ANALYSIS OF AEROSOL COAGULATION
IN A SOUND FIELD

by

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SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF BACHELOR OF SCIENCE in the MASSACHUSETTS INSTITUTE OF TECHNOLOGY (1951)

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Dear Sir:

This thesis, entitled "Analysis of Aerosol Coagulation in a Sound Field", is respectfully submitted in partial fulfillment of the requirements for the Bachelor of Science Degree.

Yours very truly,

Signature redacted

Earl G. Cole
ABSTRACT

A statistical analysis was made of the coagulation phenomenon of an aerosol in a sound field. The effect of changing important variables has been determined. Quantitative calculations have been made and the results give trends that agree with the physical facts; however, the calculations have not been extended to a point where satisfactory comparison with experimental data can be made.
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Symbol | Definition | Units  
---|---|---  
V | Total volume of particles per unit volume | cm$^3$  
W | Grain loading | gm/cm$^3$  
x | Volume of particle | cm$^3$  
y | Volume of particle (variable of integration) | cm$^3$  

Greek Letters

\( \rho \) | Density of particle | gm/cm$^3$  
\( \rho_g \) | Density of gas | gm/cm$^3$  
\( \mu_g \) | Viscosity of Gas | dyne-sec/cm$^2$  
\( \alpha^* \) | Dimensionless velocity parameter |  

Superscripts

* | Dimensionless quantities |  
' | Change in variable |  

Subscripts

g | pertains to gas |  
p | pertains to particle |  
0 | reference variable |  
1 | denotes a particular volume |  
2 | denotes a particular volume |  

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INTRODUCTION

The object of this thesis is to present an analysis of the phenomenon of aerosol coagulation in a sound field. An attempt has been made to develop a theory that will predict the principle variables involved and determine their relationship to the coagulation rate. The main application of the phenomenon is as an intermediate step in the process of industrial collection of aerosols. The large Brownian motion of extremely small particles (less than one micron in diameter) causes these particles to agglomerate; therefore, sizes less than one micron are seldom encountered in practice. There are centrifugal machines which can collect particles larger than ten microns in diameter with an efficiency of about 90 percent. Sonic application is in the range of particle sizes from one to ten microns where an attempt is made to make small particles grow into larger particles by agglomeration.

The term aerosol is used to define particles that are dispersed in a gas. The term pertains to both solid and liquid substances and covers the familiar types which are called smoke, dust, fume, etc. If the aerosol is a liquid, the particles will be spheres; small solid particles, however, will also behave essentially as a sphere.
It was observed over fifty years ago that sound waves would cause particles of an aerosol to agglomerate. Numerous experiments were carried out in an attempt to determine the cause of the coagulation and to determine the forces acting on the particles. The general conclusions arrived at from the results of these early experiments are that there are three main effects that could cause agglomeration. They are:

1. If a distribution of small spherical particles are introduced into a sound field, they will be caused to oscillate with different amplitudes depending upon their size. This relatively increased motion will result in more collisions per unit time than by normal thermal agitation and will greatly increase the agglomeration rate. Ref. (5,7)

2. A hydrodynamic analysis of the forces between two spheres which are at rest in a vibrating medium results in both attractive and repulsive forces which are inversely proportional to the fourth power of distance between particles. Ref. (6,3)

3. The third effect is the phenomenon of acoustical radiation pressure. An analysis of the forces on a small sphere in a sound field of standing
waves shows that the sphere migrates toward the node. Particles, therefore, will tend to collect at nodes and agglomerate. This effect is easily observed in a sound tube with standing waves using a stationary aerosol. In the case of a moving aerosol passing through a sound chamber with standing waves, it is reasonable to assume that this effect will only increase the turbulence of the flow and aid in the coagulation process by maintaining a random distribution of the particles in space.

It is the desire of the author to formulate a theory that will aid in the design of more efficient industrial collectors. In this analysis, a mathematical representation of the coagulation phenomenon of an aerosol flowing through a sound chamber with standing waves is formulated. The stipulation of a flowing aerosol is made since this condition tends to maintain a more random distribution of the particles in space and supports one of the basic assumptions made in the analysis presented in the next section.
STATISTICAL ANALYSIS

The present analysis is based on a simple statistical interpretation of the collision phenomenon of small particles in a moving gas exposed to a sound field. The most important assumption which we have to make is that of the laws of motion which the particles in the sound field obey. In this analysis, the assumption is made that the motion of different-sized particles relative to each other is the dominant one in the range of particle sizes in which we are interested. We will neglect hydrodynamic forces and the influence of radiation pressure, except to the extent that it will produce a more random distribution of particle position. For those who feel that this may be too strong an assumption, let us point out that the hydrodynamic forces could be included by considering the effective cross-section of each particle as being enlarged slightly, since the hydrodynamic forces are proportional to the reciprocal of the fourth power of the distance between two particles and do not become significant except at small distances.

We, therefore, make the following assumptions:

(1) Particles move with the root mean square value of the following relation

\[ u_p = (u_p)_{\text{max}} \sin (w t - \phi) \]
where
\[
(u_p)_{\text{max}} = \frac{(u_g)_{\text{max}}}{\sqrt{\left[\frac{4\pi r^2 \rho}{9 \mu_g}\right]^2 + 1}}
\]  
(1)

(See Appendix 2)
(2) The particles are essentially spherical
(3) The particles are independent, that is electrical, gravitational, and magnetic forces are neglected.
(4) Collisions result in particles whose volumes are the sum of the volumes of the colliding particles.
(5) The distribution of particles in space remains random.

We consider a non-homogenous aerosol with particles of various sizes given by a distribution function.

Let \( x \) = volume of a particle
\( g(x,t) = \text{distribution function} = \text{number of particle per unit volume per volume element } dx \)
\( n(t) = \text{number of particles per unit volume at any time } (t) \)
\( dn(x,t) = \text{number of particles per unit volume having their volume between } x \text{ and } x + dx \)

Therefore \( dn(x,t) = g(x,t) \, dx \)  
\[ \therefore \int_0^\infty dn(x,t) = n(t) = \int_0^\infty g(x,t)dx \]  
(3)
The invariant in the problem is the total mass or rather the volume since the density is a constant. Therefore

$$v = \int_{0}^{\infty} x g(x, t) \, dx = \text{constant.} \quad (4)$$

The mechanism of coagulation is a collision phenomenon; therefore, we must calculate a rate of collision between the particles of different sizes. We assume that the particles can be considered as having a velocity given by equation (1). Equation (1) represents the velocity amplitude of a sinusoidal varying velocity, therefore we want to use its effective value.

$$(u_p)_{\text{eff}} = (u_p)_{\text{max}} x \sqrt{E/2}$$

Brownian motion of particles, ineffective collisions, and turbulence of the flowing gas in the sound chamber will tend to maintain a random distribution of particles in space.

We define the following quantities:

$$Q = \text{collision efficiency}$$

- It will probably be a function of a number of particles per unit volume \(n(t)\), the maximum and minimum relative velocities, and finally a function of any given aerosol.

$$H(x_1, x_2) = \text{number of effective collisions per unit time between a particle having volume } x_1$$

with all the particles having volumes \(x_2\) and \(x_2 + dx_2\).
\[ dn(x_2, t) = g(x_2, t) \, dx_2 = \text{number of particles per unit volume having volumes between } x_2 \text{ and } x_2 + dx_2 \] (5)

We assume that the particles of volume \( x_2 \) are stationary and let particles of volume \( x_1 \) move with velocity \( u_1 - u_2 \). Then the effective volume for collision

\[ \simeq \pi (r_1 + r_2)^2 |u_1 - u_2|_{\text{eff}}. \] (6)

Therefore

\[ H(x_1, x_2) \equiv Q \pi (r_1 + r_2)^2 |u_1 - u_2|_{\text{eff}} \] (7)

using

\[ r = \left[ \frac{3}{4 \pi} \right]^{1/3} x^{1/3} \] (8)

We have

\[ H(x_1, x_2) = Q \pi \left[ \frac{3}{4 \pi} \right]^{2/3} (x_1^{1/3} + x_2^{1/3})^2 |u_1 - u_2|_{\text{eff}} \, dn(x_2, t) \] (9)

Let us examine more closely the types of phenomena that the collision efficiency \( (Q) \) would have to represent. The amplitude of a vibrating particle is given by \( S = \frac{u_1 - u_2}{2 \pi \sqrt{\frac{\alpha^*}{\rho}}} \) where \( \alpha^* \) depends on the mass of a particle and varies from 1 to zero and is defined more explicitly later in the text. When the mean distance between particles \( \bar{l} = \frac{1}{\sqrt{dn(t)}} \) increases to a value larger than the vibrating amplitude of the particle, the number of collisions per unit time will become small. Therefore as the number of particles decreases, the collision efficiency \( (Q) \) will decrease as a function of time. If the parameter \( S - \bar{l} \) is large, the collision efficiency will be high.
As the parameter \((S - \bar{l})\) approaches zero, the collision efficiency will decrease rapidly. If the intensity and frequency are such that the quantity \((S-\bar{l})\) is large at the initial time \((t = 0)\), then \(Q \approx 1\) and will decrease slowly with time (See Appendix 1). The collision efficiency could also contain the influence of a maximum and minimum velocity. There may be a minimum velocity below which there will be no effective collisions resulting in agglomeration. There may also be a maximum velocity above which collisions will result in disintegration instead of agglomeration. The exact nature of this phenomenon is unpredictable.

Finally, the collision efficiency will depend upon the type of aerosol. Attractive forces that hold the particles together will depend largely upon the physical and chemical properties of the aerosol.

It might seem at first sight that we have included too many unknowns into a single parameter \((Q)\), but if the intensity, frequency, and grain loading are not too unfavorable, the collision efficiency should remain almost constant over a time interval of the order of several seconds.

We now proceed to calculate the variation in the number of particles of volume \(x\) in a time \(\delta t\).

Let \(y\) = variable of integration

\[
y = x_1 \quad x - y = x_2 \quad x_1 + x_2 = x
\]
The first integral in equation (10) represents the collisions between particles that tend to increase \( dn(x,t) \), while the second integral represents the collisions between particles of volumes \( x \) with all of the other particles, thereby leading to a decrease in \( dn(x,t) \). After combining equations 5, 9, 10, we pass to the limit, and cancel common terms. Therefore, the governing equation has the form

\[
\frac{\partial}{\partial t} q(x,t) = \int [y^{\text{eff}} + (x-y)^{\text{eff}}] u_y - u_{x-y} q(x,t) q(x-y,t) \, dy
\]

\[
- \int [x^{\text{eff}} + y^{\text{eff}}] u_x - u_{x-y} q(y,t) q(x,t) \, dy
\]  

(11)
In order to generalize the theory and make the equations more simple for computation, we introduce a dimensionless volume ratio into our equation.

Let \( x^* = \frac{X}{x_0} = \left[ \frac{d}{d_0} \right]^3 \) and \( y^* = \frac{Y}{x_0} \) where \( x_0 \) will be the volume of a particle with some reference diameter \( d_0 \).

\[
x_0 = \frac{\pi}{6} d_0^3
\]  

(12)

It is also desirable to have the velocity dependence on a dimensionless basis.

Thus

\[
\text{Therefore } (u_p)^{\text{eff}} = \frac{.707 \ (u_g)^{\text{max}}}{\sqrt{\left[ \frac{4\pi f \rho}{9 \mu_g} \right]^2 \left[ \frac{3}{4\pi} \ x^* + 1 \right]^4}}
\]

(13)

where \( r^4 = \left[ \frac{3}{4\pi} \right] x^* \)

Now, if we let

\[
C = \frac{\rho^2 f^2 d_0^4}{\mu_g^2} \ \frac{\pi^2}{31}
\]

(14)

where \( x_0^{4/3} = \left[ \frac{\pi}{6} \right]^{4/3} d_0^4 \)

we can write

\[
u_p^{\text{eff}} = \frac{.707 \ (u_g)^{\text{max}}}{\sqrt{C x^*^{4/3} + 1}^{1/2}}
\]

(15)
Since \(0.707 (u_g)_{\text{max}}\) is a constant for any given intensity, we can take it out of the integral sign. For computation purposes, we then define \(\alpha^* = \frac{1}{\sqrt{C x^{*4/3} + 1}}\) (16).

The velocity then has form \((u_p)_{\text{eff.}} = 0.707 (u_g)_{\text{max}} \alpha^*\).

Combining these dimensionless terms, we can transform equation (11) into the following form:

\[
\frac{3}{4} t [g(x^*, t)] = Q \left[ \frac{3}{4 \pi} \right]^{2/3} x_o^{2/3} (0.707) (u_g)_{\text{max}} \left[ I_1 - g(x^*, t) I_2 \right] (17)
\]

where

\[
I_1 = \int_{x^*_{\text{min}}}^{x^*_{\text{max}}} \left[ \left( y^{*4/3} + (x^* - y^*)^{4/3} \right)^2 \right] \left| \alpha^*(y) - \alpha^*(x-y) \right| g(y^*, t) g(x^* - y^*, t) dy^* (18)
\]

and

\[
I_2 = \int_{x^*_{\text{min}}}^{x^*_{\text{max}}} \left[ \left( y^{*4/3} + y^{*4/3} \right)^2 \right] \left| \alpha^*(x) - \alpha^*(y) \right| g(y^*, t) dy^* (19)
\]

By use of the well known relation between velocity amplitude of particles in a gas and the intensity of the sound field, we can reduce the terms preceeding the bracket in equation (17).

\[
u_{u_{\text{max}}} = \sqrt{\frac{2}{\int g^2}} (20)
\]
where

\[ J = \text{sound intensity} \]
\[ \rho_g = \text{density of gas} \]
\[ C_o = \text{velocity of sound} \]

Let \( K \) represent this collection of terms. After simplification, we get

\[
K = \frac{\pi Q d_o^2 \sqrt{J}}{4 \sqrt{\rho_g C_o}}
\]

where now we can see the dependence of the important variables.

Equation (14) can now be written in the simple form

\[
\frac{\partial}{\partial t} \left( g(x^*, t) \right) = K \left( I_1 - g(x^*, t) I_2 \right)
\]

The general equation is very difficult to solve since it is a non-linear partial differential integral equation. However, with certain assumptions it is possible to solve it numerically. We can consider it essentially as an initial value problem where we are given the distribution function \( g(x^*, 0) \) at time \( t = 0 \). Then we can proceed step by step to the new distribution by using the series expansion, neglecting terms of higher order.

\[
g(x^*, t_2) = g(x^*, t_1) + \Delta t_{e1} \left[ \frac{\partial g(x^*, t)}{\partial t} \right]_{t = t_1} + \ldots
\]

Application of equation (23) to a given initial distribution
will allow us to follow the agglomeration process as a function of the exposure time in the sound field.

The magnitude of the distribution function is determined essentially by the grain loading of aerosol and the reference volume.

Let

\[ W = \text{grain loading} \ (\text{gm/cm}^3) \]

then

\[ W = \int_{0}^{\infty} x_{0} \int_{x_{0}}^{\infty} g(x^*,0) dx^* \] \hspace{1cm} (24)

For the present calculations, \( g(x^*,0) \) was assumed to be of the following form:

\[ g(x^*,0) = A x^{*2} e^{-x^{*2}} \] \hspace{1cm} (25)

where \( x^* = 1 \) was taken to be most probable size. Therefore since \( x^* = \frac{x}{X_0} = 1 \) we can pick the reference volume \( X_0 \) to be the probable particle size in the distribution. The constant \( A \) is determined by equation (24) with a given \( W, \rho, X_0 \).

The variation of grain loading will be reflected in the following manner. Suppose we have the results for a given initial function \( g(x^*,0) \). Then we change the grain loading keeping the density and \( X_0 \) constant. That is, \( g'(x,0) = B \ g(x,0) \) where \( B = \text{const.} \)

Then

\[ g'(x^*,t_1) = g'(x^*,0) + \Delta t \ \frac{3}{3t} \left[ g'(x^*,0) \right] \] \hspace{1cm} (26)
\[ g'(x^*, t_1) = B g(x^*, 0) + \Delta t' \cdot B^2 \frac{\partial}{\partial t} g'(x^*, 0) \]

where

\[ \frac{\partial g'(x^*, 0)}{\partial t} = k \left[ I_1' - g'(x^*, 0) I_2' \right] = B^2 k \left[ I_1 - g(x^*, 0) I_2 \right] \]

\[ = B^2 \frac{\partial g(x^*, 0)}{\partial t} \]

since \( I_1 \) contains \( g(x^*, 0) \) squared and \( I_2 \) contains \( g(x^*, 0) \).

Therefore \( g'(x^*, t) = B g(x^*, 0) \) if we let \( \Delta t' = \frac{\Delta t}{B} \). Then the general effect of a change in grain loading is to change the value of the distribution function and the time scale by a constant. If we decrease the grain loading, that is \( B \) is less than one, then we increase the time scale and decrease the distribution function \( g(x^*, t) \).

The effect of an intensity change can be seen by considering equation (23) in a slightly different form.

\[ g(x^*, t_2) = g(x^*, t_1) + \frac{\Delta t}{\sqrt{\mathcal{J}}} \cdot \frac{Q \cdot d_o g}{\sqrt{g_{C_0}}} \cdot \frac{4}{\mathcal{J}} \left[ I_1 - g(x^*, t_1) I_2 \right] \] (26)

If we change the intensity, holding the other variables constant, we can get the same result by considering that the time scale is changed by the square root of the intensity ratio.

Let \( J' = RJ \)

then \( \Delta t' = \frac{\Delta t}{\sqrt{R}} \)
If we, therefore, increase the intensity, we increase the coagulation rate and need less time to obtain any desired amount of agglomeration.

The effect of a frequency change is best represented by selecting different values of the parameter $C$, which is a major factor in determining the relative velocity of the particles. If we try to change the frequency, holding the parameter $C$ constant, we have to change the particle density, the reference volume, or the temperature of the gas. These changes would be reflected in the parameter $K$ and also in the distribution function through a change in grain loading. If we change the particle density to compensate for a frequency change, holding $C$ constant, we get a change in grain loading. The effect of a change in grain loading has been discussed previously. Changing the reference diameter to compensate for a frequency change will result in a change in $K$ and the grain loading. Any change in the parameter $K$ can be compensated by a change in time scale for results that have been calculated for a constant $K$.

Attempts to compensate for frequency change by a temperature change in the gas results in a variation in gas viscosity, gas density, and velocity of sound. Neglecting the change in particle density, these effects produce a change in the parameter $K$ and can be calculated by a time scale shift.

Results for different particle densities can be obtained in the following manner. An increase in particle density can be adjusted by a frequency change in the parameter $C$ and
and by a grain loading change if we hold the distribution function constant. If we want to keep the grain loading constant, then we must reduce the distribution function and shift the time scale accordingly.

A change in the reference volume is felt in all parameters, C, K, W. To obtain the results of a change in $d_0$ with given values for C, K, W, we can hold C constant by a frequency variation, let the time scale shift for the K change and let the magnitude of distribution function change with another time scale shift to account for holding W constant.

Thus we see that if we have the computations for a particular case of a given C, K, and initial distribution function, we can determine the results of changing different variables by shifting the time scale or making compensating changes in other variables. The fact that a few computations will lead to a large volume of results for different conditions makes the analysis very versatile.
COMPUTATIONS

Since the general analysis predicted that the various parameters varied qualitatively in agreement with physical observations, some calculations were made for comparison with experimental data. The initial distribution was assumed to have the form of Maxwell's thermal velocity distribution.

\[ g(x^*, 0) = A x^{*2} e^{-bx^{*2}} \]

If we take \( x^* = 1 \) to be the most probable particle size, we get \( b = 1 \).

Next, we must specify the various parameters and independent variables, letting the time be the dependent variable. Calculations were made using the following values.

- \( d_0 = 3 \text{ microns} \ (10^{-4} \text{cm}) \)
- \( \Phi = \text{particle density} = 1 \text{ gm/cm}^3 \)
- \( J_{\text{max}} = 2 \text{ watt/cm}^2 = 2 \times 10^7 \text{ ergs/cm}^2 \)
- \( W = 3 \text{ grain/ft}^3 = 6.87 \times 10^{-6} \text{ gm/cm}^3 \)
- \( \rho_g = 0.001205 \text{ gm/cm}^3 \ T = 20^\circ \text{C}, 1 \text{ atm.} \)
- \( \mu_g = 181 \times 10^{-6} \text{ dyne-sec/cm}^2 \ T = 20^\circ \text{C} \)
- \( Q = \text{collision efficiency} = 1 \ (100\%) \)
- \( C_0 = 3.452 \times 10^4 \text{ cm/sec} \ T = 20^\circ \text{C} \)

With these values the parameter \( K \) has value \( K = 5.44 \times 10^{-6} \text{ cm}^3/\text{sec} \). The coefficient \( A \) (the amplitude of distribution function) is determined by equation (24)
where $x^*_{\text{max}} = 3$ was taken as the upper limit of integration. $x^* = 3$ corresponds to a diameter of $d = \frac{3}{\sqrt{3}} d_0 = 4.32$ microns. Since $\int_0^3 x^* e^{-x^*^2} \, dx^* \approx \int_0^\infty x^* e^{-x^*^2} \, dx^* = \frac{1}{2}$, we get $A = 9.72 \times 10^5 \text{ l/cm}^3$. Therefore, $g(x^*, 0) = 9.72 \times 10^5 x^* e^{-x^*^2}$

Calculations were made for two values of the parameter $C$, which is dimensionless.

<table>
<thead>
<tr>
<th>Case</th>
<th>$C$</th>
<th>Frequency with above conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.0135</td>
<td>669 c.p.s.</td>
</tr>
<tr>
<td>2</td>
<td>1.35</td>
<td>6,690 c.p.s.</td>
</tr>
</tbody>
</table>

The initial plan was to extend both calculations to a point where they could be compared with experimental data. After the calculations were begun, however, they proved to be more complex than was anticipated and there was not sufficient time to extend the calculations to the desired goal.

The first step in the calculations is to compute the initial rate of change of the distribution function. Then several values of $g(x^*, t_1)$ are plotted with different time intervals in an attempt to determine an appropriate time scale.
The distribution function must not change too much in any one step or there will be too much distortion in the results. As the calculations proceed, it may be necessary to change the time interval for physical reasons. For example, if the distribution function should become negative at any time, it indicates that too large a time interval was used.

The distribution function is then computed as a function of time using equation (23). The two integrals $I_1$ and $I_2$ were computed by using the trapezoidal rule where computation time dominated accuracy. Even with the approximate procedure, it was estimated that each integral was accurate to $\pm 3\%$; therefore the distribution functions were accurate to $\pm 6\%$.

Finally, the results must be put in a form comparable with experimental data. For this purpose, a theoretical efficiency was defined for a hypothetical collection system. The system is composed of a sound chamber and a cyclone collector. We assume that the cyclone efficiency curve is that of the 3-inch cyclone efficiency presented in Fig. (2) and that the performance is essentially independent of grain loading.
We define the following collection efficiency.

\[ N = \frac{(\text{Mass})_{\text{in}} - (\text{Mass})_{\text{out}} - (\text{Mass})_{\text{collected}}}{(\text{Mass})_{\text{in}}} \]

Since the density is constant, we have

\[ N = \frac{(\text{Volume})_{\text{collected}}}{(\text{Volume})_{\text{in}}} \]

Now

\[ x^* x_0 = \text{volume of particle at } x^* \]

and

\[ \frac{V}{x_0} = V^* = \int_{x_{\text{min}}}^{x_{\text{max}}} x^* g(x^*, t) dx^* , \]

so that we have

\[ N = \frac{V^*_{\text{collected}}}{V^*_{\text{in}}} \]

\[ V^*_{\text{collected}} = \int_{x_{\text{min}}}^{x_{\text{max}}} E(x^*) x^* g(x^*, t) dx^* \]

where \( E(x^*) \) is the cyclone efficiency.

Finally

\[ N = \frac{\int_{x_{\text{min}}}^{x_{\text{max}}} E(x^*) x^* g(x^*, t) dx^*}{\int_{x_{\text{min}}}^{x_{\text{max}}} x^* g(x^*, t) dx^*} \]

The collection efficiency was calculated for each distribution function and is plotted in Fig. 7.
DISCUSSION OF RESULTS

In Figures 4 and 5, the change of the distribution function with time is presented. The actual parameter plotted is $x^* g(x^*, t)$ which represents the amount of the particle volume per unit volume between $x^*$ and $x^* + dx^*$. The reason for plotting this parameter is that the area under the curve remains constant and equal to the initial volume $V^*$. To get the actual volume, multiply $V^*$ by $x_0$. Since the density is constant, these curves also represent the amount of mass at any particular volume. By computing area ratios, we can determine what fraction of the total mass is contained in any volume interval.

The influence of the relative velocity of the particles, which is expressed by dimensionless parameter $\alpha^*$, on the rate of change of the distribution function is demonstrated in Figure 6. The rate of change curve for case 2 ($C = 1.35$) is roughly ten times in magnitude that for Case 1 ($C = 0.0135$). The reason for this difference is easily seen by considering the $\alpha^*$ parameter plotted in Figure 3. In case 1, we are operating on the flat portion of the curve and the relative velocity between particles of different sizes is small; therefore the rate of change of the distribution function is small.

It is obvious that if the distribution function is the range
where $\alpha^*$ is changing rapidly, then the agglomeration rate will be high. Although the rate of change curve is initially large for case 2, it will decrease as the distribution function moves toward larger volumes. With the present initial distribution function, the agglomeration process would proceed more rapidly with time if a combination of two frequencies were used. First, use a frequency corresponding to $C = 1.35$ and then when the mean volume has reached $x^* = 10$, switch to a frequency corresponding to the $C = .0135$ curve.

The result of taking too large a time interval is demonstrated by considering Figures 5 and 6. If too large a time interval is taken, the next distribution function will have too many large particles and a hump will appear in the distribution function. If we assume this to be physically correct and calculate the next step using this function, we get the obvious result that the hump will move toward large volumes. However, these results are distorted in that we are apt to get too high an collection efficiency and too short a time scale. Case 1 is an example of selecting too large a time interval, while case 2 is closer to a continuous process. There will always be some error because of a finite time interval. Further evidence that case 2 has a more accurate time interval is shown in Figure 8, where the number of particles per unit volume is plotted as a function of time. These results obey the law of a exponential decay which is what we would expect.
from a statistical phenomenon of this kind.

Finally, the collection efficiencies presented in Figure 7 are calculated by multiplying the curves in Figures 4 and 5 by the cyclone efficiency and integrating.

At the present time, these curves have not been extended into the range where they can be compared with experimental data. However, they show the correct trends and calculations are being continued to extend their application.
BIBLIOGRAPHY


INITIAL DISTRIBUTION FUNCTION

\[ G(X,0) = 9.71 \times 10^5 \ x^2 \ e^{-x^2} \]

\[ G(X^*,0) \times 10^{-4} \]

FIGURE 1
FIG. 2

TYPICAL EFFICIENCY CURVE
3 INCH CYCLONE

$d_0 = 3$ MICRONS
FIGURE 4

VOLUME OF PARTICLES PER VOLUME INTERVAL $d\xi^*$ VS. $\xi^*$

CASE 2 - $c = 1.35$

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>EXPOSURE TIME (SEC.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>0.094</td>
</tr>
<tr>
<td></td>
<td>0.141</td>
</tr>
<tr>
<td></td>
<td>0.188</td>
</tr>
</tbody>
</table>

$\chi^* G(\xi^*, t) \times 10^4$
FIGURE 5

VOLUME OF PARTICLES PER VOLUME INTERVAL $d x^*$ VS. $x^*$

CASE I - $C = 0.0135$

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>EXPOSURE TIME (SEC.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.00</td>
</tr>
<tr>
<td></td>
<td>.47</td>
</tr>
<tr>
<td></td>
<td>.94</td>
</tr>
<tr>
<td></td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>1.64</td>
</tr>
<tr>
<td></td>
<td>1.88</td>
</tr>
</tbody>
</table>
RATE OF CHANGE CURVE FOR
INITIAL DISTRIBUTION

FIGURE 6

- - - - - CASE 1 - C = 0.0135

CASE 2 - C = 1.35
COLLECTION EFFICIENCY VS. TIME

CASE 2

W = 3.0 gr/ft³

W = 1.5 gr/ft³

CASE 1

W = 3.0 gr/ft³

INTENSITY = 2.0 watts/cm²

FIGURE 7
NUMBER OF PARTICLES/cm³ VS. TIME

\[ n(t) = n(0) e^{-\frac{t}{16.3}} \]

FIGURE 8
APPENDIX I

VARIABLES EFFECTING COLLISION EFFICIENCY

The collision efficiency $Q$ may depend upon the type of aerosol and a possible maximum and minimum velocity of the particles. The only effect considered here, however, is the dependence on the relation between the vibrating amplitude and the mean distance between particles.

Let

$$\bar{l} = \frac{l}{\sqrt[3]{n(t)}} = \text{mean distance between particles}$$

$$S = \frac{u_{\text{max}} \alpha^*}{2\pi f} = \sqrt{\frac{2J}{\int_0^{\infty} c_0}} \frac{\alpha^*}{2\pi f} = \text{particle amplitude.}$$

If the particle amplitude is greater than $\bar{l}$, there will be many collisions which will maintain a random distribution of the particles in space. As the parameter $(S-\bar{l})$ approaches zero, the effective number of collisions is reduced. It may be possible to handle this effect with a variation in collision efficiency.

Consider a specific case. Let us take the following conditions:

$$J = 2 \text{ watt/cm}^2$$

$$f = 10^3, 5 \times 10^3, 10^4$$
\[
\rho_g = 1.205 \times 10^{-3} \text{ (air } T = 20^\circ \text{ C) } \text{g/cm}^3
\]
\[
c = 3.452 \times 10^4 \text{ cm/sec}
\]
\[
\alpha^* = 1
\]
\[
n(o) = 4.3 \times 10^5 \text{ particles/cm}^3
\]
\[
W = 3 \text{ grain/ft}^3
\]

where \( n(o) \) was computed from the distribution function used in our calculations

\[
h(o) = \int_0^{x_{\text{max}}} q(x_o^*) \, dx^* = 4.30 \times 10^5 \text{ number/cm}^3
\]

For this case we have the following table:

<table>
<thead>
<tr>
<th>Frequency (cps)</th>
<th>(S) (cm) max.</th>
<th>n(t)</th>
<th>( \bar{t} ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>.156</td>
<td>n(o)</td>
<td>.0132</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>.078</td>
<td>n(o)/2</td>
<td>.0167</td>
</tr>
<tr>
<td>( 10^4 )</td>
<td>.0156</td>
<td>n(o)/10</td>
<td>.0285</td>
</tr>
<tr>
<td></td>
<td></td>
<td>n(o)/100</td>
<td>.0614</td>
</tr>
</tbody>
</table>

If the frequency is low enough, and the intensity is high, we have \((S-\bar{t}) \approx 10\). Under these conditions, we should have random motion if \( n(t) \) does not decrease too rapidly. In case I, \((f = 669 \text{ cps})\) for which we have made calculations, \( n(t) \) has not dropped to \( n(o)/2 \) in one and a half seconds; therefore our assumption of random motion and a constant \( Q \) seem to be fundamentally sound.
If we had considered a case where $n(t)$ was changing rapidly so that $(S-\bar{l})$ was approaching zero, we would have had to consider $Q$ changing with time. It is reasonable to assume that $Q$ could be represented by some function ($f$) which is a function of parameter $(S-\bar{l})$.

$$Q = f(S-\bar{l}) = f\left[\sqrt{\frac{2J}{\rho c}} \frac{a}{2\pi f} - \frac{1}{\sqrt{h(t)}}\right]$$

The function $f$ could have the form

$$f = 1 - H \frac{S-\bar{l}}{\bar{l}_0}$$

where $H$ is constant and is practically equal to one. $\bar{l}_0$ is the mean distance at time $t = 0$.

Then $Q$ would have the form shown in the plot below.
For any specific case $\bar{S}$ and $\bar{l} = \bar{l}(t) = \frac{1}{\sqrt[3]{n(t)}}$ would be a function of time. We would have to calculate a mean amplitude $\bar{S}$ since not all the particles have the same amplitude and the mean amplitude would decrease with time as the particles grow larger. We can calculate $\bar{S}$ in the following manner:

$$\bar{S}(t) = \frac{\int_{x_m}^{x_{max}} S(x) q(x^*, t) \, dx^*}{\int_{x_m}^{x_{max}} q(x^*, t) \, dx^*}$$

This particular form of $q$ would seem to satisfy all of the physical conditions in the problem but it would have to be verified by actual experimental data and an attempt made to determine the constant H.
APPENDIX II

DERIVATION OF LAW OF MOTION

We assume that a small sphere moving in a gas has a force on it given by Stokes' Law

\[ F = 6\pi \mu r \bar{V} \]

where \( \mu \) = viscosity of medium
\( \bar{V} \) = relative velocity of particle and medium
\( r \) = PARTICLE RADIUS

\( \bar{V} \) has the form \((u_g - u)\) where \( u_g \) is the gas velocity and \( u \) is particle velocity.

\[ u_g = u_0 \sin wt \quad u_0 = \text{velocity amplitude} \]

The equations of motion are

\[ m \frac{du}{dt} = 4\pi r^2 \rho \frac{du}{dt} = 6\pi \mu gr(u_0 \sin wt - u) \]

if we let \( z = 9/2 \frac{\mu g}{r^2 \rho} \)

we get

\[ \frac{du}{dt} = z (u_0 \sin wt - u) \]

Solution of this equation is

\[ u = \frac{z u_0}{z^2 + w^2} (z \sin wt - w \cos wt) \]
By using the trigometric identity,
\[ \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta, \]
we can transform the above equation into the following form:
\[ u = \frac{zu_0}{z^2 + \omega^2} \sin(wt - \frac{\pi}{4}) \text{ where } \frac{\pi}{4} = \tan^{-1}\frac{\omega}{z}, \]

If we consider only the velocity amplitude, we get
\[
\left( u_p \right)_{\text{max}} = \frac{\left( u_g \right)_{\text{max}}}{\sqrt{1 + \left[\frac{\omega}{z}\right]^2}} = \frac{\left( u_g \right)_{\text{max}}}{\left[ \frac{4 \pi fr^2 \rho}{9 \mu g} \right]^{1/2} + 1}^{1/2}
\]

However, the important quantity is the effective value of the velocity or rather the root mean square velocity.
\[
\therefore \left( u_p \right)_{\text{eff}} = \sqrt{\left( u_p \right)_{\text{max}}^2}
\]

It is a simple integration to show that
\[
\left( u_p \right)_{\text{eff}} = \frac{\left( u_p \right)_{\text{max}}}{\sqrt{2}} \text{ when the velocity is a sinusoidal function.}
\]