STANDING WAVES

IN

SUPERCRITICAL FLOW OF WATER

by

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Dear Sir:

In partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering, this thesis, entitled "Standing Waves in Supercritical Flow of Water", is submitted.

Respectfully yours,

Milton P. Barschdorf

Harry G. Woodbury, Jr.

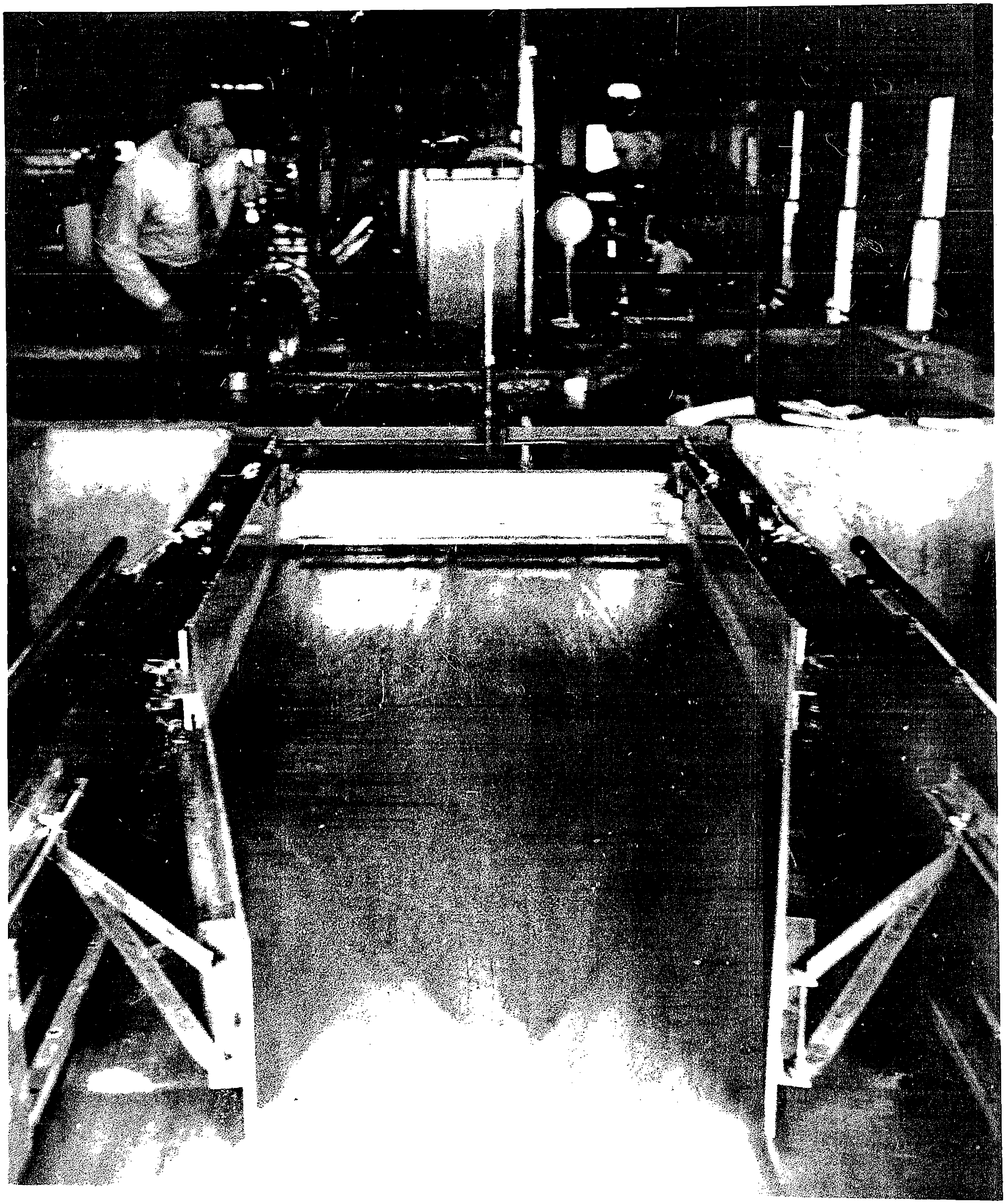

PLATE 1

ACKNOWLEDGEMENTS:

The writers are deeply grateful to Dr. Arthur T. Ippen and his associates, Dr. James W. Daily and Mr. Donald R. F. Harleman, whose keen interest and timely suggestions proved a real source of encouragement; to Mr. Herbert C. Johnston and Mr. Roy G. Milley for their able assistance in the construction of the apparatus; to the New England Division Engineer and Staff for their willing cooperation and fine work of photo reproduction; and, finally, to our wives, for their gracious assistance, patience, and understanding.

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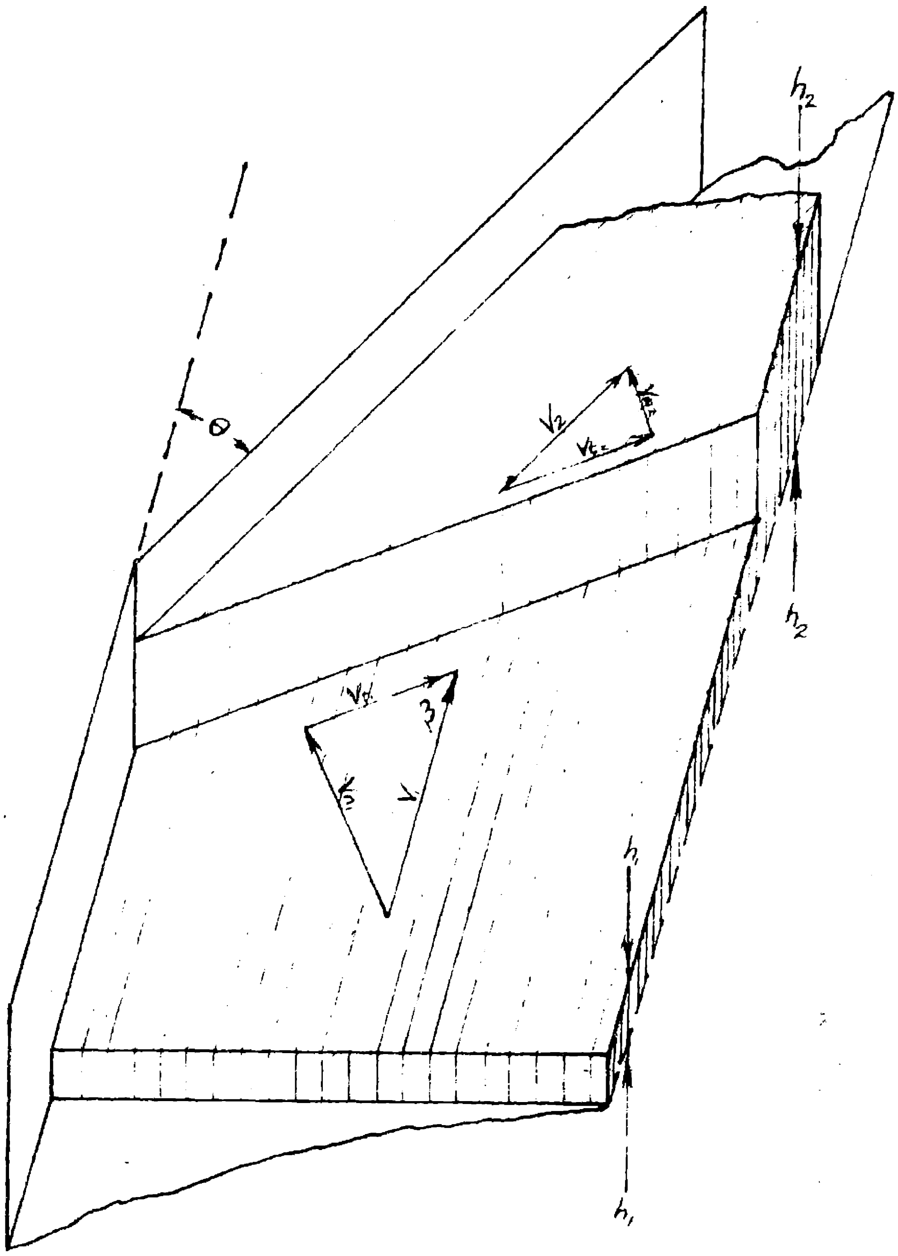


Figure 1

The following symbols and abbreviations will be used in this paper, some of which are identified in Figure 1 above:

b - channel width

C - wave celerity

cfs - cubic feet per second

F_1 - Froude number upstream of transition $= \frac{V_1}{\sqrt{g h_1}}$

g - acceleration of gravity

h - depth of flow

h_1 - depth of flow above transition

h_2 - depth of flow downstream of standing wave

H - specific energy

s - channel slope or gradient

t - time in seconds

V - velocity of flow feet/second

V_t - velocity component parallel to wave front

V_1 - velocity upstream of standing wave

V_{n_1} - upstream velocity component perpendicular to wave front

V_{n_2} - downstream velocity component perpendicular to wave front

V_2 - velocity downstream of standing wave

β - angle of wave measured from direction of flow above transition degrees

θ - angle of transition measured from original direction of flow degrees

SUMMARY

The recent tendency towards handling large volumes of water at higher and higher velocities has introduced a new problem for the designing engineer. Structures designed from a consideration of flow phenomena in natural streams have often proved unsatisfactory because of surprising standing waves generated in curved sections and expanding and contracting transitions. An analytical investigation of these standing waves was presented by Dr. A. T. Ippen at the Second Hydraulic Conference at Iowa University in 1942. In this investigation Dr. Ippen presented a relationship for the determination of wave angle and depth in terms of boundary and flow conditions. It is the purpose of this thesis to investigate the relationship between wave location and depth with changing boundary conditions for a limited variation in flow conditions and compare the results obtained with analytical results based on Dr. Ippen's paper.

An aluminum flume, three feet wide and twelve feet long, was designed and constructed to fit the bed of an existing wooden flume with an adjustable slope. Walls were provided with 40 inch machine fitted wall sections on each side of the flume that were adjustable to any desired transition angle between zero and eighteen degrees. Wave profiles were then measured at sections through the transition for wall angles of three

to twenty-four degrees in three degree increments for a Froude number of 3.86 and the wave angle and depth determined. When compared with the wave angle and depth computed for the same boundary conditions from Dr. Ippen's relationship, an agreement within three per cent was found for the depth ratios. The agreement of measured and computed wall angles varied consistently from a measured angle, being about fifteen per cent too small for transition angles of three degrees to absolute agreement between twelve and fifteen degrees. For transition angles greater than fifteen degrees, the measured angle was about five per cent too great.

It is concluded, therefore, that for Froude numbers between 3.5 and 4.1 the analytical determination of depth ratios is of sufficient accuracy for design purposes and that the analytical determination of wave angles when connected by a value K, which is apparently a function of wall angle and flow conditions, can be considered accurate to within one degree in designing a contracting transition.

INTRODUCTION

STATEMENT OF PROBLEM

The fundamental characteristics of the flow of water in open channels has, up until the last decade or so, been associated with the flow phenomena in natural streams or artificial channels at relatively low velocities. Subsequent large scale developments of irrigation and flood control projects involving the dispatch of large quantities of water at high velocities, resulted in many cases of flow behavior quite different from that predicated upon the low velocity flow theory. Channel curves and contractions designed to impart smooth, streamlined flow transitions now simply induced the formation of standing waves of such magnitude that the channel had to be operated at greatly reduced depths in order to prevent overflowing - or the walls had to be increased in height well out of reason for the quantity of flow handled. The analysis of this phenomena led to the present theory of standing waves in supercritical flow.

The fundamental difference between water flowing at "high" and "low" velocities is in the relationship of potential and kinetic energy possessed by the water. The specific energy of flowing water is made up of two components: depth and velocity heads. These may be thought of as the potential and kinetic energies, re-spectively, of the water in foot pounds per pound. When the ratio of potential to kinetic energy is two to one (2 / 1), the water is flowing at minimum energy and the depth and velocity are said to be "critical". When the ratio is greater than two to one (2 / 1), the flow is known as "streaming", the velocity as "sub-critical", and the flow conditions are those normally encountered in most rivers. When the ratio is less than two to one (2 / 1), The flow is known as "shooting" and the velocity as "supercritical". This type of flow in open channels is less common, existing over spillways, in some sluiceways, and in some diversion channels. Supercritical flow can occur in a street gutter on a really steep hill. It is with supercritical flow characteristics that this paper is concerned, and particularly with the formation of standing waves in supercritical flow. These waves are generated at points of channel contraction, expansion, or curvature, introducing a pronounced influence on downstream conditions of flow for great distances. Their amplitude is often several times the depth of flow upstream of the transition thereby causing overtopping of the channel at capacities far below design capacities.

HISTORY OF PREVIOUS RESEARCH

The first known attempt at understanding the cause of standing waves in open channels and of developing a relationship between flow conditions and boundary conditions such that the location and amplitude of these waves could be predicted was undertaken by Arthur T. Ippen and Robert T. Knapp in work done for the Los Angeles County Flood Control District. The results of their work was embodied in a report entitled: "A Study of High Velocity Flow in Curved Sections of Open Channels" (1) and is supplemented by a later report entitled "Experimental Investigations of Flow in Curved Channels" (2). A summary of the above reports was presented by Knapp and Ippen to the Fifth International Congress of Applied Mechanics, 1938, in a paper entitled: "Curvilinear Flow of Liquids with Free Surfaces at Velocities above that of Wave Propagation" (3). At the Second Hydraulics Conference, University of Iowa, 1942, Ippen presented a paper entitled "Gas Wave Analogies in Open Channel Flow" (4). A summary of this paper is given in the appendices. In this paper Ippen discusses standing waves in hydraulics and develops the following relationships between boundary and flow conditions (See figure 1):

$$\sin \beta = \frac{1}{F_1} \sqrt{\frac{1}{2}\left(\frac{h_2}{h_1}\right)\left(1+\frac{h_2}{h_1}\right)}$$

$$\tan\theta = \frac{\left(1-\frac{h_1}{h_2}\right)\tan\beta_1}{1+\frac{h_1}{h_2}\tan^2\beta}$$

Hunter Rouse in Chapter XVI of his book, "Fluid Mechanics for Hydraulic Engineers" (5), presents an excellent discussion of "Gravity Waves in Open Channels" covering briefly "Wave effects at boundary changes in the horizontal plane".

PURPOSE AND SCOPE

It is the purpose of this paper to present the results of an experimental investigation of the relationship between flow and boundary conditions as they affect the location and magnitude of standing waves, and to correlate these findings with the analytical investigation of similar conditions based on Dr. Ippen's paper "Gas Wave Analogies in Open Channel Flow" (4). The investigation includes conditions of flow with Froude numbers ranging between two (2) and five (5) and wall angles varying from zero (0) to twenty-four (24) degrees.

INVESTIGATION OF STANDING WAVES IN SUPERCRITICAL FLOW OF WATER

ANALYTICAL CONSIDERATIONS

Analysis of standing waves in supercritical flow is based upon a consideration of momentum, continuity, and geometry of flow lines. As in the development of the relationship for depths in a hydraulic jump, some assumptions are made. These are listed herewith so that a clear understanding of the basis and limitations of the analysis may be had.

1. Uniform rectilinear flow (parallel flow lines).
2. Flow downstream of the wave is parallel to the transition walls.
3. Bottom is level.
4. Viscous effects are negligible.
5. Normal hydrostatic pressure distribution.

Assumptions three (3.) and four (4.) are complimentary, the bottom slope being adjusted to balance friction effects and insure uniform flow conditions. Based on these assumptions the relationships between flow and boundary conditions in a sharp angle transition as presented by Dr. Ippen (4) are (see Appendix A):

$$\sin \beta = \frac{1}{F_1}\sqrt{\frac{1}{2}\left(\frac{h_2}{h_1}\right)\left(1+\frac{h_2}{h_1}\right)} \qquad (1)$$

$$\tan\theta = \frac{\left(1-\frac{h_1}{h_2}\right)\tan\beta}{1+\frac{h_1}{h_2}\tan^2\beta} \qquad (2)$$

With given initial conditions of Froude number F_1, upstream depth h_1, transition angle θ, the wave angle β, and depth h_2 downstream of the wave front can be determined. By a solution of the simultaneous equations (1) and (2), the following is obtained:

From equation (1)

$$2F_1^2\sin^2\beta = \frac{h_2}{h_1} + \left(\frac{h_2}{h_1}\right)^2$$

$$\frac{h_2}{h_1} = \frac{\sqrt{1+8F_1^2\sin^2\beta}-1}{2}$$

$$\frac{h_1}{h_2} = \frac{2}{\sqrt{1+8F_1^2\sin^2\beta}-1} \qquad (3)$$

From equation (2)

$$\tan\theta + \frac{h_1}{h_2}\tan^2\beta\tan\theta = \tan\beta - \frac{h_1}{h_2}\tan\beta$$

$$\frac{h_1}{h_2} = \frac{\tan\beta - \tan\theta}{\tan\beta + \tan^2\beta\tan\theta} \qquad (4)$$

Equating equations (3) and (4) and solving for tan θ, the following relationship is obtained:

$$\tan\theta = \frac{\tan\beta\left(\sqrt{1+8F_1^2\sin^2\beta}-3\right)}{\sqrt{1+8F_1^2\sin^2\beta}+2\tan^2\beta-1} \qquad (5)$$

A graphical solution of this relationship for angles of θ between zero and twenty degrees and for Froude numbers between one (1) and five (5) is illustrated on Plate 5 of Appendix C. Thus, for any wall angle θ, β is determined. Then from equation (1), a graphical solution of which appears on Plate 6 of Appendix C, for known values of F_1 and β, the ratio of depths, $\frac{h_2}{h_1}$ is determined. Thus, for given boundary and flow conditions, the wave angle and depth of wave can be determined analytically.

DESCRIPTION OF APPARATUS

In an effort to secure a flume with the greatest versatility yet at minimum expense and one which could be constructed in the allotted time, a flume three feet wide, twelve feet long and nine inches high was designed of aluminum to be inserted in an existing flume with adjustable slope. At the head of the flume an adjustable sluice gate was constructed with maximum variation in head of 0.75 feet. This limitation was imposed by the height of wall of the existing upstream stilling pool. Four feet downstream from the sluice gate, a 40 inch section of adjustable wall was machine-fitted into the fixed aluminum wall on each side of the flume, providing a contracting transition with vertical walls capable of any angular adjustment between zero and eighteen degrees. Possible disturbing effects due to non-continuity at the origin of the wall angle were minimized by use of machine fitted joints. The original flume was equipped with steel rails on which a point gauge was mounted for measuring depths and surface profiles. A lucite panel 3X3½ feet, adjustable to any position over the transition, was provided to enable experimenters to readily determine wave angle with respect to wall angle by direct measurement.

The flow in the channel is maintained by two pumps rated at 600 and 1000 g.p.m., respectively. The water is pumped from a large sump into two constant head tanks. A six inch pipe from each of these tanks carries the water by gravity flow to the stilling basin of the apparatus from whence it flows through the sluice into the channel. One of the six inch pipe inlets leads to a calibrated weir for measuring the quantity of flow. The other six inch line is equipped with an orifice for the same purpose. The original installation consisted of the one six inch line to the weir. The maximum flow over the weir was too small to furnish the desired range of depths and Froude numbers in the channel, so the second six inch line with the orifice device was added. The orifice plate was machined with an inside diameter of 4.282 inches and inserted into the six inch pipe at a flange connection. Piezometer connections were tapped into the six inch pipe radially at the one-quarter points, six inches above the orifice plate, and three inches below the plate. Both the upper and lower group of piezometer taps were joined by external ring connectors. Each ring connector was provided with a bleeding valve and a pressure connection to a four foot water manometer. This installation connected to the constant head tank supplied by the 600 gpm. pump added about one (1.0) cfs to the discharge. The capacity of the orifice was considerably greater than this but the increase of one (1.0) was found to be adequate. The design and construction of the apparatus consumed a major part of the time allocated for thesis work: thus the apparatus was not exploited fully and has many possibilities for further investigations. Suggestions for minor modifications are set forth under a later paragraph covering recommendations.

PROCEDURE

The slope of the flume was set at maximum gradient of approximately 1.68 per cent, determined by level readings along the bed and along the rails. The sluice gate was then set with spacing blocks for an opening of 0.75 inches and sufficient discharge obtained to insure maximum head on the sluice opening, the discharge being measured with a calibrated 30 cm. weir. When steady conditions were attained, and with θ equal to zero, surface and bottom profile readings were taken by the point gauge (measuring to hundredths of a centimeter) at stations -0.3, 0.0, 0.4, 0.8, 1.2, 1.6, 2.4, 2.8, and 3.35 measured in feet in the direction of flow with the origin of the stationing at the apex of the transition angle. The direction of flow is designated the X direction and is positive in the direction of flow. The Y coordinates are transverse to the direction of flow and measured from an origin on the left side of the flume with respect to the direction of flow. Depth measurements were all vertical and accurate to one half a millimeter. The accuracy of depth measurements was established by the degree of accuracy possible in fixing the water surface with the point gauge. Thus the uniformity of flow was determined with the wall angle θ equal to zero and Froude numbers were established at each station.

Knowing the desired wall angle, θ, and the length of transition section, 40 inches, the wall angles were set by means of the established X and Y coordinate system. Three degree transition angles were set on each wall and transverse surface profiles taken at stations 0.4, 0.8, 1.2, 1.6, 2.0, and 2.4. The wave angle was measured directly by tracing the wave front on the lucite panel with a grease pencil and measuring the tangent of the angle so traced. Surface and bottom profiles were taken along each wall from station -0.3 to station 2.4. These measurements were taken as close as possible to the wall but were actually about 1/8 inch from the wall because of interference of the wall with the point gauge. Wall angles were then changed to: 6°, 9°, 12°, 15°, 18°, 21°, and 24° degrees, in turn, and the above profiles and measurements taken for each angle. A plot of profiles for Run #5 of wall angles 3°, 6°, 9°, 12°, and 15° is contained in the appendices.

The above procedure was then repeated for a similar set of wall angles but with successive increases of sluice gate opening to a maximum opening of two inches. With each increase in sluice gate opening there was a corresponding decrease in Froude number as the velocity, V_1, remained about constant and the depth, h_1, increased. It was then the intention to continue the measurements with identical sluice gate openings but reduced head on the sluice gate, thereby getting another series of lower Froude numbers but with depths h_1 the same as in the previous series. For the lower Froude numbers the slope of the flume could be reduced to insure essentially uniform flow conditions. Time did not permit completion of the runs with the decreased head on the sluice gate.

All profiles were then plotted in a manner similar to the plots in the appendices for Run #5 and the wave angle β determined by striking average depths h_1 and h_2 and locating a representative point on each wave profile by balancing the two areas contained between a vertical line, the actual wave profile and the average depths h_1 and h_2. This vertical line was then projected to a point on the plane of the bottom of the flume. The best straight line joining these projected points established the wave angle, β_a. The depth h_2 (for the same boundary and flow conditions) was determined by averaging the mean depths, $\overline{h_2}$, taken from the profiles at each station.

Thus a wave depth and a wave angle has been measured for known boundary conditions and comparisons of these measurements can be made with analytical solutions for the same boundary and flow conditions.

METHOD AND SCOPE OF INVESTIGATION

The great amount of time consumed in the construction of the apparatus left only a short period in which to obtain the data necessary to analyze the standing waves. It was therefore considered best to study completely the condition for one Froude number through the range of minimum to maximum wall angle. If time then allowed, runs could be made at other Froude numbers for comparison of results. A preliminary run was taken and profiles plotted. This preliminary run was analyzed to determine a graphical procedure for measuring the wall angle. The method finally adopted is the one described elsewhere in this paper and indicated on the profile points in the appendix. The main advantage to this method is in the consistency with which representative points on the wave can be located. Large variations in the personal error involved in arbitrarily locating the average h_1 and h_2 lines reflect only a very small difference in location of the vertical line balancing the triangle areas. Furthermore, the portion of the wave front immediately at the beginning of the wave is the most accurately measured portion of the wave. The velocities are high and the position of the point gauge is readily observed. At the crest of the wave, and back of the wave, the point gauge position is more difficult to observe accurately due to irregularities of the surface. Attempts to obtain an average h_2 (directly) by averaging the measurements introduced difficulties of determining just where, beyond the crest, the h_2 readings should be taken for computing the average depth. Hence, the balanced area method was adopted and proved to be very consistent, giving results within the range of normal personal errors. The effect of the transition wall upon the wave profile clearly indicated that several more cross sections than were taken on the preliminary run would be necessary and that these sections should be taken at regular intervals. Run #5 represents a complete run at average Froude number of 3.86 with wall angles from zero to twenty-four degrees. Above twenty-four degrees the wave pattern became quite turbulent and the position of the point gauge very indeterminate. Runs #2, #3, and #4 were not complete runs but enough data were taken for comparison of results.

FLOW CONDITIONS IN THE FLUME

In designing the flume, emphasis was placed upon obtaining smooth undisturbed flow above the transition. The sluice gate was carefully fitted and machined. The entrance to the flume above the sluice gate in the region of subcritical flow was provided with curved transitions. Along the flume walls bolt heads below the water line were counter sunk, soldered, and smoothed down to assure an even, continuous wall boundary. Without the use of the sluice gate, a very smooth, undisturbed flow could be obtained. Introducing the vertical sluice gate, however, caused a series of vortices which created small transverse irregularities in the water surface. The effect of these vortices are indicated by variations of depth measurements along the cross sections at zero wall angle. It is believed that these vortices might be partially eliminated by the use of a nozzle type of entrance where the transition from large to small depth is very gradual. Time was not available to permit investigation of the nozzle type entrance. Flow conditions in the flume could be further remedied by using longer transition walls, thus giving a greater portion of the wave front that is representative of normal conditions. Actually, the 40 inch length provided only a short distance where the condition might be said to be normal. Above this normal stretch the depth of flow is affected by the excessive vertical velocities. As the upstream flow reaches the wave front the proximity of the wall behind the wave causes nearly an instantaneous change in direction and depth. This change is accompanied by high vertical velocities and pressures varying appreciably from the normal hydrostatic pressures. The depth behind the wave in this area is therefore greater than at points further downstream where the wave front is an appreciable distance from the wall and the streamlines can expand gradually to the h_2 depth. This increased depth in the vicinity of the initial transition is clearly shown in the wall profiles in the appendix. Below this normal stretch, the effect of the sudden end of the transition introduces a sudden drop to atmospheric pressure along the wall which affects the depths to a minor degree at the lower Froude numbers for a short distance upstream. Another condition of the flow in this flume is the gradual increase in depth, longitudinally, thereby causing a different Froude number at each cross section. This condition results in a slightly curved wave front, β being greater for lower Froude numbers. Hence, the use of an average Froude number, average heights, and average wave angles. If the slope of the flume could have been adjusted to equal the energy gradient, this condition of changing Froude number would have been avoided, but the maximum slope of the flume was insufficient to maintain uniform flow at the higher Froude numbers.

WAVE PATTERN

At a wall angle of three degrees the wave pattern and the location of the wave front were barely discernable. At greater angles the wave front became more pronounced and the pattern more distinct. However, with increase in wall angle, the wave distortion at the beginning of the transition increased until at the larger wall angles (18 to 24 degrees) there was a definite shooting up of the water to an exaggerated height as it hit the wall at the beginning of the transition. The water then appeared to curl over and fall along the wave front causing a roller similar to that of a hydraulic jump. The wall profiles indicate that the greatest depth occurs at the beginning of the transition and diminishes gradually to a constant value. This can be verified by the fact that the high velocity flow shooting against the wall will tend to rise higher on the wall than it would when shooting into the wave front where a gradual expansion of the streamlines can take place. As the high velocity flow approaches the wave front, the flow lines at high velocity gradually curve into the flow line pattern at the greater depth and lower velocity in back of the wave front. The "hump" or crest to the wave shown by the profiles indicates non-uniformity of pressure and velocity distribution in the vicinity of the wave. This deviation from the theoretical is exaggerated at the beginning of the transition where the horizontal flow lines near the wall are deflected almost vertically, the greater portion of the velocity head being transformed into static head. The greater departure from the theoretical conditions at the larger wall angles is in part due to the departure from the assumed of the pressure and velocity distributions. The change of Froude number resulting from the change of velocity from high to low above and below the wave front increases with increased wall angle. The Froude number below the wave front approaches one (1.0) and the specific energy the minimum. As the mimimum specific energy is approached, large deviations in depth result from small variations in velocity causing a distorted water surface.

WAVE ANGLE COMPARISON

The graphical determination of the wave angle by the balanced area method gave results which showed a consistent trend for each of the four different Froude numbers when compared to the β_o computed for the same boundary and flow conditions by equation (5). Identically the same trend was indicated by the values obtained by direct measurement of the wave angle. For the smallest wall angle (three degrees) the difference between the theoretical β_o and the β_a measured from the profiles is a maximum, the measured β_a being smaller than the computed β_o. As the angle is increased, the difference decreases until at some point between a wall angle of twelve and fifteen degrees the measured β_a coincides with the theoretical β_o. As the wall angle is further increased, the difference again becomes greater, only now the measured β_a gives larger values than the theoretical β_o. A plot of the ratio of measured to theoretical β is shown on Plate 2. A tabulation of this

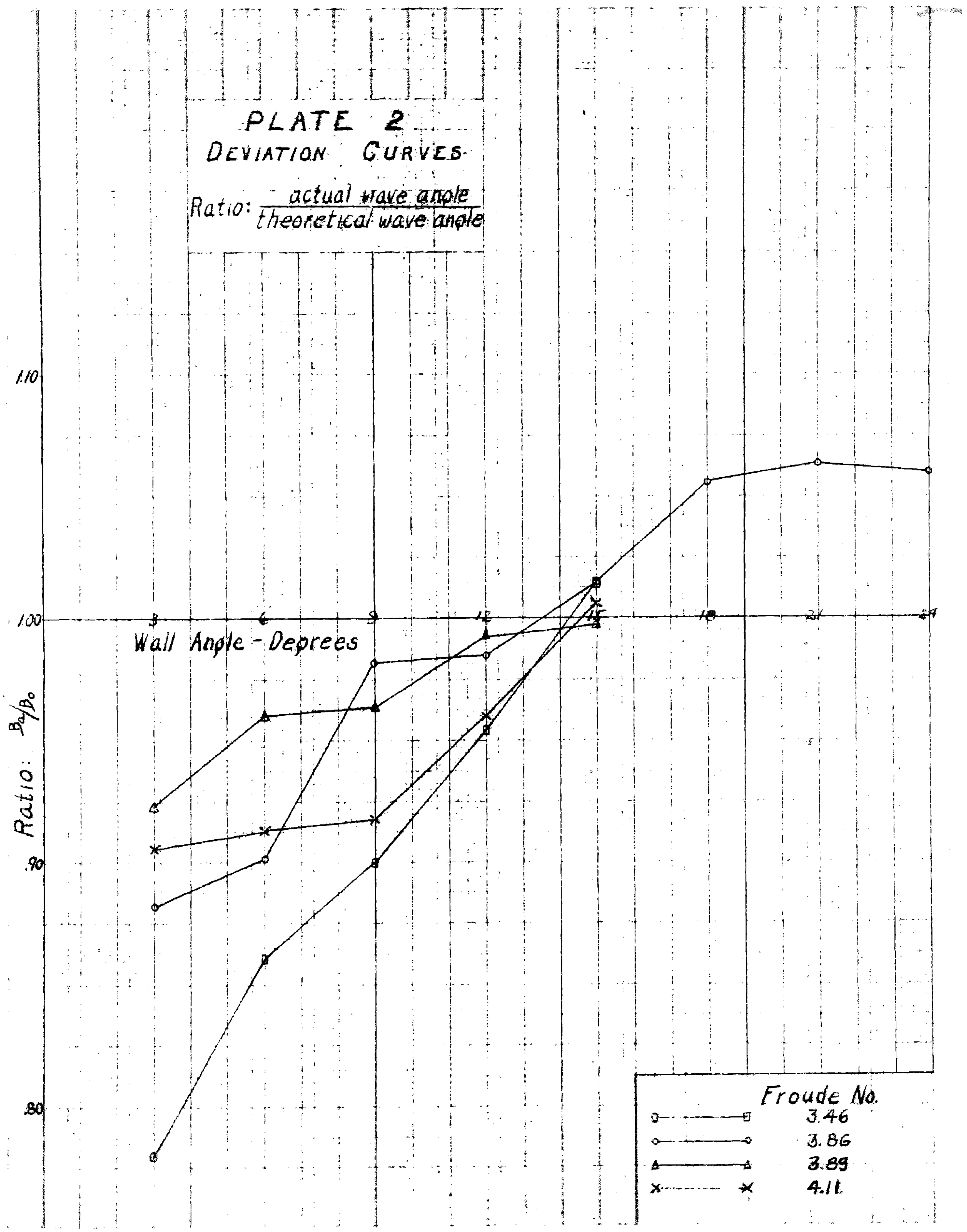


PLATE 2
DEVIATION CURVES
Ratio: actual wave angle / theoretical wave angle
1.10
1.00
.90
.80
3
6
9
12
15
18
21
24
Wall Angle - Degrees
Ratio: B_a/B_o
Froude No.
3.46
3.86
3.89
4.11

comparison is given in Appendix C. Since the scope of this investigation was limited to the one complete run at a Froude number of 3.86, a comparison of the β differences for the same wall angle at different Froude numbers is not possible. The runs which were made with different Froude numbers were incomplete and the Froude number variation insufficient to support any general statement of a comparative nature. Whether or not the ratio of measured to computed wave angle at any one wall angle approaches unity with increasing Froude numbers, as indicated by these results, might well be a subject for further investigation. However, the trend at any one Froude number with increase of wall angle is very definite as indicated in Plate 2.

COMPARISON OF h_2 / h_1 RATIO

Plate 3 illustrates the comparison between the h_2 / h_1 ratio as computed by equation (1) using the theoretical β computed from equation (5), and the h_2 / h_1 ratio as measured. The agreement is well within the range of accuracy of point gauge measurements. A list of the measured and calculated values of h_2 / h_1 is included in Appendix C as Table III.

CORRECTION FACTOR

The above analysis of the results leads to the possibility of applying a factor to equation (1) in

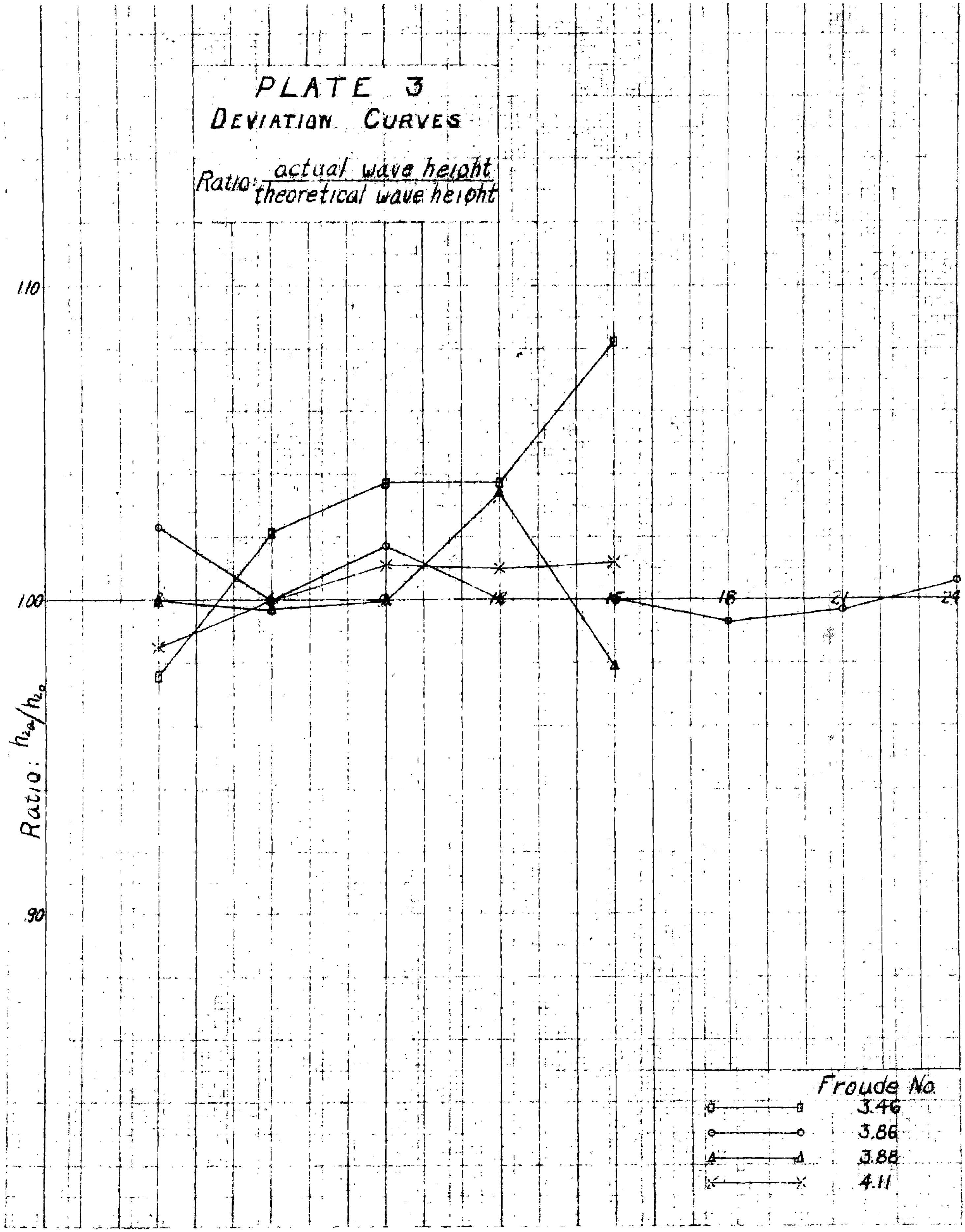


PLATE 3
DEVIATION CURVES
Ratio: actual wave height / theoretical wave height
110
100
.90
Ratio: h_{2a}/h_{2t}
18
21
24
Froude No.
3.46
3.86
3.88
4.11

the following form:

$$\sin\beta = \frac{K}{F_1}\sqrt{\frac{1}{2}\left(\frac{h_2}{h_1}\right)\left(1+\frac{h_2}{h_1}\right)}$$

Since the comparison of the h_2 / h_1 ratios are in close agreement for all values of wall angle, the factor need only to be applied to the value of $\sin\beta$ as shown above. The factor K thus becomes the ratio of the actual $\sin\beta_a$ to the theoretical $\sin\beta_o$. Values of K can be obtained for each of the wall angles used in this investigation. By plotting these values of K against the wall angles, a curve is obtained from which a value of K can be obtained for any selected wall angle. The value of K for the range of Froude numbers used in this investigation varies from approximately 0.85 at a wall angle of three degrees to 1.05 at a wall angle of twenty-one degrees, being unity at a wall angle between twelve and fifteen degrees. Application of this factor enables β determinations that are within zero to one degree of accuracy when the ratio of h_2 / h_1 is given. The K curve is shown on Plate 4.

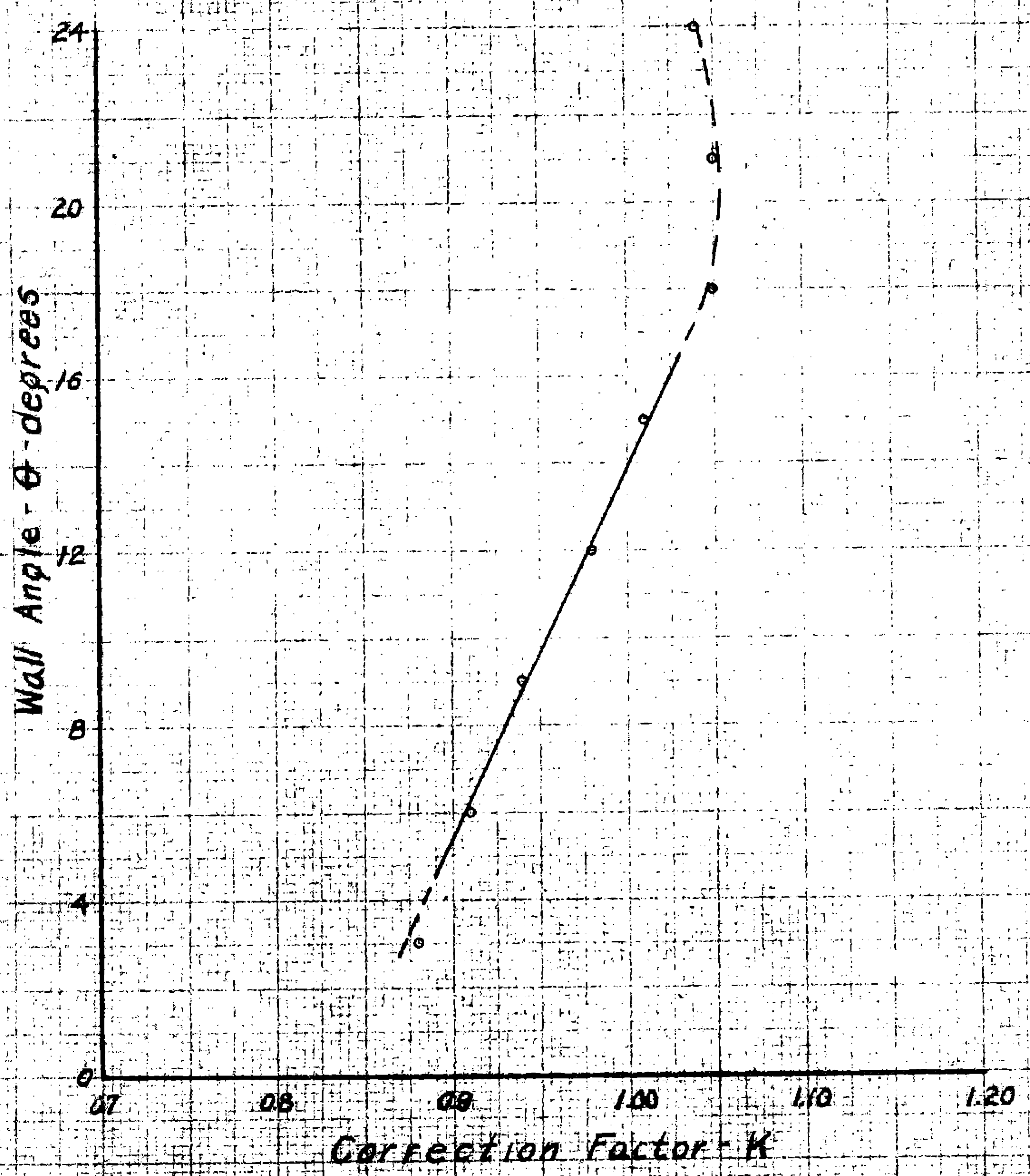
CONCLUSIONS

1. For the small range of Froude numbers used in this investigation, the value of the wave angle as computed from the theoretical equation compares very well with the measured wave angle when the wall angle is between twelve and fifteen degrees. For wall angles below six degrees and above eighteen degrees, the difference

PLATE 4

CORRECTION FACTOR CURVE

$$\sin\theta = \frac{K}{F}\sqrt{\frac{1}{2}\frac{h_2}{h_1}\left(1+\frac{h_2}{h_1}\right)}$$



between the theoretical wave angle and the actual wave angle becomes significant.

2. The agreement between the actual and theoretical h_2 / h_1 ratio is within the accuracy of point gauge measurements and personal error.

3. Introduction of a factor, K, into equation (1) enables theoretical wave angle determinations that are within zero to one degree of accuracy. The factor K can be determined for any range of Froude numbers by experiment similar to that performed in this investigation.

RECOMMENDATIONS

The results of this investigation indicate, for Froude numbers between 3.46 and 4.11, excellent agreement between measured and computed depth ratios and an agreement between measured and computed wave angles that varies consistently with wall angle variations. Indications are that the wave angle agreement improves with higher Froude numbers. This trend towards better agreement with higher Froude numbers should be investigated further such that a definite relationship between the ratio of measured to computed wave angle, Froude number, and wall angle can be ascertained. This further investigation can be carried on in the flume constructed for this investigation by increasing the available slope on the flume to insure uniform flow conditions and by increasing the available head on the sluice gate.

Should additional investigations be undertaken, transition angles should be set on but one wall at a time with the other wall angle equal to zero, thereby obtaining a more extensive wave area free from disturbances caused by opposite wall. It is further recommended that the transition section be lengthened from the present 40 inches to the maximum allowed by the flume width. A 60 inch transition could effectively be used for wall angles up to 18 degrees and it could then be replaced by a shorter section for larger wall angles. The longer transition section will produce a wave with a greater extent free from the high vertical velocities at the origin of the transition, thereby making possible a greater number of more accurate profiles.

APPENDICES

APPENDIX A

SUMMARY OF PAPER ENTITLED "GAS WAVE ANALOGIES IN OPEN CHANNEL FLOW" Presented by Dr. Arthur T. Ippen at Second Hydraulic Conference, Iowa University, 1942. (4)

In recent years high velocities of flow have been employed for the efficient handling of large discharges in channels with a free surface. The performance of many of these high velocity structures did not come up to expectations because of surprising standing wave patterns which affected flow in curved sections and in channel contractions and expansions. Some research had been done in the field of "shooting" flow and the purpose of this paper was to summarize the results of the theoretical and experimental investigations. First the paper presents and analogy between supersonic flow of gases and supercritical flow of water and then discusses standing waves in hydraulics. This discussion will be summarized here in detail.

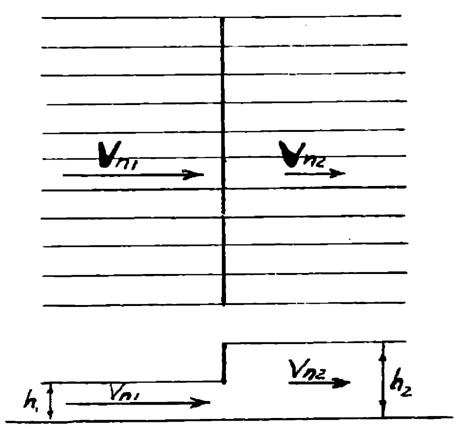


Figure 2

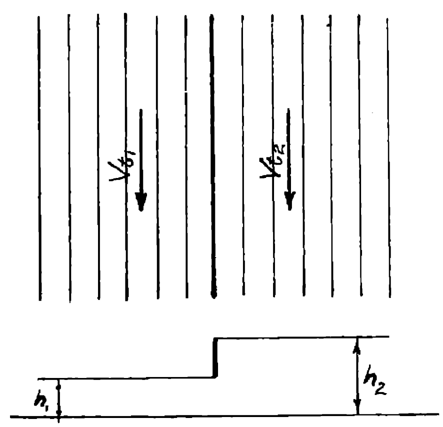


Figure 3

Figure 2 above represents the plan and elevation of a standing jump at right angles to the flow, the jump front being represented by a single line. Velocities are V_{n_1} and V_{n_2} and corresponding depths h_1 and h_2. With the jump stationary, V_{n_1} is also the velocity of the jump or wave front.

Figure 3 represents two fields of flow with parallel stream lines, equal velocities V_{t_1} and V_{t_2} but different depths h_1 and h_2 with no flow across the front.

Superimposing the two flow conditions the velocities V_1 and V_2 are obtained composed of components normal and parallel to the wave front V_{n_1} and V_{n_2}, V_{t_1} and V_{t_2}. This superposition is indicated in Figure 4.

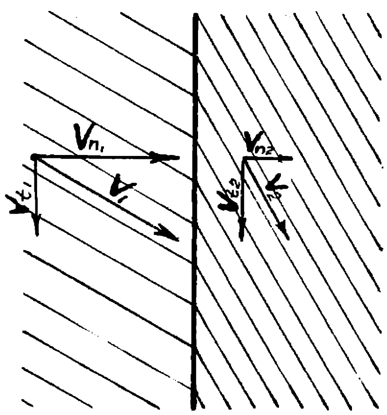


Figure 4

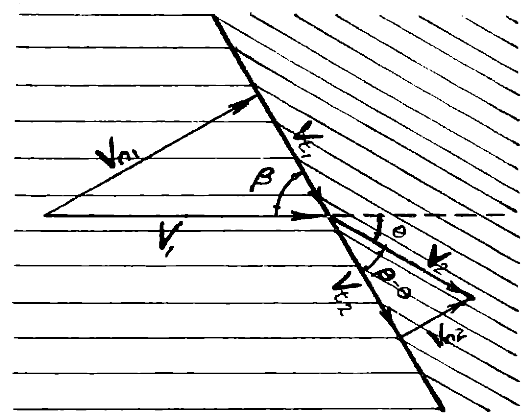


Figure 5

Figure 5 reorients conditions shown in Figure 4 (See also Figure 1). The fact that V_{n_1} changes to V_{n_2} while V_{t_1} and V_{t_2} remain equal means in effect a change in direction by an angle θ for the velocity of flow from V_1 to V_2.

From a consideration of continuity:

$$h_1 V_{n_1} = h_2 V_{n_2} \quad \text{or} \quad h_1 V_1 \sin\beta = h_2 V_2 \sin(\beta-\theta) \qquad (8)$$

From a momentum consideration:

$$\frac{\gamma b h_2^2}{2} - \frac{\gamma b h_1^2}{2} = \frac{\gamma b h_1 V_{n_1} t}{g} \cdot \frac{V_{n_1} - V_{n_2}}{t}$$

$$\frac{h_2 - h_1}{2h_1} = \frac{V_{n_1}(V_{n_1} - V_{n_2})}{g} \qquad (9)$$

Solving (8) and (9) simultaneously for V

$$V_{n_1} = V_1 \sin\beta = C = \sqrt{\frac{g(h_2^2 - h_1^2)(h_1)}{2h_1(h_2 - h_1)}}$$

$$\sin\beta = \frac{1}{F_1}\sqrt{\frac{1}{2}\left(\frac{h_2}{h_1}\right)\left(1 - \frac{h_2}{h_1}\right)} \qquad (11)$$

Thus we have an expression for β in terms of h_1, h_2, and V_1. But h_2 is dependent on the change in direction of flow, θ, thus from the geometry of Figure 5

$$V_{t_1} = V_{t_2} = \frac{V_{n_1}}{\tan\beta} = \frac{V_{n_2}}{\tan(\beta - \theta)}$$

and from continuity

$$\frac{h_1}{h_2}\tan\beta = \frac{\tan\beta - \tan\theta}{1 + \tan\beta\tan\theta}$$

$$\frac{h_1}{h_2}\tan\beta + \frac{h_1}{h_2}\tan^2\beta\tan\theta + \tan\theta = \tan\beta$$

$$\tan\theta = \frac{\tan\beta\left(1 - \frac{h_1}{h_2}\right)}{1 + \frac{h_1}{h_2}\tan^2\beta} \qquad (13)$$

The paper goes on to discuss "Wave Fronts of Small Height--Assumption of Constant Energy", the "'Method of Characteristics' for Supercritical Flow with Constant Energy", and "the 'Method of Shock Polars' for Supercritical Flow with High Wave Fronts and Energy Dissipation". These subjects are not considered in this thesis and are not included in this summary.

APPENDIX B

SAMPLE DATA SHEET

Run # 5

Wall angle $\theta = 12^\circ$
Sluice opening = 1.25"

Date: May 2, 1947
Recorder: Woodbury

Hook gauge reading	46.78 cm	Orifice h	
Hook gauge zero	31.44 cm	h	____
Head on weir	15.34 cm	Head on orifice	Zero
Weir discharge	1.3426 cfs.	Orifice discharge	None

$\tan \beta_b = \frac{1.30}{2.62}$ $\beta_b = 26^\circ\ 25'$

X Station	0.40			X Station	0.40		
Y	Surface	Bottom	Depth	Y	Surface	Bottom	Depth
.83	Wall			2.00	4.96	2.72	2.24
.108	7.32	2.65	4.67	2.50	4.93	2.68	2.25
.12	7.25	2.65	4.60	2.60	4.94	2.67	2.27
.14	6.99	2.64	4.35	2.70	5.09	2.66	2.43
.16	6.54	2.64	3.90	2.74	5.15	2.66	2.49
.19	5.54	2.64	3.90	2.78	5.37	2.66	2.71
.22	5.27	2.64	2.63	2.80	5.44	2.66	2.78
.26	5.10	2.65	2.45	2.82	5.74	2.66	3.08
.30	4.96	2.65	2.31	2.84	6.26	2.66	3.60
.40	4.88	2.65	2.23	2.86	6.79	2.66	4.13
.50	4.88	2.66	2.22	2.88	7.20	2.66	4.54
1.00	4.97	2.69	2.28	2.897	7.35	2.66	4.69
1.50	4.90	2.72	2.18	2.923	Wall		

x measured in feet y measured in feet

depth measurements in centimeters

APPENDIX C

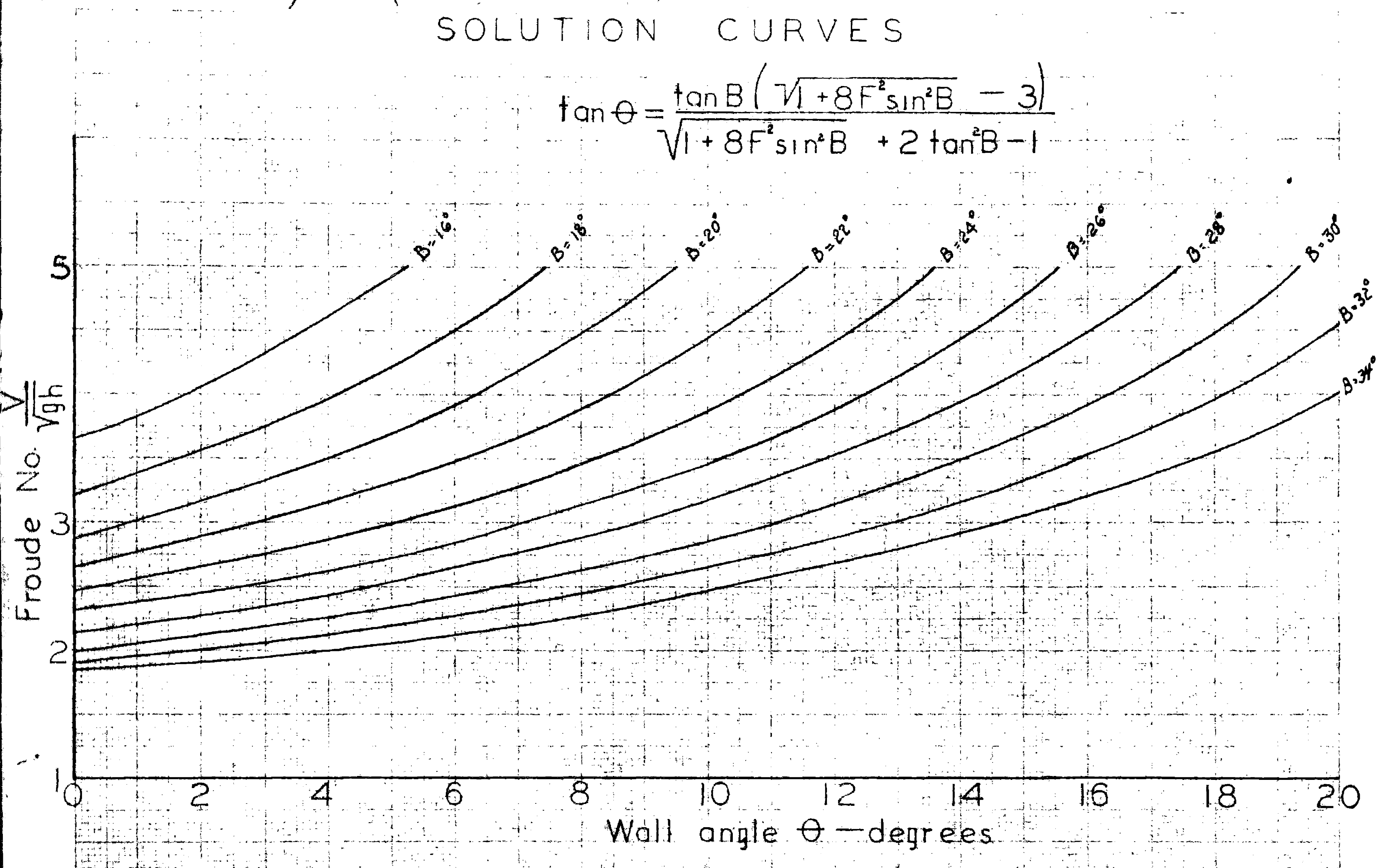

SOLUTION CURVES
$\tan\theta = \dfrac{\tan B\left(\sqrt{1+8F^2\sin^2 B} - 3\right)}{\sqrt{1+8F^2\sin^2 B} + 2\tan^2 B - 1}$
Froude No. $\frac{V}{\sqrt{gh}}$
1
2
3
5
0
2
4
6
8
10
12
14
16
18
20
Wall angle θ — degrees
B=16°
B=18°
B=20°
B=22°
B=24°
B=26°
B=28°
B=30°
B=32°
B=34°

Plate 5

TABLE I

Computations of θ using equation (5)

(Slide rule accuracy)

	Froude Numbers						
β	2.0	2.5	3.0	3.5	4.0	4.5	5.0
16°					1° 50'	3° 47'	5° 15'
18°				1° 41'	4° 8'	6° 00'	7° 26'
20°			0° 56'	3° 58'	6° 20'		9° 32'
22°			2° 52'	6° 7'	8° 27'		11° 34'
24°		0° 28'	5° 3'	8° 12'	10° 31'		13° 36'
26°		2° 36'	7° 5'	10° 12'	12° 28'		15° 35'
28°		4° 40'	8° 52'	11° 45'	14° 25'		17° 27'
30°	0° 00'	6° 38'	11° 2'	14° 5'	16° 18'		19° 22'
32°	1° 55'	8° 29'	12° 52'	15° 53'	18° 17'	19° 53'	21° 4'
34°	3° 47'	10° 19'	14° 40'	17° 45'	19° 58'	21° 42'	23° 5'
36°				19° 30'	21° 48'	23° 29'	24° 52'
38°				21° 13'	23° 29'	25° 8'	26° 38'
40°				22° 37'	25° 9'	26° 57'	

SOLUTION CURVES

$$\sin B_1 = \frac{1}{F_1}\sqrt{\frac{1}{2}\,\frac{h_2}{h_1}\left(1+\frac{h_2}{h_1}\right)}$$

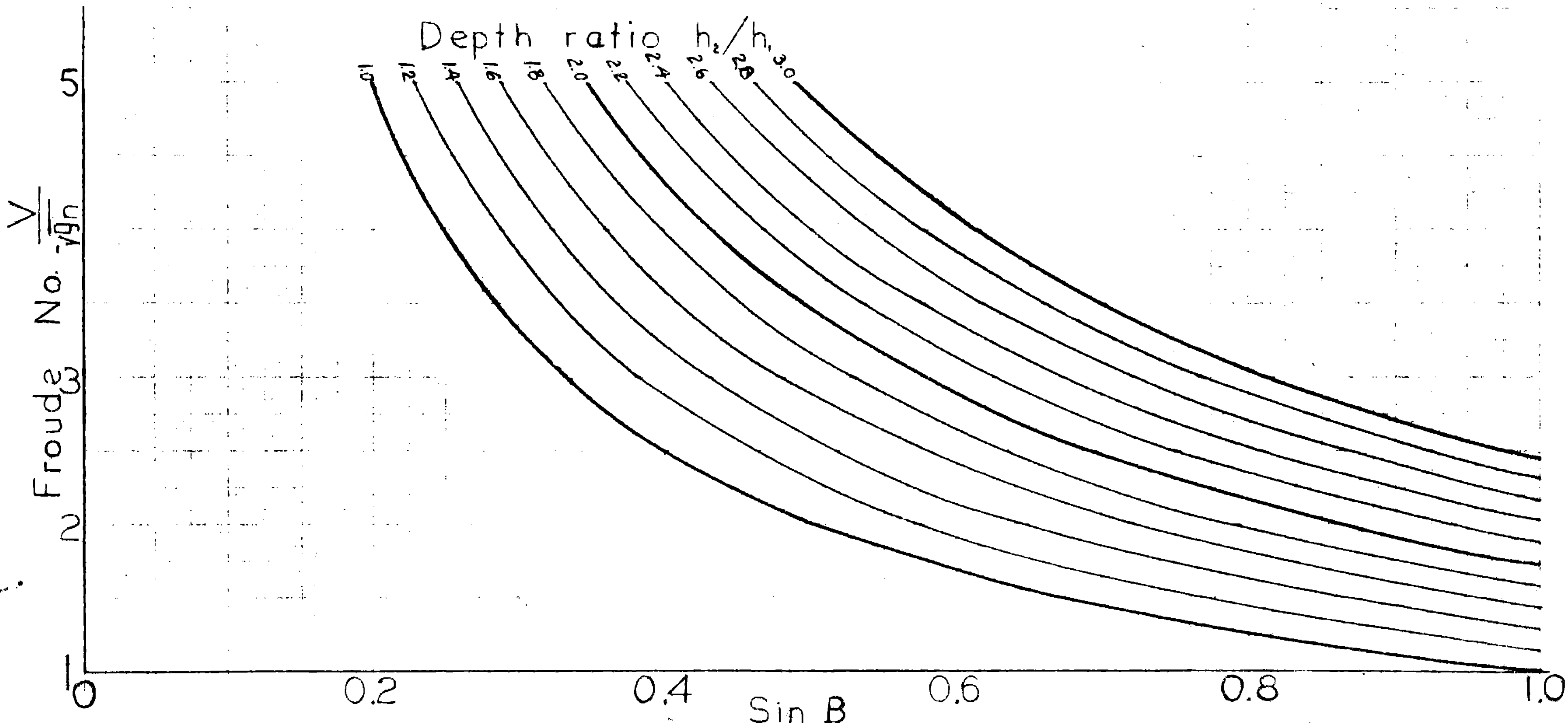


Plate 6

TABLE II
A comparison of measured and computed wave angles

F_1	θ	β_o	β_a	β_b	$\beta_a - \beta_o$	$\frac{\Delta\beta}{\beta_o}$
3.46	3°	19° 20'	15° 10'	11° 48'	-4° 10'	-21.6
	6°	22° 4'	19° 00'	17° 37'	-3° 4'	-13.9
	9°	25° 00'	22° 30'	22° 27'	-2° 30'	-10.0
	12°	28° 25'	27° 7'	27° 16'	-1° 18'	- 4.6
	15°	30° 50'	31° 15'	32° 2'	0° 25'	1.4
3.86	3°	17° 35'	15° 30'	14° 1'	-2° 5'	-11.8
	6°	20° 15'	18° 15'	17° 28'	-2° 00'	- 9.9
	9°	23° 10'	22° 45'	21° 39'	-0° 25'	- 1.8
	12°	26° 30'	26° 7'	26° 28'	-0° 23'	- 1.5
	15°	29° 30'	29° 55'	30° 23'	-0° 25'	1.4
	18°	32° 25'	34° 15'	34° 16'	1° 50'	5.6
	21°	35° 45'	38° 00'	39° 19'	2° 15'	6.3
	24°	39° 18'	41° 20'	44° 13'	2° 2'	5.17
3.89	3°	17° 30'	16° 20'	12° 27'	-1° 10'	- 6.70
	6°	20° 7'	19° 20'	17° 3'	-0° 47'	- 3.89
	9°	23° 00'	22° 10'	21° 52'	-0° 50'	- 3.62
	12°	26° 00'	25° 50'	25° 51'	-0° 10'	- 0.64
	15°	29° 10'	29° 5'	30° 6'	-0° 5'	- 0.28
4.11	3°	16° 45'	15° 10'	10° 38'	-1° 35'	- 9.45
	6°	19° 20'	17° 40'	16° 20'	-1° 40'	- 8.62
	9°	22° 10'	20° 20'	20° 8'	-1° 50'	- 8.24
	12°	25° 10'	24° 10'	23° 52'	-1° 00'	- 3.97
	15°	28° 20'	28° 30'	28° 12'	0° 10'	0.59

F_1 - Froude number
θ - Wall angle
β_o - Computed wave angle

β_a - Wave angle measured from profile
β_b - Wave angle by direct measurement with lucite panel

TABLE III
A comparison of measured and computed depth ratios

F_1	θ	$\sin \beta_0$	$\left(\frac{h_2}{h_1}\right)_0$	$\bar{h}_{1a}$	$\bar{h}_{2a}$	$\left(\frac{h_2}{h_1}\right)_a$	$\frac{h_{2a}}{h_{2o}}$
3.46	3°	.3311	1.20	3.28	3.89	1.17	.976
	6°	.3757	1.41		4.74	1.44	1.021
	9°	.4226	1.63		5.56	1.69	1.037
	12°	.4759	1.89		6.45	1.96	1.037
	15°	.5125	2.05		7.28	2.22	1.082
3.86	3°	.3021	1.21	2.28	2.84	1.24	1.023
	6°	.3461	1.47		3.36	1.47	1.000
	9°	.3934	1.70		3.95	1.73	1.017
	12°	.4462	2.00		4.56	2.00	1.000
	15°	.4924	2.22		5.08	2.22	1.000
	18°	.5361	2.49		5.64	2.47	.993
	21°	.5843	2.72		6.18	2.71	.997
	24°	.6334	2.99		6.88	3.01	1.006
3.89	3°	.3007	1.22	2.25	2.76	1.22	1.000
	6°	.3439	1.47		3.30	1.46	.997
	9°	.3907	1.71		3.84	1.71	1.000
	12°	.4384	1.97		4.60	2.04	1.034
	15°	.4874	2.23		5.15	2.28	.979
4.11	3°	.2882	1.24	1.55	1.89	1.22	.985
	6°	.3311	1.50		2.33	1.50	1.000
	9°	.3773	1.75		2.75	1.77	1.011
	12°	.4253	2.01		3.15	2.03	1.010
	15°	.4746	2.30		3.61	2.33	1.012

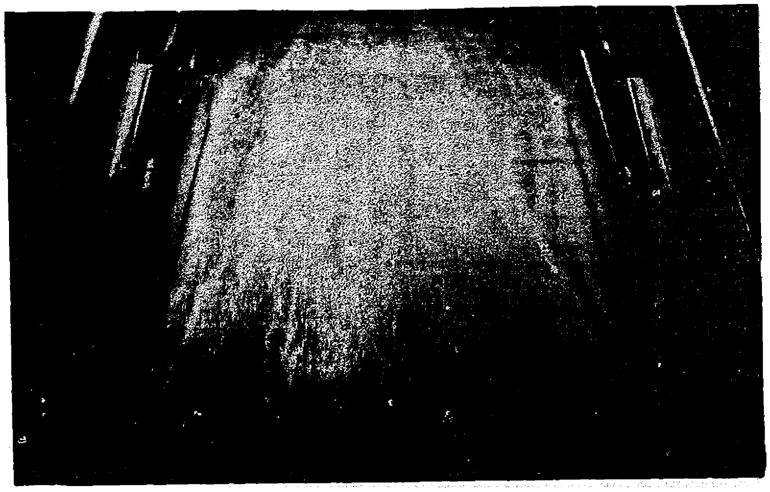
$\left(\frac{h_2}{h_1}\right)_0$ - Computed depth ratio $\left(\frac{h_2}{h_1}\right)_a$ - Measured depth ratio

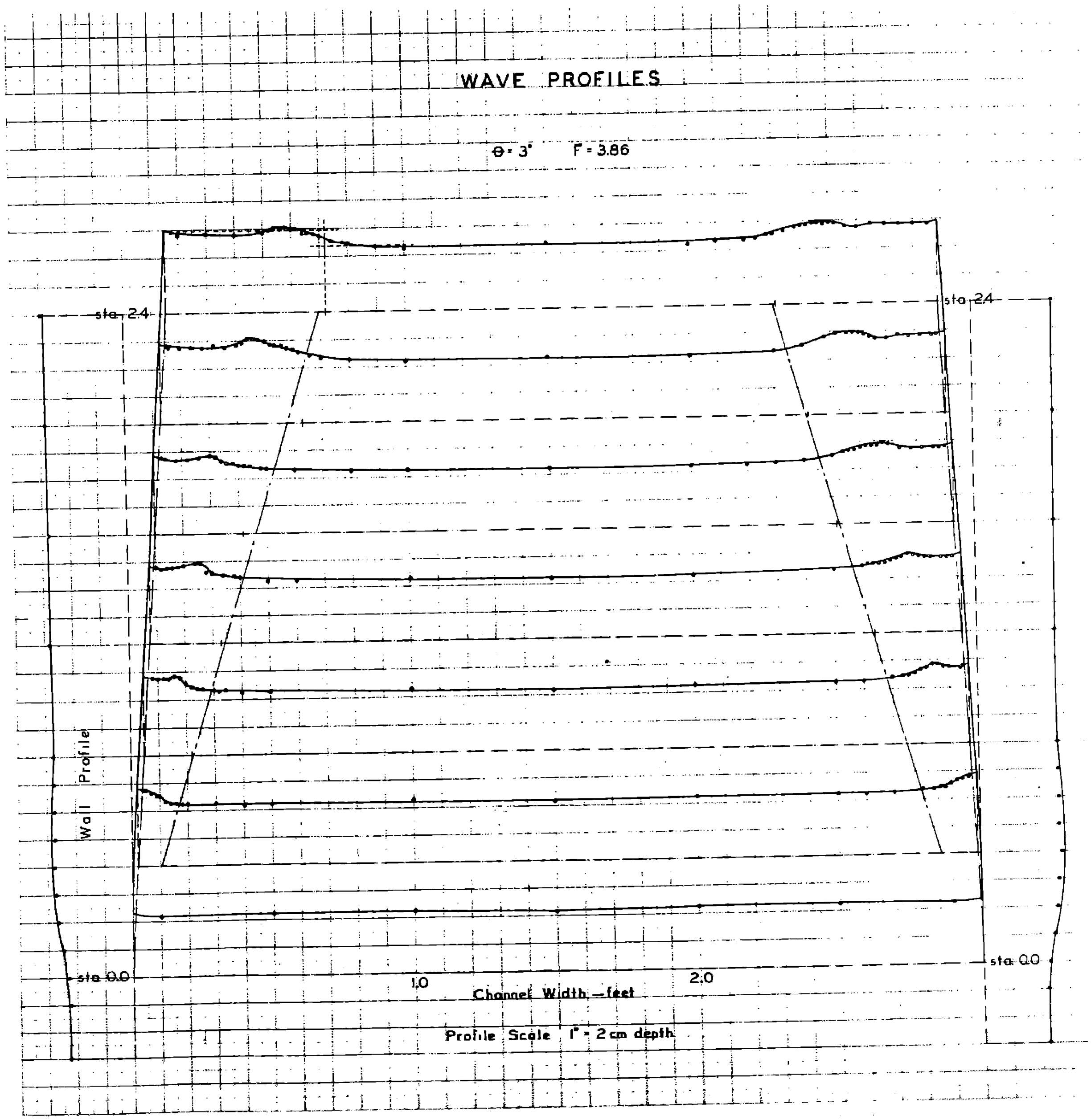
$\frac{h_{2a}}{h_{2o}}$ - Ratio of measured to computed depth ratios

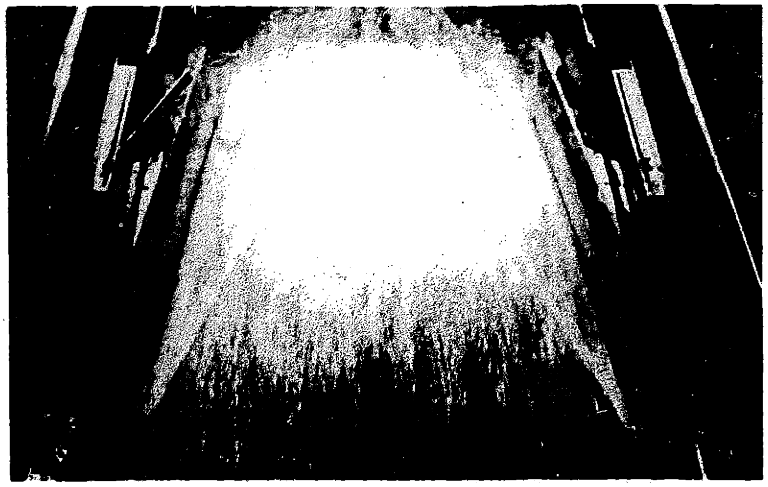
APPENDIX D

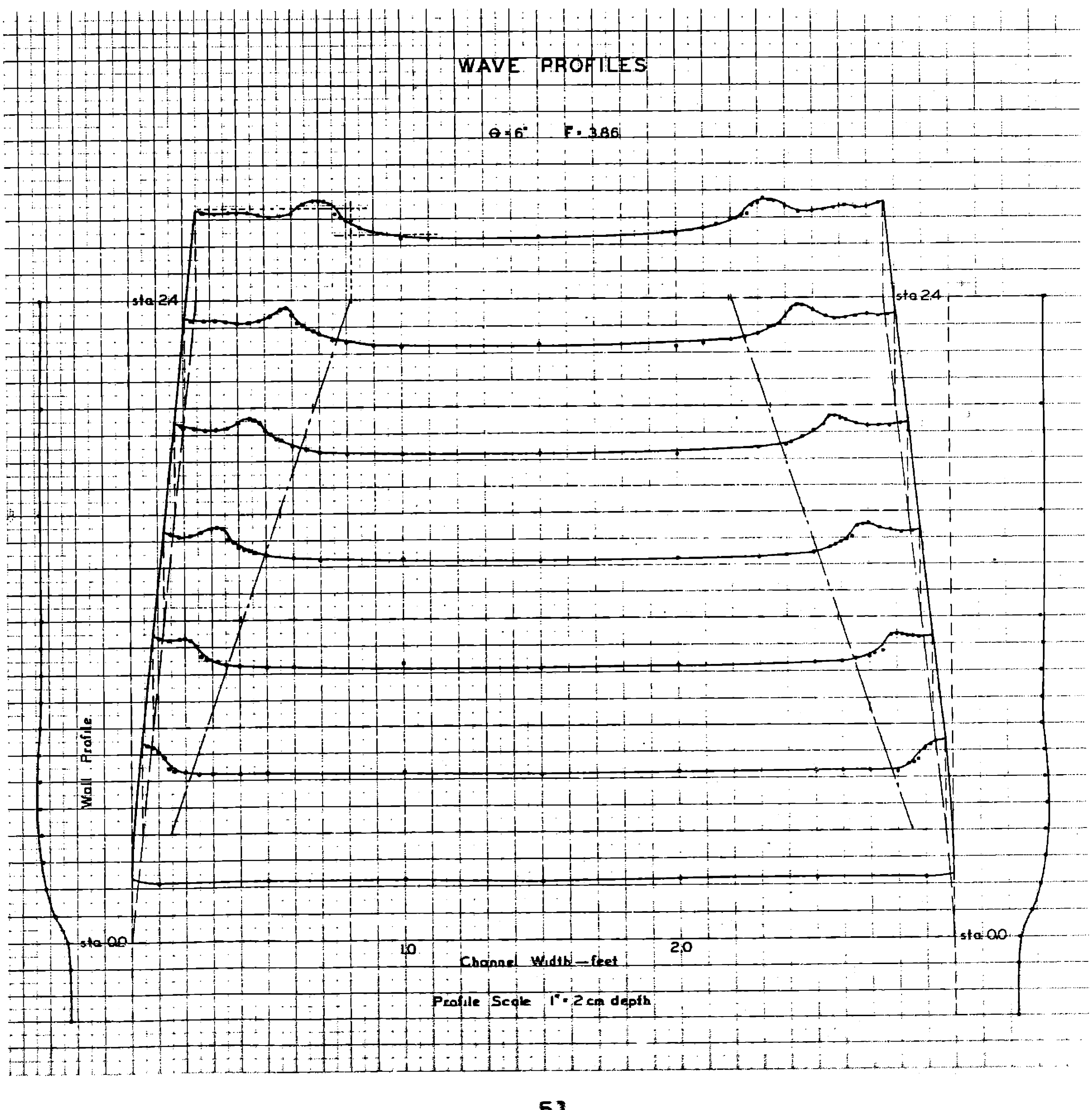
Surface profiles

Photographs

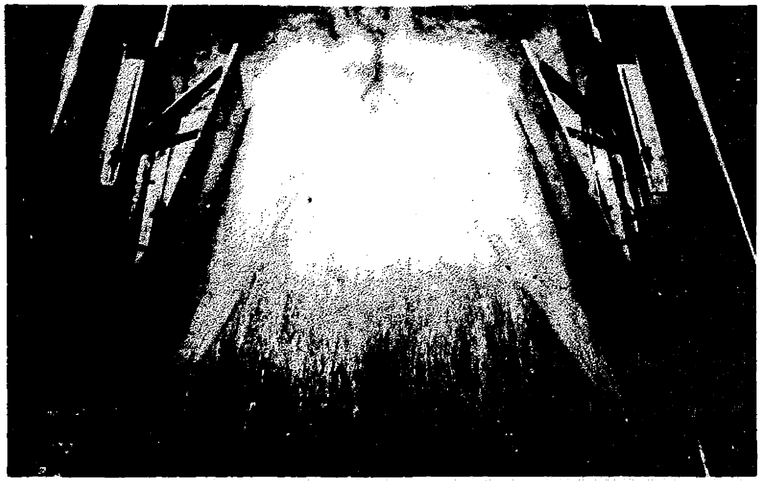


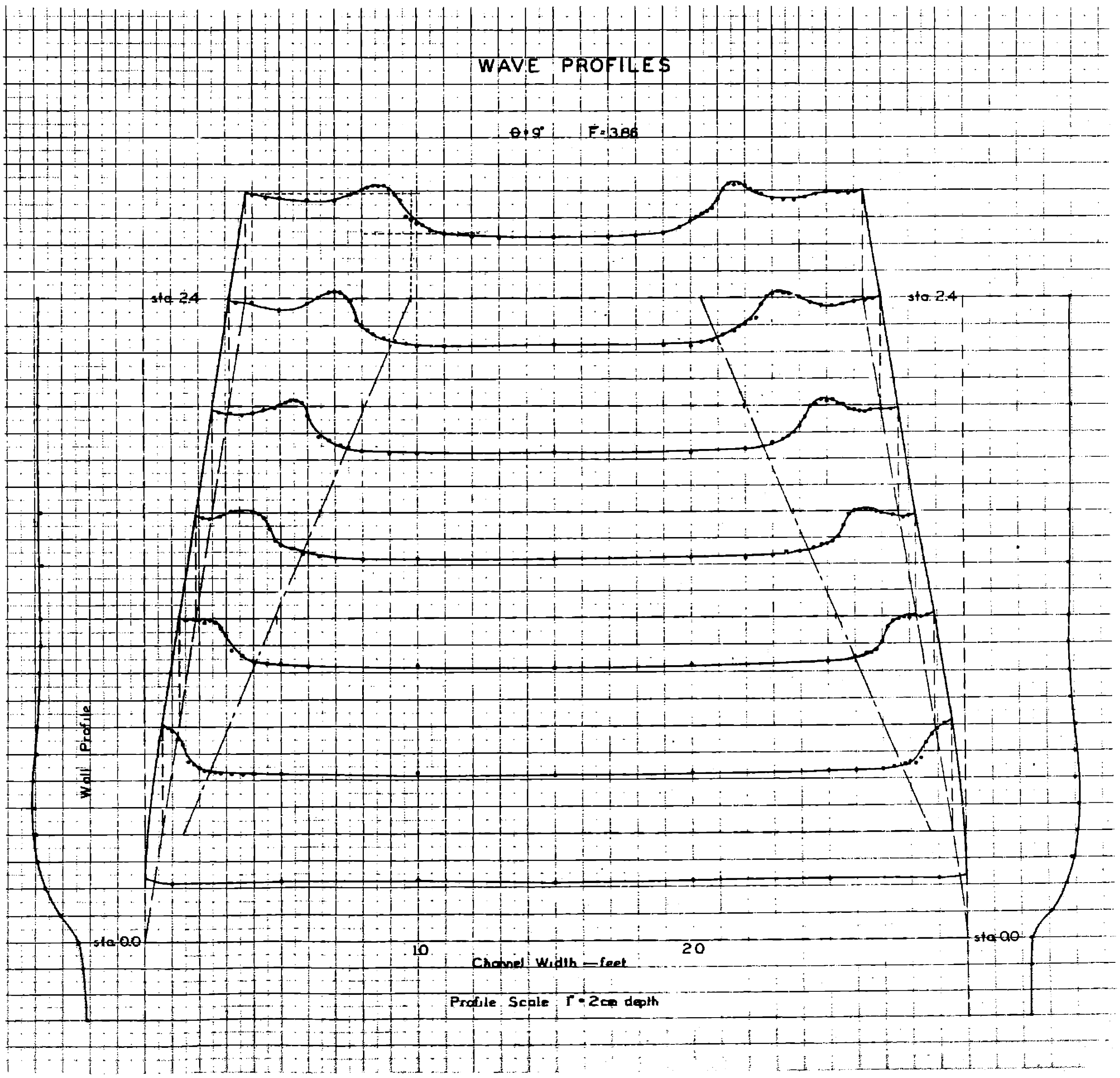

WAVE PROFILES
θ = 3°   F = 3.86
sta 2.4
sta 2.4
Wall Profile
sta 0.0
sta 0.0
1.0
2.0
Channel Width — feet
Profile Scale 1" = 2 cm depth

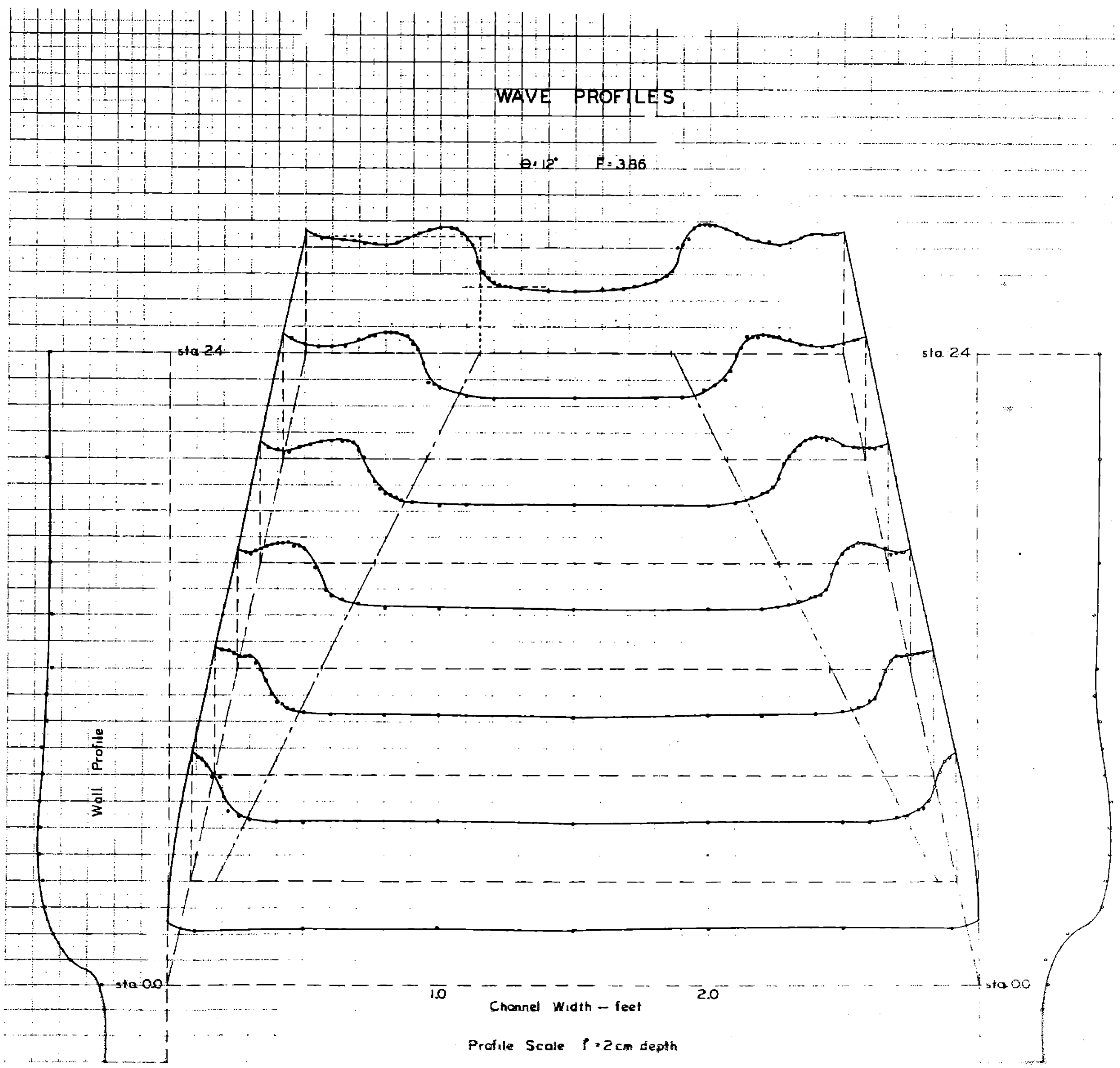




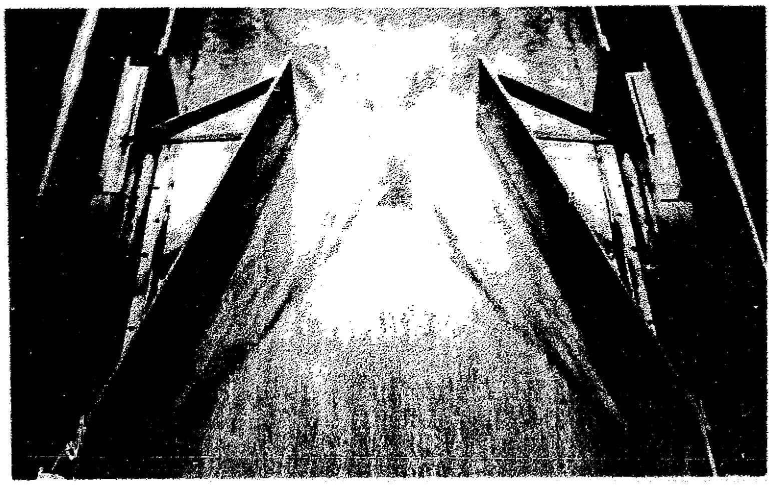
WAVE PROFILES
θ = 6° F = 386
sta 24
sta 24
Wall Profile
sta 0.0
sta 0.0
1.0
2.0
Channel Width — feet
Profile Scale 1" = 2 cm depth

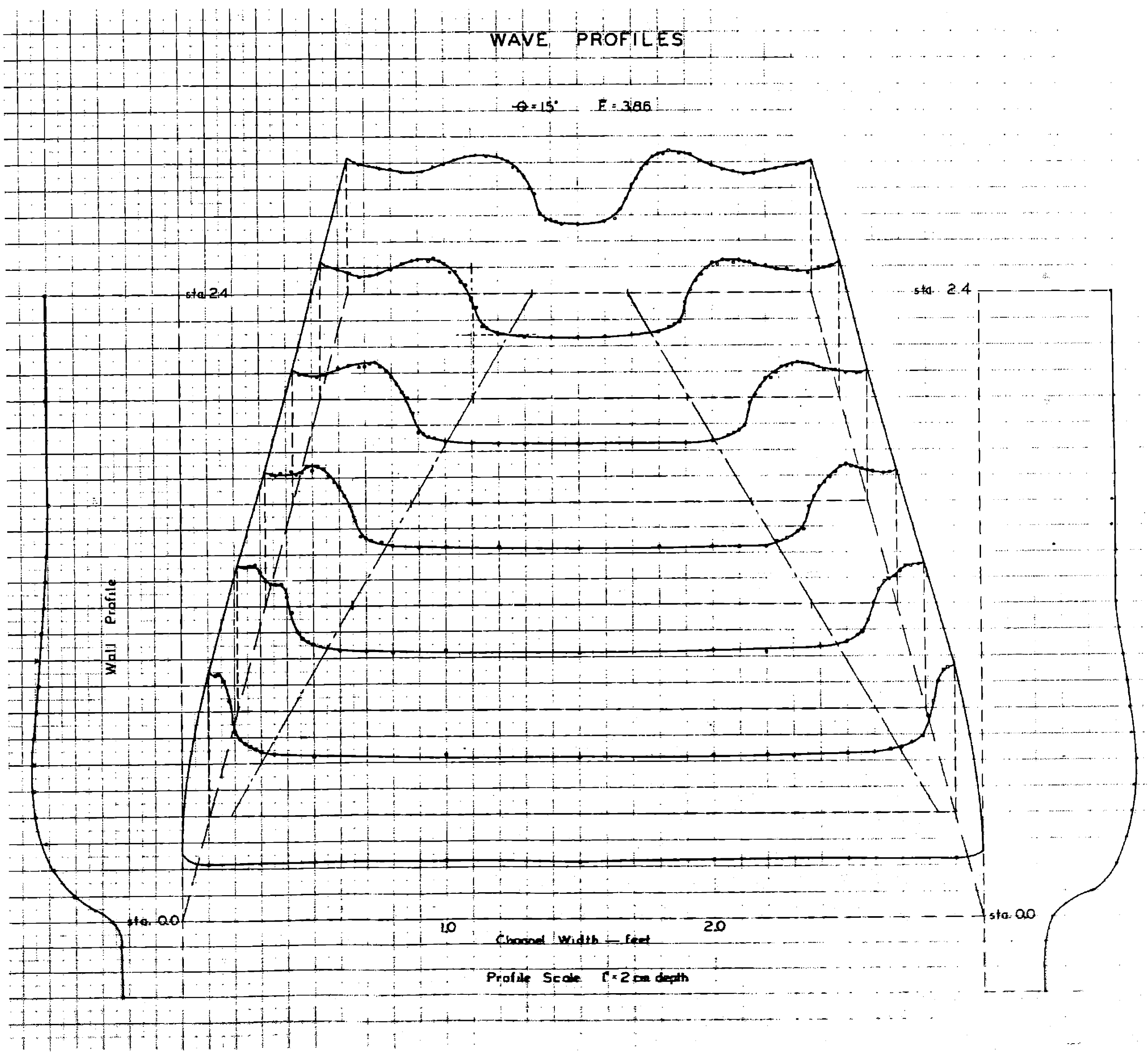



WAVE PROFILES
θ = 9° F = 3.86
sta 2.4
sta 2.4
Wall Profile
sta 0.0
sta 0.0
10
20
Channel Width — feet
Profile Scale 1" = 2cm depth

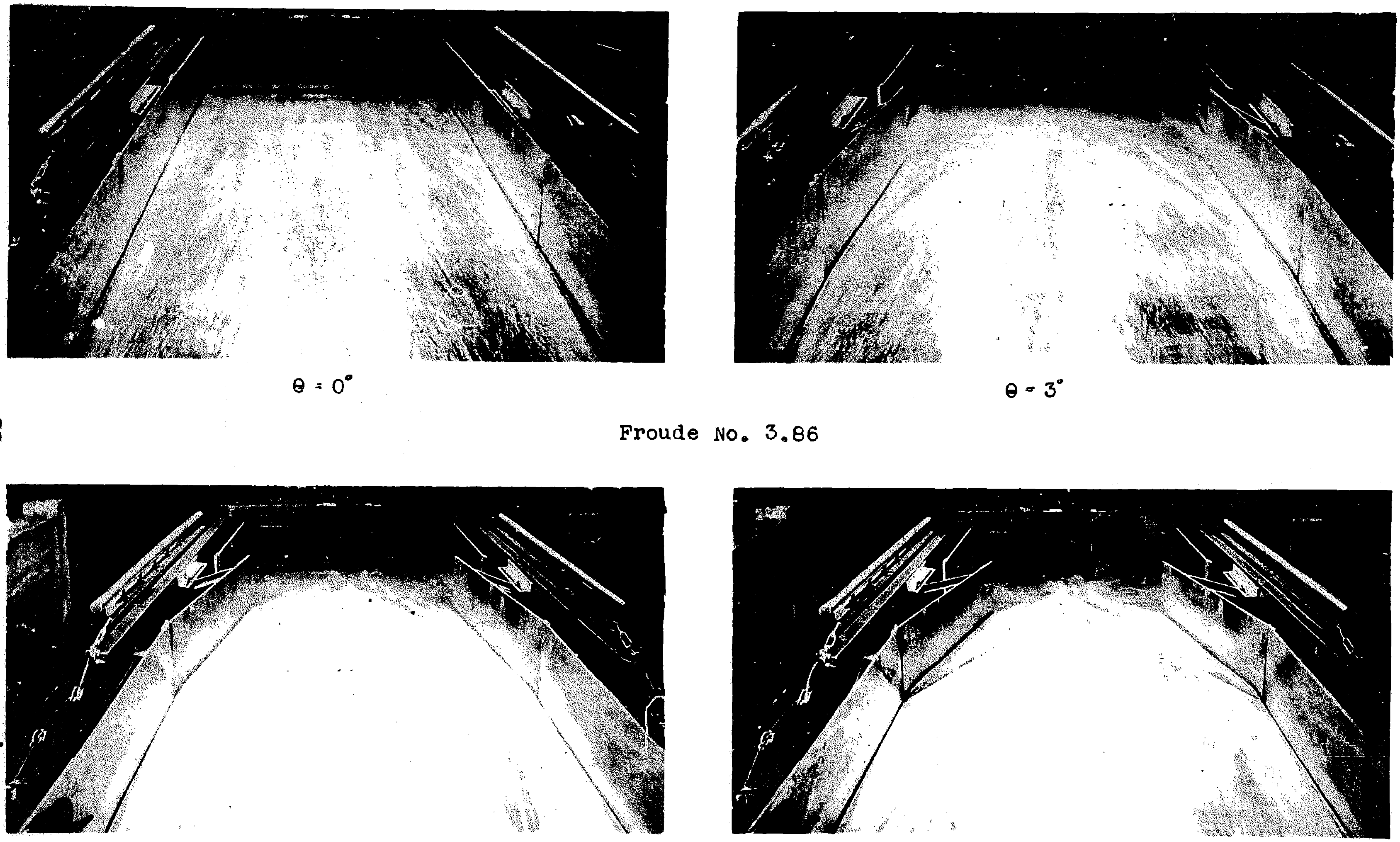


WAVE PROFILES
θ=12° F=3.86
sta 24
sta 24
Wall Profile
sta 0.0
sta 0.0
1.0
2.0
Channel Width – feet
Profile Scale f = 2cm depth





WAVE PROFILES
θ = 15° F = 3.86
sta. 2.4
sta. 2.4
Wall Profile
sta. 0.0
sta. 0.0
1.0
2.0
Channel Width — feet
Profile Scale 1" = 2 cm depth



$\theta = 0°$

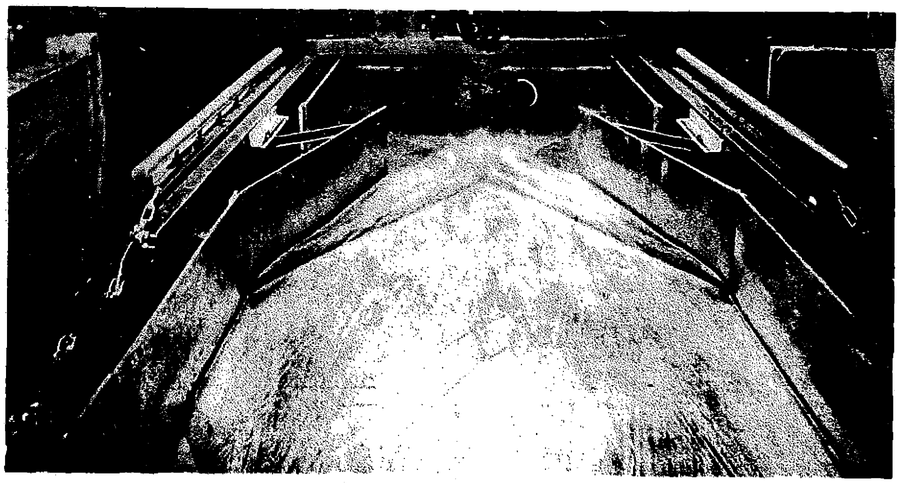
$\theta = 3°$

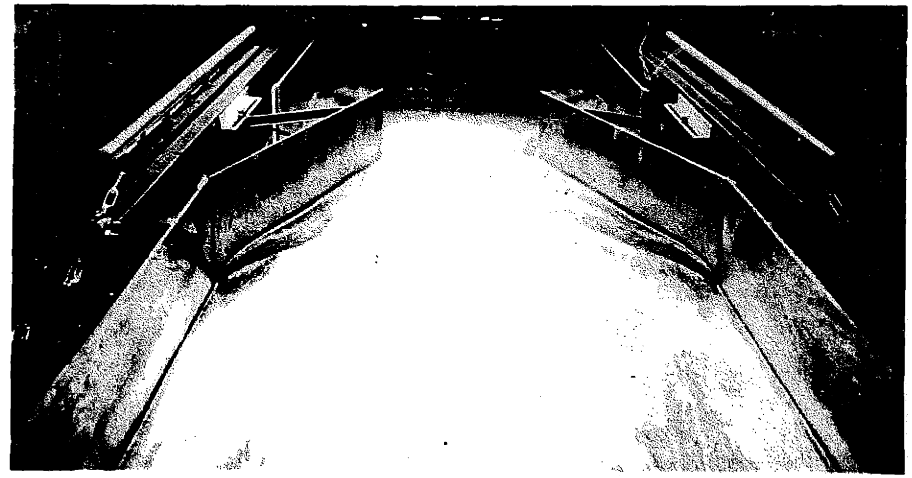
Froude No. 3.86

$\theta = 6°$

$\theta = 9°$

PLATE 7





$\theta = 12°$

$\theta = 15°$

Froude No. 3.86

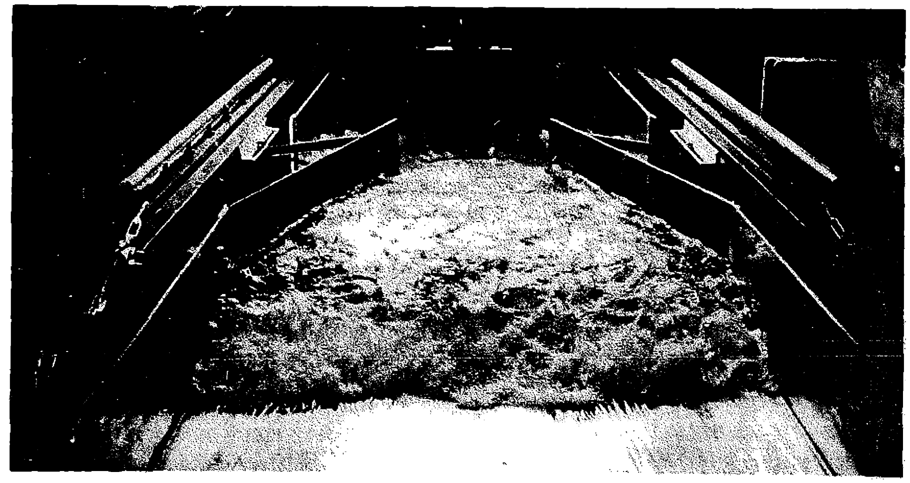


PLATE 8

APPENDIX E

BIBLIOGRAPHY

(1) Ippen, Arthur T. and Knapp, Robert T.: "A STUDY OF HIGH VELOCITY FLOW IN CURVED SECTIONS OF OPEN CHANNELS", March, 1936, Los Angeles County Flood Control District.

(2) Ippen, Arthur T. and Knapp, Robert T.: EXPERIMENTAL INVESTIGATIONS OF FLOW IN CURVED CHANNELS", July, 1938, reproduced by U.S. Engineer Office, Los Angeles, California.

(3) Knapp, Robert T. and Ippen, Arthur T.: "CURVILINEAR FLOW OF LIQUIDS WITH FREE SURFACES AT VELOCITIES ABOVE THAT OF WAVE PROPAGATION", Proceedings of the Fifth International Congress of Applied Mechanics, 1938.

(4) Ippen, Arthur T.: "GAS WAVE ANALOGIES IN OPEN CHANNEL FLOW", Proceedings of the Second Hydraulics Conference, University of Iowa, 1942.

(5) Rouse, Hunter: "FLUID MECHANICS FOR HYDRAULIC ENGINEERS", McGraw Hill Book Co., 1938.