Optimal Chartering and Investment Policies for Bulk Shipping

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Abstract

This research develops a model for the determination of optimal policies for ship chartering (i.e., whether to accept a spot or term charter contract or to lay-up a vessel) and investment (i.e., the optimal timing for the purchase or sale a vessel) in the market for ocean transportation services of bulk commodities applying the methodology of contingent claims valuation of financial economics and the theory of stochastic optimal control taking advantage of the market for freight futures.

The model constructs a continuous-time arbitrage condition between freight futures contracts and the value of ship operations by assuming that freight rates dynamics follows a Brownian Motion/Wiener stochastic process. From this arbitrage condition a partial differential equation is derived for the valuation of ship operations. It is then possible to determine the optimal chartering policies, which are characterized by trigger freight rates, from stochastic optimal control theory.

After solving the optimal chartering problem it is then possible to solve the optimal investment decision as an analogous problem to the optimal strategy to exercise an option on a common stock. Similarly to the chartering case, partial differential equations are derived characterizing the optimal timing for ship investments.

A discrete-time model is also developed assuming that freight rates follows a binomial process. In this case recursive formulas are derived by dynamic programming.

A solution for the optimal policies and ship value for the perpetuity case in continuous-time is presented and shipping markets parameters (i.e., means, variances and the risk premia) are estimated for the spot and term charter markets for grain routes.

Numerical results for the optimal policies show the hysteresis effect; where the ship will only exit the market when freight rates are well below costs and will only re-enter when freight rates are well above costs. These results confirm the existence of "an option value to wait" in the bulk shipping market due to costs to move a ship in and out of operations and market volatility. Several sensitivity analysis results are also presented for the optimal policies.

Finally, in relation to the decision between spot and term charter contracts, the results for the grain trades indicate that this decision will also depend on ship specific characteristics (i.e., costs) rather than only the state of the shipping market.

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À Virginia .

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Chapter 1

Introduction

1.1 Objectives and Review of Previous Research

This thesis develops a model for the determination of optimal policies for chartering (i.e., leasing) and investment (i.e., sale and purchase) in ships in the market for ocean transportation services of bulk commodities applying the methodology of contingent claims valuation and the theory of stochastic optimal control. Advantage is taken of a recently established market for freight futures.

The ocean shipping market can be divided into two major markets: (i) the *liner* market and (ii) the *bulk* market. The former is the market for regular transportation services and transports mostly manufactured cargo in containers. The liner market is oligopolistic and resembles the airline market. Since this is a market that deviates from the competitive case, my research is not going to cover this sector. For this market, strategic competitive analysis using game-theoretical models would be appropriate[1].

On the other hand, the bulk[2] market is close to pure competition and in fact, the bulk shipping market is among the few close to perfect competitive markets that we

[1]Recently Huang and Li (1992) extended the methodologies used in this research for the case of oligopolistic competition, where the decision variable is the demand level rather than prices. Therefore the approach developed here can also be extended to the liner shipping case.

[2]The bulk shipping market is also called charter market.

encounter in the real world [3]. Therefore this research will focus on the development of optimal ship operational and investment policies for this market. The bulk market has two major submarkets: (i) the *liquid bulk* (mostly crude oil), and (ii) the *dry bulk* (ores and grains), cargo markets. Those markets are also very volatile, where rate fluctuations of 800 % have been recorded in the past. See Chang (1991) for recent empirical analysis.

The owner of a ship, the shipowner (or ship operating company) in the bulk market can face the following OPERATIONAL / CHARTERING (resource utilization) alternatives[4]:

- Charter the ship in the *spot* market (i.e., for one voyage).
- Charter the ship in the *term charter* market[5] (i.e., for multiple voyages).
- *Lay-up* the vessel[6](i.e., out of operation).

And the following INVESTMENT (resource allocation) alternatives:

- *Sell* a vessel in the second-hand market.
- *Purchase* a vessel in the second-hand market.
- *Order* a new vessel (i.e., buy a newbuilding).
- *Scrap* the vessel (i.e., sell for demolition).

[3]For a recent discussion of bulk shipping markets and perfect competition see Kreps (1990) Ch.8, for a detailed empirical analysis, see Zannetos (1966), and for a general analysis of the shipping industry, see Frankel (1987). Note that when I claim that bulk shipping markets are competitive is in the same sense that security and commodity markets are considered competitive.

[4]The thesis will cover the shipowner case. However, I believe that, the extension of the model to the shipper case (i.e., the firms that demand transportation services like oil companies, grain traders and steel mills), is somewhat straightforward.

[5]The term charter possibilities are quite wide. It varies from 1 year (short-term charter), to 2-3 years (medium-term charter), to 5-8 years (long term charter). For the purposes of this model we will assume that we have only one term charter rate. The term charter is also known as time charter or period charter.

[6]To lay-up the ship is to stop to operate the ship for a reasonable long period of time (e.g., from 1 to 5 years), in order to wait for better market conditions. When the owner decides to lay-up the ship he incurs some costs "in" and "out" of lay-up and some minimal maintenance costs during the lay-up period. Therefore, when the ship is laid-up variable costs becomes very small.

Even though the objective of his study was to understand the price formation mechanisms in bulk shipping, Zannetos (1966), was perhaps the first to quantify (via econometrics) the operation and investment alternatives facing the bulk shipping firm. However, he did not develop an optimization procedure for those decisions. His criteria was based on the ability of his model to predict prices and market conditions[7]. The first published work with a mathematical formalization of ship chartering decisions was a paper by Mossin (1968). Mossin's model considers only the possibility of "in" and "out" of operation, that is, when to lay-up the ship. He assumes that revenues (i.e., freight rates) follows a random walk and the underlying stochastic process is stationary. Due to the stationarity of the process the optimal policies are given by fixed thresholds; that is, if revenues fall to some level y the ship should be laid-up; if revenues rise to some level z the ship should be put back into operation. Those optimal stationary policies (under the assumption of constant costs), are determined by assuming the system behaves as a discrete state Markov chain following a Bernoulli process without discounting. Mossin's work was important however, because he considered the cost of moving into and out of lay-up, a feature of our analysis.

Devanney (1971), develops a discrete-time finite-horizon dynamic programming algorithm for ship chartering decisions. The basic model takes transition probabilities as exogenous variables (based on shipowner's beliefs about future freight rates), a single ship in a single route (e.g., US Gulf - Northern Europe), and then each stage is a round trip. At each stage of the algorithm the chartering possibilities are the ones given above. The shipowner beliefs about future freight rates are based on the following variables: (i) the current spot charter rate, (ii) the rate of change of spot rates and (iii) the amount of transportation capacity on order. Devanney also considers the term charter rate, the multiple voyage rate. His objective function is the maximum expected present value profit up to the end of the operational life of the ship (unless we scrap it, a termination state in the algorithm), measured in terms of round trip voyages. A limitation of Devanney's work, is that it is necessary to estimate freight rates and to make assumptions about risk preferences of investors/operators.

[7]See also Diniz(1986) and Hale and Vanags (1989) for recent work in this area.

Devanney expanded his model to consider "fleet policies" (i.e., for several ships), but then he encountered the curse of dimensionality: the state space becomes too large. Optimal fleet policies according to Devanney requires consideration of mixed policies, i.e., $\alpha\%$ of the fleet would be in term charter and the remaining (i.e., $(1-\alpha)\%$), in the spot. The reason that the optimal policy for a single ship is different from that for a fleet is due to capital market imperfections, since long term charters can be mortgaged but spot contracts can not[8].

A more detailed but less formal development of chartering strategies is presented in Norman (1981). Norman proposes two approaches: (i) *"portfolio of charters"*, and (ii) *"chartering timing policies"*.

For the *portfolio of charters*, Norman determines, based on historical data, the shipowner price of risk against shipowners risk preferences. Then the optimal mix of ships on the spot and term charter can be determined.

In the case of *chartering timing*, Norman considers a relation between spot charter and term charter rates such as $S = A + Bq$, where S is the spot rate and q the term charter rate. Thus if $S \geq A + Bq$, accept the spot contract and otherwise accept the term charter contract. The limitation of Norman's work is that he does not devise any methodology to determine an optimal strategy. His results are based only on time-series analysis of past rates showing that many of the "rule of thumb" operational policies used by the shipping industry are not optimal[9]. Norman does not consider the lay-up case. However he also studies the sale and purchase decisions.

[8]In Devanney's analysis a single ship policy is the policy prescribed by the algorithm while the "optimal" fleet policy is the result of empirical observation, that is, for a newly (not yet paid) vessel the owner would be "forced" to go to the term charter *always* in order to assure a steady cash flow to pay back the ship loan while for an older vessel (i.e., already paid) he would be free to choose and then be able to apply the policies prescribed by the algorithm. As will be stated below, we will assume perfect capital markets when modelling the chartering and investment decisions, hence those imperfections despite their importance are not considered.

[9]One of the "rules of thumb" of ship chartering is to deploy most vessels of a fleet in long term charters in order to guarantee cash flows to pay back ship loans. In this type of policy the shipowner is substantially risk-averse as far as the shipping market is concerned but less so as far as capital investments are concerned. According to Norman (1981), for the period 1963-1979 a better policy would have been to have smaller investments in vessels (lower financial exposure), and be more aggressive in the shipping market (the case studied was the tanker market), that is, charter most vessels for most of the time in the spot market, taking advantage of the substantially higher rates in this market.

Another study of chartering strategy is given in the paper by Taylor (1981). Taylor proposes a computer driven simulation model to determine the optimal "fleet mix". A novel feature of Taylor's study is the possibility of including combined carries (ships that can carry both dry or liquid bulk cargoes). Then the shipowner can operate with added flexibility in both submarkets. Taylor analysis assumes the existence of a so called "chartering preference function" that shows the fraction of long term charters shipowners are willing to take as a function of a freight index. Taylor's work, however, does not show how to determine those preference functions nor does his methodology guarantees optimality.

Based on the above review it seems clear that up to this date a logically complete rational decision support system to optimize chartering and investment strategies has not yet been developed. The shipping industry continues to develop econometric forecasting, but forecasting alone will not solve the problem. Forecasting is just a first step in a more rational decision making process. The question that remains to be answered is: if you have the best available econometric model, what is the OPTIMAL CHARTERING AND INVESTMENT POLICY based on a rational decision making model?

An answer, as shown in this thesis, is to employ stochastic optimal control theory and financial economics, in particular capital markets theory, that have already solved many real world speculative markets decision problems. With the development of the freight futures market in the London Baltic Exchange in 1985, the shipping industry can take advantage of the methods developed in the 1970's to achieve rational pricing of derivative securities (i.e., options, forward and futures contracts), this in turn leads to the determination of *optimal* operational and investment policies with the methodology of contingent claims valuation.

1.2 Brief Overview of the Freight Futures Market

Since the existence of the freight futures market makes possible the application of contingent claims valuation methodology[10], a brief description of this market will be made.

The first freight futures market to be developed was the Baltic Freight Futures Contract (BFFC), which was introduced by the Baltic International Freight Futures Exchange (BIFFEX), in London in 1985. The BFFC is traded as an INDEX, the Baltic Freight Index (BFI), which does not involve any standardized ship or cargo, but a "basket" of voyage freight rates. The BFI is calculated from each day's freight rates for a set of 12 dry bulk cargo voyages, and each carries a different "weighting" calculated according to its historical data and importance on the whole dry bulk market.

Because the BFI is an index market, it may not be a perfect hedge to all routes; some routes are well correlated and others are not. Thus not all shipowners (or routes) can take advantage of freight futures to devise optimal chartering (or hedging) policies[11].

Chang (1991) provides a detailed statistical analysis of the freight futures market including a study of the correlations between the BFI and each individual route[12]. He does not reject the hypothesis that freight futures prices are unbiased estimators of future spot prices. These are important empirical findings that give support to assumptions considered later. Moreover, some preliminary tests indicate that freight futures are reasonably informational efficient, in the sense that current information that affects prices are included in prices.

For a detailed description and contractual details of the freight futures markets see Chang (1991). For hedging strategies see also Tang (1990).

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[10]Contingent claims valuation can be viewed as a generalisation of the option pricing approach developed by Black and Scholes (1973) for more general financial instruments and real assets. For a good review of contingent claims applications and its role as a tool for capital budgeting decisions see Mason and Merton (1985) and Pindyck (1991).

[11]With the rapid expansion of capital markets in most countries, perhaps in the near future more routes and cargoes will have their freight futures markets.

[12]He also describes how to compute implied rates an how to hedge a particular route.

1.3 Thesis Organization

The thesis is organized as follows. Chapter 2 presents a general continuous-time model for ship value and optimal policies determination for both chartering and investment decisions. An important step in this model is the determination of the freight futures pricing and dynamics, which due to the non-storability of transportation services, results from the theory of bond pricing was used instead of usual direct arbitrage arguments. Partial differential equations are then derived from an arbitrage condition between freight futures contracts and the value of ship operations for the determination of optimal policies, which are characterized by trigger freight rates.

Chapter 3 presents a discrete-time model where the economic intuition behind the replicating portfolio (the "synthetic ship"), becomes easier to understand. In this case recursive formulas are derived for the chartering and investment decisions by dynamic programming. A preliminary model for the term structure of freight rates is also presented.

In Chapter 4 a solution for the optimal policies and ship value for the perpetuity case in continuous-time is presented where the partial differential equations of Chapter 2 are transformed into ordinary differential equations and then simple numerical procedures are sufficient to implement a solution. Chapter 5 presents the shipping market parameter estimation; means and variances for the Brownian Motion process assumed for freight rates and the risk premia for freight futures for some important grain routes.

In Chapter 6 the optimal policies determined in Chapter 4 together with the shipping market parameters estimated in Chapter 5 are implemented for the grain routes. Numerical results from the optimal policies then show the hysteresis effects where a ship will only exit the market when revenues are well below costs and will only re-enter when revenues are well above costs. Several sensitivity analysis results are also presented for the optimal policies and ship value.

Finally, in Chapter 7 the implications of this thesis results in relation to ship design, to the management of the shipping firm and as a possible explanation to shipping market cycles are discussed and proposals for further research and concluding remarks are presented.

Chapter 2

The Continuous-Time Model

In this chapter the contingent claims valuation and the subsequent optimal control problem will be developed in continuous-time. An alternative discrete-time formulation is presented in Chap. 3. Continuous-time models are extensively used in financial economics, the basic idea is to develop a *continuous-time arbitrage condition* and then use the tools of stochastic control theory to determine optimal policies for managerial decision making.

The advantage of the arbitrage condition is that it makes possible to avoid the specification of a utility function specifying the decision maker preferences and in certain cases the need to estimate the expected rates of change of the underlying cash flows and output prices (in our case freight rates)[1]. Therefore we only need "observable"[2] variables such as variances of returns and interest rates.

The idea of constructing a continuous-time arbitrage condition for pricing securities was first developed by Black and Scholes (1973) and Merton (1973)[3]. The basic structure of the riskless arbitrage condition is the following: take a *long* position in the security (e.g., a commodity or as in our case, a cargo vessel) and a *short* position

[1]This is the case for stock options, where the drift term of the stochastic process driving the price of the underlying security drops out as we construct the arbitrage condition. Unfortunately, this does not always occurs, and this will be the case for the ship chartering decision as will be shown bellow.

[2]The meaning of observable here is that we can get robust statistical estimates. This is the case for the variance but not for the mean of a stochastic process. See chapter 5 for further discussions.

[3]See Hull (1990) for an introductory but comprehensive exposition of the possibilities of derivative securities pricing, and Vila (1990b) for general continuous-time models in finance.

in a *suitable* amount of the derivative security (e.g., a futures contract) such that, the instantaneous change in prices of the two are identical and finance the difference at the riskless interest rates. Since a change in the long position is offset by a change in the opposite direction in the short position, the instantaneous return of this portfolio is riskless and is given by the risk-free interest rate. A partial differential equation is then derived that is satisfied by the derivative security value.

To develop this idea we have to construct a self-financing portfolio of riskless bills and futures contracts that REPLICATES the value of the ship[4]. Therefore this model pressuposes the existence of a *futures market in "output" prices*. Thus, before 1985 the shipping industry couldn't take advantage of an arbitrage between freight futures and the value of ship operations[5].

2.1 The Chartering Decision

The ideas of constructing an arbitrage condition and of portfolio replication have been extended to price financial instruments other than options and warrants to "real assets" (also called real options) as well; for instance the pricing of corporate liabilities and of managerial flexibility in capital investments. The basic methodology that encompasses such applications is called contingent claims valuation[6].

The model we develop for the determination of optimal policies for bulk shipping chartering (operations) and later for investments is similar to a natural resource investment model developed by Brennan and Schwartz (1985) (B&S)[7]. However, the ship chartering decision is not only a decision of "in" and "out" of operation (like opening, closing and reopening a mine), but also the decision as to the type of contract to accept: a *spot* (one voyage) or *term charter* (multiple voyages), contract[8].

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[4]The idea of portfolio replication ("synthetic ship") is presented in a simpler setting in the discrete-time model in Chap. 3.

[5]Indeed, it seems that we are the first to do so.

[6]Also known as derivative securities analysis or options pricing theory.

[7]See also Dixit (1989) for a similar model.

[8]That is, long vs. short term commitment. Even though for the case of natural resources there is the possibility of long term contracts and B&S price these contracts in their paper, it is not to my knowledge a continuously available possibility quoted in an open market as it is the case for term

In order to be able to model simultaneously spot versus term charter decision, I intend to approach the problem as given by the following conjecture:

Conjecture 2.1.1 *First, optimal policies (and pricing) for the spot/lay-up condition will be computed as if there was no term charter option. Second, optimal policies for the term charter/lay-up condition will also be computed as if there was no spot charter option. Then the "final" optimal policy will be given by the comparison between the optimal spot charter policy against the optimal term charter policy. The policy that attains the highest value on the ship valuation function given by the partial differential equation from the arbitrage condition will be chosen.*

This formulation requires that two optimal control problems be solved simultaneously in a "parallel" fashion[9].

The problem may be formulated differently. For instance, assume that spot rates and term charter rates are priced in equilibrium such that the shipowner is indifferent between the two types of contracts. Hence, the problem would collapse to one of a decision between spot versus lay-up only, that is, the ship is "in" or "out" of operation. In equilibrium, there would be no gain by choosing spot or term charter contracts. However, based on actual shipowner/operator behavior, there is reason to believe , that there are different incomes to be generated in each of those markets since it is a well known fact that there are operators who only operate in one of those markets most of the time making the indifference assumption very weak. I can not prove rigorously that partitioning is best, hence the above conjecture.

Remark 2.1.1 *The model to be developed here considers only the case of a single ship in a single route. However, according to Zannetos (1966), in the bulk trades for most cases each ship is operated as if the ship is the firm. That I am not considering the fleet of ships and the multiple routes case may be not after all a very restrictive*

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charter contracts in bulk shipping.

[9]I suspect that, when such a system will be implemented for actual applications, the best way to implement a solution would be through some sort of parallel computation. Moreover, by approaching the problem in this way we rule out a closed form solution for optimal policies which would include the spot and term charter contracts at the same time.

assumption. Due to the increased dimensionality of the problem, the mathematical complexity of the model would increase substantially.

The following assumptions are made in modelling the chartering decisions:

1. Markets are considered competitive and "quasi-frictionless". By competitive I mean producers and consumers in the bulk shipping markets are price takers. By "quasi-frictionless" I mean transaction costs only occur when the ship moves in and out of lay-up and taxes are limited to a fixed amount on shipping revenues. Trading can take place continuously and there are no restrictions on borrowing and short selling with full use of proceeds.

2. The spot freight rate S follows a continuous-time stochastic process:

$$dS = S\mu dt + S\sigma dZ \tag{2.1}$$

   Where dZ is a Brownian Motion/Wiener process, σ the standard deviation of spot rates and μ the mean or local trend in prices. Both σ and μ are assumed to be non-stochastic and known.

3. Borrowing and lending rates are equal to the interest rate r which is known and non-stochastic.

4. No assumption is made about investors/operators preferences. However, it is assumed that they agree on the estimators of the parameters of the stochastic process for the spot rate S[10].

Before developing the continuous-time model further a discussion about the above assumed spot rate process is perhaps in order. While for modelling security markets price processes Geometric Brownian Motion is considered a "standard assumption", for commodity markets (e.g., natural resources), empirical evidence suggests that

[10]These assumptions are considered "standard" in the contingent claims valuation literature. However, for many applications the results are quite robust and quite often used for practical purposes. Hopefully, this will be also the case for the shipping problem.

those markets have lagged correlation effects and hence are not memoryless. Zannetos (1966) discusses extensively the role of expectations in the shipping markets and the lagged effects that shipbuilding can exert in the future supply of shipping services[11].

However, as can be seen from the study of Stensland and Tjostheim (1989) (S&T), in which they model the natural resources problem of B&S with a correlated stochastic process (by fitting auto-regressive models to the data), there is a price to be paid in terms of complexity when solving the optimal control problem for the correlated case. As a result they only determine threshold prices (i.e., an optimal policy) for opening the mine once, and thus there is no possibility of closing and reopening the mine.

Therefore, my observation is that by having a more realistic price process we may lose the possibility of getting a richer set of optimal policies. Note also that, for the chartering problem, the state space is already larger than the mine problem as discussed above. Thus I will develop the continuous-time model further and determine optimal policies under the Wiener process price model since this will be a less complex mathematical problem and will perhaps give still realistic results. After this is done and having a better understanding of the mathematical structure of the problem it will then be possible to explore the correlated case either by considering some mean-reverting process in continuous time or some AR process in discrete-time.

2.1.1 The Pricing and Dynamics of Freight Futures

The next step in the derivation of the optimal chartering policies and ship valuation is the determination of the freight futures pricing and dynamics.

Let $F(S,\tau)$ denote the freight futures rate[12], which is the implied freight futures rate for a specific route, that is a function of the spot rate S and time to maturity τ[13]. Now, since $F(S,\tau)$ is a function of a Wiener process, the *instantaneous* change in

[11]That is, as freight rates go up more ships are ordered and then at a later period freight rates will decline due to the increased capacity available. Therefore, it may matter for a policy decision whether a certain freight rate was reached on an up-going or down-going trend.

[12]Or an "equivalent spot rate" taking into account the freight futures index composition as discussed above. Recall that the freight futures is given by an index, the BFI, however we can still compute the value of the futures for a specific route. See Chang (1991) and Tang (1990) for details.

[13]That is, a freight futures at time t for a contract to be closed at time T, where $\tau = T - t$. Note

futures prices is given from Itô's Lemma[14] by:

$$dF = -F_\tau dt + F_S dS + \frac{1}{2}(F_{SS}(dS)^2) \tag{2.2}$$

Now, substituting for dS given by (2.1) above and $(dS)^2$ which by Itô's differentiation rules is given by $\sigma^2 S^2 dt$ into (2.2) and rearranging we get the following dynamics for the futures price:

$$dF = [\mu S F_S - F_\tau + \frac{1}{2}(F_{SS}\sigma^2 S^2)]dt + S\sigma F_S dZ \tag{2.3}$$

Remark 2.1.2 *At this point of the derivation the chartering problem becomes again different from the natural resources problem . The distinction lies in the fact that natural resources are storable and then it is possible to develop an arbitrage condition between futures contracts and the natural resource in order to price the futures as it is the case in B&S. Freight rates on the other hand are "non-storable"*[15] *thus it is not possible to value the futures by usual arbitrage arguments. An alternative approach is presented below.*

Since we can't use a direct arbitrage argument to price the freight futures as in the case for storable goods, where it is possible to develop an arbitrage condition between the good and its futures contracts, we have instead to determine the freight futures price of risk or risk premium. That is, how much investors are willing to pay above or below the risk-free rate to hold positions in freight futures.

The approach to follow is similar to the case of pricing bonds and interest rate sensitive securities. This is because bonds can be viewed as contingent claims on

that by changing time I am considering the "backward operator".

[14]Since a Wiener process (which is the process followed by the dynamics of the spot rate S) is continuous but almost nowhere differentiable, in order to take derivatives we have to resort to stochastic calculus, and Itô's Lemma is the equivalent to the fundamental theorem of calculus for this process. For further details on stochastic calculus and its applications see Merton (1990), Harrison (1985) and Duffie (1988).

[15]In the sense that if you have a contract to transport cargo between port A and port B and if you don't use it in the specified period, it became valueless in the future. More generally, this will be the case for any contingent claim on *services* which are non-storable goods.

interest rates which, as freight rates, are "non-storable assets"[16]. In the pricing of bonds (and for fixed-income securities in general) it is also necessary to determine their risk premia.

In the financial economics literature there are two approaches to determine bond pricing and its risk premium: (i) general equilibrium models (ii) partial equilibrium/arbitrage models.

The general equilibrium approach is developed and used in Cox, Ingersoll and Ross (1985a,b) CIRa / CIRb for pricing bonds and for the determination of the term structure of interest rates. In CIRa a complete general equilibrium model for asset pricing is developed in which the possibility of investing in contingent claims is included in the investor choice set. They then derive the risk premia (excess returns) for contingent claims in their theorem 2. Since this is an important result to be used to price the freight futures I will state below a simplified version (one dimension) of this theorem and I refer to the paper for proofs and details of the model.

Theorem 2.1.1 *(Cox, Ingersoll and Ross 1985a) The equilibrium expected excess return on any contingent claim is given by:*

$$(\beta - r)F(Y,t) = [\varphi_W \varphi_Y] F_Y(Y,t) \tag{2.4}$$

Where: F(Y,t) is the value of contingent claim, W investor wealth, Y state variable driving the economy, φ asset prices, β return on contingent claim and subscripts denotes partial derivatives.

For contingent claims that are contractually independent of wealth the above simplifies to:

$$(\beta - r)F(Y,t) = \varphi_Y F_Y(Y,t) \tag{2.5}$$

The alternative approach to derive the risk premia for bonds is by a partial equilibrium/arbitrage argument. In this case the risk premium is determined by imposing assumptions on the variables the contingent claim depends and on the stochastic

[16]In the sense that you can not hold them between periods to create an arbitrage.

properties of the underlying variables and then application of Itô's lemma will lead to an excess returns formula equivalent to the one given by theorem 2 above. This approach was initiated by Merton (1970) and is used by Richard (1978) among others for bond pricing.

The basic idea of the arbitrage approach is as follows. Let $\Upsilon(Y,t)$ denote the risk premium of the contingent claim. That is, the equilibrium expected return is given by:

$$(\beta - r)F(Y,t) = \Upsilon(Y,t) \tag{2.6}$$

Then portfolios of contingent claims with different maturities are created and using a replication argument it is shown (see e.g., Merton (1991)) that in order to prevent arbitrage opportunities $\Upsilon(Y,t)$ must have the form:

$$\Upsilon(Y,t) = \gamma(Y,t)F_Y(Y,t) \tag{2.7}$$

Now, if we can postulate a functional form for $\gamma(Y,t)$ that prevents arbitrage opportunities, in equilibrium it must be the case that $\varphi_Y(Y,t) = \gamma(Y,t)$ and then both approaches are equivalent. However as discussed and showed in CIRb, closing the model by assuming a functional form for $\gamma(Y,t)$ can lead to internal inconsistencies that will not prevent arbitrage opportunities. Therefore care should be taken when formulating $\gamma(Y,t)$ [17].

Specializing the above models and results developed for generic contingent claims and bond pricing for the case of freight futures, a first feature to be noticed is the fact that for futures contracts in general, expected returns are not well defined. This is because at initiation no payment is made for futures contracts (i.e., at $t = 0$, $F(S,0) = 0$), apart from margin requirements. Therefore, for the case of futures contract instead of:

$$E[dF/F] = (r + \Upsilon(S,t))dt \tag{2.8}$$

[17] See also Ingersoll (1987) Ch. 17 for further discussions on this issue.

We have:

$$E[dF] = (\Upsilon(S,t))dt = (\gamma(S,t)F_S(S,t))dt \tag{2.9}$$

Since no money change hands at initiation ($F(S,0) = 0$), it is as if there is no risk free investment opportunity and the investor only gets the premium $\Upsilon(S,t)$ and not $r + \Upsilon(S,t)$ as it is the case for bond pricing.

In order to compute $\varphi(S,t)$ it is necessary to develop an equilibrium model as it is the case in CIRa,b . This equilibrium model requires the specification of a functional form to investors utility. On the other hand, the partial equilibrium / arbitrage approach requires an assumption about the functional form for $\gamma(S,t)$. Therefore, for the case of interest and freight rates, an assumption in relation to a functional form will be needed. In order to avoid the development of a general equilibrium model, in this thesis I will follow the arbitrage approach and I will assume a functional form for $\gamma(S,t)$ that will be equivalent to the ones used in bond pricing (see e.g., Merton(1991)) , which is to assume that $\gamma(S,t)$ is linear in S, the spot freight rate, that is:

$$\gamma(S,t) = \lambda S \tag{2.10}$$

Where λ is a risk premium parameter to be estimated from freight futures data.

Then from (2.3) which gives the dynamics for F by applying Ito's lemma to $F(S,\tau)$ and from (2.9) and (2.10) above we get a partial differential equation that should be satisfied by the value of the freight futures contract, that is:

$$-F_\tau + \frac{1}{2}(F_{SS}\sigma^2 S^2) + S\mu F_S = excess\, returns = \lambda S F_S \tag{2.11}$$

Then the freight futures valuation equation becomes:

$$\frac{1}{2}(F_{SS}\sigma^2 S^2) + (\mu - \lambda)SF_S - F_\tau = 0 \tag{2.12}$$

And the freight futures is given by the solution to (2.12) subject to the initial condition $F(S,0) = S$, which is a well known result given by:

$$F(S,\tau) = Se^{(\mu-\lambda)\tau} \qquad (2.13)$$

An equation equivalent to (2.12) is derived in Cox, Ingersoll and Ross (1981) for futures contracts in a different context[18].

Substituting the PDE (2.12) into equation (2.3) and rearranging, the instantaneous change in the futures price is given by:

$$dF = SF_S\lambda dt + F_S S\sigma dZ \qquad (2.14)$$

Equation (2.14) gives the dynamics followed by the freight futures prices. The next step is to derive the partial differential equation that must be satisfied by the ship value and to determine the optimal ship operational policy.

There is another distinction apart from the storability question that differentiates the shipping problem from the mine problem of B&S. In the mine case we have a continuum of possible output rates while in the ship case we have only the possibility of operating or not operating and if operating we can operate for a voyage (spot charter) or multiple voyages (term charter)[19]. However we can also see the problem as one of "ship cash flow" valuation and hence instead of determining "optimal output policies", determine *"optimal freight rates"/"trigger rates"* in order to accept one of the above contracts or eventually, if the rates are sufficient low, lay-up the ship (i.e., out of operation for a period of time).

[18]Notice that the above PDE for freight futures does not have the $-rF$ term that appears in the bond equation (see e.g., equation (22) p.393 of CIRb).

[19]Of course, we can also vary the ship speed in a continuum fashion and then have a continuum of "outputs"(note that ship output is given by ton-miles, that is, the amount of cargo carried times the distance traveled, for details see Appendix C). Even though "low steaming"(i.e., to operate the ship in a low speed in order to save fuel costs) is sometimes used in practice, for most cases charter contracts cover a specific amount of cargo to be transported in a specific period of time. Thus the scope to vary ship velocity and then output is very limited leading to a fixed operating speed, hence output, which on a first approximation, is not an important control and will not be considered in the present model.

2.1.2 Determination of the Ship Value

In this section we solve the ship valuation problem and the determination of optimal policies for the *spot vs. lay-up condition only.* A later section develops the term charter vs. lay-up problem.

Now th e have developed the dynamics for the freight futures we are ready to determine shi value. The ship value will be assumed to depend on the current spot rate S, the ship second-hand market value Q, calendar time t and ship operating policy ϕ. The value of the ship is then given by :

$$H = H(S, Q, t; j, \phi) \tag{2.15}$$

Where j = 1 if the ship is operating in the spot market and j = 0 if the ship is laid-up.

The ship operating policy will then depend on the critical freight rates:

- $S_1^*(Q,t)$: freight rate level at which the ship is laid-up if it was operating.

- $S_2^*(Q,t)$: freight rate at which the ship is put back into operation being laid-up.

Since H is a function of S we can again apply Ito's Lemma to (2.15), and then the instantaneous change in the value of the ship is given by:

$$dH = H_S dS + H_Q dQ + H_t dt + \frac{1}{2}(H_{SS}(dS)^2) \tag{2.16}$$

The instantaneous change in the value of the ship as a capital equipment, is given by:

$$dQ = -qQdt \tag{2.17}$$

where q is the rate of depletion of the ship, which may be a function of the speed.

The after-tax cash flow, or continuous dividend rate of the ship is given by[20]:

$$D = [S - A - T]j - M(1 - j) \tag{2.18}$$

Where:

- A(q,Q,t) : average cash cost of ship operations (per ton of cargo).
- M : fixed cost of maintaining the ship laid-up.
- T : income taxes.

To derive the differential equations governing the value of the ship under operating policy ϕ we have to construct a replicating portfolio made of the following[21] :

TAKE A LONG POSITION ON : H (i.e., the physical ship)

TAKE A SHORT POSITION ON : XF (i.e., an amount X of freight futures contracts)

Where X is the amount of freight futures to be determined such that the instantaneous return on the portfolio is riskless. Letting V be the value of this portfolio we get:

$$V = H - XF \tag{2.19}$$

By differentiating, the dynamics of the portfolio is given by:

$$dV = dH - XdF \tag{2.20}$$

Now, if we substitute dH by its expression given by equation (2.16) and dF by its expression given by equation (2.14) in (2.20) we get :

$$dV = H_S dS + H_Q dQ + H_t dt + \frac{1}{2}(H_{SS}(dS)^2) - X[F_S S\lambda dt + F_S S\sigma dZ] \tag{2.21}$$

[20]Note that we are determining the value of the ship under various operating conditions. The meaning of value here is the instantaneous value of all future (and uncertain) cash flows/income that the ownership of a ship generates. Note also that, under the continuous-time model, at each t the ship has a value of $H(S, Q, t, ; j, \phi)$ and also generates at each dt an instantaneous cash flow D, which may be positive or negative.

[21]This is now a standard procedure in financial economics developed for the first time by Black and Scholes(1973).

From equation (2.1), we have that:

$$(dS)^2 = (\mu S dt + \sigma S dZ)^2 = \mu^2 S^2 (dt)^2 + 2\mu S dt \sigma S dZ + \sigma^2 S^2 (dZ)^2 = \sigma^2 S^2 dt \quad (2.22)$$

where the last equality comes from Itô's differentiation rules that states that :

$$(dt)^2 = 0,\ dtdZ = 0 \, and \, (dZ)^2 = dt \quad (2.23)$$

Finally by substituting dS and $(dS)^2$ by their expressions given by equations (2.1) and (2.22) respectively in equation (2.21) and rearranging we get :

$$dV = [\frac{1}{2}(\sigma^2 S^2 H_{SS}) + \mu S H_S + H_t]dt + H_S \sigma S dZ - X[S F_S \lambda dt + F_S S \sigma dZ] - H_Q dQ \quad (2.24)$$

Now, in order to make the above portfolio riskless the stochastic component of the portfolio dV should vanish, that is:

$$H_S \sigma S - X F_S S \sigma = 0 \quad (2.25)$$

Which leads to:

$$X = \frac{H_S}{F_S} \quad (2.26)$$

Equation (2.26) above give the amount of freight futures in the short position in order to make the portfolio V instantaneously riskless. Since this return is nonstochastic (because any fluctuation on the long position is offset by the short position), and to avoid riskless arbitrage opportunities the return in this portfolio must be equal to the riskless return on the value of the investment plus dividends/cash flows, that is:

$$dV + Ddt = rVdt \quad (2.27)$$

Now, since at initiation the value of the freight futures contract is zero,we have that:

$$V(S,0) = H(S,0) + 0 = H \Rightarrow dV + Ddt = rHdt \quad (2.28)$$

Where D is given by (2.18), the after tax cash flow (or instantaneous dividend). Substituting the value of X given by (2.26) and dV given by (2.28) into (2.24) and rearranging we get:

$$[\frac{1}{2}(H_{SS}\sigma^2 S^2) + (\mu - \lambda)SH_S - H_Q Qq + H_t + D]dt = rHdt \qquad (2.29)$$

By rearranging (2.29) and substituting D by (2.18) we get the partial differential equation that the value of the ship must satisfy :

$$\frac{1}{2}(\sigma^2 S^2 H_{SS}) + (\mu - \lambda)SH_S - qQH_Q + H_t + (S - A - T)j - M(1 - j) - rH = 0(j = 0, 1) \qquad (2.30)$$

Equation (2.30) is a central result in this thesis. It characterizes the value of the ship at all times for both operations in the spot market ($j = 1$) and out of operations in the laid-up condition ($j = 0$), that is, for any operating policy $\phi = (S_1, S_2)$.

Now, under the value maximizing operating policy $\phi^* = (S_1^*, S_2^*)$, the value of the ship when operating denoted by $\Pi(S, Q, t)$, and not operating denoted by $\Omega(S, Q, t)$ is given by :

$$\Pi(S, Q, t) = Max_\phi H(S, Q, t, 1, \phi) \qquad (2.31)$$

$$\Omega(S, Q, t) = Max_\phi H(S, Q, t, 0, \phi) \qquad (2.32)$$

That is, the value maximizing policies satisfy the following equations:

(i) For the ship in a spot contract (i.e., operating), $j = 1$:

$$\frac{1}{2}(\sigma^2 S^2 \Pi_{SS}) + (\mu - \lambda)S\Pi_S - qQ\Pi_Q + \Pi_t + (S - A - T) - r\Pi = 0 \qquad (2.33)$$

(ii) For the ship in laid-up (i.e., out of operation), $j = 0$:

$$\frac{1}{2}(\sigma^2 S^2 \Omega_{SS}) + (\mu - \lambda)S\Omega_S + \Omega_t - M - r\Omega = 0 \qquad (2.34)$$

So far no switching costs were considered in the model. However in order to move a ship into and out of the lay-up condition the shipowner/operator incurs non trivial costs[22]. Therefore those "lay-up switching costs" should be considered in the model. I will denote by $K_1(Q,t)$ the costs to transfer the ship *into* lay-up, that is, the ship was previously operating, and by $K_2(Q,t)$ the costs to move the ship *out* of lay-up.

2.1.3 The Bellman Equations and Optimal Controls

At this point this optimization problem becomes one of optimal stochastic control[23], and more precisely one of singular/impulse control in which the singular controls are given by the discrete binary variable: $j = 1$ or $j = 0$. This mean that controls are not continuously applied, as for instance in the portfolio selection problem[24] where adjustment in consumption and portfolios proportions are made continuously in time, but only after the state of the system crosses a certain boundary are controls applied. A typical case of singular control is the cash-management problem with transaction costs, where the optimal amount of cash is to be maintained between upper and lower boundaries[25].

Note also that the determination of the ship operational policies, even though is an instance of singular or impulse control, it does not as in the case of the cash-management problem, applies the control to bring the system to within bounds, but it triggers a change of REGIME, that is, from a condition of *operation* to a condition of *no-operation* and vice-versa. Thus, the cash-management and other related singular control methodologies does not apply. A rigorous mathematical derivation would involve methods from the field of variational inequalities in order to derive the Bellman equation characterizing the optimal change of regime[26].

[22]These cost normally include: the job relocation/firing of the crew, the transfer of the ship to a location of deep and protected waters (e.g., the Norwegian fjords), the preparation/maintenance of the vessel machinery and equipment for long term out of operations and other legal and administrative costs.

[23]See Fleming and Rishel (1975) for the general theory of stochastic optimal control in continuous-time.

[24]See Merton (1971) for the initial paper and Fleming, Grossman, Vila and Zariphopoulou (1990) and Davis and Norman (1990) for recent developments including transaction costs.

[25]See Constantinides (1976).

[26]For a full treatment of impulse control and variational inequalities see Bensoussan and Lions (1982) and for an application in the context of the portfolio problem see Fleming et al. (1990). See also Tuckman and Vila (1991) for a control problem of regime change in the context of security taxation.

However, it is possible without those more advanced methods to develop the optimality conditions using stochastic dynamic programming arguments. First, suppose we start from a condition in which the ship is in lay-up (out of operation). In order to remain in the lay-up regime the value of the ship in this regime should be greater than the value of the ship moving out of lay-up (and incurring costs $K_2(Q,t)$) and accepting a spot charter contract. From the principle of optimality of dynamic programming we have the following:

$$\Omega(S,Q,t) = Max\{\Pi(S,Q,t) - K_2,$$
$$\Omega(S,Q,t) + [\frac{1}{2}(\sigma^2 S^2 \Omega_{SS}) + (\mu - \lambda)S\Omega_S + \Omega_t - M - r\Omega]dt\} \qquad (2.35)$$

Then we have the following conditions:

(i) If it is optimal to remain laid-up, that is apply control j=0, we have the following value maximizing or Bellman equation for the ship:

$$\frac{1}{2}(\sigma^2 S^2 \Omega_{SS}) + (\mu - \lambda)S\Omega_S + \Omega_t - M - r\Omega = 0 \qquad (2.36)$$

which was generated by the arbitrage condition between freight futures and ship value.

And the inequality:

$$\Omega(S,Q,t) \geq \Pi(S,Q,t) - K_2(Q,t) \qquad (2.37)$$

should hold, which means that in order not to change regime the value of the ship in the current regime should be greater than the value of the ship in the new regime less the costs of regime change.

(ii) If it is optimal to change regime, that is apply control j=1, then we have the inequality:

$$\frac{1}{2}(\sigma^2 S^2 \Omega_{SS}) + (\mu - \lambda)S\Omega_S + \Omega_t - M - r\Omega \leq 0 \qquad (2.38)$$

which means that the no arbitrage condition will not hold for lay-up.

And the value matching condition:

$$\Omega(S,Q,t) = \Pi(S,Q,t) - K_2(Q,t) \tag{2.39}$$

That is, the value of the ship in the new regime should cover the costs to change regime.

(iii) At the trigger rate $S_2^*(Q,t)$, that characterizes the regime change from the lay-up to the spot contract condition, the "smooth pasting" condition (also known as Merton-Samuelson high contact condition), must be satisfied, that is:

$$\Omega_S(S_2^*,Q,t) = \Pi_S(S_2^*,Q,t) \tag{2.40}$$

Heuristically, the "smooth pasting" condition means that if $\Omega(S,Q,t)$ is differentiable at S_2^* then its derivative there should be equal to the derivative of $\Pi(s,Q,t) - K_2(Q,t)$, otherwise we would have a kink that would lead to an improvement in the ship value by deviating from the optimal policies. See Dixit (1989) appendix A and Dixit (1991a,b) for further results including existence and uniqueness for the perpetuity case.

Now we can characterize the conditions for moving *into* lay-up (out of operation) from a condition in which the ship is currently on a spot charter contract[27]. In order to do so optimally the value of the ship operating in the spot market should be greater than the value of the ship out of operations (and incurring costs $K_1(Q,t)$). Again from the principle of optimality of dynamic programming we have the following:

$$\begin{aligned}&\Pi(S,Q,t) = Max\{\Omega - K_1,\\ &\Pi + [\frac{1}{2}(\sigma^2 S^2 \Pi_{SS}) + (\mu - \lambda)S\Pi_S - q\Pi_Q + \Pi_t + (S - A - T) - r\Pi]dt\}\end{aligned} \tag{2.41}$$

Then we have the following conditions:

(i) If it is optimal to remain in the spot market (i.e., accept a new spot contract),

[27]More precisely, at the end of a spot contract voyage where the ship is in a position to eventually accept a new spot contract.

that is apply control j=1, we have the following value maximizing or Bellman equation for the ship:

$$\frac{1}{2}(\sigma^2 S^2 \Pi_{SS}) + (\mu - \lambda) S \Pi_S - q\Pi_Q + \Pi_t + (S - A - T) - r\Pi = 0 \qquad (2.42)$$

which was generated by the arbitrage condition between freight futures and the ship value under spot contracts.

And the inequality:

$$\Pi(S, Q, t) \geq \Omega(S, Q, t) - K_1(Q, t) \qquad (2.43)$$

which means that the value of the ship in the current regime should be greater than the ship value in the new regime less the costs of regime change.

(ii) If it is optimal to change regime, that is apply the control j=0, then we have the inequality:

$$\frac{1}{2}(\sigma^2 S^2 \Pi_{SS}) + (\mu - \lambda) S \Pi_S - q\Pi_Q + \Pi_t + (S - A - T) - r\Pi \leq 0 \qquad (2.44)$$

which means that the no arbitrage condition will not hold for spot contracts.

And the value matching condition:

$$\Pi(S, Q, t) = \Omega(S, Q, t) - K_1(Q, t) \qquad (2.45)$$

where again the value of the ship in the new regime should cover the costs to change regime.

(iii) At the trigger rate $S_1^*(Q, t)$ that characterizes the regime change from spot contracts to lay-up, the "smooth pasting" condition is given by:

$$\Pi_S(S_1^*, Q, t) = \Omega_S(S_1^*, Q, t) \qquad (2.46)$$

Finally by solving the partial differential equations (2.36) and (2.42) with boundary conditions (2.39),(2.40),(2.45) and (2.46) we can determine the value of the ship

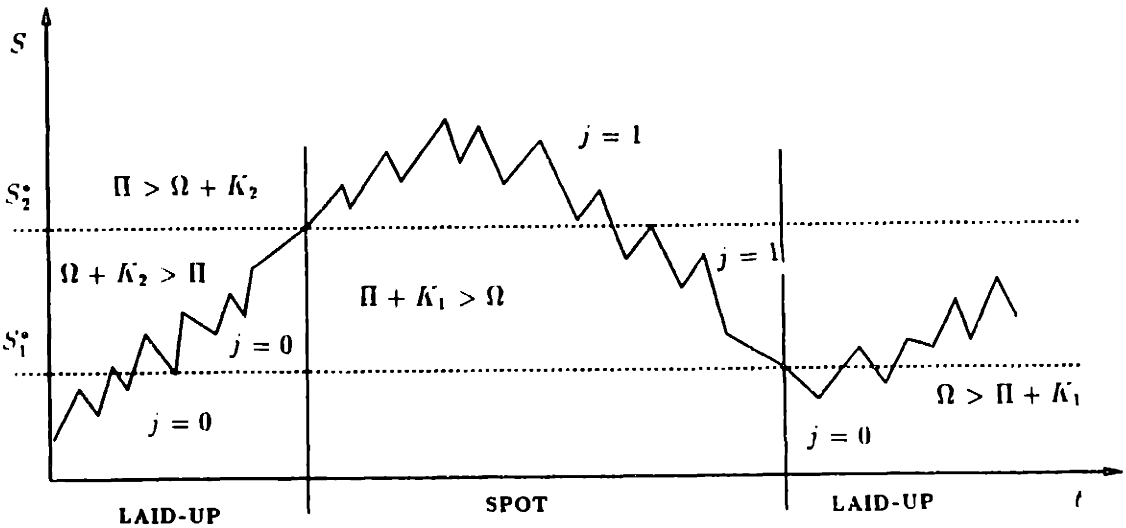

Figure 2-1: The optimal spot chartering decision characterization

under each regime and the critical (optimal) freight rates S_1^* and S_2^*.

Figure 2.1 illustrates the trigger rates and optimal controls for the spot contracts versus the lay-up decision.

2.1.4 Optimal Policies for Term Charter Contracts

Based on conjecture 2.1.1, we have now to solve for the decision between term charter versus lay-up condition only, as if there is no spot contract available. Next section the "final" optimal policy will be determined.

There are two approaches to determine the ship value and optimal operating policies in the term charter condition. One is to price the term charter contract by considering the term charter as equivalent to a number of forward contracts, that is, the length of the term charter equals the sum of the differences between the times to maturity of the forward contracts[28]. The other is to take advantage of the existence

[28] Another way to see it, is that, after each round voyage a new forward contract is available at

of a term charter market directly and to assume, as was the case for spot charter, that term charter rates follow a Brownian Motion / Wiener stochastic process[29].

Pricing Term Charter Contracts

Now, let Y(S,t;p,T) be the value at time t of a contract to purchase ship capacity (i.e., transportation services), up to time T at the term charter rate p when the current spot charter rate is S.

The instantaneous benefit of owning such a contract is :

$$q^*(S-p) \tag{2.47}$$

where q^* is the pay-load of the vessel.

Applying Ito's lemma to Y(S,t;p,T) and since S follows a Wiener stochastic process, given by equation (2.1), the instantaneous change in the value of the term contract is given by:

$$dY = [\frac{1}{2}(\sigma^2 S^2 Y_{SS}) + Y_t]dt + Y_S dS \tag{2.48}$$

Now we can use an arbitrage argument similar to above, for the case of the spot charter, and get the following partial differential equation to the value of the term contract :

$$\frac{1}{2}(\sigma^2 S^2 Y_{SS}) + (\mu S - \lambda)Y_S + Y_t + q^*(S-p) = 0 \tag{2.49}$$

As a boundary condition we have that the value of the contract at maturity, t = T is equal to zero, so that $Y(S,T;p,T) = 0$.

Since, as discussed previously, the term charter contract has a fixed price and also it normally involves a fixed quantity, that is, the full pay-load of the vessel (i.e., q^*), we can consider the term charter contract as a series of forward contracts with a fixed price. Moreover, since forward and futures prices are equivalent when the interest rate is non-stochastic (see, e.g., Cox et al. (1981) and Black (1976)), this implies

the same price. Notice that since we are assuming constant interest rates the pricing of futures and forward contracts are equivalent. See Cox, Ingersol and Ross (1981) and Black (1976).

[29]Recall that there is also a market for term charter rates which can be considered competitive.

that:

$$Y(S,t;p,T) = \int_0^{T-t} q^{\circ} F(S,\tau) d\tau \tag{2.50}$$

where $F(S,\tau)$ is the futures price for delivery in τ periods as defined previously.

The equilibrium contract price (or price schedule), is that price which makes the value of the contract at inception equal to zero, given the prevailing spot price S, and maturity T. Taking the equilibrium price as $p^*(S,T)$, we have an additional boundary condition :

$$Y(S,0;p^*(S,T),T) = 0 \tag{2.51}$$

The next step of the derivation is to solve the above PDE (2.49) and get an expression for Y(S,t;p,T), which then will give us the equilibrium term contract price $p^{\circ}(S,T)$.

However, in addition we have a ship valuation problem to solve which requires the development of a second arbitrage condition in order to price the ship and to determine the optimal operating policies. In this regard, the second approach to be developed below is more convenient. However, this pricing methodology can be used for one type of contract that is available to shipowners/operators, which is called "contract of afreightment" which specifies a certain quantity of cargo to be transported in a fixed route without specifying a time dimension. Since those contracts are not quoted in a market as for the case of the spot and term charter contracts, the above approach should be used instead.

Term Charter Policies

If we denote by S_T the term charter rate, its dynamics is given by:

$$dS_T = \mu_T S_T dt + \sigma_T S_T dZ \tag{2.52}$$

Where as before, μ_T the mean and σ_T the standard deviation are parameters to be estimated (now from term charter rates markets data), and are considered to be constant.

Since now there are term charter rates included in the freight futures index, the BFI[30], we can determine as in the case for the spot contracts, the implied future term charter rate and then determine its dynamics. Following the same analysis as for the case of spot rates, we get the freight futures dynamics for the term charter case as:

$$dF_T = S_T F_{T_{S_T}} \lambda_T dt + S_T F_{T_{S_T}} \sigma_T dZ \tag{2.53}$$

Now, denoting the value of the ship under term charter contracts by:

$$L = L(S_T, Q, t, i, \psi) \tag{2.54}$$

where : i=0 ship is laid-up, i=1 otherwise.

and the ship operating policy now will depend on the following critical freight rates:

$S^*_{T1}(Q,t)$: freight rate level at which the ship is laid-up if it was operating.

$S^*_{T2}(Q,t)$: freight rate at which the ship is put back into operation being laid-up.

Next, we develop an arbitrage condition by taking a short position on $\frac{L_{S_T}}{F_{T_{S_T}}}$ freight futures contracts and a long position in the ship L, and by applying Itô's lemma to (2.54) above. Then we would get the PDE characterizing the value of the ship under term charter, which is the counterpart of the spot contract case given by (2.30) above, as:

$$\frac{1}{2}(\sigma_T^2 S_T^2 L_{S_T S_T}) + (\mu_T - \lambda_T) S_T L_{S_T} - qQL_Q + L_t + (S_T - A - T)i - M(1-i) - rL = 0 \tag{2.55}$$

For $i = 0, 1$.

Now under the value maximizing operating policy $\psi^*(S^*_{T1}, S^*_{T2})$ the value of the ship when operating $\Gamma(S_T, Q, t)$ and not operating $\Theta(S_T, Q, t)$ is given by :

$$\Gamma(S_T, Q, t) = Max_{\psi} L(S_T, Q, t, 1, \psi) \tag{2.56}$$

[30]The term charter rates were included in the Baltic freight index in 1990. Hence before that date, this procedure would not be valid.

$$\Theta(S_T, Q, t) = Max_{\psi} L(S_T, Q, t, 0, \psi) \tag{2.57}$$

As before, we then have that:

(i) For the ship in the term charter contract (i.e., operating), i=1:

$$\frac{1}{2}(\sigma_T^2 S_T^2 \Gamma_{S_T S_T}) + (\mu_T - \lambda_T) S_T \Gamma_{S_T} - q\Gamma_Q + \Gamma_t + (S_T - A - T) - r\Gamma = 0 \tag{2.58}$$

(ii) For the ship in lay-up (i.e., out of operation), i=0:

$$\frac{1}{2}(\sigma_T^2 S_T^2 \Theta_{S_T S_T}) + (\mu_T - \lambda_T) S_T \Theta_{S_T} + \Theta_t - M - r\Theta = 0 \tag{2.59}$$

And the boundary (value matching) conditions that as before include the "switching costs" $K_1(Q,t)$ and $K_2(Q,t)$, that are the same as for the case of spot contracts, are given by:

$$\Gamma(S_{T1}^*, Q, t) = \Theta(S_{T1}^*, Q, t) - K_1(Q, t) \tag{2.60}$$

$$\Theta(S_{T2}^*, Q, t) = \Gamma(S_{T2}^*, Q, t) - K_2(Q, t) \tag{2.61}$$

And the "smooth pasting" conditions that characterizes the trigger rates S_{T1}^* and S_{T2}^* are given by:

$$\Gamma_{S_T}(S_{T1}^*, Q, t) = \Theta_{S_T}(S_{T1}^*, Q, t) \tag{2.62}$$

$$\Theta_{S_T}(S_{T2}^*, Q, t) = \Gamma_{S_T}(S_{T2}^*, Q, t) \tag{2.63}$$

Again, by solving (2.58) and (2.59) with boundary conditions (2.60) to (2.63) we can determine the value of the ship under each regime and the critical freight rates characterizing the regime change.

Similarly to last section, figure 2.2 illustrates the trigger rates and the optimal controls for the term charter against the lay-up decision.

2.1.5 Determination of the "Final" Optimal Policies

In the last two subsections the optimal policies for the spot/lay-up and for the term charter/lay-up conditions were determined as if they were two separated problems.

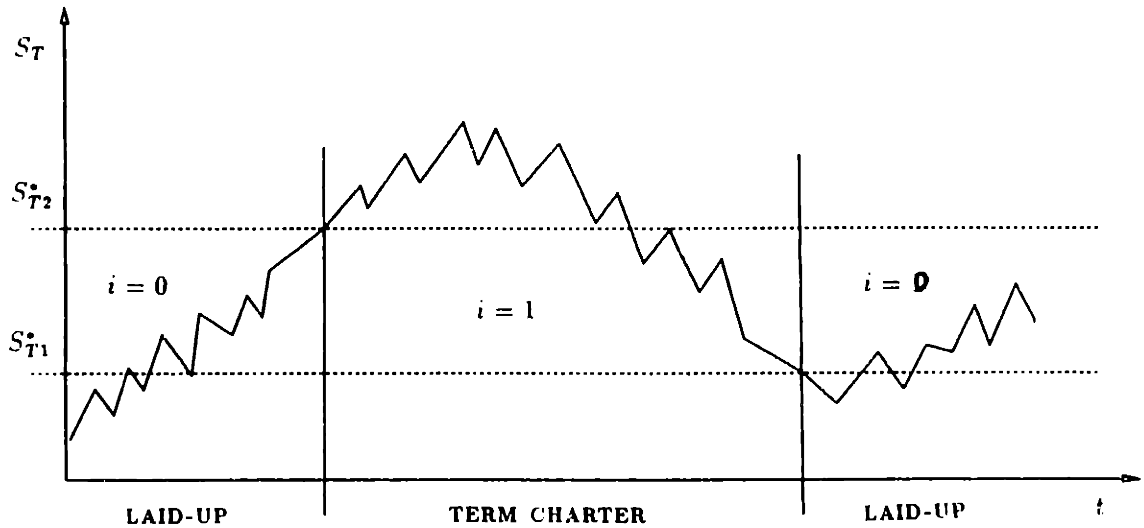

Figure 2-2: The optimal term charter decision characterization

Now we need to determine the "FINAL" DECISION, that is, if the shipowner should accept a spot contract, a term charter contract or if the ship should go to the lay-up condition.

Suppose first that the the ship is laid-up (i.e, j=0 and i=0). In order for the ship to move out of lay-up and into a spot contract regime (j=1) it must be the case that spot freight rates have increased such that:

$$S > S_2^* \Rightarrow j = 1 \tag{2.64}$$

Now suppose further that the term charter rates are such that:

$$S_T < S_{T2}^* \Rightarrow i = 0 \tag{2.65}$$

That is, there is no regime change in the term charter "environment" and the control is still i=0.

The question now is, should the shipowner accept the spot contract or keep the ship in lay-up as prescribed by the term charter, that is, what is the "final" optimal policy ?

Since from the conjecture I am partitioning the problem, the criteria in order to accept the SPOT CONTRACT, that is apply control j=1, is given by:

$$\Pi(S,Q,t) \geq \Theta(S_T,Q,t) + K_2(Q,t) \tag{2.66}$$

That is, the value of the ship under spot contract should be greater than the value of the laid-up ship in the term charter environment plus the costs of moving out of lay-up. Otherwise, the ship is more valuable by continuing LAID-UP for both "environments"(i.e., i=0 and j=0 and not j=1).

Similarly, if we have:

$$S < S_2^* \Rightarrow j = 0 \tag{2.67}$$

$$S_T > S_{T2}^* \Rightarrow i = 1 \tag{2.68}$$

We have in addition to verify if:

$$\Gamma(S_T,Q,t) \geq \Omega(S,Q,t) + K_2(Q,t) \tag{2.69}$$

holds in order to make also j=1 and move the ship out of lay-up in the spot environment and accept the TERM CHARTER contract. Otherwise it is optimal to keep the ship LAID-UP (i.e., j=0 and i=0 and not i=1).

Suppose now that rates have increased in both markets such that the following conditions holds:

$$S > S_2^* \Rightarrow j = 1 \tag{2.70}$$

$$S_T > S_{T2}^* \Rightarrow i = 1 \tag{2.71}$$

What is the optimal "final" policy ?

Again from the conjecture and partitioning argument in order to accept a SPOT

CONTRACT it must be the case that:

$$\Pi(S,Q,t) \geq \Gamma(S_T,Q,t) \tag{2.72}$$

And in order to accept a TERM CHARTER contract it must be the case that:

$$\Pi(S,Q,t) \leq \Gamma(S_T,Q,t) \tag{2.73}$$

Now suppose the ship is operating in the spot market and the freight rates drop such that:

$$S < S_1^* \Rightarrow j = 0 \tag{2.74}$$

but

$$S_T > S_{T1}^* \Rightarrow i = 1 \tag{2.75}$$

Similarly, the condition to move the ship into LAY-UP and to make control i=0 is given by:

$$\Omega(S,Q,t) \geq \Gamma(S_T,Q,t) + K_2(Q,t) \tag{2.76}$$

That is, the ship is more valuable laid-up than under a term charter contract inclusive of lay-up transfer costs.

If the last inequality is reversed then the operator should accept the TERM CHARTER contract and not lay-up the vessel and then the control becomes i=1 even though j=0.

Similarly we can have the ship in the term charter contract and freight rates decline such that:

$$S_T < S_{T1}^* \Rightarrow i = 0 \tag{2.77}$$

but

$$S > S_1^* \Rightarrow j = 1 \tag{2.78}$$

Similarly the condition to move the ship into LAY-UP and make control j=0 is :

$$\Theta(S_T, Q, t) \geq \Pi(S, Q, t) + K_2(Q, t) \tag{2.79}$$

That is, the ship is more valuable laid-up than accepting a spot charter contract.

Again, if the last inequality is reversed then spot contract is optimal and the ship should not move into lay-up, that is, the optimal control became j=1 even though i=0. That is, we transfer the ship from the term charter contract to a SPOT CONTRACT.

The last possibility is if the ship is operating either in the spot or term charter market and freight rates drop such that:

$$S_T < S^*_{T1} \Rightarrow i = 0 \tag{2.80}$$

$$S < S^*_1 \Rightarrow j = 0 \tag{2.81}$$

Then the optimal "final" policy is obviously to LAY-UP the vessel.

More generally, the ship value at time t given by the optimal "final" policy is:

$$\Phi(S, Q, t) = Max(\Omega, \Pi, \Theta, \Gamma) \tag{2.82}$$

That is, $\Phi(S, Q, t)$ is the value of the ship "operating" optimally. However, the above conditions for regime change should be satisfied at all times.

2.2 The Investment Decision

After solving the operational (chartering) problem, we now can solve the investment problem. As discussed in the introduction, the shipowner/investor can either sell or purchase a vessel in the second-hand market, scrap the vessel or order a new vessel. First, we solve the second-hand vessel and scraping cases because there is no construction lag, that is, at any time t the vessel is available for selling or scraping or in the case of purchase the vessel is available to immediate use. The newbuilding case, which includes the construction lag, will be solved subsequently.

Now, in order to make the decision of investing[31] in a new vessel (be it a new-building or a second-hand ship), we need to determine the net present value, NPV of the investment. For the investment problem I am also considering the one ship case. This mean that the shipowner owns one ship (following the optimal policies) or he has a budget constraint that only one ship can be purchased[32]. In the previous section we computed the (net) value of a ship operating in the bulk trades which is given by $\Phi(S,Q,t)$[33] at time t, under the optimal operational policy for the ship.

Now, if we let I(S,Q,t) be the investment necessary to acquire a vessel and R(S,Q,t) be the market price of a second hand vessel, the NPV at time t of purchasing a ship is given by:

$$NPV^P(S,Q,t) = \Phi(S,Q,t) - I(S,Q,t) \tag{2.83}$$

And the NPV of selling a ship is given by:

$$NPV^S(S,Q,t) = R(S,Q,t) - \Phi(S,Q,t) \tag{2.84}$$

However, the investment problem is not only determined by having a positive NPV for the investment considered, the investor has also a temporal decision to make, that is, WHEN it is optimal to purchase or sell a vessel, which of course, includes the decision of *not* investing. This is what is called in the financial economics literature the "timing option"[34] because it may pay to wait to invest in the expectation that the NPV of the investment (i.e., purchasing or selling) will increase.

The optimal "timing to invest" problem in recent years have been studied as an analogous problem to the optimal strategy to exercise a stock option. For the ship purchase case the analogy would be the following: the exercise price of the "option"

[31]Note that what I am considering the investment problem, includes both the purchase and sale of vessels. The distinction will became more apparent later in this section.

[32]This is because the ship operator/investor is in fact taking advantage of an arbitrage opportunity between the market value of a ship and the value computed from the optimal policies. This arbitrage opportunity exists only in a limited sense.

[33]Note that if term charter is optimal, we have instead $\Phi(S_T,Q,t)$. To simplify notation I am using S throughout.

[34]See for instance, McDonald and Siegel (1986), Majd and Pindyck (1987) and Dixit (1989).

is the investment to be made I(S,Q,t). The market price of the underlying "security" up until the expiration date is the optimal net cash flow determined above, $\Phi(S,Q,t)$. Finally, if we make the assumption that the investor has a finite horizon to decide to purchase the ship, then this date in the future say T, is the expiration date of the "option". Note that since the investor can decide to invest any time between his initial time t and the expiration date T, this "option" is equivalent to an (American) *call option.* Since the ship pays cash flows from operations, the analogy is with dividend paying options in which may be optimal to exercise early.

Therefore we have all the ingredients of a call option, that is, at any time between dates t and T the investor gets:

$$Max\{\Phi(.) - I(.); 0\} \tag{2.85}$$

That is, he gets $\Phi(.) - I(.)$ if $\Phi(.) > I(.)$ meaning that he exercises the "option" to invest and 0 if $\Phi(.) < I(.)$ meaning that he does not make the investment (purchase).

The value of the payoff $Max\{\Phi - I; 0\}$ will be denoted by $C(\Phi, t; I, t)$ which is the "call option" value at time t, with exercise price I, underlying asset price Φ and expiration date at T. Then (2.83) becomes:

$$NPV^P(S,Q,t) = C(\Phi, t; I, T) \tag{2.86}$$

which is the net present value of the ship purchasing opportunity inclusive of the flexibility of waiting and the option of *not* purchasing the vessel.

The next step in the strategy of the investor is to determine at what point in time between t (present time) and T (the expiration date of the investment opportunity) the investment should be made, that is at what time the "option" should be exercised.

Letting C(S,Q,t) be the value at time t of the above option to invest. Since C(.) depends on S, the stochastic process for C(.) can be determined by Ito's lemma and using an arbitrage argument similar to the one used in the derivation of (2.30), the value of the option to invest, C(.) satisfies the partial differential equation[35]:

$$\frac{1}{2}(\sigma^2 S^2 C_{SS}) + (\mu - \lambda) S C_S + C_t = 0 \tag{2.87}$$

Subject to the following boundary conditions:

$$C(0,t) = 0 \tag{2.88}$$

That is, the origin is an absorbing barrier, and

$$C(S,T) = Max\{\Phi(.) - I(.); 0\} \tag{2.89}$$

That is, the payoff of the "option" at expiration.

In addition we have the following conditions due to the time dependent optimal spot rate $S^P(t)$ [36] which characterizes the optimal investment (acquisition) strategy:

$$C(S^P, t) = \Phi(S^P, t) - I(S^P, t) \tag{2.90}$$

$$C_S(S^P, t) = \Phi_S(S^P, t) \tag{2.91}$$

Equation (2.90) is the NPV equation (i.e., the option value C(.) is the NPV^P) and (2.91) is the "smooth pasting" condition for the maximizing choice $S^P(t)$, which is equivalent to the conditions given above to determine the optimal chartering policies.

The optimal investment policy is then determined by solving the above PDE (2.87) subject to the above boundary conditions and the optimal timing to invest is determined by the "trigger" rate $S^P(t)$.

Note that the ship value $\Phi(.)$ was necessary in order to determine the optimal investment strategy. Therefore solving the chartering problem is a pre-requisite to

[35]Note again that, to simplify notation I am not distinguishing between S and S_T. This analysis assumes that before computing the value of the investment in a vessel the shipowner/investor will determine the optimal operating policy which will determine if the ship will be in the spot (S) or term charter (S_T) market.

[36]$S^P(t)$ will be the "trigger" freight rate which characterises the optimal time for the investor to purchase the vessel.

solve the ship investment problem.

The optimal sale strategy would be similar to the ship purchase problem but instead of a (American) call option on the value of the investment I(.), we have a (American) *put option* on the cash flow from the ship, that is, the pay off of this "option" is:

$$Max\{R(.) - \Phi(.); 0\} \tag{2.92}$$

Which, as above, can be denoted by $P(\Phi, t; R, T)$, and is the value of the put option at time t with exercise price R, underlying asset price (value) Φ and expiration date T. Then the net present value of the ship sale is given by:

$$NPV^S(S, Q, t) = P(\Phi, t; R, T) \tag{2.93}$$

which as before, includes the possibility of waiting and the option of not selling the vessel.

That is, the investor only sells the vessel (optimally) if the net cash flow from the ship $\Phi(.)$ falls bellow the ship market price R(.) between t and the expiration date T. Similarly to the acquisition problem we would determine a "trigger" freight rate $S^S(t)$ that characterizes the optimal time to sell.

Another possible decision apart from selling or purchase in the second-hand market, is to scrap the vessel (i.e., sell for demolition). In this case if we denote the scrap value of the ship by $W(.)$ and given the current operating value of ship $\Phi(.)$, the scrap decision will also be given by an (American) put option, that is:

$$Max\{W(.) - \Phi(.); 0\} \tag{2.94}$$

Now, in the newbuilding case due to contractual provisions, in most cases, once the shipowner/investor place an order for a vessel and sign the contract[37], if he decides not to proceed with the construction, he will be charged by the shipyard a penalty

[37] I assume that at this point the shipowner/investor have computed $\Phi(.)$ even at a design stage, and is only ordering the vessel because the condition $C(\Phi, t; I, T) \geq 0$ holds.

fee. Therefore, for the newbuilding case the analogy with the option will lead to a negative lower boundary. That is, if we let $I(.)$ be the purchasing cost (or newbuilding price) of the ship and as before $\Phi(.)$ is the optimum value of the ship and if we denote the penalty fee for breaking the contract by $U(.)$, which can be stochastic, the value of the newbuilding contract will be given by:

$$Max\{\Phi(.) - I(.); -U\} \tag{2.95}$$

And we can rewrite as:

$$\begin{aligned} -U \quad + \quad & Max\{\Phi - I + U; 0\} = \\ = \quad & Max\{\Phi - (I - U); 0\} - U = \\ = \quad & C(\Phi, t; I - U, T) - U \end{aligned} \tag{2.96}$$

Thus the NPV of a newbuilding contract is equivalent to a (American) call option with a lower exercise price (I-U) minus the contract breaking fee.

That is, the investor only breaks the contract after the lower boundary $-U$ is reached. This is again an option that can be solved similarly to the second hand market case[38]. Therefore, the construction lag can be disregarded by considering the contract breaking penalty fee.

[38]To my knowledge, it is not uncommon for shipowners/investors to break a newbuilding contract, in particular when freight rates reach very low levels. Now it will be possible to compute this low level that will be given by the "trigger rate for contract breaking". Another common alternative is to delay construction in the expectation of better rates. See Majd and Pindyck(1987) for the case of increasing or decreasing the rate of investment outlays in a project.

Chapter 3

The Discrete-Time Model

This is an alternative model to Chapter 2's continuous-time formulation. The discrete-time model is an explanatory model, as the economic ideas behind contingent claims valuation became more intuitive in this setting. Moreover, some preliminary extensions to Chap. 2 are developed as the possibility of "mixed" policies and a simple model to the term structure of freight rates. However, as will be shown below, what we gain in terms of economic intuition and simplification of the mathematics we loose in richness of the model. Many features of ship operations are lost, in particular in the process of discretization we loose some analytical structures[1] present in continuous-time that affects the optimal control analysis.

Nevertheless, I believe, that even with less power, the study of the discrete-time model will enrich our understanding of the continuous-time model and the results to be presented in Chap. 6 making it worthwhile to study this alternative formulation.

An interesting discussion of continuous versus discrete time models in the field of finance is presented in Merton (1975) and Sun (1987). As those authors emphasize, the distinction between discrete and continuous-time models in financial economics is the fact that in discrete-time the trading and decision intervals are implicitly assumed (say h) while in continuous-time they are explicitly assumed to be infinitesimal (i.e., $h = 0$). While in the real world our ability to observe and record events is in

[1]The "smooth pasting" condition for instance.

discrete-time, nothing prevents the phenomena under observation from occurring in continuous-time. Therefore, by modeling in discrete-time we make our model compatible with observational reality. However, the assumption of a fixed interval between observations the model does not capture events in between observations, while in continuous-time since the interval is infinitesimal the interval between events do not matter.

Perhaps the best way to compare those formulations would be by asking a practical question: why study a model in discrete-time if you have already studied it in continuous-time ? Reasons are:

1. A crucial argument in the continuous-time model of Chap. 2 is the replication of the ship value by an amount of freight futures and riskless bonds. This replicating portfolio however, must be adjusted every time the freight rate changes[2]. While in continuous-time this adjustment is to be made at every instant of time, in the real world the price change of the underlying security is only observed[3] (and sampled) at discrete points in time (say, every hour or day). Thus the issue is, if we include this discreteness of the replicating portfolio updating what happens to previous continuous-time results ? (e.g., optimal policies, values of trigger freight rates, etc...).

2. In many instances in finance and in economics the discrete-time models are more intuitive models to understand (e.g., the replication argument) than the continuous-time analog. Thus in developing the discrete-time counterpart we may be able to enhance our understanding of the economics and the mathematics involved which will make the theoretical results of the model and the eventual practical applications more robust. As the presentation will show, the discrete-time model is a very good pedagogical tool.

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[2]As it is known for the case of options this adjustment only takes place after the change in prices reach a certain sise. See Rubinstein and Leland (1981) and Boyle and Emanuel (1980) for further details.

[3]Note that this is a technological constraint. In fact, noting prevents information to be transmitted and price change to occur continuously but we are not able to record it. See Huang (1992), chap. 2 for a discussion on information transmission and price formation in continuous-time.

3. The discrete-time model is believed to be a natural setting for relaxation of the random walk assumption (memoryless property) embedded in the Brownian Motion / Wiener process assumed for the dynamics followed by the freight rates in the continuous-time case. It will then be possible to model a correlated process for freight rates (i.e., previous history will matter.).

We can now concentrate on the specific issues regarding ship valuation, chartering and investment[4]. As discussed in Chap. 1, Devanney (1971) developed a discrete-time model for the ship chartering problem but since he does not use contingent claims valuation his model does not account fully to the risks involved in the shipping market. Another discrete-time model related to this research, but not in shipping, is the dynamic programming (D.P.) approach to natural resources development of Stensland and Tjostheim (1989) (S&T). S&T paper is a discrete-time analog to Brennan and Schwartz (1985) (B&S) continuous-time mine operations and investment problem. However S&T also does not use contingent claims[5] but instead they estimate transition probabilities from historical data (using time-series analysis) and then solve the mine problem of B&S in manner similar to Devanney's model. They also consider the correlated price case and compare with B&S memoryless results, but as discussed in Chap. 2, their optimal operational policies does not allow for re-entry.

In this research the discrete-time model to be developed will make use of the contingent claims methodology[6][7].

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[4]For further discussions about the connections between discrete and continuous-time models see Sun (1987), Merton (1975) and Duffie (1988).

[5]Recall that contingent claims means that we will create an arbitrage condition (or replicating portfolio) between a "security" and its "derivative" that will avoid the need to estimate the probabilities of future prices of the "security".

[6]For a complete treatment of the theory of finance in discrete-time see Huang and Litzenberger (1988) and Vila (1990).

[7]Most of the application of contingent claims outside securities markets in what is usually called "real options" have used continuous-time models as for instance Dixit (1989), Majd and Pindyck (1987) and McDonald and Siegel (1986) in addition to B&S.

3.1 The Chartering Problem

In order to apply contingent claims methodology a "twin" or derivative security must exist. For the case of ship valuation the "twin" security is an amount of freight futures contracts. That is, it will be possible to replicate the value of the ship at each period with a specific amount of freight futures contracts and a specific amount of riskless bonds.

The model to be developed below is based on the simplified approach to option pricing developed by Cox, Ross and Rubinstein (1979) (CRR), a discrete-time analog of Black and Scholes (1973) option pricing model.

3.1.1 Freight Futures Pricing and Dynamics

As in the continuous-time model, the first step in the valuation of the ship is the determination of the price and dynamics of freight futures. This was accomplished in continuous-time using general equilibrium / arbitrage results used to price bonds and interest rates sensitive securities[8], which as was discussed, is conceptually similar to freight futures.

From Chap. 2 we have that the price of the futures in continuous-time is given by (2.13) as:

$$F_\tau = Se^{(\mu-\lambda)\tau} \tag{3.1}$$

Where F is the futures price, S the freight rate, μ is the drift of the Brownian Motion for freight rates, λ is the risk premium for futures and $\tau = (T-t)$ is the time to maturity of the futures.

Now we want to determine the discrete-time analog to (3.1), we rearrange it as:

$$(\mu - \lambda) = \frac{1}{(T-t)} log\left(\frac{F}{S}\right) \tag{3.2}$$

The above, is the continuously compounded excess rate of return, which is the

[8]See Ho and Lee (1986) for a model of bond pricing in discrete-time.

limit, applying L'Hopital's rule, of the discrete-time formulation given by:

$$\gamma_d = \gamma(T-t,h) = (\mu-\lambda)_d = \frac{1}{h}\left[\left(\frac{F}{S}\right)^{\frac{h}{T-t}} - 1\right] \tag{3.3}$$

Denoting $T-t=nh$, we get the following discrete-time equivalent for the price of the freight futures:

$$F_{\tau,d} = S(1+\gamma_d h)^n \tag{3.4}$$

The above formula gives the futures price for a time to maturity T-t, with $\gamma_d = \mu - \lambda$ as a parameter to be estimated for an interval of length h, n is the number of intervals to maturity and S is the current freight rate.

3.1.2 The Construction of the "Synthetic Ship"

It is intuitively appealing to describe/construct the discrete-time model in small steps. The stochastic process to be assumed is the *binomial random walk process*[9]. Start with two periods (0 and 1), two states of nature model and let:

- F_n be the value of freight futures contracts at time 0[10].
- u and d be the rates of return for period 1 if the freight futures goes up or down respectively.
- q be the probability of movement "up" in period 1.

We can then construct an event-tree for the freight futures as shown in figure 3.1

.

As was the case in continuous-time, the following assumptions are made:

1. The interest rate r[11], per period is constant.
2. Borrowing and lending is unconstrained at this rate.

[9]For an extension of the binomial model see Boyle (1988).
[10]Recall that we have first to price the freight futures as given by (3.4) above.
[11]That is, r is the per period return on a riskless bond.

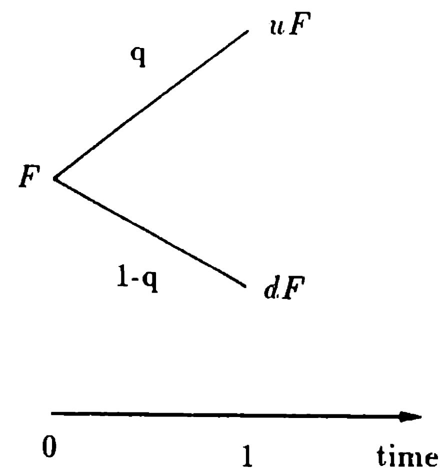

Figure 3-1: Event-tree for freight futures

3. There are no taxes or transactions costs[12].

4. Short selling is allowed[13].

Now, since the ship value is also a function of freight rates[14], when freight rates (S) and freight futures (F) go up the value of the ship (H) will also go up and vice-versa.

In addition, the ship generate cash flows during operations which can be seen as equivalent to dividends payment of a security. This cash flow is assumed to be proportional to the value of the ship. This makes intuitive sense because both ship values and cash flows will be contingent on the value of freight rates. Denoting the per period revenues by S, the costs as in the continuous-time model as (A+T) constant, the value of the ship by H and denoting the per period proportional cash flows by δ gives the following:

$$\delta = \frac{S - (A + T)}{H} \tag{3.5}$$

Thus the per-period after cash-flows value of the ship is given by:$H(1 - \delta)$.

[12]For an extension of option pricing with transaction costs see Leland (1985).

[13]These assumptions are very much used in financial economics. Even though practical applications require relaxations and/or adjustments, in many cases of derivative security pricing the "frictionless" results are quite realistic benchmarks. Hopefully the same is true for the shipping case.

[14]As was the case in the previous chapter, the spot chartering case will be developed first.

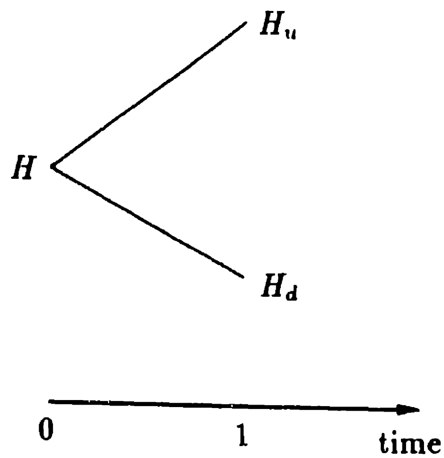

Figure 3-2: Event-tree for ship value

Letting $H_u(1-\delta)$ be the up state and $H_d(1-\delta)$ be the down state values of the ship after cash flows, figure 3.2 presents the ship value event-tree.

The next step in the contingent claims methodology is to construct the replicating portfolio which will be denoted by V. This portfolio contains an amount X of freight futures contracts and an amount B of riskless bonds.

That is, at time zero we have:

$$V = B \tag{3.6}$$

As was the case in continuous-time, at initiation there is no payment for a futures contract. At time 1 we have:

$$V_u = X[(u-1)F_n] + rB \tag{3.7}$$

$$V_d = X[(d-1)F_n] + rB \tag{3.8}$$

Where V_u is the up state and V_d the down state values of the replicating portfolio.

Now we can determine X and B such that the value of the portfolio V "matches" the value of the ship for each state of nature at period 1, that is:

$$V_u = H_u(1-\delta) \; and \; V_d = H_d(1-\delta) \tag{3.9}$$

Therefore:

$$X[(u-1)F_n] + rB = (1-\delta)H_u \tag{3.10}$$

$$X[(d-1)F_n] + rB = (1-\delta)H_d \tag{3.11}$$

Since we have two equations and two unknowns, X and B, we can compute their unique values including the value of the freight futures given in (3.4) we get the following:

$$X = \frac{(H_u - H_d)(1-\delta)}{(u-d)S(1+\gamma_d h)^n} \tag{3.12}$$

$$B = \left[\frac{1-d}{u-d}H_u + \frac{u-1}{u-d}H_d\right]\frac{1-\delta}{r} \tag{3.13}$$

The portfolio V constructed using the above values for X and B is called a *replicating (or hedging) portfolio.* And because V replicates the value of the ship, V can be called the "SYNTHETIC" SHIP.

As for the case of options and other derivatives[15] the argument given to sustain the above values for X and B (i.e., the fact that V replicates H), is the possibility of arbitrage between the option (or derivative) and its replicating portfolio which is made possible by trading in security markets[16]. Since we are also considering frictionless shipping markets, in order to sustain this no arbitrage condition, the shipowner, in case of mispricing, $V > H$ say, would sell his "physical" ship (H) and buy the "synthetic" ship (V) and would made a sure profit (i.e., $V - H$) or the other way around if $H > V$. Therefore, $H = V$. At all times the physical and synthetic ship must have the same value.

At time 0 we get:

[15]See Rubinstein (1987) for other examples of derivative securities.
[16]See CRR for the detail arguments of this no arbitrage condition for options.

$$V = B = H = \left[\left(\frac{1-d}{u-d}\right) H_u + \left(\frac{u-1}{u-d}\right) H_d\right] \frac{(1-\delta)}{r} \tag{3.14}$$

Now, if we define:

$$p \equiv \frac{1-d}{u-d} \tag{3.15}$$

$$1 - p \equiv \frac{u-1}{u-d} \tag{3.16}$$

Then we can rewrite (3.9) as:

$$H = [pH_u + (1-p)H_d] \frac{(1-\delta)}{r} \tag{3.17}$$

As in the case of options[17] which is discussed by CRR, the above formula for the value of the ship H, has many interesting features:

- The probability q of the original binomial tree does not appear in the above formula for H. This means that even if investors have different beliefs about the probabilities of an up going or down going event they would agree on the relation between H, F, u, d and r[18]. However, they may disagree on the value of F since this involves the estimation of γ_d

- Also the above formula does not involve investor's risk preferences. Both risk averse and risk preferring investors would agree on the value of H given above.

- The only random variable that the value of the ship depends on is the freight futures which is computed from freight rates as indicated in (3.4).

- Finally, notice that $p \equiv (1-d)/(u-d)$ is always greater than zero and less than one[19], hence it has the properties of a probability. Actually from (3.14) we can see that H is given by the expected discounted value of H using a discount

---

[17]Notice that there is no interest rate in the above formulas because we are dealing with futures contracts.

[18]Note that typical values for u, d and r would be: u = 1.5 for a 50 % increase in value, d = 0.5 for a 50 % decrease in value and r = 1.1 for a 10 % interest rate per period.

[19]Since u ¿ 1 and d ¡ 1.

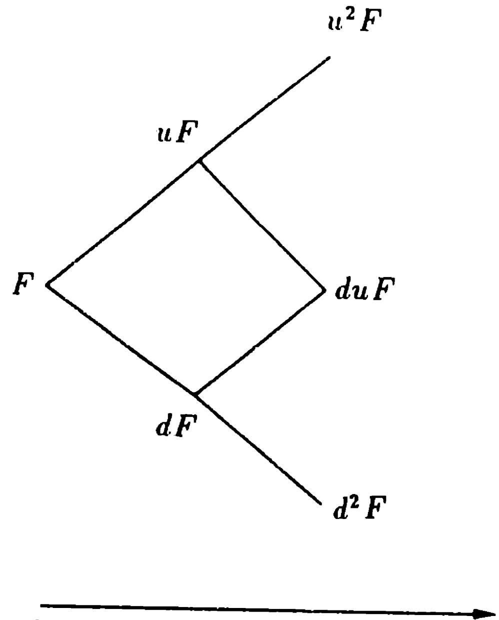

Figure 3-3: Three periods event-tree for freight futures

factor of $\frac{1-\delta}{r}$. Therefore (3.14) is equivalent to the determination of H in a risk neutral world[20].

Now we can move our model one step further, and consider a three period model. For the freight futures we have a three periods event-tree shown in figure 3.3 and for the ship value in figure 3.4.

Working backwards from period 2 to period 1, we can determine H_u and H_d at period 1 as was the case in the two period model, that is:

$$H_u = [pH_{uu} + (1-p)H_{ud}]\frac{(1-\delta)}{r} \tag{3.18}$$

$$H_d = [pH_{ud} + (1-p)H_{dd}]\frac{(1-\delta)}{r} \tag{3.19}$$

Then we can move one more period from period 1 to period 0 and compute H as equation (3.14) with the values of H_u and H_d given by (3.18) and (3.19) respectively. We can continue with the binomial model for many periods and construct a binomial lattice for the value of the ship and by moving backwards we can compute the value

[20] In fact, we can see that H is a Martingale, i.e., $H_t = E^*[H_s]$ for $s > t$, when we apply the probability p, which is called *risk adjusted probability*. See Vila (1990a) for a complete treatment of Martingales and discrete-time models in finance.

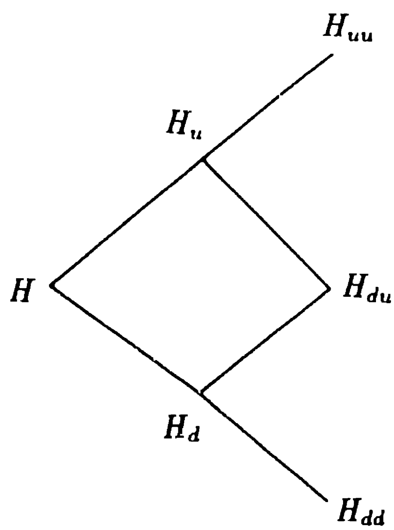

Figure 3-4: Three periods event-tree for ship value

of the ship at time zero by starting the recursion some time into the future.

With the above procedure we created a "synthetic ship" with freight futures and riskless bonds. However, as already developed in continuous-time, we have also to include the option that the ship operator has of moving the ship in and out of operations. The procedure developed so far is only applicable for the case of a ship that is either operating or we would sell it. This is analogous to the case of a "European" type option, a derivative security that can be "exercised"(i.e., claim a payoff) only at a fixed date in the future (the "expiration" date). However for other derivatives securities like "American" options on dividend paying stocks it is allowable and it may be optimal to exercise the option at any time before the expiration date. This will is the case for the ship chartering problem where we have the possibility of exiting or re-entering the market as developed in Chap. 2.

The issue here is the fact that the payoff can be claimed any time between present and future, thus at each period of the binomial model described above (which end up being a lattice from the future to the present), we have to decide if we exercise the derivative and claim the payoff or if we wait one more period and get a larger payoff. In the case of the ship chartering problem the "exercise" decision is analogous with the situation where the ship operator accepts a spot contract or lay-up the vessel.

At this point, it is important to notice that in studying "real options"[21] we can have two problems:

1. The *operational* problem ("chartering" in the shipping case). A problem of optimization of asset utilization.

2. The *investment* problem: optimization of the sale or purchase of the asset.

Most of the literature on "real options" cover the *investment* problem in which, as will be shown later in this chapter, the analogy with options is quite apparent and most options results follows (becoming an easier problem). In the *operational* problem the analogy is less immediate since operations in general requires some kind of "repetitive action" during the life of the asset. It is not a one time decision within a short horizon.

3.1.3 Optimal Policies for Spot Contracts

As was the case in Chap. 2, we will start by determining the optimal policies for the spot contract vs. lay-up condition only. Next section develops the term charter decision problem.

The following simplifying assumptions regarding the ship operation will be made:

1. The ship will operate in one single route (e.g., grain transport from US Gulf to Japan).

2. If the ship operator accepts a spot contract (for one voyage) the revenues is S per period (h) and the costs (assumed to be constant) A. Thus net revenues are: $S - A$ per period. The costs during the lay-up condition are M per period (of course during laid-up there are no revenues)[22].

[21]Note that for the case of shipping, since there is a competitive market for freight rates and also a futures market, the shipping case is an instance of "real options" closer to security markets options than many other applications like a generic factory shut-down in which the market for its outputs may not be competitive and the "twin" security non existent.

[22]Note that in the discrete-time model the trading and decision interval, h makes the values of S,A and M to vary accordingly.

3. The ship considered will have a finite operational life and at the end (time T) it is assumed that the ship is sold for scrap with salvage value equal to zero.

Based on the above operational payoffs, a ship operating in the spot market is an asset that entitles its owner between time t (the time that the ship is acquired) and time T the following cash-flow:

$$\left\{H_t, (S_{t+1} - A, -M)^+, ..., (S_{T-1} - A, -M)^+, (S_T - A, -M)^+\right\} \tag{3.20}$$

Where $(S_T - A, -M)^+ = Max\,(S_T - A, -M)$

By the no arbitrage condition (and recall that we are considering deterministic interest rates), H_t is to be determined such that:

$$H_t = \sum_{k=t+1}^{T} E_t^* \left[\left(\frac{1}{r}\right)^{k-t} (S_k(\theta_k) - A; -M)^+\right] \tag{3.21}$$

Where θ_k is the state of nature at time k and E_t^* is the expected value under the risk adjusted probabilities computed from the binomial model with the replicating portfolio V and given by (3.15) and (3.16) above.

Since at each stage of the recursive formula given by (3.21) different "actions" are available, that is, either accept a spot charter contract and get $[S_k - A]$ or lay-up the ship and incur the lay-up cost of M, it is not possible to get, as is the case for European options, a close-formula for H_t.

Notice also that the ship valuation is also different from American options on dividend paying stocks because while in the latter we have only *one action* to take between times t and T (i.e., once we exercise the option the process ends), for the former at each period we have an action to take and a payoff is claimed since the ship after a number of periods laid-up can be brought back into operations again (it is as if the option becomes alive again).

Therefore what we have for the ship valuation is equivalent to a value function of a dynamic programming algorithm in discrete-time[23] in a finite horizon (from t to T)

[23]See Bertsekas (1987) for the theory of optimal control in discrete-time.

with a discounting factor of $\frac{1}{r}$[24].

The next step in the discrete-time model is the construction of the dynamic programming algorithm in order to determine the optimal chartering policies. I will make one further simplifying assumption in discrete-time that was not made in continuous-time, I will not consider the costs of moving *in* (K_1) and *out* (K_2) of lay-up[25]. Thus the "trigger" freight rates in order to move the ship into laid-up or accept a spot charter contract will collapse to one rate S^*, instead of two as was the case in continuous-time.

Considering that the last voyage that the ship can make takes place at period T[26], the value of the ship at this period is:

$$H(n,0,j) = Max\left[u^j d^{n-j} S - A, -M\right] \; for\; j = 0,1,...,n \tag{3.22}$$

Moving one more period backwards we have for T-1, the value of the ship given by:

$$H(n,1,j) = Max\left\{\left[u^j d^{n-1-j} S - A, -M\right] + \left[pH(n,0,j+1) + (1-p)H(n,0,j)\right]\frac{1}{r}\right\} \tag{3.23}$$

$for\; j = 0,1,...,n-1$

More generally for i periods before the end of the life of the ship (i.e., T) we have:

$$\begin{aligned} H(n,i,j) &= \\ &= Max\left\{\left[u^j d^{n-i-j} S - A, -M\right] + \right. \\ &+ \left.\left[\sum_{k=i+1}^{n}\left[pH(n,k,j+1) + (1-p)H(n,k,j)\right]\left(\frac{1}{r}\right)^{k-i}\right]\right\} \end{aligned} \tag{3.24}$$

[24]In the sense that we may see the operational problem as a mix of option pricing and the optimal portfolio selection problem since at each period we make a decision (which type of contracts to accept) and the value (utility) function is time additive (i.e., we have an accumulation of "payoffs"). See Samuelson (1969) for optimal portfolio selection in discrete-time.

[25]This will make the model simpler but more pedagogical. This simplification will also allow for the implementation of "mixed" policies for spot and term charter a feature not included in continuous-time.

[26]Notice that the discretisation interval ,h , can have any size not necessary the length of a voyage, with S, A and M adjusted accordingly.

$for\ j = 0, 1, ..., n - i$

Considering the present time t, as having n periods before T we have the following recursion formula for the value of the ship:

$$H_t = H(n, n, 0) =$$
$$= \quad Max\left\{[S - A, -M] + \sum_{k=2}^{n} [pH(n, k, 1) + (1 - p)H(n, k, 0)] \left(\frac{1}{r}\right)^{k-1}\right\} \quad (3.25)$$

Therefore since at each period (or stage of the D.P. algorithm) we can make a different decision (i.e., apply a different control), it is necessary to solve the recursion from T to t, determine the optimal policy (spot or laid-up) each period, compute the income generated by that policy and add to previous periods in order to determine the value of the ship. A byproduct of the computation of the ship value and the optimal policies is the determination of the "trigger" freight rate S^*, which is the value of S that makes the value of the ship operating equal to the value of the ship laid-up.

Pictorially the lattice generated by the binomial model and the D.P. algorithm would be like figure 3.5.

3.1.4 Optimal Policies for Term Charter Contracts

In continuous-time (Chap. 2) the term charter policies were solved independently from the spot policies (by partitioning the problem). In discrete-time I will take a different approach and I will compute by arbitrage the term charter rate that will make the ship value under "spot only" policies to be the same as a one term charter period deviation from spot[27] and then continuing on spot market only policies. From the previous section the value of the ship under spot only policies is given by:

$$H_t = \sum_{k=t+1}^{T} E_t^* \left[\left(\frac{1}{r}\right)^{k-t} (S_k(\theta_k) - A; -M)^+\right] \quad (3.26)$$

[27]Note that the period covered by the term charter contract will be equivalent to a finite number of spot contracts "periods".

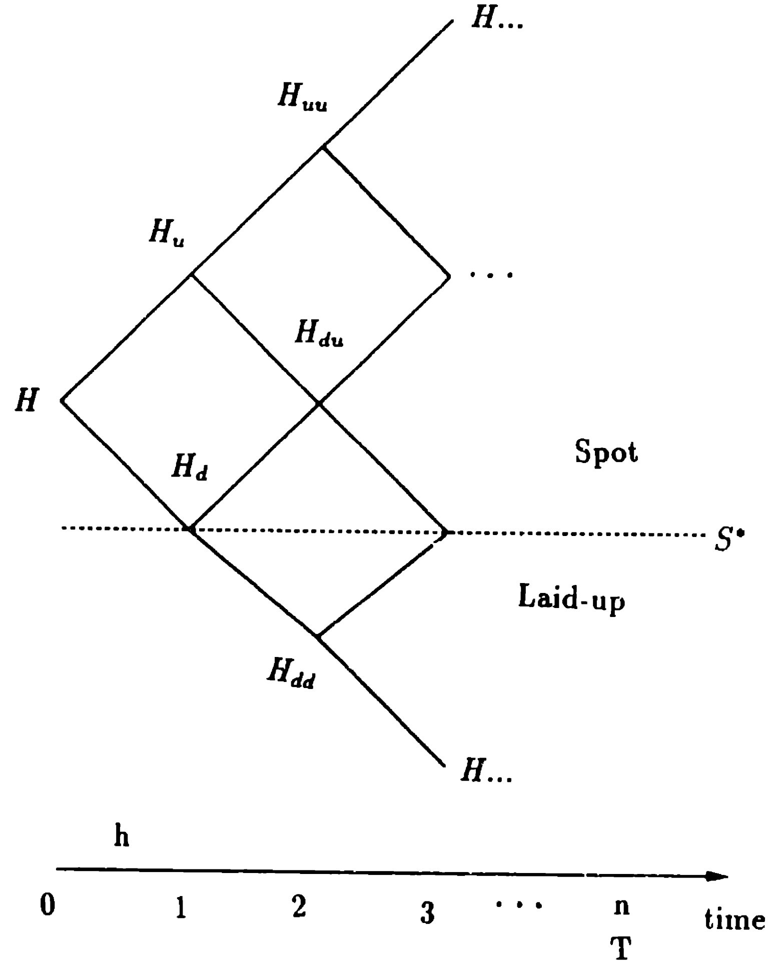

Figure 3-5: D.P./binomial lattice for chartering policies

The above recursion means that the ship is either operating in the spot market or is laid-up. Next, we are going to deviate from spot policies for one term charter period[28] and then return to spot policies until T. That is:

$$L_t = (S_T - A') + \sum_{k=t_n+1}^{T} E^*_{t_n}\left[\left(\frac{1}{r}\right)^{k-t_n}(S_k(\theta_k) - A; -M)^+\right] \qquad (3.27)$$

Where L_t is the value of the ship at time t under the one term charter period deviation, S_T is the term charter rate, A' is the operating costs during the term charter period and t_n is the time when the term charter contract expires and the ship returns to spot only policies.

From the above relations we can determine the term charter rate that will make the ship operator indifferent between staying in the spot only policies or deviating for one term charter period to term charter contracts. That is, find S_T such that $H_t = L_t$. Denoting this equilibrium rate by S_T^E, we have that:

$$\begin{aligned} S_T^E &= \sum_{k=t+1}^{T} E^*_t\left[\left(\frac{1}{r}\right)^{k-t}(S_k(\theta_k) - A; -M)^+\right] \\ &- \sum_{k=t_n+1}^{T} E^*_{t_n}\left[\left(\frac{1}{r}\right)^{k-t_n}(S_k(\theta_k) - A; -M)^+\right] + A' \end{aligned} \qquad (3.28)$$

Since we have that:

$$\begin{aligned} H_t &= \sum_{k=t+1}^{t_n} E^*_t\left[\left(\frac{1}{r}\right)^{k-t}(S_k(\theta_k) - A; -M)^+\right] \\ &+ \sum_{k=t_n+1}^{T} E^*_{t_n}\left[\left(\frac{1}{r}\right)^{k-t_n}(S_k(\theta_k) - A; -M)^+\right] \end{aligned} \qquad (3.29)$$

Substituting in the expression for S_T^E above we get:

$$S_T^E = \sum_{k=t+1}^{tn} E^*_t\left[\left(\frac{1}{r}\right)^{k-t}(S_k(\theta_k) - A; -M)^+\right] + A' \qquad (3.30)$$

Finally, under this one period deviation the optimal "final" policy will be given

[28]Recall that the the term charter period can vary depending on the contract length.

by:

If $S_T > S_T^E$ accept term charter contract.

If $S_T < S_T^E$ accept optimal spot policy (which includes the possibility of the vessel being laid-up).

Therefore, the simplifying assumption of no switching cost when moving the ship in and out of laid-up made possible the "mixing" of spot and term charter contracts when valuing the ship.

3.2 The Term Structure of Freight Rates

As an extension of the above model for term charter policies we can study the term structure of freight rates. The term structure is defined to be the function that maps the time to maturity to freight rates, i.e., $f(S_T, t_T)$, since long term contracts in shipping can have different lengths/maturities.

To study the term structure is important because it conveys long term expectations about the shipping market and from its shape operators/investors can extract relevant information for decision making.

As a preliminary and simple model, if we assume that the state variable that determine long term rates is the spot rate (i.e., the shortest maturity contract available for the shipowner), the equilibrium relationship between time to maturity T_M and long term contracts rates at time t is given by:

$$S_T(t, T_M) = \sum_{k=t+1}^{T_M} E_t^* \left[\left(\frac{1}{r} \right)^{k-t} (S_k(\theta_k) - A; -M)^+ \right] + A'(T_M) \qquad (3.31)$$

That is, from the above expression we can determine for each time to maturity T_M the value of the term charter rate for that maturity $S_T(t, T_M)$ that makes the shipowner indifferent between the spot and term charter markets. With the pair $(S(t, T_M), T_M)$ we can construct the term structure of freight rates at time t and infer from its shape ship operators/investors expectations about long term contracts in bulk shipping.

3.3 The Investment Problem

As discussed in the context of the continuous-time model, at each trading interval h the ship operator/investor has the option, in addition to operate his vessel in one of the above contracts, to sell a vessel or to buy a new one[29].

From the determination of the optimal chartering policies we can compute the optimal value of the ship at each trading interval h and determine W, the optimal ship value at each interval h.

Using the same notation of Chap. 2, we denote by $I(S,t)$ the investment necessary to acquire a vessel and by $R(S,t)$ the market price of a vessel[30], then the NPV at time t for the case of ship acquisition is given by:

$$NPV^A(S,T) = W(S,t) - I(S,t) \tag{3.32}$$

And for the case of ship selling:

$$NPV^S(S,t) = R(S,t) - W(S,t) \tag{3.33}$$

As discussed in Chap. 2, from the above relations we can see that the value of ship investments (both acquisition and selling of vessels) is analogous to an option on a common stock. Moreover, if it is assumed that the ship operator/investor has a specific period of time in order to decide about the investment, and his action can take place any time in between, the type of option in the analysis is of the American type[31].

Therefore, for the case of ship acquisition we have an American call option, that is:

$$NPV^A = Max\,[W(.) - I(.); 0] \tag{3.34}$$

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[29]As discussed in Chap. 2, we are assuming the one ship case or that the shipowner has a budget constraint that he can purchase only one ship.

[30]The newbuilding case can be treated similarly to the continuous-time case.

[31]A more complete discussion of the analogy between ship investments and options is provided in the previous chapter.

And for the case of ship selling we have an American put option, that is:

$$NPV^S = Max\,[R(.) - W(.); 0] \tag{3.35}$$

Since the ship pays dividends, i.e., generates a cash flow during operations, in addition to determine the value of the option to invest, we have to compute the "optimal timing" that makes this investment reach its maximum value up until the expiration of the investment opportunity. This optimum timing is characterized by a trigger freight rate S_A^* and S_S^* for optimum acquisition and selling respectively which are computed from the D.P. algorithm for ship investment which will be developed below.

Denoting by $Z(S,t)$ the value of the option to acquire the vessel, we can develop a discrete-time dynamic programming recursion formula to compute its value using the same ideas of portfolio replication that was developed for the chartering problem[32].

Assuming a finite horizon for the operator/investor to decide if acquire or not the vessel, say between t and T(the expiration date of the acquisition opportunity[33]). As before, starting the algorithm backwards in time, the value of the option to invest at date T is:

$$Z(n,0,j) = Max\,[0, W(S,0,j) - I]\ for\ j = 0,1,...,n \tag{3.36}$$

One period before expiration:

$$Z(n,1,j) = Max\left[W(S,1,j) - I, [pZ(n,0,j+1) + (1-p)Z(n,0,j)]\,\frac{1-\delta}{r}\right] \tag{3.37}$$

$for\ j = 0,1,...,n-1$

[32]Notice that for the case of the investment problem the analysis will be somewhat similar to the procedure developed in CRR for the pricing of American type options on dividends paying stocks. That is, it is a "one time" decision.

[33]Notice that we are also using T to denote the end of the life of the vessel. Hopefully it will became clear from the context when T denotes the expiration time of an investment opportunity and when it denotes the end of the operational life of the ship.

More generally, i periods before expiration:

$$Z(n,i,j) = Max\left[W(S,i,j) - I, [pZ(n,i-1,j+1) + (1-p)Z(n,i-1,j)]\frac{1-\delta}{r}\right] \tag{3.38}$$

$for j = 0, 1, ..., n-i$

Finally, with n periods before expiration, that is at time t, we have:

$$NPV^A = Z(n,n,0) = Max\left[W(S,t) - I, [pZ(n,n-1,1) + (1-p)Z(n,n-1,0)]\frac{1-\delta}{r}\right] \tag{3.39}$$

The following remarks about the above recursion formulas are important:

1. For each trading interval h, it is necessary to compute the optimal operational policy algorithm in order to update the value of $W(S,i,j)$; therefore we have a recursion inside a recursion.

2. To simplify matters , I assumed the acquisition price $I(S,t)$ to be constant I, between t and T. It is however possible to solve for a stochastic price at a cost of more complex computations within the same framework.

3. The risk adjusted probabilities p and $(1-p)$ are to be determined by a replicating argument similar to the chartering policies, by trading with a certain amount of freight futures and riskless bonds.

4. The optimal timing to make the vessel acquisition will be given by the period i in which it is more valuable to invest $Z(.)$ and get $W(.)-I(.)$ than to continue to hold it for some period until expiration T. It is then also possible to determine the acquisition trigger freight rate S_A^*.

Finally a similar procedure would be developed for the option to sell the vessel in which the analogy would be with the American put option.

3.4 Final Discussion

A discrete-time counterpart to the continuous-time model developed in chapter 2 was formulated. One advantage of the discrete time model is that it is an easier model to understand the arguments behind the replicating portfolio, that is, the creation of the synthetic ship, which is a critical step in the contingent claims methodology.

In discrete-time, it was also possible to model a simple case of "mixed" policies, in which the value of the ship includes the option to deviate from "spot only" policies without the partition used in continuous-time. As an extension of the mixed policies it was possible to derive a preliminary model for the term structure of freight rates.

Chapter 4

Optimal Policies for the Perpetuity Case in Continuous-Time

This chapter presents a solution to the ship valuation equations and optimal policies developed in Chap. 2. Since no closed form solution exists for those partial differential equations, two approaches may be taken: (i) solve by numerical methods (ii) solve for the simplified perpetuity case where a simpler numerical method is needed and we get "close" to closed form solution. The second approach will be used in this research, first because by being simpler we will be able not only to compute the optimal policies and ship values but also due to its simplicity it will make possible detailed sensitivity analysis of results (presented in Chap. 6). Moreover, since the life of a ship is around 10 to 15 years the overvaluation generated by perpetuity will not be so large[1] as to make the results presented in Chap. 6 unrealistic[2].

[1]This is because the present value of the cash flows from year 10 say, to infinity after discounting, will be small. Moreover, for the optimal policies, since it is a relative value criteria, the effects of overvaluation will be even smaller.

[2]An alternative for the perpetuity case would be to consider the exponential ship decay. In this case the value of the ship operating is given by $\Pi(S,t) = E[(1-qdt)e^{(r+\lambda)dt}\Pi(S+dS,t+dt)+Ddt]$. Thus we can replace $r+\lambda$ by $r+\lambda+q$ when computing the value of the ship, where 1/q = expected life of the ship.

4.1 Solution for the Spot Contract Case

From conjecture 2.1.1, the chartering decision was partitioned in two optimal control problems, one for the decision spot contract versus lay-up and the other for the decision term charter versus lay-up. Subsequently the optimal "final" policy is determined as described in Chap. 2. Therefore for each contract a partial differential equation is solved and specific stochastic process parameters estimated (see Chap. 5).

In order to determine the optimal policies under spot contracts we have to solve partial differential equations (2.36) and (2.42) with boundary conditions (2.39), (2.40), (2.45) and (2.46) as derived in Chap. 2. Solving for the perpetuity case implies that the value of the ship and the optimal operating policies will be independent of time, t. In addition, since we are assuming that the ship will live for ever, it seems reasonable also to disregard the depletion of the ship as a capital equipment and then the value of the ship will also be independent of Q. Therefore, instead of the value of the ship being given by $H(S,Q,t)$ it will be given by $H(S)$.

With the above simplifications the partial differential equations of Chap. 2 reduce to ordinary differential equations (2nd order, linear and nonhomogeneous) and together with the boundary conditions we can solve and determine the optimal policies for the spot versus lay-up condition.

The value of the ship and the optimal policies are then characterized by the following equations:

$$\frac{1}{2}(\sigma^2 S^2 \Pi_{SS}) + (\mu - \lambda) S \Pi_S - r\Pi + (S - A - T) = 0 \tag{4.1}$$

Which is the ODE for the value of the ship operating in the spot contract regime. And

$$\frac{1}{2}(\sigma^2 S^2 \Omega_{SS}) + (\mu - \lambda) S \Omega_S - r\Omega - M = 0 \tag{4.2}$$

Which is the ODE for the value of the ship in the lay-up condition.

And the following boundary conditions:

$$\Pi(S) = \Omega(S) - K_1 \tag{4.3}$$

$$\Pi_S(S_1^*) = \Omega_S(S_1^*) \tag{4.4}$$

$$\Omega(S) = \Pi(S) - K_2 \tag{4.5}$$

$$\Omega_S(S_2^*) = \Pi_S(S_2^*) \tag{4.6}$$

Boundary conditions (4.3) and (4.5) give the indifference points between changing regimes and incur the costs of regime change, and (4.4) and (4.6) are the "smooth pasting" conditions that determine the trigger rates S_1^* and S_2^* in order to change regime optimally from spot contracts to lay-up and vice-versa.

Equations (4.1) and (4.2) above have the same homogeneous part and it is a mathematical theorem, that all solutions can be represented as a linear combination of any two linearly independent solutions. That is, for the case of spot contracts we have:

$$\Pi(S) = C_1\Pi^1(S) + C_2\Pi^2(S) \tag{4.7}$$

Where:

- C_1 and C_2 are appropriately chosen constants

-$\Pi^i(S)$, $i = 1, 2$ are any two linearly independent solutions.

Similarly, for the laid-up equation we have:

$$\Omega(S) = C_3\Omega^1(S) + C_4\Omega^2(S) \tag{4.8}$$

Now, let's try a solution of the form:

$$\Pi(S) = S^\gamma \tag{4.9}$$

Substituting (4.9) and its derivatives in (4.1) gives:

$$\frac{1}{2}\sigma^2\gamma(\gamma-1)S^\gamma + (\mu-\lambda)S^\gamma\gamma - S^\gamma r = 0$$

$$(\frac{1}{2}\sigma^2\gamma(\gamma-1) + (\mu-\lambda)\gamma - r)S^\gamma = 0$$

$$\frac{1}{2}\sigma^2\gamma(\gamma-1) + (\mu-\lambda)\gamma - r = 0 \tag{4.10}$$

Solving (4.10) we get:

$$\gamma_1 = \frac{-(\mu-\lambda-\frac{1}{2}\sigma^2) + \sqrt{(\mu-\lambda-\frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} \tag{4.11}$$

$$\gamma_2 = \frac{-(\mu-\lambda-\frac{1}{2}\sigma^2) - \sqrt{(\mu-\lambda-\frac{1}{2}\sigma^2)^2 + 2\sigma^2 r}}{\sigma^2} \tag{4.12}$$

Notice that $\gamma_1 > 0$ and $\gamma_2 < 0$ for $r > 0$.

The above homogeneous solutions are the same for both regimes (spot or lay-up). We next compute the particular solution for each case.

(i) Ship operating in the spot, equation (4.1):

Try the following linear form:

$$\Pi^P(S) = B_1 S + B_2 \tag{4.13}$$

We then get: $\Pi_S^P = B_1$ and $\Pi_{SS}^P = 0$. Substituting in (4.1) we get:

$$0 + (\mu-\lambda)SB_1 - r(B_1 S + B_2) = -S + (A+T)$$

$$(\mu-\lambda-r)SB_1 - rB_2 = -S + (A+T) \tag{4.14}$$

Equating the coefficients we get:

$$(\mu-\lambda-r)B_1 = -1 \Rightarrow B_1 = \frac{1}{(\lambda+r-\mu)} \tag{4.15}$$

$$-rB_2 = A+T \Rightarrow B_2 = \frac{-(A+T)}{r} \tag{4.16}$$

Substituting the above constants in the particular form (4.13) we get:

$$\Pi^P(S) = \frac{S}{(\lambda + r - \mu)} - \frac{(A+T)}{r} \tag{4.17}$$

(ii) Ship in Lay-up,equation (4.2):

Try: $\Omega^P = B_3$, then we have, $\Omega^P_S(S) = 0$ and $\Omega^P_{SS}(S) = 0$.

Substituting in (4.2) we get:

$$0 + 0 - B_3 r = M \Rightarrow B_3 = \frac{-M}{r} \tag{4.18}$$

Therefore:

$$\Omega^P(S) = \frac{-M}{r} \tag{4.19}$$

We can now assemble the general solution of (4.1) (i.e., the value of the ship under spot contract) as:

$$\Pi(S) = C_1 S^{\gamma_1} + C_2 S^{\gamma_2} + \frac{S}{(\lambda + r - \mu)} - \frac{(A+T)}{r} \tag{4.20}$$

And the general solution of (4.2) (i.e., the value of the ship in lay-up) as:

$$\Omega(S) = C_3 S^{\gamma_1} + C_4 S^{\gamma_2} - \frac{M}{r} \tag{4.21}$$

Where C_1,C_2, C_3, C_4 are constants to be determined from boundary conditions.

Following Brennan and Schwartz (1985) and Dixit (1989), since $\gamma_1 > 0$ for $r > 0$ and $\frac{\Pi(S)}{S}$ should remain finite as $S \to \infty$, this implies that $C_1 = 0$. Thus the expression for the value of the ship operating in the spot contract becomes:

$$\Pi(S) = C_2 S^{\gamma_2} + \frac{S}{(\lambda + r - \mu)} - \frac{(A+T)}{r}\ for\ S \geq S_1^* \tag{4.22}$$

Similarly, since $\gamma_2 < 0$ for $r > 0$ and $\Omega(S)$ should remain finite as S approaches zero, this implies that $C_4 = 0$. Thus the expression for the value of the ship laid-up is given by:

$$\Omega(S) = C_3 S^{\gamma_1} - \frac{M}{r} \; for \; S \leq S_2^* \tag{4.23}$$

Now, what remains to be determined are the constants C_2 and C_3 and the trigger rates S_1^* and S_2^* in order to move the ship in and out of operations. They can be solved using boundary conditions (4.3) - (4.6) above.

Boundary condition (4.3) give the indifference condition between staying out of operations (lay-up) and moving into operations (spot contracts). The freight rate that makes this indifference condition to hold is S_2^*. We then get the condition:

$$\Omega(S_2^*) = \Pi(S_2^*) - K_2 \tag{4.24}$$

Similarly, when moving from operations (spot contracts) to non-operations (lay-up) we have:

$$\Pi(S_1^*) = \Omega(S_1^*) - K_1 \tag{4.25}$$

Now, together with the "smooth pasting" conditions (4.4) and (4.6) and substituting for the expressions for the ship value under spot contracts and lay-up given by (4.22) and (4.23) respectively, we get:

$$C_3 S_2^{*\gamma_1} - \frac{M}{r} = C_2 S_2^{*\gamma_2} + \frac{S_2^*}{(\lambda + r - \mu)} - \frac{(A+T)}{r} - K_2 \tag{4.26}$$

$$C_2 S_1^{*\gamma_2} + \frac{S_1^*}{(\lambda + r - \mu)} - \frac{(A+T)}{r} = C_3 S_1^{*\gamma_1} - \frac{M}{r} - K_1 \tag{4.27}$$

From the expressions for the value of the ship (4.22) and (4.23) we get:

$$\Pi_S(S) = C_2 \gamma_2 S^{\gamma_2 - 1} + \frac{1}{(\lambda + r - \mu)}$$
$$\Omega_S(S) = C_3 \gamma_1 S^{\gamma_1 - 1}$$

Substituting into the "smooth pasting" conditions we get two additional equations:

$$C_3 \gamma_1 S_2^{*\gamma_1 - 1} = C_2 \gamma_2 S_2^{*\gamma_2 - 1} + \frac{1}{(\lambda + r - \mu)} \tag{4.28}$$

$$C_2\gamma_2 S_1^{*\gamma_2-1} + \frac{1}{(\lambda + r - \mu)} = C_3\gamma_1 S_1^{*\gamma_1-1} \quad (4.29)$$

At this point we have to solve four equations, (4.26) to (4.29), which is a system of nonlinear equations, in order to determine the the four unknowns C_2, C_3, S_1^* and S_2^*. However, we can make further simplifications that will make easier the solution and implementation of optimal policies.

Define:

$$B_1 = \frac{(A+T)}{r} - \frac{M}{r} - K_1 \quad (4.30)$$

$$B_2 = \frac{(A+T)}{r} - \frac{M}{r} + K_2 \quad (4.31)$$

$$T_1 = \frac{1}{(\lambda + r - \mu)} \quad (4.32)$$

Substituting the above expressions into equations (4.26) to (4.29) we get the following system:

$$C_2 S_1^{*\gamma_2} + S_1^* T_1 = C_3 S_1^{*\gamma_1} + B_1 \quad (4.33)$$

$$C_3 S_2^{*\gamma_1} = S_2^* T_1 + C_2 S_2^{*\gamma_2} - B_2 \quad (4.34)$$

$$C_2\gamma_2 S_1^{*\gamma_2-1} + T_1 = C_3\gamma_1 S_1^{*\gamma_1-1} \quad (4.35)$$

$$C_2\gamma_2 S_2^{*\gamma_2-1} + T_1 = C_3\gamma_1 S_2^{*\gamma_1-1} \quad (4.36)$$

From (4.36) we have:

$$C_2 = \frac{\gamma_1}{\gamma_2} S_2^{*\gamma_1-\gamma_2} C_3 - \frac{T_1}{\gamma_2} S_2^{*-\gamma_2+1} \quad (4.37)$$

By substituting (4.37) into (4.34) we can get an expression for C_3 as a function of S_2^* and parameters as:

$$C_3 = \frac{T_1 S_2^*(\gamma_2 - 1) - \gamma_2 B_2}{S_2^{*\gamma_1}(\gamma_2 - \gamma_1)} \quad (4.38)$$

Next, substitute (4.38) into (4.37) and get also an expression for C_2 as a function of S_2^* and parameters as:

$$C_2 = \frac{T_1 S_2^*(\gamma_1 - 1) - \gamma_1 B_2}{S_2^{*\gamma_2}(\gamma_2 - \gamma_1)} \tag{4.39}$$

Now, substitute (4.38) and (4.39) into (4.33) and define $x = \frac{S_1^*}{S_2^*}$, then the constants are eliminated giving an expression for S_2^* as:

$$S_2^* = \frac{B_1(\gamma_2 - \gamma_1) + B_2(\gamma_1 x^{\gamma_2} - \gamma_2 x^{\gamma_1})}{T_1[(\gamma_1 - 1)x^{\gamma_2} + x(\gamma_2 - \gamma_1) - (\gamma_2 - 1)x^{\gamma_1}]} \tag{4.40}$$

Similarly, substitute (4.38) and (4.39) into (4.35) with $x = \frac{S_1^*}{S_2^*}$, and then the constants are again eliminated giving a second expression for S_2^* as:

$$S_2^* = \frac{\gamma_2\gamma_1 B_2(x^{\gamma_2} - x^{\gamma_1})}{T_1[\gamma_2(\gamma_1 - 1)x^{\gamma_2} + x(\gamma_2 - \gamma_1) - \gamma_1(\gamma_2 - 1)x^{\gamma_1}]} \tag{4.41}$$

Finally, with (4.40) and (4.41) we get an equation for x , the ratio of trigger rates, as:

$$\frac{B_1(\gamma_2 - \gamma_1) + B_2(\gamma_1 x^{\gamma_2} - \gamma_2 x^{\gamma_1})}{(\gamma_1 - 1)x^{\gamma_2} + x(\gamma_2 - \gamma_1) - (\gamma_2 - 1)x^{\gamma_1}} = \frac{\gamma_2\gamma_1 B_2(x^{\gamma_2} - x^{\gamma_1})}{\gamma_2(\gamma_1 - 1)x^{\gamma_2} + x(\gamma_2 - \gamma_1) - \gamma_1(\gamma_2 - 1)x^{\gamma_1}} \tag{4.42}$$

The above is a non-linear equation in x which can be solved with standard numerical procedures. A a subroutine from the NAG library of numerical subroutines, "C05AJF"[3] was used in this case. After solving for x we can compute S_2^* with equation (4.41) then compute S_1^* with S_2^* and x, and the constants C_2 and C_3 and finally the value of the ship operating, $\Pi(S)$ with (4.22) and in lay-up, $\Omega(S)$ with (4.23) for any freight rate S. The above formulas for the computation of policies and ship values were implemented into a FORTRAN program called "OPTPOL" which includes the solution of the non-linear equation (4.42) with the NAG subroutine. Appendix A presents "OPTPOL" FORTRAN code together with a sample input and output. "OPTPOL" will be extensively used in Chap. 6 for the computation of optimal

[3] C05AJF attempts to locate a zero of a continuous function by a continuation method using secant iteration. For further details see NAG Fortran Library Manual (1990).

policies and ship value[4].

4.2 Solution for the Term Charter Contract Case

As discussed in chapter 2, the structure of the term charter contract case is similar to the spot case hence their solutions for the valuation equations will be identical, however the parameters of the stochastic process will be different leading to different numerical results for the ship value and optimal policies.

The value of the ship and the optimal policies are then characterized by the following equations:

$$\frac{1}{2}(\sigma_T^2 S_T^2 \Gamma_{S_T S_T}) + (\mu_T - \lambda_T) S_T \Gamma_{S_T} - r\Gamma + (S_T - A - T) = 0 \tag{4.43}$$

Which is the ODE for the value of the ship operating in the term charter contract regime. And

$$\frac{1}{2}(\sigma_T^2 S_T^2 \Theta_{S_T S_T}) + (\mu_T - \lambda_T) S_T \Theta_{S_T} - r\Theta - M = 0 \tag{4.44}$$

Which is the ODE for the value of the ship in the laid-up condition.

And the following boundary conditions:

$$\Gamma(S_T) = \Theta(S_T) - K_1 \tag{4.45}$$

$$\Gamma_{S_T}(S_{T1}^*) = \Theta_{S_T}(S_{T1}^*) \tag{4.46}$$

$$\Theta(S_T) = \Gamma(S_T) - K_2 \tag{4.47}$$

$$\Theta_{S_T}(S_{T2}^*) = \Gamma_{S_T}(S_{T2}^*) \tag{4.48}$$

Following the same steps as the spot contracts case, the value of the ship operating

[4] "OPTPOL" was implemented in the IBM mainframe computer of the Sloan School where the NAG library is available.

in the term charter contract is given by:

$$\Gamma(S_T) = D_2 S_T^{\gamma_2^T} + \frac{S_T}{(\lambda_T + r - \mu_T)} - \frac{(A+T)}{r} \; for \; S \geq S_{T1}^* \tag{4.49}$$

And the expression for the value of the ship in laid-up is given by:

$$\Theta(S_T) = D_3 S_T^{\gamma_1^T} - \frac{M}{r} \; for \; S \leq S_{T2}^* \tag{4.50}$$

As in the spot case, with the indifference conditions for regime change and the "smooth pasting" conditions given above, we can determine the constants D_2 and D_3 and the trigger rates S_{T1}^* and S_{T2}^* with the system:

$$D_3 S_{T2}^{*\gamma_1^T} - \frac{M}{r} = D_2 S_{T2}^{*\gamma_2^T} + \frac{S_{T2}^*}{(\lambda_T + r - \mu_T)} - \frac{(A+T)}{r} - K_2 \tag{4.51}$$

$$D_2 S_{T1}^{*\gamma_2^T} + \frac{S_{T1}^*}{(\lambda_T + r - \mu_T)} - \frac{(A+T)}{r} = D_3 S_{T1}^{*\gamma_1^T} - \frac{M}{r} - K_1 \tag{4.52}$$

$$D_3 \gamma_1^T S_{T2}^{*\gamma_1^T} = D_2 \gamma_2^T S_{T2}^{*-\gamma_2^T - 1} + \frac{1}{(\lambda_T + r - \mu_T)} \tag{4.53}$$

$$D_2 \gamma_2^T S_{T1}^{*\gamma_2^T - 1} + \frac{1}{(\lambda_T + r - \mu_T)} = D_3 \gamma_1^T S_{T1}^{*\gamma_1^T - 1} \tag{4.54}$$

Again, the above is a system identical to the spot case and can also be computed with program "OPTPOL" with the term charter market parameters as computed in Chap. 5.

Chapter 5

Shipping Markets Parameter Estimation

In this chapter parameters of the geometric Brownian Motion stochastic process assumed for freight rates dynamics and of the risk premium for freight futures are estimated, for some representative routes in the transportation of grains in bulk. The routes considered are: US Gulf - Japan and US Gulf - Northern Europe (ARA : Antwerp, Rotterdam and Amsterdam). These routes are heavy traffic routes being well represented in the freight futures market and are also often used as benchmarks for practical decision-making in shipping.

5.1 The Parameter Estimators

The following parameters from the stochastic process and from the freight futures risk premia are to be estimated in order to determine the numerical values for the optimal policies (trigger rates) for ship operations:

- μ - the drift term of the Wiener process for the freight rates S.
- σ - the standard deviation for the Wiener process for the freight rates S.
- λ - the freight futures risk premium parameter.

5.1.1 Estimators for the Parameters for the Stochastic Process for Freight Rates

We assumed in Chap. 2 that freight rates follows the continuous-time stochastic process:

$$dS = S\mu dt + S\sigma dZ \tag{5.1}$$

The usual approach to estimate the parameters of the above process is the method of maximum likelihood (ML)[1]. The advantage of ML estimators is that they have the desirable properties of being consistent, asymptotically normal and asymptotically efficient (i.e., they have the smallest asymptotical variance of all consistent uniformly asymptotically normal estimators[2]). The disadvantages of ML estimators are that they are sometimes hard to compute (we have to solve an optimization problem and determine a probability density) and their small sample properties are unknown.

Formally a ML estimator $\hat{\theta}_{ML}$ is any element of Θ such that:

$$\hat{\theta}_{ML} = arg\ Max\ log\ p(S,\theta) \tag{5.2}$$

Where $\hat{\theta}_{ML} = (\hat{\mu}_{ML}, \hat{\sigma}_{ML})$ and $p(S,\theta)$ is the probability density function for the random variable S.

For this particular case, since S follows a geometric Brownian Motion, we know that S has a Lognormal distribution, and hence $logS$ is Normal. Thus for the geometric BM case we have:

$$\begin{aligned} logS_{t+1} &= \mu + logS_t + \epsilon_{t+1} \\ logS_{t+1} - logS_t &= \mu + \epsilon \end{aligned} \tag{5.3}$$

[1] The discussion and results to follow about ML estimation for continuous-time process is based on Lo (1988) and Lo (1992).

[2] See Silvey(1970) Ch. 4 and Lo (1992) for a complete discussion on ML estimation. See also Lo (1988) for an application of ML estimators for generalised Itô processes and Lo(1986) for an application of ML in the context of option pricing. For further references in the context of bond pricing see March and Rosenfeld (1983) and Pierson and Sun (1990).

Denoting $X_i = logS_i$, and since the data is sampled discretely we can define Z_k as the iid first differences, that is:

$$Z_k = X(kh) - X((k-1)h) = log\left(\frac{S(kh)}{S((k-1)h)}\right) \tag{5.4}$$

Also from the properties of a BM (see e.g., Vila (1990b)), Z_k is distributed as $N(\mu h, \sigma^2 h)$. Then we can get the expression for the p.d.f. for Z_k and the likelihood function to determine the ML estimators which will be compatible with the discretely sampled data. From Lo (1992), the likelihood function is given by:

$$\mathcal{L} = -\frac{n}{2}log(2\pi\sigma^2 h) - \sum_{k=1}^{n}\frac{(Z_k - \mu h)}{2\sigma^2 h} \tag{5.5}$$

In order to get $\hat{\theta}_{ML}$ we have the first order conditions:

$$\frac{\partial\mathcal{L}(\hat{\mu},\hat{\sigma^2})}{\partial\mu} = 0 \tag{5.6}$$

$$\frac{\partial\mathcal{L}(\hat{\mu},\hat{\sigma^2})}{\partial\sigma^2} = 0 \tag{5.7}$$

And by solving (5.6) and (5.7) we get ML estimators for freight rate dynamics parameters:

$$\hat{\mu} = \frac{1}{T}\sum_{k=1}^{n} Z_k \tag{5.8}$$

$$\hat{\sigma^2} = \frac{1}{T}\sum_{k=1}^{n}(Z_k - \hat{\mu}h)^2 \tag{5.9}$$

Also from Lo (1992) we get the following asymptotic (large n) results for $\hat{\theta}_{ML}$:

$$\sqrt{n}(\hat{\mu} - \mu) \overset{a}{\sim} N(0, \frac{n\sigma^2}{T}) \tag{5.10}$$

$$\sqrt{n}(\hat{\sigma^2} - \sigma^2) \overset{a}{\sim} N(0, 2\sigma^4) \tag{5.11}$$

The asymptotic precision of the above ML estimators are:

$$Var(\hat{\mu}) \overset{a}{\approx} \frac{\sigma^2}{T} \tag{5.12}$$

$$Var(\hat{\sigma^2}) \overset{a}{\approx} \frac{2\sigma^4}{n} \tag{5.13}$$

Therefore:

$$\hat{\mu} \xrightarrow{d} N(\mu, \frac{\sigma^2}{T}) \tag{5.14}$$

$$\hat{\sigma^2} \xrightarrow{p} \sigma^2 \tag{5.15}$$

From the above notice that even as $n \to \infty$, for fixed T, $\hat{\mu}$ is inconsistent, that is we can not do better than $\frac{\sigma^2}{T}$. While, $\hat{\sigma^2}$ is consistent we can do no better than $\frac{2\sigma^4}{n}$ for fixed n. Therefore an increaee in the frequency of observations (fixing T) will improve our estimators for the variance but not for the mean. To improve the estimator of the mean we have to increase the total length of sampling period T, independent of the frequency of observations.

5.1.2 Freight Futures Risk Premia Estimator

In Chap. 2 the PDE for the value of the freight futures contract was derived as:

$$\frac{1}{2}(F_{SS}\sigma^2 S^2) + (\mu - \lambda)SF_S - F_\tau = 0 \tag{5.16}$$

With the boundary condition: $F(S, 0) = S$

The solution to the above is given in (2.13) by:

$$F(S, \tau) = Se^{(\mu-\lambda)\tau} \tag{5.17}$$

With the above solution we can next determine an estimator for λ. There are several ways to do it. One approach would be to use as previously, ML estimation. It would then be necessary to derive the density function for $F(S, \tau, \theta)$ and as above determine $\hat{\lambda}_{ML}$. Another approach would be to use the above solution directly and run a regression of the logaritimic version, that is:

$$logF = (\mu - \lambda)\tau + logS \tag{5.18}$$

From the above regression we can compute the intercept term $\hat{\alpha} = (\hat{\mu} - \hat{\lambda})\tau$ and from above we estimated $\hat{\mu}$ with ML, then we can compute $\hat{\lambda}$ as:

$$\hat{\lambda} = \hat{\mu} - \frac{\hat{\alpha}}{\tau} \tag{5.19}$$

The time to maturity, τ, for freight futures usually varies from three to nine months.

5.2 The Estimation of Parameters

5.2.1 Description of The Data

The data set used is presented in Appendix B. The data includes the US Gulf - Japan and ARA routes for the spot market, the implied freight futures rate for each route for the next maturity date (i.e, end of the quarter) and the two subsequent quarters, and the term charter rates for one and three years contracts.

At this point, a digression is necessary in order to explain the terms of the spot and term charter contracts. The spot contract is denominated in \$/ton, the ship operator pays all operating costs and charges and the contract is for one trip for a specific loading and unloading ports. The term charter contract is denominated usually in \$/day, and the ship operator does not pay for fuel costs and port and canal charges. Therefore, a transformation should be made in the data to transform the term charter rates into spot equivalent or vice-versa to make possible the computation of "final" policies. In Appendix C the details of this transformation is presented together with ship operating costs and outputs to be used in the empirical analysis of policies and ship value reported in Chap. 6.

The data was sampled weekly from January 1985 to May 1992. The starting date, 1985, coincides with the beginning of the freight futures market.

5.2.2 Estimation of the Stochastic Process Parameters

In this section the estimates for rate of change of freight rates $\hat{\mu}$ (the drift of the Brownian Motion), and the market volatility $\hat{\sigma^2}$ (the variance of the Brownian Motion) will be computed with the estimators given by equations (5.8) and (5.9) above. These estimators which are basically sample means and sample variances of transformed data, can be computed with standard statistical software like SYSTAT or spreadsheet programs like LOTUS 123. A statistical software package that computes skewness and kurtosis will give us an idea of how far from normality we are. These statistics are usually not available in spreadsheet programs.

Spot Market Parameters

Table 5.1 presents the estimates for the means and variances together with other sample statistics for spot contracts for the US Gulf to Japan and ARA routes for the combined period 1985-1992 on a weekly basis.

Table 5.1:

Parameters and Sample Statistics for Spot Market

Results for weekly sampled data for the combined period 1985 to 1992 in $/ton.

Statistic	US Gulf - Japan	US Gulf - ARA
Observations	375	375
Minimum	-0.174353	-0.257829
Maximum	0.192904	0.360003
Mean	0.001277	0.000474
St. Error	0.017042	0.022610
Variance	0.002094	0.003686
St. Error	0.000153	0.000269
Standard Dev.	0.045761	0.060713
Skewness	-0.014684	0.260114
Kurtosis	2.188771	5.127846

The above results show the high standard errors for the mean while for the case of the variance they are quite low. It then becomes quite difficult to have good estimates for the mean as discussed above but for the variance they are quite good. Table 5.2 shows the estimates for means and standard deviations on a yearly basis for the combined period and for each year in percentage points.

Table 5.2:

Means and St. Deviations for US Gulf to Japan and ARA

Annual values in percentage points.

Period	USG-Japan Mean	USG-Japan S D	USG-ARA Mean	USG-ARA S D
1985-1992	6.64	33.00	2.46	43.78
1985	-7.27	31.69	-11.78	38.24
1986	-14.16	37.05	-16.99	56.51
1987	60.24	35.90	62.40	44.64
1988	22.08	28.50	17.90	35.97
1989	4.79	34.97	-1.72	42.70
1990	6.78	33.06	-3.53	46.35
1991	-6.18	31.21	-1.81	40.93

Term Charter Parameters

Table 5.3 presents the estimates for the means and variances together with other sample statistics for the term charter rates in spot rate equivalent, $/ton for the US Gulf to Japan route[3] for the 1 year and 3 year contracts for the combined period 1985-1992 on a weekly basis. As shown in appendix B there is no 1987 data available for the 3 year rate. Since the one and three years rate do not vary much,I used the 1 year 1987 data for the computation of the combined statistics for the 3 year rate. For the empirical testing of policies we use only the 1 year contract.

[3]See appendix C for the details of this transformation with includes the addition of fuel costs and port and canal charges.

Table 5.3:

Parameters and Sample Statistics Term Charter US Gulf to Japan

Results for weekly sampled data for the combined period 1985 to 1992 in spot equivalent $/ton .

Statistic	TC - 1 year	TC - 3 year
Observations	371	371
Minimum	-0.106386	-0.089730
Maximum	0.101258	0.101258
Mean	0.000472	0.000403
St. Error	0.008324	0.006986
Variance	0.000494	0.000348
St. Error	0.000036	0.000026
Standard Dev.	0.022218	0.018654
Skewness	0.689092	0.959566
Kurtosis	5.552269	8.743651

As expected, even with the 1987 data, the volatility and rates of change of the 3 year contract is smaller than the 1 year contract. Table 5.4 reports the estimates for means and standard deviations on a yearly basis for the combined period and for each year in percentage points.

Table 5.4:

Means and St. Deviations for US Gulf-Japan Term Charter

Annual values in percentage points, spot equivalent.

Period	1 year Mean	1 year S D	3 year Mean	3 year Var.
1985-1992	2.46	16.00	2.08	13.30
1985	-19.13	12.37	-18.20	10.20
1936	-3.33	16.31	-17.16	14.59
1987	43.91	20.02	43.91*	20.02*
1988	22.02	19.10	16.17	12.29
1989	-4.08	11.79	-4.16	6.48
1990	-20.03	16.28	-15.08	12.08
1991	17.07	11.92	18.04	12.88

5.2.3 Estimates for Risk Premium

The risk premium estimates, $\hat{\lambda}$ are computed using the estimator given by regression (5.18) and equation (5.19) above. Results for the regression and estimates for the US Gulf to Japan and ARA routes for the spot market are given in tables 5.5 and 5.6. The data set is given at Appendix B and is the value of the spot and futures with one, two and three quarters to maturity, because those longer times to maturity occur only few times a year the data set becomes small, even though it originates from the 1985 to 1992 period data set[4].

[4]The dates considered are the first week of February, May, August and November. See Appendix B.

Table 5.5:

Regression Results and Risk Premia for US Gulf to ARA

Results	USG-ARA 1Q	USG-ARA 2Q	USG-ARA 3Q
$\hat{\alpha}$	0.28076	0.40762	0.57061
SE of log F	0.11259	0.13089	0.12694
R^2	0.88169	0.82706	0.81033
Observ.	27	27	27
Deg Freed.	25	25	25
log S coef.	0.89029	0.82912	0.75998
SE of coef.	0.06522	0.07582	0.07353
τ (weeks)	13	26	39
$\hat{\mu}$	0.00047	0.00047	0.00047
$\hat{\lambda}$	-0.02113	-0.01526	-0.01416

Table 5.6:

Regression Results and Risk Premia for US Gulf to Japan

Results	USG-JAP 1Q	USG-JAP 2Q	USG-JAP 3Q
$\hat{\alpha}$	0.75616	0.88098	1.04475
SE of log F	0.15067	0.16109	0.16396
R^2	0.82036	0.77752	0.74080
Observ.	27	27	27
Deg Freed.	25	25	25
log S coef.	0.93265	0.87233	0.80292
SE of coef.	0.08728	0.09332	0.09498
τ (weeks)	13	26	39
$\hat{\mu}$	0.00128	0.00128	0.00128
$\hat{\lambda}$	-0.05689	-0.0326	-0.02551

The above estimates have very high negative values (around -70 to -120 %/year) and are well above the ranges that we would expect the risk premium to be (something around the value of the mean and probably positive). There are possibly several explanations for those poor results. First, the risk premium, λ has the characteristics of a "mean" which as discussed above for the stochastic process, is quite hard to estimate, with the very high standard errors which will induce very high errors in our risk premia estimates. Second, the quality of the freight futures data is lower than the quality of the freight rates data, in particular for longer maturity periods, because the freight futures market is a relatively new and rather specialized market which makes it a low volume ("thin") market. Finally, the estimating procedure used, given by equations (5.18) and (5.19) is rather simple and therefore further research on the statistical estimation would be necessary, even though the regression results points to the right direction, since the coefficient of log S is not far from one, confirming the validity of equation (5.17) as the pricing function for the freight futures.

However, I believe there is a way to get better estimates for the risk premium without too much cost. The alternative is to use the program "OPTPOL" to compute the value of the ship using the parameters for freight rates and the base case for ship costs and then determine the risk premium that makes the value of the ship lie in the range of observable ship prices.

Table 5.7 reports the implied risk premium computed for the base case (see Appendix C and Chap. 6) of a PANAMAX bulkcarrier for the US Gulf to ARA and Japan spot and US Gulf to Japan term charter contracts. The assumed price/value of the ship was \$ 15 M for a low level of freight rates (i.e., 15 \$/ton US Gulf - Japan and 9 \$/ton US Gulf - ARA) and \$ 30 M for moderate high levels of freight rates (i.e., 20 \$/ton US Gulf - Japan and 14 \$/ton US Gulf - ARA).

Table 5.7:

Implied Risk Premium from Ship Prices

Annual values in percentage points.

US Gulf - ARA Spot	US Gulf - Japan Spot	US Gulf - Japan Term
3.00	6.00	1.50

The above implied risk premia values even though difficult to evaluate precisely, are of the same order of magnitude as the means. Notice that the risk premia for the spot market is higher than the term charter as a percentage of their mean, as expected. Those risk premium values are used in the computation of the optimal policies and ship values in the empirical analysis presented in Chap. 6.

Chapter 6

Empirical Analysis

In this chapter the optimal policies determined in Chap. 4 together with the shipping markets parameters estimated in Chap. 5 will be implemented and tested for some realistic bulk shipping routes. In the process of simulating ship operations and implementing policies an effort was made to be as realistic as possible with respect to practical ship operations procedures and data. However, the objective of the thesis is the understanding and control of behavior rather than actual implementation, and hence some approximations were made to facilitate the testing of the model. Nevertheless, as will be seen shortly, the results seems to indicate that we are not very far from reality.

As already used for the estimation of parameters in Chap. 5, the shipping routes considered are U.S. Gulf to Japan and U.S. Gulf to Northern Europe (ARA, Antwerp, Rotterdam and Amsterdam) for the transportation of grains. Those are heavy traffic routes with good correlation/representation in the freight futures market. Moreover, they are often used as benchmarks for practical bulk shipping decisions. The ship considered is the PANAMAX bulkcarrier[1] of 60,000 dwt, which is the most used ship in these routes.

[1]A bulkcarrier is a ship specially designed to carry bulk commodities and in particular dry bulk like grains. A PANAMAX is a type of bulkcarrier such that its width make it possible to pass through the Panama canal, that is , it is the largest ship able to pass through the Panama canal.

6.1 Ship Costs and Output

The first step in the computation of the optimal policies is the determination of ship operating costs. As discussed in Chap. 5, the costs involved in term charter contracts are different from spot contracts, however a transformation is possible to be made to move from term charter ($/day) to spot equivalent ($/ton) and vice-versa and this is presented in Appendix C with the detail description of ship operating conditions, costs and outputs for both U.S. to Japan and ARA routes. With ship costs from Appendix C and shipping markets parameters from Chap. 5 we are ready to compute the optimal policies with the FORTRAN program "OPTPOL".

6.2 Optimal Policies for the US Gulf - Japan Case

6.2.1 Optimal Policies for the Spot Market

The ("realistic") base case considered for this route is given by:

1. Ship type : PANAMAX bulkcarrier, 60,000 dwt / 50,000 tons of cargo capacity.
2. Ship output : 282,000 tons/year.
3. Ship operational costs : A = 12 $/ton = 9,400 $/day.
4. Taxes : T = 0.26 $/ton = 73,320 $/year.
5. Costs during lay-up : M = 783 $/day.
6. Cost to move *into* lay-up : K_1 = 564,000 $.
7. Cost to move *out* of lay-up : K_2 = 1,694,000 $.
8. Estimated rate of change in freight rates (drift of Brownian Motion) for spot market (1985-92) : $\hat{\mu}$ = 6.64 %/year.
9. Estimated volatility (variance of Brownian Motion) for spot market (1985-92) : $\hat{\sigma^2}$ = 0.1089 ($\hat{\sigma}$ = 33.00 %/year).

10. Estimated risk premium for spot market : $\hat{\lambda} = 6.00$ %/year.

11. Interest rate : r = 9.00 %/year.

For the above case the optimal policies computed with "OPTPOL" are:

- Trigger rate to go into lay-up and out of operations (i.e., exit the market): $S_1^* = 7.81$ \$/ton.

- Trigger rate to go out of lay-up and back into operations (i.e., re-enter the market): $S_2^* = 17.24$ \$/ton.

The above results show clearly the hysteresis[2] effect; freight rates should go well below operating costs, down to 7.81 \$/ton for the value of the ship in the lay-up condition to cover the lay-up costs to exit the market. Once the ship is laid-up, freight rates must go well above operating costs, up to 17.24 \$/ton for the value of the ship in operations to cover the costs to re-enter the market. The reason for this phenomenon is the presence of costs of changing the operating regime induce an *option value to waiting* to change regime, since there is the potential (because of market volatility) that rates will go back up after a decline and vice-versa after an increase in freight rates.

Without the contingent claims methodology, which makes possible to take into account market dynamics, an alternative ("myopic") policy would be to exit the market when the decline in rates is enough to cover the exiting costs (K_1) and to re-enter when the increase in rates is enough to cover the costs of re-entering (K_2). These "myopic" policies are determined as the values of S_1^m and S_2^m such that :

$$\frac{S - S_1^m}{\lambda + r - \mu} = K_1 \; and \; \frac{S_2^m - S}{\lambda + r - \mu} = K_2 \tag{6.1}$$

The "myopic" policies computed for the above route is given by :

- $S_1^m = 11.83$ \$/ton

[2] Hysteresis is defined as the failure of an effect to reverse itself as its underlying cause is reversed.

- S_2^{in} = 12.50 $/ton

Following the "myopic" policies a shipowner would wait considerably less to exit or to re-enter the market. The reason for this is the absence of volatility effects in the computation of "myopic" policies. For this case the hysteresis effects are negligible.

6.2.2 Sensitivity Analysis for the Spot Case

In this section a sensitivity analysis is presented where ship specific parameters (i.e., ship operating costs, ship costs during lay-up and costs to go in and out of lay-up) and market parameters (i.e., rates of change and volatility of freight rates, risk premium and interest rate) from the base case above will vary and its effects on the optimal policies will be studied.

Variations in Ship Operational Costs (A)

Figure 6.1 shows the values of trigger rates for several values of ship operational costs[3]. The costs vary from a very low case of 4 $/ton to a high of 24 $/ton and the base case is 12 $/ton. As shown in the figure, as the ship operational cost *increases* both trigger rates increase meaning that the ship will move into lay-up *earlier*, if it was operating, and will move out of lay-up *later*, if it was laid-up and vice-versa for a decrease in costs. As costs increases, the distance between trigger rates also increases indicating that the lay-up time will increase. The reason for this, is that, as ship operating costs increases the option value to wait to exit the market decreases and to re-enter increases.

Variations in Costs During Lay-up (M)

Figure 6.2 shows the effects on the optimal policies for variations in the costs during lay-up. We can see that as the costs during lay-up *increase* both trigger rates *decrease*, meaning that the ship will exit the market *later* and re-enter the market *earlier*. The

[3]The interpolation was made with a spline routine available at the MATLAB program at project Athena/MIT.

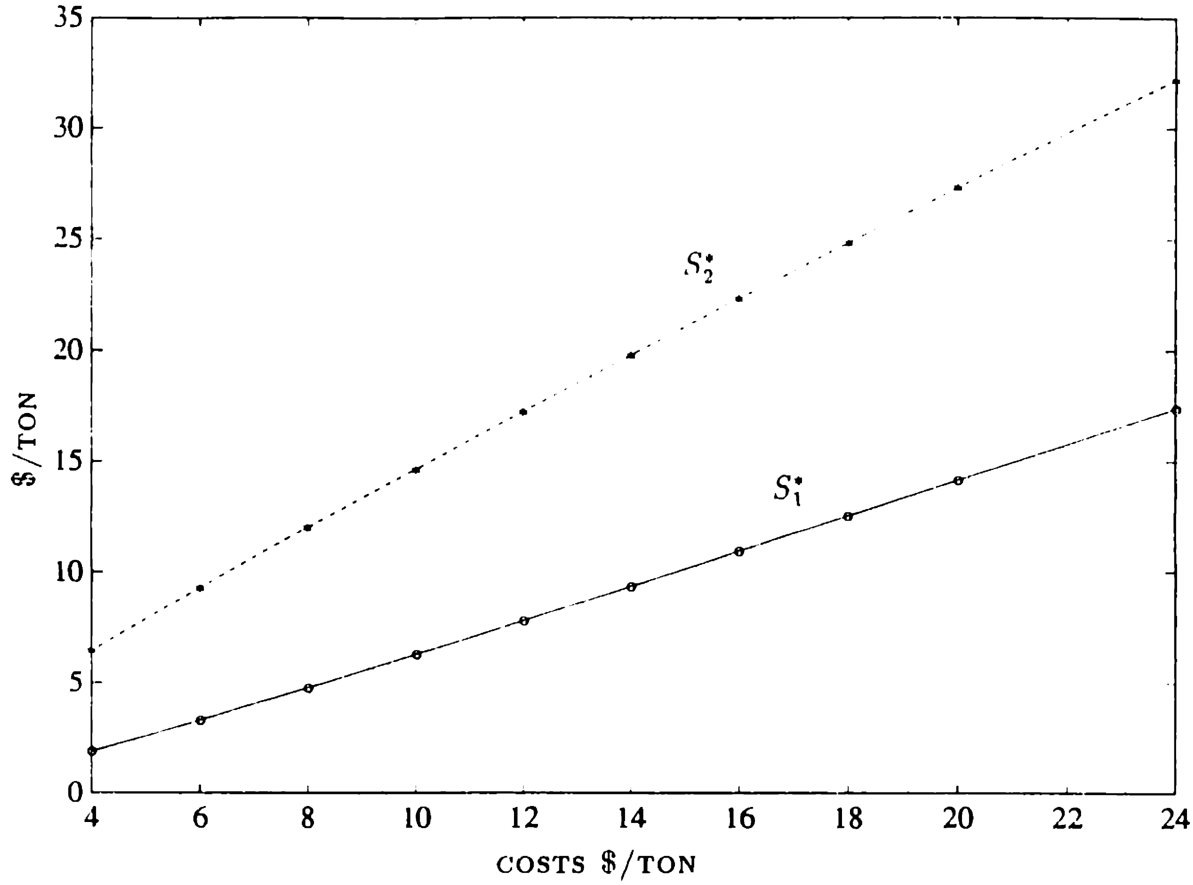

Figure 6-1: Effects on Optimal Policies due to variations in ship operating costs.

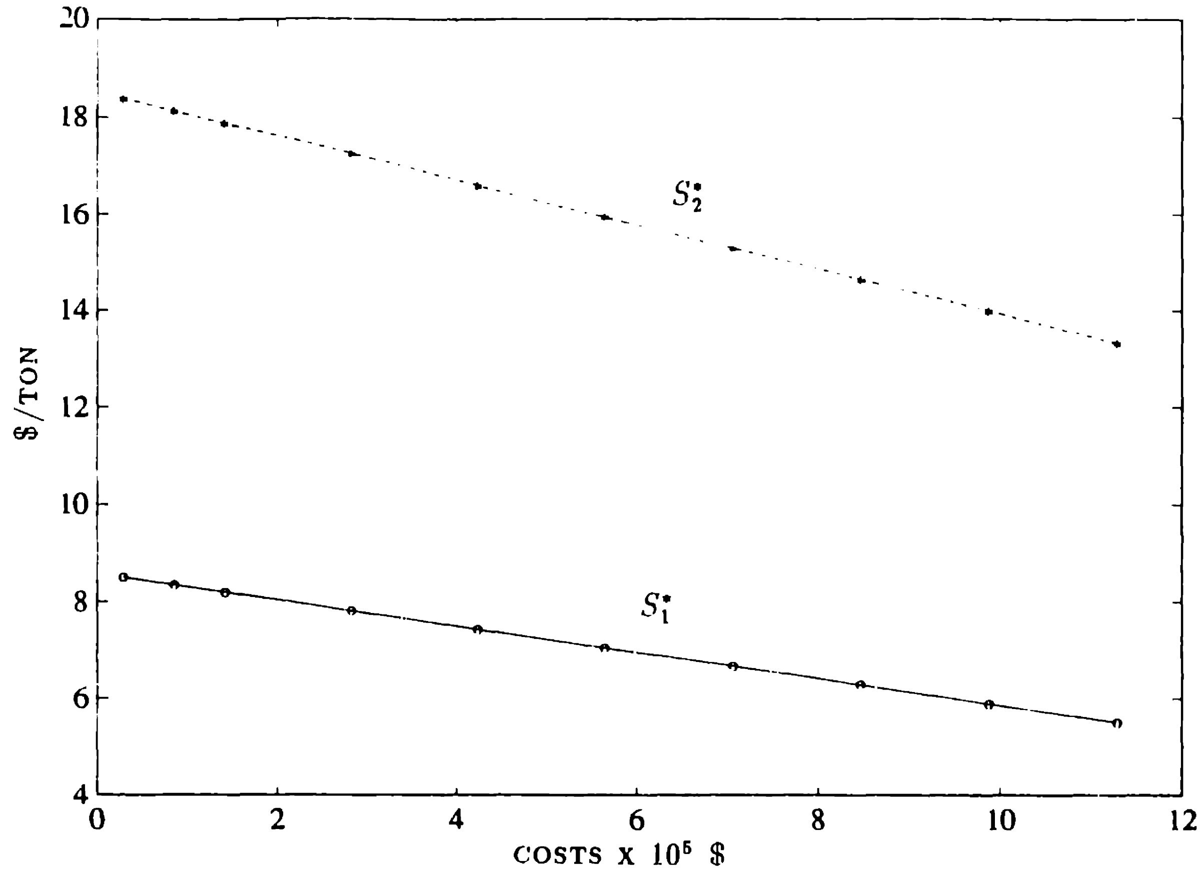

Figure 6-2: Effects on Optimal Policies due to variations in ship costs during lay-up.

intuition is that as it becomes expensive to be in lay-up the ship "tries" to avoid this regime as it "tried" to avoid the operating regime when operating costs increased above. For this case the option to wait to exit the market increases and the option to wait to re-enter decreases as lay-up costs increases.

Variations in Costs to move into, K_1, and out, K_2, of Lay-up

As figure 6.3 shows, as the costs to move in and out of lay-up increases, the trigger rate to move the ship into lay-up decreases meaning that the ship will move into lay-up later and the trigger rate to move out of lay-up increases meaning that the ship will re-enter the market also later. Due to the increase in cost to change regime the option value to wait will increase for both lay-up and operating conditions. The

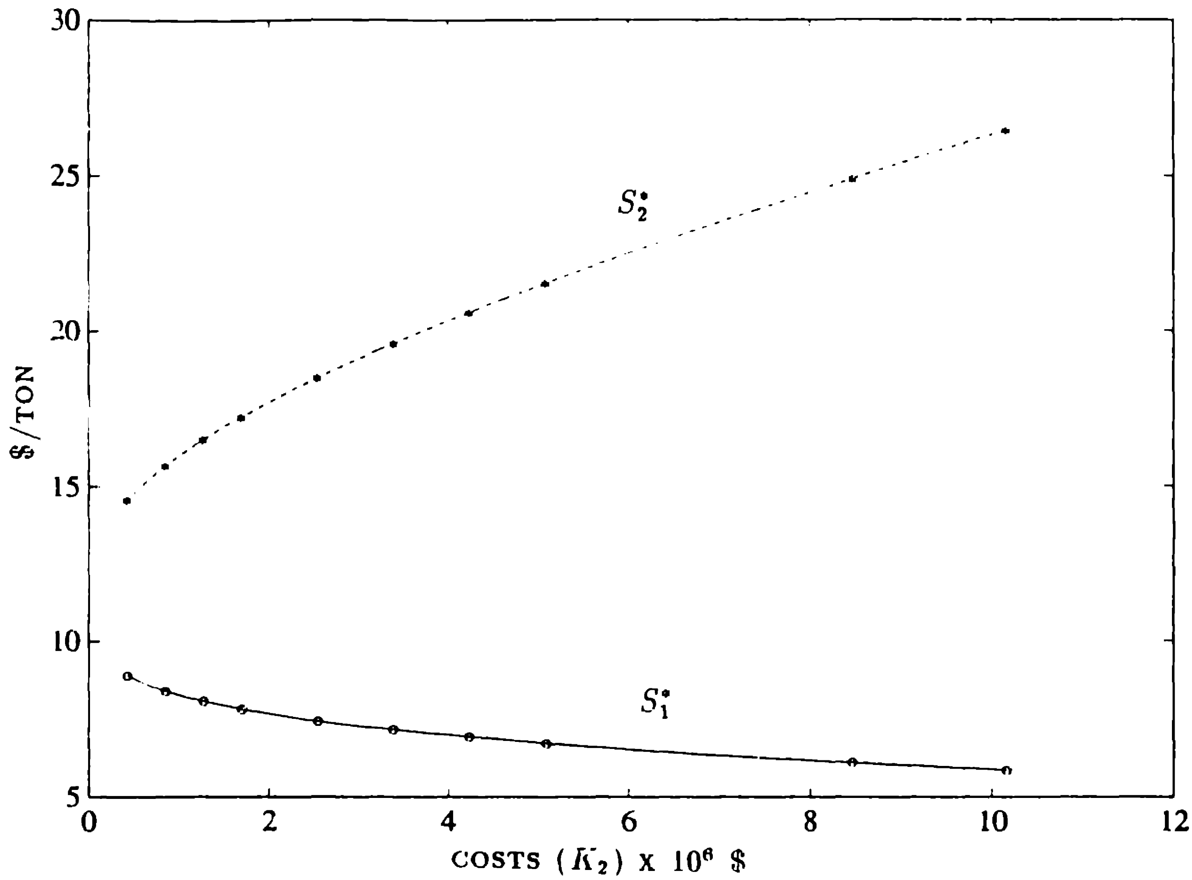

Figure 6-3: Effects on Optimal Policies due to variations in costs to move in and out of lay-up.

intuition is that, since it is costly to change regime the ship will "try" to avoid regime change as they will became less frequent.

Variations in the Rate of Change of Freight Rates (μ)

In figure 6.4 we see that as the rate of change in freight rates (drift of Brownian Motion) increases both the trigger rate to move into and out of lay-up decreases, indicating that the ship will wait longer to exit the market and will re-enter the market sooner. The intuition for this is that as the rate of change in freight rates increases the market becomes more attractive and lay-up less attractive, that is, the ship will "try" to avoid the lay-up regime as the market becomes a more advantageous regime. This effect is similar to the increase in costs during lay-up where the option

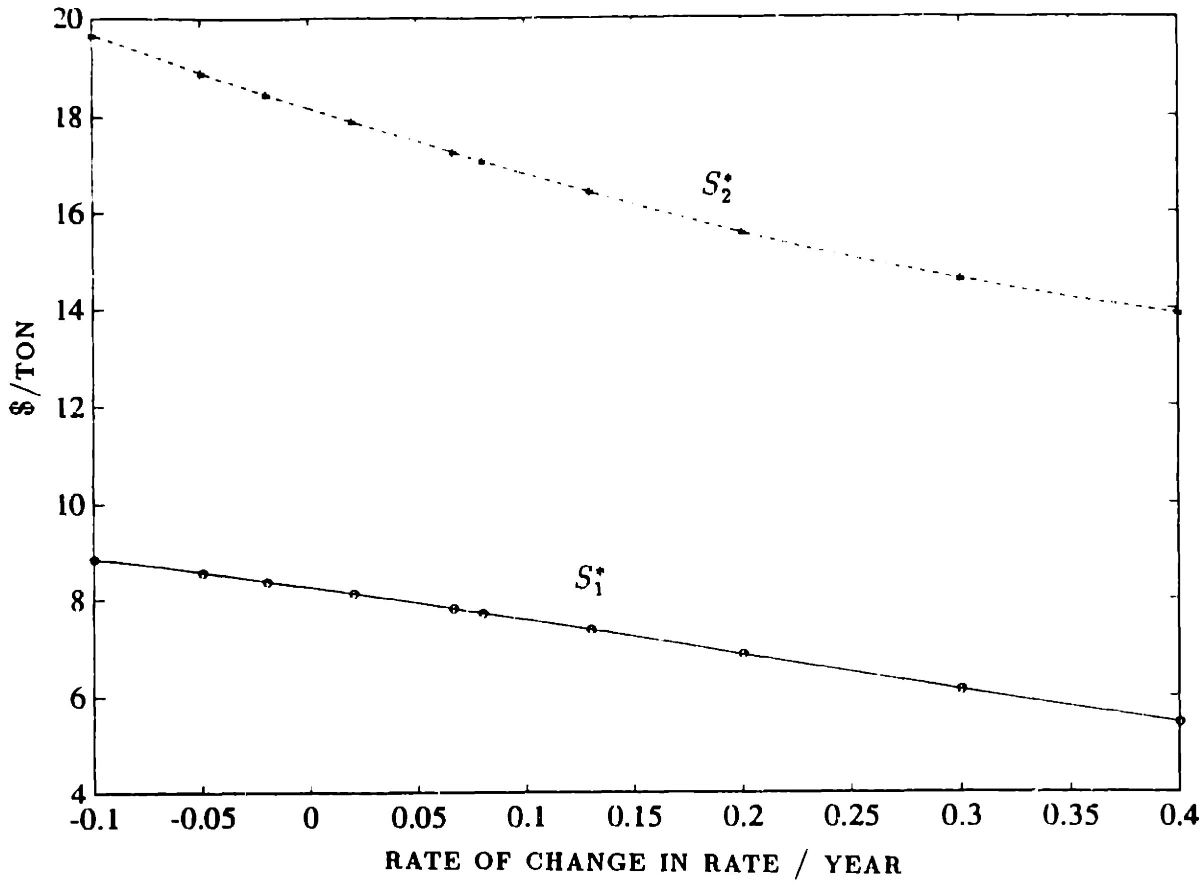

Figure 6-4: Effects on Optimal Policies due to variations in the rate of change in freight rates

to wait to exit increases and to re-enter decreases.

Variations in Market Volatility (σ^2)

As figure 6.5 shows, the effect on trigger rates due to an increase (decrease) in market volatility (the variance of Brownian Motion) are similar to the effects caused by an increase (decrease) in the costs to move in and out of lay-up. That is, as volatility increases the trigger rates to exit the market decreases and to re-enter increases, meaning that the option to wait increases for both regimes leading to a *less frequent* change of regime and vice-versa for a decrease in volatility.

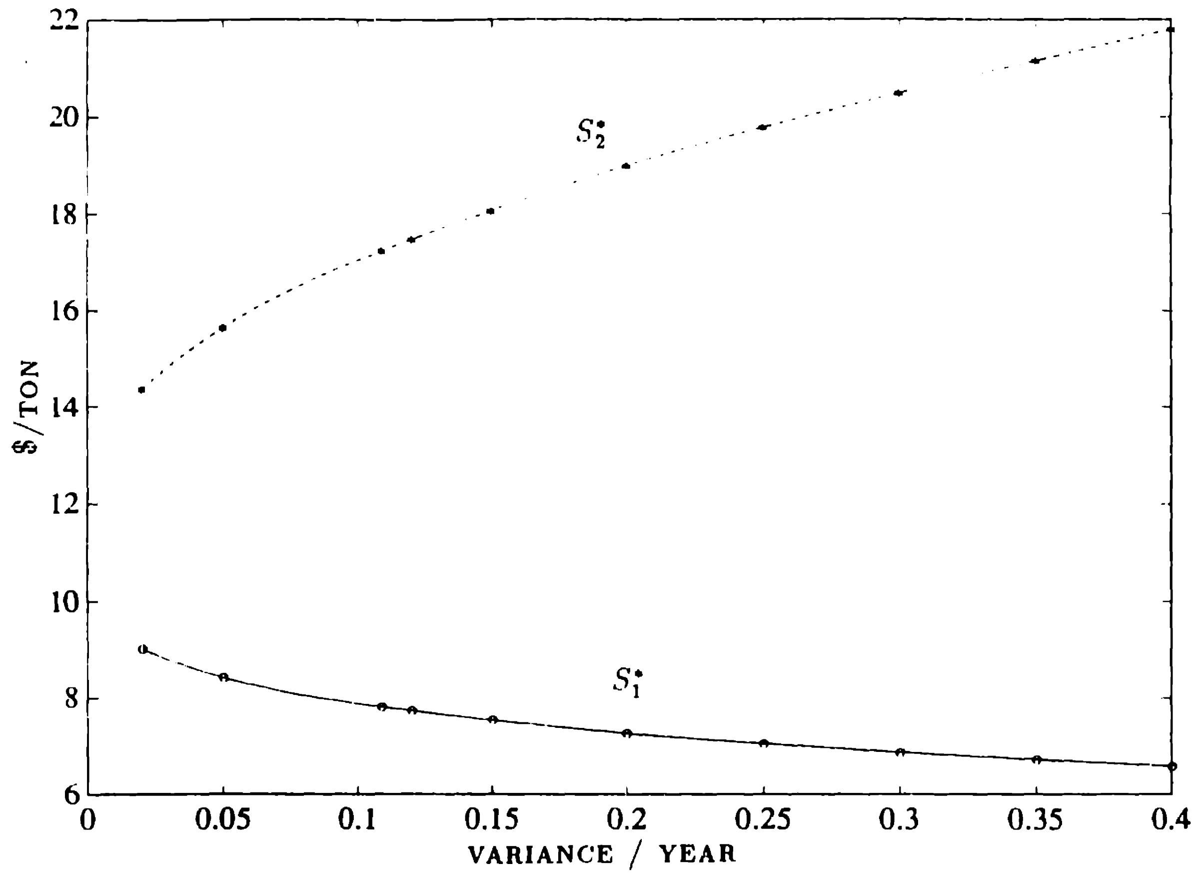

Figure 6-5: Effects on Optimal Policies due to variations in market volatility.

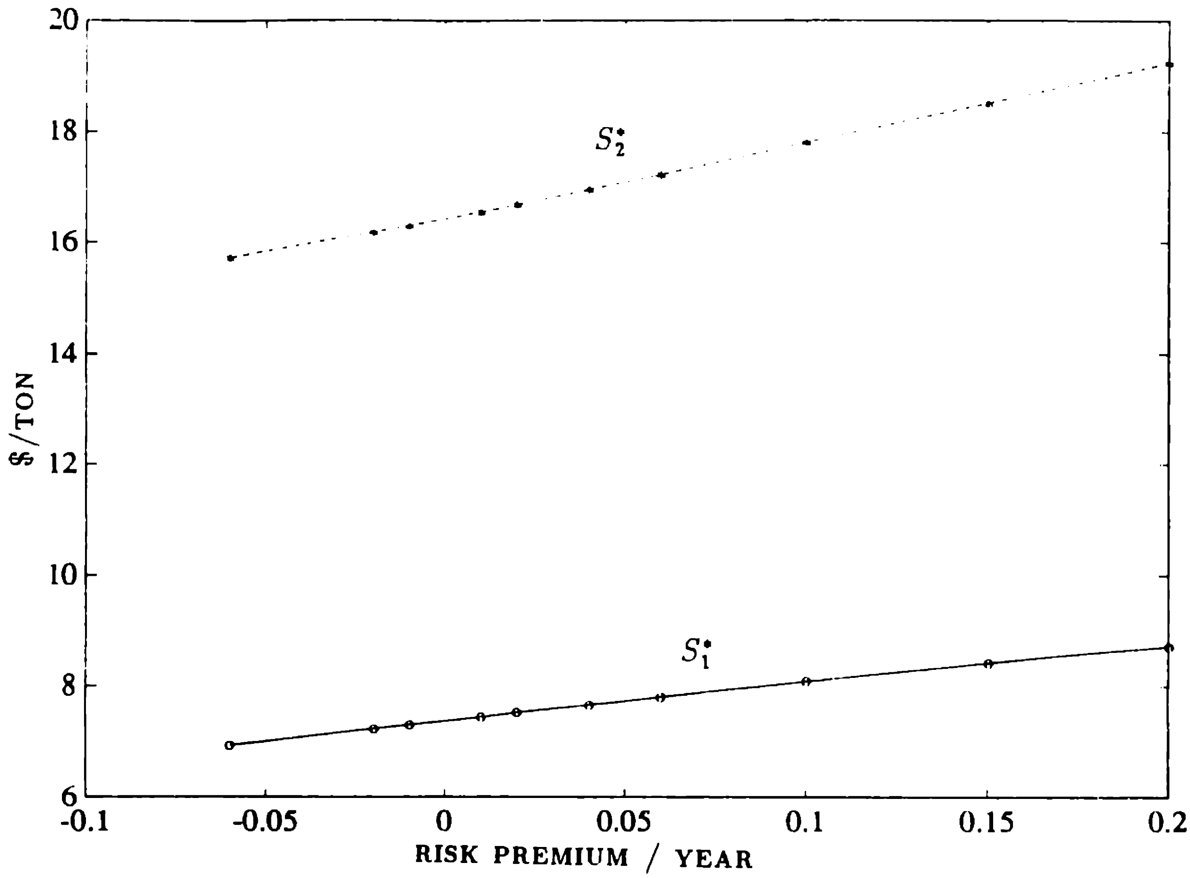

Figure 6-6: Effects on Optimal Policies due to variations in market risk premium.

Variations in Risk Premium (λ)

In figure 6.6 we see that an increase (decrease) in the risk premium will lead to an increase (decrease) in both trigger rates, meaning that the ship will wait less (longer) to exit and wait longer (less) re-enter the market. The intuition is that, as the risk premium increases the market becomes "riskier" and hence the ship will take longer to re-enter and will exit sooner.

Variations in Interest Rates (r)

Finally, figure 6.7 shows the effects on trigger rates due to changes in interest rates. As the figure shows, the effects on the exit rate is nearly non-existent, leading to a constant exit trigger rate as interest rates change. The effect on the re-entry rates

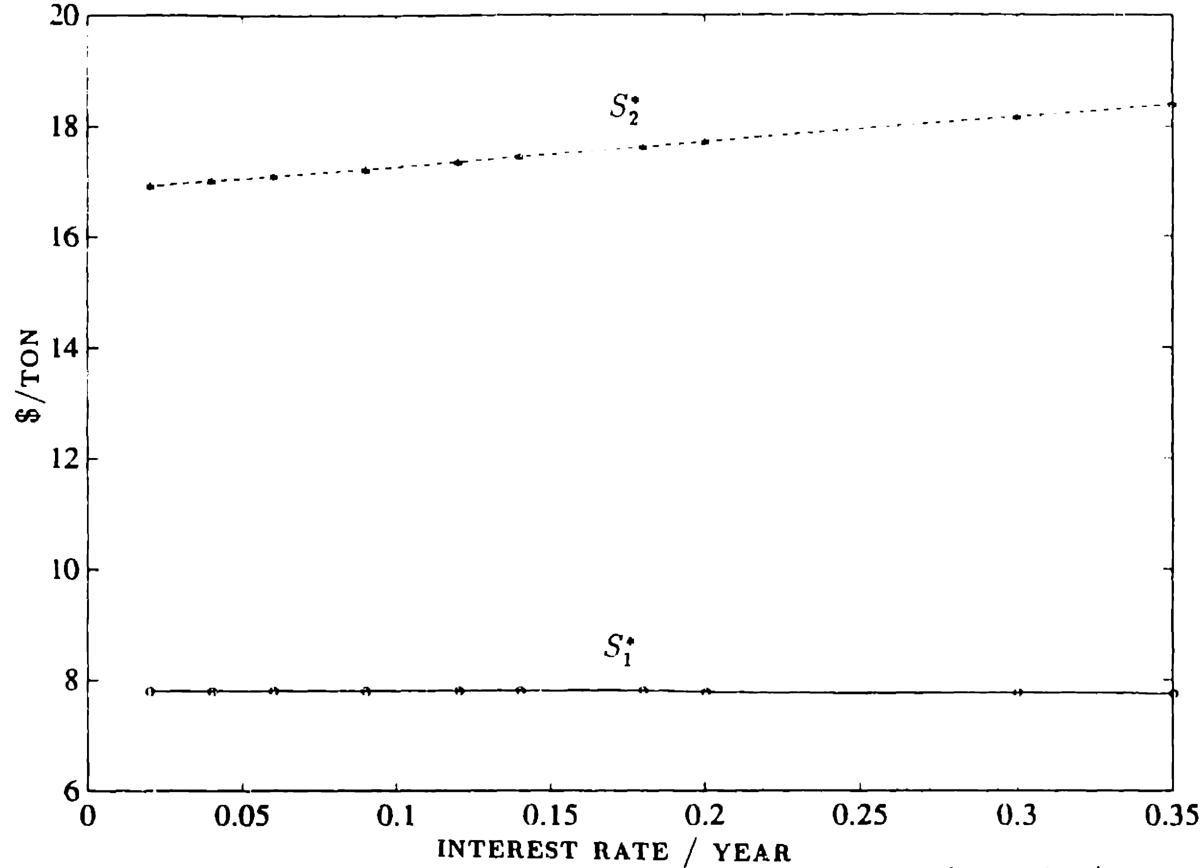

Figure 6-7: Effects on Optimal Policies due to variations in interest rates.

is small but as interest rate increases the re-entry rates increase leading to a longer lay-up time.

6.2.3 Optimal Policies for the Term Charter Market

The option to go to the term charter market has no effect on the ship specific parameters, only on market parameters exclusive of interest rates. The contract considered is the 1 year contract. Thus the base case for the term charter market will be as for the spot market apart from:

1. Estimate for the rate of change in freight rates (drift of Brownian Motion) for the term charter market (1985-92): $\hat{\mu}$ = 2.46 %/year.

2. Estimate of market volatility (variance of Brownian Motion) for term charter market (1985-92): $\hat{\sigma}^2 = 0.0256$ ($\hat{\sigma} = 16.00$ %/year).

3. Estimate for term charter risk premium: $\lambda = 1.5$ %/year.

For the above case the optimal policies computed with fortran program "OPT-POL" are:

- Trigger rate to move into laid-up and out of operations (i.e., exit the market): $S^*_{T1} = 8.84$ \$/ton.

- Trigger rate to move out of laid-up and into operations (i.e., re-entry the market) : $S^*_{T2} = 14.61$ \$/ton.

From the above policies we see that for the term charter case the ship will exit the market earlier and re-enter also earlier than for the case of the spot market. Therefore the option value to wait to change regime in the term charter market will be lower than the spot case. The explanation for this difference in trigger rates is the lower volatily of the term charter market (16 % against 33 %). As discussed and shown in figure 6.5 above, a decrease in volatility decreases the option value to wait to change regime. It seems that the effects of a lower rate of change in freight rates and lower risk premium are canceling each other out since they affect trigger rates in opposite directions (see figures 6.4 and 6.6 above).

6.2.4 Comparison of Optimal Policies for Spot and Term Charter

In this section some combined sensitivity analysis for the spot and term charter are made.

Figure 6.8 shows the superposition of the optimal policies for spot and term charter markets as the ship operational costs vary. Due to the lower volatility of the term charter the trigger rates to exit the market are higher (i.e., waiting less to exit) and the trigger rates to re-enter are lower (i.e., also waiting less to re-enter) than the spot

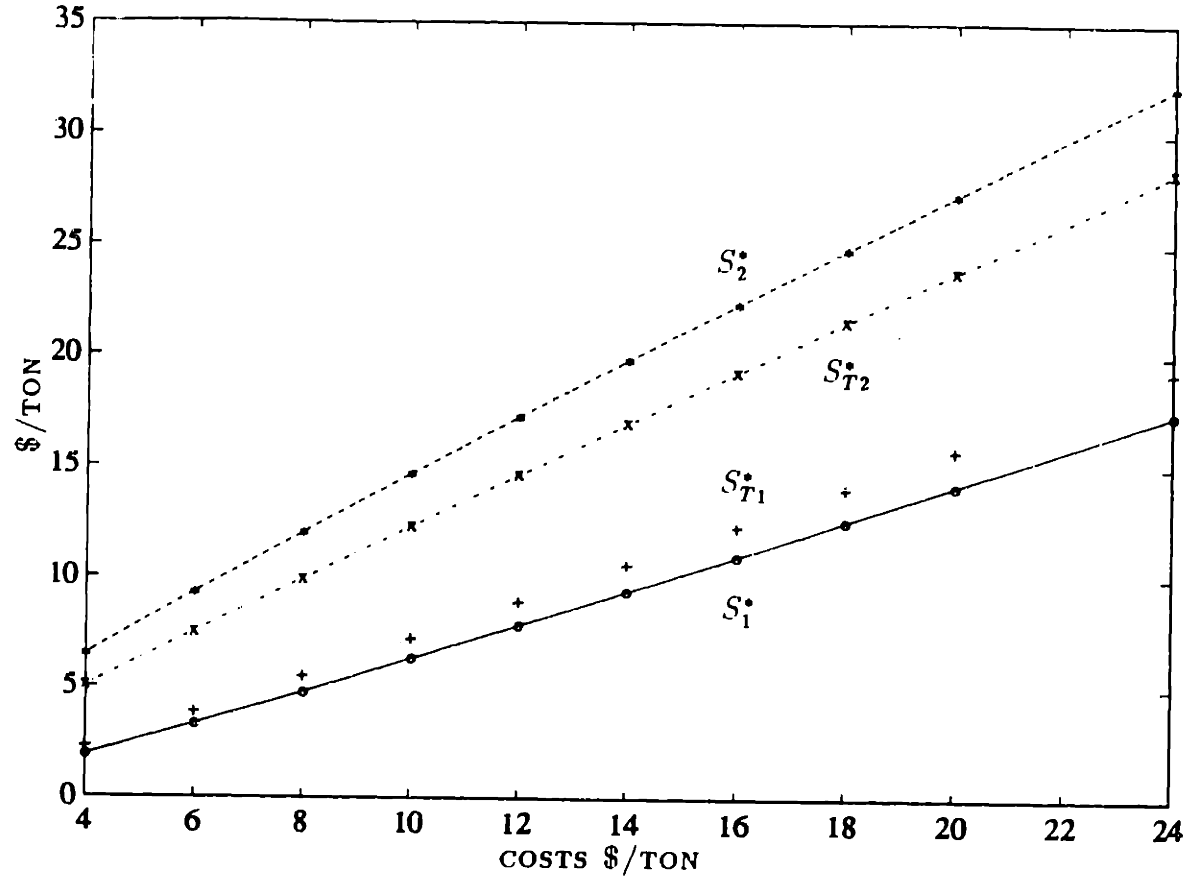

Figure 6-8: Effects on the optimal policies for the term charter and spot market for variations in ship operating costs.

case. This generalizes the lower option value to wait for the term charter market for a given ship.

Figure 6.9 shows the effects in the optimal policies for variations in the cost to move in and out of lay-up. Again the option value to wait in the term charter market is lower than for the spot case.

6.3 Optimal Policies for the US Gulf - ARA Case

A ship operating in the bulk trades has the option of not only choose between types of contracts, spot versus term charter, but also between routes. In this section results for an alternative route, US Gulf - ARA (Northern Europe), is presented together

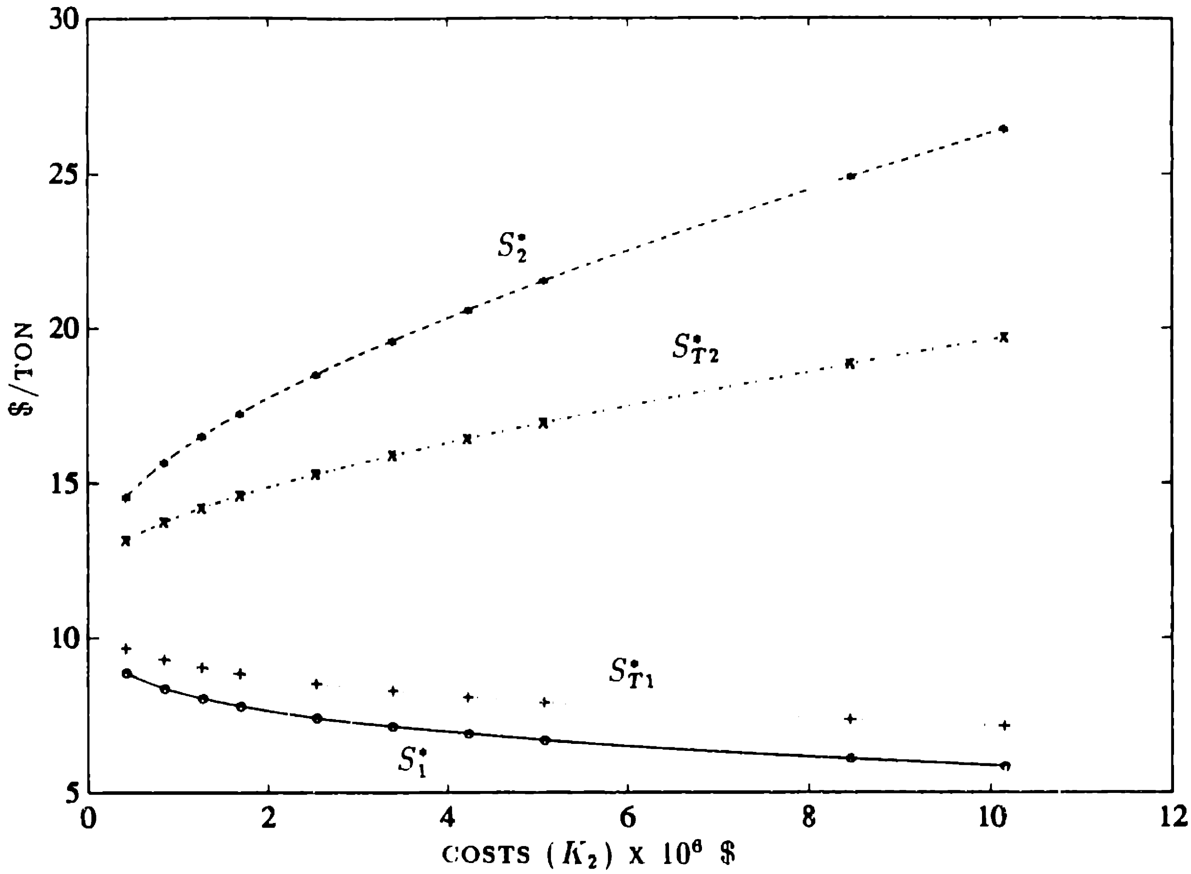

Figure 6-9: Effects on the optimal policies for the term charter and spot market for variations in costs to move in and out of lay-up.

with comparisons with the US Gulf - Japan case.

From the ship costs computations in appendix C and shipping markets parameters estimation from chapter 5 we have the following base case for the spot market in the US Gulf - ARA route:

1. Ship Type : PANAMAX bulkcarrier 60,000 dwt/ 50,000 tons of cargo capacity.
2. Ship Output : 400,000 tons/year.
3. Ship Operating Costs : A = 8 \$/ton = 8,900 \$/day.
4. Taxes : T = 0.23 \$/ton = 92,000 \$/year.
5. Costs during lay-up : M = 778 \$/day.
6. Cost to move into lay-up : K_1 = 560,000 \$.
7. Cost to go out of lay-up : K_2 = 1,630,000 \$.
8. Estimated rate of change in freight rates : $\hat{\mu}$ = 2.46 %/year.
9. Estimated volatility : $\hat{\sigma^2}$ = 0.1917 ($\hat{\sigma}$ = 43.78 %/year).
10. Estimated risk premium : λ = 3 %/year.
11. Interest Rate : r = 9 %/year.

The optimal policies computed with program "OPTPOL" are:

- S^*_{A1} = 4.90 \$/ton.
- S^*_{A2} = 12.81 \$/ton.

Since the distance between ports for this route is smaller the freight rates and trigger rates are smaller than the US Gulf - Japan case. As before the option value to wait (hysteresis effect) is present making the trigger rate to exit the market well below and the trigger rates to re-enter the market well above ship operating costs.

We can now compare the optimal policies of the US Gulf - Japan and US Gulf - ARA routes by computing them as a percentage of ship operating costs (the deterministic trigger rate) and their values are presented in table 6.1 .

Table 6.1:

Optimal Policies for US Gulf to Japan and ARA % of Base Case

Trigger Rate	US Gulf - ARA Spot	US Gulf - Japan Spot	US Gulf - Japan Term
S_1^* (exit)	61.25	65.08	73.67
S_2^* (re-entry)	160.13	143.67	121.75

The above results show that the option value to wait is higher for the Northern Europe route for spot contracts followed by the Japan route for spot contracts and the term charter with the lower option value. Again, the higher the volatility of the route, the higher the option value to wait.

As before, figures 6.10, 6.11 and 6.12 shows the effects on optimal policies for variations in ship operating costs, costs to go in and out of operations and market volatility respectively for the Northern Europe route. Those results agree with the ones presented for the Japan route above.

6.4 Ship Value and Optimal "Final" Policies

As discussed in Chap 2, a ship operator has not only to decide if the vessel should exit or re-enter the market but also once in the market which type of contract to accept: a spot or term charter contract. The procedure developed in Chap. 2, called optimal "final" policies, to decide between the two types of contracts is based on the maximum value the ship achieves under those contracts and Chap. 4 shows how to compute the value of the ship once we determine the exit/re-entry policies (i.e., equations (4.22) and (4.23)).

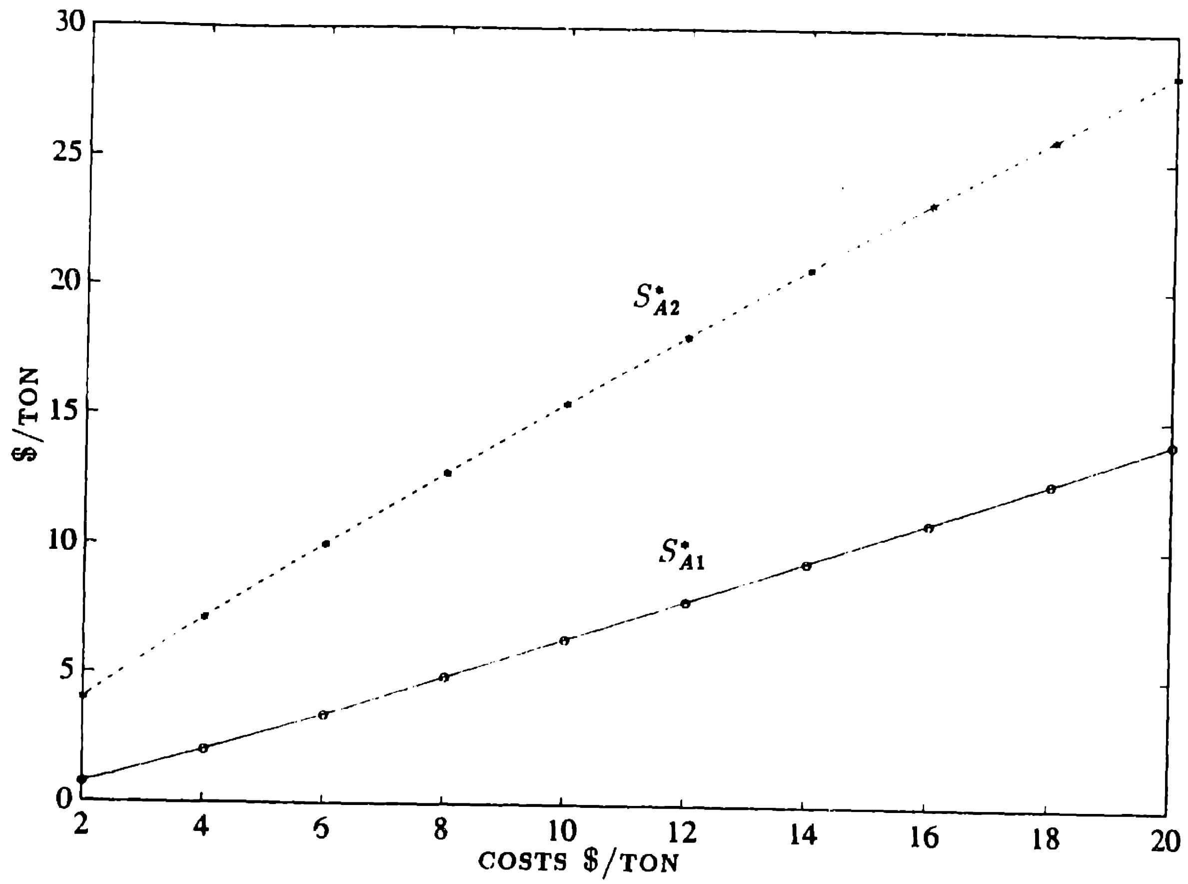

Figure 6-10: Effects on the optimal policies for the ARA route / spot market for variations in ship operating costs.

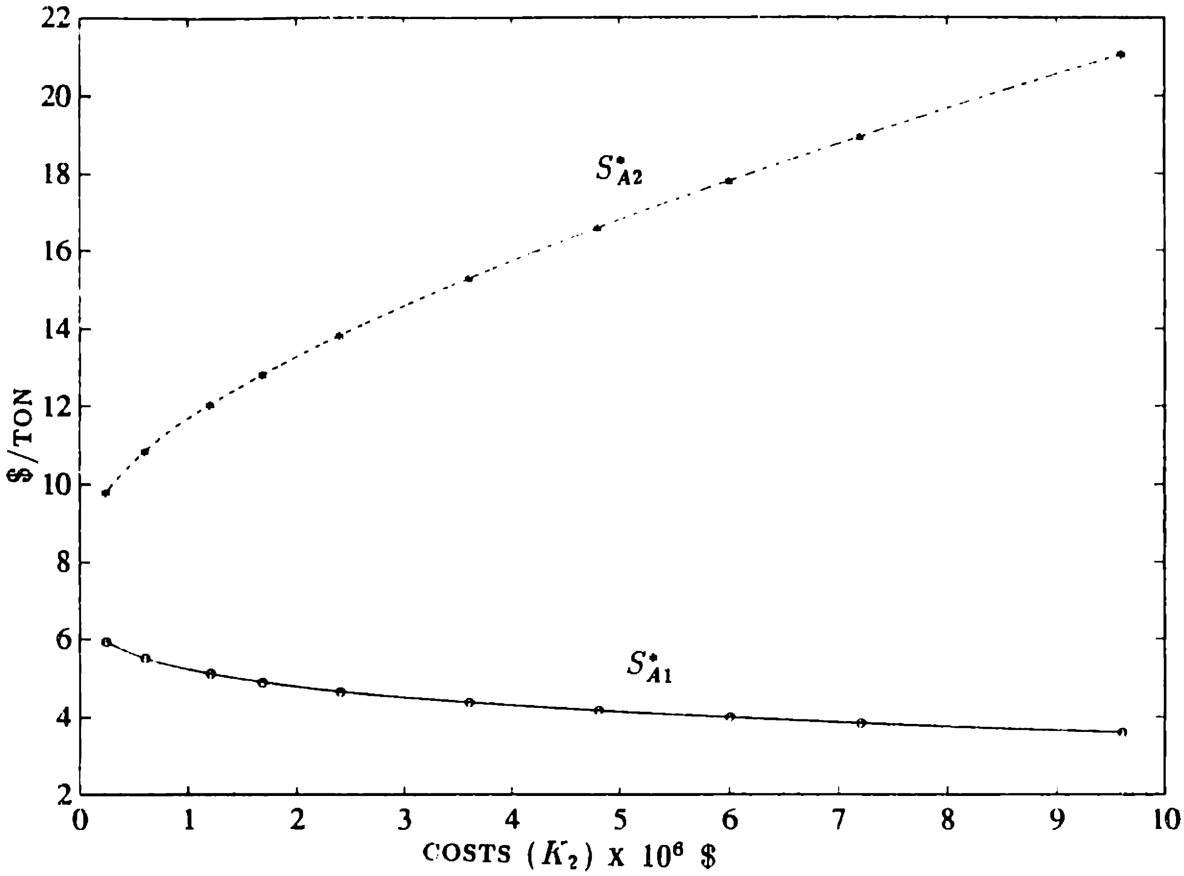

Figure 6-11: Effects on the optimal policies for the ARA route / spot market for variations in costs to move in and out of lay-up.

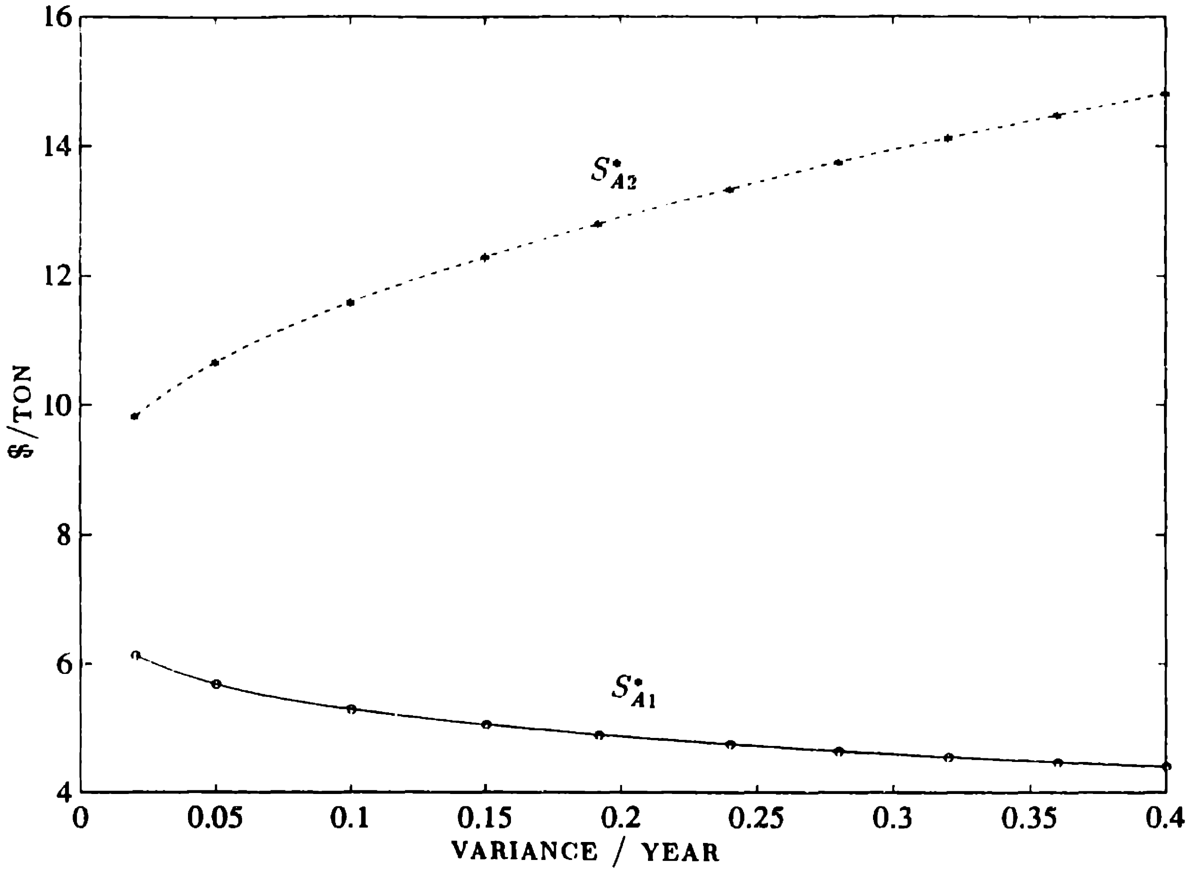

Figure 6-12: Effects on the optimal policies for the ARA route / spot market for variations in market volatility.

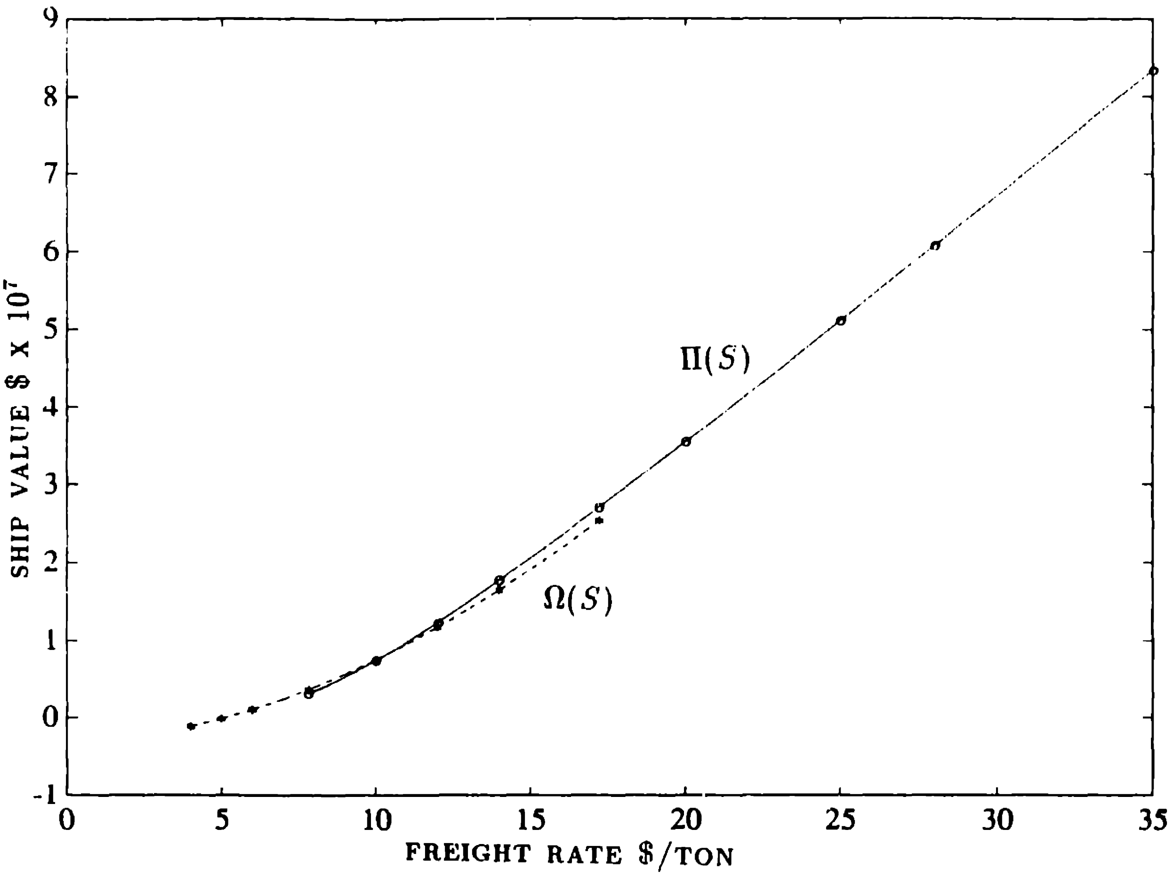

Figure 6-13: Value of the ship operating in the US Gulf - Japan route under spot contracts.

6.4.1 The Ship Value

With "OPTPOL" it is possible to compute the value of the ship for a range of freight rates, and figure 6.13 shows the value of the ship for the spot market base case in the US Gulf - Japan route.

Note that for low freight rates the value of the ship in lay-up (dotted line) is higher than the value of the ship operating in the spot market. As freight rates increase the operating value approaches the lay-up value, then crosses it but because there is the cost to move out of lay-up freight rates should increase even more until the value of the ship operating matches the value of the ship in lay-up plus the switching costs K_2, and this occur at a rate of 17.24 $/ton ($S_2^*$). After the 17.24 $/ton rate the ship is operating in the spot market. With an analogous reasoning, as freight rates decreases, the ship is still operating until rates reach the value of 7.81 \$/ton (S_1^*) when the value of the ship in lay-up matches the value of the ship operating plus the switching costs K_1.

Figure 6.14 is an extreme case of in and out of lay-up costs where the difference in value of the ship in and out of operations became much larger making it easier to see the variations in ship value and the hysteresis phenomenon, and as discussed previously, the fact that the switching costs are higher the the ship will wait longer to change regime. Figure 6.15 shows two cases for the value of the ship for the same route and also for the spot market but with different ship operating costs (base case and 33% higher). As expected, the higher cost ship is the less valuable and as discussed before, the higher cost ship takes longer to re-enter the market and will exit the market sooner.

Finally figure 6.16 shows the value of the ship for the base case for the US Gulf - ARA route.

6.4.2 The Optimal "Final" Policies

Now we have to answer the question if the ship, when operating, should accept a spot or term charter contract ? The final decision will be made with the value of the ship for each contract based on the current rate, that is, we compute the today's value of the ship in the spot market with today's spot rate and the same for the term charter market case.

A convenient way to study the behavior of optimal "final" policies is by looking at figure 6.17 where the value of the ship as a function of freight rates for the base case for the US Gulf - Japan route for both spot and term charter (dotted lines) are presented. An interesting result emerges in this figure, note that for freight rates above 30 \$/ton the value of the ship in the term charter market for the same value of freight rates, is higher than the value of the ship under spot contacts. Therefore for high levels of freight rates the term charter market will tend to dominate the spot market as an optimal "final" policy.

Interesting results are also presented in figures 6.18 and 6.19 where variations in

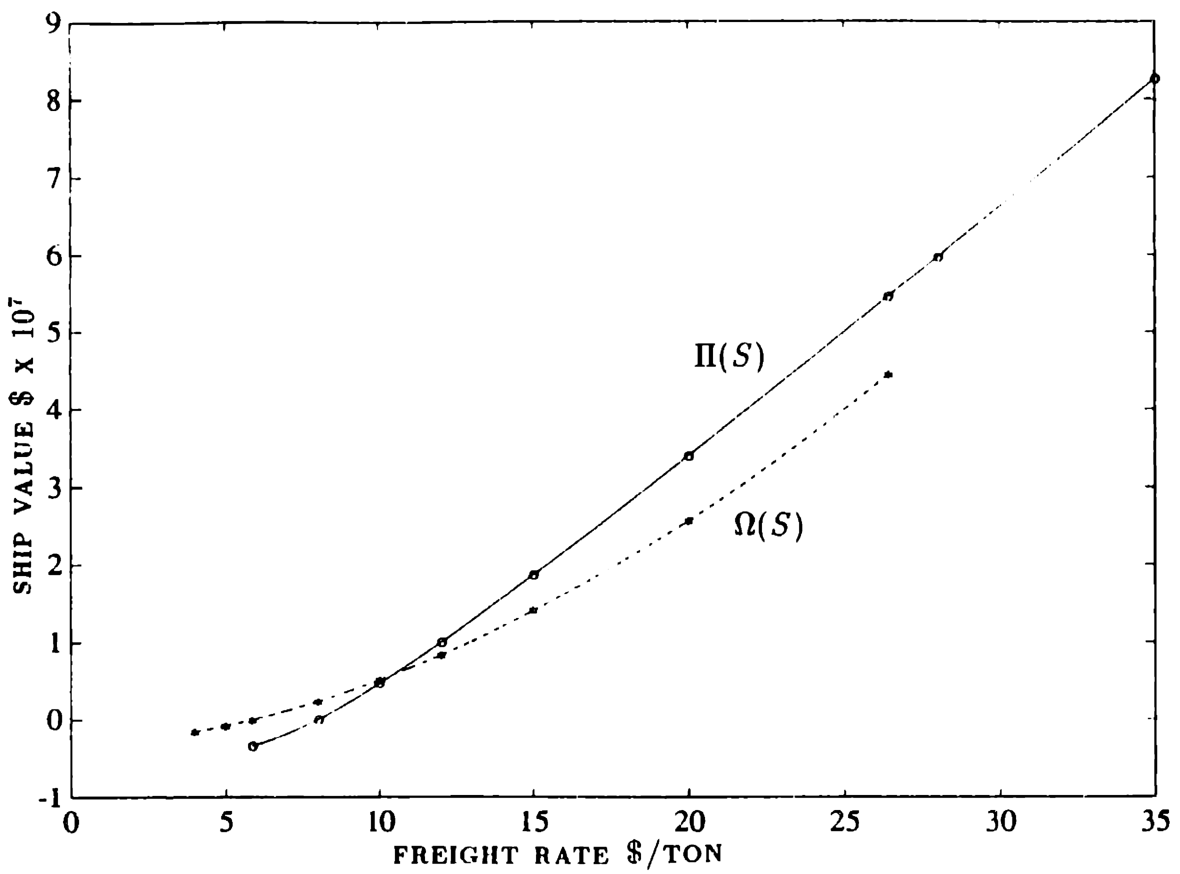

Figure 6-14: Value of the ship operating in the US Gulf - Japan route under spot contracts for large switching costs.

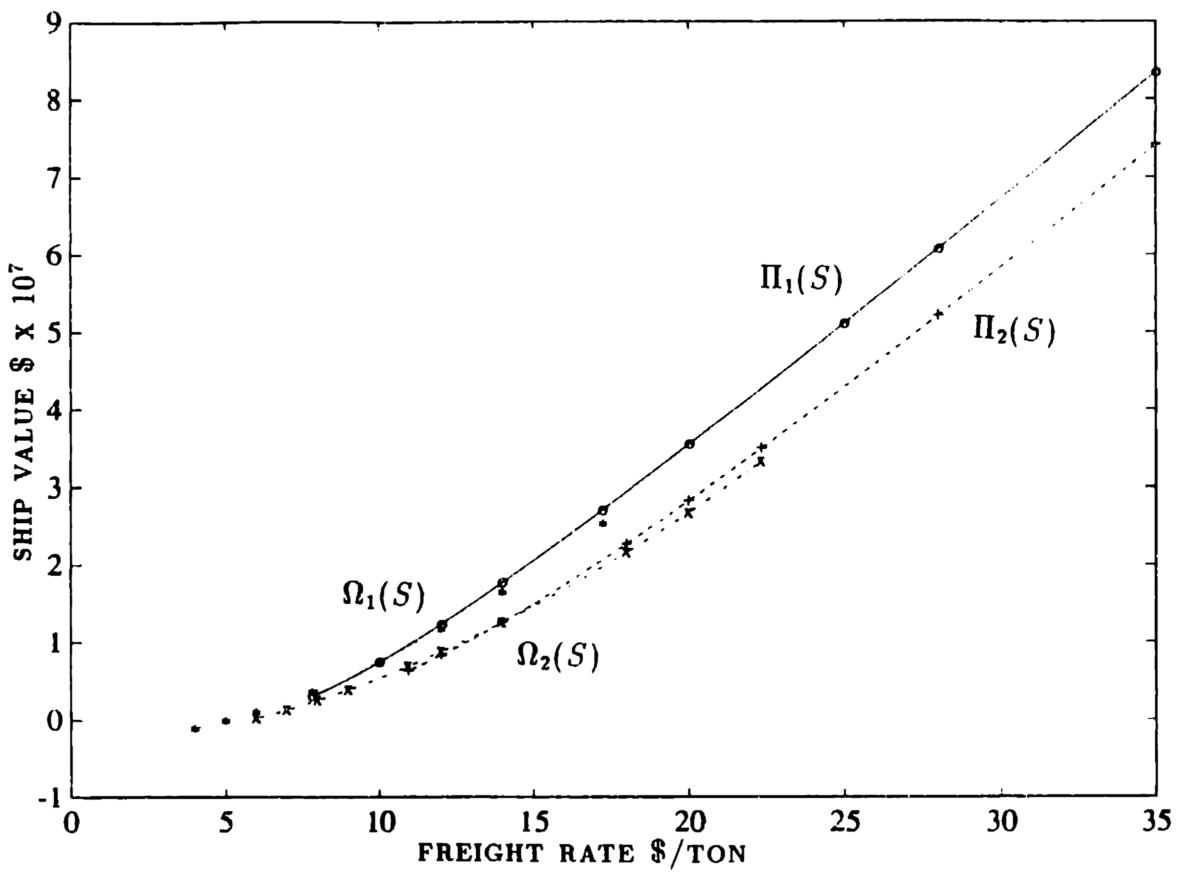

Figure 6-15: Value of the ship operating in the US Gulf - Japan route under spot contracts for base case and large operating costs.

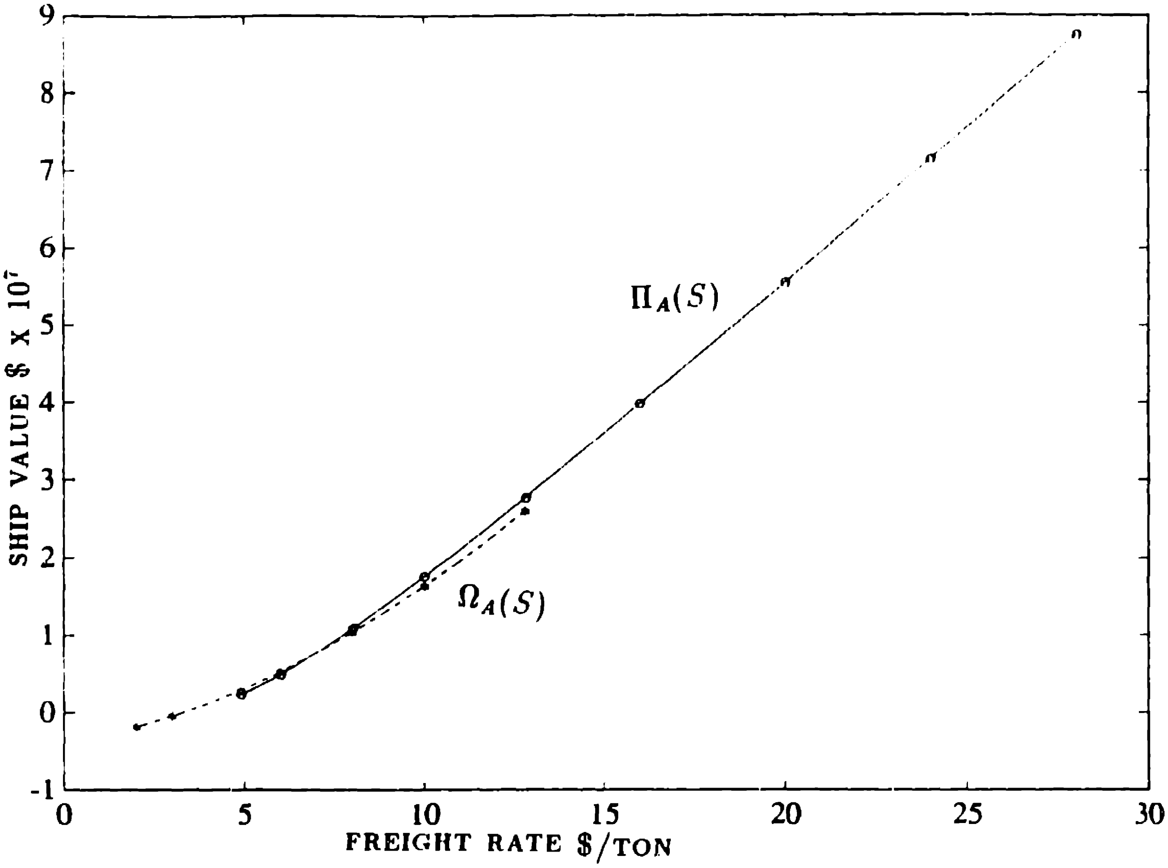

Figure 6-16: Value of the ship operating in the US Gulf - Japan route under spot contracts.

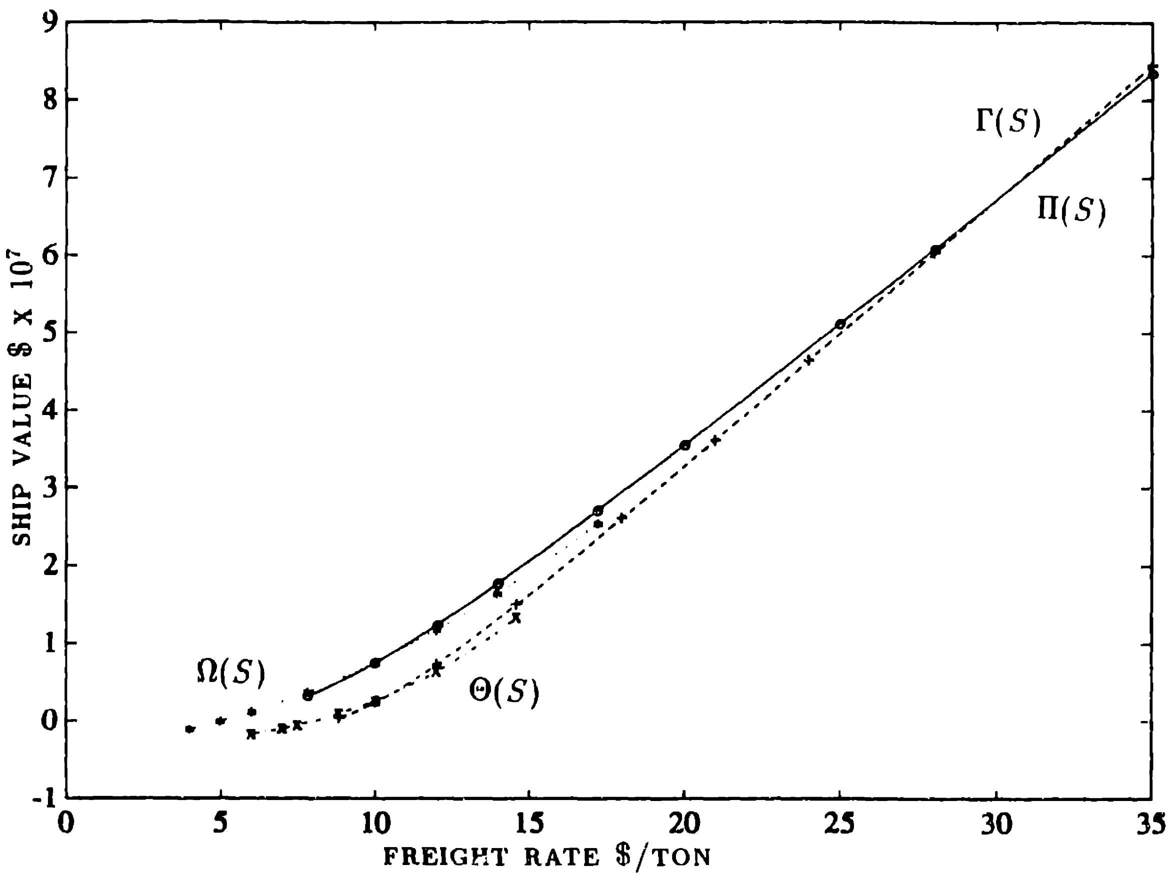

Figure 6-17: Value of the ship operating in the US Gulf - Japan route for spot and term charter contracts, ship costs of 12 $/ton.

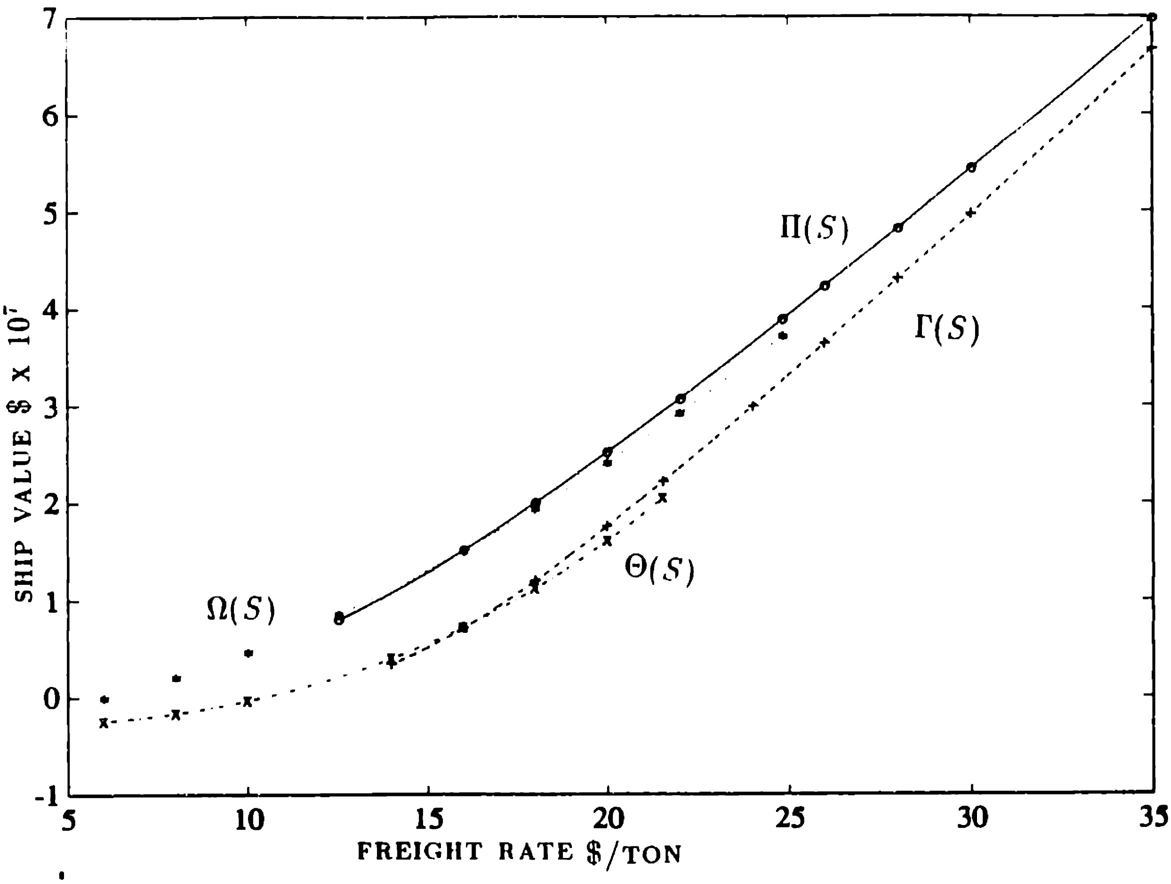

Figure 6-18: Value of the ship operating in the US Gulf - Japan route for spot and term charter contracts, ship costs of 18 $/ton.

ship costs are made in relation to the base case presented in figure 6.17. At figure 6.18 a higher cost ship (50 % higher than base case, 18 $/ton) will have the spot market as the dominant optimal "final" policy while at figure 6.19 a lower cost ship (50 % lower than base case, 6 $/ton) will have the term charter market as the dominant optimal "final" policy for rates above 15 $/ton which is a somewhat low rate indicating that for this low cost ship the most likely "final" policy will be term charter contracts.

This is a surprising result because it indicates that the optimal "final" policy will be both *ship and market specific* rather than ONLY *market specific*, as most of industrial practice assumes to be the case in the sense that decisions are made based on beliefs about market behavior and expectations rather than the specific characteristics of the ship. Therefore, according to above results, a low cost operator

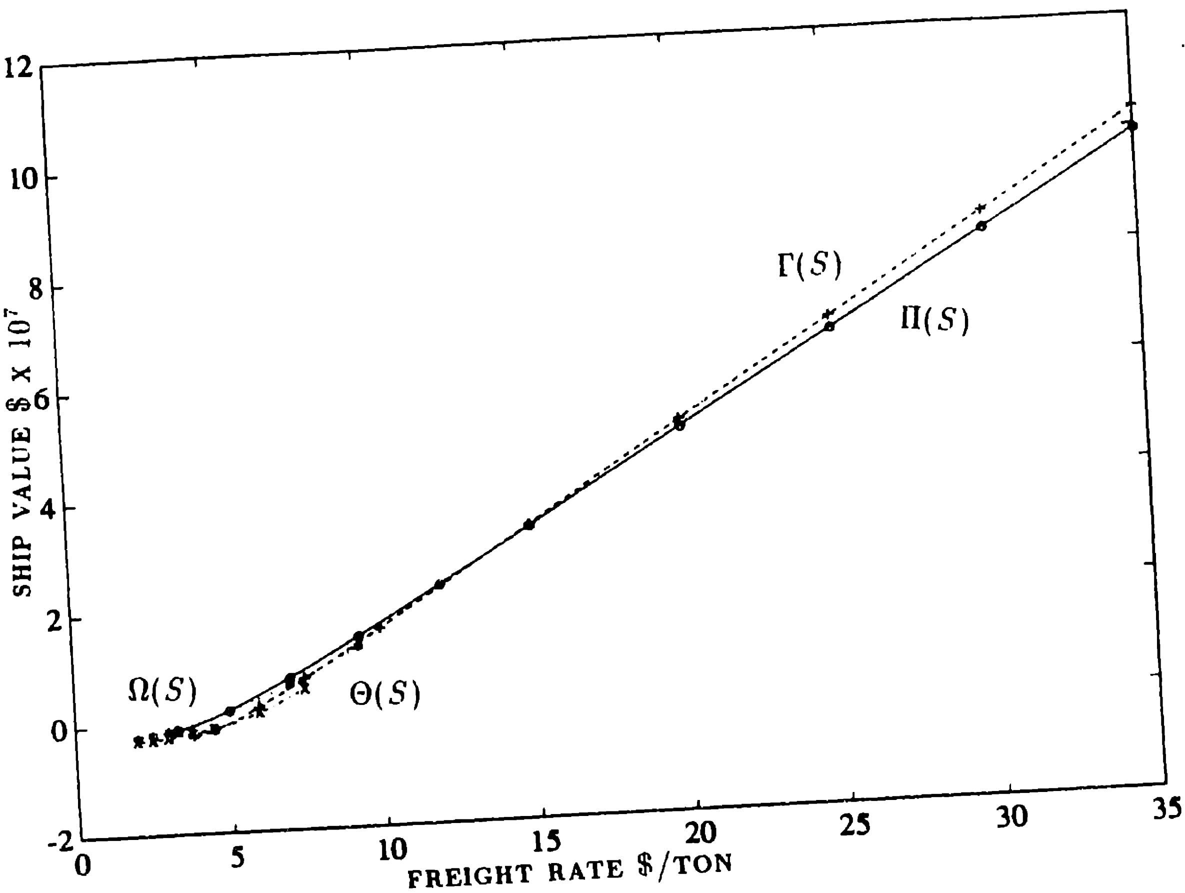

Figure 6-19: Value of the ship operating in the US Gulf - Japan route for spot and term charter contracts, ship costs of 8 $/ton.

will tend to be better off (most of the time) in the term charter market and a high cost operator will tend to be better off (most of the time) in the spot market.

The reason for this phenomenon is that the volatility effects (higher in the spot market; $\sigma_S = 33$ %, $\sigma_T = 16$ %) will affect more the value of the high cost ship than the value of the low cost ship while the rate of change effects, drift of the Brownian Motion (higher in the term charter market due to, relatively to the mean, lower risk premium, i.e., $\mu_S - \lambda_S$ = 6.64-6 = 0.64 %/year, and $\mu_T - \lambda_T$ = 2.46-1.5 = 0.96 %/year) will affect more the value of the low cost ship than the value of the high cost ship.

6.5 Optimal Policies and Ship Value for the Tankship Sector

At the moment there is no freight futures market in the very important market for the transportation of crude oil and its derivative products[4]. However, even without an oil freight futures, the methodologies of this thesis can still be applied. One immediate way would be to compute implied risk premium ($\hat{\lambda}$) from observable tankship prices and to estimate freight rates parameters ($\hat{\mu}$ and $\hat{\sigma}$) from the spot and term charter market.

An alternative, would be to synthesize the ship with stocks of tankship firms, as a substitute for the freight futures. However, this approach will be limited because most tankship operators are also dry cargo ship operators and hence the correlations between the value of the firm and the tankship market may not be good enough for replication purposes.

[4]In the beginning, 1985, there was a market but it was discontinued.

Chapter 7

Research Implications and Conclusions

7.1 Implications for Ship Design

The empirical results of Chap. 6 show the impact of several costs on the flexibility of a ship in relation to the bulk shipping market, which ultimately affects its value as a capital investment. Results also show that the best type of contract to accept is also dependent on ship costs rather than only the state of the market or operators beliefs.

Some of the costs a ship face are dependent on its design and construction while other depends on the way the ship is managed, that is, on the shipping firm. Lower operating costs, costs during lay-up and costs to move in and out of lay-up will lead to a more valuable ship. However, it may be the case that a design feature that decrease operating costs may lead to an increase in costs during lay-up or in the costs to move in and out of lay-up and vice-versa. Therefore, with this research (and "OPTPOL" in particular), ship designers will be able to evaluate their design not only in terms of sea and port time performance but also in relation to lay-up time and the transitions in and out of lay-up. From this research, perhaps new "lay-up features" will be included in the next generation of bulk ships.

7.2 Implications for Bulk Shipping Firms

The way a shipping firm is managed will also affect the costs of a ship. Given an optimally designed ship, the shipping firm will also affect costs by choosing the crew, modes of operation and maintenance, by how much it want to invest in lay-up features, etc... . Therefore, the optimization of the ship design already expanded to include lay-up features will also include the way those features interact with the firm. For instance, a high wage operator will find advantageous laid-up features that compensates high wages , while for a low wage operator, lay-up features may not be optimal.

Other managerial incentive schemes and procedures for investment decisions may also affect the flexibility of ship operations and hence its value and optimal policies.

7.3 As an Explanation for Shipping Markets Cycles

The optimal policies results can also be viewed as an explanation for the persistence of shipping and shipbuilding cycles. This is due to the existence of the already mentioned hysteresis effect that creates an option value to wait, either to exit or to re-entry the market that induce shipowners to continue to place orders for ships even though rates are below costs or to postpone orders even when rates are well above costs. Some time in the future those new orders or lack of, generate market shocks in terms of long periods of low rates and overcapacity and periods of very high rates and expensive transportation costs[1].

In this sense, the results of this research need continual updating since there is a conflict in seeing the optimal policies developed above as a criteria for operational decisions and as a tool to explain reality. It is an open question at this point whether the application of the policies developed in this thesis, if adopted by most of the industry, will lead to an increase or decrease in shipping market cycles.

[1]See Grenadier(1991) for a similar model explaining real estate cycles.

7.4 Proposals for Further Research

There are several interesting topics/areas that deserve further study and research.

Extensions in the Model

An interesting case to expand this thesis results would be to consider a stochastic process for freight rates with some sort of mean reversion. In this way, the effects of shipping cycles and shorter-term seasonality would be included. For this case, I expect a more complex mathematical development in particular with respect to optimal policies and parameter estimation. Possibly, the discrete-time version with simpler policies will be a starting point.

Extensions in Computation

The ship value and optimal policies were computed for the perpetuity case and an interesting extension would be to solve for the partial differential equations directly, that is, for a finite life ship. Due to the increased complexity in the numerical procedure for this case, more computer power will be needed to compute policies, ship values and to perform sensitivity analysis. The discrete-time model can also be a starting point for a numerical method.

Extensions in Optimal Policies

Two areas could be pursued in this case. One would be to include in one optimal control problem the term charter case and hence avoid the partition. For this case it seems that further results in the theory of optimal singular control will be needed. The "mixed" policies of discrete-time may be a starting point.

Another issue is the consideration of fleet policies, which seems to be a very hard problem, since we then have a multidimensional singular control problem.

Extensions for Liner Shipping

As discussed in the introduction, the approach presented in this thesis can also include the liner case. However, since liner shipping is a case of oligopolistic competition, other firms actions will need to be taken into account in the development optimal policies. Moreover, due to the absence of price taking behavior, the decision variable will be the demand level rather than prices[2]. A particular interesting problem would be to study the fleet capacity expansion/contraction decision.

7.5 Concluding Remarks

This thesis applied the methodology of contingent claims valuation of financial economics and the theory of optimal control to develop optimal policies for ship chartering and investment in the bulk shipping market.

A general continuous-time model was developed which includes the costs to move the ship in and out of operations. An important conceptual aspect in the continuous-time model was the pricing of the freight futures, in which non-storability, was critical in modelling. This is an issue that will apply to any contingent claim on services.

The model was solved for the perpetuity case and an extensive sensitivity analysis of ship costs and market conditions/parameters were made. Results shows that low cost and more flexible ships take greater advantage of the market and hence are more valuable.

With this thesis the shipping firm will be able to compute the optimal contract to accept, if the vessel should exit or re-enter the market and what is the optimal timing to purchase or sell a ship in the bulk trades with a rational, optimality based model.

A surprising result of the thesis was that the type of contract to accept , spot or term charter, was also ship cost dependent rather than only market dependent. This implies that decisions in the bulk trades should be made taking into account the specific characteristics (i.e., costs) of the ship and the firm rather than the usual

[2]See Huang and Li (1992) for further details.

industrial practice of considering only market conditions and or beliefs.

Appendix A

OPTPOL Fortran Code

```fortran
      DEBUG UNIT (19) , SUBCHK
      ENDDEBUG
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCC     OPTPOL COMPUTES THE VALUES FOR THE TRIGGER RATES FOR
CCCCC     DECIDING IF A SHIP OPERATING IN THE BULK TRADES WILL MOVE
CCCCC     IN OR OUT OF LAID-UP. THE MODEL WAS DEVELOPED USING THE
CCCCC     CONTINGENT CLAIMS VALUATION METHODOLOGY AND OPTIMAL CONTROL
CCCCC     THEORY TAKING ADVANTAGE OF THE FREIGHT FUTURES MARKETS.
CCCCC     THE INPUTS FOR THE PROGRAM ARE SHIP OPERATING COSTS AND
CCCCC     TAXES, MAINTENANCE COSTS DURING LAID-UP, THE COSTS TO MOVE
CCCCC     IN AND OUT OF LAID-UP, THE PARAMETERS OF THE WIENER PROCESS
CCCCC     FOR FREIGHT RATES, THE RISK PREMIUM PARAMETER FOR FREIGHT
CCCCC     FUTURES CONTRACTS, THE INITIAL VALUE FOR THE TRIGGER RATES
CCCCC     RATIO FOR THE NAG SUBROUTINE AND FREIGHT RATE VALUES FOR
CCCCC     THE COMPUTATION OF THE SHIP VALUE AFTER THE DETERMINATION
CCCCC     OF THE OPTIMAL CHARTERING POLICIES.
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
      PROGRAM OPTPOL
      IMPLICIT NONE
      COMMON /FDATA/ GAM1, GAM2, BE1, BE2
      REAL*8 ACO(20), TCO(20), CML(20), RAT(20), SKI(20), SKO(20),
     *       SMU(20), SLA(20), SIG(20), SRTE(20), XOO(20), ST1(20),
     *       ST2(20), CON2(20), CON3(20), VSOP(20,20), VSOU(20,20),
     *       XF, EPS, ETA,
     *       GML, GMR, GAM1, GAM2, BE1, BE2, RLM, TE1, XTC, DTFX,
     *       DTGX, DTHX, DTIX, QTA, QTB, QTC, QTD, DTJX, VTB,
     *       DTCX, DTDX, DTEX, VTBX
      INTEGER NA, NB, I, J, IFAIL, NFMAX, PREC
      EXTERNAL F
      DATA NA, NB, PREC / 3, 3, 3 /

CCCCC   EXPLANATION OF VARIABLES:
CCCCC    ACO(I): SHIP OPERATIONAL COSTS - CAPITAL + RUNNING COSTS IN
CCCCC            US$ PER TON. THE INDEX I IS TO ALLOW FOR SEVERAL
CCCCC            POSSIBILITIES OF INPUT FOR SENSITIVITY ANALYSIS.
CCCCC    TCO(I): TAXES ON REVENUES - CONSIDERED AS ADDITIONAL COSTS
CCCCC            INDEX I AS ABOVE.
CCCCC    CML(I): MAINTENANCE COSTS DURING LAID-UP IN US$ PER TON.
CCCCC            INDEX I AS ABOVE.
CCCCC    SKI(I): LUMP-SUM COSTS TO MOVE A SHIP INTO LAID-UP IN US$.
CCCCC            INDEX I AS ABOVE.
CCCCC    SKO(I): LUMP-SUM COSTS TO MOVE A SHIP OUT OF LAID-UP IN US$.
CCCCC            INDEX I AS ABOVE.
CCCCC    SMU(I): DRIFT OF WIENER PROCESS ASSUMED FOR FREIGHT RATES
CCCCC            CONTINUOUSLY COMPOUNDED ON YEARLY BASIS. INDEX AS ABOVE
CCCCC    SIG(I): VARIANCE OF WIENER PROCESS ASSUMED FOR FREIGHT RATES
CCCCC            CONTINUOUSLY COMPOUNDED ON YEARLY BASIS. INDEX AS ABOVE
CCCCC    SLA(I): RISK PREMIUM PARAMETER FOR FREIGHT FUTURES CONTINUOUSLY
CCCCC            COMPOUNDED ON YEARLY BASIS. INDEX AS ABOVE.
CCCCC    RAT(I): INTEREST RATE ASSUMED TO BE CONSTANT CONTINUOSLY
CCCCC            COMPOUNDED ON YEARLY BASIS. INDEX AS ABOVE.
CCCCC    SRTE(I): VALUE OF CURRENT FREIGHT RATE FOR DETERMINATION OF
CCCCC            SHIP VALUE. INDEX AS ABOVE.
CCCCC    VSOP(I): OPTIMAL VALUE OF THE SHIP IN OPERATION FOR CURRENT
CCCCC             FREIGHT RATES AND PARAMETERS. INDEX AS ABOVE.
```

```
CCCCC    VSOU(I): OPTIMAL VALUE OF THE SHIP IN LAID-UP FOR CURRENT
CCCCC             FREIGHT RATES AND PARAMETERS. INDEX AS ABOVE.
CCCCC    XOO:     INITIAL VALUE FOR THE RATIO BETWEEN TRIGGER RATES
CCCCC

CCCCC    READING INPUT DATA FOR OPTIMAL POLICIES COMPUTATION
CCCCC
      DO  I = 1,NA
         READ(8,*) ACO(I), TCO(I), CML(I), SKI(I), SKO(I), SMU(I),
     *             SIG(I), SLA(I), RAT(I), XOO(I)
CCCCC    WRITE(9,10010) ACO(I), TCO(I), CML(I), SKI(I), SKO(I),
CCCCC*                  SMU(I), SIG(I), SLA(I), RAT(I), XOO(I)
      END DO

      DO I=1,NP
         READ(8,*) SRTE(I)
CCCCC    WRITE(9,10015) SRTE(I)
      END DO

CCCCC
CCCCC   INITIAL LOOP FOR THE COMPUTATION OF SEVERAL CASES/SENS. ANAL
CCCCC
      DO 5000 I = 1,NA
         WRITE(9,10010) ACO(I), TCO(I), CML(I), SKI(I), SKO(I),
     *                  SMU(I), SIG(I), SLA(I), RAT(I), XOO(I)
CCCCC    WRITE(9,10015) I, SRTE(I)
CCCCC
CCCCC    COMPUTATION OF COEFFICIENTS GAM1 AND GAM2
CCCCC
         GML = SMU(I) - SLA(I) - (0.5*SIG(I))
         GMR = SQRT((DABS(GML))**2. + (2.*SIG(I)*RAT(I)))
         GAM1 = ((-GML)+GMR)/SIG(I)
         GAM2 = ((-GML)-GMR)/SIG(I)
CCCCC
CCCCC    COMPUTATION OF COEFFICIENTS B1, B2 AND T1
CCCCC
         BE1 =((ACO(I) + TCO(I))/RAT(I)) - (CML(I)/RAT(I)) - SKI(I)
         BE2 =((ACO(I) + TCO(I))/RAT(I)) - (CML(I)/RAT(I)) + SKO(I)
         RLM = SLA(I)+RAT(I)-SMU(I)
         TE1 = 1./RLM
CCCCC
CCCCC    COMPUTATION OF XF USING NAG SUBROUTINE
CCCCC
         EPS = 10.0**(-PREC)
         ETA = 0.0
         NFMAX = 1000
         IFAIL = 0
         XF = XOO(I)
         CALL C05AJF(XF,EPS,ETA,F,NFMAX,IFAIL)

         IF ( IFAIL.GT.0) THEN
            WRITE (9,99990) I, 'C05AJF', IFAIL
            STOP
         END IF
         WRITE (9,99980) I, XF
```

```
CCCCC
CCCCC   DETERMINATION OF OPTIMAL POLICIES S*1 AND S*2
CCCCC
        XTC = XF
CCC     DTCX = (GAM1 - 1.)*(XTC**(GAM2))
CCC     DTDX = XTC*(GAM2-GAM1)
CCC     DTEX = (GAM2-1.)*(XF**(GAM1))
CCC     VTBX = DTCX + DTDX - DTEX
        DTFX = (DABS(XTC)**(GAM2)-DABS(XTC)**(GAM1))*(BE2*GAM1*GAM2)
        DTGX = GAM2*(GAM1-1.)*(DABS(XTC)**(GAM2))
        DTHX = GAM1*(GAM2-1.)*(DABS(XTC)**(GAM1))
        DTIX = (GAM2-GAM1)*XTC
        DTJX = ((DTGX - DTHX + DTIX)*TE1)
        ST2(I) =  DTFX / DTJX
        ST1(I) = XTC*ST2(I)
CCCCC
CCCCC   DETERMINATION OF CONSTANTS C2 AND C3
CCCCC
        QTA = (TE1*ST2(I)*(GAM1-1.))-(GAM1*BE2)
        QTB = (DABS(ST2(I))**(GAM2))*(GAM2-GAM1)
        CON2(I) = QTA/QTB

        QTC = (TE1*ST2(I)*(GAM2-1.))-(GAM2*BE2)
        QTD = (DABS(ST2(I))**(GAM1))*(GAM2-GAM1)
        CON3(I) = QTC/QTD
CCCCC
CCCCC   DETERMINATION OF SHIP VALUE IN AND OUT OF OPERATIONS
CCCCC
        DO J=1,NB
        VSOP(I,J) = CON2(I)*(DABS(SRTE(J))**(GAM2))+(SRTE(J)*TE1)
     *        -((ACO(I)+TCO(I))/ RAT(I))
        VSOU(I,J) = CON3(I)*(DABS(SRTE(J))**(GAM1))-(CML(I)/RAT(I))
        END DO
CCCCC
CCCCC   WRITING RESULTS
CCCCC
        WRITE(9,10020) ST2(I), ST1(I), CON2(I), CON3(I)
CCCCC   WRITE(9,10100) DTJX, VTB, VTBX
        DO J=1,NB
           WRITE(9,10025) SRTE(J), VSOP(I,J), VSOU(I,J)
        END DO
5000  CONTINUE
10010 FORMAT (5X, 'INPUT DATA : SHIP COSTS AND PARAMETERS' /
     *        5X, 'SHIP COSTS, ACO =', F7.2 /
     *        5X, 'TAXES ON REVENUES, TCO =', F7.2 /
     *        5X, 'COSTS DURING LAID-UP, CML =', F7.2 /
     *        5X, 'COSTS TO GO INTO LAID-UP, SKI =', F9.2 /
     *        5X, 'COSTS TO GO OUT OF LAID-UP, SKO =', F9.2 /
     *        5X, 'DRIFT OF WIENER PROCESS, SMU =', F8.6 /
     *        5X, 'VARIANCE OF WIENER PROCESS, SIG =', F8.6 /
     *        5X, 'RISK PREMIUM ON FUTURES, SLA =', F8.6 /
     *        5X, 'INTEREST RATE, RAT =', F8.6 /
     *        5X, 'INITIAL VALUE FOR XF, XOO =', F8.6 /)

CCCC  FORMAT (5X,'VALUE  OF FREIGHT RATES FOR SHIP VALUE' /
```

```
CCCC *          5X, 'STRE',I3,'=', F9.3 /)
CCCCC FORMAT (5X, 'DTJX=', F11.6, 'VTB=', F11.6, 'VTBX=', F11.6 /)
10020 FORMAT (5X, 'RESULTS FOR THE OPTIMAL POLICIES'/
     *          5X, 'TRIGGER RATE TO GO INTO OPERATIONS, S*2 =', F11.2 /
     *          5X, 'TRIGGER RATE TO GO INTO LAID-UP, S*1 =', F11.2 /
     *          5X, 'CONSTANT OF ODE, C2 =', F11.3 /
     *          5X, 'CONSTANT OF ODE, C3 =', F11.3 /)
10025 FORMAT (5X, 'VALUE OF THE SHIP FOR FREIGHT RATE =', F13.2 /
     *          5X, 'SHIP OPERATING, VSOP=', F13.2 /
     *          5X, 'SHIP IN LAID-UP, VSOU=', F13.2 /)

99990 FORMAT ('CASE', I3  , 'IFAIL =', I2)
99980 FORMAT ('CASE', I3  , '  SOLUTION TO NAG SUBROUTINE C05AJF' /
     *          10X, 'ROOT =', F14.8/)
      END

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

      REAL FUNCTION F*8(XX)
      IMPLICIT NONE
      COMMON /FDATA/ GAM1, GAM2, BE1, BE2
      REAL*8 DTA, DTB, DTC, DTD, DTE, DTF, DTG, DTH, DTI, VTA,
     *       VTB, VTC, GAM1, GAM2, BE1, BE2, XX

      DTA = BE1 * (GAM2 - GAM1)
      DTB = BE2 *((GAM1*(DABS(XX)**(GAM2)))-(GAM2*(DABS(XX)**(GAM1))))
      DTC = (GAM1 - 1.)*(DABS(XX)**(GAM2))
      DTD = XX*(GAM2-GAM1)
      DTE = (GAM2-1.)*(DABS(XX)**(GAM1))
      DTF = (DABS(XX)**(GAM2)-DABS(XX)**(GAM1))*GAM1*GAM2*BE2
      DTG = GAM2*(GAM1-1.)*(DABS(XX)**(GAM2))
      DTH = GAM1*(GAM2-1.)*(DABS(XX)**(GAM1))
      DTI = (GAM2-GAM1)*XX
      VTA = DTA+DTB
      VTB = DTC - DTE + DTD
      VTC = DTG - DTH + DTI
      F = (VTA/VTB) - (DTF/VTC)
      RETURN
      END
```

```
12. 0.26 1. 2. 6. 0.0664 0.1089 0.06 0.09 0.5
 8. 0.26 1. 2. 6. 0.0664 0.1089 0.06 0.09 0.5
16. 0.26 1. 2. 6. 0.0664 0.1089 0.06 0.09 0.5
15.
20.
25.
```

```
     INPUT DATA : SHIP COSTS AND PARAMETERS
     SHIP COSTS, ACO =  12.00
     TAXES ON REVENUES, TCO =   0.26
     COSTS DURING LAID-UP, CML =   1.00
     COSTS TO GO INTO LAID-UP, SKI =     2.00
     COSTS TO GO OUT OF LAID-UP, SKO =     6.00
     DRIFT OF WIENER PROCESS, SMU =0.066400
     VARIANCE OF WIENER PROCESS, SIG =0.108900
     RISK PREMIUM ON FUTURES, SLA =0.060000
     INTEREST RATE, RAT =0.090000
     INITIAL VALUE FOR XF, XOO =0.500000

CASE  1  SOLUTION TO NAG SUBROUTINE C05AJF
          ROOT =     0.45300684

     RESULTS FOR THE OPTIMAL POLICIES
     TRIGGER RATE TO GO INTO OPERATIONS, S*2 =       17.24
     TRIGGER RATE TO GO INTO LAID-UP, S*1 =       7.81
     CONSTANT OF ODE, C2 =     356.560
     CONSTANT OF ODE, C3 =       0.601

     VALUE OF THE SHIP FOR FREIGHT RATE =        15.00
     SHIP OPERATING, VSOP=        72.88
     SHIP IN LAID-UP, VSOU=        67.68

     VALUE OF THE SHIP FOR FREIGHT RATE =        20.00
     SHIP OPERATING, VSOP=       125.80
     SHIP IN LAID-UP, VSOU=       121.15

     VALUE OF THE SHIP FOR FREIGHT RATE =        25.00
     SHIP OPERATING, VSOP=       181.39
     SHIP IN LAID-UP, VSOU=       186.54

     INPUT DATA : SHIP COSTS AND PARAMETERS
     SHIP COSTS, ACO =   8.00
     TAXES ON REVENUES, TCO =   0.26
     COSTS DURING LAID-UP, CML =   1.00
     COSTS TO GO INTO LAID-UP, SKI =     2.00
     COSTS TO GO OUT OF LAID-UP, SKO =     6.00
     DRIFT OF WIENER PROCESS, SMU =0.066400
     VARIANCE OF WIENER PROCESS, SIG =0.108900
     RISK PREMIUM ON FUTURES, SLA =0.060000
     INTEREST RATE, RAT =0.090000
     INITIAL VALUE FOR XF, XOO =0.500000

CASE  2  SOLUTION TO NAG SUBROUTINE C05AJF
          ROOT =     0.39708893

     RESULTS FOR THE OPTIMAL POLICIES
     TRIGGER RATE TO GO INTO OPERATIONS, S*2 =       12.00
     TRIGGER RATE TO GO INTO LAID-UP, S*1 =       4.77
     CONSTANT OF ODE, C2 =     148.092
     CONSTANT OF ODE, C3 =       0.821

     VALUE OF THE SHIP FOR FREIGHT RATE =        15.00
```

```
     SHIP OPERATING, VSOP=          99.97
     SHIP IN LAID-UP, VSOU=          96.49

     VALUE OF THE SHIP FOR FREIGHT RATE =          20.00
     SHIP OPERATING, VSOP=         156.92
     SHIP IN LAID-UP, VSOU=         169.52

     VALUE OF THE SHIP FOR FREIGHT RATE =          25.00
     SHIP OPERATING, VSOP=         214.98
     SHIP IN LAID-UP, VSOU=         258.83

     INPUT DATA : SHIP COSTS AND PARAMETERS
     SHIP COSTS, ACO =  16.00
     TAXES ON REVENUES, TCO =   0.26
     COSTS DURING LAID-UP, CML =   1.00
     COSTS TO GO INTO LAID-UP, SKI =      2.00
     COSTS TO GO OUT OF LAID-UP, SKO =     6.00
     DRIFT OF WIENER PROCESS, SMU =0.066400
     VARIANCE OF WIENER PROCESS, SIG =0.108900
     RISK PREMIUM ON FUTURES, SLA =0.060000
     INTEREST RATE, RAT =0.090000
     INITIAL VALUE FOR XF, XOO =0.500000

CASE  3  SOLUTION TO NAG SUBROUTINE C05AJF
          ROOT =     0.49010809

     RESULTS FOR THE OPTIMAL POLICIES
     TRIGGER RATE TO GO INTO OPERATIONS, S*2 =       22.33
     TRIGGER RATE TO GO INTO LAID-UP, S*1 =      10.94
     CONSTANT OF ODE, C2 =     651.402
     CONSTANT OF ODE, C3 =       0.481

     VALUE OF THE SHIP FOR FREIGHT RATE =          15.00
     SHIP OPERATING, VSOP=          52.98
     SHIP IN LAID-UP, VSOU=          52.00

     VALUE OF THE SHIP FOR FREIGHT RATE =          20.00
     SHIP OPERATING, VSOP=         100.20
     SHIP IN LAID-UP, VSOU=          94.82

     VALUE OF THE SHIP FOR FREIGHT RATE =          25.00
     SHIP OPERATING, VSOP=         152.30
     SHIP IN LAID-UP, VSOU=         147.20
```

Appendix B

Shipping Market Data Set

- USARA : Spot freight rate for US Gulf to ARA , $/ton.
- USJAP : Spot freight rate for US Gulf to Japan , $/ton.
- FUA1 : Implied freight futures US Gulf to ARA , $/ton for the next quarter.
- FUA2 : Implied freight futures US Gulf to ARA , $/ton for the 2nd quarter.
- FUA3 : Implied freight futures US Gulf to ARA , $/ton for the 3rd quarter.
- FUJ1 : Implied freight futures US Gulf to Japan , $/ton for the next quarter.
- FUJ2 : Implied freight futures US Gulf to Japan , $/ton for the 2nd quarter.
- FUJ3 : Implied freight futures US Gulf to Japan , $/ton for the 3rd quarter.
- TSOJ1 : Term charter rate 1 year contract in spot equivalent, $/ton for US Gulf to Japan.
- TSOJ3 : Term charter rate 3 years contract in spot equivalent, $/ton for US Gulf to Japan.

WEEK	DATE	USARA	USJAP	FUA1	FUA2	FUA3	FUJ1	FUJ2	FUJ3	TSPOJ1	TSPOJ3
85.01	04-Jan-85	9	14.25							14.7306	16.7182
85.02	11-Jan-85	8.4	14.25							14.7306	16.7182
85.03	18-Jan-85	8.75	14.25							14.7306	16.7182
85.04	25-Jan-85	9	14.25							14.6064	16.7182
85.05	01-Feb-85	9	14.5							14.4822	16.6561
85.06	08-Feb-85	9	14.25							14.4822	16.6561
85.07	15-Feb-85	8.75	14.8							14.358	16.6561
85.08	22-Feb-85	8.75	14.75							14.358	16.6561
85.09	01-Mar-85	8.75	14.75							14.4822	16.6561
85.10	08-Mar-85	9	15							15.1033	16.6561
85.11	15-Mar-85	9.25	15.75							15.4138	16.6561
85.12	22-Mar-85	9.5	15.75							15.4138	16.6561
85.13	29-Mar-85	9.75	16							15.4138	16.6561
85.14	05-Apr-85	9.75	16							14.5438	15.7861
85.15	12-Apr-85	11.5	16							14.5438	15.7861
85.16	19-Apr-85	11	16.25							14.5438	15.7861
85.17	26-Apr-85	11	16.6							14.5438	15.7861
85.18	03-May-85	11	16.6							14.5438	15.7861
85.19	10-May-85	10.5	15.5							14.2333	15.7861
85.20	17-May-85	9.75	15.25							14.2333	15.7861
85.21	24-May-85	9.25	14							14.2333	15.7861
85.22	31-May-85	8.5	14							14.2333	15.7861
85.23	07-Jun-85	8.5	14							14.2333	15.7861
85.24	14-Jun-85	8.25	14.75							13.9227	15.7861
85.25	21-Jun-85	8	13.5							13.6122	15.4755
85.26	28-Jun-85	7.5	13							13.3016	15.4755
85.27	05-Jul-85	6.25	12.25							13.1516	15.3255
85.28	12-Jul-85	6	11.25							12.8411	15.0771
85.29	19-Jul-85	6	11							13.1516	15.0771
85.30	26-Jul-85	5.8	10.25							13.1516	15.0771
85.31	02-Aug-85	5.8	10.15							12.8411	15.0771
85.32	09-Aug-85	5.65	10.25							12.8411	15.0771
85.33	16-Aug-85	6	10.5							12.8411	15.0771
85.34	23-Aug-85	6.25	10.75							12.8411	14.7044
85.35	30-Aug-85	6.25	10.75							12.4684	14.7044
85.36	06-Sep-85	6.25	10.75							12.3442	14.7044
85.37	13-Sep-85	6.6	11							12.5305	14.7044
85.38	20-Sep-85	7.2	11.75							12.5305	14.7044
85.39	27-Sep-85	8.25	14.25							12.8411	14.7044
85.40	04-Oct-85	8.25	13.25							12.8411	14.7044
85.41	11-Oct-85	8	13.5							12.8411	14.7044
85.42	18-Oct-85	8	13.25							13.0895	14.7044
85.43	25-Oct-85	8	13.5							13.0895	14.7044
85.44	01-Nov-85	8	13.5	8.1	8.53	7.65	13.55	14.28	12.81	13.0895	14.0833
85.45	08-Nov-85	8	13.25							13.0895	14.7044
85.46	15-Nov-85	7.8	12.75							13.0895	14.7044
85.47	22-Nov-85	8	12.75							13.0895	14.7044

85.48	29-Nov-85	8.5	13							13.0895	14.7044
85.49	06-Dec-85	8.5	13							13.0895	14.7044
85.50	13-Dec-85	8.15	13							12.9653	14.7044
85.51	20-Dec-85	7.9	13							12.9653	14.7044
85.52	27-Dec-85	7.9	13							12.9653	14.7044
86.01	03-Jan-86	8	13.25							12.1653	13.9044
86.02	10-Jan-86	8.5	13.5							12.1653	13.9044
86.03	17-Jan-86	8	13.5							12.1653	13.9044
86.04	24-Jan-86	7.75	13.25							12.0411	13.9044
86.05	31-Jan-86	7	12							12.0411	13.5938
86.06	07-Feb-86	6.6	11	7.54	6.56	7.33	13.17	11.45	12.79	12.0411	13.5938
86.07	14-Feb-86	6.7	10.75							12.0411	13.5938
86.08	21-Feb-86	5.75	10.25							11.9168	13.5938
86.09	28-Feb-86	6	10.25							11.9168	13.5938
86.10	07-Mar-86	6	10							11.9168	13.5938
86.11	14-Mar-86	5.75	10.5							11.6684	13.5938
86.12	21-Mar-86	5.75	10.25							11.42	13.5938
86.13	28-Mar-86	5.75	10.25							11.2957	13.2833
86.14	04-Apr-86	5.7	10.25							10.1557	12.1433
86.15	11-Apr-86	5.7	9.5							10.1557	12.1433
86.16	18-Apr-86	5.5	9							10.1557	11.6464
86.17	25-Apr-86	5.75	8.5							10.1557	11.6464
86.18	02-May-86	5.75	8.5	5.88	6.53	6.67	8.54	9.48	9.68	10.28	11.6464
86.19	09-May-86	5.75	8.7							10.4042	11.6464
86.20	16-May-86	5.75	9.25							10.4042	11.6464
86.21	23-May-86	5.75	9.25							10.4042	11.6464
86.22	30-May-86	5.75	9							10.4042	11.6464
86.23	06-Jun-86	5.75	9.5							10.4042	11.6464
86.24	13-Jun-86	5.5	6.5							10.28	11.6464
86.25	20-Jun-86	5.5	8.5							10.1557	11.6464
86.26	27-Jun-86	4.25	7.75							10.0315	11.6464
86.27	04-Jul-86	4.5	7.75							9.91944	11.4722
86.28	11-Jul-86	4.5	7.5							9.91944	11.4722
86.29	18-Jul-86	4.5	7.5							9.91944	11.4722
86.30	25-Jul-86	4.5	7.4							9.91944	11.4722
86.31	01-Aug-86	4.25	7.75	5.03	5.3	5.76	8.92	9.4	10.23	9.60888	11.0995
86.32	08-Aug-86	4.25	8							9.60888	10.5405
86.33	15-Aug-86	4.5	8.5							9.60888	10.5405
86.34	22-Aug-86	6.45	9.5							10.1057	11.0995
86.35	29-Aug-86	6.5	10.5							10.8511	11.0995
86.36	05-Sep-86	6.5	12.25							10.8511	11.4722
86.37	12-Sep-86	6.25	12.25							10.8511	11.4722
86.38	19-Sep-86	6.5	12							10.8511	11.4722
86.39	26-Sep-86	7.5	12							10.8511	11.4722
86.40	03-Oct-86	7.1	12.25							11.4111	12.0322
86.41	10-Oct-86	6.9	12.25							11.4111	12.0322
86.42	17-Oct-86	7	12							11.4111	12.0322
86.43	24-Oct-86	7	12							11.4111	12.0322

86.44	31-Oct-86	7	12							11.4111	12.0322
86.45	07-Nov-86	7.25	12.5	6.76	7.21	6.31	11.65	12.42	10.87	11.4111	12.0322
86.46	14-Nov-86	7.25	12.5							11.4111	12.0322
86.47	21-Nov-86	7.1	12							11.4111	11.908
86.48	28-Nov-86	6.75	11.5							11.4111	11.908
86.49	05-Dec-86	6.25	10.75							11.4111	11.908
86.50	12-Dec-86	6.25	10.75							11.4111	11.908
86.51	19-Dec-86	6	10.75								
86.52	26-Dec-86	6	10.75								
87.01	02-Jan-87	6.75	11.5							11.7811	
87.02	09-Jan-87	7.75	13.15							11.7811	
87.03	16-Jan-87	9	13.85							11.7811	
87.04	23-Jan-87	8.25	14.25							11.7811	
87.05	30-Jan-87	8	13.75							11.7811	
87.06	06-Feb-87	8	13.75	7.54	6.36	7.44	12.63	10.64	12.45	11.7811	
87.07	13-Feb-87	8.5	14							11.7811	
87.08	20-Feb-87	8	13.75							11.7811	
87.09	27-Feb-87	7.65	14							12.4022	
87.10	06-Mar-87	8	14.25							12.4022	
87.11	13-Mar-87	8.5	14.9							12.4022	
87.12	20-Mar-87	9	15.5							12.4022	
87.13	27-Mar-87	9.25	16							13.0233	
87.14	03-Apr-87	9.25	17							13.3733	
87.15	10-Apr-87	8.75	16							13.3733	
87.16	17-Apr-87	8.75	16							13.3733	
87.17	24-Apr-87	9	16.5							13.3733	
87.18	01-May-87	9	16.75	8.19	8.56	8.56	15.35	16.03	16.03	14.6155	
87.19	08-May-87	9.25	18.5							14.6155	
87.20	15-May-87	9.5	18.5							14.6155	
87.21	22-May-87	9.75	18.5							14.6155	
87.22	29-May-87	8.75	17.5							13.9944	
87.23	05-Jun-87	8	15							13.9944	
87.24	12-Jun-87	7.75	14.75							13.9944	
87.25	19-Jun-87	7	14							13.9944	
87.26	26-Jun-87	7	13							13.9886	
87.27	03-Jul-87	7.5	13.25							13.9886	
87.28	10-Jul-87	8	13.5							13.9886	
87.29	17-Jul-87	8.5	15							13.9886	
87.30	24-Jul-87	8.5	16							13.9886	
87.31	31-Jul-87	8.25	15.5							15.4793	
87.32	07-Aug-87	10	17.25	9.23	9.25	9.33	16.9	16.93	17.09	15.4793	
87.33	14-Aug-87	10	17.25							15.4793	
87.34	21-Aug-87	10	16.75							15.4793	
87.35	28-Aug-87	9.5	16.75							15.2308	
87.36	04-Sep-87	9	16.25							15.2308	
87.37	11-Sep-87	9	15.75							15.2308	
87.38	18-Sep-87	8.75	15.25							15.2308	
87.39	25-Sep-87	8.5	16.25							14.9108	

87.40	02-Oct-87	9.15	16.5							16.4015	
87.41	09-Oct-87	9.25	17.25							16.4015	
87.42	16-Oct-87	9.5	17.5							16.4015	
87.43	23-Oct-87	10	18							16.4015	
87.44	30-Oct-87	10	18							16.65	
87.45	06-Nov-87	10	17.75	10.57	10.74	9.71	19.08	19.37	17.52	16.65	
87.46	13-Nov-87	9.75	17.75							16.65	
87.47	20-Nov-87	9.5	17.25							16.65	
87.48	27-Nov-87	10	18							16.65	
87.49	04-Dec-87	11.5	19							16.65	
87.50	11-Dec-87	12	19.95							16.65	
87.51	18-Dec-87	11.5	19.5							16.65	
87.52	25-Dec-87	11.5	19.5								
87.53	01-Jan-88	11.75	19.75								
88.01	08-Jan-88	12.75	21.25	12.65	13.21	12.01	22.84	23.84	21.68	18.1233	18.1233
88.02	15-Jan-88	12.75	22	13.2	13.74	12.38	22.23	23.44	21.13	19.9866	18.1233
88.03	22-Jan-88	13	22	12.81	13.09	11.73	21.85	22.33	20.02	19.9866	18.1233
88.04	29-Jan-88	12.6	22.5	12.71	13.66	12.43	21.69	23.31	21.21	19.9866	18.7444
88.05	05-Feb-88	14	25	14.26	12.98	13.25	24.32	22.14	22.61	20.6077	18.7444
88.06	12-Feb-88	14.5	27.25	14.44	13.07	13.25	24.64	22.3	22.61	21.2288	18.7444
88.07	19-Feb-88	14.5	26.75	14.26	12.77	12.89	24.32	21.78	21.99	21.2288	18.7444
88.08	26-Feb-88	14.75	26	14.38	12.94	13.02	24.53	22.08	22.22	21.5394	18.7444
88.09	04-Mar-88	14.75	26	14.49	13.02	13.21	26.15	23.51	23.84	21.85	19.9866
88.10	11-Mar-88	14.25	26	14.12	13.89	14.09	25.49	25.08	25.43	21.85	20.9183
88.11	18-Mar-88	14.75	28	15.77	14.62	14.85	28.46	26.4	26.81	23.0922	20.9183
88.12	25-Mar-88	14	26	15.17	14.03	14.08	27.39	25.33	25.41	23.7133	20.9183
88.13	01-Apr-88	14	26							23.7133	20.6077
88.14	08-Apr-88	13	26.5	14.62	13.85	14.21	26.4	25	25.66	23.7633	20.6577
88.15	15-Apr-88	12.75	25	13.86	12.66	13.12	25.01	22.85	23.68	23.1422	20.6577
88.16	22-Apr-88	12	23							22.2105	20.0366
88.17	29-Apr-88	11.5	21.5							21.5894	20.0366
88.18	06-May-88	11.75	23.5							22.2105	20.0366
88.19	13-May-88	12	23.5	13.57	14.08	14.04	24.5	25.41	25.34	22.2105	20.0366
88.20	20-May-88	11.75	23	11.69	12.32	12.32	21.1	22.28	22.28	21.9	20.6577
88.21	27-May-88	11.5	22	11.24	11.88	11.97	20.3	21.45	21.62	21.2788	20.3472
88.22	03-Jun-88	11	20	10.97	11.97	11.97	19.8	21.62	21.62	20.6577	20.3472
88.23	10-Jun-88	11	19.75							20.0366	20.0366
88.24	17-Jun-88	11	19.25							19.7261	20.0366
88.25	24-Jun-88	9.75	19	11.11	12.48	12.8	20.05	22.52	23.1	19.105	20.0366
88.26	01-Jul-88	9.75	19	11.06	12.47	12.96	19.97	22.51	23.4	18.7944	20.0366
88.27	08-Jul-88	9	19	10.88	12.45	12.75	19.64	22.47	23.02	18.6044	19.6603
88.28	15-Jul-88	9	19.5	10.97	13.02	13.21	19.8	23.5	23.84	18.6044	19.6603
88.29	22-Jul-88	9	19.75	10.98	12.88	12.89	19.82	23.25	23.25	18.6044	19.6603
88.30	29-Jul-88	9.25	19.65	10.95	12.8	13.02	19.77	23.1	23.51	18.6044	19.6603
88.31	05-Aug-88	9.5	19.5	12.91	13.16	13.48	23.3	23.76	24.34	19.2255	19.6603
88.32	12-Aug-88	9.25	19.75	13.39	13.66	14.08	24.17	24.67	25.41	19.4118	19.8466
88.33	19-Aug-88	10.5	20.5	13.59	13.8	14.12	24.54	24.92	25.49	19.4118	19.8466
88.34	26-Aug-88	11.5	20.75	13.66	13.89	14.17	24.67	25.08	25.59	19.5361	19.8466

88.35	02-Sep-88	11.5	21	13.71	13.98	14.26	24.75	25.25	25.74	20.4677	20.4677
88.36	09-Sep-88	11.5	21	13.34	13.8	14.17	24.09	24.92	25.58	20.8404	20.4677
88.37	16-Sep-88	11	21.5	13.02	13.57	13.94	23.51	24.5	25.16	19.8466	20.4677
88.38	23-Sep-88	11.75	21.5	13.13	13.72	14.02	23.71	24.75	25.31	19.8466	20.1572
88.39	30-Sep-88	11.75	21	12.29	13.07	13.48	22.19	23.6	24.34	19.5361	20.1572
88.40	07-Oct-88	10.75	21.5	12.34	13.25	13.71	22.28	23.93	24.75	19.2861	19.9072
88.41	14-Oct-88	11.25	22	12.57	13.77	14.17	22.69	24.85	25.58	19.5966	19.9072
88.42	21-Oct-88	13	22.75	12.32	13.65	13.98	22.24	24.63	25.25	19.4103	19.2861
88.43	28-Oct-88	13.5	23.5	12.51	13.62	13.94	22.59	24.59	25.16	19.9072	19.2861
88.44	04-Nov-88	13.25	23.5	13.69	13.98	12.16	24.72	25.25	21.95	20.2177	19.5966
88.45	11-Nov-88	13.25	24.5	14.09	14.4	12.34	25.43	25.99	22.28	20.8388	20.2177
88.46	18-Nov-88	13.5	24.5	14.98	14.98	12.84	26.39	26.39	23.18	20.8388	20.2177
88.47	25-Nov-88	14	24.5	14.58	14.86	12.75	25.28	26.2	23.02	20.8388	20.2177
88.48	02-Dec-88	13.5	24	13.84	14.17	12.25	24.07	24.63	22.11	20.8388	20.2177
88.49	09-Dec-88	14	24	14.24	14.57	12.66	25.72	26.32	22.85	20.8388	20.2177
88.50	16-Dec-88	14.5	24.5	14.67	15	12.7	25.15	25.72	22.94	20.8388	20.4041
88.51	23-Dec-88	14.65	24.9	15.37	16.03	13.28	25.89	27	23.97	21.1494	20.4041
88.52	30-Dec-88	14.75	25.25								
89.01	06-Jan-89	15.25	26.5	15.78	16.55	14.65	26.73	28.03	24.69	22.4911	21.2488
89.02	13-Jan-89	15.5	27	15.83	16.3	14.24	27.27	28.8	24.53	22.1805	21.2488
89.03	20-Jan-89	15.5	27	15.39	15.71	13.72	26.93	27.47	24	22.1805	21.2488
89.04	27-Jan-89	14.5	26	14.94	14.94	13.14	26.65	26.65	23.46	21.87	21.2488
89.05	03-Feb-89	14	24.5	14.35	12.72	13.87	25.73	22.8	24.88	21.2488	21.2488
89.06	10-Feb-89	13	23.5	14.28	12.8	13.89	24.99	22.39	24.3	20.9383	21.2488
89.07	17-Feb-89	13	24	14.15	12.5	13.53	25.75	22.74	24.62	21.2488	21.2488
89.08	24-Feb-89	13.5	25.5	14.58	12.82	13.92	27.2	23.9	25.8	21.87	21.5594
89.09	03-Mar-89	14.25	26.5	15.24	13.3	14.39	28.63	25	27.05	22.4911	21.2488
89.10	10-Mar-89	14.5	27	15.19	13	14.19	28.19	24.12	26.33	22.4911	21.2488
89.11	17-Mar-89	14.5	26.5	14.87	12.25	13.19	27.35	22.5	24.22	22.4911	21.2488
89.12	24-Mar-89	14.65	26.5							21.87	21.2488
89.13	31-Mar-89	14.75	26.5	14.31	12.67	13.03	24.58	22.91	22.37	21.87	21.2488
89.14	07-Apr-89	15	26.5	15.2	12.91	13.69	27.22	23.11	24.52	22.27	21.3383
89.15	14-Apr-89	15	26.5	15.27	12.92	13.83	27.68	23.43	25.08	22.27	21.3383
89.16	21-Apr-89	15	26	15	12.72	13.77	26.85	22.78	24.65	21.6488	21.0277
89.17	28-Apr-89	15	27.5	15.1	13.36	14.22	27.02	23.88	25.42	22.27	21.0277
89.18	05-May-89	15.75	29.5	13.87	14.82	15.07	24.89	26.6	27.04	22.5805	21.3383
89.19	12-May-89	16	29.5	13.98	14.62	14.95	25.5	26.69	27.3	22.5805	21.3383
89.20	19-May-89	16	29	13.4	14.22	14.45	23.64	25.08	25.49	22.5805	21.3383
89.21	26-May-89	14	25	12.86	13.67	13.92	22	23.4	23.83	21.6488	21.0277
89.22	02-Jun-89	11	21	11.72	12.71	12.75	21.33	23.14	23.67	21.3383	20.4066
89.23	09-Jun-89	11	20	10.53	11.41	11.77	19.47	21.12	21.79	21.3383	20.4066
89.24	16-Jun-89	11.5	21	10.97	12.1	12.38	20.5	22.59	23.11	21.0277	20.4066
89.25	23-Jun-89	12.5	21	11.07	12.23	12.78	19.59	21.65	22.35	21.3383	20.7172
89.26	30-Jun-89	12	22.5	11.43	12.47	12.88	21.69	23.65	24.44	21.3383	20.7172
89.27	07-Jul-89	11.5	22.5	11.02	12.15	12.44	21.79	24.03	24.61	21.4888	20.5572
89.28	14-Jul-89	11	21.25	10.63	11.57	11.9	21.09	22.94	23.6	21.4888	20.8677
89.29	21-Jul-89	11	21	10.95	12.06	12.27	20.58	22.65	23.08	21.4888	20.8677
89.30	28-Jul-89	10.75	22	11	12.19	12.37	21.83	24.2	24.56	21.4888	20.8677

89.31	04-Aug-89	10.75	22.5	12.18	12.31	12.57	25.42	25.7	26.24	20.8677	20.2466
89.32	11-Aug-89	11	24	11.96	12.29	12.47	26.38	27.11	27.5	20.8677	20.2466
89.33	18-Aug-89	11	24.5	12.18	12.52	12.69	26.57	27.3	27.67	20.8677	20.2466
89.34	25-Aug-89	11	24.25	12.06	12.39	11.83	26.35	27.08	25.85	21.7994	20.2466
89.35	01-Sep-89	11.5	24.5	11.88	12.11	12.38	26.03	26.51	27.1	21.7994	20.2466
89.36	08-Sep-89	11.75	25	11.92	12.25	12.46	26.18	26.91	27.37	21.4888	20.1224
89.37	15-Sep-89	12	24.5	11.92	12.25	12.46	26.18	26.91	27.37	21.1783	19.9361
89.38	22-Sep-89	11.75	23.5	12.13	12.76	13.07	24.6	25.87	26.49	20.8677	19.9361
89.39	29-Sep-89	12	23.5	12.72	13.44	13.8	24.76	26.17	26.87	20.8677	19.9361
89.40	06-Oct-89									21.2277	20.2961
89.41	13-Oct-89	12.75	25.5	13.12	13.95	14.18	25.72	27.35	27.8	21.2277	20.2961
89.42	20-Oct-89	15	26.5	15	15.82	15.89	26.21	27.64	27.77	21.2277	20.2961
89.43	27-Oct-89	16	27.5	15.48	16.09	16.09	27.25	28.34	28.34	21.8488	20.6066
89.44	03-Nov-89	16	27.9	16.22	16.33	13.58	28.46	28.64	23.82	21.8488	20.6066
89.45	10-Nov-89	15.25	27	16.11	16.11	13.48	28.46	28.46	23.8	21.8488	20.6066
89.46	17-Nov-89	15.5	28	15.79	15.81	13.22	28.11	28.14	23.52	21.8488	20.6066
89.47	24-Nov-89	15.5	28	15.86	15.86	13.29	28.33	28.33	23.74	21.8488	20.6066
89.48	01-Dec-89	14.5	26.25	14.88	15.01	12.61	27.4	27.63	23.21	22.1594	20.6066
89.49	08-Dec-89	14.5	26.5	14.88	15.01	12.55	26.94	27.19	22.74	21.8488	20.6066
89.50	15-Dec-89	13.5	26.75	14.81	14.99	12.63	27.44	27.77	23.4	21.8488	20.6066
89.51	22-Dec-89	13.5	26.75	14.53	14.58	12.36	28.01	28.11	23.83	21.8488	20.6066
89.52	29-Dec-89										
90.01	01.05	15	27.75	15	15.17	12.77	28.32	28.64	24.11	21.6088	20.3666
90.02	01.12	15	28	15.2	15.31	13	28.23	28.43	24.13	21.6088	20.3666
90.03	01.19	15	27.5	15.17	15.33	13	27.98	28.28	23.89	21.9194	20.3666
90.04	01.26	15	27	14.91	14.9	12.58	27.5	27.48	23.2	21.9194	20.3666
90.05	02.02	14	26.5	14.58	12.44	13.5	26.34	22.5	24.42	21.9194	20.3666
90.06	02.09	14	25.5	14.41	12.41	13.5	26.18	22.31	24.26	21.6088	20.3666
90.07	02.16	14.5	25.5	14.41	12.29	13.28	26.64	22.72	24.55	21.9194	20.3666
90.08	02.23	14.5	26	15.16	12.71	13.65	26.66	22.36	24.01	21.9194	20.6772
90.09	03.02	14.5	27	15.11	12.59	13.49	27.67	23.06	24.7	22.23	20.6151
90.1	03.09	14.5	27.5	14.83	12.52	13.34	28.08	23.79	25.35	22.23	20.6151
90.11	03.16	14	27.25	13.97	11.96	12.73	27.55	23.57	25.08	22.23	20.6151
90.12	03.23	13.5	26.25	13.3	11.51	12.57	25.96	22.47	24.54	21.9194	20.6151
90.13	03.30	13.25	25.75	13.47	11.64	12.45	26	22.48	24.04	21.6088	20.6151
90.14	04.06	13	24	12.93	11.67	12.4	23.52	21.24	22.56	20.5777	19.6461
90.15	04.13										
90.16	04.20	13	23.75	13.02	11.18	11.96	23.41	19.98	21.5	19.9566	19.025
90.17	04.27	12.5	23	1377	10.08	11.03	1377	18.2	19.91	19.6461	19.025
90.18	05.04	12	22	10.35	11.24	11.28	18.74	20.4	20.47	19.3355	19.025
90.19	05.11	12	21.5	10.16	11.04	11.23	18.49	20.1	20.45	19.025	18.7144
90.2	05.18	12	21.75	10.31	11.27	11.53	18.71	20.2	20.66	19.3355	18.7144
90.21	05.25	12	20.75	10.08	10.98	11.15	1105	19.86	20.17	19.3355	19.025
90.22	06.01	11.9	21.5	10.21	10.98	11.17	17.08	18.38	18.68	18.7144	19.025
90.23	06.08	10	21	9.28	10.1	10.18	18.03	19.61	19.78	18.7144	19.025
90.24	06.15	9.75	18.5	8.45	9.17	9.35	16.52	17.92	18.29	18.4038	19.025
90.25	06.22	9.5	18	8.49	9.07	9.26	15.89	16.98	17.34	17.2858	18.7144
90.26	06.29	7.7	16.75	7.45	8.09	8.27	15.5	16.81	17.2	16.8511	18.7144

90.27	07.06	7.7	16.75	7.76	8.48	8.54	16.54	18.06	18.2	17.5611	19.4244
90.28	07.13	8	18	7.81	8.7	8.78	18.33	20.42	20.62	17.8716	18.8033
90.29	07.20	9	19	9.32	10.38	10.56	18.49	20.58	20.95	18.1822	19.1138
90.3	07.27	10.15	18.75	10.05	10.98	11.16	18.6	20.34	20.66	18.1822	19.1138
90.31	08.03	10.5	19.75	12.35	12.35	12.52	22.03	22.03	22.33	18.1822	19.1138
90.32	08.10	12.5	21.25	13.66	13.74	13.83	23.41	23.54	23.69	18.1822	18.8033
90.33	08.17	11.5	21.25	12.24	12.34	12.36	22.5	22.7	22.74	18.1822	18.8033
90.34	08.24	10.95	21.5	11.68	11.68	11.68	21.6	21.6	21.6	17.8716	18.4927
90.35	08.31	10.95	21	10.85	10.97	10.97	21.12	21.18	21.18	17.5611	18.4927
90.36	09.07	10.75	20.5	10.94	11.08	11.08	20.62	20.9	20.9	17.5611	18.4927
90.37	09.14	11	20.75	10.96	10.96	10.96	21.1	21.1	21.1	17.5611	18.4927
90.38	09.21	11.5	20.5	11.06	11.06	11.06	19.57	19.57	19.57	17.2505	18.1822
90.39	09.28	11.5	20	11.45	11.66	11.5	20.42	20.81	20.52	16.94	18.1822
90.4	10.05	11.5	21.5	11.64	12.17	12.17	21.93	22.95	22.95	18.03	18.9616
90.41	10.12	12.25	22.5	12.38	12.91	12.74	22.83	23.81	23.5	18.03	18.9616
90.42	10.19	12.75	26.25	12.73	12.73	12.57	24.08	24.03	23.79	18.3405	18.9616
90.43	10.26	12.5	24.75	12.85	12.11	11.88	25.58	24.1	23.64	18.03	18.9616
90.44	11.02	13.75	24.5	13.21	12.78	10.91	24.15	23.2	19.8	17.7194	18.6511
90.45	11.09	13.25	23	12.27	12	10.38	21.65	21.21	18.35	17.4088	18.3405
90.46	11.16	12.5	23.75	12.3	11.97	10.11	23.03	22.31	18.85	17.0983	18.03
90.47	11.23	13.25	23.75	12.58	12.16	10.02	23.39	22.61	18.62	17.4088	18.03
90.48	11.30	14.5	24.75	13.42	12.71	10.4	24.13	22.85	18.69	17.7194	18.1542
90.49	12.07	15.75	25.5	14.92	14	11.51	24.4	22.89	18.82	18.03	18.3405
90.5	12.14	15.5	25.25	14.67	13.83	11.16	24.2	22.8	18.4	18.6511	18.4647
90.51	12.21	15	26	14.85	13.63	10.92	25.74	23.61	18.92	18.9616	18.6511
90.52	12.28										
91.01	01.04	14.5	26	14.53	13.61	10.49	25.55	23.93	18.44	17.8216	17.5511
91.02	01.11	15	25.75	14.95	13.87	10.87	26.08	24.21	18.97	17.8216	17.5511
91.03	01.18	15	25.75	14.78	13.35	1074	25.22	22.78	18.33	17.8216	17.5511
91.04	01.25	15.15	25.75	15.16	13.83	10.88	25.92	23.63	18.6	17.8216	17.5511
91.05	02.01	15.15	26	14.25	11.06	12.13	24.23	18.3	20.62	17.8216	17.8616
91.06	02.08	16	26.75	14.11	10.94	11.97	24.56	19.04	20.83	17.8216	17.8616
91.07	02.15	17	27	15.11	11.6	12.64	24.77	19.02	20.73	18.7533	18.1722
91.08	02.22	17.25	27.5	14.64	11.9	13.1	23.51	17.96	19.78	19.0638	19.1038
91.09	03.01	18.25	29	15.51	12.27	13.26	23.53	18.62	20.12	19.0017	19.4144
91.1	03.08	18.75	30	16.39	12.34	13.34	26.99	20.33	21.97	19.0638	19.4144
91.11	03.15	18	26.5	15.15	11.26	12.52	24.13	17.93	19.93	19.3744	20.0355
91.12	03.22	17.25	26	14.7	11.21	12.49	21.93	16.72	18.63	19.0638	19.725
91.13	03.29										
91.14	04.05	16.5	25	15.68	11.95	13.01	23.2	17.67	19.26	18.4138	20.2772
91.15	04.12	14.75	24	14.35	10.95	12.03	23.98	18.31	20.1	18.4138	20.2772
91.16	04.19	15	25	15.06	11.73	12.54	24.6	19.17	20.49	18.4138	19.035
91.17	04.26	15.75	25.75	16.17	12.86	13.84	25.78	20.5	22.06	18.7244	19.3455
91.18	05.03	15.5	25	11.98	13.11	13.15	19.32	21.31	21.38	18.8486	19.4697
91.19	05.10	15.5	25	13.1	13.96	14.15	21.08	22.45	22.76	19.035	19.4697
91.2	05.17	16	25.25	13.77	14.83	14.8	21.99	23.68	23.63	19.6561	20.2772
91.21	05.24	17.5	28	15.82	16.56	16.58	24.94	26.11	26.14	20.2772	21.2088
91.22	05.31	18	28.5	15.55	16.37	16.52	24.8	26.11	26.36	20.5877	21.2088

91.23	06.07	15.9	26	13.89	14.85	14.9	22.98	24.57	24.65	20.5877	20.8983
91.24	06.14	16	28.5	15.18	15.94	15.88	26.33	27.65	27.55	20.5877	20.8983
91.25	06.21	16.25	28.5	15.25	15.72	15.72	27.12	27.95	27.95	20.8983	21.2088
91.26	06.28	15.75	28	15.29	15.62	15.66	26.58	27.16	27.23	21.2088	21.5194
91.27	07.05	15.35	27.5	14.82	14.81	14.85	26.17	26.16	26.22	21.1688	21.4794
91.28	07.12	13.25	24.25	12.84	13.44	13.44	22.99	24.08	24.08	20.8583	21.4794
91.29	07.19	13	23.5	13.02	13.19	13.82	23.4	23.69	24.83	20.5477	21.4794
91.3	07.26	12.75	24.5	12.76	12.88	13.83	24.18	24.41	26.2	20.5477	21.4794
91.31	08.02	12	23.5	13.39	13.47	13.52	25.74	25.9	25.99	20.2372	21.1688
91.32	08.09	11.75	24	13.73	13.77	13.8	27.86	27.94	28.01	20.8583	21.4794
91.33	08.16	13	25.5	14.13	14.11	14.05	27.69	27.64	27.53	21.4794	21.4794
91.34	08.23	12.25	24.5	14.33	14.29	14.12	27.36	27.28	26.96	21.4794	21.4794
91.35	08.30	12.9	25.25	14.4	14.4	14.35	28.65	28.65	28.6	21.79	21.79
91.36	09.06	13.5	26	14.31	14.31	14.33	28.56	28.56	28.6	21.79	22.1005
91.37	09.13	13.5	26.25	14.2	14.47	14.41	28.47	29.01	28.95	21.79	22.1005
91.38	09.20	12.25	26	13.68	13.98	13.98	28.51	29.14	29.14	21.4794	21.8521
91.39	09.27	11.5	26.5	12.42	12.93	12.95	28.29	29.45	29.48	21.4794	21.8521
91.4	10.04	13	26.5	13.96	14.45	14.45	28.89	29.89	29.89	21.8015	22.1742
91.41	10.11	14.5	26.75	14.31	14.95	14.95	28.26	29.53	29.53	21.9257	22.1742
91.42	10.18	15.5	27.75	16.25	16.79	16.79	27.94	28.86	28.86	22.05	22.2363
91.43	10.25	16.5	28	16.45	17.06	17.06	28.03	29.08	29.08	22.6711	22.2363
91.44	11.01	16.5	27.75	16.99	16.99	14.63	28.77	28.77	24.78	22.6711	22.2363
91.45	11.08	16.25	27.75	16.52	16.52	14.34	27.93	27.93	24.24	22.3605	22.2363
91.46	11.15	15.75	26	15.58	15.58	13.31	26.25	26.25	23.44	21.4288	21.4288
91.47	11.22	15.5	26	16.07	16.07	13.92	27.2	27.2	24.51	21.1183	21.4288
91.48	11.29	15.5	26	15.69	15.69	13.54	26.91	26.91	23.84	21.1183	21.4288
91.49	12.06	15	25.5	15.88	15.97	13.74	27.09	27.24	24.19	20.9941	21.4288
91.5	12.13	15	25.5	15.1	15.11	12.98	25.88	25.9	22.86	20.932	21.3046
91.51	12.20	14.5	24.75	13.95	14.71	11.97	25.19	25.58	21.61	20.932	21.3046
91.52	12.27										
92.1	01.10	14.25	24.5	13.86	14.38	12.42	25.85	26.82	23.18	20.5077	20.8804
92.2	01.17	14.5	25.5	14.47	15.08	12.93	26.02	27.1	23.25	20.5077	20.8804
92.3	01.24	14	25	13.78	14.23	12.1	24.79	25.6	21.77	20.3835	20.8804
92.4	01.31	14	23.5	13.53	14.18	12.17	23.78	24.92	21.38	20.2593	20.8183
92.5	02.07	12.25	23.5	13.28	11.32	12.47	25.77	21.97	24.22	19.8866	20.1972
92.6	02.14	11.75	23	12.29	10.65	11.88	24.16	20.94	23.36	19.5761	20.1972
92.7	02.21	10.25	22	11.24	9.59	10.69	22.58	19.26	21.48	19.2655	19.8866
92.8	02.28	10.5	20.75	10.99	9.49	10.57	22.2	19.17	21.34	18.6444	19.8866
92.9	03.06	9.75	20.5	10.51	9.09	10.22	22.72	19.65	22.1	17.4022	19.5761
92.1	03.13	9.5	21	9.88	8.53	9.58	22.78	19.67	22.1	17.4022	19.5761
92.11	03.20	10.25	20.5	10.67	9.28	10.41	21.96	19.09	21.42	17.4022	19.5761
92.12	03.27	10.5	20	10.67	9.44	10.63	20.93	18.53	20.87	17.4022	19.2655
92.13	04.03	10.5	20	10.34	9.3	10.53	19.96	17.96	20.32	17.4022	18.955
92.14	04.10	10.25	20	10.35	9.41	10.54	20.08	18.26	20.45	17.5522	19.105
92.15	04.17										
92.16	04.24	10.5	20	11	10.16	11.22	20.29	18.74	20.7	17.5522	19.105
92.17	05.01	10.75	21.25	10.67	10.15	11.14	20.7	19.68	21.59	17.5522	19.4155
92.18	05.08	10.75	23	10.94	10.4	11.41	21.21	20.16	22.13	17.5522	19.4155

Appendix C

Ship Costs and Output

C.1 The US Gulf to Japan Route

C.1.1 Ship Costs and Outputs

1. Round Trip Distance:
Cargo : ...... 9,400 nautical miles
Ballast : ...... 7,000 nautical miles

2. Port and Canal Charges:
Canal : ...... 65,000 $/round trip
Port : ...... 130,000 $/round trip

3. Port Time: ...... 18 days/round trip

4. Round Trip Time: $\frac{9,400*2}{13*24} = 60.26$ days.

5. Total Round Trip Time: $60.26+18 = 78.26$ days.

6. Days "off hire" per year : 15 days.

7. Number of Round Trips per year : $\frac{350}{78.26} = 4.47$ round trips/year.

8. Ship Type : PANAMAX bulkcarrier, 60,000 dwt, speed : 13 knots.

9. Pay load : 50,000 tons.

Note that since the ballast leg is only 7,000 nautical miles the equivalent pay load for this route is $50,000 * (1 + \frac{7000}{9400}) = 63,000$ ton/round trip.

10. Ship Output: 63,000 x 4.47 = 281,610 ton/year.

11. Voyage Related Costs:
Bunker Costs: 26 ton/day x 60.26 days x 150 $/ton = 235,000 $/ round trip.
Canal & Port: 195,000 $/round trip.
Total Voyage Costs: 195,000 + 235,000 = 430,000 $/round trip.
Voyage Costs per day : $\frac{430,000}{78.26}$ = 5,494 $/day.
Voyage Costs per ton : $\frac{430,000}{63,000}$ = 6.83 $/ton.

12. Non-Voyage Related Costs:
Non-Voyage Operating Costs[1]: 4,370 $/day.
Non-Voyage Operating Costs per ton : $\frac{4,370 * 78.26}{63,000}$ = 5.43 $/ton.

13. Ship Operating Costs per ton : 6.83 + 5.43 = 12.26 $/ton.

C.1.2 Transformation from Term Charter to Spot Equivalent

With the above ship related costs we can develop the formulas to transform the term charter rate from $/day to spot denominated in $/ton for the US Gulf - Japan route and vice-versa.

1. Transformation term charter $/day to spot equivalent in $/ton:

$$\frac{TCR * 78.26}{63,000} + 6.83 = ...\$/ton \tag{C.1}$$

2. Transformation spot rate $/ton to term charter in $/day:

$$\frac{SPOT * 63,000}{78.26} - 5,500 = ...\$/day \tag{C.2}$$

3. Bunker prices considered for the transformation from term charter rates to spot equivalent that is reported in Appendix B:

[1] This include: crew, ship maintenance and lubricants and administration.

Table C.1:

Average Bunker Prices per Quarter 1985-1992 $/ton

Year	Q1	Q2	Q3	Q4
1985	183	148	142	142
1986	110	64	62	84.5
1987	99.5	113.7	108.2	95.4
1988	80	81.8	74	64.2
1989	80.6	96.5	90.1	104.6
1990	95	78.5	107.1	151
1991	105.1	79	77.5	87.9
1992	75.9	82		

C.2 The US Gulf to ARA Route

C.2.1 Ship Costs and Outputs

1. Round Trip Distance:
Cargo : ...... 4,800 nautical miles
Ballast : ...... 4,800 nautical miles

2. **Port Charges : ...... 100,000 $/round trip.**

3. **Port Time : ...... 13 days/round trip.**

4. **Round Trip Time : $\frac{4,800*2}{13*24}$ = 30.77 days.**

5. **Total Round Trip Time : 30.77 + 13 = 43.77 days.**

6. **Days "off hire" per year : 15 days.**

7. **Number of Round Trips per year : $\frac{350}{43.77}$ = 7.99 round trips/year.**

8. **Ship Type : PANAMAX bulkcarrier, 60,000 dwt, speed : 13 knots.**

9. **Pay Load : 50,000 tons.**

10. **Ship Output: 50,000 x 7.99 = 399,817 ton/year.**

11. Voyage Related Costs:

Bunker Costs : 26 ton/day x 30.77 days x 150 \$/ton = 120,000 \$/round trip.
Port Costs : 100,000 \$/round trip.
Total Voyage Costs: 220,000 \$/round trip.
Voyage Costs per day : $\frac{220,000}{43.77}$ = 5,026 \$/day.
Voyage Costs per ton : $\frac{220,000}{50,000}$ = 4.4 \$/ton.

12. Non-Voyage Related Costs:

Non-Voyage Operating Costs: 4,370 \$/day.
Non-Voyage Operating Costs per ton : $\frac{4,370 \cdot 43.77}{50,000}$ = 3.83 \$/ton.

13. Ship Operating Costs per ton : 8.23 \$/ton.

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