

MIT Open Access Articles

*From Music to Mathematics: Exploring
the Connections by Gareth E. Roberts*

The MIT Faculty has made this article openly available. **Please share** how this access benefits you. Your story matters.

Citation: Del Cerro Santamaría, Gerardo. "From Music to Mathematics: Exploring the Connections by Gareth E. Roberts." *Mathematical Intelligencer* (September 2019): 85–88 © 2019 Springer Science Business Media

As Published: <https://doi.org/10.1007/s00283-019-09933-4>

Publisher: Springer Science and Business Media LLC

Persistent URL: <https://hdl.handle.net/1721.1/128612>

Version: Author's final manuscript: final author's manuscript post peer review, without publisher's formatting or copy editing

Terms of Use: Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.



From Music to Mathematics: Exploring the Connections *by Gareth E. Roberts*

Cite this article as: Gerardo Del Cerro Santamaría, From Music to Mathematics: Exploring the Connections *by Gareth E. Roberts*, The Mathematical Intelligencer <https://doi.org/10.1007/s00283-019-09933-4>

This Author Accepted Manuscript is a PDF file of an unedited peer-reviewed manuscript that has been accepted for publication but has not been copyedited or corrected. The official version of record that is published in the journal is kept up to date and so may therefore differ from this version.

Terms of use and reuse: academic research for non-commercial purposes, see here for full terms. <https://www.springer.com/aam-terms-v1>

Author accepted manuscript

From Music to Mathematics: Exploring the Connections

by Gareth E. Roberts

BALTIMORE: JOHNS HOPKINS UNIVERSITY PRESS, 2016, 301 PP., US \$41.88,
ISBN-10: 1421419181, ISBN-13: 978-1421419183

REVIEWED BY GERARDO DEL CERRO SANTAMARÍA

The connections, similarities, and relationships between music and mathematics are deep, multiple, and complex, and include not only ways to mathematically express music, but also linkages and associations around matters of logic (Hofstadter, 1999); aesthetics (Kung, 2013); form, geometry, and architecture (Menhinick, 2015); cosmology (James, 1995); cognition, mind, and brain (Mazzola, 2018); computing (Müller, 2016); historiography and biography (Maor, 2018); research, improvisation, and creativity (Schneiderman, 2011); phenomenology (del Cerro Santamaría, 2015); and representation, metaphor, and methods of abstraction (Rothstein, 2006), among others.

Exploring and studying the relationships between the two fields seems to be attracting an increasing number of researchers, students, and readers, and the number of publications on the subject keeps growing. A couple of decades ago, Hofstadter's *Gödel, Escher, Bach* (1999) explored the connections of form, geometry, logic, formal systems, and artificial intelligence. And James's *The Music of the Spheres* (1995) offered a broad historical exploration of cosmology, science, and music, showing the harmony of sound and number in "our mathematical universe," as Max Tegmark (2014) would have put it.

To these classics we must add *The Topos of Music*, by Mazzola (2018), a book that establishes connections between music, cognition, composition, and deep mathematical concepts from algebra and category theory. We also have *Musica Mathematica* (Povilioniene, 2017), a splendid work about how the classical connections between music and math are also visible in other fields (architecture, painting, poetry, literature) and contemporary music.

From another angle, *Emblems of Mind* (Rothstein, 2006) does an excellent job in showing the connections between mathematics and music at a philosophical and metaphorical level. The book describes the architecture of musical space, its tensions and relations, its resonances and proportions. It also deals with how both music and math use comparable methods and have similar concerns as they create their abstractions.

Fundamentals of Music Processing, by Meinard Müller (2016), contributes to an understanding of the linkages between music, mathematics, and computer science. These ties have become relevant in fields such as music information retrieval, which systematically deals with a wide range of topics in computer-based musical analysis, processing, and retrieval.

There are, of course, other important books adding to what is a growing field: *From Pythagoras to Fractals* (Fauvel et al., 2006), *Mathematics and Music* (Wright, 2009), *Musimatics: The Mathematical Foundations of Music* (Loy, 2011), *How Music and Mathematics Relate* (Kung, 2013), and *Music by the Numbers* (Maor, 2018), to name just a few.

From Music to Mathematics

In this expanding transdisciplinary field—one that can be approached from a wide variety of perspectives—Gareth E. Roberts's work *From Music to Mathematics: Exploring*

the Connections, which is intended to serve as a university textbook, offers the content of a two-semester course on music and mathematics taught by the author. Thus the book is “field-tested,” and it includes examples, problems, and exercises (200 of them) that Roberts’s students have been exposed to and asked to solve. The text is intended for undergraduate students with no advanced knowledge of mathematics or music.

In the preface we learn that the author’s intention is to “explore the connections, while also fostering the growth of the mathematical (and musical) abilities of the student.” For this pedagogical purpose, “it is important to show how composers have used mathematical ideas in their compositions,” to which the book devotes examples in virtually every chapter. In addition, there is a website designed to accompany the text with useful resources for instructors.

In the introduction, Roberts explains that his book deals with the “connections from a structural perspective” between music and mathematics, though there are also “many aesthetic and artistic links” between the two disciplines. The author wants to have readers “dive deep into the essence” of the connections while learning the mathematics needed to understand them. For Roberts, one of the most gratifying aspects of mathematics is “its ability to get to the heart of a problem, to describe its essential features.”

So here we have a book that explores the connections between mathematics and music, according to the book’s subtitle. But what is it meant by those connections and from what angle are they approached in the book? Aiming at semantic accuracy and specificity, we would say that the book deals with how some musical patterns and ideas translate into mathematical language. Roberts’s work is about finding the mathematics in certain music and discovering how we can mathematically express and explain the structure of musical ideas and language.

Roberts explicitly mentions in the introduction that his book is about the “structural” connections between the two fields. “Structural connections” imply finding mathematical structures in the structures of musical language and providing the mathematical expression of those musical statements and structures, and this is what the book is devoted to. Precision, clarity, and a sound pedagogical thrust are three key characteristics of Roberts’s work, which provides a good educational tool that will be helpful in particular to instructors wishing to teach a college-level class on music and math.

The book may be roughly divided into two parts: Chapters 1–4 are rather standard descriptions of the main mathematical aspects of music. These chapters focus on “Rhythm” (Chapter 1), “Introduction to Music Theory” (Chapter 2), “The Science of Sound” (Chapter 3), and “Tuning and Temperament” (Chapter 4). Chapters 5–8 constitute a more original approach to the connections between mathematics and music. They deal with “Musical Group Theory” (Chapter 5), “Change Ringing” (Chapter 6), “Twelve-Tone Music” (Chapter 7) and “Mathematical Modern Music” (Chapter 8).

Chapter 1 explores musical notation, geometric properties, time signatures, polyrhythmic music, and some mathematical properties in Sanskrit poetry. In this chapter, the author relates the musical notions of beat and rhythm to the fundamental mathematical concept of counting. It is refreshing to find a reference to poetry in a non-Western context such as Sanskrit poetry:

one interesting connection between math and poetry involves the subject of number theory and a famous sequence of numbers first described by the Indian scholar Hemachandra (1089–1172),

who, together with the scholar Gopala (c. 1135), was “interested in analyzing the rhythmic

structure in Sanskrit poetry,” typically divided “into long two-beat pulses (L), and short one-beat pulses (S).” Hemachandra studied “the number of different ways to subdivide n beats into long and short pulses” (p. 28).

Chapter 2 is a relatively standard account of music theory: scales, keys, intervals, chords, harmony, counterpoint, and the basics of reading music. The chapter’s sections are as follows: musical notation, scales, intervals and chords, tonality, key signatures, and the circle of fifths. Based on the circle of fifths, the author discusses “tonal proximity” and the relation between major and minor, as well as various musical keys. The chapter also includes a helpful summary of the evolution of polyphony in the Western musical tradition.

Chapter 3, on the science of sound, comprises sections on how we hear, attributes of sounds, sine waves, understanding pitch, and the monochord lab: length versus pitch, a practical experiment exploring the connection between the length of a string and the note sounded when the string is plucked. In this chapter, Roberts explains the attributes of loudness, pitch, duration, and timbre from both a physical and a perceptual point of view. We measure frequency of sound (the number of cycles a wave makes in a second) in hertz. These waves are graphically expressed in mathematical trigonometry, and their representation can lead to partial differential equations, which, in turn, help us explain the concept of overtones. Cognition and perception play a role in some musical aspects such as pitch, which is not an objectively measurable aspect of sound waves. This illustrates some of the challenges involved in trying to use mathematics to describe music, given that there are many connections between the two fields but it is impossible to establish a full equivalence or complete translatability from one to the other, as shall be explained below.

Chapter 4 is on tuning and temperament, and it is devoted to the Pythagorean scale, just intonation, equal temperament, Strähle’s guitar, and alternative tuning systems. According to the author, a comparison between the Pythagorean, just intonation, and equal temperament systems “demonstrates some of the strengths and weaknesses of equal temperament as a tuning system.” And he adds that “an excellent source for hearing some of the key differences between tuning systems is the Wolfram Demonstrations Project” (p. 143), whose URL is provided. The twelve-tone equal-tempered scale, in which an octave is subdivided into twelve equal scale steps, can be considered a compromise to address the deficiencies in the Pythagorean tuning and just intonation.

Chapter 5, on musical group theory, includes sections on symmetry in music, the Bartók controversy, and group theory. While Roberts’s book generally offers musical examples whose mathematical meaning or intent is quite clear, in this chapter the author explores an exception to this rule: the so-called Bartók controversy (pp. 182–190), referring to Béla Bartók’s *Music for Strings, Percussion and Celesta* (1936). The controversy has to do with the alleged existence of the Fibonacci series and the golden ratio in Bartók’s compositions. Such existence was mistakenly established by the Hungarian musical theorist Ernő Lendvai based on Bartók’s compositional traits and his love of nature and organic folk music. Roberts reasonably takes sides with Roy Howat’s analysis, which revealed Lendvai’s mistakes. This discussion takes place within the context of an analysis of symmetry in music (Bach, Hindemith) and its transformations (translation, retrograde, inversion, and retrograde inversion). Symmetry procedures, and pattern-making more generally, are a prominent convergence area between mathematics and music.

Chapter 6 is devoted to change ringing, its theory, practice, and examples. The art of change ringing refers to ringing a set of particular bells in a systematic manner to produce variations in their sounding order. A “change” is a specific arrangement of the bells in which each bell is rung exactly once. On page 203, Roberts states,

the intriguing activity of change ringing has evolved into a special blend of music, mathematics, and precision. The specific rules and requirements of change ringing are best understood in the mathematical language of permutations.

This leads the author to discuss combinatorial mathematics and group theory as the mathematical expressions of change ringing.

Chapter 7 discusses Schoenberg's twelve-tone method of composition, in particular his *Suite für Klavier*, op. 25, and tone row invariance. First proposed by Arnold Schoenberg, twelve-tone music has had a great influence on many musical styles in the twentieth and twenty-first centuries. Roberts introduces the main characteristics of the twelve-tone technique, in which a tone row (a specific arrangement of the twelve chromatic pitch classes) serves as the basic building block for a composition, and the twelve-tone matrix is the tool used to generate different transformations in tone rows. According to Roberts, the example of Schoenberg's suite demonstrates that "there is much more to the 12-tone method than a strict mathematical application of symmetry" (e.g., equidistance, inversion, linearity, grouping, repetition, variance). "The real challenge lies in turning the mathematical method into a creative and fluid compositional technique" (p. 236). Roberts points here to the high complexity and multiplicity of the connections between mathematics and music and also to the potential openness of mathematics to indeterminacy and choice.

Chapter 8, on mathematical modern music, explains the mathematical compositional methods of three contemporary composers: Peter Maxwell Davies, Steve Reich, and Iannis Xenakis. Davies has used the concept of magic squares in his compositions; these are special arrays of numbers in which each column, row, and diagonal sums to the same number (the "magic constant"). The composer himself has said this about magic squares:

it is not a block of numbers but a generating principle ... an orbiting dynamo of musical images, glowing with numen and lumen (pp. 253–254).

Steve Reich has used the idea of "phase shifting," which consists in

a translation or shift, by taking the graph of a function and shifting it in either the vertical or horizontal direction, [which] changes when the wave starts (p. 269).

Xenakis introduced the notion of probability in his "stochastic" music, which the composer himself describes as a

statistical mean of isolated states and transformation of sonic components ... the result is the introduction of the notion of probability ... combinatory calculus ... here is the possible escape route from the "linear category" in musical thought (p. 278).

In *From Music to Mathematics*, Roberts does a good job of presenting the mathematical expression of some specific musical ideas and compositional styles. The outcome is a neat and precise portrayal, with very clear contours, of the connections between mathematics and music. There are also a few hints at areas that remain unknown or open to fuzziness. As a whole, the book is a helpful tool for both instructors and students.

Ars Mathematica

Just as notes and rhythms are not all there is to music, so arithmetic and counting are not all there is to mathematics. For example, cognitive scientists have become very interested in the ways in which the brain grasps musical patterns. And mathematics is about structure and pattern, groups and symmetries, and chaos and complexity. Mathematicians are in search

of patterns, and music is all about pattern-making, of which Roberts is very well aware, as shown in some of his book's illustrations.

However, such patterns and symmetries cannot just be perfect or presented as complete. Artists and musicians have been aware that a too perfect symmetry is fatal to art, and this includes the art of mathematics. In literature, we find perfect symmetry as a metaphorical prelude of death, and it was Stravinsky who said that "to be perfectly symmetrical is to be perfectly dead."

In the end, both mathematics and music converge into a realm that is characterized by incompleteness and indeterminacy, where philosophical notions of aesthetics, perception, design, beauty, and taste (the knowing subject's perspective) play a fundamental role. Concerning this idea, Du Sautoy (2011) has stated:

What people don't realise about mathematics is that it involves a lot of choice: not about what is true or false (I can't make the Riemann hypothesis false if it's true), but from deciding what piece of mathematics is worth "listening to." The art of the mathematician lies in picking out what mathematics will excite the soul. Most mathematicians are driven to create not for utilitarian goals, but by a sense of aesthetics.

Mapping Connections

Further, when we consider the connections, similarities, linkages, associations, and interrelationships between any established fields of endeavor or disciplines, as is the case here with mathematics and music, we are in fact dealing with a transdisciplinary approach to knowledge involving—in an incomplete and indeterminate manner—materiality, symbolism, and perception as continuous and fluctuating manifestations of the experiential reality captured by consciousness, to use Heisenberg's own conceptualization (2007).

What is at stake here is not only the grasping of intrinsic structures, or the "inner life" of music and mathematics (Rothstein, 2006). What matters is rather the ability to map common intrinsic structures between different fields of knowledge. Put differently, the focus ought to be on crafting a symbolic display of transactions, exchanges, and equivalences of models, codes, and meanings. Schneiderman (2011, p. 929) highlights the importance and "common strength" of intrinsic structure:

While mathematics provides satisfying analyses of sound and useful parameterizations of musical choices, deeper scientific relationships between mathematics and music remain largely beyond reach. But the adoption of a more metaphorical point of view will uncover support for a return of music and mathematics to a quadrivium-like partnership in education that is based on a common strength of intrinsic structure.

This key aspect of any investigation into the connections between music and mathematics becomes apparent if we focus on the creative and innovative dimensions of research (in mathematics) and improvisation (in music)—with important implications for education—rather than attempting to map the interrelationships between established corpora of knowledge in each field. In Schneiderman's own words (2011, p. 937):

The key point here is that the intrinsic nature of mathematics and music suggests that the studies of both research mathematics and improvisational music could play valuable roles in modern education, as their *abstract yet cohesive structures serve as models* for developing flexible skills and the ability to generate spontaneous constructive thought. What is important here is that the *goals of research and improvisation guide the pedagogy*. The challenge is to develop courses, programs,

and teaching conceptions with these goals in mind and to incorporate them into the curriculum.

Our understanding of mathematics and music is both intuitive and logical, but also imaginative, experiential, and emotional through and through: the perspective of the knowing subject is fundamental and cannot be overlooked. Yet besides the workings of the human mind, an objective reality of mathematics and music—with their systems of representation, logics, and structures—exists independently and beyond these imaginaries.

There will always be a degree of ambiguity in what we know about the “betweenness” of mathematics and music and a limit to what we can know (du Sautoy, 2016). In Roberts’s *From Music to Mathematics*, both aspects (indeterminacy and clear structural relationships) help establish the connections between the two fields.

7025 Yellowstone Blvd.
Forest Hills, NY 11375
e-mail: delcerro@mit.edu

References

- G. del Cerro Santamaría (2015). Some Considerations Regarding the Phenomenological Relationship Between Music and Mathematics, *Transdisciplinary Journal of Engineering and Science* 6 (2015), 111–116.
- M. du Sautoy (2011). Listen by Numbers: Music and Maths. *The Guardian*, June 27, 2011. Available at <https://www.theguardian.com/music/2011/jun/27/music-mathematics-fibonacci>.
- M. du Sautoy (2016). *What We Cannot Know. Explorations at the Edge of Knowledge*. Fourth Estate, 2016.
- J. Fauvel et al., eds. (2006). *Music and Mathematics: From Pythagoras to Fractals*. Oxford University Press, 2006.
- W. Heisenberg (2007). *Physics and Philosophy. The Revolution in Modern Science*. Harper, 2007.
- D. R. Hofstadter (1979). *Gödel, Escher, Bach: An Eternal Golden Braid*. Basic Books, 1979.
- J. James (1995). *The Music of the Spheres: Music, Science and the Natural Order of the Universe*. Springer, 1995.
- D. Kung (2013). *How Music and Mathematics Relate*. The Teaching Company Great Courses, 2013.
- G. Loy (2011). *Musimatics: The Mathematical Foundations of Music*. MIT Press, 2011.
- E. Maor (2018). *Music by the Numbers: From Pythagoras to Schoenberg*. Princeton University Press, 2018.
- G. Mazzola (2018). *The Topos of Music I: Theory*, 2nd ed. Springer, 2018.
- C. N. Menhinick (2015). *The Fibonacci Resonance and Other New Golden Ratio Discoveries*. OnPerson International Ltd., 2015.

- M. Müller (2016). *Fundamentals of Music Processing: Audio, Analysis, Algorithms, Applications*. Springer, 2016.
- R. Povilioniene (2017). *Musica Mathematica. Traditions and Innovations in Contemporary Music*. Peter Lang, 2017.
- E. Rothstein (2006). *Emblems of Mind: The Inner Life of Music and Mathematics*. University of Chicago Press, 2006.
- R. Schneiderman (2011). Can You Hear the Sound of a Theorem? *Notices of the American Mathematical Society* 58:7 (2011), 929–937.
- M. Tegmark (2014). *Our Mathematical Universe: My Quest for the Ultimate Nature of Reality*. Knopf, 2014.
- D. Wright (2009). *Mathematics and Music*. American Mathematical Society, 2009.

Author accepted manuscript