Electron Density and Average Electron Energy in the D.C. Glow Discharge

by

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ABSTRACT

This thesis reports an attempt to develop a method of measuring the structure of the D.C. glow discharge. The interaction between the free electrons in the discharge, and an external micro-wave electromagnetic field is used to study electron density and average energy. Measurements of density have been made as a function of position along the discharge tube, for four different currents and a fixed pressure. Average energy has been measured in the Faraday dark space and positive column, using the assumption of constant mean free path.

INTRODUCTION

The study of the conduction of electricity through rarefied gases led to the discovery of the electron, the X-ray, the positive ion, and numerous other phenomena, most of which we now view with more understanding than the conduction process itself. The slow progress in this field seems not connected with any search for a fundamental law of nature, but rather to be the result of enormous experimental obstacles blocking the way to reliable data. The experimental, rather than the theoretical difficulties, should be stressed in this respect, because a theory can never be expected to make real headway when the data is in confusion. Consider for example, the case at hand: A D.C. glow discharge which is to be reproduced in several laboratories, and which is to serve as a basis for a successful theory, depends for its operation on pure electrodes, a pure gas, a pure glass discharge tube, and a vacuum which can be accurately measured and maintained. The matter of purity alone causes us to regard all of pre-1930 data with grave suspicion, and explains in part, the lack of a successful theory for a phenomena which was discovered fifty years ago. There are of course other factors as well. The methods of investigation have been difficult to devise. Probes have been inserted into the discharge to measure potential, density and average energy.

Electron beams have been directed through the discharge to measure electric field strengths, and spectroscopic means have been employed to analyze the inelastic collision processes. The most important of these, the Langmuir probe method, is suspected of altering the discharge considerably, and the data it produces has not really been checked by any other means. For this reason alone, a new type of investigation is very desirable.

The trend in the laboratory in which this work was carried out, has been to separate and examine by microwave methods, the various processes present in a complicated glow discharge, in the hope that they will be understood when they are again combined to form a glow discharge. This trend is justifiable insofar as we should certainly try to understand separate phenomena first, but there is no assurance that the tie-up will result in an adequate explanation of the more complicated phenomena. There exists, therefore, good reason to begin the study of the glow discharge anew, utilizing a method that could check the Langmuir probe method, and which may perhaps prove to be more powerful.

A DESCRIPTION OF THE INTERACTION BETWEEN A MICRO-WAVE ELECTRIC FIELD AND THE GAS DISCHARGE

The macroscopic behavior of a gas discharge is determined by a knowledge at each instant of the positions and momenta of each electron, ion, atom, and molecule in the discharge. In practice, a determination of such information would be quite impossible, because of the enormous numbers involved. The treatment, as in all phenomena of matter, must be a statistical one. Plausible assumptions must be made about the processes occuring in the discharge, and these must be translated into the mathmatical language of distribution function theory. Thus it is assumed that there exists a function $F(\vec{r},\vec{v},t)$ for each type of particle, such that $F d^3r\, d^3v$ gives the number of particles both in the ordinary volume element $d^3\vec{r} = dx\,dy\,dz$ and in the element of velocity space $d^3v = dv_x\, dv_y\, dv_z$ at any time t. In theory, the distribution $F(\vec{r},\vec{v},t)$ can then be found by using physical assumptions to formulate an equation of continuity for the particles in both ordinary and velocity space. This equation takes the following form:

$$\frac{\partial F}{\partial t} = -\vec{\nabla}_r \cdot \vec{v} F - \vec{\nabla}_v \cdot \vec{a} F + c$$

where $\vec{\nabla}_r$ is the gradient operator in ordinary space, where $\vec{\nabla}_v$ is the gradient operator in velocity space, and where C takes account of collisions. The solution to this equation will be $F(\vec{r}\vec{v}t)$. Having obtained the distribution function it is then possible to express in terms of it, several quantities which can be directly measured. For example:

$$n(r,t) = \text{density} = \int F\, d^3v$$

$$\text{current density} = \vec{\Gamma} = \int F \vec{v}\, d^3v$$

In general, the differential equation which expresses continuity, is very difficult to solve, and some simplifying assumptions must be made at the outset. In the case of a gas discharge where we deal with ordinary electric field strengths, and where the pressure is such that collisions between electrons and molecules, randomizes the velocity of the electrons, we assume that $F(\vec{r}\vec{v}t)$ is almost spherically symmetrical in velocity space, and that it is necessary to include only the first order asymetric correction term.

Allis[1] has done this by expanding $F(\vec{r}\vec{v}t)$ in spherical harmonics, and neglecting second order terms.

$$F(\vec{r}\vec{v}t) = \sum_{n=0}^{\infty} \sum_{m=-n}^{+n} \left[F'_{mn} \cos m\phi + F''_{mn} \sin m\phi \right] P_n^m(\cos\theta)$$

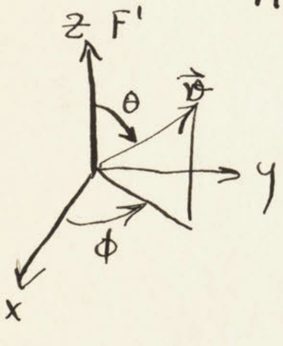
With a slight change in notation, the zeroth and first order terms are given by

$$F = F^0 + \frac{v_x}{v} F_x + \frac{v_y}{v} F_y + \frac{v_z}{v} F_z = F^0 + \frac{\vec{v} \cdot \vec{F}'}{v}$$

where F^0 and F' are functions of $|\vec{v}|$, $\vec{r}$, and t .

Fortunately, density and current density do not depend on higher terms anyway, as can be seen by carrying out the integration

$$n = \iiint \left[F^0 + F' \cos\theta \right] d^3v \, d^3r = \int F^0 4\pi v^2 dv$$



$$\Gamma_x = 0 \qquad \vec{\Gamma} = \vec{i}\,\Gamma_x + \vec{j}\,\Gamma_y + \vec{k}\,\Gamma_z$$

$$\Gamma_y = 0$$

$$\Gamma_z = \int F' \frac{4\pi}{3} v^3 dv$$

where external field determines direction of Γ_z.

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1. For all numbered references consult the bibliography at the end of the thesis.

Allis[1] has formulated the equation of continuity in terms of F^0 and F'; taking into account elastic collisions, this becomes

$$\frac{\partial F}{\partial t} = \frac{\partial F^0}{\partial t} + \frac{\vec{v}\cdot\partial\vec{F'}}{v\,\partial t} = -\vec{v}\cdot\vec{\nabla}F - \frac{v}{3}\vec{\nabla}\cdot\vec{F'} - \frac{\vec{a}\cdot\vec{v}}{v}\frac{\partial F^0}{\partial v} - \frac{1}{3v^2}\frac{\partial}{\partial v}\left(v^2\vec{a}\cdot\vec{F'}\right)$$

$$-\nu_c\frac{\vec{v}\cdot\vec{F'}}{v} + \frac{m}{M}\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^3\nu_c F^0\right)$$

The acceleration is due to an external alternating electric field

$$a = \frac{-e}{m}E_0 e^{j\omega t} = -\frac{e}{m}E$$

and it is assumed that the distribution function would therefore have both D.C. and A.C. components

$$F^0 = F_0^0 + F_1^0 e^{j\omega t}$$

$$F' = F_0' + F_1' e^{j\omega t}$$

The equation of continuity is therefore then expressed in terms of these, and F_1^0, is assumed small compared to F_0^0. For reasons of angular dependence, certain terms can be separately equated. Real and imaginary terms can also be separately equated, and three relations result

$$0 = \frac{1}{3v^2}\frac{\partial}{\partial v}\left(v^2\frac{e\vec{E}}{2m}\cdot\vec{F_1'} + \frac{3m}{M}v^3\nu_c F^0\right) - \frac{v}{3}\vec{\nabla}\cdot\vec{F_0'}$$

we have let $F_0^0 = F^0$

$$\nu_c F_0' = -v \nabla F^\circ$$

$$(\nu_c + j\omega) F_1' = \frac{eE}{m} \frac{\partial F^\circ}{\partial v}$$

The last two can easily be solved for F_0' and F_1' and be used to express the AC and DC current densities.

$$\Gamma_0 = -\int \frac{4\pi}{3\nu_c} v^4 \nabla F^\circ dv$$

$$\Gamma_1 = \int \frac{4\pi}{3} \frac{eE}{\nu_c + j\omega} \frac{\partial F^\circ}{\partial v} v^3 dv$$

It should be noted that the distribution function F° has not been solved for explicitly, but remains in differential form. At this point F° can be assumed, or its solution can be pursued further.

In this paper we shall be particularly concerned with the last equation, since it can be used to describe the AC current density due to the external micro-wave electric field with which we shall examine the discharge. Since Γ_1 is due to the AC electric field, we can ascribe a conductivity to the electrons, and define it as

$$\sigma_c \bar{E} = \sigma_c E_0 e^{j\omega t} = \Gamma_1$$

$$E_0 = |E_0| e^{j\delta}$$

This conductivity will, in general, have a real and imaginary part, and it is easily seen what the physical meaning of this is, by examining the causes of energy dissipation. Poynting's theorem tells us that the energy dissipated per unit volume is $E \cdot J$. J will consist of both electron current density Γ_1 and displacement current density, $\varepsilon_0 \frac{\partial E}{\partial t}$. Since $E = E_0 e^{j\omega t}$ we see immediately that $\varepsilon_0 \int E \cdot \frac{\partial E}{\partial t} dt$ integrated over a cycle is zero, and that

$$\int_0^T \sigma_c E^2 dt = \int_0^T \sigma_r E^2 dt$$

where

$$\sigma_c = \sigma_r + i\sigma_i$$

which yields as the average rate of energy dissipation $\frac{\sigma_r |E_0|^2}{2}$ This means that energy is drained from the electric field only by the real part of the current. We must now attempt to reconcile this with the behavior of the particles in the discharge. One statement can be made immediately: The motion of the ions, because their mass is (atomic number) × (1830) as large as that of the electrons, and because we are considering electric fields at 3000 Mc., can be neglected compared to that of the electrons. The ions never have a chance to gain energy before the direction of the field reverses (at ordinary fields). Therefore the energy must be directly attributed to an electron mechanism.

Consider a single electron in the discharge. During the course of its motion, it will find itself at times in free space, far away from other particles, and at times in the force fields of other particles. In free space§ it gains a certain energy from the electric field in one half cycle, and returns it to the field in the next half cycle. This represents no net energy dissipation, and can be associated with the imaginary component of the current $\sigma_i E$. When, however, the electron enters the force field of another particle, the interaction demands that energy and momentum be conserved in the encounter, and often the electron must give up some of its energy. The electric field, therefore, cannot always regain all of the energy which it has supplied. An extreme case is the following: An electron gains energy from the field for half a cycle, makes a collision, the duration of which is small, compared to the period of the electric field, and then is projected at right angles to the direction of the field. In this case, the field regains no energy from the electron. Although such an occurance would be rare, the

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§ Free space is here defined as the space where force fields, due to particles in the discharge, are essentially zero.

conditions can be satisfied by two electrons, when the impact parameter χ is one half of the collision diameter b, according to thefamiliar Rutherford scattering law $\chi = \frac{b}{2} \cot \frac{\theta}{2}$
This theory has postulated no other loss mechanism for the electrons than elastic collisions§, and we can therefor identify the energy loss with the real part of the electron current, which represents that part of their motion which takes place under the influence of the force fields of other particles.

Basically then, if we are to utilize the interaction between an external electric field and the electrons, our fundamental measurements must be:

1) A measurement of the energy stored and dissipated in the field.

2) A measurement of the wavelength of the external field.

A rough order of magnitude calculation using mean free path and average energy of the electrons in the glow discharge, results in the frequency of collision at a pressure of a few mm Hg, which is for helium about 5×10^{9} sec^{-1} and for mercury about 5×10^{10} sec^{-1}, and shows immediately why the 10 cm. microwave region is well suited for experiments of this kind. If the frequency of the external field were too high, then the electrons would act like

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§ Inelastic collisions will not change the argument.

ions, they would never be able to follow the field, and there would be no measurable interaction. If the frequency were too low, we would be altering the discharge, as the electrons would "see" a DC field. It must be strongly emphasized that the microwave field can also alter the nature of the discharge, if the magnitude of oscillation is too large. Margenau[2.] has developed a criteria for this, and it is found that the 10 milli-watt maximum output power of a Klystron oscillator, padded with a 10 db attenuator, satisfies this criteria well.

THE APPLICATION OF MICROWAVE TECHNIQUES TO THE STUDY OF THE DISCHARGE

The microwave study of the discharge is carried out most conveniently with the use of cavity resonators. This means that the field is localized in the form of standing waves, and an accounting of the energy can be made at any time. A further advantage is gained from the resonant properties of the cavity, because these depend on its shape, and the dielectric coefficient of the matter in the cavity, two physical quantities of which experimental use can be made.

A simple calculation will illustrate the effect of a variation in dielectric coefficient upon the resonant frequency of a cavity.[3]

Let a cavity have a dielectric constant

$$\mathcal{E} = \mathcal{E}_0 - \Delta \mathcal{E}$$

which is a function of position. We shall assume that $\Delta\mathcal{E}/\mathcal{E}_0$ is small compared to unity. The problem is to compute $\Delta\lambda/\lambda$ as a function of $\Delta\mathcal{E}/\mathcal{E}_0$ when λ_0 is the free space wavelength of the empty cavity, and $\Delta\lambda$ is the shift in wavelength, caused by changing the dielectric constant by an amount $\Delta\mathcal{E}$. For any cavity λ_0 can be expressed in terms of the dimensions of the cavity. In a cylinderical cavity, for example, $\lambda \sim \text{radius}$ and $d\lambda \sim dr$.

Since total stored energy is

$$W = \frac{1}{2}\int r\left(\varepsilon E^2 + \mu H^2\right) dr\,dx$$

we see that

$$\frac{dW}{W} = \frac{d\lambda}{\lambda}$$

writing H in terms of E, we get

$$W = \frac{\cancel{2}}{\cancel{2}}\int \varepsilon E^2 dv$$

and substituting from above, we have

$$W = W_0 + \Delta W = \left\{\frac{1}{2}\int \varepsilon_0 E_0^2 dv - \frac{1}{2}\int(\Delta\varepsilon) E_0^2 dv\right\}\times 2$$

Thus we see that

$$\Delta W = -\frac{\cancel{2}}{\cancel{2}}\int \Delta\varepsilon\, E_0^2 dv$$

and assuming in the denominator that W is approximately equal to W_0, we get

$$\frac{\Delta\lambda}{\lambda} = -\frac{\cancel{2}}{\cancel{2}}\int\left(\frac{\Delta\varepsilon}{\varepsilon_0}\right) E_0^2 dv \Big/ \int E_0^2 dv$$

where a knowledge of $\frac{\Delta\varepsilon}{\varepsilon_0}$ as a function of position yields the shift in resonant wavelength.

Slater[4.] has considered this problem for an electron current in a cavity, assuming that the imaginary part of the current density $\sigma_i E$ is much smaller than the displacement current. This assumption allows a first order perturbation calculation to be made, which results in

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_0 \int_{Vol'} \sigma_i E^2 dv'}{4\pi\varepsilon_0 c \int_{Vol} E^2 dV}$$

λ =perturbed resonant wavelength

λ_0 =unperturbed resonant wavelength

dv' =volume elements where current flows

From the complex conductivity developed in the last section , we see that σ_i involves the electron density n. Now if

1)The energy dependence of ν_c is known, or $\frac{\nu_c}{\omega}$ is small, and if

2)The form of the distribution function F^0 is known

then we can make use of Slater's formula to find the electron density in the cavity.

As was mentioned in the first section, it is expected that basic information can also be obtained from measurements of the AC field energy stored and dissipated in the discharge. Since the electric field is confined to the cavity, we expect that there will be additional energy stored in the cavity, and coupling loop. Energy is also dissipated in the cavity walls, in the coupling loop, and in the glass seals, etc., so that measurements taken on an empty cavity, will have to be compared with measurements made on the cavity when the discharge is going, in order to cancel effects not due to the discharge. Attention, however, must be called to the fact that the discharge may alter the cavity properties, to the extent that a simple cancellation is not possible. High electron density will alter the cavity electric field, and the I^2R losses in the walls of the cavity, for example, will change when the discharge is turned on.

A concept completely equivalent to the concept of power measurement when the E-field is known, is that of impedance, since power is given in terms of impedance in the following form

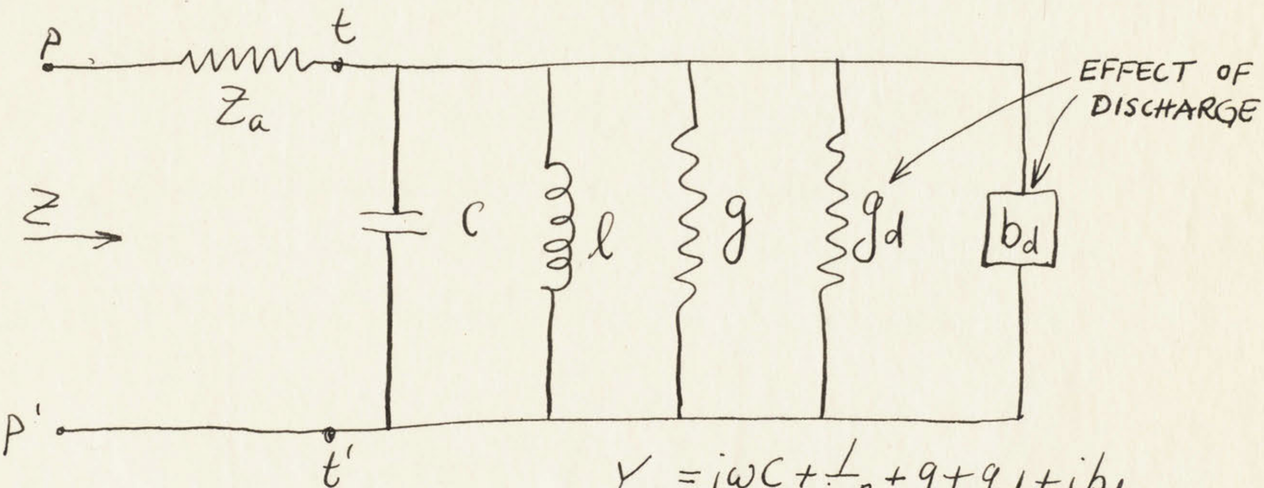
$$P = \frac{V^2}{Z}$$

and V is known if the field strength configuration is known. Slater[4] has shown that if a cavity is provided with only a single input, and if the system is supplied by only a single generator of power, operating at a single frequency, then the impedance of a cavity functioning near the ath normal mode, is given by

$$Z = Z_a + \frac{1/Q_{ext,a}}{j\left(\frac{\omega}{\omega_a} - \frac{\omega_a}{\omega}\right) + \frac{1}{Q_a} + \frac{1}{\varepsilon_0 \omega_a}\frac{\int J\cdot E_a\,dv}{\int E\cdot E_a\,dv}}$$

ω = applied frequency
ω_a = resonant " " of ath mode without electrons
E_a = unperturbed E-field E = perturbed E-field

For convenience in handling the theory and calculations, workers in this laboratory[6] have adopted the lumped circuit representation, for the cavity and the discharge, a notation which enjoys its greatest success when the Q- of the cavity is very large. Using this representation, lumped impedances are assigned to the various regions of the system, where energy storage and/ or dissipation takes place. Combined, these impedances form the following equivalent circuit.



$$Y_{tt} = j\omega C + \frac{1}{j\omega l} + g + g_d + j b_d$$

$$Z = Z_a + \frac{1}{Y_{tt'}} \qquad \frac{1}{\sqrt{lc}} = \omega_0 \qquad \sqrt{\frac{c}{l}} = \beta$$

All impedances have been normalized to Z_0, the characteristic impedance of the circuit at PP'.

By analogy with Slater's formula, we can pair off the following expressions

$$\beta = Q_{ext} \qquad g = \beta / Q_a$$

$$\frac{1}{\beta}\left(g_d + j b_d\right) = \frac{1}{\varepsilon_0 \omega} \frac{\int J \cdot E_a \, dv}{\int E \cdot E_a \, dv}$$

Above we have defined complex conductivity, such that $\sigma_c E = \Gamma_1$ where E is the electric field in the cavity with the discharge going, and Γ_1 represents the electron current density. The calculation of E is extremely difficult, and has not been achieved[5]. We must therefore confine the electron density to values which will have negligible effect on the E field in the cavity (density $= n \ll 10^{11} cm^{-3}$). Then we can set $E_a = E$ and

$$\frac{1}{\beta}\left(g_d + j b_d\right) = \frac{1}{\varepsilon_0 \omega} \frac{\int \sigma_c E_a^2 \, dv}{\int E_a^2 \, dv}$$

A little manipulation yields the real and complex conductivity per electron σ_r' and σ_i'

$$g_d = \frac{\beta \bar{n} \sigma_r'}{\varepsilon_0 \omega} \qquad b_d = \frac{\beta \bar{n} \sigma_i'}{\varepsilon_0 \omega}$$

where $\bar{n}$ is mean electron density averaged over E_a^2.

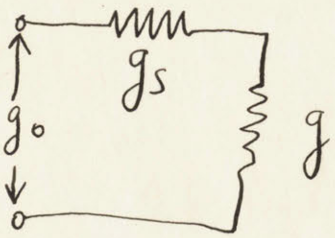
Making use of the relationship between b_d and frequency

$$b_d = \frac{2 \Delta \lambda}{\lambda_0}$$

which is obtained from an analysis of the equivalent circuit on resonance, we finally obtain the ratio $\frac{\sigma_r}{\sigma_i}$ namely:

$$\frac{\sigma_r}{\sigma_i} = \frac{\lambda_0 g d}{2\beta \Delta\lambda}$$

We have assumed that Z_a includes the series resistances and reactances between the point where measurement is made on the line and in the cavity. The major contribution will be a term due to the coupling loop, since the waveguide has very few losses. The further assumption is now made that the reactive part of the coupling loop impedance is negligible, compared to the resistive part. This means, in effect, that very little energy is stored in the loop, as compared to the cavity.§ On resonance, then, the empty cavity will be represented by the following equivalent circuit:

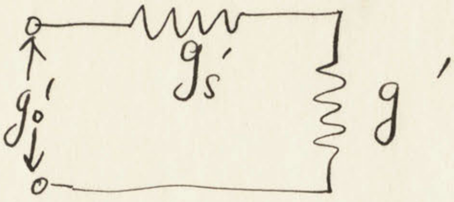


where the relation between conductances is

$$g = g_0 \frac{1}{1 - g_0/g_s}$$

§ Of late, this assumption has been seriously questioned in this laboratory, and the issue has not yet been settled.

When the discharge is turned on, the impedance is again purely resistive on resonance



and the relation between conductances is

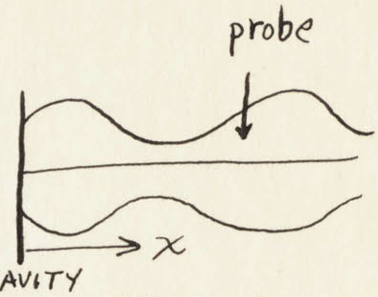
$$g' = g + g_d = g_0' \frac{1}{1 - g_0'/g_s'}$$

Since we wish experimentally to determine the real and imaginary conductivities, it is necessary to measure the following quantities

$$\Delta\lambda, \lambda_0, g_0, g_s, g_0', g_s'$$

Except for the first two quantities, which are easily measured by taking Q curves of the cavity, with and without discharge, we are concerned with the measurement of cavity impedances. Although one could without difficulty give interpretations to these impedances from the lumped parameter standpoint, it may perhaps be worthwhile to remind ourselves that we are actually dealing with a problem in electromagnetic theory, and that we can give very good physical explanations of the necessary measurements, from this standpoint also. Thus consider a coaxial cylinderical waveguide terminated in a coupling loop and cavity. Somewhere ahead of the cavity, we insert a slotted section and electric probe. At any point in the waveguide, we can then express the electric field as the sum of two traveling waves

$$E = Ae^{j\omega t - \frac{x}{\lambda}} + \Gamma A e^{j\omega t + \frac{x}{\lambda}}$$



where $\Gamma = \rho e^{j\theta}$ is the complex reflection coefficient which determines the ratio and the phase of the incident to the reflected electric field. As the probe of the slotted section is moved from one extremity to the other, it will pass through the minima and maxima characteristic of standing waves. The difference between the magnitude of the electric field at the minimum and maximum of its value can be conveniently measured as a function of frequency and will result in the familiar SWR-Q- curve. At each frequency this voltage SWR will be the following function of ρ

$$\text{Voltage SWR} = \frac{1+\rho}{1-\rho} = R_v$$

Resonance can then definedas the frequency at which ρ is a minimum. Writing Γ in terms of the wave impedance Z normalized to thecharacteristic wave impedance

$$\Gamma = \frac{1-Z}{1+Z}$$

we can get an expression for the impedance of the cavity in terms of the measurable quantities Voltage SWR ~~and~~

and phase Θ, and we can establish the values g_0, g_0', g_s, g_s' in terms of these quantities. It is a simple matter to see which measurements will yield these values. On resonance, the suseptance goes to zero, and the empty cavity has an admittance given by the pure conductance g_0, which is given by the voltage SWR on resonance (R_{V_0}). Similarly, g_0' is given by R_{V_0} with discharge going. Far off resonance, the admittance of the cavity must go to zero, and the reflection coefficient approaches minus one. Here the Voltage SWR must give the only non-zero real admittance in the circuit, which is g_s and g_s' for empty cavity, and cavity with discharge going respectively.

We have shown that the comlex conductivity can be obtained experimentally. We have also obtained a theoretical expression for it from the solution to the Boltzman transport equation, in the following form

$$\Gamma_1 = \sigma_c E = \int \frac{4\pi}{3} \frac{eE}{\nu_c + j\omega} \frac{\partial F^0}{\partial v} v^3 dv$$

In order to be able to integrate this expression, we assume that the electrons are in thermal equilibrium with the gas and F^0 is Maxwellian

$$dF^0 = -\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-u/kT} \frac{du}{kT}$$

Then, if we assume that ν_c is constant, and if the integral can be evaluated, there is a good possibility that the electron temperature will appear implicitly in the result. Margenau[2] has carried through this calculation, and has found that average energy does indeed appear in the result. His solution takes the following form

$$\sigma_r = \frac{ne^2}{m\omega}\frac{4}{3\sqrt{\pi}}\sqrt{x}\left[1 - x - x^2 e^x E_i(-x)\right]$$

$$\sigma_i = \frac{ne^2}{m\omega}\frac{4}{3\sqrt{\pi}}x\left[\left(\tfrac{1}{2} - x\right)\pi^{1/2} + \pi x^{3/2} e^x \left(1 - Erf[-x]\right)\right]$$

$$x = \frac{1}{\gamma_0^2} \qquad \gamma_0 = \frac{P_c p}{\omega}\sqrt{\frac{2kT}{m}} \qquad Ei(-x)\ \&\ Erf(-x)$$

are tabulated in Jahnke & Emde: Funktionentafeln

from which it has been found convenient to plot σ_r/σ_i versus γ_0. The experimental determination of σ_r/σ_i thus allows us to find γ_0 which in turn permits a solution for average energy. An example has been carried out in the section on Calculations. Using this procedure, electron temperatures have been found for several currents in the positive column, and in the Faraday Dark Space, and are discussed in the section on Results.

One more point is best discussed here. We remember that the density formula involves a knowledge of σ_i. The best approximation one can make for σ_i without measuring it is $\frac{ne^2}{m\omega}\frac{1}{\left(\frac{P_c}{\omega}\right)^2+1}$ (see Appendix II). Now, however, we have also an experimental determination of this quantity, which should be used instead.

EXPERIMENTAL TECHNIQUE

The object of this experiment was to measure electron density and average energy, as a function of position in a steady glow discharge. Of special interest were the cathode fall regions: Aston Dark Space, Cathode Glow, Negative Glow, Cathode Dark Space, and Faraday Dark Space, because their theory is very incomplete, and because measurements have been difficult to perform in these regions. In particular, since it is not possible to use the Lagmuir probe in the Faraday Dark Space, it was felt that the micro-wave data would be of real value there.

The basic plan of the experiment was to surround the discharge tube with a cavity excited by a micro-wave oscillator. In designing the equiptment, four important points had to be kept in mind:

1) The measurements are space dependent, and therefore have to be localized as much as possible.

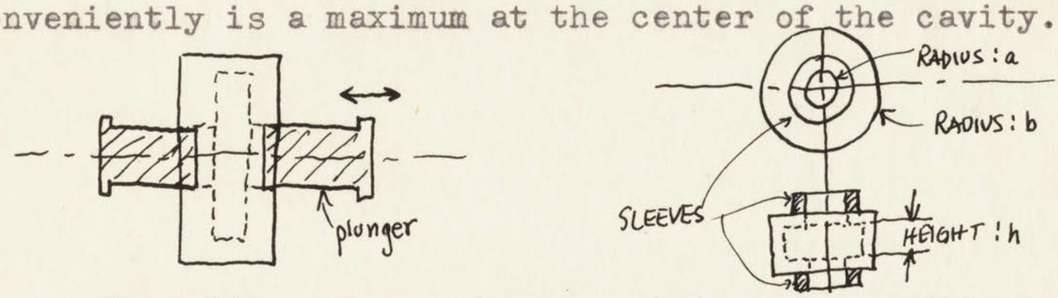
2) The electric field configuration must be accurately known, since it enters into all calculations.

3) Pressure must be variable to permit the maximization in size, of the cathode fall regions.

4) The Q of the cavity must be reasonably large if the measurements are to be performed within the minimum of error.

I. <u>The Cavity Design and the Measurement of Field Strengths</u>

The cavity presented the most important design problem; a problem which has not yet been solved with satisfaction, as shall be pointed out. In order to favor diffusion equally in all directions perpendicular to the axis, a glass discharge tube, of circular cross-section, was chosen. This immediately dictated a cylinderical shape for the cavity, if non-symmetrical distortion of the electric field, by the glass tubing, was to be avoided. At the same time, it was decided to use, if possible, the TM_{010} mode, since its field configuration conveniently is a maximum at the center of the cavity.



To achieve the maximum resolution in position, or in other words, the maximum localization of the electric field, it was found necessary to actually measure the electric field, in cavities ranging in the dimension h from 1/8 to 1/2 inch. This was first attempted qualitatively with the use of plungers, by noting the change in resonant frequency against position of the plunger. (See above figure) These measurements exposed serious fringing near thecircular apertures, and it was decided to add cylinderical sleeves to the cavity, at these points, in the hope that they would act like wave guides below cut-off, and reduce the electric field in some sort of exponential way, with position along the axis(See above figure). With fringing neglected, the cylinderical cavity with two circular aperatures, has been treated theoretically by Schelkunoff.[8] He has shown that the resonant frequqncy of such a cavity, differs only by a few % from that of a regular cylinderical geometry operated in the TM_{010} mode. Thus it was possible to calculate the resonant frequency, within 5%, for the cavity designed.

It was desireable to reduce the radius of the apertures, as much as possible, in order to save as much of the known TM_{010} electric field configuration as possible. However, such a decision had to consider the discharge as well. Reducing the radius of the discharge, increases the effects of diffusion, and presents a special case. In addition, because diffusion is increased, the self-maintaining mechanisms in the discharge, have to produce ions and electrons at an increased rate, and a larger maintaining voltage becomes necessary. Such a constricted tube also limits the current strengths by limiting the size of the electrodes, and the danger exists that the electron density will be too small to cause a measur~~e~~able shift. Special low dielectric pyrex glass tubing of outer diameter 5/8 inch was finally chosen. Now the resolution of the cavity was again considered. In the reduction of the height of the cavity, two opposing tendencies come into play. Increased resolution also means lower experimental accuracy, because the Q of the cavity is proportional to the quantity $\frac{1}{1+b/h}$. To find the proper height, which would balance these opposing factors to best advantage, the electric field was measured, and Q observed visually with the aid of a scope, for cavities with the dimentions "h" ranging 1/8 to 1/2 inch, as before. Now however, the use of the plungers was abandoned, instead the fieldstrength measurements were based upon a formula given by Slater,[4] which relates the shift of cavity resonant frequency, caused by the introduction of a small metallic perturbation, to the volume integral of the field removed from the cavity, by the perturbation. The shift in resonant frequency is given in the following form

$$\frac{\omega_a^2 - \omega^2}{\omega_a^2} = \int_{\Delta V} \left(E_a^2 - H_a^2\right) dv$$

where ω_a = unperturbed resonant frequency $\times 2\pi$

ω = perturbed resonant frequency $\times 2\pi$

where E_a and H_a are the time independent field quantities of the unperturbed cavity and where ΔV is the volume which was removed from the cavity by the metallic perturbation. This perturbation formula can be applied only for infinitesmal perturbations, and to situations where the unperturbed fields satisfy boundary conditions before the infinitesmal change is made. That is, this formula will not hold for cases where the infinitesmal perturbation is made from a situation not originally satisfying boundary conditions. The simplest perturbation, that of the metallic sphere, has been used in this experiment, and derivation for this case[9] will do much to clear up some of the subtle points of this method. The method of application of the above formula, is to assume that the metallic sphere is placed into a constant electro (E_o)- and magneto (H_o)-static field. This assumption can be made because the perturbation formula is concerned only with the volume integral, over the space variation of the fields. Furthermore, since the sphere can be made quite small, compared to the wavelength, the approximation is satisfied very well.§ The resulting E and H fields are then solved for, subject to the usual boundary conditions. The result is

$$E_r = E_0\left(1+\frac{2r_0^3}{r^3}\right)\cos\theta \qquad H_r = H_0\left(1-\frac{r_0^3}{r^3}\right)\cos\theta$$

$$E_\theta = E_0\left(-1+\frac{r_0^3}{r^3}\right)\sin\theta \qquad H_\theta = -H_0\left(1+\frac{r_0^3}{2r^3}\right)\sin\theta$$

$$E_\psi = 0 \qquad H_\psi = 0$$

where r_o is the radius of the sphere. Setting $r_o = 0$, we see that $E=E_o$ and $H=H_o$, so that the original field satisfies boundary conditions even before the sphere is introduced. Electric field measurements in this experiment were made only inside of the empty glass tubing, running thru the cavity.

---

§ For example, the sphere used in this experiment had a diameter of 0.127 inch and caused frequency shifts of about 1% in high field strength regions. Since smaller frequency shifts can be measured without too much trouble the constant field approximation can be even better satisfied.

Since this region is very close to the axis, the H field was assumed to be negligible, and I shall neglect it in this derivation also. The perturbation formula becomes in this case

$$\frac{\omega_a^2 - \omega^2}{\omega_a^2} = \int_{\Delta V} E_a^2 \, dv$$

since

$$E_a^2 = E_r^2 + E_\theta^2 + E_\psi^2$$

and

$$dv = r^2 \sin\theta \, dr \, d\theta \, d\psi$$

integrating the field over an infinitesmal expansion from $r = r_0$ to $r = r_0 + dr_0$ gives

$$\left\{ \int_{r_0}^{r_0+dr_0} \left[1 + \frac{2r_0^3}{r^3}\right] r^2 dr \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta + \int_{r_0}^{r_0+dr_0} \left[1 - \frac{r_0^3}{r^3}\right] r^2 dr \int_0^{\pi} \sin^3\theta \, d\theta \right\} =$$

$$= \int_V^{V+dV} E_a^2 \, dV$$

Since for any function $f(x)$

$$\int_{x_0}^{x_0+dx_0} f(x)\,dx = f(x_0)\,dx_0$$

for $\int_V^{V+dV} E_a^2 dV$ we get analogously that

$$\int_V^{V+dV} E_a^2 \, dV = 12\pi E_0^2 r_0^2 \, dr_0$$

For the total perturbation of the sphere, this must be integrated from r=0 to $r = r_0$, and then substituted into the original formula. Then the result is

$$\frac{\omega_a^2 - \omega^2}{\omega_a^2} = 3E_0^2 \, \frac{4\pi}{3} r_0^3$$

To carry out this measurement, a metallic sphere of diameter 0.127 inches, with a tiny hole along the diameter was attached to a silk thread, which alone had no effect upon the resonant frequency. Thread and sphere were given two degrees of freedom by means of two screws which controlled the radial motion, and by means of a pulley which wound up the thread and controlled axial motion. See Fig 1.

Largest errors occured in the positioning of the sphere, but these are not critical if one believes in the symmetry of the field,and shifts the data until it is attained. Largest errors of this type were of the order of 5-8%. Error in the field itself, due to the finite size of the sphere, is much smaller than this according to Maier[9] who compared his measure - ments in a standard cavity with the theory, and found an error of only 1.3%, using a metal sphere of diameter twice as large as the one used in this experiment.

Results showed that the 1/8 inch cavity gave the best resolution, and the Q that was not very much different than the Q's of the larger cavities. Results showed also that the use of plungers in determining the fringing near a cavity is not justified. The reason for this is to be found in the fact that the plunger was not an infinitesmal perturbation, and therefore the shift in frequency could not be simply related to the electric field strength.

The electric field measured in the 1/8 inch cavity is shown in Figures (2,3,4,&5) together with the best fitted curves. Qualitatively the effect of the apertures in the cavity, is to let the energy density in the electric field expand from the high value at the center of the regular TM_{010} mode, into the fringing region near the apertures. No measurements were made for radii larger than the inner radius of the glass tube. For larger radii it is assumed that the field turns to a Bessel function dependence, and satisfies boundary conditions at the cavity radius as usual. This is explained in more detail under Calculations.

II The Glow Discharge

To provide an easy method for variation in pressure, mercury was chosen for this study. The discharge tube shown in Fig(7) was heated for 4 hours at 450 $^\circ$ c. to drive off im - purities; then mercury was distilled into it. The pressure was read by an ionization guage which levelled off and held steady at about 2.7×10^{-7} mm Hg before the tube was sealed off. The nickel electrodes were not outgassed, and a certain amount of impurity was therefore present in the tube.

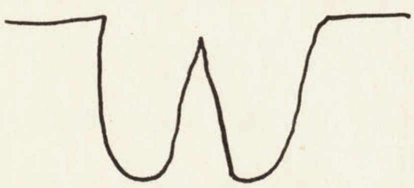
Under these conditions the pressure of mercury was easily varied by controlling the temperature of the mercury pool. To this end two separate furnaces were constructed as shown in Fig.(7). The mercury pool furnace consisted of a quenching oil bath surrounded by a ceramic element which was heated electrically by means of a coil of nichrome wire, controlled by a variac. The temperature was measured by an iron-constantan thermo-couple, and potentiometer which read directly in degrees for this type of couple, and also compensated automatically for reference point temperature changes. The accuracy of this com - bination was checked against a standard ice mixture and the boiling point of water, and was found to be in error 0.1 %; in any case much less than the error due to the drifting of the temperature during a run. This type of variation introduced a maximumerror into pressure readings of 10%. The second oven, a much larger one enclosed the entire discharge tube with the exception of the tube containing the mercury pool. Its purpose was to prevent the condensation of mercury in the main portion of the tube, since this would have affected the discharge, and would also have made the measurement of a Q curve impossible. Both ovens were surrounded with fire brick which acts as a heat insulator. Temperature in the main oven was measured with two thermometers at widely separated points. The temperature in the oven as read by these usually differed by 5-10 $^\circ$ C. This means that at worst the density in the discharge varied by 6% from cathode to anode since the temperature was kept in the vicinity of 180° C.

The breakdown characteristics of this discharge tube were found to be very poor. No breakdown would occur at all untill a pressure of about 3mm Hg was reached. The inducement of breakdown with the aid of a spark coil did not work with success either. This bears out the views stated before , that a constricted discharge has difficulty maintaining itself, and DC voltages larger than the 2000 volts available for this experiment are necessary to supply the extra energy needed for larger rates of ionization and ion bombardment of the cathode. Because such power was not available the work had to be carried out on a discharge in which the cathode fall region occupied only a few centimeters, the positive column taking up the rest of the space. The discharge was run at 3.75 mm Hg and was steady in its behavior.

III Microwave Measurements

The schematic diagram of the apparatus is shown in fig.(7). To make a run the cavity was adjusted around the discharge tube, and both were inserted into the ovens. A coaxial waveguide coupled into the cavity and passed thru the oven wall to the measuring apparatus as shown in the diagram. The coax and slotted section were supported on a movable, wooden carriage. To change the position of the cavity the entire wall of the oven was moved in the desired direction, and the micro - wave equipment external to the oven followed this motion easily. A measuring device which allowed reproducibility of data within 1/2 mm along the discharge was used.

To perform the density measurement, the reflector voltage of a klystron oscillator (Max. power output = 10 milliwatts) was amplitude modulated at 60cycles by a saw tooth oscillator. The power was fed through a 10 db at - tenuator, and a slotted section into the cavity. An electric probe on the slotted section detected the sum of the incident and reflected signals, fed it into a crystal, which rectified and presented it to the amplifier of the vertical deflecting plates of a scope. The resulting trace was a klystron mode minus the absorption in the cavity.



A calibrated echo box placed between slotted section and crystal produced a small pip on the trace by means of which frequency was measured.

In the cathode fall region measurements were made at one millimeter intervals, the discharge being turned on and off at each position to determine the frequency shift with maximum accuracy. The reproducibility of the discharge was checked both in the positive column, and the cathode fall region by measuring frequency shift many times at a fixed position and constant current, turning discharge on and off. These shifts indicated a maximum deviation, about an average, of 6%; Most deviations being closer to 2.5%. This was true as well of the potential across the tube.

Density measurements were performed for four different currents: 0.3, 0.5, 0.7, and 0.9 ma. at each position of the cavity. Sputtering at the cathode prevented an examination of the region close to the cathode. Near the anode, measurements became impossible to perform when the electrode entered the cavity within one centimeter of the center, shifted its resonance, and destroyed its Q.

The controlling factor in the accuracy of these measurements is by far the determination of the electric field. It should have been measured much more extensively in all regions of the cavity. The theoretical expressions fitted to the data, in addition to the approximation made in joining a Bessel to the field in the central region, may well cause the densities found to be too large by a factor of ten.

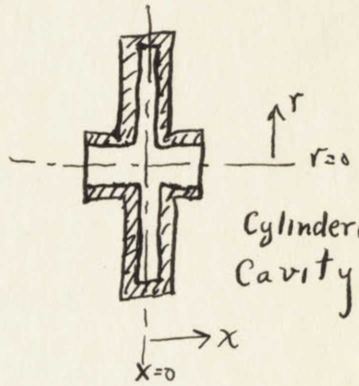
The determinations of $\frac{\sigma_r}{\sigma_i}$ were carried out in two regions of the discharge: The positive column, and the Faraday Dark Space. Decibel SWR versus frequency curves were taken for four different currents, 0.5, 0.7, 0.9, & 1.5 ma., see Figs.(10&11), by displaying the signal on a spectrum analyzer. The analyzer had a conveniently adjustable at - tenuator with a range that was sufficient to handle all standing wave ratios. The error in the attenuator calibration was less than 0.2 db per 10 db, and thus was smaller than the lowest SWR on resonance encountered. During these measurements the klystron was of course not frequency modulated; the scope was disconnected; and the echo box was used to measure the frequency. Largest error is due to variation in pressure which as mentioned above had a maximum value of 10%. In most cases this error was smaller, and was distributed slowly over a whole run.

CALCULATIONS

1) Calculation of electron density

Slater gives

$$\frac{\lambda-\lambda_0}{\lambda} = \frac{\lambda_0}{4\pi\varepsilon_0 c} \frac{\int \sigma_i \bar{E}^2 dv}{\int E^2 dv}$$



where $\sigma_i = -\frac{ne^2\lambda}{2\pi mc} f(\gamma_0) = \sigma_{i_0} f(\gamma_0)$

$f(\gamma_0)$ can be obtained from Margenau's solution for σ_c , and is conveniently given by Phelps[10] in a plot of $\frac{\sigma_i}{\sigma_{i_0}}$ versus γ_0 .

Thus $$\Delta\lambda = \underbrace{\frac{-\lambda_0 e^2\lambda^2}{8\pi^2 m\varepsilon_0 c}}_{const} f(\gamma_0) \frac{\int n E^2 dv}{\int E^2 dv}$$

where n , the electron density, is a function of axial and radial distance, x and r, repectively. It is seen then that $n(x,r)$ is actually the solution to an integral equation. I have indicated in Appendix I how this solution is obtained, but here we assume a certain radial dependence for n(x,r), and find the average n(x) in the cavity at each position x.

Von Engel and Steenbeck[7A] have shown that $n(x,r) = N_0(x) J(2.4\frac{r}{R})$ where R is the radius of the discharge tube is a very good assumption. Using this we can write

$$\Delta\lambda = \text{const.} \times f(\gamma_0) \frac{N_0 \iint r J_0(2.4 \frac{r}{R}) E^2 \, dr\,dx}{\iint E^2 r \, dr\,dx}$$

where we have already integrated over the angular dependence, and take N_0 to be the average density amplitude in the cavity.

Figures (2,3,4,& 5) give the configuration of $E^2(x,r)$ in the discharge part of the cavity. From these it can be shown that $E^2(0,r)$ is given to a good approximation by $15.3 \cosh a(x)r$, $E^2(x,0)$ by $15.3\ e^{-4x^2}$, and $E^2(x,0.28)$ by $24\ e^{-6.4x^2}$. Thus a general form for $E^2(x,r)$ is

$$E^2(x,r) = A\ e^{-g(r)\ x^2} \cosh(a(x)\ r)$$

Using figures (2,3,4&5) it can be shown that a(x) has an average value of 2.52, and g(r) is best taken to be 6.40. This results in the following expression for $E^2(x,r)$

$$E^2(x,r) = 15.3\ e^{-6.4x^2} \cosh 2.52r$$

$$0 \leq r \leq 0.65 \qquad -\infty < x < +\infty$$

Some sort of E - field must be assumed for $3.4 \geq r \geq 0.65$ and $0.16 \geq x \geq -0.16$ since no measurements were made in this region. We reason that the field is not very different in this region from the configuration in a TM_{010} mode, and that the zeroth order Bessel function can be joined to $E^2(x,r) = 15.3\ e^{-6.4\ x^2} \cosh 2.52r$ at the inside wall of the glass tube. The amplitude B of this Bessel function at r=0.65 is taken to be

$$J_0\left(2.4\ \frac{0.65}{3.4}\right) A = \frac{\int_{-0.16}^{+0.16} 15.3\ e^{-6.4x^2} \cosh(2.52 \times 0.65)\, dx}{\int_{-0.16}^{0.16} dx} = 40.8$$

$$\therefore A = 42.8$$

and finally $E^2(x,r)$ everywhere is given by

$$E^2(x,r)= \begin{cases} 15.3\, e^{-6.4x^2} \cosh 2.52r \\ 0 \leq r \leq 0.65 \quad -\infty < x < +\infty \\ 42.8\, (\, J_0(0.71r)\,)^2 \\ 0.16 \geq x \geq -0.16 \qquad 0.65 \leq r \leq 3.4 \end{cases}$$

and the density $n(x,r)$ by

$$n(x,r) = \begin{cases} N_0(x)\, J_0(3.89r) & 0 \leq r \leq 0.65 \\ 0 & r > 0.65 \end{cases}$$

The integrals are easily solved with the exception of $\int_0^{0.65} rJ_0(3.89r) \cosh 2.52r\, dr$ which has been solved by numerical methods, and is equal to 0.11.

Thus finally we get for the density N_0

$$N_0 = \frac{\Delta\lambda(\text{cm})}{f(\gamma_0)}\, 6.18 \times 10^{11} \text{ electrons per cm}^3$$

For the measurements taken in this experiment the average value of $f(\gamma_0)$ in the Faraday Dark Space is 0.29 and in the positive column is 0.49 .

2) Calculation of electron temperature

Margenau's solution of the conductivity integrals involves the parameter x

$$x = \frac{1}{\gamma_0^2}$$

where

$$\gamma_0 = \frac{P_c\, p \sqrt{\frac{2KT}{m}}}{\omega}$$

p= pressure in mm Hg; ω is the radian frequency of the electric field; P_c = effective crossection for collision assumed to equal $\frac{1}{9.5} \times 10^3$ cm^{-1} mm Hg^{-1} for energies less than about 1.8 electron volts; $k = 1.38 \times 10^{-16}$ ergs deg.$^{-1}$; T = temperature of the electrons in degrees Kelvin; m = electron mass = 9.11×10^{-28} gm.

Substitution of these quantities gives T in terms of

$$T = 3.4 \times 10^4 \ \gamma_0^{\,2} \text{ deg. Kelvin}$$

Discussion of Results

Electron density has been measure as a function of position, at a constant pressure, for 4 different currents, as shown in Fig.(8 & 9). Unfortunately the poor operating characteristics of the discharge prevented a detailed examination of the cathode fall regions. It was possible to study only one half of the Faraday Dark Space, a portion of the anode fall, and the positive column. The results are as expected. The positive column showed a constant density, and the anode fall a decreasing density. Some interesting structure is to be noticed near the cathode. Whereas density is linearly related to current, see Fig.(14), in the positive column, no such simple re - lationship is found to hold in the Faraday Dark Space, where density increases much faster than current. Moving towards the positive column we see that there is a minimum in the density, quickly followed by a maximum, before the density levels off. The minimum can be associated with an increasing energy. The maximum probably denotes the region where excitation and ionization have slowed the electrons. For 0.9 Ma. this behavior of alternating minimums and maximums is to be noticed for quite a distance, and one can not help thinking of a possible connection with the well known phenomena of striations. Striations were,however, not noticed visually. The values calculated for the electron density are in doubt mainly, because the electric field configuration is based on too many assumptions. Thus, it may very well be that the densities are too large by a factor of 10.

Electron temperatures measured in the positive column and Faraday Dark Space are given below

Current	Pos. Col. Temp.	Faraday D.S. Temp	Ave. ν_c
0.5 Ma	9.55 10^3 ^oK 1.23 e.v.	78.2 10^3 ^oK 10.2 ev	Pos.C. 2.6×10^{10} sec^{-1} FDS 7.5×10^{10}
0.7	13.9 10^3 1.79 ev	43.5 10^3 5.6 ev	P.C. 3.14×10^{10} FDS 5.5×10^{10}
0.9	11.8 10^3 1.52 ev	35 10^3 4.51 ev	P.C. 2.8×10^{10} FDS 4.9×10^{10}
1.5	31.3 10^3 4.04 ev	20.7 10^3 2.67 ev	P.C. 4.6 10^{10} F.D.S. 3.8×10^{10}

The measurements were taken at the following two position:

Positive column: 25.8 cm from the cathode
Faraday D. S. : 2.15 cm " " "

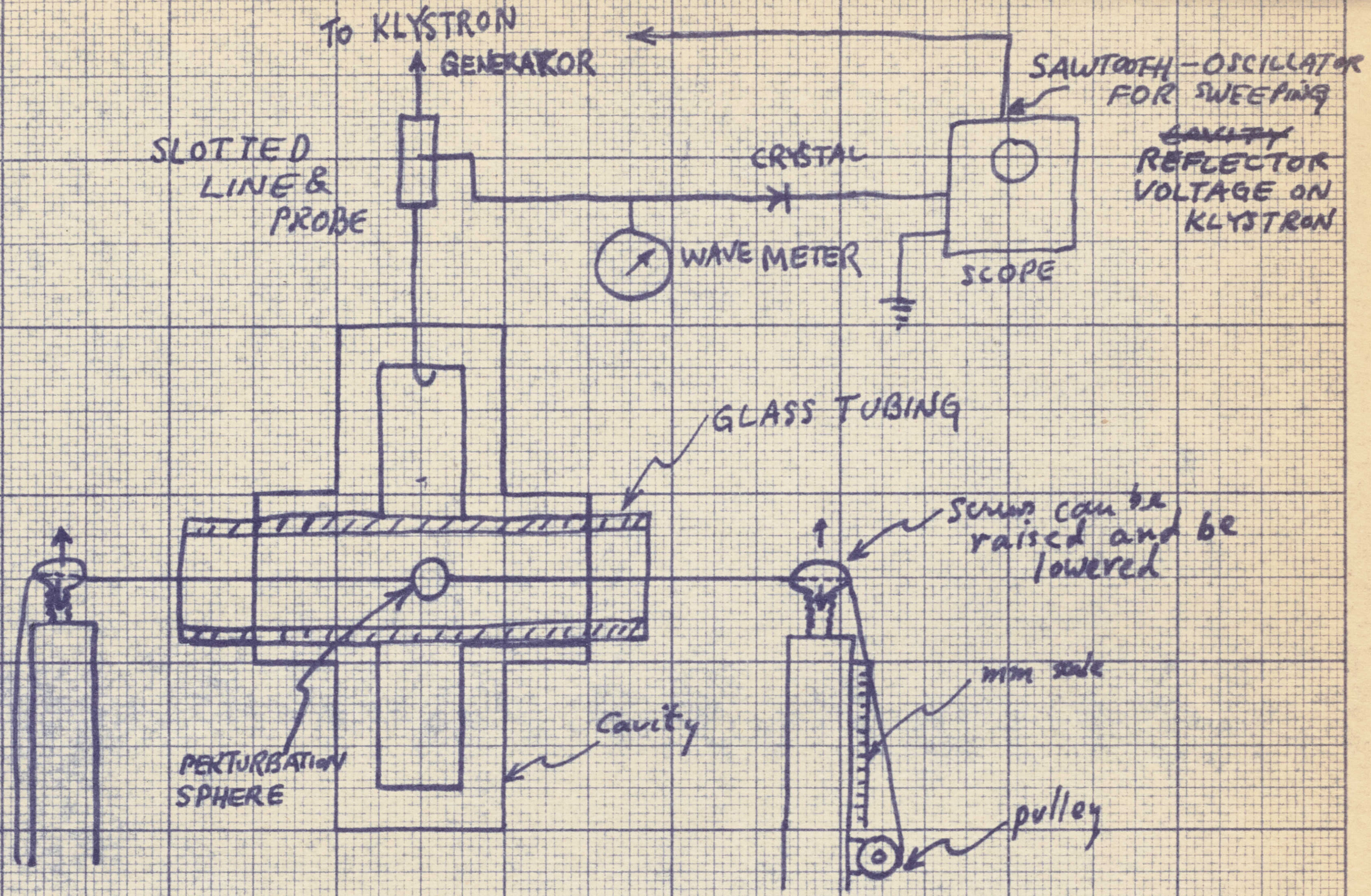
Since the position of the Faraday Dark Space shifts with current in the manner indicated on Fig.(8), temperatures can not be directly compared for the various currents.

The temperatures in the positive column have been compared to Von Engel and Steenbeck's[7] theory, which predicts that temperature is functionally related to the product (pressure times radius) of discharge tube. This relation is shown in Fig.(13) together with the experimental points. They are seen in each case to be larger than predicted by the theory. The temperatures in the Faraday Dark Space to are much too large to explain the darkness of the region, or the high density of electrons. These discrepancies seem to point to the fact that the value of P_c assumed is too small; on the other hand a plot of the product, pressure times mean free time which can be written

$$pt = \frac{1}{P_c p \bar{v}}$$ see Fig. 12

where $\bar{v}$ is the average velocity is nearly linear, and does indeed point to a constant P_c. The arguments are therefore conflicting, and unfortunately have not yet been resolved.

The error in carrying out this portion of the experi - ment is mainly due to the fluctuations in pressure, which were as high as 10%.



MEASUREMENT of E

Fig. 1

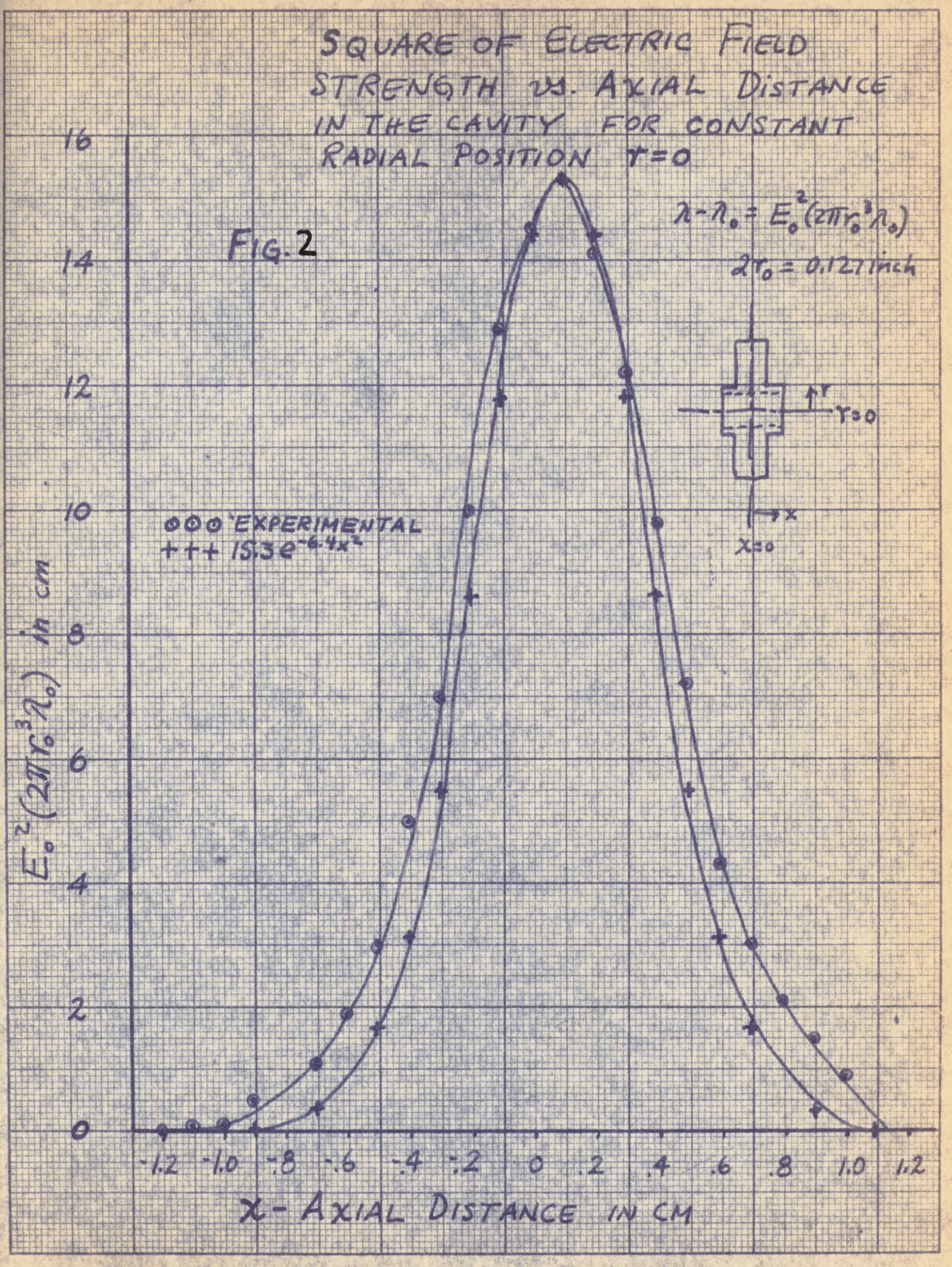

SQUARE OF ELECTRIC FIELD STRENGTH vs. AXIAL DISTANCE IN THE CAVITY FOR CONSTANT RADIAL POSITION r=0
$\lambda - \lambda_0 = E_0^2(2\pi r_0^3 \lambda_0)$
$2r_0 = 0.1271$ inch
r
r=0
x
x=0
ooo EXPERIMENTAL
+++ $15.3e^{-6.4x^2}$
$E_0^2(2\pi r_0^3 \lambda_0)$ in cm
x - AXIAL DISTANCE IN CM
16
14
12
10
8
6
4
2
0
-1.2
-1.0
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
1.2

FIG. 2

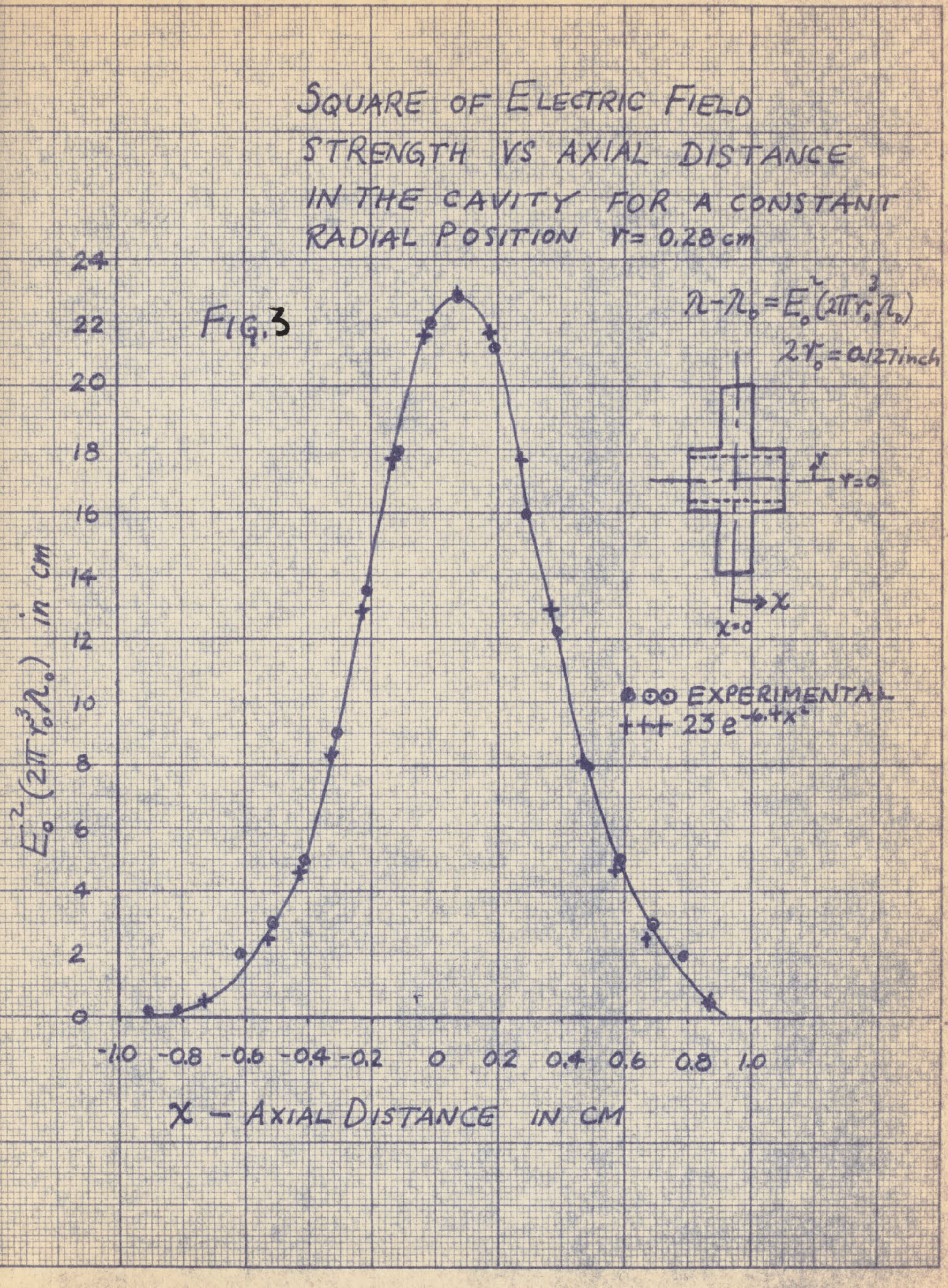
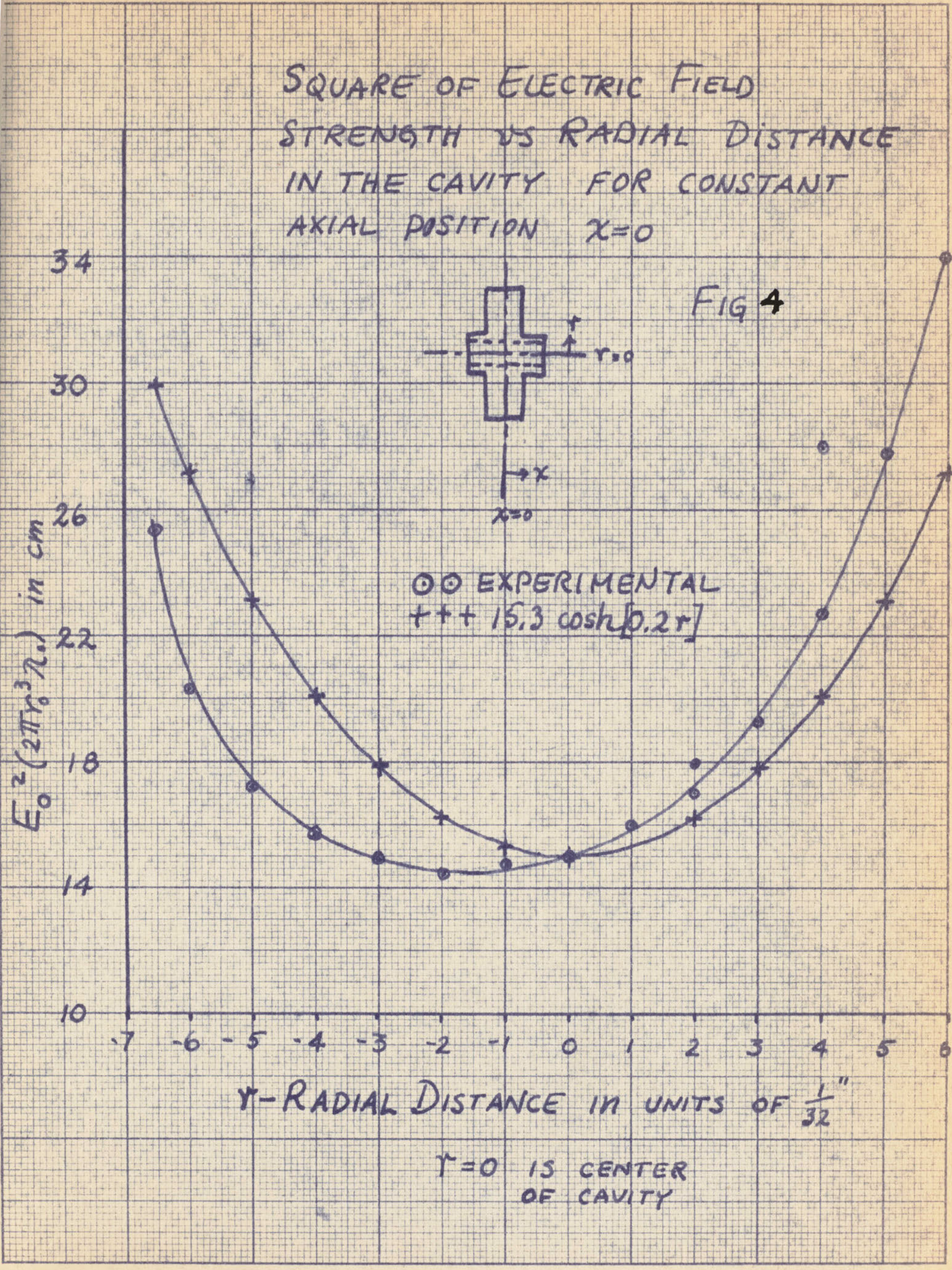
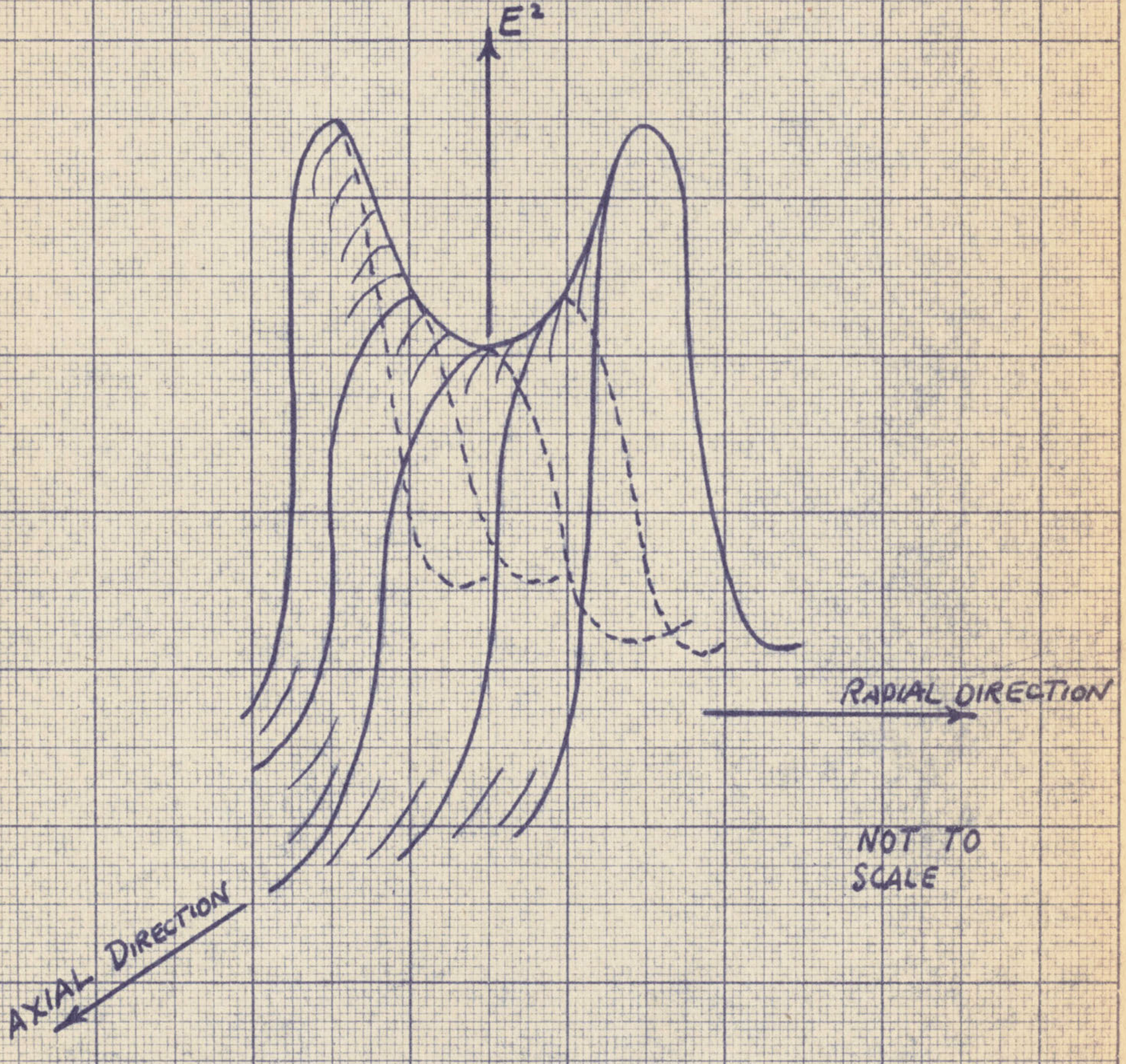

SQUARE OF ELECTRIC FIELD STRENGTH VS AXIAL DISTANCE IN THE CAVITY FOR A CONSTANT RADIAL POSITION $r = 0.28$ cm
$\lambda - \lambda_0 = E_0^2 (2\pi r_0^3 \lambda_0)$
$2r_0 = 0.127$ inch
r
r=0
x
x=0
●○○ EXPERIMENTAL
+++ $23 e^{-6.4 x^2}$
$E_0^2 (2\pi r_0^3 \lambda_0)$ in cm
X – AXIAL DISTANCE IN CM
0
2
4
6
8
10
12
14
16
18
20
22
24
-1.0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1.0

FIG. 3

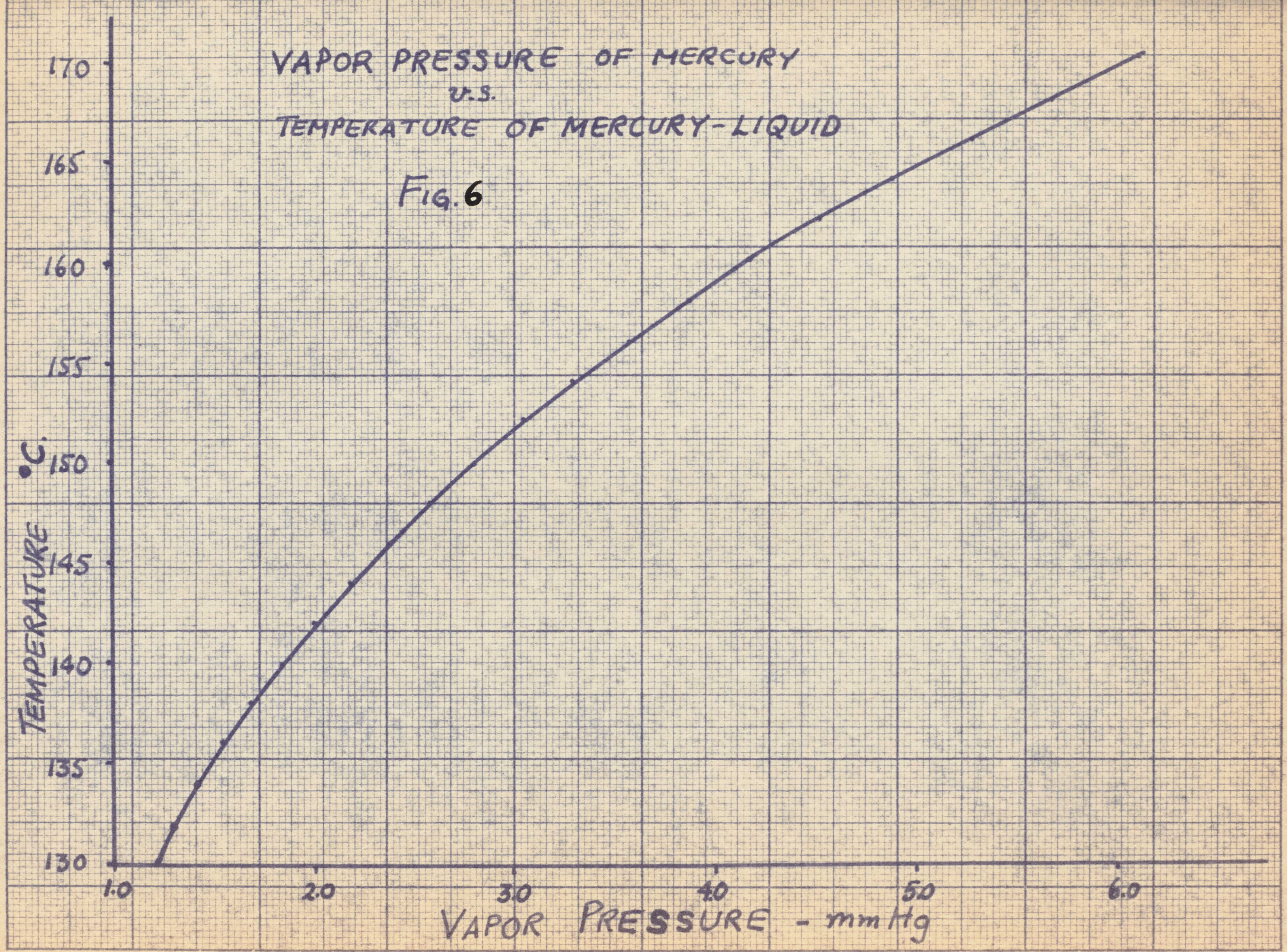


SQUARE OF ELECTRIC FIELD STRENGTH vs RADIAL DISTANCE IN THE CAVITY FOR CONSTANT AXIAL POSITION x=0
FIG 4
r=0
x
x=0
⊙⊙ EXPERIMENTAL
+++ 15.3 cosh [0.2r]
$E_0^2 (2\pi r_0^3 \lambda_0)$ in cm
34
30
26
22
18
14
10
-7
-6
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
r - RADIAL DISTANCE in UNITS OF 1/32"
r=0 IS CENTER OF CAVITY



MICRO-WAVE ELECTRIC FIELD-STRENGTH IN DISCHARGE

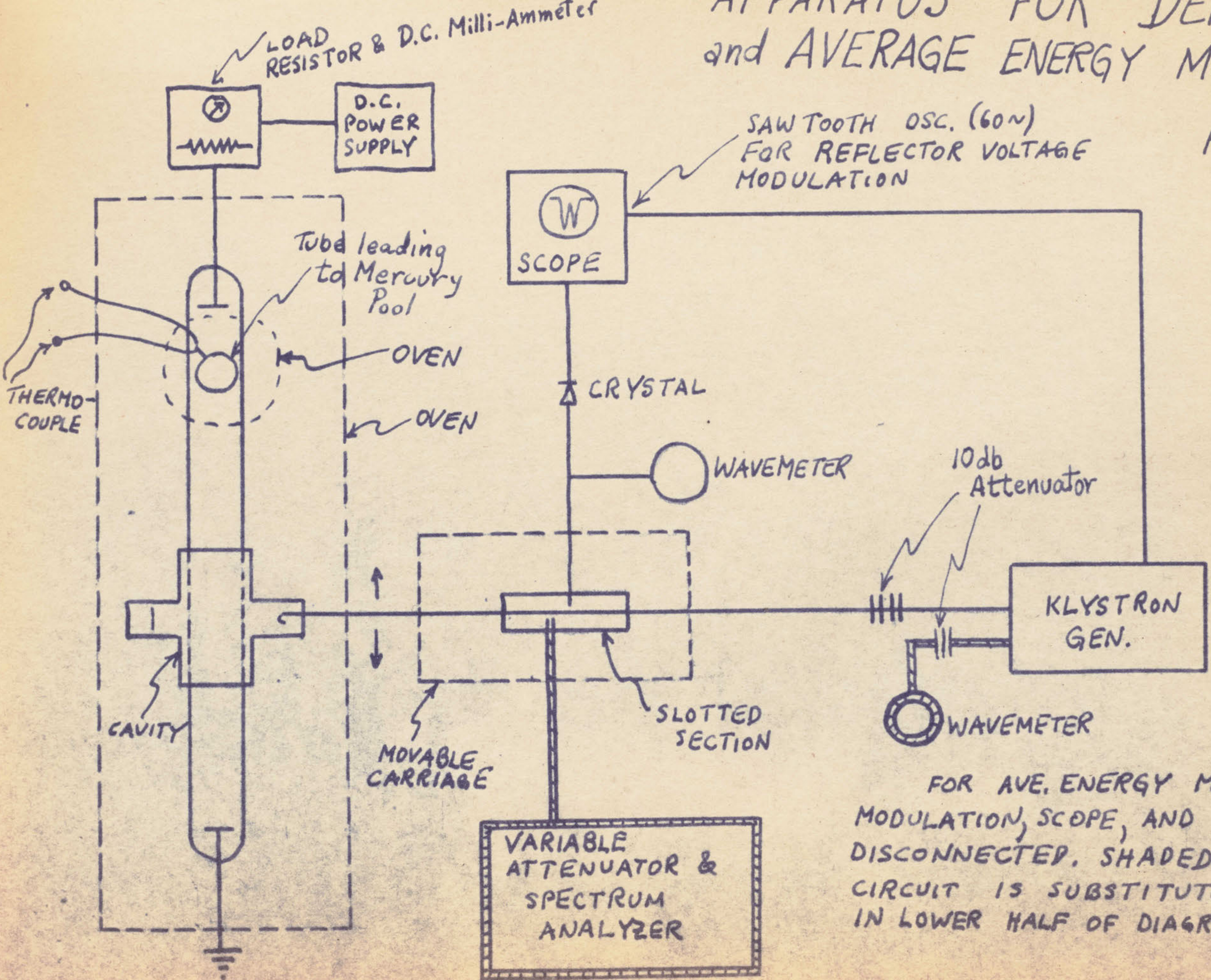
FIG. 5



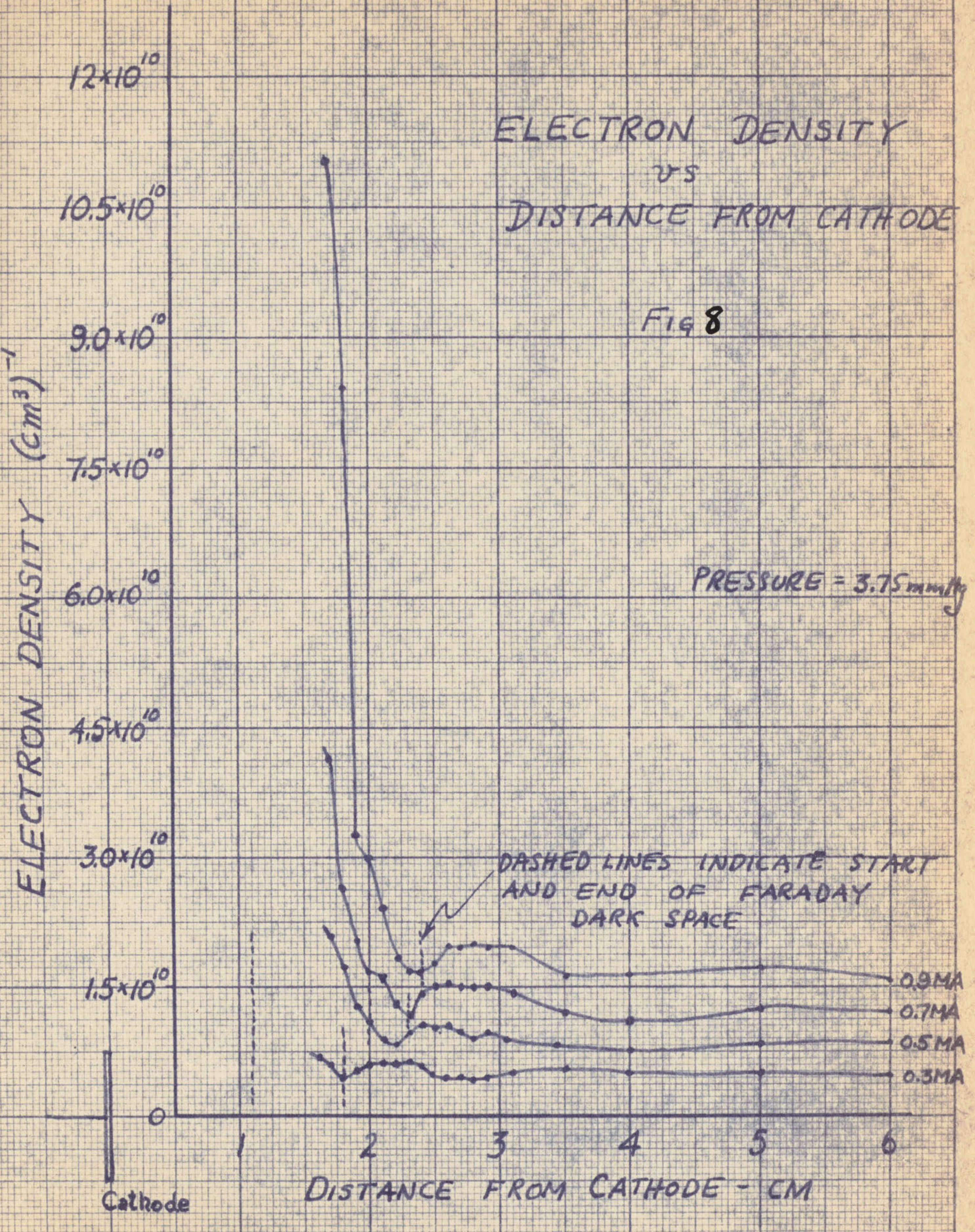
VAPOR PRESSURE OF MERCURY
V.S.
TEMPERATURE OF MERCURY-LIQUID
FIG. 6
TEMPERATURE °C.
170
165
160
155
150
145
140
135
130
1.0
2.0
3.0
4.0
5.0
6.0
VAPOR PRESSURE - mm Hg

APPARATUS FOR DENSITY and AVERAGE ENERGY Measurement

FIG. 7

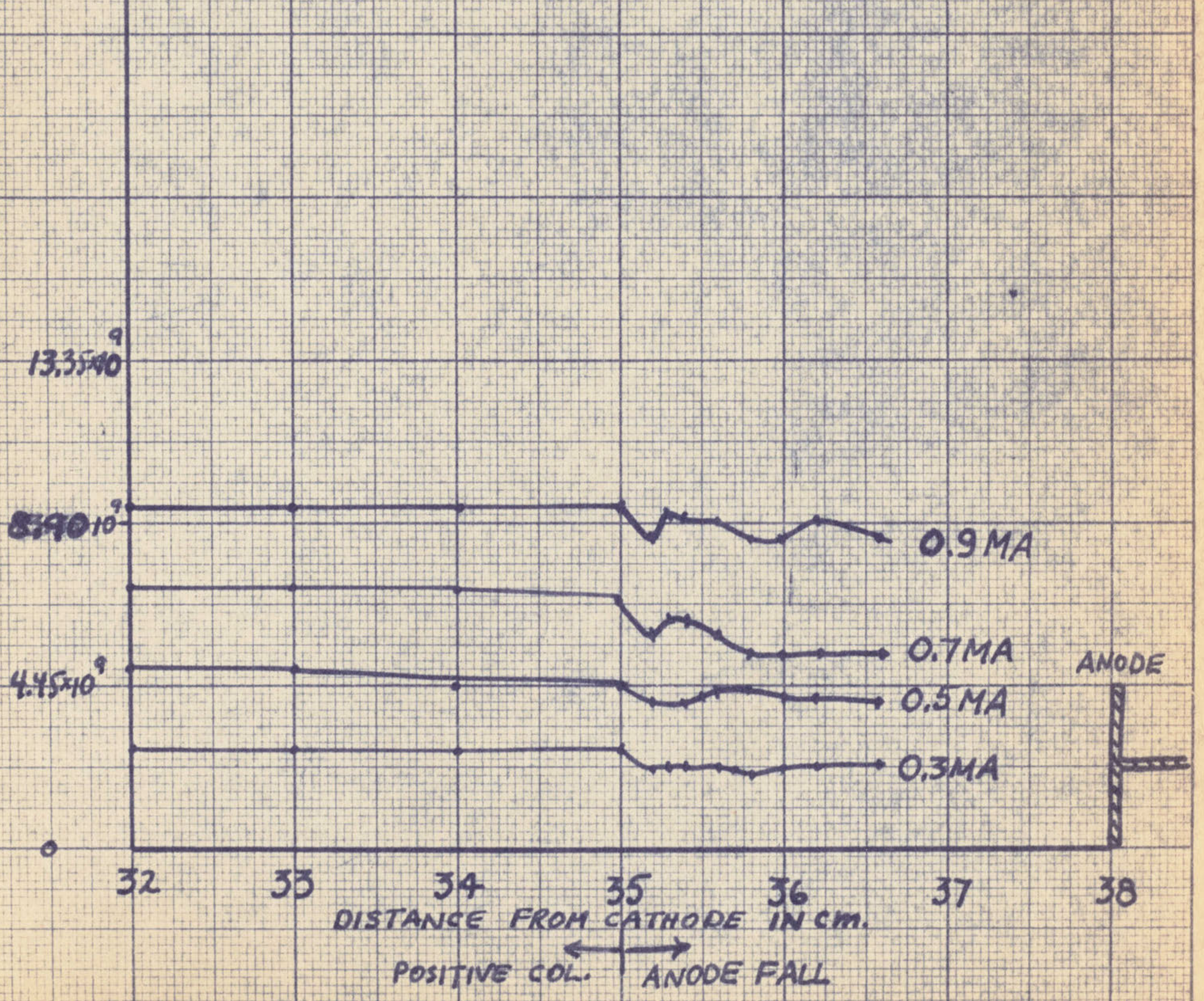


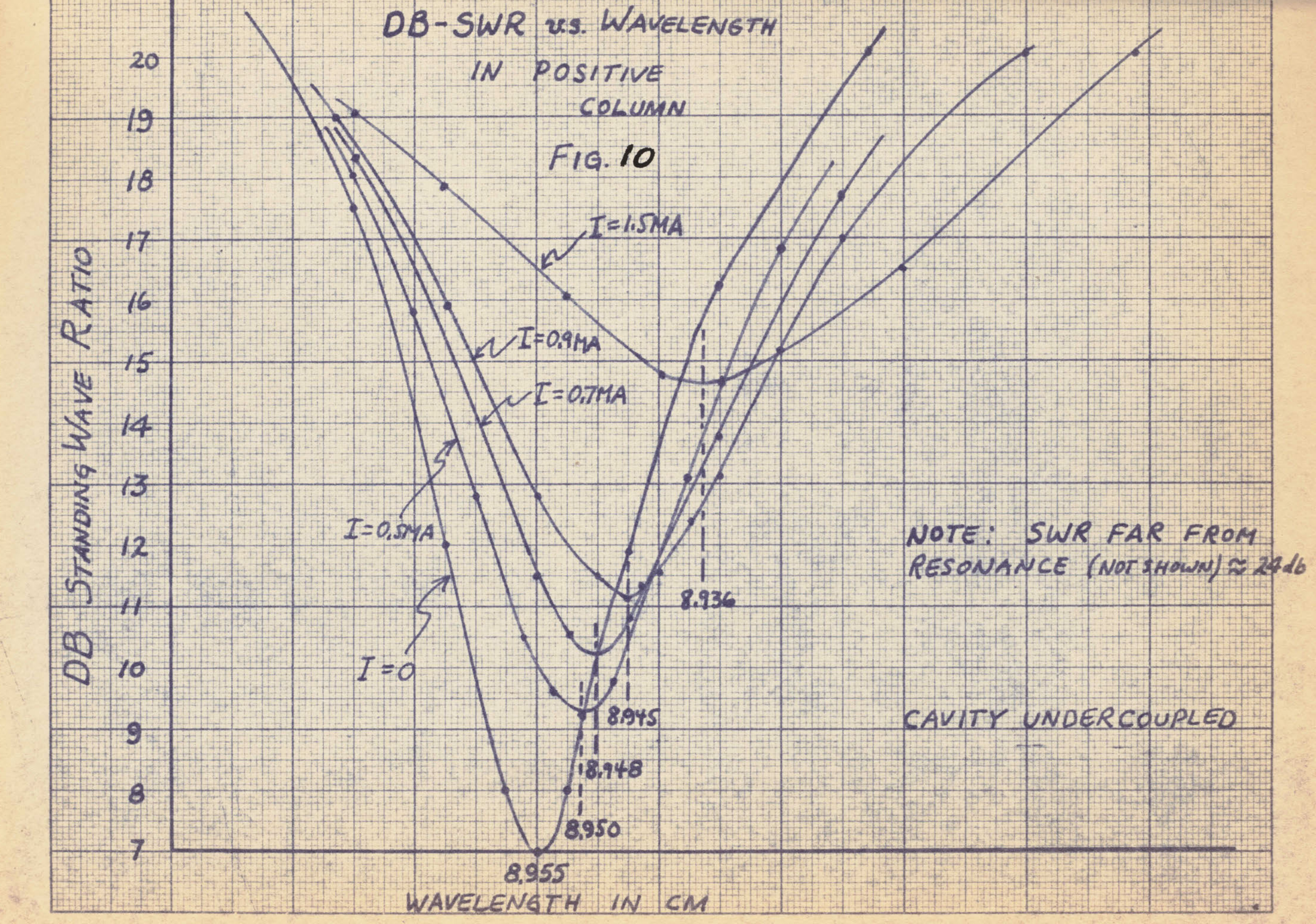
FOR AVE. ENERGY MEASUREMENT MODULATION, SCOPE, AND CRYSTAL ARE DISCONNECTED. SHADED ( ) CIRCUIT IS SUBSTITUTED AS SHOWN IN LOWER HALF OF DIAGRAM

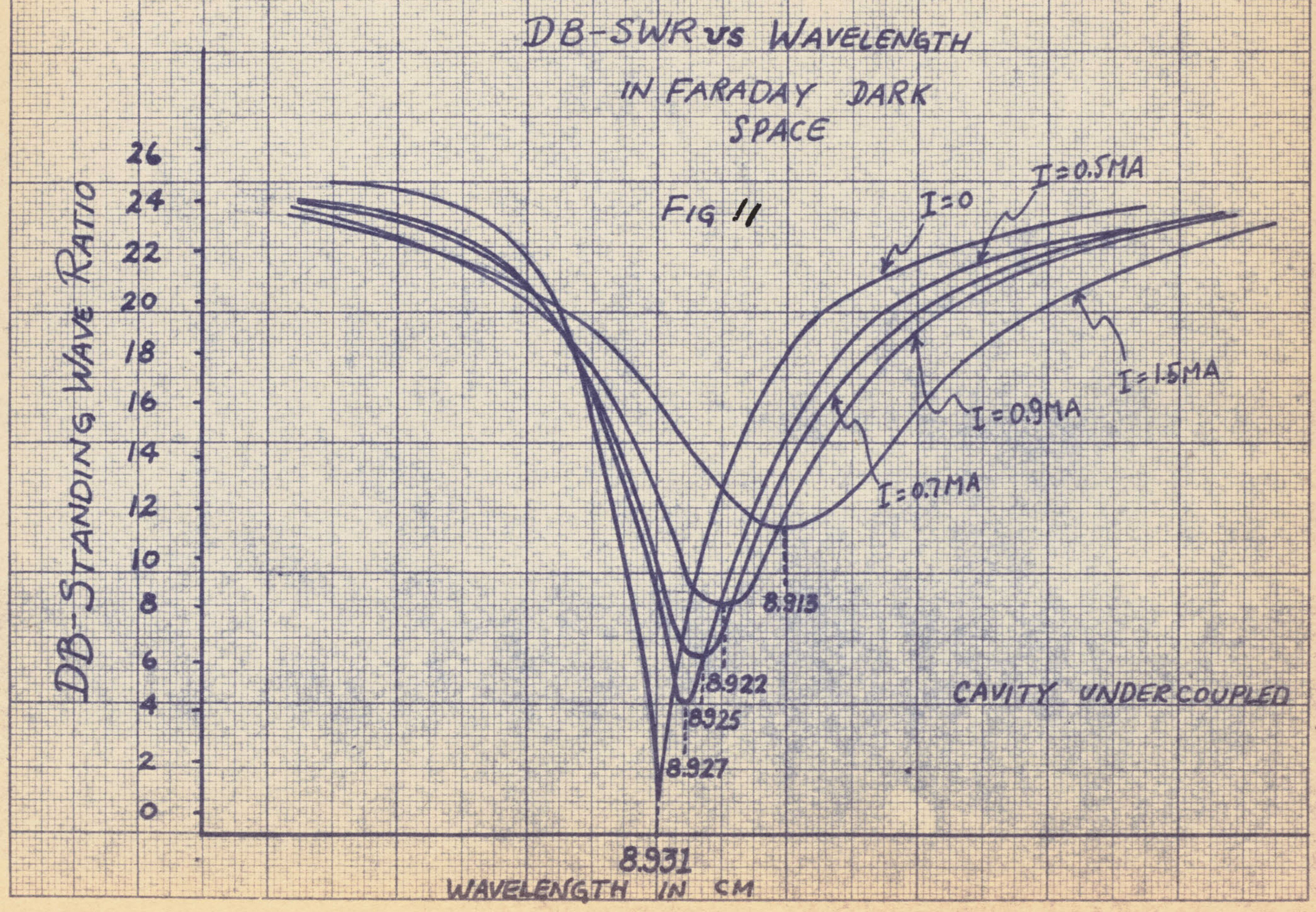

ELECTRON DENSITY
vs
DISTANCE FROM CATHODE
FIG 8
PRESSURE = 3.75 mm Hg
DASHED LINES INDICATE START AND END OF FARADAY DARK SPACE
ELECTRON DENSITY (cm³)⁻¹
12×10¹⁰
10.5×10¹⁰
9.0×10¹⁰
7.5×10¹⁰
6.0×10¹⁰
4.5×10¹⁰
3.0×10¹⁰
1.5×10¹⁰
0
1
2
3
4
5
6
0.9MA
0.7MA
0.5MA
0.3MA
Cathode
DISTANCE FROM CATHODE - CM

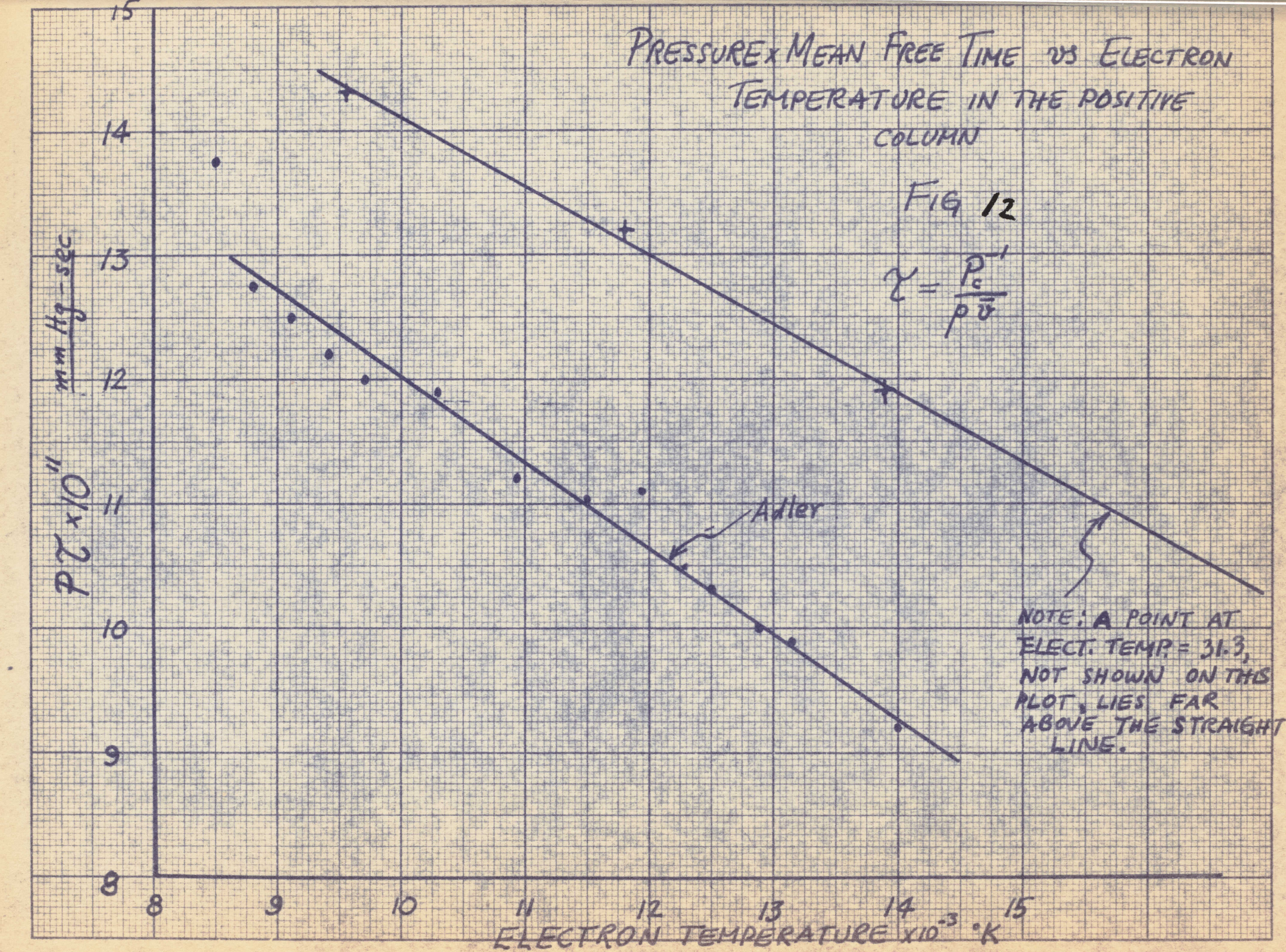
ELECTRON DENSITY
NEAR THE ANODE

FIG 9

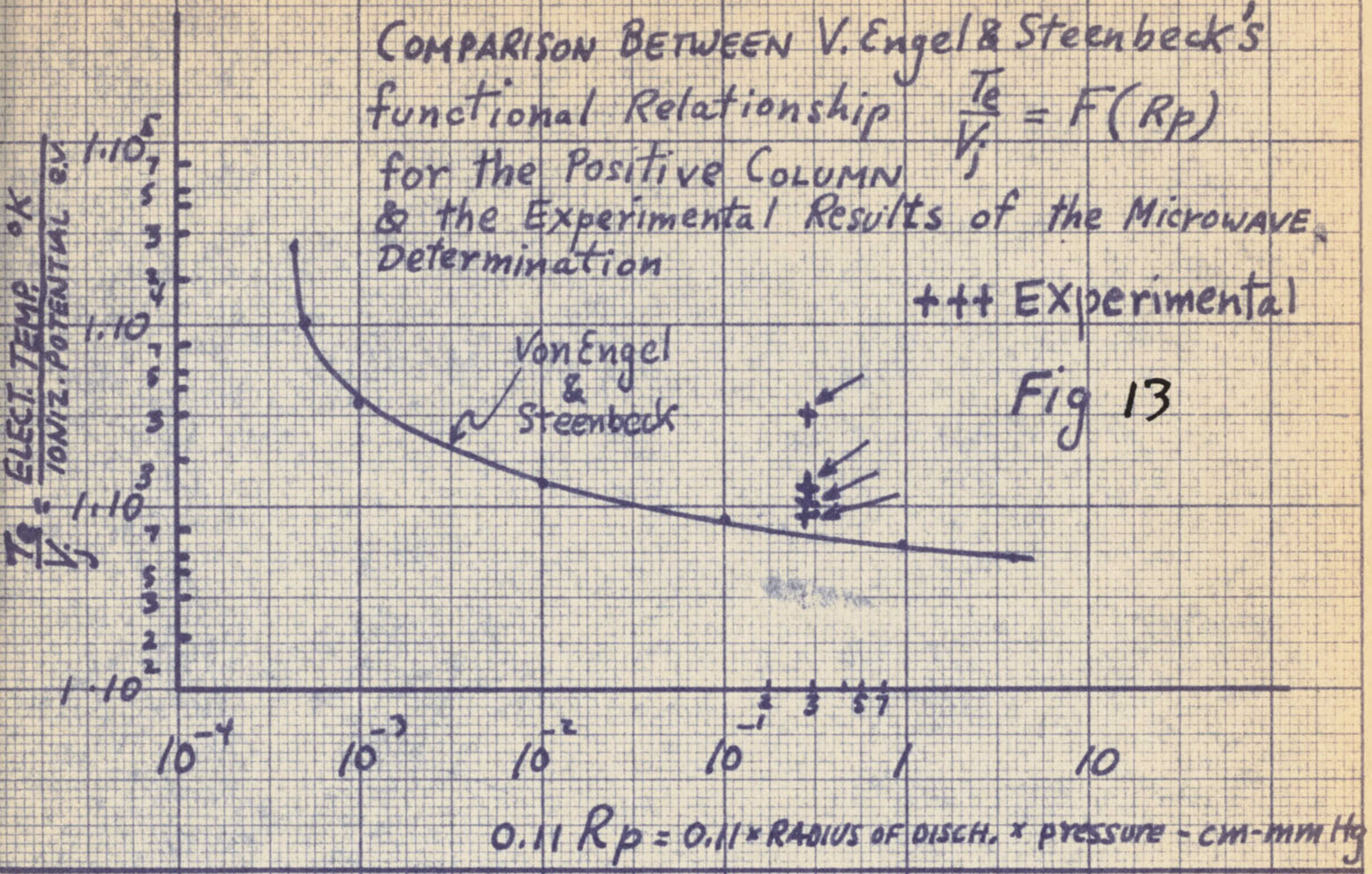


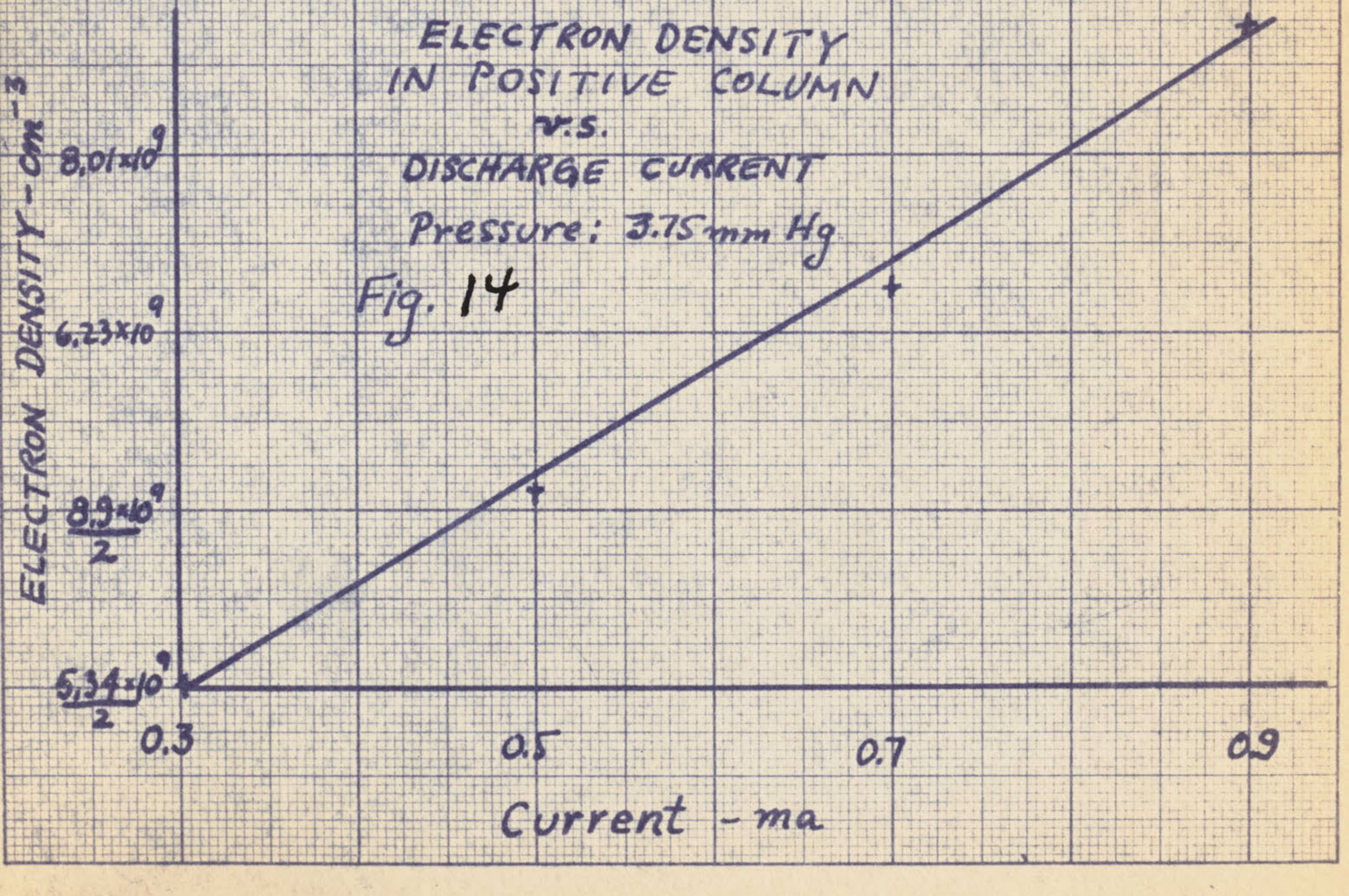

DB-SWR vs. WAVELENGTH
IN POSITIVE
COLUMN
FIG. 10
I=1.5MA
I=0.9MA
I=0.7MA
I=0.5MA
I=0
DB STANDING WAVE RATIO
20
19
18
17
16
15
14
13
12
11
10
9
8
7
8.936
8.945
8.948
8.950
8.955
WAVELENGTH IN CM
NOTE: SWR FAR FROM
RESONANCE (NOT SHOWN) ≈ 24db
CAVITY UNDERCOUPLED


DB-SWR vs WAVELENGTH
IN FARADAY DARK SPACE
FIG 11
DB-STANDING WAVE RATIO
26
24
22
20
18
16
14
12
10
8
6
4
2
0
I=0
I=0.5MA
I=0.7MA
I=0.9MA
I=1.5MA
8.913
8.922
8.925
8.927
CAVITY UNDERCOUPLED
8.931
WAVELENGTH IN CM



PRESSURE x MEAN FREE TIME vs ELECTRON TEMPERATURE IN THE POSITIVE COLUMN
FIG 12
$\tau = \frac{P_c^{-1}}{p\bar{v}}$
Adler
NOTE: A POINT AT ELECT. TEMP. = 31.3, NOT SHOWN ON THIS PLOT, LIES FAR ABOVE THE STRAIGHT LINE.
$P\tau \times 10^{11}$ $\frac{mm\ Hg - sec}{}$
ELECTRON TEMPERATURE $\times 10^{-3}$ °K
8
9
10
11
12
13
14
15


COMPARISON BETWEEN V. Engel & Steenbeck's
functional Relationship $\frac{T_e}{V_j} = F(Rp)$
for the Positive COLUMN
& the Experimental Results of the MICROWAVE Determination
+++ Experimental
Fig 13
Von Engel & Steenbeck
$\frac{T_e}{V_j} = \frac{\text{ELECT. TEMP. } °K}{\text{IONIZ. POTENTIAL e.V.}}$
$0.11\,Rp = 0.11 \times$ RADIUS OF DISCH. $\times$ pressure - cm-mm Hg

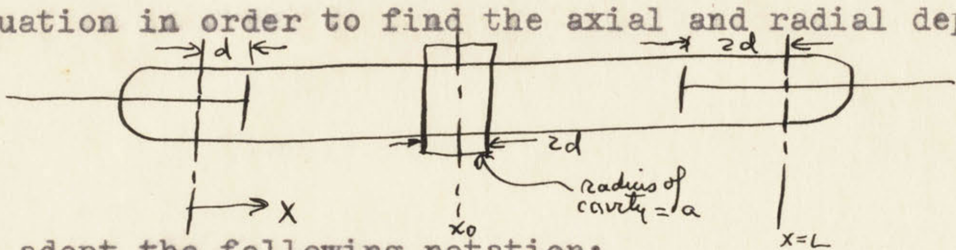

ELECTRON DENSITY
IN POSITIVE COLUMN
v.s.
DISCHARGE CURRENT
Pressure: 3.75 mm Hg
Fig. 14
ELECTRON DENSITY - cm^{-3}
Current - ma

APPENDIX I

Electron density in Slater's formula is un - fortunately the solution to an integral equation.

$$\Delta \lambda(x_0, a) = \text{const.} \iint n(x,r)\, E^2(x_0 - x, r, a)\, r\,dr\,dx$$

Suppose we want the density distribution in a cylinderical discharge which has no angular dependence; then we see that we must really solve a double integral equation in order to find the axial and radial dependence.



We adopt the following notation:

x = axial distance; x_0 = position of cavity at any time;

r= radial distance; a= radius of cavity at any time;

$\Delta\lambda(x_0,a) = u(x_0,a)$ = frequency shift at cavity position x_0 using a cavity of radius a.

The method of solution adopted here is to expand the kernel $E^2(x_0 - x, r, a)$, the density $n(x,r)$, and the measured frequency shift $u(x_0, a)$ in terms of certain functions which when substituted into the original equation yield sets of linear algebraic equations for the coefficients in the expansion for $n(x,r)$. One method of finding these functions consists of solving

$n = e\rho$

$$\nabla^2 u(x_0,a) + k^2 u(x_0,a) = \rho(x_0,a)$$

e = elect. charge

In order to do this we must solve the homogeneous problem first

$$\nabla^2 u(x_0, a) + k_m^2 u(x_0, a) = 0$$

subject to the boundary conditions $u(0,r) = u(L,r) = 0$ and $u(x_0, a_0) = 0$ (ie. we choose $x_0 = 0$ & L, and $a = a_0$ to be experimental reference points). The solution is a set of eigenfunctions $u_m(x_0, a)$ such that

$$\nabla^2 u_m(x_0, a) = -k_m^2 u_m(x_0, a)$$

and

$$u(x_0, a) = \sum_m A_m u_m$$

Expanding $\rho(x,r)$ in terms of these functions : $\rho(x,r) = \sum_m B_m u_m$ we find that

$$A_m = \frac{B_m}{k^2 - k_m^2}$$

Substitution of these expansions into the integral equation yields the kernel

$$E^2(x_0 - x, r, a) = \sum_m \frac{u_m(x_0, a)\, u_m(x, r)}{\text{const} \times (k^2 - k_m^2) \iint^{a_0} r u_m^2 \, dr\, dx}$$

Since $E^2(x_0 - x, r, a)$ is known experimentally, orthogonality conditions yield the k's, and we can put the integral equation into the form of a series involving the coefficients in the expansion of the density

$$u(x_0, a) = \Delta\lambda(x_0, a) = \sum_m B_m \frac{u_m(x_0, a)}{\text{const} \times (k^2 - k_m^2)}$$

To find the axial distribution of density in a discharge, $u(x_0, a)$ must be measured for various x_0 and constant a. To find the radial distribution, $u(x_0, a)$ must be measured at constant x_0 for various sized cavities. It must be remembered, however, that the expansion for the density will be valid only in the region under examination. The determination of the radial dependence could be easily carried out by varying both the frequency and the modes used.

APPENDIX II

The assumption made by Slater in the derivation of the density formula was that the absolute magnitude of $|\sigma_i|$ be much smaller than $\omega\varepsilon_0$.

ω = radian frequency of the microwave electric field

ε_0 = dielectric constant for free space

An order of magnitude calculation can be made to see what these terms depend on, by making ~~uses~~ use of a simplified equation of motion for the electrons: We assume that the electrons are acted on by an electric field, but instead of considering a distribution function we write the equation of motion of each electron separately, considering collisions to act like viscous damping forces. This gives us the familiar equation

$$\frac{dv}{dt} + \frac{k}{m} v = \frac{eE}{m}$$

where we call k/m, for dimensional reasons, the frequency of collision. The current density is then given in terms of the solution of this equation as

$$J = \frac{ne^2 E_0 e^{j\omega t}}{j\omega m + k}$$

and the imaginary conductivity becomes

$$\sigma_i = \frac{|\sigma_i|}{\left(\frac{\nu_c^2}{\omega^2}+1\right)^{1/2}} = \frac{ne^2}{m\omega} \frac{1}{\left(\frac{\nu_c}{\omega}\right)^2+1}$$

The frequency of collision of an electron is defined as the ratio of its velocity to its mean free path, ℓ. The mean free path in turn is defined in terms of an effective cross-section P_c which is normalized in pressure to 1 mm Hg., so that

$$\ell = \frac{1}{P_c p}$$

Adler and Margenau have shown that for mercury, P_c is independent of energy at low energies ($u < 1.4$ ev), and has the value $\frac{1}{9.5} \times 10^{3}\ cm^{-1}\,mmHg^{-1}$. Using the average energies found in this experiment together with Adler and Margenau's P_c, it is found that n varies from a maximum value of $8.2 \times 10^{9}\ cm^{-3}$ in the positive column to $1.86 \times 10^{10}\ cm^{-3}$ in the Faraday Dark Space, if the ratio $\frac{|\sigma_c|}{\omega \varepsilon_0}$ is less than 0.01.

Fig. (8 & 9) shows that the density indeed satisfies these limits, although for 1.5 Ma. indications are that a density of 10^{11} cm^{-3} is reached in the Faraday Dark Space. Again, it must be remembered that the use of Adler and Margenau's P_c has not been fully justified in this experiment, and in a rigerous examination one would need P_c as a function of energy, and one would need a clear knowledge of the form of the distribution function.

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