The Effect of Cash Constraints on Smallholder Farmer Revenue

by

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Abstract

Many smallholder farmers in developing countries struggle to make ends meet. We develop a model that examines how markets catering to numerous smallholder farmers reach an equilibrium, while incorporating real world challenges that smallholder farmers face, namely a lack of long term planning and cash constraints. Through this, we analyze the effectiveness of two common forms of government intervention, storage and loan provision. We fully characterize market equilibrium conditions under the base scenario of no government intervention, analyzing how price conditions, number of farmers, and severity of cash constraints impact farmer behaviour. We then illustrate how these results change when storage and loans are integrated into the model. The analysis demonstrates that myopic optimization and cash constraints induce farmers to make sub-optimal decisions, resulting in farmers not receiving the full benefit of government interventions. We show that while storage is always useful in situations where farmers have excess quantity, providing overly generous loan terms can negatively impact farmer revenue by disincentivizing farmers from selling their produce on the market. We also show that attempting to improve equality by alleviating farmer cash constraints can result in negative externalities like increased wastage. Empirical analysis with Bengal gram farmers in India shows that farmers are in dire need of government assistance to meet their cash constraints. However, improving loan terms only boosts farmer revenue up to a point, after which revenue declines. The analysis shows that while loan schemes are widely popular and sometimes necessary in aiding struggling farmers, governments should be aware that the strategic response of different farmers can result in adverse effects.

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Chapter 1

Introduction

Smallholder farmers are an integral part of the agriculture industry, accounting for 84% of farms and 2 billion people worldwide [15]. Even as they provide over 80% of the food consumed in developing countries, they also account for most of the 1.4 billion people living in poverty [21]. Therefore, improving revenue outcomes for smallholder farmers is an important and relevant problem for governments worldwide.

One reason that smallholder farmers struggle to generate revenue is that they are forced to sell most or all of their produce immediately after harvest at depressed prices. This is done firstly because of a lack of storage infrastructure. Smallholder farmers do not have the capital necessary to invest in high quality storage facilities, and as a result there are significant post-harvest crop losses from storage due to decay, physical shocks, pests and disease [7]. A second reason is the need for immediate cash. It is estimated that fewer than 10% of smallholder farmers have access to finance [4], and as such most farmers rely on revenue generated through selling produce to prepare for the next harvest. As a result, smallholder farmers flood the market with produce during the harvest season and are forced to accept the low prices offered by traders [20].

An illustration of how devastating this phenomenon can be for farmers was seen recently in India, where the price of the khatif onion dropped below the cost of production [11]. Without the means to store their produce, farmers were forced to sell at a loss while traders and stockists with access to warehouses were able to store the produce for sale during the lean season, where prices are typically higher.

This paper evaluates the effectiveness of commonly observed government interventions in improving farmer revenue. We use a 2 period model, representing the harvest and lean season, and simulate how a market with numerous farmers selling small quantities of produce reaches an equilibrium in each season. This model is unique in two ways: first, farmers are assumed to optimize myopically; second, farmers are heterogeneous, each constrained by their need for different amounts of cash. We believe that these conditions accurately depict the realities faced by smallholder farmers - the threat of poverty induces farmers to prioritize immediate financial benefits over long-term strategy [3], and the need to purchase inputs like seeds and fertilizer for the next harvest ensures that farmers face cash constraints. However, given that farmers have multiple sources of income and some will be better off than others, we incorporate heterogeneous cash constraints into the model.

Since cash constraints and myopic optimization results in farmers making long-term sub-optimal decisions, government interventions designed to boost farmer revenue may be rendered less effective. Furthermore, heterogeneity of cash constraints allows us to analyze how the benefits of government intervention may be unevenly distributed across farmers. In particular, this paper considers the effect of providing storage infrastructure and short-term loans to farmers. These measures are of particular relevance in India as the government has looked to invest in these areas. Recently, the Indian government pledged 1 trillion rupees for investment in cold storage facilities and post-harvest storage centers, for the benefit of smallholder farmers that are unable to afford such services [25]. There have also been numerous policies aimed at improving access to credit, such as requiring banks to meet credit targets for agricultural loans every year, and introducing schemes to lower the effective rate of interest on agricultural loans [17].

We evaluate the effectiveness of storage and loans by examining their impact on farmer revenue and inequality amongst farmers. We also consider the cost of such schemes, both in terms of wasted produce as well as government expenditure. We find that provision of storage always improves revenue and reduces wastage if farmers have excess quantity in the harvest season. In contrast, loans have mixed effects - while offering more generous loans reduces inequality amongst farmers, it also encourages wastage and higher government expenditure. Furthermore, loans can backfire and reduce farmer revenue.

We illustrate these findings using field data from the Bengal gram market in India. Without government intervention, we find that farmers are unable to meet their cash constraints. We then consider the introduction of storage and loans, calibrating loan terms to government data. We find that farmers can now meet their cash constraints in the harvest season and earn additional revenue in the lean season. However, optimizing loan terms sees a 12.6% increase in total revenue for farmers, as well as reduced revenue disparity. Improving loan terms beyond the optimal level results in lower revenue for farmers, on top of higher government expenditure.

The remainder of the paper is structured as follows. Chapter 2 reviews the related literature. Chapter 3 characterizes the base model with no government intervention. Chapter 4 explains how the introduction of storage infrastructure and loans influences the model. Chapter 5 examines how the base model compares to the model under government intervention. Chapter 6 considers how the government can determine the optimal level of intervention, and discusses the policy implications. Chapter 7 presents the empirical analysis. Chapter 8 concludes the paper.

Chapter 2

Literature Review

Studying the effect of storage on revenue is closely related to the classical warehouse problem in operations management, where aspects of warehouses such as location or size are optimized to maximize sales. Existing literature in the agriculture industry includes studies by Jasinska and Wojtych [10] and Monteroso et al. [16] that examined the warehouse location problem from the government's perspective in the sugar beet and grain industry respectively. Research in this area tends to view the storage problem from a macro perspective, modelling the total flow of produce between areas. This paper differs by approaching the storage problem from the farmer's perspective - rather than treating farmers as a homogeneous whole, we consider the reality that individual farmers may have differing cash constraints that influence their decisions. This adds value to the current discourse by enabling us to model how the benefits of government intervention may be unevenly distributed to farmers.

Loans in the agriculture industry have also been well-studied. For example, studies by Zelenovic, Vojinovic and Cvijanovic [28] and Sharifat et al. [24] examine agricultural loans in Serbia and Iran respectively. The existing literature is largely focused on understanding the underlying factors affecting credit access and default rates for farmers. In contrast, this paper examines the effect of loans on farmer welfare, modelling the balance between offering farmers loans large enough to meaningfully improve their revenue, while also ensuring loans are not so large as to disincentivize farmers from selling their produce on the market.

In markets catering to smallholder farmers, we have hundreds of farmers selling their produce simultaneously. Since market price is a function of total quantity sold, the individual farmer's decision on quantity to sell is clearly influenced by his peers. However, due to the large number of farmers, it is unlikely that the individual farmer considers the strategies of each of his peers separately. Using traditional dynamic game theory is infeasible and implausible when analyzing dynamic systems with a large number of agents [12]. Beyond computational limitations, as the number of players increases, 'interindividual complex strategies can no longer by implemented...each player is progressively lost in the crowd in the eyes of other players' [9]. Therefore, we use the concept of mean field equilibria to model his decision making process, where we assume each farmer bases his decision on the long run average behaviour of all other farmers. We believe that the mean field approach provides a more realistic approximation to farmer behavior in reality, compared to traditional dynamic game theory.

Finally, we refer to a study by Liao, Chen and Tang [14] which modelled the responses of smallholder farmers to information provision policies. They accounted for heterogeneity amongst farmers in terms of their distance from markets, and modelled this by assuming farmers were distributed uniformly over a 2D space representing distance from the market. We adopt a similar approach in modelling heterogeneous cash constraints.

Chapter 3

Base Model

In the base case, we model the existing situation for farmers - that is, without access to storage or loans. Note that although we have a two period model, without storage farmers are unable to sell produce in the lean season. Therefore, farmers earn no revenue in the lean season, and we focus on the harvest season market equilibrium.

We consider an agricultural market with N farmers. Since smallholder farmers have limited access to finance [4], farmers rely on revenue generated through selling produce to purchase inputs for the next harvest season. However, since farmers can have multiple sources of income [23], different farmers require different amounts of cash. Therefore, we assume heterogeneous cash constraints distributed uniformly over $[0, C_{max}]$. We model market price using the same method as Liao, Chen and Tang [14], as is commonly seen in operations research literature. Market price is given by the equation $\alpha - \beta \sum_{i=1}^{N} q_i$, where α is the intercept, β is price elasticity, and $\sum_{i=1}^{N} q_i$ is the total quantity sold by the N farmers. Since smallholder farmers operate farms of 2 hectares or less, the output level of each farm is similar. Therefore, we assume homogeneous production of 1 unit per farmer during the harvest season. Each farmer is aware of his individual cash constraint, the total number of farmers, and the distribution of cash constraints across farmers.

3.1 Characterization of equilibria

Using the principle of mean field equilibria, we assume individual farmers make quantity decisions based on the average quantity sold by other farmers. We call this their **best-response quantity**. We characterize an equilibrium by computing the best-response quantity for each farmer as a function of their cash constraint, and we say an equilibrium is feasible if it meets the following conditions: (i) Farmers satisfy their cash constraint by selling their best-response quantity; (ii) No farmer is selling more than 1 unit. In this section, we begin by defining the parameter region in which feasible equilibria exist. Thereafter, we show the derivation of best-response quantities for feasible equilibria. Finally, we consider the sensitivity of the equilibrium to changes in parameter values.

We begin by considering how the individual farmer computes his best-response quantity. The ith farmer chooses quantity $q_i \in [0, 1]$ to sell in order to maximize revenue. The farmer has cash constraint C_i and solves the following problem:

$$\begin{aligned} max_{q_i}\ Revenue &= max_{q_i}\ (\alpha - \beta(q_i + (N-1)\bar{q}_{-i}))q_i \qquad (3.1) \\ s.t.\ & Revenue \geq C_i \\ & 0 \leq q_i \leq 1 \end{aligned}$$

where $\bar{q}_{-i}$ is the average quantity sold by the other $N-1$ farmers. There are two possibilities: (i) The cash constraint is not tight for any farmer. (ii) The cash constraint is tight for some farmers. We refer to the former as an **unconstrained equilibrium**, and the latter as a **partially constrained equilibrium**. Note that the cash constraint cannot be tight for all farmers because cash constraints start at 0.

Before presenting the theorem, we introduce some terminology. In the context of partially constrained equilibria, we separate the farmers into two groups: unconstrained (Revenue $> C$) and cash constrained (Revenue $= C$). As seen in problem 3.1, the individual farmer's decision is dependent on the average quantity sold by other farmers, $\bar{q}_{-i}$. We use the following terms to express $\bar{q}_{-i}$:

1. $\hat{c}$: The boundary cash constraint between unconstrained and cash constrained farmers.

2. F: The average quantity sold by unconstrained farmers

3. $\tilde{f}$: The average quantity sold by cash constrained farmers, weighted by the proportion of cash constrained farmers

The average quantity sold by the other farmers can thus be written $\bar{q}_{-i} = \frac{\hat{c}}{C_{max}}F + \tilde{f}$.

Lemma 1 *Let $\alpha_1, \tilde{f}_1$ and $\alpha_2, \tilde{f}_2$ be the solutions to the system of equations 3.2 and 3.3 respectively. At least one of the systems of equations has a solution. If only one has a solution, denote it α^*. Else, let $\alpha^* = min\{\alpha_1, \alpha_2\}$.*

$$\begin{aligned}
P &= \alpha - \beta(N-1)(\frac{\hat{c}}{C_{max}}F + \tilde{f}) \geq \beta + C_{max} \\
g(\tilde{f}) &= \frac{1}{2\beta}(1 - \frac{\hat{c}}{C_{max}})P - \frac{1}{12\beta^2 C_{max}}((P^2 - 4\beta\hat{c})^{1.5} - (P^2 - 4\beta C_{max})^{1.5}) - \tilde{f} \\
&= 0 \\
g'(\tilde{f}) &= 0
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
P &= \beta + C_{max} \\
g(\tilde{f}) &= 0
\end{aligned} \tag{3.3}$$

where $F = \frac{C_{max}(\alpha - \beta\tilde{f}(N-1))}{2\beta(C_{max} + \hat{c}(N-1))}$ *and* $\hat{c} = \frac{-\beta C_{max} + \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta\tilde{f}(N-1))^2}}{2\beta(N-1)}$.

Theorem 1 *Let $q = min\{\frac{\alpha}{2\beta N}, 1\}$ and $R = (\alpha - \beta N q)q$.*

1. *If $\alpha \geq \alpha^*$, then if*

   (a) If $R \geq C_{max}$, $q^* = min\{\frac{\alpha}{2\beta N}, 1\}$

   (b) If $R < C_{max}$, q^* is a piece-wise function of the following form:

$$q(C) = \begin{cases} F & C \leq \hat{c} \\ \frac{1}{2\beta}(P - \sqrt{P^2 - 4\beta C}) & \hat{c} < C \leq C_{max} \end{cases}$$

2. *If* $\alpha < \alpha^*$ *then the problem 3.1 is infeasible and* $\exists c^* > 0$ *s.t.* $\forall C_i > c^*$, $Revenue < C_i$, .

To interpret Lemma 1, we begin by explaining the derivation and intuition behind the formulas for $F, \hat{c}$ and $g(\tilde{f})$. As mentioned earlier, F and $\hat{c}$ exist in the context of a partially constrained equilibria where some farmers are cash constrained and others are unconstrained. We consider problem 3.1 from the perspective of both groups of farmers. Since unconstrained farmers' quantity decision is unaffected by their cash constraint, we can remove it from their revenue maximization problem. As a result, all unconstrained farmers solve identical problems and hence have the same best-response quantity, which we denote as F. By solving the problem below, we can express F in terms of $\hat{c}$ and $\tilde{f}$.

$$
\begin{aligned}
max_F \qquad & (\alpha - \beta(F + (N-1)(\frac{\hat{c}}{C_{max}}F + \tilde{f})))F \\
F \quad = \quad & \frac{C_{max}(\alpha - \beta\tilde{f}(N-1))}{2\beta(C_{max} + \hat{c}(N-1))}
\end{aligned}
\tag{3.4}
$$

Since cash constrained farmers make their cash constraint exactly, we can determine their best-response quantity q in terms of $\hat{c}$ and $\tilde{f}$ by equating revenue to their cash constraint C. Note that q is increasing in C, as more severely cash constrained farmers will have to sell larger quantities to meet their cash constraint.

$$
\begin{aligned}
C \quad &= \quad (\alpha - \beta(q + (N-1)(\frac{\hat{c}}{C_{max}}F + \tilde{f})))q \\
q \quad &= \quad \frac{1}{2\beta}(P - \sqrt{P^2 - 4\beta C})
\end{aligned}
\tag{3.5}
$$

where $P = \alpha - \beta(N-1)(\frac{\hat{c}}{C_{max}}F + \tilde{f})$. P can be interpreted as the market price observed by the farmer before making a quantity decision.

We can express $\hat{c}$ in terms of $\tilde{f}$ by noting that the farmer with cash constraint $\hat{c}$ belongs to both the unconstrained and cash constrained groups. Therefore, his cash constraint is tight, but he also sells quantity F:

$$\begin{aligned}\hat{c} &= (\alpha - \beta(F + (N-1)(\frac{\hat{c}}{C_{max}}F + \tilde{f})))F \\ \hat{c} &= \frac{-\beta C_{max} + \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta\tilde{f}(N-1))^2}}{2\beta(N-1)} \end{aligned} \tag{3.6}$$

We are then left with one unknown $\tilde{f}$. By construction, equations 3.4, 3.5 and 3.6 guarantee that farmers satisfy their cash constraint. For a given value of $\tilde{f}$, we can compute the realized weighted average quantity sold by cash constrained farmers, $\tilde{f}_{real} = \int_{\hat{c}}^{C_{max}} \frac{1}{C_{max}} \frac{1}{2\beta}(P - \sqrt{P^2 - 4\beta C})dC$. The fixed point equation $g(\tilde{f})$ checks if $\tilde{f}_{real} = \tilde{f}$. When $g(\tilde{f}) = 0$, we know that farmers are selling their best-response quantity, and all farmers satisfy their cash constraint, meeting the first condition of feasibility.

To check the second condition of feasibility, that no farmer is selling more than 1 unit, we check that $q(C_{max}) \leq 1$, since the most cash constrained farmer sells the greatest quantity. We find that this equivalent to the condition $P \geq \beta + C_{max}$. When $P = \beta + C_{max}$, the most cash constrained farmer is forced to sell 1 unit, since the market price then decreases by β and he earns exactly C_{max}. Therefore, as long as $P \geq \beta + C_{max}$, the most cash constrained farmer will not sell more than 1 unit.

The systems of equations 3.2 and 3.3 allow us to find the boundary parameter values for each of the feasibility conditions. System of equations 3.2 finds the value of α such that if α decreases, condition (i) will fail. System of equations 3.3 does the same for condition (ii).

Before explaining the systems of equations, we prove the following properties of $\tilde{f}_{real}$:

Proposition 1 *1.* $\tilde{f}_{real} > 0$ *when* $\tilde{f} = 0$

2. $\frac{d\tilde{f}_{real}}{d\tilde{f}} > 0$

3. $\frac{d\tilde{f}_{real}}{d\alpha} < 0$ *and* $\frac{d\tilde{f}_{real}}{d\beta}, \frac{d\tilde{f}_{real}}{dN}, \frac{d\tilde{f}_{real}}{dC_{max}} > 0$

To provide some intuition for Proposition 1, note that feasibility is related to how easily farmers can meet their cash constraint. In terms of model parameters, we say that it is easier for farmers to meet their cash constraint if α increases or β, N, or C_{max} decreases. Increasing α raises the price intercept, while decreasing β reduces the sensitivity of price to quantity sold. Therefore, a farmer will earn more revenue for the same quantity sold. Decreasing N reduces the number of competing farmers, effectively reducing the total supply to the market and making it easier for farmers to get a better price. Finally, reducing C_{max} not only makes it easier for the most cash constrained farmer to meet his cash constraint, but also improves price for other farmers, since the most cash constrained farmer no longer has to sell as much quantity. If we adjust parameters to make it easier for farmers to meet their cash constraint, we will eventually allow all farmers to meet their cash constraint by selling 1 unit or less. Conversely, if we make it more difficult for farmers to meet their cash constraint, we will reach an infeasible situation where it is impossible for every farmer to satisfy his cash constraint.

We start by explaining system of equations 3.2. As explained earlier, $g(\tilde{f})$ has two components - the given value of $\tilde{f}$ and the realized best-response quantity $\tilde{f}_{real}$. Taking these components separately and plotting them as functions of $\tilde{f}$, we see a solution to $g(\tilde{f})$ exists when there is an intersection between the components. The equations $g(\tilde{f}) = 0$ and $g'(\tilde{f}) = 0$ find the value of $\tilde{f}$ such that the line $y = \tilde{f}_{real}$ is tangent to $y = \tilde{f}$, as illustrated in Figure 3-1. The inequality $P \geq \beta + C_{max}$ ensures that farmers are selling feasible quantities at this value of $\tilde{f}$. By Proposition 2, decreasing α or increasing β, N, C_{max} causes $\tilde{f}_{real}$ to shift upwards, hence there is no intersection point and $g(\tilde{f}) = 0$ becomes infeasible. This means that farmers cannot sell their best-response quantity and meet their cash constraint.

For system of equations 3.3, $P = \beta + C_{max}$ ensures that the farmer with cash constraint C_{max} is selling 1 unit, and the second equation ensures the fixed point equation is solved. Decreasing α or increasing N, β, C_{max} causes $\tilde{f}_{real}$ to shift upwards. From Figure 3-1, we see that the equilibrium value of $\tilde{f}$ increases. This reflects the fact that cash constrained farmers are selling greater quantity, since it is now more

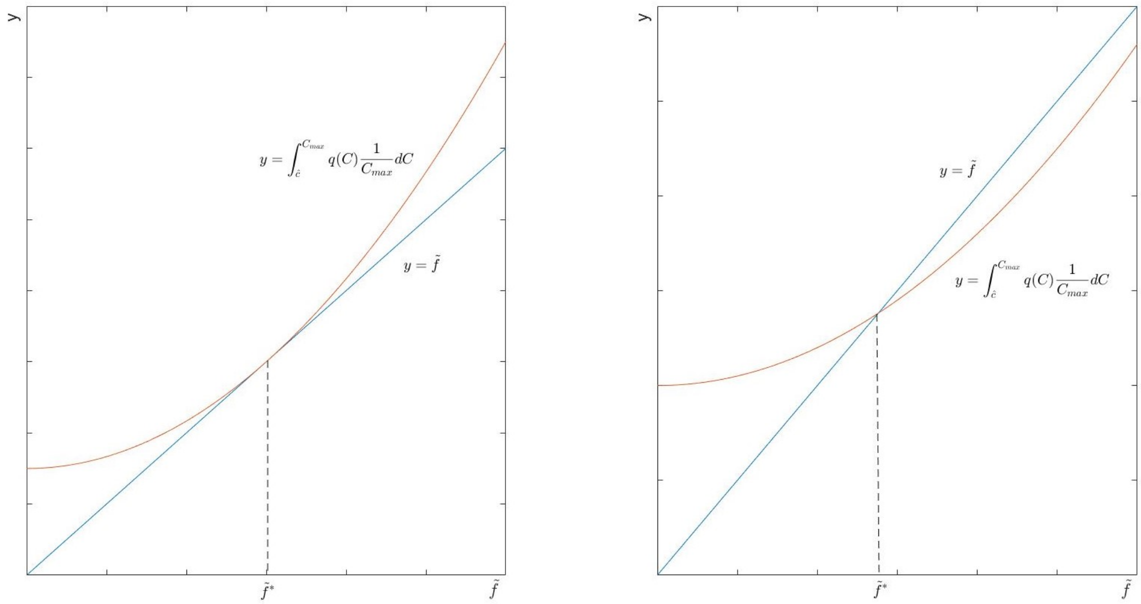

Figure 3-1: Illustration of Eqn. 3.2 (left) and 3.3 (right).

difficult for them to meet their cash constraint. Market price decreases as a result of greater quantity being sold, meaning the most cash constrained farmer now has to sell more than 1 unit to earn C_{max}.

We now explain the characterization of unconstrained and partially constrained equilibria. For unconstrained equilibria, since the cash constraint is not tight for any farmer, we can remove it from the farmers' revenue maximization problem. Hence, all farmers solve identical problems, given by

$$max_{q_i}\ Revenue = max_{q_i}\ (\alpha - \beta(q_i + (N-1)\bar{q}_{-i}))q_i$$
$$s.t.\quad 0 \leq q_i \leq 1$$

All farmers therefore have the same optimal quantity, given by $\frac{\alpha}{2\beta N}$. We refer to this as the **optimal unconstrained quantity**. Note that $\frac{\alpha}{2\beta N}$ can be greater than 1, so farmers' best-response quantity is actually $min\{\frac{\alpha}{2\beta N}, 1\}$. If farmers sell $\frac{\alpha}{2\beta N}$, each farmer earns $\frac{\alpha^2}{4\beta N}$. Else if farmers sell 1 unit, each farmer earns $\alpha - \beta N$. If individual farmer revenue is greater than C_{max}, then every farmer meets his cash constraint, and therefore the unconstrained equilibrium is feasible.

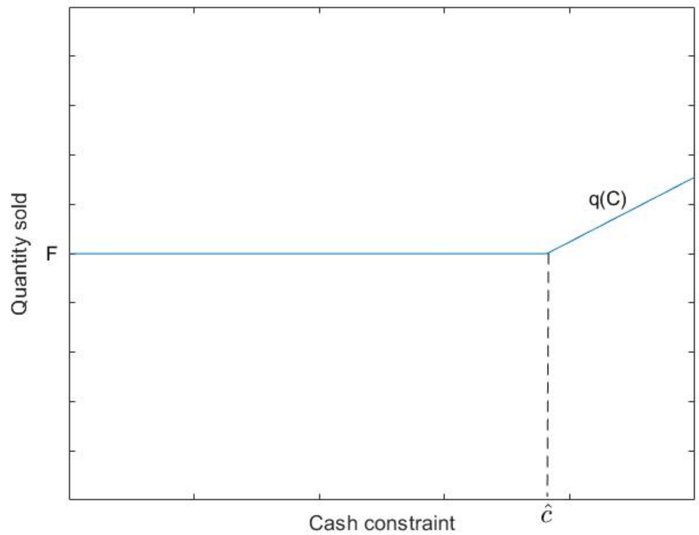

Figure 3-2: Harvest season partially constrained equilibrium.

For partially constrained equilibria, given the formulas for F and $\hat{c}$ from equations 3.4 and 3.6, we can characterize the best-response quantity for all farmers in terms of $\tilde{f}$. Let $q(C)$ denote the best-response quantity sold by a farmer with cash constraint C.

$$q(C) = \begin{cases} F & C \leq \hat{c} \\ \frac{1}{2\beta}(P - \sqrt{P^2 - 4\beta C}) & \hat{c} < C \leq C_{max} \end{cases}$$

By solving systems of equations 3.2 and 3.3 to obtain α^*, we guarantee that for $\alpha \geq \alpha^*$, there exists a solution to $g(\tilde{f}) = 0$ where all farmers are selling 1 unit or less. Therefore, we only need to solve the fixed point equation to find the equilibrium value of $\tilde{f}$. Figure 3-2 illustrates a typical harvest season partially constrained equilibrium. Observe that unconstrained farmers sell the same quantity while increasingly cash constrained farmers are forced to sell greater amounts.

Note that while unconstrained equilibria are guaranteed to be unique, it is possible for multiple feasible partially constrained equilibria to exist for a single parameter set. However, we can rank them in terms of farmer revenue. We find that the equilibrium with the lowest $\tilde{f}$ has the smallest proportion of cash constrained farmers, and all farmers earn equal or better revenue than the other equilibria.

Proposition 2 *Suppose there* $\exists \tilde{f}_1 < \tilde{f}_2$ *corresponding to feasible partially constrained equilibria. Then* $(\alpha - \beta(q_1(C) + (N-1)(\frac{\hat{c}_1}{C_{max}} F_1 + \tilde{f}_1))q_1(C) \geq (\alpha - \beta(q_2(C) + (N-1)(\frac{\hat{c}_2}{C_{max}} F_2 + \tilde{f}_2))q_2(C) \forall C \in [0, C_{max}]$.

Finally, we examine how the equilibrium value of $\tilde{f}$ shifts in response to changes in the parameters $\alpha, \beta, N, C_{max}$ and how this affects farmer revenue:

Proposition 3 $\frac{d\tilde{f}}{d\alpha} < 0$ *and* $\frac{d\tilde{f}}{d\beta}, \frac{d\tilde{f}}{dN}, \frac{d\tilde{f}}{dC_{max}} > 0$.

Proposition 3 reflects the intuitive result that as α increases or β, N, C_{max} decreases, cash constrained farmers will sell less quantity since it is easier for them to meet their cash constraint. As $\tilde{f}$ decreases, the proportion of cash constrained farmers decreases, and unconstrained farmers earn more revenue. From a government standpoint, this means that making it easier for farmers to meet their cash constraints improves farmer welfare.

Chapter 4

Government Intervention

We now introduce two forms of government intervention into the model - provision of storage infrastructure and loans. We select these forms of government intervention because they align with government policy in reality. In India, access to storage and finance are key components of the government's approach to helping smallholder farmers [25][17].

4.1 Storage

We assume that farmers are now able to store their unsold quantity from the harvest season to the lean season. We further assume that there is no limit to the quantity that can be stored, there is no cost of storage, and there is no depreciation of quality over time. We make the first two assumptions with the view that it is in the government's interest to reduce wastage, and that the government is not trying to profit through providing storage. We ignore the effect of depreciation for the sake of model simplicity. Under these assumptions, farmers will store all of their unsold quantity since there is no downside. During the lean season, we also assume that the price parameters α and β are unchanged from the harvest season. This is a conservative take, given that seasonal variation in prices of staple crops has been observed in India, where prices fall during harvest months and increase in the lean season [26]. Finally, we assume that farmers are not cash constrained in the lean season, since they would have already purchased new inputs post harvest.

Note that because farmers still optimize myopically, the harvest season equilibrium is unchanged from the base case. We now characterize the equilibrium in the lean season. Moving forward, we introduce subscripts to differentiate the harvest and lean season, with 1 referring to the harvest season and 2 the lean season. Similar to the base case, we begin by considering how the individual farmer computes his best-response quantity. A farmer with cash constraint C_i solves the following problem:

$$\begin{aligned} max_{q_i} \; Revenue &= max_{q_i} \; (\alpha - \beta(q_i + (N-1)\bar{q}_{-i}))q_i \\ s.t. &\quad q_i \leq 1 - q_1(C_i) \end{aligned} \tag{4.1}$$

where $\bar{q}_{-i}$ is the average quantity sold by the other $N-1$ farmers, and $q_1(C_i)$ is the farmer's quantity sold in the harvest season. Note that instead of a cash constraint, farmers now face a quantity constraint in $q_i \leq 1 - q_1(C_i)$. There are three cases: (i) The quantity constraint is not tight for any farmer. (ii) The quantity constraint is tight for some farmers. (iii) The quantity constraint is tight for all farmers. We henceforth refer to (i) as an **unconstrained equilibrium**; (ii) as a **partially constrained equilibrium**; and (iii) as a **fully constrained equilibrium**. For the following analysis, we assume that a feasible equilibrium exists in the harvest season.

In case (ii), we can separate the farmers into two groups: unconstrained ($q_i < 1 - q_1(C_i)$) and quantity constrained ($q_i = 1 - q_1(C_i)$). As seen in problem 4.1, the farmer's decision is dependent on $\bar{q}_{-i}$, so we introduce the following terminology:

1. $\hat{c}_2$: Boundary cash constraint between unconstrained and quantity constrained farmers

2. F_2: Average quantity sold by unconstrained farmers

3. $\tilde{f}_2$: Average quantity sold by quantity constrained farmers, weighted by the proportion of quantity constrained farmers

We can write $\bar{q}_{-i} = \frac{\hat{c}_2}{C_{max}} F_2 + \tilde{f}_2$.

Theorem 2 *Let* $q_1 = min\{\frac{\alpha}{2\beta N}, 1\}$ *and* $R = (\alpha - \beta N q_1)q_1$.

1. *If* $R \geq C_{max}$, $q_2^* = min\{\frac{\alpha}{2\beta N}, 1 - q_1\}$

2. *If* $R < C_{max}$, *a partially constrained equilibrium exists in the harvest season, and we obtain* $F_1, \hat{c}_1, \tilde{f}_1, q_1(C_{max})$ *and* P_1.

   (a) *If* $1 - q_1(C_{max}) \geq \frac{\alpha}{2\beta N}$, $q_2^* = \frac{\alpha}{2\beta N}$

   (b) *If* $\frac{\alpha}{2\beta N} \geq 1 - F_1$ *and* $\frac{C_{max}(\alpha-\beta(N-1)(1-\frac{\hat{c}_1}{C_{max}}-\tilde{f}_1)}{2\beta(C_{max}+(N-1)\hat{c}_1)} \geq 1 - F_1$, q_2^* *is a piece-wise function of the following form:*

$$q_2(C) = \begin{cases} 1 - F_1 & C \leq \hat{c}_1 \\ 1 - \frac{1}{2\beta}(P_1 - \sqrt{P_1^2 - 4\beta C}) & \hat{c}_1 < C \leq C_{max} \end{cases}$$

   (c) *Else,* q_2^* *is a piece-wise function of the following form:*

$$q_2(C) = \begin{cases} F_2 & C \leq \hat{c}_2 \\ 1 - \frac{1}{2\beta}(P_1 - \sqrt{P_1^2 - 4\beta C}) & \hat{c}_2 < C \leq C_{max} \end{cases}$$

where $F_2 = \frac{C_{max}(\alpha-\beta(N-1)\tilde{f}_2)}{2\beta(C_{max}+(N-1)\hat{c}_2)}$.

Recall that if $R \geq C_{max}$, an unconstrained equilibrium is feasible in the harvest season, and all farmers store the same quantity $1 - min\{\frac{\alpha}{2\beta N}, 1\}$, so stored quantity is uniform. Therefore, in the lean season, the quantity constraint is identical for farmers, and we either have an unconstrained or fully constrained equilibrium. In an unconstrained equilibrium, we remove the quantity constraint from problem 4.1 and find that the optimal quantity for farmers is $\frac{\alpha}{2\beta N}$. In a fully constrained equilibrium farmers will sell $1 - min\{\frac{\alpha}{2\beta N}, 1\}$. Therefore, the best-response quantity for farmers is $min\{\frac{\alpha}{2\beta N}, 1 - min\{\frac{\alpha}{2\beta N}, 1\}\}$.

If $R < C_{max}$, a feasible partially constrained equilibrium exists in the harvest season. In this case, stored quantity differs between farmers. Due to heterogeneity in the quantity constraint, unconstrained, partially constrained, and fully constrained equilibria are possible. From the harvest season, we have equilibrium values of $\tilde{f}_1, \hat{c}_1$ and F_1. Recall that $\tilde{f}_1$ is the weighted average quantity sold by cash constrained farmers, $\hat{c}_1$ is the boundary cash constraint between cash constrained and unconstrained farmers, and F_1 is the quantity sold by unconstrained farmers.

For unconstrained equilibria, as explained above the optimal unconstrained quantity is $\frac{\alpha}{2\beta N}$. Since the most cash-constrained farmer sells the greatest quantity in the harvest season, he consequently has the least quantity in the lean season. Therefore, to ensure that no farmer is violating his quantity constraint we check if $1-q_1(C_{max}) \geq \frac{\alpha}{2\beta N}$.

In a fully constrained equilibrium, all farmers are quantity constrained. We consider the conditions under which this is optimal for farmers. First, if the optimal unconstrained quantity is greater than quantity stored for all farmers, it must be optimal for all farmers to be quantity constrained. The inequality $\frac{\alpha}{2\beta N} \geq 1 - F_1$ checks for this condition. Second, note that cash constrained farmers from the harvest season store less than unconstrained farmers. Take the cash constrained farmers and assume they sell all of their stored quantity in the lean season. We can then compute the best-response quantity for unconstrained farmers. If their best-response quantity is greater than their stored quantity, a fully constrained equilibrium exists. The inequality $\frac{C_{max}(\alpha-\beta(N-1)(1-\frac{\hat{c}_1}{C_{max}}-\tilde{f}_1)}{2\beta(C_{max}+(N-1)\hat{c}_1)} \geq 1 - F_1$ checks for this condition.

We show in the proof of Theorem 2 that a partially constrained equilibrium is guaranteed to exist if the conditions for unconstrained and fully constrained equilibria are not met. To characterize the quantity sold in a partially constrained equilibrium, we separate farmers into unconstrained and quantity constrained groups, and solve problem 4.1 from both perspectives. Since unconstrained farmers are unaffected by their quantity constraint, we can remove it from their revenue maximization problem. Hence all unconstrained farmers sell the same quantity, and we can express their best-response quantity F_2 in terms of $\tilde{f}_2$ and $\hat{c}_2$

$$F_2 = \frac{C_{max}(\alpha - \beta(N-1)\tilde{f}_2)}{2\beta(C_{max} + (N-1)\hat{c}_2)}$$

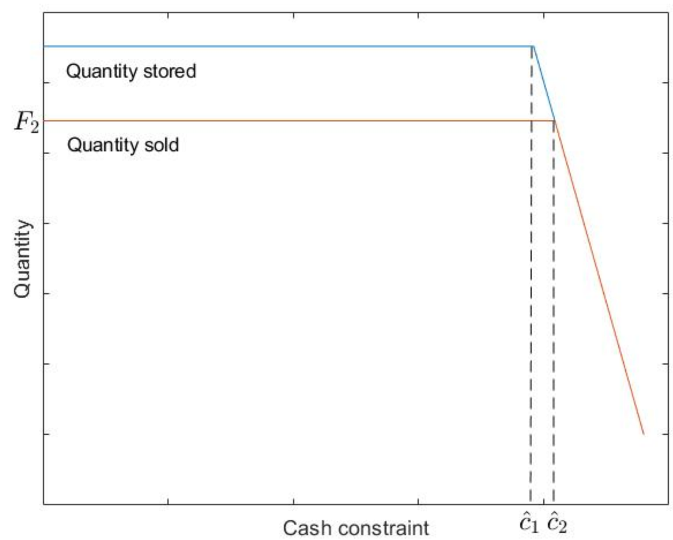

Figure 4-1: Lean season partially constrained equilibrium.

Since all other farmers are quantity constrained, we know that they sell $1 - q_1(C)$. Let $q_2(C)$ be the best-response quantity sold by a farmer with cash constraint C.

$$q_2(C) = \begin{cases} F_2 & C \leq \hat{c}_2 \\ 1 - q_1(C) & \hat{c}_2 < C \leq C_{max} \end{cases}$$

The fact that a fully constrained equilibrium does not exist guarantees that $q_2(C)$ does not violate farmers' quantity constraint. To find the equilibrium values of $\hat{c}_2$ and $\tilde{f}_2$, two conditions must be fulfilled - first, the farmer with cash constraint $\hat{c}_2$ should be quantity constrained but also selling F_2; second, for a given value of $\tilde{f}_2$, the realized weighted average quantity sold by cash constrained farmers must be equal to $\tilde{f}_2$. Such an equilibrium is depicted in Figure 4-1, and expressed mathematically as follows:

$$\begin{aligned} F_2 &= 1 - q_1(\hat{c}_2) \\ \tilde{f}_2 &= \int_{\hat{c}_2}^{C_{max}} \frac{1}{C_{max}}(1 - q_1(C))dC \end{aligned}$$

4.2 Loans

Although introducing storage provides farmers with revenue in the lean season, it does not address the problem of farmers struggling to meet their cash constraints in the harvest season. Therefore, we suppose the government offers farmers a loan in the harvest season, using the quantity stored by farmers as collateral. We introduce two new parameters, L and r. Let L be the maximum loan quantum, and assume that the quantum that farmers are eligible for scales linearly with quantity stored. A farmer that sells $q_1(C)$ in the harvest season is thus eligible for a loan of size $(1 - q_1(C))L$. Let r be the proportion of the loan to be repaid. In our analysis, we assume that r takes on values between 0 and 1. Since we contextualize our analysis to India, this is justified by the extensive history of borrower bailouts in India. In 2009, the Agricultural Debt Waiver and Debt Relief Scheme distributed more than Rs.520 billion in debt waivers. This was followed by state level loan waivers in 2014, 2016, and 2017, amounting to more than Rs.1 trillion [19]. The efficacy of such loans have been subject to debate for many years, with studies finding that loan performance declines in districts with greater exposure to loan waivers, and that farmers may anticipate credit market interventions, resulting in more loan defaults [8] [13]. Therefore, it is certainly possible that many farmers take on loans with the intention of only repaying a fraction of the principal.

Since farmers optimize myopically, the introduction of loans only affects the harvest season equilibrium, where farmers can now use the loan to meet their cash constraints. We reformulate the individual farmer's maximization problem in the harvest season:

$$\begin{aligned} max_{q_i} \; Revenue + Loan \quad &= \quad max_{q_i} \; (\alpha - \beta(q_i + (N-1)\bar{q}_{-i}))q_i + (1-r)(1-q_i)L \\ s.t. \quad & Revenue + Loan \geq C_i \\ & 0 \leq q_i \leq 1 \end{aligned}$$

Note that we differentiate between **income** and **revenue**: income is the sum of revenue, the money earned from selling produce on the market, and the loan received from the government. Also note that the farmer has factored the loan repayment into his optimization problem - even though he receives a loan of size $(1-q_i)L$, he accounts for the fact that he has to repay $r(1-q_i)L$ and therefore only gains $(1-r)(1-q_i)L$ in income. As in Chapter 3, we have unconstrained equilibria where no farmers are cash constrained, and partially constrained equilibria where some farmers are cash constrained.

The characterization of the equilibrium remains very similar to the base case, except farmers now incorporate the loan into their income calculations. Therefore, we retain the definitions of $F_1, \hat{c}_1$ and $\tilde{f}_1$ for partially constrained equilibria, as in Chapter 3.

Lemma 2 *Let $\alpha_1, \tilde{f}_1$ and $\alpha_2, \tilde{f}_2$ be the solutions to the system of equations 4.2 and 4.3 respectively. At least one of the systems of equations has a solution. If only one has a solution, denote it α^*. Else, let $\alpha^* = min\{\alpha_1, \alpha_2\}$.*

$$\begin{aligned}
P_1 &= \alpha - \beta(N-1)(\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1) \geq \beta + C_{max} \\
g(\tilde{f}_1) &= \frac{1}{2\beta}(1 - \frac{\hat{c}_1}{C_{max}})(P_1 - (1-r)L) - \frac{1}{12\beta^2 C_{max}}(((P_1 - (1-r)L)^2 \\
&\quad -4\beta(\hat{c}_1 - (1-r)L))^{1.5} - ((P_1 - (1-r)L)^2 - 4\beta(C_{max} - (1-r)L))^{1.5}) \\
&\quad -\tilde{f}_1 \\
&= 0 \\
g'(\tilde{f}_1) &= 0
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
P_1 &= \beta + C_{max} \\
g(\tilde{f}_1) &= 0
\end{aligned} \tag{4.3}$$

where $F_1 = \frac{C_{max}(\alpha - \beta\tilde{f}_1(N-1)-(1-r)L)}{2\beta(C_{max}+\hat{c}_1(N-1))}$ *and*
$\hat{c}_1 = \frac{-\beta(C_{max}-(N-1)(1-r)L)+\sqrt{\beta^2(C_{max}+(N-1)(1-r)L)^2+\beta C_{max}(N-1)(\alpha-\beta(N-1)\tilde{f}_1-(1-r)L)^2}}{2\beta(N-1)}$.

Theorem 3 *Let $q = min\{\frac{\alpha-(1-r)L}{2\beta N}, 1\}$ and $R = (\alpha - \beta N q)q + (1-r)(1-q)L$.*

1. *If* $\alpha \geq \alpha^*$*, then if*

   (a) *If* $R \geq C_{max}$*,* $q_1^* = min\{\frac{\alpha-(1-r)L}{2\beta N}, 1\}$

   (b) *If* $R < C_{max}$*,* q_1^* *is a piece-wise function of the following form:*

$$q_1(C) = \begin{cases} F_1 & C \leq \hat{c}_1 \\ \frac{1}{2\beta}(P_1 - (1-r)L - \\ \qquad \sqrt{(P_1 - (1-r)L^2 - 4\beta(C-(1-r)L))} & \hat{c}_1 < C \leq C_{max} \end{cases}$$

2. *If* $\alpha < \alpha^*$ *then the problem 4.1 is infeasible and* $\exists c^* > 0$ *s.t.* $\forall C_i > c^*$*,* $Revenue + Loan < C_i$*.*

We begin by explaining the derivation of formulas for F_1 and $\hat{c}_1$. In a partially constrained equilibrium, we have unconstrained and cash constrained farmers. As in Chapter 3, we remove the quantity constraint from the income maximization problem of unconstrained farmers, to obtain their best response quantity F_1.

$$\begin{aligned} max_{F_1} \quad & (\alpha - \beta(F_1 + (N-1)(\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1)))F_1 + (1-r)(1-F_1)L \\ F_1 \quad &= \quad \frac{C_{max}(\alpha - \beta(N-1)\tilde{f}_1 - (1-r)L)}{2\beta(C_{max} + \hat{c}_1(N-1))} \end{aligned} \tag{4.4}$$

Similarly, cash-constrained farmers now use the loan to meet their cash constraint C:

$$\begin{aligned} C &= (\alpha - \beta(q + (N-1)(\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1)))q + (1-r)(1-q)L \\ q &= \frac{1}{2\beta}(P_1 - (1-r)L - \sqrt{(P_1 - (1-r)L)^2 - 4\beta(C-(1-r)L)}) \end{aligned} \tag{4.5}$$

where $P_1 = \alpha - \beta(N-1)(\frac{\hat{c}_1}{C_{max}} + \tilde{f}_1)$.

Finally, the farmer with cash constraint $\hat{c}_1$ sells quantity F_1, and combining his revenue and loan income makes just enough to meet his cash constraint:

$$\hat{c}_1 = (\alpha - \beta(F_1 + (N-1)(\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1)))F_1 + (1-r)(1-F_1)L$$

$$\hat{c}_1 = \frac{-\beta(C_{max} - (N-1)(1-r)L) + \sqrt{\begin{array}{l}\beta^2(C_{max} + (N-1)(1-r)L)^2 + \beta C_{max} \\ (N-1)(\alpha - \beta(N-1)\tilde{f}_1 - (1-r)L)^2\end{array}}}{2\beta(N-1)}$$

Since the intuition behind the fixed point equation $g(\tilde{f}_1)$ and the systems of equations 4.2 and 4.3 is identical to the base case, we do not reproduce the explanations here.

For unconstrained equilibria, we remove the cash constraint from the income maximization problem. Farmers solve:

$$\begin{aligned} max_{q_i}\ Revenue + Loan &= max_{q_i}\ (\alpha - \beta(q_i + (N-1)\bar{q}_{-i}))q_i + (1-r)(1-q_i)L \\ s.t.\ & \ 0 \leq q_i \leq 1 \end{aligned}$$

The optimal quantity is $\frac{\alpha-(1-r)L}{2\beta N}$, and the best-response quantity is $min\{\frac{\alpha-(1-r)L}{2\beta N}, 1\}$ due to farmers' quantity constraint. Intuitively, $\frac{\alpha-(1-r)L}{2\beta N} < \frac{\alpha}{2\beta N}$ because farmers now derive income from unsold quantity and are hence incentivized to sell less.

For partially constrained equilibria, as in Chapter 3 we use the formulas for F_1 and $\hat{c}_1$ to find $q_1(C)$, the best-response quantity sold by a farmer with cash constraint C.

$$q_1(C) = \begin{cases} F_1 & C \leq \hat{c}_1 \\ \frac{1}{2\beta}(P_1 - (1-r)L - \\ \qquad \sqrt{(P_1 - (1-r)L^2 - 4\beta(C - (1-r)L)}) & \hat{c}_1 < C \leq C_{max} \end{cases}$$

The findings on ranking feasible partially constrained equilibria and the sensitivity of $\tilde{f}_1$ to α, β, N and C_{max} remain identical to those expressed in Propositions 2 and 3. We examine how the equilibrium shifts in response to changes in the new parameters L and r:

Proposition 4 $\frac{d\tilde{f}_1}{dL} < 0$ *and* $\frac{d\tilde{f}_1}{dr} > 0$. *Also,* $\frac{dq_1(C)}{dL} < 0$ *and* $\frac{dq_1(C)}{dr} > 0$.

Improving loan terms by increasing L or decreasing r causes the equilibrium value of $\tilde{f}_1$ to decrease, and induces all farmers to increase stored quantity. The proposition reflects the intuitive result that as loan terms are made more generous, fewer farmers are cash constrained and farmers that remain cash constrained sell less quantity. Furthermore, since storing produce now allows farmers to take a larger loan, farmers will opt to store more produce.

Chapter 5

Comparison of Models

We now consider 3 scenarios: (i) No storage or loans; (ii) Storage but no loan; (iii) Storage and loan. We use the following metrics for comparison: First, total net revenue, which we define as harvest season and lean season revenue less cash constraint, across all farmers. Second, wastage, defined as unsold quantity after the lean season. Third, inequality, defined as the proportion of cash constrained farmers. We use this as a metric for inequality because cash constraints are the key driver of sub-optimal decision making in the model, so equality is achieved when no farmers are cash constrained. This is also justified from a monetary perspective, since all farmers earn equal revenue when no farmers are cash constrained.

We conduct a theoretical analysis to examine the sensitivity of total net revenue, wastage, and inequality to the repayment rate r. We use r because from a government policy perspective, r is the means by which the government can control how much money is disbursed to farmers. By setting $r = 1$, the loan model reduces to the base case of no loan, since farmers will opt not to take the loan if they have to repay the full amount. Following the theoretical analysis, we conduct numerical simulations to show how different parameter sets can affect the benefits of storage and loans.

Proposition 5 *Define* $R_1(r) = \int_0^{C_{max}} (\alpha - \beta(q_1(C) + (N-1)(\frac{\hat{c}_1}{C_{max}} F_1 + \tilde{f}_1)))q_1(C) - C dC$ *and* $R_2(r) = \int_0^{C_{max}} (\alpha - \beta(q_2(C) + (N-1)(\frac{\hat{c}_2}{C_{max}} F_2 + \tilde{f}_2)))q_2(C)\, dC$ *as harvest season and lean season net revenue respectively, as functions of repayment rate* r.

The following equations have unique solutions, denoted as r_1, r_2 and r_3 respectively:

$$\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1 = \frac{\alpha}{2\beta N} \tag{5.1}$$

$$\begin{aligned}(\alpha - \beta(1 - \frac{\alpha}{2\beta N} + (N-1)(\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1)))\\ (1 - \frac{\alpha}{2\beta N}) + (1-r)\frac{\alpha}{2\beta N}L = C_{max}\end{aligned} \tag{5.2}$$

$$1 - \frac{\hat{c}_1}{C_{max}}F_1 - \tilde{f}_1 = \frac{\alpha}{2\beta N} \tag{5.3}$$

$R_1(r)$ is unimodal, and $\exists r_1^* \in [0,1]$ s.t. $R_1(r_1^*) \geq R_1(r) \forall r \in [0,1]$, where r_1^* is expressed by the following piece-wise equation:

$$r_1^* = \begin{cases} 0 & r_1 \leq 0 \\ r_1 & 0 < r_1 < 1 \\ 1 & r_1 \geq 1 \end{cases}$$

For $R_2(r)$,

1. *If $r_2 \in [0,1]$ and $r_3 \notin [0,1]$, $R_2(r_2) \geq R_2(r) \forall r \in [0,1]$.*

2. *If $r_2 \notin [0,1]$ and $r_3 \in [0,1]$, $R_2(r_3) \geq R_2(r) \forall r \in [0,1]$.*

3. *If $r_2, r_3 \notin [0,1]$, $max\{R_2(0), R_2(1)\} \geq R_2(r) \forall r \in [0,1]$.*

4. *If $r_2, r_3 \in [0,1]$, $R_2(r_2) = R_2(r_3) \geq R_2(r) \forall r \in [0,1]$.*

Define $W(r) = \int_0^{C_{max}} 1 - q_1(C) - q_2(C)\, dC$ as wastage. The following system of equations has a unique solution, r_4.

$$\begin{aligned} F_2 &= 1 - F_1 \\ \hat{c}_2 &= \hat{c}_1 \\ \tilde{f}_2 &= 1 - \frac{\hat{c}_1}{C_{max}} - \tilde{f}_1 \\ \hat{c}_1 &= \frac{C_{max}}{\beta(N-1)}\{2\alpha - 2\beta - \beta(N-1) - (1-r)L\} \end{aligned} \tag{5.4}$$

$W(r)$ is decreasing in r for $r \in [0, max\{r_4, 0\}]$, and $W(r) = 0$ for $r \in (min\{r_4, 1\}, 1]$.

Define $I(r) = 1 - \frac{\hat{c}_1}{C_{max}}$ and let $q = \frac{\alpha-(1-r)L}{2\beta N}$. The following equation has a unique solution r_5.

$$(\alpha - \beta N q)q + (1 - r)(1 - q)L = C_{max} \tag{5.5}$$

$I(r) = 0$ for $r \in [0, max\{r_5, 0\}]$ and increasing for $r \in (min\{r_5, 1\}, 1]$

From Proposition 5, we see that harvest season revenue is guaranteed to have a unique maximizer, while lean season revenue can have up to 2 maximizers.

The LHS of equation 5.1 is the average quantity sold in the harvest season, while the RHS is the optimal unconstrained quantity $\frac{\alpha}{2\beta N}$. Note that we do not use $\frac{\alpha-(1-r)L}{2\beta N}$ because we want to maximize revenue, not income. When they are equal, harvest season net revenue is maximized. By Proposition 4, we know that a unique solution exists because as r decreases, average quantity sold decreases. However, it is possible that the solution is not within the interval $[0, 1]$, in which case the maximizer of $R_1(r)$ is at the boundary.

Equations 5.2 and 5.3 correspond to the two possible maximizers of $R_2(r)$. Like the harvest season, lean season revenue is maximized when the average quantity sold is equal to the optimal unconstrained quantity. There are two cases when this may occur: First, when all farmers are able to sell $\frac{\alpha}{2\beta N}$. By Proposition 4, since stored quantity is increasing for all farmers as r decreases, there is some unique value where the most cash-constrained farmer stores exactly $\frac{\alpha}{2\beta N}$. This is expressed in equation 5.2, where the LHS is the farmer's harvest season revenue when selling quantity $1 - \frac{\alpha}{2\beta N}$. Therefore, the most cash constrained farmer will sell this quantity to meet his cash constraint. Second, in a fully constrained equilibrium, due to differences in quantity stored some farmers are forced to sell less than $\frac{\alpha}{2\beta N}$ while others can sell more. As a result, it is possible for the overall average quantity to equal $\frac{\alpha}{2\beta N}$. By Proposition 4, there is a unique value r_3 corresponding to this equilibrium, characterized by equation 5.3. The LHS is the average quantity stored, while the RHS is the optimal unconstrained quantity. It is not guaranteed that r_2 or r_3 is within the interval $[0, 1]$, so we consider all possibilities in Proposition 5.

We now move on to the analysis of wastage. From Proposition 4, it is clear that wastage is non-increasing in r. Since increasing r results in decreased stored quantity, wastage cannot increase. In fact, wastage is strictly decreasing in r up to the point where the lean season equilibrium transitions from partially constrained to fully constrained, which is characterized by equation 5.4. Recall that we check for the existence of a fully constrained equilibrium by assuming all cash constrained farmers from the harvest season sell all of their stored quantity, then computing the best-response quantity of the remaining unconstrained farmers. The boundary between partially constrained and fully constrained equilibrium is therefore when the best-response quantity of unconstrained farmers is equal to their stored quantity. If r is increased, all farmers store less and will therefore sell their maximum quantity, resulting in a fully constrained equilibrium. Hence, wastage is 0 for all greater values of r. Conversely, if r is decreased all farmers store more, and there are fewer cash constrained farmers from the harvest season. As a result, the best-response quantity of the unconstrained farmers decreases, and we have a partially constrained equilibrium. Hence, wastage increases.

Finally, with regard to inequality, from Proposition 4 we know that inequality is non-decreasing in r. In fact, inequality is constant at 0 to the point where the harvest season equilibrium transitions from unconstrained to partially constrained, after which inequality is strictly increasing. To find the boundary between unconstrained and partially constrained equilibria, we use the result from Theorem 3 to find the value of r such that the most cash constrained farmer just meets his cash constraint by selling the optimal unconstrained quantity. This condition is expressed in equation 5.5.

For the numerical simulations, Figures 5-1 and 5-2 illustrate how total net revenue, wastage, and inequality change in response to changes in α and C_{max}, under 3 scenarios: (i) no storage or loan, (ii) storage but no loan, (iii) storage and loan. We exclude β and N since the results are similar. Note that scenarios (i) and (ii) are unaffected by r, so the lines are constant. Also note that providing storage does not affect inequality, so we only show scenarios (i) and (iii) in the inequality plot.

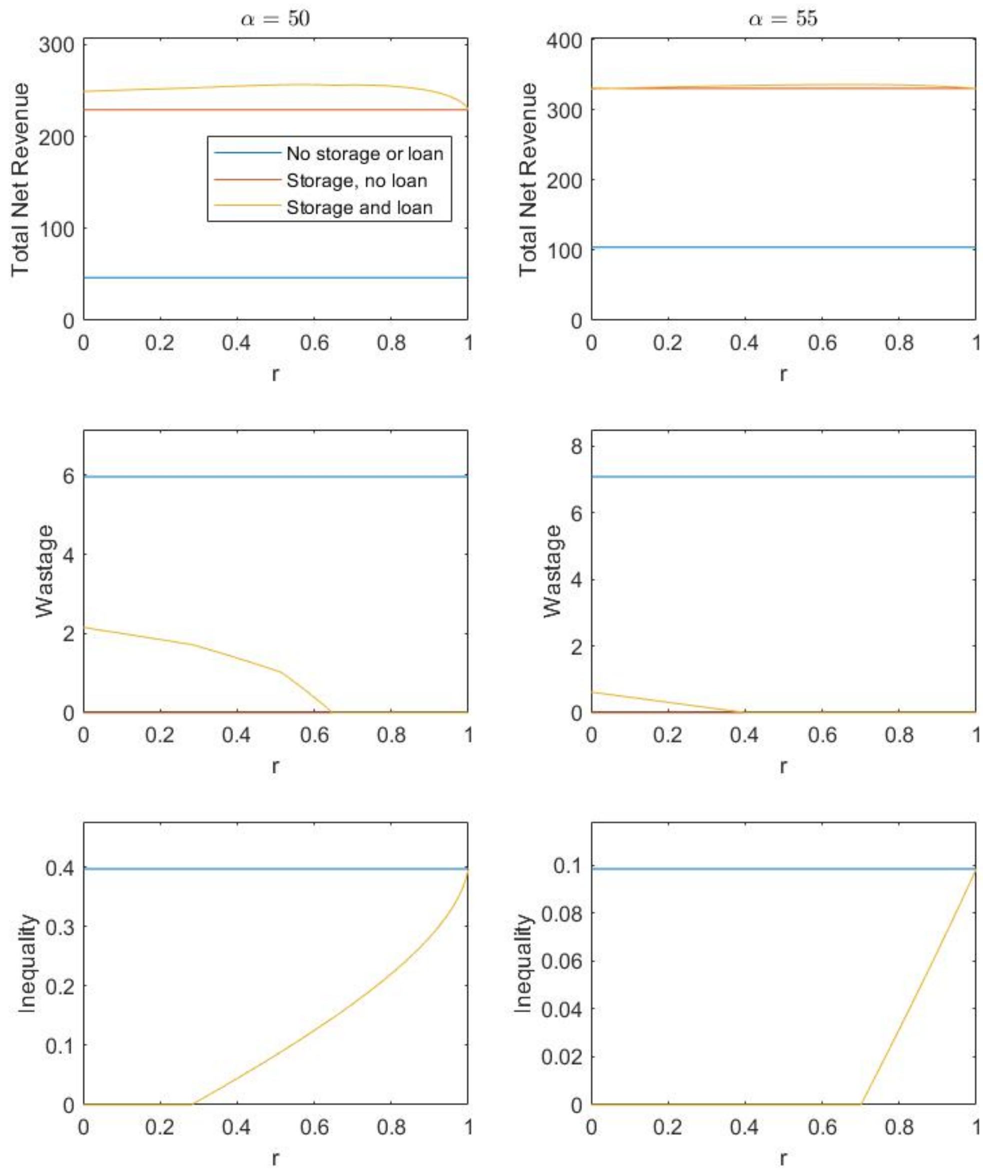

Figure 5-1: Simulation results for the following parameter values: $\beta = 0.1, N = 520, C_{max} = 16, L = 10$.

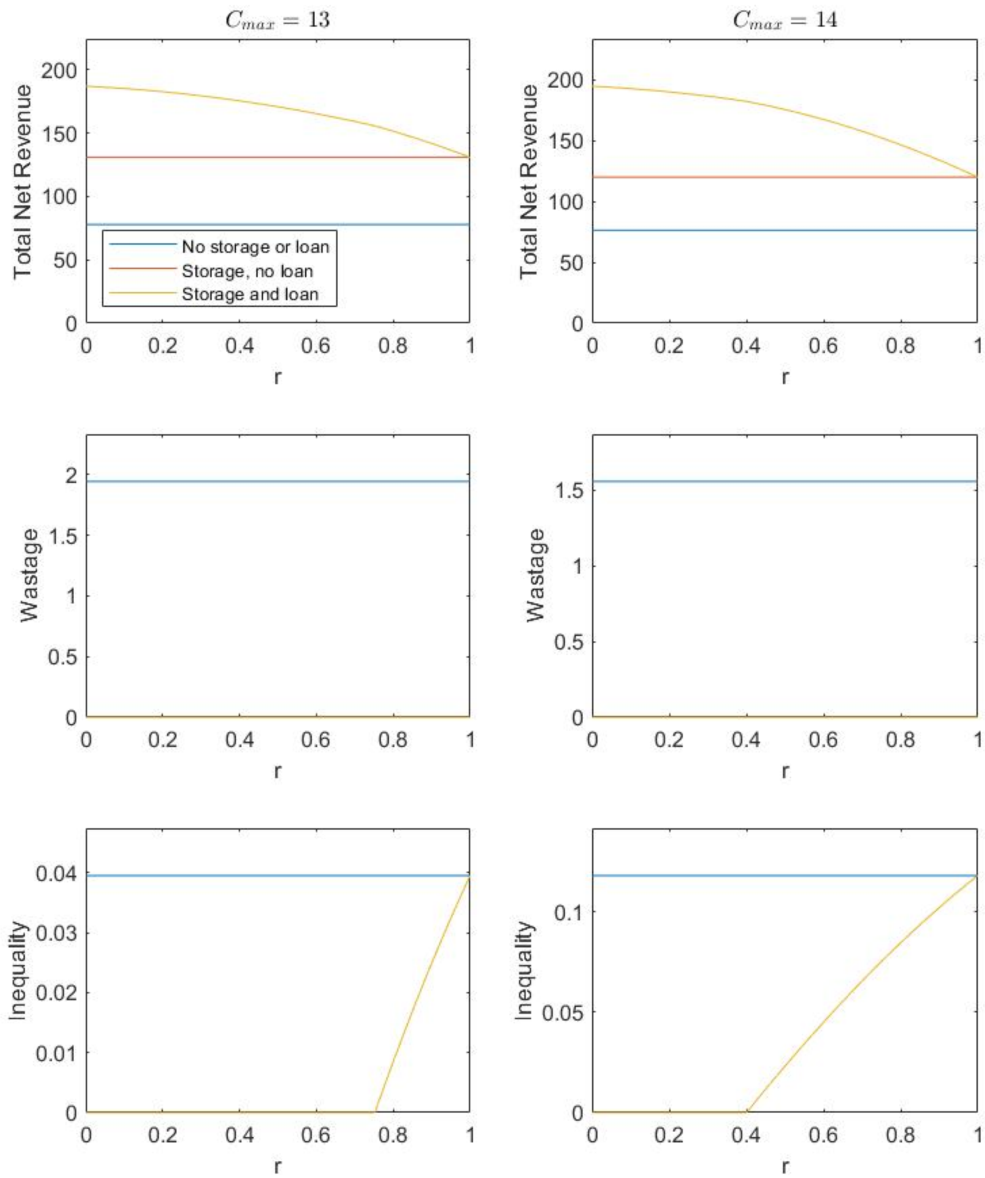

Figure 5-2: Simulation results for the following parameter values: $\alpha = 30, \beta = 0.01, N = 1800, L = 10$.

We begin by analyzing Figure 5-1. For this parameter set, note that optimal unconstrained quantity $\frac{\alpha}{2\beta N} = 0.48$ when $\alpha = 50$. This indicates that if farmers were not cash constrained, it would be optimal for them to waste a significant amount of produce in the harvest season. Therefore, if storage were provided, we expect a large quantity to be stored, and farmers would see a large increase in revenue. This is reflected in the total net revenue plot for $\alpha = 50$, where farmers saw a 393% increase in net revenue from storage. In comparison, loans provide a relatively small improvement in net revenue. The wastage plot highlights one of the drawbacks of loan provision, as it becomes optimal for farmers to waste produce as r decreases. Decreasing r also serves to reduce inequality, establishing the trade-off between reducing wastage and inequality. However, for $\alpha = 55$, there is a range of r values where we have zero wastage and inequality simultaneously. In increasing $\alpha = 50$ to $\alpha = 55$, we improve market price conditions and increase the optimal unconstrained quantity. Storage is therefore utilized less, but still provides a 184% increase in revenue. Loans provide even smaller benefit than before, because taking a loan is now a less attractive option for farmers compared to selling their produce on the market. With better prices, more farmers can meet their cash constraint and there is less reason to take a loan. The reduction in cash constrained farmers is clear in the inequality plot. Finally, wastage is reduced because better prices incentivize farmers to sell more of their produce on the market.

For Figure 5-2, the optimal unconstrained quantity is now 0.83. As a result, it is optimal for farmers to sell almost all of their produce in the harvest season. Therefore, storage has less of an impact on net revenue, which only increases 68%. This parameter set demonstrates how loans can be used to encourage usage of storage and thereby boost revenue. For $C_{max} = 13$ and $r = 0$, the loan improves revenue by a further 43%, relative to the storage and no loan case. Because of the higher optimal unconstrained quantity, it is optimal for farmers to sell all of their produce over the harvest and lean seasons. This is reflected in the wastage plots, where we have zero wastage for scenarios (ii) and (iii). Note that the effect on net revenue and inequality when reducing C_{max} from 14 to 13 is very similar to that of increasing α from 50 to

55. This reflects the point in Chapter 3 that both of these parameter shifts have the effect of making it easier for farmers to meet their cash constraints. Therefore, we intuitively expect these changes to have similar results.

Chapter 6

Government Optimization Problem

We consider the scenario where the government has provided storage and determined L, and now wants to find the optimal r that maximizes government utility. We propose the following objective function:

$$
\begin{aligned}
max_r \qquad & w_1\{\int_0^{C_{max}} (\alpha - \beta(q_1(C) + (N-1)(\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1)))q_1(C) - C\,dC \\
& + \int_0^{C_{max}} (\alpha - \beta(q_2(C) + (N-1)(\frac{\hat{c}_2}{C_{max}}F_2 + \tilde{f}_2)))q_2(C)\,dC\} \\
& -w_2\{\int_0^{C_{max}} 1 - q_1(C) - q_2(C)\,dC\} \\
& -w_3\{1 - \frac{\hat{c}_1}{C_{max}}\}
\end{aligned}
$$

The objective function is the weighted sum of total farmer net revenue, wastage, and inequality, as defined in Chapter 5. To characterize the objective function maximizer r^*, we begin by considering the edge case where $w_2 = w_3 = 0$. Following that, we examine how the inclusion of wastage and inequality affects r^*. Finally, we conclude by summarizing the implications on government policy.

Recall from Proposition 5 that harvest season net revenue has a unique maximizer, while lean season net revenue has up to two maximizers. For the following analysis, we assume that all three maximizers are in the interval $[0, 1]$. Denote the maximizers r_1, r_2 and r_3, as defined in Proposition 5. Let the total net revenue maximizer be r^*_{rev}.

Proposition 6 *1. If* $\frac{\alpha}{2\beta N} \geq 1$*,* $r^*_{rev} \in (0, \frac{2\beta N-(\alpha-L)}{L})$

2. If $\frac{1}{2} < \frac{\alpha}{2\beta N} < 1$*,* $r_2 \leq r_3 < r^*_{rev} < r_1$

3. If $\frac{\alpha}{2\beta N} = \frac{1}{2}$*,* $r_2 \leq r_3 = r_1 = r^*_{rev}$

4. If $0 < \frac{\alpha}{2\beta N} < \frac{1}{2}$*, then for* $r = r_1$*,*

(a) If $1 - q_1(C_{max}) > \frac{\alpha}{2\beta N}$*,* $r_1 < r^*_{rev} < r_2 < r_3$

(b) If $1 - q_1(C_{max}) = \frac{\alpha}{2\beta}$*,* $r^*_{rev} = r_1 = r_2 < r_3$

(c) Else, $r_2 < r^*_{rev} < r_3$

If $\frac{\alpha}{2\beta N} \geq 1$, it is optimal for farmers to sell 1 unit in the harvest season until r decreases to the point that $\frac{\alpha-(1-r)L}{2\beta N} < 1$. Since farmers will earn maximum revenue by selling 1 unit, harvest season net revenue is maximized for all $r \geq \frac{2\beta N-(\alpha-L)}{L}$. For the lean season, it is always optimal for farmers to sell all of their stored quantity. Lean season revenue is thus maximized for $r = 0$, where quantity stored is maximized. r^*_{rev} is thus in the interval $(0, \frac{2\beta N-(\alpha-L)}{L})$.

If $\frac{1}{2} < \frac{\alpha}{2\beta N} < 1$, since farmers only have 1 unit to sell over the harvest and lean season, if harvest season net revenue is maximized, farmers will store insufficient quantity to maximize lean season revenue. Hence we have $r_2 \leq r_3 < r_1$, and since harvest season net revenue is unimodal by Proposition 5, r^*_{rev} is in the interval (r_3, r_1).

If $\frac{\alpha}{2\beta N} = \frac{1}{2}$, farmers will maximize harvest season and lean season net revenue at the same time, hence we have $r_2 \leq r_3 = r_1 = r^*_{rev}$.

If $0 < \frac{\alpha}{2\beta N} < \frac{1}{2}$, when farmers maximize harvest season net revenue, they will store too much to maximize lean season revenue with a fully constrained equilibrium. Hence we have $r_1 < r_3$. However, the relationship between r_1 and r_2 is dependent on the quantity stored by the farmer with cash constraint C_{max}, $1 - q_1(C_{max})$. If, at $r = r_1$, $1 - q_1(C_{max}) > \frac{\alpha}{2\beta N}$, then the farmer is storing more than enough to sell the optimal unconstrained quantity. Hence $r_1 < r_2 < r_3$, and r^*_{rev} is in the interval (r_1, r_2). If $1 - q_1(C_{max}) = \frac{\alpha}{2\beta N}$, we have $r_1 = r_2 < r_3$, and $r^*_{rev} = r_1$. Finally, if $1 - q_1(C_{max}) < \frac{\alpha}{2\beta N}$, then $r_2 < r_1 < r_3$, then r^*_{rev} is in the interval (r_2, r_3).

From Proposition 5, wastage is non-increasing in r. Therefore, as w_2 increases, we expect r^* to increase. However, note that if $r^*_{rev} \geq r_4$, where r_4 is the solution to equation 5.4, then $r^* = r^*_{rev}$ since there is zero wastage at r^*_{rev}. Conversely, inequality is non-decreasing in r, and we expect r^* to decrease as w_3 increases. However, if $r^*_{rev} \leq r_5$, where r_5 is the solution to equation 5.5, then $r^* = r^*_{rev}$ since there is zero inequality at r^*_{rev}. Note that due to the multimodal nature of net revenue, we cannot guarantee the existence of a unique r^*.

As a robustness check, we also consider adding an additional term to the objective function for government expenditure, computed as the proportion of the loan that is not paid back by farmers.

$$\int_0^{C_{max}} (1-r)(1-q_1(C))L\, dC$$

Similar to wastage, we find that government expenditure is decreasing in r, and becomes constant at 0 when it is optimal for farmers to sell everything in the harvest season. We find that the inclusion of government expenditure in the objective function does not affect our prior findings on r^*.

We now examine policy insights that can be drawn from the analysis. Since the provision of storage infrastructure gives farmers a mechanism to transfer quantity across periods, it is clear that all farmers with excess quantity in the harvest season will obtain greater revenue, as well as reduced wastage. Whether the investment in storage facilities will be worthwhile is strongly dependent on the nature of the harvest season market. If demand outpaces supply, storage facilities may be underutilized. However, the government must be cognizant that farmers may be forced to sell larger quantities than they would prefer in the harvest season due to their cash constraints. In this case, offering a loan can be an effective mechanism to increase the benefits from storage. The crux of the optimization problem then lies in the loan quantum and repayment rate, which allows the government to control how much quantity farmers store.

In determining the optimal value of r, there is a clear trade-off between equality

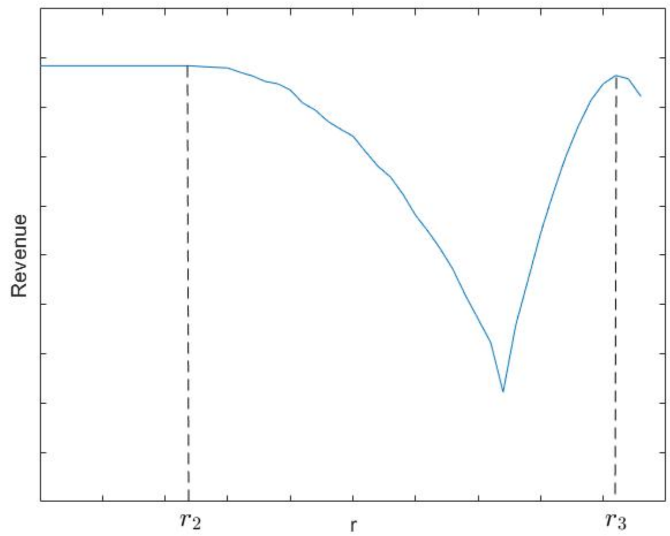

Figure 6-1: Lean season total revenue.

and wastage. On one hand, improving equality requires decreasing r so that cash constrained farmers can afford to store more. However, generous loan terms can create a wastage problem as it disincentivizes unconstrained farmers from selling their produce on the market during the harvest season. They would rather store excess quantity to take a larger loan, even if much of that stored quantity goes unsold in the lean season. On the other hand, by increasing r, it is possible to achieve zero wastage. We plot lean season total revenue as a function of r in Figure 6-1. We have zero wastage at r_3 where we have a fully constrained equilibrium, and we achieve the same total revenue as at r_2, where we have an unconstrained equilibrium and total lean season revenue is maximized. However, total revenue as a metric fails to reflect the distribution of revenue amongst farmers. At r_3, cash-constrained farmers are in fact doubly worse off relative to their peers - not only do they have less net revenue in the harvest season, they also have less quantity and therefore less revenue in the lean season.

Another crucial insight is that decreasing r does not necessarily improve total revenue in the first or lean season. In the harvest season, this occurs when average quantity sold declines below $\frac{\alpha}{2\beta N}$. Since the loan offering can be interpreted as an alternative 'market' where the farmer can sell his produce for a price of $(1-r)L$, the government should be cautious of offering terms that overly disincentivize farmers from selling their produce on the actual market.

In the lean season, total revenue can decline if the average quantity sold exceeds $\frac{\alpha}{2\beta N}$. This is seen in Figure 6-1 where total revenue initially decreases as r is decreased past r_3. This is a result of the uneven distribution of quantity amongst farmers. Farmers with greater quantity know that their peers are quantity constrained and therefore flood the market, driving prices down. As r is decreased further, total revenue gradually reverts to the optimal level as stored quantity becomes increasingly uniform across farmers. Total revenue in the lean season can also remain constant if all farmers are able to sell $\frac{\alpha}{2\beta N}$. On Figure 6-1, this is represented by $r < r_2$. Further reducing the repayment rate will increase the quantity stored but not the quantity sold, resulting in increased wastage.

Therefore, on top of balancing the need to improve farmer revenue and equality amongst farmers with the desire to limit wastage, the government must also be aware that offering improved loan terms can backfire by reducing farmer revenue.

Chapter 7

Relating Model Predictions to Empirical Observations

In this chapter, we calibrate our model parameters with field data to examine the extent to which storage and loan provision can improve farmer outcomes, as measured by total net revenue, wastage, and equality. We consider 4 scenarios: (i) the base case without storage or loans; (ii) intervention using calibrated values of L and r; (iii) intervention using the calibrated value of L and the optimized value of r; (iv) intervention using the calibrated value of L and $r = 0$. The last case effectively means the government provides a subsidy for farmers, and we include it because of the numerous instances of Indian state governments offering loan waivers to farmers [19]. Given the popularity of such schemes, we feel that it is worth analyzing.

We use field data from Bengal gram farmers in Karnataka state. We use Karnataka because agriculture is the dominant industry for the rural population, supporting over 60% of the workforce and occupying over 64% of state land [2]. Furthermore, 79% of farmer households are smallholders occupying less than 2 hectares of land [22]. Karnataka is also India's fourth largest producer of Bengal gram, which is cultivated in 70% of the land in North Karnataka during the dry season [27]. Despite the Bengal gram's popularity, many Bengal gram farmers suffer from poverty. In early 2020, farmers launched a state-wide protest demanding increased government assistance for farmers. A key complaint was that due to a lack of storage facilities for grams,

α	β	N	C_{max}	L	r
85960.91	215.89	248	44580.87	44500	0.62

Table 7.1: Calibrated parameter values.

farmers were forced to accept low prices offered by traders, negatively influencing farmer revenue [1]. Therefore, there is certainly a pressing need to help farmers, and we believe that storage and loan provision are feasible interventions to be considered.

We calibrate the price parameters α and β, as well as the number of farmers N using data from the Unified Market Platform as well as demographic data from the Karnataka state government. We estimate C_{max} using cost of production and income data from the Ministry of Agriculture and Farmers Welfare [23][6]. Finally, we estimate L and r using information from the National Bank for Agriculture and Rural Development [18] and state data [5].

Table 7.1 summarizes the parameter values used in our analysis. α and β are obtained by linear regression, using weekly price and quantity data for markets across Karnataka. N is obtained by averaging the number of farmers that sold produce during the harvest season over the number of markets. C_{max} is obtained from cost of production data from the Commission for Agricultural Costs and prices, less the average farming households' monthly income from non-agricultural sources. We estimate L to be slightly lower than C_{max}, using the state government's guidelines for determining the loan quantum that farmers are eligible for, and r is obtained from data on loan repayment rates.

We find that without storage and loans, some farmers are unable to meet their cash constraint, and consequently there is no feasible solution. We find that the maximum value of C_{max} for a feasible equilibrium to exist is 39450. Consequently, assuming farmers have uniformly distributed cash constraints, at least 11.5% of farmers are unable to meet their cash constraints without government assistance. This is consistent with reports of farmers struggling to recoup their cost of production.

Using the calibrated value $r = 0.62$, all farmers can meet their cash constraints. The equilibrium value of $\hat{c}_1$ is 38636, indicating that 13.3% of farmers are cash constrained. Unconstrained farmers sell 68% of their produce in the harvest season, while the most cash constrained farmer sells 87%. We find that the optimal value of r using the government objective function is 0.271, which is also the total net revenue maximizer. There is zero wastage for all values of r since the optimal unconstrained quantity $\frac{\alpha}{2\beta N} = 0.8$. We therefore expect that farmers will sell all of their quantity over the two seasons. Furthermore, we find that when total net revenue is maximized, no farmers are cash constrained and inequality is therefore minimized. As a result, changing the weight values do not affect r^*. At this equilibrium, all farmers sell exactly 50% of their produce in the harvest and lean season. Finally, for $r = 0$, while there are also no cash constrained farmers, farmers now choose to sell just 39% of their produce in the harvest season and 61% in the lean season.

The comparison of equilibria from $r = 0.62$, $r = 0.271$, and $r = 0$ is shown in in Table 7.2, and the equilibrium quantity sold is depicted in Figure 7-1. Note that although government expenditure is not included in the government objective function, we include it in the results to illustrate the efficacy of government spending.

Comparing $r = 0.62$ to $r = 0.271$, we observe that a 216% increase in government expenditure results in a 12.6% increase in total revenue. In absolute terms, this represents a gain of 37 cents for every additional rupee in government expenditure. The gain in revenue is a product of increased utilization of storage - in the calibrated case, stored quantity is low, with the most cash constrained farmer only able to store 13% of his produce. By optimizing loan terms, we increase stored quantity to 50% for all farmers, while also seeing a significant reduction in inequality, from 13.3% of farmers being cash constrained to zero cash constrained farmers. It should be noted that if farmers were not cash constrained and had the ability to plan long-term, this would be their optimal quantity. However, due to myopic optimization, the government must step in to help them reach this equilibrium.

		Total Net Revenue	Wastage	Inequality	Govt. Exp.
Optimized	r = 0.271	1,645,039.1	0	0	723,387.2
Calibrated	r = 0.62	1,460,553.3	0	0.133	228,718.5
Subsidy	r = 0	1,584,288.1	0	0	1,215,721.5

Table 7.2: Simulation Results.

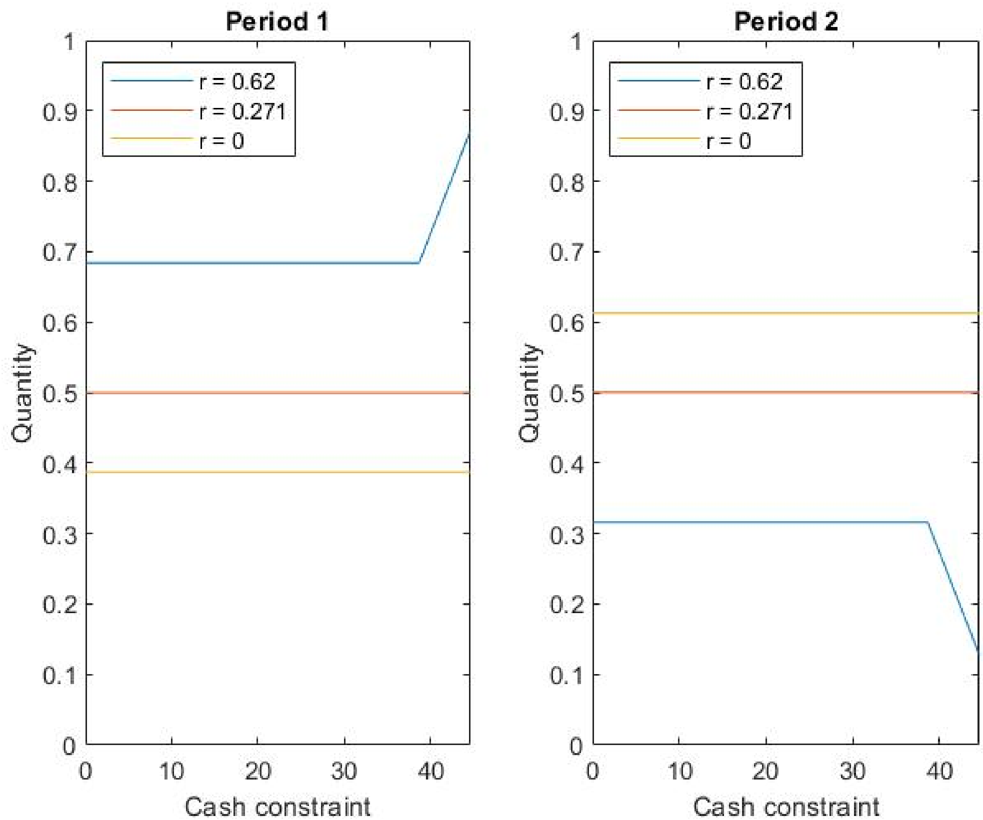

Figure 7-1: Quantity sold in the harvest and lean seasons.

However, the results also make clear that helping farmers is more complex than simply giving them money. When $r = 0$, total revenue decreases while government expenditure increases 68% from the optimized case. Although there is zero inequality, as in the optimized case, farmers have grown increasingly reliant on the government for their income. This occurs because the government intervention disincentivizes farmers from selling their produce on the market. Instead, they would rather store their produce to qualify for a larger loan, as seen in Figure 7-1. Even though farmers earn greater income, the gain from the loan now negatively affects revenue, rendering government intervention less effective.

This underlines the fact that governments need to consider the strategic nature of farmers when implementing loan schemes. While offering generous loan terms is undoubtedly the popular decision, it can result in excessively high government expenditure that does not translate into gains for farmers.

Chapter 8

Conclusion

This paper develops a model to examine how markets catering to smallholder farmers in developing countries reach an equilibrium. To capture the real world challenges that smallholder farmers face, we incorporate the fact that many smallholders lack the information to make long term plans, as well as the fact that farmers often need immediate cash to survive. The analysis suggests that cash constraints have a significant effect on sales decisions, inducing farmers to sell more than they would prefer to raise funds. The model then allows us to analyze the effectiveness of storage and loan schemes, both of which are popular forms of government intervention.

We demonstrate that government interventions can have a varied effect on farmer outcomes. Storage always benefits farmers by providing farmers with access to a previously untapped market in the lean season. In contrast, loans can backfire by reducing farmer revenue, as they can disincentivize farmers from selling produce on the market. We also show that due to heterogeneous cash constraints, there exists a trade-off between improving equality and reducing wastage. Helping cash constrained farmers by offering more generous loans can result in their unconstrained peers preferring to waste their produce to qualify for larger loans, instead of selling their produce on the market. These findings were backed up by an empirical analysis using data from the Bengal gram market in Karnataka, India. We found that farmers are unable to meet their cash constraints without government intervention, and would benefit strongly in terms of revenue and equality from storage and more generous loan terms.

However, we also find that offering farmers an outright cash subsidy has a negative effect on farmer revenue. These results highlight that while poverty amongst smallholder farmers is of acute concern, government interventions must be constructed while accounting for the strategic behavior of farmers.

Our results have important practical implications and opens up new areas for future research. For example, in our analysis we assumed that government interventions were available to all farmers, when in fact it might be preferable for the government to prioritize aid to poorer and more severely cash constrained farmers. Therefore, it would be valuable to consider extensions such as a limit on quantity stored per farmer or heterogeneous access to loans, to determine if these adaptations can effectively mitigate the negative externalities of government intervention.

Appendix A

Summary of Notation Used

Notation	Interpretation
α	Market price intercept
β	Market price elasticity
N	Number of farmers
C_{max}	Maximum cash constraint
L	Maximum loan quantum
r	Loan repayment rate
F_1	Average quantity sold by non-cash constrained farmers
$\hat{c}_1$	Boundary cash constraint between cash constrained and non-cash constrained farmers
$\tilde{f}_1$	Average quantity sold by cash constrained farmers, weighted by the proportion of cash constrained farmers
P_1	Market price observed by the farmer before making a quantity decision
F_2	Average quantity sold by non-quantity constrained farmers
$\hat{c}_2$	Boundary cash constraint between quantity constrained and non-quantity constrained farmers
$\tilde{f}_2$	Average quantity sold by quantity constrained farmers, weighted by the proportion of quantity constrained farmers
$q_1(C), q_2(C)$	Quantity sold by a farmer with cash constraint C in the harvest and lean seasons respectively
$g(\tilde{f})$	Fixed point equation comparing a given value of $\tilde{f}$ to the realized value $\tilde{f}_{real}$

Table A.1: Summary of Notation Used.

Appendix B

Proofs

Lemma 1: Want to show that at least one of the systems of equations 3.2 and 3.3 has a solution. First, claim that for any value of α, $\exists \tilde{f}$ such that $P = \beta + C_{max}$. For a fixed value of α, $\lim_{\tilde{f} \to -\infty} P = \infty$ and $P = 0$ when $\tilde{f} = \frac{\alpha}{\beta(N-1)}$. Thus by IVT, there must exist $\tilde{f}$ such that $P = \beta + C_{max}$.

$$\frac{dP}{d\tilde{f}} = -\beta(N-1)(1 - \frac{A + (N-1)(\alpha - \beta(N-1)\tilde{f})^2 - \beta C_{max}}{2(A + (N-1)(\alpha - \beta(N-1)\tilde{f})^2 + \beta C_{max})}) < 0$$

Where $A = \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta(N-1)\tilde{f})^2}$. Since $\frac{dP}{d\tilde{f}} < 0$, the solution is unique. Lastly, claim that $\frac{dP}{d\alpha} > 0$.

$$\frac{dP}{d\alpha} = 1 - \frac{\beta C_{max}(N-1)(\alpha - \beta(N-1)\tilde{f})^2(2\beta C_{max} + A)}{2(\beta C_{max} + A)^2 A}$$

where $A = \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta(N-1)\tilde{f})^2}$. Showing $\frac{dP}{d\alpha} > 0$ simplifies to showing

$$\begin{aligned} \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta(N-1)\tilde{f})^2} &> 2\beta C_{max} \\ (N-1)(\alpha - \beta(N-1)\tilde{f})^2 &> 3\beta C_{max} \end{aligned}$$

Let $P(\alpha, \tilde{f})$ denote P for given values of α and $\tilde{f}$. Note that $\alpha - \beta(N-1)\tilde{f} > P(\alpha_{min}, \tilde{f}_{max}) = \beta + C_{max}$ for $\tilde{f} \leq \tilde{f}_{max}$ and $\alpha \geq \alpha_{min}$. Then

$$\begin{aligned}(N-1)(\alpha - \beta(N-1)\tilde{f})^2 &\geq (N-1)(\beta + C_{max})^2 \\ &= (N-1)(\beta^2 + C_{max}^2 + 2\beta C_{max})\end{aligned}$$

Thus $\frac{dP}{d\alpha} > 0$ if $N \geq 3$. Therefore, the corresponding value of $\tilde{f}$ increases as α increases. Let $\alpha^*, \tilde{f}^*$ be one such solution.

Suppose there exists one or more values of $\tilde{f} < \tilde{f}^*$ such that $g(\alpha^*, \tilde{f}) = 0$. If the minimum point of g is at $\tilde{f}^*$, we have $g(\alpha^*, \tilde{f}^*) < 0$. Denote one of the equilibrium values $\tilde{f}_*$ such that $g(\alpha^*, \tilde{f}_*) = 0$. Now find $\alpha_* < \alpha^*$ such that $P(\alpha_*, \tilde{f}_*) = \beta + C_{max}$. As shown above, $g(\alpha_*, \tilde{f}_*) > 0$. Then by IVT, there must exist a solution to Eqn 3.2.

Now suppose the minimum point of g is at an interior point. Clearly α can be decreased until $g > 0$, otherwise an equilibrium would always exist. Thus there must exist $\alpha_* < \alpha^*$ such that $g(\alpha_*, \tilde{f}_*) = g'(\alpha_*, \tilde{f}_*) = 0$. If the corresponding $\tilde{f}$ such that $P(\alpha_*, \tilde{f}) = \beta + C_{max}$ is greater than $\tilde{f}_*$, then we take $\alpha_*, \tilde{f}_*$ as the solution to Eqn 3.3. If it is smaller than $\tilde{f}_*$, then there must exist a solution to Eqn 3.2.

Suppose there does not exist $\tilde{f} \leq \tilde{f}^*$ such that $g(\alpha^*, \tilde{f}) = 0$. Then since $g(\alpha^*, 0) > 0$, $g(\alpha^*, \tilde{f}^*) > 0$. Now increase α^*. As α^* increases, $\tilde{f}^*$ increases to maintain $P(\tilde{f}^*) = \beta + C_{max}$. Either we increase α^* until an equilibrium exists for $\tilde{f} < \tilde{f}^*$, in which case we use the logic above, or no equilibrium exists and α^* increases until an unconstrained equilibrium exists, in which case we use that value of α as the minimum.

Theorem 1: Begin by showing that a feasible equilibrium exists for $\alpha \geq \alpha^*$, and no feasible equilibrium exists for $\alpha < \alpha^*$.

Let $\alpha \geq \alpha^*$.

$$\frac{d\hat{c}}{d\alpha} = \frac{C_{max}(\alpha - \beta(N-1)\tilde{f})}{2\sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta(N-1)\tilde{f})^2}} > 0$$

Thus $\hat{c}(\alpha, \tilde{f}_{max}) > \hat{c}(\alpha_{min}, \tilde{f}_{max})$. Now consider the cash-constrained quantity equation:

$$\begin{aligned} q(C) &= \frac{1}{2\beta}(P - \sqrt{P^2 - 4\beta C}) \\ \frac{dq}{dP} &= \frac{1}{2\beta}(1 - \frac{P}{\sqrt{P^2 - 4\beta C}}) < 0 \end{aligned}$$

Note that $g(\tilde{f}) = \int_{\hat{c}}^{C_{max}} q(C)dC - \tilde{f}$. Want to show that as α increases, $q(C)$ decreases. Since q is decreasing in P and $\frac{dP}{d\alpha} > 0$, then $q(\alpha^*, \tilde{f}_{max}) > q(\alpha, \tilde{f}_{max}) \forall C \in [\hat{c}(\alpha, \tilde{f}_{max}), C_{max}]$. Then $g(\alpha, \tilde{f}_{max}) < 0$, and since $g(0) > 0$, there exists an equilibrium $\tilde{f} \in (0, \tilde{f}_{max})$.

Let $\alpha < \alpha^*$. Note that we have that $g(\tilde{f}) > 0$ for $\tilde{f} \in (0, \tilde{f}_{max})$. From the argument above, we know that decreasing α decreases P, therefore increasing $q(C)$. Since this is true for all values of $\tilde{f}$, we have that $g(\alpha, \tilde{f}) > g(\alpha^*, \tilde{f})$ for $\tilde{f} \in (0, \tilde{f}_{max})$. Thus, no equilibrium can exist for α.

The proof of the characterization of q^* is presented in Chapter 3.

Proposition 1: Since quantity sold is non-negative, $\int_{\hat{c}}^{C_{max}} q(C)\frac{1}{C_{max}}dC \leq 0$ when $\hat{c} \geq C_{max}$. Suppose for contradiction this is true.

$$\begin{aligned} \frac{-\beta C_{max} + \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)\alpha^2}}{2\beta(N-1)} &\geq C_{max} \\ \beta^2 C_{max}^2 + \beta C_{max}(N-1)\alpha^2 &\geq \beta^2 C_{max}^2(4N^2 - 4N + 1) \\ \frac{\alpha^2}{4\beta N} &\geq C_{max} \end{aligned}$$

Note that $\frac{\alpha^2}{4\beta N}$ is the revenue earned per farmer if everyone sells $\frac{\alpha}{2\beta N}$. Therefore, if an unconstrained equilibrium does not exist then $\hat{c} < C_{max}$ and $\int_{\hat{c}}^{C_{max}} q(C)\frac{1}{C_{max}}dC > 0$.

Since $\frac{dq}{d\tilde{f}} = \frac{dq}{dP}\frac{dP}{d\tilde{f}} > 0$, $\int_{\hat{c}}^{C_{max}} q(C)\frac{1}{C_{max}}dC$ is strictly increasing in $\tilde{f}$.

For proof of the third statement, see Proposition 3.

Proposition 2: Suppose that we have two equilibrium values of $\tilde{f}$, $\tilde{f}_1 < \tilde{f}_2$. Claim that for both equilibria to be valid, $\hat{c}_1 \neq \hat{c}_2$. Suppose for contradiction that $\hat{c}_1 = \hat{c}_2$.

$$\frac{d\hat{c}}{d\tilde{f}} = -\frac{2\beta C_{max}(N-1)(\alpha-\beta(N-1)\tilde{f})}{4\sqrt{\beta^2 C_{max}^2+\beta C_{max}(N-1)(\alpha-\beta\tilde{f}(N-1))^2}}$$

Then it must be that $\alpha-\beta(N-1)\tilde{f}_2 < 0$. However, then $F(\tilde{f}_2) < 0$, so the equilibrium is invalid. Therefore we must have $\hat{c}_1 > \hat{c}_2$. Comparing the revenue of farmers with cash constraint C:

1. $C \geq \hat{c}_1$

   Farmers in equilibrium 1 and 2 will both be making their cash constraint, so revenue is equal.

2. $C \leq \hat{c}_2$

   Farmers in equilibrium 1 will be making $\hat{c}_1$ while farmers in equilibrium 2 will be making $\hat{c}_2$, so equilibrium 1 is better.

3. $\hat{c}_2 < C < \hat{c}_1$

   Farmers in equilibrium 1 will be making $\hat{c}_1$ while farmers in equilibrium 2 will be making C, so equilibrium 1 is better.

Proposition 3: We prove the claim for each parameter separately:

1. α**:**

   Claim that since $\frac{d\hat{c}}{d\alpha} > 0$, and $\frac{dq}{d\alpha} = \frac{dq}{dP}\frac{dP}{d\alpha} < 0$, $g(\tilde{f})$ shifts downwards for all values of $\tilde{f}$ as α increases. Hence $\tilde{f}^*_{new} < \tilde{f}^*$.

$$\begin{aligned}
\frac{d\hat{c}}{d\alpha} &= \frac{C_{max}(\alpha-\beta(N-1)\tilde{f})}{2\sqrt{\beta^2 C_{max}^2+\beta C_{max}(N-1)(\alpha-\beta(N-1)\tilde{f})^2}} > 0 \\
\frac{dq}{dP} &= \frac{1}{2\beta}(1-\frac{P}{\sqrt{P^2-4\beta C}}) < 0 \\
\frac{dP}{d\alpha} &= 1-\frac{\beta C_{max}(N-1)(\alpha-\beta(N-1)\tilde{f})^2(2\beta C_{max}+A)}{2(\beta C_{max}+A)^2 A}
\end{aligned}$$

Showing $\frac{dP}{d\alpha} > 0$ simplifies to showing

$$\begin{aligned}\sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha-\beta(N-1)\tilde{f})^2} &> 2\beta C_{max} \\ (N-1)(\alpha-\beta(N-1)\tilde{f})^2 &> 3\beta C_{max}\end{aligned}$$

Note that $\alpha - \beta(N-1)\tilde{f} \geq \beta + C_{max}$. Then

$$\begin{aligned}(N-1)(\alpha-\beta(N-1)\tilde{f})^2 &\geq (N-1)(\beta + C_{max})^2 \\ &= (N-1)(\beta^2 + C_{max}^2 + 2\beta C_{max})\end{aligned}$$

Thus $\frac{dP}{d\alpha} > 0$ if $N \geq 3$. Finally,

$$\begin{aligned}\frac{d\hat{c}}{d\tilde{f}} &= -\frac{2\beta C_{max}(N-1)(\alpha-\beta(N-1)\tilde{f})}{4\sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha-\beta\tilde{f}(N-1))^2}} < 0 \\ \frac{dF}{d\tilde{f}} &= -\frac{\beta^2 C_{max}^2(N-1)}{(\beta C_{max}+A)A} < 0 \\ \frac{dP}{d\tilde{f}} &= -\beta(N-1)(1 - \frac{A + (N-1)(\alpha-\beta(N-1)\tilde{f})^2 - \beta C_{max}}{2(A + (N-1)(\alpha-\beta(N-1)\tilde{f})^2 + \beta C_{max})}) < 0\end{aligned}$$

2. β:

   Claim that since $\frac{d\hat{c}}{d\beta} < 0$, and $\frac{dq}{d\beta} = \frac{\partial q}{\partial \beta} + \frac{\partial q}{\partial P}\frac{\partial P}{\partial \beta} > 0$, $g(\tilde{f})$ shifts downwards for all values of $\tilde{f}$ as β decreases. Hence $\tilde{f}^*_{new} < \tilde{f}^*$.

$$\begin{aligned}\frac{d\hat{c}}{d\beta} &= -\frac{C_{max}(\alpha^2 - (\beta(N-1)\tilde{f})^2)}{4\beta\sqrt{\beta C_{max}(\beta(C_{max} + \beta(N-1)^3\tilde{f}^2) + \alpha^2(N-1) - 2\alpha\beta(N-1)^2\tilde{f})}} \\ &< 0 \\ \frac{\partial q}{\partial \beta} &= \frac{1}{\beta}\{\frac{1}{2\beta}(\sqrt{P^2 - 4\beta C} - P) + \frac{C}{\sqrt{P^2-4\beta C}}\} \\ \frac{\partial P}{\partial \beta} &= -B\{(\alpha-\beta(N-1)\tilde{f})^2[\tilde{f}(N-1)A + C_{max}(3\beta(N-1)\tilde{f} - \alpha)] \\ &\quad +4\beta C_{max}\tilde{f}(\beta C_{max} + A)\}\end{aligned}$$

   where $B = \frac{\beta(N-1)C_{max}}{2(\beta C_{max}+A)^2 A}$.

We bound $\frac{\partial P}{\partial \beta}$ from above by $\sqrt{\frac{C_{max}}{4\beta(N-1)}}$. Denote this quantity D. Want to show:

$$\begin{aligned}\frac{dq}{d\beta} &\geq \frac{1}{\beta}\{\frac{1}{2\beta}(\sqrt{P^2-4\beta C}-P)+\frac{C}{\sqrt{P^2-4\beta C}}\} \\ &+\frac{1}{2\beta}(1-\frac{P}{\sqrt{P^2-4\beta C}})D \\ &> 0\end{aligned}$$

$$\frac{2\beta C-\beta PT}{\sqrt{P^2-4\beta C}} > P-\sqrt{P^2-4\beta C}-\beta T$$

$$2\beta C-\beta PT > (P-\beta T)\sqrt{P^2-4\beta C}-(P^2-4\beta C)$$

$$P^2-\beta PT-2\beta C > (P-\beta T)\sqrt{P^2-4\beta C}$$

$$4\beta^2C^2-4\beta^2TCP+4\beta^3T^2C > 0 \tag{B.1}$$

$$C-PT+\beta T^2 > 0 \tag{B.2}$$

Note that in B.1 we square both sides because the LHS is guaranteed to be positive if N, the number of farmers, is greater than 2. We can lower bound the LHS of B.2 by setting $T=\sqrt{\frac{C_{max}}{4\beta(N-1)}}$, and letting $C=\hat{c}$. We obtain the following:

$$\begin{aligned}&P\sqrt{\frac{C_{max}}{4\beta(N-1)}} < \hat{c}+\frac{C_{max}}{4(N-1)} \\ &(\alpha-\beta(N-1)(\frac{\hat{c}}{C_{max}}F+\tilde{f}))\sqrt{\frac{C_{max}}{4\beta(N-1)}} < \\ &\qquad -\frac{C_{max}}{4(N-1)}+(\alpha-\beta(N-1)\tilde{f})\sqrt{\frac{C_{max}}{4\beta(N-1)}} \qquad \text{(B.3)} \\ &(N-1)\frac{\hat{c}}{C_{max}}F\sqrt{\frac{C_{max}}{4\beta(N-1)}} > \frac{C_{max}}{4\beta(N-1)} \\ &\frac{(\alpha-\beta(N-1)\tilde{f})(-\beta C_{max}+A)}{2\beta(\beta C_{max}+A)} > \sqrt{\frac{C_{max}}{4\beta(N-1)}}\end{aligned}$$

$$
\begin{aligned}
&(\alpha - \beta(N-1)\tilde{f})(-\beta C_{max} + A) > \sqrt{\frac{\beta C_{max}}{(N-1)}}(\beta C_{max} + A) \\
&((\alpha - \beta(N-1)\tilde{f}) - 1)A > ((\alpha - \beta(N-1)\tilde{f}) + 1)\beta C_{max} \qquad \text{(B.4)} \\
&((\alpha - \beta(N-1)\tilde{f}) - 1)^2 \beta C_{max}(N-1)(\alpha - \beta(N-1)\tilde{f}) > \\
&\qquad 4(\alpha - \beta(N-1)\tilde{f})\beta^2 C_{max}^2 \\
&((\alpha - \beta(N-1)\tilde{f}) - 1)^2(N-1) > 4\beta C_{max} \qquad \text{(B.5)}
\end{aligned}
$$

Note that in B.3, we take a lower bound of $\hat{c}$. In B.4, we use the fact that N, the number of farmers, is greater than βC_{max}, the maximum cash constraint multiplied by the price sensitivity. We also use the fact that $\alpha - \beta(N-1)\tilde{f} > P$. B.5 is true given that N is large and that $C_{max} \geq 2$ (hence $(\alpha - \beta(N-1)\tilde{f}) - 1 \geq 1$). Thus we have $\frac{dq}{d\beta} > 0$.

3. N:

   Claim that since $\frac{d\hat{c}}{dN} < 0$, and $\frac{dq(\tilde{f},C)}{dN} = \frac{dq(\tilde{f},C)}{dP}\frac{dP}{dN} > 0$, $g(\tilde{f})$ shifts downwards for all values of $\tilde{f}$ as N decreases. Hence $\tilde{f}^*_{new} < \tilde{f}^*$.

$$
\begin{aligned}
\frac{d\hat{c}}{dN} &= \frac{C_{max}}{4(N-1)^2\sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta\tilde{f}(N-1))^2}} \\
&\quad \{2(\sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha - \beta\tilde{f}(N-1))^2} - \beta C_{max}) \\
&\quad -(N-1)(\alpha^2 - (\beta(N-1)\tilde{f})^2)\}
\end{aligned}
$$

$$
\begin{aligned}
&2(A - \beta C_{max}) - (N-1)(\alpha^2 - (\beta(N-1)\tilde{f})^2) < 0 \\
&2A < (N-1)(\alpha - \beta(N-1)\tilde{f})(\alpha + \beta(N-1)\tilde{f}) + 2\beta C_{max} \\
&(\alpha - \beta(N-1)\tilde{f})^2 < (\alpha - \beta(N-1)\tilde{f})(\alpha + \beta(N-1)\tilde{f})
\end{aligned}
$$

Since $(\alpha + \beta(N-1)\tilde{f}) > (\alpha - \beta(N-1)\tilde{f}) > 0$, $\frac{d\hat{c}}{dN} < 0$.

Now want to show $\frac{dP}{dN} < 0$.

$$
\frac{dP}{dN} = C\{-\alpha^3 C_{max} - \alpha^2(N-1)\tilde{f}(-\beta C_{max} + A)
$$

$$
\begin{aligned}
& -\beta\tilde{f}(4C_{max}+\beta(N-1)^3\tilde{f}^2)(\beta C_{max}+A) \\
& +\alpha\beta(N-1)^2\tilde{f}^2(\beta C_{max}+2A)\} \\
= \; & C\{-\alpha^2 C_{max}(\alpha-\beta(N-1)\tilde{f}) \\
& -\alpha(N-1)\tilde{f}A(\alpha-\beta(N-1)\tilde{f}) \\
& -(\beta C_{max}+A)(4\beta C_{max}\tilde{f}-\beta(N-1)^2\tilde{f}^2(\alpha-\beta(N-1)\tilde{f}))\} \\
= \; & C\{(\alpha-\beta(N-1)\tilde{f})[-C_{max}(\alpha^2-(\beta(N-1)\tilde{f})^2) \\
& -(N-1)\tilde{f}A(\alpha-\beta(N-1)\tilde{f})] \\
& -4\beta C_{max}\tilde{f}(\beta C_{max}+A)\} \\
< \; & 0
\end{aligned}
$$

where $C = \frac{\beta^2 C_{max}}{2(\beta C_{max}+A)^2 A}$.

Then since $\frac{dq}{dP} < 0$, we have $\frac{dq}{dN} = \frac{dq}{dP}\frac{dP}{dN} > 0$ and we are done.

4. C_{max}: Claim that since $\frac{dg}{dC_{max}} > 0$, $g(\tilde{f})$ shifts downwards for all values of $\tilde{f}$ as C_{max} decreases. Hence $\tilde{f}^*_{new} < \tilde{f}^*$.

Lemma 3 $\frac{d\hat{c}}{dC_{max}} - \frac{N-1}{2C_{max}}P(F(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}}) - \hat{c}\frac{dF}{dC_{max}}) > 0$

$$\frac{\beta C_{max}(-\beta C_{max}+A)(\alpha-\beta(N-1)\tilde{f})^4(N-1)}{8A(\beta C_{max}+A)^3} > 0$$

Lemma 4 $\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}} > 0$

$$\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}} = \frac{(\alpha-\beta(N-1)\tilde{f})^2}{4D} > 0$$

Where $D = \sqrt{\beta^2 C_{max}^2 + \beta C_{max}(N-1)(\alpha-\beta\tilde{f}(N-1))^2} > 0$

Lemma 5 $((\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}})F - \hat{c}\frac{dF}{dC_{max}}) > 0$

$$((\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}})F - \hat{c}\frac{dF}{dC_{max}}) = \frac{\beta C_{max}^2(\alpha-\beta(N-1)\tilde{f})^3}{2(\beta C_{max}+D)^2 D} > 0$$

$$
\begin{aligned}
\frac{dg}{dC_{max}} &= \frac{\partial g}{\partial C_{max}} + \frac{\partial g}{\partial F}\frac{\partial F}{\partial C_{max}} + \frac{\partial g}{\partial \hat{c}}\frac{\partial \hat{c}}{\partial C_{max}} \\
&= \frac{1}{2\beta C_{max}}(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}})(P + \beta(N-1)(1 - \frac{\hat{c}}{C_{max}})F) \\
&\quad + \frac{1}{12\beta^2 C_{max}^2}\{(P^2 - 4\beta\hat{c})^{1.5} - P^2 - 4\beta C_{max})^{1.5}\} \\
&\quad - \frac{N-1}{4\beta C_{max}^2}P\{\sqrt{P^2 - 4\beta\hat{c}} - \sqrt{P^2 - 4\beta C_{max}}\} \\
&\quad (F(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}}) - \hat{c}\frac{dF}{dC_{max}}) \\
&\quad - \frac{1}{2\beta C_{max}}\{\sqrt{P^2 - 4\beta C_{max}} - \frac{d\hat{c}}{dC_{max}}\sqrt{P^2 - 4\beta\hat{c}}\} \\
&\quad - \frac{\hat{c}}{2C_{max}}(N-1)(1 - \frac{\hat{c}}{C_{max}})\frac{dF}{dC_{max}} \\
&\geq \frac{1}{2\beta C_{max}}(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}})(P + \beta(N-1)(1 - \frac{\hat{c}}{C_{max}})F) \\
&\quad + \frac{1}{2\beta C_{max}}\sqrt{P^2 - 4\beta\hat{c}}\{\frac{d\hat{c}}{dC_{max}} \\
&\quad - \frac{N-1}{2C_{max}}P(F(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}}) - \hat{c}\frac{dF}{dC_{max}})\} \\
&\quad + \frac{1}{2\beta C_{max}}\sqrt{P^2 - 4\beta C_{max}}\{\frac{N-1}{2C_{max}}P(F(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}}) \\
&\quad - \hat{c}\frac{dF}{dC_{max}}) - \frac{\hat{c}}{C_{max}}\} - \frac{\hat{c}}{2C_{max}}(N-1)(1 - \frac{\hat{c}}{C_{max}})\frac{dF}{dC_{max}} \\
&> \frac{1}{2\beta C_{max}}\{(\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}})(P - \sqrt{P^2 - 4\beta C_{max}}) \\
&\quad + \beta(N-1)(1 - \frac{\hat{c}}{C_{max}})((\frac{\hat{c}}{C_{max}} - \frac{d\hat{c}}{dC_{max}})F - \hat{c}\frac{dF}{dC_{max}})\} \\
&> 0
\end{aligned}
$$

Theorem 2: The proof for unconstrained and fully constrained equilibria are given in Chapter 4. We now show that a partially constrained equilibrium is feasible if unconstrained and fully constrained equilibria are infeasible. Consider the pairs of $(\tilde{f}_2, \hat{c}_2)$ values that fulfill the mean-field equation $\int_{\hat{c}_2}^{C_{max}} \frac{1}{C_{max}}(1 - q_1^*(C))dC = \tilde{f}_2$. Know that $(0, C_{max})$ and $(1 - \frac{\hat{c}_1}{C_{max}} - \tilde{f}_1^*, \hat{c}_1^*)$ are two such pairs, and that $\hat{c}_2$ is strictly decreasing as $\tilde{f}_2$ increases.

$F_2(\hat{c}_2 = C_{max}, \tilde{f}_2 = 0) > 1 - q_1^*(C_{max})$, else an unconstrained equilibrium exists since $F_2(\hat{c}_2 = C_{max}, \tilde{f}_2 = 0) = \frac{\alpha}{2\beta N}$. We also know that $F_2(\hat{c}_2 = \hat{c}_1, \tilde{f}_2 = 1 - \frac{\hat{c}_1}{C_{max}} - \tilde{f}_1^*) < 1 - F_1^* = 1 - q_1^*(\hat{c}_1^*)$. Therefore, by IVT, there must exist some $(\tilde{f}_2, \hat{c}_2)$ pair that fulfills the mean-field equation, while also fulfilling $F_2(\hat{c}_2, \tilde{f}_2) = 1 - q_1^*(\hat{c}_2)$. Since this pair is between $(0, C_{max})$ and $(1 - \frac{\hat{c}_1}{C_{max}} - \tilde{f}_1^*, \hat{c}_1^*)$, we know that it is feasible. Thus a partially constrained equilibrium is feasible.

Lemma 2: Refer to the proof of Lemma 1.

Theorem 3: The proof that no feasible equilibrium exists for $\alpha < \alpha^*$ and that a feasible equilibrium exists for $\alpha \geq \alpha^*$ is similar to that of Theorem 1. The characterization of $q_1(C)$ is given in Chapter 4.

Proposition 4: Note that because L and r are always combined in the form $(1-r)L$ for all equations in the model, increasing L is equivalent to decreasing r. Therefore, we only consider the sensitivity of $\tilde{f}$ and $q_1(C)$ with regard to L.

For $\tilde{f}$, want to show that since $\frac{d\hat{c}}{dL} > 0$, and $\frac{dq}{dL} < 0$, $g(\tilde{f})$ shifts downwards for all values of $\tilde{f}$ as L increases. Hence $\tilde{f}^*_{new} < \tilde{f}^*$.

$$\begin{aligned}\frac{d\hat{c}}{dL} &= \frac{1}{2}(1-r) \\ &\quad (1 - \frac{C_{max}(\alpha - \beta(N-1)\tilde{f} - (1-r)L) - \beta(C_{max} + (1-r)(N-1)L)}{A})\end{aligned}$$

Want to show $A > C_{max}(\alpha - \beta(N-1)\tilde{f} - (1-r)L) - \beta(C_{max} + (1-r)(N-1)L)$. If the RHS is negative then we are done, so we assume it is positive. Squaring both sides, we obtain

$$(C_{max} - \beta(N-1))(\alpha - \beta(N-1)\tilde{f} - (1-r)L) < 2\beta(C_{max} + (1-r)(N-1)L)$$

If $\frac{\alpha-(1-r)L}{2\beta N} \geq 1$ an unconstrained equilibrium exists, so we set $\tilde{f} = 0$ and bound the LHS from above:

$$(C_{max} - \beta(N-1))2\beta N \quad < \quad 2\beta C_{max} + 2\beta(1-r)(N-1)L$$

$$C_{max} - (1-r)L \quad < \quad \beta N$$

To show why this inequality is true, consider the price in an unconstrained equilibrium, $\frac{\alpha+(1-r)L}{2}$.

Lemma 6 *The equilibrium price in a partially constrained equilibrium must be less than that in an unconstrained equilibrium.*

Suppose for contradiction that we have a partially constrained equilibrium with a higher price than $\frac{\alpha+(1-r)L}{2}$. Then the total quantity being sold must be less than the quantity in the unconstrained case. Then there must exist some $\epsilon > 0$ such that the non cash constrained farmers can increase their revenue by supplying $F + \epsilon$. Thus the non-cash constrained farmers are not behaving optimally.

Thus $\frac{\alpha+(1-r)L}{2} > C_{max}$, else no partially constrained equilibrium would be feasible. Now suppose for contradiction that $C_{max} - (1-r)L > \beta N$. Then since $\frac{\alpha-(1-r)L}{2\beta N} > \frac{\alpha-(1-r)L}{2(C_{max}-(1-r)L)}$, we must have $\frac{\alpha-(1-r)L}{2(C_{max}-(1-r)L)} < 1$. But this means that $\frac{\alpha+(1-r)L}{2} < C_{max}$, hence no partially constrained equilibrium can exist. Therefore we have that $\frac{d\hat{c}}{dL} > 0$.

$$\begin{aligned}
\frac{dq}{dL} &= \frac{\partial q}{\partial L} + \frac{\partial q}{\partial \hat{c}}\frac{d\hat{c}}{dL} + \frac{\partial q}{\partial F}\frac{dF}{dL} \\
\frac{\partial q}{\partial L} &= \frac{1}{2\beta}(1-r)(-1 + \frac{P-(1-r)L-2\beta}{\sqrt{(P-(1-r)L)^2 - 4\beta(C-(1-r)L)}})
\end{aligned}$$

Want to show $\frac{\partial q}{\partial L} < 0$.

$$\begin{aligned}
P-(1-r)L-2\beta &< \sqrt{(P-(1-r)L)^2 - 4\beta(C-(1-r)L)} \\
C &< P - \beta
\end{aligned}$$

Since we know that $P \geq \beta + C_{max}$, we have $\frac{\partial q}{\partial L} < 0$.

Want to show $\frac{\partial q}{\partial \hat{c}}\frac{d\hat{c}}{dL} + \frac{\partial q}{\partial F}\frac{dF}{dL} < 0$.

$$\begin{aligned}
\frac{\partial q}{\partial \hat{c}}\frac{d\hat{c}}{dL} + \frac{\partial q}{\partial F}\frac{dF}{dL} &= \frac{N-1}{2C_{max}}(-1 + \frac{P-(1-r)L}{\sqrt{(P-(1-r)L)^2 - 4\beta(C-(1-r)L)}}) \\
&\quad (F\frac{d\hat{c}}{dL} + \hat{c}\frac{dF}{dL})
\end{aligned}$$

$$
\begin{aligned}
F\frac{d\hat{c}}{dL} + \hat{c}\frac{dF}{dL} &= B\{(N-1)(\alpha - \beta(N-1)\tilde{f} - (1-r)L) \\
&(-C_{max}(\alpha - \beta(N-1)\tilde{f} - (1-r)L) \\
&+\beta(C_{max} + (1-r)L(N-1)) + A) - \\
&((N-1)(\alpha - \beta(N-1)\tilde{f}) + C_{max}) \\
&(-\beta(C_{max} - (1-r)L(N-1)) + A)\}
\end{aligned}
$$

Where $B = \frac{C_{max}(1-r)}{2(N-1)A(\beta(C_{max}+(1-r)L(N-1))+A)}$. Since $B > 0$, we want to show that the term in the curly brackets is negative. We can simplify it to obtain

$$
\begin{aligned}
&-(N-1)C_{max}(\alpha - \beta(N-1)\tilde{f} - (1-r)L)^2 - ((1-r)L(N-1) + C_{max})A \\
&+\beta(N-1)(C_{max} + (1-r)L(N-1))(\alpha - \beta(N-1)\tilde{f} - (1-r)L) \\
&-\beta((1-r)L(N-1) - C_{max})((N-1)(\alpha - \beta(N-1)\tilde{f}) + C_{max}) < 0
\end{aligned}
$$

If $(1-r)L(N-1) > C_{max}$, we can simplify the equation

$$
\begin{aligned}
&2\beta - (\alpha - \beta(N-1)\tilde{f} - (1-r)L) < 0 \\
&C_{max} > \beta + (1-r)L
\end{aligned}
$$

If $(1-r)L(N-1) \leq C_{max}$, expand the expression fully and compare positive and negative terms

$$
\begin{aligned}
&(C_{max} + (N-1)(1-r)L)A + \beta((N-1)(1-r)L)^2 > \\
&\quad \beta(C_{max}^2 - ((N-1)(1-r)L)^2) + \beta((N-1)(1-r)L)^2 = \beta C_{max}^2 \\
&(N-1)C_{max}(\alpha - \beta(N-1)\tilde{f} - (1-r)L)^2 > \\
&\quad 2\beta(N-1)C_{max}(\alpha - \beta(N-1)\tilde{f}) \\
&(\alpha - \beta(N-1)\tilde{f})^2 - 2(1-r)L(\alpha - \beta(N-1)\tilde{f}) + ((1-r)L)^2 \geq \\
&\quad (\alpha - \beta(N-1)\tilde{f})^2 - 2(1-r)L(\alpha - \beta(N-1)\tilde{f}) > 2\beta(\alpha - \beta(N-1)\tilde{f}) \\
&(\alpha - \beta(N-1)\tilde{f}) > 2\beta + 2(1-r)L \\
&C_{max} > \beta + \frac{2C_{max}}{N-1}
\end{aligned}
$$

$$C_{max} > \frac{N-1}{N-3}\beta$$

For large N, this approaches $C_{max} > \beta$. Therefore, if $C_{max} > \beta + (1-r)L$, we have that $\frac{dq}{dL} < 0$. We can interpret this condition as the maximum cash constraint being greater than the net present value of the loan plus the price sensitivity to quantity.

For $q_1(C)$, we know that cash constrained farmers increase stored quantity if L increases. Hence all that remains to show is that unconstrained farmers increase stored quantity as well.

Lemma 7 $F_1 > \frac{\alpha-(1-r)L}{2\beta N}$

$$\begin{aligned}
\frac{C_{max}(\alpha - \beta(N-1)\tilde{f}_1 - (1-r)L)}{2\beta(C_{max} + (N-1)\hat{c}_1)} &> \frac{\alpha-(1-r)L}{2\beta N} \\
(\alpha - (1-r)L)(1 - \frac{\hat{c}_1}{C_{max}}) &> \beta N \tilde{f}_1 \\
\alpha - \beta N \bar{q}_1 &> (1-r)L
\end{aligned}$$

This condition can be interpreted as follows: If every farmer sells the average quantity sold by cash-constrained farmers, the market price must be greater than the net present value of taking the full loan. This is true because we have assumed that L is lower than the market price to encourage farmers to sell their goods on the market.

Note that the unconstrained optimal quantity $\frac{\alpha-(1-r)L}{2\beta N}$ decreases as r decreases. Furthermore, note that as L increases, we showed previously that $\hat{c}_1$ increases and $\tilde{f}_1$ decreases (ie. as the loan terms become more favourable for farmers, the proportion of cash-constrained farmers decreases). Therefore, we approach the unconstrained equilibrium as L increases.

Now suppose for contradiction that F_1 does not decrease monotonically as L increases. Then there exists some value of L where increasing L causes F_1 to diverge from unconstrained optimum, a contradiction. By similar argument for decreasing r, we are done.

Proposition 5: Begin by showing $R_1(r)$ is unimodal. Net revenue is maximized when the average quantity sold is $\frac{\alpha}{2\beta N}$, the unconstrained optimal quantity. With loans, the optimal quantity becomes $\frac{\alpha-(1-r)L}{2\beta N}$.

1. $\frac{\alpha}{2\beta N} \geq 1$

   Farmers will all sell 1 unit for $r \geq \frac{2\beta N-(\alpha-L)}{L}$, therefore net revenue is constant. For $r < \frac{2\beta N-(\alpha-L)}{L}$, as r decreases farmers sell less quantity, moving away from the optimal. Therefore, net revenue decreases.

2. $\frac{\alpha}{2\beta N} < 1, \frac{\alpha^2}{4\beta N} \geq C_{max}$

   All farmers meet their cash constraint by selling the optimal quantity, so maximum market revenue is achieved at $r = 1$. As r decreases farmers sell less quantity, moving away from the optimal. Therefore, net revenue decreases.

3. $\frac{\alpha}{2\beta N} < 1, \frac{\alpha^2}{4\beta N} < C_{max}$

   Farmers cannot meet their cash constraint by selling the optimal quantity, so there is either a partially constrained equilibrium or no equilibrium. For this analysis we assume a partially constrained equilibrium exists at $r = 1$.

   The average quantity sold in a partially constrained equilibrium is given by $\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1$. Maximum net revenue is achieved when $\frac{\hat{c}_1}{C_{max}}F_1 + \tilde{f}_1 = \frac{\alpha}{2\beta N}$. In Proposition 3, we proved that $\frac{dP}{d\tilde{f}} < 0$, which is equivalent to showing $\frac{\hat{c}_1}{C_{max}}F_1+\tilde{f}_1$ decreases as $\tilde{f}_1$ decreases. Furthermore, from Proposition 4 we know that $\tilde{f}_1$ decreases as r decreases. Hence there can only be a maximum of one value of r corresponding to the maximum net revenue.

For $R_2(r)$, the uniqueness of r_2 and r_3 are explained in Chapter 5. We prove the claim that maximum revenue is never achieved in a partially constrained equilibrium. We know that the average quantity sold by quantity constrained farmers must be less than $\frac{\alpha}{2\beta N}$. Let $\bar{q}$ be the average quantity sold by quantity constrained farmers, and suppose that there are N_1 and N_2 unconstrained and constrained farmers respectively, $N_1 + N_2 = N$. Want to show that the unconstrained farmers will always prefer to raise the average quantity sold above $\frac{\alpha}{2\beta N}$. Unconstrained farmers solve the problem

$$max_q \qquad (\alpha - \beta(N_1 q + N_2 \bar{q}))q$$

$$q^* = \frac{\alpha}{2\beta N_1} - \frac{N_2}{2N_1}\bar{q}$$

The average quantity is given by

$$\begin{aligned}\frac{1}{N}(N_1 q + N_2 \bar{q}) &= \frac{1}{N}(\frac{\alpha}{2\beta} + \frac{N_2}{2}\bar{q}) \\ &> \frac{\alpha}{2\beta N}\end{aligned}$$

Hence the average quantity in a partially constrained equilibrium cannot be $\frac{\alpha}{2\beta N}$, since unconstrained farmers will always prefer to sell more.

For $W(r)$, consider the lean season equilibrium for some value of r. By Proposition 4, we know that as r decreases, stored quantity increases for all farmers. Therefore, if an unconstrained equilibrium exists, it will also exist for lower values of r. Since the quantity sold remains the same, wastage must increase. If a fully constrained equilibrium exists, wastage is 0 so it is certainly non-decreasing as r decreases.

Now consider the case if a partially constrained equilibrium exists. Claim that as r decreases, equilibrium F_2 decreases. As r decreases, increased quantity becomes available for all farmers, and therefore $\hat{c}_2$ increases. Note that as equilibrium $\hat{c}_2$ increases, F_2 approaches $\frac{\alpha}{2\beta N}$ (ie. as the number of quantity-constrained farmers decreases, quantity sold by unconstrained farmers approaches the fully unconstrained optimal quantity). Therefore, since $F_2 > \frac{\alpha}{2\beta N}$, F_2 must decrease.

$$\begin{aligned}\frac{C_{max}(\alpha - \beta(N-1)\tilde{f}_2)}{2\beta(C_{max} + (N-1)\hat{c}_2} &> \frac{\alpha}{2\beta N} \\ \alpha - \beta N \bar{q}_2 &> 0\end{aligned}$$

where $\bar{q}_2$ is the average quantity sold by quantity constrained farmers. Since quantity constrained farmers sell less than their peers, this is guaranteed to be true. Given that F_2 decreases as r decreases, wastage increases as r decreases.

Finally, the result for $I(r)$ follows directly from Proposition 4.

Proposition 6: Proof is presented in Chapter 6.

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