution of earnings observed in the data. I describe the environment below. The full analysis is presented in Zorzi (2020b).

3.4.1 Environment

The environment is identical to the one presented in Section 3.2.1, except for a few changes that I highlight below.

Occupations. Occupations are now indexed by an extra idiosyncratic state: their stock of (effective) labor (n). The representative firm in each occupation faces convex labor adjustment costs. Let

$$\lambda (z^\star; \hat{\chi}) \equiv \int \mathbf{1}_{\{\chi = \hat{\chi}\}} \zeta^\star (\xi, \theta, \theta^\star) \Lambda (dz, z^\star) \ , \ \hat{\chi} \in \{e, u\}$$

denote the mass of households employed and unemployed in an occupation, respectively. Here, $\zeta^\star (\xi, \theta, \theta^\star)$ denotes the effective productivity of households' unit of labor supply (defined below). The change in the firm's labor demand is

$$H (z^\star) \equiv f^{-1} (y (z^\star)) - \lambda (z^\star; e)$$

The firm incurs a non-convex labor adjustment cost. It consists of two components. The first one corresponds a discrete severance payment $S > 0$ to each worker that the firm separates from.[43] The second one takes the form of a convex technological cost

$$\Gamma (z^\star) \equiv \kappa \left| \frac{H (z^\star)}{\lambda (z^\star; e)} \right|^\gamma \tag{3.4.1}$$

paid in terms of the final good, for some $\kappa, \gamma > 0$. A mutual fund claims firms' profits,

[42] However, I suppose that these mobility costs *not* indexed by the occupation of origin. As a result, I cannot match empirical gross flows between two occupations.

[43] That is, the process for firms' cumulated payments is discontinuous at the time of separation.

and sells a diversified equity to the households.[44] The dividend on this equity is[45]

$$\Pi \equiv \int \left[\left(\frac{y(z^\star)}{Q} \right)^{-\frac{1}{\nu}} y(z^\star) - W(z^\star) f^{-1}(y(z^\star)) - \Gamma(z^\star) \right] d\pi$$
$$- S \int \mathbf{1}_{\{H(z^\star)<0\}} H(z^\star)\, d\pi \tag{3.4.2}$$

Households. Households are now indexed by one extra idiosyncratic states: their persistent productivity shock (ξ). Furthermore, household's employment status (χ) becomes a state variable and replaces their mobility status (m).[46] Households can be either employed $(\chi = e)$, unemployed $(\chi = u)$, moving between occupations $(\chi = m)$. Households' productivity follows a first-order autoregressive process

$$d\xi_t = -\eta_\xi \left(\xi_t - \bar{\xi} \right) dt + \sigma_\xi B_t \tag{3.4.3}$$

for some (common) mean and persistence $\bar{\xi} \equiv 1$ and $\eta > 0$, with independence across households. The productivity of a household's unit of labor is now $\zeta^\star(\xi, \theta, \theta^\star) \equiv \exp(\xi) \times \zeta(\theta, \theta^\star)$. Total incomes become

$$\mathcal{Y}(z, z^\star) = \begin{cases} \zeta^\star(\xi, \theta, \theta^\star) W(z^\star) & \text{if } \chi = e \\ b^u + B^u & \text{if } \chi = u \\ b^m(\theta) + B^m & \text{otherwise} \end{cases} \tag{3.4.4}$$

where $B^u, B^m \geq 0$ denotes unemployment insurance and a subsidy to mobility, respectively. Households allocate their savings between the two riskless financial assets they have access to: bonds b_t (in zero net supply); th diversified equity e_t (in unitary net supply). Households are subject to the following flow borrowing constraint

$$a(z, z^\star) \equiv b(z, z^\star) + qe(z, z^\star) \geq -\underline{\phi} \mathcal{Y}(z, z^\star) \tag{3.4.5}$$

where q denotes the price of equity. By no arbitrage, the nominal return on the two assets

[44] That is, I suppose that households cannot directly invest in the equity of a particular occupation. Otherwise, they would be able to diversify away their occupation-specific risk. Empirically, households fail to hedge against sector-specific aggregate risk (Massa and Simonov (2006)).

[45] Note that the term in brackets on the right-hand side of (3.4.2) corresponds to the continuous part of occupation-specific dividends, while the second term correspond to the discountinuous part due to severance payments.

[46] That is, households' employment status $(\chi \in \{e, u\})$ is a state variable in this richer version of the model, instead of a choice variable as in the benchmark model.

is equalized (r).[47] The households' flow budget constraint (3.2.5) is

$$da_t(z,z^\star) = \left[(1-\tau)\mathcal{Y}(z,z^\star) + ra_t(z,z^\star) + T(z,z^\star) - c_t(z,z^\star)\right]dt \tag{3.4.6}$$

where τ denotes a distorsionary tax on incomes.

Labor markets. Labor markets are now subject to search frictions, so that there is unvoluntary unemployment. Nominal wages are sticky and firms face convex labor adjustment costs.

The following occurs in period $t \geq 0$.[48] First, households decide whether to search within their occupation or move to another one (when allowed to), and the firm chooses its labor demand $f^{-1}(y(z^\star))$. Second, some unemployed workers are matched to the firm in their occupation or endogeneous separation occurs, and production occurs. Specifically, if the excess labor demand satisfies $H(z^\star) \geq 0$, i.e. the firm hires, then each unemployed household $(\chi = u)$ becomes employed $(\chi = e)$ with intensity $h(z^\star; e) \equiv H(z^\star)/\lambda(z^\star; u)$. Otherwise, i.e. the firm lays off, then each employed household $(\chi = e)$ becomes unemployed $(\chi = u)$ with intensity $h(z^\star; u) \equiv H(z^\star)/\lambda(z^\star; e)$. Third, exogeneous separation between workers and firms takes place with intensity $s > 0$ in any case. Finally, households who search are (stochastically) assigned to occupations with type-specific intensities $\lambda_0(\theta^\star)$, and draw a mobility cost as in the benchmark model.

Wage setting. Nominal wages are sticky. Each occupation resets its wage with intensity $\kappa > 0$. At that point, wages are Nash-bargained (collectively) between the firm and the households who are present in that occupation. Households have bargaining power $\varsigma \in (0,1)$. The firms' and households' threat points are the values of delaying the wage reset until they next have the opportunity to do so. I consider the limit as $\kappa \to 0$, i.e. wages are rigid.[49] Therefore, wages are only indexed by the occupations' type: $W(z^\star) = \bar{W}(\theta^\star)$ for some wage schedule $\bar{W}(\cdot)$.

Market clearing. The good and labor market clearing conditions in each occupation are

$$y(z^\star) = \int\int f\left(\mathbf{1}_{\{\chi(z)=e\}}\zeta^\star(\xi,\theta,\theta^\star)\Lambda(dz,z^\star)\right)d\pi, \tag{3.4.7}$$

[47] That is, the price of equity satisfies $q = r\Pi$.

[48] Again, time is continuous so that the exact timing of actions within each mobility state is irrelevant.

[49] The evidence suggests that wages are very sticky, both for incumbents and new hires (Grigsby et al. (2019)).

The market clearing condition for the final good is[50]

$$\int c(z,z^\star)\,\Lambda(dz,z^\star)\,d\pi = Q + \int \left[b^u \mathbf{1}_{\{\chi(z)=u\}} + b^m \mathbf{1}_{\{m=1\}}\right] \Lambda(dz,z^\star)\,d\pi + \int \Gamma(z^\star)\,d\pi \tag{3.4.8}$$

3.4.2 Policy

Fiscal policy. Fiscal policy chooses the level of severance payments $S > 0$, and finances unemployment insurance and its subsidy to mobility using the distorsionary tax on incomes. The government's budget is balanced:

$$\int \tau \mathcal{Y}(z,z^\star)\,\Lambda(dz,z^\star)\,dg = \int \left(\mathbf{1}_{\{\chi=u\}} B^u + \mathbf{1}_{\{\chi=m\}} B^m\right) \Lambda(dz,z^\star)\,dg$$

Monetary policy. Monetary policy sets the nominal interest rate r to implement (uniquely) a non-inflationary, stationary equilibrium for this economy. That is aggregate prices and allocations are constant. The price of the final good is still normalized to 1 (*numéraire*).

3.4.3 Discussion

Inefficiency. The complete markets allocation is now typically inefficient due to search frictions and nominal rigidities. In particular, search frictions contribute to excess unemployment compared to the first best allocation. Furthermore, occupation-specific shocks lead to an inefficient dispersion in labor market tightness across occupations.[51] In this richer, a new source of inefficiency emerges in general equilibrium. By depressing aggregate productivity, labor misallocation lowers the value of firms' equity below its efficient level, i.e. a pecuniary externality. As a result, households hold inefficiently low liquidity buffers, which makes incomplete markets and borrowing constraints even more acute.

Severance payments. The presence of labor adjustment costs leads to labor hoarding in this richer model.[52] In turn, the effective degree of market incompleteness depends on the importance of labor hoarding.[53] Severance playments (S) play an important role in this

50 The market clearing condition for assets $\int\int a(z,z^\star)\,\Lambda(dz,z^\star)\,d\pi = 1$ is redundant.

51 Shimer (2007) refers to this inefficient dispersion as "mismatch." Sahin et al. (2014) assess the importance of this mismatch to explain the rise is unemployment during the Great Recession.

52 Bertola and Caballero (1994) investigate the role of labor hoarding in a Lucas and Prescott (1974) economy with complete markets.

53 Guerrieri et al. (2020) provide an example where, despite incomplete markets, the insurance provided by labor hoarding allows to implement the complete markets allocation.

setup with occasionally-binding borrowing constraints. These payments are fully front-loaded, as opposed to unemployment insurance (B^u) which is claimed linearly over time. Therefore, severance payments allow workers to partially undo the financial frictions that they potentially face in the short-term. This effect is absent from earlier work on severance payments, including Alvarez and Veracierto (2000) and Alvarez and Veracierto (2001).

Job ladder. Workers now search on-the-job even when markets are complete.[54] This effectively creates a job-ladder for workers,[55] who transition from occupation to occupation until they are matched with one where they have a strong comparative advantage. I explore whether workers fall short on climbing their job ladder when markets are incomplete and workers face borrowing constraints. The presene of on-the-job search allows me to speak to an extensive literature on gross flows across sectors and occupations.[56]

3.5 Next Steps

In the previous sections, I presented a variant of the Lucas and Prescott (1974) model augmented with two-sided heterogeneity between workers and occupations and incomplete markets. Using this model, I clarified how imperfect risk sharing can hinder the efficient assignment of workers to firms. Taking stock, I presented a quantitative version of this model. The next steps consist of calibrating this model to match empirical moments of the micro and aggregate U.S. data, simulate this calibrated model, and solve numerically for optimal social insurance. The full analysis is presented in Zorzi (2020b).

[54] The reason is that I rule out voluntary employment in this richer model.

[55] Burdett (1978) is a seminal contribution on on-the-job search. Moscarini and Postel-Vinay (2018) provide a review of the literature on job-ladders.

[56] Davis and Haltiwanger (1999) survey the evidence on gross flows across occupations and sectors. Moscarini and Vella (2008) document the cyclical properties of gross flows across occupations.

Appendix to Chapter 3

3.A Quantitative Appendix

Complete Markets

In this appendix, I state the planner's problem with complete markets.

Planner's problem. The planner maximizes

$$\mathcal{U}^{\star} \equiv \int u_{\theta}\left(c\left(\theta\right)\right) d\mu$$

by choosing households' type-specific consumption $c\left(\theta\right)$, output of the final sector Q, occupation-specific output $y\left(\theta\right)$, joint measure $g\left(z^{\star}\right)$ of households' idiosyncratic states within each occupation, and the (household) type-specific mass of households $\hat{g}\left(\theta\right)$ moving between occupations, and (household) type- and occupation-specific cutoffs for rest $\underline{\theta}^{r}\left(z^{\star}\right)$ and mobility $\underline{\theta}^{m}\left(z^{\star}\right)$, subject to the resource constraint

$$\int c\left(\theta\right) d\mu = Y \equiv Q + \int \left[b^{u}\mathbf{1}_{\{\chi(z)=u\}} + b^{m}\mathbf{1}_{\{m=1\}}\right] \Lambda\left(dz, z^{\star}\right) d\pi,$$

the technological constraints

$$Q = \left[\int \exp\left(\varphi\right)^{\frac{1}{\nu}} y\left(z^{\star}\right)^{\frac{\nu-1}{\nu}} d\pi\right]^{\frac{\nu}{\nu-1}}$$

and

$$y\left(z^{\star}\right) = b^{e} \int \int \mathbf{1}_{\{\theta \geq \underline{\theta}^{r}(z^{\star})\}} \zeta\left(\theta, \theta^{\star}\right) \Lambda\left(dz, z^{\star}\right) dg,$$

together with the restrictions

$$\int \int d\Lambda dg + \int d\hat{g} = 1 \text{ and } g, \hat{g} \text{ non-decreasing}$$

and the laws of motion for the distributions of households who stay in their occupation and move, respectively. Therefore, the planner's problem consists of maximizing aggregate output Y, subject to the above constraints. The solution to this problem is characterized in Section 3.3.2.

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