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Last time: the practically rational thing to do is the thing with the highest expected value. Is this really true? The expected value for an act A is defined as follows, where the O s are the potential outcomes of the act:

$$(EV) \quad \text{Expected Value}(A) = \sum_O \text{Value}(A \ \& \ O) \cdot \text{Probability}(O \ \text{if } A)$$

1 Newcomb's Paradox

Priscilla is a near-perfect predictor of human action. She sets you the following problem. There are two boxes, one opaque, one translucent. Your options: take the money in both, or just the money in the opaque box. You can see that there is \$1,000 in the translucent box. In the opaque box, Priscilla has placed either \$1 million or nothing at all. Her decision was determined as follows: if she predicts that you will take only the opaque box, she puts \$1 million in it; if she predicts you will be greedy and take both, she places nothing inside the opaque box. Should you take just the opaque box, or both boxes? It might seem both, for the money is there or it isn't. But let G = greedy, M = there's a million in the opaque box ("V" stands for value, and "P" for probability):
 $EV(G) = V(M \& G)P(M \ \text{if } G) + V(\sim M \& G)P(\sim M \ \text{if } G) = \$1,001,000\epsilon + \$1,000(1 - \epsilon) \approx \$1,000$
 $EV(\sim G) = V(M \& \sim G)P(M \ \text{if } G) + V(\sim M \& \sim G)P(\sim M \ \text{if } \sim G) = \$1,000,000\epsilon + \$0\epsilon \approx \$1,000,000$

2 The Saint Petersburg Paradox

The game: a fair coin will be tossed repeatedly until a tail first appears, ending the game. The pot starts at 1 dollar and is doubled every time a head appears. You win whatever is in the pot after the game ends. Thus you win 1 dollar if a tail appears on the first toss, 2 dollars if on the second, 4 dollars if on the third, 8 dollars if on the fourth, etc. In short, you win 2^k dollars if the sequence of tosses begins with k heads (followed by a tail). Would you rather have a billion dollars or the chance to play this game? Let's consider the expected payout. With probability $\frac{1}{2}$, you win 2 dollars; with probability $\frac{1}{4}$, you win 4 dollars; with probability $\frac{1}{8}$, you win 8 dollars etc. The expected value is thus $1 + 1 + 1 + \dots = \infty$. You should therefore play the game rather than take the billion; indeed you should prefer the game to a sure $\$n$ for every n whatsoever.

3 The Two Envelope Paradox

You are given two indistinguishable envelopes, each containing a positive sum of money. One envelope contains twice as much as the other. You may select either envelope and keep whatever amount it contains; I get the contents of the other envelope. But before opening the envelope, you are given the chance to trade yours for mine. Should you trade? Apparently so!

1. Let x be the amount in the selected envelope.
2. The other envelope may contain either $2x$ or $\frac{x}{2}$
3. The probability that x is the smaller amount is $\frac{1}{2}$, as is the probability that it's the larger.

4. So the expected value of trading is $\frac{1}{2} \cdot \frac{x}{2} + \frac{1}{2} \cdot 2x = \frac{5x}{4}$
5. This is greater than x , so the rational option is to trade.
6. The same argument shows that you should trade back!

4 The Monty Hall Problem (*Not* a Paradox)

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, and the host, who knows what's behind the doors, opens another door, which has a goat. [He's always in a position to do this, since at least one of the other doors will have a goat.] He then says to you, "Do you want to stick with your initial choice, or switch to the other closed door?" Should you stay? Switch? Does it matter? (Formulation adapted from vos Savant 1990)

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