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Title Page

Paper Title:

A Puzzle About Rates of Change

Authors (alphabetical):

David Builes

Massachusetts Institute of Technology

dbuiles@mit.edu

Trevor Teitel

New York University

trevor.teitel@nyu.edu

A Puzzle About Rates of Change*

David Builes and Trevor Teitel

Abstract. Most of our best scientific descriptions of the world employ rates of change of some continuous quantity with respect to some other continuous quantity. For instance, in classical physics we arrive at a particle's velocity by taking the time-derivative of its position, and we arrive at a particle's acceleration by taking the time-derivative of its velocity. Because rates of change are defined in terms of other continuous quantities, most think that facts about some rate of change obtain in virtue of facts about those other continuous quantities. For example, on this view facts about a particle's velocity at a time obtain in virtue of facts about how that particle's position is changing at that time. In this paper we raise a puzzle for this orthodox reductionist account of rate of change quantities and evaluate some possible replies. We don't decisively come down in favour of one reply over the others, though we say some things to support taking our puzzle to cast doubt on the standard view that spacetime is continuous.

1. Introduction

Most of our best scientific descriptions of the world employ rates of change of some continuous quantity with respect to some other continuous quantity. For instance, in classical physics we arrive at a particle's velocity by taking the time-derivative of its position, and we arrive at the particle's acceleration by taking the time-derivative of its velocity. Other such rates of change abound across physics: for example, gradients and Laplacians of a scalar field, and indeed time-derivatives appear in every equation of motion ranging from classical particle mechanics to quantum field theories. Moreover, the methods of differential calculus (and thereby rate of change quantities) pervade the sciences generally, including physics, biology, economics, and the list goes on.

Because rates of change are defined in terms of some other continuous quantities, it's natural to think that facts about some rate of change obtain in virtue of facts about those other continuous quantities. Using velocity as an example, it's natural to think that, at Newtonian worlds (where absolute velocities are well-defined), facts about a particle's velocity at some time obtain in virtue of facts about how its position is changing at that time (via the standard definition of a derivative from elementary calculus). This is the widely held *at-at theory*, according to which velocity facts aren't fundamental, but rather obtain in virtue of facts about position developments across time. The alternative view denies that velocity facts obtain in virtue of facts about position developments, and instead holds that velocities are additional intrinsic properties that are (perhaps

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necessarily) *correlated* with position developments via their standard calculus definition.¹ An analogous choice-point arises for any rate of change quantity: does it obtain in virtue of facts about the quantities at issue in its standard calculus definition? In each case, there's prima facie reason to answer in the affirmative, and thereby adopt the analogue of the at-at theory. Doing so explains why the quantity correlates with those that appear in its standard calculus definition, and avoids the need to countenance numerous additional properties and relations.

Our goal in this paper is to raise a puzzle about reductionist "at-at" accounts of rate of change quantities, and to explore some possible resolutions. Reductionist accounts of some rate of change R must isolate some facts in virtue of which facts about R obtain. But we'll see that it's not clear how there could be any such facts without accepting other seemingly unpalatable assumptions. We'll argue that some resolutions of our puzzle are more attractive than others, but our main goal here is to chart the territory.

Two preliminaries before we get started. First, to simplify the exposition we'll state our puzzle in terms of velocity and its definition as the time-derivative of position, and this will be our paradigm rate of change quantity throughout. However, we want to emphasize that the puzzle applies to rates of change generally (acceleration, purely spatial derivatives, etc.), including those that appear in our best contemporary scientific theories.

Second, we'll cash out the 'in virtue of' talk we've been using so far in terms of grounding, which makes things concrete. We want to emphasize, however, that an analogue of our puzzle could be stated using any of the other standard frameworks for cashing out 'in virtue of' talk. For instance, we could state an exact analogue of our puzzle in terms of truthmaking, or Sider's (2011) notion of a "metaphysical semantics".

Grounding has been pervasive across recent metaphysics, so we'll only quickly review the basic background.² Intuitively, some fact A is grounded in another B just in case A obtains in virtue of B , or B makes it the case that A .³ Ground is sometimes taken to embody a "vertical" or "metaphysical" sense of explanation, because one way to explain

¹ For a classic defense of the reductionist at-at theory of velocity, see Russell (1937). See Tooley (1998), Arntzenius (2000; 2012, ch.2), Carroll (2002), Lange (2005), and Easwaran (2014) for further discussion on both sides of the debate.

² For the classic papers on grounding, see Schaffer (2009), Rosen (2010), and Fine (2012). For more discussion, see the papers collected in Correia and Schnieder (2012).

³ We don't intend anything metaphysically significant by our 'fact' talk throughout. Indeed everything we say is neutral on whether grounding is an operator or a relation, so we'll just ignore this issue here.

a phenomenon is to cite the underlying facts that make it the case. For example, when asking why there's a party in my neighbor's apartment, you might be interested in something like a causal explanation ('because my neighbor just had a birthday'), but another sense of explanation you might be interested in cites some more fundamental facts that ground the fact that there's a party in the apartment ('because there are many people in the apartment, loud music is playing, etc.').

One distinction we'll need is between full grounding and partial grounding. What we just described (and what we'll mean by unqualified uses of 'grounding' throughout) was full grounding. By contrast, B is a partial ground of A iff B together with some other facts fully ground A. Suppose Bob is at the party in the example above. Then the fact that Bob is dancing in my neighbor's apartment is a partial ground of the fact that there's a party in my neighbor's apartment, but not a full ground.

2. The Puzzle

Let's use [A] throughout to abbreviate the fact that A. Now consider some object o , and some fact of the form [o has velocity v at time t]. If the at-at theory is true, then this fact must be partially grounded in some collection of facts of the form [o has position p at time t]. Question: which such position facts are the partial grounds? Our contention is that every way of answering this question faces difficulties. Again, we're phrasing our puzzle in terms of velocities for simplicity, but an analogous question arises for any rate of change.

The naive answer is that the relevant position facts are those about o 's position "immediately before" and "immediately after" t . But of course, given standard continuity assumptions there are no such facts: for every fact about o 's position at some time before or after t , there's another fact about o 's position at a time even closer to t .⁴

So why not pick some ε and claim that the relevant facts that ground [o has velocity v at t] are all and only the facts of the form [o has position p at t^*] where t^* is in $(t - \varepsilon, t + \varepsilon)$? Call such a view the *Precise Cut-Off View*. The main problem with such a view is that any particular choice of ε is completely arbitrary: what could motivate any such choice over the rest? Moreover, there is no shortest interval of time containing the relevant position facts, yet for any choice of interval, we could choose a smaller interval and intuitively not leave anything relevant out.

⁴ Of course this points to a quite radical reaction to our puzzle; namely, reject the continuity of spacetime in favour of a gunky or discrete view. We'll discuss this reaction in section 7, but until then we'll take the standard view for granted.

The only remaining answer is to claim that facts of the form [o has position p at t] for every t partially ground [o has velocity v at t]. Call this view the *Egalitarian View*.⁵ The Egalitarian View seems to us a non-starter. The view predicts that facts about some particle's position a billion years from now are partial grounds of facts about how fast that particle is moving here and now. Yet such temporally distant facts seem totally irrelevant to the particle's velocity now, and indeed this relevance condition is commonly taken to be definitional of grounding.⁶ Here's a case to drive this point home. Suppose I throw a frisbee to my friend across the hall, and years later I throw the frisbee into the East River. According to the Egalitarian View, facts about the frisbee being located at the bottom of the East River turn out to be partial grounds of its velocity when I threw it to my friend years earlier. We doubt anyone would want to embrace such a view. Nevertheless, the Egalitarian View is precisely what comes out if you identify velocity with the time-derivative of position using the standard definition of a derivative from elementary calculus. We'll spell this out in more detail in section 4, and in doing so further justify our pessimistic assessment of the Egalitarian view.

So we see that proponents of the at-at theory face a challenge, namely to isolate the relevant position facts that ground each velocity fact. And indeed the same challenge generalizes to any rate of change. In each case, an analogue of our puzzle arises for those who want to ground each fact about the rate of change in terms of facts about the quantities that appear in its standard calculus definition.

In the rest of the paper we'll investigate various reactions to our puzzle, and evaluate some of their pros and cons. We won't decisively come down in support of one reaction over the others, though we won't shy away from revealing where our sympathies lie as we go.

3. Option I: Deny At-At

One straightforward reaction to our puzzle is to reject orthodox at-at reductionist accounts of any rate of change. Notice that on at-at accounts rates of change are extrinsic. For instance, velocity and acceleration facts at a time hold in virtue of position facts at distinct times. Similarly, the gradient of a field at a single point holds in virtue of the

⁵ In fact there are two other remaining answers: partially ground [o has velocity v at t] in o 's position facts only before t , or alternatively in o 's position facts only after t . However, the issues we'll raise for the Egalitarian View carry over straightforwardly to these two alternatives, so we'll set them aside in the main text.

⁶ This relevance criterion is explicit in both Rosen (2010) and Fine (2012). See Raven (2013) and Dasgupta (2014) for further discussion of the relevance condition. It's also worth noting that a similar relevance condition would hold if we were running the puzzle using the framework of metaphysical semantics or truthmaking.

field's values at other points. Let the *Wholesale Intrinsic View* be the view that no rates of change are extrinsic. Rather, an object has its velocity and acceleration at a time in virtue of how that object is at that very time. Similarly, the gradient of a field at a single point has the value it does purely in virtue of what that very point is like. And so on for every rate of change.

While some philosophers have defended the view that some rates of change are intrinsic, we regard the view that *all* rates of change are intrinsic in this way as a fairly radical solution to our puzzle. The main worry for such a view is that it seems to be committed to a proliferation of new fundamental rate of change properties. True, not all rates of change need to be treated as fundamental according to this view (e.g. presumably biological and economic rates of change can be grounded in more fundamental physical rates of change), but at least *prima facie* such a view seems committed to far more fundamental properties than we'd like. Moreover, although there may be some independent motivation to regard rates of change *with respect to time* as intrinsic (such as velocities and accelerations), our puzzle arises for *purely spatial* derivatives, like spatial curvature or slope, where there's even more pressure to adopt an at-at theory.⁷ We'll continue to use velocity as our central example, and proceed on the assumption that denying at-at isn't a live option. We now see that, given our puzzle's generality, even those sympathetic to viewing velocities as intrinsic should want a solution to our puzzle consistent with the at-at theory.

4. Option II: The Egalitarian View Revisited

We were quick to dismiss the Egalitarian View above, but there are two important considerations in its favour that are worth discussing.

First, notice that if one identifies an object's velocity at a time with the time-derivative of its position at that time (as most physics textbooks do), then the Egalitarian View seems to follow from relatively uncontroversial principles about grounding together with the standard epsilon-delta definition of a derivative. The two widely accepted principles about grounding are as follows:

- (\forall) For any φ , $[\varphi(c)]$ is a partial ground for $[\forall x \varphi(x)]$
- (\exists) For any φ , $[\varphi(c)]$ is a partial (as well as full) ground for $[\exists x \varphi(x)]$

⁷ One of us (TT) is sympathetic to the at-at theory even in the case of temporal derivatives like velocity. The other (DB) is more sympathetic to intrinsic temporal derivatives for independent reasons (see Builes (MS)). Nevertheless, we both agree that our puzzle isn't a good reason to reject the at-at theory for any rate of change, even those with respect to time.

According to the standard definition of derivative, v is the velocity of some object o at time t iff it satisfies the following formula:

$$(1) \forall(\varepsilon > 0)\exists(\delta > 0)\forall t'(|t' - t| < \delta \rightarrow |(p' - p)/(t' - t) - v| < \varepsilon)$$

where t' ranges over times, and p' and p are the positions of o at times t' and t , respectively.⁸

We will now show that any fact of the form [o has position p_0 at t_0] is a partial ground for [o has velocity v at t] using (\forall) , (\exists) , (1), and the identification of velocity with the time-derivative of position. First, we need to choose appropriate values ε_0 and δ_0 to instantiate in the first two quantifiers of (1). For δ_0 , we only need to choose some value greater than the distance between t and t_0 . For ε_0 , we need to choose some value that is greater than the distance between v and the average velocity of o between t' and t , for every time t' within δ_0 of t .⁹ Then, by (\forall) , (1), and our choice of ε_0 , v has the value it does partly in virtue of the following fact:

$$(2) \exists(\delta > 0)\forall t'(|t' - t| < \delta \rightarrow |(p' - p)/(t' - t) - v| < \varepsilon_0)$$

Using (\exists) and our definition of δ_0 , (2) holds partly in virtue of the following fact:

$$(3) \forall t'(|t' - t| < \delta_0 \rightarrow |(p' - p)/(t' - t) - v| < \varepsilon_0)$$

By (\forall) we have that (3) holds partly in virtue of the following fact:

$$(4) |t_0 - t| < \delta_0 \rightarrow |(p_0 - p) / (t_0 - t) - v| < \varepsilon_0$$

Lastly, it is clear that this arithmetical fact between the positions of o at times t and t_0 holds partly in virtue of the fact that [o has position p_0 at t_0], as desired.¹⁰

We're inclined to respond to this argument with a *modus tollens* rather than a *modus ponens*. Since we accept that every fact of the form [o has position p_0 at t_0] is a partial ground for the time-derivative of o 's position at t , and we reject the Egalitarian View about velocity, we see this as a novel ground-theoretic argument against the identification of velocity with the time-derivative of position. We remain convinced by

⁸ You might worry that the standard definition of a derivative predicts that facts about real numbers are amongst the partial grounds for facts about objects' velocities at times. We'll continue working with the standard definition in the main text, but we want to emphasize that every point we'll make has an analogue in terms of nominalist-friendly definitions of a derivative, which don't quantify over real numbers (in particular, we have in mind the definition offered by Field (1980)).

⁹ For example we may set $\delta_0 = |t_0 - t| + 1$ and $\varepsilon_0 = \max\{|(p' - p)/(t' - t) - v| + 1 : \forall t' (|t' - t| < \delta_0)\}$.

¹⁰ In order to draw our conclusion that [o has position p_0 at t_0] is a partial ground for [the time-derivative of o 's position at t is v] we are tacitly appealing to the transitivity of partial ground. While transitivity is widely endorsed, there has been some push-back (e.g. Schaffer (2012)). We're inclined to side with the orthodoxy in endorsing the transitivity of partial ground, but we think that even those who are skeptical of it in full generality should endorse it in this case. However, see Raven (2013) for a full defense of transitivity.

cases like the frisbee case above. Here's another case of the same sort. Suppose Bob is swimming in a race. We don't believe that how fast Bob is swimming one minute into the race is metaphysically dependent, *in any way at all*, on exactly where Bob is eventually buried when he dies. Since the Egalitarian View has this consequence, we reject the Egalitarian View. Since the identification of velocity with the time-derivative of position implies the Egalitarian View, we reject the identification.¹¹

Of course, this isn't to deny that there's a necessary connection between the velocity facts and the time-derivative of position facts. It's commonplace for grounding theorists to make hyperintensional distinctions, and we think this is one of them. Compare, for example, the following modified version of (1):

$$(5) \forall(\epsilon \in (0,2))\exists(\delta > 0)\forall t'(|t' - t| < \delta \rightarrow |(p' - p)/(t' - t) - v| < \epsilon)$$

It's easy to see that (1) and (5) are necessarily equivalent, yet they have different grounds.¹² The argument for why every position fact is a partial ground for (1) fails to show that every position fact is a partial ground for (5). Indeed, any position fact [*o* has position p_1 at time t_1] that does not satisfy the arithmetical relationship $|(p_1 - p) / (t_1 - t) - v| < 2$ will *not* be a partial ground for (5).

Notice that it won't do to reject the standard identification of [*o* has velocity v at t] with (1) in favor of an identification with any fact like (5), whose initial universal quantifier ranges over a more restricted domain than the initial universal quantifier in (1). Any such identification will then imply the Precise Cut-Off View, for exactly analogous reasons that the standard identification implies the Egalitarian View. As a result, any such identification will be vulnerable to the worries we raised for the Precise Cut-Off View above.

The second defense of the Egalitarian View attempts to give an error theory about the relevance intuitions we're using to reject the view in cases like our frisbee case or Bob's swim race. The defense goes via other cases of ground. Suppose, for example, that the

¹¹ Although we reject the Egalitarian View for velocity, we remain open to it being the correct account of any derivative-like quantities in pure mathematics that are stipulatively defined using the standard epsilon-delta definition. Moreover, we suspect that the relevance intuitions we used to argue against the Egalitarian View in the case of velocity will be less strong as concerns these purely mathematical quantities. We'll discuss a further disanalogy between the cases in footnote 16. Thanks to two anonymous referees for encouraging us to say more about this issue.

¹² Notice that this claim requires reading the initial universal quantifier in (5) as a binary quantifier ranging only over reals in the open interval (0,2), rather than as a unary quantifier ranging over all objects whatsoever, which is then further restricted by some conditional antecedent. We intend a similar reading of the initial quantifiers in (1), though we'll generally elide the difference in the main text.

following is a fact: [there are infinitely many electrons]. Let e_1, e_2, e_3, \dots be these electrons. Prima facie, it seems that for each i , [e_i is an electron] is a partial ground for [there are infinitely many electrons]. However, someone might say that [e_i is an electron] is irrelevant to the fact that [there are infinitely many electrons]. After all, even if e_i did not exist, there would still be infinitely many electrons. However, this relevancy argument clearly gives the wrong result, since if no fact of the form [e_i is an electron] is a partial ground for [there are infinitely many electrons], then it seems like there would be no ground for [there are infinitely many electrons]. But this conclusion is clearly untenable. These sorts of examples need not rely on infinity in any way. For example, take the fact that [there are more than two cats]. This fact seems to be partially grounded in, say, [Tibbles is a cat]. According to the relevance objection, there would still be more than two cats regardless of whether Tibbles existed. So, it seems like any fact about the existence of any cat is not a partial ground for [there are more than two cats]. Again, however, this seems to over-generate.

Our response is to diagnose the “irrelevance” in these sorts of cases to harmless overdetermination. It does not suffice to establish that [p] is irrelevant to [q] merely to say that [q] would have obtained even if [p] did not obtain. If [p] and [q] are both true facts, then of course they are both grounds of [p or q], even if [p or q] would have obtained regardless of whether one of the disjuncts failed to obtain. In the above cases, we think that [e_1 is an electron] is clearly relevant to the number of electrons in the world, even if that number is infinite. Similarly, [Tibbles is a cat] is clearly relevant to how many cats there are in the world, even if the number is far greater than two. However, it seems to us, facts about, say, exactly where Bob ends up being buried when he dies several decades later are not relevant in any sense to facts about how fast he’s swimming one minute into his race.

5. Option III: Indeterminate Grounds

Another reaction to our puzzle is to think that there’s some indeterminacy in which position facts ground the velocity facts at any given time. Intuitively, the position facts closer to the relevant time are “more relevant” than the position facts further away from the relevant time. Perhaps, then, the position facts sufficiently close to the relevant time are *determinately* partial grounds, the position facts sufficiently far away from the relevant time are *determinately not* partial grounds, and the position facts in the middle are such that it’s *indeterminate* whether they count as partial grounds.¹³

¹³ There are several variants of indeterminacy-based reactions. For instance, perhaps while position facts sufficiently far away from the relevant time are determinately not partial grounds, *no* position facts are determinately partial grounds. Instead it simply becomes indeterminate what the partial grounds are once you get close enough to the relevant time. We’ll discuss only

We think that most attempts to avoid our puzzle by pleading indeterminacy will face the same sort of worries as the Precise Cut-Off View. Remember that any choice of precise cut-off seems completely unmotivated, and also that, for any such choice, we could choose a closer cut-off and intuitively not leave anything relevant out. Appealing to indeterminacy leaves one with a similar issue: which position facts are close enough to the relevant time to qualify as determinate partial grounds (or as not determinately not partial grounds)? Any such choice seems completely unmotivated, and moreover, given any such choice, it seems we could draw the line closer without leaving anything relevant out. Here we're coming up against familiar problems of higher-order vagueness, where it seems like pleading indeterminacy still saddles one with an obligation to draw cut-offs. So it's no surprise that the appeal to indeterminacy doesn't seem to rid one of the problems for the Precise Cut-Off View.

Another worry for appeals to indeterminacy arises when we ask why it's indeterminate which position facts ground the velocity facts at some time. We see two options here: either blame the indeterminacy on the vagueness of 'grounding' or on the vagueness of something's 'velocity at a time'. Both options face problems; let's discuss them in turn.¹⁴

The problem with claiming that 'grounding' itself is vague is that fans of grounding regard it as a central piece of ideology in which to couch interesting metaphysical debates (see the works cited in footnote 2 for some examples). Yet if 'ground' is vague, then at best there are numerous reasonably unnatural grounding relations in the vicinity, each of which implies some version of the Precise Cut-Off View, with its attendant problems. (We doubt any admissible precisification of 'ground' will imply the egalitarian view, given the relevance condition on grounding. And even if one did, it would still face the objection from relevance.) Questions about what grounds what seem to lose their interest if they involve some such precisification that implies the Precise Cut-Off View. Any such relation draws distinctions that seem completely arbitrary from

certain paradigm examples of appeals to indeterminacy in the main text, but the points we make apply generally to indeterminacy-based replies to our puzzle.

¹⁴ In the main text we're taking for granted that there isn't any genuinely metaphysical indeterminacy (following, e.g., Lewis (1986)). While some philosophers accept the possibility of metaphysical indeterminacy, we count ourselves amongst those who find the notion mysterious. Nevertheless, many of the points we make could also be phrased in terms of metaphysical indeterminacy, given the main candidate accounts of metaphysical indeterminacy on offer. For a survey of some arguments against metaphysical indeterminacy together with possible responses, see Barnes (2010). For some of the main options in the area, see Barnes and Williams (2011) and Wilson (2013).

the perspective of reality itself, not distinctions that cull nature's joints. The picture of ground that results is one that's fairly hostile to fans of grounding.

What about the second option, according to which the notion of something's 'velocity at a time' is vague? Here's how we think this option should be developed. Consider some utterance of the form ' o has velocity v at t' '. Recall (1) above—the standard calculus account of this utterance—and (5) above, which is just the standard calculus account yet with an initial universal quantifier that ranges over a more restricted domain. We also saw that there are infinitely many facts akin to (5), for every way of putting some positive upper bound on the range of the initial universal quantifier in the standard calculus definition of a derivative. We also saw that all such facts will be necessarily equivalent, albeit hyperintensionally distinct. Now the proponent of the vagueness of ' o has velocity v at t' ' may claim that this utterance doesn't privilege any of these infinitely many facts akin to (5) over the others. Each such fact is an admissible precisification of ' o has velocity v at t' ', yet none is determinately privileged over the others.

Although initially attractive, we also don't think this option stands up to scrutiny. First off, some may worry that this proposal amounts to an error theory about the ordinary concept of an object's velocity at a time. For reasons that motivated our rejection of the Precise Cut-Off View, perhaps given the ordinary notion it's determinately the case that facts about an object's position at times any finite distance from t are irrelevant to the object's velocity at t . Yet if that's right then each of these infinitely many facts akin to (5) is determinately *not* an admissible precisification of an object having some velocity at a time, because it brings in irrelevant information about the object's position at other times.

In response, one might simply jettison our pre-theoretic concept of rates of change and insist that the only relevant facts in the vicinity are facts like (5). We see two worries with this response. First, we experience the "inertial effects" of things like acceleration every time we go on a rollercoaster, or get in an elevator, or take off in an airplane, and so on. Perhaps the proponent of the error-theoretic response will claim that in each of these cases the effects we experience somehow depend only on infinitely many facts like (5). Yet we don't think these inertial effects depend on the temporally distant facts smuggled in by the relevant derivatives. My present experience of being thrown to the back of my seat when my airplane takes off doesn't seem in any way metaphysically dependent on where the airplane was yesterday. The error-theorist might complain that here we're just restating the relevance intuitions built into our pre-theoretical concepts. Still, we think the connection between our pre-theoretical concept of some rate of change and our direct experience of the physical world renders the error-theoretic option harder to swallow.

But even setting that first worry aside, there's a bigger problem. Rates of change have figured in all of our best guesses for the laws of fundamental physics, from Newton's laws to those of our current best physical theories. As a result, it's reasonable to expect that the true fundamental laws, whatever they ultimately turn out to be, will also contain some rates of change. Yet on the present proposal there's nothing privileged on the side of the world in the vicinity of our talk about some rate of change; instead there are just numerous admissible precisifications that draw different seemingly arbitrary cut-offs, with none privileged over the rest. However, we're reticent to accept that the fundamental laws of physics are similarly indeterminate, and that there's a law corresponding to each admissible precisification of the relevant rates of change. A common picture is that the fundamental laws are in some robust sense the reason why things behave the way they do. But if such a view is correct, which of the precisifications governs the evolution of the universe? We're both loathe to accept that the answer to this question is simply indeterminate.¹⁵

Putting our points in this section together, we're not optimistic about the prospects of replying to our puzzle by appealing to indeterminacy, however this strategy is ultimately implemented.¹⁶

6. Option IV: Holistic Grounds

A more revisionary reaction to our puzzle takes its cue from Dasgupta (2014). Dasgupta argues that different views of a structuralist bent motivate a non-standard view about the logic of grounding. According to the standard position, grounding always relates a single fact, the grounded, to some plurality of one or more facts, the grounds. According to Dasgupta's "pluralist" position, by contrast, grounding can be irreducibly plural on *both* sides. So on this position, the most fundamental grounding facts in some domain can take the form *these* facts are grounded in *those* facts, with no further story about the grounds of particular facts to be told.

¹⁵ One response to this objection would be to adopt a Humean conception of laws, according to which laws are certain especially simple and informative summaries of the facts at a world. Humeans may embrace indeterminacy in what the fundamental laws are, since it can be explained in terms of there being "ties" amongst which laws are the simplest and most informative. Neither of us is particularly sympathetic to the Humean conception, so we leave it to Humeans to adjudicate whether to simply accept this widespread indeterminacy in what the fundamental laws are. We'd regard it as an interesting result in its own right if our puzzle shows that the metaphysics of rates of change has this striking implication for the Humean view.

¹⁶ Note that our final objection to the indeterminacy-based approach applies only to rates of change that appear in the laws of our best fundamental physical theories. We thus remain more open to the indeterminacy-based approach as applied to other rates of change (see also footnote 11). Thanks to both anonymous referees here.

One view that Dasgupta uses to motivate his pluralist position is “comparativism” about physical quantities like mass, charge, and so on (see Field (1980) for the classic statement of comparativism). In the case of mass, for example, comparativists deny that the most fundamental facts are about the individual masses of things (e.g. [my laptop has determinate mass m]). Instead, for comparativists the most fundamental facts about mass describe the comparative masses of things (e.g. [x is twice as massive as y], [the mass of x is between the mass of y and the mass of z], and the options go on). Comparativists then claim that these comparative mass facts *taken altogether* ground (and hence necessitate) facts about the individual masses of things (usually up to some conventional choice of zero and scale). As a result, given comparativism and the standard singularist conception of grounding, facts about the individual mass of any object are grounded in that object’s mass relations to every other object in the universe. For example, [my laptop has determinate mass m] turns out to be partially grounded in some fact about the comparative mass between my laptop and some electron in Alpha Centauri. Dasgupta argues that this violates the relevance condition on grounding. How my laptop relates in mass to an arbitrary electron in Alpha Centauri seems irrelevant to how massive it is.

This problem is dissolved if one moves to Dasgupta’s pluralist position. This position allows that the most fundamental grounding fact in the vicinity says that *all* of the facts about the individual masses of things are (plurally) grounded in *all* of the facts about the comparative masses of things. On this view, there is no violation of the relevance condition. The fact about the comparative mass between my laptop and an arbitrary electron in Alpha Centauri *is* relevant, on a comparativist view, to the mass of my laptop and the mass of that arbitrary electron, taken together. The view also better captures the spirit of the comparativist view: the individual mass facts arise only altogether from all of the comparative mass facts.

A similar move can be made in response to our puzzle. Proponents of the pluralist position can claim that, for any object o , all facts of the form [o has velocity v at t] are (plurally) grounded in all facts of the form [o has position p at t], yet there’s no further grounding story to be told about o ’s velocity facts taken individually. The picture is that facts about an object’s velocity throughout all time arise only altogether from facts about the object’s position throughout all time. This option avoids the objections we raised to the Egalitarian view and the Precise Cut-Off View, and also isn’t vulnerable to any of the problems for indeterminacy-based replies that we saw in the previous section.

Nevertheless, we don’t consider this holistic reply a plausible solution to our puzzle. In the case of various structuralist views, like comparativism, one can plausibly appeal to Dasgupta’s pluralist position. Comparativism makes plausible that the individual mass

facts will be metaphysically interconnected: they arise out of the comparative masses between all of these individuals, and hence it's natural to expect these comparative mass facts to fix the individual mass facts only altogether. However, we don't see anything particularly "structuralist" about an electron taking a particular trajectory across spacetime. Why should the electron's velocity eons in the future be somehow metaphysically interconnected with its velocity here and now, such that these velocities arise only altogether from the electron's positions throughout its career? The resulting holism looks even more radical when we turn our attention to other rates of change. Consider some scalar field on spacetime together with the gradient of that field at every point. On the reply under consideration, the facts about the gradient of the field *at every point in spacetime* are plurally grounded in the facts about the value of that field at every point in spacetime, yet we can't ask about the grounds of the gradient of the field at some point in isolation from its gradient at every other point. This implies that facts about how the field is changing at any single spacetime point are for some reason metaphysically interconnected with how the field is changing at every other spacetime point.

We claim, then, that replying to our puzzle by appealing to plural grounding seems to be an *ad hoc* fix with no independent motivation. We believe everyone should hope to find a more principled reply to our puzzle.

7. Two More Radical Options

In this section we'll look at two more radical reactions to our puzzle.

The first is to claim that there's something problematic about the very notion of grounding that we used to state our puzzle. It's easy to see the attraction of blaming our puzzle on grounding from the point of view of the at-at theorist. At first blush, it seems like there's nothing at all puzzling or mysterious about the at-at view. In the Newtonian case, amongst the fundamental facts are those detailing the positions of each object at each time. Velocity facts then simply summarize certain features of these position facts, in accordance with the standard definition of a derivative from elementary calculus. Indeed many have worried, for reasons independent of our puzzle, that the notion of grounding is too unclear or abstract to do interesting theoretical work.¹⁷ That being said, we aren't as pessimistic about grounding as some of these authors. Moreover, this response to our puzzle is independently problematic. As we stressed at the outset, our puzzle can be raised using a variety of different metaphysical frameworks for cashing out 'in virtue of' talk, and so isn't peculiar to grounding. Eschewing all of these other

¹⁷ The details of the criticisms differ, but for a sampling see Hofweber (2009), Daly (2012), Wilson (2014), Koslicki (2015), and Miller and Norton (2017).

notions as well comes at a steep cost, and one we're not willing to pay. Still, we certainly see how our puzzle might lend credence to such an outlook.

Finally, perhaps the most radical reply to our puzzle is to reject the underlying presupposition that spacetime is a continuum composed of extensionless points, in favour of a gunky or discrete view. It's easy to see how this would immediately dissolve our puzzle. If time is gunky then there are no instants of time, and hence there are no instantaneous rates of change. If time is discrete then there are times immediately before and immediately after any particular time, so our puzzle has an obvious solution. It's worth emphasizing that rejecting the continuity of spacetime isn't an *ad hoc* move. First, many philosophers have advocated this position in light of various puzzles that arise for the orthodox continuum view (see, e.g., Arntzenius (2003; 2008; 2012, ch.4), Forrest (2004), Russell (2008), and Segal (2017)). Some also take the position to garner support from some of the main current options for a theory of quantum gravity (see Rovelli (2001) for relevant discussion), though whether anything even analogously spatiotemporal will remain at the fundamental level in such a theory still remains to be seen (see Huggett and Wüthrich (2013)). We think our puzzle adds some fuel to this fire, and lends credence to regarding the continuity of spacetime as a dogma that it's time to jettison.¹⁸

8. Conclusion

Our main goal here has been to state our puzzle about orthodox at-at accounts of rates of change, and evaluate some potential replies. We've seen that certain natural replies to our puzzle don't survive scrutiny, and other potential replies engender quite radical revisions in our metaphysical picture of the world. Both of us have some sympathy for blaming our puzzle on the continuity of spacetime, but we haven't tried to argue that this reply is more plausible than any other, and we hold out hope that a more

¹⁸ An anonymous referee wonders whether proponents of the continuity of spacetime can reply by adopting whatever fundamental physical properties proponents of gunk embrace to replace standard rate of change quantities (which presuppose continuity). In fact there is nothing inconsistent about this package: the main replacements proffered by proponents of gunk, such as fundamental average values, are also well-defined in standard continuous spacetimes (see Arntzenius and Hawthorne (2005) for a survey of the options available to gunk-theorists here). Nevertheless, we think the combination is an unhappy one: abandoning the standard calculus treatment of rates of change undermines a central motivation for believing in the continuity of spacetime in the first place. Given the various problems that arise for the orthodox continuum view, we doubt anyone willing to concede the gunk theorist's conception of the fundamental physical properties (which carries costs of its own) would nevertheless still cling to orthodoxy.

conservative satisfactory reply can be found. Nonetheless, we're confident that the problem of exactly how to ground rates of change is ripe for further investigation.

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